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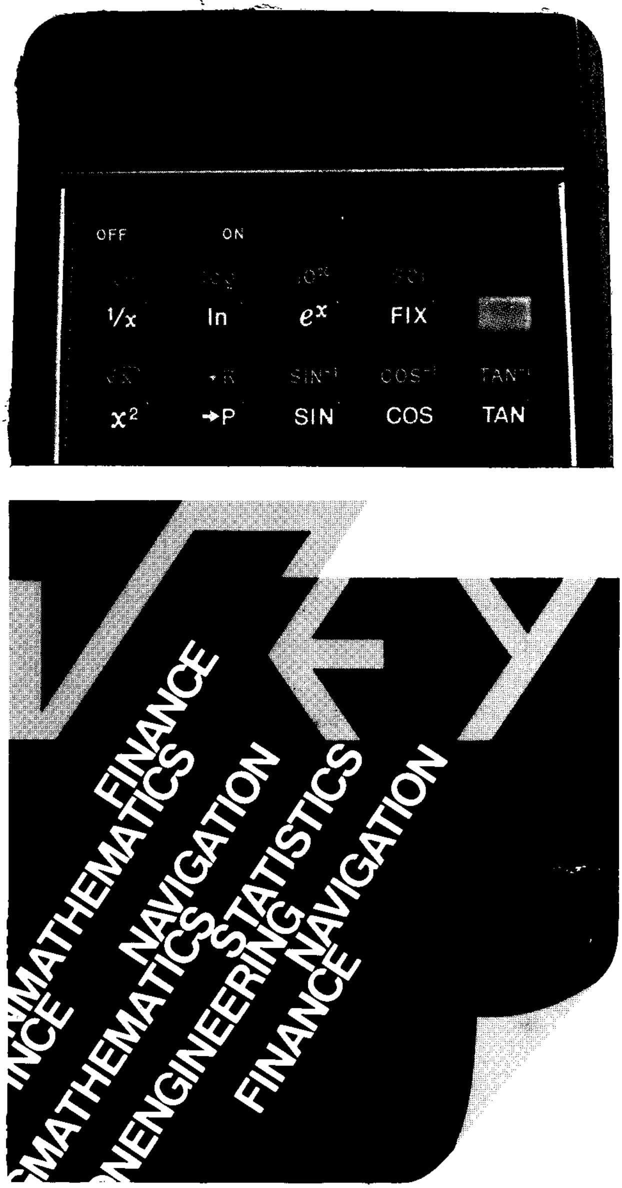

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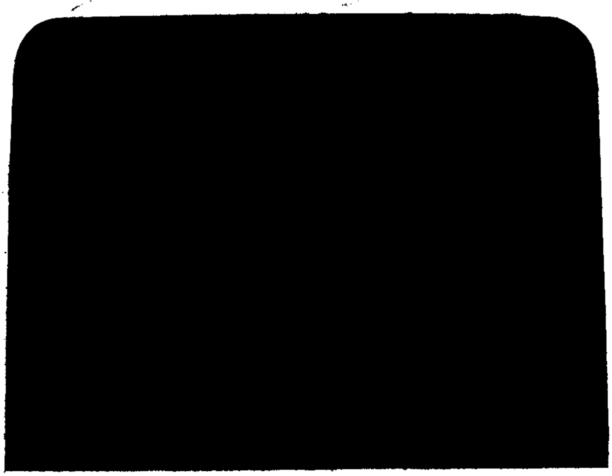

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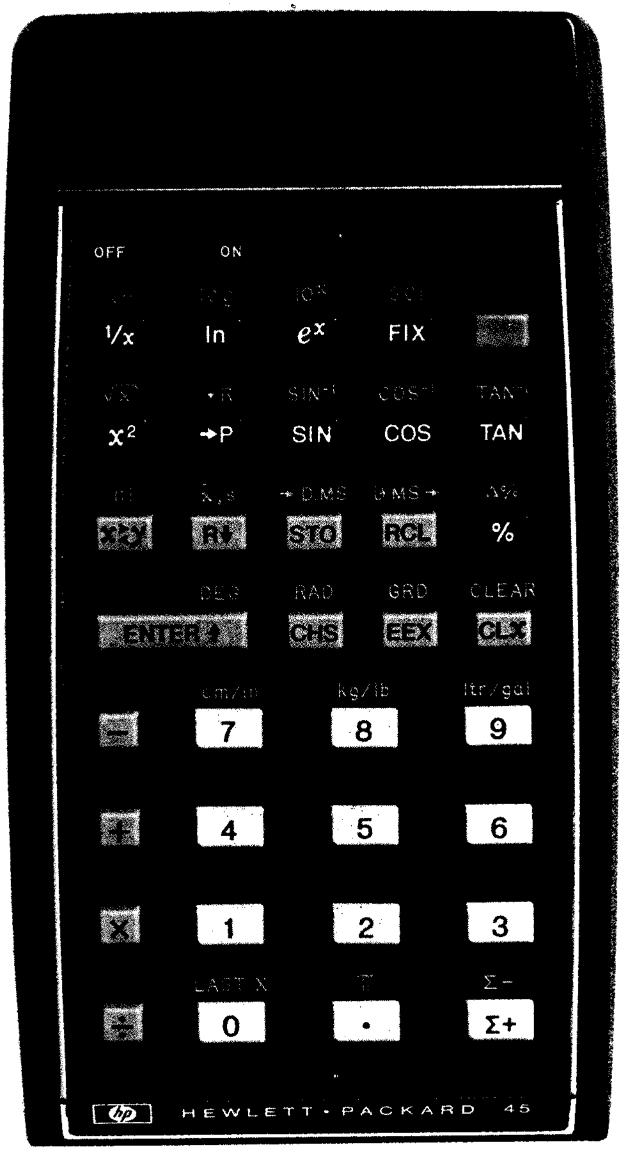

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HP-45

APPLICATIONS BOOK

Hewlett Packard
Advanced Products Division

Your Applications Book

The Applications Book is a representative collection of key sequence routines for solving problems with your HP-45 Pocket Calculator.

We suggest that you first read the introductory material explaining the Standard Key Sequence Format. Then find the routine you want, and use it. Numerical examples are provided to enable you to try out routines before using them. An understanding of the HP-45 Owner's Handbook is also required if, in addition, you wish to track the changes in the calculator's memory on a step-by-step basis.

The body of the book is arranged in alphabetic sequence of topics—those you are likely to think of when you want a routine. Thus, Complex Number Operations are presented in the "C" section; Progressions are presented in the "P" section, including arithmetic, geometric, and harmonic progressions. In addition to such main entries, cross-reference entries enable the reader to find a routine by an alternate route. Thus, in the "A" section (between "Arithmetic Mean" and "Average") a cross-reference entry, "Arithmetic Progressions," refers the reader to the page under "Progressions" where the routine is presented. Similarly, cross-references are provided for "Geometric Progressions," and "Harmonic Progressions," etc. The contents section at the front is arranged logically (instead of by page order) to show all the routines available under each of seven broad categories. The back cover contains an index.

Two key sequence forms are included inside the back cover of this manual. You may wish to duplicate these forms and record your own key step programs.

Standard Key Sequence Format

Shown below is the key sequence routine for computing the roots of the Quadratic Equation:

$$AX^2 + BX + C = 0$$

Using A, B, and C, which you supply as data, the routine produces an intermediate result D. If $D < 0$, one root is the complex conjugate of the other; the real and imaginary parts of one complex root develop on lines 10 and 11. Except for the opposite sign of the imaginary part, the other complex root is identical. If $D \geqslant 0$, the roots are real and develop on lines 6 and 9 (if $-B/2A \geqslant 0$) or on lines 7 and 9 (if $-B/2A < 0$).

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	A	STO	1	↑	+			
2	B	x⇄y	÷	CHS	↑	x^2		
3	C	STO	2	RCL	1	÷		
4		−					D	If D < 0, go to 10
5		■	$\sqrt{x}$	x⇄y			−B/2A	If −B/2A < 0, go to 7
6		+					x_1	Go to 8
7		−	CHS				x_1	
8		RCL	1	x	RCL	2		
9		x⇄y	÷				x_2	Stop
10		CHS	■	$\sqrt{x}$	x⇄y		u	
11		x⇄y					v	

To execute the sequence, start with line 1 and read from left to right, making the appropriate keystrokes as you proceed. Interpret the respective columns as follows:

Data: Information to be supplied by you, the user. In the sample case, lines 1, 2 and 3 prompt the reader to enter coefficients A, B and C. To enter negative data, it is merely necessary to press CHS after pressing the data value.

Operations: The keys to be pressed after you enter any requested data item for the line. [↑] is the symbol used to denote the [ENTER↑] key of the HP-45. All other key designations are identical to the HP-45 keys. Ignore any blank positions in the operations column. The gold prefix key is represented as a solid key with no lettering (e.g., the first stroke of line 5). The next key to be pressed is denoted by the corresponding functional name (e.g., $\sqrt{x}$, second stroke, line 5) which, on the keyboard is printed in gold.

Display: Intermediate or final results which you should, in most cases, jot down. In the sample case, D is developed so that the reader can decide which line (5 or 10) to execute next.

Remarks: Conditional and unconditional jumps to specified lines or other information for the reader. In the sample case, the reader is prompted to continue with line 10 (ignoring lines 5 through 9) if D is negative. If the condition fails, execution continues on the next line. In the sample case, the reader proceeds to line 5, if D is zero or positive.

Thus, lines are read in sequential order except where the remarks column directs otherwise (as in line 4 of the sample case). To assist the reader in distinguishing lines to be repeated, a sequence of lines making up an iterative process is outlined with a bold border. The following sequence for computing chi-square statistic for goodness of fit illustrates this convention.

Formula:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequency

E_i = expected frequency

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		CLX	↑					
2	O_i	↑						Perform 2–4 for i = 1, 2, ..., n
3	E_i	STO	1	–	x^2	RCL		
4		1	÷	+				

In a few cases, an iterative process is embedded in a series of lines which are themselves iterated.

Contents

GEOMETRY AND TRIGONOMETRY

Amortization

See page 90

Analysis of Variance (One Way)

The one-way analysis of variance tests the differences between means of k treatment groups, group i ($i = 1, 2, ..., k$) has n_i observations (treatment may have equal or unequal number of observations).

The following keysequence yields the analysis of variance table, sum of squares, mean squares, degrees of freedom, and the F ratio.

Formulas:

$$\text{Total SS} = \sum_{i=1}^{k}\sum_{j=1}^{n_i} x_{ij}^2 - \frac{\left(\sum_{i=1}^{k}\sum_{j=1}^{n_i} x_{ij}\right)^2}{\sum_{i=1}^{k} n_i}$$

$$\text{Treat SS} = \sum_{i=1}^{k} \frac{\left(\sum_{j=1}^{n_i} x_{ij}\right)^2}{n_i} - \frac{\left(\sum_{i=1}^{k}\sum_{j=1}^{n_i} x_{ij}\right)^2}{\sum_{i=1}^{k} n_i}$$

$$\text{Error SS} = \text{Total SS} - \text{Treat SS}$$

$$\text{Treat df} = k - 1$$

$$\text{Error df} = \sum_{i=1}^{k} n_i - k$$

$$\text{Treat MS} = \frac{\text{Treat SS}}{\text{Treat df}}$$

$$\text{Error MS} = \frac{\text{Error SS}}{\text{Error df}}$$

$$F = \frac{\text{Treat MS}}{\text{Error MS}} \quad \text{(with } k-1 \text{ and } \sum_{i=1}^{k} n_i - k \text{ degrees of freedom)}$$

Example:

	i \ j	1	2	3	4	5	6
	1	10	8	5	12	14	11
Treatment	2	6	9	8	13		
	3	14	13	10	17	16	

Answers: Total SS = 172.93
Treat SS = 66.93
Error SS = 106.00
Treat df = 2.00
Treat MS = 33.47
Error df = 12.00
Error MS = 8.83
F = 3.79 (with 2 and 12 degrees of freedom)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR	STO	1	STO		Or turn machine off and on.
2		2	STO	3	STO	4		
3		1	STO	+	4			Perform 3–8 for i=1, 2, ..., k
4	x_{ij}	STO	+	1	Σ+	1		Perform 4–5 for j=1, 2, ..., n_i
5		STO	+	2				
6		RCL	1	x^2	RCL	÷		
7		2	STO	+	3	0		
8		STO	1	STO	2			
9		RCL	6	RCL	7	x^2		
10		RCL	÷	5	–		Total SS	
11		RCL	3	RCL	7	x^2		
12		RCL	÷	5	–		Treat SS	
13		–					Error SS	
14		■	LAST x	RCL	4	1		
15		–					Treat df	
16		÷					Treat MS	
17		x⇄y	RCL	5	RCL	–		
18		4					Error df	
19		÷					Error MS	
20		÷					F	

Angle Conversions

Bearing to azimuth

Note: x = bearing

Example:

S 42.6° E = 137.40° = 137°24′

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	■ DEG		If x is in decimal degrees,
				go to 3.
2		■ D.MS→		
3				If Sx°E, go to 4,
				If Sx°W, go to 5,
				If Nx°W, go to 6,
				If Nx°E, go to 7.
4		1 8 0 x⇄y -		Go to 7
5		1 8 0 +		Go to 7
6		CHS ↑ 3 6 0		
		+		
7				For degrees, minutes,
				seconds, go to 8.
				Otherwise, stop.
8		■ →D.MS		

Azimuth to bearing

Note: x = azimuth

Method:

If $0 < x < 90°$, convert to ND°E

If $90° < x < 180°$, convert to SD°E

If $180° < x < 270°$, convert to SD°W

If $270° < x < 360°$, convert to ND°W

Example:

226°23′ = S46.38°W = S46°23′ W

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	■ DEG		If x is in decimal degrees,
				go to 3.
2		■ D.MS→	D	
3				If 0 < x < 90, go to 7,
				If 90 < x < 180, go to 4.
				If 180 < x < 270, go to 5.
				If 270 < x < 360, go to 6.
4		1 8 0 x⇄y –	D	Go to 7
5		1 8 0 –	D	Go to 7
6		3 6 0 – CHS	D	
7				For degrees, minutes,
				seconds, go to 8.
				Otherwise, stop.
8		■ →D.MS		

Radians to degrees

Note: x = input data in radians

Examples:

1. 1 radian = 57.30° (To see full display, press FIX 9.)
2. $\frac{3}{4}\pi$ radians = 135°

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		1 8 0 ■ π		
2		÷ ↑ ↑ ↑		
3		CLX		
4	x	x		Stop. For new case, go to 3.

Degrees to radians

Note: x = input data in degrees

Examples:

1. 1° = 0.02 radians (To see full display, press FIX 9.)
2. 266° = 4.64 radians

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		■ π 1 8 0		
2		÷ ↑ ↑ ↑		
3		CLX		
4	x	x		Stop. For new case, go to 3.

Mils to degrees

Example: 1600 mils = 90°

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	mils	↑ 9 x 1 6		
2		0 ÷		

Degrees to mils

Note: x = input data in degrees

Example: 90° = 1600 mils

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	↑ 1 6 0 x		
2		9 ÷		

Grads to degrees

Note: x = input data in grads

Example: 300 grads = 270°

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[↑] [.] [9] [×] [ ]		

Degrees to grads

Note: x = input data in degrees

Example: 360° = 400 grads

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[↑] [.] [9] [÷] [ ]		

Angles of Triangles

See page 190

Appendix

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Arithmetic Mean

See page 148

Arithmetic Progressions

See page 157

Bartlett's Chi-Square

See page 36

Base Conversions

Note:

Base conversion algorithms are given for positive values only. To convert a negative number, change sign, convert, and change sign of result.

Decimal integer to integer in any base

$$I_{10} \rightarrow J_b$$

In the following key sequence, f + 1 is the number of digits in J_b.

d_i $(i = 1, ..., f + 1)$ represents the i^{th} digit in J_b, counting from left to right, i.e.

$$J_b = (d_1 d_2 \text{ --- } d_{f+1})_b$$

For large numbers, $J_b = (d_1 . d_2 \text{ --- } d_{f+1})_b \cdot b^f$, see example 3.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑	↑					
2	I	STO	1	ln	x⇄y	ln		
3		÷					D	Let f be the largest
								integer ≤ D
4		CLX						
5	f	x⇄y	↑	↑	RCL	1		
6		R↓	R↓	x⇄y	■	y^x		
7		÷					E_1	d_i = integer part of E_i
								(i=1, ..., f)
8	d_1	−	x				E_2	
9	d_i	−	x				E_{i+1}	Perform 9 for i=2, ..., f
10		FIX	0				d_{f+1}	

Example 1:

Convert 1206 to hexadecimal (base 16).

(The hexadecimal digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)

Answer:

$1206_{10} = 4B6_{16}$ (f = 2)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	16	↑	↑					
2	1206	STO	1	ln	x⇄y	ln		
3		÷					2.56	f = 2
4		CLX						
5	2	x⇄y	↑	↑	RCL	1		
6		R↓	R↓	x⇄y		y^x		
7		÷					4.71	$d_1 = 4$
8	4	−	x				11.38	$d_2 = 11$
9	11	−	x				6.00	$d_3 = 6$

Example 2:

Convert 513 to octal (base 8).

Answer:

$513_{10} = 1001_8$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	8	↑	↑					
2	513	STO	1	ln	x⇄y	ln		
3		÷					3.00	f = 3
4		CLX						
5	3	x⇄y	↑	↑	RCL	1		
6		R↓	R↓	x⇄y		y^x		
7		÷					1.00	$d_1 = 1$
8	1	−	x				0.02	$d_2 = 0$
9	0	−	x				0.12	$d_3 = 0$
10	0	−	x				1.00	$d_4 = 1$

Example 3:

Convert 6.023×10^{23} to octal.

Answer:

$6.023 \times 10^{23} = 1.7743_8 \times 8^{26}$

Note:

If we consider 6.023×10^{23} to be a scientific measurement good only to 4 significant digits, it is meaningless for the octal representation to contain more than 5 significant digits. Therefore, we stop before the loop is completed.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	8	↑	↑	6	.	0		
2		2	3	EEX	2	3		
3		STO	1	ln	x⇄y	ln		
4		÷					26.33	f = 26 (Note: this gives
5		CLX						the exponent in base 8)
6	26	x⇄y	↑	↑	RCL	1		
7		R↓	R↓	x⇄y	■	y^x		
8		÷					1.99	$d_1 = 1$
9	1	−	x				7.94	$d_2 = 7$
10	7	−	x				7.54	$d_3 = 7$
11	7	−	x				4.34	$d_4 = 4$
12	4	−	x				2.69	$d_5 = 3$ (rounded), stop

Integer without exponent in base b to decimal

$$(d_1 d_2 \cdots d_{n-1} d_n)_b \rightarrow I_{10}$$

Examples:

1. $730020461_8 = 123740465_{10}$
2. $7D0F_{16} = 32015_{10}$

   (A = 10, B = 11, C = 12, D = 13, E = 14, F = 15 in the hexadecimal system)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑	↑	↑				
2	d_1	x						
3	d_i	+	x					Perform 3 for i=2, ..., n−1
4	d_n	+						

Integer with exponent in base b to decimal

$$d_1 . d_2 \text{---} d_n \times b^{Exp} \rightarrow I_{10}$$

Examples:

1. $3.0002_8 \times 8^{11} = 2.577399803 \times 10^{10}$
2. $D2EE4_{16} \times 16^{28} = D.2EE4_{16} \times 16^{32}$

   $= 4.485999088 \times 10^{39}$

   $(D_{16} = 13, E_{16} = 14)$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑	↑					
2	Exp	↑						
3	n	−	1	+	■	y^x		
4		STO	1					
5	b	↑	↑	↑				
6	d_1	x						
7	d_i	+	x					Perform 7 for i=2, ..., n−1
8	d_n	+	RCL	1	x			

Fractional decimal number to base b

$$x_{10} \rightarrow y_b$$

Method:

In the following algorithm, c is the number of significant digits in x. d_i (i = 1, ..., c) represents the i^{th} digit in y_b, counting from left to right.

$$y_b = (d_1 . d_2 \text{---} d_c)_b \times b^{-H}$$

Examples:

1. $0.2937_{10} = 2.263_8 \times 8^{-1} = .2263_8$ (c = 4)
2. $3.688 \times 10^{-54} = 5.A6E_{16} \times 16^{-45}$ ($c = 4, A_{16} = 10, E_{16} = 14$)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑	↑					
2	x	STO	1	1/x	ln	x⇄y		
3		ln	÷				D	Let H be the smallest
								integer ⩾ D
4		CLX						
5	H	x⇄y	↑	↑	RCL	1		
6		R↓	R↓	x⇄y	■	y^x		
7		x					E_1	d_i = integer part of
								E_i (i=1, ..., c−1)
8	d_1	−	x				E_2	
9	d_i	−	x				E_{i+1}	Perform 9 for i=2, ..., c−1
10		FIX	0				d_c	

Fractional number in base b to decimal

$$(.\,d_1 d_2 \cdots d_n)_b \rightarrow a_{10}$$

or

$$d_1\,.\,d_2 \cdots d_n \times b^{-Exp} \rightarrow a_{10}$$

Examples:

1. $.0E728_{16} = 0.056434631\ (E_{16} = 14)$
2. $7.200067_8 \times 8^{-29} = 4.685338214 \times 10^{-26}$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	1/x	↑	↑	↑			
2	d_n	x	FIX	9				
3	d_i	+	x					Perform 3 for i=n−1, ..., 1
4								If there is no exponent,
								stop. Otherwise go to 5.
5	b	↑	■	SCI	9			
6	Exp	↑	1	−	■	y^x		
7		1/x	x					

Bearing to Azimuth

See page 17

Bernoulli Numbers

The Bernoulli numbers B_1, B_2, B_3 ... are defined by

$$B_n = \frac{2(2n)!}{(2^{2n}-1)\,\pi^{2n}} \left[1 + \frac{1}{3^{2n}} + \frac{1}{5^{2n}} + \ldots\right]$$

specifically

$$\frac{1}{6}, \frac{1}{30}, \frac{1}{42}, \frac{1}{30}, \frac{5}{66}, \frac{691}{2730}, \frac{7}{6}, \ldots$$

Example:

The 8^{th} Bernoulli number = 7.09 (i takes the values 1, 2).

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	↑	2	x	↑	↑		
2		↑	1					
3		STO	1	CLX				Perform 3–6 for i=1,2, ...,
4	i	↑	2	x	1	+		until A_i does not change
5		x⇄y	■	y^x	1/x	RCL		
6		1	+				A_i	
7		R↓	■	n!	2	x		
8		RCL	1	x	R↓	R↓		
9		2	x⇄y	■	y^x	1		
10		–	x⇄y	■	π	x⇄y		
11		■	y^x	x	÷			

Bessel Function of First Kind , Order n

Formula:

$$J_n(x) = \left(\frac{x}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(-\frac{x^2}{4}\right)^k}{k!\,(n+k)!}$$

$$x \geqslant 0$$

Note:

This routine is only good for small values of the argument x.

Example:

$J_4(2) = 0.033995720$ (i = 2, ..., 6, 7)
(Press [FIX] [9] to see the whole display.)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	x^2	4	÷	STO	1		Set machine to desired
2	n	STO	2	■	n!	1/x		decimal setting
3		RCL	1	RCL	2	1		
4		+	■	n!	÷	−		
5		RCL	1					Perform 5–12 for i=2, 3,
6	i	STO	3	■	y^x	■		4, ..., until D_i does not
7		LAST x	■	n!	÷	RCL		change
8		2	RCL	3	+	■		
9		n!	÷					If i is odd, go to 11.
10		+						Go to 12
11		−						
12		STO	4				D_i	
13		RCL	1	■	$\sqrt{x}$	RCL		
14		2	■	y^x	×		$J_n(x)$	

Binomial Distribution

Formula:

$$f(x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

where n and x are integers,

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!},$$

$$0 \leqslant x \leqslant n \leqslant 69, \text{ and}$$

$$0 < p < 1.$$

Example:

If n = 6, p = 0.49 then

Answer:

f(4) = 0.224913711

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	p	STO	1					
2	x	STO	2	■	y^x			
3		1	RCL	−	1			
4	n	STO	3	RCL	−	2		
5		■	y^x	■	LAST x	■		
6		n!	÷	×	RCL	3		
7		■	n!	×	RCL	2		
8		■	n!	÷	FIX	9		

Bivariate Normal Distribution

Formula:

$$f(x,y) = \frac{1}{2\pi\,\sigma_1\sigma_2\sqrt{1-\rho^2}}\;e^{-P(x,y)}$$

where

$$P(x,y) = \frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_1)^2}{\sigma_1{}^2} - 2\rho\,\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} + \frac{(y-\mu_2)^2}{\sigma_2{}^2}\right]$$

Example:

If $\mu_1 = -1$, $\mu_2 = 1$, $\sigma_1 = 1.5$, $\sigma_2 = .5$, $\rho = .7$, find f (1, 2).

Answer:

f(1, 2) = 0.04

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	↑						
2	μ_1	−						
3	σ_1	STO	1	÷	STO	2		
4		x^2						
5	y	↑						
6	μ_2	−						
7	σ_2	STO	3	÷	STO	4		
8		x^2	+	RCL	2	RCL		
9		4	x	2	x			
10	ρ	STO	5	x	−	1		
11		RCL	5	x^2	−	STO		
12		5	2	x	÷	CHS		
13		e^x	RCL	5		$\sqrt{x}$		
14		RCL	1	x	RCL	3		
15		x	2	x		π		
16		x	÷				f(x,y)	

Bonds

Formulas:

n = total number of days between purchase and maturity

C = decimal coupon rate (on an annual basis)

i = annual yield to maturity (as a decimal)

PV = bond price

If $n < 182.5$,

$$PV = \frac{200 + 100C}{2 + \frac{n}{180} \cdot i} - \left(1 - \frac{n}{180}\right) \frac{100C}{2}$$

If $n > 182.5$,

$$PV = 100\left(1 + \frac{i}{2}\right)^{\frac{-n}{182.5}} + 100\left(\frac{C}{i}\right)\left\{\left(1 + \frac{i}{2}\right)^{j} - \left(1 + \frac{i}{2}\right)^{\frac{-n}{182.5}}\right\} - \frac{100Cj}{2}$$

where $j = 1 -$ fractional part of $\dfrac{n}{182.5}$

Example 1:

What is the price of a 4% bond yielding 3% and maturing in 99 days?

Answer:

100.27

Example 2:

What is the price of a bond which has a coupon rate of 4.5%, a yield of 3.22% to maturity, and the number of days between purchase and maturity is 1868?

Answer:

105.99

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	C	STO	1					If n > 182.5, go to 9.
2		2	+					
3	n	↑	1	8	0	÷		
4		↑	↑	R↓	R↓			
5	i	x	2	+	÷	x⇄y		
6		1	–	RCL	1	x		
7		2	÷	+	1	0		
8		0	x				PV	Stop
9	n	STO	2	1	8	2		
10		·	5	÷	STO	3	D	Let f = integer part of D
11	f	–	1	x⇄y	–	STO		
12		4	1	↑				
13	i	STO	7	2	÷	+		
14		STO	5	RCL	3	■		
15		y^x	$1/x$	STO	6	RCL		
16		5	RCL	4	■	y^x		
17		x⇄y	–	RCL	1	x		
18		RCL	7	÷	RCL	6		
19		+	RCL	4	RCL	1		
20		x	2	÷	–	1		
21		0	0	x			PV	

Cash Flow Analysis (Discounted)

Formula:

PV_0 = original investment

PV_k = cash flow of the k^{th} period

i = discount rate per period (as a decimal)

C_k = net present value at period k

$$C_k = -PV_0 + \sum_{j=1}^{n} \frac{PV_j}{(1+i)^j}$$

Example:

You are offered an investment opportunity for $100,000 at a capital cost of 10% after taxes. Will this investment be profitable based on the following cash flows?

Year	Cash flow
1	$34,000
2	$27,500
3	$59,700
4	$ 7,800

Answer:

Final value C_4 = $3817.36 is positive, so the investment will be profitable.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	↑	1	+	STO	1		
2	PV_1	x⇄y	÷					
3	PV_0	−					C_1	
4	PV_j	RCL	1					Perform 4–5 for j=2, 3, ..., n
5	j		y^x	÷	+		C_j	

Centigrade to Fahrenheit

See page 65

Chi–Square Statistics

Evaluation of Chi-Square for goodness of fit

Formula:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequency

E_i = expected frequency

Example:

O_i	8	50	47	56	5	14
E_i	9.6	46.75	51.85	54.4	8.25	9.15

Answer:

$\chi^2 = 4.84$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		CLX	↑					
2	O_i	↑						Perform 2–4 for i=1,2,...,n
3	E_i	STO	1	–	x^2	RCL		
4		1	÷	+				

2 x 2 contingency table

	observed results		
	I	II	Totals
Group A	a	b	a + b
Group B	c	d	c + d

Formula:

$$\chi^2 = \frac{(ad - bc)^2 \, (a + b + c + d)}{(a + b)\,(c + d)\,(a + c)\,(b + d)}$$

(Degrees of freedom d.f. = 1)

Example:

	I	II
A	75	25
B	65	35

Answer:

$\chi^2 = 2.38$ (d.f. = 1)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1					
2	b	STO	2	+				
3	c	STO	3	+				
4	d	STO	4	+				
5		RCL	1	RCL	4	x		
6		RCL	2	RCL	3	x		
7		–	x^2	x	RCL	1		
8		RCL	2	+	RCL	3		
9		RCL	4	+	x	RCL		
10		1	RCL	3	+	x		
11		RCL	2	RCL	4	+		
12		x	÷				χ^2	

2 x k contingency table

	1	2	3	...	k	Totals
A	a_1	a_2	a_3	...	a_k	N_A
B	b_1	b_2	b_3	...	b_k	N_B
Totals	N_1	N_2	N_3	...	N_k	N

Formula:

$$\chi^2 = \frac{N}{N_A} \sum_{i=1}^{k} \frac{a_i^2}{N_i} + \frac{N}{N_B} \sum_{i=1}^{k} \frac{b_i^2}{N_i} - N$$

Degrees of freedom = k − 1

Example:

	1	2	3
A	2	5	4
B	3	8	7

Answer:

$\chi^2 = 0.02$ (d.f. = 2)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1			CLEAR	STO	1	STO		
2		2						
3	b_i	STO	+	2	↑	↑		Perform 3–7 for i=1,2,...,k
4	a_i	STO	+	1	↑	R↓		
5		+	x⇄y	x^2	x⇄y	÷		
6			LAST x	R↓	R↓	R↓		
7		x^2	x⇄y	÷	Σ+			
8		RCL	1	RCL	2	+		
9		STO	4	RCL	1	÷		
10		RCL	7	x	RCL	4		
11		RCL	2	÷	RCL	8		
12		x	+	RCL	4	−	x^2	

Testing a population variance

Given a random sample of size n from a normal population (variance σ^2 is unknown), we can use

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^{\,2}}$$

(where s^2 is the sample variance) to test the null hypothesis

$$H_0 : \sigma^2 = \sigma_0^{\,2}$$

Degrees of freedom = n − 1.

Example:

Given a sample $\{2.1, 0.5, -3.1, 1.4, -0.92, -1.35, 1.2\}$ and $\sigma_0^{\,2} = 2.5$, find chi-square.

Answer:

$\chi^2 = 8.13$ (d.f. = 6)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR					
2	x_i	Σ+						Perform 2 for i=1,2,...,n
3		■	$\bar{x}$,s					
4		x⇄y	x^2	RCL	5	1		
5		−	×					
6	$\sigma_0^{\,2}$	÷						

Bartlett's chi-square

Formula:

$$\chi^2 = \frac{f \ln S^2 - \sum_{i=1}^{k} f_i \ln S_i^{\,2}}{1 + \frac{1}{3(k-1)}\left[\left(\sum_{i=1}^{k}\frac{1}{f_i}\right) - \frac{1}{f}\right]}$$

where S_i^2 = sample variance of the i^{th} sample

f_i = degrees of freedom associated with S_i^2

$i = 1, 2, ..., k$

k = number of samples

$$S^2 = \frac{\sum_{i=1}^{k} f_i S_i^2}{f} \qquad f = \sum_{i=1}^{k} f_i$$

This χ^2 has a chi-square distribution (approximately) with $k - 1$ degrees of freedom which can be used to test the null hypothesis that $S_1^2, S_2^2, ..., S_k^2$ are all estimates of the same population variance σ^2.

H_0: Each of $S_1^2, S_2^2, ..., S_k^2$ is an estimate of σ^2.

Example:

i	1	2	3	4	5	6
f_i	10	20	17	18	8	15
S_i^2	5.5	5.1	5.2	4.7	4.8	4.3

Answer:

$\chi^2 = 0.25$ (d.f. = 5)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1			CLEAR	STO	3	STO		
2		4						
3	f_i	STO	1	STO	+	3		Perform 3–7 for i=1,2,...,k
4		1/x	STO	+	4			
5	S_i^2	↑	↑	RCL	1	x		
6		x⇄y	ln	RCL	1	x		
7		Σ+						
8		RCL	8	RCL	÷	3		
9		ln	RCL	3	x	RCL		
10		–	7	RCL	4	RCL		
11		3	1/x	–	RCL	5		
12		1	–	3	x	÷		
13		1	+	÷			χ^2	

Circle

See page 95

Combinations

Combinations of a objects taken b at a time (binomial coefficient)

Formula:

$$\binom{a}{b} = {}_aC_b = C_b^a = C(a, b) = \frac{a!}{b!\,(a-b)!}$$

Example:

$${}_7C_5 = 21.00$$

Note:

$${}_aC_0 = {}_aC_a = 1,$$

$${}_aC_1 = {}_aC_{a-1} = a$$

Program requires $a \leqslant 69$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a	■ [n!] ■ [LAST x] [ ]		
2	b	[–] ■ [LAST x] ■ [n!]		
3		[x⇄y] ■ [n!] [x] [÷]		

Complex Hyperbolic Functions

Note:

In this section, all angles in the equations are in radians.

Complex hyperbolic sine

Formula:

$$\sinh(a + ib) = -i \sin i(a + ib) = u + iv$$

Example:

$$\sinh(3 - 2i) = -4.17 - 9.15\, i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	STO	1		RAD		
2		SIN						
3	a	STO	2	e^x	↑	1/x		
4		+	2	÷	x	CHS		
5		RCL	1	COS	RCL	2		
6		e^x	↑	1/x	–	2		
7		÷	x				u	
8		x⇄y					v	

Complex hyperbolic cosine

Formula:

$$\cosh(a + ib) = \cos i(a + ib) = u + iv$$

Example:

$$\cosh(1 + 2i) = -0.64 + 1.07\, i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	STO	1		RAD		
2		COS						
3	a	STO	2	e^x	↑	1/x		
4		+	2	÷	x		u	
5		RCL	1	SIN	RCL	2		
6		e^x	↑	1/x	–	2		
7		÷	x	CHS			v	

Complex hyperbolic tangent

Formula:

$$\tanh (a + ib) = -i \tan i\,(a + ib) = u + iv$$

Example:

$$\tanh (1 + 2i) = 1.17 - 0.24\, i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	■	RAD	2	x		
2		SIN	■	LAST x	COS			
3	a	↑	2	x	STO	1		
4		e^x	↑	$1/x$	+	2		
5		÷	+	÷	CHS	■		
6		LAST x	RCL	1	e^x	↑		
7		$1/x$	–	2	÷	x⇄y		
8		÷					u	
9		x⇄y					v	

Complex hyperbolic cotangent

Formula:

$$\coth (a + ib) = i \cot i\,(a + ib) = u + iv$$

Example:

$$\coth (1 + 2i) = 0.82 + 0.17\, i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	■	RAD	2	x		
2		SIN	■	LAST x	COS	CHS		
3	a	↑	2	x	STO	1		
4		e^x	↑	$1/x$	+	2		
5		÷	+	÷	■	LAST x		
6		RCL	1	e^x	↑	$1/x$		
7		–	2	÷	x⇄y	÷	u	
8		x⇄y					v	

Complex hyperbolic cosecant

Formula:

$$\operatorname{csch}(a + ib) = i \csc i(a + ib) = u + iv$$

Example:

$$\operatorname{csch}(1 + 2i) = -0.22 - 0.64\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	STO	1	■	RAD		
2		SIN						
3	a	STO	2	e^x	↑	$1/x$		
4		+	2	÷	x	RCL		
5		1	COS	RCL	2	e^x		
6		↑	$1/x$	−	2	÷		
7		x	STO	1	x^2	x⇄y		
8		x^2	■	LAST x	↑	R↓		
9		R↓	+	÷	RCL	1		
10		■	LAST x	÷			u	
11		x⇄y					v	

Complex hyperbolic secant

Formula:

$$\operatorname{sech}(a + ib) = \sec i(a + ib) = u + iv$$

Example:

$$\operatorname{sech}(1 + 2i) = -0.41 - 0.69\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	STO	1	■	RAD		
2		COS						
3	a	STO	2	e^x	↑	$1/x$		
4		+	2	÷	x	RCL		
5		1	SIN	RCL	2	e^x		
6		↑	$1/x$	−	2	÷		
7		x	STO	1	x^2	x⇄y		
8		x^2	■	LAST x	↑	R↓		
9		R↓	+	÷			u	
10		RCL	1	■	LAST x	÷	v	

Complex inverse hyperbolic sine

Formula:

$$\sinh^{-1}(a + ib) = -i \sin^{-1} i\,(a + ib) = u + iv$$

Example:

$$\sinh^{-1}(8 - 5i) = 2.94 - 0.56\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	↑	1	+	↑		
2		↑	2	−	x^2			
3	a	x^2	+	x⇄y	■	LAST x		
4		x⇄y	x^2	+	■	$\sqrt{x}$		
5		STO	1	x⇄y	■	$\sqrt{x}$		
6		−	■	LAST x	RCL	1		
7		+	2	÷	↑	x^2		
8		1	−	■	$\sqrt{x}$	+		
9		ln						If a ⩾ 0, go to 11
10		CHS						
11							u	
12		x⇄y	2	÷	■	RAD		
13		■	SIN^{-1}	CHS			v	

Complex inverse hyperbolic cosine

Formula:

$$\cosh^{-1}(a + ib) = i \cos^{-1}(a + ib) = u + iv$$

Example:

$$\cosh^{-1}(5 + 8i) = 2.94 + 1.01\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	1	+	↑	↑		
2		2	–	x^2	■	RAD		
3	b	x^2	+	x⇄y	■	LAST x		
4		x⇄y	x^2	+	■	$\sqrt{x}$		
5		STO	1	x⇄y	■	$\sqrt{x}$		
6		–	■	LAST x	RCL	1		
7		+	2	÷	↑	x^2		
8		1	–	■	$\sqrt{x}$	+		
9		ln						If $b \geqslant 0$, go to 11
10		CHS						
11							u	
12		x⇄y	2	÷	■	COS^{-1}	v	

Complex inverse hyperbolic tangent

Formula:

$$\tanh^{-1} (a + ib) = -i \tan^{-1} i (a + ib) = u + iv$$

Example:

$$\tanh^{-1} (8 - 5i) = 0.09 - 1.51 i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	π	1	↑			
2	a	STO	1	+	■	RAD		
3	b	CHS	STO	2	÷	■		
4		TAN^{-1}	–	1	RCL	–		
5		1	RCL	÷	2	■		
6		TAN^{-1}	–	2	÷	CHS		
7		RCL	1	1	+	x^2		
8		RCL	2	x^2	+	■		
9		LAST x	1	RCL	–	1		
10		x^2	+	÷	ln	4		
11		÷					u	
12		x⇄y					v	

Complex inverse hyperbolic cotangent

Formula:

$$\coth^{-1}(a + ib) = i \cot^{-1} i(a + ib) = u + iv$$

Example:

$$\coth^{-1}(8 - 5i) = 0.09 + 0.06\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1	1	+			
2	b	CHS	STO	2	÷	■		
3		RAD	■	TAN^{-1}	1	RCL		
4		–	1	RCL	÷	2		
5		■	TAN^{-1}	+	2	÷		
6		RCL	1	1	+	x^2		
7		RCL	2	x^2	+	■		
8		LAST x	1	RCL	–	1		
9		x^2	+	÷	ln	4		
10		÷					u	
11		x⇄y					v	

Complex inverse hyperbolic cosecant

Formula:

$$\operatorname{csch}^{-1}(a + ib) = i \csc^{-1} i(a + ib) = u + iv$$

Example:

$$\operatorname{csch}^{-1}(8 - 5i) = 0.09 + 0.06\,i$$

$$(D = -0.09)$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	CHS	STO	1	x^2			
2	b	CHS	x^2	■	LAST x	↑		
3		R↓	R↓	+	÷	RCL		
4		1	■	LAST x	÷	STO		
5		1	■	RAD			D	
6		x⇄y	↑	1	+	↑		
7		↑	2	–	x^2	RCL		
8		1	x^2	+	x⇄y	■		
9		LAST x	x⇄y	x^2	+	■		
10		$\sqrt{x}$	STO	1	x⇄y	■		
11		$\sqrt{x}$	–	■	LAST x	RCL		
12		1	+	2	÷	↑		
13		x^2	1	–	■	$\sqrt{x}$		
14		+	ln					If D < 0, go to 16
15		CHS						
16							u	
17		x⇄y	2	÷	■	SIN^{-1}	v	

Complex inverse hyperbolic secant

Formula:

$$\operatorname{sech}^{-1}(a + ib) = i \sec^{-1}(a + ib) = u + iv$$

Example:

$$\operatorname{sech}^{-1}(5 + 8i) = -0.09 + 1.51\, i$$

$$(D = -0.09)$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	STO	1	x^2			
2	a	x^2	■	LAST x	↑	R↓		
3		R↓	+	÷	RCL	1		
4		■	LAST x	÷	STO	1	D	
5		$x \rightleftarrows y$	↑	1	+	↑		
6		↑	2	−	x^2	■		
7		RAD	RCL	1	x^2	+		
8		$x \rightleftarrows y$	■	LAST x	$x \rightleftarrows y$	x^2		
9		+	■	$\sqrt{x}$	STO	1		
10		$x \rightleftarrows y$	■	$\sqrt{x}$	−	■		
11		LAST x	RCL	1	+	2		
12		÷	↑	x^2	1	−		
13		■	$\sqrt{x}$	+	ln			If D ⩾ 0, go to 15
14		CHS						
15							u	
16		$x \rightleftarrows y$	2	÷	■	$\cos^{-1}$	v	

Complex Number Operations

Complex add

Formula:

$$(a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i\,(b_1 + b_2) = u + iv$$

Example:

$$(3 + 4i) + (7.4 - 5.6i) = 10.40 - 1.60i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a_1	↑						
2	a_2	+					u	
3	b_1	↑						
4	b_2	+					v	

Complex subtract

Formula:

$$(a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + i\,(b_1 - b_2) = u + iv$$

Example:

$$(3 + 4i) - (7.4 - 5.6i) = -4.4 + 9.6\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a_1	↑						
2	a_2	−					u	
3	b_1	↑						
4	b_2	−					v	

Complex multiply

Formula:

$$(a_1 + ib_1)(a_2 + ib_2) = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1) = u + iv$$

Example:

$$(3 + 4i)(7 - 2i) = 29.00 + 22.00\, i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a_1	STO	1					
2	b_2	STO	2	x				
3	b_1	STO	3					
4	a_2	x	■	LAST x	RCL	x		
5		1	RCL	2	RCL	x		
6		3	-				u	
7		R↓	+				v	

Multiplication of n complex numbers

Formula:

$$\prod_{k=1}^{n} (a_k + ib_k) = \left(\prod_{k=1}^{n} r_k \right) e^{\,i \sum_{k=1}^{n} \theta_k} = u + iv$$

where $a_k + ib_k = r_k\, e^{i\theta_k}$.

Example:

$$(3 + 4i)(7 - 2i)(4.38 + 7i)(12.3 - 5.44i) = 1296.66 + 3828.90\, i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR					
2	b_k	↑						Perform 2–3 for k=1,2,...,n
3	a_k	→P	ln	Σ+				
4		RCL	Σ+	e^x	■	→R	u	
5		x⇄y					v	

Complex divide

Formula:

$$(a_1 + ib_1) \div (a_2 + ib_2) = \frac{r_1}{r_2}\, e^{i(\theta_1 - \theta_2)} = u + iv$$

where

$$a_1 + ib_1 = r_1\, e^{i\theta_1}$$

$$a_2 + ib_2 = r_2\, e^{i\theta_2} \neq 0$$

Example:

$$\frac{3 + 4i}{7 - 2i} = 0.25 + 0.64\, i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b_2	↑	■	RAD				
2	a_2	→P						
3	b_1	↑						
4	a_1	→P	x⇄y	R↓	x⇄y	÷		
5		R↓	–	R↓	R↓	R↓		
6		■	→R				u	
7		x⇄y					v	

Complex reciprocal

Formula:

$$\frac{1}{a + ib} = \frac{a - ib}{a^2 + b^2} = u + iv$$

Example:

$$\frac{1}{2 + 3i} = 0.15 - 0.23\, i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	STO	1	x^2			
2	a	x^2	■	LAST x	↑	R↓		
3		R↓	+	÷			u	
4		RCL	1	■	LAST x	÷	v	

Complex absolute value

Formula:

$$|a + ib| = \sqrt{a^2 + b^2}$$

Example:

$$|3 + 4i| = 5.00$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	→P						

Complex square

Formula:

$$(a + ib)^2 = (a^2 - b^2) + i\,(2ab) = u + iv$$

Example:

$$(7 - 2i)^2 = 45.00 - 28.00\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	↑	x				
2	b	STO	1	↑	x	−	u	
3		x⇄y	RCL	1	x	2		
4		x					v	

Complex square root

Formula:

$$\sqrt{a + ib} = \pm\, r^{\frac{1}{2}}\, e^{i\frac{\theta}{2}}$$

where

$$a + ib = re^{i\theta}$$

Example:

$$\sqrt{7 + 6i} = \pm(2.85 + 1.05i)$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	→P	■	$\sqrt{x}$	x⇄y	2		
3		÷	x⇄y	■	→R		u	
4		x⇄y					v	

Complex natural logarithm (base e)

Formula:

$$\ln(a + ib) = \ln\left(\sqrt{a^2 + b^2}\right) + i\left(\tan^{-1}\frac{b}{a}\right)$$

$$= \ln r + i\theta = u + iv \quad (\theta \text{ is in radians})$$

Example:

$\ln i = 1.57\,i$

Note: $a + ib = r\,e^{i\theta} = r(\cos\theta + i\sin\theta)$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	■	RAD	→P	ln		u	
3		x⇄y					v	

Complex exponential

Formula:

$$e^{(a+ib)} = e^a\,e^{ib} = e^a(\cos b + i\sin b) = u + iv$$

Example:

$$e^{1.57i} = 1.00\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	■	RAD					
2	a	e^x	■	→R			u	
3		x⇄y					v	

Complex exponential (t^{a+ib})

Formula:

$$t^{a+ib} = e^{(a+ib)\ln t} = u + iv \quad (t > 0)$$

Example:

$$2^{3+4i} = -7.46 + 2.89\,i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b	↑ ■ RAD		
2	t	ln × ■ LAST x		
3	a	× e^x ■ →R	u	
4		x⇄y	v	

Integral power of a complex number

Formula:

$$(a + ib)^n = r^n (\cos n\theta + i \sin n\theta) = u + iv$$

where $r = \sqrt{a^2 + b^2}$, $\theta = \tan^{-1} \dfrac{b}{a}$ and

n is a positive integer.

Example:

$$(3 + 4.5i)^5 = 926.44 - 4533.47\,i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b	↑		
2	a	→P		
3	n	■ y^x x⇄y ■ LAST x		
4		× x⇄y ■ →R	u	
5		x⇄y	v	

Integral roots of a complex number

Formula:

$$(a + b)^{\frac{1}{n}} = r^{\frac{1}{n}} \left(\cos \frac{\theta + 360k}{n} + i \sin \frac{\theta + 360k}{n} \right)$$

$$= u_k + iv_k$$

where n is a positive integer and k = 0, 1, ..., n − 1. (θ is in degrees)

Example:

5 + 3 i has three cube roots:

$$u_0 + iv_0 = 1.77 + 0.32i$$
$$u_1 + iv_1 = -1.16 + 1.37i$$
$$u_2 + iv_2 = -0.61 - 1.69i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑	■	DEG				
2	a	→P						
3	n	$^1/_x$	■	y^x	x⇄y	■		
4		LAST x	x	x⇄y	3	6		
5		0	■	LAST x	x	STO		
6		2	R↓	■	→R		u_0	
7		x⇄y					v_0	
8		x⇄y	→P	x⇄y	RCL	+		Perform 8–10 for k=1,2,...,
9		2	x⇄y	■	→R		u_k	n−1
10		x⇄y					v_k	

Complex number to a complex power

Formula:

$$(a_1 + ib_1)^{(a_2+ib_2)} = e^{(a_2+ib_2)\ln(a_1 + ib_1)} = u + iv$$

where $a_1 + ib_1 \neq 0$

Example:

$$(1 + i)^{(2-i)} = 1.49 + 4.13\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b_1	↑	■	RAD				
2	a_1	→P	ln	STO	1			
3	b_2	STO	2	x	x⇄y	STO		
4		3						
5	a_2	x	■	LAST x	RCL	x		
6		1	RCL	2	RCL	x		
7		3	–	STO	1	R↓		
8		+	RCL	1	e^x	■		
9		→R					u	
10		x⇄y					v	

Complex root of a complex number

Formula:

$$(a_1 + ib_1)^{\frac{1}{a_2+ib_2}} = e^{[\ln(a_1 + ib_1) / (a_2 + ib_2)]} = u + iv$$

where $a_1 + ib_1 \neq 0$

Example:

Find the $(2 - i)^{th}$ root of $1.49 + 4.13i$.

Answer:

$1.00 + 1.00i$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b_1	↑	■	RAD				
2	a_1	→P	ln	STO	1	x⇄y		
3		STO	2					
4	b_2	↑						
5	a_2	→P	RCL	2	RCL	1		
6		→P	x⇄y	R↓	x⇄y	÷		
7		R↓	–	R↓	R↓	R↓		
8		■	→R	e^x	■	→R	u	
9		x⇄y					v	

Logarithm of a complex number to a complex base

Formula:

$$\log_{(a_1+ib_1)}(a_2+ib_2) = \frac{\ln(a_2+ib_2)}{\ln(a_1+ib_1)} = u + iv$$

$$a_1 + ib_1 \neq 0$$

Example:

$$\log_{(1+i)}(1.49 + 4.13i) = 2.00 - 1.00i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b_1	↑	■	RAD				
2	a_1	→P	ln	→P				
3	b_2	↑						
4	a_2	→P	ln	→P	x⇄y	R↓		
5		x⇄y	÷	R↓	–	R↓		
6		R↓	R↓	■	→R		u	
7		x⇄y					v	

Complex Trigonometric Functions

Note:

In this section, all angles in the equations are in radians.

Complex sine

Formula:

$$\sin(a + ib) = \sin a \cosh b + i \cos a \sinh b = u + iv$$

Example:

$$\sin(2 + 3i) = 9.15 - 4.17i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1		RAD	SIN		
2	b	STO	2	e^x	↑	$1/x$		
3		+	2	÷	x		u	
4		RCL	1	COS	RCL	2		
5		e^x	↑	$1/x$	−	2		
6		÷	x				v	

Complex cosine

Formula:

$$\cos(a + ib) = \cos a \cosh b - i \sin a \sinh b = u + iv$$

Example:

$$\cos(2 + 3i) = -4.19 - 9.11i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1		RAD	COS		
2	b	STO	2	e^x	↑	$1/x$		
3		+	2	÷	x		u	
4		RCL	1	SIN	RCL	2		
5		e^x	↑	$1/x$	−	2		
6		÷	x	CHS			v	

Complex tangent

Formula:

$$\tan(a + ib) = \frac{\sin 2a + i \sinh 2b}{\cos 2a + \cosh 2b} = u + iv$$

Example:

$$\tan(2 + 3i) = -0.004 + 1.003i$$

(Press FIX 3 to see the answers.)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	■	RAD	2	x	SIN		
2		■	LAST x	COS				
3	b	↑	2	x	STO	1		
4		e^x	↑	1/x	+	2		
5		÷	+	÷			u	
6		■	LAST x	RCL	1	e^x		
7		↑	1/x	–	2	÷		
8		x⇄y	÷				v	

Complex cotangent

Formula:

$$\cot(a + ib) = \frac{\sin 2a - i \sinh 2b}{\cosh 2b - \cos 2a} = u + iv$$

Example:

$$\cot(2 + 3i) = -0.004 - 0.997i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	■	RAD	2	x	SIN		
2		■	LAST x	COS	CHS			
3	b	↑	2	x	STO	1		
4		e^x	↑	1/x	+	2		
5		÷	+	÷			u	
6		■	LAST x	RCL	1	e^x		
7		↑	1/x	–	2	÷		
8		x⇄y	÷	CHS			v	

Complex cosecant

Formula:

$$\csc(a + ib) = \frac{1}{\sin(a + ib)} = u + iv$$

Example:

$$\csc(2 + 3i) = 0.09 + 0.04i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1	■	RAD	SIN		
2	b	STO	2	e^x	↑	1/x		
3		+	2	÷	x	RCL		
4		1	COS	RCL	2	e^x		
5		↑	1/x	–	2	÷		
6		x	CHS	STO	1	x^2		
7		x⇄y	x^2	■	LAST x	↑		
8		R↓	R↓	+	÷		u	
9		RCL	1	■	LAST x	÷	v	

Complex secant

Formula:

$$\sec(a + ib) = \frac{1}{\cos(a + ib)} = u + iv$$

Example:

$$\sec(2 + 3i) = -0.04 + 0.09i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1	■	RAD	COS		
2	b	STO	2	e^x	↑	1/x		
3		+	2	÷	x	RCL		
4		1	SIN	RCL	2	e^x		
5		↑	1/x	–	2	÷		
6		x	STO	1	x^2	x⇄y		
7		x^2	■	LAST x	↑	R↓		
8		R↓	+	÷			u	
9		RCL	1	■	LAST x	÷	v	

Complex arc sine

Formula:

$$\sin^{-1}(a+ib) = \sin^{-1}\beta + i\,\mathrm{sgn}(b)\ln(\alpha + \sqrt{\alpha^2 - 1}\,) = u + iv$$

where $\alpha = \frac{1}{2}\sqrt{(a+1)^2 + b^2} + \frac{1}{2}\sqrt{(a-1)^2 + b^2}$

$$\beta = \frac{1}{2}\sqrt{(a+1)^2 + b^2} - \frac{1}{2}\sqrt{(a-1)^2 + b^2}$$

$$\mathrm{sgn}(b) = \begin{cases} 1 & \text{if } b \geqslant 0 \\ -1 & \text{if } b < 0 \end{cases}$$

Example:

$$\sin^{-1}(5 + 8i) = 0.56 + 2.94i$$

Note:

Inverse trigonometric and inverse hyperbolic functions are multiple-valued functions, but only one answer is given for each.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	1	+	↑	↑		
2		2	−	x^2	■	RAD		
3	b	x^2	+	x⇄y	■	LAST x		
4		x⇄y	x^2	+	■	$\sqrt{x}$		
5		STO	1	x⇄y	■	$\sqrt{x}$		
6		−	■	LAST x	RCL	1		
7		+	2	÷	↑	x^2		
8		1	−	■	$\sqrt{x}$	+		
9		ln	x⇄y	2	÷	■		
10		SIN^{-1}					u	
11		x⇄y						If b ⩾ 0, go to 13
12		CHS						
13							v	

Complex arc cosine

Formula:

$$\cos^{-1}(a+ib) = \cos^{-1}\beta - i\,\mathrm{sgn}(b)\ln(\alpha + \sqrt{\alpha^2 - 1})$$

where $\alpha = \frac{1}{2}\sqrt{(a+1)^2 + b^2} + \frac{1}{2}\sqrt{(a-1)^2 + b^2}$

$$\beta = \frac{1}{2}\sqrt{(a+1)^2 + b^2} - \frac{1}{2}\sqrt{(a-1)^2 + b^2}$$

$$\mathrm{sgn}(b) = \begin{cases} 1 & \text{if } b \geqslant 0 \\ -1 & \text{if } b < 0 \end{cases}$$

Example:

$$\cos^{-1}(5 + 8i) = 1.01 - 2.94i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	1	+	↑	↑		
2		2	–	x^2	■	RAD		
3	b	x^2	+	x⇄y	■	LAST x		
4		x⇄y	x^2	+	■	$\sqrt{x}$		
5		STO	1	x⇄y	■	$\sqrt{x}$		
6		–	■	LAST x	RCL	1		
7		+	2	÷	↑	x^2		
8		1	–	■	$\sqrt{x}$	+		
9		ln	x⇄y	2	÷	■		
10		$\cos^{-1}$					u	
11		x⇄y						If b < 0, go to 13
12		CHS						
13							v	

Complex arc tangent

Formula:

$$\tan^{-1}(a+ib)=\frac{1}{2}\left[\pi-\tan^{-1}\left(\frac{1+b}{a}\right)-\tan^{-1}\left(\frac{1-b}{a}\right)\right]+\frac{i}{4}\ln\left[\frac{a^2+(1+b)^2}{a^2+(1-b)^2}\right]$$

$$= u + iv$$

where $(a+ib)^2 \neq -1$

Example:

$$\tan^{-1}(5+8i) = 1.51 + 0.09\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	π	1	↑			
2	b	STO	1	+	■	RAD		
3	a	STO	2	÷	■	TAN⁻¹		
4		−	1	RCL	−	1		
5		RCL	÷	2	■	TAN⁻¹		
6		−	2	÷			u	
7		RCL	1	1	+	x^2		
8		RCL	2	x^2	+	■		
9		LAST x	1	RCL	−	1		
10		x^2	+	÷	ln	4		
11		÷					v	

Complex arc cotangent

Formula:

$$\cot^{-1}(a + ib) = \frac{\pi}{2} - \tan^{-1}(a + ib)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{1 + b}{a}\right) + \frac{1}{2}\tan^{-1}\left(\frac{1 - b}{a}\right) - \frac{i}{4}\ln\left[\frac{a^2 + (1 + b)^2}{a^2 + (1 - b)^2}\right]$$

$$= u + iv$$

Example:

$$\cot^{-1}(5 + 8i) = 0.06 - 0.09\,i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	STO	1	1	+			
2	a	STO	2	÷	■	RAD		
3		■	TAN^{-1}	1	RCL	−		
4		1	RCL	÷	2	■		
5		TAN^{-1}	+	2	÷		u	
6		RCL	1	1	+	x^2		
7		RCL	2	x^2	+	■		
8		LAST x	1	RCL	−	1		
9		x^2	+	÷	ln	4		
10		÷	CHS				v	

Complex arc cosecant

Formula:

$$\csc^{-1}(a + ib) = \sin^{-1}\left(\frac{1}{a + ib}\right) = u + iv$$

Example:

$$\csc^{-1}(5 + 8i) = 0.06 - 0.09\,i$$

$$(D = -0.09)$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	STO	1	x^2			
2	a	x^2	■	LAST x	↑	R↓		
3		R↓	+	÷	RCL	1		
4		■	LAST x	÷	STO	1	D	
5		x⇄y	↑	1	+	↑		
6		↑	2	–	x^2	■		
7		RAD	RCL	1	x^2	+		
8		x⇄y	■	LAST x	x⇄y	x^2		
9		+	■	$\sqrt{x}$	STO	1		
10		x⇄y	■	$\sqrt{x}$	–	■		
11		LAST x	RCL	1	+	2		
12		÷	↑	x^2	1	–		
13		■	$\sqrt{x}$	+	ln	x⇄y		
14		2	÷	■	SIN^{-1}		u	
15		x⇄y						If $D \geqslant 0$, go to 17
16		CHS						
17							v	

Complex arc secant

Formula:

$$\sec^{-1}(a + ib) = \cos^{-1}\left(\frac{1}{a + ib}\right) = u + iv$$

Example:

$$\sec^{-1}(5 + 8i) = 1.51 + 0.09\,i$$

$$(D = -0.09)$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	CHS	STO	1	x^2			
2	a	x^2	■	LAST x	↑	R↓		
3		R↓	+	÷	RCL	1		
4		■	LAST x	÷	STO	1	D	
5		x⇄y	↑	1	+	↑		
6		↑	2	–	x^2	■		
7		RAD	RCL	1	x^2	+		
8		x⇄y	■	LAST x	x⇄y	x^2		
9		+	■	$\sqrt{x}$	STO	1		
10		x⇄y	■	$\sqrt{x}$	–	■		
11		LAST x	RCL	1	+	2		
12		÷	↑	x^2	1	–		
13		■	$\sqrt{x}$	+	ln	x⇄y		
14		2	÷	■	$\cos^{-1}$		u	
15		x⇄y						If D < 0, go to 17
16		CHS						
17							v	

Compound Interest

See page 132

Contingency Table

See page 34

Conversions

Repetitive use of formulas with two constants

Note:

The technique used here is storing both constants (one in register R_1, the other in T), after which the formulas can be used repeatedly.

Centigrade to Fahrenheit

Formula:

$$a°C \rightarrow b°F$$

where $b = \frac{9}{5} a + 32$

Examples:

a	–30	0	28	100	539
b	–22.00	32.00	82.40	212.00	1002.20

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		3	2	STO	1	9		
2		↑	5	÷	↑	↑		
3		↑						
4		CLX						
5	a	x	RCL	1	+		b	Stop. For new case,
								go to 4

Fahrenheit to centigrade

Formula:

$$b^\circ F \rightarrow a^\circ C$$

where $a = \frac{5}{9}(b - 32)$

Examples:

b	−460	−40
a	−273.33	−40.00

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		3	2	STO	1	5		
2		↑	9	÷	↑	↑		
3		↑						
4		CLX						
5	b	RCL	1	−	x		a	Stop. For new case,
								go to 4

Feet and inches to centimeters

Formula:

$$1 \text{ foot} = 12 \text{ inches},$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

Examples:

$4'8'' = 142.24$ cm

$5'5'' = 165.10$ cm

$6'3'' = 190.50$ cm

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		1	2	↑	↑	↑		
2		CLX						
3	Ft	x						
4	In	+		cm/in	x		Cm	Stop. For new case,
								go to 2

Centimeters to feet and inches

Examples:

164 cm = 5′4.57″

180 cm = 5′10.87″

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		1	2	STO	1			
2		cm/in	1/x	↑	↑	↑		
3		CLX						
4	Cm	x	RCL	1	÷		D	Let f = integer part of D
5	f						Feet	
6		–	RCL	1	x		Inches	Stop. For new case,
								go to 3

Gallons to liters

Formula:

$$1 \text{ gallon} = 3.785411784 \text{ liters}$$

Notation:

gal = gallons

ltr = liters

Examples:

5.3 gal = 20.06 ltr

61.55 gal = 232.99 ltr

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1			ltr/gal	↑	↑	↑		
2		CLX						
3	gal	x					ltr	Stop. For new case,
								go to 2

Liters to gallons

Notation:

gal = gallons

ltr = liters

Examples:

20 ltr = 5.28 gal

232.99 ltr = 61.55 gal

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	ltr/gal	$1/x$	↑	↑		
2		↑						
3		CLX						
4	ltr	x					gal	Stop. For new case,
								go to 3

Miles to kilometers

Formula:

1 mile = 1.609344 kilometers

Notation:

mi = miles

km = kilometers

Examples:

40 mi = 64.37 km

12.34 mi = 19.86 km

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		1	·	6	0	9		
2		3	4	4	↑	↑		
3		↑						
4		CLX						
5	mi	x					km	Stop. For new case,
								go to 4

Kilometers to miles

Notation:

mi = miles

km = kilometers

Examples:

65 km = 40.39 mi

24.44 km = 15.19 mi

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		1	·	6	0	9		
2		3	4	4	$1/x$	↑		
3		↑	↑					
4		CLX						
5	km	×					mi	Stop. For new case,
								go to 4

Pounds to kilograms

Formula:

1 pound = 0.45359237 kilograms

Notations:

lb = pounds

kg = kilograms

Examples:

150 lb = 68.04 kg

120.5 lb = 54.66 kg

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1			kg/lb	↑	↑	↑		
2		CLX						
3	lb	×					kg	Stop. For new case,
								go to 2

Kilograms to pounds

Notation:

lb = pounds

kg = kilograms

Examples:

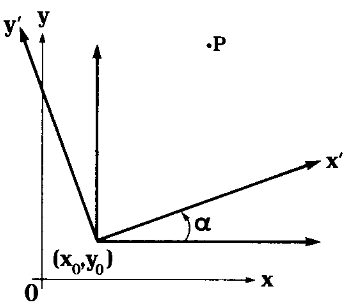
60 kg = 132.28 lb

51.34 kg = 113.19 lb

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	kg/lb	$^1/_x$	↑	↑		
2		↑						
3		CLX						
4	kg	×					lb	Stop. For new case,
								go to 3

Coordinate Translation and Rotation (Rectangular)

Suppose point P has coordinates (x, y) with respect to a coordinate system having x, y axes. After translating the origin to (x_0, y_0), rotate the two axes through an angle α. Find the coordinates (x', y') of P with respect to the new system having x', y' axes.



Formulas:

$$x' = (x - x_0)\cos\alpha + (y - y_0)\sin\alpha$$

$$y' = -(x - x_0)\sin\alpha + (y - y_0)\cos\alpha$$

Example:

If $(x_0, y_0) = (3,2)$, $(x, y) = (5,5)$, $\alpha = 20°$

then $(x', y') = (2.91, 2.14)$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	↑						
2	x_0	–	STO	1				
3	α	STO	2	COS	x			
4	y	↑						
5	y_0	–	STO	3	RCL	2		
6		SIN	x	+			x'	
7		RCL	1	RCL	2	SIN		
8		x	CHS	RCL	3	RCL		
9		2	COS	x	+		y'	

Correlation Coefficient

See page 72

Cosecant

See page 201

Cotangent

See page 200

Covariance and Correlation Coefficient

Formulas:

Covariance

$$Sxy = \frac{1}{n-1}\left[\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i\right]$$

Correlation coefficient

$$r = \frac{Sxy}{Sx\,Sy}$$

where

$$Sx = \left[\frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1}\right]^{1/2}$$

$$Sy = \left[\frac{\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}}{n-1}\right]^{1/2}$$

n = number of data points

Example:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Answers:

Sxy = –354.14

r = –0.96

(Sx = 18.50, Sy = 20.00)

Note:

Also see *t statistic for correlation coefficient.*

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1			CLEAR	STO	3	STO		
2		4						
3	y_i	x^2	STO	+	3			Perform 3–6 for i=1,2,...,n
4		LAST x	↑	↑				
5	x_i	x	STO	+	4	CLX		
6			LAST x	Σ+				
7			$\bar{x}$,s	x⇄y	STO	1	Sx	
8		RCL	4	RCL	7	RCL		
9		8	x	RCL	5	÷		
10		–	RCL	5	1	–		
11		÷					Sxy	
12		RCL	3	RCL	8	x^2		
13		RCL	5	÷	–	RCL		
14		5	1	–	÷			
15		$\sqrt{x}$					Sy	
16		RCL	1	x	÷		r	

Coversine

See page 202

Cross Product (Vector)

See page 211

Cubic Equation

Formulas:

The general cubic equation

$$x^3 + ax^2 + bx + c = 0 \tag{1}$$

can be reduced to the form

$$y^3 + py + q = 0 \tag{2}$$

by letting

$$x = y - \frac{a}{3}$$

where

$$p = b - \frac{a^2}{3}$$

$$q = 2\left(\frac{a}{3}\right)^3 - \frac{ab}{3} + c.$$

The reduced equation (2) has solutions

$$y_1 = A + B$$

$$y_2 = \frac{-(A+B)}{2} + i\,\frac{\sqrt{3}\,(A-B)}{2}$$

$$y_3 = \frac{-(A+B)}{2} - i\,\frac{\sqrt{3}\,(A-B)}{2}$$

where

$$A = \sqrt[3]{\frac{-q}{2} + \sqrt{d}}$$

$$B = \sqrt[3]{\frac{-q}{2} - \sqrt{d}}$$

$$d = \frac{q^2}{4} + \frac{p^3}{27}$$

Case 1. $d > 0$: there exist one real and two complex roots.

Case 2. $d = 0$: there exist three real roots of which at least two are equal.

Case 3. $d < 0$: there exist three real and distinct roots.

Equation (1) has solutions

$$x_i = y_i - \frac{a}{3}, \quad i = 1, 2, 3 .$$

Example 1:

Solve $$x^3 + 2x^2 - 5x - 6 = 0$$

Answers:

$x_1 = 2.00$

$x_2 = -3.00$

$x_3 = -1.00$

$(d = -8.33)$.

Example 2:

Solve $$x^3 - 4x^2 + 8x - 8 = 0$$

Answers:

$x_1 = 2.00$

$x_2 = 1 + 1.73i$

$x_3 = 1 - 1.73i$

$(d = 1.78)$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	STO	7					
2	a	STO	8	↑	x	3		
3		÷	–	STO	1	RCL		
4		8	3	÷	↑	↑		
5		↑	x	x	2	x		
6		$x \rightleftarrows y$	RCL	7	x	–		
7	c	+	STO	2	↑	x		
8		4	÷	RCL	1	↑		
9		↑	x	x	2	7		
10		÷	+	STO	3			
11		DEG					d	If $d < 0$, go to 31.
12			$\sqrt{x}$	STO	3	RCL		
13		2	CHS	2	÷	STO		
14		2	+				D_1	If $D_1 > 0$, go to 16.
								If $D_1 = 0$, go to 18.
15		CHS						
16		3	$1/x$		y^x			If $D_1 > 0$, go to 18.
17		CHS						
18		STO	4	RCL	2	RCL		
19		3	–				D_2	If $D_2 > 0$, go to 21.
								If $D_2 = 0$, go to 23.
20		CHS						
21		3	$1/x$		y^x			If $D_2 > 0$, go to 23.
22		CHS						
23		STO	5	RCL	4	+		
24		STO	4	RCL	8	3		
25		÷	STO	8	–		x_1	
26		RCL	4	2	÷	CHS		
27		STO	6	RCL	8	–	u	If $d = 0$, $x_2 = x_3 = u$, stop
28		RCL	4	RCL	5	2		
29		x	–	2	÷	3		
30			$\sqrt{x}$	x	STO	5	v	$x_2 = u + iv$,
								$x_3 = u - iv$, stop
31		RCL	2	RCL	1	↑		
32		↑	x	x	CHS	2		
33		7	÷		$\sqrt{x}$	2		

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
34		x	÷	CHS	■	$\cos^{-1}$		
35		3	÷	STO	2	COS		
36		RCL	1	3	÷	x^2		
37		■	$\sqrt{x}$	■	$\sqrt{x}$	2		
38		x	STO	1	x	RCL		
39		8	3	÷	STO	8		
40		–					x_1	
41		RCL	2	1	2	0		
42		+	COS	RCL	1	x		
43		RCL	8	–			x_2	
44		RCL	2	2	4	0		
45		+	COS	RCL	1	x		
46		RCL	8	–			x_3	

Curve Fitting

Linear regression and correlation coefficient

Formulas:

This routine fits a straight line

$$y = ax + b$$

to a set of data points

$$\left\{ (x_i, y_i), i = 1, \ldots, n \right\}$$

by the least squares method.

$$a = \frac{\Sigma x_i y_i - \dfrac{\Sigma x_i \Sigma y_i}{n}}{\Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n}}$$

$$b = \bar{y} - a\bar{x}$$

where

$$\bar{x} = \frac{\Sigma x_i}{n} \qquad \bar{y} = \frac{\Sigma y_i}{n}$$

coefficient of determination:

$$r^2 = \frac{\left[\Sigma x_i y_i - \frac{\Sigma x \Sigma y}{n}\right]^2}{\left[\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right]\left[\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right]}$$

r^2 can be interpreted as the proportion of total variation about the mean $\overline{y}$ explained by the regression. In other words, r^2 measures the "goodness of fit" of the regression line. Note that $0 \leqslant r^2 \leqslant 1$, and if $r^2 = 1$, we have a perfect fit.

Correlation coefficient

$$r = \sqrt{r^2}$$

(r takes the sign of a)

Σy_i^2, $\Sigma x_i y_i$, n, Σx_i^2, Σx_i, Σy_i are in storage registers R_3 through R_8.

Example:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Answers:

$y = -1.03x + 121.04$

$r^2 = 0.92$

$r = -0.96$

$n = 7$

$\Sigma x_i = 354$

$\Sigma x_i^2 = 19956$

$\Sigma y_i = 481$

$\Sigma y_i^2 = 35451$

$\Sigma x_i y_i = 22200$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR	STO	3	STO		
2		4						
3	y_i	x^2	STO	+	3	■		Perform 3–6 for i=1, 2, ..., n
4		LAST x	↑	↑				
5	x_i	x	STO	+	4	CLX		
6		■	LAST x	Σ+				
7		RCL	4	RCL	Σ+	x		
8		RCL	5	÷	−	STO		
9		2	RCL	6	RCL	7		
10		x^2	RCL	5	÷	−		
11		÷	STO	1			a	
12		RCL	7	x	RCL	8		
13		x⇄y	−	RCL	5	÷	b	
14		RCL	1	RCL	2	x		
15		RCL	3	RCL	8	x^2		
16		RCL	5	÷	−	÷	r^2	
17		■	$\sqrt{x}$				r	If a > 0, stop
18		CHS					r	

Multiple linear regression (three variables)

For the set of data points (x_i, y_i, z_i), this key sequence fits a linear equation of the form

$$z = a_0 + a_1 x + a_2 y$$

by the method of least squares.

Formulas:

Regression coefficients a_0, a_1, a_2 can be found by solving the normal equations:

$$\begin{cases} \Sigma z_i = a_0 n + a_1\ \Sigma x_i + a_2\ \Sigma y_i \\ \Sigma x_i z_i = a_0\ \Sigma x_i + a_1\ \Sigma x_i^2 + a_2\ \Sigma x_i y_i \\ \Sigma y_i z_i = a_0\ \Sigma y_i + a_1\ \Sigma x_i y_i + a_2\ \Sigma y_i^2 \end{cases}$$

$$i = 1, 2, \ldots, n$$

$$a_2 = \frac{A - B}{[n\Sigma x_i^2 - (\Sigma x_i)^2]\ [n\Sigma y_i^2 - (\Sigma y_i)^2] - [n\Sigma x_i y_i - (\Sigma x_i)(\Sigma y_i)]^2}$$

where $A = [n\Sigma x_i^2 - (\Sigma x_i)^2]\ [n\Sigma y_i z_i - (\Sigma y_i)(\Sigma z_i)]$

$B = [n\Sigma x_i y_i - (\Sigma x_i)(\Sigma y_i)]\ [n\Sigma x_i z_i - (\Sigma x_i)(\Sigma z_i)]$

$$a_1 = \frac{[n\Sigma x_i z_i - (\Sigma x_i)(\Sigma z_i)] - a_2\ [n\Sigma x_i y_i - (\Sigma x_i)(\Sigma y_i)]}{n\Sigma x_i^2 - (\Sigma x_i)^2}$$

$$a_0 = \frac{1}{n}\ (\Sigma z_i - a_2 \Sigma y_i - a_1 \Sigma x_i)$$

$\Sigma x_i y_i$, $\Sigma x_i z_i$, $\Sigma y_i z_i$, Σy_i^2, n, Σx_i^2, Σx_i, Σy_i, Σz_i are in storage registers R_1 through R_9.

Example:

x	1.5	0.45	1.8	2.8
y	0.7	2.3	1.6	4.5
z	2.1	4.0	4.1	9.4

Answer:

$z = -0.10 + 0.79x + 1.63y$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR	STO	1	STO		Or turn machine off and on
2		2	STO	3	STO	4		
3		STO	9					
4	z_i	STO	+	9				Perform 4–12 for i=1,2,...,n
5	x_i	↑	R↓	x⇄y	x	STO		
6		+	2	CLX	■	LAST x		
7	y_i	x	STO	+	3	CLX		
8		■	LAST x	x^2	STO	+		
9		4	CLX	■	LAST x	R↓		
10		R↓	R↓	x⇄y	x	STO		
11		+	1	■	LAST x	R↓		
12		R↓	R↓	Σ+				
13		RCL	5	RCL	x	6		
14		RCL	7	x^2	–	RCL		
15		5	RCL	x	3	RCL		
16		8	RCL	x	9	–		
17		x	RCL	5	RCL	x		
18		2	RCL	7	RCL	x		
19		9	–	RCL	5	RCL		
20		1	x	RCL	7	RCL		
21		x	8	–	x	–		
22		RCL	5	RCL	x	6		
23		RCL	7	x^2	–	RCL		
24		5	RCL	x	4	RCL		
25		8	x^2	–	x	RCL		
26		1	RCL	5	x	RCL		
27		7	RCL	x	8	–		
28		x^2	–	÷			a_2	
29		RCL	5	RCL	x	2		
30		RCL	7	RCL	x	9		
31		–	RCL	5	RCL	1		
32		x	RCL	7	RCL	x		
33		8	–	R↓	R↓	R↓		
34		x	–	RCL	5	RCL		
35		6	x	RCL	7	x^2		
36		–	÷				a_1	
37		RCL	9	RCL	8	R↓		
38		R↓	R↓	x	–	RCL		
39		7	R↓	R↓	R↓	x		
40		–	RCL	5	÷		a_0	

Parabola (least squares fit)

Formula:

For a set of data points $\{(x_i, y_i), i = 1, 2, ..., n\}$, this routine fits a parabola

$$y = a_0 + a_1 x + a_2 x^2$$

which is a special case of the three variable multiple regression model

$$z = a_0 + a_1 x + a_2 y$$

(if we replace y by x^2 and z by y).

Σx_i^3, $\Sigma x_i y_i$, $\Sigma x_i^2 y_i$, Σx_i^4, n, Σx_i^2, Σx_i, Σy_i are in registers R_1 through R_8.

Example:

x_i	0	1	1.5	3	5
y_i	2.1	2	-5	-24.5	-80

Answer:

$y = 2.28 + 1.85x - 3.66x^2$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR	STO	1	STO		
2		2	STO	3	STO	4		
3	x_i	↑	↑	↑	x^2	×		Perform 3–8 for i=1,2,...,n
4		STO	+	1	×	STO		
5		+	4	R↓				
6	y_i	STO	9	×	STO	+		
7		2	×	STO	+	3		
8		CLX	RCL	9	x⇄y	Σ+		
9		RCL	5	RCL	×	6		
10		RCL	7	x^2	−	STO		
11		9	RCL	5	RCL	×		
12		3	RCL	6	RCL	×		
13		8	−	×	RCL	5		
14		RCL	×	2	RCL	7		
15		RCL	×	8	−	RCL		
16		5	RCL	1	×	RCL		
17		7	RCL	×	6	−		
18		×	−	RCL	9	RCL		
19		5	RCL	×	4	RCL		
20		6	x^2	−	×	RCL		
21		1	RCL	5	×	RCL		
22		7	RCL	×	6	−		
23		x^2	−	÷			a_2	
24		RCL	5	RCL	×	2		
25		RCL	7	RCL	×	8		
26		−	RCL	5	RCL	1		
27		×	RCL	7	RCL	×		
28		6	−	R↓	R↓	R↓		
29		×	−	RCL	9	÷	a_1	
30		RCL	8	RCL	6	R↓		
31		R↓	R↓	×	−	RCL		
32		7	R↓	R↓	R↓	×		
33		−	RCL	5	÷		a_0	

Least squares regression of $y = cx^a + dx^b$

Formula:

This key sequence determines the coefficients c, d of the equation

$$y = cx^a + dx^b$$

for a set of data points

$$\{(x_i, y_i), i = 1, 2, \ldots, n\},$$

where a, b are any given real numbers.

$$d = \frac{(\Sigma x_i^{2a})(\Sigma x_i^b y_i) - (\Sigma x_i^a y_i)(\Sigma x_i^{a+b})}{(\Sigma x_i^{2b})(\Sigma x_i^{2a}) - (\Sigma x_i^{a+b})^2}$$

$$c = \frac{\Sigma x_i^a y_i - d\Sigma x_i^{a+b}}{\Sigma x_i^{2a}}$$

where $x_i > 0$ for $i = 1, 2, \ldots, n$.

Example:

$$a = \frac{1}{2}, \quad b = 3$$

x_i	1	4	9	16
y_i	9	-44	-699	-4056

Answer:

$$y = 10x^{1/2} - x^3$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR	STO	4			
2	a	STO	1					
3	b	STO	2					
4	x_i	STO	3	RCL	1	■		Perform 4–13 for i=1,2,...,n
5		y^x	↑	↑	↑			
6	y_i	x	STO	+	6	CLX		
7		■	LAST x	RCL	3	RCL		
8		2	■	y^x	STO	9		
9		x	STO	+	4	CLX		
10		RCL	9	x	STO	+		
11		5	R↓	x^2	STO	+		
12		8	RCL	9	x^2	STO		
13		+	7					
14		RCL	8	RCL	x	4		
15		RCL	6	RCL	x	5		
16		–	RCL	7	RCL	x		
17		8	RCL	5	x^2	–		
18		÷					d	
19		RCL	5	x	RCL	6		
20		x⇄y	–	RCL	8	÷	c	

Power curve (least squares fit)

Formula:

For a given set of data points

$$\{(x_i, y_i), i = 1, 2, ..., n\}$$

fit a power curve of the form

$$y = ax^b$$

$$(a > 0)$$

By writing this equation as

$$\ln y = b \ln x + \ln a$$

the problem can be solved as a linear regression problem.

Example:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Answers:

$y = 987.66x^{-0.7}$

$r^2 = 0.80$

Note:

Compare results with those of the example for "Linear regression and correlation coefficient". Since in that case $r^2 = 0.92 > 0.80$, we know that the linear regression line

$$y = -1.03x + 121.04$$

fits the data points better than the power curve

$$y = 987.66x^{-0.7}$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR	STO	3	STO		
2		4						
3	y_i	ln	x^2	STO	+	3		Perform 3–6 for i=1,2,...,n
4		■	LAST x	↑	↑			
5	x_i	ln	x	STO	+	4		
6		CLX	■	LAST x	Σ+			
7		RCL	4	RCL	7	RCL		
8		8	x	RCL	5	÷		
9		–	STO	2	RCL	6		
10		RCL	7	x^2	RCL	5		
11		÷	–	÷	STO	1	b	
12		RCL	7	x	RCL	8		
13		x⇄y	–	RCL	5	÷		
14		e^x					a	
15		RCL	1	RCL	2	x		
16		RCL	3	RCL	8	x^2		
17		RCL	5	÷	–	÷	r^2	

Exponential curve (least squares fit)

Formula:

For a given set of data points

$$\{(x_i, y_i), i = 1, 2, ..., n\}$$

this algorithm fits an exponential curve of the form

$$y = ae^{bx}$$

$$(a > 0)$$

By writing this equation as

$$\ln y = bx + \ln a$$

the problem can be solved as a linear regression problem (y_i must be positive for all $i = 1, 2, ..., n$).

Example:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Answers:

$y = 149.07e^{-0.02x}$

$r^2 = 0.89$

Note:

Compare results with those of the example for "Linear regression and correlation coefficient". Since in that case $r^2 = 0.92 > 0.89$, we know that the linear regression line

$$y = -1.03x + 121.04$$

fits the data points better than the exponential curve

$$y = 149.07e^{-0.02x}$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		[redacted]	CLEAR	STO	3	STO		
2		4						
3	y_i	ln	x^2	STO	+	3		Perform 3–6 for i=1,2,...,n
4		[redacted]	LAST x	↑	↑			
5	x_i	x	STO	+	4	CLX		
6		[redacted]	LAST x	Σ+				
7		RCL	4	RCL	7	RCL		
8		8	x	RCL	5	÷		
9		–	STO	2	RCL	6		
10		RCL	7	x^2	RCL	5		
11		÷	–	÷	STO	1	b	
12		RCL	7	x	RCL	8		
13		x⇄y	–	RCL	5	÷		
14		e^x					a	
15		RCL	1	RCL	2	x		
16		RCL	3	RCL	8	x^2		
17		RCL	5	÷	–	÷	r^2	

Declining Balance Depreciation

See page 91

Depreciation Amortization

Straight line depreciation

Formulas:

$$D = \frac{PV}{n}$$

$$B_k = PV - kD$$

where PV = original value of asset (less salvage value)

n = lifetime periods of asset

D = each year's depreciation

B_k = book value at time period k

Example:

A fleet car has a value of \$2100 and a life expectancy of six years. Using the straight line method, what is the amount of depreciation and what is the book value after two years?

Answers:

$D = \$350.00$

$B_2 = \$1400.00$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	PV	↑	↑					
2	n	÷					D	
3	k	x	−				B_k	

Variable rate declining balance

Formulas:

$$D_k = PV \cdot \frac{R}{n}\left(1 - \frac{R}{n}\right)^{k-1}$$

$$B_k = PV\left(1 - \frac{R}{n}\right)^{k}$$

where PV = original value of asset

n = lifetime periods of asset

R = depreciation rate (given by user)

D_k = depreciation at time period k

B_k = book value at time period k

Example:

A fleet car has a value of \$2500 and a life expectancy of six years. Use the double declining balance method (R = 2) to find the amount of depreciation and book value after four years.

Answers:

B_4 = \$ 493.83

D_4 = \$246.91

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	k	↑	1	↑				
2	R	↑						
3	n	÷	−	STO	1	x⇄y		
4		■	y^x					
5	PV	x					B_k	
6		RCL	1	÷	1	RCL		
7		1	−	x			D_k	

Sum of the year's digits depreciation (SOD)

Formula:

$$D_k = \frac{2(n - k + 1)}{n(n + 1)} PV$$

$$B_k = S + (n - k) D_k/2$$

where PV = original value of asset

n = life time periods of asset

S = salvage value

D_k = depreciation at time period k

B_k = book value at time period k

Example:

A car has a value (less salvage value – $800) of $2100 and a life expectancy of 6 years. Using the SOD method, what is the amount of depreciation and what is the book value after 2 years?

Answers:

$D_2 = \$500.00$

$B_2 = \$1800.00$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	STO	1					
2	k	STO	2	−	STO	3		
3		1	+	RCL	1	↑		
4		↑	×	+	÷	2		
5		×						
6	PV	×					D_k	
7		RCL	3	×	2	÷		
8	S	+					B_k	

Diminishing balance depreciation

Formulas:

$$D_k = PV_{k-1}\left[1 - \left(\frac{S}{PV_0}\right)^{1/n}\right]$$

$$PV_k = PV_{k-1} - D_k$$

where PV_0 = beginning value of asset

S = salvage value (> 0)

PV_k = book value at time period k ($k = 1, 2, \ldots, n$)

D_k = depreciation at time period k

Example:

A car has a value of $2500, a salvage value of $400, and a life expectancy of six years. Find the amount of depreciation and book value for each of the first three years by using the diminishing balance method.

Answers:

D_1 = $657.98

PV_1 = $1842.02

D_2 = $484.81

PV_2 = $1357.21

D_3 = $357.21

PV_3 = $1000.00

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	S	↑						
2	PV_0	STO	1	÷				
3	n	1/x	■	y^x	1	x⇄y		
4		−	RCL	1	↑	↑		
5		R↓	R↓	R↓	STO	1		
6		×					D_1	
7		−					PV_1	
8		↑	↑	RCL	1	×	D_j	Perform 8–9 for j=2, ..., k
9		−					PV_j	

DETERMINANT OF A 3x3 MATRIX

Formula:

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

Example:

$$\begin{vmatrix} 1.3 & -4.5 & 25 \\ 2.9 & 3.3 & -7.8 \\ 2.2 & 4.1 & -2.5 \end{vmatrix} = 191.19$$

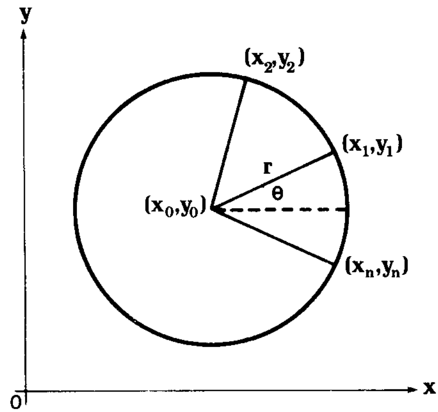
LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a_1	STO	1					
2	b_2	STO	2	x				
3	c_3	STO	3	x				
4	b_1	STO	4					
5	c_2	STO	5	x				
6	a_3	STO	6	x	+			
7	c_1	STO	7					
8	a_2	STO	8	x				
9	b_3	STO	9	x	+			
10		RCL	6	RCL	2	x		
11		RCL	7	x	–			
12		RCL	9	RCL	5	x		
13		RCL	1	x	–			
14		RCL	3	RCL	8	x		
15		RCL	4	x	–			

DOT PRODUCT

See page 212

EQUALLY SPACED POINTS ON A CIRCLE

Given a circle with radius r and center (x_0, y_0), the routine computes the rectangular coordinates of equally spaced points (x_i, y_i), $(i = 1, 2, ..., n)$ on the circle if angle θ and number of points n are known. The position of the first point (x_1, y_1) on the circle is determined by the angle θ.



$$x_{k+1} = x_0 + r\cos\left(\theta + \frac{360}{n}k\right)$$

$$y_{k+1} = y_0 + r\sin\left(\theta + \frac{360}{n}k\right)$$

where $k = 0, 1, 2, ..., n-1$

Note: θ is in degrees.

Example:

$(x_0, y_0) = (1, 1)$

$\theta = 90°$

$r = 1$

$n = 4$

Answers:

$$(x_1, y_1) = (1, 2)$$
$$(x_2, y_2) = (0, 1)$$
$$(x_3, y_3) = (1, 0)$$
$$(x_4, y_4) = (2, 1) .$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	θ	■	DEG	STO	1			
2	r	STO	2	■	→R			
3	x_0	STO	3	+			x_1	
4		x⇄y						
5	y_0	STO	4	+			y_1	
6		3	6	0	↑			
7	n	÷	STO	5				
8		RCL	1	RCL	5	+		Perform 8–11 for i=2,...,n
9		STO	1	RCL	2	■		
10		→R	RCL	3	+		x_i	
11		x⇄y	RCL	4	+		y_i	

Equation Solving (Iterative Techniques)

Note:

We will deal here with equations of the form

$$x = f(x)$$

for cases where it is difficult to separate all x's to one side of the equal sign. The iterative approach is illustrated through the solution of selected equations.

Example 1: Find x such that $x = e^{-x}$

Method:

Choose $x_a = 5$ as an approximation for the solution. Then after 44 iterations, the answer is x = 0.567143290.

Note:

The algorithm will converge to 0.567143290 in about 50 iterations for any value of x_a.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	5	CHS	e^x	FIX	9		0.006737947	
2		CHS	e^x					Perform 2 forty-three times
3							0.567143290	

Example 2: $4 = x - \frac{1}{x}$

Method:

Rewrite the equation as

$$x = \frac{1}{x} + 4.$$

Choose an approximate solution for x, say $x_a = 4$.

Answer:

4.236067978

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	4	$1/x$	4	+	FIX	9		
2		$1/x$	4	+				Perform 2 seven times
3							4.236067978	

Example 3: $x^x = 1000$

Method:

Rewrite the equation in the form

$$x = \ln 1000/\ln x\,.$$

Pick an approximation x_a for x, say $x_a = 4$. If we use the following algorithm, convergence is from both sides, and takes a long time.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	1000	ln	↑	↑	↑			
2	4	ln	÷	FIX	9		4.982892143	
3		ln	÷				4.301189432	
4		ln	÷				4.734933900	
5		ln	÷				4.442378437	
6		ln	÷				4.632377942	
7		ln	÷				4.505830645	
8		ln	÷				4.588735608	
9		ln	÷				4.533824354	
10		ln	÷				4.569933525	etc.

To hasten convergence, modify the loop and use the average of the last two approximations as the new approximation.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	1000	ln	↑	↑	↑			
2	4	STO	1	ln	÷	RCL		
3		1	+	2	÷	STO		
4		1					4.491446072	
5		ln	÷	RCL	+	1		Perform 5–6 twelve times
6		2	÷	STO	1			
7							4.555535705	

Example 4: Largest x^{x^x}

What is the largest value of x such that x^{x^x} does not overflow in the HP-45 (i.e., $x^{x^x} < 9.999999999 \times 10^{99}$)?

Method:

Since 9.999999999 x 10^{99} is only slightly less than a *googol* (10^{100}), let us call this constant G.

Then $x^{x^x} = G$

$$x^x \ln x = \ln G$$

$$x^x = \ln G/\ln x$$

$$x \ln x = \ln (\ln G/\ln x)$$

$$x = [\ln (\ln G/\ln x)] \ / \ln x$$

Use 3 as an initial approximation for the solution. The loop in the following algorithm eventually alternates between the last two answers, 3.830482865 and 3.830482864. Upon substitution of both values in x^{x^x}, 3.830482865 gives an overflow. Thus the answer is 3.830482864.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		9	e^x	e^x	ln	↑		
2		↑	↑	FIX	9		2.302585093 02	← ln G
3		3	ln	÷	ln	3		
4		ln	÷	STO	1		4.865369574	
5		ln	÷	ln	RCL	1		
6		ln	÷	RCL	1	+		
7		2	÷	STO	1		4.006633409	
8		ln	÷	ln	RCL	1		
9		ln	÷	RCL	1	+		
10		2	÷	STO	1		3.844654250	
↕							↕	6 iterations
29		ln	÷	ln	RCL	1		
30		ln	÷	RCL	1	+		
31		2	÷	STO	1		3.830482865	
32		ln	÷	ln	RCL	1		
33		ln	÷	RCL	1	+		
34		2	÷	STO	1		3.830482864	etc.

Example 5: Solve $x^2 + 4 \sin x = 0$

Method:

Rewrite the equation as

$$x = \pm \sqrt{-4 \sin x} \quad \text{(x is in radians)}$$

upon plotting this curve, we see that there is a root near −2, so take $x_a = -2$ as an approximation of the solution, and substitute in

$$x = - \sqrt{-4 \sin x}\,.$$

Note:

Since this algorithm converges from both sides, we modified our loop to average two approximations to hasten convergence. It is only necessary to do the loop 5 times to get 4 digits of accuracy.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	2	CHS	STO	1		RAD		
2		FIX	9					
3		SIN	CHS	4	x			Perform 3–5 seventeen times
4		$\sqrt{x}$	RCL	1	−	2		
5		÷	CHS	STO	1			
6							−1.933753764	

A method for faster convergence

Some equations f(x) = 0 converge very slowly by the above methods, however, the following method gives faster convergence.

Formula:

$$x_{i+1} = x_i - \frac{E_i\,(x_i - x_{i-1})}{E_i - E_{i-1}}$$

where $i = 1, 2, 3, \ldots$

x_i = current trial value

x_{i+1} = next trial value

x_{i-1} = previous trial value

E_i = current error = $f(x_i)$

E_{i-1} = last error = $f(x_{i-1})$

L = lower bound for the solution

U = upper bound for the solution

Example 6: Solve $x^3 = 3^x$ where $1 < x < e$

Method:

Rewrite equation in the form

$$x^3 - 3^x = 0$$

Replace f(x) in the program by

"[↑] [↑] 3 ■ [y^x] [x⇄y] 3 [x⇄y] ■ [y^x] [−]"

Let $L = 1$,
$U = e$.

Answer:

2.478052679

($E_0 = -2.000000001$, $E_1 = 0.272546170$, $E_2 = 0.056084610$,
$E_3 = -0.033421420$, $E_4 = 0.001191760$, $E_5 = 0.000022540$,
$E_6 = E_7 = -1 \times 10^{-8}$)

Example 7: Find a root of a polynomial

$$f(x) = x^4 - 4x^3 + 8x^2 + 20x - 65.$$

Method:

Replace f(x) in the program by

"[↑] [↑] [↑] 4 [−] [×] 8 [+] [×] 20 [+] [×] 65 [−] ".

Note that $f(2) = -9 < 0$, $f(3) = 40 > 0$, so there is a root between 2 and 3.

Let $L = 2$
$U = 3$.

Answer:

2.236067977 (or $\sqrt{5}$)

($x_0 = 2, x_1 = 3, x_2 = 2.183673469, x_3 = 2.224244398,$
$x_4 = 2.236236914, x_5 = 2.236067428, x_6 = x_7 = 2.236067977$)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	L	STO	1	FIX	9			
2		f(x)	STO	2			E_0	Replace f(x) by proper
								keystroke(s)
3	U	STO	3					
4		f(x)	STO	4			E_i	Perform 4–11 for i=1,2,...
5		RCL	3	RCL	–	1		until either the last two
6		x	RCL	4	RCL	2		trial values are the same
7		–	÷	RCL	3	x⇄y		or the last two errors
8		–					x_{i+1}	are the same
9		RCL	3	STO	1	RCL		
10		4	STO	2	R↓	R↓		
11		STO	3					

Error Function

Formula:

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du \qquad (x \geqslant 0)$$

$$\operatorname{erf} x \cong 1 - (a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5)\, e^{-x^2}$$

where $t = \dfrac{1}{1 + px}$

$p = 0.3275911$ $\qquad a_1 = 0.254829592$

$a_2 = -0.284496736$ $\qquad a_3 = 1.421413741$

$a_4 = -1.453152027$ $\qquad a_5 = 1.061405429$

Example:

$\operatorname{erf} 1.34 = 0.9419138$

Note:

$\operatorname{erf}(-x) = -\operatorname{erf} x$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a_1	STO	1	FIX	7			
2	a_2	STO	2					
3	a_3	STO	3					
4	a_4	STO	4					
5	a_5	STO	5					
6	p	STO	6					
7	x	STO	7	RCL	6	x		
8		1	+	1/x	↑	↑		
9		↑	RCL	5	x	RCL		
10		4	+	x	RCL	3		
11		+	x	RCL	2	+		
12		x	RCL	1	+	x		
13		RCL	7	x^2	CHS	e^x		
14		x	1	x⇄y	−			Stop; for new case,
								go to 7

Euler Numbers

Compute the nth Euler number

Note:

The Euler numbers are 1, 5, 61, 1385, 50521, ...

Formula:

The Euler numbers $E_1, E_2, E_3, \ldots$ are defined by

$$E_n = \frac{2^{2n+2}\,(2n)!}{\pi^{2n+1}}\left[1 - \frac{1}{3^{2n+1}} + \frac{1}{5^{2n+1}} - \frac{1}{7^{2n+1}} + \ldots\right]$$

Example:

The 5th Euler number = 50521.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	↑	2	×	↑	↑		
2		↑	■	n!	x⇄y	2		
3		+	2	x⇄y	■	y^x		
4		×	x⇄y	1	+	■		
5		π	x⇄y	■	y^x	÷		
6		FIX	0					

Exponential Curve (Least Squares Fit)

See page 88

Exponentiation

Multiple successive power operations

As written, these terms must be executed from right to left. For example,

$$e^{x^{1.5}}$$

means

$$e^{(x^{1.5})}, \text{ not } (e^x)^{1.5},$$

$$a^{b^c}$$

means

$$a^{(b^c)}, \text{ not } (a^b)^c.$$

Example:

Evaluate $S = t^{t^{t^t}}$ where $t = e^{\frac{1}{e}}$.

Answer:

1.99

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		1	e^x	$1/x$	e^x	↑	1.44	← t
2		↑	↑	■	y^x	■		
3		y^x	■	y^x			1.99	

Example:

Find the limit of $s^{s^{s^{\cdot^{\cdot^{\cdot}}}}}$ where $s = \sqrt{3}$.

Answer:

∞ (Display = 9.999999999 *99*)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	3	■	$\sqrt{x}$	↑	↑	↑		
2		■	y^x					Perform 2 six times

Example:

Find the limit of $s^{s^{s^{\cdot^{\cdot^{\cdot}}}}}$, where $s = \sqrt{2}$.

Answer:

2 (rounded)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		FIX	9	2	■	$\sqrt{x}$		
2		↑	↑	↑				
3		■	y^x					Perform 3 fifty-six times
4							1.999999995	Display would not change
								any more.

e^x for large positive x (x > 230)

Formula:

Suppose $e^x = a \times 10^b$

where $a < 10$

Find a, b for a given x.

Example:

$e^{300} = 1.942426525 \times 10^{130}$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		FIX	9					
2	x	↑	1	0	ln	STO		
3		1	÷	↑	↑	↑		
4		.	5	–	EEX	9		
5		+	EEX	9	–	↑		
6		R↓	–	RCL	1	x		
7		e^x					a	
8		R↓	R↓				b	

y^x for large x and/or y (x ln y > 230)

Formula:

Suppose $y^x = a \times 10^b$

where $a < 10$

Find a, b for given x, y.

Example:

$75^{100} = 3.207202635 \times 10^{187}$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		FIX	9					
2	x	↑						
3	y	ln	x	1	0	ln		
4		STO	1	÷	↑	↑		
5		↑	.	5	–	EEX		
6		9	+	EEX	9	–		
7		↑	R↓	–	RCL	1		
8		x	e^x				a	
9		R↓	R↓				b	

Example:

The largest number that can be written with three digits and no other symbol is 9^{9^9}. How big is this number?

Answer:

$9^{9^9} = 3.981071706 \times 10^{369693099}$

(Due to machine accuracy limitations, this is only an approximate answer.)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		FIX	9	9	↑	↑		
2		↑	x	x	x	x		
3		x	x	x	x	9		
4		ln	x	1	0	ln		
5		STO	1	÷	↑	↑		
6		↑	·	5	−	EEX		
7		9	+	EEX	9	−		
8		↑	R↓	−	RCL	1		
9		x	e^x				3.981071706	
10		R↓	R↓				3.696930990 08	

Converging $u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$

Formula:

If $0 < x < e$, and $u = x^{\frac{1}{x}}$, then $u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$ will converge at x.

Example:

Let x = 1.5, and $u = x^{\frac{1}{x}}$, find $u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$

Answer:

$u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$ converges at 1.5 in 21 iterations

(display shows 1.499999999).

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		FIX	9					
2	x	↑	1/x		y^x	↑		
3		↑	↑					
4			y^x					Perform 4 as many times as
								necessary to see
								convergence.

Diverging $u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$

Formula:

If $u > e^{\frac{1}{e}}$ (= 1.444667861), $u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$ will diverge.

Example:

If u = 1.45 then $u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$ diverges

(overflows after 43 iterations).

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	u	↑	↑	↑				
2		■	y^x					Perform 2 as many times
								as necessary to see
								divergence.

Calculator limits for y^x

1. For a positive value of y, how big can x be so that y^x will not overflow in HP-45?

Example:

If y = 50, then x can be as large as around 58 (50^{59} will overflow).

Note: The following gives an approximate, not exact answer.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		2	3	0	↑			
2	y	ln	÷				D	Answer is the largest
								integer ⩽ D

2. For a positive value of x, how big can y be without causing y^x to overflow?

Example:

If x = 50, then we can take y as large as around 99 (100^{50} will overflow).

Note: The following gives an approximate, not exact answer.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		2	3	0	↑			
2	x	÷	e^x				D	Answer is the largest
								integer ⩽ D.

Factorial and Gamma Function

Stirling's approximation

Notes:

This approximation can be used for positive $x \leqslant 69$ (otherwise the answer is $> 10^{100}$).

This approximation is good for large x.

For $x < 1$, use polynomial approximation.

To compute Gamma Function $\Gamma(x) = (x - 1)!$

Formula:

$$x! \cong \sqrt{2\pi x}\ x^x e^{-x} \left(1 + \frac{1}{12x} + \frac{1}{288x^2} - \frac{139}{51840x^3} - \frac{571}{2488320x^4}\right)$$

Example:

$$4.25! \cong 35.21$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		5	7	1	↑	2		
2		4	8	8	3	2		
3		0	÷	CHS	STO	1		
4		1	3	9	↑	5		
5		1	8	4	0	÷		
6		CHS	STO	2	2	8		
7		8	$1/x$	STO	3	1		
8		2	$1/x$	STO	4			
9	x	STO	5	2	x			
10		π	x		$\sqrt{x}$	RCL		
11		5	RCL	5	2	÷		
12			y^x	STO	6	x		
13		RCL	5	e^x	$1/x$	x		
14		STO	x	6	RCL	5		
15		$1/x$	↑	↑	↑	RCL		
16		1	x	RCL	2	+		
17		x	RCL	3	+	x		
18		RCL	4	+	x	1		
19		+	RCL	6	x			For new case, go to 9

Polynomial approximation

Notes:

This approximation can be used for positive $x < 69$.

This is a more accurate method (to 6 or 7 decimal places), but longer than Stirling's approximation.

Formula: $x! \cong 1 + b_1 x + b_2 x^2 + \ldots + b_8 x^8$

for $0 \leqslant x \leqslant 1$

where $b_1 = -.577191652$, $b_2 = .988205891$

$b_3 = -.897056937$, $b_4 = .918206857$

$b_5 = -.756704078$, $b_6 = .482199394$

$b_7 = -.193527818$, $b_8 = .035868343$

Example: $4.25! \cong 35.2116196$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b_8	STO	1	FIX	7			
2	b_7	STO	2					
3	b_6	STO	3					
4	b_5	STO	4					
5	b_4	STO	5					
6	b_3	STO	6					
7	b_2	STO	7					
8	b_1	STO	8					
9	x	↑						If x > 1, go to 11
10		1	STO	9	R↓			Go to 17
11								If x > 2, go to 13
12		STO	9	1	−			Go to 17
13		↑	1	−				
14		STO	9	x	RCL	9		
15		1	−				D	If D > 1, go to 14
16		x⇄y	STO	9	x⇄y			
17		↑	↑	↑	RCL	1		
18		x	RCL	2	+	x		
19		RCL	3	+	x	RCL		
20		4	+	x	RCL	5		
21		+	x	RCL	6	+		
22		x	RCL	7	+	x		
23		RCL	8	+	x	1		
24		+	RCL	9	x			For new case, go to 9

Factoring Integers and Determining Primes

Prime Numbers under 100

2	13	31	53	73
3	17	37	59	79
5	19	41	61	83
7	23	43	67	89
11	29	47	71	97

With the list memorized or in sight, it is easy to factor any integer x less than 10000 (and many other integers even greater). In the following program, omit the numbers 2 and 5 from the list of primes if the integer ends in 1, 3, 7 or 9.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	FIX	5					If x is even, let P=2;
								otherwise P=3
2		↑	↑	↑	■	$\sqrt{x}$	Max	
3								If P ⩾ Max, stop
4		R↓						
5	P	÷					Q	Read note

Note: If Q is not an integer, let P = next prime number, go to 3. If Q is a prime, then both P and Q are factors, stop. Otherwise P is a factor, let P = current prime, go to 2.

Example:

Factor 4807.

Answer:

4807 = 11 x 19 x 23

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	4807	FIX	5	↑	↑	↑		
2			$\sqrt{x}$				69.33253	←MAX, P = 3
3		R↓	3	÷			1602.33333	P = 7
4		R↓	7	÷			686.71429	P = 11
5		R↓	1	1	÷		437.00000	←Q is an integer,
								so 11 is a factor
6		↑	↑	↑		$\sqrt{x}$	20.90454	P = 11
7		R↓	1	1	÷		39.72727	P = 13
8		R↓	1	3	÷		33.61538	P = 17
9		R↓	1	7	÷		25.70588	P = 19
10		R↓	1	9	÷		23.00000	Q = 23 is a prime, 19 and
								23 are factors, stop

Example:

Factor 2909.

Answer:

2909 is a prime.

Fibonacci Numbers

Formula:

In a Fibonacci sequence, each term is the sum of the two preceding terms.

$$f_i = f_{i-1} + f_{i-2}$$

f_i represents the i^{th} term in the sequence.

Example:

Develop the Fibonacci sequence with $f_1 = 1$, $f_2 = 1$.

Answer:

1, 1, 2, 3, 5, 8, 13, 21, ...

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	f_1	↑						
2	f_2							
3		↑	↑	R↓	R↓	+	f_i	Perform 3 for i=3,4,...

Finding the n^{th} Fibonacci number

Example:

Find the 12^{th} Fibonacci number in the sequence 1, 1, 2, 3, 5, 8 . . .

Answer:

144

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	↑	5	■	$\sqrt{x}$	STO		
2		1	↑	1	+	2		
3		÷	x⇄y	■	y^x	RCL		
4		1	÷	FIX	0			

F Statistic

Testing two population variances

Given independent random samples $\{x_i, i = 1.2, ..., n_x\}$ and $\{y_i, i = 1.2, ..., n_y\}$ taken from two normal populations whose variances are σ_x^2 and σ_y^2, the F statistic (with $n_x - 1$ and $n_y - 1$ degrees of freedom) can be used to test the null hypothesis

$$H_0: \sigma_x^2 = \sigma_y^2$$

F is computed from the following:

$$F = \frac{s_x^2}{s_y^2}$$

where s_x^2 = sample variance of x
s_y^2 = sample variance of y

Example:

x: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54 ($n_x = 10$)

y: 79, 84, 108, 114, 120, 103, 122, 120 ($n_y = 8$)

Answer:

F = 1.02 (d.f. = 9 and 7)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1			CLEAR					
2	x_i	Σ+						Perform 2 for $i=1,2,...,n_x$
3			$\bar{x}$,s	x⇄y	STO	1		
4			CLEAR					
5	y_i	Σ+						Perform 5 for $i=1,2,...,n_y$
6			$\bar{x}$,s	x⇄y	x^2	RCL		
7		1	x^2	x⇄y	÷			

Future Value

See pages 135, 139

Gamma Function

See page 109

Gaussian Probability Function

Formula:

$$\varphi(x) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{x^2}{2}}$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\, dt$$

where $x \geqslant 0$

$$\Phi(x) \cong 1 - \varphi(x)\,(a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5)$$

where $t = \dfrac{1}{1 + px}$

$p = 0.2316419$	$a_1 = 0.31938153$
$a_2 = -0.356563782$	$a_3 = 1.781477937$
$a_4 = -1.821255978$	$a_5 = 1.330274429$

Example:

$$\varphi(2.22) = 0.033940763$$

$$\Phi(2.22) = 0.9867907$$

Note:

$$\varphi(-x) = \varphi(x), \Phi(-x) = 1 - \Phi(x)$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	p	STO	3					
2	a_5	STO	4					
3	a_4	STO	5					
4	a_3	STO	6					
5	a_2	STO	7					
6	a_1	STO	8					
7	x	STO	1	x^2	2	÷		
8		CHS	e^x		π	2		
9		x		$\sqrt{x}$	÷	STO		
10		2	FIX	9			$\varphi(x)$	
11		1	↑	RCL	1	RCL		
12		3	x	+	$1/x$	↑		
13		↑	↑	RCL	4	x		
14		RCL	5	+	x	RCL		
15		6	+	x	RCL	7		
16		+	x	RCL	8	+		
17		x	RCL	2	x	1		
18		x⇄y	−	FIX	7		$\Phi(x)$	For new case, go to 7

Gaussian Quadratures

Gaussian quadrature for $\int_a^b f(x)\,dx$ or $\int_a^\infty f(x)\,dx$

Formula:

We estimate the value

$$I_1 = \int_a^b f(x)\,dx$$

or

$$I_2 = \int_a^{\infty} f(x)\,dx$$

by the six point Gauss-Legendre quadrature formula,

$$\int_a^b f(x)\,dx \cong \frac{b-a}{2} \sum_{i=1}^{6} w_i f\left(\frac{z_i\,(b-a)+b+a}{2}\right)$$

$$\text{or} \int_a^{\infty} f(x)\,dx \cong \frac{1}{2} \sum_{i=1}^{6} \frac{4\,w_i}{(1+z_i)^2}\; f\left(\frac{2}{1+z_i} + a - 1\right)$$

where

i	z_i	w_i
1	.2386191861	.4679139346
2	−.2386191861	.4679139346
3	.6612093865	.360761573
4	−.6612093865	.360761573
5	.9324695142	.1713244924
6	−.9324695142	.1713244924

f(x) is any single-valued function and a, b are finite.

Example:

Evaluate $\ln 10 = \int_1^{10} \frac{dx}{x}$.

Answer:

2.30 (replace f(x) by " [1/x] ")

Example:

Evaluate $\int_e^{e^2} \frac{dx}{x(\ln x)^3}$

Answer:

0.37 (replace f(x) by " [↑] [ln] 3 [■] [y^x] [×] [1/x] ")

Example:

Evaluate $\Gamma(1.8) = \int_0^\infty e^{-x}\, x^{0.8}\, dx$.

Answer:

0.92

(replace f(x) by " [CHS] e^x ■ [LAST x] [CHS] 0.8 ■ [y^x] [×]

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	z_3	STO	2					
2	z_5	STO	3					
3	w_1	STO	4					
4	w_3	STO	5					
5	w_5	STO	6					If b = ∞, go to 31
6	b	↑	↑					
7	a	+	STO	7	x⇄y	■		
8		LAST x	–	STO	8			
9	z_1	x	+	2	÷			
10		f(x)						Replace f(x) by proper
								keystroke(s)
11		RCL	4	x	STO	1		
12		RCL	8	RCL				Perform 12–17 for i=2,3
13	i	x	RCL	7	+	2		
14		÷						
15		f(x)						
16		RCL						
17	i+3	x	STO	+	1			
18	z_1	RCL	8	x	CHS	RCL		
19		7	+	2	÷			
20		f(x)						
21		RCL	4	x	STO	+		
22		1						
23		RCL	8	RCL				Perform 23–28 for i=2,3
24	i	x	CHS	RCL	7	+		
25		2	÷					
26		f(x)						
27		RCL						
28	i+3	x	STO	+	1			

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
29		RCL	1	RCL	8	2		
30		÷	x				I_1	Stop, for new case, go to
								6 or 31
31	a	↑	1	–	STO	7		
32	z_1	STO	1	0	STO	8		
33		2	RCL					Perform 33–48 for i=1,2,3
34	i	1	+	÷	RCL	7		
35		+						
36		f(x)						
37		2	RCL					
38	i	1	+	÷	x^2	x		
39		RCL						
40	i+3	x	STO	+	8	2		
41		RCL						
42	i	CHS	1	+	÷	RCL		
43		7	+					
44		f(x)						
45		2	RCL					
46	i	CHS	1	+	÷	x^2		
47		x	RCL					
48	i+3	x	STO	+	8			
49		RCL	8	2	÷		I_2	Stop, for new case,
								go to 6 or 31

Gaussian quadrature for $\int_a^\infty e^{-x} f(x)\,dx$

We estimate the value

$$I = \int_a^\infty e^{-x} f(x)\,dx$$

by the three-point Gauss-Laguerre quadrature formula:

$$\int_a^\infty e^{-x} f(x)\,dx \cong e^{-a} \sum_{i=1}^{3} w_i f(z_i + a)$$

where

i	z_i	w_i
1	.4157745568	.7110930099
2	2.29428036	.2785177336
3	6.289945083	.0103892565

Example:

Approximate the Gamma function

$$I = \Gamma(\alpha) = \int_0^{\infty} e^{-x}\, x^{\alpha-1}\, dx \quad \text{for } \alpha = 5.25\,.$$

Answer:

35.27

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	z_1	STO	1					
2	z_2	STO	2					
3	z_3	STO	3					
4	w_1	STO	4					
5	w_2	STO	5					
6	w_3	STO	6					
7	a	STO	7	RCL	1	+		
8		f(x)						Replace f(x) by proper
								keystrokes
9		RCL	4	x	STO	8		
10		RCL	7	RCL	2	+		
11		f(x)						
12		RCL	5	x	STO	+		
13		8	RCL	3	RCL	7		
14		+						
15		f(x)						
16		RCL	6	x	RCL	8		
17		+	RCL	7	CHS	e^x		
18		x						Stop, for new case, go to 7

Geometric Mean

See page 148

Geometric Progressions

See page 157

Goodness of Fit

See page 33

Harmonic Mean

See page 149

Harmonic Numbers

Formula:

The Harmonic numbers H_i $(i = 1, 2, \ldots)$ are

$$1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{3}, 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}, \ldots$$

or

$$1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \ldots$$

Example:

Display the sequence in decimal form.

Answer:

1.00, 1.50, 1.83, 2.08, ...

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		[redacted]						
2		CLEAR						Perform 2–4 for i=1,2,...
3		1	+	↑	$1/x$	RCL		
4		8	+	STO	8		H_i	

n^{th} Harmonic number

Example:

Find the 7^{th} Harmonic number.

Answer:

2.59

Note: E = .5772156649 is Euler's constant.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	STO	1	$1/x$	↑	↑		
2		↑	1	2	0	$1/x$		
3		×	×	1	2	$1/x$		
4		−	×	2	$1/x$	+		
5		×						
6	E	+	RCL	1	ln	+		

Harmonic Progressions

See page 157

Haversine

See page 203

Highest Common Factor

Definition:

The highest common factor (or greatest common divisor) of two positive integers a and b is the largest integer which divides both a and b. We write it as HCF(a, b).

Example:

$$\text{HCF}(51, 119) = 17.00$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1					
2	b							
3		↑	↑	RCL	1	÷	D	Let f be the largest
								integer ≤ D
4	f	x⇄y	CLX	RCL	1	x		
5		−					E	If E = 0, go to 8
6		RCL	1	x⇄y	STO	1		
7		CLX	+					Go to 3
8		RCL	1				HCF(a, b)	

Hyperbolic Functions

Hyperbolic sine

Formula:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

Example:

$$\sinh 3.2 = 12.25$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	e^x	↑	$1/x$	−	2		
2		÷						

Hyperbolic cosine

Formula:

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Example:

$$\cosh 3.2 = 12.29$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	e^x	↑	$1/x$	+	2		
2		÷						

Hyperbolic tangent

Formula:

$$\tanh x = \frac{\sinh x}{\cosh x}$$

Example:

$$\tanh 3.2 = 1.00$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	e^x	STO	1	↑	$1/x$		
2		−	RCL	1		LAST x		
3		+	÷					

Hyperbolic cotangent

Formula:

$$\coth x = \frac{1}{\tanh x}$$

Example:

$$\coth 3.2 = 1.00$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	e^x	STO	1	↑	1/x		
2		−	RCL	1		LAST x		
3		+	÷	1/x				

Hyperbolic cosecant

Formula:

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

Example:

$$\operatorname{csch} 3.2 = 0.08$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	e^x	↑	1/x	−	2		
2		÷	1/x					

Hyperbolic secant

Formula:

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

Example:

$$\operatorname{sech} 3.2 = 0.08$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	e^x	↑	1/x	+	2		
2		÷	1/x					

Inverse hyperbolic sine

Formula:

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

Example:

$$\sinh^{-1} 51.777 = 4.64$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	↑ ↑ ↑ × 1		
2		+ ■ $\sqrt{x}$ + ln		

Inverse hyperbolic cosine

Formula:

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right) \quad (x \geqslant 1)$$

Example:

$$\cosh^{-1} 51.777 = 4.64$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	↑ x^2 1 − ■		
2		$\sqrt{x}$ + ln		

Inverse hyperbolic tangent

Formula:

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1 + x}{1 - x}$$

$$(-1 < x < 1)$$

Example:

$$\tanh^{-1} 0.777 = 1.04$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		1 ↑		
2	x	+ 1 ■ LAST x −		
3		÷ ln 2 ÷		

Inverse hyperbolic secant

Formula:

$$\operatorname{sech}^{-1} x = \cosh^{-1} \frac{1}{x}$$

$$(0 < x < 1)$$

Example:

$$\operatorname{sech}^{-1} 0.777 = 0.74$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	$^1/_x$	↑	x^2	1	−		
2		■	$\sqrt{x}$	+	ln			

Inverse hyperbolic cotangent

Formula:

$$\coth^{-1} x = \tanh^{-1} \frac{1}{x}$$

$$(x^2 > 1)$$

Example:

$$\coth^{-1} 51.777 = 0.02$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		1	↑					
2	x	$^1/_x$	+	1	■	LAST x		
3		−	÷	ln	2	÷		

Inverse hyperbolic cosecant

Formula:

$$\operatorname{csch}^{-1} x = \sinh^{-1} \frac{1}{x}$$

Example:

$$\operatorname{csch}^{-1} 0.777 = 1.07$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	$^1/_x$	↑	x^2	1	+		
2		■	$\sqrt{x}$	+	ln			

Hypergeometric Distribution

Formula:

$$f(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$x = 0, 1, 2, ..., h$$

where h = min (n, k).

If a population consists of k elements of one kind and N–k elements of another kind, then f(x) represents the probability of getting exactly x elements of the first kind in a random sample of size n.

Restriction: $N < 69$

Example:

If k = 2, n = 3, N = 5 then

f(0) = 0.10
f(1) = 0.60
f(2) = 0.30

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	k	STO	1	■	n!			
2	N	STO	2					
3	n	STO	3	–	■	n!		
4		x	RCL	3	■	n!		
5		x	RCL	2	RCL	1		
6		–	STO	4	■	n!		
7		x	RCL	2	■	n!		
8		÷	STO	5				
9	x	STO	6	■	n!	RCL		
10		1	RCL	6	–	■		
11		n!	x	RCL	3	RCL		
12		6	–	STO	7	■		
13		n!	x	RCL	4	RCL		
14		7	–	■	n!	x		
15		RCL	5	x⇄y	÷		f(x)	Stop; for new value
								of x, go to 9

Interpolation

Aiken's formula

Given a set of n + 1 data points

$$\{(x_i, y_i), i = 1, 2, \ldots, n + 1\}$$

where the x_i are distinct, a unique polynomial $P_n(x)$ of degree n exists which passes through those points.

We generate a table (using Aiken's formula) to evaluate $P_n(x)$ at a given point x_0.

$$\begin{array}{ccccc}
x_1 & y_1 & & & \\
 & & y_1{}^1 & & \\
x_2 & y_2 & & y_2{}^1 & \\
| & | & y_1{}^2 & & \\
| & | & & y_2{}^2 & \\
| & | & & & y_n{}^1 = P_n(x_0) \\
| & | & & y_2{}^{n-2} & \\
| & | & y_1{}^{n-1} & & \\
x_n & y_n & & y_2{}^{n-1} & \\
 & & y_1{}^n & & \\
x_{n+1} & y_{n+1} & & &
\end{array}$$

where

$$y_1{}^k = [y_k \cdot (x_0 - x_{k+1}) - y_{k+1} \cdot (x_0 - x_k)] / (x_k - x_{k+1})$$

$$k = 1, 2, \ldots, n$$

$$y_m{}^k = [y^k_{m-1} \cdot (x_0 - x_{k+m}) - y^{k+1}_{m-1} \cdot (x_0 - x_k)] / (x_k - x_{k+m})$$

$$m = 2, \ldots, n, \text{ and } k = 1, 2, \ldots, n+1-m$$

Superscripts of $y_m{}^k$ denote the index value of the left hand data point used in an interpolation, *subscripts* indicate the degree of the iterated interpolating polynomial at the current stage of the procedure.

Example:

Use Aiken's formula to approximate P_4 (0.25) if five data points (0, 1), (0.1, 1.105171), (0.2, 1.221403), (0.3, 1.349859), (0.4, 1.491825) are given.

Answer:

$x_1 = 0 \quad y_1 = 1$

$y_1{}^1 = 1.262928$

$x_2 = 0.1\ y_2 = 1.105171 \quad y_2{}^1 = 1.283667$

$y_1{}^2 = 1.279519 \quad y_3{}^1 = 1.284030$

$x_3 = 0.2\ y_3 = 1.221403 \quad y_2{}^2 = 1.284103 \quad y_4{}^1 = 1.284026 = P_4(0.25)$

$y_1{}^3 = 1.285631 \quad y_3{}^2 = 1.284023$

$x_4 = 0.3\ y_4 = 1.349859 \quad y_2{}^3 = 1.283942$

$y_1{}^4 = 1.278876$

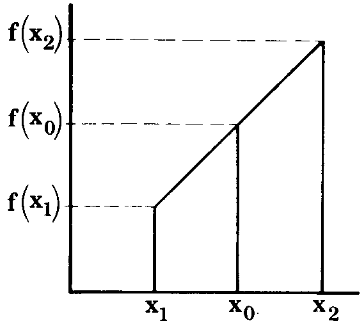
$x_5 = 0.4\ y_5 = 1.491825$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x_0	STO	1	FIX	6			
2	x_1	STO	2					
3	y_1	STO	4					
4		RCL	4	RCL	1			Perform 4–9 for k=1,2,...,n
5	x_{k+1}	STO	3	–	x			
6	y_{k+1}	STO	4	RCL	1	RCL		
7		2	–	x	–	RCL		
8		2	RCL	3	–	÷	$y_1{}^k$	
9		RCL	3	STO	2			
10	y^k_{m-1}	STO	4					Perform 10–16 for m=2,...,n
11		RCL	4	RCL	1			Perform. 11–16 for k = 1, ..
								n+1–m
12	x_{k+m}	STO	2	–	x			
13	y^{k+1}_{m-1}	STO	4	RCL	1			
14	x_k	STO	3	–	x	–		
15		RCL	3	RCL	2	–		
16		÷					$y_m{}^k$	$P_n(x_0) = y_n{}^1$

Linear interpolation

Assume f(x) is a function of x, for given x_1, x_2, $f(x_1)$, $f(x_2)$ and $x_1 < x_0 < x_1$, we can approximate $f(x_0)$ by

$$f(x_0) = \frac{(x_2 - x_0)\, f(x_1) + (x_0 - x_1)\, f(x_2)}{x_2 - x_1}$$



Example:

Suppose a table shows

x	f(x)
1.2	0.30119
1.3	0.27253

Interpolate f to 5 decimal places for x = 1.27.

Answer:

0.28113

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x_2	STO	1					
2	x_0	STO	2	−				
3	$f(x_1)$	×	RCL	2				
4	x_1	STO	3	−				
5	$f(x_2)$	×	+	RCL	1	RCL		
6		3	−	÷				

Intersections of Straight Line and Conic

Formula:

Find the intersections (x_1, y_1), (x_2, y_2) of the equations

$$ax + by + c = 0 \qquad (1)$$

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \qquad (2)$$

if there are any real intersections.

We can solve equation (1) for x (if $a \neq 0$), then substitute in equation (2) and solve the new quadratic equation in y (this program does not work if the new equation in y is linear or constant, in that case display will show flashing zeros).

Example:

Find intersections of

$$2x - y - 2 = 0$$

and

$$4x^2 + 16y^2 - 8x - 32y - 44 = 0$$

Answers:

$(x_1, y_1) = (2.43, 2.87)$, $(x_2, y_2) = (0.51, -0.98)$ $\qquad (Q = 3.71)$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	A	STO	1					
2	b	STO	2	x^2	×			
3	a	STO	3	x^2	÷			
4	B	STO	4	RCL	3	÷		
5		RCL	2	×	−			
6	C	+	↑	+	STO	5		
7		RCL	1	2	×	RCL		
8		2	×	RCL	3	x^2		
9		÷						
10	c	STO	6	×	RCL	4		
11		RCL	6	×	RCL	3		
12		÷	−					
13	D	STO	7	RCL	2	×		
14		RCL	3	÷	−			

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
15	E	+	STO	8	RCL	1		
16		RCL	6	x^2	x	RCL		
17		3	x^2	÷	RCL	7		
18		RCL	6	x	RCL	3		
19		÷	−					
20	F	+	STO	1	RCL	8		
21		RCL	5	÷	CHS	↑		
22		x^2	RCL	1	↑	+		
23		RCL	5	÷	−		Q	If Q < 0, there are no
								real intersections, stop.
24		■	$\sqrt{x}$	STO	1	+	y_1	
25		STO	4	x⇄y	RCL	1		
26		−					y_2	
27		RCL	2	x	RCL	6		
28		+	RCL	3	÷	CHS	x_2	
29		RCL	4	RCL	2	x		
30		RCL	6	+	RCL	3		
31		÷	CHS				x_1	

Interest (Compound)

Notation:

n = number of time periods

i = periodic interest rate expressed as a decimal

PV = present value or principal

FV = future value or amount

I = interest amount

Interest amount

Formula:

$$I = PV\ [(1 + i)^n - 1]$$

Example:

Find the compound interest on \$1500 for 5 years if interest at 6% is compounded annually.

Answer:

\$507.34 (Note: i = 0.06)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	↑	1	+				
2	n	■	y^x	1	−			
3	PV	x						

Number of time periods

Formula:

$$n = \frac{\ln\left(\dfrac{FV}{PV}\right)}{\ln (1 + i)}$$

Example:

How long does it take to yield \$2007.34 at 6% compounded annually if the principal is \$1500?

Answer:

5 years (Note: i = 0.06)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	FV	↑						
2	PV	÷	ln	1	↑			
3	i	+	ln	÷				

Rate of return

Formula:

$$i = \left(\frac{FV}{PV}\right)^{1/n} - 1$$

Example:

Find the rate of return if \$1500 invested today compounded annually will amount to \$2007.34 in 5 years?

Answer:

0.06 (6%)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	FV	↑						
2	PV	÷						
3	n	1/x	■	y^x	1	−		

Present value

Formula:

$$PV = \frac{FV}{(1+i)^n}$$

Example:

What sum invested today at 6% compounded annually will amount to \$2007.34 in 5 years?

Answer:

\$1500.00 (Note: i = 0.06)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	FV	↑						
2	i	↑	1	+				
3	n	■	y^x	÷				

Future value

Formula:

$$FV = PV\,(1 + i)^n$$

Example:

Find the future amount of \$1500 invested at 6% compounded annually for 5 years.

Answer:

\$2007.34 (Note: i = 0.06)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	↑	1	+				
2	n		y^x					
3	PV	x						

Compound continuously

Formula:

$$FV = PV \cdot e^{in}$$

Example:

Determine the value of \$50 deposited at 6% for 5 years, compounded continuously?

Answer:

\$67.49 (Note: i = 0.06)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	↑						
2	n	x	e^x					
3	PV	x						

Nominal rate converted to effective annual rate

Formula:

$$\text{Effective rate} = (1 + i)^n - 1$$

Example:

What is the effective annual rate of interest if the nominal (annual) rate of 12% is compounded quarterly? ($n = 4$, $i = 0.12/4$).

Answer:

0.1255 (12.55%)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	↑	1	+	FIX	4		
2	n	■	y^x	1	−			

Add-on rate converted to true annual percentage rate (APR)

Formula:

$$\text{APR} \cong \frac{600\, ni}{3\,(n+1) + [(n-1)\, ni/m]}$$

where n = number of payments

m = number of payments in one year

i = add-on interest rate

Note: This formula will give an approximate, not exact answer.

Example:

What is the true rate of interest (APR) on an 18-month, 5% add-on loan?

Answer:

9.27 (%)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	↑	STO	1				
2	n	STO	2	x	RCL	2		
3		1	−	x				
4	m	÷	RCL	2	1	+		
5		3	x	+	6	0		
6		0	RCL	2	x	RCL		
7		1	x	x⇄y	÷			

Interest (Simple)

Notation:

n = number of time periods

i = periodic interest rate expressed as a decimal

PV = present value or principal

FV = future value or amount

I = interest amount

Interest amount

Formula:

$$I = PV \cdot n \cdot i$$

Example:

Find the interest payment due of $1500 on a 360-day basis at 6% simple interest for 200 days.

Answer:

$50.00 (Note: i = 0.06/360)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	↑						
2	n	x						
3	PV	x						

Number of time periods

Formula:

$$n = \frac{FV - PV}{PV \cdot i}$$

Example:

How long does it take to yield \$1950 at 6% simple interest if the present value is \$1500?

Answer:

5 years (Note: $i = 0.06$)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	FV	↑						
2	PV	STO	1	–				
3	i	RCL	1	x	÷			

Interest rate

Formula:

$$i = \frac{FV - PV}{PV \cdot n}$$

Example:

Find the simple interest rate if \$1500 invested today will amount to \$1950 in 5 years.

Answer:

0.06 (6%)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	FV	↑						
2	PV	STO	1	–				
3	n	RCL	1	x	÷			

Present value

Formula:

$$PV = \frac{FV}{1 + ni}$$

Example:

What sum invested today at 6% simple interest will amount to $1950 in 5 years?

Answer:

$1500 (Note: i = 0.06)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	FV	↑						
2	n	↑						
3	i	x	1	+	÷			

Future value

Formula:

$$FV = PV\,(1 + ni)$$

Example:

Find the future value of $1500 invested at 6% simple interest for 5 years.

Answer:

$1950.00 (Note: i = 0.06)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	↑						
2	n	x	1	+				
3	PV	x						

Interest Rebate (Rule of 78's)

See page 145

Inverse Hyperbolic Functions

See page 125

Iterative Solution of Equations

See page 96

Least Common Multiple

Formula:

The least common multiple of two positive integers a and b is the smallest positive integer that both a and b can divide.

$$\mathrm{LCM}\,(a, b) = \frac{a \cdot b}{\mathrm{HCF}\,(a, b)}$$

Example:

$$\mathrm{LCM}\,(51, 119) = 357.00$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1	STO	3			
2	b	STO	2					
3		↑	↑	RCL	1	÷	D	Let f be the largest
								integer ≤ D
4	f	x⇄y	CLX	RCL	1	×		
5		−					E	If E = 0, go to 8
6		RCL	1	x⇄y	STO	1		
7		CLX	+					Go to 3
8		RCL	1	↑	RCL	3		
9		↑	RCL	2	×	x⇄y		
10		÷					LCM(a, b)	

Least Squares Regression

See page 77

Linear Regression

See page 77

Loan Repayments

Notations:

n = number of payments

i = periodic interest rate expressed as a decimal

PMT = payment

PV = present value or principal

Number of time periods

Formula:

$$n = \log_{1+i}\left(\frac{1}{1 - \dfrac{PV \cdot i}{PMT}}\right)$$

Example:

How many payments does it take to pay off a loan of \$4000 at 9.5% annual rate, with payments close to \$150 per month?

Answer:

30.07 payments (Note: $i = 0.095/12$)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	STO	1					
2	PV	x						
3	PMT	÷	1	x⇄y	−	1/x		
4		ln	RCL	1	1	+		
5		ln	÷					

Interest rate

Formula:

$$\text{Monthly interest rate } i = \frac{\text{PMT}\left[1-\left(\frac{1}{1+i}\right)^{n}\right]}{\text{PV}}$$

$$\text{Annual interest rate} = \text{monthly rate} \times 12$$

Example:

If n = 360, monthly payment PMT = 179.86, PV = 30000, find the annual interest rate.

Answer:

6.00% (8 iterations)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	↑	↑					
2	PMT	↑	FIX	9				
3	PV	÷	STO	1	CLX	+		
4		1	+	1/x	x⇄y			Perform 4–6 for k=1,2,...
5		y^x	1	x⇄y	–	RCL		until D_k converges (to
6		1	x				D_k	desired decimal place)
7		EEX	2	x	1	2		
8		x	FIX	2				Answer is in %

Payment amount

Formula:

$$\text{PMT} = \frac{\text{PV} \cdot i}{1-(1+i)^{-n}}$$

Example:

To pay off a loan of $4000 at 9.5% interest in 30 months, what monthly payment is required?

Answer:

$150.32 (Note: i = 0.095/12)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	STO	1					
2	PV	x	RCL	1	1	+		
3	n		y^x	1/x	1	x⇄y		
4		–	÷					

Present value

Formula:

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i}\right]$$

Example:

A person is willing to pay $150 per month for 30 months for a loan at 9.5%, how much can be borrowed?

Answer:

$3991.55 (Note: $i = 0.095/12$)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	STO	1	1	+			
2	n	■	y^x	1/x	1	x⇄y		
3		−	RCL	1	÷			
4	PMT	x						

Accumulated interest

Formula:

The interest paid from payment j to payment k is

$$I_{j-k} = PMT \left[k-j - \frac{(1 + i)^{k-n}}{i}\left(1 - (1 + i)^{j-k}\right)\right]$$

Compute the monthly payment, PMT, by the formula given above under "Payment Amount."

Example:

Consider a house costing \$30,000 with a mortgage life of 30 years at 8% yearly interest. Find the interest paid on the first 36 monthly payments ($i = 0.08/12, j = 0, k = 36, n = 360$).

Answer:

PMT = \$220.13

$I_{0\text{-}36} = \$7108.72$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	STO	1	1	+			
2	j	STO	2					
3	k	STO	3	−	■	y^x		
4		1	x⇄y	−	RCL	1		
5		÷	1	RCL	1	+		
6		RCL	3					
7	n	−	■	y^x	x	RCL		
8		2	+	RCL	3	x⇄y		
9		−						
10	PMT	x						

Remaining balance

Formula:

The remaining balance at payment k (k = 1, 2, 3, ..., n) is

$$PV_k = \frac{PMT}{i}\left[1 - (1 + i)^{k-n}\right]$$

Example:

Using the previous example, find the remaining balance at payment 36.

Answer:

\$29184.13

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	STO	1	1	+			
2	k	↑						
3	n	−		y^x	1	x⇄y		
4		−						
5	PMT	×	RCL	1	÷			

Interest rebate (Rule of 78's)

Formula:

$$F = \text{finance charge}$$

$$I_k = \text{interest charged at month } k = \frac{2\,(n - k + 1)}{n\,(n + 1)}\,F$$

$$\text{rebate} = \frac{(n - k)\,I_k}{2}$$

Example:

A 30-month, \$1000 loan having a finance charge of \$180.00 is being repaid at \$39.33 per month. What is the interest portion of the 25^{th} payment? What is the interest rebate at that point?

Answers:

Interest portion of the 25^{th} payment = \$2.32

Rebate = \$5.81

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	STO	1					
2	k	STO	2	−	STO	3		
3		1	+	RCL	1	÷		
4		RCL	1	1	+	÷		
5		2	×					
6	F	×					I_k	
7		RCL	3	×	2	÷		

Logarithms

Logarithm of a to base b ($\log_b$ a)

Formula:

$$\log_b a = \frac{\ln a}{\ln b}$$

Example:

$\log_7 5 = 0.83$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	ln						
2	b	ln	÷					

Means

Mean, standard deviation and sums of grouped data

Formulas:

Given a set of data points

$$x_1, x_2, \ldots, x_n$$

with respective frequencies

$$f_1, f_2, \ldots, f_n$$

Let $k = \sum_{i=1}^{n} f_i$.

Then

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n} f_i x_i^2 - \frac{\left(\sum_{i=1}^{n} f_i x_i\right)^2}{k}}{k-1}}$$

$$Sx = \sum_{i=1}^{n} f_i x_i$$

$$SSx = \sum_{i=1}^{n} f_i x_i^2.$$

Example:

f_i	30	13	4	22	7
x_i	1	2	3	4	5

Answers: $\bar{x} = 2.51$

$s = 1.48$

$Sx = 191.00$

$SSx = 645.00$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR					
2	f_i	STO	+	5				Perform 2–4 for i=1,2,...,n
3	x_i	×	STO	+	7	■		
4		LAST x	×	STO	+	6		
5		■	$\bar{x}$,s				$\bar{x}$	
6		x⇄y					s	
7		RCL	7					Sx
8		RCL	6					SSx

Arithmetic mean

Formula:

The arithmetic mean (average) of a set of numbers

$$\{a_1, a_2, ..., a_n\}$$

is

$$A = \frac{1}{n}\sum_{i=1}^{n} a_i$$

Example:

Compute the arithmetic mean of

2, 3.4, 3.41, 7, 11, 23

Answer:

8.30

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a_1	↑						
2	a_i	+						Perform 2 for i=2,3,...,n
3	n	÷						

Geometric mean

Formula:

The geometric mean of a set of numbers

$$\{a_1, a_2, ..., a_n\}$$

is

$$G = \sqrt[n]{a_1 \cdot a_2 \cdot ... \cdot a_n}$$

Example:

Compute the geometric mean of

2, 3.4, 3.41, 7, 11, 23

Answer:

5.87

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a_1	↑						
2	a_i	x						Perform 2 for i=2,3,...,n
3	n	$^1/_x$		y^x				

Harmonic mean

Formula:

The harmonic mean of a set of numbers

$$\left\{a_1, a_2, ..., a_n\right\}$$

is

$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{a_i}}$$

Example:

Find the harmonic mean of

2, 3.4, 3.41, 7, 11, 23

Answer:

4.40

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	↑						
2	a_1	$^1/_x$						
3	a_i	$^1/_x$	+					Perform 3 for i=2,3,...,n
4		÷						

Mils to Degrees

See page 19

Multiple Linear Regression

See page 79

Navigation

See page 186

Negative Binomial Distribution

Formula:

$$f(x) = \binom{-r}{x} p^r (1-p)^x = \binom{r+x-1}{x} p^r (1-p)^x$$

where $x = 0, 1, 2, \ldots$

$r = 1, 2, \ldots$

$0 \leqslant p \leqslant 1$

Example:

If $r = 4$, $p = 0.9$ then

$f(0) = 0.66$

$f(1) = 0.26$

$f(2) = 0.07$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	r	STO	1	1	↑			
2	p	STO	2	−	STO	3		
3	x	STO	4	RCL	1	+		
4		1	−	■	n!	RCL		
5		4	■	n!	÷	RCL		
6		1	1	−	■	n!		
7		÷	RCL	2	RCL	1		
8		■	y^x	x	RCL	3		
9		RCL	4	■	y^x	x		Stop; for new value of x,
								go to 3

Normal Distribution

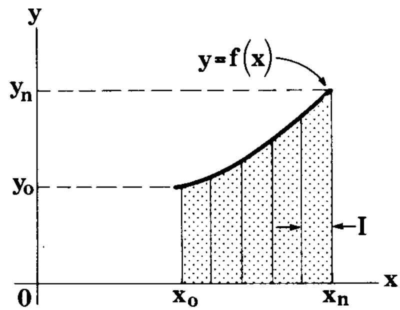
See page 115

Numerical Integration

Method:

To approximate the area A under a curve, sum the areas of the consituent trapezoids of width I. Each trapezoid has the area

$$I \times \frac{y_i + y_{i-1}}{2}$$



Example:

Find the area bounded by

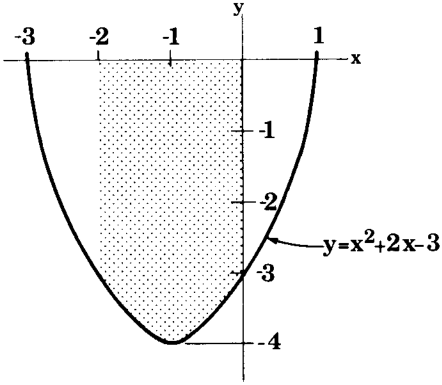
$$y = x^2 + 2x - 3,$$

$$x = -2,$$

$$x = 0 \text{ (the y axis)}$$

and

$$y = 0 \text{ (the x axis)}.$$



In this case

$$x_0 = -2, x_n = 0, n = 4, I = 0.5$$

Replace f(x) by "[↑] [↑] [↑] 2 [+] [×] 3 [−]"

Answer:

7.25

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR					
2	x_n	↑						
3	x_0	STO	1	−				
4	n	÷	STO	2			I	
5		RCL	1					
6		f(x)	STO	3			y_0	
7		RCL	1	RCL	+	2		Perform 7−12 for i=1,2,...,n
8		STO	1					
9		f(x)	STO	4			y_i	
10		RCL	3	+	x^2	■		
11		$\sqrt{x}$	Σ+	RCL	4	STO		
12		3						
13		RCL	7	RCL	2	×		
14		2	÷					

Parabola (Least Squares Fit)

See page 82

Payments

See page 142

Percent

Markup percent

Formula:

To make a gross profit of G%, add A% to the cost price. To find A for a given G.

$$A = \frac{100G}{100 - G}$$

Example:

To make a profit of 30%, what is the percentage of markup?

Answer:

42.86%

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	G	1/x	EEX	CHS	2	−		
2		1/x						

Gross % profit

Formula:

If A% is added to the cost price, the profit will be G% of the selling price.

$$G = \frac{100A}{A + 100}$$

Example:

If we add 30% to our cost price, what percent of the selling price will be the profit?

Answer:

23.08%

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	A	1/x	EEX	CHS	2	+		
2		1/x						

Permutations

Permutations of a objects taken b at a time

Formula:

$$_aP_b = P\,(a, b) = \frac{a!}{(a-b)!}$$

Example:

$$_7P_5 = 2520.00$$

Notes:

$_aP_0 = 1,$

$_aP_1 = a,$

$_aP_a = a!$

Program requires $a \leqslant 69$.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a	■ [n!] ■ [LAST x] [ ]		
2	b	[−] ■ [n!] [÷] [ ]		

Plotting Curves

Objective:

The following routines give values of $y = f(x)$ in increments of I for values of x between $x_0 = a$, and $x = b$ where $b > a$. f(x) should be replaced by appropriate sequence of keystrokes. I is saved in register R_1, so f(x) cannot use that register.

Examples of f(x): (assuming x is in the X register)

For	*Replace f(x) by*
$y = x^2$	↑ ×
$y = \ln \sin x$	SIN ln
$y = 3\sqrt[x]{x}$	↑ 1/x ■ y^x 3 ×
$y = x^4 - 2x^2 + 3x - 7$	↑ ↑ ↑ × 2 − × 3 + × 7 −

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	I	STO	1					
2	x_0	↑						
3		f(x)					y_0	
4		CLX	+	RCL	+	1	x_i	Perform 4–6 for i=1,2,...,k,
								until x_k = b
5		↑						
6		f(x)					y_i	

Example:

Plot $$y = \sqrt{\ln \frac{x}{2}}$$

from x = 3 to x = 5 at intervals of 0.5.

Replace f(x) in the program by ↑ 2 ÷ ln ■ $\sqrt{x}$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	0.5	STO	1					
2	3	↑					3.00	← x_0
3		↑	2	÷	ln	■		
4		$\sqrt{x}$					0.64	← $f(x_0)$
5		CLX	+	RCL	+	1	3.50	← x_1
6		↑	↑	2	÷	ln		
7		■	$\sqrt{x}$				0.75	← $f(x_1)$
8		CLX	+	RCL	+	1	4.00	← x_2
9		↑	↑	2	÷	ln		
10		■	$\sqrt{x}$				0.83	← $f(x_2)$
11		CLX	+	RCL	+	1	4.50	← x_3
12		↑	↑	2	÷	ln		
13		■	$\sqrt{x}$				0.90	← $f(x_3)$
14		CLX	+	RCL	+	1	5.00	← x_4
15		↑	↑	2	÷	ln		
16		■	$\sqrt{x}$				0.96	← $f(x_4)$

Poisson Distribution

Formula:

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

where x is a positive integer and $\lambda > 0$.

Example:

Suppose $\lambda = 2.8$; $f(7) = 0.016279878$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	λ	CHS	e^x	■	LAST x	CHS		
2	x	■	y^x	■	LAST x	■		
3		n!	÷	×	FIX	9		

Polynomial Evaluation

Formulas:

$$f(x) = c_0x^n + c_1x^{n-1} + \ldots + c_{n-1}x + c_n$$

write

$$f(x_0) = (\ldots (((c_0x_0 + c_1)\,x_0 + c_2)\,x_0 + c_3)\,x_0 + \ldots)\,x_0 + c_n$$

Example:

If $f(x) = x^5 + 5x^4 - 3x^2 - 7x + 11$, find f(2.5).

Answer:

267.72

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x_0	↑	↑	↑				
2	c_0							
3		×						Perform 3–4 for i=1,2,...,n
4	c_i	+						

Power Curve (Least Squares Fit)

See page 86

Present Value

See pages, 134, 139

Primes

See page 111

Progressions

Formulas:

Arithmetic Progression

$$a, a + d, a + 2d, \ldots, a + (n - 1)d$$

Geometric Progression

$$a, ar, ar^2, \ldots, ar^{n-1}$$

Harmonic Progression

$$\frac{a}{b}, \frac{a}{b + c}, \frac{a}{b + 2c}, \ldots, \frac{a}{b + (n - 1)c}$$

n = number of terms

a = first term in arithmetic and geometric progressions

l = last term

d = difference between two successive terms in an arithmetic progression

r = ratio between two successive terms in a geometric progression

S = sum of a progression

Step through an arithmetic progression

Formula:

$$a, a + d, a + 2d, \ldots, a + (n - 1)d$$

Example:

Display the progression with a = 0, d = 17.

Answer:

0.00, 17.00, 34.00, 51.00, 68.00, 85.00, 102.00, 119.00, ...

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	d	↑	↑	↑				
2	a							
3		+						Perform 3 as many times
								as desired

Step through a geometric progression

Formula:

$$a, ar, ar^2, \ldots, ar^{n-1}$$

Example:

Step through the powers of 8.

Answers:

8.00, 64.00, 512.00, 4096.00, 32768.00, ...

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	r	↑	↑	↑				
2	a							
3		x						Perform 3 as many times
								as desired

Step through a harmonic progression

Formula:

$$\frac{a}{b}, \frac{a}{b+c}, \frac{a}{b+2c}, \ldots, \frac{a}{b+(n-1)c}$$

Note: A harmonic progression can be obtained by multiplying the constant a by the reciprocals of the terms of the arithmetic progression b, b + c, b + 2c, ..., b + (n − 1) c. In the following algorithm, x_i (i = 1, 2, ...) represents the i^{th} term of the progression.

Example:

Step through the harmonic progression where a = 1, b = 2, and c = 3.

Answers:

0.50, 0.20, 0.13, 0.09, ...

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1					
2	c	↑	↑	↑				
3	b	↑	1/x	RCL	1	×	x_1	
4		CLX	+	+	↑	1/x		Perform 4–5 for i=2,3,...
5		RCL	1	×			x_i	

n^{th} term of an arithmetic progression

Formula:

Given the number of terms, the last term of an arithmetic progression is given by

$$n^{th}\text{ term} = a + (n-1)d$$

Example:

Find the 25^{th} term of the arithmetic progression with a = 2, d = 3.14.

Answer:

77.36

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	↑	1	−				
2	d	×						
3	a	+						

n^{th} term of a geometric progression

Formula:

Given the number of terms, the last term of a geometric progression is given by

$$n^{th} \text{ term} = ar^{n-1}$$

Example:

Find the 14^{th} term of the geometric progression with a = 2, r = 3.14.

Answer:

5769197.69

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	r	↑		
2	n	↑ 1 −		If r > 0, go to 4
3		x⇄y CHS x⇄y ■ y^x		If n is even, go to 5.
				Otherwise, go to 6
4		■ y^x		Go to 6
5		CHS		
6	a	x		

Arithmetic progression sum (given the last term)

Formula:

Given the last term, the sum of an arithmetic progression to n terms is

$$S = \frac{n}{2}(a + l)$$

Example:

If a = 3.5, l = 25, and n = 11, find the sum.

Answer:

S = 156.75

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	ℓ	+						
3	n	×	2	÷				

Arithmetic progression sum (given the difference)

Formula:

Given the first term and the difference between two successive terms, the sum of an arithmetic progression to n terms is:

$$S = na + \frac{n(n-1)d}{2}$$

Example:

If a = 3.5, n = 11, and d = 2.15, find the sum of 11 terms.

Answer:

S = 156.75

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	↑	↑	1	−			
2	d	×						
3	a	↑	2	×	+	×		
4		2	÷					

Sum of a geometric progression (r < 1)

Formula:

The sum of a geometric progression to n terms with r < 1 is

$$S = \frac{a(1 - r^n)}{1 - r}$$

Example:

If a = 1, r = –2.1, and n = 6, find the sum of 6 terms.

Answer:

S = –27.34

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	↑					
2	n	↑						
3	r	STO	1					If r > 0, go to 5
4		CHS	x⇄y	■	y^x			If n is odd, go to 6;
								Otherwise, go to 7
5		x⇄y	■	y^x				Go to 7
6		CHS						
7		x	–	1	RCL	1		
8		–	÷					

Sum of a geometric progression (r > 1)

Formula:

The sum of geometric progression to n terms with r > 1 is

$$S = \frac{a\,(r^n - 1)}{r - 1}$$

Example:

If a = 1, r = 2.1, n = 6, find the sum.

Answer:

S = 77.06

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	↑					
2	r	↑	STO	1				
3	n	■	y^x	x	x⇄y	–		
4		RCL	1	1	–	÷		

Sum of an infinite geometric progression ($-1 < r < 1$)

Formula:

$$S = \frac{a}{1 - r}$$

Example:

If a = 2 and r = .5, find the sum.

Answer:

S = 4.00

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	1	↑				
2	r	−	÷					

Quadratic Equation

Formula:

The roots of $$Ax^2 + Bx + C = 0$$

are $$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

If $$D = (B^2 - 4AC)/4A^2$$

is positive, the roots are real. The root with larger absolute value x_1 is computed first to obtain better significance. The second real root x_2 is found by

$$x_2 = \frac{C}{Ax_1}$$

If D is negative, roots are complex (one root is the complex conjugate of the other), being

$$u \pm iv = \frac{-B}{2A} \pm \frac{\sqrt{4AC - B^2}}{2A}\, i$$

Example 1:

Solve $3.142958x^2 - 6.987122x + 1.001976 = 0$

Answers: $x_1 = 2.07,\ x_2 = 0.15$

$$\left(D = 0.92,\ \frac{-B}{2A} = 1.11\right)$$

Example 2:

Solve $-7.23x^2 + 2.67x - 3.17 = 0$

Answers: $u \pm iv = 0.18 \pm 0.64i$

$(D = -0.40)$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	A	STO	1	↑	+			
2	B	$x \rightleftarrows y$	÷	CHS	↑	x^2		
3	C	STO	2	RCL	1	÷		
4		−					D	If D < 0, go to 10.
5			$\sqrt{x}$	$x \rightleftarrows y$			−B/2A	If −B/2A < 0, go to 7.
6		+					x_1	Go to 8.
7		−	CHS				x_1	
8		RCL	1	×	RCL	2		
9		$x \rightleftarrows y$	÷				x_2	Stop
10		CHS		$\sqrt{x}$	$x \rightleftarrows y$		u	
11		$x \rightleftarrows y$					v	

Radians to Degrees

See page 19

Random Numbers

Objective:

This routine will use a "seed" S to generate a sequence of pseudo random numbers R_i in either of two ranges -1 to 0 (if $S < 0$) or 0 to 1 (if $S > 0$). For best results, the seed must be a ten digit decimal fraction containing all digits 0 through 9 in an arbitrary order.

Example:

If S = .2510637948, generate a random series.

Answers:

0.28, 0.14, 0.19, 0.65, 0.90, 0.20, 0.85, etc.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		2	9	↑	↑	↑		
2	S							
3		x					D_i	Perform 3–4 for i=1,2,3,...
4	f_i	−					R_i	Let f_i = integer part
								of D_i

Rank Correlation (Spearman's Coefficient)

Formula:

$$r_s = 1 - \frac{6 \sum_{i=1}^{n} D_i^2}{n\,(n^2 - 1)}$$

where n = number of paired observations (x_i, y_i)

$D_i = \text{rank}(x_i) - \text{rank}(y_i) = R_i - S_i$

If the X and Y random variables from which these n pairs of observations are derived are independent, then r_s has a mean of zero and a variance

$$\frac{1}{n-1}$$

An approximate test for the null hypothesis

$$Ho: \ X, Y \text{ are independent}$$

is

$$Z = r_s \sqrt{n-1}$$

which is approximately a standardized normal variable (for large n, say $n \geqslant 10$).

If the null hypothesis of independence is not rejected, we can infer that the population correlation coefficient $\rho(x, y) = 0$, but dependence between the variables does not necessarily imply that $\rho(x, y) \neq 0$.

Note: $-1 \leqslant r_s \leqslant 1$

Example:

Student	X Math Grade	Y Stat Grade	R Rank of X	S Rank of Y
1	82	81	6	7
2	67	75	14	11
3	91	85	3	4
4	98	90	1	2
5	74	80	11	8
6	52	60	15	15
7	86	94	4	1
8	95	78	2	9
9	79	83	9	6
10	78	76	10	10
11	84	84	5	5
12	80	69	8	13
13	69	72	13	12
14	81	88	7	3
15	73	61	12	14

Answers:

$r_s = 0.76$

$Z = 2.85$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1			CLEAR					
2	R_i	↑						Perform 2–3 for i=1,2,...,n
3	S_i	–	Σ+					
4		RCL	6	6	x	RCL		
5		5	↑	x^2	1	–		
6		x	÷	1	x⇄y	–	r_s	
7		RCL	5	1	–			
8		$\sqrt{x}$	x				Z	

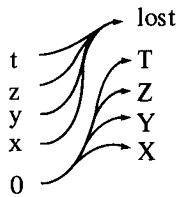
Rate of Return

See page 134

Register Operations

In the following eighteen routines, x, y, z, t and r_k denote the contents of registers X, Y, Z, T and R_k, respectively. ($k = 1, 2, ..., 9$)

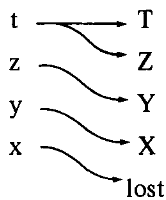
1. Clear stack; retain all storage registers



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		CLX ↑ ↑ ↑		

2. Delete x

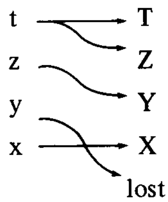
(Lower the stack.)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		CLX +		

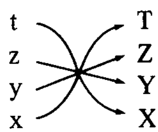
3. Delete y

(Lower that part of the stack above X.)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		x⇄y CLX +		

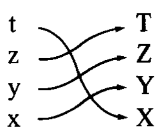
4. Reverse the stack



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		x⇄y R↓ R↓ x⇄y		

5. Fetch t or roll up

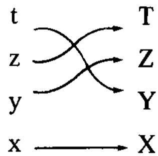
(Bring t to X, keeping the other operands in the same order).



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		R↓ R↓ R↓		

6. Fetch t to Y

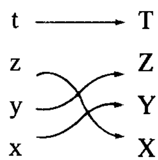
(Bring t to Y, keeping the other operands in the same order).



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[R↓] [R↓] [R↓] [x⇄y] [ ]		

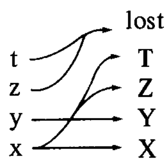
7. Fetch z

(Bring z to X, keeping the other operands in the same order).



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[R↓] [R↓] [x⇄y] [R↓] [ ]		

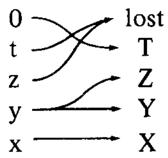
8. Copy x into Z and T



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[↑] [↑] [R↓] [R↓] [ ]		

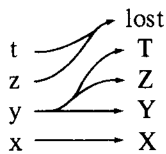
9. Copy y into Z

(T is cleared).



LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		↑	↑	–	R↓			

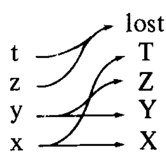
10. Copy y into Z and T



LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		x⇄y	↑	↑	R↓	R↓		
2		R↓						

11. Copy y and x into Z and T, respectively

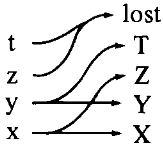
(Copy x and y in reverse stack order, but this is the shortest way to save both x and y in the stack)



LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		↑	↑	CLX	+	R↓		

12. Copy y and x into T and Z, respectively

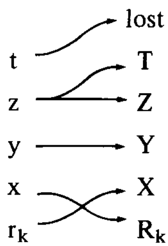
(Copy x and y in the same stack order to Z and T).



LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		x⇄y	↑	↑	CLX	+		
2		R↓	x⇄y					

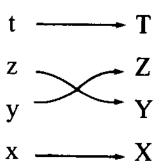
13. Swap x and r_k

(Exchange x and r_k, t is lost), where k = 1, 2, ..., 9.



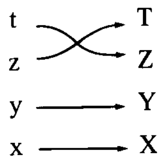
LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		RCL						k is an integer and
2	k	x⇄y	STO					$1 \leqslant k \leqslant 9$
3	k	CLX	+					

14. Swap y and z



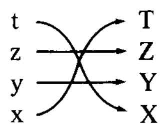
LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		R↓	x⇄y	R↓	R↓	R↓		

15. Swap z and t



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		R↓ R↓ x⇄y R↓ R↓		

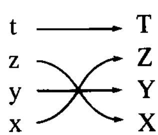
16. Swap x and t



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		R↓ R↓ R↓ x⇄y R↓		

17. Swap x and z

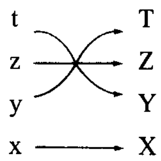
(Reverse contents of X, Y, Z)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		x⇄y R↓ R↓ x⇄y R↓		

18. Swap y and t

(Reverse contents of Y, Z, T).



LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		R↓	x⇄y	R↓	R↓	x⇄y		

Resistance

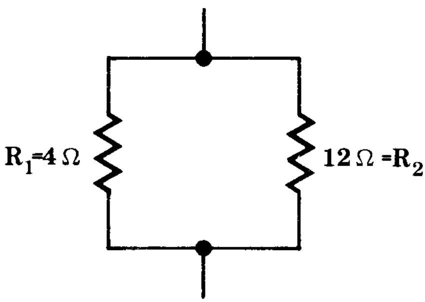
Formula:

The equivalent resistence R of a parallel combination of resistors is

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}$$

Example:

Find R.



Answer:

R = 3.00

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	R_1	1/x						
2	R_i	1/x	+					Perform 2 for i=2,...,n
3		1/x						

Roots of a Polynomial

Formula:

The Newton-Raphson iterative method can be used to compute a root of the polynomial equation

$$f(x) \equiv a_0 x^n + a_1 x^{n-1} + \ldots + a_{n-1} x + a_n = 0$$

by using

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

where f' is the derivative of f.

Due to storage limitations, the following key sequence is limited to handling polynomials with $n \leqslant 9$. The routine can be modified (to record intermediate results instead of storing them) to solve polynomials with $n > 9$.

Example:

Given an initial estimate $x_0 = 0.6875$ of a zero of the polynomial

$$f(x) = x^3 - 11x^2 + 32x - 22$$

improve the estimate so that it is accurate to the 5^{th} decimal place.

Answer:

$$x_1 = 0.95396, x_2 = 0.99875, x_3 = x_4 = 1.00000$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x_0	↑	↑	↑				
2	a_0	x						Perform 2–13 for i=1,2,...
3	a_j	+	STO					Perform 3–4 for j=1,...,
								n −1
4	j	x						
5	a_n	+	STO					
6	n	x⇄y	↑	↑	↑			
7	a_0	x						
8		RCL						Perform 8–9 for k=1,...,
								n − 2
9	k	+	x					
10		RCL						
11	n−1	+	RCL					
12	n	x⇄y	÷	−	↑	↑		
13		↑					x_{i+1}	

Secant

See page 201

Simple Interest

See page 137

Simultaneous Linear Equations

Two unknowns

Formula:

Solve for x and y

$$\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$$

Determinant solutions are:

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{D}$$

$$y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{D}$$

where

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0 .$$

Example:

Solve $\begin{cases} 7.32x - 9.08y = 3.14 \\ 12.39x + 7y = 0.05 \end{cases}$

Answers:

x = 0.14

y = −0.24

(D = 163.74)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	e	STO	1					
2	d	STO	2	x				
3	b	STO	3					
4	f	STO	4	x	−			
5	a	STO	5	RCL	2	x		
6	c	STO	6	RCL	3	x		
7		−	STO	7			D	
8		÷					x	
9		RCL	5	RCL	4	x		
10		RCL	1	RCL	6	x		
11		−	RCL	7	÷		y	

Three unknowns

Formulas:

$$\begin{cases} a_1 x + b_1 y + c_1 z = d_1 \\ a_2 x + b_2 y + c_2 z = d_2 \\ a_3 x + b_3 y + c_3 z = d_3 \end{cases}$$

Determinant solution to these simultaneous equations:

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D}$$

$$y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D}$$

$$z = \frac{d_1 - b_1 y - a_1 x}{c_1}$$

where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$$

Example:

Solve

$$\begin{cases} 3.14x + 10.02y - 7z = 1 \\ 0.25x + 30.3y - 9.1z = 2 \\ -3.5x + 27.4y + 8z = 3 \end{cases}$$

Answers:

x = 0.29

y = 0.11

z = 0.14

(D = 1052.86)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a_1	↑						
2	b_2	STO	5	×				
3	c_3	STO	9	×				
4	b_1	STO	4					
5	c_2	STO	8	×				
6	a_3	×	+					
7	c_1	STO	7					
8	a_2	×						
9	b_3	STO	6	×	+			
10	a_3	RCL	5	×	RCL	7		
11		×	–	RCL	6	RCL		
12		8	×					
13	a_1	×	–	RCL	9			
14	a_2	×	RCL	4	×	–	D	
15		↑	↑	↑				
16	d_1	STO	1	RCL	5	×		
17		RCL	9	×	RCL	4		
18		RCL	8	×				
19	d_3	STO	3	×	+	RCL		
20		7	RCL	6	×			
21	d_2	STO	2	×	+	RCL		
22		3	RCL	5	×	RCL		
23		7	×	–	RCL	6		
24		RCL	8	×	RCL	1		
25		×	–	RCL	9	RCL		

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
26		2	x	RCL	4	x		
27		−	x⇄y	÷	STO	5	x	
28		CLX						
29	a_1	STO	6	RCL	2	x		
30		RCL	9	x	RCL	1		
31		RCL	8	x				
32	a_3	x	+	RCL	7			
33	a_2	x	RCL	3	x	+		
34	a_3	RCL	2	x	RCL	7		
35		x	−	RCL	3	RCL		
36		8	x	RCL	6	x		
37		−	RCL	9				
38	a_2	x	RCL	1	x	−		
39		x⇄y	÷				y	
40		RCL	4	x	CHS	RCL		
41		1	+	RCL	6	RCL		
42		5	x	−	RCL	7		
43		÷					z	

Sinking Fund

Notation:

n = number of time periods

i = periodic interest rate, expressed as a decimal

PMT = payment

FV = future value

Number of periods

Formula:

$$n = \frac{\ln\left(\frac{i \cdot FV}{PMT} + 1\right)}{\ln(1 + i)}$$

Example:

If you put $100 into a savings account every month, how long does it take to save $5000, if interest is earned at 5% compounded monthly?

Answer:

45.51 months (Note: $i = 0.05/12$)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	STO	1					
2	FV	x						
3	PMT	÷	1	+	ln	1		
4		RCL	1	+	ln	÷		

Payment amount

Formula:

$$PMT = \frac{FV \cdot i}{(1+i)^n - 1}$$

Example:

To save $5000 in 45 months in a savings account paying 5%, compounded monthly, how much should you deposit each month?

Answer:

$101.25 (Note: $i = 0.05/12$)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	STO	1					
2	FV	x	1	RCL	1	+		
3	n	[redacted]	y^x	1	−	÷		

Future value

Formula:

$$FV = PMT \left[\frac{(1+i)^n - 1}{i}\right]$$

Example:

If you deposit $100 every month in a savings account at 5% compounded monthly, how much will you accumulate after 45 months?

Answer:

$4938.25 (Note: i = 0.05/12)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	STO	1	1	+			
2	n		y^x	1	−	RCL		
3		1	÷					
4	PMT	×						

Skewness and Kurtosis

(for grouped or ungrouped data)

Formulas:

mean:

$$\bar{x} = \frac{1}{n} \Sigma x_i$$

2^{nd} moment:

$$m_2 = \frac{1}{n} \Sigma x_i^2 - \bar{x}^2$$

3^{rd} moment:

$$m_3 = \frac{1}{n} \Sigma x_i^3 - \frac{3}{n} \bar{x} \Sigma x_i^2 + 2\bar{x}^3$$

4^{th} moment:

$$m_4 = \frac{1}{n} \Sigma x_i^4 - \frac{4}{n} \bar{x} \Sigma x_i^3 + \frac{6}{n} \bar{x}^2 \Sigma x_i^2 - 3\bar{x}^4$$

Skewness:

$$\gamma_1 = \frac{m_3}{m_2^{3/2}}$$

Kurtosis (excess):

$$\gamma_2 = \frac{m_4}{{m_2}^2} - 3$$

Example 1: Ungrouped data

i	1	2	3	4	5	6	7	8	9
x_i	2.1	3.5	4.2	6.5	4.1	3.6	5.3	3.7	4.9

Answers:

$\gamma_1 = 0.24,\ \gamma_2 = -0.16$

$(m_2 = 1.39, m_3 = 0.39, m_4 = 5.49)$

Example 2: Grouped data

x_i	3	2	4	6	1
f_i	4	5	3	2	1

Answers:

$\gamma_1 = 0.77,\ \gamma_2 = -0.19$

$(m_2 = 1.98, m_3 = 2.14, m_4 = 11.05)$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR	STO	4			For grouped data, go to 7
2	x_i	↑	↑	4	■	y^x		Perform 2–5 for i=1,2,...,n
3		STO	+	4	x⇄y	↑		
4		↑	3	■	y^x	x⇄y		
5		Σ+						
6								Go to 16
7	x_i	↑	↑	4	■	y^x		Perform 7–15 for i=1,2,...,n
8	f_i	STO	1	x	STO	+		f_i is the frequency of x_i
9		4	x⇄y	↑	↑	3		
10		■	y^x	RCL	1	x		
11		STO	+	8	x⇄y	↑		
12		x^2	RCL	1	x	STO		
13		+	6	x⇄y	RCL	1		
14		x	STO	+	7	RCL		
15		1	STO	+	5			

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
16		■	$\bar{x},s$	STO	1	RCL		
17		6	RCL	5	÷	x⇄y		
18		x^2	−				m_2	
19		■	$\sqrt{x}$	STO	2	RCL		
20		8	RCL	1	RCL	6		
21		×	3	×	−	RCL		
22		5	÷	2	RCL	1		
23		3	■	y^x	×	+	m_3	
24		RCL	2	3	■	y^x		
25		÷					γ_1	
26		RCL	4	RCL	1	RCL		
27		8	×	4	×	−		
28		RCL	1	x^2	6	×		
29		RCL	6	×	+	RCL		
30		5	÷	RCL	1	x^2		
31		x^2	3	×	−		m_4	
32		RCL	2	4	■	y^x		
33		÷	3	−			γ_2	

SOD Depreciation

See page 92

Speedometer/Odometer Adjustments

Objective:

Assume that you are an automobile passenger approaching a speedometer test section with an HP-45 in your hand. You want to calculate

TS – true speed of the car

RS – what your speedometer will register at a posted speed

DS – distance traveled after a trip

RO – the reading of the odometer after a specific distance d

Notation:

a_1 = mileage read at beginning of trip

a_2 = mileage read at end of trip

b_1 = mileage read at beginning of test section (to nearest 20^{th} of a mile, if possible)

b_2 = mileage read at end of test section

L = length of test section

s = speedometer value

p = posted speed

q = present milage

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b_1	CHS	↑					
2	b_2	+						
3	L	÷	STO	1				
4								For TS, go to 5
								For RS, go to 6
								For DS, go to 7
								For RD, go to 9
5	s	RCL	1	÷			TS	Go to 4
6	p	RCL	1	x			RS	Go to 4
7	a_2	↑						
8	a_1	−	RCL	1	÷		DS	Go to 4
9	d	RCL	1	x				
10	q	+					RD	Go to 4

Example:

Assume the following:

a_1 = 2185.2	Mileage at start of trip
b_1 = 2219.4	Mileage at start of test section
b_2 = 2224.15	Mileage at end of test section
L = 5 miles	Length of test section

1. How fast is the car actually traveling when the speedometer registers (a) 50 mph? (b) 70 mph? (c) 30 mph?

2. What will the speedometer register if the true speed is (a) 65 mph? (b) 55 mph?

3. When the odometer reads 2236.3, how many miles have you traveled since the start of your trip.

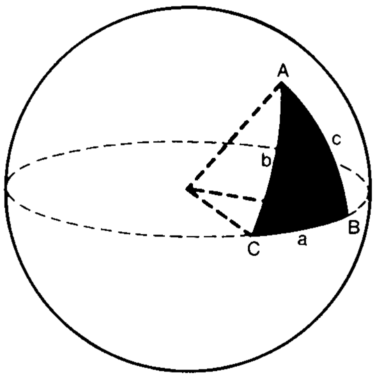
4. You see a sign saying "DOEVILLE 48". If your odometer now reads 2241.5, what will it register when you arrive at DOEVILLE?

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		2	2	1	9	.		
2		4	CHS	↑	2	2		
3		2	4	.	1	5		
4		+	5	÷	STO	1		
5		5	0	RCL	1	÷	52.63	Answer for 1(a)
6		7	0	RCL	1	÷	73.68	Answer for 1(b)
7		3	0	RCL	1	÷	31.58	Answer for 1(c)
8		6	5	RCL	1	x	61.75	Answer for 2(a)
9		5	5	RCL	1	x	52.25	Answer for 2(b)
10		2	2	3	6	.		
11		3	↑	2	1	8		
12		5	.	2	−	RCL		
13		1	÷				53.79	Answer for 3
14		4	8	RCL	1	x		
15		2	2	4	1	.		
16		5	+				2287.10	Answer for 4

Spherical Triangle

Formulas:

Suppose A, B, C are the three angles of a spherical triangle and a, b, c are the opposite sides.



If A, b, c are given, then

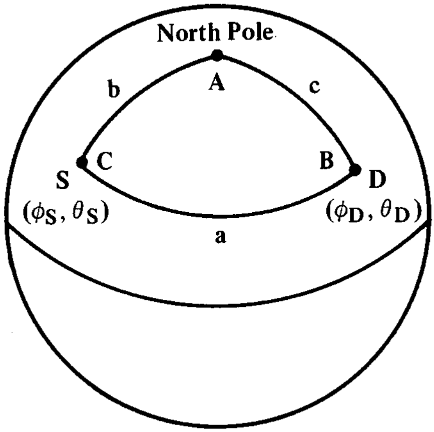
$$a = \cos^{-1}\ [\cos b \cos c + \sin b \sin c \cos A]$$

$$C = \cos^{-1}\left[\frac{\cos c - \cos a \cos b}{\sin a \sin b}\right]$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	STO	1	COS				
2	c	STO	2	COS	x	RCL		
3		1	SIN	RCL	2	SIN		
4		x						
5	A	COS	x	+	■	COS^{-1}	a	
6		STO	3	RCL	2	COS		
7		RCL	3	COS	RCL	1		
8		COS	x	–	RCL	3		
9		SIN	RCL	1	SIN	x		
10		÷	■	COS^{-1}			C	

Navigational course

The above routine can be used to find the great circle route a and the true course C if the coordinates of the source (ϕ_s, θ_s) and destination (ϕ_D, θ_D) are known.



S = Source D = Destination

Notes:

1. $A = \theta_s - \theta_D$, $b = 90° - \phi_s$, $c = 90° - \phi_D$
2. Northern hemisphere latitudes and Western hemisphere longitudes are indicated as positive numbers, Southern and Eastern coordinates are indicated as negative numbers.
3. 1° (spherical coordinate) = 60 nautical miles
4. True course = 360° – C if sin A < 0 (i.e., going west).

Example:

Find the great circle distance and true course from San Francisco (37° 37′ N, 122° 23′ W) to Monterey (36° 35′ N, 121° 51′ W).

Answers:

Distance = 67.05 nautical miles

True course = 157.46°

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	DEG	9	0	↑		
2		3	7	.	3	7		
3		■	D.MS→	−	STO	1	52.38	← b
4		COS	9	0	↑	3		
5		6	.	3	5	■		
6		D.MS→	−	STO	2		53.42	← c
7		COS	x	RCL	1	SIN		
8		RCL	2	SIN	x	1		
9		2	2	.	2	3		
10		■	D.MS→	1	2	1		
11		.	5	1	■	D.MS→		
12		−					0.53	← A
13		COS	x	+	■	COS^{-1}	1.12	← a (in decimal degrees)
14		STO	3	6	0	x	67.05	← a (in nautical miles)
15		RCL	2	COS	RCL	3		
16		COS	RCL	1	COS	x		
17		−	RCL	3	SIN	RCL		
18		1	SIN	x	÷	■		
19		COS^{-1}					157.46	← C (in decimal degrees)

Stack Operations

See page 168

Stirling's Approximation

See page 109

Synthetic Division

Formula:

This program performs synthetic division on a polynomial of degree n (with real coefficients)

$$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

by $x - x_0$ so that

$$\frac{a_n x^n + \ldots + a_1 x + a_0}{x - x_0} = b_{n-1} x^{n-1} + b_{n-2} x^{n-2} + \ldots + b_1 x + b_0 + \frac{R}{x - x_0}$$

Example:

Divide $x^5 - 4x^4 + 7x^3 - 10x^2 + 8$ by $x - 2$.

Answer:

$$(x^4 - 2x^3 + 3x^2 - 4x - 8) + \frac{(-8)}{x - 2}$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x_0	↑	↑	↑				
2	a_n						b_{n-1}	
3		×						Perform 3–4 for i=n−1,
								n−2, ..., 1
4	a_i	+					b_{i-1}	
5		×						
6	a_0	+					R	

Triangles (Oblique)

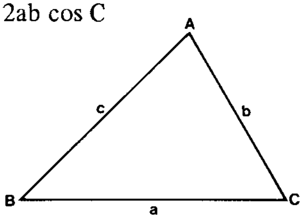
The basic formulas used to solve a triangle are:

1. law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

2. law of cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Note: Triangle solution routines work in any angular mode. When the calculator is in DEG mode, all angles are in decimal degrees.

Given a, b, C; find A, B, c

Formulas:

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

$$A = \tan^{-1}\left(\frac{a \sin C}{b - a \cos C}\right)$$

$$B = \cos^{-1}\,[-\cos(A + C)]$$

Example:

Given $b = 224$
$C = 28°40'$
$a = 132$

Find c, A, B

(*Note:* C must be converted to decimal degrees before calculation.)

Answer:

$c = 125.35$
$A = 30.34°$
$B = 120.99°$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	STO	1					
2	c	STO	2					
3	a	■	→R	RCL	1	x⇄y		
4		−	→P				c	
5		x⇄y					A	
6		RCL	2	+	COS	CHS		
7		■	$\cos^{-1}$				B	

Given a, b, c; find A, B, C

Formulas:

$$A = 2\cos^{-1}\left(\sqrt{\frac{S(S-a)}{bc}}\right)$$

$$\text{where } S = (a + b + c)/2$$

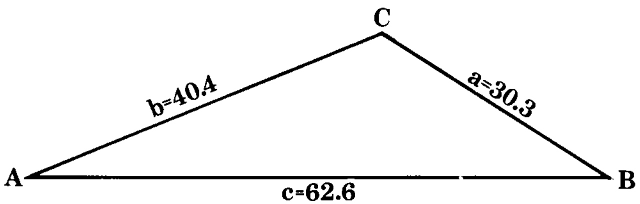
$$B = \tan^{-1}\left(\frac{b \sin A}{c - b \cos A}\right)$$

$$C = \cos^{-1}\,[-\cos\,(A + B)]$$

Example:

Given $a = 30.3$
$b = 40.4$
$c = 62.6$

Find A, B, C.



Answer:

$A = 23.66^\circ = 0.41$ radians $= 26.29$ grads
$B = 32.35^\circ = 0.56$ radians $= 35.95$ grads
$C = 123.99^\circ = 2.16$ radians $= 137.76$ grads

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1					
2	b	STO	2					
3	c	STO	3	+	+	2		
4		÷	↑	↑	RCL	1		
5		−	x	RCL	2	÷		
6		RCL	3	÷	■	$\sqrt{x}$		
7		■	$\cos^{-1}$	2	x	STO		
8		1					A	
9		RCL	2	■	→R	RCL		
10		3	x⇄y	−	→P	x⇄y	B	
11		RCL	1	+	COS	CHS		
12		■	$\cos^{-1}$				C	

Given a, A, C; find B, b, c

Formulas:

$$b = \frac{a \sin (A + C)}{\sin A}$$

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

$$B = \tan^{-1}\left(\frac{b \sin C}{a - b \cos C}\right)$$

Example:

Given $a = 17.5$
$C = 1.09$ radians
$A = 0.72$ radians

Find B, b, c.

Answer:

$b = 25.78$
$c = 23.53$
$B = 1.33$ radians

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1					
2	C	STO	2					
3	A	STO	3	+	SIN	RCL		
4		3	SIN	÷	RCL	1		
5		x					b	
6		RCL	2	x⇄y	■	→R		
7		RCL	1	x⇄y	–	→P	c	
8		x⇄y					B	

Given a, B, C; find A, b, c

Formulas:

$$c = \frac{a \sin C}{\sin (B + C)}$$

$$b = \sqrt{a^2 + c^2 - 2\, ac \cos B}$$

$$A = \cos^{-1}\ [-\cos (B + C)]$$

Example:

Given a = 25.2
B = 39.26 grads
C = 76.11 grads

Find A, b, c.

Answer:

c = 24.15
b = 15.01
A = 84.63 grads

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1					
2	B	STO	2					
3	C	STO	3	SIN	RCL	2		
4		RCL	3	+	SIN	÷		
5		RCL	1	x	STO	3	c	
6		RCL	2	RCL	3	■		
7		→R	RCL	1	x⇄y	–		
8		→P					b	
9		x⇄y	RCL	2	+	COS		
10		CHS	■	COS^{-1}			A	

Given B, b, c; find a, A, C

Formulas:

$$a = \frac{c \sin (B + C_1)}{\sin C_1}$$

where

$$C_1 = \begin{cases} \sin^{-1}\left(\dfrac{c \sin B}{b}\right) & \text{or} \\ \sin^{-1}\left(-\dfrac{c \sin B}{b}\right) \end{cases}$$

$$A = \tan^{-1}\left(\frac{a \sin B}{c - a \sin B}\right)$$

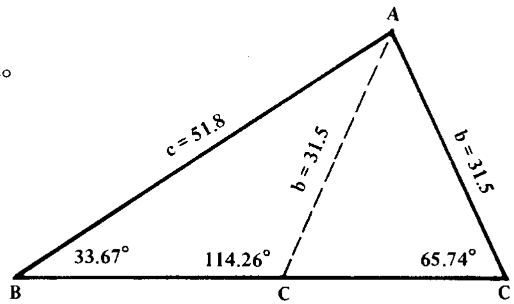
$$C = \cos^{-1}\ [-\cos (A + B)]$$

Note: If B is acute and $b < c$, two solutions exist.

Example:

Given $b = 31.5$
$c = 51.8$
$B = 33.67°$

Find a, A, C.



Answer:

$a = 56.05$
$A = 80.59°$
$C = 65.74°$

Alternate answer:

$a = 30.17$
$A = 32.07°$
$C = 114.26°$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	STO	1					
2	c	STO	2					
3	B	STO	3	SIN	x	RCL		
4		1	÷	■	SIN^{-1}	STO		
5		4					C_1	
6		SIN	RCL	3	RCL	4		
7		+	SIN	x⇄y	÷	RCL		
8		2	x				a	
9		RCL	3	x⇄y	■	→R		
10		RCL	2	x⇄y	–	→P		
11		x⇄y					A	
12		RCL	3	+	COS	CHS		
13		■	COS^{-1}				C	If b ≥ c, stop
14		RCL	4	CHS	STO	4		Go to 6 for alternate solution

Given a, b, c; find area

Formula:

$$\text{area} = \sqrt{S(S-a)(S-b)(S-c)}$$

where

$$S = \frac{1}{2}(a+b+c).$$

Example:

a = 5.317361553

b = 7.089815404

c = 8.862269255

Answer:

area = 18.85

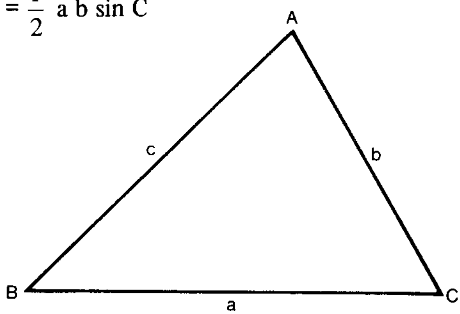
(S = 10.63).

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO	1					
2	b	STO	2	+				
3	c	STO	3	+	2	÷	S	
4		↑	↑	↑	RCL	1		
5		−	x	x⇄y	RCL	2		
6		−	x	x⇄y	RCL	3		
7		−	x		$\sqrt{x}$		area	

Given a, b, C; find area

Formula:

$$\text{area} = \frac{1}{2}\ a\ b\ \sin C$$



Example:

If a = 5.3174,

b = 7.0898

$C = \frac{\pi}{4}$

Answer:

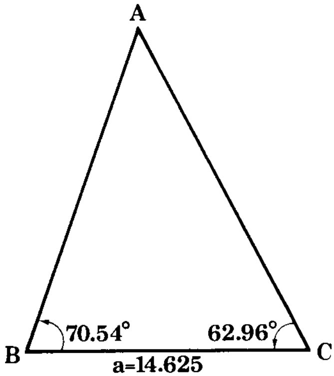
area = 13.33

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						Set machine to any desired
2	b	x	2	÷				mode (DEG, RAD, or GRD).
3	C	SIN	x					

Given a, B, C; find area

Formula:

$$\text{area} = \frac{a^2}{2} \frac{\sin B \sin C}{\sin (B + C)}$$



Example:

If $B = 70° \; 32' \; 12''$
$C = 62° \; 57' \; 28''$
$a = 14.625$

Answer:

area = 123.80

Note:

In this example, convert angles to decimal degrees before using trigonometric function keys.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	x^2						Set machine to any desired
2	B	STO	1	SIN	x			mode (DEG, RAD, or GRD).
3	C	STO	+	1	SIN	x		
4		RCL	1	SIN	2	x		
5		÷						

Given vertices; find area

Formula:

Given the three vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) of a triangle

$$\text{area} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \left[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)\right]$$

Example:

Compute the area of the triangle with vertices (0,0), (4,0), (4,3).

Answer:

6.00

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	y_2	STO	1					
2	y_3	STO	2	−				
3	x_1	x	RCL	2				
4	y_1	STO	3	−				
5	x_2	x	+	RCL	3	RCL		
6		1	−					
7	x_3	x	+	2	÷			

Trigonometric Functions

Let p = principal value

q = secondary value.

We set the calculator to DEG, RAD, or GRD mode, as desired.

Secondary value of arc sin x

Example:

x = −0.77, find secondary value of arc sin x.

Answer:

q = 230.35° = 4.02 radians = 255.95 grads.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x							If in RAD mode, go to 4
								If in GRD mode, go to 6
2		■	SIN^{-1}				p	
3		1	8	0	x⇄y	–	q	Stop
4		■	SIN^{-1}				p	
5		■	π	x⇄y	–		q	Stop
6		■	SIN^{-1}				p	
7		2	0	0	x⇄y	–	q	

Secondary value of arc cos x

Example:

x = 0.76, find secondary value of arc cos x.

Answer:

q = 319.46° = 5.58 radians = 354.96 grads.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x							If in RAD mode, go to 4
								If in GRD mode, go to 6
2		■	COS^{-1}				p	
3		3	6	0	x⇄y	–	q	Stop
4		■	COS^{-1}				p	
5		■	π	2	x	÷	q	Stop
6		■	COS^{-1}				p	
7		4	0	0	x⇄y	–	q	

Secondary value of arc tan x

Example:

x = 2, find secondary value of arc tan x.

Answer:

q = 243.43° = 4.25 radians = 270.48 grads.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x							If in RAD mode, go to 4
								If in GRD mode, go to 6
2		■	TAN⁻¹				p	
3		1	8	0	+		q	Stop
4		■	TAN⁻¹				p	
5		■	π	+			q	Stop
6		■	TAN⁻¹				p	
7		2	0	0	+		q	

Cotangent

Formula:

$$\cot x = \frac{1}{\tan x}$$

Example:

x = 37

Answer:

cot x = 1.33 (in DEG mode) or
−1.19 (in RAD mode) or
1.52 (in GRD mode).

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	TAN	1/x					

Cosecant

Formula:

$$\csc x = \frac{1}{\sin x}$$

Example:

x = 30

Answer:

csc x = 2.00 (in DEG mode) or
–1.01 (in RAD mode) or
2.20 (in GRD mode).

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	SIN $^1/_x$		

Secant

Formula:

$$\sec x = \frac{1}{\cos x}$$

Example:

x = 45

Answer:

sec x = 1.41 (in DEG mode) or
1.90 (in RAD mode) or
1.32 (in GRD mode).

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	COS $^1/_x$		

Versine

Formula:

$$\text{vers } x = 1 - \cos x$$

Example:

x = 38

Answer:

vers x = 0.21 (in DEG mode) or
0.04 (in RAD mode) or
0.17 (in GRD mode).

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	COS	1	x⇄y	−			

Coversine

Formula:

$$\text{covers } x = 1 - \sin x$$

Example:

x = 38

Answer:

covers x = 0.38 (in DEG mode) or
0.70 (in RAD mode) or
0.44 (in GRD mode).

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	SIN	1	x⇄y	−			

Haversine

Formula:

$$\text{hav x} = \frac{1 - \cos x}{2}$$

Example:

x = 42.3

Answer:

hav x = 0.13 (in DEG mode) or
0.56 (in RAD mode) or
0.11 (in GRD mode).

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	COS	1	x⇄y	–	2		
2		÷						

Arc cotangent

Formula:

$$\cot^{-1} x = \tan^{-1} \frac{1}{x}$$

Example:

x = 0.35

Answer:

$\cot^{-1}$ x = 70.71° or 1.23 radians or 78.57 grads.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	$1/x$		TAN^{-1}				

Arc cosecant

Formula:

$$\csc^{-1} x = \sin^{-1} \frac{1}{x}$$

Example:

x = 3.45

Answer:

$\csc^{-1} x = 16.85°$ or 0.29 radians or 18.72 grads.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	$\boxed{1/x}$ $\blacksquare$ $\boxed{\mathrm{SIN}^{-1}}$ $\boxed{}$ $\boxed{}$		

Arc secant

Formula:

$$\sec^{-1} x = \cos^{-1} \frac{1}{x}$$

Example:

x = 1.1547

Answer:

$\sec^{-1} x = 30°$ or 0.52 radians or 33.33 grads.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	$\boxed{1/x}$ $\blacksquare$ $\boxed{\mathrm{COS}^{-1}}$ $\boxed{}$ $\boxed{}$		

T Statistics

t for paired observations

Formulas:

Given a set of paired observations:

x_i	x_1	x_2	x_3	...	x_n
y_i	y_1	y_2	y_3	...	y_n

let $$D_i = x_i - y_i$$

$$\bar{D} = \frac{1}{n}\sum_{i=1}^{n} D_i$$

$$S_D = \sqrt{\frac{\Sigma D_i^2 - \frac{(\Sigma D_i)^2}{n}}{n-1}}$$

$$S_{\bar{D}} = \frac{S_D}{\sqrt{n}}$$

test statistic

$$t = \frac{\bar{D}}{S_{\bar{D}}}$$

Example:

Compute t for the following:

x_i	14	17.5	17	17.5	15.4
y_i	17	20.7	21.6	20.9	17.2

Answer:

t = –7.16

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR					
2	x_i	↑						Perform 2–3 for i=1,2,...,n
3	y_i	–	Σ+					
4		■	$\bar{x}$,s	x⇄y	÷	RCL		
5		5	■	$\sqrt{x}$	×			

t for population mean

Formula:

Suppose $\{x_1 \ldots x_n\}$ is a random sample from a normal population (mean μ and variance are unknown).

$$\bar{x} = \frac{1}{n} \Sigma x_i$$

$$S_x = \left(\frac{\Sigma x_i^2 - n\bar{x}^2}{n - 1} \right)^{1/2}$$

$$t = \frac{\bar{x} - \mu_0}{S_x}$$

Degrees of freedom = n - 1

We can use this t statistic to test the null hypothesis $H_0: \mu = \mu_0$.

Example:

Compute t from the sample:

$$\{2.1, 0.5, -3.1, 1.4, -0.92, -1.35, 1.2\}$$

for testing H_0: $\mu = 0.2$.

Answer:

$t = -0.12$
(6 degrees of freedom)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR					
2	x_i	Σ+						Perform 2 for i=1,2,...,n
3		■	$\bar{x}$,s					
4	μ_0	−	x⇄y	÷				

t for correlation coefficient

Formula:

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

where n = sample size

r = estimate of the correlation coefficient.

This t statistic has n – 2 degrees of freedom and can be used to test the null hypothesis that the correlation coefficient ρ is zero.

$$H_0:\ \rho = 0$$

Example:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

n = 7

r = –0.96 (See *Covariance and Correlation Coefficient*)

Answer:

t = –7.67
(Degrees of freedom = 5)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	r	STO	1					
2	n	↑	2	–	■	$\sqrt{x}$		
3		×	1	RCL	1	x^2		
4		–	■	$\sqrt{x}$	÷		t	

t for two sample means

Formula:

Suppose $x_1, x_2, \ldots, x_{n_1}$ and $y_1, y_2, \ldots, y_{n_2}$ are samples from normal populations with means μ_1 and μ_2, both with variance σ^2 (unknown).

$$t = \frac{\bar{x} - \bar{y} - d}{\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}\ \sqrt{\dfrac{\Sigma x_i^2 - n_1\bar{x}^2 + \Sigma y_i^2 - n_2\bar{y}^2}{n_1 + n_2 - 2}}}$$

where $\bar{x} = \dfrac{1}{n_1}\Sigma x_i \qquad \bar{y} = \dfrac{1}{n_2}\Sigma y_i$

We can use this t to test the null hypothesis

$$H_0 : \mu_1 - \mu_2 = d$$

t has $n_1 + n_2 - 2$ degrees of freedom.

Example:

x: 79, 84, 108, 114, 120, 103, 122, 120 ($n_1 = 8$)

y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54 ($n_2 = 10$)

Answer:

If d = 0, then t = 1.73
degrees of freedom = 16

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR					
2	x_i	Σ+						Perform 2 for i=1,2,...,n_1
3		■	$\bar{x}$,s	STO	1	x⇄y		
4		STO	2	RCL	5	STO		
5		3	■	CLEAR				
6	y_i	Σ+						Perform 6 for i=1,2,...,n_2
7		■	$\bar{x}$,s	CHS	RCL	1		
8		+						
9	d	−	STO	1	x⇄y	x^2		
10		RCL	5	1	−	x		
11		RCL	2	x^2	RCL	3		
12		1	−	x	+	RCL		
13		3	RCL	5	+	2		
14		−	÷	RCL	3	1/x		
15		RCL	5	1/x	+	x		
16		■	$\sqrt{x}$	RCL	1	x⇄y		
17		÷					t	

Variance, Analysis of

See page 15

Vector Operations

Vector addition

Suppose vector V_k (in 2-dimensional space) has magnitude m_k and direction θ_k ($k = 1, 2, ..., n$). Find the sum

$$V = \sum_{k=1}^{n} V_k = x\vec{i} + y\vec{j}$$

Example:

m_k	θ_k
2	30°
6.2	-45°
7.6	125°
10.7	232°

Answer:

$$V = -4.83\vec{i} - 5.59\vec{j}$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR					
2	θ_k	↑						Perform 2–3 for k=1,2,...,n
3	m_k	■	→R	Σ+				
4		RCL	Σ+				x	
5		x⇄y					y	

Vector angles

Suppose

$$\vec{x} = (x_1, x_2, x_3)$$

$$\vec{y} = (y_1, y_2, y_3)$$

then the angle between these two vectors is

$$\theta = \cos^{-1}\left[\frac{x_1 y_1 + x_2 y_2 + x_3 y_3}{\sqrt{x_1^2 + x_2^2 + x_3^2}\ \sqrt{y_1^2 + y_2^2 + y_3^2}}\right]$$

Example:

Find the angle between

$$\vec{x} = (5, -6.2, -7),$$

$$\vec{y} = (3.15, 2.22, -0.3)$$

Answer:

θ= 84.28 degrees = 1.47 radians = 93.64 grads.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		■	CLEAR					
2	x_i	↑	x^2	STO	+	5		Perform 2–5 for i=1,2,3
3		R↓						
4	y_i	↑	x^2	STO	+	6		
5		R↓	×	+				
6		RCL	5	■	$\sqrt{x}$	RCL		
7		6	■	$\sqrt{x}$	×	÷		
8		■	$\cos^{-1}$					

Vector cross product

Formula:

If $\vec{x} = (x_1, x_2, x_3)$ and $\vec{y} = (y_1, y_2, y_3)$ are two vectors, then cross product $\vec{z}$ is also a vector.

$$\vec{z} = \vec{x} \times \vec{y}$$
$$= (x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1)$$
$$= (z_1, z_2, z_3)$$

Example:

If $\vec{x} = (2.34, 5.17, 7.43)$
$\vec{y} = (.072, .231, .409)$

Find $\vec{x} \times \vec{y}$

Answer:

$\vec{x} \times \vec{y} = (0.40, -0.42, 0.17)$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x_2	STO	1					
2	y_3	STO	2	×				
3	x_3	STO	3					
4	y_2	STO	4	×	−		z_1	
5	y_1	STO	5	RCL	×	3		
6	x_1	STO	6	RCL	×	2		
7		−					z_2	
8		RCL	6	RCL	4	×		
9		RCL	1	RCL	5	×		
10		−					z_3	

Vector dot product

Formulas:

Given two vectors $\vec{x}$, $\vec{y}$ in an n-dimensional vector space

$$\vec{x} = (x_1, x_2, ..., x_n)$$
$$\vec{y} = (y_1, y_2, ..., y_n)$$

the dot product is

$$\vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2 + ... + x_n y_n$$

Example:

If $\vec{x} = (2.34, 5.17, 7.43, 9.11, 11.41)$
$\vec{y} = (.072, .231, .409, .703, .891)$

then $\vec{x} \cdot \vec{y} = 20.97$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x_1	↑						
2	y_1	x						
3	x_i	↑						Perform 3–4 for i=2,3,...,n
4	y_i	x	+					

Versine

See page 202

Weekday

Day of the week for any date since September 14, 1752

d = day of month

m = month, with January and February being the 13^{th} and 14^{th} months of the previous year.

y = year (4 digits)

$$\text{Weekday} = d + n_1 + n_2 - n_3 + n_4 \pmod 7$$

where

$$n_1 = \text{Int}\left(\frac{13}{5}(m+1)\right)$$

$$n_2 = \text{Int}\left(\frac{5}{4}y\right)$$

$$n_3 = \text{Int}\left(\frac{y}{100}\right)$$

$$n_4 = \text{Int}\left(\frac{y}{400}\right)$$

Int is "integer part of".

Output is read as follows:

0 – Saturday
1 – Sunday
2 – Monday
3 – Tuesday
4 – Wednesday
5 – Thursday
6 – Friday

Example:

On what day was February 29, 1972?

Answer:

Tuesday (d = 29, m = 14, y = 1971)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	d	↑						
2	m	↑						
3	y	STO	1	R↓	1	+		
4		1	3	x	5	÷	E_1	Let e_1 = integer part of E_1
5		CLX						
6	e_1	+	RCL	1	x⇄y	STO		
7		1	x⇄y	↑	↑	↑		
8		5	x	4	÷		E_2	Let e_2 = integer part of E_2
9		CLX						
10	e_2	RCL	1	+				For 20th century date,
								go to 18
11		STO	1	R↓	EEX	2		
12		÷					E_3	Let e_3 = integer part of E_3
13		CLX						
14	e_3	CHS	STO	+	1	R↓		
15		4	0	0	÷		E_4	Let e_4 = integer part of E_4
16		CLX						
17	e_4	RCL	1	+				Go to 19
18		6	+					
19		↑	↑	7	÷		E_5	Let e_5 = integer part of E_5
20		CLX						
21	e_5	↑	7	x	−			

Appendix

Questions you may have wanted answered about your HP-45.

1. **Question:** What are the maximum and minimum light displays on the HP-45?

   Answer: Maximum: -8.888888888-88 after all decimal points light up on low battery power.

   Minimum: •

2. **Question:** A googol is 10^{100}. We can key this in as 10×10^{99} on the HP-45, right? Its reciprocal can be keyed in as $.1 \times 10^{-99}$, right?

   Answer: Wrong. 10 $\boxed{\text{EEX}}$ 99 → 9.999999999 99
   .1 $\boxed{\text{EEX}}$ $\boxed{\text{CHS}}$ 99 → 0.00

3. **Question:** The display is blanked out during computation. Is the keyboard also locked out?

   Answer: Yes

4. **Question:** What calculation takes the longest response on the HP-45?

   Answer: 99! It's about 2 seconds.

   $\boxed{\text{SIN}}$ or $\boxed{\text{COS}}$ of 9.999999999 99. It's about 2 seconds.

5. **Question:** Do 1 $\boxed{\uparrow}$ 2 $\boxed{\uparrow}$ 3 $\boxed{\uparrow}$ $\boxed{\text{CLX}}$. Now if we press $\boxed{\div}$ or $\boxed{x{\rightleftarrows}y}$ $\blacksquare$ $\boxed{y^x}$ $\left(\text{that is } \frac{3}{0} \text{ or } 0^3\right)$ which will result in flashing zeros, will the stack be dropped?

   Answer: Yes for $\frac{3}{0}$: $\boxed{\text{R}\downarrow}$ → 2.00 $\boxed{\text{R}\downarrow}$ → 1.00 $\boxed{\text{R}\downarrow}$ → 1.00

   No for 0^3: $\boxed{\text{R}\downarrow}$ → 3.00 $\boxed{\text{R}\downarrow}$ → 2.00 $\boxed{\text{R}\downarrow}$ → 1.00

6. **Question:** Suppose we pressed the gold key by mistake. Which key will cancel it, causing no other operations?

Answer: Any one of [1], [2], [3], [4], [5], [6].

7. **Question:** Do: 6 [EEX] 16. How can we make the whole number negative?

Answer: [x⇄y] [x⇄y] [CHS]

8. **Question:** How can three successive [TAN] presses be used to calculate the tangent of 30° (accurate to about 7 decimal places)?

Answer: 30 [■] [DEG] [■] [TAN^{-1}] [TAN] [TAN] → 0.58

9. **Question:** When may it be desirable to press [CLX] if the display is already zero?

Answer: When we get zero as the result of an operation, but don't want to raise it in the stack upon entering data.

10. **Question:** When may it be desirable to enter 0 if x is already zero?

Answer: If an algorithm requires the integer part of a number (already in the X register) such as 0.41, we would do: [CLX] 0, which would insure that this zero would be raised in the stack, if the algorithm requires it.

11. **Question:** Enter any three digits in the form d.dd x 10^{-7}:
d.dd [EEX] [CHS] 7.

How do you add 1 to this number with one keystroke? Your number now has the form 1.000000ddd, how do you subtract 1 from it with one keystroke?

Answer: [e^x] and [ln] (All this means is that ln (1 + A) approaches A as A approaches 0.)

12. Question: What numbers will the HP-45 not give reciprocals to?

Answer: 0 and any number with an exponent of +99 having a value other than ±1.

13. Question: How many decimal points light up on low battery power?

Answer: Only 14; the point following the true decimal point does not come on!

14. Question: To divide x by 2: [↑] 2 [÷]. What other keystrokes will do the same?

Answer: [e^x] [■] [√x] [ln] for $|x| \leqslant 227$.

15. Question: 9 [↑] 8 [÷] [CHS] [EEX] 97 [×] [1/x] [■] [SCI] 9

→ −8.888888889-98

Change this number to −8.888888888−98 in a shorter number of keystrokes than keying in the new value. (Note: we must add 1 x 10^{-107} to the number in the display. If we try to add .000000001 x 10^{-98}, we also get into trouble.)

Answer: [EEX] 98 [↑] [EEX] 89 [−] [1/x] [+] [EEX] [CHS] 98 [−]

16. Question: How would you display the number e(2.718281828):

(a) in the shortest number of keystrokes? (2 keystrokes)

(b) as the limit of $\left(1 + \frac{1}{n}\right)^n$ as n gets large? (9 keystrokes)

(c) as the function of a digit? (6 keystrokes)

(d) using the [π] key, but not [e^x] or [y^x]? (14 keystrokes)

(e) as a continued fraction? (21 keystrokes)

(Press [FIX] [9] to see the whole number.)

16. Cont'd

Answer: (a) 1 [e^x]

(b) [EEX] 6 [↑] [1/x] 1 [+] [x⇄y] [■] [y^x]

(c) 5 [e^x] 5 [1/x] [■] [y^x]

(d) [■] [π] [↑] [ln] [÷] [↑] [ln] [÷] [↑] [ln] [÷]
[↑] [ln] [÷]

(e) 18 [1/x] 14 [+] [1/x] 10 [+] [1/x] 6 [+] [1/x]
1 [+] [1/x] 2 [×] 1 [+]

17 Question: Can you display all ten L.E. D. digits in an orderly sequence?

Answer: 80 [↑] 81 [÷] [FIX] 9 → 0.987654321

18. Question: Determine the Golden Ratio ϕ to 11 significant digits, given $\phi = \left(\sqrt{5}+1\right)\Big/2$ and $\phi - \frac{1}{\phi} = 1$.

Answer: 5 [■] [$\sqrt{x}$] 1 [+] 2 [÷] [■] [SCI] 9 → 1.618033989
[↑] [1/x] → 6.180339887–01
[−] [FIX] 2 → 1.00

$\phi = 1.6180339887$

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