Hewlett-Packard

HP-91

Owner's Handbook

with applications section

"The success and prosperity of our company will be assured only if we offer our customers superior products that fill real needs and provide lasting value, and that are supported by a wide variety of useful services, both before and after sale."

Statement of Corporate Objectives.
Hewlett-Packard

When Messrs. Hewlett and Packard founded our company in 1939, we offered one superior product, an audio oscillator. Today, we offer more than 3,000 quality products, designed and built for some of the world's most discerning customers.

Since we introduced our first scientific calculator in 1967, we've sold over a million worldwide, both pocket and desktop models. Their owners include Nobel laureates, astronauts, mountain climbers, businessmen, doctors, students, and housewives.

Each of our calculators is precision crafted and designed to solve the problems its owner can expect to encounter throughout a working lifetime.

HP calculators fill real needs. And they provide lasting value.

HEWLETT PACKARD

HP-91 Scientific Portable Printing Calculator

Owner's Handbook

February 1976

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Printed in U.S.A.

How to Use This Handbook

New user? If you're a new calculator user, or even new to the many advantages offered by Hewlett-Packard calculators, you'll appreciate the step-by-step explanations in this handbook. After you have learned how to use the HP-91 by reading sections 1 and 2, *Getting Started* and *Printer and Display Control*, you will probably want to look at section 3, *The Automatic Memory Stack*, to see how the HP-91 is able to work through difficult problems quickly, easily, and accurately. In section 4 you will learn, via text and examples, how to use each of the many *Function Keys*.

Experienced on other Hewlett-Packard calculators? If you have used other portable HP calculators, you will find that many features of the HP-91 are old friends—the automatic memory stack, the storage registers, and most of the mathematical functions. But you'll find some new highlights on the HP-91 too. Eventually you will want to look over this entire handbook, but to maximize the usability and power of the calculator immediately, you will especially want to refer to the pages dealing with the many features of the special thermal printer. And be sure to read about the powerful statistical capabilities of the HP-91.

Novice or expert, you will find that the *Function and Key Index* on pages 7–9 packs a lot of information about the HP-91 into two pages. Use the index as a quick reference guide, as a handy page index to the operation of any key, or even to show your friends the many features available on your HP-91 portable calculator.

Nor should you overlook the *HP-91 Applications Routines* in section 5. Here are step-by-step solutions to important problems from the areas of mathematics, statistics, finance, surveying, and navigation. Whether knowledgeable or a neophyte in these fields, you will find it a simple matter to solve common problems by following the keystroke lists—you don't have to remember formulas or evaluate expressions. And you can pick up some hints to help use your HP-91.

Whether your interest lies in learning to use your calculator completely, or merely in solving a particular type of problem, we hope that this handbook will help you get the most from your HP-91.

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The HP-91
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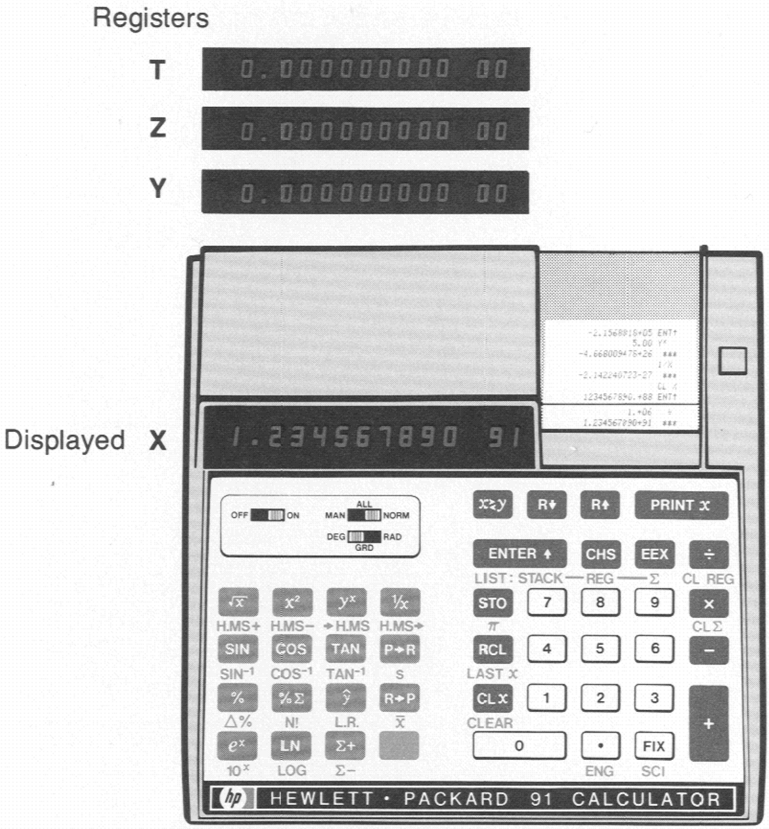
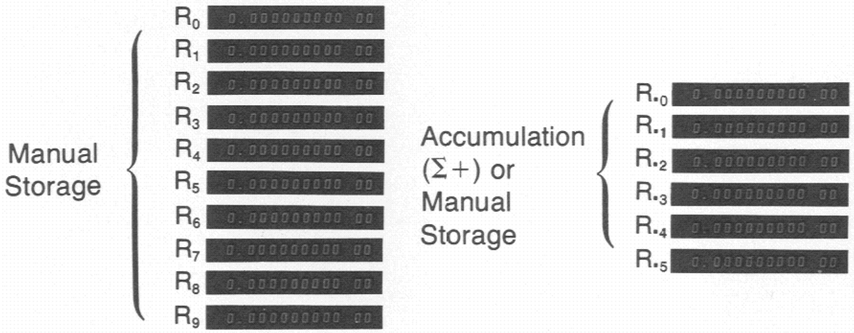
HP-91 Memory

Automatic Memory Stack



Storage Registers



Function and Key Index

☐ Paper advance pushbutton. Press to advance paper without printing **(page 13)**.

OFF ▥ ON Power switch **(page 11)**.

DEG ▥ RAD GRD Trigonometric Mode switch. Selects degrees, grads, or radians for trigonometric functions **(page 63)**.

ALL MAN ▥ NORM Print Mode switch. Controls extent of printing of keyboard operations **(page 12)**.

■ Prefix key. Press before function key to select function printed above that key **(page 11)**.

Mathematics

$\sqrt{x}$ Computes square root of number in displayed X-register **(page 55)**.

x^2 Computes square of number in displayed X-register **(page 56)**.

$1/x$ Computes reciprocal of number in displayed X-register **(page 54)**.

π Places value of pi (3.141592654) into displayed X-register **(page 56)**.

+ − × ÷ Arithmetic operators **(page 16)**.

Digit Entry

ENTER↑ Enters a copy of number in displayed X-register into Y-register. Used to separate numbers **(page 41)**.

CHS Changes sign of number or exponent of 10 in displayed X-register **(page 12)**.

EEX Enter exponent. After pressing, next numbers keyed in are exponents of 10 **(page 32)**.

x⇄y Exchanges contents of X- and Y-registers of stack **(page 39)**.

R↓ Rolls down contents of stack for viewing in displayed X-register **(page 38)**.

R↑ Rolls up contents of stack for viewing in displayed X-register **(page 39)**.

CLx Clears contents of displayed X-register to zero. **(page 13)**.

CLEAR Clears contents of stack (X,Y,Z,T) and all storage registers (R_0 through R_9; $R_{.0}$ through $R_{.5}$) to zero **(page 40)**.

PRINTx Prints contents of displayed X-register **(page 19)**.

LIST: STACK Causes printer to list contents of stack **(page 37)**.

0 through 9 Digits used for keying in numbers and display formatting **(page 12)**.

Logarithmic and Exponential

y^x Raises number in Y-register to power of number in displayed X-register **(page 72)**.

10^x Common antilogarithm. Raises 10 to power of number in displayed X-register **(page 71)**.

e^x Natural antilogarithm. Raises e (2.718281828) to power of number in displayed X-register **(page 71)**.

LOG Computes common logarithm (base 10) of number in displayed X-register **(page 71)**.

LN Computes natural logarithm (base e, 2.718281828) of number in displayed X-register **(page 71)**.

Display Control

FIX Fixed point display. Followed by a number key, selects fixed point notation display **(page 26)**.

SCI Scientific display. Followed by a number key, selects scientific notation display **(page 27)**.

ENG Engineering display. Followed by a number key, selects engineering notation display **(page 28)**.

Manual Storage

STO Store. Followed by number key, or decimal point and number key, stores displayed number in storage register specified (R_0 through R_9; $R_{.0}$ through $R_{.5}$). Also used to perform storage register arithmetic **(page 58)**.

RCL Recall. Followed by number key or decimal point and number key, recalls value from storage register specified (R_0 through R_9; $R_{.0}$ through $R_{.5}$) into the displayed X-register **(page 59)**.

CL REG Clears contents of storage registers **(page 61)**.

LIST: REG Causes printer to list contents of all storage registers **(page 60)**.

LAST x Recalls number displayed before the previous operation back into the displayed X-register **(page 53)**.

Trigonometry

H.MS+ Adds *hours, minutes, seconds* or *degrees, minutes, seconds* in X-register to those in Y-register **(page 65)**.

H.MS− Subtracts *hours, minutes, seconds* or *degrees, minutes, seconds* in displayed X-register from those in Y-register **(page 65)**.

→H.MS Converts decimal hours or degrees to *hours, minutes, seconds* or *degrees, minutes, seconds* **(page 64)**.

H.MS→ Converts *hours, minutes, seconds* or *degrees, minutes, seconds* to decimal hours or degrees **(page 64)**.

SIN COS TAN Compute sine, cosine, or tangent of value in displayed X-register **(page 63)**.

SIN^{-1} COS^{-1} TAN^{-1} Compute arc sine, arc cosine, or arc tangent of number in displayed X-register **(page 63)**.

Statistics

Σ+ Accumulates numbers from X- and Y-registers into storage registers $R_{.0}$ through $R_{.5}$ **(page 74)**.

Σ− Subtracts x and y values from storage registers $R_{.0}$ through $R_{.5}$ for correcting Σ+ accumulations **(page 82)**.

CLΣ Clears storage registers used for accumulations ($R_{.0}$ through $R_{.5}$) to zero **(page 61)**.

LIST: Σ Causes printer to list contents of accumulation registers (storage registers $R_{.0}$ through $R_{.5}$) **(page 76)**.

N! Computes factorial of number in displayed X-register **(page 55)**.

$\bar{x}$ Computes mean (average) of x and of y values accumulated by Σ+ **(page 77)**.

S Computes sample standard deviations of x and y values accumulated by Σ+ **(page 79)**.

L.R. Linear regression. Computes y-intercept (A) and slope (B) for x and y data points accumulated using Σ+ **(page 83)**.

$\hat{y}$ Linear estimate. With set of x, y data points accumulated using Σ+, computes estimated y for new x **(page 85)**.

Polar/Rectangular Conversion

R→P Converts x, y rectangular coordinates placed in X- and Y-registers to polar magnitude r and angle θ **(page 67)**.

P→R Converts polar magnitude r and angle θ in X- and Y-registers to rectangular x and y coordinates **(page 68)**.

Percentage

% Computes x% of y **(page 57)**.

Δ% Computes percent of change from number in Y-register to number in displayed X-register **(page 58)**.

%Σ Computes percent that x is of the number (Σx) in storage register $R_{.1}$ **(page 76)**.


34740A DISPLAY
HEWLETT · PACKARD 34702A MULTIMETER
1707B OSCILLOSCOPE
HEWLETT · PACKARD
DUAL DC POWER SUPPLY
78075
HEWLETT · PACKARD 91 CALCULATOR
1200.00 1/X
470.00 1/X
2200.00 1/X
2.2+02 1/X
1/X

Section 1

Getting Started

Congratulations!

Your HP-91 is a professional-quality instrument from the Hewlett-Packard line of calculators, calculators whose durability and ease of operation have made them famous around the world. Besides the HP logic system that lets you slice with ease through the most difficult equations, the HP-91 includes:

- Dozens of scientific, mathematical, and statistical functions.
- 16 storage registers for unparalleled computing power.
- Whisper-quiet printer to give enhanced usability and archival permanence to your answers.
- Rechargeable batteries for completely portable operation.
- AC adapter/recharger for desktop use.

In addition, each HP calculator is backed up by continuing support in accessories, maintenance, and applications from the worldwide Hewlett-Packard network of sales and service facilities. You're in good company with HP!

Power On

Your HP-91 is shipped fully equipped, including a battery pack.

Although the calculator is completely portable, if you want to use your HP-91 on battery power alone, you should connect the ac adapter/recharger and charge the battery for 7–10 hours first. Whether you operate from battery power or from the ac adapter/recharger, *the battery pack must always be in the calculator*. The battery pack is never in danger of being overcharged.

To begin: **Slide the OFF-ON switch** OFF ▮▯ ON **to ON.**

Slide the Print Mode switch MAN ▯▮ NORM (ALL) **to MAN.**

Display

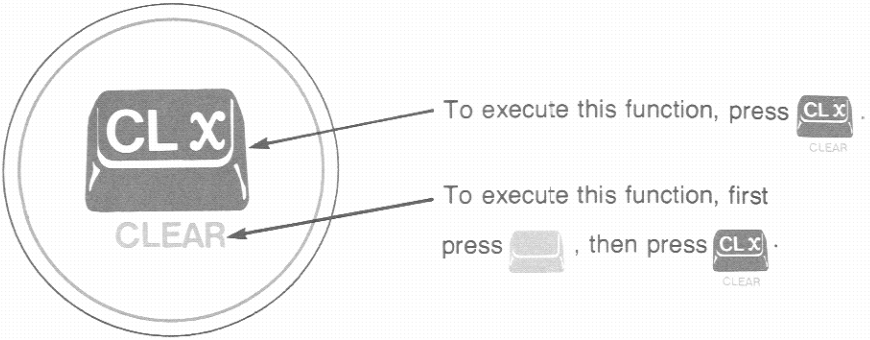
Numbers that you key into the calculator and intermediate and final answers are always seen in the bright red display. When you first turn the calculator ON, the display is set to 0.00 to show you that all zeros are present there.

Keyboard

Most keys on the keyboard perform two functions. One function is indicated by the symbol on the face of the key, while another function is indicated by the gold symbol written below the key.

To select the function printed on the face of the key, press the key.

To select the function printed in gold below the key, press the gold prefix key ▭ then press the function key.



To execute this function, press CL x CLEAR .

To execute this function, first press ▭ , then press CL x CLEAR .

In this handbook, the selected key function will appear in the appropriate color outlined by a box, like this: CL x , CLEAR.

Keying in Numbers

Key in numbers by pressing the number keys in sequence, just as though you were writing on a piece of paper. The decimal point must be keyed in if it is part of the number (unless it is to the right of the last digit).

For example, to key in 148.84:

Press	**Display**
1 4 8 • 8 4	148.84

The resultant number 148.84 is seen in the display.

Negative Numbers

To key in a negative number, press the keys for the number, then press CHS *(change sign)*. The number, preceded by a minus (–) sign, will appear in the display. For example, to change the sign of the number now in the display:

Press	**Display**
CHS	–148.84

You can change the sign of either a negative or a positive nonzero number in the display. For example, to change the sign of the –148.84 now in the display back to positive:

Press	**Display**
CHS	148.84

Notice that only negative numbers are given a sign in the display.

Clearing

You can clear any numbers that are in the display by pressing **CLx** *(clear x)*. This key erases the number in the display and replaces it with `0.00`.

Press	Display
CLx	`0.00`

If you make a mistake while keying in a number, clear the entire number string by pressing **CLx**. Then key in the correct number.

Printer

The printer has three modes of operation, which you control using the Print Mode switch MAN ALL NORM:

With the Print Mode switch MAN ALL NORM set to MAN *(manual)*, the printer is idle and does not print unless you press the **PRINTx** key or one of the LIST functions. This mode gives greatest economy of paper and battery power.

With the Print Mode switch MAN ALL NORM set to NORM *(normal)*, the calculator records a history of the calculation sequence so that you can reconstruct your problem. In this mode you see digit entries and functions, but intermediate and final answers are not printed unless you press the **PRINTx** key.

With the Print Mode switch MAN ALL NORM set to ALL, the calculator prints numbers, functions, and intermediate and final answers, just as they are seen in the display. The results of functions are printed with the symbol *** to the right of the number.

To advance the printer paper, press the paper advance pushbutton that is to the right of the paper output. Don't worry if the display blanks out while the paper advance is operating—this is normal. To advance the paper more than one space, simply hold the pushbutton down until the paper has advanced the desired amount. To replace the paper roll, refer to Using Your HP-91 Printer in appendix A of this handbook.

No matter what print mode you choose, you seldom have to worry about "overrunning" the printer when you are calculating. Your HP-91 contains a key buffer that "remembers" up to seven keystrokes—no matter how fast you press the keys.

Print Mode Switch

ALL
MAN ▨ NORM

Manual. Printer operates only when you press PRINT x or one of the LIST functions.

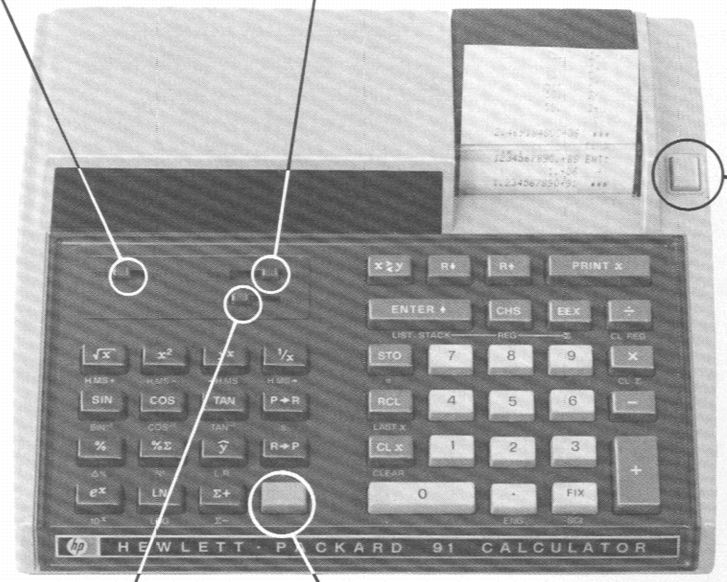
ALL
MAN ▨ NORM

Normal. Printer records history of calculation sequence, showing inputs and function keys pressed.

ALL
MAN ▨ NORM

All. Printer duplicates display changes, showing all functions and intermediate and final answers.

Calculator Power Switch



DEG ▨ RAD
GRD

Trigonometric Mode Switch. Lets you assume angles in degrees, grads, or radians.

Shift Key

Paper Advance

Functions

The best way to see how simple functions operate on your HP-91 is with the Print Mode switch set to ALL to give you a complete record of inputs, functions, and answers.

Slide the Print Mode switch MAN ALL NORM to ALL now.

In spite of the dozens of functions available on the HP-91 keyboard, you will find the calculator functions simple to operate by using a single, all-encompassing rule: *When you press a function key, the calculator immediately executes the function written on the key.*

Pressing a function key causes the calculator to immediately perform that function.

For example, to calculate the square root of 148.84, merely:

Press	Display
148.84	148.84
√x	12.20

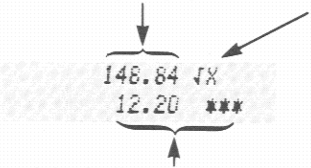
```
148.84 √X
 12.20  ***
```

Let's look briefly at the printed copy of that problem to see the simple way that the HP-91 printer duplicates your calculations.

The paper tapes are printed just as you read, from left to right and top to bottom. The number, 148.84, is printed exactly as you keyed it in. A symbol for the function performed, √x, is printed next to it. The answer, 12.20, is printed with a three-asterisk label to its right, indicating that the HP-91 performed some operation in order to obtain the number as it is printed.

Number keyed in—no asterisks.

Function performed.



```
148.84 √X
 12.20  ***
```

Asterisks indicate this number as printed is the result of some operation.

Now let's continue. To square the result of the previous calculation:

Press	Display
x²	148.84

```
             X²
148.84 ***
```

√x and x² are examples of one-number function keys; that is, keys that execute upon a single number. All function keys in the HP-91 operate upon either one number or two numbers at a time (except for statistics keys like Σ+ and S—more about these later).

Function keys operate upon either one number or two numbers.

One-Number Functions

To use any one-number function key:

1. Key in the number.
2. Press the function key (or press the prefix key, then the function key).

For example, to use the one-number function **1/x** key, you first key in the number represented by x, then press the function key. To calculate ¼, key in 4 (the x-number) and press **1/x**.

Press	Display
4	4.
1/x	0.25

```
4.00 1/X
0.25 ***
```

Now try these other one-number function problems. Remember, *first key in the number, then press the function:*

$\frac{1}{25} = \boxed{0.04}$

$\sqrt{2500} = \boxed{50.00}$

$10^5 = \boxed{100000.00}$ (Use the **10ˣ** key.)

$\sqrt{3204100} = \boxed{1790.00}$

$\log 12.58925411 = \boxed{1.10}$

$71^2 = \boxed{5041.00}$

Two-Number Functions

Two-number functions are functions that must have two numbers present in order for the operation to be performed. **+**, **−**, **×**, and **÷** are examples of two-number function keys. You cannot add, subtract, multiply, or divide unless there are two numbers present in the calculator. Two-number functions work the same way as one-number functions—that is, the operation occurs when the function key is pressed. Therefore, *both numbers must be in the calculator before the function key is pressed.*

When more than one number must be keyed into the calculator before performing an operation, the **ENTER↑** key is used to separate the two numbers.

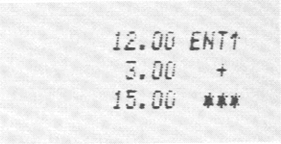
> Use the **ENTER↑** key whenever more than one number must be keyed into the calculator before pressing a function.

If you key in only one number, you never need to press **ENTER↑**. To place two numbers into the calculator and perform an operation:

1. Key in the first number.
2. Press **ENTER↑** to separate the first number from the second.
3. Key in the second number.
4. Press the function key to perform the operation.

For example, to add 12 and 3:

Press	
12	The first number.
ENTER↑	Separates the first number from the second.
3	The second number.
+	The function.

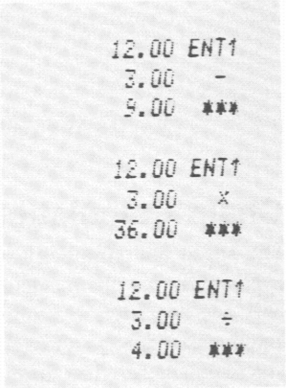


```
12.00 ENT↑
 3.00    +
15.00  ***
```

The answer, 15.00 , is displayed and printed.

Other arithmetic functions are performed the same way:

To perform	Press	Display
12 − 3	12 ENTER↑ 3 −	9.00
12 × 3	12 ENTER↑ 3 ×	36.00
12 ÷ 3	12 ENTER↑ 3 ÷	4.00



```
12.00 ENT↑
 3.00    -
 9.00  ***

12.00 ENT↑
 3.00    x
36.00  ***

12.00 ENT↑
 3.00    ÷
 4.00  ***
```

The y^x key is also a two-number operation. It is used to raise numbers to powers, and you can use it in the same simple way that you use every other two-number function key:

1. Key in the first number.
2. Press ENTER↑ to separate the first number from the second.
3. Key in the second number (power).
4. Perform the operation (press y^x).

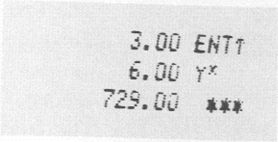
When working with any function key (including y^x), you should remember that the displayed number is always designated by *x* on the function key symbols.

The number displayed is always *x*.

So $\sqrt{x}$ means square root of the displayed number, $1/x$ means $\frac{1}{\text{displayed number}}$, etc.

Thus, to calculate 3^6:

Press	Display	
3	3.	
ENTER↑	3.00	
6	6.	x, the displayed number, is now 6.
y^x	729.00	The answer.



```
 3.00 ENT↑
 6.00 Y^x
729.00 ***
```

Now try the following problems using the y^x key, keeping in mind the simple rules for two-number functions:

16^4	(16 to the 4th power)=	65536.00	
81^2	(81 squared)=	6561.00	(You could also have done this as a one-number function using x^2.)
$225^{.5}$	(Square root of 225)=	15.00	(You could also have done this as a one-number function using $\sqrt{x}$.)
2^{16}	(2 to the 16th power)=	65536.00	
$16^{.25}$	(4th root of 16)=	2.00	

Chain Calculations

The speed and simplicity of operation of the Hewlett-Packard logic system become most apparent during chain calculations. Even during the longest of calculations, you still perform only one operation at a time, and you see the results as you calculate—the Hewlett-Packard automatic memory stack stores up to four intermediate results inside the calculator until you need them, then inserts them into the calculation. This system makes the process of working through a problem as natural as it would be if you were working it out with pencil and paper, but the calculator takes care of the hard part.

For example, solve (12 + 3) × 7.

If you were working the problem with a pencil and paper, you would first calculate the intermediate result of (12 + 3).....

(~~12 + 3~~) × 7 =
15

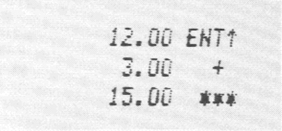
.....and then you would multiply the intermediate result by 7.

(~~12 + 3~~) × 7 = 105
15 × 7

You work through the problem exactly the same way with the HP-91, one operation at a time. You solve for the intermediate result first.....

(12 + 3)

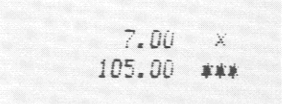
Press	Display	
12	12.	
ENTER↑	12.00	
3	3.	
+	15.00	Intermediate result.



```
 12.00 ENT↑
  3.00   +
 15.00  ***
```

.....and then solve for the final answer. You don't need to press ENTER↑ to store the intermediate result—the HP-91 automatically stores it inside the calculator when you key in the next number. To continue.....

Press	Display	
7	7.	The intermediate result from the preceding operation is automatically stored inside the calculator when you key in this number.
×	105.00	Pressing the function key multiplies the new number and the intermediate result, giving you the final answer.



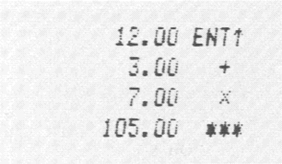
```
   7.00   ×
 105.00  ***
```

Because the HP-91 stores intermediate results automatically, you don't need to print them. You can slide the Print Mode switch to NORM to preserve a record of your calculations, and then press PRINT x to print the final answer.

For example, when you solved the above problem in ALL mode, you preserved *all* intermediate and final results. To solve the same problem and preserve only a history of the calculation:

Slide the Print Mode switch MAN ALL NORM to NORM.

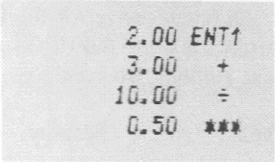
Press	Display	
12	12.	
ENTER↑	12.00	
3	3.	
+	15.00	
7	7.	
×	105.00	
PRINT x	105.00	Preserves the final answer in your printed record.



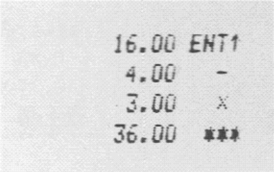
```
  12.00 ENT↑
   3.00   +
   7.00   ×
 105.00  ***
```

Now try these problems. Notice that for each problem you only have to press ENTER↑ to insert a pair of numbers into the calculator—each subsequent operation is performed using a new number and an automatically stored intermediate result.

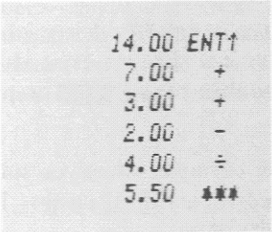
To solve	Press	Display
$\frac{(2 + 3)}{10}$	2	
	ENTER↑	
	3	
	+	
	10	
	÷	0.50
	PRINTx	0.50
3 (16 − 4)	16	
	ENTER↑	
	4	
	−	
	3	
	×	36.00
	PRINTx	36.00
$\frac{14 + 7 + 3 - 2}{4}$	14	
	ENTER↑	
	7	
	+	
	3	
	+	
	2	
	−	
	4	
	÷	5.50
	PRINTx	5.50



```
 2.00 ENT↑
 3.00    +
10.00    ÷
 0.50  ***
```



```
16.00 ENT↑
 4.00    -
 3.00    ×
36.00  ***
```



```
14.00 ENT↑
 7.00    +
 3.00    +
 2.00    -
 4.00    ÷
 5.50  ***
```

Problems that are even more complicated can be solved in the same simple manner, using the automatic storage of intermediate results. For example, to solve (2 + 3) × (4 + 5) with a pencil and paper, you would:

$$\underbrace{(2 + 3)} \times \underbrace{(4 + 5)}$$

First solve for the contents of these parentheses...

...and then for these parentheses...

...and then you would multiply the two intermediate answers together.

You work through the problem the same way with the HP-91. First you solve for the intermediate result of (2 + 3).....

Press	Display	
2	2.	
ENTER↑	2.00	
3	3	
+	5.00	Intermediate result.

```
2.00 ENT↑
3.00   +
```

Then add 4 and 5:

(Since you must now key in another *pair* of numbers before you can perform a function, you use the ENTER↑ key again to separate the first number of the pair from the second.)

Procedure	Press	Display
$(\underset{5}{\cancel{2+3}}) \times (\underset{9}{\cancel{4+5}})$	4 ENTER↑ 5 +	9.00

```
4.00 ENT↑
5.00   +
```

Then multiply the intermediate answers together for the final answer:

Procedure	Press	Display
$(\underset{5}{\cancel{2+3}}) \times (\underset{9}{\cancel{4+5}})$	×	45.00
	PRINT x	

```
          ×
45.00   ***
```

Notice that you didn't need to write down or key in the intermediate answers from inside the parentheses before you multiplied—the HP-91 automatically stacked up the intermediate results inside the calculator for you and brought them out on a last-in, first-out basis when it was time to multiply.

No matter how complicated a problem may look, it can always be reduced to a series of one- and two-number operations. Just work through the problem in the same logical order you would use if you were working it with a pencil and paper.

For example, to solve:

$$\frac{(9 + 8) \times (7 + 2)}{(4 \times 5)}$$

Press	Display	
9 ENTER↑ 8 +	17.00	Intermediate result of (9 + 8).
7 ENTER↑ 2 +	9.00	Intermediate result of (7 + 2).
×	153.00	(9 + 8) multiplied by (7 + 2).
4 ENTER↑ 5 ×	20.00	Intermediate result of (4 × 5).
÷	7.65	The final answer.
PRINT x	7.65	

```
9.00 ENT↑
8.00   +
7.00 ENT↑
2.00   +
       ×
4.00 ENT↑
5.00   ×
       ÷
7.65  ***
```

Now try these problems. Remember to work through them as you would with a pencil and paper, but don't worry about intermediate answers—they're handled automatically by the calculator.

$$(2 \times 3) + (4 \times 5) = \boxed{26.00}$$

$$\frac{(14 + 12) \times (18 - 12)}{(9 - 7)} = \boxed{78.00}$$

$$\frac{\sqrt{16.3805 \times 5}}{.05} = \boxed{181.00}$$

$$4 \times (17 - 12) \div (10 - 5) = \boxed{4.00}$$

$$\sqrt{(2 + 3) \times (4 + 5)} + \sqrt{(6 + 7) \times (8 + 9)} = \boxed{21.57}$$

A Word about the HP-91

Now that you've learned how to use the calculator, you can begin to fully appreciate the benefits of the Hewlett-Packard logic system. With this system, you enter numbers using a parenthesis-free, unambiguous method called RPN (Reverse Polish Notation).

It is this unique system that gives you all these calculating advantages:

- You never have to work with more than one function at a time. The HP-91 cuts problems down to size instead of making them more complex.
- Pressing a function key immediately executes the function. You work naturally through complicated problems, with fewer keystrokes and less time spent.
- Intermediate results appear as they are calculated. There are no "hidden" calculations, and you can check each step as you go.
- Intermediate results are automatically handled. You don't even have to print out long intermediate answers when you work a problem. (Of course, if you want intermediate answers, the HP-91 printer will record them in ALL mode.)
- Intermediate answers are automatically inserted into the problem on a last-in, first-out basis. You don't have to remember where they are and then summon them.
- You can calculate in the same order that you do with pencil and paper. You don't have to think the problem through ahead of time.

The HP system takes a few minutes to learn. But you'll be amply rewarded by the ease with which the HP-91 solves the longest, most complex equations. With HP, the investment of a few moments of learning yields a lifetime of mathematical dividends.


HEWLETT · PACKARD 91 CALCULATOR
PRINT x
ENTER ↑
CHS
EEX
STO
RCL
CL x
FIX
SIN
COS
TAN
LN
OFF ON
MAN ALL NORM
DEG GRD RAD

Printer and Display Control

In the HP-91, you can select many different rounding options for display of numbers. When you first turn on the HP-91, for example, the calculator "wakes up" with numbers appearing rounded to two decimal places. Thus, the fixed constant π, which is actually in the calculator as 3.141592654, will *appear in the display* as 3.14 (unless you tell the calculator to display the number rounded to a greater or lesser number of decimal places).

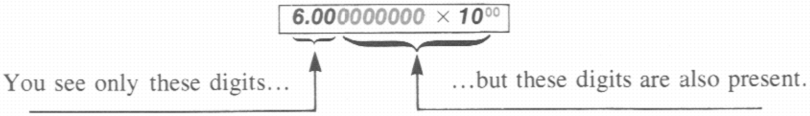
Although a number is normally shown to only two decimal places, the HP-91 always computes internally using each number as a 10-digit mantissa and a two-digit exponent of 10. For example, when you compute 2 × 3, you *see* the answer to only two decimal places:

Press	**Display**
2 ENTER↑ 3 ×	6.00

However, inside the calculator all numbers have 10-digit mantissas and two-digit exponents of 10. So the HP-91 *actually* calculates using full 10-digit numbers:

$$2.000000000 \times 10^{00} \text{ ENTER↑ } 3.000000000 \times 10^{00} \times$$

yields an answer that is actually carried to full 10 digits internally:



Display Control Keys

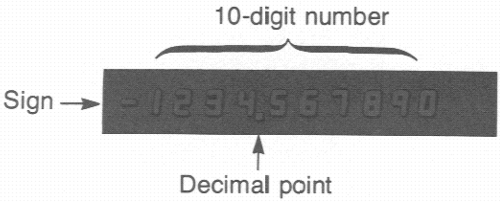
There are four keys, FIX, SCI, ENG, and the prefix key ■, that allow you to control the manner in which numbers appear in the display in the HP-91.

FIX displays and prints numbers in fixed decimal point format, while SCI permits you to view numbers in a scientific notation format. ENG displays numbers in engineering notation, with exponents of 10 shown in multiples of three (e.g., 10^3, 10^{-6}, 10^{12}). ■ followed by a number key (0 through 9) changes the number of displayed digits without changing the format.

No matter which format or how many displayed digits you choose, display control alters only the manner in which a number is displayed and printed in the HP-91. The actual number itself is not altered by any of the print options or the display control keys. No matter what type of display you select, the HP-91 always calculates internally with a full 10-digit number, multiplied by 10 raised to a two-digit exponent.

The printer does not immediately indicate when you change display formats, but any new results will be shown in the new format.

Fixed Point Display



Using fixed point display, you can specify the number of places to be shown after the decimal point. It is selected by pressing [FIX] followed by a number key to specify the number of decimal places (0 through 9) to which the display is to be rounded. The displayed number begins at the left side of the display (or the right side of the printed tape) and includes trailing zeros within the setting selected. When the calculator is turned OFF, then ON, it "wakes up" in fixed point notation with the display rounded to two decimal places.

For example:

Slide the Print Mode switch MAN ALL NORM to MAN now so that you can concentrate on the display changes.

Press	Display	
(Turn the calculator OFF, then ON.)	0.00	Calculator "wakes up" in [FIX] 2 display format.
123.4567	123.4567	
[FIX] 0	123.	Display is rounded off to 0 decimal places. Internally, however, the number maintains its original value of 123.4567.
[FIX] 4	123.4567	
[FIX] 7	123.4567000	
[FIX] 1	123.5	Notice that the display rounds if the first *hidden* digit is 5 or greater.
[FIX] 2	123.46	Normal [FIX] 2 display.

Scientific Notation Display

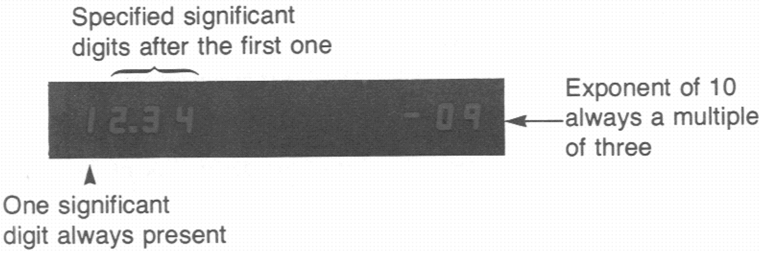
In scientific notation each number is displayed with a single digit to the left of the decimal point followed by a specified number of digits (up to nine) to the right of the decimal point and multiplied by a power of 10. Scientific notation is particularly useful when working with very large or small numbers.

Scientific notation is selected by pressing ▨ SCI followed by a digit key to specify the number of decimal places to which the number is rounded. The display is left-justified and includes trailing zeros within the selected setting. The printed copy is right-justified, with a sign to identify the exponent of 10. For example:

Press	**Display**	
123.4567	123.4567	
▨ SCI 2	1.23 02	Indicates 1.23 × 10^2.
▨ SCI 4	1.2346 02	Indicates 1.2346 × 10^2. Notice that the display rounds if the first *hidden* mantissa digit is 5 or greater.
▨ SCI 7	1.2345670 02	Indicates 1.2345670 × 10^2.
▨ SCI 9	1.234567000 02	Indicates 1.234567000 × 10^2.

Note: You can easily key in numbers in scientific notation format by using the EEX *(enter exponent)* key—more about this later.

Engineering Notation Display



Engineering notation allows all numbers to be shown with exponents of 10 that are multiples of three (e.g., 10^3, 10^{-6}, 10^{12}).

This is particularly useful in scientific and engineering calculations, where units of measure are often specified in multiples of three. Refer to the prefix chart below.

Multiplier	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Engineering notation is selected by pressing ■ [ENG] followed by a number key. The first significant digit is always present in the display, and the number key specifies the number of additional significant digits to which the display is rounded. The decimal point always appears in the display. For example:

Press	Display	
.000012345	.000012345	
■ [ENG] 1	12. -06	Engineering notation display. Number appears in the display rounded off to one significant digit after the omnipresent first one. Power of 10 is proper multiple of three.
■ [ENG] 3	12.35 -06	Display is rounded off to third significant digit after the first one.
■ [ENG] 9	12.34500000-06	
■ [ENG] 0	10. -06	Display rounded off to first significant digit.

Notice that rounding can occur to the *left* of the decimal point, as in the case of [ENG] 0 specified above.

When engineering notation has been selected, the decimal point shifts to show the mantissa as units, tens, or hundreds in order to maintain the exponent of 10 as a multiple of three. For example, multiplying the number now in the calculator by 10 causes the decimal point to shift to the right without altering the exponent of 10:

Press	Display
■ [ENG] 2	12.3 -06
10 ×	123. -06

However, multiplying again by 10 causes the exponent to shift to another multiple of three and the decimal point to move to the units position. Since you specified [ENG] 2 earlier, the HP-91 maintains two significant digits after the first one when you multiply by 10.

Press	Display	
10 ×	1.23 -03	Decimal point shifts. Power of 10 shifts to 10^{-3}. Display maintains two significant digits after the first one.

Display Number Changes

You have seen how you can change the HP-91 display to show numbers in fixed, scientific notation, or engineering notation format. When you have specified any of these formats, the HP-91 permits you to change the *number* of displayed digits by simply pressing the prefix key ■ followed by the desired number key. For example:

Press	Display	
12345	12345.	
■ [SCI] 3	1.235 04	Scientific notation format selected.
■ 1	1.2 04	The HP-91 remains in scientific notation mode; only the number of displayed digits is changed.
■ 6	1.234500 04	
■ 2	1.23 04	
■ [ENG] 2	12.3 03	Engineering notation format selected.
■ 3	12.35 03	Number of displayed digits changes, but calculator remains in engineering notation mode.
■ 7	12.345000 03	
■ 1	12. 03	
[FIX] 5	12345.00000	Fixed format selected.
■ 3	12345.000	Number of displayed digits changes, but calculator remains in fixed mode.
■ 0	12345.	
■ 2	12345.00	

Format of Printed Numbers

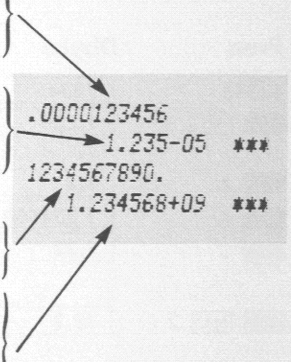
When using the printer, whether you are in MAN or NORM mode (where you must press **PRINTx** to see answers) or in ALL (where the HP-91 automatically prints answers as they are calculated), printed numbers can be shown in any display format—fixed point, scientific notation, or engineering notation. By selecting the display format, you also select the print format.

Results from your HP-91 are always displayed and printed in the format that you have chosen. The three-asterisk label that you see printed next to a result is a guarantee that it is in the chosen display format. Although numbers in the display are left-justified, printed numbers are right-justified.

Numbers that you key in—that is, numbers that are *not* the results of operations—are also printed by the HP-91. When you key in a number with the Print Mode switch set to NORM or ALL, the HP-91 does not print it until you change display format or press a function key. Then the number is printed exactly *as you keyed it in*. (One case is an exception to this rule—more about that later.) A number that you keyed in is not the result of an operation, and no asterisks are printed to its right. Subsequent *results*, of course, are printed in the selected format with a three-asterisk label. For example:

Slide the Print Mode switch MAN ALL NORM to NORM.

Press	Display	
.0000123456	.0000123456	
■ SCI 3	1.235 -05	When you press any function, the number is first printed just as you keyed it in.
PRINTx	1.235 -05	Results of functions, including display formatting, are printed in the selected format.
1234567890	1234567890.	
■ ENG 6	1.234568 09	The number is printed as you keyed it in.
PRINTx	1.234568 09	The three-asterisk label guarantees that the number is now in the selected format.



```
 .0000123456
   1.235-05  ***
 1234567890.
 1.234568+09  ***
```

Notice that the HP-91 *prints* a + sign to show you positive exponents of 10.

Thus, whenever you key in a number, the HP-91 prints it just as you keyed it in; *then* the format is changed. It is easy for you to reconstruct your calculation because your exact inputs are identifiable from your printed copy.

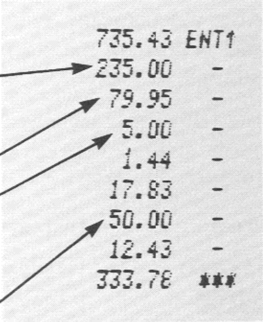
When you have keyed in a number, there is one time that the HP-91 will change its format *before* printing. If you have specified fixed point notation (by turning the calculator OFF, then ON, or by pressing FIX followed by a number key) and the number keyed in is also in fixed point format (i.e., you have not pressed EEX), the HP-91 will attempt to align the decimal points for easy readibility on your printed copy. It will do this in fixed point notation by printing the number that you keyed in in the *specified* format (if the number can be printed without truncating), adding trailing zeros if necessary.

This feature permits you to key in numbers in fixed point notation and line up the decimal points in the printed record of your calculations.

Example: You begin the month with a balance of $735.43 in your checking account. During the month, you write checks for $235, $79.95, $5, $1.44, $17.83, $50, and $12.43. Calculate the closing balance for the account and preserve a printed record of your calculations.

First, ensure that the Print Mode switch MAN ALL NORM is set to NORM.

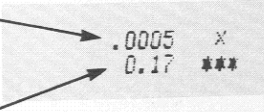
Press	**Display**	
FIX 2	0.00	Sets FIX 2 display mode. (Display shown assumes that no results remain from previous example.)
735.43 ENTER↑	735.43	
235 −	500.43	Two extra zeros printed so that decimal points will line up.
79.95 −	420.48	The number is printed exactly as you keyed it in.
5 −	415.48	Two extra zeros printed.
1.44 −	414.04	
17.83 −	396.21	
50 −	346.21	Two extra zeros printed.
12.43 −	333.78	
PRINT x	333.78	Closing balance.



```
735.43 ENT↑
235.00  -
 79.95  -
  5.00  -
  1.44  -
 17.83  -
 50.00  -
 12.43  -
333.78 ***
```

You need not worry about "losing" digits on the printed copy. The HP-91 printer will never truncate digits (not even extra zeros) that you have keyed in. For example, if you wanted to set aside 5/10000 of the closing balance of your account for a present for your sister-in-law:

Press	**Display**	
.0005	0.0005	
×	0.17	Entire number is printed—not rounded to FIX 2.
PRINT x	0.17	Amount set aside for sister-in-law's gift. Result of function is rounded to FIX 2.

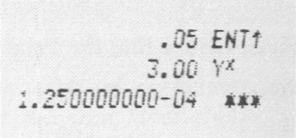


```
.0005   x
 0.17 ***
```

Automatic Display Switching

The HP-91 switches the display from fixed point notation to full scientific notation (SCI 9) whenever the number is too large or too small to be seen with a fixed decimal point. This feature keeps you from missing unexpectedly large or small answers. For example, if you try to solve (.05)^3 in normal FIX 2 display, the answer is automatically shown in scientific notation.

Press	Display	
FIX 2	0.00	Normal FIX 2 display. (Display shown assumes no results remain from previous example.)
.05 **ENTER↑**	0.05	
3 **yˣ** **PRINTx**	1.250000000-04	Display automatically switched to SCI 9 to show answer.

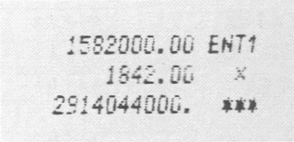


```
          .05 ENT↑
         3.00 Yx
1.250000000-04 ***
```

After automatically switching from fixed to scientific, and a new number is keyed in or **CLx** is pressed, the display automatically reverts back to the fixed point display originally selected.

The HP-91 also switches to scientific notation if the answer is too large ($\geqslant 10^{10}$) for fixed point display. For example, the display will not switch from fixed if you solve 1582000 × 1842:

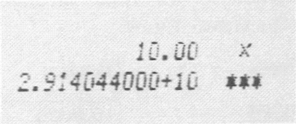
Press	Display	
1582000	1582000.	
ENTER↑	1582000.00	
1842 **×**	2914044000.	Fixed point format.
PRINTx	2914044000.	



```
1582000.00 ENT↑
   1842.00    ×
2914044000.  ***
```

However, if you multiply the result by 10, the answer is too large for fixed point notation, and the calculator display switches automatically to scientific notation:

Press	Display	
10 **×** **PRINTx**	2.914044000 10	Scientific notation format.



```
         10.00    ×
2.914044000+10  ***
```

Notice that automatic switching is between fixed and scientific notation display modes only—engineering notation display must be selected from the keyboard.

Keying in Exponents of Ten

You can key in numbers multiplied by powers of 10 by pressing **EEX** *(enter exponent of 10)* followed by number keys to specify the exponent of 10. For example, to key in 15.6 trillion (15.6 × 10^{12}), and multiply it by 25:

Press	Display	
15.6	15.6	
EEX	15.6 00	
12	15.6 12	This means 15.6 × 10^{12}.

Now Press	Display
ENTER↑	1.560000000 13
25 × PRINT x	3.900000000 14

```
   15.6+12 ENT↑
     25.00   ×
3.900000000+14 ***
```

You can save time when keying in exact powers of 10 by merely pressing EEX and then pressing the desired power of 10. For example, key in 1 million (10^6) and divide by 52.

Press	Display	
EEX	1. 00	You do not have to key in the number 1 before pressing EEX when the number is an exact power of 10.
6	1. 06	
ENTER↑	1000000.00	Since you have not specified scientific notation, the display reverts to fixed point notation when you press ENTER↑.
52 ÷ PRINT x	19230.77	

```
  1.+06 ENT↑
  52.00   ÷
19230.77 ***
```

To see your answer in scientific notation with six decimal places:

Press	Display
■ SCI 6	1.923077 04
PRINT x	1.923077 04

```
1.923077+04 ***
```

To key in negative exponents of 10, key in the number, press EEX, press CHS to make the exponent negative, then key in the power of 10. For example, key in Planck's constant (h)—roughly, 6.625 × 10^{-27} erg sec.—and multiply it by 50.

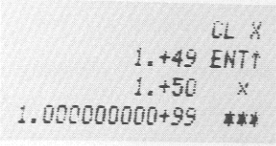
Press	Display	
CL x	0.000000 00	
FIX 2	0.00	
6.625 EEX	6.625 00	
CHS	6.625 -00	
27	6.625 -27	
ENTER↑	6.625000000 -27	
50 × PRINT x	3.312500000 -25	Erg sec.

```
                CL X
     6.625-27 ENT↑
         50.00   ×
3.312500000-25 ***
```

Calculator Overflow

When the number in the display would be greater than 9.999999999 × 10^{99}, the HP-91 displays all 9's to indicate that the problem has exceeded the calculator's range. For example, if you solve (1 × 10^{49}) × (1 × 10^{50}), the HP-91 will display the answer:

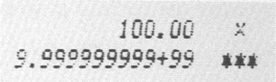
Press	**Display**
CL x	0.00
EEX 49 **ENTER↑**	1.000000000 49
EEX 50 **×**	1.000000000 99
PRINT x	1.000000000 99



```
          CL X
    1.+49 ENT↑
    1.+50    ×
1.000000000+99 ***
```

But if you attempt to multiply the above result by 100, the HP-91 display indicates overflow by showing you all 9's:

Press	**Display**	
100 **×** **PRINT x**	9.999999999 99	Overflow indication.



```
        100.00   ×
9.999999999+99 ***
```

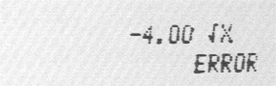
Error Display

If you happen to key in an improper operation, the word *Error* will appear in the display. In addition, if the Print Mode switch MAN ALL NORM is set to NORM or ALL, the printer will print *Error*.

For example, if you attempt to calculate the square root of −4, the HP-91 will recognize it as an improper operation:

Ensure that the Print Mode switch MAN ALL NORM is set to NORM.

Press	**Display**
4 **CHS**	−4.
√x	Error



```
-4.00 √X
   ERROR
```

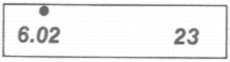
Pressing any key clears the error and is *not* executed. (Pressing the paper advance push-button clears the error and *is* executed.) The number that was in the display before the error-causing function is returned to the display so that you can see it.

Press	**Display**
CL x	−4.00

All those operations that cause an error condition are listed in appendix B.

Low Power Display

When you are operating the HP-91 from battery power, a red lamp inside the display will glow to warn you that the battery is close to discharge.



Low Power Display

You must then connect the ac adapter/recharger to the calculator and operate from ac power, or you must substitute a fully charged battery pack for the one that is in the calculator. Refer to appendix A for a description of these operations.


15.00 ENT↑
78.00 Σ+
25.00 ENT↑
PRINT x
x⇄y
R↓
R↑
ENTER ↑
CHS
EEX
STO
RCL
CL x
FIX
√x
x²
yˣ
1/x
SIN
COS
TAN
P→R
%
%Σ
ŷ
R→P
eˣ
LN
Σ+
HEWLETT · PACKARD 91 CALCULATOR
f(x)
f(x₀)
P(x)

Section 3

The Automatic Memory Stack

The Stack

Automatic storage of intermediate results is the reason that the HP-91 slides so easily through the most complex equations. And automatic storage is made possible by the Hewlett-Packard automatic memory stack.

Initial Display

Turn the HP-91 OFF, then ON.

You can work through this section with the Print Mode switch at any setting you desire. The printed tapes that illustrate the problems in this handbook were created with the Print Mode switch **MAN** **ALL** **NORM** set to NORM.

When you first switch the calculator ON, the display shows 0.00 . This represents the contents of the display, or "X-register."

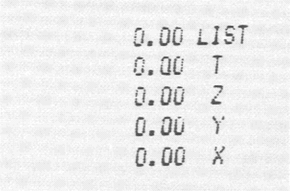
Basically, numbers are stored and manipulated in the machine "registers." Each number, no matter how few digits (e.g., 0, 1, or 5) or how many (e.g., 3.141592654, −23.28362, or 2.87148907 × 10^{27}), occupies one entire register.

The displayed X-register, which is the only visible register, is one of four registers inside the calculator that are positioned to form the automatic memory stack. We label these registers X, Y, Z, and T. They are "stacked" one on top of the other with the displayed X-register on the bottom. When the calculator is switched ON, these four registers are cleared to 0.00.

Name	Register	
T	0.00	
Z	0.00	
Y	0.00	
X	0.00	Always displayed.

You can view the contents of the entire stack at any time by printing them using the LIST: STACK *(list stack)* key.

Press	Display
■ LIST: STACK	0.00



```
0.00 LIST
0.00  T
0.00  Z
0.00  Y
0.00  X
```

Notice that LIST: STACK, like PRINT x and the other LIST functions, operates regardless of the position of the Print Mode switch.

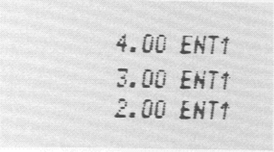
Manipulating Stack Contents

The R↓ *(roll down)*, R↑ *(roll up)*, and x⇄y *(x exchange y)* keys allow you to review the stack contents or to shift data within the stack for computation at any time.

Reviewing the Stack

To see how the R↓ key works, first load the stack with numbers 1 through 4 by pressing:

4 ENTER↑ 3 ENTER↑ 2 ENTER↑ 1



```
4.00 ENT↑
3.00 ENT↑
2.00 ENT↑
```

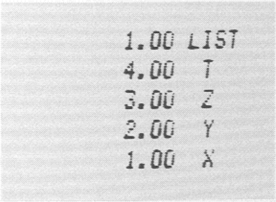
The numbers that you keyed in are now loaded into the stack, and its contents look like this:

T	4.00	
Z	3.00	
Y	2.00	
X	1.	Display

To see the contents of the stack now, press:

Press

■ LIST: STACK

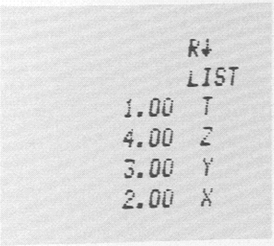


```
1.00 LIST
4.00  T
3.00  Z
2.00  Y
1.00  X
```

When you press the R↓ key, the stack contents shift downward one register. So the last number that you have keyed in will be rotated around to the T-register when you press R↓ . When you press R↓ again, the stack contents again roll downward one register.

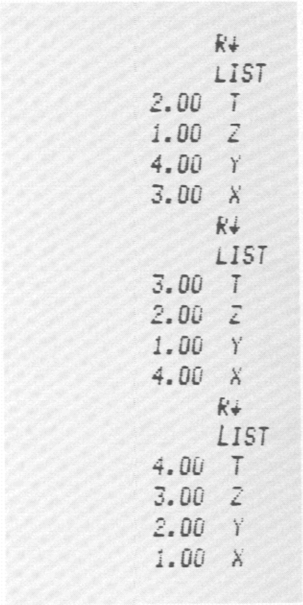
To see how the R↓ key operates, press ■ LIST: STACK to list the stack contents after each press of the R↓ key:

Press	Display
R↓	
■ LIST: STACK	2.00



```
     R↓
     LIST
1.00  T
4.00  Z
3.00  Y
2.00  X
```

Press	Display
R↓	
■ LIST: STACK	3.00
R↓	
■ LIST: STACK	4.00
R↓	
■ LIST: STACK	1.00



```
          R↓
          LIST
     2.00  T
     1.00  Z
     4.00  Y
     3.00  X
          R↓
          LIST
     3.00  T
     2.00  Z
     1.00  Y
     4.00  X
          R↓
          LIST
     4.00  T
     3.00  Z
     2.00  Y
     1.00  X
```

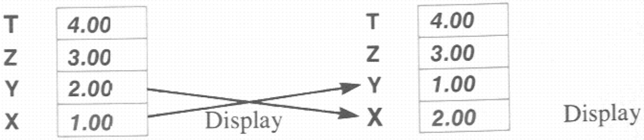
Once again the number 1.00 is in the displayed X-register. Four presses of the R↓ key roll the stack down four times, returning the contents of the stack to their original registers.

You can also manipulate the stack contents using the R↑ *(roll up)* key. This key rolls the stack contents *up* instead of down, but it otherwise operates in the same manner as the R↓ key.

Exchanging x and y

The x≷y *(x exchange y)* key exchanges the contents of the X- and the Y-registers without affecting the Z- and T-registers. If you press x≷y with data intact from the previous example, the numbers in the X- and Y-registers will be changed...

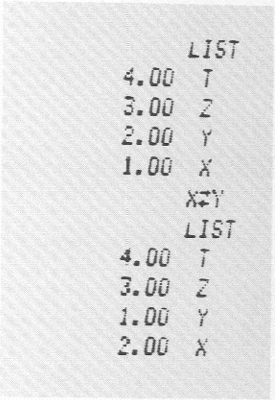
...from this... **...to this.**



T	4.00	T	4.00
Z	3.00	Z	3.00
Y	2.00	Y	1.00
X	1.00	X	2.00

You can verify this by first listing the stack contents and then pressing x≷y. To see the results, list the stack contents again:

Press	Display
■ LIST: STACK	1.00
x≷y	2.00
■ LIST: STACK	2.00



```
       LIST
4.00   T
3.00   Z
2.00   Y
1.00   X
        X⇄Y
       LIST
4.00   T
3.00   Z
1.00   Y
2.00   X
```

Notice that whenever you move numbers in the stack using one of the data manipulation keys, the actual stack registers maintain their positions. Only the *contents* of the registers are shifted. The contents of the X-register are always displayed.

Clearing the Stack

To clear the displayed X-register only, press CLx. To clear the entire automatic memory stack, including the displayed X-register, press ■ CLEAR. This replaces all numbers in the stack with zeros. (It also clears all manual storage registers—more about these later.) When you turn the calculator OFF, then ON, it "wakes up" with all zeros in the stack registers.

Although it may be comforting, *it is never necessary to clear the stack or the displayed X-register when starting a new calculation*. This will become obvious when you see how old results in the stack are automatically lifted by new entries.

Press CLx now, and the stack contents are changed...

	...from this...			...to this.	
T	4.00		T	4.00	
Z	3.00		Z	3.00	
Y	1.00		Y	1.00	
X	2.00	Display	X	0.00	Display

You can verify that only the X-register contents are affected by listing the stack contents after you have pressed [CL x]:

Press	Display
■ LIST: [STACK]	0.00

```
        CL X
        LIST
4.00  T
3.00  Z
1.00  Y
0.00  X
```

Now press ■ [CLEAR]. The contents of the stack are changed…

	…from this…		…to this.
T	4.00	T	0.00
Z	3.00	Z	0.00
Y	1.00	Y	0.00
X	0.00	X	0.00

```
       CLEAR
```

You can verify that the stack has been cleared completely and now contains all zeros by listing the stack contents:

Press	Display
■ LIST: [STACK]	0.00

```
        LIST
0.00  T
0.00  Z
0.00  Y
0.00  X
```

The [ENTER↑] Key

When you key a number into the calculator, its contents are written into the displayed X-register. For example, if you key in the number 314.32 now, you can see that the display contents are altered.

When you key in 314.32, the contents of the stack registers are changed…

	…from this		…to this.
T	0.00	T	0.00
Z	0.00	Z	0.00
Y	0.00	Y	0.00
X	0.00	X	314.32

In order to key in another number at this point, you must first terminate digit entry—i.e., you must indicate to the calculator that you have completed keying in the first number and that any new digits you key in are part of a new number.

Use the ENTER↑ key to separate the digits of the first number from the digits of the second.

When you press the ENTER↑ key, the contents of the stack registers are changed…

	…from this…			…to this.	
T	0.00		T	0.00	
Z	0.00		Z	0.00	
Y	0.00		Y	314.32	
X	314.32	Display	X	314.32	Display

As you can see, the number in the displayed X-register is copied into Y. The numbers in Y and Z have also been transferred to Z and T, respectively, and the number in T has been lost off the top of the stack. But this will be more apparent when we have different numbers in all four registers.

Immediately after pressing ENTER↑, the X-register is prepared for a new number, and that new number writes over the number in X. For example, key in the number 543.28 and the contents of the stack registers change…

	…from this…			…to this.	
T	0.00		T	0.00	
Z	0.00		Z	0.00	
Y	314.32		Y	314.32	
X	314.32	Display	X	543.28	Display

CLx replaces any number in the display with zero. Any new number then writes over the zero in X.

For example, if you had meant to key in 689.4 instead of 543.28, you would press CLx now to change the stack…

	…from this…			…to this.	
T	0.00		T	0.00	
Z	0.00		Z	0.00	
Y	314.32		Y	314.32	
X	543.28	Display	X	0.00	Display

and then key in 689.4 to change the stack...

...from this...

T	0.00	
Z	0.00	
Y	314.32	
X	0.00	Display

...to this.

T	0.00	
Z	0.00	
Y	314.32	
X	689.4	Display

Notice that numbers in the stack do not move when a new number is keyed in immediately after you press LIST: STACK, PRINT x, ENTER↑, or CL x. However, numbers in the stack *do* lift upward when a new number is keyed in immediately after you press most other functions, including R↓, R↑, and x≷y.

One-Number Functions and the Stack

One-number functions execute upon the number in the X-register only, and the contents of the Y-, Z-, and T-registers are unaffected when a one-number function key is pressed.

For example, with numbers positioned in the stack as in the earlier example, pressing the √x key changes the stack contents...

...from this...

T	0.00	
Z	0.00	
Y	314.32	
X	689.4	Display

...to this.

T	0.00	
Z	0.00	
Y	314.32	
X	26.26	Display

The one-number function executes upon only the number in the displayed X-register, and the answer writes over the number that was in the X-register. No other register is affected by a one-number function.

Two-Number Functions and the Stack

Hewlett-Packard calculators do arithmetic by positioning the numbers in the stack the same way you would on paper. For instance, if you wanted to add 34 and 21 you would write 34 on a piece of paper and then write 21 underneath it, like this:

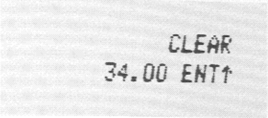
$$\begin{array}{r} 34 \\ \underline{21} \end{array}$$

and then you would add, like this:

$$\begin{array}{r} 34 \\ \underline{+21} \\ 55 \end{array}$$

Numbers are positioned the same way in the HP-91. Here's how it is done. (If you clear the stack first by pressing ■ CLEAR, the numbers in the stack will correspond to those shown here in the example.)

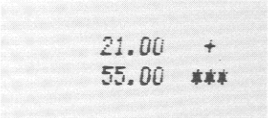
Press	**Display**	
■ CLEAR	0.00	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.


CLEAR
34.00 ENT↑

Now 34 and 21 are sitting vertically in the stack as shown below, so we can add.

T	0.00	
Z	0.00	
Y	34.00	
X	21.	Display

Press	**Display**	
+ PRINTx	55.00	The answer.

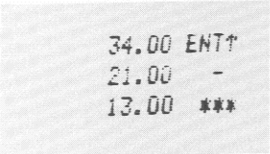

21.00 +
55.00 ***

The simple old-fashioned math notation helps explain how to use your calculator. Both numbers are always positioned in the stack in the natural order first; then the operation is executed when the function key is pressed. *There are no exceptions to this rule*. Subtraction, multiplication, and division work the same way. In each case, the data must be in the proper position before the operation can be performed.

To subtract 21 from 34:

$$\begin{array}{r} 34 \\ -21 \\ \hline \end{array}$$

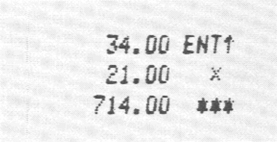
Press	**Display**	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.
− PRINTx	13.00	The answer.


34.00 ENT↑
21.00 −
13.00 ***

To multiply 34 by 21:

$$\begin{array}{r} 34 \\ \times 21 \\ \hline \end{array}$$

Press	Display	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.
× PRINTx	714.00	The answer.

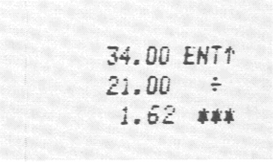


```
34.00 ENT↑
21.00    ×
714.00 ***
```

To divide 34 by 21:

$$\frac{34}{21}$$

Press	Display	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.
÷ PRINTx	1.62	The answer.



```
34.00 ENT↑
21.00    ÷
 1.62  ***
```

Chain Arithmetic

You've already learned how to key numbers into the calculator and perform calculations with them. In each case you first needed to position the numbers in the stack manually using the ENTER↑ key. However, the stack also performs many movements automatically. These automatic movements add to its computing efficiency and ease of use, and it is these movements that automatically store intermediate results. The stack automatically "lifts" every calculated number in the stack when a new number is keyed in because it knows that after it completes a calculation, any new digits you key in are a part of a new number. Also, the stack automatically "drops" when you perform a two-number operation.

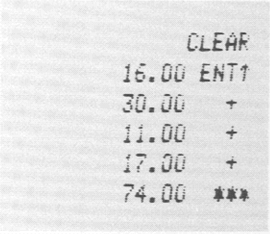
To see how it works, let's solve

$$16 + 30 + 11 + 17 = ?$$

If you press ■ CLEAR first, you will begin with zeros in all of the stack registers, as in the example below; but of course, you can also do the calculation without first clearing the stack.

Note: You can use the LIST: STACK function to monitor the changes in the stack contents.

Press	Stack Contents		
■ CLEAR	T	0.00	
16	Z	0.00	16 is keyed into the displayed X-register.
	Y	0.00	
	X	16.	
	T	0.00	
ENTER↑	Z	0.00	16 is copied into Y.
	Y	16.00	
	X	16.00	
	T	0.00	
30	Z	0.00	30 writes over the 16 in X.
	Y	16.00	
	X	30.	
	T	0.00	16 and 30 are added together. The answer, 46, is displayed.
	Z	0.00	
+	Y	0.00	
	X	46.00	
	T	0.00	11 is keyed into the displayed X-register. The 46 in the stack is automatically raised.
11	Z	0.00	
	Y	46.00	
	X	11.	
	T	0.00	
+	Z	0.00	46 and 11 are added together. The answer, 57, is displayed.
	Y	0.00	
	X	57.00	
	T	0.00	17 is keyed into the X-register. 57 is automatically entered into Y.
17	Z	0.00	
	Y	57.00	
	X	17.	



```
          CLEAR
16.00 ENT↑
30.00     +
11.00     +
17.00     +
74.00   ***
```

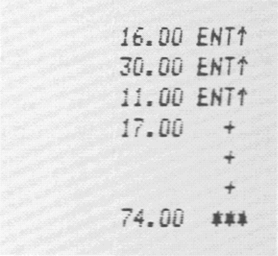
	T	0.00	
+	Z	0.00	57 and 17 are added together for the final answer.
PRINT x	Y	0.00	
	X	74.00	

After any calculation or number manipulation, the stack automatically lifts when a new number is keyed in. Because operations are performed when the operations are pressed, the length of such chain problems is unlimited unless a number in one of the stack registers exceeds the range of the calculator (up to 9.999999999 × 10^{99}).

In addition to the automatic stack lift after a calculation, the stack automatically drops during calculations involving both the X- and Y-registers. It happened in the above example, but let's do the problem differently to see this feature more clearly. First press CLx to clear the X-register. Now, again solve 16 + 30 + 11 + 17 = ?

Press	**Stack Contents**		
	T	0.00	
16	Z	0.00	16 is keyed into the displayed X-register.
	Y	0.00	
	X	16.	
	T	0.00	
ENTER↑	Z	0.00	16 is copied into Y.
	Y	16.00	
	X	16.00	
	T	0.00	
30	Z	0.00	30 is written over the 16 in X.
	Y	16.00	
	X	30.	
	T	0.00	
ENTER↑	Z	16.00	30 is entered into Y. 16 is lifted up to Z.
	Y	30.00	
	X	30.00	
	T	0.00	
11	Z	16.00	11 is keyed into the displayed X-register.
	Y	30.00	
	X	11.	

Press	Stack Contents		
	T	16.00	11 is copied into Y. 16 and 30 are lifted up to T and Z respectively.
ENTER↑	Z	30.00	
	Y	11.00	
	X	11.00	
	T	16.00	17 is written over the 11 in X.
17	Z	30.00	
	Y	11.00	
	X	17.	
	T	16.00	17 and 11 are added together and the rest of the stack drops. 16 drops to Z and is also duplicated in T. 30 and 28 are ready to be added.
+	Z	16.00	
	Y	30.00	
	X	28.00	
	T	16.00	30 and 28 are added together and the stack drops again. Now 16 and 58 are ready to be added.
+	Z	16.00	
	Y	16.00	
	X	58.00	
	T	16.00	16 and 58 are added together for the final answer and the stack continues to drop.
+	Z	16.00	
PRINT x	Y	16.00	
	X	74.00	



```
16.00 ENT↑
30.00 ENT↑
11.00 ENT↑
17.00    +
         +
         +
74.00  ***
```

The same dropping action also occurs with −, × and ÷. The number in T is duplicated in T and drops to Z, the number in Z drops to Y, and the numbers in Y and X combine to give the answer, which is visible in the X-register.

This automatic lift and drop of the stack give you tremendous computing power since you can retain and position intermediate results in long calculations without the necessity of reentering the numbers.

Order of Execution

When you see a problem like this one:

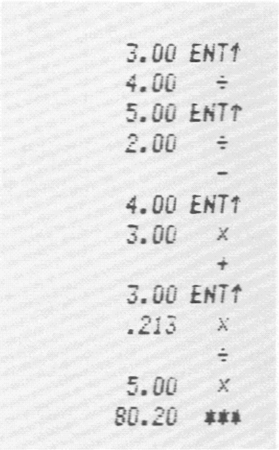
$$5 \times \left[(3 \div 4) - (5 \div 2) + (4 \times 3)\right] \div (3 \times .213)$$

you must decide where to begin before you ever press a key.

Experienced HP calculator users have determined that by starting every problem at its innermost number or parentheses and working outward, just as you would with paper and pencil, you maximize the efficiency and power of your HP calculator. Of course, with the HP-91 you have tremendous versatility in the order of execution.

For example, you could work the problem above by beginning at the left side of the equation and simply working through it in left-to-right order. All problems cannot be solved using left-to-right order, however, and the best order for solving any problem is to begin with the innermost parentheses and work outward. So, to solve the problem above:

Press	Display	
3	3.	
ENTER↑	3.00	
4	4.	
÷	0.75	Intermediate answer for (3 ÷ 4).
5	5.	
ENTER↑	5.00	
2	2.	
÷	2.50	Intermediate answer for (5 ÷ 2).
−	-1.75	Intermediate answer for (3 ÷ 4) − (5 ÷ 2).
4	4.	
ENTER↑	4.00	
3	3.	
×	12.00	Intermediate answer for (4 × 3).
+	10.25	Intermediate answer for (3 ÷ 4) − (5 ÷ 2) + (4 × 3).
3	3.	
ENTER↑	3.00	
.213	.213	
×	0.64	Intermediate answer for (3 × .213).
÷	16.04	
5	5.	The first number is keyed in.
×	80.20	
PRINT x	80.20	



```
 3.00 ENT↑
 4.00   ÷
 5.00 ENT↑
 2.00   ÷
        -
 4.00 ENT↑
 3.00   x
        +
 3.00 ENT↑
 .213   x
        ÷
 5.00   x
80.20 ***
```

Constant Arithmetic

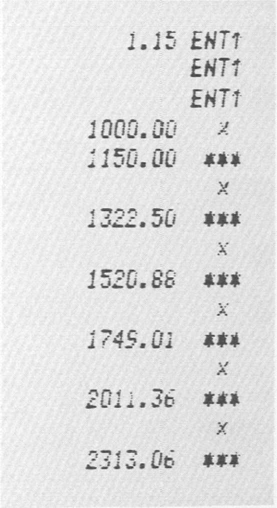
You may have noticed that whenever the stack drops because of a two-number operation (not because of R↓), the number in the T-register is reproduced there. This stack operation can be used to insert a constant into a problem.

Example: A bacteriologist tests a certain strain whose population typically increases by 15% each day. If he starts a sample culture of 1000, what will be the bacteria population at the end of each day for 6 consecutive days?

Method: Put the growth factor (1.15) in the Y-, Z-, and T-registers and put the original population (1000) in the X-register. Thereafter, you get the new population whenever you press ×. Try working this problem with the Print Mode switch set to ALL so that you'll have a record of all the answers without pressing PRINTx each time.

Slide the Print Mode switch MAN ALL NORM **to ALL.**

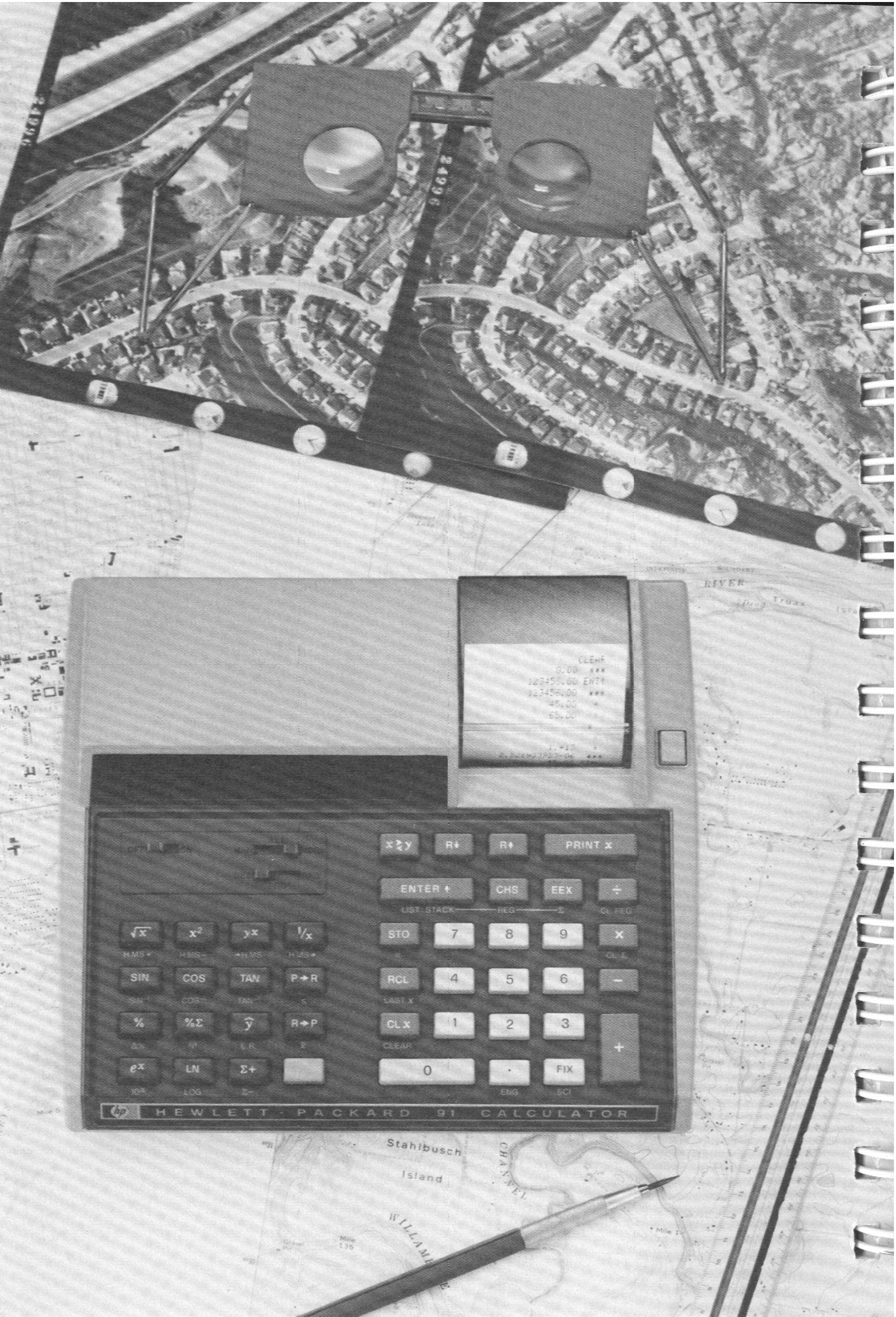
Press	Display	
1.15	1.15	Growth factor.
ENTER↑	1.15	
ENTER↑	1.15	
ENTER↑	1.15	Growth factor now in T.
1000	1000.	Starting population.
×	1150.00	Population after 1st day.
×	1322.50	Population after 2nd day.
×	1520.88	Population after 3rd day.
×	1749.01	Population after 4th day.
×	2011.36	Population after 5th day.
×	2313.06	Population after 6th day.



```
      1.15 ENT↑
           ENT↑
           ENT↑
   1000.00   ×
   1150.00 ***
             ×
   1322.50 ***
             ×
   1520.88 ***
             ×
   1749.01 ***
             ×
   2011.36 ***
             ×
   2313.06 ***
```

When you press × the first time, you calculate 1.15 × 1000. The result (1150.00) is displayed in the X-register and a new copy of the growth factor drops into the Y-register. Since a new copy of the growth factor is duplicated from the T-register each time the stack drops, you never have to reenter it.

Notice that performing a two-number operation such as × causes the number in the T-register to be duplicated there each time the stack is dropped. However, the R↓ key, since it rotates the contents of the stack registers, does not rewrite any number, but merely shifts the numbers that are already in the stack.


x⇄y
R↓
R↑
PRINT x
ENTER↑
CHS
EEX
÷
STO
7
8
9
×
RCL
4
5
6
−
CL x
1
2
3
+
0
·
FIX
√x
x²
yˣ
1/x
SIN
COS
TAN
P→R
%
%Σ
ŷ
R→P
eˣ
LN
Σ+
HEWLETT · PACKARD 91 CALCULATOR
Stahlbusch
Island
CHANNEL
WILLAME
RIVER

Section 4

Function Keys

The HP-91 has dozens of internal functions that allow you to compute answers to problems quickly and accurately. Each function operates immediately when the function key is pressed. To save printing time and paper, you might wish to learn how to use the functions with the Print Mode switch set to MAN. Or you might want to see every intermediate and final answer by setting the switch to ALL. Except when indicated, however, all examples in this section are illustrated with the Print Mode switch MAN ALL NORM set to NORM.

LAST X

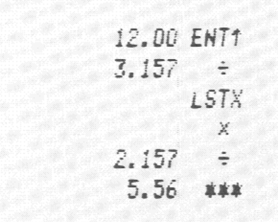
In addition to the four stack registers that automatically store intermediate results, the HP-91 also contains a separate automatic register, the LAST X register. This register preserves the value that was in the displayed X-register before the performance of a function. To place the contents of the LAST X register into the display again, press ■ LAST X.

Recovering from Mistakes

LAST X makes it easy to recover from keystroke mistakes, such as pressing the wrong function key or keying in the wrong number.

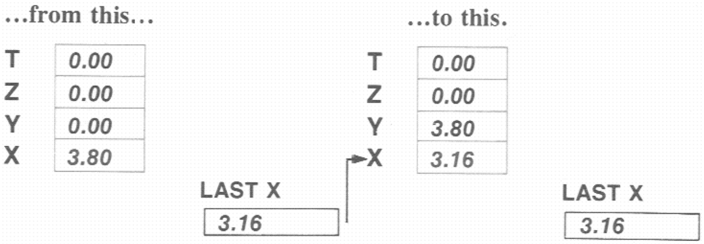
Example: Divide 12 by 2.157 after you have mistakenly divided by 3.157.

Press	Display	
12	12.	
ENTER↑	12.00	
3.157 ÷	3.80	Oops! You made a mistake.
■ LAST X	3.16	Retrieves that last entry (3.157).
×	12.00	You're back at the beginning.
2.157 ÷	5.56	The correct answer.
PRINT x	5.56	



```
12.00 ENT↑
3.157    ÷
      LSTX
         ×
2.157    ÷
 5.56  ***
```

In this example, when LAST X is pressed, the contents of the stack and LAST X register are changed…



This makes possible the correction illustrated in the example above.

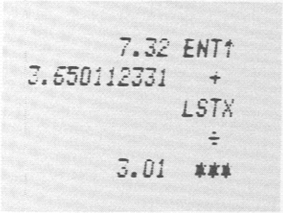
Recovering a Number

The LAST X register is useful in calculations where a number occurs more than once. By recovering a number using LAST X, you do not have to key that number into the calculator again.

Example: Calculate

$$\frac{7.32 + 3.650112331}{3.650112331}$$

Press	Display	
7.32	7.32	
ENTER↑	7.32	
3.650112331	3.650112331	
+	10.97	Intermediate answer.
■ LAST X	3.65	Recalls 3.650112331 to X-register.
÷	3.01	The answer.
PRINT x	3.01	

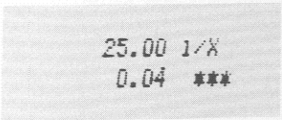


```
       7.32 ENT↑
3.650112331    +
            LSTX
               ÷
       3.01  ***
```

Reciprocals

To calculate the reciprocal of a number in the displayed X-register, key in the number, then press 1/x. For example, to calculate the reciprocal of 25:

Press	Display
25 1/x	0.04
PRINT x	0.04



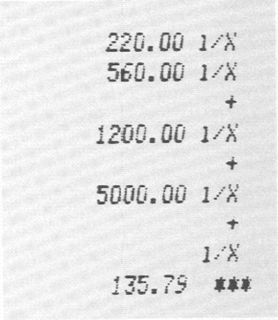
```
25.00  1/X
 0.04  ***
```

You can also calculate the reciprocal of a value in a previous calculation without reentering the number.

Example: In an electrical circuit, four resistors are connected in parallel. Their values are 220 ohms, 560 ohms, 1.2 kilohms, and 5 kilohms. What is the total resistance of the circuit?

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{1}{\frac{1}{220} + \frac{1}{560} + \frac{1}{1200} + \frac{1}{5000}}$$

Press	Display	
220 1/x	4.545454545–03	
560 1/x	1.785714286–03	
+	0.01	
1200 1/x	8.333333333–03	
+	0.01	
5000 1/x	2.000000000–04	
+	0.01	
1/x	135.79	The reciprocal of the sum of reciprocals yields the answer in ohms.
PRINT x	135.79	



```
      220.00 1/X
      560.00 1/X
               +
     1200.00 1/X
               +
     5000.00 1/X
               +
             1/X
     135.79  ***
```

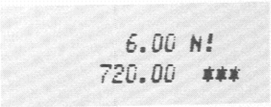
Factorials

The N! *(factorial)* key permits you to handle permutations and combinations with ease. To calculate the factorial of a positive integer in the displayed X-register, press ■ N!.

Example: Calculate the number of ways that six people can line up for a photograph.

Method: $P_6^6 = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1.$

Press	Display	
6	6.	
■ N!	720.00	The answer.
PRINT x	720.00	



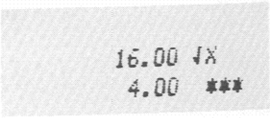
```
    6.00 N!
  720.00  ***
```

The calculator overflows for factorials of numbers greater than 69.

Square Roots

To calculate the square root of a number in the displayed X-register, press √x. For example, to find the square root of 16:

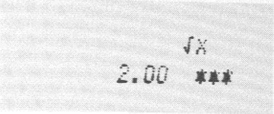
Press	Display
16 √x	4.00
PRINT x	4.00



```
   16.00 √X
    4.00  ***
```

To find the square root of the result:

Press	Display
√x	2.00
PRINT x	2.00

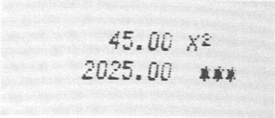


```
        √X
2.00   ***
```

Squaring

To square a number in the displayed X-register, press x². For example, to find the square of 45:

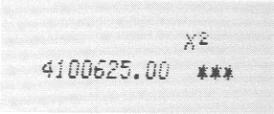
Press	Display
45 x²	2025.00
PRINT x	2025.00



```
  45.00 X²
2025.00  ***
```

To find the square of the result:

Press	Display
x²	4100625.00
PRINT x	4100625.00

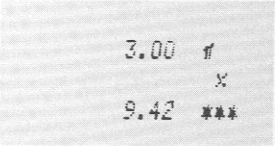


```
                X²
4100625.00     ***
```

Using Pi

The value π accurate to 10 places (3.141592654) is provided as a fixed constant in the HP-91. Merely press ■ π whenever you need it in a calculation. For example, to calculate 3π:

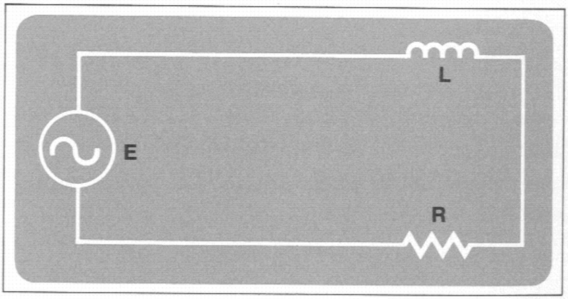
Press	Display
3 ■ π ×	9.42
PRINT x	9.42



```
3.00   π
       ×
9.42  ***
```

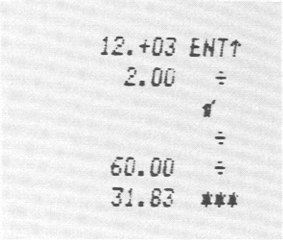
Example: In the schematic diagram below, X_L is 12 kilohms, R is 7 kilohms, E is 120 volts, and f is 60 Hz. Find the inductance of the coil L in henries according to the formula:

$$L = \frac{X_L}{2\pi f}.$$



$$L = \frac{X_L}{2\pi f} = \frac{12,000}{2 \times \pi \times 60}$$

Press	Display	
12 EEX 3	12. 03	
ENTER↑	12000.00	
2 ÷	6000.00	
■ π ÷	1909.86	
60 ÷	31.83	Henries.
PRINTx	31.83	



```
12.+03 ENT↑
  2.00    ÷
          π
          ÷
 60.00    ÷
 31.83  ***
```

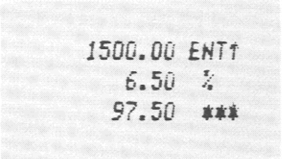
Percentages

The % key is a two-number function which allows you to compute percentages. To find the percentage of a number:

1. Key in the base number.
2. Press ENTER↑.
3. Key in the number representing percent rate.
4. Press %.

For example, to calculate a sales tax of 6.5% on a purchase of $1500:

Press	Display	
1500 ENTER↑	1500.00	Base number.
6.5	6.5	Percent rate.
%	97.50	The answer.
PRINTx	97.50	



```
1500.00 ENT↑
   6.50    %
  97.50  ***
```

6.5% of $1500 is $97.50.

In the above example, when the % key is pressed, the calculated answer writes over the percentage rate in the X-register, and the base number is preserved in the Y-register.

When you press %, the stack contents were changed…

…from this…

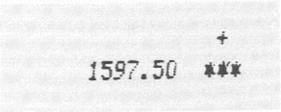
T	0.00
Z	0.00
Y	1500.00
X	6.5

…to this.

T	0.00
Z	0.00
Y	1500.00
X	97.50

Since the purchase price is now in the Y-register and the amount of tax is in the X-register, the total amount can be obtained by simply adding:

Press	Display	
+	1597.50	Total of price and sales tax combined.
PRINTx	1597.50	



```
           +
1597.50  ***
```

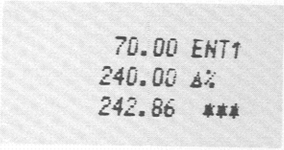
Percent of Change

The [△%] *(percent of change)* key is a two-number function that gives the percent increase or decrease from Y to X. To find the percent of change:

1. Key in the base number (usually, the number that happens first in time).
2. Press ENTER↑.
3. Key in the second number.
4. Press ■ [△%].

Example: Find the percent of increase of your rent 10 years ago (($70 per month) to today ($240 per month).

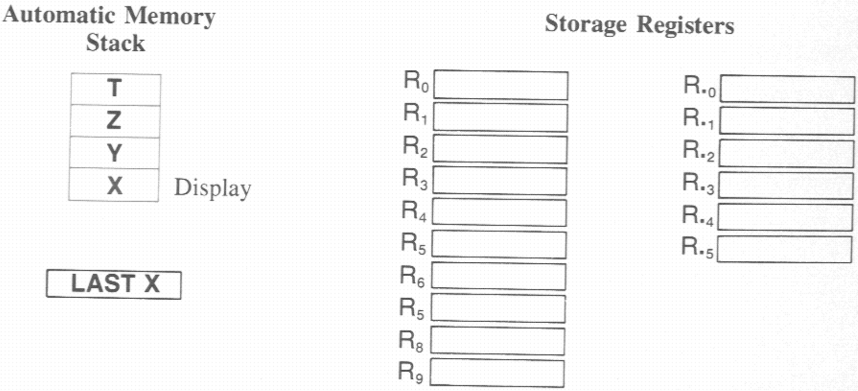
Press	**Display**	
70 ENTER↑	70.00	
240 ■ [△%]	242.86	Percent increase.
PRINT x	242.86	



70.00 ENT↑
240.00 Δ%
242.86 ***

Storage Registers

In addition to automatic storage of intermediate results that is provided by the four-register automatic memory stack, the HP-91 also has 16 addressable storage registers that are unaffected by operations within the stack. These storage registers allow you to set aside numbers as constants or for use in later calculations.



The addresses of the storage registers are indicated by number keys [0] through [9], and by [•] [0] through [•] [5].

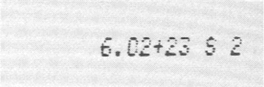
Storing Numbers

To store a displayed number in any of storage registers R_0 through R_9:

1. Press STO *(store)*.
2. Press the number key of the applicable register address ([0] through [9]).

For example, to store Avogadro's number (approximately 6.02×10^{23}) in register R_2:

Press	Display
6.02 EEX 23	6.02 23
STO 2	6.020000000 23



```
6.02+23 S 2
```

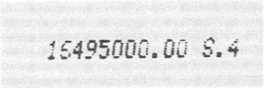
Avogadro's number is now stored in register R_2. Notice that when a number is stored, it is merely copied into the storage register, so 6.02×10^{23} also remains in the displayed X-register.

To store a displayed number in any of storage registers $R_{.0}$ through $R_{.5}$:

1. Press STO.
2. Press the decimal point key ⊡.
3. Press the number key of the applicable register address (0 through 5).

For example, to store 16,495,000 (the number of persons carried daily by the Japanese National Railway) in register $R_{.4}$:

Press	Display
16495000	16495000.
STO ⊡ 4	16495000.00



```
16495000.00 S.4
```

The number has been copied into storage register $R_{.4}$ and also remains in the displayed X-register.

Recalling Numbers

Numbers are recalled from storage registers back into the displayed X-register in much the same way as they are stored. To recall a number from any of storage registers R_0 through R_9:

1. Press RCL *(recall)*.
2. Press the number key of the applicable register address (0 through 9).

For example, to recall Avogadro's number from register R_2:

Press	Display
RCL 2	6.020000000 23

```
R 2
```

To recall a number from any of registers $R_{.0}$ through $R_{.5}$:

1. Press RCL.
2. Press the decimal point key ⊡.
3. Press the number key of the applicable register address (0 through 5).

For example, to recall the number of persons carried daily by the Japanese National Railway:

Press	Display
RCL ⊡ 4	16495000.00

```
R.4
```

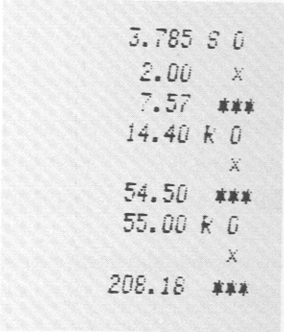
Recalling a number causes the stack to lift unless the preceding keystroke was ENTER↑, CLX, or Σ+ (more about Σ+ later).

When you recall a number, it is copied from the storage register into the display, and it also remains in the storage register. You can recall a number from a storage register any number of times without altering it—the number will remain in the storage register as a 10-digit number with a two-digit exponent of 10 until you overwrite it by storing another number there, or until you clear the storage registers.

Example: Three tanks have capacities in U.S. units of 2.0, 14.4, and 55.0 gallons, respectively. If 1 U.S. gallon is equivalent to 3.785 liters, what is the capacity in liters of each of the tanks?

Method: Place the conversion constant in one of the storage registers and bring it out as required.

Press	**Display**	
3.785 STO 0	3.79	Constant placed in register R_0.
2 ×	7.57	Capacity in liters of 1st tank.
PRINTx	7.57	
14.4 RCL 0 ×	54.50	Capacity in liters of 2nd tank.
PRINTx	54.50	
55 RCL 0 ×	208.18	Capacity in liters of 3rd tank.
PRINTx	208.18	

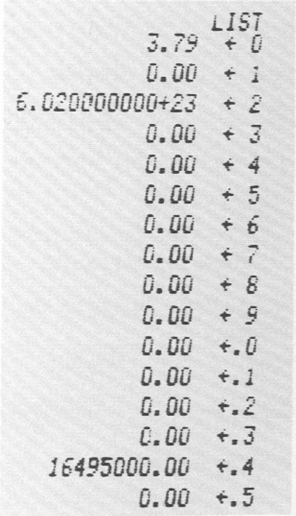


```
  3.785 S 0
   2.00   x
   7.57 ***
  14.40 R 0
          x
  54.50 ***
  55.00 R 0
          x
 208.18 ***
```

Listing the Storage Registers

You can see the contents of all of the storage registers at any time by pressing ■ LIST: REG to print the contents of all storage registers. If you have worked through the examples above, a listing of storage contents should look like the one shown here.

Press	**Display**
■ LIST: REG	208.18



```
                LIST
          3.79  ← 0
          0.00  ← 1
6.020000000+23  ← 2
          0.00  ← 3
          0.00  ← 4
          0.00  ← 5
          0.00  ← 6
          0.00  ← 7
          0.00  ← 8
          0.00  ← 9
          0.00  ←.0
          0.00  ←.1
          0.00  ←.2
          0.00  ←.3
   16495000.00  ←.4
          0.00  ←.5
```

If you want only a partial listing of storage registers, you can stop the printing of them at any time by holding down the paper advance pushbutton for about one second, then releasing it.

Clearing Storage Registers

Notice that even though you have recalled the numbers from storage registers R_0, R_2, and $R_{.4}$, the numbers remain in the registers. A number stored in one of the storage registers may be recalled into the display any number of times, and it will still remain in the storage register. Storage registers may be cleared in any of three ways:

- To replace a number in a storage register, merely store another number there. To clear a storage register, replace the number in it with zero. For example, to clear storage register R_2, press 0 STO 2.
- To clear *all* storage registers back to zero at one time, press ■ CLEAR. Besides replacing the contents of each storage register with zero, this also replaces the contents of the automatic memory stack with zeros as well.
- When the HP-91 is first turned ON, it "wakes up" with the quantity zero in each of the storage registers and in each of the automatic stack registers. Thus, turning the calculator OFF, then ON, also clears the storage registers and the stack.

You can also clear storage registers R_0 through R_9 or registers $R_{.0}$ through $R_{.5}$ while leaving the remaining registers and the stack intact.

- Press ■ CL REG to clear only storage registers R_0 through R_9 while preserving the contents of the stack and storage registers $R_{.0}$ through $R_{.5}$.
- Press ■ CLΣ to clear only storage registers $R_{.0}$ through $R_{.5}$ while preserving the contents of the stack and storage registers R_0 through R_9.

Storage Register Arithmetic

Arithmetic can be performed *upon* the contents of storage registers R_0 through R_9 by pressing STO followed by the arithmetic function key followed in turn by the register address. For example:

Press	**Result**
STO + 1	Number in displayed X-register added to contents of storage register R_1, and sum placed into R_1; $(r_1 + x \rightarrow R_1)$.
STO − 2	Number in displayed X-register subtracted from contents of storage register R_2, and difference placed into R_2; $(r_2 - x \rightarrow R_2)$.
STO × 3	Number in displayed X-register multiplied by contents of storage register R_3, and the product placed into R_3; $\left[(r_3)x \rightarrow R_3\right]$.
STO ÷ 4	Contents of storage register R_4 divided by number in displayed X-register, and quotient placed into register R_4; $(r_4 \div x \rightarrow R_4)$.

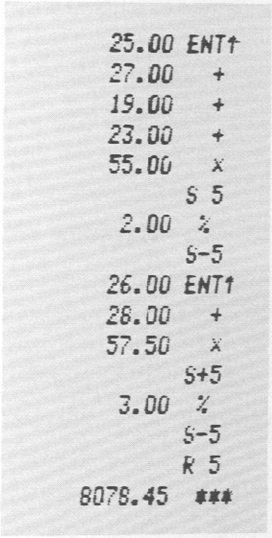
When storage register arithmetic operations are performed, the answer is written into the selected storage register, while the contents of the displayed X-register and the rest of the stack remain unchanged.

Notice that you can perform storage register arithmetic upon the contents of *only* storage registers R_0 through R_9. You *cannot* perform storage register arithmetic upon storage registers $R_{.0}$ through $R_{.5}$.

Example: During harvest, farmer Flem Snopes trucks tomatoes to the cannery for three days. On Monday and Tuesday he hauls loads of 25 tons, 27 tons, 19 tons, and 23 tons, for which the cannery pays him $55 per ton. On Wednesday the price rises to $57.50 per ton, and Snopes ships loads of 26 tons and 28 tons. If the cannery deducts 2% of the price on Monday and Tuesday because of blight on the tomatoes, and 3% of the price on Wednesday, what is Snopes' total net income?

Method: Keep total amount in a storage register while using the stack to add tonnages and calculate amounts of loss.

Press	**Display**	
25 ENTER↑	25.00	
27 +	52.00	
19 + 23 +	94.00	Total of Monday's and Tuesday's tonnage.
55 ×	5170.00	Gross amount for Monday and Tuesday.
STO 5	5170.00	Gross placed in storage register R_5.
2 %	103.40	Deductions for Monday and Tuesday.
STO − 5	103.40	Deductions subtracted from total in storage register R_5.
26 ENTER↑	26.00	
28 +	54.00	Wednesday's tonnage.
57.50 ×	3105.00	Gross amount for Wednesday.
STO + 5	3105.00	Wednesday's gross amount added to total in storage register R_5.
3 %	93.15	Deduction for Wednesday.
STO − 5	93.15	Wednesday deduction subtracted from total in storage register R_5.
RCL 5	8078.45	Snopes' total net income from his tomatoes.
PRINT x	8078.45	



```
  25.00 ENT↑
  27.00   +
  19.00   +
  23.00   +
  55.00   ×
        S 5
   2.00   %
        S-5
  26.00 ENT↑
  28.00   +
  57.50   ×
        S+5
   3.00   %
        S-5
        R 5
8078.45 ***
```

(You could also work this problem using the stack alone, but doing it as shown here illustrates how storage register arithmetic can be used to maintain and update different running totals.)

Trigonometric Functions

Your HP-91 provides you with six trigonometric functions, which operate in decimal degrees, radians, or grads. You can convert angles between decimal degrees and *degrees minutes, seconds,* and you can add and subtract angles in any of these forms without converting them.

Trigonometric Modes

The Trigonometric Mode switch 

is used to select whether angles are assumed by the calculator to be specified in decimal degrees, radians, or grads.

Note: 360 degrees = 400 grads = 2π radians.

Functions

The six trigonometric functions provided by the calculator are:

SIN	(sine)
■ SIN^{-1}	(arc sine)
COS	(cosine)
■ COS^{-1}	(arc cosine)
TAN	(tangent)
■ TAN^{-1}	(arc tangent)

Each trigonometric function assumes that angles are in decimal degrees, radians, or grads, depending upon the position of the Trigonometric Mode switch.

All trigonometric functions are one-number functions, so to use them, you key in the number, then press the function key(s).

Example 1: Find the cosine of 35°.

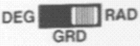
First, specify degrees mode by sliding the Trigonometric Mode switch 

to DEG.

Press	Display
35	35.00
COS	0.82
PRINT x	0.82

```
35.00 COS
 0.82  ***
```

Example 2: Find the arc sine in radians of .964.

First, specify radians mode by sliding the Trigonometric Mode switch 

to RAD.

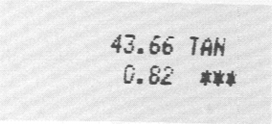
Press	Display	
.964	.964	
■ SIN^{-1}	1.30	Radians.
PRINT x	1.30	

```
.964 SIN-1
1.30  ***
```

Example 3: Find the tangent of 43.66 grads.

Slide the Trigonometric Mode switch DEG ▥ RAD GRD to GRD.

Press	Display
43.66	43.66
TAN	0.82
PRINTx	0.82



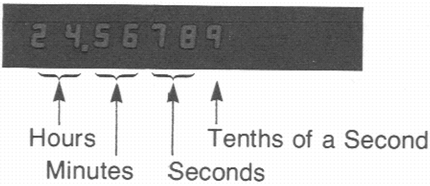
```
43.66 TAN
 0.82  ***
```

Hours, Minutes, Seconds/Decimal Hours Conversions

Using the HP-91, you can change time specified in decimal hours to *hours, minutes, seconds* format by using the →H.MS *(to hours, minutes, seconds)* key; you can also change from *hours, minutes, seconds* to decimal hours by using the H.MS→ *(from hours, minutes, seconds)* key.

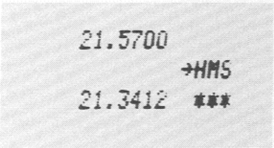
When a time is displayed or printed in *hours, minutes, seconds* format, the digits specifying *hours* occur to the left of the decimal point, while the digits specifying *minutes, seconds,* and *fractions of seconds* occur to the right of the decimal point.

Hours, Minutes, Seconds Display



To convert from decimal hours to *hours, minutes, seconds,* simply key in the value for decimal hours and press ■ →H.MS. For example, to change 21.57 hours to *hours, minutes, seconds:*

Press	Display	
21.57	21.57	Key in the decimal time.
■ 4	21.5700	Reset display format.
■ →H.MS	21.3412	This is 21 hours, 34 minutes, 12 seconds.
PRINTx	21.3412	

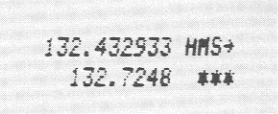


```
21.5700
        →HMS
21.3412  ***
```

Notice that the display is *not* automatically switched to show you more than the normal two digits after the decimal point (FIX 2), so to see the digits for *seconds,* you had to reset the display format to FIX 4.

To convert from *hours, minutes, seconds* to decimal hours, simply key in the value for *hours, minutes, seconds* in that format and press ■ [H.MS→]. For example, to convert 132 hours, 43 minutes, and 29.33 seconds to its decimal degree equivalent:

Press	Display	
132.432933	132.432933	This is 132 hours, 43 minutes, 29.33 seconds.
■ [H.MS→]	132.7248	This is 132.7248 hours.
PRINTx	132.7248	

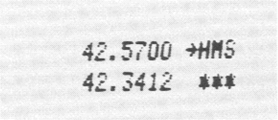


```
132.432933 HMS→
  132.7248  ***
```

Using the [→H.MS] and [H.MS→] operations, you can also convert angles specified in decimal degrees to *degrees, minutes, seconds,* and vice versa. The format for *degrees, minutes, seconds* is the same as for *hours, minutes, seconds.*

Example: Convert 42.57 decimal degrees to *degrees, minutes, seconds.*

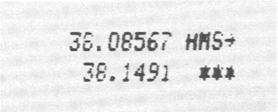
Press	Display	
42.57	42.57	Key in the angle.
■ [→H.MS]	42.3412	This means 42°34′ 12″. (Display assumes [FIX] 4 notation remains specified from previous example.)
PRINTx	42.3412	



```
42.5700 →HMS
42.3412  ***
```

Example: Convert 38°8′56.7″ to its decimal equivalent.

Press	Display	
38.08567	38.08567	Key in the angle.
■ [H.MS→]	38.1491	Answer in decimal degrees. ([FIX] 4 display specified from previous example.)
PRINTx	38.1491	



```
38.08567 HMS→
 38.1491  ***
```

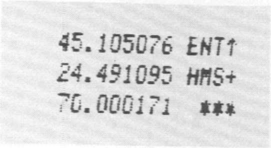
Adding and Subtracting Time and Angles

To add or subtract decimal hours, merely key in the numbers for the decimal hours and press [+] or [−]. To add or subtract *hours, minutes, seconds,* use the [H.MS+] *(add hours, minutes, seconds)* and [H.MS−] *(subtract hours, minutes, seconds)* keys.

Likewise, angles specified in *degrees, minutes, seconds* are added by pressing ■ [H.MS+] and subtracted by pressing ■ [H.MS−].

Example: Find the sum of 45 hours, 10 minutes, 50.76 seconds and 24 hours, 49 minutes, 10.95 seconds.

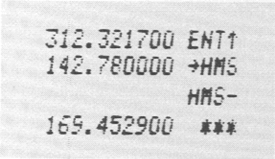
Press	Display	
45.105076	45.105076	
ENTER↑	45.1051	FIX 4 notation from previous example.
24.491095	24.491095	
■ H.MS+	70.0002	
■ 6	70.000171	
PRINT x	70.000171	



```
45.105076 ENT↑
24.491095 HMS+
70.000171  ***
```

Example: Subtract 142.78° from 312°32′17″, with the answer in *degrees, minutes, seconds* format.

Press	Display	
312.3217	312.3217	
ENTER↑	312.321700	FIX 6 from previous example.
142.78	142.78	Decimal degrees.
■ →H.MS	142.464800	To degrees, minutes, seconds.
■ H.MS−	169.452900	This is 169°45′29″.
PRINT x FIX 2	169.45	Display mode reset.



```
312.321700 ENT↑
142.780000 →HMS
            HMS-
169.452900  ***
```

In the HP-91, trigonometric functions assume angles in decimal degrees, decimal radians, or decimal grads, so if you want to compute any trigonometric functions of an angle given in *degrees, minutes, and seconds,* you must first convert the angle to decimal degrees.

Example: Lovesick sailor Oscar Odysseus dwells on the island of Tristan da Cunha (37°03′S, 12°18′W), and his sweetheart, Penelope, lives on the nearest island. Unfortunately for the course of true love, however, Tristan da Cunha is the most isolated inhabited spot in the world. If Penelope lives on the island of St. Helena (15°55′S, 5°43′W), use the following formula to calculate the great circle distance that Odysseus must sail in order to court her.

$$\text{Distance} = \cos^{-1}\left[\sin(LAT_s)\sin(LAT_d) + \cos(LAT_s)\cos(LAT_d)\cos(LNG_d - LNG_s)\right] \times 60.$$

Where: LAT_s and LNG_s = latitude and longitude of the source (Tristan da Cunha).

LAT_d and LNG_d = latitude and longitude of the destination.

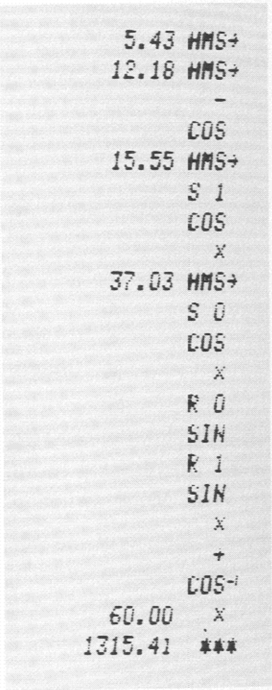
Solution: Convert all *degrees, minutes, seconds* entries into decimal degrees as you key them in. The equation for the great circle distance from Tristan da Cunha to the nearest inhabited land is:

$$\text{Distance} = \cos^{-1}\left[\sin(37°03') \sin(15°55') + \cos(37°03') \cos(15°55') \cos(5°43'\text{W} - 12°18''\text{W})\right] \times 60$$

First, ensure that the Trigonometric Mode switch DEG ▮ RAD GRD **is set to DEG.**

Press	Display
5.43	5.43
■ H.MS→	5.72
12.18	12.18
■ H.MS→ −	-6.58
COS	0.99
15.55	15.55
■ H.MS→	15.92
STO 1	15.92
COS	0.96
×	0.96
37.03 ■	
H.MS→ STO 0	37.05
COS	0.80
×	0.76
RCL 0 SIN	0.60
RCL 1 SIN	0.27
×	0.17
+	0.93
■ COS⁻¹	21.92
60 × PRINT x	1315.41

Distance in nautical miles that Odysseus must sail to visit Penelope.



```
   5.43 HMS→
  12.18 HMS→
           -
        COS
  15.55 HMS→
        S 1
        COS
          x
  37.03 HMS→
        S 0
        COS
          x
        R 0
        SIN
        R 1
        SIN
          x
          +
        COS⁻¹
  60.00   x
1315.41  ***
```

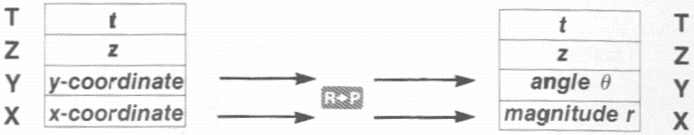
Polar/Rectangular Coordinate Conversions

Two functions are provided for polar/rectangular coordinate conversions. Angle θ is assumed in decimal degrees, radians, or grads, depending upon the position of the Trigonometric Mode switch.

To convert from rectangular x, y coordinates to polar r, θ coordinates (magnitude and angle, respectively):

1. Key in the y-coordinate.
2. Press ENTER↑ to raise the y-coordinate value to the Y-register of the stack.
3. Key in the x-coordinate.
4. Press the R→P *(rectangular to polar)* key. Magnitude r then appears in the X-register and angle θ is placed in the Y-register. (To display the value for θ, you press x⇄y.)

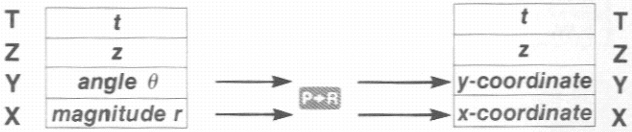
The following diagram shows how the stack contents change when you press R→P.



To convert from polar r, θ, coordinates to rectangular x, y, coordinates:

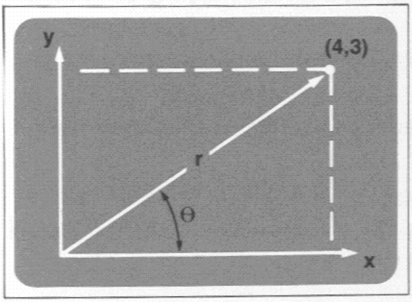
1. Key in the value for the angle θ.
2. Press ENTER↑ to raise the value for θ to the Y-register of the stack.
3. Key in the value for magnitude r.
4. Press the P→R *(polar to rectangular)* key. The x-coordinate then appears in the displayed X-register and the y-coordinate is placed in the Y-register. (To display the value for the y-coordinate, you can press x≷y.)

The following diagram shows how the stack contents change when you press P→R.



After you have pressed R→P or P→R, you can use the x≷y key to bring the calculated angle θ or the calculated y-coordinate into the X-register for viewing or further calculation. With the Print Mode switch set to MAN or NORM, you must also use the x≷y key to *print* these values. With the Print Mode switch MAN ALL NORM set to ALL, however, both computed values are printed automatically when you press R→P or P→R. (No three-asterisk label is printed next to these results in ALL. Instead, they are conveniently labled with symbols for x and y or for r and θ. These results are printed in the specified display format.)

Example 1: Convert rectangular coordinates (4, 3) to polar form with the angle expressed in radians.



Slide the Trigonometric Mode switch DEG GRD RAD to RAD.

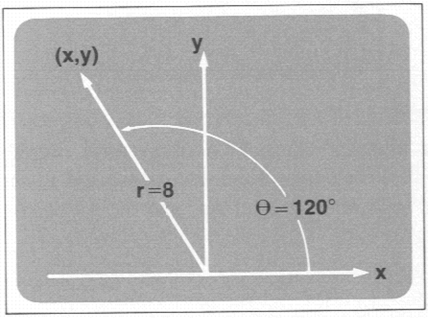
Press	Display	
3 ENTER↑	3.00	y-coordinate entered into the Y-register.
4	4.	x-coordinate keyed into the X-register.
R→P	5.00	Magnitude *r*.
PRINTx	5.00	
x⇄y	0.64	Angle θ in radians.
PRINTx	0.64	

```
 3.00 ENT↑
 4.00 R→P
 5.00  ***
       X⇄Y
 0.64  ***
```

Now slide the Print Mode switch MAN ALL NORM to ALL and work the problem again.

```
 3.00 ENT↑
 4.00 R→P
 0.64  θ
 5.00  R
```

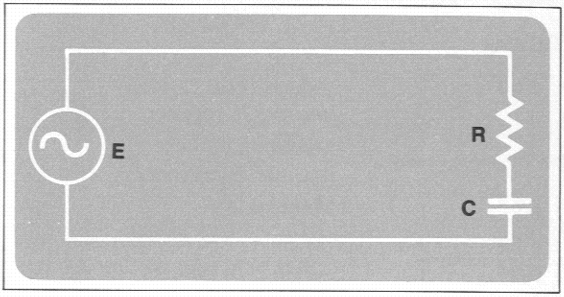
Example 2: Convert polar coordinates (8, 120°) to rectangular coordinates.



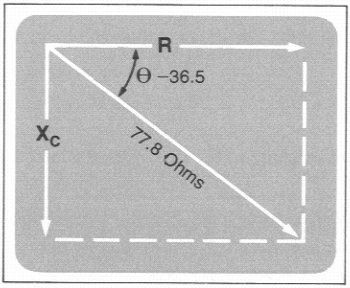
Slide the Trigonometric Mode switch DEG GRD RAD to DEG.
Ensure that the Print Mode switch MAN ALL NORM is set to ALL.

Press	Display	
120 ENTER↑	120.00	Angle θ entered into the Y-register.
8	8.	Magnitude *r* placed in displayed X-register.
P→R	–4.00	x-coordinate.
x⇄y	6.93	y-coordinate brought into displayed X-register for use, if desired.

```
120.00 ENT↑
  8.00 P→R
  6.93  Y
 -4.00  X
       X⇄Y
  6.93  ***
```



Example 3: Engineer Tobias Slothrop has determined that in the RC circuit shown above, the total impedance is 77.8 ohms and voltage lags current by 36.5°. What are the values of resistance R and capacitive reactance X_c in the circuit?



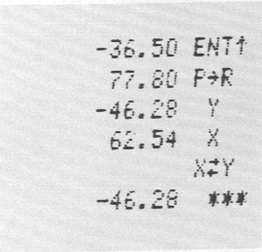
Method: Draw a vector diagram using 77.8 ohms total impedance for polar magnitude r and −36.5° for angle θ. When the values are converted to rectangular coordinates, the x-coordinate value yields resistance R in ohms, and the y-coordinate value yields reactance X_c in ohms.

Solution:

Ensure that the Trigonometric Mode switch DEG GRD RAD **is set to DEG.**

Ensure that the Print Mode switch MAN ALL NORM **is set to ALL.**

Press	**Display**	
36.5 CHS	−36.5	
ENTER↑	−36.50	
77.8	77.8	
P→R	62.54	Resistance R in ohms.
x⇄y	−46.28	Reactance X_c, 46.28 ohms, available in displayed X-register.



```
-36.50 ENT↑
 77.80 P→R
-46.28  Y
 62.54  X
       X⇄Y
-46.28 ***
```

Logarithmic and Exponential Functions

Logarithms

The HP-91 computes both natural and common logarithms as well as their inverse functions (antilogarithms):

LN	is $\log_e$ (natural log). It takes the log of the value in the X-register to base e (2.718. . .).
e^x	is antilog_e (natural antilog). It raises e (2.718. . .) to the power of the value in the X-register. (To display the value of e, press 1 e^x.)
LOG	is $\log_{10}$ (common log). It computes the log of the value in the X-register to base 10.
10^x	is antilog_{10} (common antilog). It raises 10 to the power of the value in the X-register.

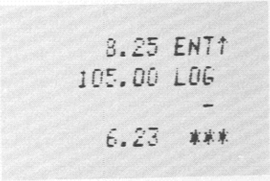
Example 1: The 1906 San Francisco earthquake, with a magnitude of 8.25 on the Richter Scale is estimated to be 105 times greater than the Nicaragua quake of 1972. What would be the magnitude of the latter on the Richter Scale?

The equation is:

$$R_1 = R_2 - \log\frac{M_2}{M_1} = 8.25 - \left(\log\frac{105}{1}\right)$$

Solution:
(If you want your printed copy to match the one shown here, slide the Print Mode switch MAN ALL NORM **to NORM.)**

Press	Display	
8.25 ENTER↑	8.25	
105 ■ LOG	2.02	
−	6.23	Rating on Richter scale.
PRINT X	6.23	



```
 8.25 ENT↑
105.00 LOG
         -
  6.23  ***
```

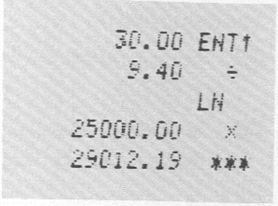
Example 2: Having lost most of his equipment in a blinding snowstorm, ace explorer Jason Quarmorte is using an ordinary barometer as an altimeter. After measuring the sea level pressure (30 inches of mercury) he climbs until the barometer indicates 9.4 inches of mercury. Although the exact relationship of pressure and altitude is a function of many factors, Quarmorte knows that an *approximation* is given by the formula:

$$\text{Altitude (feet)} = 25{,}000\, ln\frac{30}{\text{Pressure}} = 25{,}000\, ln\frac{30}{9.4}$$

Where is Jason Quarmorte?

Solution:

Press	Display	
30 ENTER↑	30.00	
9.4 ÷	3.19	
LN	1.16	
25000	25000	
×	29012.19	Altitude in feet.
PRINT x	29012.19	



```
   30.00 ENT↑
    9.40  ÷
          LN
25000.00  ×
29012.19 ***
```

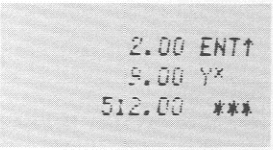
Quarmorte is probably near the summit of Mount Everest (29,028 feet).

Raising Numbers to Powers

The y^x key is used to raise numbers to powers. Using the y^x permits you to raise a positive real number to any real power—that is, the power may be positive or negative, and it may be an integer, a fraction, or a mixed number. y^x also permits you to raise any negative real number to the power of any integer (within the calculating range of the HP-91, of course).

For example, to calculate 2^9 (that is, 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2):

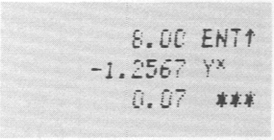
Press	Display
2 ENTER↑ 9	9.
y^x	512.00
PRINT x	512.00



```
  2.00 ENT↑
  9.00 Yˣ
512.00 ***
```

To calculate $8^{-1.2567}$:

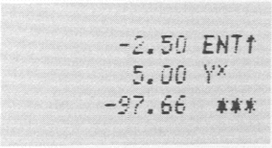
Press	Display
8 ENTER↑	8.00
1.2567 CHS	-1.2567
y^x	0.07
PRINT x	0.07



```
   8.00 ENT↑
-1.2567 Yˣ
   0.07 ***
```

To calculate $(-2.5)^5$:

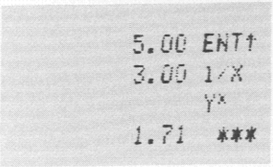
Press	Display
2.5 CHS	-2.5
ENTER↑	-2.50
5 y^x	-97.66
PRINT x	-97.66



```
 -2.50 ENT↑
  5.00 Yˣ
-97.66 ***
```

In conjunction with $1/x$, y^x provides a simple way to extract roots. For example, find the cube root of 5. (This is equivalent to $5^{1/3}$.)

Press	Display	
5 ENTER↑	5.00	
3 $1/x$	0.33	Reciprocal of 3.
y^x PRINT x	1.71	Cube root of 5.



```
5.00 ENT↑
3.00 1/X
     Yˣ
1.71 ***
```

Example: In a rather overoptimistic effort to break the speed of sound, highflying pilot Ike Daedalus cranks open the throttle on his surplus Hawker Siddeley Harrier aircraft. From his instruments he reads a pressure altitude (PALT) of 25,500 feet with a calibrated airspeed (CAS) of 350 knots. What is the flight mach number

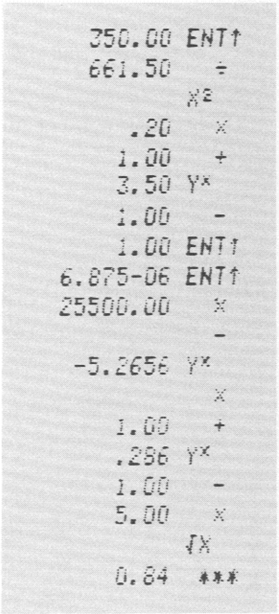
$$M = \frac{\text{speed of aircraft}}{\text{speed of sound}}$$

if the following formula is applicable?

$$M = \sqrt{5\left[\left(\left\{\left[\left(1+0.2\left[\frac{350}{661.5}\right]^{2}\right)^{3.5}-1\right]\left[1-\left(6.875\times10^{-6}\right)25{,}500\right]^{-5.2656}\right\}+1\right)^{0.286}-1\right]}$$

Method: The most efficient place to begin work on this problem is at the innermost set of brackets. So begin by solving for the quantity $\left[\frac{350}{661.5}\right]^{2}$ and proceed outward from there.

Press	**Display**	
350 ENTER↑	350.00	
661.5 ÷	0.53	
x^2	0.28	Square of bracketed quantity.
.2 × 1 +	1.06	
3.5 y^x 1 −	0.21	Contents of left-hand set of brackets are in the stack.
1 ENTER↑	1.00	
6.875 EEX	6.875 00	
CHS 6 ENTER↑	6.875000000−06	
25500 × −	0.82	
5.2656 CHS	−5.2656	Contents of right-hand set of brackets are in the stack.
y^x	2.76	
× 1 +	1.58	
.286 y^x	1.14	
1 −	0.14	
5 × $\sqrt{x}$	0.84	Mach number of Daedalus' Harrier.
PRINTx	0.84	



```
  350.00 ENT↑
  661.50    ÷
          X²
     .20    ×
    1.00    +
    3.50 Yˣ
    1.00    -
    1.00 ENT↑
6.875-06 ENT↑
25500.00    ×
            -
 -5.2656 Yˣ
            ×
    1.00    +
    .286 Yˣ
    1.00    -
    5.00    ×
         √X
    0.84  ***
```

In working through complex equations like the one containing six levels of parentheses above, you really appreciate the value of the Hewlett-Packard logic system. Because you calculate one step at a time, you don't get "lost" within the problem. You see every intermediate result, and you emerge from the calculation confident of your final answer.

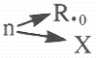
Statistical Functions

Accumulations

Pressing the Σ+ key automatically gives you several different sums and products of the values in the X- and Y-registers at once. In order to make these values accessible for sophisticated statistics problems, they are automatically placed by the calculator into storage registers $R_{.0}$ through $R_{.5}$. *The only time that information is automatically accumulated in the storage registers is when the* Σ+ *key is used*. Before you begin any calculations using the Σ+ key, you should first clear the storage registers used in accumulations by pressing ■ CLΣ.

When you key a number into the display and press the Σ+ key, each of the following operations is performed:

1. The number that you keyed into the X-register is added to the contents of storage register $R_{.1}$. ($\Sigma x \rightarrow R_{.1}$)
2. The square of the number that you keyed into the X-register is added to the contents of storage register $R_{.2}$. ($\Sigma x^2 \rightarrow R_{.2}$)
3. The number in the Y-register of the stack is added to the contents of storage register $R_{.3}$. ($\Sigma y \rightarrow R_{.3}$)
4. The square of the number in the Y-register of the stack is added to the contents of storage register $R_{.4}$. ($\Sigma y^2 \rightarrow R_{.4}$)
5. The number that you keyed into the X-register is multiplied by the contents of the Y-register, and the product added to storage register $R_{.5}$. ($\Sigma xy \rightarrow R_{.5}$)
6. The number 1 is added to storage register $R_{.0}$, and the total number in $R_{.0}$ then writes over the number in the displayed X-register of the stack. The stack does not lift.



The number that you keyed into the X-register is preserved in the LAST X register, while the number in the stack Y-register remains in the Y-register.

Thus, when you press Σ+, the stack and storage register contents are changed...

...from this...

T	t
Z	z
Y	y
X	x

	$R_{.0}$
	$R_{.1}$
	$R_{.2}$
	$R_{.3}$
	$R_{.4}$
	$R_{.5}$

LAST X

...to this.

T	t
Z	z
Y	y
X	n

n	$R_{.0}$
Σx	$R_{.1}$
Σx^2	$R_{.2}$
Σy	$R_{.3}$
Σy^2	$R_{.4}$
Σxy	$R_{.5}$

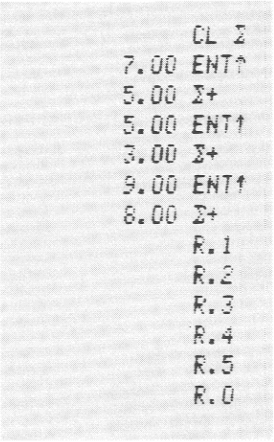
x
LAST X

To use any of the summations individually at any time, you can recall the contents of the desired storage register into the displayed X-register by pressing RCL ⊡ followed by the number key of the storage register address. (After you have pressed Σ+, recalling storage register contents or keying in another number writes over the number of entries *(n)* that is displayed. The stack does not lift.)

Example: Find Σx, Σx², Σy, Σy², and Σxy for the paired values of x and y listed below.

y	7	5	9
x	5	3	8

Press	**Display**	
■ CLΣ	0.00	Ensures that storage registers $R_{.0}$ through $R_{.5}$ are cleared to zero initially. Display assumes no results remain from previous example.
7 ENTER↑	7.00	
5 Σ+	1.00	First pair is accumulated; n = 1.
5 ENTER↑	5.00	
3 Σ+	2.00	Second pair is accumulated; n = 2.
9 ENTER↑	9.00	
8 Σ+	3.00	Third pair is accumulated; n = 3.
RCL ⊡ 1	16.00	Sum of x values from register $R_{.1}$.
RCL ⊡ 2	98.00	Sum of squares of x values from register $R_{.2}$.
RCL ⊡ 3	21.00	Sum of y values from register $R_{.3}$.
RCL ⊡ 4	155.00	Sum of squares of y values from register $R_{.4}$.
RCL ⊡ 5	122.00	Sum of products of x and y values from register $R_{.5}$.
RCL ⊡ 0	3.00	Number of entries (n = 3) from register $R_{.0}$.



```
          CL Σ
     7.00 ENT↑
     5.00 Σ+
     5.00 ENT↑
     3.00 Σ+
     9.00 ENT↑
     8.00 Σ+
          R.1
          R.2
          R.3
          R.4
          R.5
          R.0
```

Listing Accumulations

You can see *all* of the values accumulated by the Σ+ key at any time. Simply press ■ LIST: Σ, and the printer will print out the contents of the storage registers used for summations along with a description for each summation.

For example, to list *all* of the accumulations that are now in the storage registers from the previous example:

Press	**Display**
■ LIST: Σ	3.00

```
           LIST
     3.00   N
    16.00 ΣX
    98.00 ΣX²
    21.00 ΣY
   155.00 ΣY²
   122.00 ΣXY
```

Percent of Sum

The %Σ *(percent of sum)* key permits you to compute the percentage that several values are of a total, while leaving the total intact. The %Σ key computes the percentage the number in the X-register is of the value in storage register $R_{.1}$. The formula used is:

$$\frac{x}{\Sigma x} \times 100 = \%\Sigma$$

The computed value for %Σ writes over the number in the X-register, and the rest of the stack remains unchanged. (*x* is, of course, preserved in the LAST X register.)

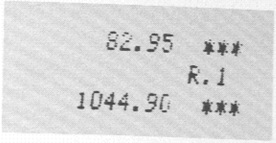
You will probably want to accumulate the total value in register $R_{.1}$ using the Σ+ key before you press %Σ. (You could also accumulate a value in register $R_{.1}$ manually, by simply storing the value there using the STO key.)

Example: A compound is made up of 5.4 grams of hydrogen (H), 172.8 grams of oxygen (O) and 866.7 grams of sulfur (S). What is the percentage by weight of each chemical in the compound, and what is the total weight of the compound?

Press	**Display**	
■ CLΣ	0.00	Display assumes no results remain from previous example.
5.4 STO 1 Σ+	1.00	
172.8 STO 2 Σ+	2.00	
866.7 STO 3 Σ+	3.00	
RCL 1 %Σ	0.52	Percent H is of total weight.
PRINT x	0.52	
RCL 2 %Σ	16.54	Percent O is of total weight.
PRINT x	16.54	
RCL 3 %Σ	82.95	Percent S is of total weight.

```
            CL Σ
     5.40   S 1
            Σ+
   172.80   S 2
            Σ+
   866.70   S 3
            Σ+
            R 1
            %Σ
     0.52    ***
            R 2
            %Σ
    16.54    ***
            R 3
            %Σ
```

PRINT x	82.95	
RCL ⊡ 1	1044.90	Total weight of the compound.
PRINT x	1044.90	



```
 82.95  ***
        R.1
1044.90 ***
```

Mean

The $\boxed{\bar{x}}$ *(mean)* key is the key you use to calculate the mean (arithmetic average) of x and y accumulated in registers $R_{\cdot 1}$ and $R_{\cdot 3}$, respectively.

When you press ■ $\boxed{\bar{x}}$:

1. The mean ($\bar{x}$) of x is calculated using the data accumulated in register $R_{\cdot 1}$ (Σx) and $R_{\cdot 0}$ (n) according to the formula:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \left(\text{That is, } \frac{R_{\cdot 1}}{R_{\cdot 0}} = \bar{x}\right)$$

   The resultant value for $\bar{x}$ is seen in the displayed X-register.

2. The mean ($\bar{y}$) of y is calculated using the data accumulated in register $R_{\cdot 3}$ (Σy) and register $R_{\cdot 0}$ (n) according to the formula:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad \left(\text{That is, } \frac{R_{\cdot 3}}{R_{\cdot 0}} = \bar{y}\right)$$

   The resultant value for $\bar{y}$ is available in the Y-register of the stack.

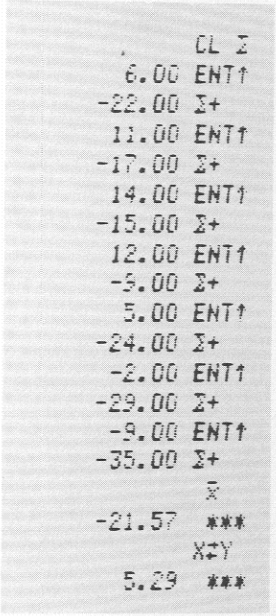
The easiest way to accumulate the required data in the applicable registers is through the use of the **Σ+** key as described above.

In order to *see* the calculated values for $\bar{x}$ and $\bar{y}$, you can slide the Print Mode switch ALL MAN ▥ NORM to ALL before pressing $\boxed{\bar{x}}$. The HP-91 will compute and print both the value for $\bar{x}$ and the value for $\bar{y}$. To *use* either of these values, of course, it must be summoned into the displayed X-register if it is not already present there.

Example: Below is a chart of a daily high and low temperatures for a winter week in Fairbanks, Alaska. What are the *average* high and low temperatures for the week selected?

	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
High	6	11	14	12	5	−2	−9
Low	−22	−17	−15	−9	−24	−29	−35

Press	Display	
■ CLΣ	0.00	Accumulation registers cleared. (Display assumes no results remain from previous calculations.)
6 ENTER↑ 22 CHS Σ+	1.00	Number of data pairs (n) is now 1.
11 ENTER↑ 17 CHS Σ+	2.00	Number of data pairs (n) is now 2.
14 ENTER↑ 15 CHS Σ+	3.00	
12 ENTER↑ 9 CHS Σ+	4.00	
5 ENTER↑ 24 CHS Σ+	5.00	
2 CHS ENTER↑	−2.00	
29 CHS Σ+	6.00	
9 CHS ENTER↑	−9.00	
35 CHS Σ+	7.00	Number of data pairs (n) is now 7.
■ x̄	−21.57	Average low temperature.
PRINT x	−21.57	
x⇄y	5.29	Average high temperature.
PRINT x	5.29	

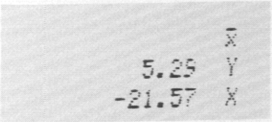


```
              CL Σ
      6.00 ENT↑
    -22.00 Σ+
     11.00 ENT↑
    -17.00 Σ+
     14.00 ENT↑
    -15.00 Σ+
     12.00 ENT↑
     -9.00 Σ+
      5.00 ENT↑
    -24.00 Σ+
     -2.00 ENT↑
    -29.00 Σ+
     -9.00 ENT↑
    -35.00 Σ+
              x̄
    -21.57  ***
            X⇄Y
      5.29  ***
```

As shown, you can use the PRINT x and x⇄y keys to print the values for x̄ and ȳ, but another method is:

Slide the Print Mode switch MAN ALL NORM to ALL.

Press	Display
■ x̄	−21.57

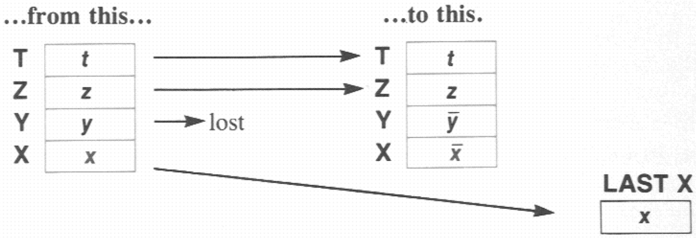


```
           x̄
     5.29  Y
   -21.57  X
```

Notice that instead of a three-asterisk label, the HP-91 conveniently prints Y and X labels when you press ■ x̄ in ALL mode.

The illustrations below represent what happens in the stack when you press ▨ $\boxed{\bar{x}}$.

Press ▨ $\boxed{\bar{x}}$ and the contents of the stack registers are changed…



Standard Deviation

The $\boxed{s}$ *(standard deviation)* key is the key you use to calculate the standard deviation (a measure of dispersion around the mean) of data accumulated in storage registers $R_{.0}$ through $R_{.5}$.

When you press ▨ $\boxed{s}$:

1. Sample x standard deviation (s_x) is calculated using the data accumulated in storage register $R_{.2}$ (Σx^2), $R_{.1}$ (Σx), and $R_{.0}$ (n) according to the formula:

$$s_x = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n - 1}}$$

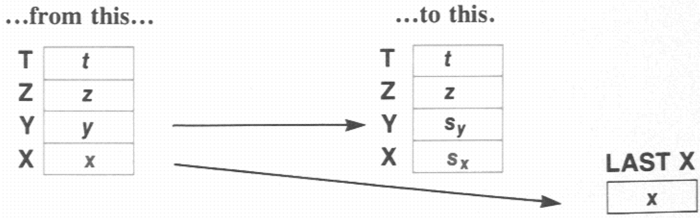
The resultant value for standard deviation of x (s_x) is seen in the displayed X-register.

2 Sample y standard deviation (s_y) is calculated using the data accumulated in storage registers $R_{.4}$ (Σy^2), $R_{.3}$ (Σy), and $R_{.0}$ (n) according to the formula:

$$s_y = \sqrt{\frac{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}{n - 1}}$$

The resultant value for standard deviation of y (s_y) is available in the Y-register of the stack.

Thus, with data accumulated in registers $R_{.0}$ through $R_{.5}$, when you press ▨ $\boxed{s}$, the contents of the stack registers are changed…



To *see* both of the values returned by the calculator when you press ■ S, you can print out the values for s_x and s_y automatically by first sliding the Print Mode switch MAN ALL NORM to ALL.

To *use* the value for standard deviation of y (s_y) simply use the x⇄y key to bring that value into the displayed X-register of the stack.

Example: In a recent survey to determine the age and net worth (in millions of dollars) of six of the 50 wealthiest persons in the United States, the following data were obtained (sampled). Calculate the average age and net worth of the sample, and calculate the standard deviations for these two sets of data.

Age	62	58	62	73	84	68
Value	1200	1500	1450	1950	1000	1750

If you want your printed copy to match the one shown here, begin with the Print Mode switch MAN ALL NORM set to NORM.

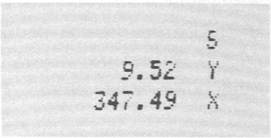
Press	**Display**	
■ CL Σ	0.00	Clears storage registers used for Σ+. (Display assumes no results remain from previous examples.)
62 ENTER↑	62.00	
1200 Σ+	1.00	Number of data pairs (n) is 1.
58 ENTER↑	58.00	
1500 Σ+	2.00	
62 ENTER↑	62.00	
1450 Σ+	3.00	
73 ENTER↑	73.00	
1950 Σ+	4.00	
84 ENTER↑	84.00	
1000 Σ+	5.00	
68 ENTER↑	68.00	
1750 Σ+	6.00	Number of data pairs (n) is 6.
■ $\bar{x}$	1475.00	Average value of net worth.
x⇄y	67.83	Average age of the sample.
■ S	347.49	Standard deviation (s_x) of net worth of sample.
x⇄y	9.52	Standard deviation (s_y) of age of sample.

```
                CL Σ
        62.00 ENT↑
      1200.00 Σ+
        58.00 ENT↑
      1500.00 Σ+
        62.00 ENT↑
      1450.00 Σ+
        73.00 ENT↑
      1950.00 Σ+
        84.00 ENT↑
      1000.00 Σ+
        68.00 ENT↑
      1750.00 Σ+
                x̄
                X⇄Y
                S
                X⇄Y
```

To see how the HP-91 prints both s_x and s_y:

Slide the Print Mode switch MAN ALL ▥ NORM to ALL.

Press	Display
■ [S]	347.49



```
        S
  9.52  Y
347.49  X
```

Notice that instead of a three-asterisk label, in ALL mode the HP-91 identifies these results with Y and X labels when you press [S].

If the six persons used in the sample were actually the *six wealthiest persons*, the data would have to be considered as a population rather than as a sample. The relationship between sample standard deviation (s) and the population standard deviation (σ) is illustrated by the following equation.

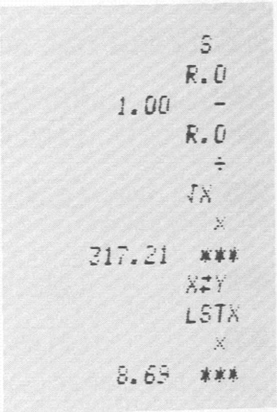
$$\sigma = s\sqrt{\frac{n-1}{n}}$$

Since *n* is automatically accumulated in register $R_{.0}$ when data is accumulated, it is a simple matter to convert the sample standard deviations that have already been calculated to population standard deviations.

If the accumulations are still intact from the previous example in registers $R_{.0}$ through $R_{.5}$, you can calculate the population standard deviations this way:

If you want your printed copy to match the one shown here, slide the Print Mode switch MAN ALL ▥ NORM back to NORM.

Press	Display	
■ [S]	347.49	Calculate s_x and s_y.
RCL [·] 0	6.00	Recall n.
1 [−]	5.00	Calculate n − 1.
RCL [·] 0 [÷]	0.83	Divide n − 1 by n.
√x [×]	317.21	Population standard deviation σ_x.
PRINT x	317.21	
x⇄y	9.52	Brings s_y to the X-register.
■ [LAST X]	0.91	Recall conversion factor.
[×]	8.69	Population standard deviation σ_y.
PRINT x	8.69	



```
          S
        R.0
  1.00    -
        R.0
          ÷
         √X
          ×
317.21  ***
        X⇄Y
       LSTX
          ×
  8.69  ***
```

Deleting and Correcting Data

If you key in an incorrect value and have not pressed Σ+, press CLx and key in the correct value.

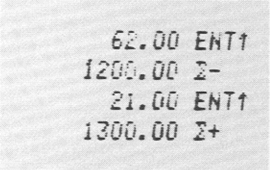
If one of the values is changed, or if you discover after you have pressed the Σ+ key that one of the values is in error, you can correct the summations by using the Σ- *(summation minus)* key as follows:

1. Key in the *incorrect* data pair into the X- and Y-registers. (You can use LAST X to return a single incorrect data value to the displayed X-register.)
2. Press ■ Σ- to delete the incorrect data.
3. Key in the correct values for x and y. (If one value of an x, y data pair is incorrect, both values must be deleted and reentered.)
4. Press Σ+.

The correct values for mean and standard deviation are now obtainable by pressing ■ x̄ and ■ s.

For example, suppose the 62-year old member of the *sample* as given above were to lose his position as one of the wealthiest persons because of a series of ill-advised investments in cocoa futures. To account for the change in data if he were replaced in the sample by a 21-year old rock musician who is worth 1300 million dollars:

Press	**Display**	
62 ENTER↑	62.00	Data to be replaced.
1200	1200.	
■ Σ-	5.00	Number of entries (n) is now five.
21 ENTER↑	21.00	The new data.
1300	1300.	
Σ+	6.00	Number of entries (n) is six again.

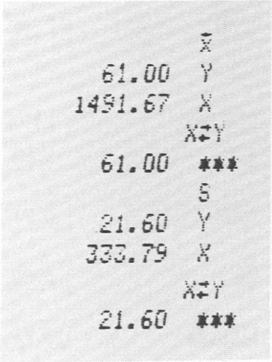


```
   62.00 ENT↑
 1200.00 Σ-
   21.00 ENT↑
 1300.00 Σ+
```

The new data has been calculated into each of the summations present in the storage registers. To see the new mean and standard deviation:

Slide the Print Mode switch MAN ALL NORM to ALL.

Press	**Display**	
■ x̄	1491.67	The new average (mean) worth.
x⇄y	61.00	The new average (mean) age available in X-register for use.
■ s	333.79	The new standard deviation for worth.
x⇄y	21.60	The new standard deviation for age available in X-register for use.



```
            x̄
   61.00    Y
 1491.67    X
           X⇄Y
   61.00   ***
            S
   21.60    Y
  333.79    X
           X⇄Y
   21.60   ***
```

Linear Regression

Linear regression is a statistical method for finding a straight line that best fits a set of data points, thus providing a relationship between two variables. After a group of data points has been totaled in registers $R_{\cdot 0}$ through $R_{\cdot 5}$, you can calculate the coefficients of the linear equation y = A + Bx using the least squares method by pressing ■ [L.R.]. (Naturally, at least two data points must be in the calculator before a least squares line can be fitted to them.)

To use the linear regression function on your HP-91, first key in a series of data points using the [Σ+] key. Then press ■ [L.R.].

When you press ■ [L.R.], two values are calculated:

1. The y-intercept (A) of the least squares line of the data is calculated using the equation:

$$A = \frac{\Sigma y\ \Sigma x^2 - \Sigma x\ \Sigma xy}{n\ \Sigma x^2 - (\Sigma x)^2}$$

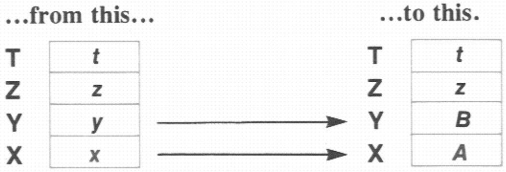
   The y-intercept (A) appears in the displayed X-register of the stack.

2. The slope (B) of the least squares line of the data is calculated using the equation:

$$B = \frac{n\ \Sigma xy - \Sigma x\ \Sigma y}{n\ \Sigma x^2 - (\Sigma x)^2}$$

   The slope (B) is available in the Y-register of the stack.

Thus, when you press ■ [L.R.], the contents of the stack registers change...



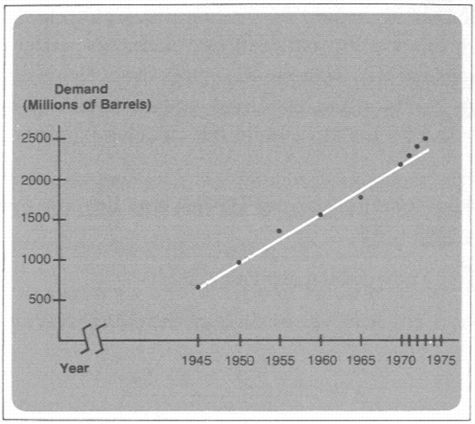
With the Print Mode switch MAN ALL ▥ NORM set to ALL, the HP-91 automatically prints values for both A and B whenever you press ■ [L.R.].

To *use* the value for B or to bring it into the displayed X-register, simply shift the stack contents with the [x⇄y] key.

Example: Big Lyle Hephaestus, owner-operator of the Hephaestus Oil Company, wishes to know the slope and y-intercept of a least squares line for the consumption of motor fuel in the United States against time since 1945. He knows the data given in the table.

Motor Fuel Demand (Millions of Barrels)	969	994	1330	1512	1750	2162	2243	2382	2484
Year	1945	1950	1955	1960	1965	1970	1971	1972	1973

Solution: Hephaestus *could* draw a plot of motor fuel demand against time like the one shown below.



However, with his HP-91, Hephaestus has only to key the data into the calculator using the Σ+ key, then press L.R. .

(If you want your printed copy to match the one shown here, begin with the Print Mode switch MAN ALL NORM set to NORM.)

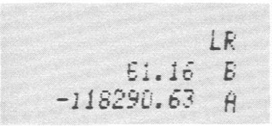
Press	**Display**	
■ CLEAR	0.00	Stack, summation, and storage registers all cleared to zero.
696 ENTER↑	696.00	
1945 Σ+	1.00	
994 ENTER↑	994.00	
1950 Σ+	2.00	
1330 ENTER↑	1330.00	
1955 Σ+	3.00	
1512 ENTER↑	1512.00	
1960 Σ+	4.00	
1750 ENTER↑	1750.00	
1965 Σ+	5.00	
2162 ENTER↑	2162.00	
1970 Σ+	6.00	
2243 ENTER↑	2243.00	
1971 Σ+	7.00	
2382 ENTER↑	2382.00	
1972 Σ+	8.00	
2484 ENTER↑	2484.00	
1973 Σ+	9.00	All data pairs have been keyed in.
■ L.R.	-118290.63	The y-intercept of the line.
x⇄y	61.16	Slope of the line.

```
           CLEAR
   696.00 ENT↑
  1945.00 Σ+
   994.00 ENT↑
  1950.00 Σ+
  1330.00 ENT↑
  1955.00 Σ+
  1512.00 ENT↑
  1960.00 Σ+
  1750.00 ENT↑
  1965.00 Σ+
  2162.00 ENT↑
  1970.00 Σ+
  2243.00 ENT↑
  1971.00 Σ+
  2382.00 ENT↑
  1972.00 Σ+
  2484.00 ENT↑
  1973.00 Σ+
            LR
           X⇄Y
```

To see how the HP-91 automatically prints the y-intercept A and the slope B of the line:

Slide the Print Mode switch MAN ALL NORM to ALL.

Press	Display
■ L.R.	-118290.63



```
            LR
     61.16  B
-118290.63  A
```

In ALL mode, the HP-91 identifies these results with labels for A and B instead of with a three-asterisk label.

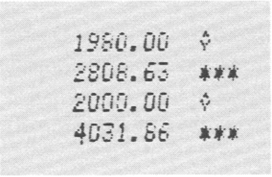
Linear Estimate

With the data totaled in registers $R_{.0}$ through $R_{.5}$, a predicted y (that is, a $\hat{y}$) can be calculated by keying in a new x-value and pressing ŷ.

For example, with data intact from the previous example in registers $R_{.0}$ through $R_{.5}$, if Hephaestus wishes to predict the demand for oil for the years 1980 and 2000, he has only to key in the new x-values and press ŷ.

(If you want your printed copy to match the one shown here, ensure that the Print Mode switch MAN ALL NORM is set to ALL.)

Press	Display	
1980 ŷ	2808.63	Predicted demand in millions of barrels for the year 1980.
2000 ŷ	4031.86	Predicted demand in millions of barrels for the year 2000.



```
1980.00  ŷ
2808.63  ***
2000.00  ŷ
4031.86  ***
```

Coefficient of Determination

To establish how well the data fits the linear regression, you may want to calculate the coefficient of determination (r^2). The coefficient of determination is a value between 0 and 1. At $r = 0$ you have no fit, while at $r^2 = 1$ you have a perfect fit. The traditional equation for r^2 is:

$$r^2 = \frac{\left[\Sigma\,(x - \bar{x})\,(y - \bar{y})\right]^2}{\left[\Sigma\,(x - \bar{x})^2\right]\left[\Sigma\,(y - \bar{y})^2\right]}$$

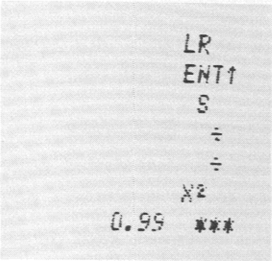
On your HP-91, however, the most efficient way to calculate r^2 is to use this equivalent equation:

$$r^2 = \left[\frac{n\,\Sigma xy - \Sigma x\,\Sigma y}{n\,(n - 1)\,s_x\,s_y}\right]^2$$

Example: Calculate r² for the previously calculated linear regression.

Slide the Print Mode switch MAN ▮ ALL ▯ NORM **to NORM if you want your printed copy to match the one shown here.**

Press	Display
■ L.R.	-118290.63
ENTER↑	-118290.63
■ s	10.37
÷	61.59
÷	0.99
x² PRINTx	0.99



```
        LR
      ENT↑
         S
         ÷
         ÷
        X²
0.99    ***
```

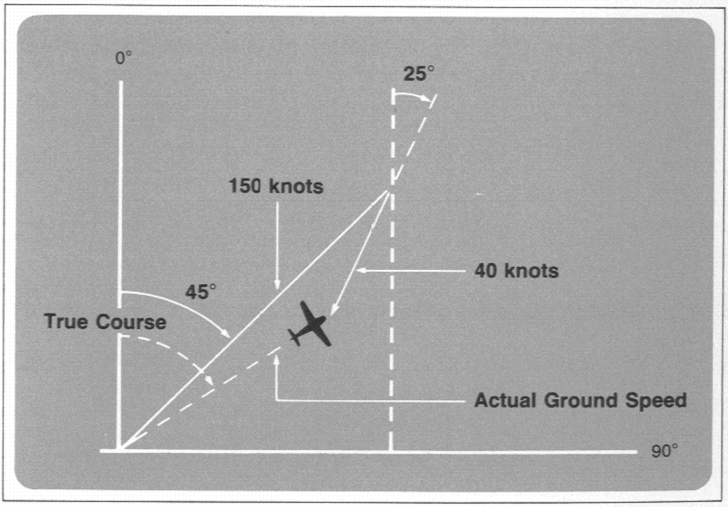
Since the correlation coefficient is 0.99, you can assume the fit of the line is excellent.

Vector Arithmetic

You can use your HP-91 to add or subtract vectors by combining the polar/rectangular conversion functions (the R→P and P→R keys) with the summation functions (the Σ+ and Σ− keys).

Example: Grizzled bush pilot Apeneck Sweeney's converted Swordfish aircraft has a true air speed of 150 knots and an estimated heading of 45°. The Swordfish is also being buffeted by a headwind of 40 knots from a bearing of 25°. What is the actual ground speed and course of the Swordfish?

Method: The course and ground speed are equal to the difference of the vectors. (North becomes the x-coordinate so that the problem corresponds with navigational convention.)



Slide the Trigonometric Mode switch DEG ▮ RAD GRD to DEG.

Press	Display	
■ CLΣ	0.00	Clears summation registers $R_{\cdot 0}$ through $R_{\cdot 5}$. (Display assumes no results remain from previous examples.)
45 ENTER↑	45.00	θ for 1st vector is entered to Y-register.
150	150.	r for 1st vector is keyed in.
P→R	106.07	Converted to rectangular coordinates.
Σ+	1.00	1st vector coordinates accumulated in storage registers $R_{\cdot 1}$ and $R_{\cdot 3}$.
25 ENTER↑	25.00	θ for 2nd vector is entered to Y-register.
40	40.	r for 2nd vector is keyed in.
P→R	36.25	2nd vector is converted to rectangular coordinates.
■ Σ-	0.00	2nd vector rectangular coordinates subtracted from those of 1st vector.
RCL Σ+	69.81	Recalls both $R_{\cdot 1}$ and $R_{\cdot 3}$.

```
         CL Σ
  45.00 ENT↑
 150.00 P→R
         Σ+
  25.00 ENT↑
  40.00 P→R
         Σ-
         R Σ
```

Slide the Print Mode switch MAN ALL ▮ NORM to ALL now, so that the HP-91 will automatically print both desired values.

Press	Display	
		Course in degrees of the Swordfish.
R→P	113.24	Actual ground speed in knots of the Swordfish.

```
          R→P
   51.94  θ
  113.24  R
```


Over-the-Counter Quotations
Mutual Funds
Stock Exchange
Industrials
U.V.W.X.Y.Z.
HEWLETT · PACKARD 91 CALCULATOR
PRINT x
ENTER ↑
CHS
EEX
STO
RCL
CL X
FIX
SIN
COS
TAN
LN
Σ+

Section 5

HP-91 Applications Routines

In order to further enhance the usability of your HP-91, we have included in your *HP-91 Owner's Handbook* dozens of keystroke routines to solve problems in several scientific disciplines. In the next pages are routines to use your HP-91 to solve common problems from the areas of mathematics, statistics, navigation, surveying, and finance.

To use any of the routines:

1. Begin at line #1 of the keystroke list.
2. Key in the information called for under DATA at line #1.
3. Press in left-to-right order the keys called for under OPERATIONS for line #1.
4. If specified under RESULTS, read the answer from the display or the paper tape.
5. Note any REMARKS.
6. Continue with line #2, reading from left to right.

You can place the Print Mode switch MAN ALL NORM in any of its three positions when using the routines shown here. Of course, in MAN *(manual)*, the printer will be idle and will only print if you press PRINT x or one of the LIST functions. In NORM (*normal*), the printer will record your inputs and the function keys you press—to record your results, press PRINT x . In ALL, the HP-91 prints inputs, functions, and the result of each function. Regardless of the position of the Print Mode switch, you will find that you can press keys quite rapidly—the internal key buffer in the HP-91 "remembers" up to seven keystrokes, even though you seem to be outrunning the printer.

Within each application area, we've tried to arrange the routines in the order of use, with the most common routines from each discipline at the beginning.

Don't be afraid to rearrange and experiment with any of the routines. The HP-91 is a tremendously powerful and versatile calculating instrument, and with a little practice, you'll soon be writing keystroke procedures of your own to solve the most complicated of problems within your field.

Mathematical Applications

Quadratic Equation

Formula: A general quadratic equation is of the form

$$Ax^2 + Bx + C = 0.$$

The equation has two roots, x_1 and x_2.

Let $$D = \frac{B^2 - 4AC}{4A^2}$$

If $$D \geqslant 0, \text{ then } x_1 = \begin{cases} -\frac{B}{2A} + \sqrt{\frac{B^2 - 4AC}{4A^2}} & \text{if } \frac{-B}{2A} \geqslant 0 \\ -\frac{B}{2A} - \sqrt{\frac{B^2 - 4AC}{4A^2}} & \text{if } -\frac{B}{2A} < 0 \end{cases}$$

$$\text{and } x_2 = \frac{C}{AX_1}$$

If $$D < 0, \text{ then } x_1, x_2 = -\frac{B}{2A} \pm i\sqrt{\frac{4AC - B^2}{4A^2}}$$

$$= u \pm iv$$

The coefficient A cannot be zero.

Examples: Find the solutions to the following equations:

1. $x^2 - 3x - 4 = 0$
2. $2x^2 + 3x + 4 = 0$

Answers:

1. $D = 6.25$ $x_1 = 4, x_2 = -1$
2. $D = -1.44$ $x_1, x_2 = -0.75 \pm 1.20i$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	C	STO 3		
2	B	STO 2		
3	A	STO 1 ENTER↑ R↓ ÷		
4		2 ÷ CHS ENTER↑ x^2		
5		R↓ R↓ x⇄y ÷ STO		
6		1 −	D	If D < 0, go to 11.
7		$\sqrt{x}$ x⇄y	−B/2A	If −B/2A < 0, go to 9.
8		+	x_1	Go to 10.
9		x⇄y −	x_1	
10		$1/x$ RCL 1 ×	x_2	Stop.
11		CHS $\sqrt{x}$ x⇄y	u	
12		x⇄y	v	

Simultaneous Linear Equations in Two Unknowns

Formula: Solve for x and y given the following:

$$ax + by = e$$

$$cx + dy = f$$

Cramer's Rule is used to find the solution.

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{ed - bf}{ad - bc}$$

$$y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{af - ec}{ad - bc}$$

where $ad - bc \neq 0$.

Example:

$$\text{Solve} \quad \begin{cases} 7.32x - 9.08y = 3.14 \\ 12.39x + 7y = 0.05 \end{cases}$$

Answer:

$x = 0.14$

$y = -0.24$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	e	STO 1		
2	d	STO 2 ×		
3	b	STO 3		
4	f	STO 4 × −		
5	a	STO 5 RCL 2 ×		
6	c	STO 6 RCL 3 ×		
7		− STO 7 ÷	x	
8		RCL 5 RCL 4 ×		
9		RCL 1 RCL 6 ×		
10		− RCL 7 ÷	y	

Determinant of a 3 × 3 Matrix

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \text{ be a } 3 \times 3 \text{ matrix.}$$

The determinant of D is calculated by expanding D by minors about the first column. The formula is:

$$\text{Det } D = a_{11} (a_{22} a_{33} - a_{23} a_{32}) - a_{21} (a_{12} a_{33} - a_{13} a_{32}) + a_{31} (a_{12} a_{23} - a_{13} a_{22})$$

Example:

$$D = \begin{vmatrix} -1 & 3 & 2 \\ 2 & 1 & -1 \\ 4 & 2 & 3 \end{vmatrix} = -35$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a_{11}	STO 1		
2	a_{22}	STO 2 ×		
3	a_{33}	STO 3 ×		
4	a_{12}	STO 4		
5	a_{23}	STO 5 ×		
6	a_{31}	STO 6 × +		
7	a_{13}	STO 7		
8	a_{21}	STO 8 ×		
9	a_{32}	STO 9 × +		
10		RCL 6 RCL 2 ×		
11		RCL 7 × −		
12		RCL 9 RCL 5 ×		
13		RCL 1 × −		
14		RCL 3 RCL 8 ×		
15		RCL 4 × −	D	

Hyperbolic Functions

These procedures evaluate three hyperbolic functions and their inverses.

Hyperbolic Sine

Formula:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

Example:

$$\sinh 3.2 = 12.25$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	x	e^x ENTER↑ $1/x$ − 2		
2		÷	sinh x	

Hyperbolic Cosine

Formula:

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Example:

$$\cosh 3.2 = 12.29$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	x	e^x ENTER↑ $1/x$ + 2		
2		÷	cosh x	

Hyperbolic Tangent

Formula:

$$\tanh x = \frac{\sinh x}{\cosh x}$$

Example:

$$\tanh 3.2 = 1.00$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	x	e^x ENTER↑ $1/x$ − ■		
2		LAST X ENTER↑ $1/x$ + ÷	tanh x	

Inverse Hyperbolic Sine

Formula:

$$\sinh^{-1} x = \ln (x + \sqrt{x^2 + 1})$$

Example:

$$\sinh^{-1} 51.777 = 4.64$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	x	ENTER↑ x^2 1 + $\sqrt{x}$		
2		+ LN	$\sinh^{-1} x$	

Inverse Hyperbolic Cosine

Formula:

$$\cosh^{-1} x = \ln (x + \sqrt{x^2 - 1}) \qquad (x \geqslant 1)$$

Example:

$$\cosh^{-1} 51.777 = 4.64$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	x	ENTER↑ x^2 1 − $\sqrt{x}$		
2		+ LN	$\cosh^{-1} x$	

Inverse Hyperbolic Tangent

Formula:

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1 + x}{1 - x}$$

$$(-1 < x < 1)$$

Example:

$$\tanh^{-1} 0.777 = 1.04$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		1 ENTER↑		
2	x	+ 1 ■ LAST X −		
3		÷ LN 2 ÷	$\tanh^{-1} x$	

Complex Number Operations

These procedures evaluate the basic complex number operations.

Complex Addition

Formula:

$$(a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2) = u + iv$$

Example:

$$(3 + 4i) + (7.4 - 5.6i) = 10.40 - 1.60i$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a_1	ENTER↑		
2	a_2	+	u	
3	b_1	ENTER↑		
4	b_2	+	v	

Complex Subtraction

Formula:

$$(a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + i(b_1 - b_2) = u + iv$$

Example:

$$(3 + 4i) - (7.4 - 5.6i) = -4.40 + 9.60i$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a_1	ENTER↑		
2	a_2	−	u	
3	b_1	ENTER↑		
4	b_2	−	v	

Multiplication of n Complex Numbers

Formula:

$$\prod_{k=1}^{n}(a_k + ib_k) = \left(\prod_{k=1}^{n} r_k\right) e^{i\sum_{k=1}^{n}\theta_k} = u + iv$$

$$\text{where} \quad a_k + ib_k = r_k e^{i\theta_k}$$

Examples:

$(3.1 + 4.6i) \times (5 - 12i) = 70.70 - 14.20i$

$(3 + 4i)(7 - 2i)(4.38 + 7i)(12.3 - 5.44i) = 1296.66 + 3828.90i$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLΣ		
2	b_k	ENTER↑		Perform 2-3 for
3	a_k	R→P LN Σ+	k	k = 1, 2, ..., n.
4		RCL Σ+ e^x P→R	u	
5		x⇄y	v	

Complex Division

Formula:

$$\frac{(a_1 + ib_1)}{(a_2 + ib_2)} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)} = u + iv$$

$$\text{where} \quad a_1 + ib_1 = r_1 e^{i\theta_1}$$

$$a_2 + ib_2 = r_2 e^{i\theta_2} \neq 0$$

Example:

$$\frac{(3 + 4i)}{7 - 2i} = 0.25 + 0.64i$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	b_2	ENTER↑		
2	a_2	R→P		
3	b_1	ENTER↑		
4	a_1	R→P x⇄y R↓ x⇄y ÷		
5		R↓ − R↑ P→R	u	
6		x⇄y	v	

Complex Reciprocal

Formula:

$$\frac{1}{a+ib} = \frac{1}{r} e^{-i\theta}, z \neq 0$$

$$= u + iv$$

Example:

$$\frac{1}{2+3i} = 0.15 - 0.23i$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	b	CHS ENTER↑		
2	a	R→P 1/x P→R	u	
3		x⇄y	v	

Complex Square

Formula:

$$(a+ib)^2 = r^2 e^{i2\theta}$$

Example:

$$(7-2i)^2 = 45.00 - 28.00i$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	b	ENTER↑		
2	a	R→P x⇄y 2 × x⇄y		
3		x² P→R	u	
4		x⇄y	v	

Complex Square Root

Formula:

$$\sqrt{a+ib} = \pm(\sqrt{r}\,e^{i\theta/2}) = \pm(u+iv)$$

Example:

$$\sqrt{7 + 6i} = \pm\,(2.85 + 1.05i)$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	b	ENTER↑		
2	a	R→P √x x⇄y 2 ÷		
3		x⇄y P→R	u	
4		x⇄y	v	

Vector Operations

Vector Addition

Suppose vector V_k (in two-dimensional space) has magnitude m_k and direction $\theta_k (k = 1, 2, \ldots, n)$. Find the sum

$$V = \sum_{k=1}^{n} V_k = x\vec{i} + x\vec{j}$$

Example:

m_k	θ_k
2	30°
6.2	−45°
7.6	125°
10.7	232°

Answer:

$$V = -4.83\vec{i} - 5.59\vec{j}$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLΣ		
2	θ_k	ENTER↑		Perform 2-3
3	m_k	P↔R Σ+	k	for k = 1, 2, ..., n.
4		RCL Σ+	x	
5		x≷y	y	

Vector Angles

Suppose

$$\vec{x} = (x_1, x_2, x_3)$$

$$\vec{y} = (y_1, y_2, y_3)$$

then the angle between these two vectors is

$$\theta = \cos^{-1}\left[\frac{x_1 y_1 + x_2 y_2 + x_3 y_3}{\sqrt{x_1^2 + x_2^2 + x_3^2} \quad \sqrt{y_1^2 + y_2^2 + y_3^2}}\right]$$

Example: Find the angle between

$$\vec{x} = (5,\ -6.2,\ -7)$$

$$\vec{y} = (3.15,\ 2.22,\ -0.3)$$

Answer:

$$\theta = 84.28 \text{ degrees} = 1.47 \text{ radians}$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2	x_i	ENTER↑ x^2 STO + 1		Perform 2-5
3		R↓		for i = 1, 2, 3
4	y_i	ENTER↑ x^2 STO + 2		
5		R↓ × +		
6		RCL 1 $\sqrt{x}$ RCL 2		
7		$\sqrt{x}$ × ÷ ■ $\cos^{-1}$	θ	

Vector Cross Product

Formula:

If $\vec{x} = (x_1, x_2, x_3)$ and $\vec{y} = (y_1, y_2, y_3)$ are two vectors, then the cross product $\vec{z}$ is also a vector.

$$\begin{aligned}\vec{z} &= \vec{x} \times \vec{y} \\ &= (x_2 y_3 - x_3 y_2,\ x_3 y_1 - x_1 y_3,\ x_1 y_2 - x_2 y_1) \\ &= (z_1,\ z_2,\ z_3)\end{aligned}$$

Example:

If $\vec{x} = (2.34,\ 5.17,\ 7.43)$

$\vec{y} = (.072,\ .231,\ .409)$

Find $\vec{x} \times \vec{y}$

Answer:

$$\vec{x} \times \vec{y} = (0.40,\ -0.42,\ 0.17)$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	x_2	STO 1		
2	y_3	STO 2 ×		
3	x_3	STO 3		
4	y_2	STO 4 × −	z_1	
5	y_1	STO 5 RCL 3 ×		
6	x_1	STO 6 RCL 2 ×		
7		−	z_2	
8		RCL 6 RCL 4 ×		
9		RCL 1 RCL 5 ×		
10		−	z_3	

Vector Dot Product

Formulas:

Given two vectors $\vec{x}$, $\vec{y}$ in an n-dimensional vector space

$$\vec{x} = (x_1, x_2, \ldots, x_n)$$

$$\vec{y} = (y_1, y_2, \ldots, y_n)$$

the dot product is

$$x \cdot \vec{y} = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n$$

Example:

If $\vec{x} = (2.34, 5.17, 7.43, 9.11, 11.41)$

$\vec{y} = (.072, .231, .409, .703, .891)$

then $\vec{x} \cdot \vec{y} = 20.97$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	x_1	ENTER↑		
2	y_1	×		
3	x_i	ENTER↑		Perform 3-4
4	y_i	× +		for i = 2, 3, ..., n.

Triangle Solutions

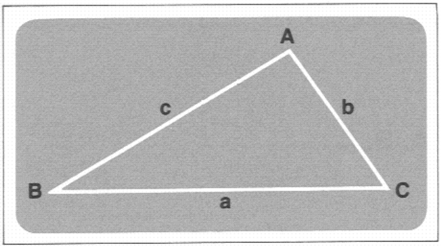
The basic formulas used to solve a triangle are:

1. law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

2. law of cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Note: Triangle solution routines work in any of the three angular modes. When the calculator is in DEG mode, all angles are in decimal degrees.

Given a, b, C; Find A, B, c

Formulas:

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

$$A = \tan^{-1}\left(\frac{a \sin C}{b - a \cos C}\right)$$

$$B = \cos^{-1}\left[-\cos(A + C)\right]$$

Example:

Given $C = 28°40'$ (convert angle C to decimal degrees)

$a = 132$

$b = 224$

Find c, A, B

Answer:

c = 125.35

A = 30.34°

B = 120.99°

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	C	ENTER↑ ENTER↑		
2	a	P→R		
3	b	x⇄y − R→P	c	
4		R↓	A	
5		+ COS CHS ■ $\cos^{-1}$	B	

Given a, b, c; Find A, B, C

Formulas:

$$A = 2\cos^{-1}\left(\sqrt{\frac{S(S-a)}{bc}}\right)$$

$$B = \tan^{-1}\left(\frac{b \sin A}{c - b\cos A}\right)$$

$$\text{where } S = (a + b + c)/2$$

$$C = \cos^{-1}\left[-\cos(A + B)\right]$$

Example:

Given a = 30.3
b = 40.4
c = 62.6

find A, B, C

Answer:

A = 23.66° = 0.41 radians = 26.29 grads

B = 32.35° = 0.56 radians = 35.95 grads

C = 123.99° = 2.16 radians = 137.76 grads

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a	STO 1		
2	b	STO 2		
3	c	STO 3 + + 2		
4		÷ ENTER↑ ENTER↑ RCL 1		
5		− × RCL 2 ÷		
6		RCL 3 ÷ $\sqrt{x}$ ■		
7		$\cos^{-1}$ 2 × STO 1	A	
8		RCL 2 P→R RCL 3		
9		x⇄y − R→P x⇄y	B	
10		RCL 1 + COS CHS		
11		■ $\cos^{-1}$	C	

Given a, A, C; Find B, b, c

Formulas:

$$b = \frac{a \sin (A + C)}{\sin A}$$

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

$$B = \tan^{-1} \left(\frac{b \sin C}{a - b \cos C} \right)$$

Example:

Given $a = 17.5$
$C = 1.09$ radians
$A = 0.72$ radians

Find b, c, B

Answer:

$b = 25.78$
$c = 23.53$
$B = 1.33$ radians

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a	STO 1		
2	C	STO 2		
3	A	STO 3 + SIN RCL		
4		3 SIN ÷ RCL 1		
5		×	b	
6		RCL 2 x⇄y P→R RCL		
7		1 x⇄y − R→P	c	
8		x⇄y	B	

Given a, B, C; Find A, b, c

Formulas:

$$c = \frac{a \sin C}{\sin (B + C)}$$

$$b = \sqrt{a^2 + c^2 - 2ac \cos B}$$

$$A = \cos^{-1} \left[-\cos (B + C) \right]$$

Example:

Given $a = 25.2$
$B = 35.3°$
$C = 68.5°$

Find c, b, A

Answer:

$c = 24.14$
$b = 14.99$
$A = 76.20°$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a	STO 1		
2	B	STO 2		
3	C	STO 3 SIN RCL 2		
4		RCL 3 + SIN ÷		
5		RCL 1 × STO 3	c	
6		RCL 2 RCL 3 P→R		
7		RCL 1 x⇄y − R→P	b	
8		x⇄y RCL 2 + COS		
9		CHS ■ COS⁻¹	A	

Given B, b, c; Find a, A, C

Formulas:

$$a = \frac{c \sin (B + C_1)}{\sin C_1}$$

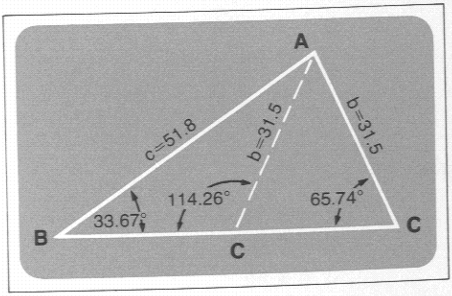
where

$$C_1 = \begin{cases} \sin^{-1}\left(\dfrac{c \sin B}{b}\right) \text{ or} \\ \sin^{-1}\left(-\dfrac{c \sin B}{b}\right) \end{cases}$$

$$A = \tan^{-1}\left(\frac{a \sin B}{c - a \sin B}\right)$$

$$C = \cos^{-1}\left[-\cos (A + B)\right]$$

Note: If B is acute and $b < c$, two solutions exist.



Example:

Given $b = 31.5$
$c = 51.8$
$B = 33.67°$

Find a, A, C

Answer:

C = 65.74°
a = 56.05
A = 80.59°

Alternate answer:

C = 114.26°
a = 30.17
A = 32.07°

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	b	STO 1		
2	c	STO 2		
3	B	STO 3 SIN × RCL		
4		1 ÷ ■ sin⁻¹ STO		
5		4	C_1	
6		SIN RCL 3 RCL 4		
7		+ SIN x⇄y ÷ RCL		
8		2 ×	a	
9		RCL 3 x⇄y P→R RCL		
10		2 x⇄y − R→P x⇄y	A	
11		RCL 3 + COS CHS		
12		■ cos⁻¹	C	If b ⩾ c, stop.
13		RCL 4 CHS STO 4		Go to 6 for alternate
				solution

Given a, b, c; Find Area

Formula:

$$\text{area} = \sqrt{S\,(S - a)\,(S - b)\,(S - c)}$$

$$\text{where } S = 1/2\,(a + b + c)$$

Example:

a = 5.31
b = 7.09
c = 8.86

Answer:

area = 18.82
(S = 10.63)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a	STO 1		
2	b	STO 2 +		
3	c	STO 3 + 2 ÷	S	
4		ENTER↑ ENTER↑ ENTER↑ RCL 1		
5		− × x⇄y RCL 2		
6		− × x⇄y RCL 3		
7		− × √x	area	

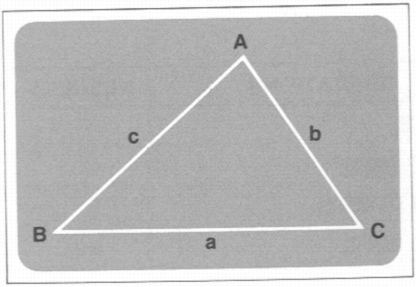
Given a, b, C; Find Area

Formula:

area = 1/2 a b sin C

Example:

a = 5.3174
b = 7.0898
$C = \frac{\pi}{4} = 45°$



Answer:

area = 13.33

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a	ENTER↑		Set machine to
2	b	× 2 ÷		desired mode
3	C	SIN ×	area	(DEG, RAD, or GRD).

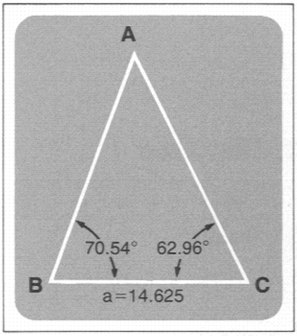
Given a, B, C; Find Area

Formula:

$$\text{area} = \frac{a^2}{2} \; \frac{\sin B \sin C}{\sin (B + C)}$$

Example:

$a = 14.625$
$B = 70° \, 32' \, 12''$
$C = 62° \, 57' \, 28''$



Answer:

area = 123.80

Note: In this example, convert angles to decimal degrees before using trigonometric function keys.

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a	x^2 2 ÷		Set machine to
2	B	STO 1 SIN ×		desired mode
3	C	STO + 1 SIN ×		(DEG, RAD, or GRD).
4		RCL 1 SIN ÷	area	

Given Vertices; Find Area

Formula: Given the three vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) of a triangle,

$$\text{area} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \left[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) \right]$$

Example: Compute the area of a triangle with vertices (0, 0), (4, 0), (4, 3).

Answer:

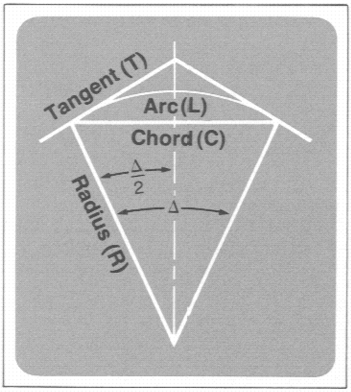
6.00

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	y_2	STO 1		
2	y_3	STO 2 −		
3	x_1	× RCL 2		
4	y_1	STO 3 −		
5	x_2	× + RCL 3 RCL		
6		1 −		
7	x_3	× + 2 ÷	Area	

Curve Solutions

Notation:

T = Tangent distance
C = Chord length
L = Arc length
R = Radius
Δ = Central angle



Given Δ and R, Find Remaining Parameters Plus Sector and Segment Area

Formulas:

$T = R \tan(1/2\ \Delta)$
$C = 2\ R \sin(1/2\ \Delta)$
$L = \Delta\ \pi\ R/180$
Sector area $= LR/2$
Segment area $=$ Sector area $- 1/2\ R^2 \sin(\Delta)$

Example:

$\Delta = 45^\circ\ 30'\ 23''$
$R = 223.181$

Answers:

$1/2\ \Delta = 22^\circ\ 45'\ 12''$
$T = 93.602$
$C = 172.636$
$L = 177.258$
Sector area (▽) = 19,780
Segment area (⌓) = 2,015

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2	Δ	■ H.MS→ 2 ÷ STO		
3		1 ■ →H.MS	1/2 Δ	
4		RCL 1 TAN		
5	R	STO 2 ×	T	
6		RCL 1 SIN RCL 2		
7		× 2 ×	C	
8		RCL 1 RCL 2 ×		
9		■ π × 9 0		
10		÷	L	
11		RCL 2 × 2 ÷	Sector area	
12		RCL 2 x^2 RCL 1		
13		2 × SIN × 2		
14		÷ −	Segment area	

Given R and C, Find Remaining Parameters Plus Sector and Segment Area

Formulas:

R = C/(2 sin (1/2 Δ))
Δ = 2 $\sin^{-1}$ (1/2 C/R)
T = R tan (1/2 Δ)
L = Δ π R/180
Sector area = LR/2
Segment area = Sector area − 1/2 R^2 sin Δ

Example:

C = 172.636
R = 223.181

Answers:

Δ = 45° 30′ 23″
1/2 Δ = 22° 45′ 11″
T = 93.602
L = 177.258
Sector area (▽) = 19,780
Segment area (⌓) = 2,015

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2	C	ENTER↑ 2 ÷		
3	R	STO 2 ÷ ■ $\sin^{-1}$		
4		STO 1 ■ →H.MS	1/2 Δ	
5		RCL 1 TAN RCL 2		
6		×	T	
7		RCL 2 ■ π ×		
8		RCL 1 × 9 0		
9		÷	L	
10		RCL 2 × 2 ÷	Sector area	
11		RCL 2 x^2 RCL 1		
12		2 × SIN × 2		
13		÷ −	Segment area	

Given Δ and C, Find Remaining Parameters Plus Sector and Segment Area

Formulas:

$R = C/[2 \sin (1/2\ \Delta)]$
$T = R \tan (1/2\ \Delta)$
$L = \Delta\ \pi\ R/180$
Sector area $= LR/2$
Segment area $=$ Sector area $- 1/2\ R^2 \sin \Delta$

Example:

$C = 172.636$
$\Delta = 45° 30' 23''$

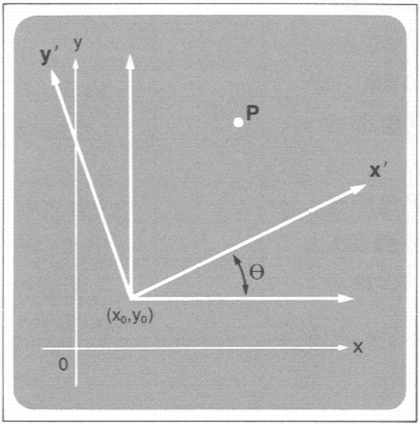
Answers:

$1/2\ \Delta = 22° 45' 12''$
$R = 223.181$
$T = 93.602$
$L = 177.258$
Sector area (▽) = 19,780
Segment area (⌓) = 2,015

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2	C	ENTER↑ 2 ÷		
3	Δ	■ H.MS→ 2 ÷ STO		
4		1 ENTER↑ ■ →H.MS	1/2 Δ	
5		R↓ SIN ÷ STO 2	R	
6		RCL 1 TAN ×	T	
7		RCL 2 ■ π ×		
8		RCL 1 × 9 0		
9		÷	L	
10		RCL 2 × 2 ÷	Sector area	
11		RCL 2 x^2 RCL 1		
12		2 × SIN × 2		
13		÷ −	Segment area	

Coordinate Translation and Rotation

Suppose point P has coordinates (x, y) with respect to the rectangular coordinate system (x, y axes). Let (x_o, y_o) be the center of a new coordinate system rotated through an angle θ. Find the new coordinates (x′, y′) of P with respect to the new system having x′, y′ axes.



Formulas:

$$x' = (x - x_o)\cos\theta + (y - y_o)\sin\theta$$

$$y' = -(x - x_o)\sin\theta + (y - y_o)\cos\theta$$

Example: Translate the point (1, 3) to a new coordinate system with center at (−1, 4) at 30° to the old system.

Answer:

$x' = 1.23$
$y' = -1.87$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	y	ENTER↑		
2	y_o	−		
3	x	ENTER↑		
4	x_o	− R→P x⇄y		
5	θ	− x⇄y P→R	x′	
6		x⇄y	y′	In ALL, this
				is not needed.

Base Conversions

Note: Base conversion algorithms are given for positive values only. To convert a negative number, change sign, convert, and change sign of result.

Decimal Integer to Integer in Any Base

$$I_{10} \rightarrow J_b$$

In the following key sequence, f + 1 is the number of digits in J_b. d_i (i = 1, ..., f + 1) represents the i^{th} digit in J_b, counting from left to right;

$$\text{i.e.} \quad J_b = (d_1\ d_2\ \text{---}\ d_{f+1})_b$$

For large numbers, $J_b = (d_1\ d_2\ \text{---}\ d_{f+1})_b \cdot b^f$. See example 3.

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	b	ENTER↑ ENTER↑		
2	I	STO 1 LN x≷y LN		
3		÷	D	Let f be the largest
				integer ≤ D.
4		CL X		
5	f	x≷y ENTER↑ ENTER↑ RCL 1		
6		R↓ R↓ x≷y y^x ÷	E_1	d_i = integer part of
				E_i (i = 1, ..., f).
7	d_1	− ×	E_2	
8	d_i	− ×	E_{i+1}	Perform 8 for i = 2, ..., f.
9		FIX 0	d_{f+1}	

Example 1: Convert 1206 to hexadecimal (base 16).
(The hexadecimal digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.)

Answer:

$1206_{10} = 4B6_{16}$ (f = 2)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	16	ENTER↑ ENTER↑		
2	1206	STO 1 LN x⇄y LN		
3		÷	2.56	f = 2
4		CLX		
5	2	x⇄y ENTER↑ ENTER↑ RCL 1		
6		R↓ R↓ x⇄y y^x ÷	4.71	d_1 = 4
7	4	− ×	11.38	d_2 = 11
8	11	− ×	6.00	d_3 = 6

Example 2: Convert 513 to octal (base 8).

Answer:

$513_{10} = 1001_8$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	8	ENTER↑ ENTER↑		
2	513	STO 1 LN x⇄y LN		
3		÷	3.00	f = 3
4		CLX		
5	3	x⇄y ENTER↑ ENTER↑ RCL 1		
6		R↓ R↓ x⇄y y^x ÷	1.00	d_1 = 1
7	1	− ×	0.02	d_2 = 0
8	0	− ×	0.13	d_3 = 0
9	0	− ×	1.00	d_4 = 1

Example 3: Convert 6.023×10^{23} to octal.

Answer:

$6.023 \times 10^{23} = 1.7743_8 \times 8^{26}$

Note: If we consider 6.023×10^{23} to be a scientific measurement good only to four significant digits, it is meaningless for the octal representation to contain more than 5 significant digits. Therefore, we stop before the loop is completed.

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	8	ENTER↑ ENTER↑ 6 • 0		
2		2 3 EEX 2 3		
3		STO 1 LN x⇄y LN		
4		÷	26.33	f = 26 (Note: this gives
				the exponent in base 8.)
5		CL X		
6	26	x⇄y ENTER↑ ENTER↑ RCL 1		
7		R↓ R↓ x⇄y y^x ÷	1.99	$d_1 = 1$
8	1	− ×	7.94	$d_2 = 7$
9	7	− ×	7.54	$d_3 = 7$
10	7	− ×	4.34	$d_4 = 4$
11	4	− ×	2.69	$d_5 = 3$ (rounded), stop.

Integer in Base b to Decimal

$$(d_1\ d_2\ \text{---}\ d_{n-1}\ d_n)_b \rightarrow I_{10}$$

Examples:

1. $730020461_8 = 123740465_{10}$
2. $7DOF_{16} = 32015_{10}$
(A = 10, B = 11, C = 12, D = 13, E = 14, F = 15 in the hexadecimal system.)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	b	ENTER↑ ENTER↑ ENTER↑		
2	d_1	×		
3	d_i	+ ×		Perform 3 for
				i = 2, ..., n − 1.
4	d_n	+		

Highest Common Factor

The highest common factor (or greatest common divisor) of two positive integers a and b is the largest integer which divides both a and b. We write it as HCF(a, b).

Example:

$$\text{HCF}\ (51,\ 119)\ =\ 17.00$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a	STO 1		
2	b			
3		ENTER↑ ENTER↑ RCL 1 ÷	D	Let f be the largest integer ≤ D.
4	f	x⇄y CL X RCL 1 ×		
5		−	E	If E = 0, go to 8. Otherwise, continue with line 6.
6		RCL 1 x⇄y STO 1		
7		CL X +		Go to 3.
8		RCL 1	HCF(a, b)	

Least Common Multiple

The least common multiple of two positive integers a and b is the smallest positive integer that both a and b can divide.

$$\mathrm{LCM}(a, b) = \frac{a \cdot b}{\mathrm{HCF}(a, b)}$$

Example:

$$\mathrm{LCM}(51, 119) = 357.00$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a	STO 1 STO 3		
2	b	STO 2		
3		ENTER↑ ENTER↑ RCL 1 ÷	D	Let f be the largest
				integer ≤ D.
4	f	x⇄y CLx RCL 1 ×		
5		−	E	If E = 0, go to 8.
				Otherwise, continue with
				line 6.
6		RCL 1 x⇄y STO 1		
7		CLx +		Go to 3.
8		RCL 1 RCL 3 RCL		
		2 × x⇄y ÷	LCM (a,b)	

Statistical Applications

Exponential Curve Fit

Formulas: For a given set of data points

$$\{(x_i, y_i), i = 1, 2, \ldots, n\}$$

this procedure fits an exponential curve of the form

$$y = ae^{bx}$$

$$(a > 0)$$

The equation is linearized into

$$\ln y = \ln a + bx$$

The following statistics are computed:

1. Coefficients a, b

$$b = \frac{\Sigma x_i \ln y_i - \frac{1}{n} (\Sigma x_i)(\Sigma \ln y_i)}{\Sigma x_i^2 - \frac{1}{n} (\Sigma x_i)^2}$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\frac{\Sigma x_i}{n}\right]$$

2. Coefficient of determination

$$r^2 = \frac{\left[\Sigma x_i \ln y_i - \frac{1}{n} \Sigma x_i \Sigma \ln y_i\right]^2}{\left[\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right]\left[\Sigma(\ln y_i)^2 - \frac{(\Sigma \ln y_i)^2}{n}\right]}$$

3. Estimated value $\hat{y}$ for a given x

$$\hat{y} = ae^{bx}$$

Note: n is a positive integer and $n \neq 1$.

Example:

x_i	.72	1.31	1.95	2.58	3.14
y_i	2.16	1.61	1.16	.85	.50

Answer:

$a = 3.45$
$b = -0.58$
$y = 3.45e^{-0.58x}$
$r^2 = 0.98$
For $x = 1.5$, $\hat{y} = 1.44$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLΣ		
2	y_i	LN		Perform 2-3 for
3	x_i	Σ+	i	i = 1, 2, ..., n.
2′	y_k	LN		Perform 2′-3′ to delete
3′	x_k	■ Σ−		erroneous data x_k, y_k.
4		■ L.R. e^x STO 1	a	
5		x⇄y STO 2	b	
6		ENTER↑ ENTER↑ ■ s −		
7		÷ x^2	r^2	
8	x	RCL 2 × e^x RCL		
9		1 ×	$\hat{y}$	For a new x, go to step 8.
				For a new case, go to
				step 1.

Power Curve Fit

Formula: For a given set of data points

$$\left\{(x_i, y_i), i = 1, 2, \ldots, n\right\}$$

where $x_i > 0$ and $y_i > 0$

this procedure fits a power curve of the form

$$y = ax^b$$
$$(a > 0)$$

By writing this equation as

$$\ln y = b \ln x + \ln a$$

the problem can be solved as a linear regression problem.

Output statistics are:

1. Regression coefficients

$$b = \frac{\Sigma(\ln x_i)(\ln y_i) - \dfrac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}}{\Sigma(\ln x_i)^2 - \dfrac{(\Sigma \ln x_i)^2}{n}}$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\frac{\Sigma \ln x_i}{n}\right]$$

2. Coefficient of determination

$$r^2 = \frac{\left[\Sigma(\ln x_i)(\ln y_i) - \dfrac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}\right]^2}{\left[\Sigma(\ln x_i)^2 - \dfrac{(\Sigma \ln x_i)^2}{n}\right]\left[\Sigma(\ln y_i)^2 - \dfrac{(\Sigma \ln y_i)^2}{n}\right]}$$

3. Estimated value $\hat{y}$ for given x

$$\hat{y} = ax^b$$

Note: m is a positive integer and $n \neq 1$.

Example:

x_i	10	12	15	17	20	22	25	27	30	32	35
y_i	0.95	1.05	1.25	1.41	1.73	2.00	2.53	2.98	3.85	4.59	6.02

Answer:

$a = 0.03$
$b = 1.46$
$y = 0.03x^{1.46}$
$r^2 = 0.94$
For $x = 18$, $\hat{y} = 1.76$
$x = 23$, $\grave{y} = 2.52$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLΣ		
2	x_i	LN		Perform 2-3 for
3	y_i	LN x≷y Σ+	i	i = 1, 2, ..., n.
2′	x_k	LN		Perform 2′ -3′ to delete
3′	y_k	LN x≷y ■ Σ−		erroneous data x_k, y_k.
4		■ L.R. e^x STO 1	a	
5		x≷y STO 2	b	
6		ENTER↑ ENTER↑ ■ s ÷		
7		÷ x^2	r^2	
8	x	RCL 2 y^x RCL 1		
9		×	ŷ	For a new x, go to step 8.
				For a new case,
				go to step 1.

Analysis of Variance (One Way)

The one-way analysis of variance tests the differences between the population means of k treatment groups. Group i (i = 1, 2, ..., k) has n_i observations (treatment group may have equal or unequal number of observations).

The key sequence yields the analysis of variance table: sum of squares (SS), mean squares (MS), degrees of freedom (df), and the F ratio.

Formulas:

$$\text{Total SS} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}^2 - \frac{\left(\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^{k} n^i}$$

$$\text{Treat SS} = \sum_{i=1}^{k} \frac{\left(\sum_{j=1}^{n_i} x_{ij} \right)^2}{n_i} - \frac{\left(\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^{k} n_i}$$

$$\text{Error SS} = \text{Total SS} - \text{Treat SS}$$

$$\text{Treat df} = k - 1$$

$$\text{Error df} = \sum_{i=1}^{k} n_i - k$$

$$\text{Treat MS} = \frac{\text{Treat SS}}{\text{Treat df}}$$

$$\text{Error MS} = \frac{\text{Error SS}}{\text{Error df}}$$

$$F = \frac{\text{Treat MS}}{\text{Error MS}} \left(\text{with } k - 1 \text{ and } \sum_{i=1}^{k} n_i - k \text{ degrees of freedom} \right)$$

Example:

	i \ j	1	2	3	4	5	6
	1	10	8	5	12	14	11
Treatment	2	6	9	8	13		
	3	14	13	10	17	16	

Answer:

Total SS = 172.93
Treat SS = 66.93
Error SS = 106.00
Treat df = 2.00
Treat MS = 33.47
Error df = 12.00
Error MS = 8.83
F = 3.79 (with 2 and 12 degrees of freedom)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2		1 STO + 4		Perform 2-8 for i = 1, 2, ..., k.
3	x_{ij}	Σ+	j	Perform 3 for j = 1, 2, ..., n_i.
4		STO + 0 RCL ·		
5		1 STO + 1 x^2		
6		x≷y ÷ STO + 3		
7		RCL · 2 STO +		
8		2 ■ CLΣ		
9		RCL 2 RCL 1 x^2		
10		RCL 0 ÷ −	Total SS	
11		■ LAST X RCL 3 x≷y		
12		−	Treat SS	
13		−	Error SS	
14		■ LAST X RCL 4 1		
15		−	Treat df	
16		÷	Treat MS	
17		x≷y RCL 0 RCL 4		
18		−	Error df	
19		÷	Error MS	
20		÷	F	

Covariance and Correlation Coefficient

Formulas: For a set of given data points $\{(x_i, y_i), i = 1, 2, \ldots, n\}$, the covariance and the correlation coefficient are defined as:

$$\text{covariance } s_{xy} = \frac{1}{n-1}\left(\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \,\Sigma y_i\right)$$

$$\text{or } s_{xy}' = \frac{1}{n}\left(\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \,\Sigma y_i\right)$$

$$\text{correlation coefficient } r = \frac{s_{xy}}{s_x\, s_y}$$

where s_x and s_y are standard deviations

$$s_x = \sqrt{\frac{\Sigma x_i^2 - (\Sigma x_i)^2/n}{n-1}}$$

$$s_y = \sqrt{\frac{\Sigma y_i^2 - (\Sigma y_i)^2/n}{n-1}}$$

Note: $-1 \leqslant r \leqslant 1$

Example:

y_i	92	85	78	81	54	51	40
x_i	26	30	44	50	62	68	74

Answer:

$r = -0.96$
$s_{xy} = -354.14$
$s_{xy}' = -303.55$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CL Σ		
2	y_i	ENTER↑		Perform 2-3 for
				i = 1, 2, ..., n.
3	x_i	Σ+	i	
2′	y_k	ENTER↑		Perform 2′-3′ to delete
3′	x_k	■ Σ−		erroneous data x_k, y_k.
4		■ L.R. ENTER↑ ■ s		
5		÷ ÷	r	
6		ENTER↑ ENTER↑ ■ s ×		
7		×	s_{xy}	
8		RCL · 0 ENTER↑ ENTER↑		
9		1 − ÷ ÷	s_{xy}	

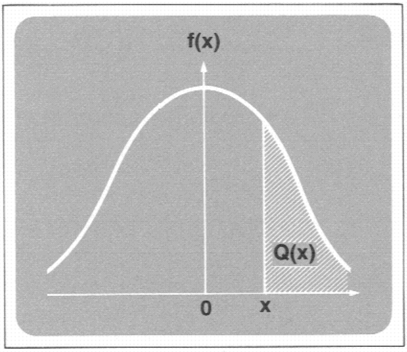
Normal Distribution

Formulas: The density function for a standard normal variable is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

The upper tail area is

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$$



For $x \geqslant 0$, polynomial approximation is used to compute Q(x):

$$Q(x) = f(x)\,(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + \epsilon(x)$$

$$\text{where } |\epsilon(x)| < 7.5 \times 10^{-8}$$

$$t = \frac{1}{1 + rx}, r = 0.2316419$$

$b_1 = .31938153$ $b_2 = -.356563782$
$b_3 = 1.781477937$ $b_4 = -1.821255978$
$b_5 = 1.330274429$

Note: The routine only works for $x \geqslant 0$. Equations $f(-x) = f(x)$, $Q(-x) = 1 - Q(x)$, where $x \geqslant 0$, can be used to find f and Q for negative numbers.

Examples:

1. $x = 1.18$
$f(x) = 0.20$
$Q(x) = 0.12$

2. $x = 2.28$
$f(x) = 0.03$
$Q(x) = 0.01$

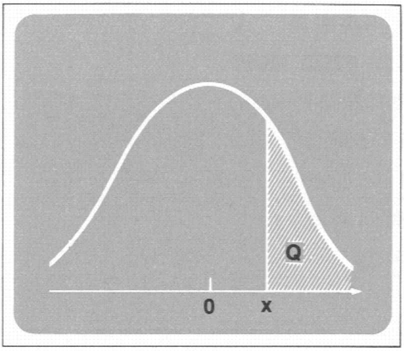
LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	r	STO 0		
2	b_1	STO 1		
3	b_2	STO 2		
4	b_3	STO 3		
5	b_4	STO 4		
6	b_5	STO 5		
7	x	STO 6 x^2 2 ÷		
8		CHS e^x ■ π 2		
9		× $\sqrt{x}$ ÷ STO 7	f(x)	
10		RCL 0 RCL 6 ×		
11		1 + $1/x$ ENTER↑ ENTER↑		
12		ENTER↑ RCL 5 × RCL		
13		4 + × RCL 3		
14		+ × RCL 2 +		
15		× RCL 1 + ×		
16		RCL 7 ×	Q(x)	Go to 7 for new x.

Inverse Normal integral

Formulas: This procedure determines the value of x such that

$$Q = \int_x^\infty \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt$$

where Q is given and $0 < Q \leqslant 0.5$.



The following rational approximation is used:

$$x = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

where $|\epsilon(Q)| < 4.5 \times 10^{-4}$

$$t = \sqrt{\ln \frac{1}{Q^2}}$$

$c_0 = 2.515517$ $d_1 = 1.432788$
$c_1 = 0.802853$ $d_2 = 0.189269$
$c_2 = 0.010328$ $d_3 = 0.001308$

Examples:

$Q = 0.12$ $Q = 0.05$
$x = 1.18$ $x = 1.65$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	c_0	STO 0		
2	c_1	STO 1		
3	c_2	STO 2		
4	d_1	STO 3		
5	d_2	STO 4		
6	d_3	STO 5		
7	Q	x^2 $1/x$ LN $\sqrt{x}$ STO		
8		6 ENTER↑ ENTER↑ ENTER↑ RCL		
9		5 × RCL 4 +		
10		× RCL 3 + ×		
11		1 + STO 7 CL X		
12		RCL 2 × RCL 1		
13		+ × RCL 0 +		
14		RCL 7 ÷ −	x	Go to 7 for new Q.

Permutations

A permutation is an ordered subset of a set of distinct objects. This procedure calculates the number of possible permutations of "a" objects taken "b" at a time.

Formula:

$$_aP_b = P(a, b) = \frac{a!}{(a - b)!}$$

where a, b are integers and $0 \leqslant b \leqslant a$.

Example:

$$_7P_5 = 2520.00$$

Notes:

$_aP_0 = 1$
$_aP_1 = a$
$_aP_a = a!$
Procedure requires $a \leqslant 69$.

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a	▨ N! ▨ LAST X		
2	b	− ▨ N! ÷	$_aP_b$	

Combinations

A combination is a selection of one or more of a set of distinct objects without regard to order. This procedure calculates the number of possible combinations of "a" objects taken "b" at a time (binomial coefficient).

Formula:

$$\binom{a}{b} = {}_aC_b = C^a_b = C(a, b) = \frac{a!}{b!(a-b)!}$$

Example:

$${}_7C_5 = 21.00$$

Notes:

${}_aC_0 = {}_aC_a = 1$
${}_aC_1 = {}_aC_{a-1} = a$
Procedure requires $a \leqslant 69$.

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	a	■ N! ■ LAST X		
2	b	− ■ LAST X ■ N!		
3		x⇄y ■ N! × ÷	${}_aC_b$	

Random Number Generator

Formulas: This procedure calculates uniformly distributed pseudo random numbers u_i in the range

$$0 \leqslant u_i \leqslant 1$$

using the following formula:

$$u_i = \text{fractional part of } [997\ u_{i-1}]$$

$$\text{where } u_0 = 0.5284163.$$

Example: Using the above u_0, generate a series of uniformly distributed random numbers.

0.83, 0.56, 0.27, 0.04, 0.20, 0.75, 0.83, 0.95,...

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	997	ENTER↑ ENTER↑ ENTER↑		
2	.5284163			
3		×	D_i	Perform 3-4 for $i = 1, 2, 3,$ ....
4	f_i	−	u_i	Let f_i = integer part of D_i.

Mean, Standard Deviation, Standard Error for Grouped Data

Formulas: Given a set of data points

$$x_1, x_2, \ldots, x_n$$

with respective frequencies

$$f_1, f_2, \ldots, f_n$$

the following statistics are computed:

$$\text{mean } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\text{standard deviation } s = \sqrt{\frac{\Sigma f_i x_i^2 - (\Sigma f_i) x^2}{\Sigma f_i - 1}}$$

$$\text{standard error } s_{\bar{x}} = \frac{s_x}{\sqrt{\Sigma f_i}}$$

Example:

x_i	2	3.4	7	11	23	3.41
f_i	5	3	4	2	3	1

Answer:

$\bar{x} = 7.92$
$s = 7.52$
$s_{\bar{x}} = 1.77$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2	x_i	ENTER↑		Perform 2-5 for i = 1, 2, ..., n.
3	f_i	STO + 0 x⇄y ×		
4		■ LAST x x⇄y × ■		
5		LAST x Σ+	i	
6		RCL 0 STO • 0		
7		RCL • 3 STO •		
8		2 ■ $\bar{x}$	$\bar{x}$	
9		■ s	s	
10		RCL 0 √x ÷	$s_{\bar{x}}$	

Chi-Square Statistics

Chi-Square Evaluation

This routine calculates the value of the χ^2 statistic for the goodness of fit test by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequency
E_i = expected frequency

Example:

O_i	8	50	47	56	5	14
E_i	9.6	46.75	51.85	54.4	8.25	9.15

Answer:

$\chi^2 = 4.84$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		▨ CLEAR		
2	E_i	ENTER↑ ENTER↑		Perform 2-3 for
3	O_i	− x^2 x⇄y ÷ +		i = 1, 2, ..., n.

2 × k Contingency Table

Formulas: Contingency tables can be used to test the null hypothesis that two variables are independent.

	1	2	3	…	k	Totals
A	a_1	a_2	a_3	…	a_k	N_A
B	b_1	b_2	b_3	…	b_k	N_B
Totals	N_1	N_2	N_3	…	N_k	N

Test statistic χ^2 has k − 1 degrees of freedom.

$$\chi^2 = \frac{N}{N_A} \sum_{i=1}^{k} \frac{a_i^2}{N_i} + \frac{N}{N_B} \sum_{i=1}^{k} \frac{b_i^2}{N_i} - N$$

Pearson's coefficient of contingency C measures the degree of association between the two variables.

$$C = \sqrt{\frac{\chi^2}{N + \chi^2}}$$

Example:

	1	2	3
A	2	5	4
B	3	8	7

Answer:

$\chi^2 = 0.02$
$C = 0.03$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2	a_i	ENTER↑		Perform 2-7 for
3	b_i	STO 3 STO + 1		i = 1, 2, ..., k.
4		x⇄y STO 2 STO +		
5		0 + √x RCL 3		
6		x⇄y ÷ RCL 2 ■		
7		LAST X ÷ Σ+	i	
8		RCL • 2 RCL 0		
9		÷ RCL • 4 RCL		
10		1 ÷ + 1 −		
11		RCL 0 RCL 1 +		
12		×	χ^2	
13		ENTER↑ ENTER↑ RCL 0 RCL		
14		1 + + ÷ √x	C	

F Statistic

This procedure is used for testing two population variances.

Formulas: Given independent random samples $\{x_i, i = 1, 2, \ldots, n_x\}$ and $\{y_i, i = 1,2, \ldots, n_y\}$ taken from two normal populations whose variances are σ_x^2 and σ_y^2, the F statistic, with $n_x - 1$ and $n_y - 1$ degrees of freedom (df), can be used to test the null hypothesis

$$H_0: \sigma_x^2 = \sigma_y^2$$

F is computed from the following:

$$F = \frac{s_x^2}{s_y^2}$$

where s_x^2 = sample variance of x
s_y^2 = sample variance of y

Example:

x: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54 $(n_x = 10)$

y: 79, 84, 108, 114, 120, 103, 122, 120 $(n_y = 8)$

Answer:

F = 1.02 (df = 9 and 7)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CL Σ		
2	x_i	Σ+	i	Perform 2 for i = 1, 2, ..., n_x.
3		■ s STO 1		
4		■ CL Σ		
5	y_i	Σ+	i	Perform 5 for i = 1, 2, ..., n_y.
6		■ s x^2 RCL 1		
7		x^2 x≷y ÷	F	

Paired t Statistic

Formulas: Given a set of paired observations from two normal populations with means μ_1, μ_2 (unknown)

x_i	x_1	x_2	...	x_n
y_i	y_1	y_2	...	y_n

let

$$D_i = x_i - y_i$$

$$\overline{D} = \frac{1}{n} \sum_{i=1}^{n} D_i$$

$$s_D = \sqrt{\frac{\Sigma D_i^2 - \frac{1}{n} (\Sigma D_i)^2}{n - 1}}$$

$$s_{\overline{D}} = \frac{s_D}{\sqrt{n}}$$

The test statistic

$$t = \frac{\overline{D}}{s_{\overline{D}}}$$

which has n − 1 degrees of freedom (df), can be used to test the null hypothesis

$$H_0: \mu_1 = \mu_2.$$

Example:

x_i	14	17.5	17	17.5	15.4
y_i	17	20.7	21.6	20.9	17.2

Answer:

$t = -7.16$
$df = 4.00$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLΣ		
2	x_i	ENTER↑		Perform 2-3 for i = 1, 2, ..., n.
3	y_i	− Σ+	i	
4		■ s RCL · 0		
5		$\sqrt{x}$ ÷ STO 0 ■		
6		$\bar{x}$ RCL 0 ÷	t	
7		RCL · 0 1 −	df	

t Statistic for Two Means

Formulas: Suppose $\{x_1, x_2, \ldots, x_{n_1}\}$ and $\{y_1, y_2, \ldots, y_{n_2}\}$ are independent random samples from two normal populations having means μ_1, μ_2 (unknown) and the same unknown variance σ^2.

We want to test the null hypothesis

$$H_0\colon \mu_1 - \mu_2 = D$$

where D is a given number.

Define

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - D}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{\Sigma x_i^2 - n_1 \bar{x}^2 + \Sigma y_i^2 - n_2 \bar{y}^2}{n_1 + n_2 - 2}}}$$

We can use this t statistic, which has the t distribution with $n_1 + n_2 - 2$ degrees of freedom, to test the null hypothesis H_0.

Example:

x: 79, 84, 108, 114, 120, 103, 122, 120

y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54

$n_1 = 8$
$n_2 = 10$

If $D = 0$ (i.e., $H_0\colon \mu_1 = \mu_2$)

then

$\bar{x} = 106.25$
$\bar{y} = 92.5$
$t = 1.73$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLΣ		
2	x_i	Σ+	i	Perform 2 for
				i = 1, 2, ..., n_1.
3		RCL • 0 STO 0		
4		RCL • 1 STO 1		
5		RCL • 2 STO 2		
6		■ $\bar{x}$ STO 3	$\bar{x}$	
7		■ CLΣ		
8	y_j	Σ+	j	Perform 8 for
				j = 1, 2, ..., n_2.
9		■ $\bar{x}$ STO 4	$\bar{y}$	
10	D	RCL 4 + RCL 3		
11		x⇄y − RCL 0 1/x		
12		RCL • 0 1/x +		
13		$\sqrt{x}$ ÷ RCL 2 RCL		
14		3 x^2 RCL 0 ×		
15		− RCL • 2 +		
16		RCL 4 x^2 RCL •		
17		0 × − RCL 0		
18		RCL • 0 + 2		
19		− ÷ $\sqrt{x}$ ÷	t	

Factorial and Gamma Function

This procedure uses Stirling's approximation to compute factorial. From factorial, Gamma function can easily be calculated.

Notes: This approximation can be used for positive $x \leqslant 69$ (otherwise the answer is $> 10^{100}$).

This approximation is good for large x.

For $x < 1$, use polynomial approximation.

To compute Gamma function, $\Gamma(x) = (x - 1)!$

Formula:

$$x! \cong \sqrt{2\pi x}\, x^x e^{-x} \left(1 + \frac{1}{12x} + \frac{1}{288x^2} - \frac{139}{51840x^3} - \frac{571}{2488320x^4}\right)$$

Example:

$$4.25! \cong 35.21$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CL Σ		
2	x	ENTER↑ ENTER↑ ENTER↑ y^x 2		
3		■ π × R↑ ×		
4		$\sqrt{x}$ × R↑ CHS e^x		
5		× STO 0 R↑ $1/x$		
6		ENTER↑ ENTER↑ ENTER↑		
7	12	$1/x$ Σ+		
8	288	$1/x$ × Σ+		
9	139	ENTER↑		
10	51840	÷ × × CHS Σ+		
11	571	ENTER↑		
12	2488320	÷ × × × CHS		
13		Σ+ RCL • 5 1		
14		+ RCL 0 ×	x!	

Financial Applications

Interest (Compound)

This procedure applies to an amount of principal that has been placed into an account and compounded periodically with no further deposits. With any three of the important variables given, a fourth may easily be calculated.

Notation:

n = number of time periods
i = periodic interest rate expressed as a decimal
PV = present value or principal
FV = future value or amount
I = interest amount

Future Value

Formula:

$$FV = PV\ (1 + i)^n$$

Example: Find the future amount of $1000 invested at 6% compounded annually for 5 years.

Answer:

$1338.23 (Note: i = 0.06)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	ENTER↑ 1 +		
2	n	y^x		
3	PV	×	FV	

Present Value

Formula:

$$PV = \frac{FV}{(1 + i)^n}$$

Example: What sum invested today, at 6% compounded annually, will amount to $1500 in 5 years?

Answer:

$1120.89 (Note: i = 0.06)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	FV	ENTER↑		
2	i	ENTER↑ 1 +		
3	n	y^x ÷	PV	

Number of Time Periods

Formula:

$$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1 + i)}$$

Example: If you deposit $250 in a savings account at 6% interest, compounded annually, how long will it take for your money to double?

Answer:

11.90 years (Note: i = 0.06)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	FV	ENTER↑		
2	PV	÷ LN 1 ENTER↑		
3	i	+ LN ÷	n	

Rate of Return

Formula:

$$i = \left(\frac{FV}{PV}\right)^{1/n} - 1$$

Example: Find the annual rate of return if $2000 doubles in 10 years when compounded monthly.

Answer:

6.95% (0.0695) annually

(Note: n = 120 months; FV = 4000; answer must be multiplied by 12 to find an annual rate of return.)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	FV	ENTER↑		
2	PV	÷		
3	n	$1/x$ y^x 1 −	i	

Interest Amount

Formula:

$$I = PV\left[(1 + i)^n - 1\right]$$

Example: Find the compounded interest on $2000 for 5 years if interest at 5.5% is compounded monthly.

Answer:

$631.41

(Note: n = 60 months; i = 0.055/12)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	ENTER↑ 1 +		
2	n	y^x 1 −		
3	PV	×	I	

Nominal Rate Converted to Effective Annual Interest Rate

This procedure calculates the effective or compounded annual interest rate when the number of periods per year and the nominal annual interest rate are known.

Finite Compounding

Formula:

$$\text{Effective} = (1 + i)^n - 1$$

Example: What is the effective annual rate of interest if the nominal (annual) rate of 6% is compounded monthly?

Answer:

6.17% (0.0617)
(Note: n = 12, i = .06/12)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		FIX 4		
2	i	ENTER↑ 1 +		
3	n	y^x 1 −	Effective	

Continuous Compounding

Formula:

$$\text{Effective} = e^i - 1$$

Example: A bank offers a savings plan with a 5.75% annual nominal interest rate. What is the annual effective rate if compounding is continuous?

Answer:

5.92% (0.0592)
(Note: i = .0575)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	e^x 1 −		
2		FIX 4	Effective	

Note: Some banks offer 365 days of continuous compounding on a 360-day basis. To find the effective interest rate, use the following procedure.

$$e^{i\left(\frac{365}{360}\right)}-1$$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	ENTER↑ 3 6 5 ×		
2		3 6 0 ÷		
3		e^x 1 − FIX 4	Effective	

Add-On Rate to Annual Percentage Rate (APR)

This procedure converts add-on interest rate (when a percentage of the loan amount is added on as a finance charge) to the true rate of interest (annual percentage rate).

Formula:

$$\mathrm{APR} \cong \frac{600\,\mathrm{ni}}{3(\mathrm{n}+1) + \left[(\mathrm{n}-1)\,\mathrm{ni/m}\right]}$$

where n = number of payments
m = number of payments in one year
i = add-on interest rate

Note: This formula will give an approximate, not an exect, answer.

Example: What is the true rate of interest (APR) on an 18-month, 5% add-on loan?

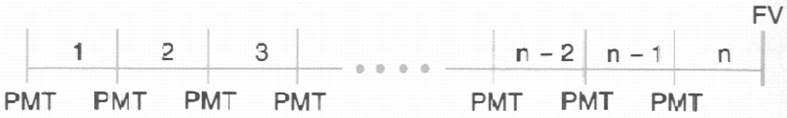
Answer:

9.27%
(Note: i = 0.05)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	ENTER↑ STO 1		
2	n	STO 2 × RCL 2		
3		1 − ×		
4	m	÷ RCL 2 1 +		
5		3 × + 6 0		
6		0 RCL 2 × RCL		
7		1 × x≷y ÷	APR	

Periodic Savings

This procedure calculates payment, future value, or number of time periods for a schedule of periodic payments into a savings account, given the interest rate and two of the other three variables.



Notation:

n = number of payments
i = periodic interest rate expressed as a decimal
PMT = payment (at the beginning of the period)
FV = future value

Note: Payments are assumed to occur at the beginning of the time period (annuity due or "payments in advance").

Number of Periods

Formula:

$$n = \frac{\ln\left[\dfrac{FV \cdot i}{PMT} + (1 + i)\right]}{\ln(1 + i)} - 1$$

Example: If you deposit $100 a month in a savings account which earns 5½% interest (compounded monthly), how long will it take to accumulate $2000?

Answer:

19.10 months
(Note: i = 0.055/12)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	STO 1		
2	FV	×		
3	PMT	÷ RCL 1 1 +		
4		STO 2 + LN RCL		
5		2 LN ÷ 1 −	n	

Payment Amount

Formula:

$$PMT = \frac{FV \cdot i}{(1 + i)^{n+1} - (1 + i)}$$

Example: In 3 years you will need $5000. How much should you deposit each month, if you will receive 6% annual interest, compounded monthly?

Answer:

$126.48
(Note: n = 36, i = .06/12)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	STO 1		
2	FV	× 1 RCL 1 +		
3		STO 2		
4	n	ENTER↑ 1 + y^x RCL		
5		2 − ÷	PMT	

Future Value

Formula:

$$FV = \frac{PMT}{i}\left[(1 + i)^{n+1} - (1 + i)\right]$$

Example: You are depositing $1000 per year in a savings account earning 7.5% interest compounded annually. How much will you have in 10 years?

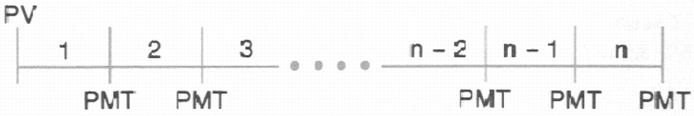
Answer:

$15,208.12
(Note: i = 0.075)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	STO 1 1 + ENTER↑		
2		ENTER↑		
3	n	ENTER↑ 1 + y^x x⇄y		
4		−		
5	PMT	× RCL 1 ÷	FV	

Direct Reduction Loan

Given any three of the variables listed below, these procedures calculate the fourth for a direct reduction loan (the type of loan commonly used for mortgages).



Notation:

n = number of payments
i = periodic interest rate expressed as a decimal
PMT = payment
PV = present value or principal

Payment Amount

Formula:

$$\mathrm{PMT} = \frac{\mathrm{PV} \cdot i}{1 - (1 + i)^{-n}}$$

Example: What monthly payment is required to pay off a $5000 loan at 9.5% interest in 36 months?

Answer:

$160.16
(Note: i = 0.095/12)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	STO 1		
2	PV	× RCL 1 1 +		
3	n	CHS y^x 1 x≷y −		
4		÷	PMT	

Present Value

Formula:

$$\mathrm{PV} = \mathrm{PMT} \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

Example: You are willing to pay $125 per month for 36 months. If the current interest rate is 9.5%, how much can you borrow?

Answer:

$3902.23
(Note: i = 0.095/12)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	STO 1 1 +		
2	n	CHS y^x 1 x≷y −		
3		RCL 1 ÷		
4	PMT	×	PV	

Number of Time Periods

Formula:

$$n = -\frac{\ln(1 - i\,PV/PMT)}{\ln(1 + i)}$$

Example: How many payments are required to pay off a loan of $4000 at 9.5% annual interest, with payments of $175 per month?

Answer:

25.31 months
(Note: i = 0.095/12)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	STO 1		
2	PV	×		
3	PMT	÷ 1 x≷y − LN		
4		RCL 1 1 + LN		
5		÷ CHS	n	

Interest Rate

It is not possible to write a closed equation for i in a direct reduction loan. Hence, it is necessary to use an iterative process like the one below.

Formula:

$$\text{Periodic interest rate } i = \frac{\text{PMT}\left[1 - \left(\frac{1}{1+i}\right)\right]^{n}}{\text{PV}}$$

$$\text{Annual interest rate } = i \times A$$

where A = number of payments per year

Example: You have a $30,000 mortgage on which you will make 360 monthly payments of $179.86. What interest rate are you paying?

Answer:

6.00% (8 iterations)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		FIX 9		
2	n	ENTER↑ ENTER↑		
3	PMT	ENTER↑ FIX 9		
4	PV	÷ STO 1 CLx +		
5		1 + $1/x$ $x⇄y$ y^x		Perform 5-7 for k = 1, 2, ...,
6		1 $x⇄y$ − RCL 1		until i_k converges (to
7		×	i_k	desired decimal place).
8		EEX 2 ×		
9	A	× FIX 2		Answer is in %.

Amortization Schedule

I_k = interest paid in kth payment
PMT = payment
PP_k = payment to principal of kth payment
PV_k = remaining balance after kth payment
PV_0 = amount of loan
i = periodic interest rate expressed as a percent

An amortization schedule consists of the interest paid, the payment to principal, and the remaining balance for each payment k = 1, 2, ...

Formulas:

$I_k = iPV_{k-1}/100$
$PP_k = PMT - I_k$
$PV_k = PV_{k-1} - PP_k$

Example: Generate an amortization schedule for the first two months of a $40,000 loan at 9% (i = 9/12) with monthly payments of $321.85.

Answer:

I_1 = $300.00	I_2 = $299.84
PP_1 = $21.85	PP_2 = S22.01
PV_1 = S39978.15	PV_2 = $39956.14

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	PMT	STO 1		
2	i	ENTER↑ ENTER↑		i is expressed as a percent.
3	PV_0			
4		x≷y %	I_k	Repeat for subsequent payments.
5		RCL 1 x≷y −	PP_k	
6		−	PV_k	

Discounted Cash Flow Analysis

The primary purpose of this procedure is to compute the net present value (NPV) of a series of cash flows. The NPV is found by discounting the cash flows at the desired rate of return, and then subtracting the initial investment.

In general, an initial investment is made in some enterprise that is expected to bring in periodic cash flows. After discounting, a negative value for NPV indicates that the enterprise would not be profitable, while a positive value for NPV means that the enterprise will show a profit to the extent that a rate of return i on the initial investment has been exceeded.

Notation:

PV_0 = original investment
PV_k = net cash flow of the k^{th} period
i = discount rate per period (as a decimal)
NPV_k = net present value at period k

Formula:

$$NPV_k = -PV_0 + \sum_{j=1}^{k} \frac{PV_j}{(1+i)^j}$$

Example: A small shopping complex, which costs $137,000, is estimated to have annual cash flows as follows:

Year	Cash Flow ($)
1	10,000
2	13,000
3	19,000
4	152,000 (property sold in 4th year)

The desired minimum yield is 10%. Will this rate be achieved by the above cash flows?

Answer:

$NPV_1 = -127909.09$
$NPV_2 = -117165.29$
$NPV_3 = -102890.31$
$NPV_4 = 927.74$

Because the final NPV is positive, the investment more than achieves the desired yield.

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	i	ENTER↑ 1 + STO 1		
2	PV_1	x⇄y ÷		
3	PV_0	−	NPV_1	
4	PV_j	RCL 1		Perform 4-5 for
				j = 2, 3, ..., k.
5	j	y^x ÷ +	NPV_j	

Depreciation

These procedures can be used to calculate depreciation of assets using straight line, declining balance, or sum of the years' digits method.

Straight Line Depreciation

Formulas:

$$D = \frac{PV}{n}$$

$$RDV_k = PV - kD$$

where PV = original value of asset (less salvage value)
n = depreciable life of asset
D = depreciation per year
RDV_k = remaining depreciable value at time period k

Example: A duplex costing $40,000 (exclusive of land) is depreciated over 25 years, using the straight line method. What is the depreciation amount and remaining depreciable value after 5 years?

Answer:

$D = \$1600$
$RDV_5 = \$32,000$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	PV	ENTER↑ ENTER↑		
2	n	÷	D	
3	k	× −	RDV_k	

Declining Balance Depreciation

Formulas:

$$D_k = PV \cdot \frac{R}{n} \left(1 - \frac{R}{n}\right)^{k-1}$$

$$RDV_k = PV \left(1 - \frac{R}{n}\right)^{k}$$

where PV = original value of asset
n = depreciable life of asset
R = depreciation rate (given by user)
D_k = depreciation at time period k
RDV_k = remaining depreciable value at time period k

Example: A fleet car has a value of $2500 and a life expectancy of 6 years. Use the double declining balance method ($R = 2$) to find the amount of depreciation and remaining depreciable value after 4 years.

Answer:

$RDV_4 = \$493.83$
$D_4 = \$246.91$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	k	ENTER↑ 1 ENTER↑		
2	R	ENTER↑		
3	n	÷ − STO 1 x⇄y		
4		y^x		
5	PV	×	RDV_k	
6		RCL 1 ÷ 1 RCL		
		1 − ×	D_k	

Sum of the Years' Digits Depreciation (SOYD)

Formula:

$$D_k = \frac{2(n - k + 1)}{n(n + 1)} PV$$

$$RDV_k = S + (n - k)\, D_k/2$$

where PV = original value of asset
n = depreciable life of asset
S = salvage value
D_k = depreciation at time period k
RDV_k = remaining depreciable value at time period k

Example: Apartments valued at $88,000 are depreciated over 25 years using SOYD depreciation. What is the depreciation amount and remaining depreciable value after 10 years? Assume a salvage value of zero.

Answer:

$D_{10} = \$4332.31$
$RDV_{10} = \$32492.31$

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	n	STO 1		
2	k	STO 2 − STO 3		
3		1 + RCL 1 ENTER↑		
4		ENTER↑ × + ÷ 2		
5		×		
6	PV	×	D_k	
7		RCL 3 × 2 ÷		
8	S	+	RDV_k	

Calendar Routine

Weekday

This procedure finds the day of the week for any date since September 14, 1752.

d = day of month
m = month, with January and February being the 13th and 14th months of the previous year
y = year (4 digits)

$$\text{Weekday} = \left[d + e_1 + e_2 - e_3 + e_4\right] \pmod{7}$$

$$\text{where} \quad e_1 = \text{INT}\left(\frac{13}{5}(m+1)\right)$$

$$e_2 = \text{INT}\left(\frac{5}{4}\,y\right)$$

$$e_3 = \text{INT}\left(\frac{y}{100}\right)$$

$$e_4 = \text{INT}\left(\frac{y}{400}\right)$$

INT is ''integer part of''.

Output is read as follows:

0 – Saturday
1 – Sunday
2 – Monday
3 – Tuesday
4 – Wednesday
5 – Thursday
6 – Friday

Example: On what day was February 29, 1972?

Answer:

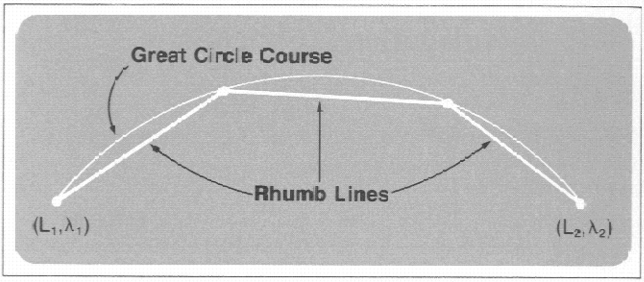
Tuesday (d = 29, m = 14, y = 1971)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	d	ENTER↑		
2	m	ENTER↑		
3	y	STO 1 R↓ 1 +		
4		1 3 × 5 ÷	E_1	Let e_1 = integer part of E_1.
5		CLx		
6	e_1	+ RCL 1 x⇄y STO		
7		1 x⇄y ENTER↑ ENTER↑ ENTER↑		
8		5 × 4 ÷	E_2	Let e_2 = integer part of E_2.
9		CLx		
10	e_2	RCL 1 +		For 20th century date,
				go to 18.
11		STO 1 R↓ EEX 2		
12		÷	E_3	Let e_3 = integer part of E_3.
13		CLx		
14	e_3	CHS STO + 1 R↓		
15		4 0 0 ÷	E_4	Let e_4 = integer part of E_4.
16		CLx		
17	e_4	RCL 1 +		Go to 19.
18		6 +		
19		ENTER↑ ENTER↑ 7 ÷	E_5	Let e_5 = integer part of E_5.
20		CLx		
21	e_5	ENTER↑ 7 × −		

Navigation Applications

Rhumb Line Navigation

This procedure calculates the rhumb line distance and course for the rhumb line between two points on the earth (a spherical earth is assumed). Successive legs can be linked without reentry of initial latitude and longitude.



Notation:

L_1 = latitude of initial point
λ_1 = longitude of initial point
L_2 = latitude of final point
λ_2 = longitude of final point
C = rhumb line course
DIST = rhumb line distance

Formulas:

$$C = \left| \tan^{-1} \left(\frac{\pi \sin^{-1} \left\{ \sin \left[(\lambda_1 - \lambda_2)/2 \right] \right\}}{90 \ln \dfrac{\tan (45 + L_2/2)}{\tan (45 + L_1/2)}} \right) \right|$$

$$\text{If } \sin^{-1} \left[\sin (\lambda_1 - \lambda_2) \right] < 0, \text{ then } C = 360 - C$$

$$\text{DIST} = \begin{cases} 60 (\lambda_2 - \lambda_1) \cos (L), \text{if} \cos (C) = 0 \\ 60 \dfrac{L_2 - L_1}{\cos (C)}, \text{ if } \cos (C) \neq 0 \end{cases}$$

Notes: No course should pass through the North or South Pole.

This procedure gives incorrect results when computing distances due east or due west across the dateline. To obtain correct results, compute up to the dateline and then proceed on the other side.

Errors in distance calculations may be encountered as cos(C) approaches zero.

Accuracy deteriorates for very short legs.

Northern latitudes and western longitudes are input and output as positive values; southern latitudes and eastern longitudes are input and output as negative values.

Example: Find the distances and headings for a flight from Anchorage, Alaska, to Juneau, Alaska, to Seattle, Washington.

Anchorage	L61°13′N	λ149°54′W
Juneau	L58°18′N	λ134°25′W
Seattle	L47°36′N	λ122°20′W

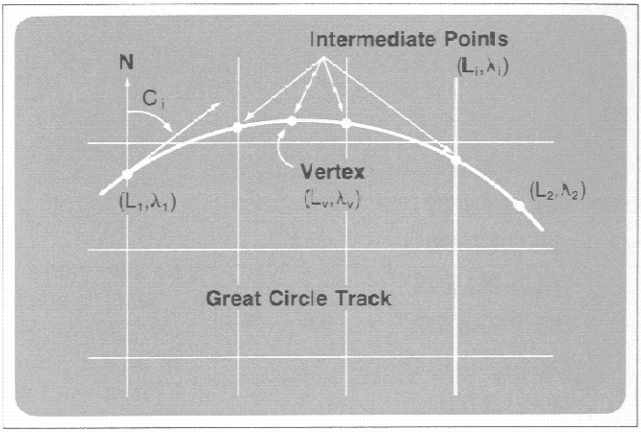
Answer:

Anchorage – Juneau	C = 110.52°	DIST = 499.22 nautical miles
Juneau – Seattle	C = 145.94°	DIST = 774.90 nautical miles

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	L_1	■ H.MS+ STO 1		
2	λ_1	■ H.MS+ STO 3		
3	Next L	■ H.MS+ STO 2		
4	Next λ	■ H.MS+ STO 4		
5		RCL 3 RCL 4 −		
6		STO 7 2 ÷ SIN		
7		■ SIN⁻¹ 9 0 ÷		
8		■ π × RCL 2		
9		2 ÷ 4 5 +		
10		TAN RCL 1 2 ÷		
11		4 5 + TAN ÷		
12		LN R→P x² √x x⇄y		
13		RCL 7 SIN ■ SIN⁻¹		If negative, go to line 15.
14		x⇄y	C	C in decimal degrees. Go to
				line 16.
15		x⇄y 3 6 0 +	C	C in decimal degrees.
16		COS	cos (C)	If zero, go to line 19.
17		RCL 2 RCL 1 −		
18		x⇄y ÷ 6 0 ×	DIST	DIST in nautical miles. Go
				to line 21.
19		RCL 7 RCL 2 COS		
20		× 6 0 ×	DIST	DIST in nautical miles.
21		RCL 2 STO 1 RCL		
22		4 STO 3		Go to line 3 for next leg.

Great Circle Navigation

This procedure calculates the great circle distance and initial course for the great circle track between two points on the earth (a spherical earth is assumed). The coordinates of the vertex and the distance from the initial point to the vertex can be calculated. The latitude of a point of intersection of a great circle track with an intermediate longitude can also be calculated.



Notation:

L_1 = latitude of initial point
λ_1 = longitude of initial point
L_2 = latitude of final point
λ_2 = longitude of final point
DIST = great circle distance
C_i = initial course
L_v = latitude of vertex
λ_v = longitude of vertex
λ_v' = longitude of alternate vertex
DIST_v = distance from initial point to vertex
L_i = latitude of intermediate point
λ_i = longitude of intermediate point

Formulas:

$$\text{DIST} = 60 \cos^{-1} \left[\sin(L_1) \sin(L_2) + \cos(L_1) \cos(L_2) \cos(\lambda_1 - \lambda_2)\right]$$

$$C_i = \cos^{-1} \left(\frac{\sin(L_2) - \cos(\text{DIST}/60) \sin(L_1)}{\sin(\text{DIST}/60) \cos(L_1)} \right)$$

$$\text{If } \sin(\lambda_1 - \lambda_2) < 0, \text{ then } C_i = 360 - C_i$$

$$L_i = \tan^{-1}\left(\frac{\tan(L_1)\sin(\lambda_i - \lambda_2) - \tan(L_2)\sin(\lambda_i - \lambda_1)}{\sin(\lambda_1 - \lambda_2)}\right)$$

$$\lambda_v = \tan^{-1}\left(\frac{\tan(L_2)\cos(\lambda_1) - \tan(L_1)\cos(\lambda_2)}{\tan(L_1)\sin(\lambda_2) - \tan(L_2)\sin(\lambda_1)}\right)$$

$$\lambda_v' = \lambda_v = 180^\circ$$

$$DIST_v = 60\sin^{-1}\left(\frac{\cos(C_i)\cos(L_1)}{\sin(L_v)}\right)$$

Notes: No point on a leg should be at either the North or South Pole.

No leg should pass more than halfway around the earth.

Points located at diametrically opposite sides of the earth should not be used since there are an infinite number of great circle courses through such points.

C_i cannot always be calculated along lines of longitude ($\lambda_1 = \lambda_2$).

Equator crossings are at $\lambda = \lambda_v \pm 90°$.

Northern latitudes and western longitudes are input and output as positive values; southern latitudes and eastern longitudes are input and output as negative values.

Example: A ship is proceeding from Manila to Los Angeles. The captain wishes to sail a great circle course from L12°45′12″N, λ124°20′06″E (input as negative), off the entrance to San Bernardino Strait, to L33°48′48″N, λ120°07′06″W, five miles south of Santa Rosa Island.

Find the initial great circle course and great circle distance; the latitude and longitude of the vertex and the distance from the initial point to the vertex; the latitude at λ180°.

Answer:

DIST = 6185.88 nautical miles
C_i = 50.32°
λ_v = 19°26′00″E (output as negative)
λ_v' = 160°34′00″W
L_v = 41°21′08″N
$DIST_v$ = 4228.83 nautical miles
L_i = 39°41′33″N (longitude at λ_i = 180°)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	L_2	■ H.MS→ STO 2 1		
2		P←R		
3	L_1	■ H.MS→ STO 1 1		
4		P←R STO 5 x⇄y STO		
5		6 R↓ × R↓ STO		
6		7 × R↑		
7	λ_1	■ H.MS→ STO 3		
8	λ_2	■ H.MS→ STO 4 −		
9		STO 0 COS × +		
10		■ COS⁻¹ STO 8 6		
11		0 ×	DIST	DIST in nautical miles.
12		RCL 7 RCL 6 RCL		
13		8 COS × − RCL		
14		8 SIN ÷ RCL 5		
15		÷ ■ COS⁻¹ RCL 0		
16		SIN STO 0		If negative, go to line 18.
17		x⇄y STO 9	C_i	C_i in decimal degrees.
				Go to line 20.
18		x⇄y 3 6 0 x⇄y		
19		− STO 9	C_i	C_i in decimal degrees.
20		RCL 1 TAN STO 1		
21		RCL 2 TAN STO 2		If the coordinates of the
				vertex are not desired,
				go to line 32.
22		RCL 4 RCL 1 P→R		
23		x⇄y RCL 3 RCL 2		
24		P→R R↓ − R↓ −		
25		R↑ ÷ ■ TAN⁻¹ ENTER↑		
26		■ →H.MS	λ_v	If negative, go to line 29.
27		x⇄y 1 8 0 −		
28		■ →H.MS	λ_v'	Go to line 31.
29		x⇄y 1 8 0 +		

LINE	DATA	OPERATIONS	RESULTS	REMARKS
30		▨ →H.MS	λ_v'	
31				To calculate L_v, let
				$\lambda_i = \lambda_v$ or λ_v' and proceed
				to line 32.
32	λ_i	▨ H.MS→ ENTER↑ ENTER↑ RCL		
33		4 − SIN RCL 1		
34		× x⇄y RCL 3 −		
35		SIN RCL 2 × −		
36		RCL 0 ÷ ▨ TAN⁻¹		
37		▨ →H.MS	L_i	Goto line 32 for next λ_i; or,
				if L_v was just calculated,
				continue with line 38 to
				calculate **DIST**$_v$.
38		▨ H.MS→ SIN RCL 9		
39		COS RCL 5 × ÷		
40		√x ▨ SIN⁻¹ 6 0		
41		×	DIST$_v$	DIST$_v$ in nautical miles.
42				Go to line 32 for
				intermediate points.

Sight Reduction Table

This procedure calculates the computed altitude and azimuth of a celestial body given the observer's latitude and the declination and local hour angle of the body.

Notation:

DEC = declination of celestial body
LHA = local hour angle of body
L = observer's latitude
Zn = azimuth of body
Hc = computed altitude of body

Formulas:

$$Hc = \sin^{-1}\left[\sin(DEC)\sin(L) + \cos(DEC)\cos(L)\cos(LHA)\right]$$

$$Z = \cos^{-1}\left(\frac{\sin(DEC) - \sin(L)\sin(Hc)}{\cos(Hc)\cos(L)}\right)$$

$$Zn = \begin{cases} Z, \text{ if } \sin(LHA) < 0 \\ 360 - Z, \text{if} \sin(LHA) > 0 \end{cases}$$

Notes: Northern latitudes, northern declinations, and western hour angles are input as positive values; southern latitudes, southern declinations and eastern hour angles are input as negative values.

This procedure may also be used for star identification by entering the observed azimuth in place of local hour angle and observed altitude in place of declination. The outputs are then declination and local hour angle, respectively, instead of altitude and azimuth. The star may be identified by comparing this computed declination to the list of stars in *The Nautical Almanac*.

Example: Compute the altitude and azimuth of the Sun if its LHA is 333°01′54″W and its declination is 12°28′06″S (input as negative). The assumed latitude is 34°11′06″S (input as negative).

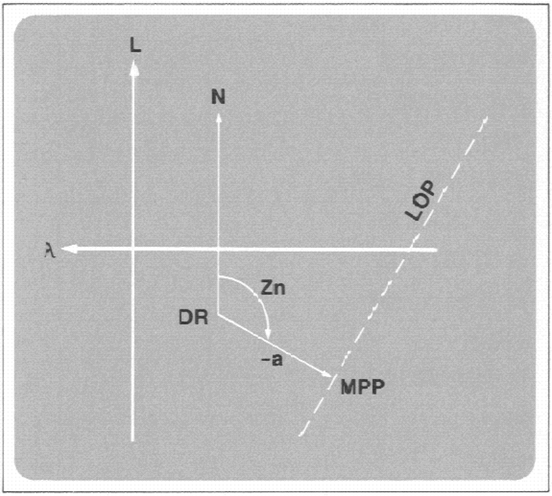
Answer:

Hc = 57°15′58″
Zn = 54.97°

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	DEC	■ H.MS→ 1 P→R		
2	L	■ H.MS→ 1 P→R STO		
3		1 x⇄y STO 2 R↓		
4		× R↓ STO 3 ×		
5		R↑		
6	LHA	■ H.MS→ 1 P→R x⇄y		
7		STO 4 R↓ × +		
8		■ SIN⁻¹ STO 5		
9		■ →H.MS	H_c	
10		RCL 3 RCL 2 RCL		
11		5 1 P→R R↓ ×		
12		− R↑ RCL 1 ×		
13		÷ ■ COS⁻¹ STO 6	Z	
14		RCL 4		If negative, go to line 17.
15		3 6 0 RCL 6		
16		−	Z_n	Z_n in decimal degrees.
17		RCL 6	Z_n	Z_n in decimal degrees.

Most Probable Position

This procedure computes the most probable position (MPP) from a single observation of a celestial object by dropping a perpendicular from the dead reckoning position (DR) to the line of position (LOP) of the object.



Notation:

L_1 = latitude of observer's DR
λ_1 = longitude of observer's DR
L_{mpp} = latitude of most probable position
λ_{mpp} = longitude of most probable position
Hc = computed altitude of object
Ho = corrected sextant height
a = altitude intercept: (−) = toward, (+) = away
Zn = azimuth of object

Formulas:

$$a = Hc - Ho$$

$$\lambda_{mpp} = \lambda_1 + \frac{a \sin(Zn)}{\cos(L_1)}$$

$$L_{mpp} = L_1 - a \cos(Zn)$$

Notes: Northern latitudes and western longitudes are input and output as positive values; southern latitudes and eastern longitudes are input and output as negative values.

Example: A navigator determines his DR to be L40°12'S (input as negative), λ159°57'E (input as negative). He observes Procyon to be 11°11'18" above the horizon. The computed altitude is 10°57' at azimuth 73.4°. What is his MPP?

Answer:

L_{mpp} = 40°07'55"S (output as negative)
λ_{mpp} = 160°14'56"E (output as negative)

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1	Zn	ENTER↑		Z_n in decimal degrees.
2	Hc	ENTER↑		
3	Ho	■ H.MS→ −	a	
4		P→R CHS		
5	L_1	■ H.MS→ STO 1 +		
6		■ →H.MS	L_{mpp}	
7		R↓ RCL 1 COS ÷		
8		■ →H.MS		
9	λ_1	■ H.MS+	λ_{mpp}	

Surveying Applications

Note: Several procedures from the section on Mathematical Applications will also be of interest to surveyors, such as:

Triangle Solutions
Curve Solutions
Coordinate Translation and Rotation

Bearing Traverse

This procedure uses bearings and distances to calculate coordinates of successive points in a traverse. Area, closing distance, and closing azimuth can be calculated for a closed traverse.

Notation:

HD = horizontal distance
SD = slope distance
ZA = zenith angle
AZ = azimuth
BRG = bearing
N_i = northing of point i
E_i = easting of point i
AREA = area of traverse in square feet
CL HD = closing horizontal distance
CL AZ = closing azimuth

Formulas:

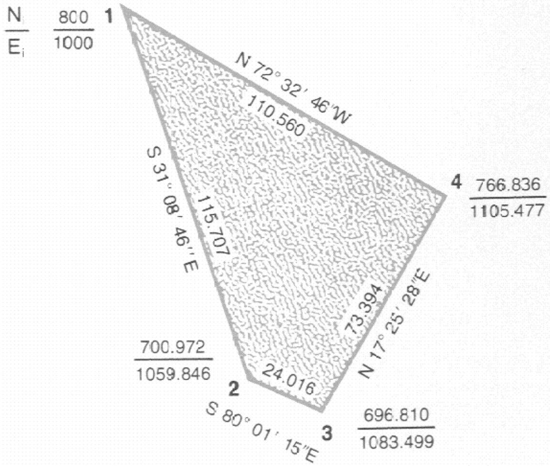
$$HD = SD \sin(ZA)$$

$$N_{i+1} = N_i + HD \sin(AZ)$$

$$E_{i+1} = E_i + HD \cos(AZ)$$

$$\begin{aligned} AREA = {} & (E_2 - E_1)\left[(N_2 - N_1)/2\right] + (E_3 - E_2)\left[(N_3 - N_1) + (N_3 - N_2)/2\right] \\ & + \ldots + (E_k - E_{k-1})\left[(N_{k-1} - N_1) + (N_k - N_{k-1})/2\right] \\ & + \ldots + \ (E_1 \ - E_n)\left[(N_n - N_1) + (N_1 - N_n)/2\right] \end{aligned}$$

Example: Traverse the figure shown below using the bearing and distance for each side to calculate the coordinates of the points. At the end of the traverse, calculate area and closure.



Answer:

CL N = 799.998
CL E = 1000.007
AREA = 5104 sq. ft.
CL HD = 0.007 ft.
CL AZ = 109°05′49″

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2	N_1	STO 1		
3	E_1	STO 2 x⇄y Σ+		
4	BRG NE	■ H.MS→		
	BRG SE	■ H.MS→ CHS 1 8		
		0 +		
	BRG NW	■ H.MS→ CHS		
	BRG SW	■ H.MS→ 1 8 0		
		+		
5	HD or SD			
6		ENTER↑		Omit lines 6-7 if distance
7	ZA	■ H.MS→ SIN ×		is horizontal.
8		P→R Σ+		
9		RCL 3 ■ LAST X +		Omit lines 9-12 if
10		STO 3 ■ LAST X 2		AREA is not calculated.
11		÷ − × STO +		
12		4		
13		RCL Σ+	N	
14		x⇄y	E	Go to line 4 for next course.
15		RCL 4	AREA	Ignore sign if negative.
16		RCL 2 RCL 1 ■		
17		Σ− RCL Σ+ R→P	CL HD	
18		x⇄y ■ →H.MS	CL AZ	If negative, add 360°
				(■ H.MS+).

Field Angle Traverse

This procedure uses angles or deflections and distances to calculate coordinates of successive points in a traverse. Area, closing distance, and closing azimuth can be calculated for a closed traverse.

Notation:

HD = horizontal distance
SD = slope distance
ZA = zenith angle
AZ = azimuth
REF AZ = reference azimuth
ANG RT = angle right
ANG LT = angle left
DEF RT = deflection right
DEF LT = deflection left
N_i = northing of point i
E_i = easting of point i
AREA = area of traverse in square feet
CL HD = closing horizontal distance
CL AZ = closing azimuth

Formulas:

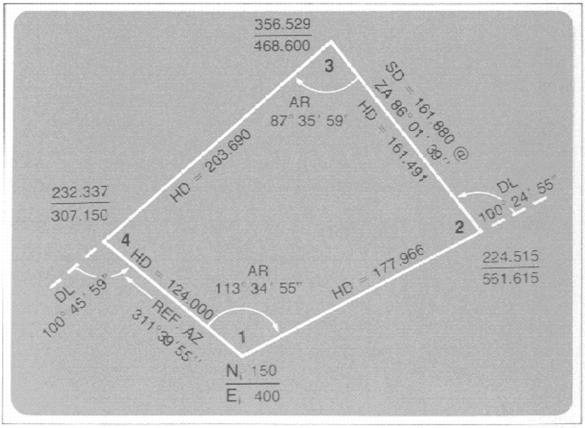
$$HD = SD \sin(ZA) \qquad N_{i-1} = N_i + HD \cos(AZ)$$

$$E_{i+1} = E_i + HD \sin(AZ)$$

$$AREA = (N_2 - N_1)\left[(E_2 - E_1)/2\right] + (N_3 - N_2)\left[(E_3 - E_1) + (E_3 - E_2)/2\right]$$

$$+ \ldots + (N_n - N_{n-1})\left[(E_{n-1} - E_1) + (E_n - E_{n-1})/2\right]$$

Example: Traverse the figure shown below using the field angle traverse for each side to calculate the coordinates of the points. At the completion of the traverse, calculate area and closure.



Answer:

CL N = 149.905
CL E = 399.783
AREA = 26559 sq. ft.
CL HD = 0.237 ft.
CL AZ = 246° 19′ 43″

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2	N₁	STO 1		
3	E₁	STO 2 x⇄y Σ+		
4	REF AZ	■ H.MS→ 1 8 0		
5		+ STO 5		
6	ANG RT	■ H.MS→ 1 8 0		
		+		
	ANG LT	■ H.MS→ CHS 1 8		
		0 +		
	DEF RT	■ H.MS→		
	DEF LT	■ H.MS→ CHS		
7		RCL 5 + STO 5		
8	HD or SD			
9		ENTER↑		Omit lines 9-10 if distance
10	ZA	■ H.MS→ SIN ×		is horizontal.
11		P→R Σ+		
12		RCL 3 ■ LAST X		Omit lines 12-15 if AREA
13		STO 3 ■ LAST X 2		is not calculated.
14		÷ − × STO +		
15		4		
16		RCL Σ+	N	
17		x⇄y	E	Go to line 6 for next course.
18		RCL 4	AREA	Ignore sign if negative.
19		RCL 2 RCL 1 ■		
20		Σ− RCL Σ+ R→P	CL HD	
21		x⇄y ■ →H.MS	CL AZ	If negative, add
				360°. (■ H.MS→).

Inverse from Coordinates

This procedure uses coordinates to calculate distance and azimuth between points of a traverse. Area can be calculated for a closed traverse.

Notation:

HD = horizontal distance
AZ = azimuth
N_i = northing of point i
E_i = easting of point i
AREA = area of traverse in square feet

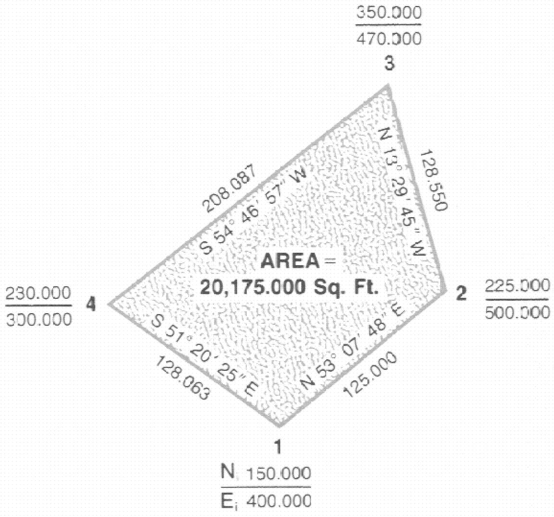
Formulas:

$$HD = \sqrt{(E_i - E_{i-1})^2 + (N_i - N_{i-1})^2}$$

$$AZ = \tan^{-1}\left(\frac{E_i - E_{i-1}}{N_i - N_{i-1}}\right)$$

$$\begin{aligned} AREA = {} & (N_2 - N_1)\left[(E_2 - E_1)/2\right] + (N_3 - N_2)\left[(E_3 - E_1) + (E_3 - E_2)/2\right] \\ & + \ldots + (N_k - N_{k-1})\left[(E_{k-1} - E_1) + (E_k - E_{k-1})/2\right] \\ & + \ldots + (N_1 - N_n)\left[(E_n - E_1) + (E_1 - E_n)/2\right] \end{aligned}$$

Example: Traverse the figure shown below using coordinates to calculate the azimuth and distance for each side. At the completion of the traverse, calculate area.



Answer:

AREA = 20175 sq. ft.

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ [CLEAR]		
2	N_1	[ENTER↑]		
3	E_1	[Σ+]		
4		[RCL] [Σ+]		
5	Next N	[ENTER↑]		
6	Next E	[R↑] [x⇄y] [R↓] [−] [R↓]		
7		[−] [STO] [2] [R↑] [STO]		
8		[1] [Σ+]		
9		[RCL] [3] ■ [LAST x] [+]		Omit lines 9-15 if AREA
10		[STO] [3] ■ [LAST x] [2]		is not calculated.
11		[÷] [−] [×] [STO] [+]		
12		[4] [RCL] [1] [RCL] [2]		
13		[R→P]	HD	
14		[x⇄y] ■ [→HMS]	AZ	If negative, add 360°
				(■ [HMS+]).
15				Go to line 18.
16		■ [LAST x] [x⇄y] [R→P]	HD	
17		[x⇄y] ■ [→H.MS]	AZ	If negative, add 360°
				(■ [H.MS+]).
18				Go to line 4 for next N
				and next E. Continue
				until N_1 and E_1 have
				been reentered.
19		[RCL] [4]	AREA	

Convert azimuth (in DMS) to bearing (in DMS) as follows:

AZ	BRG
0° to 90°	NE
90° to 180°	SE, press [CHS] [1] [8] [0] ■ [H.MS+]
180° to 270°	SW, press [1] [8] [0] [−]
270° to 360°	NW, press [CHS] [3] [6] [0] ■ [H.MS+]

Horizontal Curve Layout

Given the radius of the curve, this procedure calculates short and long chords and deflection angles for successive arcs along the curve.

Notation:

R = radius of curve
ARC = length of curve between stations
SHT CHD = length of short chord
LNG CHD = length of long chord
DFL ANG = deflection angle
PC = station at start of curve
PT = station at end of curve

Formulas:

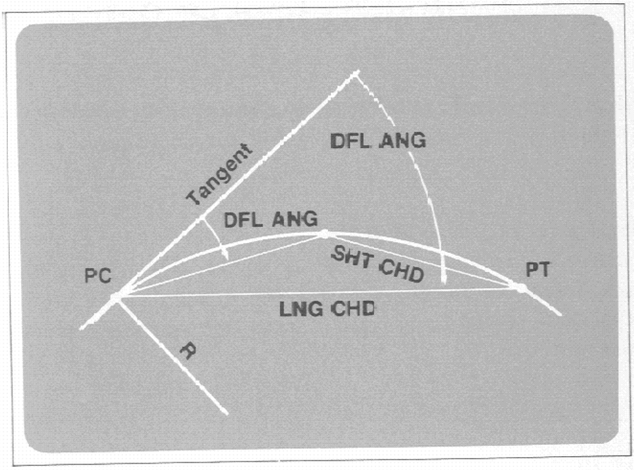
$$\text{SHT CHD} = 2R \sin\left(\text{ARC}\frac{180}{2\pi R}\right)$$

$$\text{LNG CHD} = 2R \sin(\text{DFL ANG})$$

$$\text{DFL ANG} = \frac{180}{2\pi R}(\text{ARC}_1 + \text{ARC}_2 + \ldots + \text{ARC}_n)$$

Example: R = 900 feet

Station	ARC	SHT CHD	LNG CHD	DFL ANG
PC 12 + 57.00				
12 + 75.00	18	18.000	18.000	00°34′23″
13 + 00.00	25	24.999	42.996	1°22′07″
13 + 43.00	43	42.996	85.967	2°44′15″
13 + 75.00	32	31.998	117.916	3°45′22″
PT 13 + 89.00	14	14.000	131.882	4°12′06″



LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2	R	ENTER↑ 2 × ENTER↑ STO		
3		1 1/x ■ π 1		
4		8 0 ÷ ÷ Σ+		
5		RCL Σ+		
6	ARC	× STO + 3 SIN		
7		×	SHT CHD	
8		RCL 3 SIN RCL 1		
9		×	LNG CHD	
10		RCL 3 ■ →H.MS	DFL ANG	Go to line 5 for next ARC.

Elevations along Straight Grades

This procedure calculates elevations of specified stations along straight grades.

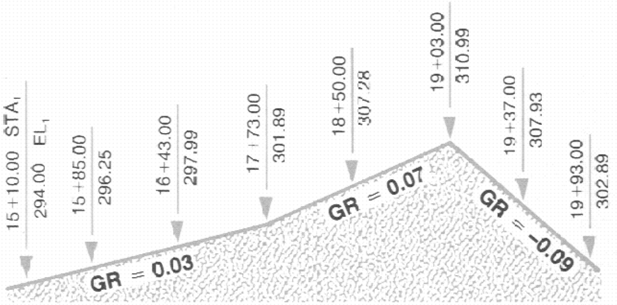
Notation:

EL_1 = elevation at beginning of grade
STA_1 = station at beginning of grade
GR = grade in feet per foot
EL_i = elevation at station i
STA_i = station i

Formula:

$$EL_i = (STA_i)(GR) + EL_1 - (STA_1)(GR)$$

Example:



LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLΣ		
2	EL_1	ENTER↑		
3	STA_1	ENTER↑		
4	GR	× ■ LAST X R↓ −		
5		R↓ Σ+		
6		RCL Σ+		
7	STA	× +	EL	Repeat lines 6-7 for next
				STA. For a new GR, go to
				line 1.

Elevations along a Vertical Curve

This procedure calculates elevation at any specified station along a vertical curve. The elevation and station at the highest or lowest point can also be calculated.

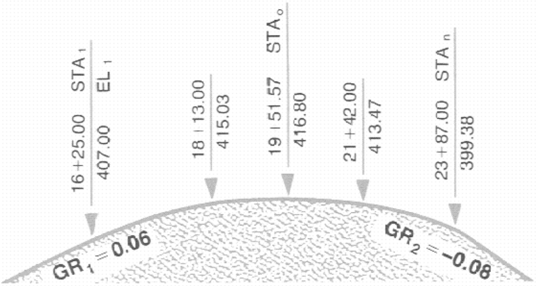
Nomenclature:

GR_1 = grade at beginning of curve in feet per foot
GR_n = grade at end of curve in feet per foot
STA_1 = station at beginning of curve
STA_n = station at end of curve
EL_1 = elevation at beginning of curve
STA_i = station i
EL_i = elevation at station i
STA_0 = station on curve where grade is zero

Formula:

$$EL_i = \frac{(GR_n - GR_1)}{2(STA_n - STA_1)} (STA_i - STA_1)^2 + GR_1 (STA_i - STA_1) + EL_1$$

Example:



LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLΣ		
2	GR$_1$	ENTER↑ ENTER↑		
3	GR$_n$	x⇄y −		
4	STA$_n$	ENTER↑		
5	STA$_1$	STO 1 − ÷ 2		
6		÷ x⇄y Σ+		
7	EL$_1$	STO 2		
8		RCL Σ+		
9	STA	RCL 1 − × x⇄y		
10		■ LAST x x² × +		
11		RCL 2 +	EL	Repeat lines 8-11 for
				next STA.
12		RCL 1 ENTER↑ ENTER↑ RCL		Lines 12-14 may be per-
13		Σ+ x⇄y ÷ 2 ÷		formed any time after
14		−	STA$_0$	line 6.

Volume by Average End Area

This procedure calculates the end area for a station, volume from the previous station, and accumulated volume to the present station.

Nomenclature:

INT_i = interval between stations **i** and **i** + 1
EL_i = elevation from datum of point i in cross-section
HD_i = horizontal distance from centerline or baseline to point i in cross-section
$AREA_i$ = end area of cross-section at station i
VOL_i = volume between stations i and i + 1
TOT VOL = total volume between all stations

Formulas:

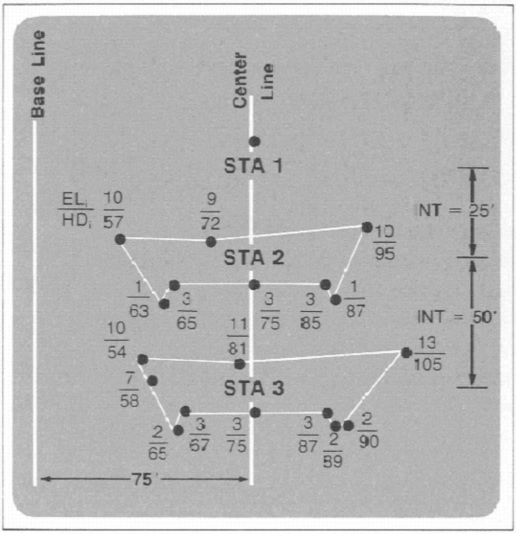
$$AREA_i = 1/2 \left[(EL_2 + EL_1)(HD_2 - HD_1) + (EL_3 + EL_2)(HD_3 - HD_2) + \ldots + (EL_1 + EL_n)(HD_1 - HD_n)\right]$$

$$VOL_i = 1/2\ INT_i\ (AREA_i + AREA_{i+1}),\ AREA_i \neq 0 \text{ and } AREA_{i+1} \neq 0$$

$$VOL_i = 1/3\ INT_i\ (AREA_i + AREA_{i+1}),\ AREA_i = 0 \text{ or } AREA_{i+1} = 0$$

$$TOT\ VOL = VOL_1 + VOL_2 + \ldots + VOL_n$$

Example:



Answer:

STA	AREA	VOL	TOT VOL
1	0	0	0
2	216	1800	1800
3	321.5	13437.5	15237.5

LINE	DATA	OPERATIONS	RESULTS	REMARKS
1		■ CLEAR		
2	*EL_1	ENTER↑		If END AREA of station is
3	HD_1	STO 2 x⇄y		zero, go to line 7.
4	NEXT EL	STO 1 + RCL 2		
5	NEXT HD	STO 2 − Σ+ RCL		
6		1		Go to line 4 for next EL and
				HD. Continue until EL_1 and
				HD_1 have been reentered.
7		RCL 3 RCL · 5		
8		2 ÷ x² √x STO		
9		3	AREA	
10		+ 2 **		
11		÷ ■ CLΣ		
12	INT	× STO + 5	VOL cu. ft.	
13		2 7 ÷	VOL cu. yd.	Go to line 2 for next END
				AREA.
14		RCL 5	TOT VOL cu. ft.	
15		2 7 ÷	TOT VOL cu. yd.	

* If first station has zero end area, start with second station.

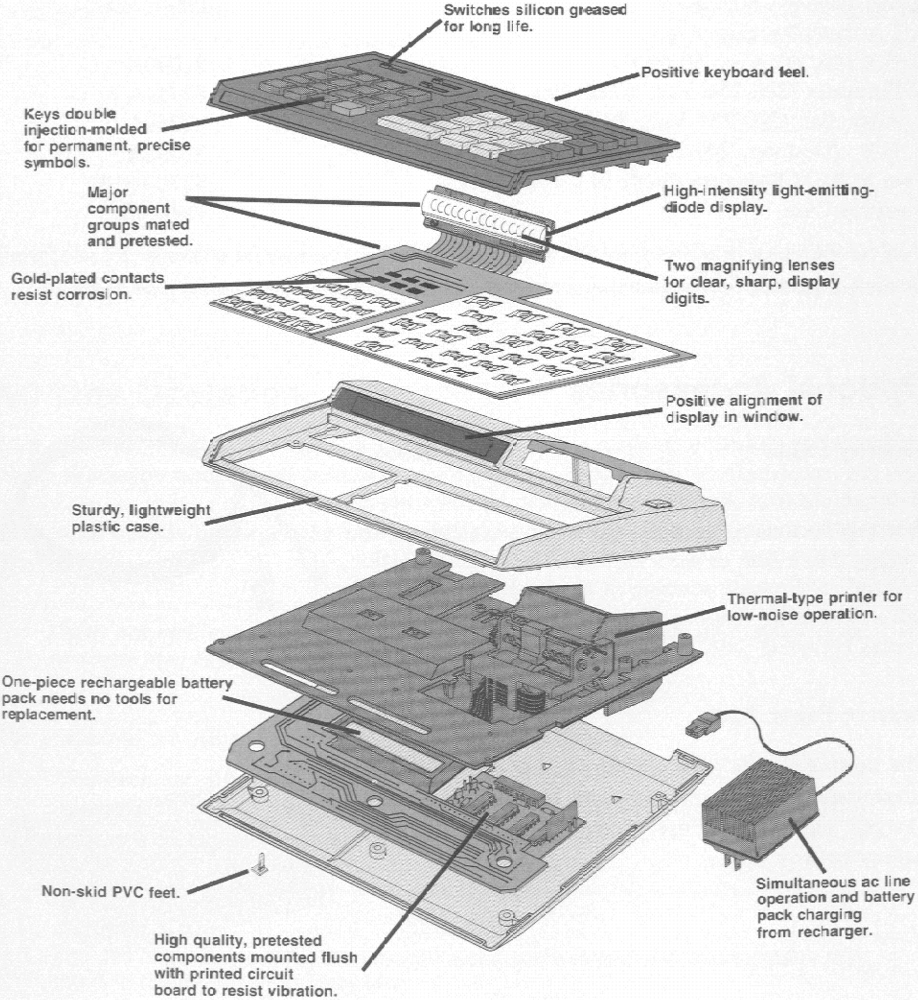
** Change 2 to 3 if previous station area was zero.

Appendix A

Accessories, Service, and Maintenance

Your Hewlett-Packard Calculator

Your HP-91 is another example of the award-winning design, superior quality, and attention to detail in engineering and construction that have marked Hewlett-Packard electronic instruments for more than 30 years. Each Hewlett-Packard calculator is precision crafted by people who are dedicated to giving you the best possible product at any price.



After construction, every calculator is thoroughly inspected for electrical or mechanical flaws, and each function is checked for proper operation.

When you purchase a Hewlett-Packard calculator, you deal with a company that stands behind its products. Besides an instrument of unmatched professional quality, you have at your disposal many extras, including a host of accessories to make your calculator more usable and service that is available worldwide.

Standard Accessories

Your HP-91 comes complete with the following standard accessories:

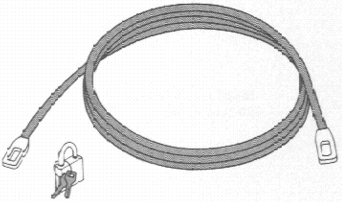
Accessory	**HP Number**
Battery Pack (installed in calculator before packaging)	1420-0227
HP-91 Owner's Handbook	00091-90001
AC Adapter/Recharger (one of the following)	
U.S.(90-127 Vac, 50-60 Hz)	82040A
European (200-254 Vac, 50-60 Hz)	82031A
Australian (200-254 Vac, 50-60 Hz)	82039A
U.K. (Desktop, 200-254 Vac, 50-60 Hz)	82032A
Two Rolls of Paper (available in six-roll packs)	9270-0513
Carrying Case	1540-0383

You can purchase additional standard accessories from your nearest dealer or by mail from Hewlett-Packard. See Optional Accessories below for information on how to order.

Optional Accessories

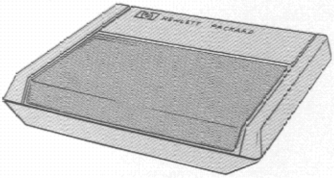
Security Cable **82044A**

A tough six-foot long steel cable that prevents unauthorized borrowing or pilferage of your calculator by locking it to a desk or work surface. The cable is plastic-covered to eliminate scarring of furniture, and you have full access to all features of your HP-91 at all times. Comes complete with lock.



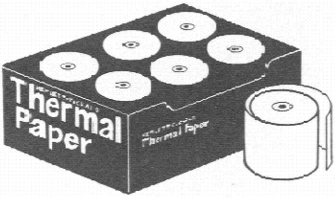
Reserve Power Pack **82037A**

The reserve power pack attaches to the calculator's ac adapter/recharger to keep an extra battery pack freshly charged and ready for use. Comes complete with extra battery pack.



Paper Rolls **9270-0513**

Each pack gives you six rolls of special Hewlett-Packard thermal paper for your HP-91 printer.



To order additional standard or optional accessories for your HP-91 see your nearest dealer or fill out an Accessory Order Form and return it with check or money order to:

Hewlett-Packard
Advanced Products Division
19310 Pruneridge Avenue
Cupertino, CA 95014

If you are outside the U.S., please contact the Hewlett-Packard Sales Office nearest you.

Availability of all accessories, standard or optional, is subject to change without notice.

AC Line Operation

Your calculator contains a rechargeable battery pack that is made up of nickel-cadmium batteries. When you receive your calculator, the battery pack inside may be discharged, but you can operate the calculator immediately by using the ac adapter/recharger. Even though you are using the ac adapter/recharger, the batteries must remain in the calculator whenever the calculator is used.

> **Note:** Attempting to operate the HP-91 from the ac line with the battery pack removed may result in wrong or improper displays.

The procedure for using the ac adapter/recharger is as follows:

1. You need not turn the HP-91 OFF.
2. Insert the female ac adapter/recharger plug into the rear connector of the HP-91.
3. Insert the power plug into a live ac power outlet.

CAUTION

The use of a charger other than the HP recharger supplied with the calculator may result in damage to your calculator.

Battery Charging

The rechargeable batteries in the battery pack are being charged when you are operating the calculator from the ac adapter/recharger. With the batteries in the calculator and the recharger connected, the batteries will charge with the calculator OFF or ON. Normal charging times from fully discharged battery pack to full charge are:

Calculator OFF: 7–10 hours
Calculator ON: 17 hours

Shorter charging periods will reduce the operating time you can expect from a single battery charge. Whether the calculator is OFF or ON, the HP-91 battery pack is never in danger of becoming overcharged.

> **Note:** It is normal for the ac adapter/recharger to be warm to the touch when it is plugged into an ac outlet.

Battery Operation

To operate the HP-91 from battery power alone, simply disconnect the female recharger plug from the rear of the calculator. (Even when not connected to the calculator, the ac adapter/recharger may be left plugged into the ac outlet.)

Using the HP-91 on battery power gives the calculator full portability, allowing you to carry it nearly anywhere. A fully charged battery pack provides approximately 3 to 6 hours of continuous operation. By turning the power OFF when the calculator is not in use, the charge on the HP-91 battery pack should easily last throughout a normal working day.

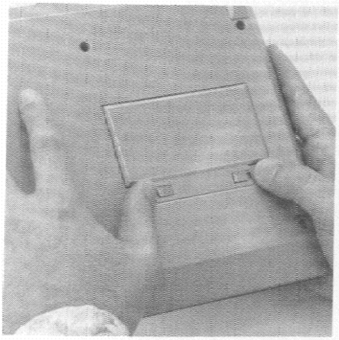
The printer is the most power-consuming part of your HP-91, and you can maximize battery operating time by leaving the calculator in MANUAL MAN ALL NORM printing mode when printing is not necessary.

Battery Pack Replacement

If it becomes necessary to replace the battery pack, use only another Hewlett-Packard battery pack like the one shipped with your calculator.

> **CAUTION**
> Use of any batteries other than the Hewlett-Packard battery pack may result in damage to your calculator.

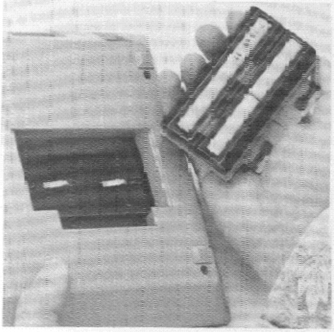
To replace your battery pack, use the following procedure:



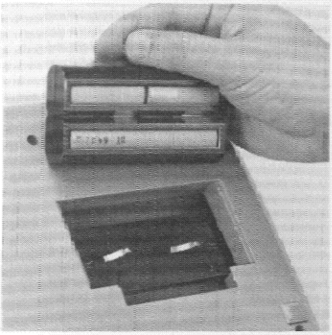
1. Turn the ON-OFF switch to OFF and disconnect the ac adapter/recharger from the calculator.
2. Slide the two battery door latches inward.

3. Let the battery door and battery pack fall into the palm of your hand.

4. If the battery connector springs have been flattened inward, bend them slightly outward again.

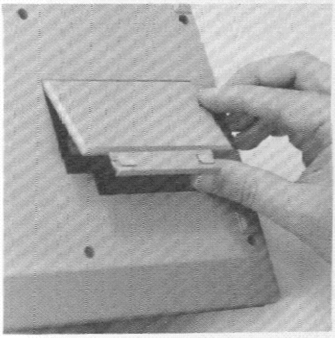


5. Insert the new battery pack so that its contacts face the calculator and line up with the connector springs.



6. Insert the end of the battery door opposite the latches behind the retaining groove and close the door.

7. Secure the battery door by pressing it gently while sliding the two battery door latches outward.



Battery Care

When not being used, the batteries in your HP-91 have a self-discharge rate of approximately 1% of available charge per day. After 30 days, a battery pack could have only 50 to 75% of its charge remaining, and the calculator might not even turn on. If a calculator fails to turn on, you should substitute a charged battery pack, if available, for the one in the calculator. The discharged battery pack should be charged for at least 14 hours.

If a battery pack will not hold a charge and seems to discharge very quickly in use, it may be defective. The battery pack is warranted for one year, and if the warranty is in effect, return the defective pack to Hewlett-Packard according to the shipping instructions. (If you are in doubt about the cause of the problem, return the complete HP-91 along with its battery pack and ac adapter/recharger.) If the battery pack is out of warranty, see your nearest dealer or use the Accessory Order Form provided with your HP-91 to order a replacement.

WARNING

Do not attempt to incinerate or mutilate your HP-91 battery pack—the pack may burst or release toxic materials.

Do not connect together or otherwise short circuit the battery terminals—the pack may melt or cause serious burns.

To maximize the life you get from your battery pack, keep printing to a minimum and display only the fewest number of digits necessary during portable operation.

Your HP-91 Printer

The printing device in your HP-91 is a thermal printer that uses a moving print head to print upon a special heat-sensitive paper. When the print head is energized, it heats the paper beneath it. The heat causes a chemical change in the paper, which then changes color. The printer in your HP-91 prints answers quickly and quietly, and has been expressly designed to give you a permanent record of your computations in a portable scientific calculator.

Paper for your HP-91

Because the printer in your HP-91 is a thermal printer, it requires special heat-sensitive paper. You should use only the Hewlett-Packard thermal paper available in 80-foot rolls from your nearest HP distributor or sales office, or by mail from:

Hewlett-Packard
Advanced Products Division
19310 Pruneridge Avenue
Cupertino, CA 95014

Because of the special heat-sensitive requirements of the paper, standard adding machine paper will *not* work in the HP-91. Also, since different types of thermal paper vary in their sensitivities, the use of thermal paper other than that available from Hewlett-Packard may result in poor print quality or even in damage to your calculator.

CAUTION

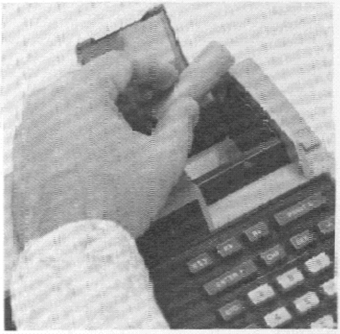
Use only Hewlett-Packard paper in your HP-91.

The heat-sensitive paper used in your HP-91 should be stored in a cool, dark place. Discoloration of paper may occur if it is exposed to direct sunlight for long periods of time, if storage temperatures rise above 50°C (122°F), or if the paper is exposed to excessive humidity or to acetone, ammonia, or other organic compounds. (Exposure to gasoline or oil fumes will not harm your HP-91 paper supply.)

Printed tapes from your HP-91 will last 30 days or more without fading under fluorescent light, but to ensure the permanence of your records, you should store printed tapes at room temperature in a dark place away from direct sunlight, heat, or fumes from organic compounds. (For added permanence, you can copy tapes with a suitable office copier.)

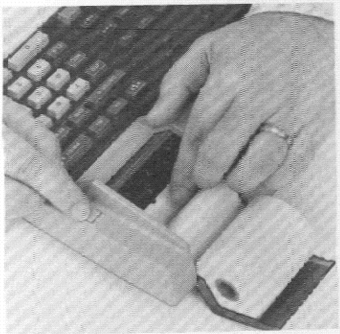
Replacing Paper

To replace the paper roll in your HP-91, proceed as follows:

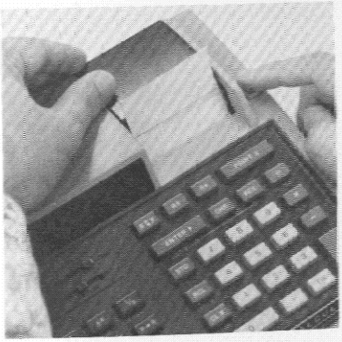


1. Open the paper roll cover and remove the empty core from the paper well.

2. Before inserting the new roll of paper into the calculator, discard the first 2/3 turn to ensure that no glue, tape, or other foreign matter is on the paper.
3. Fold the leading edge of the paper and crease the fold with your fingernail.



4. Temporarily place the paper roll into the paper roll cover and insert the leading edge of paper into the slot near the bottom of the paper well.



5. Turn the calculator ON-OFF switch to ON and press the paper advance pushbutton several times until the leading edge of paper becomes visible beneath the clear plastic tear bar. You can remove the tear bar for accessibility, if desired.
6. Drop the roll of paper into the paper well and close the paper roll cover.

When there is no paper in the calculator, the paper advance pushbutton operates, but the printer does not.

Printer Maintenance

The printer in your HP-91, like the rest of the calculator, is crafted for engineering excellence and is designed to give trouble-free operation with a minimum of maintenance. All moving parts in the printer mechanism contain self-lubricating compound, and no lubrication, cleaning, or servicing of the mechanism is ever required. You may want to occasionally remove the clear plastic tear bar and clean it with mild soap and water solution. (Do not use acetone or alcohol to clean the tear bar.)

You should *never* attempt to insert a tool, such as a screwdriver, or pencil into the printer or its mechanism. If the paper tape should become jammed and fail to feed properly, clear it by grasping the tape and pulling it forward or backward through the printer mechanism. (You can remove the plastic tear bar for accessibility.)

If the paper is feeding properly through the printer mechanism, but no printing appears on the tape, the paper roll is probably inserted backwards. (The paper is chemically treated, and will print on only one side.) Tear off the leading edge of paper, open the paper roll cover and grasp the paper roll, and pull it backward to remove the paper tape that is in the print mechanism. Reverse the paper roll and feed it back into the printing mechanism as described earlier under Replacing Paper.

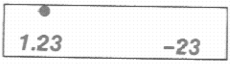
If, after reversing, there is still no printing on the tape when you press **PRINTx** or other print functions, remove the paper roll and insert a roll of Hewlett-Packard thermal paper.

> **Note:** Printer operation may be affected if the printer is in close proximity to a strong magnetic field. Normal operation can be restored by removing the calculator from the vicinity of the magnetic field. No permanent damage will result.

Service

Low Power

When you are operating from battery power, a bright red lamp inside the display will glow to warn you that the battery is close to discharge.



Low Power Display

You must then either connect the ac adapter/recharger to the calculator as described under AC Line Operation, or you must substitute a fully charged battery pack for the one in the calculator.

Blank Display

If the display blanks out, turn the HP-91 OFF, then ON. If 0.00 does not appear in the display, check the following:

1. If the ac adapter/recharger is attached to the HP-91, make sure it is plugged into an ac outlet.
2. Examine the battery pack to see if the contacts are dirty.
3. Substitute a fully charged battery pack, if available, for the one that was in the calculator.
4. If the display is still blank, try operating the HP-91 using the recharger (with the batteries in the calculator).
5. If, after step 4, the display is still blank, service is required. (Refer to Warranty paragraphs.)

Temperature Range

Temperature ranges for the calculator are:

Operating	0° to 45°C	32° to 113°F
Charging	15° to 40°C	59° to 104°F
Storage	-40° to +55°C	−40° to +131°F

Warranty

Full One-Year Warranty

The HP-91 is warranted against defects in materials and workmanship for one (1) year from the date of delivery. During the warranty period, Hewlett-Packard will repair or, at its option, replace at no charge components that prove to be defective, provided the calculator is returned, shipping prepaid, to Hewlett-Packard's Customer Service Facility. (Refer to Shipping Instructions.)

This warranty does not apply if the calculator has been damaged by accident or misuse, or as a result of service or modification by other than an authorized Hewlett-Packard Customer Service Facility. **Hewlett-Packard shall not be liable for consequential damages.**

Obligation to Make Changes

Products are sold on the basis of specifications applicable at the time of sale. Hewlett-Packard shall have no obligation to modify or update products once sold.

Repair Policy

Repair Time

Hewlett-Packard calculators are normally repaired and reshipped within five (5) working days of receipt at any Customer Service Facility. This is an average time and could possibly vary depending upon time of year and work load at the Customer Service Facility.

Shipping Instructions

The calculator should be returned, along with completed Service Card, in its shipping case (or other protective package) to avoid in-transit damage. Such damage is not covered by warranty, and Hewlett-Packard suggests that the customer insure shipments to the Customer Service Facility. A calculator returned for repair should include the ac adapter/recharger and the battery pack. Send these items to the address shown on the Service Card.

Shipping Charges

Whether the calculator is in-warranty or out-of-warranty, the customer should prepay shipment to the Hewlett-Packard Customer Service Facility. During warranty, Hewlett-Packard will prepay shipment back to the customer.

Further Information

Service contracts are not available. Calculator circuitry and design are proprietary to Hewlett-Packard, and Service Manuals are not available to customers.

Should other problems or questions arise regarding repairs, please call your nearest Hewlett-Packard Sales Office or Customer Service Facility.

Appendix B

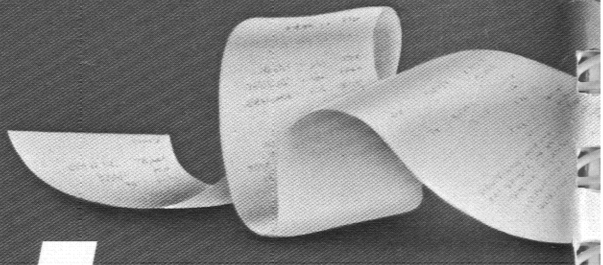
Improper Operations

If you attempt a calculation containing an improper operation—say, division by zero—the calculator display will show Error . In addition, if the Print Mode switch MAN ALL NORM is set to NORM or ALL, the word ERROR will be printed.

The following are improper operations:

÷	where x = 0
y^x	where y = 0 and x $\leq$ 0
y^x	where y $<$ 0 and x is non-integer
$\sqrt{x}$	where x $<$ 0
$1/x$	where x = 0
LOG	where x $\leq$ 0
LN	where x $\leq$ 0
SIN^{-1}	where $\lvert x \rvert$ is $>$ 1
COS^{-1}	where $\lvert x \rvert$ is $>$ 1
STO ÷	where x = 0
$\bar{x}$	where n = 0
s	where n $\leq$ 1
L.R. or $\hat{y}$	where $n \Sigma x^2 - (\Sigma x)^2 = 0$
L.R. or $\hat{y}$	where n = 0
Δ%	where y = 0
%Σ	where Σx = 0
N!	where x $<$ 0 or x is non-integer

General Index



HEWLETT hp PACKARD

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For additional Sales and Service Information contact your local Hewlett-Packard Sales Office or call 408/996-0100 (ask for Calculator Customer Service).

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