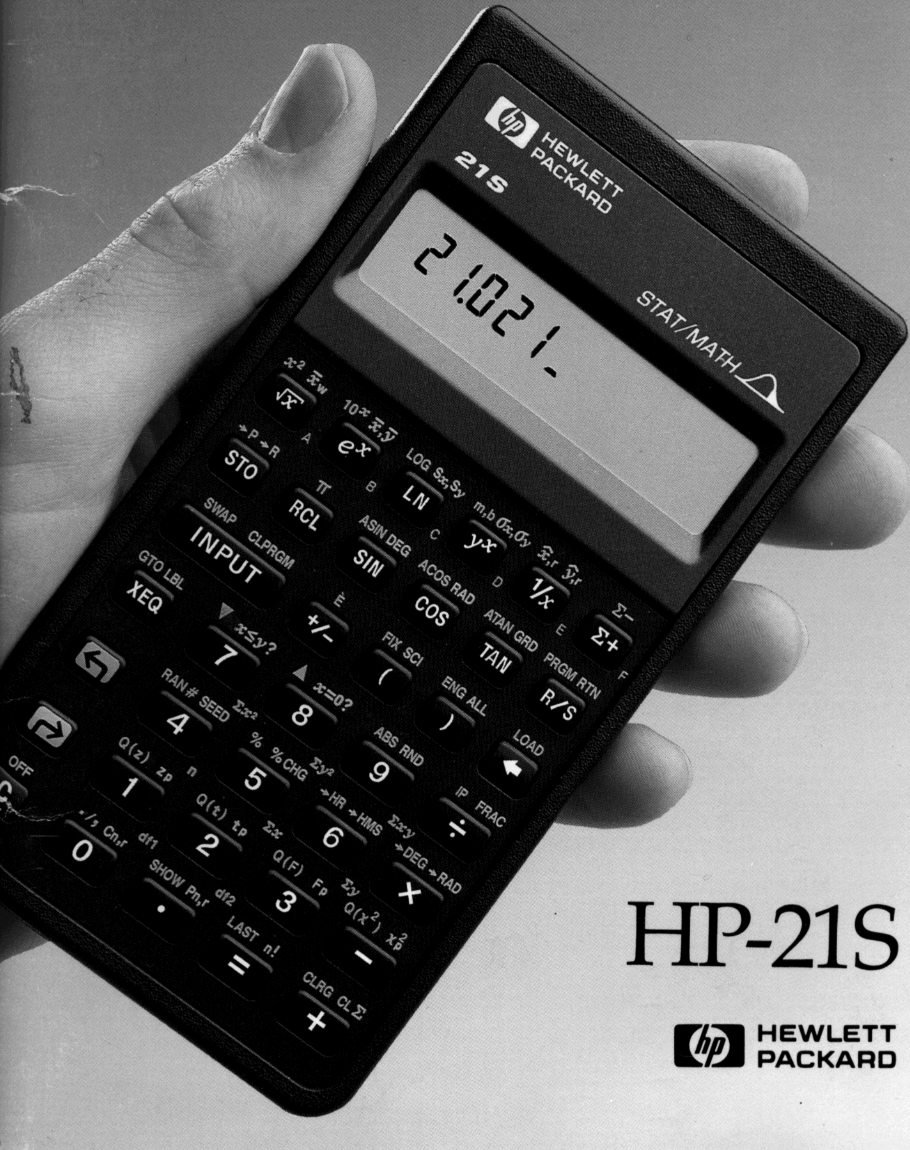
HEWLETT-PACKARD

Stat/Math Calculator

Owner's Manual



HP-21S

HEWLETT PACKARD

1. Separates two numbers.
2. Executes a program.
3. Activates blue-labeled keys.
4. Activates yellow-labeled keys.
5. On; clears display, cancels operation.
6. Distribution keys.
7. df_1 and df_2 are degrees of freedom memory aids.
8. n through Σxy are statistical summation memory aids.
9. Backspace.
10. Loads built-in programs.
11. Enters Program mode.
12. Accumulates statistical data.
13. A through F keys for labels and built in programs.
14. Annunciator Line.

Linear Regression Test Statistics (Lr-StAt)

[↰][LOAD]C
[↱][CLΣ]
x_1[INPUT]y_1[Σ+]
x_2[INPUT]y_2[Σ+]
.
.
.
x_n[INPUT]y_n[Σ+]
[XEQ]A (displays std. err. of est.)

Which test?

- slope: [XEQ]B, [↰][LOAD]A
- mean of y given x: x[XEQ]D, [↰][LOAD]A
- particular y given x: x[XEQ]E, [↰][LOAD]A

Go to "Test of Hypothesis or Confidence Interval?" on page 3 .

Note: For another test using the same data, press [↰][LOAD]C and start at "Which test".

Test(CHI-2)

[↰][LOAD]D

Expected values unequal or equal?

unequal:
O_1[INPUT]E_1[XEQ]A
O_2[INPUT]E_2[R/S]
.
.
O_n[INPUT]E_n[R/S]

equal:
[↰][CLΣ]
O_1[Σ+]
O_2[Σ+]
.
.
O_n[Σ+]
[XEQ]F

n-1 correct degrees of freedom?

no:
df[STO]1
[XEQ]B(displays χ^2)
[↰][Q(X²)](displays U_{tp})

yes:
[XEQ]B(displays χ^2)
[↰][Q(X²)](displays U_{tp})

Binomial Distribution (bin)

[↰][LOAD]E
n[STO]1
pr[STO]2
x[XEQ]A(displays pr of x successes)
x[XEQ]B(displays pr of $\leq x$ successes)

Time Value of Money (FinAnCE)

[↰][LOAD]F

n	Total number of compounding periods.
$i\%$	Periodic interest rate as a percent.
PV	Present value (the initial cash flow).
PMT	Uniform, periodic payment.
FV	Future value (the final cash flow).

Note: For PV, PMT, and FV, money flowing to you is treated as a positive value, and money flowing from you is treated as a negative value.

Variable	To Store	To Calculate	To Review
n	[STO]4	[XEQ]4	[RCL]4
$i\%$	[STO]5	$i\%$[STO]5[XEQ]5[R/S]...*	[RCL]5
PV	[STO]7	[XEQ]7	[RCL]7
PMT	[STO]8	[XEQ]8	[RCL]8
FV	[STO]9	[XEQ]9	[RCL]9

*$i\%$ is the initial guess you supply for the interest rate.

HP-21S Quick Reference Guide (for the built-in library)

How To Use This Guide

Once you've used your owner's manual to become familiar with the HP-21S built-in library, you can use this guide for "quick reference" to the logic and keystrokes used in each of the library programs.

Symbols Used in This Guide

n	Sample size.
$\bar{x}$	Sample mean.
s	Sample standard deviation.
σ	Population standard deviation.
p	Sample proportion.
π	Population proportion.
μ	Population mean.
d	Precision ($^1/_2$ width of confidence interval).
O_i	Observed value.
E_i	Expected value.
df	Degrees of freedom.
z	Standard normal random variable.
t	Student's t random variable.
χ^2	CHI-square random variable.
V_0	Hypothesized value.
U_{tp}	Upper-tail probability.
pr	Probability

One Sample Test Statistics (1-StAt)

Test of Mean

[↰][LOAD]A

Form of data?

raw data:

[↱][CLΣ]
Enter data using [Σ+]
[XEQ]A

summary statistics:

n [XEQ]D
$\bar{x}$ [R/S]
s or σ [R/S]

Go to "Test of Hypothesis or Confidence Interval?" on page 3 .

Test of Proportion

[↰][LOAD]A

Test of hypothesis or confidence interval?

test of hypothesis:

n[XEQ]D
p [R/S]
π_0[XEQ]1
[R/S]
π_0[XEQ]B (displays z)
[↰][Q(z)](displays U_{tp})

confidence interval:

n[XEQ]D
p[R/S]
[XEQ]1
[R/S]
U_{tp}[↱][zp](displays z)
[XEQ]C (displays precision)
[R/S](displays lower limit)
[R/S](displays upper limit)

Paired Difference

[↰][LOAD]A
[↱][CLΣ]
x_1[−] y_1 [Σ+]
x_2[−] y_2 [Σ+]
.
.
.
x_n[−] y_n [Σ+]
[XEQ] A

Go to "Test of Hypothesis or Confidence Interval?" below.

Sample Size Determination

[↰][LOAD]A
σ[XEQ]E
d[INPUT]
U_{tp}[↱][zp](displays z)
[R/S](displays required sample size)

Test of Hypothesis or Confidence Interval?

.
.
.

Test of hypothesis or confidence interval?

test of hypothesis:

V_0[XEQ]B (displays z or t)
[↰][Q(z)]or [↰][Q(t)]
(displays U_{tp})

confidence interval:

U_{tp}[↱][zp]or[↱][tp](displays z or t)
[XEQ]C (displays precision)
[R/S](displays lower limit)
[R/S](displays upper limit)

Two Sample Test Statistics (2-StAt)

Difference Between Means

[↰][LOAD]B

Form of data?

raw data:

[↱][CLΣ]
Enter 1st sample using [Σ+]
[XEQ]A
Enter 2nd sample using [Σ+]
[R/S]

Which distribution?

normal:
[XEQ]C
[↰][LOAD]A

t:
[↰][LOAD]A

summary statistics:

n_1[XEQ]D
$\bar{x}_1$[R/S]
s_1 or σ_1[R/S]
n_2[R/S]
$\bar{x}_2$[R/S]
s_2 or σ_2[R/S]

Which distribution?

normal:
[XEQ]C
[↰][LOAD]A

t:
[↰][LOAD]A

Go to "Test of Hypothesis or Confidence Interval?" on page 3 .

Difference Between Proportions

[↰][LOAD]B

Assumed difference non-zero or zero?

non-zero:

n_1[XEQ]D
p_1[R/S]
[XEQ]1
[R/S]
n_2[R/S]
p_2[R/S]
[XEQ]2
[R/S]
[XEQ]C
[↰][LOAD]A

Go to "Test of Hypothesis or Confidence Interval?" on page 3 .

zero:

n_1[XEQ]D
p_1[R/S]
$\frac{x_1 + x_2}{n_1 + n_2}$ [XEQ]1
[R/S]
n_2[R/S]
p_2[R/S]
[XEQ]B
[↰][LOAD]A
0[XEQ]B(displays z)
[↰][Q(z)](displays U_{tp})

HP-21S Stat/Math Calculator

Owner's Manual

HEWLETT
PACKARD

Edition 3 June 1990
Reorder Number 00021-90025

Notice

For warranty and regulatory information for this calculator, see pages 148 and 151.

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Printing History

Edition 1	March 1989	Mfg. No. 00021-90026
Edition 2	July 1989	Mfg. No. 00021-90042
Edition 3	June 1990	Mfg. No. 00021-90043

Welcome to the HP-21S

Your HP-21S reflects the superior quality and attention to detail in engineering and manufacturing that have distinguished Hewlett-Packard products for 50 years. Hewlett-Packard stands behind this calculator — we offer expertise to support its use (see inside back cover) and worldwide service.

Hewlett-Packard Quality

Our calculators are made to excel and to be easy to use.

- This calculator is designed to withstand the drops, vibrations, pollutants (smog, ozone), temperature extremes, and humidity variations that it may encounter in everyday work life.
- Both the calculator and its manual have been designed and tested for ease of use. We added many examples to highlight the varied uses of the calculator.
- Advanced materials and permanent, molded key lettering provide a long keyboard life and a positive feel to the keys.
- CMOS (low-power) electronics and a liquid-crystal display allow data to be retained indefinitely and the batteries to last a long time.
- The microprocessor has been optimized for fast and reliable computations, using 15 digits internally for precise results.
- Extensive research has created a design that has minimized the adverse effects of static electricity, a potential cause of malfunctions and data loss in calculators.

Features

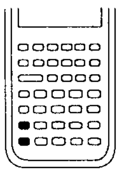
- Probability distribution functions and inverses.
  - Normal distribution.
  - Student's t distribution.
  - F distribution.
  - χ^2 (Chi-square) distribution.
- Six built-in programs.
  - One sample statistics.
  - Two sample test statistics.
  - Linear regression test statistics.
  - Chi-square test.
  - Binomial distribution.
  - Time value of money.
- One and two variable statistics with linear regression.
- Factorial, combinations, permutations.
- Essential scientific functions.
- Polar/rectangular conversions.
- Keystroke programming.
- Large 12-character display.
- Ten data registers and 99 program lines.
- Accurate math, 12-digits with a $10^{\pm 499}$ exponent range.
- Detachable quick reference guide.

Contents

1

Getting Started

On, Off, and Display Contrast

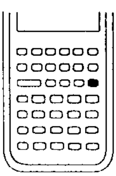


To turn on your HP-21S, press [C] (also labeled "ON"). To turn the calculator off, press [↱], then [C] (written as [↱] [OFF]).

Since the calculator has continuous memory, turning it off does not affect the information you've stored. To conserve energy, the calculator turns itself off approximately 10 minutes after you stop using it. The calculator's three alkaline batteries last approximately one year. If you see the low-battery symbol (⊏▭) in the display, replace the batteries as soon as possible (page 145).

To change the display contrast, hold down [C] and press [+] or [−].

Simple Arithmetic Calculations



If you make a typing mistake while entering a number, press [←] to erase the incorrect digits.

Arithmetic Operators. The following examples demonstrate using the arithmetic operators [+], [−], [×], [÷], and [y^x] (exponentiation). If you press more than one operator, for example [+] [−] [+] [×] [+], all are ignored except the last one.

Keys:	Display:	Description:
24.715 [+] 62.471 [=]	87.1860	Adds 24.715 and 62.471.

After a calculation has been completed (by pressing [=]), pressing a number key starts a new calculation:

Keys:	Display:	Description:
19 [×] 12.68 [=]	240.9200	Calculates 19×12.68.

The [y^x] key is the exponentiation operator:

Keys:	Display:	Description:
4.7 [y^x] 3 [=]	103.8230	Calculates 4.7^3.

To continue an operation (chain calculation), press an operator key.

Keys:	Display:	Description:
[+] 115.5	115.5_	Continues calculation.
[=]	219.3230	Completes calculation of $4.7^3 + 115.5$.

You can do chain calculations without using [=] after each step. Calculate $6.9 \times 5.35 \div 0.918$:

Keys:	Display:	Description:
6.9 [×] 5.35 [÷]	36.9150	Pressing [÷] displays intermediate answer (result of 6.9×5.35).
.918 [=]	40.2124	Completes calculation.

Chain calculations are interpreted according to the priority of the operators in the expression. Calculate $4 + (9 \times 3)$:

Keys:	Display:	Description:
4 [+] 9 [×]	9.0000	Delays addition ([×] has higher priority than [+]).
3 [=]	31.0000	Calculates $4 + (9 \times 3)$.

Negative Numbers. Enter the number and press [+/-]. Calculate $-75 \div 3$.

Keys:	**Display:**	**Description:**
75 [+/-]	-75_	Changes sign of 75.
[÷] 3 [=]	-25.0000	Calculates result.

Calculate $0.4 - e^{-1.1}$.

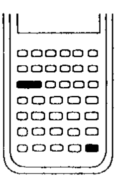
.4 [-] 1.1 [+/-]	-1.1_	
[e^x]	0.3329	Calculates $e^{-1.1}$.
[=]	0.0671	Completes calculation.

Understanding the Display and Keyboard

Cursor

The cursor (_) is visible when you are entering a number.

Clearing the Calculator



Clearing the Display. When the cursor is on, [←] erases the last digit you entered. Otherwise, [←] clears the display of its contents and cancels the calculation.

While you are entering a number, pressing [C] clears it to zero. Otherwise, [C] clears the display of its contents and cancels the current calculation.

Clearing Messages. [←] and [C] also clear messages. When the HP-21S is displaying an error message, [←] or [C] clears the message and restores the original contents of the display.

Clearing Memory. To clear portions of memory:

Keys	Description
[↱] [CLPRGM]	Clears programs when in Program mode.
[↰] [CLRG]	Clears registers R_0 through R_9.
[↱] [CLΣ]	Clears statistical registers R_4 through R_9 and display.

To clear all memory and reset the calculator, press and hold down [C], then press and hold down both [√x] and [Σ+]. When you release them, all memory is cleared. The ALL CLr message is displayed.

Annunciators

Annunciators are displayed symbols that indicate the status of the calculator.

Annunciator	Status
[↰]	Left shift is active. When you press a key, the function labeled in *blue* is executed (page 14).
[↱]	Right shift is active. When you press a key, the function labeled in *yellow* is executed (page 14).
:	[INPUT] has been pressed, or two values have been entered or returned (page 14).
PEND	An arithmetic operation is pending besides what shows in the display.
(battery symbol)	Battery power is low (page 145).

Annunciator	Status
GRAD	The calculator is in Grads mode for trigonometric calculations (page 34).
RAD	The calculator is in Radians mode for trigonometric calculations (page 34).
PRGM	The calculator is in Program mode. (Refer to chapters 6 and 7.)

Shift Keys

Most keys have blue or yellow functions printed above the key. The shift keys access these labeled operations: the blue shift key executes a blue labeled operation; the yellow shift key executes a yellow labeled operation. To perform a shifted operation, press [◄] or [►] to turn on the shift annunciator. Then, press the key that has the desired label.

For example, pressing [►] followed by [RAD] puts the calculator in Radians mode. Pressing [►][DEG] returns the calculator to Degrees mode.

If there is just one shifted function label above a key, you can press either shift key to activate it.

To perform consecutive shifted operations, hold down the shift key.

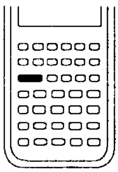
If you accidentally press [◄] or [►], just press it again to turn off the shift annunciator. If you press the wrong shift key, press the other one to correct it.

INPUT Key

The [INPUT] key is used to separate two numbers when using two-number functions or two-variable statistics.

The : annunciator is displayed if [INPUT] has been pressed. If a number is in the display, press [C] to erase the : annunciator and the display. If the cursor or an error message is visible in the display, press [C] twice to erase the : annunciator.

SWAP Key



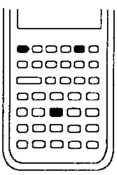
Pressing [↰][SWAP] exchanges any of the following:

- The last two numbers that you entered; for instance, the order of division or subtraction.
- The results of functions that return two values. The : annunciator indicates that two results have been returned; press [↰][SWAP] to see the hidden result.
- The *x*- and *y*-values when using statistics.

Alpha Keys

The A, B, C, D, E, and F labels to the right of the top row keys are used as program labels and to load built-in programs.

Introducing the Math Functions



One-Number Functions. Math functions involving one number use the number in the display.

Keys:	**Display:**	**Description:**
89.25 [√x]	9.4472	Calculates $\sqrt{89.25}$.
3.57 [+] 2.36 [1/x]	0.4237	1/2.36 is calculated first.
[=]	3.9937	Adds 3.57 and 1/2.36.

Two-Number Functions. When a function requires two numbers, the numbers are entered like this: *number1* [INPUT] *number2*. Pressing [INPUT] evaluates the current expression and displays **:**. For example, the following keystrokes calculate the percent change between 17 and 29:

Keys:	**Display:**	**Description:**
17 [INPUT]	17.0000	Enters *number1*, displays **:** annunciator.
29	29	Enters *number2*.
[↱] [%CHG]	70.5882	Calculates percent change.

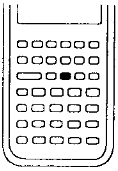
If you enter *number1*, then press a two-number function key without pressing [INPUT], the calculator supplies a zero as *number2*. If you enter a number, press [INPUT], and then press a two-number function key, the calculator uses the same number for both *number1* and *number2*.

Display Format of Numbers

When you turn on the HP-21S for the first time, numbers are displayed with four decimal places and a period as the decimal point. The *display format* controls how many digits appear in the display.

Regardless of the current display format, each number is stored as a signed, 12-digit mantissa with a signed, three-digit exponent. For example, pressing [↰] [π] in FIX 4 (four decimal places) displays 3.1416. Internally, the number is stored as $3.14159265359 \times 10^{000}$. If the result of a calculation is a number containing more significant digits than can be displayed in the current display format, the displayed number is rounded to fit, but 12 digit precision is maintained internally.

Specifying the Number of Displayed Decimal Places (FIX)

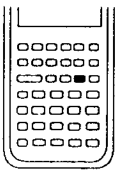


To specify the number of decimal places, press [⬅][FIX], then enter the number of digits (0 through 9) that you wish to appear after the decimal point.

Keys:	**Display:**	**Description:**
[C] [⬅][FIX] 3	0.000	Displays three decimal places.
45.6 [×] .1256 [=]	5.727	
[⬅][FIX] 9	5.727360000	Displays nine decimal places.
[⬅][FIX] 4	5.7274	Restores four decimal places.

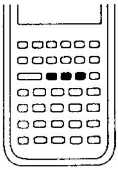
When a number is too large or too small to be displayed in FIX format, it is automatically displayed in scientific notation.

Displaying the Full Precision of Numbers (ALL)



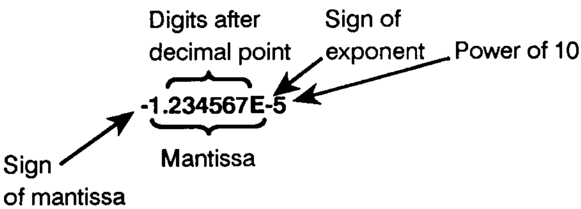
To set your calculator to display numbers as precisely as possible, press [➡][ALL]. Trailing zeros are not displayed.

Scientific and Engineering Notation



Scientific and engineering notation express the number as a mantissa multiplied by a power of 10. The letter E separates the exponent from the mantissa.

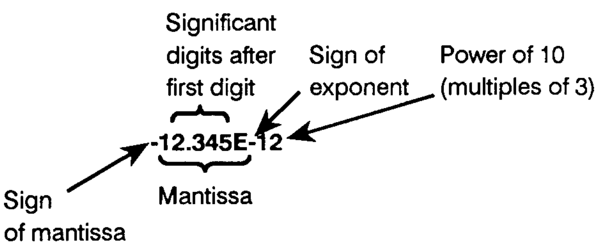
Scientific Notation (SCI). Scientific notation uses a mantissa with one digit to the left of the decimal point. For example, this is SCI 6:



To specify scientific notation,

1. Press [↱] [SCI].
2. Enter the number of digits that you wish to appear after the decimal point.

Engineering Notation (ENG). Engineering notation expresses a number as a mantissa with one, two, or three digits to the left of the decimal point, multiplied by 10 raised to a power that is a multiple of 3. For example, this is ENG 4:



To specify engineering notation,

1. Press [↰][ENG].
2. Enter the number of significant digits that you wish to appear after the first digit.

Entering Numbers With Exponents (E). Regardless of the current display format, you can always enter a number as a mantissa followed by an exponent:

1. Enter the mantissa. If the mantissa is negative, use [+/-] to change the sign.
2. Press [↰][E] to start the exponent.
3. If the exponent is negative, press [+/-] or [-].
4. Enter the exponent.

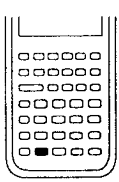
Calculate $4.78 \times 10^{13} \div 8 \times 10^{25}$.

Keys:	Display:	Description:
4.78 [↰][E] 13 [÷]	4.7800E13	
8 [↰][E] 25 [=]	5.9750E-13	5.975×10^{-13}.

Calculate $-2.36 \times 10^{-15} \times 12$.

2.36 [+/-][↰][E] 15 [+/-][×] 12[=]	-2.8320E-14	-2.832×10^{-14}.

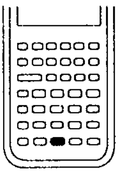
Interchanging the Period and Comma



To switch between the period and comma (United States and International display) used as the decimal point and digit separator, press [↰][·/,]. For example, one million can be displayed:

1,000,000.0000 or 1.000.000,0000

Full Precision of a Number (SHOW)



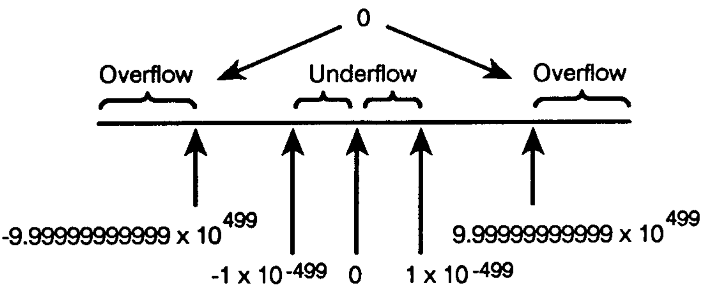
To temporarily view all 12 stored mantissa digits of the number in the display, press [⬅] and then hold down [SHOW]. The 12 digits are shown *without* the decimal point.

Keys:	Display:	Description:
10 [÷] 7 [=]	1.4286	
[⬅] [SHOW]	142857142857	Displays 12 digits.
1 [÷] 80 [+/-] [=]	-0.0125	
[⬅] [SHOW]	-125000000000	Displays 12 digits.

Range of Numbers

The range of numbers the HP-21S can store is shown below. Underflow displays zero. Overflow displays the OFLO message for a moment, then the largest positive or negative number possible.

Numbers the HP-21S Can Store



Messages

The HP-21S displays messages about the status of the calculator or informs you if you attempt an incorrect operation. To clear a message from the display, press [C] or [←]. Refer to page 152 for a list of messages and their meanings.

2

Arithmetic and Storage Registers

Chain Calculations

Chain calculations do a sequence of operations without pressing [=] after each operation. The HP-21S interprets expressions using the system of *operator priority* described below. Calculate 750 × 12 ÷ 360.

Keys:	Display:	Description:
750 [×] 12 [÷]	9,000.0000	Calculates intermediate value. **PEND** annunciator is on.
360 [=]	25.0000	Completes the calculation. **PEND** annunciator is off.

Operator Priority and Pending Operations

Some chain calculations might be interpreted several different ways. For example, 9 + 12 ÷ 3 has two interpretations:

$$\frac{9 + 12}{3} \quad \textit{or} \quad 9 + \frac{12}{3}$$

The HP-21S uses the following system of operator priority:

[y^x] (exponentiation)	Highest priority
[×] [÷]	↓
[+] [−]	Lowest priority

The HP-21S calculates an intermediate result when the next operator you enter has lower or equal priority. When the next operator has higher priority, the HP-21S retains any previous numbers. For example, in the calculation 9 [+] 12 [÷] 3 [=], division has a higher priority than addition. Thus, [+] is retained as a pending operation until the division is completed.

Keys:	**Display:**	**Description:**
9 [+] 12 [÷]	12.0000	Pressing [÷] *does not* add 9 + 12.
3 [=]	13.0000	The [=] key completes the calculation.

Calculate $4 \times 7^3 + 5 \times 7^2 + 6$.

4 [×] 7 [y^x]	7.0000	[y^x] has higher priority than [×].
3 [+]	1,372.0000	Calculates 4×7^3.
5 [×]	5.0000	[×] has higher priority than [+].
7 [y^x]	7.0000	[y^x] has higher priority than [×].
2 [+]	1,617.0000	Adds 5×7^2 to 1,372.
6 [=]	1,623.0000	Completes calculation.

If a calculation requires that operations be done in an order inconsistent with operator priority (for example, addition *before* multiplication), use parentheses. You can use a maximum of five pending operations.*

* There are fewer than five pending operations available if the calculator has more than four pending left parentheses. Example: you can calculate 1 + (2 + (3 + (4 + (5 + 6.

Using Parentheses

Use parentheses to group operations and to specify the order in which they are performed. Closing parentheses at the end of the expression can be omitted. For example, 25 ÷ (3 × (9 + 12 = is equivalent to 25 ÷ (3 × (9 + 12)) =.

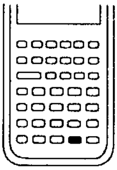
You can calculate $\frac{9 + 12}{3}$ by placing parentheses around the addition so that it is done before the division.

Keys:	Display:	Description:
[(] 9 [+] 12 [)]	21.0000	Evaluates expression inside parentheses.
[÷] 3 [=]	7.0000	Completes calculation.

Now calculate $\frac{30}{85 - 12} \sqrt{(16.9 - 8)}$

Keys:	Display:	Description:
30 [÷] [(]	30.0000	
85	85_	
[-]	85.0000	[(] prevents division of 30 by 85.
12 [)]	73.0000	Evaluates expression inside parentheses.
[×]	0.4110	Calculates 30 ÷ 73.
[(] 16.9	16.9_	
[-] 8 [)]	8.9000	Evaluates expression inside parentheses.
[√x]	2.9833	Calculates $\sqrt{8.9}$.
[=]	1.2260	Completes calculation.

Reusing the Previous Result (LAST)



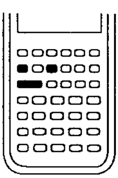
When you start a new calculation, a copy of the last result is stored in the LAST register. To recall that value to the display, press [↰] [LAST]. For example, LAST shortens the following two calculations:

$$0.0821 \times (18 + 273.1)$$

$$2 + \frac{13}{0.0821 \times (18 + 273.1)}$$

Keys:	Display:	Description:
.0821 [×] [(] 18 [+] 273.1 [=]	23.8993	Displays first result, which is stored in LAST when next calculation is started.
2 [+] 13 [÷] [↰] [LAST]	23.8993	[↰] [LAST] recalls previous result.
[=]	2.5439	Displays second result.

Exchanging Two Numbers (SWAP)



Pressing [↰] [SWAP] exchanges the last two numbers that you entered during a calculation. For example, if you have entered 44 [÷] 75, [↰] [SWAP] reverses the order of the numbers to 75 [÷] 44.

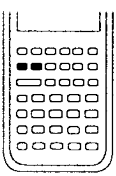
Keys:	Display:	Description:
44 [÷] 75	75_	Oops! What if you meant to enter 75 ÷ 44?
[◄] [SWAP]	44.0000	Swaps 75 and 44.
[=]	1.7045	Completes calculation.
8 [+] 4 [÷] 5	5_	What if you really wanted to add 8 + 5 ÷ 4?
[◄] [SWAP]	4.0000	Swaps 5 and 4.
[=]	9.2500	Completes calculation.

When a function returns two results, the **:** annunciator comes on. Pressing [◄] [SWAP] exchanges the two results. To convert the rectangular coordinates (10, −15) to polar coordinates:

[►] [DEG]		Sets Degrees mode.
10 [INPUT]	10.0000	Stores *x*.
15 [+/-] [◄] [→P]	−56.3099	Displays angle (**:** indicates another result).
[◄] [SWAP]	18.0278	Displays radius.
[C]	0.0000	Clears display.

You can also use [◄] [SWAP] with functions that require two numbers separated by [INPUT]. For example, to accumulate (*x*,*y*) data pairs in the statistical registers, enter *x*-value [INPUT] *y*-value [Σ+]. Pressing [◄] [SWAP] (before pressing [Σ+]) exchanges the *x*-value and *y*-value.

Using Storage Registers



Registers R_0 through R_9 are used to store numbers. They are accessed using [STO] and [RCL]. When you are using the statistics functions, R_4 through R_9 are used to store summation data.

- [STO] *n* (*n* is an integer 0 through 9) copies the number in the display to the designated register, with full 12 digit precision.
- [RCL] *n* copies the contents of R_n to the display. The number is displayed in the current display format.

The following keystrokes use R_1 and R_2 to calculate

$$\frac{(27.1 + 35.6) \times 1.0823}{(27.1 + 35.6)^{1.0823}}$$

Keys:	**Display:**	**Description:**
27.1 [+] 35.6 [=]	62.7000	
[STO] 1	62.7000	Stores 62.7 in R_1.
[×] 1.0823 [STO] 2	1.0823	Stores 1.0823 in R_2.
[÷]	67.8602	
[RCL] 1	62.7000	Recalls contents of R_1.
[y^x] [RCL] 2	1.0823	Recalls contents of R_2.
[=]	0.7699	Exponentiation is done before division.

To cancel [STO] or [RCL], press [C] or [←].

Clearing Registers. Press [⬑][CLRG] to clear all registers. To clear an individual register, store 0 in it. It is unnecessary to clear a register before storing a value since [STO] *n* replaces the previous value with the new value.

Storage Register Arithmetic. This table describes the arithmetic operations that can be performed on numbers stored in registers. The result is stored in the register.

Keys	New Number in Register *n*
[STO] [+] *n*	old number + displayed number
[STO] [−] *n*	old number − displayed number
[STO] [×] *n*	old number × displayed number
[STO] [÷] *n*	old number ÷ displayed number

The following keystrokes use two registers to calculate

$$1.097 \times 25.6671 = ?$$

$$1.097 \times 35.6671 = ?$$

Keys:	Display:	Description:
1.097 [STO] 0	1.0970	Stores 1.097 in R_0.
[×] 25.6671 [STO] 1	25.6671	Stores 25.6671 in R_1.
[=]	28.1568	Displays first answer.
[RCL] 0	1.0970	Recalls contents of R_0; starts new calculation.
[×] 10 [STO] [+] 1	10.0000	Adds 10 to contents of R_1.
[RCL] 1	35.6671	Contents of R_1 replace right-most number of pending expression.
[=]	39.1268	Displays second answer.

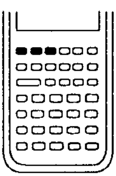
3

Numeric Functions

HP-21S functions require either one or two arguments (an argument is a number acted upon by a function):

- Functions with one argument act on the number in the display. For example, 6 [√x] calculates the square root of 6.
- Functions with two arguments use [INPUT] to separate the arguments. For example, 4 [INPUT] 5 [↱] [%CHG] calculates the percent change between 4 and 5. The arguments can be expressions. For example, 1 [+] 3 [INPUT] 2 [+] 3 [↱] [%CHG] also calculates the percent change between 4 and 5.
- Polar/rectangular coordinate conversions use two arguments and return two results.

General and Logarithmic Functions



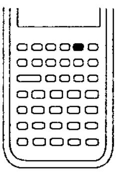
Key(s)	Description
[$\sqrt{x}$]	Square root.
[⬑] [x^2]	Square.
[e^x]	Natural antilogarithm.
[⬑] [10^x]	Base 10 antilogarithm.
[LN]	Natural logarithm.
[⬑] [LOG]	Base 10 logarithm.

Keys:	Display:	Description:
45 [$\sqrt{x}$]	6.7082	$\sqrt{45}$.

Calculate $10^{-4.5} \times 10^{-3.7}$.

Keys:	Display:	Description:
4.5 [+/-] [⬑] [10^x]	3.1623E-5	Calculates base 10 antilogarithm of -4.5.
[×] 3.7 [+/-] [⬑] [10^x]	0.0002	Calculates base 10 antilogarithm of -3.7.
[=]	6.3096E-9	Multiplies two antilogarithms.

Reciprocal



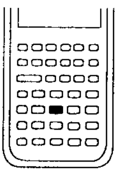
Press [$1/x$] to calculate the reciprocal of the number in the display. Calculate $^1/_3 + ^1/_4$.

Keys:	Display:	Description:
3 [1/x] [+] 4 [1/x]	0.2500	Calculates 1 ÷ 3 and 1 ÷ 4. Defers addition.
[=]	0.5833	Adds two reciprocals.

The exponentiation operator, [y^x], can also be used to find roots of positive numbers. For example, find $\sqrt[4]{3}$ (which is equivalent to $3^{1/4}$):

3 [y^x]	3.0000	Exponentiation.
4 [1/x] [=]	1.3161	Reciprocal of power calculates root.

Percent Functions



The [%] and [%CHG] keys are used to find a percent, add or subtract a percent, or calculate the percent change between two numbers.

Percent

The [◀] [%] function performs two different operations:

- When there is no pending operator, or the last operator you entered was [×], [÷], or [y^x], pressing [◀] [%] divides the displayed number by 100.
- When [+] or [−] is the pending operator, [◀] [%] interprets the displayed number as a percent and returns that percent of the number preceding the [+] or [−].

Example. Find 27% of 85.3.

Keys:	Display:	Description:
85.3 [×] 27 [◄][%]	0.2700	Divides 27 by 100.
[=]	23.0310	Calculates 27% of 85.3.

Find the number that is 25% less than 200.

200 [−] 25 [◄][%]	50.0000	Calculates 25% of 200.
[=]	150.0000	Completes calculation.

Percent Change

To calculate the percent change between two numbers, n_1 and n_2, (expressed as a percentage of n_1) enter n_1 [INPUT] n_2 [►][%CHG].

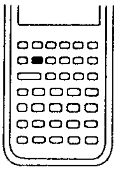
Example. Calculate the percent change between 291.7 and 316.8.

Keys:	Display:	Description:
291.7 [INPUT]	291.7000	Enters n_1.
316.8 [►][%CHG]	8.6047	Calculates percent change.

Calculate the percent change between (12 × 5) and (65 + 18).

12 [×] 5 [INPUT]	60.0000	Calculates and enters n_1.
65 [+] 18 [►][%CHG]	38.3333	Displays percent change between 60 and (65 + 18).

Pi (π)



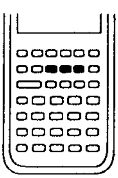
Pressing [←][π] displays the value of π. Although the displayed value is rounded to the current display format, the 12-digit value is actually used.

Example. Find the surface area of a sphere with a 4.5 inch radius ($A = 4\pi r^2$).

Keys:	**Display:**	**Description:**
4 [×] [←][π]	3.1416	Displays π.
[×] 4.5 [←][x^2]	20.2500	Displays 4.5^2.
[=]	254.4690	Calculates surface area in square inches.

Trigonometric Modes and Functions

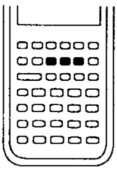
Changing the Trigonometric Mode



The trigonometric mode determines how numbers are interpreted when using the trigonometric and coordinate conversion functions. To exit one mode, press another.

Keys	Description	Annunciator
[↱] [DEG]	Sets *Degrees* mode. There are 360 degrees in a circle. Angles are measured in decimal degrees (rather than degrees-minutes-seconds).	None
[↱] [RAD]	Sets *Radians* mode. There are 2π radians in a circle.	**RAD**
[↱] [GRD]	Sets *Grads* mode. There are 400 grads in a circle.	**GRAD**

Trigonometric Functions



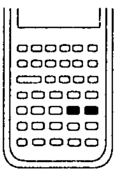
Angles are interpreted in decimal degrees, radians, or grads depending on the current trigonometric mode.

Keys	Function	Keys	Function
[SIN]	sine	[↰] [ASIN]	arc sine
[COS]	cosine	[↰] [ACOS]	arc cosine
[TAN]	tangent	[↰] [ATAN]	arc tangent

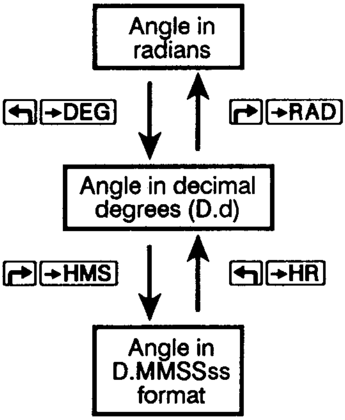
Keys:	**Display:**	**Description:**
[↱] [DEG]		Sets Degrees mode.
15 [SIN]	0.2588	Displays sine of 15°.
1 [+] 60 [TAN]	1.7321	Displays tangent of 60°.

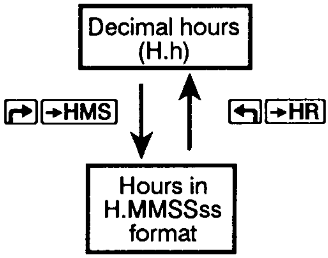
[=]	2.7321	Calculates 1 + tan 60°.
.35 [◄] [ACOS]	69.5127	Displays arc cosine of 0.35.
[−] .62 [◄] [ACOS]	51.6839	Displays arc cosine of 0.62.
[=]	17.8288	Displays arc cosine of 0.35 − arc cosine of 0.62.

Angle and Hour Conversions



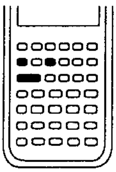
Keys	Function
[◄] [→HR]	*To hours*; converts the number from hours(degrees)-minutes-seconds-decimal seconds format (H.MMSSss or D.MMSSss) to decimal hours (or degrees) format.
[►] [→HMS]	*To hours-minutes-seconds*; converts the number from decimal hours (or degrees) to hours(degrees)-minutes-seconds-decimal seconds format (H.MMSSss or D.MMSSss).
[◄] [→DEG]	*To degrees*; converts the number from a radian value to its decimal degree equivalent.
[►] [→RAD]	*To radians*; converts the number from a decimal degree value to its radian equivalent.



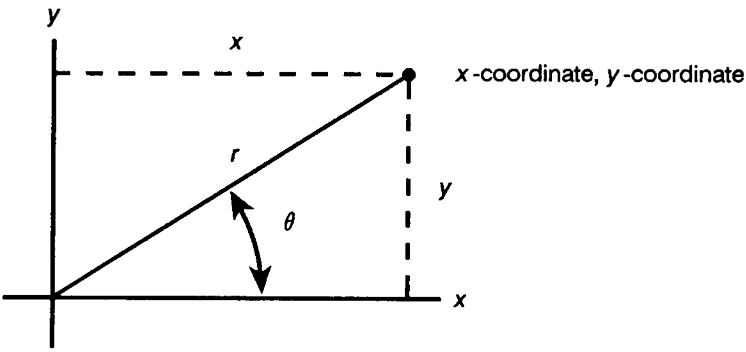


Keys:	**Display:**	**Description:**
1.79 [×] [←][π][=]	5.6235	Calculates 1.79π.
[←][→DEG]	322.2000	Converts 1.79π radians to degrees.
90.2015 [←][→HR]	90.3375	Converts 90 degrees, 20 minutes, 15 seconds to decimal degrees.
25.2589 [→][→HMS]	25.1532	Converts 25.2589 degrees to 25 degrees, 15 minutes, 32 seconds.
[←][SHOW]	251532040000	Shows decimal seconds (32.04 seconds).

Coordinate Conversions



Coordinate conversions require pairs of data separated by [INPUT]; θ is interpreted according to the current trigonometric mode.



To convert from rectangular to polar coordinates,

1. Enter x [INPUT] y [↰][→P] to display θ.
2. Press [↰][SWAP] to display r.

To convert from polar to rectangular coordinates,

1. Enter r [INPUT] θ [↱][→R] to display y.
2. Press [↰][SWAP] to display x.

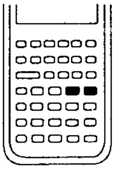
Example. Convert the rectangular coordinates (10, −15) to polar coordinates.

Keys:	**Display:**	**Description:**
[↱] [DEG]		Sets Degrees mode.
10 [INPUT]	10.0000	Enters x.
15 [+/-] [↰] [→P]	−56.3099	Enters y, calculates r and θ, and displays θ.
[↰] [SWAP]	18.0278	Displays r.

Convert the polar coordinates (7, 30°) to rectangular coordinates:

7 [INPUT]	7.0000	Enters r.
30 [↱] [→R]	3.5000	Enters θ, calculates x and y, and displays y.
[↰] [SWAP]	6.0622	Displays x.

Parts of Numbers



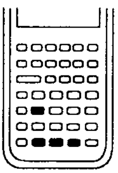
Keys	Function
[↰] [IP]	Integer part of the number.
[↱] [FRAC]	Fractional part of the number (the number without its integer part).
[↰] [ABS]	Absolute value of the number.
[↱] [RND]	Rounds the number internally to the number of digits specified in the current FIX, SCI, or ENG display format. (No rounding occurs in ALL mode.)

Keys:	**Display:**	**Description:**
12.3456789 [=]	12.3457	Enters a nine-digit number.
[↰] [SHOW]	123456789000	Displays full precision of number.
[↱] [RND] [↰] [SHOW]	123457000000	Rounds number internally.

4

Probability and Distributions

Probability



Your HP-21S provides factorials, combinations, permutations, and random numbers to help you calculate probabilities.

Factorial

Pressing [→] [n!] calculates the factorial of the number in the display. The number must be an integer in the range 0 through 253.

Combinations and Permutations

The number of *combinations* of *n* objects taken *r* at a time is the number of different sets containing *r* items, taken from a larger group of *n* items. No item occurs more than once in the set of *r* items, and different orders of the same *r* items *are not* counted separately. Use these keystrokes for calculating combinations: n-value [INPUT] r-value [→] [Cn,r].

The number of *permutations* of *n* objects taken *r* at a time is the number of different arrangements of *r* items that can be taken from a larger group of *n* items. No item can occur more than once in an arrangement, and different orders of the same *r* items *are* counted separately. Use these keystrokes for calculating permutations: n-value [INPUT] r-value [→] [Pn,r].

Keys:	Display:	Description:
5 [INPUT]	5.0000	Enters the n-value.
3 [→] [Cn,r]	10.0000	Enters the r-value; calculates combinations of 5 objects, 3 at a time.
5 [INPUT]	5.0000	Enters the n-value.
3 [→] [Pn,r]	60.0000	Enters the r-value; calculates permutations of 5 objects, 3 at a time.

Combination and Permutation Equations.

$$C_{n,r} = \frac{n!}{(n - r)!\,r!}$$

$$P_{n,r} = \frac{n!}{(n - r)!}$$

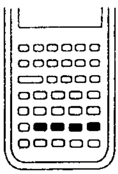
Random Number and Seed

To store a seed for the random number generator, enter a *positive* number and press [→] [SEED]. The seed is stored in R_3 and writes over the previous contents. The first random number in the sequence is displayed.

To generate a sequence of numbers from the previously stored seed, press [←] [RAN#]. This produces a pseudo-random number in the range $0 < \text{RAN\#} \leq 1$.*

* The number is part of a sequence of uniformly distributed psuedo-random numbers. This sequence passes the spectral test (D. Knuth, *Seminumerical Algorithms*, Vol. 2. London: Addison Wesley, 1981).

Distributions



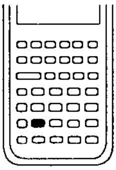
Using the four probability distribution functions on the HP-21S, you can calculate an upper tail probability or its inverse. The upper tail probability corresponds to the area under the curve to the right of the random variable value. You can calculate the area under the curve if you know the value of the random variable. Conversely, you can calculate the value of the random variable if you know the area under the curve. *The area under the curve that is calculated is the upper tail.* If you are using tables that are *other* than upper tail, use the conversion instructions at the end of this chapter.

The upper tail distribution functions are:

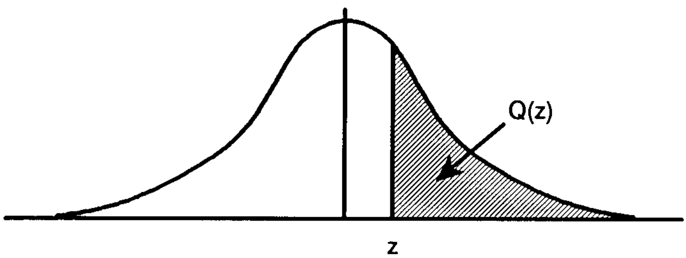
Distribution	**Keys**	**Inverse Keys**
Normal	[←] [Q(z)]	[→] [zp]
Student's t	[←] [Q(t)]	[→] [tp]
F	[←] [Q(F)]	[→] [Fp]
χ^2	[←] [Q(χ^2)]	[→] [χ^2p]

These distribution functions replace the statistical tables found in textbooks. The calculator functions expand the tables by allowing you to determine *any* upper tail probability, not just the selected values in statistical tables.

Normal Upper Tail Probability



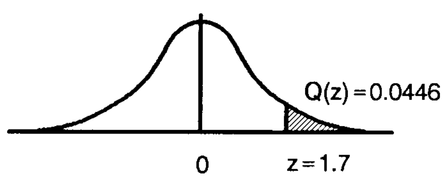
To calculate the area under the curve to the right of z (the upper tail probability), enter the z value and press [↰] [Q(z)]. Q(z) is the probability that a standard normal random variable Z is greater than z.



Example. The variable Z is a standard normal random variable. What is the probability that Z is greater than 1.7?

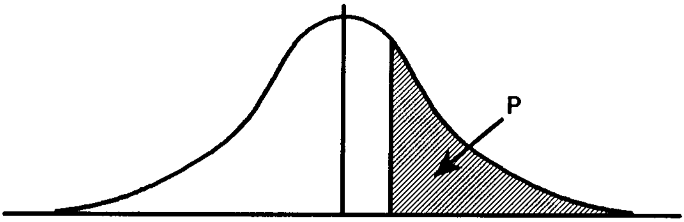
Keys:	Display:	Description:
1.7 [↰] [Q(z)]	0.0446	Calculates the upper tail probability.

1.68	.4535	.0465
1.69	.4545	.0455
1.70	.4554	.0446
1.71	.4564	.0436



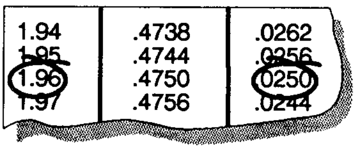
Inverse of Upper Tail Probability — Normal Distribution

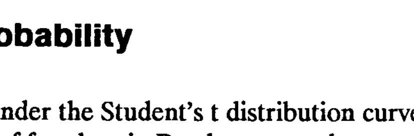
If you know the upper tail probability p (area under the curve) and want to calculate z, enter the probability and press [↱] [zp].



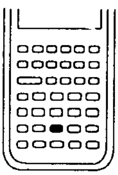
Example. What is the z value corresponding to an upper tail probability of 0.025?

Keys:	Display:	Description:
.025 [↱] [zp]	1.9600	Calculates the z value.



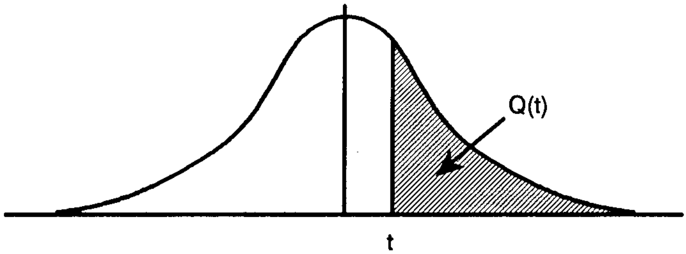


Student's t Upper Tail Probability



To calculate the area under the Student's t distribution curve, first store the degrees of freedom in R_1, then enter the t value and press [↰] [Q(t)].

The label df_1 is to the lower right of the [1] key to remind you where to store degrees of freedom. To store degrees of freedom, enter the value and press [STO] 1. The maximum number of degrees of freedom the calculator can use is 299.



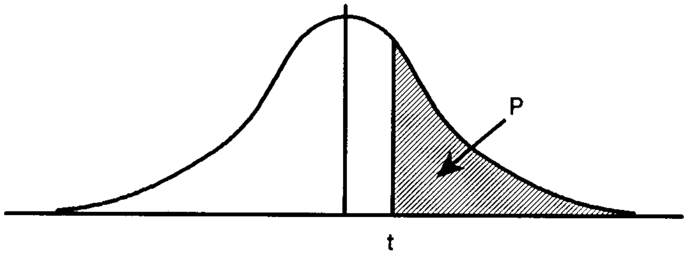
Example. What is the upper tail probability associated with a Student's t distribution with 8 degrees of freedom (df_1) for a t value of 1.86?

Keys:	**Display:**	**Description:**
8 [STO] 1	8.0000	Stores the degrees of freedom.
1.86 [◄] [Q(t)]	0.0500	Calculates the upper tail probability.

df	.05	.01	.001
6	1.943	3.143	5.208
7	1.895	2.998	4.785
8	1.860	2.896	4.501
9	1.833	2.821	4.297

Inverse of Upper Tail Probability—Student's t Distribution

If you know the upper tail probability p (area under the curve) and want to calculate t, store the degrees of freedom in R_1, then enter p, and press [►] [tp].

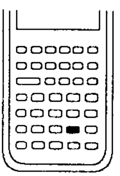


Example. A hypothesis test requires the critical t value from the Student's t distribution with 26 degrees of freedom (df_1). Find the t value for an upper tail probability of 0.05.

Keys:	**Display:**	**Description:**
26 [STO] 1	26.0000	Stores the degrees of freedom.
.05 [→] [tp]	1.7056	Calculates the critical t value.

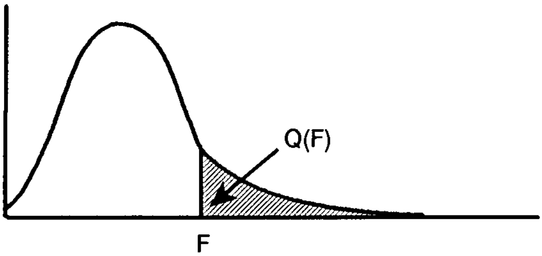
df	.05	.01	.001
24	1.711	2.492	3.467
25	1.708	2.485	3.450
26	1.706	2.479	3.435
27	1.703	2.473	3.421

F Distribution Upper Tail Probability



To calculate the area under the F distribution curve, first store the numerator degrees of freedom in R_1 and the denominator degrees of freedom in R_2. Then enter the F value and press [←] [Q(F)].

The df_1 and df_2 appear as labels to the lower right of the [1] and [2] keys to remind you where to store the degrees of freedom. To store the numerator degrees of freedom, enter the value and press [STO] 1. To store the denominator degrees of freedom, enter the value and press [STO] 2. The maximum number of degrees of freedom the calculator can use is 299.



Example. What is the upper tail probability with numerator degrees of freedom (df_1) equal to 3 and denominator degrees of freedom (df_2) equal to 16, for an F value of 5.29?

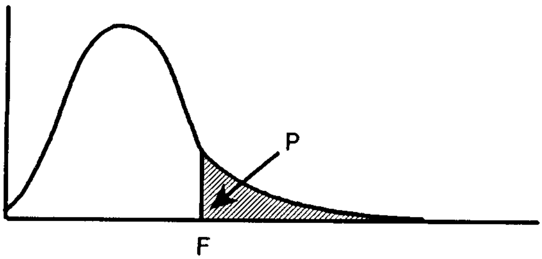
Keys:	**Display:**	**Description:**
3 [STO] 1	3.0000	Stores df_1.
16 [STO] 2	16.0000	Stores df_2.
5.29 [◄] [Q(F)]	0.0100	Calculates the upper tail probability.

.01 Level of Significance

df	1	2	3
14	8.86	6.51	5.56
15	8.68	6.36	5.42
16	8.53	6.23	5.29
17	8.40	6.11	5.18

Inverse of Upper Tail Probability—F Distribution

If you know the upper tail probability p (area under the curve) and want to calculate F, first store the numerator degrees of freedom in R_1 and the denominator degrees of freedom in R_2. Then enter p and press [►] [Fp].



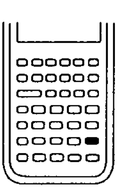
Example. A hypothesis test requires the critical F value from the F distribution with the numerator degrees of freedom (df_1) equal to 4 and the denominator degrees of freedom (df_2) equal to 8. What is the associated upper tail F value for a 0.05 level of significance?

Keys:	**Display:**	**Description:**
4 [STO] 1	4.0000	Stores df_1.
8 [STO] 2	8.0000	Stores df_2.
.05 [↱] [Fp]	3.8379	Calculates F.

.05 Level of Significance

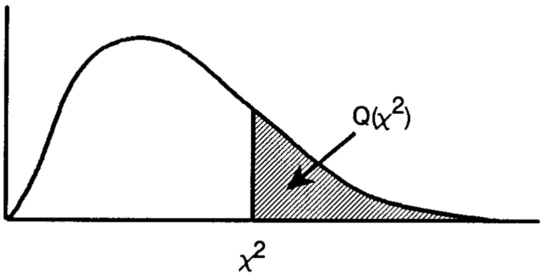
df	1	2	3	4	5
6	5.99	5.14	4.76	4.53	4.39
7	5.59	4.74	4.35	4.12	3.97
8	5.32	4.46	4.07	3.84	3.69
9	5.12	4.26	3.86	3.63	3.48

Upper Tail Probability—Chi-Square Distribution (χ^2)



To calculate the area under the χ^2 distribution curve, store the degrees of freedom in R_1, then enter the χ^2 value and press [↰] [Q(X²)].

To store degrees of freedom (df_1), enter the value and press [STO] 1. The maximum number of degrees of freedom the calculator can use is 299.



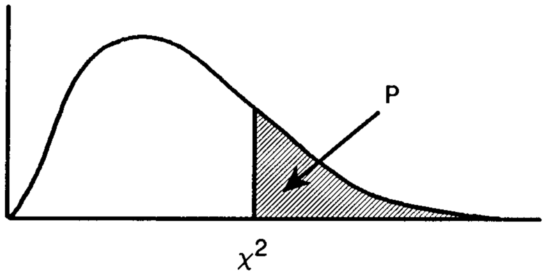
Example. The χ^2 statistic calculated from a sample is 33.41 and there are 17 degrees of freedom (df_1). What is the upper tail probability?

Keys:	**Display:**	**Description:**
17 [STO] 1	17.0000	Stores the degrees of freedom.
33.41 [◀] [Q(X²)]	0.0100	Calculates the upper tail probability.

df	.10	.05	.01
15	22.31	25.00	30.58
16	23.54	26.30	32.00
17	24.77	27.59	33.41
18	25.99	28.87	34.80

Inverse of Upper Tail Probability—Chi-Square Distribution (χ^2)

If you know the upper tail probability (area under the curve), and want to calculate χ^2, store the degrees of freedom in R_1, then enter p and press [▶] [X²p].



Example. A test sample has 19 degrees of freedom (df_1). Find the χ^2 value corresponding to an upper tail probability of 0.001.

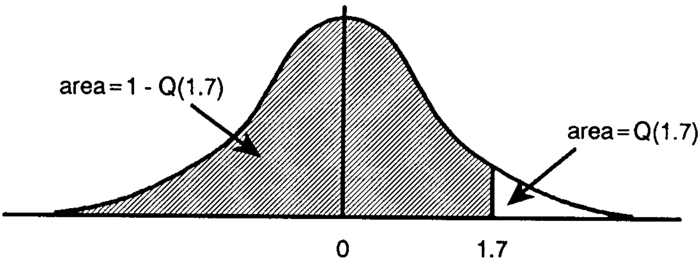
Keys:	**Display:**	**Description:**
19 [STO] 1	19.0000	Stores the degrees of freedom.
.001 [⮫] [X²p]	43.8202	Calculates the χ^2 value.

df	.05	.01	.001
17	27.59	33.41	40.79
18	28.87	34.80	42.31
19	30.14	36.19	43.82
20	31.41	37.57	45.32

How to Convert From Upper Tail

The distribution functions return values for the *upper tail* cumulative probability. The upper tail probability corresponds to the area under the curve to the right of the given value. Sometimes you will need to work with areas other than upper tail. It is easy to convert from upper tail to some other area if you remember that the total area under the curve is equal to one, and that the normal and Student's t distributions are symmetric. That is, the portion of the curve to the left of zero is the mirror image of the portion of the curve to the right of zero.

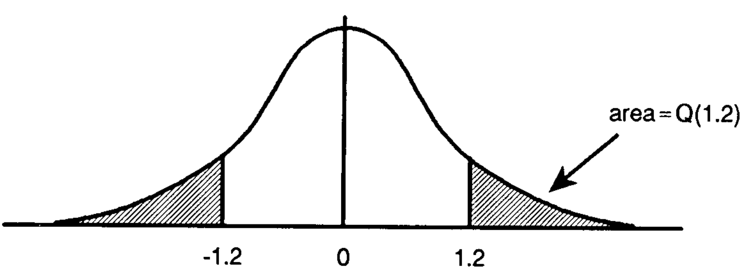
Example. The variable Z is a standard normal random variable. What is the probability that Z is less than 1.7?



The probability that Z is less than 1.7 is the area under the standard normal curve to the left of 1.7. You can calculate the area to the right of 1.7, Q(1.7), and then subtract it from 1 (the total area under the curve).

Keys:	**Display:**	**Description:**
1.7 [◄] [Q(z)]	0.0446	Calculates upper tail area.
[+/–] [+] 1 [=]	0.9554	Subtracts upper tail area from 1.

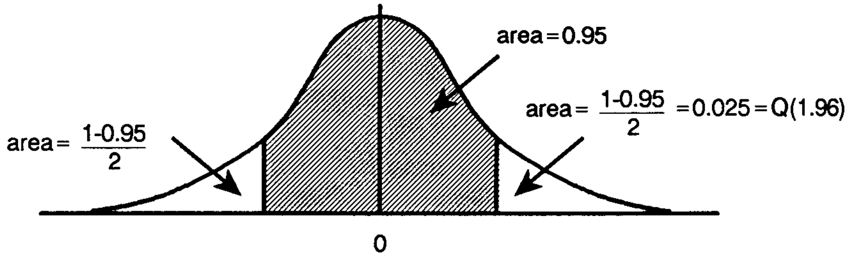
Example. The variable Z is a standard normal random variable. What is the probability that Z is greater than 1.2 or less than −1.2?



The desired area is to the right of 1.2, plus the area to the left of −1.2. Since the normal distributions are symmetric, the areas are the same, so you can calculate the upper tail area, Q(1.2), and multiply by 2.

Keys:	Display:	Description:
1.2 [⬑] [Q(z)]	0.1151	Calculates upper tail area.
[×] 2 [=]	0.2301	Calculates answer.

Example. The variable Z is a standard normal random variable. Find z so that the probability that Z is less than z and greater than −z is equal to 0.95.



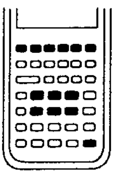
The given area is 0.95. The area not included is 1 − 0.95 = 0.05. Since the normal distribution is symmetric, half of this area is in the upper tail (0.05 ÷ 2 = 0.025). The desired z corresponds to an upper tail probability of 0.025.

Keys:	Display:	Description:
.025 [⬏] [zp]	1.9600	Desired value of z.

You will find other examples of tail conversions in chapter 6.

5

Statistical Calculations



The [Σ+] and [◀][Σ-] keys are used to enter and delete data for one- and two-variable statistics. Summation data is accumulated in registers R_4 through R_9. The register labels at the lower right of the keys indicate what statistical data is stored in each register. Once you enter the data, you can use the statistical functions to calculate the following:

- Mean and standard deviation.
- Linear regression statistics.
- Linear estimate and forecasts.
- Weighted mean.
- Summation statistics: n, Σx, Σx^2, Σy, Σy^2, and Σxy.

Clearing Statistical Data

Clear the statistical registers (R_4 through R_9) before entering new data. If you don't clear the registers, data currently stored in R_4 through R_9 is automatically included in the summation calculations. To clear the statistical registers, press [▶][CLΣ]. (The display is also cleared.)

Summary of Statistical Calculations

Some functions return two values. The : annunciator indicates that two values have been returned. Press [⬑][SWAP] to see the hidden value.

Keys	Description	[⬑][SWAP] to Display
[⮣][$\bar{x},\bar{y}$]	Arithmetic mean (average) of the *x*-values.	Arithmetic mean (average) of the *y*-values.
[⮣][$\bar{x}$w]	Mean of the *x*-values weighted by the *y*-values.	
[⮣][S*x*,S*y*]	Sample standard deviation of the *x*-values.*	Sample standard deviation of the *y*-values if you entered *y*-data.*
[⮣][$\sigma x,\sigma y$]	Population standard deviation of the *x*-values.*	Population standard deviation of the *y*-values if you entered *y*-data.*
y-value [⬑][$\hat{x}$,r]	Estimate of $\hat{x}$ for a given value of *y*.	Correlation coefficient.†
x-value [⮣][$\hat{y}$,r]	Estimate of $\hat{y}$ for a given value of *x*.	Correlation coefficient.†
[⬑][m,b]	Slope (m) of the calculated line.	*y*-intercept (b) of the calculated line.

* Sample standard deviation assumes data is a sampling of a larger, complete set. Population standard deviation assumes the data constitutes the entire population.

† The correlation coefficient is a number in the range −1 through +1 that measures how closely the data fits the calculated line. A +1 indicates a perfect positive correlation, and −1 indicates a perfect negative correlation. A value close to zero indicates non-correlated data.

Keys	Description
[RCL] 4 (n)	Number of data points entered.
[RCL] 5 (Σx)	Sum of the *x*-values.
[RCL] 6 (Σy)	Sum of the *y*-values.
[RCL] 7 (Σx^2)	Sum of the squares of the *x*-values.
[RCL] 8 (Σy^2)	Sum of the squares of the *y*-values.
[RCL] 9 (Σxy)	Sum of the products of the *x*- and *y*-values.

Entering Statistical Data

There is no limit to the number of values you can accumulate in the statistical registers. If statistical data causes the value of a register to exceed $\pm 9.99999999999 \times 10^{499}$, the HP-21S displays a temporary overflow warning (OFLO). Data remains stored until you press [→][CLΣ] or [←][CLRG].

One-Variable Statistics

To enter *x* data for one-variable statistics,

1. Clear the contents of R_4 through R_9 by pressing [→][CLΣ].
2. Enter the first value and press [Σ+]. The HP-21S displays n, the number of items accumulated.
3. Continue accumulating values by entering the numbers and pressing [Σ+]. The n-value is incremented with each entry.

Two-Variable Statistics and Weighted Mean

To enter x,y pairs of statistical data,

1. Clear the contents of R_4 through R_9 by pressing [↱] [CLΣ].
2. Enter the first x-value and press [INPUT]. The HP-21S displays the x-value and the : annunciator appears in the display.
3. Enter the corresponding y-value and press [Σ+]. The HP-21S displays n, the number of pairs of items accumulated.
4. Continue entering x,y pairs. The n-value is incremented with each entry.

To enter data for calculating the weighted mean, enter each data value as x, and its corresponding weight as y.

Correcting Statistical Data

Incorrect entries can be deleted using [↰] [Σ-]. If either value of an x,y pair is incorrect, you must delete and reenter both values.

Correcting One-Variable Data

To delete and reenter statistical data,

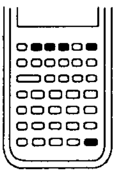
1. Key in the x-value to be deleted.
2. Press [↰] [Σ-] to delete the value. The n-value is decreased by one.
3. Enter the correct value using [Σ+].

Correcting Two-Variable Data

To delete and reenter x,y pairs of statistical data,

1. Key in the x-value, press [INPUT] and then key in the y-value.
2. Press [↰] [Σ-] to delete the values. The n-value is decreased by one.
3. Enter the correct x,y pair using [INPUT] and [Σ+].

Mean, Standard Deviations, and Summation Statistics



You can calculate the sample mean ($\bar{x}$), sample standard deviation (S_x), population standard deviation (σ_x), and summation statistics, n, Σx, and Σx^2 of x-data. For x,y data, you can also calculate the mean, sample standard deviation, population standard deviation of the y-data, and the summation statistics Σy, Σy^2, and Σxy.

Example 1. A yacht captain wants to determine how long it takes on average to change a sail. She randomly chooses six members of her crew, observes them as they carry out the sail change, and records the number of minutes required: 4.5, 4, 2, 3.25, 3.5, 3.75. Calculate the mean and sample standard deviation of the times. Also, calculate the root mean square, using the formula $\sqrt{\Sigma x^2/n}$.

Keys:	**Display:**	**Description:**
[↱] [CLΣ]	0.0000	Clears statistics registers.
4.5 [Σ+]	1.0000	Enters first value.
4 [Σ+]	2.0000	Enters second value.
2 [Σ+]	3.0000	Enters third value.
3.25 [Σ+]	4.0000	Enters fourth value.
3.6 [Σ+]	5.0000	Oops! What if you meant to enter 3.5?
3.6 [↰] [Σ-]	4.0000	Deletes 3.6 and decreases n-value by one.
3.5 [Σ+]	5.0000	Enters fifth time correctly.
3.75 [Σ+]	6.0000	Enters sixth time.
[↱] [$\bar{x}$,$\bar{y}$]	3.5000	Calculates the mean.
[↱] [Sx,Sy]	0.8515	Calculates the sample standard deviation.

[RCL] 7	77.1250	Displays Σx^2.
[÷] [RCL] 4	6.0000	Displays n.
[=] [$\sqrt{x}$]	3.5853	Calculates the root mean square.

The standard deviations calculated by [↱][Sx,Sy] and [↱][Sx,Sy] [↰][SWAP] are the sample standard deviations. They assume that the data is a sampling of a larger, complete set of data.

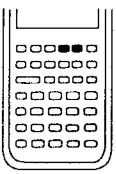
If the data constitutes the entire population, the population standard deviations can be calculated by pressing [↱][σx,σy] to display σ_x and [↱] [σx,σy] [↰][SWAP] to display σ_y.

Example 2. The coach has four new players on the team with heights of 193, 182, 177, and 185 centimeters and weights of 90, 81, 83, and 77 kilograms. Find the mean and population standard deviation of both their heights and weights, then find the total weight of the players (sum of the *y*-data).

Keys:	**Display:**	**Description:**
[↱][CLΣ]	0.0000	Clears statistics registers.
193 [INPUT] 90 [Σ+]	1.0000	Begins entry of heights and weights.
182 [INPUT] 81 [Σ+]	2.0000	
177 [INPUT] 83 [Σ+]	3.0000	
185 [INPUT] 77 [Σ+]	4.0000	
[↱][$\bar{x},\bar{y}$]	184.2500	Calculates mean of heights (x).
[↰][SWAP]	82.7500	Displays mean of weights (y).
[↱][σx,σy]	5.8041	Calculates population standard deviation for heights (x).

[◄][SWAP]	4.7104	Displays population standard deviation for weights (y).
[RCL] 6	331.0000	Displays total weight (sum of y's).

Linear Regression and Estimation



Least squares linear regression is a statistical method used for estimation and forecasting. It is used to fit a straight line through a set of x,y data. There must be at least two different x,y pairs. A straight line estimates the relationship between the x- and y-variables of the form $y = mx + b$, where m is the slope and b is the y-intercept.

Linear Regression. To do a linear regression calculation,

1. Enter the x,y-data using the instructions on page 56.
2. Press
   - [◄][$\hat{x}$,r] [◄][SWAP] (or [►][$\hat{y}$,r] [◄][SWAP]) to display r, the correlation coefficient.
   - [◄][m,b] to display m, the slope of the line, then [◄][SWAP] to display b, the y-intercept.

Linear Estimation. The straight line calculated by linear regression can be used to estimate a y-value for a given x-value, or vice versa. To do linear estimation calculations,

1. Enter the x,y-data using the instructions on page 56.
2. Enter the known x-value or y-value.
   - To estimate x for the given y, enter the y-value, then press [◄][$\hat{x}$,r].
   - To estimate y for the given x, enter the x-value, then press [►][$\hat{y}$,r].

Example. The rate of a certain chemical reaction depends on the initial concentration of one chemical. When the reaction is run repeatedly, varying only the initial concentration of the chemical, the following rates are observed:

X	0.050	0.075	0.10	0.125	0.20
Y	0.0062	0.00941	0.0140	0.0146	0.023

Calculate the slope and y-intercept of the least squares regression straight line fitted to the data. Also calculate the correlation coefficient.

Keys:	**Display:**	**Description:**
[↱] [CLΣ]	0.0000	Clears statistics registers.
.05 [INPUT] .0062 [Σ+] .075 [INPUT] .00941 [Σ+] .1 [INPUT] .014 [Σ+] .125 [INPUT] .0146 [Σ+] .2 [INPUT] .023 [Σ+]	5.0000	Enters x,y-data.
[↰] [m,b]	0.1093	Displays slope.
[↰] [SWAP]	0.0014	Displays y-intercept (: indicates another result).
[↰] [x̂,r] [↰] [SWAP]	0.9890	Displays correlation coefficient.

Estimate the rate of the reaction when the concentration equals 0.09 moles per liter.

.09 [↱] [ŷ,r]	0.0113	Calculates estimate of y for x = 0.09.

Estimate the concentration necessary for the rate equal to 0.0200.

.02 [◄] [x̂,r]	0.1700	Calculates estimate of x for $y = 0.02$.
[C]	0.0000	Clears display and : annunciator.

Weighted Mean

To calculate the weighted mean of data points $x_1, x_2, \ldots, x_n$ occurring with frequencies $y_1, y_2, \ldots, y_n$,

1. Use [INPUT] and [Σ+] to enter x,y pairs. The y-values are the frequencies of the x-values.
2. Press [►] [x̄w].

Example. A survey of 266 one-bedroom rental apartments reveals that 54 of them rent for $200 per month, 32 for $205, 88 for $210, and 92 for $216. What is the average monthly rent?

Keys:	Display:	Description:
[►] [CLΣ]	0.0000	Clears statistics registers.
200 [INPUT] 54 [Σ+]	1.0000	Begins entry of rents and number of apartments.
205 [INPUT] 32 [Σ+]	2.0000	
210 [INPUT] 88 [Σ+]	3.0000	
216 [INPUT] 92 [Σ+]	4.0000	
[►] [x̄w]	209.4436	Calculates weighted mean.

Equations:

$$\bar{x} = \frac{\Sigma x}{n}, \quad \bar{y} = \frac{\Sigma y}{n}, \quad \bar{x}_w = \frac{\Sigma xy}{\Sigma y}$$

$$Sx = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n - 1}}$$

$$Sy = \sqrt{\frac{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}{n - 1}}$$

$$\sigma x = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n}} \qquad \sigma y = \sqrt{\frac{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}{n}}$$

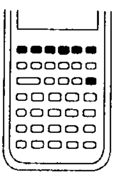
$$r = \frac{\Sigma xy - \frac{\Sigma x \, \Sigma y}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)}}$$

$$m = \frac{\Sigma xy - \frac{\Sigma x \, \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

$$b = \bar{y} - m\bar{x} \qquad \hat{x} = \frac{y - b}{m} \qquad \hat{y} = mx + b$$

6

Built-in Library



The built-in program library consists of six programs that can be accessed by pressing [↰][LOAD] followed by one of the letter keys, A through F, in the top row. (When [↰][LOAD] is pressed, the top row keys assume their letter values.) An abbreviated program name is displayed until the letter key is released.

Program Name	Title	Message
A	One Sample Test Statistics	1-StAt
B	Two Sample Test Statistics	2-StAt
C	Linear Regression Test Statistics	Lr-StAt
D	Chi-Square Test Statistic	CHI-2
E	Binomial Probability Distribution	bin
F	Time Value of Money	FinAnCE

When a program is loaded, it clears any other programs that may be in memory. (However, the storage registers are left intact.)

Once you've entered your data (either raw or summary), you can calculate the statistics and tests described in this chapter in any order. For instance, you can use Lr-StAt, load and use 1-StAt, and then reload and use Lr-StAt without affecting your data. Also, if you make a mistake calculating a statistic, you do not need to reenter your data. You simply repeat the calculation, correcting the mistake.

This table lists the symbols used in the flow diagrams that show the organization of the programs. The diagrams in this chapter make up the quick reference guide.

Symbols in Program Flow Diagrams

Symbol	Description
n	Sample size.
$\overline{x}$	Sample mean.
s	Sample standard deviation.
σ	Population standard deviation.
p	Sample proportion.
π	Population proportion.
μ	Population mean.
d	Precision ($^1/_2$ width of confidence interval).
O_i	Observed value.
E_i	Expected value.
df	Degrees of freedom.
z	Standard normal random variable.
t	Student's t random variable.
χ^2	Chi-square random variable.
V_o	Hypothesized value (e.g., π_0, μ_0, D_0,...).
U_{tp}	Upper tail probability.
pr	Probability.

The rest of this chapter describes each program, mainly by example. Find the example that resembles the problem you are trying to solve and use it as a guide to find the solution, mixing and matching the parts.

Quick Reference Guide. The diagrams in this chapter are in the quick reference guide. You can detach the quick reference guide along the perforated lines and slip it inside your calculator carrying case.

One Sample Statistics (1-StAt)

1-StAt calculates test statistics and confidence interval estimates for a variety of one-sample problems. You can calculate test statistics, confidence intervals, the sample size required for estimation test of the mean, degrees of freedom, and standard error.

The 1-StAt program is organized as follows.

Test of Mean

[↰] [LOAD] A

↓

Form of data?

raw data:

[↱] [CLΣ]
Enter data using [Σ+]
[XEQ] A

summary statistics:

n [XEQ] D
$\bar{x}$ [R/S]
s or σ [R/S]

Test of hypothesis or confidence interval?

test of hypothesis:

V_0 [XEQ] B (displays z or t)
[↰] [Q(z)] or [↰] [Q(t)]
(displays U_{tp})

confidence interval:

U_{tp} [↱] [zp] or [↱] [tp] (displays z or t)
[XEQ] C (displays precision)
[R/S] (displays lower limit)
[R/S] (displays upper limit)

Test of Proportion

[↰] [LOAD] A

↓

Test of hypothesis or confidence interval?

test of hypothesis:

n [XEQ] D
p [R/S]
π_0 [XEQ] 1
[R/S]
π_0 [XEQ] B (displays z)
[↰] [Q(z)] (displays U_{tp})

confidence interval:

n [XEQ] D
p [R/S]
[XEQ] 1
[R/S]
U_{tp} [↱] [zp] (displays z)
[XEQ] C (displays precision)
[R/S] (displays lower limit)
[R/S] (displays upper limit)

Paired Difference

[↰] [LOAD] A
[↱] [CLΣ]
x_1 [−] y_1 [Σ+]
x_2 [−] y_2 [Σ+]
.
.
.
x_n [−] y_n [Σ+]
[XEQ] A

↓

Test of hypothesis or confidence interval?

test of hypothesis ↙

V_0 [XEQ] B (displays z or t)
[↰] [Q(z)] or [↰] [Q(t)]
(displays U_{tp})

confidence interval ↘

U_{tp} [↱] [zp] or [↱] [tp] (displays z or t)
[XEQ] C (displays precision)
[R/S] (displays lower limit)
[R/S] (displays upper limit)

Sample Size Determination

[↰] [LOAD] A
σ [XEQ] E
d [INPUT]
U_{tp} [↱] [zp] (displays z)
[R/S] (displays required sample size)

1-StAt Registers and Labels

R_0	*n*, the sample size.
R_1	The degrees of freedom n − 1.
R_2	The sample mean $\overline{x}$ or proportion *p*.
R_3	The standard deviation, $\sqrt{p(1-p)}$ or $\sqrt{\pi(1-\pi)}$.
R_4 through R_9	Statistics registers (page 55).
LBL A	Processes summary statistics for 1-StAt and stores in R_0 through R_3.
LBL B	Calculates test statistic for hypothesis. Used after [XEQ] A or [XEQ] D.
LBL C	Calculates confidence interval. Used after [XEQ] A or [XEQ] D.
LBL D	Stores summary statistics into R_0 through R_3 for test of one sample mean or proportion.
LBL 7	Calculates standard error, the standard deviation of the sampling distribution.

The method for entering data depends on the data type: raw data or summary statistics.

If you are using *raw data*,

1. Clear the statistics registers by pressing [↱] [CLΣ].
2. Enter sample data using [Σ+].
3. Press [XEQ] A to process the data and display the standard error (standard deviation of the sampling distribution).
4. Optional:
   - Press [RCL] 0 to display the sample size.
   - Press [RCL] 1 to display the degrees of freedom.
   - Press [RCL] 2 to display the sample mean.
   - Press [RCL] 3 to display the sample standard deviation.
   - Press [XEQ] 7 to calculate the standard error (standard deviation of the sampling distribution).

If you are using *summary statistics*,

1. Enter the sample size and press [XEQ] D.
2. Enter the mean and press [R/S].
3. Enter the standard deviation and press [R/S]. The display shows the standard error (standard deviation of the sampling distribution).
4. Optional:
   - Press [RCL] 0 to display the sample size.
   - Press [RCL] 1 to display the degrees of freedom.
   - Press [RCL] 2 to display the sample mean.
   - Press [RCL] 3 to display the sample standard deviation.

Population Mean—Test of Hypothesis/Confidence Interval Estimate

Example: Population Standard Deviation Known. A random sample of 75 elements is taken from a population. The sample mean is 123.45 and the population standard deviation is 6.78.

Part 1. First, enter the data. In this example the data is summary statistics.

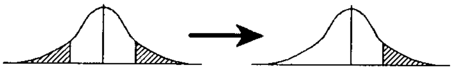
Keys:	Display:	Description:
[◄] [LOAD] A		Loads 1-StAt.
75 [XEQ] D	75.0000	Stores sample size.
123.45 [R/S]	123.4500	Stores sample mean.
6.78 [R/S]	0.7829	Stores standard deviation and calculates standard error.

Now you can either test the hypothesis (part 2) or calculate a confidence interval estimate (part 3). This example includes both.

Part 2. Calculate the test statistic for the null hypothesis that the population mean is 122. Test it against the alternative hypothesis that the mean is not equal to 122 at the 0.05 significance level.

Because the population standard deviation is known, the normal distribution will be used to make the test.

The problem is stated so that the hypothesis will be rejected if the test statistics fall in the area of the two tails of the normal distribution. To use the normal distribution key, the two-tail 0.05 significance level is translated to an upper-tail 0.025 probability.



Equation:

$$z = \frac{(\bar{x} - \mu_0)}{\sigma/\sqrt{n}}$$

Keys:	**Display:**	**Description:**
122 [XEQ] B	1.8521	Enters hypothesized population mean and calculates test statistic.
[⬅] [Q(z)]	0.0320	Calculates upper-tail probability.

Because the calculated probability is greater than 0.025 (0.05 ÷ 2), the hypothesis is not rejected at the 0.05 level in this two-tail test.

Here's an alternative procedure to test the hypothesis:

.025 [➡] [zp]	1.9600	Calculates critical z value.
122 [XEQ] B	1.8521	Calculates test statistic.

Because the test statistic is less than the critical z value, the hypothesis is not rejected.

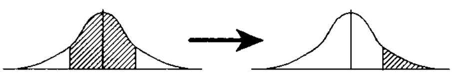
Part 3. Calculate the 90% confidence interval estimate of the population mean.

Equations:

$$precision = \frac{z\sigma}{\sqrt{n}}$$

$$confidence\ limits = \overline{x} \pm \frac{z\sigma}{\sqrt{n}}$$

The inside two-tail 90% confidence level will be translated to upper-tail, that is (1 − 0.9) ÷ 2 = 0.05 so that you can use the [⮫][zp] key.



Keys:	Display:	Description:
.05 [⮫][zp]	1.6449	Calculates standard normal variate.
[XEQ] C	1.2877	Calculates precision.
[R/S]	122.1623	Calculates lower confidence limit.
[R/S]	124.7377	Calculates upper limit.

Example: Population Standard Deviation Unknown. The following is a random sample of 8 elements from a population: 129.7, 149.8, 129.6, 109.3, 130.0, 133.4, 116.4, and 134.8.

Part 1. First, enter the data. In this example the data is raw.

Keys:	Display:	Description:
[⮪][LOAD] A		Loads 1-StAt.
[⮫][CLΣ]	0.0000	Clears statistics registers.

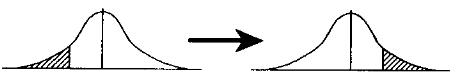
129.7 [Σ+]	1.0000	Enters raw data.
149.8 [Σ+]	2.0000	
129.6 [Σ+]	3.0000	
109.3 [Σ+]	4.0000	
130 [Σ+]	5.0000	
133.4 [Σ+]	6.0000	
116.4 [Σ+]	7.0000	
134.8 [Σ+]	8.0000	
[↱] [$\bar{x},\bar{y}$]	129.1250	Calculates mean (optional).
[XEQ] A	4.2975	Processes data and calculates standard error.

Now you can either test the hypothesis (part 2) or calculate a confidence interval estimate (part 3). This example includes both.

Part 2. If the alternative is that the population mean is lower than 137, would you reject or not reject the hypothesis that the population mean is 137, testing at the 0.1 level of significance?

Because you do not know the population standard deviation, the sampling distribution of the test statistic is t distributed. Because the sample is small, the normal distribution cannot be used as an approximation.

The alternative hypothesis implies a lower-tail test. This procedure translates to an upper-tail so you can use the [↰] [Q(t)] key.



Equations:

$$t = \frac{(\bar{x} - \mu_0)}{s / \sqrt{n}}$$

degrees of freedom $= n - 1$

Keys:	Display:	Description:
RCL 1	7.0000	Checks calculated degrees of freedom (optional).
137 XEQ B	-1.8325	Enters hypothesized population mean and calculates test statistic.
+/-	1.8325	Converts to upper tail.
← Q(t)	0.0548	Calculates upper-tail probability.

Because the calculated tail probability is less than 0.1 (the rejection level), the hypothesis is rejected.

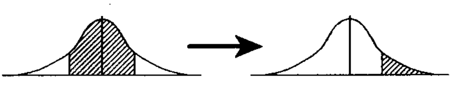
Here's an alternative procedure to test the hypothesis:

.1 → tp	1.4149	Calculates t distribution critical value.
137 XEQ B	-1.8325	Calculates test statistic.
+/-	1.8325	Converts to upper tail.

When the two values are compared, the calculated statistic is greater than the critical value. Thus, the hypothesis is rejected at the 0.1 level.

Part 3. Calculate the 95% confidence interval estimate of the population mean.

The inside two-tail 95% confidence level is translated to an upper-tail probability, that is $(1 - 0.95) \div 2 = 0.025$.



Equations:

$$precision = \frac{ts}{\sqrt{n}}$$

$$\textit{confidence limits} = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

$$\textit{degrees of freedom} = n - 1$$

Keys:	**Display:**	**Description:**
.025 [↱] [tp]	2.3646	Calculates required test statistic.
[XEQ] C	10.1619	Calculates precision of interval.
[R/S]	118.9631	Calculates lower confidence limit.
[R/S]	139.2869	Calculates upper confidence limit.

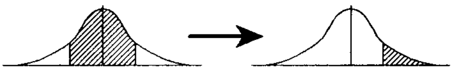
Population Proportion

Example: Confidence Interval Estimate. A sample of 65 students showed 38 favoring a proposal to raise the student fee to provide bus service to students. What is the 99% confidence interval estimate of the population proportion in favor of the proposal?

Part 1. First, enter the summary statistics.

Keys:	**Display:**	**Description:**
[↰] [LOAD] A		Loads 1-StAt.
65 [XEQ] D	65.0000	Enters sample size.
38 [÷] 65 [=] [R/S]	0.5846	Calculates and stores sample proportion.
[XEQ] 1	0.4928	Calculates $\sqrt{p\,(1-p\,)}$.
[R/S]	0.0611	Calculates standard error.

Part 2. The normal approximation method is used. To use the [↱][zp] key for the calculations, translate the 99% inside-tail confidence level implied in the problem statement to an upper-tail probability: (1 − 0.99) ÷ 2 = 0.005.



Equations:

$$precision = \frac{z\sqrt{p(1-p)}}{\sqrt{n}}$$

$$confidence\ interval = p \pm \frac{z\sqrt{p(1-p)}}{\sqrt{n}}$$

Keys:	**Display:**	**Description:**
.005 [↱][zp]	2.5758	Calculates z value.
[XEQ] C	0.1574	Calculates precision.
[R/S]	0.4272	Calculates lower limit.
[R/S]	0.7421	Calculates upper limit.

Example: Test of Hypothesis. A fraternity did a market study to test their opinion that at least 52% of their national members would buy their new logo sweatshirt. Of 75 randomly sampled members who were offered the logo sweatshirt, 31 made a purchase. To meet their fund raising goal, at least 52% of the members would have to buy.

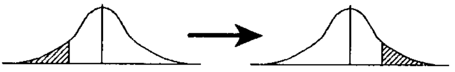
Part 1. First, enter the summary statistics.

Keys:	**Display:**	**Description:**
[↰][LOAD] A		Loads 1-StAt.
75 [XEQ] D	75.0000	Enters sample size.

31 [÷] 75 [=] [R/S]	0.4133	Calculates and stores sample proportion.
.52 [XEQ] 1	0.4996	Calculates $\sqrt{\pi(1-\pi)}$ for hypothesized proportion.
[R/S]	0.0577	Calculates standard error.

Part 2. Test the hypothesis that the proportion of buyers in the population is 0.52 against the alternative that the proportion is less than 0.52. Use the normal approximation method and test at the 0.05 level of significance.

With the statement of the alternative hypothesis, the lower-tail critical region of the normal distribution is implied. Use the [+/-] key to translate the lower-tail to the upper-tail.



This test statistic is calculated:

$$z = \frac{(p - \pi_0)}{\frac{\sqrt{\pi_0(1 - \pi_0)}}{\sqrt{n}}}$$

where p is the sample proportion and π_0 is the hypothesized population proportion.

Keys:	**Display:**	**Description:**
.52 [XEQ] B	−1.8490	Enters hypothesized population proportion and calculates test statistic.
[+/-]	1.8490	Converts to upper tail.
[◄] [Q(z)]	0.0322	Calculates upper-tail probability.

The hypothesis is rejected because the tail probability is less than the significance level.

Here's an alternative procedure to test the hypothesis:

.05 [→] [ZP]	1.6449	Calculates upper tail critical value.
.52 [XEQ] B	−1.8490	Calculates test statistic.
[+/-]	1.8490	Converts to upper tail.

Because the test statistic is greater than the critical value, the test statistic falls in the rejection area and the hypothesis is rejected.

Example: Test on Paired Differences (Paired t) of Means of Two Populations

Students have alleged that the test bank for a certain large-enrollment course has exams that vary in difficulty. In response, the instructor agreed to give six students two different exams. The scores of the two exams are in the following table. Can the hypothesis that the mean difference in scores between the two exams is zero be rejected in favor of the hypothesis that Test A is easier than B (that the mean difference, A − B, is greater than zero)? Use a 0.10 level of significance.

Student	1	2	3	4	5	6
Test A	90	85	78	75	80	98
Test B	83	93	72	68	87	92

Part 1. First, enter the difference between each pair of observations using the [Σ+] key.

Keys:	Display:	Description:
[←] [LOAD] A		Loads 1-StAt.
[→] [CLΣ]	0.0000	Clears statistics registers.
90 [−] 83 [Σ+]	1.0000	Enters raw data.

85 [-] 93 [Σ+]	2.0000	
78 [-] 72 [Σ+]	3.0000	
75 [-] 68 [Σ+]	4.0000	
80 [-] 87 [Σ+]	5.0000	
98 [-] 92 [Σ+]	6.0000	
[↱] [$\bar{x},\bar{y}$]	1.8333	Processes data and calculates mean difference (optional).
[XEQ] A	2.9599	Calculates standard error.

Now you can test the hypothesis or calculate a confidence interval estimate. This example is a test of hypothesis.

Part 2. Test the hypothesis. The problem is stated as an upper-tail test and, since the sample is small and the population standard deviation is unknown, the t distribution is used.

Equation:

$$t = \frac{\bar{d} - d_0}{s_d / \sqrt{n}}$$

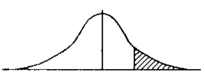
where

$\bar{d}$ is the mean of the differences of the paired values.
s_d is the sample standard deviation of the differences of the paired values.
d_0 is the hypothesized mean difference.

degrees of freedom $= n - 1$ (n is the number of differences).

Keys:	Display:	Description:
[RCL] 1	5.0000	Reviews calculated degrees of freedom (optional).
0 [XEQ] B	0.6194	Enters hypothesized mean difference and calculates test statistic.
[◄] [Q(t)]	0.2814	Calculates upper-tail probability.

Because the tail probability of the test statistic is greater than the 0.10 significance level, the hypothesis is not rejected.



Here's an alternative procedure to test the hypothesis:

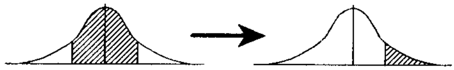
.1 [►] [tp]	1.4759	Calculates t distribution critical value.
0 [XEQ] B	0.6194	Enters hypothesized mean difference and calculates test statistic.

Because the calculated test statistic is less than the critical value, the hypothesis is not rejected.

Example: Calculating Sample Size

Suppose that the standard deviation of a population is 34.26. What simple random sample size must be taken to produce an estimate of the mean within 3 units of the true population mean (a precision of 3) with 90% confidence? Use the normal probability distribution.

The problem is stated as an inside two-tail probability. By translating this to an upper-tail probability, $(1 - 0.9) \div 2 = 0.05$, you can use the [►] [zp] key to solve the problem.



Equation:

$$n = \left(\frac{z\sigma}{precision} \right)^2$$

Keys:	**Display:**	**Description:**
[⬅] [LOAD] A		Loads program A.
34.26 [XEQ] E	34.2600	Enters standard deviation.
3 [INPUT]	3.0000	Enters precision.
.05 [➡] [zp]	1.6449	Calculates standard normal variate.
[R/S]	352.8472	Calculates required sample size.

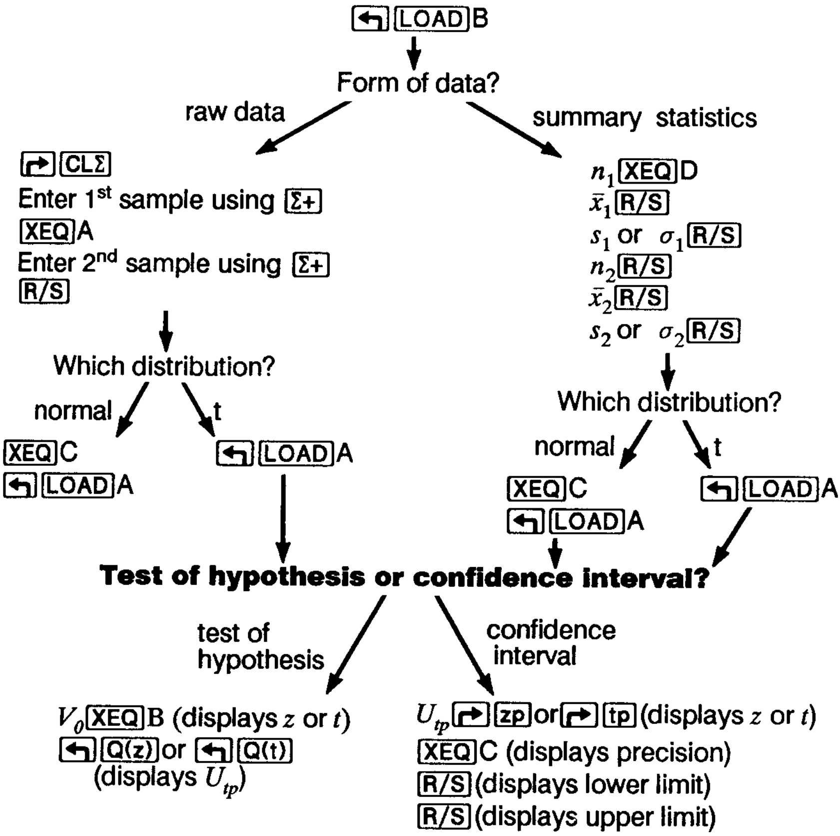
The convention is to round the calculated sample size up to the next larger integer, in this case 353.

Two Sample Test Statistics (2-StAt)

2-StAt stores the statistics needed to prepare for 1-StAt to calculate test statistics and confidence interval estimates involving the difference between population means or proportions. It also calculates the pooled standard deviation.

2-StAt is organized as follows:

Difference Between Means



Difference Between Proportions

[↰] [LOAD] B

↓

Assumed difference non-zero or zero?

non-zero:

n_1 [XEQ] D
p_1 [R/S]
[XEQ] 1
[R/S]
n_2 [R/S]
p_2 [R/S]
[XEQ] 2
[R/S]
[XEQ] C
[↰] [LOAD] A

↓

Test of hypothesis or confidence interval?

test of hypothesis:

V_0 [XEQ] B (displays z)
[↰] [Q(z)] or [↰] [Q(t)]
(displays U_{tp})

confidence interval:

U_{tp} [↱] [zp] or [↱] [tp] (displays z)
[XEQ] C (displays precision)
[R/S] (displays lower limit)
[R/S] (displays upper limit)

zero:

n_1 [XEQ] D
p_1 [R/S]
$\frac{x_1 + x_2}{n_1 + n_2}$ [XEQ] 1
[R/S]
n_2 [R/S]
p_2 [R/S]
[XEQ] B
[↰] [LOAD] A
0 [XEQ] B (displays z)
[↰] [Q(z)] (displays U_{tp})

2-StAt Registers and Labels

R_0	$\dfrac{1}{(\frac{1}{n_1} + \frac{1}{n_2})}$ *or* 1 if B or C is executed, respectively.
R_1	Degrees of freedom n_1+n_2-2.
R_2	Difference in sample means or proportions.
R_3	Pooled standard deviation.
R_4	n_2.
R_5	$\bar{x}_2$.
R_6	s_2.
R_7	n_1.
R_8	$\bar{x}_1$.
R_9	s_1.
LBL A	Calculates and moves statistics after raw data is entered.
LBL B	Calculates pooled standard deviation estimate of unknown but assumed equal population standard deviations.
LBL C	Calculates standard error of mean/proportion difference when given unequal population standard deviations.
LBL D	Stores summary statistics from two samples.
LBL 1	Calculates $\sqrt{p_1(1 - p_1)}$.
LBL 2	Calculates $\sqrt{p_2(1 - p_2)}$.

The method for entering data depends on the data type: raw data or summary statistics.

If you are using *raw data*,

1. Clear the statistics registers by pressing [➦] [CLΣ].
2. Enter sample data for sample 1 using [Σ+].
3. Press [XEQ] A.

4. Enter sample data for sample 2 using [Σ+].
5. Press [R/S] to process the data and display the standard error.
6. Optional: The values stored and calculated can be recalled from the registers shown in the table on page 83.

If you are using *summary statistics*,

1. Enter the sample size for sample 1 and press [XEQ] D.
2. Enter the mean for sample 1 and press [R/S].
3. Enter the standard deviation for sample 1 and press [R/S]. The display shows the standard error (standard deviation of the sampling distribution).
4. Enter the sample size for sample 2 and press [R/S].
5. Enter the mean for sample 2 and press [R/S].
6. Enter the standard deviation for sample 2 and press [R/S].
7. Optional: The stored and calculated values can be recalled from the registers shown in the table on page 83.

Difference Between Population Means—Test of Hypothesis/Confidence Interval Estimation

Example: Population Standard Deviations Equal and Unknown. Sample data from two populations assumed to have equal variances are given below.

A	86	109	112	91	103	121	107	100	97
B	93	101	111	117	105	97	99		

Part 1. First, enter the data and calculate the pooled standard deviation. This example uses raw data.

Equation:

$$s_P = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Keys:	Display:	Description:
[⬅][LOAD] B		Loads 2-StAt.
[➡][CLΣ]	0.0000	Clears statistics registers.
86 [Σ+]	1.0000	Enters sample A data.
109 [Σ+]	2.0000	
112 [Σ+]	3.0000	
91 [Σ+]	4.0000	
103 [Σ+]	5.0000	
121 [Σ+]	6.0000	
107 [Σ+]	7.0000	
100 [Σ+]	8.0000	
97 [Σ+]	9.0000	
[➡][$\bar{x},\bar{y}$]	102.8889	Calculates mean (optional).
[XEQ] A	0.0000	Prepares for sample B.
93 [Σ+]	1.0000	Enters sample B data.
101 [Σ+]	2.0000	
111 [Σ+]	3.0000	
117 [Σ+]	4.0000	
105 [Σ+]	5.0000	
97 [Σ+]	6.0000	
99 [Σ+]	7.0000	

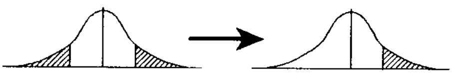
[→] [$\bar{x},\bar{y}$]	103.2857	Calculates mean (optional).
[R/S]	9.8355	Calculates pooled standard deviation.
[RCL] 1	14.0000	Recalls calculated degrees of freedom (optional).

Pressing [XEQ] B displays the pooled standard deviation again.

Now you can either test the hypothesis (part 2) or calculate a confidence interval estimate (part 3). The statistic is t distributed. This example includes both.

Part 2. Test the hypothesis that the means of the two populations are equal against the alternative hypothesis that they are not equal. Use the t distribution and a 0.10 level of significance.

The alternative hypothesis implies a two-tail rejection area. This is translated to the upper tail $0.10 \div 2 = 0.05$, so that you can use the [←] [Q(t)] key.



Equation:

$$t = \frac{((\overline{x_1} - \overline{x_2}) - D_0)}{S}$$

where D_0 is the hypothesized difference between the population means, and S is the standard error of the difference.

$$S = s_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

s_p is defined in part 1 on page 85.

$$degrees\ of\ freedom = n_1 + n_2 - 2$$

Keys:	Display:	Description:
[◄] [LOAD] A		Loads 1-StAt.
0 [XEQ] B	−0.0801	Enters hypothesized difference between population means and calculates t test statistic.
[+/-]	0.0801	Converts to upper tail.
[◄] [Q(t)]	0.4687	Calculates upper tail probability.

The probability of getting this statistic if the population means are equal is approximately 0.47, which is greater than the 0.05 significance level. The hypothesis that the two means are equal is not rejected.

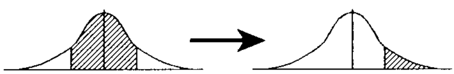
Here's an alternative procedure to test the hypothesis:

Keys:	Display:	Description:
.05 [►] [tp]	1.7613	Calculates upper tail critical value.
0 [XEQ] B	−0.0801	Enters hypothesized difference between population means and calculates t test statistic.
[+/-]	0.0801	Converts to upper tail.

Because the test statistic is less than the critical test value, the hypothesis is not rejected.

Part 3. Calculate the 90% confidence interval estimate of the difference between the two population means.

The inside two tail 90% confidence level is translated to an upper tail probability, that is $(1 - 0.9) \div 2 = 0.05$, so that you can use the [►] [tp] key.



Equations:

$$precision = ts_P\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$confidence\ interval = (\overline{x_1} - \overline{x_2}) \pm ts_P\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

s_p is defined in part 1 on page 85.

$$degrees\ of\ freedom = n_1 + n_2 - 2$$

Keys:	**Display:**	**Description:**
[⬅] [LOAD] A		Loads 1-StAt.
[RCL] 1	14.0000	Checks that degrees of freedom have been calculated and stored (optional).
.05 [➡] [tp]	1.7613	Calculates t value.
[XEQ] C	8.7302	Calculates precision.
[R/S]	-9.1270	Calculates lower confidence limit.
[R/S]	8.3333	Calculates upper limit.

Example: Population Standard Deviations Unequal and Known. Samples are taken from two populations for which the population standard deviations are known but unequal. The first sample of 75 has a mean of 43.2 with a population standard deviation of 1.34. The second sample of 125 has a mean of 38.1 and a population standard deviation of 1.45.

Enter the data and calculate the standard error. The standard error term is calculated by pressing [XEQ] C after you've entered the summary statistics.

Equation:

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Part 1. First, enter the data. This example uses summary statistics.

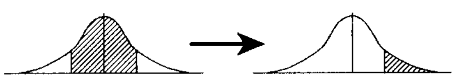
Keys:	**Display:**	**Description:**
[←] [LOAD] B		Loads 2-StAt.
75 [XEQ] D	75.0000	Enters sample size from first sample.
43.2 [R/S]	43.2000	Enters sample mean.
1.34 [R/S]	1.3400	Enters population standard deviation.
125 [R/S]	125.0000	Enters sample size of second sample.
38.1 [R/S]	38.1000	Enters sample mean.
1.45 [R/S]	1.4099	Enters standard deviation of second sample.
[XEQ] C	0.2019	Calculates standard error.

Now you can either test the hypothesis or calculate a confidence interval estimate. The statistic is normally distributed. This example calculates the confidence interval estimate.

Part 2. What is the 90% confidence interval estimate of the difference between the population means?

For these conditions, the standard normal distribution is used.

Once you've translated the inside two tail 90% confidence level to an upper-tail probability, that is $(1 - 0.9) \div 2 = 0.05$, you can use the [→] [zp] key.



Equations:

$$precision = z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$confidence\ interval = (\overline{x}_1 - \overline{x}_2) \pm z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Keys:	Display:	Description:
[⬑] [LOAD] A		Loads 1-StAt.
.05 [⮫] [zp]	1.6449	Calculates upper-tail z value.
[XEQ] C	0.3321	Calculates precision.
[R/S]	4.7679	Calculates lower limit of estimate of difference.
[R/S]	5.4321	Calculates upper limit.

Difference Between Population Proportions

Example: Difference Non-Zero. Samples were taken from two populations. The first sample size was 400 and the proportion of persons responding "yes" was 0.47. The second sample size was 500 and the proportion responding "yes" was 0.35.

Part 1. Enter the summary statistics and calculate the standard error term.

Equation:

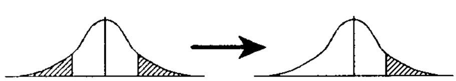
$$\sigma_{p_1 - p_2} = \sqrt{\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}}$$

Keys:	Display:	Description:
[⬏] [LOAD] B		Loads 2-StAt.
400 [XEQ] D	400.0000	Enters size of first sample.
.47 [R/S]	0.4700	Enters proportion for first sample.
[XEQ] 1	0.4991	Calculates $\sqrt{p_1(1-p_1)}$.
[R/S]	0.4991	Enters result.
500 [R/S]	500.0000	Enters size of second sample.
.35 [R/S]	0.3500	Enters proportion for second sample.
[XEQ] 2	0.4770	Calculates $\sqrt{p_2(1-p_2)}$.
[R/S]	0.4869	Enters it and processes data.
[XEQ] C	0.0328	Calculates standard error for test with difference assumed to be non-zero.

Now you can either test the hypothesis (part 2) or calculate a confidence interval estimate (part 3). This example includes both.

Part 2. Test the hypothesis that the differences between the population proportions is 0.04 against the alternative that the difference is not 0.04. Use a 0.01 level of significance and the normal approximation method.

The problem statement implies the use of the normal distribution and a two-tail rejection area. Once you've translated the two-tail significance level to upper-tail, that is, $0.01 \div 2 = 0.005$, you can use the [⬏] [Q(z)] key.



Equation:

$$z = \frac{(p_1 - p_2) - D_0}{\sqrt{\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}}}$$

D_0 is the hypothesized difference.

Keys:	**Display:**	**Description:**
[◄] [LOAD] A		Loads 1-StAt.
.04 [XEQ] B	2.4369	Enters hypothesized difference and calculates test statistic.
[◄] [Q(z)]	0.0074	Calculates upper-tail normal probability.

Because the probability of the test statistic is greater than the tail probability of 0.005, the hypothesis is not rejected.

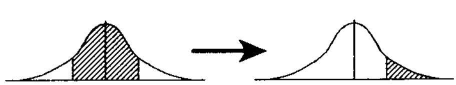
Here's an alternative procedure to test the hypothesis:

.005 [►] [zp]	2.5758	Calculates critical z value.
.04 [XEQ] B	2.4369	Calculates test statistic.

Because the test statistic is less than the critical value, the hypothesis is not rejected.

Part 3. Calculate a 95% confidence interval estimate of the difference between the population proportions.

To use [►] [zp], you must also translate the inside two-tail 95% confidence level to upper-tail: (1 − 0.95) ÷ 2 = 0.025.



Equations:

$$precision = z\sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2}}$$

$$confidence\ interval = (p_1 - p_2) \pm z\sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2}}$$

Keys:	Display:	Description:
[⬑] [LOAD] A		Loads 1-StAt.
.025 [⮫] [zp]	1.9600	Calculates z value.
[XEQ] C	0.0643	Calculates precision of interval.
[R/S]	0.0557	Calculates lower limit of difference between population proportions.
[R/S]	0.1843	Calculates upper limit.

Example: Difference Zero. Samples were taken from two populations. The first sample size was 250 and 117 respondents replied "yes" to a particular question. The second sample size was 283 and 124 replied "yes" to that question. It is hypothesized that the proportion of "yes" in the two populations is the same.

Part 1. Enter the data, including the calculation of the pooled sample proportion.

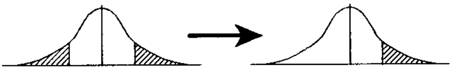
Equation:

$$\overline{p} = \frac{(x_1 + x_2)}{(n_1 + n_2)}$$

Keys:	**Display:**	**Description:**
[←] [LOAD] B		Loads 2-StAt.
250 [XEQ] D	250.0000	Enters size of first sample.
117 [÷] 250 [=] [R/S]	0.4680	Calculates and enters first sample proportion.
117 [+] 124 [=]	241.0000	Calculates sum of successes.
[÷] [(] 250 [+] 283 [=]	0.4522	Calculates pooled proportion $\overline{p}$.
[XEQ] 1	0.4977	Calculates $\sqrt{\overline{p}(1 - \overline{p})}$.
[R/S]	0.4977	Enters result.
283 [R/S]	283.0000	Enters size of second sample.
124 [÷] 283 [=] [R/S]	0.4382	Calculates and enters sample proportion of second sample.
[XEQ] B	0.4977	Processes statistics and calculates standard error term.

Part 2. Test the hypothesis that the proportions of "yes" in the two samples are equal against the alternative that they are not equal. Use a 0.10 significance level and the normal approximation method.

In this problem the normal approximation is used. And, in order to use the [←] [Q(z)] key, you must translate the two-tail rejection area to upper-tail: 0.10 ÷ 2 = 0.05.



Equation:

$$z = \frac{(p_1 - p_2) - D_0}{\sqrt{\bar{p}(1 - \bar{p})}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where D_0 is the hypothesized difference ($D_0 = 0$ in this case) between the population proportions and $\bar{p}$ is the pooled sample proportion.

Keys:	**Display:**	**Description:**
[◄] [LOAD] A		Loads 1-StAt.
0 [XEQ] B	0.6907	Enters 0 or the hypothesized difference and calculates test statistic.
[◄] [Q(z)]	0.2449	Calculates upper-tail probability.

Because the tail probability of the test statistic is greater than the tail probability of 0.05, the hypothesis is not rejected.

Here's an alternative procedure to test the hypothesis:

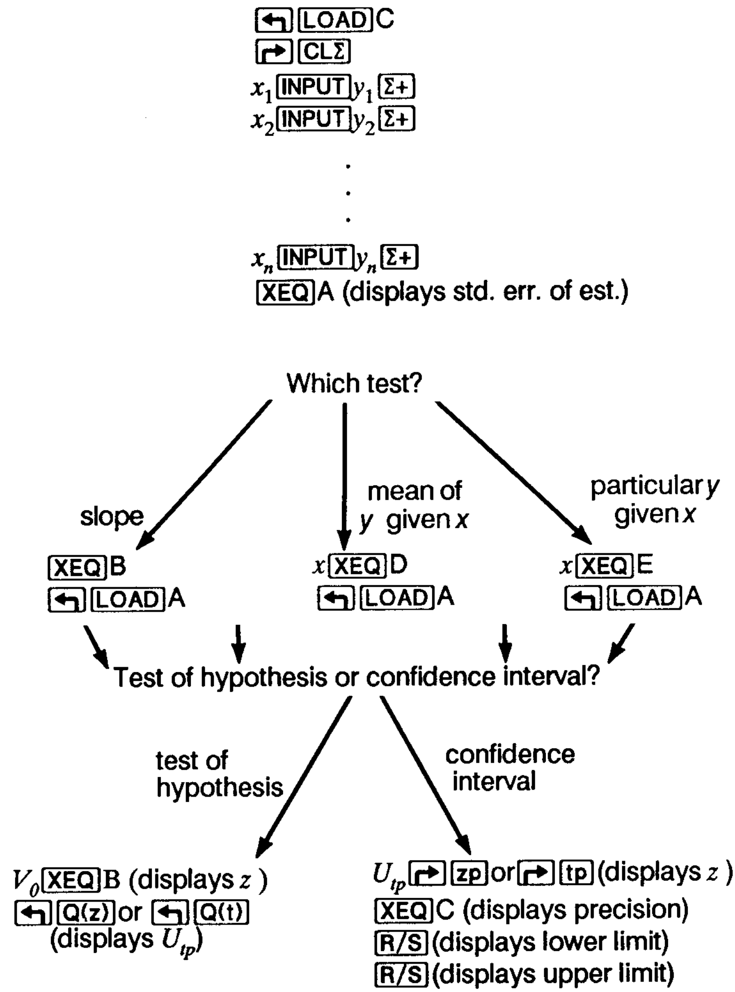
.05 [►] [zp]	1.6449	Calculates critical z value.
0 [XEQ] B	0.6907	Calculates test statistic.

Because the test statistic is less than the critical value, the hypothesis is not rejected.

Linear Regression Test Statistics (Lr-StAt)

Lr-StAt stores the statistics to prepare 1-StAt to calculate test statistics and confidence interval estimates involving the slope, estimated mean value of y, and particular estimated value of x. It also calculates the standard error of estimate.

Lr-StAt is organized as follows:



Note: For another test using the same data, press [↰] [LOAD] C and start at "Which test".

Lr-StAt Registers and Labels

R_0	Stores a number such that $\frac{S_{y\mid x}}{\sqrt{R_0}}$ produces the appropriate standard errors for several tests and estimation procedures.
R_1	Degrees of freedom $n-2$.
R_2	Slope or estimate of y, given x.
R_3	Standard error of estimate $s_{y\mid x}$.
R_4-R_9	Statistics summary registers (page 55).
LBL A	Calculates the standard error of estimate.
LBL B	Sets up registers (R_0 through R_3) for test of slope.
LBL D	Sets up registers (R_0 through R_3) for test of $\hat{y}$ as mean, given x.
LBL E	Sets up registers (R_0 through R_3) for test of $\hat{y}$ as particular value, given x.

Equations:

$$S_{y\mid x} = \sqrt{\frac{(n-1)}{(n-2)}\left(s_y^2 - m^2 s_x^2\right)}$$

$$= \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-2}}$$

Data Entry

The following table shows a sample of lengths (x) in inches and weights (y) in pounds for six animals. All the examples in this section use the same data.

	1	2	3	4	5	6
x	40.5	38.6	37.9	36.2	35.1	34.6
y	104.5	102	100	97.5	95.5	94

To enter the data use the procedure described in chapter 5.

Keys:	**Display:**	**Description:**
[↱] [CLΣ]		
40.5 [INPUT]	40.5000	Enters x_1.
104.5 [Σ+]	1.0000	Enters y_1.
38.6 [INPUT] 102 [Σ+]	2.0000	Enters data pairs.
37.9 [INPUT] 100 [Σ+]	3.0000	
36.2 [INPUT] 97.5 [Σ+]	4.0000	
35.1 [INPUT] 95.5 [Σ+]	5.0000	
34.6 [INPUT] 94 [Σ+]	6.0000	Enters last data pair.

Calculate linear regression and correlation statistics and recall the summation registers. (These steps are optional, and are not required for the examples that follow. For more information on linear regression, see chapter 5).

[↱] [$\bar{x},\bar{y}$]	37.1500	Displays $\bar{x}$.
[↰] [SWAP]	98.9167	Displays $\bar{y}$.
[↰] [SWAP]	37.1500	Redisplays $\bar{x}$.
[↱] [$\hat{y}$,r]	98.9167	Calculates $\hat{y}$ given $\bar{x}$, verifying that $(\bar{x}, \bar{y})$ is on regression line.

[↰] [SWAP]	0.9955	Displays correlation coefficient.
[↰] [x^2]	0.9909	Calculates r^2 (coefficient of determination).
[RCL] 4	6.0000	Recalls n.
[RCL] 5	222.9000	Recalls Σx.
[RCL] 6	593.5000	Recalls Σy.
[RCL] 7	8,306.2300	Recalls Σx^2.
[RCL] 8	58,786.7500	Recalls Σy^2.
[RCL] 9	22,093.4000	Recalls Σxy.

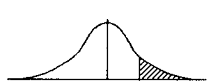
Now, if you want to see the standard error of estimate, load the program and press [XEQ] A.

Keys:	**Display:**	**Description:**
[↰] [LOAD] C		Loads Lr-StAt.
[XEQ] A	0.4247	Calculates standard error of estimate ($S_{y\,\|x}$).

Test of Hypothesis/Confidence Interval Estimation

In the next four examples you can either test the hypothesis, calculate a confidence interval estimate, or both.

Example: Slope. Part 1. Test the hypothesis that the slope is equal to zero against the alternative that it is greater than zero. Use a 0.001 significance level. The test is made using the t distribution. The problem statement implies an upper-tail test, so you can use the [↰] [Q(t)] key.



Equation:

$$t = \frac{(m - m_0)\, s_x \sqrt{n - 1}}{s_{y|x}}$$

where

m is the slope.
m_0 is the hypothesized slope.

degrees of freedom = n − 2.

Keys:	**Display:**	**Description:**
[⬑] [LOAD] C		Loads Lr-StAt.
[⬑] [m,b]	1.7601	Calculates slope (optional).
[XEQ] B	0.0000	Sets up registers for test of slope.
[⬑] [LOAD] A		Loads 1-StAt.
[RCL] 1	4.0000	Reviews calculated degrees of freedom ($n-2$) (optional).
0 [XEQ] B	20.9240	Enters hypothesized slope and calculates t.
[⬑] [Q(t)]	1.5415E-5	Calculates upper-tail probability.

Because the probability is below 0.001, the hypothesis is rejected.

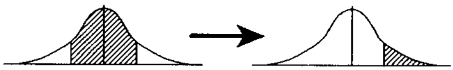
Here's an alternative procedure to test the hypothesis:

.001 [⬏] [tp]	7.1732	Calculates critical t value.
0 [XEQ] B	20.9240	Calculates t statistic.

Because the test statistic is greater than the critical t value, the hypothesis is rejected.

Part 2. Calculate the 95% confidence interval estimate of the slope parameter.

The inside two tail 95% confidence level will be translated to upper tail, that is (1 − 0.95) ÷ 2 = 0.025, so that you can use the [↱][tp] key.



Equations:

$$precision = \frac{t \times s_{y\,|\,x}}{s_x \sqrt{n - 1}}$$

$$confidence\ interval = m \pm \frac{t \times s_{y\,|\,x}}{s_x \sqrt{n - 1}}$$

degrees of freedom = n − 2.

Keys:	**Display:**	**Description:**
[↰][LOAD] C		Loads Lr-StAt.
[↰][m,b]	1.7601	Calculates slope (optional).
[XEQ] B	0.0000	Sets up registers for interval estimate of slope.
[↰][LOAD] A		Loads 1-StAt.
.025[↱][tp]	2.7764	Calculates t value.
[XEQ] C	0.2336	Calculates precision
[R/S]	1.5266	Calculates lower confidence limit.
[R/S]	1.9937	Calculates upper limit.

Part 3. Display the standard error of estimate of the data and calculate the standard error of the slope.

Equations:

$$\textit{Standard error of estimate} = S_{y\,|\,x} = \sqrt{\frac{(n-1)}{(n-2)}\left(s_y^2 - m^2 s_x^2\right)}$$

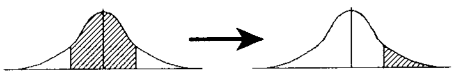
$$= \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-2}}$$

$$\textit{Standard error of slope} = \frac{s_{y\,|\,x}}{s_x \sqrt{n-1}}$$

Keys:	Display:	Description:
RCL 3	0.4247	Displays the standard error of estimate, $s_{y\,|\,x}$. (Calculated and stored automatically as part of earlier steps.)
÷RCL 0	25.4950	
$\sqrt{x}$	5.0493	
=	0.0841	Calculates the standard error of the estimate of the slope.

Example: Estimated y as a Mean Value, Given x. Calculate the 95% confidence interval estimate of the mean value of y, given $x = 39$.

The inside two tail 95% confidence level will be translated to upper tail, that is (1 − 0.95) ÷ 2 = 0.025, so that you can use the [↱] [tp] key.



Equation:

$$precision = t \times s_{y|x} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{(n - 1) s_x^2}}$$

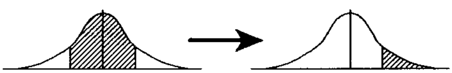
$$confidence\ interval = \hat{y}_{y|x} \pm t \times s_{y|x} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{(n - 1) s_x^2}}$$

Keys:	**Display:**	**Description:**
[↰] [LOAD] C		Loads Lr-StAt.
39 [↱] [ŷr]	102.1729	Calculates $\hat{y}$ given x (optional).
39 [XEQ] D	39.0000	Sets up registers for interval estimate of mean y given x.
[↰] [LOAD] A		Loads 1-StAt.

[RCL] 1	4.0000	Recalls degrees of freedom $(n-2)$ (optional).
.025[→][tp]	2.7764	Calculates t value.
[XEQ] C	0.6469	Calculates precision.
[R/S]	101.5260	Calculates lower confidence limit.
[R/S]	102.8198	Calculates upper limit.

Example: Estimated y as a Particular Value, Given x. Calculate the 95% confidence interval estimate of a particular value of y, given x = 39.

The inside two tail 95% confidence level will be translated to upper tail, that is (1 − 0.95) ÷ 2 = 0.025 so that you can use the [→][tp] key.



Equations:

$$precision = t \times s_{y|x} \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{(n - 1)\, s_x^2}}$$

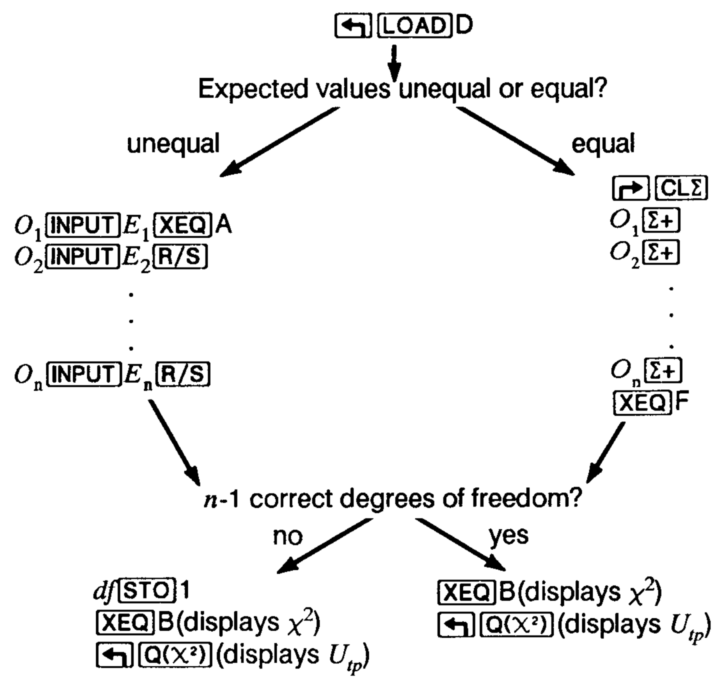
$$confidence\ interval = \hat{y}_{y|x} \pm t \times s_{y|x} \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{(n - 1) s_x^2}}$$

Keys:	**Display:**	**Description:**
[←][LOAD] C		Loads Lr-StAt.
39 [→][ŷ,r]	102.1729	Calculates $\hat{y}$ given x (optional).
39 [XEQ] E	39.0000	Sets registers for interval estimate of particular y given x.

[⬅][LOAD] A		Loads 1-StAt.
[RCL] 1	4.0000	Recalls calculated degrees of freedom $(n-2)$ (optional).
.025[➡][tp]	2.7764	Calculates t value.
[XEQ] C	1.3451	Calculates precision.
[R/S]	100.8279	Calculates lower confidence limit.
[R/S]	103.5180	Calculates upper limit.

Chi-Square Test (CHI-2)

This program calculates the χ^2 test statistic for categorical data with equal or unequal expected values. It allows you to delete data. CHI-2 is organized like this:



Chi-Square Registers and Labels

R_0	Temporary storage.
R_1	Degrees of freedom $n-1$.
R_2	The χ^2 statistic.
R_3	The numbers of classes or categories n.
LBL A	Begin data entry of expected values of pairs not equal between classes.
LBL B	Routine that recalls χ^2 test statistic.
LBL D	$\Sigma-$ routine that deletes a data pair.
LBL F	Routine that calculates χ^2 when expected pair values are equal.

Example: Single Classification (Including Goodness of Fit) With Unequal Expected Values

You conduct an experiment on the behavior of sub-atomic particles. Each particle displays one of six types of behavior. Existing theory predicts the probability of occurrence of each behavior type. A total of 180 particles are observed. The table below shows the number of occurrences of each type of behavior (observed) and the number of occurrences predicted by the theory (expected).

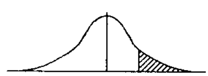
Observed	8	50	47	56	5	14
Expected	9.6	46.75	51.85	54.4	8.25	9.15

Part 1. First, enter the data.

Keys:	**Display:**	**Description:**
[⬑] [LOAD] D		Loads the CHI-2 program.
8 [INPUT] 9.6 [XEQ] A	1.0000	Initializes and stores first pair.
50 [INPUT] 46.75 [R/S]	2.0000	Stores data pairs.
47 [INPUT] 51.85 [R/S]	3.0000	
56 [INPUT] 54.4 [R/S]	4.0000	
5 [INPUT] 8.25 [R/S]	5.0000	
14 [INPUT] 9.15 [R/S]	6.0000	Enters last pair.

Part 2. Test the hypothesis that the theory adequately predicts the probabilities of particle behavior. Use the 0.10 level of significance to determine if the data invalidates the theory.

The statement of the problem implies an upper-tail probability. Use [⬑] [Q(X²)] to calculate it.



Equation:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where

χ^2 is the chi-square test statistic.
n is the number of classes or categories.
i is the index for pairs $i = 1, 2, \ldots, n$.
O_i is the observed value of pair i.
E_i is the expected value of pair i.

CHI-2 calculates $n-1$ degrees of freedom and stores the number in R_1. For situations where the appropriate degrees of freedom value is different from $n-1$, enter the correct number and press [STO] 1.

Keys:	Display:	Description:
[RCL] 3	6.0000	Recalls sample size n (optional).
[RCL] 1	5.0000	Recalls degrees of freedom $(n-1)$ (optional).
[XEQ] B	4.8444	Calculates χ^2 statistic.
[◄] [Q(X²)]	0.4352	Calculates upper-tail probability.

Because the tail probability is greater than the 0.10 level of significance, the hypothesis is not rejected.

Here is an alternative to test the hypothesis:

.1 [►] [X²p]	9.2364	Calculates the critical value of χ^2.
[XEQ] B	4.8444	Calculates χ^2 statistic.

Because the test statistic is less than the critical value of χ^2, the hypothesis is not rejected.

Example: Single Classification (Including Goodness of Fit) With Equal Expected Values Case

You used a random number generator to get 100 random numbers. The result appears in the following frequency table. The table ranges read "greater than the first number up to and including the second number." For example, the first range reads "greater than 0 up to and including 0.1."

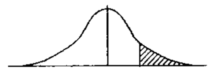
Range	0-.1	.1-.2	.2-.3	.3-.4	.4-.5
Observed	12	6	15	8	8

Range	.5-.6	.6-.7	.7-.8	.8-.9	.9-1
Observed	10	13	11	6	11

Part 1. First, enter the data:

Keys:	Display:	Description:
[◄] [LOAD] D		Loads the CHI-2 program.
[►] [CLΣ]		Clears statistics registers.
12 [Σ+]	1.0000	Enters first observed value.
6 [Σ+]	2.0000	
15 [Σ+]	3.0000	
8 [Σ+]	4.0000	
8 [Σ+]	5.0000	
10 [Σ+]	6.0000	
13 [Σ+]	7.0000	
11 [Σ+]	8.0000	
6 [Σ+]	9.0000	
11 [Σ+]	10.0000	Enters last observed value.

Part 2. Test the hypothesis that the distribution from which the random numbers were taken is uniform. Use a 0.05 level of significance. The statement of the problem implies an upper-tail probability, which is calculated by pressing [◄] [Q(X²)].



Equation:

$$\chi^2 = \frac{n\ \Sigma O_i^2}{\Sigma O_i} - \Sigma O_i$$

where:

χ^2 is the chi-square test statistic.
n is the number of pairs of data.
i is the index for pairs $i = 1, 2, \ldots, n$.
O_i is the observed value of pair i.

CHI-2 calculates $n-1$ degrees of freedom and stores the number in R_1. If the degrees of freedom for your problem is different than $n-1$, enter the correct number and press [STO] 1.

Keys:	Display:	Description:
[RCL] 4	10.0000	Recalls total number of classes (optional).
[↱] [$\bar{x},\bar{y}$]	10.0000	Displays the uniform expected in each class (optional).
[XEQ] F	8.0000	Calculates χ^2 statistic.
[↰] [Q(X²)]	0.5341	Calculates upper-tail probability.
[RCL] 1	9.0000	Recalls degrees of freedom ($n-1$) (optional).

Because the tail probability is greater than the 0.05 level of significance, the hypothesis is not rejected.

Here is an alternative to test the hypothesis:

.05 [↱][X²p]	16.9190	Calculates χ^2 critical value.
[XEQ] F	8.0000	Calculates χ^2 statistic.

Because the test statistic is less than the critical χ^2 value, the hypothesis is not rejected.

Binomial Distribution (bin)

This program calculates the probability of x successes from n trials, and the cumulative probability of x or fewer successes from n trials, given the probability of success of *pr*.

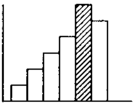
The bin program is organized like this:

[↰][LOAD]E
n[STO]1
pr[STO]2
x[XEQ]A(displays *pr* of *x* successes)
x[XEQ]B(displays *pr* of ≤ *x* successes)

bin Registers and Labels

R_0	*F(x)*, the probability that *x* is ≤ *x*.
R_1	The number of trials n.
R_2	The probability of success pr.
R_3	The number of successes x.
LBL A	Calculates the probability of x successes.
LBL B	Calculates the probability of x or fewer successes.

If the traffic light at an intersection is red 15 seconds each minute and green 45 seconds, what is the probability of making the green light four out of the five days you drive to work? Four or fewer? Four or more?

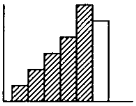


Equation:

$$P(x) = C_{n,x}\,prx\,(1 - pr)^{(n-x)}$$

where

n is the number of trials.
x is the number of successes out of n trials.
pr is the probability of success.
i is the index of summation $i = 0, 1, \ldots, n$.
$P(x)$ is the probability of x successes.

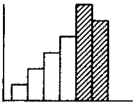


Equation:

$$F(x) = \sum_{i=1}^{n} C_{n,i}\,p_r^i\,(1 - p_r)^{(n-i)}$$

where $F(x)$ is the probability of x or fewer successes.

To calculate the probability of x or more successes, calculate the probability of $x-1$ or fewer successes and subtract it from 1.



Equation:

$$Q(x) = 1 - F(x - 1)$$

where $Q(x)$ is the probability of x or more successes.

Keys:	Display:	Description:
[←] [LOAD] E		Loads binomial distribution program.
5 [STO] 1	5.0000	Stores n, number of trials.
.75 [STO] 2	0.7500	Stores probability of green.
4 [XEQ] A	0.3955	Calculates probability of four greens.
4 [XEQ] B	0.7627	Calculates probability of four or fewer greens.
4 [-] 1 [=]	3.0000	Calculates $x - 1$.
[XEQ] B	0.3672	Calculates probability of three or fewer greens.
[+/-] [+] 1 [=]	0.6328	Subtracts probability of three or fewer greens from 1, giving probability of four or more greens.

Time Value of Money (FinAnCE)

The phrase *time value of money* (TVM) describes calculations based on uniform cash flows (money received or money paid) earning compound interest over a period of time. Many financial problems are TVM problems—for example, problems involving savings accounts, mortgages, car loans, and leases.

To solve a TVM problem, simply load the FinAnCE program, enter any four of the five TVM variables, and then execute the program to solve for the unknown. The five variables are:

n	The total number of compounding periods.
i%	The periodic interest rate (as a percentage).
PV	The present value (the initial cash flow).
PMT	The uniform, periodic payment.
FV	The future value (the final cash flow).

Variables are entered into registers using the [STO] key. They are solved for using the [XEQ] key and can be reviewed using the [RCL] key. The following table shows the keys that correspond to the TVM variables (note that the 4 key has "n" printed next to it):

Variable	To Store	To Calculate	To Review
n	[STO] 4	[XEQ] 4	[RCL] 4
i%	[STO] 5	*guess* [STO] 5 [XEQ] 5 [R/S]...	[RCL] 5
PV	[STO] 7	[XEQ] 7	[RCL] 7
PMT	[STO] 8	[XEQ] 8	[RCL] 8
FV	[STO] 9	[XEQ] 9	[RCL] 9

Calculating *i%* is an iterative process that you control with the [R/S] key. The program uses an interest rate "guess" that you supply to start the process. (If you don't supply an estimate, the program uses 1% as the guess.) Press [R/S] after each iteration until you reach the desired accuracy. Generally, you should continue to press [R/S] until two consecutive displayed values are the same. If you see the message Error-Func, you have reached the accuracy limit of the program. Press [RCL] 5 to see the value of *i%* in this case.

If pressing [R/S] does not converge, or does not converge to a reasonable value, then try a new guess and repeat the process.

The values you put in the TVM registers remain until you either replace them with new values or clear the registers.

Problems must meet the following requirements:

- The dollar amount is the same for each payment.
- Payments occur at the end of each period and none are skipped.
- Payment periods coincide with the compounding periods.
- The interest rate is periodic, coinciding with the compounding period. It is stored and calculated as a percent.
- Money flowing to you is treated as a positive value, and money flowing from you is treated as a negative value.
- Four of the five variables must have values, even if one is zero.

Equation:

$$0 = PV + PMT \times \left[\frac{1 - \left(1 + \frac{i\%}{100}\right)^{-N}}{\frac{i\%}{100}} \right] + FV \times \left(1 + \frac{i\%}{100}\right)^{-N}$$

Example: Student Loan. You receive a $10,000 student loan. The contract specifies an annual interest rate of 7% and a repayment period of 10 years from the date you finish school. What monthly payment can you look forward to?

Keys:	Display:	Description:
[⬑][FIX] 2		Displays two decimal places.
[⬑][LOAD] F		Loads TVM program.
10 [×] 12 [=] [STO] 4	120.00	Enters total number of payments.
7 [÷] 12 [=] [STO] 5	0.58	Enters periodic (monthly) interest rate (annual rate divided by 12).

10000 [STO] 7	10,000.00	Enters present value (loan amount).
0 [STO] 9	0.00	Enters zero for future value at end of loan.
[XEQ] 8	−116.11	Calculates monthly payment of −$116.11. (*PMT* is negative because the dollars flow *from* you.)

Example: Home Mortgage. You've decided that the maximum monthly mortgage payment you can afford is $730. If you make a $12,000 down payment and take out a 30-year, 11.5% mortgage, what is the maximum purchase price you can afford?

Keys:	Display:	Description:
[⬅] [FIX] 2		Displays two decimal places.
[⬅] [LOAD] F		Loads TVM program.
30 [×] 12 [=] [STO] 4	360.00	Enters *n*.
11.5 [÷] 12 [=] [STO] 5	0.96	Enters *i%*.
730 [+/-] [STO] 8	−730.00	Enters *PMT*. (The sign is negative because the dollars flow *from* you.)
0 [STO] 9	0.00	Enters *FV*.
[XEQ] 7	73,715.67	Calculates *PV*.
[+] 12000 [=]	85,715.67	Adds down payment to calculate total price.

Example: Savings Account. You opened a savings account with a deposit of $2,000. This account compounds interest quarterly.

Part 1. If you make no other deposits to the account, what interest rate would you need to double your investment in 10 years?

Keys:	Display:	Description:
[⬅][LOAD] F		Loads TVM program.
4 [×] 10 [=] [STO] 4	40.00	Enters *n*.
2000 [+/-] [STO] 7	-2,000.00	Enters *PV*. Dollars flowing *from* you are negative.
0 [STO] 8	0.00	Enters *PMT*.
4000 [STO] 9	4,000.00	Enters *FV*. (The sign is positive because the dollars will flow *to* you.)
1.5 [STO] 5	1.50	Enters a guess for *i%*.
[⬅][FIX] 6		Displays six decimal places.
[XEQ] 5	-371.963183	Displays first iteration value.
[R/S]	1.333333	Continue pressing [R/S].
[R/S]	1.768580	
[R/S]	1.746288	
[R/S]	1.747963	
[R/S]	1.747969	
[R/S]	1.747969	Two consecutive values are the same.
[R/S]	Error - Func	Displays message indicating accuracy of program has been reached.
[RCL] 5	1.747969	Displays quarterly interest rate.
[×] 4 [=]	6.991877	Displays required annual nominal interest rate.

Part 2. You decide to turn this account into an IRA. If, after your initial deposit of $2,000, you make quarterly payments of $500 for 35 years, how much money will you have in the account if the interest rate is 6.75%?

Keys:	Display:	Description:
[⬅] [FIX] 2		Displays two decimal places.
4 [×] 35 [=] [STO] 4	140.00	Reenters *n*, the total number of quarterly periods.
6.75 [÷] 4 [=] [STO] 5	1.69	Divides into quarterly interest and stores.
500 [+/-] [STO] 8	-500.00	Enters quarterly *PMT*.
[XEQ] 9	299,641.72	Calculates *FV*, the amount in your account after 35 years.

Example: Compound Interest/Discount Tables. These tables, which are found in most accounting and finance text books, can be easily duplicated using the TVM program. First enter the number of periods (in R_4) and the periodic interest rate (as a % in R_5). Then, depending on what you want to find, use one of the following sets of keystrokes:

To Find	Use These Keystrokes
Present Value of $1	1 [+/-] [STO] 9 0 [STO] 8 [XEQ] 7
Compound Amount of $1	1 [+/-] [STO] 7 0 [STO] 8 [XEQ] 9
Present Value of $1 per Period	1 [+/-] [STO] 8 0 [STO] 9 [XEQ] 7
Capital Recovery Factor	1 [+/-] [STO] 7 0 [STO] 9 [XEQ] 8
Compound Amount of $1 per Period	1 [+/-] [STO] 8 0 [STO] 7 [XEQ] 9
Sinking Fund Factor	1 [+/-] [STO] 9 0 [STO] 7 [XEQ] 8

For example, what is the present value of $1 received at the end of 10 years using a 12% discount rate and assuming annual compounding?

Keys:	**Display:**	**Description:**
[←][FIX] 4		Displays four decimal places.
[←][LOAD] F		Loads FINANCE program.
10 [STO] 4	10.0000	Enters number of compounding periods.
12 [STO] 5	12.0000	Enters periodic interest rate.
0 [STO] 8	0.0000	Enters 0 for payment.
1 [+/-] [STO] 9	-1.0000	Enters future value. (Making *FV* negative causes *PV* to be positive, matching the tables.)
[XEQ] 7	0.3220	Calculates present value.

Example: Net Present Value (NPV) of Non-Uniform Cash Flows. The typical finance or accounting text book development of net present value (NPV) asks you to calculate the present value of each cash flow in a series of unequal cash flows and then total them, usually in a column structure on your work paper. The following procedure provides the present value figures needed for the table and also lets you review the accumulated NPV at any time.

These keystrokes use TVM to calculate the present value of each individual cash flow and use memory register arithmetic to accumulate the NPV in register 0. Note that [STO] [-] 0 is used because the sign convention results in negative present values of the cash flows.

What is the NPV of the following annual cash flows if the required rate of return is 18%?

Period	Annual Cash Flow
0	−20,000
1	6,000
2	13,000
3	7,000
4	9,000

Keys:	Display:	Description:
[←] [FIX] 2		Displays two decimal places.
[←] [LOAD] F		Loads FinAnCE.
18 [STO] 5	18.00	Stores periodic interest rate as %.
0 [STO] 8	0.00	Stores 0 as PMT
20000 [+/−] [STO] 0	−20,000.00	Stores initial investment (or cash flow).
1 [STO] 4	1.00	Stores period 1.
6000 [STO] 9	6,000.00	Stores first cash flow as future value (FV).
[XEQ] 7	−5,084.75	Calculates present value (PV) of flow.
[STO] [−] 0	−5,084.75	Accumulates in register.
[RCL] 0	−14,915.25	Recalls NPV (optional).
2 [STO] 4	2.00	Stores period 2.
13000 [STO] 9	13,000.00	Stores cash flow as FV.
[XEQ] 7	−9,336.40	Calculates PV of flow.
[STO] [−] 0	−9,336.40	Accumulates in register.
[RCL] 0	−5,578.86	Recalls NPV (optional).

3 [STO] 4	3.00	Stores period 3.
7000 [STO] 9	7,000.00	Stores cash flow as FV.
[XEQ] 7	−4,260.42	Calculates PV of flow.
[STO] [−] 0	−4,260.42	Accumulates in register.
[RCL] 0	−1,318.44	Recalls accumulated NPV (optional).
4 [STO] 4	4.00	Stores period 4.
9000 [STO] 9	9,000.00	Stores cash flow as FV.
[XEQ] 7	−4,642.10	Calculates PV of flow.
[STO] [−] 0	−4,642.10	Accumulates in register.
[RCL] 0	3,323.66	Recalls accumulated NPV.

7

Programming

A program lets you repeat calculations without repeating keystrokes. To create a program, enter the series of keystrokes while you are in Program mode. Your calculator will then repeat them on command.

The HP-21S enables you to use its programming features in two ways. You can write original programs by having the calculator record and repeat your keystrokes, or you can run any one of six built-in programs.

Any program, regardless of whether you enter it yourself or load it from the built-in program library, can be run and edited. This chapter explains how to do original programming and editing. Chapter 6 gives instructions on using the built-in programs.

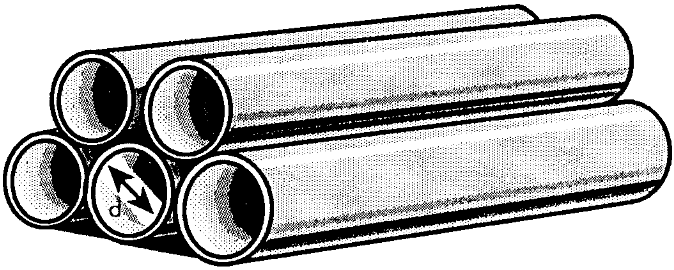
Before the programming concepts and commands are explained in detail, try this quick example. Start by writing out the formula, then solve the problem from the keyboard.

A Simple Programming Example. To find the cross-sectional area of a pipe with a diameter of 5 inches, use the formula

$$A = \frac{\pi d^2}{4}.$$

Before doing the calculation, rearrange the equation in this order: $d^2 \times \pi \div 4 = A$.

Enter 5 in the display and press the following keys: [↰] [x^2] [×] [↰][π] [÷] 4 [=]. The result given is 19.6350 square inches.



But what if you wanted to find the area of many different pipes? Rather than repeating the keystrokes each time (varying only the diameter size), you can put the repeatable keystrokes into the following program:

01 x^2

02 ×

03 π

04 ÷

05 4

06 =

This program assumes that the value for the diameter is in the display when the program starts to run. It calculates and displays the area. To enter this program into program memory, press the following keys. (Don't worry about the numbers that appear in your display—they're called keycodes and are explained later.) If you notice a mistake while typing a line, press [←] to erase the line, then type it over.

Keys:	Display:	Description:
[⬅] [PRGM]		Enters Program mode.
[➡] [CLPRGM]	00-	Clears any previously stored programs.

[⬅] [x^2]	01- 51 11	Enters keystrokes that create program.
[×]	02- 55	
[⬅] [π]	03- 51 22	
[÷]	04- 45	
4	05- 4	
[=]	06- 74	
[⬅] [PRGM]		Exits Program mode.

Now run this program to find the area of a pipe with a diameter of 5 inches.

[C]	0.0000	Clears display.
[⬅] [GTO] [·] [·]	0.0000	Goes to first line of program.
5 [R/S]	19.6350	The answer!

Creating Programs

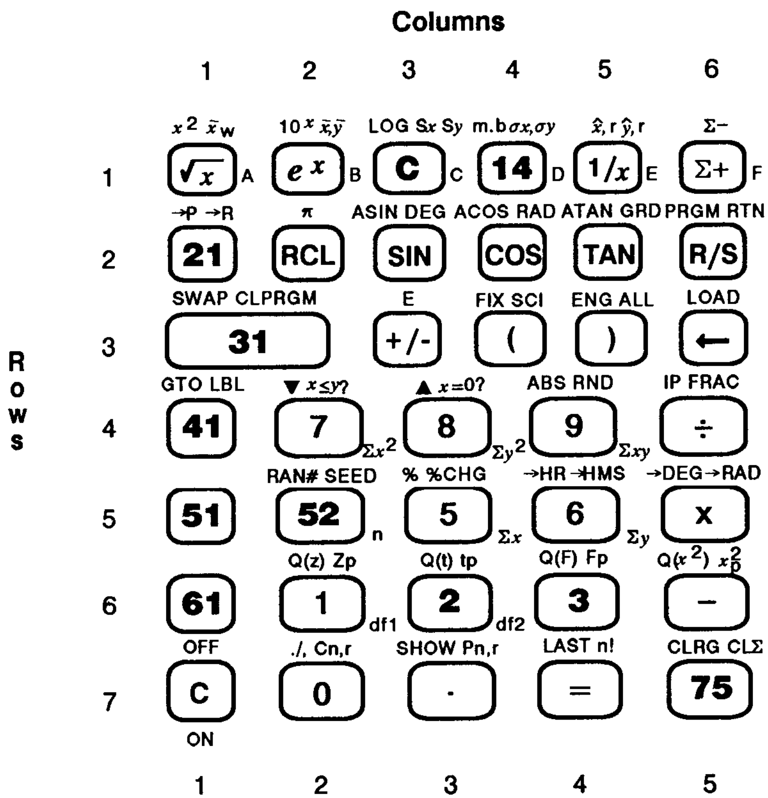
Follow these steps to create programs:

1. Enter Program mode.
2. Enter the repeatable keystrokes.
3. Exit Program mode.
4. Run the program.

We will continue using the pipe area program to illustrate programming concepts. As you were programming, you may have noticed the numbers in the display. They are line numbers and keycodes.

Line Numbers. *Line numbers* appear left-justified in the display as you are entering your program. The numbers, 00 through 99, are followed by a dash. The dash separates the line numbers from keycodes.

Keycodes. The numbers to the right of the line number are called *keycodes*. A keycode indicates which key you pressed. The first digit indicates which row on the keyboard the key is in. The second digit indicates which column the key is in. A line contains one or more keycodes that together represent a single operation. Labels and number keys don't appear as keycodes, but instead as A through F, or 0 through 9.



[INPUT] is 31

[⬑][GTO] C is 51 41 C

[⬑][RAN#] is 51 52

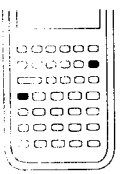
[⮣][$\sigma x,\sigma y$] is 61 14

[STO] [+] 3 is 21 75 3

2 is 2

Checksum. After you have entered a program you can check to see if the keystrokes are entered correctly by comparing the *checksum* listed in this manual to the checksum created by your program. The checksum is a unique hexadecimal value assigned to the specific keystrokes that you entered. To view the checksum, press and hold [↰][SHOW] for a moment while you are in Program mode. The checksums for the examples in this manual are valid if there are no other programs in memory. The checksum for the pipe area program on page 123 is 411A.

Program Boundaries (LBL and RTN)



If you want to store more than one program in your HP-21S, each program needs boundaries — a *label* to mark its beginning and a *return* to mark its end.

Program Labels. Programs and segments of programs (called *routines*) start with a label that acts as a name. Use a label to separate programs any time you have more than one program in memory. The keystrokes to create a label are [↱][LBL] followed by one of the keys, A through F, or 0 through 9. A label is used to execute a specific program or routine.

When you press [XEQ] *label* (after exiting Program mode), the program pointer moves to the specified label and begins execution. (The *program pointer* is an internal pointer that marks the line that is displayed while in Program mode.) All of memory is searched for the specified label, starting at the program pointer. If no label is found, the message Error - LbL is displayed.

Return. Programs end with a return ([↱][RTN]) instruction. When a program finishes running, the RTN instruction returns the program pointer to line 00. If the last line of the program is not a RTN instruction, the program pointer automatically returns to line 00. The keystrokes are [↱][RTN]. Using [↱][RTN] in subroutines is discussed on page 134.

Entering Programs

Pressing [⬑] [PRGM] toggles the calculator into and out of Program mode (**PRGM** annunciator on). While the HP-21S is in Program mode, keystrokes that you enter are stored as program lines. The calculator has enough memory for 99 program lines. Each function and each digit of a number occupy one program line.

To enter a program into memory,

1. Press [⬑] [PRGM] to enter Program mode. The **PRGM** annunciator appears in the display.
2. Press [⬑] [GTO] [·] [·] to display line 00. This sets the program pointer to line 00 without affecting other programs.

   If you don't need any other programs that might be in memory, clear program memory by pressing [⬏] [CLPRGM]. This sets the program pointer to line 00 since there are no other lines to display.
3. To start entering the program, press [⬏] [LBL] followed by the label you wish to assign: A through F, or 0 through 9.
4. To enter program instructions, press the same keys you would use to do an operation manually.
5. End the program with a return instruction by pressing [⬏] [RTN].
6. Press [⬑] [PRGM] to exit Program mode.

Data Input. There are many ways to supply a program with data. Here are two ways to supply data to a program that expects one data item:

- Enter the number in the display before you run the program.
- Store the number into a register before you run the program, then recall it from within the program.

Here are two ways to supply data to a program that expects two data items:

- Enter data in the display before you run the program by using $number_1$ [INPUT] $number_2$. The program can store $number_2$ then do a [⬑] [SWAP] to access $number_1$.
- Store both items in registers before you run the program, then recall them from within the program.

Example. In this example you will clear the pipe area program, then enter a new version of the program that includes a label and a return instruction. (Refer to page 131 if you don't want to clear all of program memory.) If you make a mistake during entry, press [◄] to delete the current program line, then reenter it correctly.

Keys:	**Display:**	**Description:**
[↰] [PRGM]		Enters Program mode (**PRGM** annunciator on).
[↱] [CLPRGM]	00-	Clears program memory.
[↱] [LBL] A	01- 61 41 A	Labels this program routine "A".
[↰] [x^2]	02- 51 11	Enters program lines.
[×]	03- 55	
[↰] [π]	04- 51 22	
[÷]	05- 45	
4	06- 4	
[=]	07- 74	
[↱] [RTN]	08- 61 26	Ends program.
[↰] [SHOW]	6F53	Checksum (page 126).
[↰] [PRGM]		Exits Program mode (**PRGM** annunciator off).

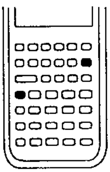
Positioning the Program Pointer

Program memory starts at line 00. The list of program lines is circular, so you can wrap the program pointer from the bottom to the top. There are several ways to move the program pointer to view different lines whether you are in Program mode or not:

- [↰] [GTO] [·] [·] to move to line 00.
- [↰] [GTO] [·] *line-number* to move to a specified line.
- [↰] [▲] or [↰] [▼] to move one line at a time.
- Hold [↰] and press [▲] or [▼] to move one line at a time.

- When in Program mode, hold [⬑][▲] or [⬑][▼] to move up or down rapidly.
- When not in Program mode, [⬑][GTO] *label* to move to a specified label.

Running Programs



While a program is running, the **PRGM** annunciator blinks on and off, and the message running appears in the display.

There are two ways to run a program:

- Use [XEQ] followed by a label.
- Use [GTO], followed by a label, and [R/S] (Run/Stop).

Starting Programs With XEQ

To run a program using [XEQ],

- Enter data required by the program, if necessary.
- Press [XEQ] *label*.
- If you hold down *label* after pressing [XEQ], the line where execution will begin is displayed. The program starts to run when *label* is released.

Example. Run the program labeled A to find the areas of three different pipes with diameters of 5, 2.5, and 2π. Remember to enter the diameter before executing label A.

Keys:	Display:	Description:
5 [XEQ] A	19.6350	Enters diameter, then starts program A. The resulting area is displayed.
2.5 [XEQ] A	4.9087	Area of second pipe.
2 [×] [←] [π]	3.1416	Displays π value.
[=]	6.2832	Diameter of third pipe.
[XEQ] A	31.0063	Area of third pipe.

Starting Programs With GTO and R/S

To execute a program using [GTO] and [R/S],

- Use [GTO] to position the program pointer where you want to begin (page 128).
- Enter data required by the program, if necessary.
- Press [R/S]. If you hold down [R/S], the line where execution will begin is displayed. The program starts to run when [R/S] is released.

Stopping Programs

When a program is running, you can stop it by pressing [R/S] or [C].

Programming a Stop. Pressing [R/S] while in Program mode inserts a STOP instruction. During execution this halts a program until you press [R/S] again. You can use [R/S] to stop a program to enter data. You can use [R/S] rather than RTN to end a program. When the program halts, the program pointer will not return to the top of the program.

Error Stops. If an error occurs while a program is running, program execution halts at the point the error occurred, and an error message is displayed. (There is a list of error messages and conditions on page 153.) Press [C] or [←] to clear the display. To see the program line containing the error-causing instruction, press [←][PRGM].

Clearing Programs

You must be in Program mode (**PRGM** annunciator on) to clear programs. Press [↱][CLPRGM] to clear all programs from memory.

To clear a specific program, you must delete each line individually. Position the pointer at the last line of the program that you want to delete and press [←] repeatedly. Refer to page 128 for more information about how to position the pointer.

Editing Programs

You can modify a program by inserting and deleting program lines. Even if a program line requires only a minor change, you must delete the old line and insert a new one.

To delete program lines,

1. Enter Program mode.
2. Position the pointer where you want to begin. (If you are deleting more than one consecutive program line, start with the *last* line in the group.)
3. Delete the line you want to change by pressing [←]. Succeeding lines are automatically renumbered.
4. To exit Program mode, press [↰][PRGM].

For example, if you want to delete lines 05 through 08, you first display line 08, then press [←] four times. Subsequent program lines are moved up and automatically renumbered.

To insert program lines,

1. Enter Program mode.
2. Position the pointer to the line preceding the new lines.
3. Enter the new lines. They are inserted after the displayed line. Succeeding lines are automatically renumbered.
4. To exit Program mode, press [↰][PRGM].

For example, if you want to insert several new lines between lines 04 and 05 of a program, you first display line 04, then enter the instructions. Subsequent program lines, starting with the original line 05, are moved down and renumbered accordingly.

Stepping Through Programs

You can test programs by stepping through them. The program executes one line at a time as you step through it. The result is displayed after each program line is executed, so you can verify the progress of calculations. To execute a program one line at a time,

1. Exit Program mode.
2. Position the pointer where you want to begin.
3. Enter data in the display, if necessary.
4. Press [↰], then press and hold [▼]. This displays the current program line. When you release [▼], the line is executed. The result of that execution is then displayed, and the program pointer moves to the next line.
5. Repeat step 4 until you find an error or reach the end of the program.

To move to the *preceding* line, you can press [↰][▲]. No execution occurs.

Example. Step through the execution of the program labeled A (the pipe program, page 123). Use a diameter of 5 for the test data. Check that the **PRGM** annunciator is off before you start.

Keys:	Display:	Description:
[↰][GTO] A		Moves program pointer to label A.
5	5_	Enters 5 in the display.
[↰][▼] *(hold)*		Displays label A.
(release)	01- 61 41 A 5.0000	

[⬑][▼] *(hold)*		Squares input.
(release)	02- 51 11 25.0000	
[⬑][▼] *(hold)*		Multiplies 25 by . . .
(release)	03- 55 25.0000	
[⬑][▼] *(hold)*		. . . π.
(release)	04- 51 22 3.1416	
[⬑][▼] *(hold)*		Calculates intermediate
(release)	05- 45 78.5398	result.
[⬑][▼] *(hold)*		. . . ÷ 4.
(release)	06- 4 4_	
[⬑][▼] *(hold)*		. . . =.
(release)	07- 74 19.6350	
[⬑][▼] *(hold)*		End of program. Result
(release)	08- 61 26 19.6350	is correct.

Sample Program: Pythagorean Theorem

You can use most of the HP-21S features in programs just like you use them manually. To illustrate how [STO] and [RCL] are used to recall data from registers in a program, enter the following Pythagorean theorem program. It calculates the length of the hypotenuse (side c) of a right triangle, given the lengths of sides a and b. The formula used is $c = \sqrt{a^2 + b^2}$. Assume that the calculation begins with side a in R_1 and side b in R_2.

Keys:	Display:	Description:
[⬑] [PRGM]		Enters Program mode.
[⮫] [CLPRGM]	00-	Clears program memory. (Skip this step to leave programs intact.)
[⮫] [LBL] E	01- 61 41 E	Labels program "E".
[RCL] 1	02- 22 1	Recalls *a* from R_1.
[⬑] [x^2]	03- 51 11	a^2.
[+]	04- 75	
[RCL] 2	05- 22 2	Recalls *b* from R_2.
[⬑] [x^2]	06- 51 11	b^2.
[=]	07- 74	$a^2 + b^2$.
[$\sqrt{x}$]	08- 11	$\sqrt{a^2 + b^2}$.
[⮫] [RTN]	09- 61 26	
[⬑] [SHOW]	99F3	Checksum (page 126).
[⬑] [PRGM]		Exits Program mode.

Now store the *a* and *b* values of 22 and 9 into R_1 and R_2, then run the program:

22 [STO] 1	22.0000	Stores *a* in R_1.
9 [STO] 2	9.0000	Stores *b* in R_2.
[XEQ] E	23.7697	Length of hypotenuse.

Subroutines

A program is composed of one or more *routines*. A routine is a functional unit that accomplishes a specific task. As programs get more complicated, it helps to break them into smaller pieces. This makes a program easier to write, read, understand, and alter.

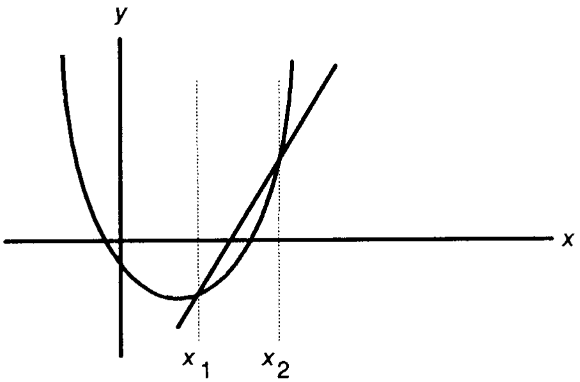
A routine typically starts with a label (LBL) and ends with an instruction that alters or stops program execution, such as RTN or GTO. A subroutine is a routine that is called from (executed by) another routine and returns control to that same routine when it finishes. The subroutine must start with a LBL and end with a RTN. A subroutine can call other subroutines.

If a subroutine is at the end of program memory and does not end with [↱] [RTN], control is still transferred to the step after the originating [XEQ] when the routine completes. It is as if the subroutine had ended with [↱] [RTN].

Calling Subroutines (XEQ). In a program, use [XEQ] *label* to call a specific subroutine. The subroutine must start with a label A through F, or 0 through 9. Searching begins at the [XEQ] and proceeds down the program, wrapping around through line 00 until the label is found. Within a program, [XEQ] *label* transfers execution of a running program to the program line containing that label, wherever it may be. The program continues running from the new location. Then, at the next [↱] [RTN] statement, execution returns to the line after the originating [XEQ] and continues.

For example, to write a program that calculates the average slope between x_1 and x_2 on the graph, where $y = x^2 - \sin x$ (where x is in radians), you would use the formula:

$$slope = \frac{(x_2^2 - \sin x_2) - (x_1^2 - \sin x_1)}{x_2 - x_1}$$



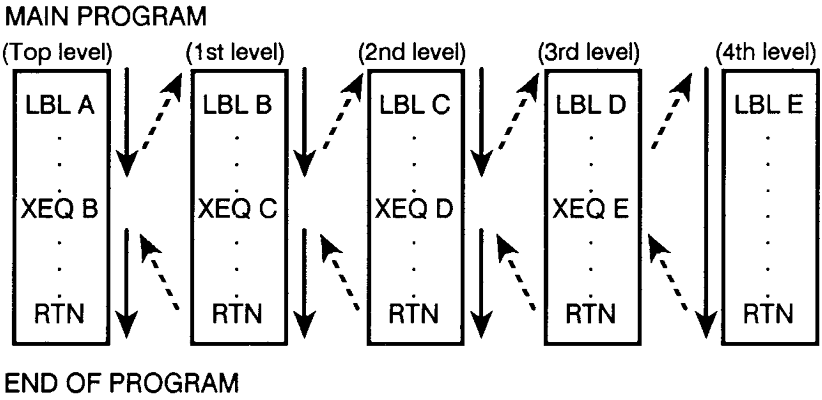
The solution requires two calculations of the expression $x^2 - \sin x$ (for $x = x_1$ and for $x = x_2$). Since the solution includes an expression that must be repeated for both values of x, you can create a subroutine to execute the repeated keystrokes and save space in program memory. The program assumes that x_1 [INPUT] x_2 has been entered before executing the program and that the calculator is in Radians mode.

Keys:	Display:	Description:
[↱] [RAD]		Puts calculator in Radians mode.
[↰] [PRGM]		Enters Program mode.
[↱] [CLPRGM]	00-	Clears program memory.
[↱] [LBL] C	01- 61 41 C	Names program "C".
[STO] 2	02- 21 2	Stores displayed value (x_2) in R_2.
[↰] [SWAP]	03- 51 31	Swaps (x_2 for x_1).
[STO] 1	04- 21 1	Stores displayed value (x_1) in R_1.
[C]	05- 71	Clears display so no hidden value or : annunciator remains when program is complete.
[RCL] 2	06- 22 2	Recalls x_2.
[XEQ] 5	07- 41 5	Executes subroutine to calculate $x_2^2 - \sin x_2$.
[−]	08- 65	$(x_2^2 - \sin x_2) - \ldots$
[RCL] 1	09- 22 1	Recalls x_1.
[XEQ] 5	10- 41 5	Executes subroutine again to calculate $x_1^2 - \sin x_1$.
[=]	11- 74	$(x_2^2 - \sin x_2) - (x_1^2 - \sin x_1)$.

[÷]	12- 45	Divides result by...
[(]	13- 33	Reorders precedence.
[RCL] 2	14- 22 2	Recalls x_2.
[−]	15- 65	$x_2 - \ldots$
[RCL] 1	16- 22 1	Recalls x_1. Closing parenthesis not required because = follows.
[=]	17- 74	$((x_2^2 - \sin x_2) - (x_1^2 - \sin x_1))/(x_2 - x_1)$.
[R/S]	18- 26	Stops.
[↱] [LBL] 5	19- 61 41 5	Label 5 starts subroutine.
[STO] 0	20- 21 0	Stores displayed value in R_0.
[(]	21- 33	Reorders operator priority.
[↰] [x^2]	22- 51 11	Squares displayed value.
[−]	23- 65	Subtracts.
[RCL] 0	24- 22 0	Recalls contents of R_0.
[SIN]	25- 23	Calculates sine.
[)]	26- 34	Closing parenthesis required to evaluate $x^2 - \sin x$.
[↱] [RTN]	27- 61 26	Ends subroutine and returns to line following originating [XEQ].
[↰] [SHOW]	Fb9b	Checksum (page 126).
[↰] [PRGM]		Exits Program mode.

To execute the program using 3 and 4 as x_1 and x_2, press 3 [INPUT] 4 [XEQ] C. The result is 7.8979. To exit Radians mode, press [↱] [DEG].

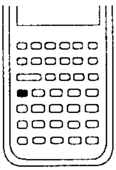
Nested Subroutines. A subroutine can call another subroutine, and that subroutine can call yet another subroutine. This "nesting" of subroutines — the calling of a subroutine from within another subroutine — is limited to four levels of subroutines. The operation of nested subroutines is shown on the following page:



If you attempt to execute a subroutine nested more than four levels deep, the message Error - Sub appears in the display.

Branching and Conditionals

Branching (GTO)

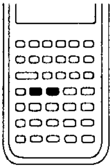


As we have seen with subroutines, it is often desirable to transfer execution to a part of the program other than the next line. This is called *branching.*

Unconditional branching uses the GTO (*go to*) instruction to branch to a program *label* ([←] [GTO] *label*).

The [←] [GTO] *label* instruction transfers the execution of a running program to the program line containing that label, wherever it may be. Searching starts at [←] [GTO] and continues throughout all of program memory. The program continues running from the new location. It does not automatically return to its point of origin when a [→] [RTN] is encountered. Consequently, [←] [GTO] is not used for subroutines.

Conditional Instructions — Decisions and Control



In addition to subroutines, another way to control program execution is with a *conditional test* — a true/false test that compares two numbers and skips the next program instruction if the comparison is false.

The HP-21S has two conditional statements: they are [↱] [x≤y?] and [↱][x=0?]. The $x \le y$? asks the question, "Is x less than or equal to y?" And $x=0$? asks the question, "Is x equal to 0?" If the answer is true, the program continues execution with the line immediately following the question. If the answer is false, the program skips one line and continues from there.

For instance, if a conditional instruction is $x=0$?, then the program compares the contents of the display to zero. If there is a zero in the display, then the program goes on to the next line. If there is *not* a zero in the display, then the program *skips* one line and continues from there. This rule is commonly known as "Do if true."

For $x \le y$?, the program compares y (the displayed value) with x (the hidden value). Use [INPUT] or any other operator (for example, [+] or [÷]) to separate x and y. If x is less than or equal to y, then the program goes on to the next line. If x is *not* less than or equal to y (that is, x is greater than y), then the program *skips* one line and continues from there.

The following example illustrates conditional branching and a GTO statement.

Example: Random Number Generator. This program uses the [↱][x≤y?] key. The program accumulates the number of times a random number occurs less than or equal to 0.5 (register 1) or more than 0.5 (register 2).

Keys:	Display:	Description:
[←] [PRGM]	00-	Enters Program mode.
[→] [CLPRGM]		Clears program memory.
[→] [LBL] A	01-61 41 A	Labels this part of program "A".
[←] [RAN#]	02- 51 52	Generates random number.
[INPUT]	03- 31	Inputs random number for test.
[.]	04- 73	Enters decimal portion of number.
5	05- 5	Enters number.
[→] [$x \leq y$?]	06- 61 42	Tests if number is ≤ 0.5.
[←] [GTO] B	07- 51 41 b	If yes, branch to label.
1	08- 1	If no, put 1 in display.
[STO] [+] 1	09- 21 75 1	Adds 1 to memory register.
[←] [GTO] A	10- 51 41 A	Goes to label A.
[→] [LBL] B	11-61 41 b	Labels this part of program "B".
1	12- 1	Puts 1 in display.
[STO] [+] 2	13- 21 75 2	Adds 1 to register 2.
[←] [GTO] A	14- 51 41 A	Repeats the program.
[←] [SHOW]	E144	Checksum (page 126).
[←] [PRGM]		Exits Program mode.

To run the program,

[←] [CLRG]		Clears contents of registers.
[XEQ] A	running	Program runs until stopped.

[R/S]	Stops program.
[RCL] 1	Displays number of times random numbers occurred ≤0.5.
[RCL] 2	Number of occurrences >0.5.

Sample Program: Standard Deviation of Grouped Data

The weighted mean is the mean of data points $x_1, x_2, \ldots x_n$ occurring with weights $w_1, w_2, \ldots w_n$. The weights (frequencies) are non-negative and can be non-integer or integer values. The standard deviation of grouped data is the standard deviation of data points $x_1, x_2, \ldots\ x_n$ occurring at nonnegative integer frequencies $f_1, f_2, \ldots f_n$.

To calculate the mean and standard deviation of grouped data, use the following keystrokes. (If you wish to calculate *only* the weighted mean, use "Weighted Mean" on page 61.)

Keys:	Display:	Description:
[◀] [PRGM]		Enters Program mode.
[▶] [CLPRGM]	00-	Clears program memory.
[▶] [LBL] A	01- 61 41 A	Names program A.
[STO] [+] 4	02- 21 75 4	Adds frequency f_i to sample size.
[◀] [LAST]	03- 51 74	Stores f_i in LAST.
[◀] [SWAP]	04- 51 31	Recalls x_i.
[◀] [LAST]	05- 51 74	Recalls f_i and stores x_i in LAST.
[×]	06- 55	
[◀] [LAST]	07- 51 74	Recalls x_i.

[=]	08- 74	Calculates $x_i f_i$.
[STO] [+] 5	09- 21 75 5	Adds $x_i f_i$ to register 5.
[×]	10- 55	
[←] [LAST]	11- 51 74	Recalls x_i.
[=]	12- 74	Calculates f_i^2.
[STO] [+] 7	13- 21 75 7	Adds $x_i^2 f_i$ to register 7.
[C]	14- 71	Clears display and : annunciator.
[RCL] 4	15- 22 4	Recalls sample size.
[→] [RTN]	16- 61 26	Returns to line 00.
[←] [SHOW]	F1bF	Checksum (page 126).
[←] [PRGM]		Exits Program mode.

Enter the following data and calculate the mean and standard deviation.

Group	1	2	3	4	5	6
x_i	5	8	13	15	22	37
f_i	17	26	37	43	73	115

Keys:	**Display:**	**Description:**
[→] [CLΣ]	0.0000	Clears statistics registers.
5 [INPUT] 17 [XEQ] A	17.0000	Enters x_1 and f_1.
8 [INPUT] 26 [R/S]	43.0000	Enters x_2 and f_2.
13 [INPUT] 37 [R/S]	80.0000	Enters x_3 and f_3.
15 [INPUT] 43 [R/S]	123.0000	Enters x_4 and f_4.
22 [INPUT] 73 [R/S]	196.0000	Enters x_5 and f_5.
37 [INPUT] 115 [R/S]	311.0000	Enters x_6 and f_6.

[→] [$\bar{x},\bar{y}$]	23.4084	Calculates weighted mean.
[→] [Sx,Sy]	11.4118	Calculates sample standard deviation of grouped data.

Available Program Memory

You can store up to 99 lines of code in program memory. If you attempt to add program lines (anywhere in program memory) after 99 lines have been entered, the message Error - FuLL is displayed.

Nonprogrammable Functions

The following HP-21S functions are not programmable:

[←]	[→] [CLPRGM]
[←] [▼]	[←] [SHOW]
[←] [▲]	[←] [LOAD]
[←] [GTO] [.] *line-number*	[←] [PRGM]
[←] [GTO] [.] [.]	[→] [OFF]

A

Support, Batteries, and Service

Calculator Support

You can obtain answers to questions about using your calculator from our Calculator Support department. Our experience has shown that many customers have similar questions about our products, so we have provided the following section, "Answers to Common Questions." If you don't find the answer to your question, contact us using the address or phone number on the inside back cover.

Answers to Common Questions

Q: *I'm not sure if the calculator is malfunctioning or if I'm doing something incorrectly. How can I determine if the calculator is operating properly?*
A: The diagnostic self-test is described on page 147.

Q: *My numbers contain commas as decimal points. How do I restore periods?*
A: Press [↰][·/,] (page 19).

Q: *How do I change the number of decimal places that the HP-21S displays?*
A: Press [↰][FIX] and the number of decimal places you want (page 17).

Q: *What does an "E" in a number (for example, 2.51E − 13) mean?*
A: Exponent of ten (for example, 2.51×10^{-13}). Refer to "Scientific and Engineering Notation" (page 18).

Q: *What does* **PEND** *in the display mean?*
A: An arithmetic operation is pending (in progress).

Q: *What does* **:** *in the display mean?*
A: Two values have been returned or [INPUT] has been pressed (page 14).

Q: *Why does* 00- *appear in the display?*
A: The calculator is in Program mode. Press [↰][PRGM] to exit.

Q: *When I take the sine of* π *in radians mode, why do I get a small number instead of zero?*
A: The calculator is *not* malfunctioning. π cannot be expressed *exactly* with the 12-digit precision of the calculator.

Q: *Why is the answer returned by my calculator different than the answer derived from the statistics table?*
A: The calculator functions expand the tables by allowing you to determine *any* upper tail probability, not just the selected values in statistical tables.

Environmental Limits

To maintain product reliability, you should avoid getting the calculator wet and observe the following temperature and humidity limits:

- Operating temperature: 0° to 45°C (32° to 113°F).
- Storage temperature: −20° to 65°C (−4° to 149°F).
- Operating and storage humidity: 90% relative humidity at 40°C (104°F) maximum.

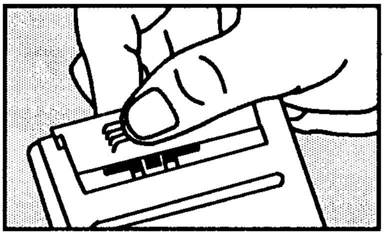
Changing the Batteries

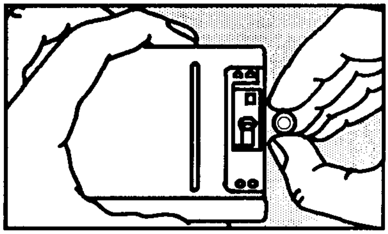
Replace the batteries as soon as possible when the low battery annunciator (⊏⊐) appears. If the battery annunciator is on and the display dims, you may lose data. If data is lost, the ALL CLr message is displayed.

Once you've removed the batteries, you should replace them within one minute to avoid losing stored information. Use any brand of I.E.C. LR44 (or manufacturer's equivalent) button-cell batteries. Do not use rechargeable batteries.

1. Have three fresh button-cell batteries at hand. (Only touch batteries by their edges.)

2. Turn the calculator off. (You will lose memory if the batteries are removed when the calculator is on.)
3. Remove the battery-compartment door by pressing down and sliding it off away from the calculator (left illustration).





4. Turn the calculator over and shake the batteries out.

Warning **Do not mutilate, puncture, or dispose of batteries in fire. The batteries can burst or explode, releasing hazardous chemicals.**

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5. Insert the new batteries (right illustration). Stack them according to the diagram inside the battery compartment.
6. Replace the battery-compartment door (slide the tab of the door back into the slot on the calculator case).

Testing Calculator Operation

Use the following guidelines to determine whether the calculator is functioning properly. Test the calculator after every step to see if operation has been restored. If your calculator requires service, refer to page 149.

- **The calculator won't turn on (steps 1-4) or doesn't respond when you press the keys (steps 1-3):**
  1. Reset the calculator. Hold down the [C] key and press [LN] at the same time. It may be necessary to repeat the reset

keystrokes several times.

2. Erase memory. Press and hold down [C], then press and hold down both [√x] and [Σ+]. Memory is cleared and the ALL CLr message is displayed when you release all three keys.
3. Remove the batteries (page 145) and lightly press a coin against both battery contacts in the calculator. Replace the batteries and turn on the calculator. It should display ALL CLr.
4. Install new batteries (page 145).

If these steps fail to restore calculator operation, it requires service.

- **The calculator responds to keystrokes, but you suspect it is malfunctioning:**
  1. Complete the self-test described next. If the calculator fails the self test, it requires service.
  2. If the calculator passes the self-test, you may have made a mistake operating the calculator. Reread portions of the manual, and check "Answers to Common Questions," page 144.
  3. Contact the Calculator Support department. The address and phone number are listed on the inside back cover.

The Self-Test

If the display can be turned on, but the calculator does not seem to be operating properly, you can run a diagnostic self-test:

1. Hold down the [C] key, then press [y^x] at the same time.
2. Press any key four times and watch the various patterns displayed. After displaying the copyright message COPr. HP 1987 momentarily and the message 01, the calculator is ready for the key test.
3. Starting at the upper left corner [√x] and moving left to right, press every key on the keyboard.
   - If you press the keys in the proper order, and they are functioning properly, the calculator displays two-digit numbers. (The calculator is counting the keys using hexadecimal base.)
   - If you press a key out of order, or if a key isn't functioning properly, the next keystroke displays 21 - FAIL, followed by a one-digit number. If the message appeared because you pressed

a key out of order, reset the calculator (hold down [C] and press [LN]) and start the self-test over. If you pressed the keys in order but got this message, the calculator requires service.

4. The calculator displays a message when the test has been completed:
   - 21 - Good if it passed the self-test.
   - 21 - FAIL followed by a hexadecimal number 1 through F if it failed the self-test and requires service (page 149). Include a copy of the message when you ship the calculator for service.
5. If the calculator failed, rerun the self-test to verify the results.
6. To exit the self-test, reset the calculator (hold down [C], press [LN]).

Pressing [C] and [1/x] starts another self-test that is used at the factory. This test will repeat various patterns until you press [C].

Limited One-Year Warranty

What Is Covered. *The calculator* (except for the batteries, or damage caused by the batteries) *is warranted by Hewlett-Packard against defects in materials and workmanship for one year from the date of original purchase.* If you sell your unit or give it as a gift, the warranty is automatically transferred to the new owner and remains in effect for the original one-year period. During the warranty period, we will repair or, at our option, replace at no charge a product that proves to be defective, provided you return the product, shipping prepaid, to a Hewlett-Packard service center. (Replacement may be with a newer model of equal or better functionality.)

This warranty gives you specific legal rights, and you may also have other rights that vary from state to state, province to province, or country to country.

What Is Not Covered. *Batteries, and damage caused by the batteries, are not covered by the Hewlett-Packard warranty. Check with the battery manufacturer about battery and battery leakage warranties.*

This warranty does not apply if the product has been damaged by accident or misuse or as the result of service or modification by other than an authorized Hewlett-Packard service center.

No other express warranty is given. The repair or replacement of a product is your exclusive remedy. **ANY OTHER IMPLIED WARRANTY OF MERCHANTABILITY OR FITNESS IS LIMITED TO THE ONE-YEAR DURATION OF THIS WRITTEN WARRANTY.** Some states, provinces, or countries do not allow limitations on how long an implied warranty lasts, so the above limitation may not apply to you. **IN NO EVENT SHALL HEWLETT-PACKARD COMPANY BE LIABLE FOR CONSEQUENTIAL DAMAGES.** Some states, provinces, or countries do not allow the exclusion or limitation of incidental or consequential damages, so the above limitation or exclusion may not apply to you.

Products are sold on the basis of specifications applicable at the time of manufacture. Hewlett-Packard shall have no obligation to modify or update products, once sold.

Consumer Transactions in the United Kingdom. This warranty shall not apply to consumer transactions and shall not affect the statutory rights of a consumer. In relation to such transactions, the rights and obligations of Seller and Buyer shall be determined by statute.

If the Calculator Requires Service

Hewlett-Packard maintains service centers in many countries. These centers will repair a calculator, or replace it with the same model or one of equal or better functionality, whether it is under warranty or not. There is a service charge for service after the warranty period. Calculators normally are serviced and reshipped within five working days.

- **In the United States:** Send the calculator to the Corvallis Service Center listed on the inside of the back cover.
- **In Europe:** Contact your Hewlett-Packard sales office or dealer, or Hewlett-Packard's European headquarters (address below) for the location of the nearest service center. *Do not ship the calculator for service without first contacting a Hewlett-Packard office.*

Hewlett-Packard S.A.
150, Route du Nant-d'Avril
P.O. Box CH 1217 Meyrin 2
Geneva, Switzerland
Telephone: (022) 83 81 11

- **In other countries:** Contact your Hewlett-Packard sales office or dealer or write to the Corvallis Service Center (listed on the inside of the back cover) for the location of other service centers. If local service is unavailable, you can ship the calculator to the Corvallis Service Center for repair.

  All shipping, reimportation arrangements, and customs costs are your responsibility.

Service Charge. Contact the Corvallis Service Center (inside back cover) for the standard out-of-warranty repair charges. This charge is subject to the customer's local sales or value-added tax wherever applicable.

Calculator products damaged by accident or misuse are not covered by the fixed charges. These charges are individually determined based on time and material.

Shipping Instructions. If your calculator requires service, ship it to the nearest authorized service center or collection point.

- Include your return address and a description of the problem.
- Include proof of purchase date if the warranty has not expired.
- Include a purchase order, check, or credit card number plus expiration date (VISA or MasterCard) to cover the standard repair charge.
- Ship your calculator postage *prepaid* in adequate protective packaging to prevent damage. Such damage is not covered by the warranty, so we recommend that you insure the shipment.

Warranty on Service. Service is warranted against defects in materials and workmanship for 90 days from the date of service.

Service Agreements. In the U.S., a support agreement is available for repair and service. Refer to the form in the front of the manual. For additional information, contact the Corvallis Service Center (see the inside of the back cover).

Regulatory Information

U.S.A. The HP-21S generates and uses radio frequency energy and may interfere with radio and television reception. The calculator complies with the limits for a Class B computing device as specified in Subpart J of Part 15 of FCC Rules, which provide reasonable protection against such interference in a residential installation. In the unlikely event that there is interference to radio or television reception (which can be determined by turning the HP-21S off and on or by removing the batteries), try the following:

- Reorienting the receiving antenna.
- Relocating the calculator with respect to the receiver.

For more information, consult your dealer, an experienced radio/television technician, or the following booklet, prepared by the Federal Communications Commission: *How to Identify and Resolve Radio-TV Interference Problems*. This booklet is available from the U.S. Government Printing Office, Washington, D.C. 20402, Stock Number 004-000-00345-4. At the first printing of this manual, the telephone number was (202) 783-3238.

West Germany. The HP-21S complies with VFG 1046/84, VDE 0871B, and similar noninterference standards. If you use equipment that is not authorized by Hewlett-Packard, that system configuration has to comply with the requirements of Paragraph 2 of the German Federal Gazette, Order (VFG) 1046/84, dated December 14, 1984.

Messages

Press [C] or [←] to clear a message from the display.

1-StAt (One Sample Statistics)
Built-in program (page 65).

2-StAt (Two Sample Test Statistics)
Built-in program (page 80).

21 - FAIL n (HP-21S Fail)
The self-test failed; *n* is the fail code (page 147).

21 - Good (HP-21S Good)
The self-test is complete (page 147).

ALL CLr (All Clear)
Continuous memory has been erased (page 13).

CHI-2 (Chi-Square Test Statistic)
Built-in program (page 105).

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Copyright is displayed during self-test.

Error - Func (Error - Function)

- Attempt to divide by zero.
- Attempt to calculate combinations or permutations with $n<r$, n or r not positive integer or $\geq 10^{12}$.
- Attempt to use a trigonometric function with an illegal argument.
- Attempt to calculate the logarithm of zero or a negative number.
- Attempt to calculate 0^0 or 0 raised to a negative power.

- Attempt to raise a negative number to a noninteger power.
- Attempt to calculate the square root of a negative number.

Error - FuLL (Error - Full)
Attempt to calculate an expression with more than five pending operations (page 23), or attempt to enter more than 99 program lines.

Error - LbL (Error - Label)
Attempt to [XEQ] or [GTO] a label that is not in the program.

Error - StAt (Error - Statistics)

- Attempt to calculate $\bar{x}_w, \hat{x}, \hat{y}$, or r with x-data only (all y-values = 0).
- Attempt to calculate $\hat{x}, \hat{y}$, r, m, or b with all x-values equal.
- Attempt to calculate with n equal to zero.
- Attempt to calculate $S_x, S_y, \hat{x}, \hat{y}$, r, m, or b, with n ≤ 1, or when a division by zero or square root of a negative number occurred. Also, attempt to calculate $\bar{x}, \bar{y}$ with n = 0, or $\hat{x}_w$ with $\Sigma y = 0$.

Error - Sub (Error - Subroutine)
Subroutines nested more than four levels deep (page 137).

LR-StAt (Linear Regression Test Statistics)
Built-in program (page 95).

OFLO (Overflow)
The magnitude of a result is too large for the calculator to handle. OFLO is displayed for a moment (or remains in the display when OFLO occurs in a running program), then the HP-21S returns ±9.99999999999E499 in the current display format. If OFLO is caused by storage register arithmetic, the display value remains unchanged. If OFLO is caused by $\Sigma+$, n appears in the display. (See "Range of Numbers" on page 20).

running (Running)
A program or a long calculation is running.

bin (Binomial Probability Distribution)
Built-in program (page 111).

FinAnCE (Time Value of Money)
Built-in program (page 113).

Index

Special Characters

A

P

Q

R

S

T

U

W

X

Y

Z

Comments on the HP-21S Owner's Manual

We welcome your evaluation of this manual. Your comments and suggestions help us improve our publications.

HP-21S Owner's Manual

Please circle a response for each of the statements below. You can use the **Comments** space to provide additional opinions.

1 = Strongly Agree
2 = Agree
3 = Neutral
4 = Disagree
5 = Strongly Disagree

■ I am satisfied with the product documentation.	1 2 3 4 5
■ The manual is well organized.	1 2 3 4 5
■ I can find the information I want.	1 2 3 4 5
■ The information in the manual is accurate.	1 2 3 4 5
■ I can easily understand the instructions.	1 2 3 4 5
■ The manual contains enough examples.	1 2 3 4 5
■ The examples are appropriate and helpful.	1 2 3 4 5
■ The illustrations are clear and helpful.	1 2 3 4 5

■ The amount of information is: too much appropriate too little

■ The parts I refer to most frequently are:

Welcome 1 2 3 4 5 6 7 A Messages

Comments: ______________________________

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Name: ______________________________
Address: ______________________________
City/State/Zip: ______________________________
Occupation: ______________________________
Phone: (______) ______________________________

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Contacting Hewlett-Packard

For Information About Using the Calculator. If you have questions about how to use the calculator, first check the table of contents, the index, and "Answers to Common Questions" in the Appendix. If you can't find an answer in the manual, you can contact the Calculator Support department:

Hewlett-Packard
Calculator Support
1000 N.E. Circle Blvd.
Corvallis, OR 97330, U.S.A.

(503) 757-2004
8:00 a.m. to 3:00 p.m. Pacific time
Monday through Friday

For Service. If your calculator doesn't seem to work properly, refer to appendix A for diagnostic instructions and information on obtaining service. If you are in the United States and your calculator requires service, mail it to the Corvallis Service Center:

Hewlett-Packard
Corvallis Service Center
1030 N.E. Circle Blvd.
Corvallis, OR 97330, U.S.A.
(503) 757-2002

If you are outside the United States, refer to appendix A for information on locating the nearest service center.

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