

HEWLETT-PACKARD

HP-27

Owner's Handbook





HP-27

Owner's Handbook

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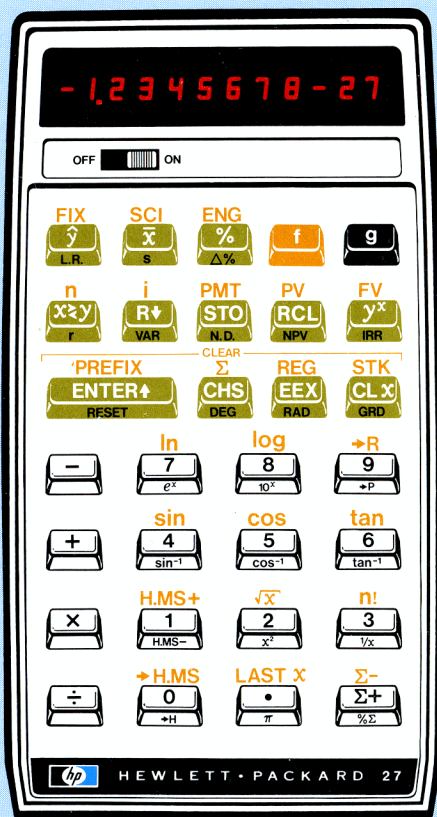
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HP-27 Functions and Keyboard Index





OFF  ON Power switch (page 11).


 Linear estimate (page 76).


 Fixed decimal display format (page 24).


 Linear regression. Calculates linear function between two or more points (page 73).


 Computes the mean or arithmetic average of two variables (page 77).


 Scientific display format (page 25).


 Calculates the standard deviation for two variables (page 79).


 Calculates x% of y (page 61).


 Engineering display format (page 26).


 Computes % difference between two numbers (page 63).


 Prefix key to select alternate function in gold above a key (page 11).


 Prefix key to select alternate function in blue on lower face of a key (page 11).


 Exchanges contents of X- and Y-registers (page 35).


 Number of periods in a financial transaction (page 66).


 Correlation coefficient for L. R. data (page 75).


 Rolls down contents of stack for viewing in display (page 33).


 Interest rate per period (page 66).


 Computes the variance (s^2) of two variables (page 80).


 Stores displayed value in one of 10 storage registers (page 45).


 Payment amount per period (page 66).


 Calculates density function and upper tail area under a normal distribution curve (page 81).


 Recalls (copies) stored number onto display (page 45).


 Present value or the amount of money at the start of financial term (page 66).


 Calculates the net present value of uneven cash flows (page 181).


 Raises the number in the Y-register to the power of the number in the display (page 51).


 Future value or amount received/paid at end of term (page 67).


 Calculates the internal rate of return for uneven cash flows (page 183).


 Copies number in displayed X-register into Y-register; also, separates successive numerical entries (page 36).


 Clears a prefix function pressed erroneously (page 13).


 Resets financial status indicators (page 198).


 Changes sign of display number (page 12).


 Clears statistical data in registers 4 thru 9 (page 71).

 Sets decimal degree mode for trigonometric functions (page 56).

 Enter exponent. After pressing, next numbers keyed in are exponents of 10 (page 30).

 Clears storage registers 0 thru 9 (page 48).

 Sets radians mode for trigonometric functions (page 56).

 Clears display (X-register) (page 12).

STK Clears the 4-register stack (page 35).

GRD Sets grads mode for trigonometric functions (page 56).

+ **-** Arithmetic operations (page 13).
x **÷**

ln Computes natural logarithm (base e) of value in display (page 55).

e^x Natural antilog. Raises e to the power of value in the display (page 55).

log Computes common logarithm (base₁₀) of value in display (page 55).

10^x Raises 10 to the power of value in display (page 55).

→R Converts polar magnitude and angle in X- and Y-registers to rectangular x and y coordinates (page 60).

→P Converts x , y rectangular coordinates to polar magnitude and angle (page 60).

sin Calculates the

cos sine, cosine or

tan tangent of value in displayed X-register (page 57).

sin⁻¹ Calculates the arc

cos⁻¹ sine, arc cosine,

tan⁻¹ or arc tangent of value in display (page 57).

H.MS+ Adds and subtracts degrees in hours, minutes, seconds (page 58).

\sqrt{x} Computes positive square root on number in display (page 53).

x² Computes square of number in display (page 52).

n! Calculates factorial of any number up to 69 (page 54).

1/x Calculates reciprocal of the number in the display (page 52).

→H.MS Converts decimal degrees to hours, minutes, seconds (page 57).

→H Converts hours, minutes, seconds to decimal hours (degrees) (page 58).

LAST X Recalls number displayed before the previous operation back into the displayed X-register (page 43).

π Places value of pi (3.14159...) into displayed X-register (page 54).

$\Sigma+$ Sums numbers and products of numbers in storage registers 4 thru 9 (page 72).

$\Sigma-$ Subtracts an incorrect value from **$\Sigma+$** entries in registers 4 thru 9 (page 73).

% Σ Finds what % one number is of another or of the total (page 64).

Introduction

At last, someone has made “a calculator for all seasons.” The HP-27 solves the problems that you, the multi-dimensional professional, encounter everyday. Whether you are juggling budgets, answering a technical question, forecasting trends, checking lab results, or analyzing market data, the HP-27 solves problems fast and accurately to make your job easier.

It's three calculators in one. Use it to solve:

- Mathematical and scientific calculations
- Business and financial problems
- Statistical calculations.

At work, at home, or on the road. . . whether you are tackling the company's problems or your own, the HP-27 puts answers at your fingertips. Over a million HP business and scientific pocket calculators are in use throughout the world, so you're in good company with HP!

To get the most from your calculator, take the time to read through this handbook and work the sample problems. The more confidence you have in your HP-27 and your own understanding of it, the more profitably and often you will use it. So, let's get started. . . .



Getting Started

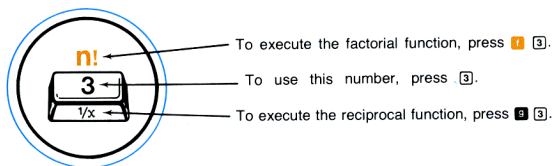
Your HP-27 is shipped fully assembled, including a battery. You can begin using your calculator immediately by connecting the cord from the ac adapter/battery charger to the calculator and plugging the charger into an ac outlet. If you want to use your HP-27 on battery power alone, you should charge the battery for 6 hours first. Whether you operate from battery power or from power supplied by the charger, the battery *must* always be in the calculator.

To begin, slide the OFF-ON switch to ON. The display reads 0.00.

Keyboard

Most keys on the keyboard perform three functions. One function is indicated by the symbol on the flat face of the key, another by the black symbol on the slanted key face, and a third by the gold symbol written above the key.

- To select the function printed in black on the slanted face of the key, first press the black prefix key **g**, then press the function key.
- To select the function printed on the flat face of the key; press the key.
- To select the function printed in gold above the key, first press the gold prefix key **f**, then press the function key.



In this handbook, the selected key function will appear in the appropriate color (gold or black) outlined by a box, like this: **sin** **sin⁻¹**.

Keying in Numbers

Key in numbers by pressing the number keys in sequence, just as though you were writing on a piece of paper. The decimal point must be keyed in if it is part of the number.

For example, key in 148.84 by pressing the keys:

Press

Display

[1] [4] [8] [.] [8] [4]

148.84

The number 148.84 appears in the display.

Negative Numbers

To key in a negative number, press the keys for the number, then press **CHS** (*change sign*). The number, preceded by a minus (-) sign, will appear in the display. For example, to change the sign of the number now in the display:

Press

Display

CHS

-148.84

You can change the sign of either a negative or a positive number in the display. For example, to change the sign of the -148.84 now in the display back to positive:

Press

Display

CHS

148.84

Notice that only negative numbers are given a sign in the display.

Clear X

You can clear any numbers that are in the display by pressing **CLx** (*clear x*). This key erases the number in the display and replaces it with 0.00.

Press

Display

CLx

0.00

If you make a mistake while keying in a number, clear the entire number string by pressing **CLx**. Then key in the correct number.

An important point about your HP-27 is that it isn't necessary to clear it between arithmetic calculations. Intermediate answers are saved, but data from previous problems is automatically pushed out of the way.

When you turn the power switch OFF, then ON again, the entire calculator is cleared.

Prefix Clear

If you inadvertently press a prefix key (**g** or **f**) when you really want a primary key, press **f** **PREFIX**, then press the correct key. If you press the wrong prefix key (**f** when you actually want **g**), just pressing **g** corrects that by overwriting the **f**.

Performing Simple Arithmetic

Whenever you add, subtract, multiply or divide, you work with two numbers and an arithmetic operation (+, −, × or ÷). Likewise, you cannot add, subtract, multiply or divide unless there are two numbers present in the calculator.

After both numbers are in the calculator, press the operation key. Your answer immediately appears on the display. To place two numbers into the calculator and perform simple arithmetic:

1. Key in the first number.
2. Press **ENTER** to separate the first number from the second.
3. Key in the second number.
4. Press **+**, **−**, **×** or **÷** to perform the operation.

For example, to add 12 and 3:

| Press | Display | |
|--------------|---------|---|
| 12 | 12. | The first number. |
| ENTER | 12.00 | Separates the first number from the second. |
| 3 | 3. | The second number. |
| + | 15.00 | The operation is executed and your answer appears on the display. |

14 Getting Started

The four arithmetic functions are all performed the same way:

| To Solve | Press | Display |
|---------------|--|---------|
| $24 + 3$ | 24 ENTER 3 + | 27.00 |
| $24 - 3$ | 24 ENTER 3 - | 21.00 |
| 24×3 | 24 ENTER 3 \times | 72.00 |
| $24 \div 3$ | 24 ENTER 3 \div | 8.00 |

In the problems above, you pressed 24 **ENTER** 3. Try the same number sequence *without* the **ENTER** step. What appears on the display? It's readily apparent that the **ENTER** key separates the first number from the second.

Sample Problems: Ready to try some problems on your own? The correct answer is given; try figuring the keystroke sequences by yourself.

| Problem | Answer Displayed |
|--|---------------------|
| $14 \times 6 =$ | 84.00 |
| $144 \div 6 =$ | 24.00 |
| $1/25 =$ | 0.04 |
| A customer buys 12 items at \$19.95 each. What is the total sale? | 239.40 |

Chain Calculations

The process of solving a chain calculation is as natural as if you were working it out with pencil and paper. The calculator itself takes care of the hard part. The HP-27 not only displays intermediate answers; but it also retains these results until you need them, then inserts them into the calculation.

For example, solve $(12 + 3) \times 7$.

If you work the problem with a pencil and paper, you would first calculate the intermediate result of $(12 + 3)$. . .

$$\begin{array}{r} 15 \\ \cancel{(12 + 3)} \times 7 = \end{array}$$

Then you would multiply 15 by 7.

$$\begin{array}{r} 15 \\ (12 + 3) \times 7 = 105 \end{array}$$

With the HP-27 you work through the problem exactly the same way—always concentrating on just two numbers at a time. First, you solve for the intermediate result. . .

| To Solve | Press | Display |
|------------|----------------------------|--------------|
| $(12 + 3)$ | 12 ENTER 3 + | 15.00 |

. . .then solve for the final answer. You don't need to press **ENTER** to store the intermediate result—the HP-27 stores it automatically when you key in the next number. To continue. . .

| To Solve | Press | Display |
|--|------------|---------------|
| $\begin{array}{r} 15 \\ (12 + 3) \times 7 \end{array}$ | 7 x | 105.00 |

Now, try this problem. Remember that you have to press **ENTER** to separate the first number from the second. After that, as the calculator performs each operation, it retains the new number.

| To Solve | Press | Display |
|--------------------|--|-------------|
| $\frac{2 + 3}{10}$ | 2 ENTER 3 + 10 ÷ | 0.50 |

Following the rules of mathematics, you solve the expressions in innermost parentheses first. Then move through the equation as you did before, one successive number and function at a time.

| To Solve | Press | Display |
|------------------------------------|---|--------------|
| $3(16 - 4)$ | 16 ENTER 4 - 3 x | 36.00 |
| $2\left(\frac{50 - 14}{12}\right)$ | 50 ENTER 14 - 12 ÷ 2 x | 6.00 |

16 Getting Started

Problems that are even more complicated can be solved in the same manner, using automatic storage of results. For example, to solve $(2 + 3) \times (4 + 5)$, you would:

First add 2 and 3:

| To Solve | Press | Display |
|--|---------------------------|-------------|
| $\overset{5}{\cancel{(2 + 3)}} \times (4 + 5)$ | 2 ENTER 3 + | 5.00 |

Then add 4 and 5:

| | | |
|--|---------------------------|-------------|
| $\overset{5}{\cancel{(2 + 3)}} \times \overset{9}{\cancel{(4 + 5)}}$ | 4 ENTER 5 + | 9.00 |
|--|---------------------------|-------------|

The 5.00 and the 9.00 are already in the calculator; so for the final answer, simply multiply them.

| | Press | Display |
|--|----------|--------------|
| $\overset{5}{\cancel{(2 + 3)}} \times \overset{9}{\cancel{(4 + 5)}}$ | × | 45.00 |

Not once did you have to write down a “subtotal” or intermediate result. The HP-27 automatically remembered them.

Now, try solving these sample problems. (If you have trouble obtaining the correct answers, review the last few pages.)

Sample Problems:

$$(2 \times 3) + (4 \times 5) = \boxed{26.00}$$

$$\frac{(14 - 12) \times (18 - 12)}{(9 - 7)} = \boxed{6.00}$$



$$(17 - 12) \times 4 \div (10 - 5) = \boxed{4.00}$$

Simple Functions



In spite of the dozens of functions available on the HP-27 keyboard, you will find the calculator simple to operate by using a single, all-encompassing rule: *When you press a function key, the calculator immediately executes the function written on that key.*



Pressing a function key causes the calculator to immediately perform that function.

For example, to calculate the square root of 148.84 merely:

| Press | Display |
|---|---------|
| 148.84 | 148.84 |
|  | 148.84 |
|  | 12.20 |

To square the result:

| Press | Display |
|--|---------|
|  | 12.20 |
|  | 148.84 |

 and  are examples of one-number function keys; that is, keys that execute upon a *single* number. Except for the financial and statistical keys, all function keys on the HP-27 operate upon either one or two numbers at a time.

One-Number Functions

To use any one-number function key:

1. Key in the number.
2. Press the function key (or press the applicable prefix key, then the function key).

18 Getting Started

For example, to use the one-number function $\boxed{1/x}$ key, you first key in the number represented by x , then press the function key. To calculate $1/4$, key in 4 (the x -number) and press **9** $\boxed{1/x}$.

| Press | Display |
|-------|---------|
|-------|---------|

| | |
|---------------|------|
| 4 | 4. |
| 9 | 4.00 |
| $\boxed{1/x}$ | 0.25 |

Now try these other one-number function problems. Remember, *first key in the number, then press the function*:

Problem

Answer

| | | | |
|---------------------|---|-----------|-------------------------------|
| $\frac{1}{36}$ | = | 0.03 | (Use the $\boxed{10^x}$ key.) |
| $\sqrt{2500}$ | = | 50.00 | |
| 10^5 | = | 100000.00 | |
| $\sqrt[3]{3204100}$ | = | 1790.00 | |
| $\log 12.58925411$ | = | 1.10 | |
| 71^2 | = | 5041.00 | |

Two-Number Functions

Two-number functions are functions that must have two numbers present in order for the operation to be performed. $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$ and $\boxed{\div}$ are examples of two-number function keys. So are $\boxed{y^x}$, $\boxed{\%}$, $\boxed{\Delta\%}$, $\boxed{\rightarrow R}$ and $\boxed{\rightarrow P}$.

Two-number functions work the same way as one-number functions—that is, the operation occurs when the function key is pressed. Therefore, *both numbers must be in the calculator before the function key is pressed*:

1. Key in the base number.
2. Press **ENTER** to separate the first number from the second.
3. Key in the second number.
4. Press the function key to perform the function.

For example, to calculate 3^4 ,

| Press | Display |
|---------------------------------------|---------|
| 3 ENTER 4 y^x | 81.00 |

When working with any function key (including **y^x**), you should remember that the displayed number is always designated by the x symbol on the function key.

The number displayed is always x .

Another two-step function is **%**. You follow the same procedure to find 25% of 167.

| Press | Display |
|------------------------------|---------|
| 167 ENTER 25 % | 41.75 |

(Percentage calculations are explained in detail in section 3.)

Now that you've learned how to use the calculator, you can begin to fully appreciate the benefits of the Hewlett-Packard logic system. With this system, you enter numbers using a parentheses-free, unambiguous method called RPN (Reverse Polish Notation).

- *You never have to work with more than one function at a time.* The HP-27 cuts problems down to size instead of making them more complex.
- *Pressing a function key immediately executes the function.* You work naturally through complicated problems, with fewer keystrokes and less time spent.
- *Intermediate results appear as they are calculated.* There are no "hidden" calculations, and you can check each step as you go.

20 Getting Started

- *Intermediate results are automatically handled.* You don't have to write down long intermediate answers when you work a problem.
- *You can calculate in the same order you do with pencil and paper.* You don't have to think the problem through ahead of time.

Before tackling more problems, take a moment to look further at your calculator. The more you understand your HP-27, the more you will use it efficiently and confidently every day.

SELECTED ASSET YIELDS (Taxable Equivalent Basis)



Money In

Money Out

ING INCOME

ing Income" figures
a total of the non-
s—on a taxable-
tailed in the Six-Year
g Statements.
deposit accounts—
individual depositors—
e non-interest
2, 1973 and

Fargo
consu
intro
Other
Wells
comp
depos
throu
met t
subst
incom

EQUIVALENT BASIS

1974 PERCENTAGE
(IN THOUSANDS) CHANGE

| | |
|-----------|-------|
| \$793,513 | 41.0% |
| 136,054 | 32.5 |
| 78,479 | 11.6 |
| 1,008,046 | 37.0 |

| | |
|---------|-------|
| 520,442 | 41.6 |
| 155,728 | 43.9 |
| 17,784 | 41.1 |
| 693,954 | 42.1 |
| 314,092 | 27.0 |
| 64,096 | (3.1) |
| 267,638 | 21.6 |
| 110,550 | 18.5 |
| 60,476 | 23.0 |
| 50,074 | 13.5 |

| | | |
|---------|-------|---------|
| 108,228 | 350.3 | 24,032 |
| 12,602 | 114.0 | 5,890 |
| 488,422 | 98.1 | 246,611 |
| 247,262 | 10.6 | 223,477 |
| 66,128 | 10.5 | 59,843 |
| 220,091 | 10.8 | 198,667 |
| 93,299 | 10.2 | 84,653 |
| 49,180 | 7.9 | 45,559 |
| 44,119 | 12.9 | 39,094 |

Trust fees—the most par
a the market value of the a
eld in trust—14.3 per ce
trust fees from 1972 to 19
—primarily from an increase in
business. While trust fees gen
1974 were depressed due to t
market value of trusts, fees fo
ust accounts a from client
through the acquisition of Be
tional Bank in primary he
improve the trust value for

TABLE II • USES

CASH AND DUE FROM
DEPOSITS PLACED BY
INVESTMENT SECUR
U.S. Treasury Securities
Securities of Other U.S.
Agencies and Corporat
Obligations of States and
Other Securities (Excludin
TRADING ACCOUNT SE
FUNDS SOLD
LOANS AND LEASES:
* Commercial Loans (Net of C
* Real Estate Loans
* Consumer Loans (Net of Unear
Loans of Overseas Offices
Direct Lease Financing
ALL OTHER ASSETS
TOTAL

*Yields computed on an all-inclus

The Display, Stack and Storage Registers

Controlling the Display

In the HP-27, numbers in the display normally appear rounded to only two decimal places. For example, the fixed constant π , which is actually in the calculator as 3.141592654, normally appears in the display as **3.14** unless you tell the calculator to show you the number rounded to a greater or lesser number of decimal places.

Although a number is normally shown to only two decimal places, the HP-27 always computes internally using each number as a 10-digit mantissa and a two-digit exponent of 10. For example, when you compute 2×3 , you see the answer to only two decimal places:

Press

2 **ENTER** 3 **x**

Display

6.00

However, inside the calculator all numbers have 10-digit mantissas and two-digit exponents of 10. So the calculator actually calculates using full 10-digit numbers:

2.000000000×10^0 **ENTER** 3.000000000×10^0 **x**

yields an answer that is actually carried to 10 digits:

6.000000000 × 10⁰

You see only
these digits. . .

but these digits
are also present.

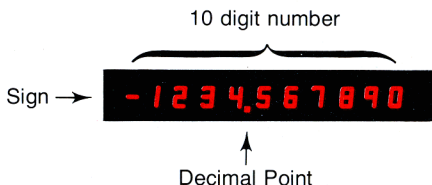
Display Formatting Keys

There are three keys, **FIX**, **SCI**, and **ENG** that allow you to control the manner in which numbers are displayed in the HP-27. **FIX** allows numbers to be displayed in “business” (fixed decimal point) format. **SCI** displays numbers in scientific notation. **ENG** displays numbers in engineering notation with exponents of 10 shown in

24 The Display, Stack and Storage Registers

multiples of three (e.g., 10^3 , 10^{-6} , 10^9). Display control alters only the *manner* in which numbers are displayed in the HP-27. The actual number itself is not altered by any of the display control keys. No matter what notation you select, these rounding options affect the display only—the HP-27 always calculates internally with a 10-digit number multiplied by 10 raised to a two-digit exponent.

Fixed Point (Business) Display



Using fixed point display, you can specify the number of places to be shown after the decimal point. It is selected by pressing **f** **FIX**, followed by a number key to specify the number of decimal places (0 thru 9) to which the display is to be rounded. The displayed number begins at the left side of the display and includes trailing zeros within the setting selected. When the calculator is turned OFF, then ON, it “wakes up” in fixed point notation with the display rounded to two decimal places. For example:

| Press | Display |
|-------|---------|
|-------|---------|

(Turn the calculator
OFF, then ON.)

123.4567 **ENTER**+

0.00

123.46

Display is rounded off to two decimal places. Internally, however, the number maintains its original value of 123.4567.

f **FIX** 4

123.4567

f **FIX** 7

123.4567000

f **FIX** 0

123.

f **FIX** 2

123.46

Normal **f** **FIX** 2 display.

Scientific Notation Display


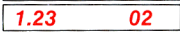
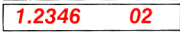



In scientific notation, each number is displayed with a single digit to the left of the decimal point followed by a specified number of digits (up to seven) to the right of the decimal point and multiplied by a power of 10. It is particularly useful when working with very large or small numbers.

Scientific notation is selected by pressing **f** **[SCI]** followed by a number key to specify the number of decimal places to which the number is rounded. The display is left-justified and includes trailing zeros within the selected setting. For example:

Press

Display

| | | |
|-------------------------|---|-----------------------------------|
| 123.4567 [ENTER] |  | Normal [FIX] 2 display. |
| f [SCI] 2 |  | Display 1.23×10^2 . |
| f [SCI] 4 |  | Display 1.2346×10^2 . |
| f [SCI] 7 |  | Display 1.2345670×10^2 . |

In scientific notation, although the calculator displays a maximum of seven digits after the decimal point, it always maintains the full 10-digit number and the two-digit exponent of 10 internally. The portion of the number that is not displayed affects the rounding of the displayed portion.

For example, if you key in 1.000000094 and specify full scientific notation display (**f** **[SCI]** 7), the calculator display rounds off to the seventh digit after the decimal point:

1.000000094



Calculator rounds to this digit in **[SCI]** 7.

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Press

1.000000094

 **SCI** 7

Display

1.000000094

1.0000001 00

In **SCI** 8, the display rounds off to the eighth digit after the decimal point, but you can see only out to seven digits after the decimal:

1.000000094
You see to here. . . _____ but the calculator display rounds to here in **SCI** 8.

Press

 **SCI** 8

Display

1.0000000 00

You can see that if you had keyed in 1.000000095, **SCI** 8 would also have caused the display to round the seventh and final digit after the decimal to a one (1).

Engineering Notation Display



Engineering notation allows all numbers to be shown with exponents of 10 that are multiples of three (e.g., 10^3 , 10^{-6} , 10^9). This is particularly useful in scientific and engineering calculations, where units of measure are often specified in powers of 10 that are multiples of three. See the following prefix chart.

| Factor by which unit is multiplied | Prefix | Symbol |
|------------------------------------|--------|--------|
| 10^{12} | tera | T |
| 10^9 | giga | G |
| 10^6 | mega | M |
| 10^3 | kilo | k |
| 10^{-3} | milli | m |
| 10^{-6} | micro | μ |
| 10^{-9} | nano | n |
| 10^{-12} | pico | p |
| 10^{-15} | femto | f |
| 10^{-18} | atto | a |

Engineering notation is selected by pressing **f** **ENG** followed by a number key. In engineering notation, the first three digits are always present, and the number key specifies the number of *additional* digits displayed after the first three. For example:

Press

Display

0.000012345

0.000012345

f **ENG** 0

12.3 -06

Engineering notation display. First three digits visible and power of 10 is the proper multiple of three.

f **ENG** 2

12.345 -06

The number key specifies the number of digits displayed *beyond* the first three.

f **ENG** 4

12.34500 -06

Notice that because the first three digits are always present, the greatest number of additional digits that can be specified in engineering notation is five.

Press

Display

f **ENG** 5

12.345000-06

Maximum number of digits displayed.

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f **ENG** 6

12.345000-06

No change in display.

f **ENG** 7

12.345000-06

No change in display.

Rounding of displayed numbers in **ENG** 5 and **ENG** 6 is similar to the rounding of numbers in **SCI** 7 and **SCI** 8, discussed earlier. As with all display formats, engineering notation display does not affect the *actual* number as it is held internally by the calculator, but only alters the manner in which the number is displayed.

When engineering notation has been selected, the decimal point shifts to show the mantissa as units, tens, or hundreds in order to maintain the exponent of 10 as a multiple of three. For example, multiplying the number now in the calculator by 10 causes the decimal point to shift to the right without altering the exponent of 10:

Press

f **ENG** 0

10 **⊗**

Display

12.3 -06

123. -06

Decimal point shifts.
Power of 10 remains
at 10^{-6} .

However, multiplying again by 10 causes the exponent to shift to another multiple of three and the decimal point to move to the units position:

Press

10 **⊗**

Display

1.23 -03

Decimal point shifts.
Power of 10 shifts to
 10^{-3} .

Automatic Display Switching

The HP-27 switches the display from fixed point notation to full scientific notation (**SCI** 7) whenever the number is too large or too small to be seen with a fixed decimal point. This feature keeps you from missing unexpectedly large or small answers. For example, if you try to solve $(.05)^3$ in normal **FIX** 2 display, the answer is automatically shown in scientific notation.

Press**Display****CLx**

0.00 00

ENG 0 from previous example.**f** **FIX** 2

0.00

Normal **FIX** 2 display..05 **ENTER**

0.05

3 **y^x**

1.250000-04

Display automatically switched to **SCI** 7 to show answer.

Another way of displaying the answer would be 0.000125, but in normal **FIX** 2 display, you would have seen only 0.00 displayed.

After automatically switching from fixed to scientific, when a new number is keyed in or **CLx** is pressed, the display automatically reverts back to the fixed point display originally selected.

The HP-27 also switches to scientific notation if the answer is too large ($> 10^{10}$) for fixed point display. The display will not switch from fixed if you solve 1582000×1842 :

Press**Display**1582000 **ENTER**

1582000.00

1842 **x**

2914044000.

Fixed decimal point display.

However, if you multiply the result by 10, the answer is too large for fixed point notation, and switches automatically to scientific notation:

Press**Display**10 **x**

2.9140440 .10

Scientific notation display.

Notice that automatic switching is between fixed and scientific notation display modes only—engineering notation display must be selected from the keyboard.

Keying in Exponents of Ten

You can key in numbers multiplied by powers of 10 by pressing **EEX** (*enter exponent of ten*). For example, to key in 15.6 trillion (15.6×10^{12}), and multiply it by 25:

| Press | Display |
|------------|--|
| 15.6 | 15.6 |
| EEX | 15.6 00 |
| 12 | 15.6 12 (This means 15.6×10^{12} .) |

| Now Press | Display |
|-------------------------|--------------|
| ENTER \uparrow | 1.5600000 13 |
| 25 \times | 3.9000000 14 |

You can save time when keying in exact powers of 10 by merely pressing **EEX** and then pressing the desired power of 10. For example, key in 1 million (10^6) and divide by 52.

| Press | Display |
|-------------------------|------------|
| EEX | 1. 00 |
| 6 | 1. 06 |
| ENTER \uparrow | 1000000.00 |
| 52 \div | 19230.77 |

You do not have to key in the number 1 before pressing **EEX** when the number is an exact power of 10.

Since you have not specified scientific notation, the answer reverts to fixed point notation when you press **ENTER** \uparrow .

To see your answer in scientific notation with six decimal places:

| Press | Display |
|-----------------------|-------------|
| f SCI 6 | 1.923077 04 |

To key in negative exponents of 10, key in the number, press **EEX**, press **CHS** to make the exponent negative, then key in the power of 10. For example, key in Planck's constant (h)—roughly, 6.625×10^{-27} erg sec.—and multiply it by 50.

| Press | Display | |
|-----------------------|---------------|----------|
| CLx | 0.000000 00 | |
| f FIX 2 | 0.00 | |
| 6.625 EEX | 6.625 00 | |
| CHS | 6.625 -00 | |
| 27 | 6.625 -27 | |
| ENTER | 6.6250000 -27 | |
| 50 x | 3.3125000 -25 | Erg sec. |

When you use the **EEX** key, the HP-27 displays each number as an eight-digit mantissa and a two-digit exponent of 10. In a few cases, a number may have to be altered slightly in form before you can key it in using the **EEX** key:

1. If you key in a number whose mantissa contains more than eight digits to the left of the decimal point, the **EEX** key is overridden and does not operate. Begin again and key in the number in a form that displays the mantissa with eight digits or less to the left of the decimal point before pressing the **EEX** key. Thus, $123456789.1 \times 10^{23}$ could be keyed in as $12345678.91 \times 10^{24}$.
2. If you key in a number whose first significant digit occurs *after* the first eight digits of the display (e.g., $0000.000025 \times 10^{55}$) all zeros are displayed but the number is operated on correctly internally. To view the number, you may prefer to key it in as $0000.000025 \times 10^{54}$ or as 0.000025×10^{55} , then proceed using the **EEX** key.

Overflow and Underflow Displays

When the number in the display is greater than 9.9999999×10^{99} , the HP-27 displays all 9's to indicate that the problem has exceeded the calculator's range:

9.9999999 99

If an excessively large negative number is calculated (greater than $-9.9999999 \times 10^{99}$), the overflow indicator is all 9's preceded by a minus sign.

-9.9999999 99

Very small numbers (less than 10^{-100}) are displayed as 0.00. If you press **f** **SCI** 7, the display reads:

00000000 00

Error Display

If you attempt an improper or impossible operation, the word **Error** appears on the display. For example, enter a number and try to divide by zero. (Go ahead, try it.) The calculator recognizes this as an illegal operation. Other examples of improper operations are square root of a negative number or 0 raised to a negative power.

To clear the error display, press **CLx**.

Low Power Display

When the batteries get low, several decimal points will appear on the display interspersed among the numbers. Where the true decimal point appeared, there will be a blank. For example, if you have the following number on the display:

Ordinary display:

17.45

the low-power indication will look like this:

Low-power display:

1.7 4.5.....

↑
 blank

This means you have approximately one minute of operating time left. Then you must either charge the battery or insert a fully-charged spare battery pack. (Refer to appendix A.)

The Automatic Memory Stack

Automatic storage of intermediate results is the reason that the HP-27 slides so easily through the most complex equations. Automatic storage is made possible by the Hewlett-Packard automatic memory stack.

The X-Register (Display)

When you first switch the calculator ON, the display shows 0.00. This represents the contents of the display, or “X-register.”

Basically, numbers are stored and manipulated in the machine “registers.” Each number, no matter how few digits (e.g., 5) or how many (e.g., $2.87148907 \times 10^{27}$), occupies one entire register.

The displayed X-register, which is the only visible register, is one of four registers inside the calculator that are positioned to form the automatic memory stack. We label these registers X, Y, Z, and T. They are “stacked” one on top of the other (like bookshelves) with the displayed X-register on the bottom. When the calculator is switched ON, these four registers are cleared to 0.00.

| Name | Register | |
|------|--|-------------------|
| T | 0.00 | |
| Z | 0.00 | |
| Y | 0.00 | |
| X | 0.00 | Always displayed. |

Manipulating Stack Contents

The R↓ (roll down) and x↔y (x exchange y) keys allow you to review the stack contents or to shift data within the stack for computation at any time.

Reviewing the Stack: To see how the R↓ key works, first load the stack with numbers 1 through 4 by pressing:

4ENTER↑ 3ENTER↑ 2ENTER↑ 1

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The numbers that you entered are loaded into the stack, and its contents look like this:

| | |
|---|------|
| T | 4.00 |
| Z | 3.00 |
| Y | 2.00 |
| X | 1. |

Display

Each time you press the **R↓** key, the stack contents shift downward one register. So the last number that you have keyed in will be rotated around to the T-register when you press **R↓**.

| | | | | | |
|---|------|---------------|---|------|---------|
| T | 4.00 | → R↓ → | T | 1.00 | Display |
| Z | 3.00 | | Z | 4.00 | |
| Y | 2.00 | | Y | 3.00 | |
| X | 1.00 | | X | 2.00 | |

Display

Notice that the *contents* of the registers are shifted. The registers (“shelves”) maintain in the same positions. The contents of the X-register are always displayed, so **2.00** is now visible.

Press **R↓** again and the stack contents are shifted:

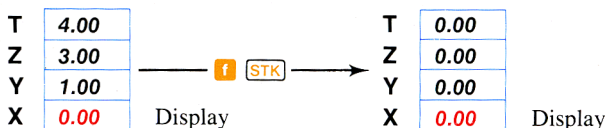
| | | | | | |
|---|------|---------------|---|------|---------|
| T | 1.00 | → R↓ → | T | 2.00 | Display |
| Z | 4.00 | | Z | 1.00 | |
| Y | 3.00 | | Y | 4.00 | |
| X | 2.00 | | X | 3.00 | |

Display

Press **R↓** twice more, and the stack shifts back to the start. Once again, the number 1.00 is in the displayed X-register. Now that you know how the stack is rotated, you can use the **R↓** key to review the contents of the stack at any time so that you can always tell what is in the calculator. Always remember, though, that it takes four presses of the **R↓** key to return the contents to their original registers.

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If you press **f** **STK**, the contents of the entire stack are cleared.



The **ENTER↓** Key

When you key a number into the calculator, its contents are written into the displayed X-register and the other registers remain unchanged. For example, if you keyed in the number 314.32, your stack registers would look like this:

| Name | Register |
|------|----------|
| T | 0.00 |
| Z | 0.00 |
| Y | 0.00 |
| X | 314.32 |

Display

In order to key in a second number at this point, you must separate the digits of the first number from the digits of the second.

One way to separate numbers is to press **ENTER↓**. Press **ENTER↓** to change the contents of the registers:



As you can see, the number in the displayed X-register is copied into Y. (The numbers in Y and Z have also been transferred to Z and T, respectively, and the number in T has been lost off the top of the stack. But this will be more apparent when we have different numbers in all four registers.)

Immediately after pressing **ENTER**, the X-register is prepared for a new number, and that new number writes over the number in X. For example, key in the number 543.28 and the contents of the stack registers change. . .

from this:

| | |
|---|--------|
| T | 0.00 |
| Z | 0.00 |
| Y | 314.32 |
| X | 314.32 |

Display

to this:

| | |
|---|--------|
| T | 0.00 |
| Z | 0.00 |
| Y | 314.32 |
| X | 543.28 |

Display

CLx replaces any number in the display with zero. You can then key in a new number and write over the zero in X. For example, if you had meant to key in 689.4 instead of 543.28, you would press **CLx**:

| | |
|---|--------|
| T | 0.00 |
| Z | 0.00 |
| Y | 314.32 |
| X | 543.28 |

Display

→ **CLx** →

| | |
|---|--------|
| T | 0.00 |
| Z | 0.00 |
| Y | 314.32 |
| X | 0.00 |

Display

and then key in 689.4 to change the stack

from this:

| | |
|---|--------|
| T | 0.00 |
| Z | 0.00 |
| Y | 314.32 |
| X | 0.00 |

Display

to this:

| | |
|---|--------|
| T | 0.00 |
| Z | 0.00 |
| Y | 314.32 |
| X | 689.4 |

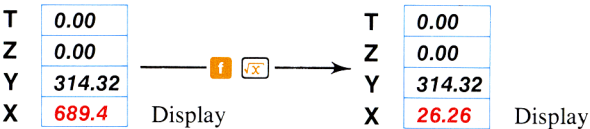
Display

Notice that numbers in the stack do not move when a number is keyed in immediately after pressing **ENTER** or **CLx**. (However, the numbers in the stack *do* lift when a new number is keyed in immediately after pressing **R↓**.)

One-Number Functions and the Stack

One-number functions execute upon the number in the X-register only, and the contents of the Y-, Z-, and T-registers are unaffected when a one-number function key is pressed.

For example, with numbers positioned in the stack as in the earlier example, pressing the **f** **√x** keys changes the stack contents, like so:



The one-number function executes upon only the number in the displayed X-register, and the answer writes over the number that was in the X-register. No other register is affected by a one-number function.

Two-Number Functions and the Stack

Hewlett-Packard calculators do arithmetic by positioning the numbers in the stack the same way you would on paper. For instance, if you wanted to add 34 and 21, you would write 34 on a piece of paper and then write 21 underneath it, like this:

$$\begin{array}{r} 34 \\ 21 \\ \hline \end{array}$$

and then you would add, like this:

$$\begin{array}{r} 34 \\ +21 \\ \hline 55 \end{array}$$

Numbers are positioned the same way in the HP-27. Here’s how it is done.

| Press | Display | |
|---------------------|---------|-----------------------------|
| f STK | 0.00 | |
| 34 | 34. | 34 is keyed into X. |
| ENTER ↵ | 34.00 | 34 is copied into Y. |
| 21 | 21. | 21 writes over the 34 in X. |

Now 34 and 21 are sitting vertically in the stack as shown below, so we can add.

| | |
|---|-------|
| T | 0.00 |
| Z | 0.00 |
| Y | 34.00 |
| X | 21. |

Display

| Press | Display | |
|---|---|-------------|
| + | 55.00 | The answer. |

The simple old-fashioned math notation helps explain how to use your calculator. Both numbers are always positioned in the stack in the natural order first; then the operation is executed when the function key is pressed. *There are no exceptions to this rule.* Subtraction, multiplication, and division work the same way. In each case, the data must be in the proper position before the operation can be performed.

To subtract 21 from 34:

$$\begin{array}{r} 34 \\ -21 \\ \hline \end{array}$$

| Press | Display | |
|---|---|-----------------------------|
| 34 | 34. | 34 is keyed into X. |
| ENTER | 34.00 | 34 is copied into Y. |
| 21 | 21. | 21 writes over the 34 in X. |
| - | 13.00 | The answer. |

To multiply 34 by 21:

$$\begin{array}{r} 34 \\ \times 21 \\ \hline \end{array}$$

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| Press | Display | |
|----------------|---------|-----------------------------|
| 34 | 34. | 34 is keyed into X. |
| ENTER ↕ | 34.00 | 34 is copied into Y. |
| 21 | 21. | 21 writes over the 34 in X. |
| \times | 714.00 | The answer. |

To divide 34 by 21:

$$\begin{array}{r} 34 \\ \hline 21 \end{array}$$

| Press | Display | |
|----------------|---------|-----------------------------|
| 34 | 34. | 34 is keyed into X. |
| ENTER ↕ | 34.00 | 34 is copied into Y. |
| 21 | 21. | 21 writes over the 34 in X. |
| \div | 1.62 | The answer. |

Chain Arithmetic

You've already learned how to key numbers into the calculator and perform calculations with them. In each case you first needed to position the numbers in the stack manually using the **ENTER**↕ key. However, the stack also performs many movements automatically. These automatic movements add to its computing efficiency and ease of use, and it is these movements that automatically store intermediate results. The stack automatically "lifts" every calculated number in the stack when a new number is keyed in because it knows that after it completes a calculation, any new digits you key in are a part of a new number. Also, the stack automatically "drops" when you perform an arithmetic operation.

Press **f** **STK** first, then calculate $16 + 30 + 11 + 17 = ?$ You've learned to solve such a problem by pressing: 16 **ENTER** 30 **+** 11 **+** 17 **+**. Now, let's try another method that utilizes the "lift" and "dropping" features of the stack.

Press

Stack Contents

| | | | |
|----|---|------|--|
| 16 | T | 0.00 | 16 is keyed into the displayed X-register. |
| | Z | 0.00 | |
| | Y | 0.00 | |
| | X | 16. | |

| | | | |
|--------------|---|-------|----------------------|
| ENTER | T | 0.00 | 16 is copied into Y. |
| | Z | 0.00 | |
| | Y | 16.00 | |
| | X | 16.00 | |

| | | | |
|----|---|-------|---------------------------------|
| 30 | T | 0.00 | 30 is written over the 16 in X. |
| | Z | 0.00 | |
| | Y | 16.00 | |
| | X | 30. | |

| | | | |
|--------------|---|-------|--|
| ENTER | T | 0.00 | 30 is entered into Y. 16 is lifted up to Z. |
| | Z | 16.00 | |
| | Y | 30.00 | |
| | X | 30.00 | |

| | | | |
|----|---|-------|--|
| 11 | T | 0.00 | 11 is keyed into the displayed register. |
| | Z | 16.00 | |
| | Y | 30.00 | |
| | X | 11. | |

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Press

Stack Contents

ENTER↵

| | |
|---|-------|
| T | 16.00 |
| Z | 30.00 |
| Y | 11.00 |
| X | 11.00 |

11 is copied into Y. 16 and 30 are lifted up to Z and T respectively.

17

| | |
|---|-------|
| T | 16.00 |
| Z | 30.00 |
| Y | 11.00 |
| X | 17. |

17 is written over the 11 in X.

+

| | |
|---|-------|
| T | 16.00 |
| Z | 16.00 |
| Y | 30.00 |
| X | 28.00 |

17 and 11 are added together and the rest of the stack drops. 16 drops to Z and is also duplicated in T. 30 and 28 are ready to be added.

+

| | |
|---|-------|
| T | 16.00 |
| Z | 16.00 |
| Y | 16.00 |
| X | 58.00 |

30 and 28 are added together and the stack drops again. Now 16 and 58 are ready to be added.

⊕

| | |
|---|-------|
| T | 16.00 |
| Z | 16.00 |
| Y | 16.00 |
| X | 74.00 |

16 and 58 are added together for the final answer and the stack continues to drop.

The same dropping also occurs with \ominus , \otimes and \div . The number in T is duplicated in T and Z, the number in Z drops to Y, and the numbers in Y and X combine to give the answer, which is visible in the X-register.

LAST X Register

In addition to the four stack registers, the HP-27 also contains a separate LAST X register that preserves the value displayed in the X-register *before* you performed a function. To place the contents of the LAST X register in the display, press **f** **LAST X**. This register is convenient and helpful when you use a number repeatedly or want to recover from mistakes.

To Avoid Reentering Numbers

The LAST X register is useful in calculations where a number occurs more than once.

For example, to calculate $\frac{7.32 + 3.65}{3.65}$

Press

Display

7.32 **ENTER**⬆

7.32

3.65

3.65

⊕

10.97

f **LAST X**

3.65

÷

3.01

Intermediate answer.
Recalls 3.65 to X.
The answer.

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Recovering from Mistakes

LAST X makes it easy to recover from keystroke mistakes, such as pressing the wrong function or keying in the wrong number.

Example: Divide 12 by 2.157 after you have mistakenly divided by 3.157.

| Press | Display | |
|------------------------|---------|--|
| 12 | 12. | |
| ENTER | 12.00 | |
| 3.157 \div | 3.80 | |
| f LAST X | 3.16 | Oops! You made a mistake. |
| \times | 12.00 | Retrieves that last entry. |
| 2.157 \div | 5.56 | You're back at the beginning. The correct answer. |

In the above example, when **f** **LAST X** is pressed, the contents of the stack and LAST X registers are changed:

| from this: | | | to this: | | |
|------------|------|--------|----------|------|--------|
| T | 0.00 | | T | 0.00 | |
| Z | 0.00 | | Z | 0.00 | |
| Y | 0.00 | LAST X | Y | 3.80 | LAST X |
| X | 3.80 | ← 3.16 | X | 3.16 | ← 3.16 |

This makes possible the correction illustrated in the example above.

Ten Storage Registers

Besides the stack, another 10 memory registers (0 thru 9) are provided for manual storage and recalling numbers.

Storing and Recalling Numbers

To store a number, press **STO** then a digit from 0 to 9 to specify the storage location. **STO** 0 places the number on the display in register R_0 ; **STO** 1, in register R_1 ; **STO** 2, in register R_2 ; etc. Data in a storage register is changed by writing over it with a new value or by storage arithmetic.

To recall the number, press **RCL**, then the address digit (0 to 9). When you press **RCL** 3, a copy of the number in register R_3 appears on the display; the original value will remain in the storage location until you write over it or clear it. Recalling data also moves the numbers in the stack upward. (If you press **CLx** or **ENTER** just prior to **RCL**, then the stack does *not* lift.)

Example 1: Suppose you want to calculate the cost of buying an item in various quantities. The unit price of the item is \$132.57 and the quantities selected are 47, 36 and 29.

One way to solve this is to store the unit price in register R_0 . Then recall it to multiply each quantity.

| Press | Display | |
|---------------------|---------|---------------|
| 132.57 STO 0 | 132.57 | |
| 47 × | 6230.79 | First total. |
| RCL 0 | 132.57 | |
| 36 × | 4772.52 | Second total. |
| RCL 0 | 132.57 | |
| 29 × | 3844.53 | Third total. |

The individual totals are still in the stack, so you can easily calculate the combined total cost by adding them.

| Press | Display | |
|----------|----------|-------------|
| + | 8617.05 | |
| + | 14847.84 | Total cost. |

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Example 2: Three tanks have capacities in U.S. units of 2.0, 14.4 and 55.0 gallons respectively. If one U.S. gallon is approximately equal to 3.785 liters, what is the capacity of each of the tanks in liters?

| Press | Display | |
|----------------------------|----------------|-------------------------------------|
| f REG | 0.000 | Display mode set. |
| f FIX 3 | 3.785 | Constant stored in register R_0 . |
| 3.785 STO 0 | 7.570 | Capacity of first tank in liters. |
| 2 × | 54.504 | Capacity of second tank in liters. |
| 14.4 RCL 0 × | 208.175 | Capacity of third tank in liters. |
| 55 RCL 0 × | 208.18 | Display mode reset. |
| f FIX 2 | | |

Storage and Recall Register Arithmetic

Arithmetic operations (+, −, ×, ÷) can be performed between a storage register and the X-register in two ways: storage arithmetic and recall arithmetic.

Storage Arithmetic— Arithmetic is performed *upon* the register contents and answers are placed in the storage register—not in the display—so you have to recall them to see or use them.

To perform storage register arithmetic using the number on the display:

1. Press **STO**.
2. Press the desired arithmetic operation (**+**, **−**, **×** or **÷**).
3. Press the storage location number (**0** thru **9**).

For example, to store 6 in register R_3 and add 5:

| Press | Display | |
|-------------------------|--------------|--|
| 6 STO 3 | 6.00 | Stores 6 in register R_3 . |
| 5 STO + 3 | 5.00 | Adds 5 to register R_3 . |
| RCL 3 | 11.00 | Confirms that 11 is stored in register R_3 . |

If you had pressed: 5 **STO** **3** **+**, that would overwrite the stored 6 with the 5—the value stored in register R_3 would be 5, not 11.

Now, subtract 4 from the number in register R_3 :

| Press | Display |
|--------------------------------|---------|
| 4 STO - 3 | 4.00 |
| RCL 3 | 7.00 |

Notice that the general rule is:

Number on display, **STO**, operation, register number.

Recall Arithmetic—Conversely, to alter the X-register (displayed value) without affecting the contents of the storage register or the stack, press **RCL**, the arithmetic operation, then the register number. For example, add the current value stored in register R_3 to a new entry (2.00).

| Press | Display |
|-------------------------|---------|
| 2 RCL + 3 | 9.00 |
| RCL 3 | 7.00 |

To subtract the contents of register R_3 from the number 11:

| Press | Display |
|--------------------------|---------|
| 11 RCL - 3 | 4.00 |
| RCL 3 | 7.00 |

Recall arithmetic does not affect any other stack register except the X-register; i.e., the contents of the stack do not lift.

Remember, like storage arithmetic, the general rule is:

Number on display, **RCL** operation, register number.

Storage Overflow Indicator

If the value in a storage register exceeds 9.9999999×10^{99} , the following indicator appears on the display:

OF

Clearing the Storage Registers

It is usually unnecessary to clear storage registers R_0 thru R_9 because you can simply write over the old number.

The **f** **REG** function clears all 10 storage registers, as well as the financial registers and **LAST x**.

Availability of Storage Registers

As a general practice, it is wise to use the four storage registers R_0 thru R_3 first to preserve your data.

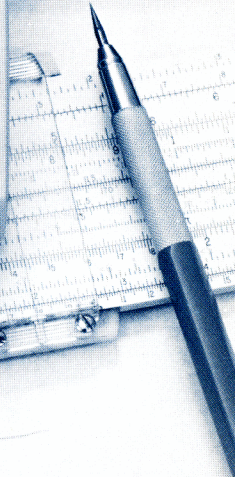
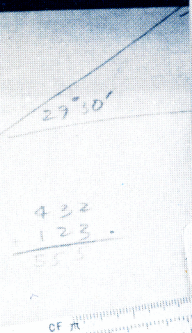
When the **Σ+** key is used in statistical problems, the HP-27 uses registers R_4 thru R_9 for its own internal data storage. Therefore, those registers are temporarily not available to you for manual storage.

During iterative **i** calculations, storage registers R_7 , R_8 , and R_9 are utilized by the calculator.

Clearing Operations

At this point, it might be useful to review the HP-27 clearing operations:

- | | |
|------------------------|---|
| CLx | Clears the display (X-register). |
| f PREFIX | Clears the previously pressed prefix key. |
| f STK | Clears the four-register stack. |
| f REG | Clears storage registers R_0 thru R_9 , the five financial registers, and LAST x . |
| f Σ | Clears storage registers R_4 thru R_9 . |



Handwritten calculations on a piece of paper:

$$\begin{array}{r} 233 \overline{) 144000000} \\ \underline{1398} \\ 420 \\ \underline{233} \\ 1870 \\ \underline{1864} \end{array}$$

Below the division, the numbers 920, 233, 1870, and 1864 are written.

Keyboard Calculations

Exponentiation:

Raising a Number to a Power

The **y^x** key raises a positive number to a positive or negative power or a negative number to an integer power. You use it the same simple way you've performed arithmetic operations; the function is executed immediately when you press the key.

1. Key in the base number. This number is designated y.
2. Press **ENTER** to separate the first number from the second.
3. Key in the second number (power). This number is designated as x.
4. Press **y^x**.

To calculate 3^6 ,

| Press | Display |
|---------------------------------------|---------------|
| 3 ENTER 6 y^x | 729.00 |

To raise a number to a negative power, follow the same procedure and change the sign (**CHS**) of your exponent before you perform the operation. To solve $4.37^{-2.5}$,

| Press | Display |
|---|-------------|
| 4.37 ENTER 2.5 CHS y^x | 0.03 |

To raise a negative number to a positive or negative integer power, key in the base number, press **CHS**, press **ENTER**, key in the integer power, and press **y^x**.

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| To Solve | Press | Display |
|-------------|---|------------------|
| -2^3 | 2 CHS ENTER 3 y^x | <div>-8.00</div> |
| $-2^{(-3)}$ | 2 CHS ENTER 3 CHS y^x | <div>-0.13</div> |

With a negative base, if the exponent is an odd number, the answer will be negative. If the exponent is an even number, your answer will be positive:

$$(-2)^2 = -2 \times -2 = 4$$

$$(-2)^3 = -2 \times -2 \times -2 = -8$$

You can also use **y^x** to raise 0 to a positive power; but of course, your answer will always be zero.

Squaring

To square a number in the displayed X-register, press **g** **x²**. For example, to find the square of 45:

| Press | Display |
|----------------------------------|--------------------|
| 45 g x² | <div>2025.00</div> |

To find the square of the result:

| Press | Display |
|-------------------------------|-----------------------|
| g x² | <div>4100625.00</div> |

Reciprocals

To calculate the reciprocal of a number in the displayed X-register, key in the number, then press **g** **1/x**. For example, to calculate the reciprocal of 25:

| Press | Display |
|------------------------|-----------------|
| 25 g 1/x | <div>0.04</div> |

You can also calculate the reciprocal of a value in a previous calculation without re-entering the number. For example, to calculate

$$\frac{1}{1/3 + 1/6}$$

| Press | Display | |
|--------------------------|---------|---------------------|
| 3 g $\frac{1}{x}$ | 0.33 | Reciprocal of 3. |
| 6 g $\frac{1}{x}$ | 0.17 | Reciprocal of 6. |
| + | 0.50 | Sum of reciprocals. |
| g $\frac{1}{x}$ | 2.00 | Reciprocal of sum. |

Extracting Roots

To calculate the square root of a number, key in the number, then press **f** \sqrt{x} .

| To Solve | Press | Display |
|-------------|------------------------|---------|
| $\sqrt{25}$ | 25 f \sqrt{x} | 5.00 |
| $\sqrt{81}$ | 81 f \sqrt{x} | 9.00 |

You must use a positive number. You cannot calculate the square root of a negative number; that's an illegal operation.

You can also extract higher roots, like cube roots and fourth roots; but you use the **y^x** key, not **f** \sqrt{x} . The cube root of a number is that number raised to the 1/3 power. Thus, $\sqrt[3]{n}$ is the same as $n^{1/3}$; the fourth root can be written as $n^{1/4}$, etc.

Use the same keystroke sequence that you learned for exponentiation to extract higher roots:

1. Key in the base number and press **ENTER**.
2. Key in the root desired and press **g** $\frac{1}{x}$.
3. Press **y^x**.

To solve $\sqrt[3]{5} = 5^{1/3}$:

| Press | Display |
|--|---------|
| 5 ENTER 3 g $\frac{1}{x}$ y^x | 1.71 |

Now, try $\sqrt[4]{81}$ or $81^{1/4}$:

| Press | Display |
|---|---------|
| 81 ENTER 4 g $\frac{1}{x}$ y^x | 3.00 |

Using PI

The value π accurate to 10 places (3.141592654) is provided as a fixed constant in the HP-27. Merely press **g** **π** whenever you need it in a calculation. For example, to calculate 3π ,

| Press | Display |
|---|---------|
| 3 g π \times | 9.42 |

Example: In the *Guinness Book of Records*, you find that the largest pizza ever baked had a diameter of 21 feet. If your appetite were equal to the task, how many square feet of pizza would you have to devour in order to consume all of the world's largest pizza?

$$\text{Area} = \pi \left(\frac{d}{2} \right)^2 = \pi \left(\frac{21}{2} \right)^2$$

| Press | Display |
|---------------------------------------|---------|
| 21 ENTER | 21.00 |
| 2 \div | 10.50 |
| g \times^2 | 110.25 |
| g π | 3.14 |
| \times | 346.36 |

Square feet of pizza.

Factorials

The keys **f** **$n!$** enable you to handle permutations and combinations with ease. To calculate the factorial of an integer, key in that number, then press **f** **$n!$** .

Example: Calculate the number of ways 6 people can line up for a photograph. ($6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$)

| Press | Display |
|---------------------------------|---------|
| 6 | 6. |
| f $n!$ | 720.00 |

The HP-27 can calculate factorials for numbers up to 69.

Logarithms and Antilogs

The HP-27 computes both natural and common logarithms as well as their inverse functions (antilogarithms):

- f** **ln** is \log_e (natural log). It takes the log of the value in the X-register to base e (2.718. . .).
- g** **e^x** is antilog_e (natural antilog). It raises e (2.718. . .) to the power of the value in the X-register.
- f** **log** is \log_{10} (common log). It computes the log of the value in the X-register to base 10.
- g** **10^x** is antilog_{10} (common antilog). It raises 10 to the power of the value in the X-register.

Example 1: The 1906 San Francisco earthquake, with a magnitude of 8.25 on the Richter Scale is estimated to be 105 times greater than the Nicaragua quake of 1972. What would be the magnitude of the latter on the Richter Scale? The equation is

$$R_1 = R_2 - \log \frac{M_2}{M_1} = 8.25 - \left(\log \frac{105}{1} \right)$$

Solution:

Press

Display

8.25 **ENTER**

8.25

105 **f** **log**

2.02

=

6.23

Rating on Richter scale.

Example 2: Ace explorer Jason Quarmorte is using an ordinary barometer as an altimeter. After measuring the sea level pressure (30 inches of mercury) he climbs until the barometer indicates 9.4 inches of mercury. Although the exact relationship of pressure and altitude is a function of many factors, Jason knows that an *approximation* is given by the formula:

$$\text{Altitude (feet)} = 25,000 \ln \frac{30}{\text{Pressure}} = 25,000 \ln \frac{30}{9.4}$$

Where is Jason?

Solution:

| Press | Display | |
|--------------------|----------|-------------------|
| 30 ENTER ↑ | 30.00 | |
| 9.4 ÷ | 3.19 | |
| f ln | 1.16 | |
| 25000 | 25000 | |
| (x) | 29012.19 | Altitude in feet. |

He's probably near the summit of Mount Everest (29,028 feet).

Example 3: Logarithms and antilogarithms are also used in continuous compound interest formulas. To compute the continuous effective rate, given the nominal rate, the formula is:

$$\text{Continuous Effective Rate} = \left(e^{\frac{\text{nominal rate}}{100}} - 1 \right) \times 100$$

If a savings institution quotes a nominal rate of 6%, compounded continuously, what is the effective rate?

| Press | Display | |
|---|---------|------------------------------|
| 6 ENTER ↑ 100 ÷ | 0.06 | |
| g (e^x) 1 = 100 (x) | 6.18 | % continuous effective rate. |

Trigonometric Functions

Your HP-27 provides you with six trigonometric functions. It calculates angles in decimal degrees, radians, or grads. It converts between decimal degrees and degrees, minutes, seconds, and lets you add or subtract degrees, minutes, seconds.

Trigonometric Modes

When the HP-27 is first turned ON, it “wakes up” with angles specified in decimal degrees. To set radians or grads mode, press the **g** shift key followed by either **(RAD)** (*radians*) or **(GRD)** (*grads*). To switch back to the decimal degrees mode again, press the **g** shift key followed by the **(DEG)** (*degrees*) key.

Note: 360 degrees = 2π radians = 400 grads.

Functions

The six trigonometric functions provided by the calculator are:

| | | |
|----------|-------------------------|---------------|
| f | sin | (sine) |
| g | sin⁻¹ | (arc sine) |
| f | cos | (cosine) |
| g | cos⁻¹ | (arc cosine) |
| f | tan | (tangent) |
| g | tan⁻¹ | (arc tangent) |

Each trigonometric function assumes angles in decimal degrees, radians, or grads. Trigonometric functions are one-number functions; so to use them, you key in the number, then press the function keys.

Example 1: Find the cosine of 35°.

| Press | Display | |
|---------------------|---------|--|
| 35 | 35. | Calculator “wakes up” in decimal degrees mode. |
| f cos | 0.82 | The answer. |

Example 2: Find the arc sine in grads of .964.

| Press | Display | |
|----------------------------------|---------|--------------------|
| CLx | 0.00 | |
| g GRD | 0.00 | Grads mode is set. |
| .964 | 0.964 | |
| g sin⁻¹ | 82.87 | Grads. |

Hours, Minutes, Seconds

The **↔HMS** (to hours, minutes, seconds) key converts decimal hours (or degrees) to the format of hours (degrees), minutes and seconds. To see the digits for seconds, you should specify **FIX** 4 display format. For example, to convert 12.56 hours to hours, minutes, seconds:

| Press | Display | |
|-----------------------|---------|---|
| f FIX 4 | 0.0000 | Sets display format. (Assumes no results remain from previous example.) |

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12.56


 

12.56

12.3336

Decimal hours.

This is read as 12 hours, 33 minutes, 36 seconds.

Conversely, the  key permits you to change hours (degrees), minutes, seconds to decimal hours (degrees). For example, to change 173°45' 12'' to decimal degrees:

Press

137.4512





Display

137.4512

137.7533

Decimal degrees.

The conversion is important because trigonometric functions in the HP-27 operate on angles in decimal degrees, but not in *hours, minutes, seconds*.

Two other keys   and  , allow you to add and subtract in hours (degrees), minutes, and seconds without converting back to decimal hours (degrees).

Example: Lovesick sailor Oscar Odysseus dwells on the island of Tristan da Cunha (37°03'S, 12°18'W), and his sweetheart, Penelope, lives on the nearest island. Unfortunately for the course of true love, however, Tristan da Cunha is the most isolated inhabited spot in the world. If Penelope lives on the island of St. Helena (15°55'S, 5°43'), calculate the great circle distance that Odysseus must sail in order to court her using the following formula:

$$\text{Distance} = \cos^{-1} \left[\sin (\text{LAT}_s) \sin (\text{LAT}_s) \right. \\ \left. \cos (\text{LAT}_d) \cos (\text{LNG}_d - \text{LNG}_s) \right] \\ \times 60.$$

Where LAT_s and LNG_s = latitude and longitude of the source
(Tristan da Cunha).

LAT_D and LNG_d = latitude and longitude of the
destination.

Solution: Convert all degrees, minutes, seconds entries into decimal degrees as you key them in. The equation for the great circle distance from Tristan da Cunha to the nearest inhabited land is:

$$\text{Distance} = \cos^{-1} \left[\sin(37^\circ 03') \sin(15^\circ 55') \right. \\ \left. + \cos(37^\circ 03') \cos(15^\circ 55') \right. \\ \left. \cos(5^\circ 43' - 12^\circ 18') \right] \times 60$$

Press

Display

g **DEG**

Sets decimal degree mode
for trigonometric functions.
(Assumes no results remain
from previous example.)
Display mode set.

f **FIX** **2**

5.43 **ENTER**

12.18 **g** **HMS** **g** **→H**

f **cos**

15.55 **g** **→H** **STO** **1**

f **cos**

x

37.03 **g** **→H** **STO** **0**

f **cos**

x

RCL **0** **f** **sin**

RCL **1** **f** **sin**

x

+

g **COS⁻¹**

60 **x**

5.43

-6.58

0.99

15.92

0.96

0.96

37.05

0.80

0.76

0.60

0.27

0.17

0.93

21.92

1315.41

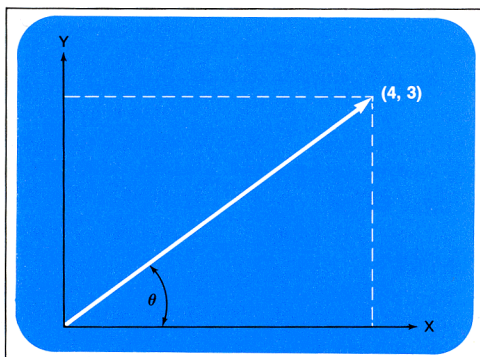
Distance in nautical miles
that Odysseus must sail to
visit Penelope.

Polar/Rectangular Coordinates Conversion

Two functions are provided for polar/rectangular coordinate conversion. To convert values in the X- and Y-registers, (representing rectangular x , y coordinates, respectively) to polar r , θ coordinates (magnitude and angle, respectively), press **9** **↔P**. Magnitude r appears in the X-register and angle θ appears in the Y-register.

Conversely, to convert values in the X- and Y-registers (representing polar r , θ , respectively) to rectangular coordinates (x , y respectively), press **f** **↔R**.

Example 1: Convert rectangular coordinates (4, 3) to polar form with the angle expressed in radians.



Press

9 **RAD**

Display

0.00

Specifies radians mode.
(Assumes no results remain
from previous example.)

3 **ENTER** 4

4.

Rectangular coordinates
placed in X- and Y-registers.

9 **↔P**

5.00

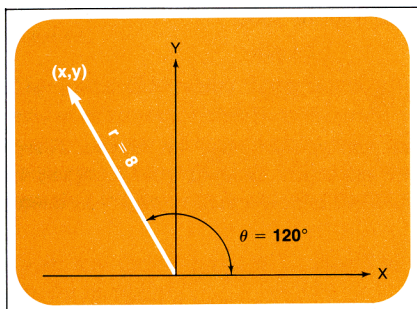
Magnitude r .

x↔y

0.64

Angle θ in radians.

Example 2: Convert polar coordinates $(8, 120^\circ)$ to rectangular coordinates



Press

Display

g **DEG**

0.64

Specifies degrees mode.
(Results from previous example.)

120 **ENTER** 8

8.

Polar coordinates θ and r
placed in Y- and X-registers,
respectively.

f **→R**

-4.00

X-coordinate.

x↔y

6.93

Y-coordinate.

Calculating Percentage Problems

There are three keys on your HP-27 for calculating percentage problems: **%** function is used to find a percentage of a number. The **Δ%** function is used to find percent differences (% increase/decrease). And the **%Σ** function is used to find what percent one number is of another or of the total sum (proportion).

With the HP-27, you don't have to convert percents to their decimal equivalents; 4% need not be changed to .04. It can be keyed in the way you see and say it, 4 **%**

Percentage **%**

To find the percentage of a number, key in that base number and

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press **ENTER** . Then key in the numerical value of the percent and press **%** .

For example, find 14% of \$300:

| Press | Display |
|------------------------------|--------------|
| 300 ENTER 14 % | 42.00 |

Example: Every year you set aside 4% of your company's profits for the employee retirement fund. If your company made a profit last year of \$1,576,432, how much money was contributed to the fund?

| Press | Display |
|---------------------------------|-----------------|
| 1576432 ENTER 4 % | 63057.28 |

Net Amount **%** **+** or **%** **-**

If you buy a new car, you have to figure the sales tax percentage, then add that to the purchase price to find the total cost of the car. It is easy to calculate this net amount with your HP-27 because the calculator holds the base number while you calculate percentages.

For example, if the sales tax on a \$6200 car is 5%, what is the amount of the tax and total cost of the car?

| Press | Display | |
|------------------------------|----------------|---|
| 6200 ENTER 5 % | 310.00 | Percentage amount (sales tax). |
| + | 6510.00 | Net amount (base plus percentage amount). |

If the dealer gives you a 10% discount on the car, what will your total cost be?

| Press | Display | |
|-------------------------------|----------------|---|
| 6200 ENTER 10 % | 620.00 | Amount of discount. |
| - | 5580.00 | Discounted price. |
| 5 % + | 5859.00 | Net amount (discounted price plus sales tax). |

Notice in the last problem that you subtracted a percentage amount and added a percentage amount, without repeating the base number. The calculator stores intermediate answers in the stack until you need them in a calculation.

Percent Difference Between Two Numbers **9** $\Delta\%$

To find the percent difference between two numbers—the ratio of increase or decrease—key in the base number, press **ENTER**, then key in the second number and press **9** $\Delta\%$. Usually, the base amount, or the first number you key in, is the one that occurred first in time.

For example, your rent jumps from \$285 a month to \$335 a month. What percent is the increase?

Press

Display

285 **ENTER** 335 **9** $\Delta\%$ 17.54 % increase.

A positive answer denotes an increase, while a negative answer denotes a decrease.

Example: You forgot to place a stop order, and your stock fell from \$57.50 to \$13.25 a share. What percent is the decrease?

Press

Display

57.50 **ENTER** 13.25 **9** $\Delta\%$ -76.96 %

Markup

Markup is a simple percentage calculation using the $\Delta\%$ function and wholesale or original purchase cost as the base number.

Example: You purchase typewriters at \$159.95 wholesale and retail them for \$195.00. What percent is your markup?

Press

Display

159.95 **ENTER** 195.00 **9** $\Delta\%$ 21.91 % markup.

Perhaps you have a fixed profit rate in mind and wish to establish your retail price. You would solve for the retail price the same as you solved for net amount.

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Example: You purchase several valves @ \$2.26 from the manufacturer and wish to sell them at 25% profit. How much should you mark them up?

Press

2.26 **ENTER** 25 **%**
+

Display

0.57
2.83

Amount of markup.
Retail price.

Margin

Margin also is a simple percentage problem with the **(Δ%)** function, only this time you use the selling price as the base number. Returning to the typewriters that you bought for \$159.95 and sold for \$195, the margin is calculated as follows:

Press

195 **ENTER** 159.95 **g** **(Δ%)** **CHS**

Display

17.97

% margin.

So, the markup is 21.91% and the margin is 17.97%.

Finding Percent of Total **g** **(%Σ)**

(Σ) is the Greek symbol “sigma” which we use to mean sum or total. To find what percentage one number is of the total, add the numbers first by keying in each number followed by **(Σ+)**. Then key in the particular number you wish to convert to a percentage and press **g** **(%Σ)**.

Example 1: You own 150 shares of stock in Sleepy-Head Waterbeds, 52 shares of Flickering Films Inc., and 200 shares of Raucous Records Company. What percent of your portfolio does each represent?

Press

150 **(Σ+)**
52 **(Σ+)**
200 **(Σ+)**
150 **g** **(%Σ)**
52 **g** **(%Σ)**
200 **g** **(%Σ)**

Display

1.00
2.00
3.00
37.31
12.94
49.75

First entry.
Second entry.
Third entry.
% Sleepy-Head Waterbeds.
% Flickering Films.
% Raucous Records.

The Σ^+ key displays the number of entries rather than the numerical value keyed in. When you use the Σ^+ key, press **f** Σ (clear Σ) between problems.

Example 2: Assume that the Sleepy-Head Waterbeds stock is worth \$450; Flickering Films stock, \$1404; and Raucous Records, \$1500. What percent of the total value of your portfolio does each represent?

Press

Display

f Σ

450 Σ^+

1.00

1404 Σ^+

2.00

1500 Σ^+

3.00

Number of entries.

450 **g** $\% \Sigma$

13.42

% Sleepy-Head Waterbeds.

1404 **g** $\% \Sigma$

41.86

% Flickering Films Inc.

1500 **g** $\% \Sigma$

44.72

% Raucous Records Co.

Proportions **g** $\% \Sigma$

To find what percent one number is of another (proportion), state the problem as “A is what percent of B?” Key in B (the base number), press Σ^+ , then key in A (the number to be converted), and press **g** $\% \Sigma$. (An alternative solution would be to key in B, press **STO** 5, key in A, and press **g** $\% \Sigma$.)

For example, 64 is what percent of 340?

Press

Display

f Σ

340 Σ^+

1.00

64 **g** $\% \Sigma$

18.82

%

Or the alternative procedure:

Press

Display

f Σ

340 **STO** 5

340.00

64 **g** $\% \Sigma$

18.82

%

Example: To purchase that \$47,000 lakefront cabin that you've had your eye on, a \$9400 down payment is required. What percentage of the price does your down payment represent?

Press

Display

47000

1.00

9400

20.00

%

The Financial Function Keys

Your HP-27 has the most frequently used business calculations (including the associated formulas) preprogrammed into the second row of keys:

The nature of these calculations is to use known data and solve for an unknown value. Three of these values must be entered to solve for one or both of the other values.

The *n* value represents the total number of compounding or payment periods. To enter 30 years, press 30 . If you wish to input that in monthly periods, press 30 12 .

The *i* value is the interest rate per period. If interest is expressed as an annual rate but your problem involves monthly payments or daily compounding, then you must convert the annual rate to rate per period. To key in 9% annual interest, press 9 . To input the monthly rate, press 9 12 . To input the daily rate, press 9 365 .

Remember that and *must* correspond to the same time frame. If *n* is months, then *i* must be the monthly interest rate.

The function stands for payment or deposit amount. It assumes equal payments and must correspond to the same time frame as or . Don't mix monthly payments with annual periods or daily interest.

The function stands for present value or the amount of money at the start of the transaction.

The **FV** function represents the future value of money or the amount you will obtain/pay at the end of the term.

Both **n** and **i** are involved in all financial calculations. The nature of financial calculations on the HP-27 requires that given three of the above values (including **n** or **i**), you can solve for a fourth value. You can enter the values in any order.

Displaying Financial Values

To see what number is in a particular financial register, press **RCL** **f** then the desired location key (**n**, **i**, **PMT**, **PV**, or **FV**).

Changing Financial Values

You may solve a problem and want to explore a different alternative by changing only one of the three known values—perhaps using a different interest rate or a greater/smaller payment amount.

With the HP-27 you don't have to re-enter the numbers all over again. Simply key in the new value and press the appropriate financial function key. The new value will be substituted for the old value.

Example: You borrow \$9000 at 9½% interest, to put a new swimming pool in the backyard. What will your monthly payments be if you pay the loan in 2 years?

Press

g **RESET**

2 **ENTER** 12 **x** **f** **n**

9.5 **ENTER** 12 **÷** **f** **i**

9000 **f** **PV**

f **PMT**

Display

24.00

0.79

9000.00

413.23

Number of months.

Interest rate/month.

Amount of your loan.

Your monthly payment.

What if you take out a 3-year loan?

Press

3 **ENTER** 12 **x** **f** **n**

f **PMT**

Display

36.00

288.30

Number of months.

Alternative monthly payment.

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This is considered an alternative of the same problem because given n , i and PV , you solved for payment more than once.

If you decide “but I can only afford \$200 a month, what would my loan be?”—that’s a new problem. It requires a different set of three input values (now n , i and PMT to solve for PV), so you solve it as a new problem.

Press **9** **RESET** between problems to reset the five financial keys for new values. Your previous answer is left on the display so that you can use it again or store it, if you wish. (For a detailed explanation, refer to appendix B.)

The following examples quickly illustrate use of the financial keys. Section 4 describes how to solve loan, savings, lease, and investment problems.

Solving for n

Given the periodic interest rate (i) and two of the following: initial principal (PV), periodic payment (PMT), or future value (FV), you can calculate the number of periods in a compound amount or a loan.

Example 1: Fur trapper Bill Buckskin wishes to buy a \$22,000 log cabin. A local bank offered to loan Bill the \$22,000 at $9\frac{1}{4}\%$ interest. Making \$200 monthly payments, how long will it take Bill to repay his mortgage?

Press

9 **RESET**

9.25 **ENTER** 12 **÷** **f** **i**

200 **f** **PMT**

22000 **f** **PV**

f **n**

12 **÷**

Display

0.77 Monthly interest rate.

200.00 Monthly payment.

22000.00 Amount of loan.

245.26 Months.

20.44 Years.

Example 2: With an eye toward the future, you put \$5,000 into an education fund for your child. If the money earns 6% interest compounded daily, how many years before the fund reaches \$10,000?

Press

g **RESET**
 5000 **f** **PV**
 6 **ENTER** 365 **÷** **f** **i**
 10000 **f** **FV**
f **n**
 365 **÷**

Display

| | |
|----------|----------------------|
| 5000.00 | Initial principal. |
| 0.02 | Daily interest rate. |
| 10000.00 | Desired amount. |
| 4216.98 | Days. |
| 11.55 | Years. |

Solving for i

Given the number of periods and two of the following: initial principal (PV), periodic payment (PMT) or future value (FV), you can calculate the periodic annual interest rate, annual yield, or rate of return.

Example: What is the annual interest rate on a 25-year \$32,500 mortgage with \$230 monthly payments?

Press

g **RESET**
 25 **ENTER** 12 **×** **f** **n**
 230 **f** **PMT**
 32500 **f** **PV**
f **i**
 12 **×**

Display

| | |
|----------|--------------------------|
| 300.00 | Number of months. |
| 230.00 | Monthly payment. |
| 32500.00 | Amount of loan. |
| 0.58 | % monthly interest rate. |
| 7.01 | % annual interest rate. |

Solving for PMT

Given the number of periods, the interest rate and either present value (PV) or future value (FV), you can calculate the payment amount.

Example: Find the monthly payments on a 30-year, \$47,000 mortgage at 8.5% annual interest rate.

Press

g **RESET**
 30 **ENTER** 12 **×** **f** **n**

Display

| | |
|--------|-------------------|
| 360.00 | Number of months. |
|--------|-------------------|

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| | | |
|--|-----------------|--------------------------|
| 8.5 ENTER 12 ÷ f i | 0.71 | % monthly interest rate. |
| 47000 f PV | 47000.00 | Principal of loan. |
| f PMT | 361.39 | Monthly payment. |

Solving for PV

Given the number of periods, the interest rate, and either payment (PMT) or future value (FV), you can calculate the initial principal.

Example: Junior Engineer wants to buy an oscilloscope. He can afford \$60 a month for 36 months and is willing to pay 9% annual interest. How much can he afford to pay for the scope?

Press

g **RESET**

36 **f** **n**

9 **ENTER** 12 **÷** **f** **i**

60 **f** **PMT**

f **PV**

Display

36.00

Number of months.

0.75

% monthly interest rate.

60.00

Monthly payment.

1886.81

Maximum he can afford.

Solving for FV

Given the number of periods, the interest rate, and either the payment (PMT) or the initial amount (PV), you can calculate the amount of money you will pay/receive in the future.

Example: A house purchased 3 years ago for \$42,500 is located in an area where property values are appreciating at 12% a year. What is the current approximate value of the house?

Press

g **RESET**

3 **f** **n**

12 **f** **i**

42500 **f** **PV**

f **FV**

Display

3.00

Number of years.

12.00

% annual interest.

42500.00

Original price.

59709.44

Current value.

There are two other financial keys on your HP-27: **g** **NPV** solves for net present value, and **g** **IRR** solves for internal rate of return. Both of these are sophisticated methods of evaluating investments and are explained in *Discounted Cash Flow Analysis*, section 4.

Statistical Functions

The following keys are used in statistical calculations:

- Σ+** Sigma plus. Totals sums, products and squares of numbers.
- f** **Σ-** Sigma minus. Subtracts a sum, product or square from the summation in **Σ+**. Useful for subtracting out an incorrect entry.
- g** **L.R.** Linear regression. Plots a straight line that best fits a set of data points by finding the slope and y-intercept.
- ŷ** Linear estimate. Given a value for x, this computes the predicted value for y or where the point will fit on a plotted linear function.
- g** **r** Correlation coefficient measures goodness of fit for linear regression data.
 $r = +1$ (perfect fit, positive slope)
 $r = -1$ (perfect fit, negative slope)
 $r = 0$ (no fit)
- g** **N.D.** Normal distribution. Calculates the density function and the upper tail area under a normal distribution curve.
- ̄x** Mean. Computes the mean or arithmetic average for both x and y.
- g** **s** Standard deviation. Calculates the dispersion around the mean for both x and y.
- g** **VAR** Variance. Computes the variances of x and y, such that:

$$\text{VAR}_x = (s_x)^2 \quad \text{and} \quad \text{VAR}_y = (s_y)^2$$

When completely finished with a statistical problem and before starting a new problem, press **f** **Σ**. This clears old data from the statistical registers.

Summations $\Sigma+$

Pressing the $\Sigma+$ key automatically gives you several different sums and products of values in the X- and Y-registers. The calculator stores these values in registers R_4 thru R_9 .

When you key in one or two numbers and press $\Sigma+$, the following happens:

1. The number of entries is stored in register R_4 .
2. The number (x) on the display is added to the contents of storage register R_5 .
3. The square (x^2) of the display number is added to the contents of register R_6 .
4. The number in the Y-register of the stack is added to the contents of storage register R_7 .
5. The square of y (y^2) is added to the contents of register R_8 .
6. The product of x and y (xy) is added to the contents of register R_9 .

When you input paired data (x and y), you must key in the y value first and separate the two numbers by $\text{ENTER}+$. The general rule is:

$$y \text{ value } \text{ENTER}+ x \text{ value } \Sigma+$$

To recap, this is where values are stored inside your calculator:

| Register | Contents |
|----------------------|--|
| R_4 | Number of entries (n). |
| R_5 | Summation of x values (Σx). |
| R_6 | Summation of x^2 values (Σx^2). |
| R_7 | Summation of y values (Σy). |
| R_8 | Summation of y^2 values (Σy^2). |
| R_9 | Summation of products of x and y values (Σxy). |
| Display (X-register) | Number of entries (n). |

Immediately, you have a powerful data bank for statistical calculations. To see any of these summations at any time, simply recall the

contents of the desired register. Remember, when you press **RCL** 4 thru 9, you will not lose that data. Only a copy of the number (sum) appears in the display. (Note that the n in register R₄ has nothing to do with the financial n.)

Deleting and Correcting Data

If you key an incorrect entry into **Σ+**, you don't have to start over again. If you keyed it before pressing **Σ+**, simply press **CLX** to clear the display, then continue on with the correct value.

If you had already added in the wrong value, simply key in that wrong number, subtract it out by pressing **f** **Σ-**, then continue with the correct number.

This applies to two variables, as well as one. Suppose you key in 10 **ENTER** 20 **Σ+** and discover that the y value is wrong. Delete the data pair by pressing 10 **ENTER** 20 **f** **Σ-**, then continue with the correct numbers.

Linear Regression **L.R.**

Linear regression is a statistical method for finding a straight line that best fits a set of data points, thus providing a relationship between two variables. If there is equal time or space between data points, then this is called a trend line.

Naturally, at least two data points must be in the machine before a line can be drawn or fitted to them. After you have totaled the data points using the **Σ+** key, you can calculate the coefficients of the linear equation

$$y = A + Bx$$

by pressing **9** **L.R.** A is the y-intercept, and appears in the display. B represents the slope of the line and is stored in the Y-register.

Example: A commercial land appraiser has examined six vacant lots in a local community, all of which have the same depths but different frontages and values. Based on the following input data, what is the relationship between frontage and lot value?

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Input Data:

| (x) Lot frontage (feet) | (y) Lot value (\$) |
|----------------------------|-----------------------|
| 70.8 | 10100 |
| 60.0 | 9000 |
| 85.0 | 12700 |
| 75.2 | 11120 |
| 69.5 | 11000 |
| 84.0 | 12500 |

Accumulate the data using Σ^+ . (Remember that when you enter two values, x and y, you must enter the y value first.)

Press

f Σ

10100 ENTER 70.8 Σ^+

9000 ENTER 60 Σ^+

12700 ENTER 85 Σ^+

11120 ENTER 75.2 Σ^+

11000 ENTER 69.5 Σ^+

12500 ENTER 84 Σ^+

g L.R.

$\text{x}\hat{y}$

Display

1.00

First entry.

2.00

Second entry.

3.00

Third entry.

4.00

Fourth entry.

5.00

Fifth entry.

6.00

Sixth entry.

393.90

A = y-intercept.

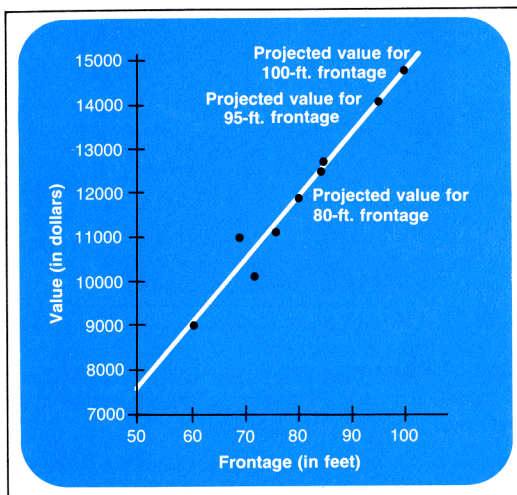
144.11

B = slope of line.

Thus, the equation of the regression line is:

$$y = 393.90 + 144.11 x$$

The y-intercept value represents the projected value for $x = 0$. The slope indicates the change in the projected value caused by an incremental change in the x value. Plotting this example, you see that a 1-foot increase in the frontage results in a projected increase in value of \$144.11.



Correlation Coefficient r

In the diagram, the solid line is the best fit for the given data points.

It's a good idea to check the "goodness of fit" of the linear function by calculating the correlation coefficient. This tells you how close to a straight line the data points lie. Since all the data is in your HP-27, to calculate the correlation coefficient,

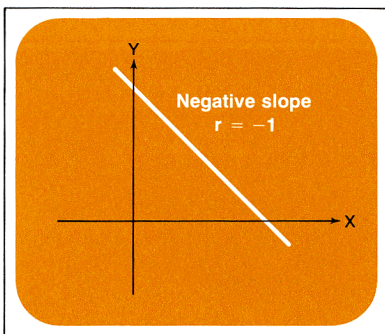
Press

g **r**

Display

0.97

If $r = +1$, then the line has a positive slope and the data fits perfectly. If $r = -1$, the data still is a perfect fit but the line has a negative slope, like so:




(An example of a negative trend line is declining property values or declining sales.) If $r = 0$, the data values are spread out all over and do not come close to a straight line.

In the example of lot frontage related to value, the correlation coefficient is close to 1, so we can feel comfortable using linear regression. Suppose, though, that the correlation coefficient was not close to 1 but instead was 0.5 or 0.6. This would indicate that a straight line is not a very good fit to the data. Then you might try to fit the data to a curve.

In the statistical applications in section 4, three different types of curve fits are described: exponential, logarithmic, and power. A correlation coefficient may be calculated for each of these curves and should be interpreted similarly: if r is close to ± 1 , the curve is a reasonable approximation to the data; if not, try a different fit.

Linear Estimate

Having plotted a line, you can quickly estimate other values. With the data totaled in registers R_4 thru R_9 , a predicted y (designated \hat{y}) can be calculated by keying in an x value and pressing .

Example 1: For the previous example, find projected values for 80-, 95-, and 100-foot frontages.

| Press | Display | |
|---------------|-----------------|------------------------------------|
| 80 \hat{y} | 11922.65 | 80-foot frontage projected value. |
| 95 \hat{y} | 14084.29 | 95-foot frontage projected value. |
| 100 \hat{y} | 14804.83 | 100-foot frontage projected value. |

To find an estimated value, it is not necessary to calculate $\boxed{\text{L.R.}}$ first. In the next example, you key in the known data and solve for a projected unknown.

Example 2: You bought a house 3 years ago for \$47,500. The first year it appreciated \$5,000. The second year its value rose to \$60,000. Today you figure the market price to be \$64,000 if you were to sell. What will your house be worth next year?

| Press | Display |
|-----------------------------------|-------------|
| f Σ | |
| 47500 ENTER $\Sigma+$ 1 $\Sigma+$ | 1.00 |
| 52500 ENTER $\Sigma+$ 2 $\Sigma+$ | 2.00 |
| 60000 ENTER $\Sigma+$ 3 $\Sigma+$ | 3.00 |
| 64000 ENTER $\Sigma+$ 4 $\Sigma+$ | 4.00 |

To make a projection for next year (year 5), simply solve for \hat{y} :

| Press | Display |
|-------------|-----------------|
| 5 \hat{y} | 70250.00 |

Mean

Your HP-27 can quickly calculate the means or arithmetic averages of two variables. Whether it's the average of tests scores or last month's sales figures, given one or two sets of numbers, your HP-27 will calculate the mean of those samples.

1. Press f Σ (Clear Σ).

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- If you are summing *one* set of numbers, key in the first number and press $\Sigma+$; then the second number and $\Sigma+$ again; the third number, etc. Continue until you have entered all the values.
- If you are summing *two* sets of numbers, key in the y value and press $\text{ENTER} \blacktriangle$; key in the x value, then press $\Sigma+$. Key in the second y value, press $\text{ENTER} \blacktriangle$, key in the second x value, and press $\Sigma+$. Continue until you have entered all the values.
- Press \bar{x} for the mean of the x values.
- Press $\bar{x} \div \bar{y}$ for the mean of the other set of values (\bar{y}).

Example: A survey of seven salesmen in your company reveals that they work the following hours a week and sell the following dollar volume each month. How many hours does the average salesman work each week? How much does the average salesman sell each month?

| Salesman | Hours/Week | Sales/Month |
|----------|------------|-------------|
| 1 | 32 | \$17,000 |
| 2 | 40 | \$25,000 |
| 3 | 45 | \$26,000 |
| 4 | 40 | \$20,000 |
| 5 | 38 | \$21,000 |
| 6 | 50 | \$28,000 |
| 7 | 35 | \$15,000 |

To find the average workweek and sales of this sample:

Press

Display

f Σ

Storage registers cleared.

32 $\text{ENTER} \blacktriangle$ 17000 $\Sigma+$
 40 $\text{ENTER} \blacktriangle$ 25000 $\Sigma+$
 45 $\text{ENTER} \blacktriangle$ 26000 $\Sigma+$
 40 $\text{ENTER} \blacktriangle$ 20000 $\Sigma+$
 38 $\text{ENTER} \blacktriangle$ 21000 $\Sigma+$
 50 $\text{ENTER} \blacktriangle$ 28000 $\Sigma+$

1.00
 2.00
 3.00
 4.00
 5.00
 6.00

First entry.

Second entry.

35 **ENTER** 15000 **Σ+**

7.00

Total number of entries in each sample.

 \bar{x}

21714.29

Mean dollar sales/month (\bar{x}). **$\bar{x}\bar{y}$**

40.00

Mean workweek in hours (\bar{y}).

Note that x and y values overwrite the X- and Y-registers.

Standard Deviation

You already have data from the previous example in **Σ+** so to calculate the standard deviation (a measure of dispersion around the mean), simply:

Press

Display

g **S**

4820.59

Dollars (s_x) **$\bar{x}\bar{y}$**

6.03

Hours (s_y)

The HP-27 calculates standard deviation according to the formulas:

$$s_x = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}} \qquad s_y = \sqrt{\frac{\sum y^2 - \frac{(\sum y)^2}{n}}{n - 1}}$$

Notice that the seven salesmen that we used is a sample. If we used *all the salesmen*, then the data would be considered a population rather than a sample.

The relationship between *sample* standard deviation (s) and *population* standard deviation (s') is given by:

$$s' = s \sqrt{\frac{n - 1}{n}}$$

Since n (number of entries) is stored in register R_4 , you can convert sample standard deviation to population standard deviation. You already have (s) **6.03** on the display, so simply:

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Press

Display

\bar{x}

21714.29

Mean (dollars).

$\Sigma+$

8.00

Number of entries + 1.

σ s

4463.00

s'_x

$x \div y$

5.58

s'_y

To continue summing data pairs, press \bar{x} f $\Sigma-$ before entering more data.

Variance

The variance of a set of data is the square of the standard deviation. Just as you can calculate the mean and standard deviation of two variables, you can compute the variances of both x and y :

$$\text{VAR}_x = (s_x)^2$$

$$\text{VAR}_y = (s_y)^2$$

Example: Eight students have the following I.Q.s and received the following grades on their chemistry final. What are the mean I.Q. and exam scores? What is the variance of each?

| I.Q. | Exam Grade |
|------|------------|
| 112 | 75 |
| 142 | 78 |
| 120 | 85 |
| 110 | 78 |
| 120 | 88 |
| 132 | 95 |
| 116 | 60 |
| 114 | 80 |

Press

Display

f Σ

112 ENTER 75 $\Sigma+$

1.00

First data pair.

142 **ENTER** 78 **Σ^+** 120 **ENTER** 85 **Σ^+** 110 **ENTER** 78 **Σ^+** 120 **ENTER** 88 **Σ^+** 132 **ENTER** 95 **Σ^+** 116 **ENTER** 60 **Σ^+** 114 **ENTER** 80 **Σ^+** **\bar{x}** **$x \div y$** **9** **VAR** **$x \div y$** **8.00**

Last data pair.

79.88

Mean (exam scores).

120.75

Mean (I.Q.)

106.70

Variance of x (exam scores).

119.93

Variance of y (I.Q.).

Just as you were able to calculate the standard deviation of a population as well as a sample, you can also calculate population variance. Press **\bar{x}** **Σ^+** **9** **VAR**.

Press**Display** **\bar{x}** **79.88**

Mean (exam scores).

 Σ^+ **9.00**

Number of entries +1.

9 **VAR****93.36**

Population variance (exam scores).

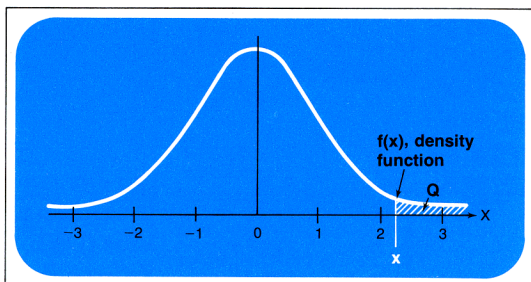
 $x \div y$ **104.94**

Population variance (I.Q.).

To continue summing data pairs, press **\bar{x}** **f** **Σ^+** before entering more data.

Normal Distribution

The **$N.D.$** key calculates the density function $f(x)$ and the upper tail area Q under a standardized normal distribution curve, given x . The density function is the relative probability that a particular x value will occur. The upper tail area signifies the probability of occurrence of all values $\geq x$. The value of Q is returned to the X-register and the value $f(x)$, to the Y-register.



The value input to **[N.D.]** is the standard variable. Given the mean (\bar{x}) and standard deviation (s) of a normal population, any value x' is converted to the standard variable by the following formula:

$$x = \frac{x' - \bar{x}}{s}$$

Example: The mean weight of 500 male students is 151 lbs. and the standard deviation is 15. Assuming that the weights are normally distributed, how many students weigh 185 or more?

First, find x according to the following formula:

$$x = \frac{x' - \bar{x}}{s} = \frac{185 - 151}{15}$$

Press

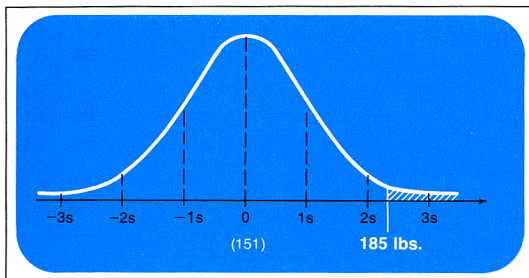
185 **[ENTER]** 151 **[=]**

15 **[÷]**

Display

2.27

This means the standard variable (185 lbs) is 2.27 standard deviations to the right of the mean.



You already have 2.27 in the display, so to continue. . .

Press

g **N.D.**

f **FIX** 4

x²y

Display

0.01

0.0117

0.0306

Upper tail area (Q).

Displayed to four places.

Density function.

To find approximately how many students weigh 185 or more, multiply the total number in the sample by the factor 0.0117.

$$500 \times 0.0117 = 5.85 \text{ students}$$

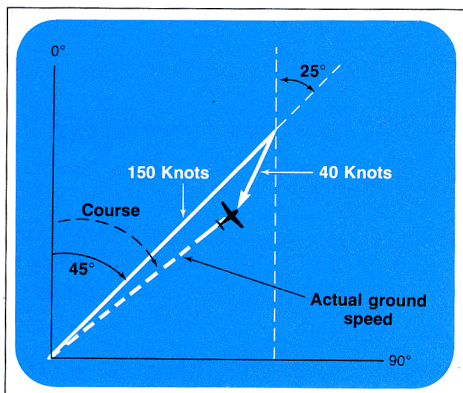
Vector Summations

You have learned rectangular to polar coordinate conversions and you have learned how to sum quantities using Σ^+ . Combining these functions, you can perform vector addition and subtraction.

Example: In his converted Swordfish aircraft, bush pilot Apeneck Sweeney reads an air speed of 150 knots and a heading of 045° from his instruments. The Swordfish is also being buffeted by a headwind of 40 knots from a bearing of 025° . What should Sweeney use to have a true heading of 45° ? What is the ground speed of Swordfish?

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Method: The true heading and ground speed are equal to the sum of the instrument vector and the wind vector. The vectors are converted to rectangular coordinates and summed using the $\Sigma+$ and $\Sigma-$ keys. Their sum is recalled by recalling the values in storage registers R_5 (Σx) and R_7 (Σy), and the new rectangular coordinates are then converted back to polar coordinates to give the vector of the actual ground speed and true heading.



Press

f **REG**

Display

0.00

Clears storage registers. (Display assumes no results remain from previous examples.)

g **DEG**

0.00

Sets degrees mode.

45 **ENTER**

45.00

θ for the Swordfish instrument vector.

150

150.

r for the Swordfish instrument vector.

f **→R**

106.07

Converted to rectangular coordinates.

$\Sigma+$

1.00

25 ENTER

25.00

40

40.

f $\rightarrow R$

36.25

f Σ^-

0.00

RCL 7

89.16

RCL 5

69.81

g $\rightarrow P$

113.24

 $x \leftrightarrow y$

51.94

Instrument coordinates accumulated in storage registers R_5 and R_7 .

θ for wind vector.

r for wind vector.

Converted to rectangular coordinates.

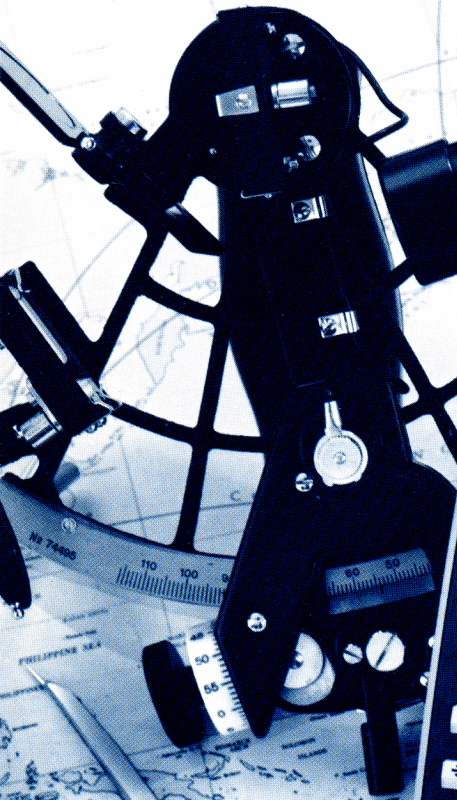
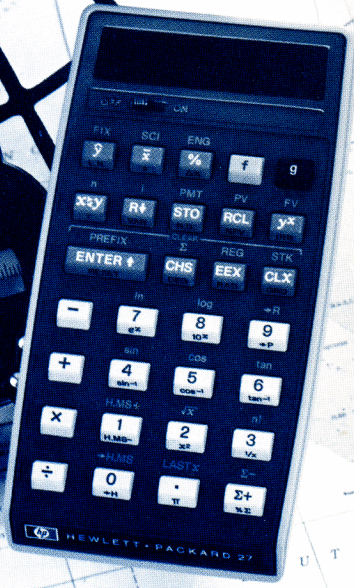
Coordinates for wind vector subtracted from coordinates for Swordfish's instrument vector.

Recalls sum of y-coordinates from register R_7 .

Recalls sum of x-coordinates from register R_5 . (Sum of y-coordinates lifted to Y-registers.)

Actual ground speed of the Swordfish in knots.

True heading of the Swordfish in degrees.



Applications

This section covers both common and complex problems encountered in the fields of statistics, mathematics, navigation, surveying, and finance. Problems, such as these, demonstrate the amazing versatility of your HP-27.

Statistical Applications

Permutations

A permutation of m different objects, taken n at a time, is an *arrangement* of n out of m objects with several possible ways of ordering the arrangement.

$${}_m\text{Permutation}_n = \frac{m!}{(m - n)!}$$

where m and n are integers and $0 \leq n \leq m \leq 69$.

For example, how many ways can 10 people be seated on a bench if only 4 seats are available?

$${}_{10}P_4 = \frac{10!}{6!} = 5040$$

Note:

$${}_mP_0 = \frac{m!}{m!} = 1$$

$${}_mP_1 = \frac{m!}{(m - 1)!} = m$$

$${}_mP_m = m!$$

The keystrokes are as follows:

1. Key in the value m , and press \boxed{f} $\boxed{n!}$ \boxed{f} $\boxed{\text{LAST } x}$.
2. Key in the value n .
3. Press $\boxed{=}$ \boxed{f} $\boxed{n!}$ $\boxed{\div}$.

Example: A child has a set of toy blocks with the 26 letters of the alphabet each appearing once. How many different three-letter permutations (${}_{26}P_3$) can be made with these blocks?

Press

26 \boxed{f} $\boxed{n!}$ \boxed{f} $\boxed{\text{LAST } x}$

3 $\boxed{=}$ \boxed{f} $\boxed{n!}$ $\boxed{\div}$

Display

15600.00

Number of
permutations.

Combinations

A combination of m different objects taken n at a time is a *selection* of n out of m objects with no attention given to the order of arrangement. The number of possible combinations is given by:

$${}_mC_n = \frac{m!}{(m - n)! n!}$$

where m and n are integers and $0 \leq n \leq m \leq 69$.

In the permutation example, you arranged 10 people in different order, four at a time. If you were to *select* four out of the same 10 people for a committee, how many combinations could you pick?

$${}_{10}C_4 = \frac{10!}{6! 4!} = 210$$

Note:

$${}_mC_0 = \frac{m!}{m!} = 1$$

$${}_mC_1 = \frac{m!}{(m - 1)! 1!} = m$$

The following keystrokes give the number of possible combinations:

1. Key in the value m . Press **f** **n!** **f** **LAST x**.
2. Key in the value n .
3. Press **=** **f** **LAST x** **f** **n!**.
4. Press **x²y** **f** **n!** **x** **÷**.

Example: A manager wants to choose a committee of three people from the seven engineers working for him. In how many different ways can the committee be selected?

Press

Display

7 **f** **n!** **f** **LAST x**

3 **=** **f** **LAST x** **f** **n!**

x²y **f** **n!** **x** **÷**

35.00

The committee may be chosen in ${}_7C_3 = 35$ different ways.

* * * * *

In section 3, linear regression was used as an analysis tool for plotting and projecting values. But perhaps your data doesn't fit a straight line. The three applications that follow are the methods of fitting your data to three different types of curves: exponential, logarithmic and power curves.

Exponential Curve Fit

The exponential curve is representative of situations where an increase or decrease in a quantity is "compounded" over time; e.g., financial growth curve, compounded amount, or radioactive decay.

Using natural logarithms, a least squares exponential curve fit may be calculated according to the equation $y = ae^{bx}$. The keystrokes given here compute the estimates of the constants a and b by rewriting the equation $y = ae^{bx}$ as

$$\ln y = bx + \ln a$$

and solving this equation as a linear regression problem. (The y values must be positive.) Here, a is the y -intercept.

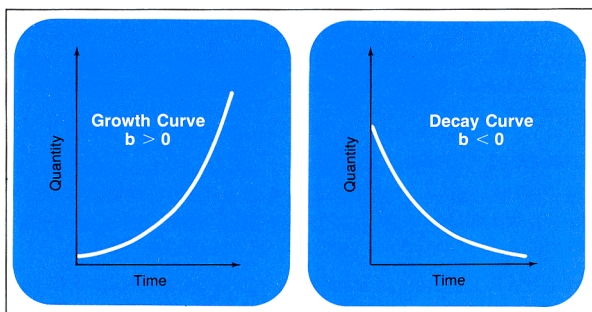
$$b = \frac{\sum x_i \ln y_i - \frac{1}{n} (\sum x_i)(\sum \ln y_i)}{\sum x_i^2 - \frac{1}{n} (\sum x_i)^2}$$

$$a = \exp \left[\frac{\sum \ln y_i}{n} - b \frac{\sum x_i}{n} \right]$$

$$\hat{y} = ae^{bx}$$

$$x = \frac{1}{b}(\ln y - \ln a)$$

If $b > 0$, you will have a growth curve. If $b < 0$, you will have a decay curve. Examples of these are given below:



The keystrokes are as follows:

1. Press **f** **Σ**.
2. For each pair of values, key in y , press **f** **ln**, key in x , press **Σ+**. Repeat this step for all data pairs.

3. Press g L.R. g e^x . The value of a is displayed.
4. Press $x \div y$ to obtain the slope of the curve.
5. Press g e^x 1 $=$ 100 \times to obtain the percentage growth rate.
6. Press g r to obtain the correlation coefficient.
7. To solve for a projected value of y , key in the x value and press \hat{y} g e^x . The value \hat{y} is displayed.
8. To find a projected x value, press g L.R. . Key in the y value, and press f \ln $x \div y$ $=$ $x \div y$ \div . The value \hat{x} is displayed.

Example 1: A stock's price history is listed below. If the stock continues this growth rate, what is the price projected to be at the end of 1976 (year 5)? (*Hint: Let y be the price of the stock and x , the year. Solve for \hat{y} .*)

| End of Year | Price |
|-------------|----------------|
| 1972 (1) | 52-1/2 |
| 1973 (2) | 55-1/4 |
| 1974 (3) | (missing data) |
| 1975 (4) | 61 |
| 1976 (5) | ? |

Press

f Σ

52.5 f \ln 1 $\Sigma+$

55.25 f \ln 2 $\Sigma+$

61 f \ln 4 $\Sigma+$

g L.R. g e^x

$x \div y$

g e^x 1 $=$ 100 \times

5 \hat{y} g e^x

Display

1.00

2.00

3.00

49.96

0.05

5.12

64.14

First data pair input.

Second data pair input.

Third data pair input.

Value of a .

Slope of curve.

% growth rate.

Projected price at the end of year 1976.

Example 2: A company's growth over a period of several years is measured by its net earnings, in thousands of dollars, at the end of each fiscal year. The data below shows this growth:

| Fiscal year | 1 | 2 | 3 | 4 | 5 | 6 | ? |
|----------------|-------|-------|-------|-------|-------|-------|------|
| Earnings (K\$) | 179.6 | 215.1 | 260.7 | 301.2 | 368.5 | 424.1 | 1000 |

If the earnings (y) are assumed to be growing exponentially with time (x), at what time in the future will earnings hit \$1 million? (Here you are solving for \hat{x} .)

Pressf Σ 179.6 f ln 1 $\Sigma+$ 215.1 f ln 2 $\Sigma+$ 260.7 f ln 3 $\Sigma+$ 301.2 f ln 4 $\Sigma+$ 368.5 f ln 5 $\Sigma+$ 424.1 f ln 6 $\Sigma+$ g L.R. g e^x $x \div y$ g e^x 1 \square 100 \square

g r

g L.R.

1000 f ln $x \div y$ \square $x \div y$ \div **Display**

1.00

First data pair.

2.00

3.00

4.00

5.00

6.00

152.40

Value of a .

0.17

Slope of curve.

18.89

% growth rate.

1.00

Correlation coefficient.

10.87

Years.

After 10.87 years, earnings will be \$1000K or a million dollars.

Logarithmic Curve Fit

If your data doesn't fit linear regression or an exponential curve, try logarithmic curve fit. This is calculated according to the equation $y = a + b (\ln x)$, and all x values must be positive.

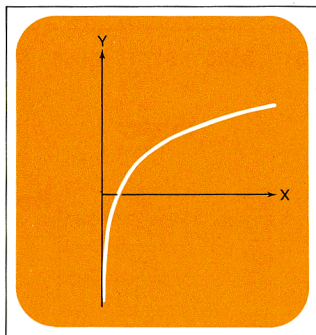
$$b = \frac{\sum y_i \ln x_i - \frac{1}{n} \sum \ln x_i \sum y_i}{\sum (\ln x_i)^2 - \frac{1}{n} (\sum \ln x_i)^2}$$

$$a = \frac{1}{n} (\sum y_i - b \sum \ln x_i)$$

$$\hat{y} = a + b (\ln x)$$

$$x = e^{[(y - a)/b]}$$

A typical logarithmic curve is shown below.



The following keystrokes calculate logarithmic curve fit according to the equation $y = a + b(\ln x)$:

1. Press **f** **Σ**.
2. Key in the first y value, press **ENTER**. Key in the first x value, press **f** **ln** **Σ+**. Repeat this step for each data pair.
3. After all data pairs are input, press **9** **L.R.** to obtain a in the equation above.
4. Press **x↔y** to obtain b.

- Press $\boxed{9}$ \boxed{r} to obtain the correlation coefficient.
- To solve for a projected y value, key in the x value, and press \boxed{f} $\boxed{\ln}$ $\boxed{\hat{y}}$.
- To obtain a projected x value, press $\boxed{9}$ $\boxed{\text{L.R.}}$. Key in the y value, and press $\boxed{x \hat{z} y}$ $\boxed{-}$ $\boxed{x \hat{z} y}$ $\boxed{\div}$ $\boxed{9}$ $\boxed{e^x}$. The value \hat{x} is displayed.

Example: A manufacturer observes declining sales of a soon-to-be obsoleted product, of which there were originally 10,000 units in inventory. The cumulative sales figures over a number of months, given below, may be fit by a logarithmic curve of the form $y = a + b \ln x$, where y represents cumulative sales in units and x the number of months since the beginning. Find what month the last unit (number 10,000) is projected to be sold.

| Month | 1 | 2 | 3 | 4 | 5 | 6 |
|--------------------------|------|------|------|------|------|------|
| Cumulative Sales (units) | 1431 | 3506 | 5177 | 6658 | 7810 | 8592 |

Press \boxed{f} $\boxed{\Sigma}$ 1431 $\boxed{\text{ENTER} \uparrow}$ 1 \boxed{f} $\boxed{\ln}$ $\boxed{\Sigma+}$ 3506 $\boxed{\text{ENTER} \uparrow}$ 2 \boxed{f} $\boxed{\ln}$ $\boxed{\Sigma+}$ 5177 $\boxed{\text{ENTER} \uparrow}$ 3 \boxed{f} $\boxed{\ln}$ $\boxed{\Sigma+}$ 6658 $\boxed{\text{ENTER} \uparrow}$ 4 \boxed{f} $\boxed{\ln}$ $\boxed{\Sigma+}$ 7810 $\boxed{\text{ENTER} \uparrow}$ 5 \boxed{f} $\boxed{\ln}$ $\boxed{\Sigma+}$ 8592 $\boxed{\text{ENTER} \uparrow}$ 6 \boxed{f} $\boxed{\ln}$ $\boxed{\Sigma+}$ $\boxed{9}$ $\boxed{\text{L.R.}}$ $\boxed{x \hat{z} y}$ $\boxed{9}$ \boxed{r} $\boxed{9}$ $\boxed{\text{L.R.}}$ 10000 $\boxed{x \hat{z} y}$ $\boxed{-}$ $\boxed{x \hat{z} y}$ $\boxed{\div}$ $\boxed{9}$ $\boxed{e^x}$ **Display**1.00 First data pair.2.003.004.005.006.001066.15 Value of a .4069.93 Value of b .0.99 Correlation coefficient.8.98 Months.

Considering that you start at 0 and 0.9 is near the end of the first month (it's like centuries), then $\hat{x} = 8.98$ means that the 10,000th unit will be sold near the end of the ninth month.

Power Curve Fit

Another method of analysis is the power curve or geometric curve. The equation $y = ax^b$ is linearized to the form

$$\ln y = \ln a + b (\ln x)$$

and values a and b are computed by calculations similar to linear regression.

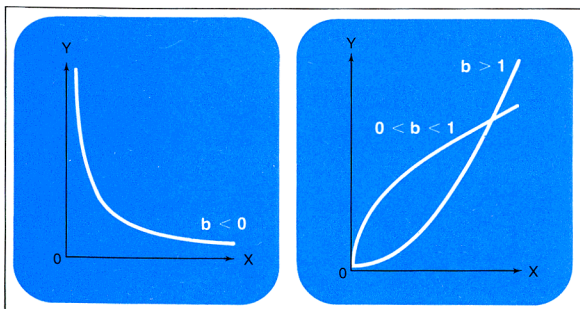
$$b = \frac{\sum (\ln x_i) (\ln y_i) - \frac{(\sum \ln x_i) (\sum \ln y_i)}{n}}{\sum (\ln x_i)^2 - \frac{(\sum \ln x_i)^2}{n}}$$

$$a = \exp \left[\frac{\sum \ln y_i}{n} - b \frac{\sum \ln x_i}{n} \right]$$

$$\hat{y} = ax^b$$

$$\hat{x} = e^{[(y - \ln a)/b]}$$

Some examples of power curves are shown below.



The following keystrokes fit a power curve according to the equation $\ln y = a + b (\ln x)$:

1. Press **f** **Σ** .
2. Key in the first y value, press **f** **\ln** . Key in the first x value, press **f** **\ln** **$\Sigma+$** . Repeat this step for all data pairs.
3. Press **g** **L.R.** **g** **e^x** to obtain a in the above equation.
4. Press **$x \div y$** to obtain b.
5. Press **g** **r** to obtain the correlation coefficient.
6. To find a projected y, key in the x value and press **\hat{y}** **g** **e^x** to obtain \hat{y} .
7. To find a projected x, press **g** **L.R.** **f** **\ln** . Key in the y value, and press **$x \div y$** **$=$** **$x \div y$** **\div** **g** **e^x** . The \hat{x} value is displayed.

Example: If Galileo had wished to investigate quantitatively the relationship between the time (t) for a falling object to hit the ground and the height (h) it has fallen, he might have released a rock from various levels of the Tower of Pisa (which was leaning even then) and timed its descent by counting his pulse. The following data are measurements Galileo might have made.

| t (pulses) | 2 | 2.5 | 3.5 | 4 | 4.5 |
|------------|----|-----|-----|-----|-----|
| h (feet) | 30 | 50 | 90 | 130 | 150 |

Find the power curve formula that best expresses h as a function of t ($h = at^b$).

Press

f **Σ**

30 **f** **\ln** 2 **f** **\ln** **$\Sigma+$**

50 **f** **\ln** 2.5 **f** **\ln** **$\Sigma+$**

90 **f** **\ln** 3.5 **f** **\ln** **$\Sigma+$**

130 **f** **\ln** 4 **f** **\ln** **$\Sigma+$**

150 **f** **\ln** 4.5 **f** **\ln** **$\Sigma+$**

Display

1.00

First data pair.

2.00

3.00

4.00

5.00

g **L.R.** **g** **e^x**

x²y

g **r**

7.72

Value of a.

1.99

Value of b.

1.00

Correlation coefficient.

So the formula that best expresses h as a function of t is

$$h = 7.72 t^{1.99}$$

We know, as Galileo did not, that in fact $h \propto t^2$.

Chi-Square Statistic

The chi-square statistic measures the goodness of fit between two sets of frequencies. It's used to test whether a set of observed frequencies differ from a set of expected frequencies sufficiently to reject the hypothesis under which the expected frequencies were obtained.

In other words, you are testing whether discrepancies between the observed frequencies (O_i) and the expected frequencies (E_i) are significant, or whether they may reasonably be attributed to chance. The formula generally used is:

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

If there is a close agreement between the observed and expected frequencies, χ^2 will be small. If the agreement is poor, χ^2 will be large.

The following keystrokes calculate the χ^2 statistic:

1. Press **CLx** **ENTER**.
2. Key in the first O_i value, press **ENTER**.
3. Key in the first E_i value, press **STO** **0** **=** **ENTER** **×** **RCL** **0** **÷** **+**.
4. Repeat steps 2 and 3 for all data pairs. The χ^2 value is displayed.

Example: A suspect die from a Las Vegas casino is brought to an independent testing firm to determine its bias, if any. The die is tossed 120 times and the following results obtained.

| Number | 1 | 2 | 3 | 4 | 5 | 6 |
|--------------------|----|----|----|----|----|----|
| Observed Frequency | 25 | 17 | 15 | 23 | 24 | 16 |

The expected frequency = 120 throws/6 sides, or $E = 20$ for each number, 1 thru 6. (Since E is a constant in this example, there's no need to store it in R_0 each time.)

Press

CLX ENTER+ 25 ENTER+ 20 STO 0

[-] ENTER+ [x] RCL 0 [÷] [+]

17 ENTER+ 20 [-]

ENTER+ [x] RCL 0 [÷] [+]

15 ENTER+ 20 [-]

ENTER+ [x] RCL 0 [÷] [+]

23 ENTER+ 20 [-]

ENTER+ [x] RCL 0 [÷] [+]

24 ENTER+ 20 [-]

ENTER+ [x] RCL 0 [÷] [+]

16 ENTER+ 20 [-]

ENTER+ [x] RCL 0 [÷] [+]

Display

1.25

1.70

2.95

3.40

4.20

5.00

χ^2 .

The number of degrees of freedom is $(n - 1)$. Since $n = 6$, the degrees of freedom = 5.

Consulting statistical tables, you look up χ^2 to a 0.05 significance level with 5 degrees of freedom, and see that $\chi^2_{0.05, 5} = 11.07$. Since $\chi^2 = 5$ is within 11.07, we may conclude that to a 0.05 significance level (Probability = .95), the die is fair.

Paired t Statistic

There are other tests of significance besides chi-square. Suppose you have two samples drawn from two normal populations. The t test is the most commonly used method of testing the differences between the two means.

An example might be the “before” and “after” characteristics of a group that has been subjected to some treatment. The t statistic tests the equality of the means, in this case, to determine whether the treatment has resulted in an “after” group whose characteristics differ significantly from the “before” group.

Let

$$D_i = x_i - y_i \text{ (the difference of all the } x\text{'s and } y\text{'s)}$$

$$\overline{D} = \frac{1}{n} \sum_{i=1}^n D_i$$

$$s_D = \sqrt{\frac{\sum D_i^2 - \frac{1}{n} (\sum D_i)^2}{n - 1}}$$

$$s_{\overline{D}} = \frac{s_D}{\sqrt{n}}$$

The test statistic

$$t = \frac{\overline{D}}{s_{\overline{D}}}$$

which has $n - 1$ degrees of freedom (df) can be used to test the null hypothesis

$$H_0: \bar{x} = \bar{y}$$

In other words, the hypothesis to be tested is that the two means are *not* significantly different. The following keystrokes calculate test statistic t and the degrees of freedom (df):

1. Press **f** **Σ**.
2. Key in the first x value, press **ENTER**. Key in the first y value, press **□** **Σ+**. Repeat this step for all data pairs. (Note that the order of entry differs from previous problems. Here you key in the x value first, then the y value.)

3. Press **9** **S** **STO** 3. The s_D value is displayed.
4. Press **\bar{x}** . The \bar{D} value is displayed.
5. Press **RCL** 3 **\div** **RCL** 4 **f** **\sqrt{x}** **\times** . The test statistic t is displayed.
6. Press **RCL** 4, press 1 **$=$** to obtain degrees of freedom (df).

Example: An electronics firm makes an efficiency study of its production line. One of the factors to be evaluated is the ambient temperature in the building. The firm drops the temperature from the standard 23°C (73.4°F) to 21.5°C (70.7°F). The total outputs at each of 7 production stations are recorded for one day with ambient temperature 23°C , and then for a day at 21.5°C , and are given below.

| | | | | | | | |
|--|-----|----|-----|-----|----|-----|----|
| 23°C | 147 | 38 | 103 | 96 | 62 | 117 | 98 |
| 21.5°C | 153 | 35 | 94 | 101 | 60 | 106 | 90 |

Press

Display

f **Σ** 147 **ENTER** 153 **$=$** **$\Sigma+$** 38 **ENTER** 35 **$=$** **$\Sigma+$** 103 **ENTER** 94 **$=$** **$\Sigma+$** 96 **ENTER** 101 **$=$** **$\Sigma+$** 62 **ENTER** 60 **$=$** **$\Sigma+$** 117 **ENTER** 106 **$=$** **$\Sigma+$** 98 **ENTER** 90 **$=$** **$\Sigma+$** **9** **S** **STO** 3 **\bar{x}** **RCL** 3 **\div** **RCL** 4 **f** **\sqrt{x}** **\times** **RCL** 41 **$=$**

1.00

2.00

3.00

4.00

5.00

6.00

7.00

6.72

3.14

1.24

7.00

6.00

First data pair.

Total number of entries.

Standard deviation is calculated and stored.

 \bar{D}
 t (test statistic).

df (degrees of freedom).

Consulting statistical tables, we find that $t_{0.1, 6} = 1.44$; i.e., t to a 0.1 significance level, with 6 degrees of freedom, equals 1.44. Since $t = 1.24$ is within that limit, we conclude that the change in temperature did not appreciably affect the production line output.

t Statistic for Two Means

Suppose there are two normal populations with unknown means and the same variance. One may wish to examine the relationship between the two means, and hypothesize that there *is* a difference:

$$H_0: \mu_1 - \mu_2 = D$$

where D is some constant, and μ_1 and μ_2 are the means of the two populations.

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - D}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{\sum x_i^2 - n_1 \bar{x}^2 + \sum y_i^2 - n_2 \bar{y}^2}{n_1 + n_2 - 2}}}$$

We can use this t statistic which has the t distribution with $n_1 + n_2 - 2$ degrees of freedom (df) to test the null hypothesis H_0 .

The procedure below calculates the t statistic for two means to measure the accuracy of this hypothesis.

1. Press **f** **Σ**.
2. Key in the first x value, press **Σ+**. Repeat this step for all x values.
3. Press **Σ** **STO** 0 **9** **VAR** **RCL** 4 **STO** 2.

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4. Press 1 \square \square STO 1 .
5. Press f \square .
6. Key in the first y value, press \square . Repeat this step for all y values.
7. Press \bar{x} CHS RCL 0 \square .
8. Key in the constant D, and press \square STO 0 g VAR RCL 4 . Press 1 \square \square RCL 1 \square .
9. Press RCL 4 RCL 2 \square 2 \square . The degrees of freedom (df) are displayed.
10. Press \square RCL 4 g \sqrt{x} RCL 2 g \sqrt{x} \square \square f \square RCL 0 $\text{x}\div\text{y}$ \square . The t statistic is displayed.

Example: Figures indicate that, at age 12, girls have a mean height that is 2.3 centimeters greater than the mean height of boys that age. A sixth-grade class measures the heights of its 12-year-olds and finds these data for 11 girls and 7 boys (all heights in cm):

| | |
|---------------|---|
| Girls: | 158, 144, 139, 154, 149, 150, 154, 160, 147, 155, 143 |
| Boys: | 152, 138, 154, 147, 158, 148, 147 |

Find the t statistic for this data to test the hypothesis that $\mu_1 - \mu_2 = 2.3$ cm.

Press

f \square

158 \square 144 \square 139 \square

154 \square 149 \square 150 \square

154 \square 160 \square 147 \square

155 \square 143 \square

\bar{x} STO 0 g VAR

RCL 4 STO 2

1 \square \square STO 1

Display

3.00

6.00

9.00

11.00

Total number of girls.

43.62

Variance s_x .

436.18

f Σ 152 $\Sigma+$ 138 $\Sigma+$ 154 $\Sigma+$ 147 $\Sigma+$ 158 $\Sigma+$ 148 $\Sigma+$ 147 $\Sigma+$

3.00

6.00

7.00

Total number of boys.

 \bar{x} CHS RCL 0 \div 2.3 \div STO 0 9 VAR

1.13

40.81

Variance s_y .

RCL 4

1 \div \times RCL 1 $+$ RCL 4RCL 2 $+$ 2 \div

16.00

df.

 \div RCL 4 9 \sqrt{x} RCL 2 9 \sqrt{x} $+$ \times f \sqrt{x} RCL 0 $\times y$ \div

-0.37

t statistic.

Consulting statistical tables, we find that $t_{0.1, 16} = 1.337$. Since $t = -0.37$ falls within that limit, the hypothesis of height difference is not rejected at the 0.1 level of significance.

Analysis of Variance (One-Way)

The one-way analysis of variance tests the differences among the means of several samples to see whether the variability is due to chance or due to real differences in the populations from which the samples were drawn.

Variability by chance is measured by the Error SS (sum of the squares). Variability caused by real differences is measured by Treat SS. Error SS and Treat SS are normalized to mean square value to obtain an F ratio.

Let

x_{ij} = j^{th} observation in i^{th} treatment group

n_i = number of observations in group i

$$\bar{x}_i = \text{mean of group } i = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij}$$

$$\bar{x} = \text{grand mean of observations} = \frac{\sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij}}{\sum_{i=1}^k n_i}$$

$$\text{Then Total SS} = \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x})^2 = \sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij}^2 - \frac{\left(\sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^k n_i}$$

$$\text{Treat SS} = \sum_{i=1}^k n_i (\bar{x}_i - \bar{x})^2 = \sum_{i=1}^k \frac{\left(\sum_{j=1}^{n_i} x_{ij} \right)^2}{n_i} = \frac{\left(\sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^k n_i}$$

$$\text{Error SS} = \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 = \text{Total SS} - \text{Treat SS}$$

$$df_1 = \text{Treat df} = k - 1$$

$$df_2 = \text{Error df} = \sum_{i=1}^k n_i - k$$

$$\text{Treat MS} = \frac{\text{Treat SS}}{\text{Treat df}}$$

$$\text{Error MS} = \frac{\text{Error SS}}{\text{Error df}}$$

$$F = \frac{\text{Treat MS}}{\text{Error MS}} \quad (\text{with } k - 1 \text{ and } \sum_{i=1}^k n_i - k \text{ degrees of freedom})$$

The following keystrokes yield the analysis of a variance table: sum of squares, mean squares, degrees of freedom, and the F ratio.

1. Press **f** **REG**.
2. Press 1 **STO** **+** 3.
3. Key in the first value in the first row across and press **STO** **+** 0 **Σ+** 1 **STO** **+** 1. Repeat this step for the second value in the first row across, the third value, etc. to the end of the first row.
4. Press **RCL** 0 **ENTER** **×** **RCL** 1 **÷** **STO** **+** 2.
5. Press 0 **STO** 0 **STO** 1.
6. Repeat steps 2 thru 5 for each row of the table.
7. Press **RCL** 6 **RCL** 5 **ENTER** **×**.
8. Press **RCL** 4 **÷** **□**. The Total SS value is displayed.
9. Press **RCL** 2 **RCL** 5 **ENTER** **×** **RCL** 4 **÷** **□**. The Treat SS value is displayed.
10. Press **□** to obtain Error SS.
11. Press **f** **LAST X** **RCL** 3, press 1 **□**. The Treat df value is displayed.
12. Press **÷** to obtain Treat MS.
13. Press **x₂y** **RCL** 4 **RCL** 3 **□**. The Error df value is displayed.
14. Press **÷** to obtain Error MS.
15. Press **÷** to obtain F.

Example: Lucy Shopper is choosing a new car from among three different makes of comparable price. One of the factors she wishes to evaluate is the expected resale value of the car after 2 years. After identifying a number of two-year-old cars of each make, she compiles the table of prices below (all prices in thousands of dollars):

| | | Cars | | | | |
|------|---|------|-----|-----|-----|-----|
| | | 1 | 2 | 3 | 4 | 5 |
| Make | 1 | 3.4 | 4.0 | 3.9 | 3.2 | 3.3 |
| | 2 | 4.1 | 4.0 | 3.7 | 4.1 | |
| | 3 | 3.6 | 2.9 | 3.0 | 3.5 | 3.4 |

It is not clear whether the variation in prices among the makes is a true difference, or due to randomness within the sample. To get a more quantitative answer to this question, Lucy runs an analysis of variance on the data.

Press

f REG

1 STO + 3

3.4 STO + 0 Σ + 1 STO + 14.0 STO + 0 Σ + 1 STO + 13.9 STO + 0 Σ + 1 STO + 13.2 STO + 0 Σ + 1 STO + 13.3 STO + 0 Σ + 1 STO + 1RCL 0 ENTER+ \times RCL 1 \div STO

+ 2

0 STO 0 STO 1

1 STO + 3

4.1 STO + 0 Σ + 1 STO + 14.0 STO + 0 Σ + 1 STO + 13.7 STO + 0 Σ + 1 STO + 14.1 STO + 0 Σ + 1 STO + 1RCL 0 ENTER+ \times RCL 1 \div STO

+ 2

0 STO 0 STO 1

1 STO + 3

3.6 STO + 0 Σ + 1 STO + 12.9 STO + 0 Σ + 1 STO + 13.0 STO + 0 Σ + 1 STO + 13.5 STO + 0 Σ + 1 STO + 13.4 STO + 0 Σ + 1 STO + 1RCL 0 ENTER+ \times RCL 1 \div STO

+ 2

0 STO 0 STO 1

RCL 6 RCL 5 ENTER+ \times RCL 4 \div =**Display**

1.00

1.00

1.00

1.00

63.37

0.00

1.00

1.00

63.20

0.00

1.00

1.00

1.00

53.79

0.00

2.10

Total SS.

RCL 2 RCL 5 ENTER+ X

RCL 4 ÷ -

-

f LAST X RCL 3

1 -

÷

x²y RCL 4 RCL 3 -

÷

÷

1.08

Treat SS.

1.03

Error SS.

2.00

Treat df.

0.54

Treat MS.

11.00

Error df.

0.09

Error MS.

5.76

F (with 2 and 11 degrees of freedom).

Consulting statistical tables, we find that F to a .05 significance level, with 2 and 11 degrees of freedom, equals 3.98 ($F_{.05, 2, 11} = 3.98$).

If the F ratio obtained above fell within this limit, the differences in resale price would not be significant. Since $F = 5.76$ exceeds the value $F = 3.98$, we conclude that the resale prices of different makes are significantly different.

Covariance

Covariance is a measure of the interdependence between paired variables (x and y). Like standard deviation, covariance may be defined for either a sample (S_{xy}) or a population (S'_{xy}) as follows:

$$S_{xy} = r \times s_x \times s_y$$

$$S'_{xy} = r \times s'_x \times s'_y$$

The following keystrokes solve for covariance of a sample (S_{xy}) and of a population (S'_{xy}):

1. Press f Σ .
2. Key in y, press ENTER+, key in x, and press $\Sigma+$. Repeat this step for all data pairs.
3. Press 9 S X ENTER+ 9 r X. The value of S_{xy} appears in the display.
4. With S_{xy} on the display, press RCL 4, press 1 - RCL 4 ÷ X to obtain S'_{xy} .

Example: Find the sample covariance (S_{xy}) and population covariance (S'_{xy}) for the following paired variables:

| | | | | | | | |
|-------|----|----|----|----|----|----|----|
| x_i | 26 | 30 | 44 | 50 | 62 | 68 | 74 |
| y_i | 92 | 85 | 78 | 81 | 54 | 51 | 40 |

Press

Display

f Σ 92 ENTER+ 26 $\Sigma+$

1.00

First entry.

85 ENTER+ 30 $\Sigma+$

2.00

78 ENTER+ 44 $\Sigma+$

3.00

81 ENTER+ 50 $\Sigma+$

4.00

54 ENTER+ 62 $\Sigma+$

5.00

51 ENTER+ 68 $\Sigma+$

6.00

40 ENTER+ 74 $\Sigma+$

7.00

Total number of entries.

g s \times ENTER+ g r \times

-354.14

 S_{xy}

RCL 4

7.00

Number of entries.

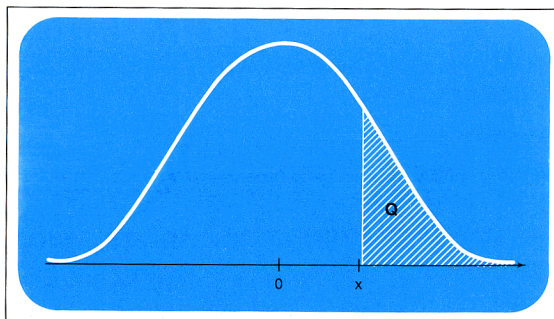
1 \div RCL 4 \div \times

-303.55

 S'_{xy}

Inverse Normal Integral

With the preprogrammed normal distribution function $\boxed{\text{N.D.}}$ in section 3, you found the upper tail area Q , given a point x .



This procedure performs the inverse or opposite operation. That is, given an upper tail area Q , you can determine the value of x . (Note: $0 < Q \leq 0.5$)

$$x = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

where $|\epsilon(Q)| < 4.5 \times 10^{-4}$

$$t = \sqrt{\ln \frac{1}{Q^2}}$$

$$\begin{array}{ll} c_0 = 2.515517 & d_1 = 1.432788 \\ c_1 = 0.802853 & d_2 = 0.189269 \\ c_2 = 0.010328 & d_3 = 0.001308 \end{array}$$

This keystroke procedure solves for point x , given the upper tail area Q :

- Input and store the following constants:
 - Key in 2.515517 (c_0) and press **STO** 0.
 - Key in 0.802853 (c_1) and press **STO** 1.
 - Key in 0.010328 (c_2) and press **STO** 2.
 - Key in 1.432788 (d_1) and press **STO** 3.
 - Key in 0.189269 (d_2) and press **STO** 4.
 - Key in 0.001308 (d_3) and press **STO** 5.
- Key in the given Q value, press **ENTER** **×** **9** **1/x** **f** **ln** **f** **√x** **STO** 6.
- Press **ENTER** **ENTER** **ENTER** **RCL** 5 **×** **RCL** 4 **+** **×** **RCL** 3 **+** **×** 1 **+** **STO** 7.
- Press **CLx** **RCL** 2 **×** **RCL** 1 **+** **×** **RCL** 0 **+** **RCL** 7 **÷** **=**.
- For a new Q , go to step 2.

If less accuracy is acceptable, it may be faster to arrive at x by successive approximations, using the **N.D.** function to calculate Q for each estimate of x . Continue to revise your estimate of x until the calculated Q is within an acceptable tolerance.

Example: Find point x if $Q = 0.12$.

Press

Display

2.515517 **STO** 0

2.52

0.802853 **STO** 1

0.80

0.010328 **STO** 2

0.01

1.432788 **STO** 3

1.43

0.189269 **STO** 4

0.19

0.001308 **STO** 5

1.308000-03

0.12 **ENTER** **×** 9 **1/x**

f **ln** **f** **√x** **STO** 6

2.06

ENTER **ENTER** **ENTER**

RCL 5 **×** **RCL** 4 **+** **×**

RCL 3 **+** **×** 1 **+** **STO** 7

4.76

CLx

0.00

RCL 2 **×** **RCL** 1 **+** **×**

RCL 0 **+** **RCL** 7 **÷** **—**

1.18

Value of x .

Mathematical Applications

The following calculations solve problems commonly encountered in mathematical and scientific applications.

Quadratic Equation

This calculation solves for the roots x_1 , x_2 of the quadratic equation

$$ax^2 + bx + c = 0$$

1. Key in a , press **STO** 0 **ENTER** **+**.
2. Key in b , press **$x \div y$** **÷** **CHS** **ENTER** **ENTER** **×**.
3. Key in c , press **RCL** 0 **÷** **STO** 1 **—**.
4. If the displayed number is positive or 0, the roots are real. Skip to step 5 for the solutions.

If the displayed number is negative, this means the roots are complex ($u \pm iv$). Press **CHS** **f** **√x** **$x \div y$** for the value of u , and press **$x \div y$** to obtain v .

5. Press \boxed{f} $\boxed{\sqrt{x}}$ $\boxed{x \div y}$. If the displayed number is ≥ 0 , press $\boxed{+}$ to obtain the first root x_1 . If the displayed number is < 0 , press $\boxed{-}$ \boxed{CHS} to obtain x_1 .
6. Press \boxed{RCL} $\boxed{1}$ $\boxed{x \div y}$ $\boxed{\div}$ to obtain the second root x_2 .

Example: A man stands at the top of a cliff 110 meters (361 feet) high. He throws a rock off the cliff with a velocity in the vertical direction of 20 m/s. How long will it take the rock to hit the ground? (Take the acceleration due to gravity as 9.8 m/s^2 .)

The equation to be solved is $s = \frac{1}{2}gt^2 + v_0t + s_0$ where $s = 0$, $g = -9.8$, $v_0 = 20$, and $s_0 = 110$. Rewrite the equation as $4.9t^2 - 20t - 110 = 0$.

| Press | Display |
|--|------------------|
| 4.9 \boxed{STO} $\boxed{0}$ \boxed{ENTER} $\boxed{+}$ | $\boxed{9.80}$ |
| 20 \boxed{CHS} $\boxed{x \div y}$ $\boxed{\div}$ \boxed{CHS} \boxed{ENTER} | |
| \boxed{ENTER} $\boxed{\times}$ | $\boxed{4.16}$ |
| 110 \boxed{CHS} \boxed{RCL} $\boxed{0}$ $\boxed{\div}$ \boxed{STO} $\boxed{1}$ $\boxed{-}$ | $\boxed{26.61}$ |
| \boxed{f} $\boxed{\sqrt{x}}$ $\boxed{x \div y}$ $\boxed{+}$ | $\boxed{7.20}$ |
| \boxed{RCL} $\boxed{1}$ $\boxed{x \div y}$ $\boxed{\div}$ | $\boxed{-3.12}$ |
| | Roots are real. |
| | Value of t_1 . |
| | Value of t_2 . |

The primary value for t is 7.20 seconds. (The secondary root, -3.12, is not meaningful here.)

Polynomial Evaluation

This calculation solves a polynomial of the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where n can be any positive integer.

The following routine calculates $f(x)$:

1. Key in the value of x . Press \boxed{ENTER} three times.
2. Key in the value of a_n and press $\boxed{\times}$.

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3. Key in the value of a_{n-1} , press $\boxed{+}$ $\boxed{\times}$. Repeat this step for all the a values thru a_1 .
4. Key in the value of a_0 and press $\boxed{+}$. The value of $f(x)$ is displayed.

Example: Let $f(x) = 4x^3 - x^2 + 17x - 5$. Find $f(7)$.

Press

Display

| | | | | |
|----|------------------|------------------|------------------|---------|
| 7 | ENTER | ENTER | ENTER | 7.00 |
| 4 | $\boxed{\times}$ | | | 28.00 |
| 1 | CHS | $\boxed{+}$ | $\boxed{\times}$ | 189.00 |
| 17 | $\boxed{+}$ | $\boxed{\times}$ | | 1442.00 |
| 5 | CHS | $\boxed{+}$ | | 1437.00 |

$f(7)$.

Complex Arithmetic

This keystroke procedure is designed to perform chain calculations on complex numbers of the form $a + ib$. The operations involved are the basic four functions: $+$, $-$, \times , \div . For a calculation such as $(a_1 + ib_1) \times (a_2 + ib_2)$, a_1 , b_1 , a_2 , and b_2 are stored in registers R_1 , R_2 , R_3 , and R_4 , respectively. At the end of a calculation, the result $u + iv$ is stored in registers R_1 (u) and R_2 (v). The next entry in the chain, $a_3 + ib_3$ is then keyed in and stored in registers R_3 (a_3) and R_4 (b_3).

Addition

$$(a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + (b_1 + b_2)i$$

Subtraction

$$(a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + (b_1 - b_2)i$$

Multiplication

$$(a_1 + ib_1) \times (a_2 + ib_2) = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

Division

$$\frac{(a_1 + ib_1)}{(a_2 + ib_2)} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}, a_2 + ib_2 \neq 0$$

where $r_1 e^{i\theta_1}$ is the polar representation of $a_1 + ib_1$, and $r_2 e^{i\theta_2}$ is the representation of $a_2 + ib_2$.

These keystrokes cover addition, subtraction, multiplication, and division of complex numbers. After the first step, skip to the desired function.

1. Input and store the following:

- Key in a_1 and press **STO** 1.
- Key in b_1 and press **STO** 2.
- Key in a_2 and press **STO** 3.
- Key in b_2 and press **STO** 4.

Addition

2. Press **RCL** 1 **RCL** 3 **+** to obtain the value of u .
3. Press **RCL** 2 **RCL** 4 **+** to obtain v .

Subtraction

2. Press **RCL** 1 **RCL** 3 **-** to obtain the value of u .
3. Press **RCL** 2 **RCL** 4 **-** to obtain v .

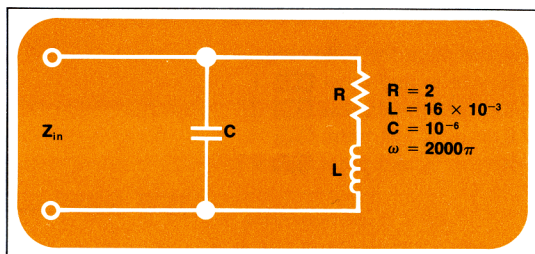
Multiplication

2. Press **RCL** 2 **RCL** 1 **g** **→P**.
3. Press **RCL** 4 **RCL** 3 **g** **→P**.
4. Press **x²y** **R+** **×** **R+** **+** **R+** **R+** **R+** **f** **→R** to obtain the value of u .
5. Press **x²y** to obtain v .

Division

2. Press **RCL** 2 **RCL** 1 **g** **→P**.
3. Press **RCL** 4 **RCL** 3 **g** **→P**.
4. Press **x²y** **R+** **÷** **R+** **-** **CHS** **R+** **R+** **R+** **f** **→R** to obtain the value of u .
5. Press **x²y** to obtain v .

To perform another operation, press **STO** 2 **x²y** **STO** 1. Key in a_3 , press **STO** 3, key in b_3 , press **STO** 4, then go back to step 2 of the appropriate function.

Example:

In the above circuit, find Z_{in} if the equation is:

$$Z_{in} = \frac{-\frac{500}{\pi}i(2 + 32\pi i)}{2 + \left(32\pi - \frac{500}{\pi}\right)i}$$

(First, multiply the complex numbers in the numerator, then divide.)

Press

0 **STO** 1
 500 **CHS** **g** **π** **÷** **STO** 2
 2 **STO** 3
 32 **g** **π** **×** **STO** 4
RCL 2 **RCL** 1 **g** **→P**
RCL 4 **RCL** 3 **g** **→P**
x²y **R[↓]** **×** **R[↓]** **+**
R[↓] **R[↓]** **R[↓]** **f** **→R**
x²y

Display

| | |
|----------|-------------|
| 0.00 | a_1 |
| -159.15 | b_1 |
| 2.00 | a_2 |
| 100.53 | b_2 |
| 159.15 | |
| 100.55 | |
| 16000.00 | Value of u. |
| -318.31 | Value of v. |

Now the problem becomes:

$$Z_{in} = \frac{16000 - 318.31 i}{2 + \left(32\pi - \frac{500}{\pi}\right) i}$$

Press

STO 2 x²y STO 1
 2 STO 3
 32 g π × 500 g π ÷
 = STO 4
 RCL 2 RCL 1 g ⇨P
 RCL 4 RCL 3 g ⇨P
 x²y R⁺ ÷ R⁺ = CHS
 R⁺ R⁺ R⁺ f ⇨R
 x²y

Display

u₁ and v₁ stored.
 2.00 a₂.
 -58.62 b₂.
 16003.17
 58.66
 14.72 Value of u.
 272.42 Value of v.

$$Z_{in} = 14.72 + 272.42 i.$$

Complex Functions

This routine calculates various functions of a complex number. Let the complex number $z = a + ib$ have polar representation $re^{i\theta}$. Then the functions and their formulas are as follows:

1. $|z| = r$.
2. $z^2 = r^2 e^{i2\theta}$.
3. $1/z = \frac{1}{r} e^{-i\theta}, z \neq 0$.
4. $\sqrt{z} = \pm (\sqrt{r} e^{i\theta/2}) = \pm (u + iv)$.

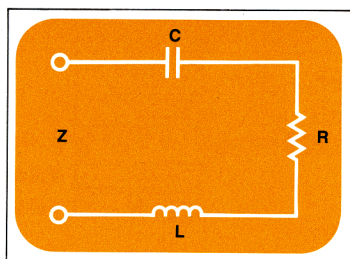
Let the answer be represented by $u + iv$.

The following procedures solve for $|z|$, z^2 , $1/z$ and \sqrt{z} .

1. Key in b, press **ENTER**. Key in a, press **g** **⇨P**. The absolute value of z is displayed.
2. To calculate z^2 , go to step 3. To calculate $1/z$, go to step 4. To calculate \sqrt{z} , go to step 5.

3. Press **ENTER** **×** **xzy** **ENTER** **+** **xzy** **f** **→R**. The value of u is displayed. Press **xzy** to obtain v .
4. Press **9** **1/x** **xzy** **CHS** **xzy** **f** **→R**. The value of u is displayed. Press **xzy** to obtain v .
5. Press **f** **√x** **xzy** **2** **÷** **xzy** **f** **→R**. The value of u is displayed. Press **xzy** to obtain v .

Example:



In the circuit above, the impedance Z is given by

$$z = 2 + i \left(\omega L - \frac{1}{\omega C} \right)$$

Given $R = 2$, $L = 16 \times 10^{-3}$, $C = 1.5 \times 10^{-6}$, $\omega = 2000\pi$, the formula becomes

$$z = 2 + i \left(32\pi - \frac{1000}{3\pi} \right)$$

The admittance Y of the circuit is defined as

$$Y = 1/z$$

Find Y and $|Y|$.

Press

32 **g** **π** **×** 1000 **ENTER** 3 **÷**
g **π** **÷** **-** 2 **g** **→P**
g **1/x** **x²y** **CHS** **x²y** **f** **→R**
x²y
x²y **g** **→P**

Display

| | |
|------|-------------|
| 5.92 | $ z $. |
| 0.06 | Value of u. |
| 0.16 | Value of v. |
| 0.17 | $ Y $. |

$$Y = 1/z = 0.06 + 0.16 i.$$

$$|Y| = 0.17.$$

Vector Cross Product

If $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ are two three-dimensional vectors then the cross product of A and B is calculated as follows:

$$A \times B = \left(\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right) = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)$$

and the solution is represented by (c_1, c_2, c_3) .

This procedure calculates the cross product (c_1, c_2, c_3) of two vectors.

1. Input and store the following:

- Key in a_1 , press **STO** 1.
- Key in a_2 , press **STO** 2.
- Key in a_3 , press **STO** 3.
- Key in b_1 , press **STO** 4.
- Key in b_2 , press **STO** 5.
- Key in b_3 , press **STO** 6.

2. Press **RCL** 2 **RCL** 6 **×** **RCL** 3 **RCL** 5 **×** **-**.

The value of c_1 is displayed.

3. Press **RCL** 3 **RCL** 4 \times **RCL** 1 **RCL** 6 \times \square .

The value of c_2 is displayed.

4. Press **RCL** 1 **RCL** 5 \times **RCL** 2 **RCL** 4 \times \square .

The value of c_3 is displayed.

Example: The force F on a particle with charge q which is moving with a velocity \vec{v} through a magnetic field \vec{B} is given by $F = q \vec{v} \times \vec{B}$. Suppose a proton ($q = 1.6 \times 10^{-19}$ coulomb) is moving with $\vec{v} = (0.4, 2.8, -1.2) \times 10^7$ m/s. A uniform magnetic field surrounding the proton is of a strength $\vec{B} = (1.3, -0.3, 0.7)$ tesla. Calculate the force on the proton.

This can be written as $F = q \vec{v} \times \vec{B} = (1.6 \times 10^{-19}) (10^7) (0.4, 2.8, -1.2) \times (1.3, -0.3, 0.7)$ N.

Press

0.4 **STO** 1

2.8 **STO** 2

1.2 **CHS** **STO** 3

1.3 **STO** 4

0.3 **CHS** **STO** 5

0.7 **STO** 6

RCL 2 **RCL** 6 \times **RCL** 3

RCL 5 \times \square

RCL 3 **RCL** 4 \times **RCL** 1

RCL 6 \times \square

RCL 1 **RCL** 5 \times **RCL** 2

RCL 4 \times \square

Display

0.40 a_1 .

2.80 a_2 .

-1.20 a_3 .

1.30 b_1 .

-0.30 b_2 .

0.70 b_3 .

1.60 c_1 .

-1.84 c_2 .

-3.76 c_3 .

The solution then becomes:

$$\begin{aligned} F &= (1.6 \times 10^{-19}) (1.60, -1.84, -3.76) \text{ N} \\ &= (2.56, -2.94, -6.02) \times 10^{-19} \text{ N.} \end{aligned}$$

This is about 4×10^{14} times the weight of the proton.

Vector Dot Product

Let $\vec{a} = (a_1, a_2, \dots, a_n)$ and $\vec{b} = (b_1, b_2, \dots, b_n)$ be two vectors. The norm of \vec{a} is denoted by $|\vec{a}|$ and the norm of \vec{b} is denoted by $|\vec{b}|$ where

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

$$|\vec{b}| = \sqrt{b_1^2 + b_2^2 + \dots + b_n^2}$$

The dot product of \vec{a} and \vec{b} ($\vec{a} \cdot \vec{b}$) is calculated by the following formula:

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n.$$

The angle between \vec{a} and \vec{b} is denoted by θ and is calculated by the following formula:

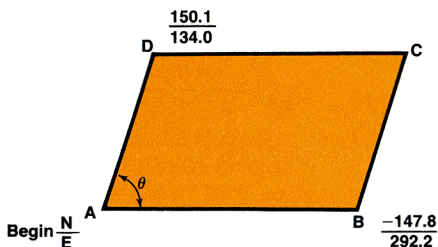
$$\theta = \cos^{-1} \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} \right)$$

The angle is calculated in any angular mode. When calculated in degrees, decimal degrees are assumed.

This procedure calculates the vector dot product and angle θ .

1. Press **f** **Σ**.
2. Key in a_1 , press **ENTER**. Key in b_1 , press **Σ+**. Repeat this step for all values of a and b .
3. Press **RCL** 8 **f** **√x** to obtain $|\vec{a}|$.
4. Press **RCL** 6 **f** **√x** to obtain $|\vec{b}|$.
5. Press **RCL** 9 for the dot product $\vec{a} \cdot \vec{b}$.
6. Press **RCL** 8 **RCL** 6 **×** **f** **√x** **÷** **9** **cos⁻¹** for angle θ .

Example: A surveyor has determined the coordinates of three corners of a plot of land in the shape of a parallelogram. He now wants to find the area of the plot, the lengths of the sides, and the angle θ .

**Solution:**

$$\text{Area} = \vec{AB} \cdot \vec{AD} = (-147.8, 292.2) \cdot (150.1, 134.0)$$

Press**Display**f Σ 147.8 CHS ENTER+ 150.1 $\Sigma+$ 292.2 ENTER+ 134 $\Sigma+$ RCL 8 f \sqrt{x} RCL 6 f \sqrt{x}

RCL 9

RCL 8 RCL 6 \times f \sqrt{x} \div 9 \cos^{-1}

1.00

First entry.

2.00

Second entry.

327.45

Side AB $|\vec{AB}|$.

201.21

Side AD $|\vec{AD}|$.

16970.02

Area of plot.

75.07

 $^\circ$ angle θ .**Determinant and Inverse of a 2×2 Matrix**

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a 2×2 matrix.

The determinant of A denoted by $\text{Det } A$ or $|A|$ is evaluated by the following formula:

$$\text{Det } A = a_{22} a_{11} - a_{12} a_{21}.$$

The HP-27 also calculates the multiplicative inverse A^{-1} of A according to the following formula:

$$A^{-1} = \begin{bmatrix} a_{22}/\text{Det } A & -a_{12}/\text{Det } A \\ -a_{21}/\text{Det } A & a_{11}/\text{Det } A \end{bmatrix}$$

This procedure calculates the determinant of a 2×2 matrix and an inverse matrix such that you can calculate A^{-1} .

- Input and store the following:
 - Key in a_{11} and press **STO** 1.
 - Key in a_{12} and press **STO** 2.
 - Key in a_{21} and press **STO** 3.
 - Key in a_{22} and press **STO** 4.
- Press **RCL** 1 **RCL** 4 **×** **RCL** 2 **RCL** 3 **×** **−** **STO** 5. The determinant of A is displayed.
- Press **RCL** 4 **RCL** 5 **÷**. The inverse of a_{11} is displayed.
- Press **RCL** 2 **RCL** 5 **÷** **CHS**. The inverse of a_{12} is displayed.
- Press **RCL** 3 **RCL** 5 **÷** **CHS**. The inverse of a_{21} is displayed.
- Press **RCL** 1 **RCL** 5 **÷**. The inverse of a_{22} is displayed.

Example: Given a pair of simultaneous linear equations in two unknowns,

$$2x + 3y = 21$$

$$4x - y = 7$$

Another way to write the equations is in terms of matrices:

$$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21 \\ 7 \end{bmatrix}$$

Rewriting the matrix equation gives

where

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} 21 \\ 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$$

and A^{-1} is the multiplicative inverse of A .

Find Det A and A^{-1} .

Press

2 **STO** 1
 3 **STO** 2
 4 **STO** 3
 1 **CHS** **STO** 4
RCL 1 **RCL** 4 **×**
RCL 2 **RCL** 3 **×** **=** **STO** 5
f **FIX** 4
RCL 4 **RCL** 5 **÷**
RCL 2 **RCL** 5 **÷** **CHS**
RCL 3 **RCL** 5 **÷** **CHS**
RCL 1 **RCL** 5 **÷**

Display

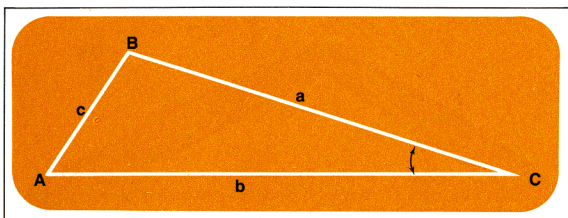
2.00
 3.00
 4.00
 -1.00
 -2.00
 -14.00
 -14.0000
 0.0714
 0.2143
 0.2857
 -0.1429

Determinant of A .
 Display to 4 places.
 Inverse of a_{11} .
 Inverse of a_{12} .
 Inverse of a_{21} .
 Inverse of a_{22} .

$$A^{-1} = \begin{bmatrix} 0.0714 & 0.2143 \\ 0.2857 & -0.1429 \end{bmatrix}$$

Triangle Solution a,b,C

Given two sides (a and b) and their included angle C , this calculation solves for the third side and other two angles.



$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

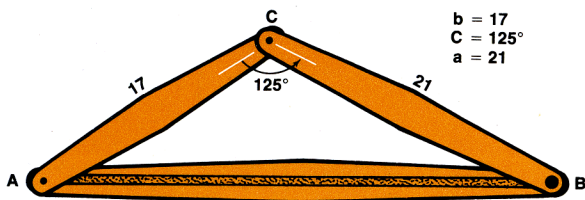
$$A = \tan^{-1} \left(\frac{a \sin C}{b - a \cos C} \right)$$

$$B = \cos^{-1} [-\cos (A + C)]$$

These keystrokes work in any angular mode. However, if in degrees, decimal degrees are assumed.

1. Key in b , press **STO** 0.
2. Key in angle C , press **STO** 1.
3. Key in a , press **f** **→R** **RCL** 0 **x↔y** **=** **g** **→P**. The length of the third side (c) is displayed.
4. Press **x↔y** to obtain angle A .
5. Press **RCL** 1 **÷** **f** **COS** **CHS** **g** **COS⁻¹** to obtain angle B .

Example: A mechanical linkage is to be designed so that arms AC and CB are pinned at point C . Point B is able to slide along the base, hence changing the angle C . At its maximum extension, angle C is to have a value of 125° . If the arms AC and BC have lengths 17 cm and 21 cm, respectively, what is the minimum length of the base (side AB)?



$$\begin{aligned} b &= 17 \\ C &= 125^\circ \\ a &= 21 \end{aligned}$$

Press

17 **STO** 0125 **STO** 121 **f** **→R** **RCL** 0 **x↔y****←** **9** **→P****x↔y****RCL** 1 **+** **f** **COS** **CHS****g** **COS⁻¹**

Display

17.00

125.00

33.76

Value of c.

30.64

°angle A.

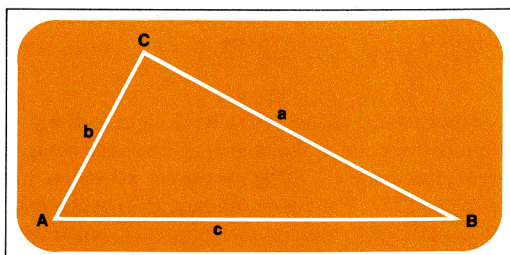
24.36

°angle B.

The side AB must be at least 33.76 cm in length.

Triangle Solution a,b,c

Given three sides (a, b and c), of a triangle, this calculation solves for the three angles.



$$A = 2 \cos^{-1} \left(\sqrt{\frac{S(S-a)}{bc}} \right)$$

$$\text{where } S = (a + b + c)/2$$

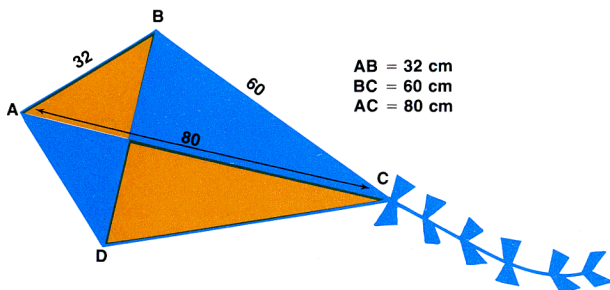
$$B = \tan^{-1} \left(\frac{b \sin A}{c - b \cos A} \right)$$

$$C = \cos^{-1} [-\cos (A + B)]$$

These keystrokes work in any angular mode. However, if in degrees, all angles are assumed to be in decimal degrees.

- Input and store the following:
 - Key in side a, press **STO** 0.
 - Key in side b, press **STO** 1.
 - Key in side c, press **STO** 2.
- Press **+** **+** 2 **÷** **ENTER** **ENTER** **RCL** 0 **=** **X**.
- Press **RCL** 1 **÷** **RCL** 2 **÷** **f** **√x** **9** **COS⁻¹** 2 **X** **STO** 3. The value of angle A is displayed.
- Press **RCL** 1 **f** **→R** **RCL** 2 **x↔y** **=** **9** **↔P** **x↔y**. The value of angle B is displayed.
- Press **RCL** 3 **+** **f** **COS** **CHS** **9** **COS⁻¹** to obtain angle C.

Example: You decide to make a kite with the dimensions shown below. Calculate the angles $\angle ACB$, $\angle BAC$, and $\angle ABC$.



Press

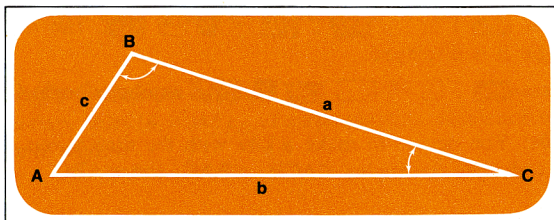
60 **STO** 0
 80 **STO** 1
 32 **STO** 2
+ **+** 2 **÷** **ENTER** **ENTER**
RCL 0 **-** **x**
RCL 1 **÷** **RCL** 2 **÷**
f **√x** **g** **COS⁻¹** 2 **x** **STO** 3
RCL 1 **f** **→R** **RCL** 2 **x²y**
- **g** **÷P** **x²y**
RCL 3 **+** **f** **COS** **CHS**
g **COS⁻¹**

Display

| | |
|---------|--------------|
| 60.00 | Side BC (a). |
| 80.00 | Side AC (b). |
| 32.00 | Side AB (c). |
| 86.00 | |
| 2236.00 | |
| 0.87 | |
| 41.68 | °<BAC (A). |
| 117.55 | °<ABC (B). |
| 20.77 | °<ACB (C). |

Triangle Solution a,A,C

Given two angles (A and C) and an opposite side (a), this calculation solves for the third angle and the other two sides.



$$B = 180^\circ - (A + C) = 200 \text{ grads} - (A + C)$$

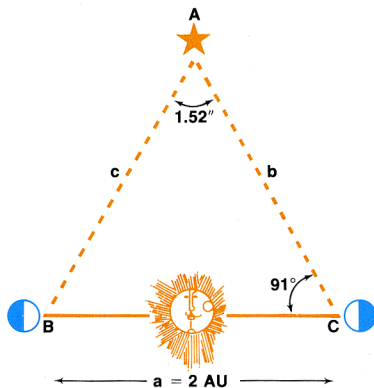
$$b = \frac{a \sin B}{\sin A}$$

$$c = \frac{a \sin C}{\sin A}$$

These keystrokes work in any angular mode. However, if in degree mode, all angles are assumed to be in decimal degrees.

- Input and store the following:
 - Key in side a , press **STO** 0.
 - Key in angle C , press **STO** 1.
 - Key in angle A , press **STO** 2.
- Press **[+]** **[f]** **[sin]** **[RCL]** 2 **[f]** **[sin]** **[÷]** **[RCL]** 0 **[×]** to obtain side B .
- Press **[RCL]** 1 **[x²y]** **[f]** **[→R]** **[RCL]** 0 **[x²y]** **[−]** **[g]** **[+P]** to obtain side c .
- Press **[x²y]** to obtain angle B .

Example: An astronomer trying to determine the distance from the earth to a nearby star, photographs the star at approximately six-month intervals and records the data as shown below.



Find the distance b from the earth to the star, given that the mean distance of the earth from the sun is 1 astronomical unit (AU).

(Note that $A = 1.52$ seconds must be converted to decimal degrees.)

Press

2 **STO** 0
 91 **STO** 1
 .000152 **g** **⇨H** **STO** 2
+ **f** **sin** **RCL** 2 **f** **sin**
÷ **RCL** 0 **x**

RCL 1 **xzy** **f** **⇨R**
RCL 0 **xzy** **-** **g** **⇨P**
xzy

Display

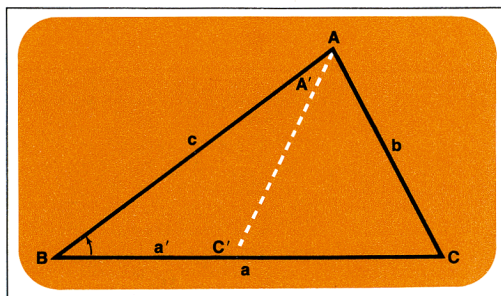
2.00 Side A.
 91.00 °angle C.
 4.2222222-04 °angle A.

271359.69 AU side b, distance to the star.

271359.73 AU side c.
 89.00 °angle B.

Triangle Solution B,b,c,

Given two sides (b and c) and a non-included angle (B), this calculation solves for the remaining two angles and the third side.



$$C = \sin^{-1} \left(\frac{c \sin B}{b} \right)$$

$$A = 2 \sin^{-1} - (B + C) = \pi \text{ radians} - (B + C) = 180^\circ - (B + C)$$

$$= 200 \text{ grads} - (B + C)$$

$$a = \frac{b \sin A}{\sin B}$$

If angle B is acute ($< 90^\circ$) and side $b < \text{side } c$, a second set of solutions exists. Therefore, you would also solve for A' , C' and a' .

$$C' = 2 \sin^{-1} 1 - C$$

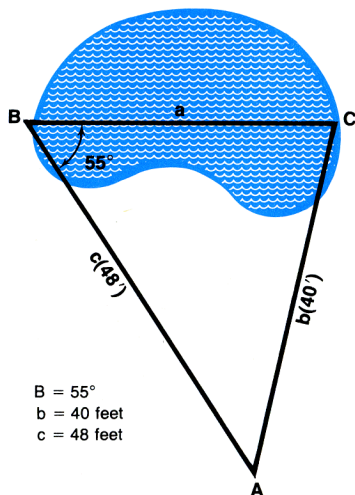
$$A' = 2 \sin^{-1} 1 - (B + C')$$

$$a' = \frac{b \sin A'}{\sin B}$$

These keystrokes work in any angular mode. However, if in degrees, decimal degrees are assumed.

1. Press **1** **9** **[sin⁻¹]** **[ENTER]** **+** **[STO]** **0**.
2. Input and store the following:
 - Key in angle B, press **[STO]** **1**.
 - Key in side b, press **[STO]** **2**.
 - Key in side c, press **[STO]** **3**.
3. Press **[x↔y]** **[÷]** **[RCL]** **1** **[f]** **[sin]** **[x]** **9** **[sin⁻¹]** **[STO]** **4**. Angle C is displayed.
4. Press **[RCL]** **1** **+** **[RCL]** **0** **[x↔y]** **[=]** to obtain angle A.
5. Press **[f]** **[sin]** **[RCL]** **2** **[x]** **[RCL]** **1** **[f]** **[sin]** **[÷]** to obtain side a.
6. If side $b < \text{side } c$, a second set of solutions exist. Press **[RCL]** **0** **[RCL]** **4** **[=]** to find C' . Repeat steps 4 and 5 to find A' and a' .

Example: A landscape designer discovers Zen and decides to transform the backyard of his palatial Beverly Hills mansion into a Japanese garden. The heart of the transformation will be the conversion of his kidney-shaped swimming pool into a goldfish pond spanned by a stone bridge. To find the length of the pool and still stay dry, the designer makes measurements as shown in the diagram.



Calculate angles A and C and find the length of the pool.

Press

Display

1 9 \sin^{-1} ENTER +

STO 0

55 STO 1

40 STO 2

48 STO 3

$x \div y$ \div RCL 1 f sin

\times 9 \sin^{-1} STO 4

RCL 1 + RCL 0 $x \div y$ =

f sin RCL 2 \times

RCL 1 f sin \div

55.00 °angle B.

40.00 Side b.

48.00 Side c.

79.41 °angle C.

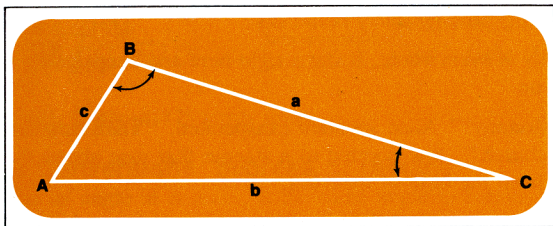
45.59 °angle A.

34.88 Feet (length of pool).

The alternate solution is $C' = 100.59^\circ$, $A' = 24.41^\circ$, and $a' = 20.18$ ft. This solution, however, does not fit the diagram.

Triangle Solution a, B, C

Given two angles (B and C) and their included side (a), this calculation solves for the third angle and the other two sides.



$$A = 180^\circ - (B + C) = 200 \text{ grads} - (B + C)$$

$$b = \frac{a \sin B}{\sin A}$$

$$c = \frac{a \sin C}{\sin A}$$

These keystrokes work in any angular mode. However, if in degrees, decimal degrees are assumed.

1. Press 1 **g** **sin⁻¹** **ENTER** **+**.
2. Key in angle B, press **STO** 0 **=**.
3. Key in angle C, press **STO** 1 **=**. Angle A is displayed.
4. Key in side a. Press **x↔y** **f** **sin** **÷** **STO** 2 **RCL** 0 **f** **sin** **=**. Side b is displayed.
5. Press **RCL** 2 **RCL** 1 **f** **sin** **=** to obtain side c.

Example: Alice and her mother were lying in the new spring grass on a hill overlooking the town. It was early morning and Venus was still visible near the Eastern horizon.

“Mommy,” Alice mused, “how far away is that star?”

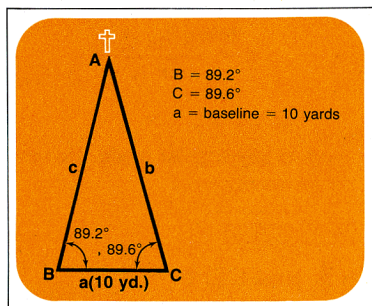
“I don’t know, dear. But it’s not really a star; it’s a planet.”

“Oh. Well, how far away is the sky?”

“The sky is everywhere. It starts right here and keeps going up and up and up.”

“Oh, Mommy!” Alice sounded exasperated. “Then how far away is that church?” She pointed to a steeple that rose above the trees not far from the base of their hill.

“Ah, that we can find,” her mother said, and jumped up. “Come on.” They ran back to the house and returned shortly with a yardstick, string, a protractor, and some stakes. Within minutes they’d staked out a baseline 10 yards long, and measured the angle from each end point of the baseline to the steeple. The measurements they made are shown below:



With these measurements, can you tell Alice how far away the steeple is?

Press**Display**1 **g** **sin⁻¹** **ENTER** **+**89.2 **STO** 0 **=**89.6 **STO** 1 **=****1.20** °angle A.10 **x²y** **f** **sin** **÷** **STO** 2**RCL** 0 **f** **sin** **×****477.45** Yards (side b).**RCL** 2 **RCL** 1 **f** **sin** **×****477.49** Yards (side c).

The steeple is about 475 yards away.

Angle Conversions: degrees \rightleftharpoons radians

Let D be an angle in degrees, R the same angle in radians.

$$D = \frac{180}{\pi} R.$$

$$R = \frac{\pi}{180} D.$$

To convert degrees to radians:

1. Press **g** **π** 180 **÷**. Press **ENTER** three times.
2. Press **CLx**, key in the degree value and press **×**. The value of the angle in radians is displayed.
3. For a new degree value, go to step 2.

To convert radians to degrees:

1. Key in 180 and press **g** **π** **÷**. Press **ENTER** three times.
2. Press **CLx**, key in the radian value, and press **×**. The value of the angle in degrees is displayed.
3. For a new radian value, go to step 2.

Examples: Convert 1° to radians.**Press****Display****g** **π** 180 **÷****ENTER** **ENTER** **ENTER** **CLx****0.00**

134 Navigation Applications

1 \times

0.02

Radians.

Convert 240° to radians.

CLx 240 \times

4.19

Radians.

Convert 1 radian to degrees.

180 g π \div

ENTER \uparrow ENTER \uparrow ENTER \uparrow CLx

0.00

1 \times

57.30

Degrees.

Convert $\frac{3}{4} \pi$ radians to degrees.

CLx 3 ENTER \uparrow 4 \div

g π \times \times

135.00

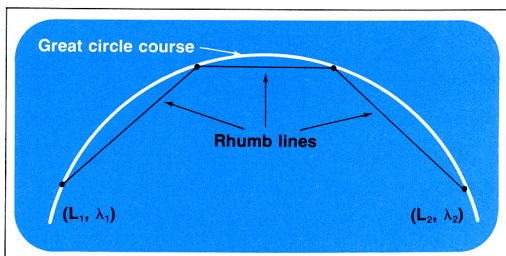
Degrees.

Navigation Applications

The following procedures describe how to calculate a rhumb line course, great circle navigation, and a sight reduction table.

Rhumb Line Navigation

A rhumb line is a curve on the surface of a sphere that cuts all meridians at the same angle.



Your HP-27 can calculate the rhumb line distance and course between two points on the earth. You can link successive legs without keying in the initial latitude and longitude again. (Accuracy deteriorates for very short legs.)

Northern latitudes and western longitudes are keyed in and displayed as positive values. Southern latitudes and eastern longitudes are keyed in and displayed as negative values.

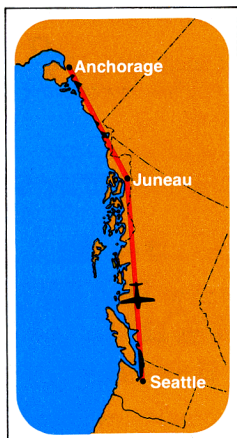
To avoid getting incorrect results computing distances due east or due west across the international dateline, compute up to the dateline, then proceed on the other side.

No course should pass through the North or South Pole. Also, as the rhumb line course approaches 90° or 270° , you may encounter errors in the distance calculations.

The following keystrokes calculate a rhumb line course (C) and rhumb line distance (DIST):

1. Key in latitude of initial point (L_1). Press **9** **⇨H** **STO** 1.
2. Key in longitude of initial point (λ_1). Press **9** **⇨H** **STO** 3.
3. Key in the next latitude, and press **9** **⇨H** **STO** 2.
4. Key in the next longitude, and press **9** **⇨H** **STO** 4.
5. Press **RCL** 3 **RCL** 4 **-** **STO** 7.
6. Press **2** **÷** **f** **sin** **9** **sin⁻¹** **90** **÷** **9** **π** **×** **RCL** 2.
7. Press **2** **÷** **45** **+** **f** **tan** **RCL** 1.
8. Press **2** **÷** **45** **+** **f** **tan** **÷** **f** **ln**.
9. Press **9** **⇨P** **x↔y** **RCL** 7 **f** **sin** **9** **sin⁻¹**.
10. If the number on the display is positive, press **x↔y** to obtain C (rhumb line course) in decimal degrees.
If the displayed number is negative, press **x↔y** **360** **+** to obtain C in decimal degrees.
11. Press **f** **cos**. If $\cos(c)$ is 0, skip to step 13.
12. Press **RCL** 2 **RCL** 1 **-** **x↔y** **÷** **6** **×** to obtain the distance (DIST) in nautical miles. Go to step 14.
13. Press **RCL** 7 **RCL** 2 **f** **cos** **×** **60** **×** to obtain the distance (DIST) in nautical miles.
14. To solve for the course and distance of another leg, press **RCL** 2 **STO** 1 **RCL** 4 **STO** 3 and return to step 3.

Example: Find the distances and headings for a flight from Anchorage, Alaska, to Juneau, Alaska, to Seattle, Washington.



| | | |
|-----------|--------------------|-----------------------------|
| Anchorage | L $61^{\circ}13'N$ | λ $149^{\circ}54'W$ |
| Juneau | L $58^{\circ}18'N$ | λ $134^{\circ}25'W$ |
| Seattle | L $47^{\circ}36'N$ | λ $122^{\circ}20'W$ |

Press

61.13 **g** **→H** **STO** 1
 149.54 **g** **→H** **STO** 3
 58.18 **g** **→H** **STO** 2
 134.25 **g** **→H** **STO** 4
RCL 3 **RCL** 4 **-** **STO** 7
 2 **÷** **f** **sin**
g **sin⁻¹** 90 **÷**
g **π** **×** **RCL** 2
 2 **÷** 45 **+** **f** **tan** **RCL** 1
 2 **÷** 45 **+** **f** **tan**
÷ **f** **ln**
g **→P** **x^zy** **RCL** 7 **f** **sin**

Display

| | |
|---------------|-----------------------|
| 61.22 | Anchorage L. |
| 149.90 | Anchorage λ . |
| 58.30 | Juneau L. |
| 134.42 | Juneau λ . |
| 15.48 | |
| 58.30 | |
| 61.22 | |
| -0.10 | |

g \sin^{-1} $\times \div y$
 f \cos RCL 2 RCL 1 \square
 $\times \div y$ \div 60 \times
 RCL 2 STO 1 RCL 4 STO 3

47.36 g $\rightarrow \text{H}$ STO 2
 122.20 g $\rightarrow \text{H}$ STO 4
 RCL 3 RCL 4 \square STO 7
 2 \div f \sin g \sin^{-1} 90 \div

g π \times RCL 2
 2 \div 45 $+$ f \tan RCL 1
 2 \div 45 $+$ f \tan \div f \ln
 g $\rightarrow \text{P}$ $\times \div y$ RCL 7 f \sin

g \sin^{-1} $\times \div y$
 f \cos RCL 2 RCL 1 \square
 $\times \div y$ \div 60 \times

110.52 °C₁

499.22 DIST₁

Prepare to solve
for next leg.

47.60 Seattle L.

122.33 Seattle λ .

12.08

47.60

58.30

-0.31

145.94 °C₂.

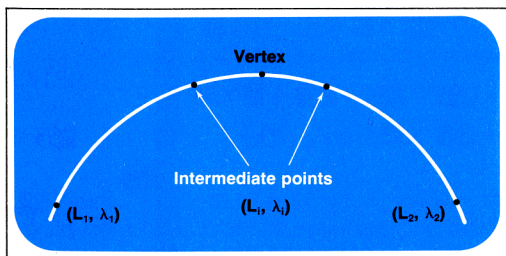
774.90 DIST₂.

Anchorage to Juneau $C = 110.52^\circ$ DIST = 499.22 naut. miles.

Juneau to Seattle $C = 145.94^\circ$ DIST = 774.90 naut. miles.

Great Circle Navigation

This method of navigation involves travelling between two points according to the arc of a great circle.



This procedure calculates the great circle distance and initial course for the great circle track between two points on the earth. You can also calculate the coordinates of the vertex, the distance from the initial point to the vertex, and the latitude where a longitude line and the great circle track intersect.

Northern latitudes and western longitudes are keyed in and displayed as positive values; southern latitudes and eastern longitudes, as negative values.

No point on a leg should be at either the North or South Pole. Nor should a leg pass more than halfway around the earth. Points located at diametrically opposite sides of the earth should not be used because there are an infinite number of great circle courses through such points.

These keystrokes solve for the great circle distance (DIST) in nautical miles; the initial course (C_i); the longitude (λ_v) and latitude (L_v) of the vertex; the distance (DIST_v) from the initial point to the vertex; and the latitude (L_i) when an intermediate longitude is known.

1. Key in the latitude of your destination. Press **g** **⇨H** **STO** 2, then press 1 **f** **⇨R**.
2. Key in the latitude of your starting point. Press **g** **⇨H** **STO** 1, then press 1 **f** **⇨R** **STO** 5.
3. Press **xzy** **STO** 6 **R↓** **×** **R↓** **STO** 7 **×** **R↓** **R↓** **R↓**.
4. Key in the longitude of the starting point, and press **f** **⇨H** **STO** 3.
5. Key in the longitude of the destination, and press **g** **⇨H** **STO** 4.
6. Press **−** **STO** 0 **f** **cos** **×** **+** **g** **cos⁻¹** **STO** 8. Press 60 **×**. The great circle distance (DIST) is displayed.
7. Press **RCL** 7 **RCL** 6 **RCL** 8 **f** **cos** **×** **−**.
8. Press **RCL** 8 **f** **sin** **÷** **RCL** 5 **÷** **g** **cos⁻¹** **RCL** 0 **f** **sin** **STO** 0.
9. If the number on the display is positive, press **xzy** **STO** 9 to obtain the great circle course (C_i). If the displayed number is negative, press **xzy** 360 **xzy** **−** **STO** 9 to obtain C_i .
10. Press **RCL** 1 **f** **tan** **STO** 1.

11. Press **RCL** 2 **f** **tan** **STO** 2. If you don't want to find the coordinates of the vertex, skip to step 17.
12. Press **RCL** 4 **RCL** 1 **f** **→R** **xzy** **RCL** 3 **RCL** 2 **f** **→R** **R↓** **−** **R↓** **−** **R↓** **R↓** **÷** **9** **tan⁻¹** **ENTER↑** **f** **→HMS**.
The longitude of the vertex (λ_v) is displayed.
13. If λ_v is negative, press **xzy** 180 **+** **f** **→HMS** to obtain the alternate vertex longitude (λ_v'). If λ_v is positive, press **xzy** 180 **−** **f** **→HMS** to obtain the alternate vertex longitude (λ_v').
14. To calculate the latitude of the vertex (L_v), let $\lambda_i = \lambda_v$ or λ_v' . Key in λ_i and press **9** **→H** **ENTER↑** **ENTER↑**.
15. Press **RCL** 4 **−** **f** **sin** **RCL** 1 **×** **xzy**.
16. Press **RCL** 3 **−** **f** **sin** **RCL** 2 **×** **−** **RCL** 0 **÷** **9** **tan⁻¹** **f** **→HMS**. The latitude of the vertex (L_v) or of any intermediate point is displayed (depending whether you keyed in λ_v or λ_i).
17. To calculate the distance to the vertex, press **9** **→H** **f** **sin** **RCL** 9 **f** **cos** **RCL** 5 **×** **÷** **9** **√x** **9** **sin⁻¹** 60 **×**. The distance from the initial point to the vertex ($DIST_v$) in nautical miles is displayed.

Example: A ship is proceeding from Manila to Los Angeles. The captain wishes to sail a great circle course from the entrance of San Bernardino Strait, $L12^\circ45'12''N$, $\lambda124^\circ20'06''E$ (input as negative), to $L33^\circ48'48''N$, $\lambda120^\circ07'06''W$, five miles south of Santa Rosa Island.

Find the initial great circle course and great circle distance, the latitude and longitude of the vertex, and the distance from the initial point to the vertex.

Press

33.4848 **9** **→H** **STO** 2

1 **f** **→R**

12.4512 **9** **→H** **STO** 1

Display

33.81

0.83

12.75

1 **f** **→R** **STO** 5
x_zy **STO** 6
R **+** **x** **R** **+** **STO** 7
x **R** **+** **R** **+**
124.2006 **CHS** **g** **→H** **STO** 3
120.0706 **g** **→H** **STO** 4
- **STO** 0
f **cos** **x** **+** **g** **cos⁻¹** **STO** 8
60 **x**

RCL 7 **RCL** 6 **RCL** 8
f **cos** **x** **-**
RCL 8 **f** **sin** **÷** **RCL** 5 **÷**
g **cos⁻¹** **RCL** 0 **f** **sin** **STO** 0
x_zy **STO** 9

RCL 1 **f** **tan** **STO** 1
RCL 2 **f** **tan** **STO** 2
RCL 4 **RCL** 1 **f** **→R**
x_zy **RCL** 3 **RCL** 2 **f** **→R**
R **+** **-** **R** **+** **-** **R** **+** **R** **+**
÷ **g** **TAN⁻¹** **ENTER** **+**
f **→HMS**
x_zy 180 **+** **f** **→HMS**

g **→H** **ENTER** **+** **ENTER** **+**
RCL 4 **-** **f** **sin** **RCL** 1
x **x_zy** **RCL** 3 **-** **f** **sin**
RCL 2
x **-** **RCL** 0 **÷** **g** **tan⁻¹**
f **→HMS**
g **→H** **f** **sin** **RCL** 9
f **cos** **RCL** 5 **x** **÷** **g** **1/x**
g **sin⁻¹** 60 **x**

0.98

0.81

-124.34

120.12

-244.45

103.10

6185.88

Great circle distance.

0.61

0.90

50.32

Great circle course (decimal degrees).

0.23

0.67

0.75

-19.26

160.34

 λ_v . λ'_v . (Leave in display for L_v calculation.)

160.57

0.23

0.67

41.21

50.32

0.94

4228.83

 L_v (L_i).Distance_v.

Sight Reduction Table

Given the observer's latitude and both the declination and local hour angle (LHA) of a celestial body, you can calculate the azimuth and altitude of the body.

Northern latitude, northern declinations and western hour angles are input as positive values. Southern latitudes, southern declinations, and eastern hour angles are input as negative values.

The keystrokes are as follows:

1. Key in the declination, and press **9** **⇨H** **1** **f** **⇨R**.
2. Key in the observer's latitude, and press **9** **⇨H** **1** **f** **⇨R**.
3. Press **STO** **1** **x↔y** **STO** **2** **R↔** **x** **R↔** **STO** **3** **x** **R↔** **R↔** **R↔**.
4. Key in the local hour angle (LHA), and press **9** **⇨H** **1** **f** **⇨R**.
5. Press **x↔y** **STO** **4** **R↔** **x** **+** **9** **sin⁻¹** **STO** **5** **f** **⇨HMS**. The computed altitude of the celestial body is displayed.
6. Press **RCL** **3** **RCL** **2** **RCL** **5**. Press **1** **f** **⇨R** **R↔** **x** **-** **x↔y**.
7. Press **RCL** **1** **x** **÷** **9** **cos⁻¹** **STO** **6** **RCL** **4**.
8. If the displayed value is positive, press **360** **RCL** **6** **-** to obtain the azimuth of the body in decimal degrees.

If the displayed value is negative, press **RCL** **6** to obtain the azimuth.

This procedure may also be used for star identification. Input the azimuth in place of LHA and the observed altitude in place of declination, and your outputs are declination and LHA respectively. You can then identify the star by comparing this computed declination to the list of stars in *The Nautical Almanac*.

Example: Compute the altitude and azimuth of the sun if its LHA is 333°01'54''W and its declination is 12°28'06''S (input as negative). The assumed latitude is 34°11'06''S (input as negative).

Press

12.2806 **CHS** **9** **⇨H** **1**

f **⇨R**

Display

0.98

| | | |
|---|----------------|---------------------------|
| 34.1106 CHS 9 ⇨H 1 f ⇨R | 0.83 | |
| STO 1 x↔y STO 2 R⇧ × | | |
| R⇧ STO 3 × R⇧ R⇧ R⇧ | 0.81 | |
| 333.0154 9 ⇨H 1 f ⇨R | 0.89 | |
| x↔y STO 4 R⇧ × + | | |
| 9 sin⁻¹ STO 5 f ⇨H.MS | 57.16 | Altitude. |
| f FIX 4 | 57.1558 | Displayed to four places. |
| f FIX 2 | 57.16 | |
| RCL 3 RCL 2 RCL 5 | 57.27 | |
| 1 f ⇨R R⇧ × - x↔y | | |
| RCL 1 × | | |
| ÷ 9 cos⁻¹ STO 6 RCL 4 | -0.45 | |
| RCL 6 | 54.97 | Azimuth. |

Altitude = $57^{\circ}15'58''$ Azimuth = 54.97°

Surveying Applications

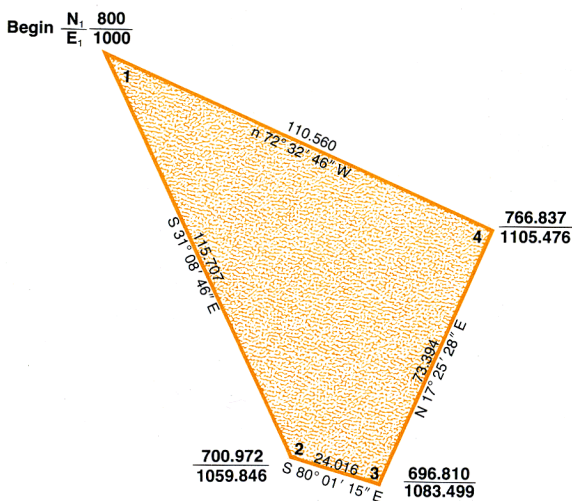
Bearing Traverse

This procedure uses bearing and distances to calculate the coordinates of successive points in a traverse. Area, closing distance, and closing azimuth can be calculated for a closed traverse.

1. Press **f** **REG**.
2. Key in the northing of your starting point (N_1) and press **STO** 1.
3. Key in the easting of the starting point (E_1) and press **STO** 2 **x↔y** **Σ+**.
4. Input one of the following:
 - Key in the NE bearing, and press **9** **⇨H**.
 - Key in the SE bearing, and press **9** **⇨H** **CHS** 180 **+**.
 - Key in the NW bearing, and press **9** **⇨H** **CHS**.
 - Key in the SW bearing, and press **9** **⇨H** 180 **+**.
5. Input either of the following:
 - Key in the horizontal distance.
 - Key in the slope distance, and press **ENTER⇩**. Next, key in the zenith angle, and press **9** **⇨H** **f** **sin** **×**.

6. Press $\boxed{f} \boxed{+R} \boxed{\Sigma+} \boxed{RCL} \boxed{3} \boxed{f} \boxed{LAST X} \boxed{+} \boxed{STO} \boxed{3}$.
7. Press $\boxed{f} \boxed{LAST X} \boxed{2} \boxed{\div} \boxed{-} \boxed{\times} \boxed{STO} \boxed{+} \boxed{0}$.
8. Press $\boxed{RCL} \boxed{5}$ to obtain northing of the second point. Press $\boxed{RCL} \boxed{7}$ to obtain the easting of the second point.
9. Repeat steps 4 thru 8 for each successive leg on the traverse.
10. Press $\boxed{RCL} \boxed{0}$ to obtain the area. (Ignore the sign if your answer is negative.)
11. Press $\boxed{RCL} \boxed{2} \boxed{RCL} \boxed{1} \boxed{f} \boxed{\Sigma-} \boxed{RCL} \boxed{7} \boxed{RCL} \boxed{5} \boxed{9} \boxed{\rightarrow P}$ to obtain the closing horizontal distance.
12. Press $\boxed{x\leftrightarrow y} \boxed{f} \boxed{\rightarrow HMS}$ to obtain the closing azimuth. (If your answer is negative, add 360° .)

Example: Given the following closed traverse, find the closing northing, easting, azimuth, and distance, as well as the area.



Press

Display

f REG

800 STO 1

1000 STO 2

x₂y Σ+

31.0846 g →H CHS 180 +

115.707 f →R Σ+

RCL 3 f LAST X + STO 3

f LAST X 2 ÷ - ×

STO + 0 RCL 5

RCL 7

80.0115 g →H CHS 180 +

24.016 f →R Σ+ RCL 3

f LAST X + STO 3

f LAST X 2 ÷ - ×

STO + 0 RCL 5

RCL 7

17.2528 g →H

73.394 f →R Σ+

RCL 3 f LAST X + STO 3

f LAST X 2 ÷ - ×

STO + 0 RCL 5

RCL 7

72.3246 g →H CHS

110.560 f →R Σ+

RCL 3 f LAST X + STO 3

f LAST X 2 ÷ - ×

STO + 0 RCL 5

RCL 7

RCL 0

RCL 2 RCL 1 f Σ-

RCL 7 RCL 5 g →P

x₂y f →H.MS

f FIX 4

800.00 N₁1000.00 E₁

1.00

148.85

-99.03

700.97 N₂1059.85 E₂

99.98

-103.19

696.81 N₃1083.50 E₃

17.42

-33.16

766.84 N₄1105.48 E₄

-72.55

5.00

-2.3909900-03

800.00 Closing N.

1000.01 Closing E.

-5104.08 Area.

4.00

0.01 Closing HD.

109.05 Closing azimuth.

109.0549

Closing N = 800

Closing E = 1000.01

Area = 5104.08 sq. feet

Closing Horizontal Distance = 0.01 feet

Closing Azimuth = $109^{\circ}05'49''$

Field Angle Traverse

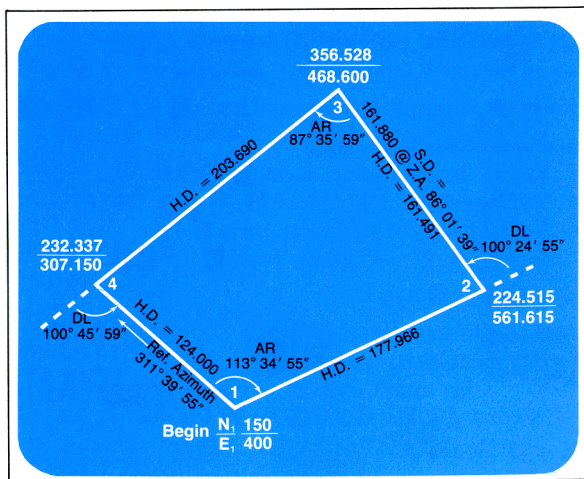
In the previous application, you used a bearing with respect to a reference line to calculate the coordinates of a point. With field angle traverse, you use an angle with respect to the previous side.

This procedure uses angles or deflections and distances to calculate coordinates of successive points in a traverse. Area, closing distance, and closing azimuth can be calculated for a closed traverse.

1. Press **f** **REG**.
2. Key in the initial easting (E_1), and press **ENTER**.
Key in the initial northing (N_1), and press **Σ +**.
3. Key in the reference azimuth, and press **g** **\rightarrow H** 180 **+** **STO** 1.
4. Input *one* of the following:
 - Key in the right angle, press **g** **\rightarrow H** 180 **+**.
 - Key in the left angle, press **g** **\rightarrow H** **CHS** 180 **+**.
 - Key in the right deflection, press **g** **\rightarrow H**.
 - Key in the left deflection, press **g** **\rightarrow H** **CHS**.
5. Press **RCL** 1 **+** **STO** 1.
6. Input *one* of the following:
 - Key in the horizontal distance.
 - Key in the slope distance, and press **ENTER**. Key in the zenith angle, and press **g** **\rightarrow H** **f** **sin** **\times** .
7. Press **f** **\rightarrow R** **Σ +**.
8. Omit this step if area is not calculated. Press **RCL** 3 **f** **LAST X** **+** **STO** 3 **f** **LAST X** 2 **\div** **$-$** **\times** **STO** **+** 2.
9. Press **RCL** 5 for N.

10. Press **RCL** 7 for E.
11. Repeat steps 4 thru 10 for all points.
12. Press **RCL** 2 for area.
13. For closing horizontal distance and azimuth, key in E_1 again and press **ENTER**. Key in N_1 again and press **f** **Σ^-** . Press **RCL** 7 **RCL** 5 **9** **\rightarrow P** to obtain closing HD.
14. Press **$x \rightarrow y$** **f** **\rightarrow HMS** to obtain the closing azimuth. (If negative, add 360° by pressing 360 **f** **\rightarrow HMS+.**)

Example: Given the following traverse, calculate the northings, eastings, closing horizontal distance, and closing azimuth.



Press

f REG
 400 ENTER 150 Σ^+
 311.3955 g \rightarrow H 180 +
 STO 1
 113.3455 g \rightarrow H 180 +
 RCL 1 + STO 1
 177.966 f \rightarrow R Σ^+
 RCL 5
 RCL 7
 100.2455 g \rightarrow H CHS
 RCL 1 + STO 1
 161.880 ENTER 86.0139
 g \rightarrow H
 f sin \times
 f \rightarrow R Σ^+
 RCL 5
 RCL 7
 87.3559 g \rightarrow H 180 +
 RCL 1 + STO 1
 203.690 f \rightarrow R Σ^+
 RCL 5
 RCL 7
 100.4559 g \rightarrow H CHS
 RCL 1 + STO 1
 124.00 f \rightarrow R Σ^+
 RCL 5
 RCL 7
 400 ENTER 150 f Σ^-
 RCL 7 RCL 5 g \rightarrow P
 $\times \div y$ f \rightarrow HMS
 360 f \rightarrow HMS+
 f FIX 4

Display

1.00
 491.67
 293.58
 785.25
 2.00
 224.52
 561.61
 -100.42
 684.83
 86.03
 161.49
 3.00
 356.53
 468.60
 267.60
 952.43
 4.00
 232.34
 307.15
 -100.77
 851.67
 5.00
 149.90
 399.78
 4.00
 0.24
 -113.40
 246.19
 246.1943

 N₂

 E₂

 N₃

 E₃

 N₄

 E₄

 N₁

 E₁

Closing HD.

Closing azimuth.

 Displayed as HH.
 MMSS.

Inverse from Coordinates

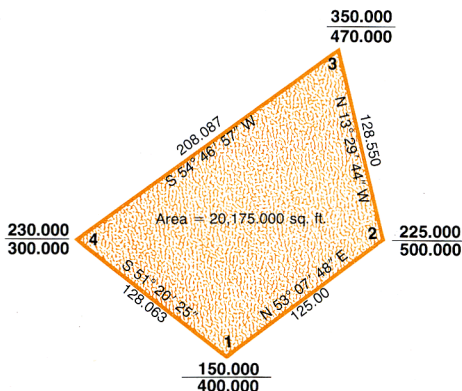
The previous two applications solved for coordinates of a traverse. This procedure uses the coordinates to calculate the distance and azimuth between points. Area can be calculated for a closed traverse.

1. Press **f** **REG**.
2. Key in N_1 , press **ENTER**, key in E_1 , and press **$\Sigma+$** .
3. Press **RCL** 7.
4. Key in the next N , and press **ENTER**. Key in the next E , and press **RCL** 5 **\square** **STO** 1 **R** **\pm** **$x\bar{z}y$** **\square** **STO** 2 **$x\bar{z}y$** **R** **\pm** **$x\bar{z}y$** **$\Sigma+$** .
5. If area is to be calculated, skip to step 6. If area is not calculated, press **f** **LAST X** **g** **$\rightarrow P$** to obtain the horizontal distance. Press **$x\bar{z}y$** **f** **$\rightarrow H.MS$** to obtain the azimuth. (If negative, add 360° .) Return to step 3 for next N and next E .
6. Press **RCL** 3 **f** **LAST X** **+** **STO** 3.
7. Press **f** **LAST X** 2 **\div** **\square** **\times** **STO** **+** 0 **RCL** 1 **RCL** 2 **g** **$\rightarrow P$** to obtain the horizontal distance.
8. Press **$x\bar{z}y$** **f** **$\rightarrow H.MS$** to obtain the azimuth. (If negative, add 360° .) Return to step 3 for next N and E , and repeat until N_1 and E_1 have been keyed in again.
9. Press **RCL** 0 to obtain the area.

Once the azimuth is calculated, you can convert azimuth (in H.MS) to bearing (in H.MS) as follows:

| Azimuth | Bearing |
|----------------------------|--|
| 0° to 90° | NE. |
| 90° to 180° | SE, press CHS 180 f H.MS+ . |
| 180° to 270° | SW, press 180 \square . |
| 270° to 360° | NW, press CHS 360 f H.MS+ . |

Example: Calculate the distance and azimuth between points for the following traverse and solve for area.



Press

f REG

f FIX 4

 150 ENTER 400 $\Sigma+$

RCL 7

 225 ENTER 500 RCL 5 \square STO 1

 R \uparrow x \uparrow y \square STO 2

 x \uparrow y R \uparrow x \uparrow y $\Sigma+$

 RCL 3 f LAST x \square STO 3

 f LAST x 2 \div \square \times STO \square 0

 RCL 1 RCL 2 9 \rightarrow P

 x \uparrow y f \rightarrow HMS

RCL 7

 350 ENTER 470 RCL 5 \square STO 1

 R \uparrow x \uparrow y \square STO 2

 x \uparrow y R \uparrow x \uparrow y $\Sigma+$

Display

0.0000

Display to 4 places.

1.0000

150.0000

100.0000

75.0000

2.0000

100.0000

3750.0000

125.0000

 HD_{1,2}.

53.0748

 Azimuth_{1,2}.

225.0000

-30.0000

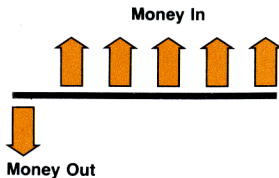
125.0000

3.0000

| | | |
|------------------------------|------------|--------------------------|
| RCL 3 f LAST X + STO 3 | 70.0000 | |
| f LAST X 2 ÷ - × STO + 0 | 10625.0000 | |
| RCL 1 RCL 2 g →P | 128.5496 | HD _{2,3} . |
| x↔y f →HMS | -13.2945 | |
| 360 f HMS+ | 346.3015 | Azimuth _{2,3} . |
| RCL 7 | 350.0000 | |
| 230 ENTER+ 300 RCL 5 - STO 1 | -170.0000 | |
| R+ x↔y - STO 2 | -120.0000 | |
| x↔y R+ x↔y Σ+ | 4.0000 | |
| RCL 3 f LAST X + STO 3 | -100.0000 | |
| f LAST X 2 ÷ - × STO + 0 | 1800.0000 | |
| RCL 1 RCL 2 g →P | 208.0865 | HD _{3,4} . |
| x↔y f →HMS | -125.1303 | |
| 360 f HMS+ | 234.4657 | Azimuth _{3,4} . |
| RCL 7 | 230.0000 | |
| 150 ENTER+ 400 RCL 5 - STO 1 | 100.0000 | |
| R+ x↔y - STO 2 | -80.0000 | |
| x↔y R+ x↔y Σ+ | 5.0000 | |
| RCL 3 f LAST X + STO 3 | 0.0000 | |
| f LAST X 2 ÷ - × STO + 0 | 4000.0000 | |
| RCL 1 RCL 2 g →P | 128.0625 | HD _{4,1} . |
| x↔y f →HMS | 128.3935 | Azimuth _{4,1} . |
| RCL 0 | 20175.0000 | Area. |

Financial Applications

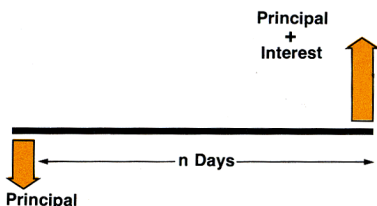
Throughout the following text are cash flow diagrams associated with the financial problems. A cash flow diagram is simply a picture of money in and money out.



If the solution to your problem isn't evident, ask yourself, "What are the cash flows?" Construct a cash flow diagram; then look for a similar diagram in this applications section.

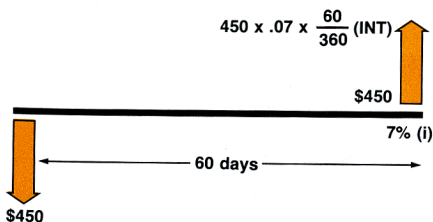
Simple Interest—360 and 365 Days

This calculation finds the amount of accrued simple interest on either a 360-day or 365-day basis when the number of days, interest rate, and principal (present value) are known.



1. Key in the number of days, press **ENTER**.
2. Key in 360 or 365 (whichever base you prefer to use), press **÷**.
3. Key in principal, press **×**.
4. Key in interest rate (as a decimal), press **×**.

Example 1: Your good friend needs a loan to start his latest enterprise. He has requested that you lend him \$450 for 60 days. You lend him the money at 7% simple interest, based on a 360-day year. What is the amount of accrued interest he will owe you in 60 days?



Press**Display**60 **ENTER** 360 **÷**

0.17

450 **×**

75.00

.07 **×**

5.25

Accrued interest.

Example 2: What is the accrued interest on \$450 for 60 days at 7%, figured on a 365-day year?

Press**Display**60 **ENTER** 365 **÷**

0.16

450 **×**

73.97

.07 **×**

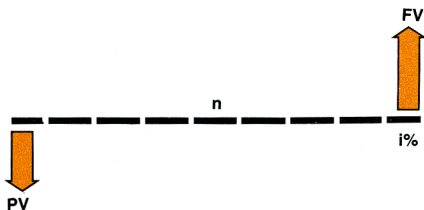
5.18

Accrued interest.

Compound Interest

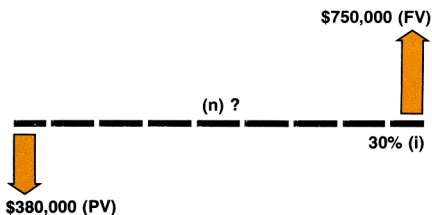
The following calculations deal with a lump sum deposit or investment, subject to multiple compounding of interest.

Number of Periods in a Compounded Amount. This calculation finds the number of compounding periods (n) when the interest rate (i), initial principal (present value) and compounded amount (future value) are given.



1. Press **9** **RESET**.
2. Input the following in any order:
 - Key in the periodic interest rate, press **f** **i**.
 - Key in the present value, press **f** **PV**.
 - Key in the future value, press **f** **FV**.
3. Press **f** **n** to obtain the number of periods.

Example: A potential development site currently appraised at \$380,000 appreciates at 30% per year. If this rate continues, how many years will it be before this land is worth \$750,000?



Press

Display

g **RESET**

30 **f** **i**

30.00

% annual interest rate.

380000 **f** **PV**

380000.00

Current value.

750000 **f** **FV**

750000.00

Future value.

f **n**

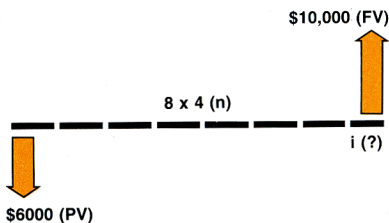
2.59

Years.

Interest Rate for Compounded Amounts. Given the initial amount, future value, and number of time periods, this calculates the interest rate:

1. Press **g** **RESET**.
2. Input the following in any order:
 - Key in the number of periods, press **f** **n**.
 - Key in the present value, press **f** **PV**.
 - Key in the future value, press **f** **FV**.
3. Press **f** **i** to obtain the periodic interest rate.
4. Key in the number of periods per year, press **ⓧ** to obtain an annual interest rate.

Example: What annual interest rate must be obtained to amass \$10,000 in 8 years on an investment of \$6000, with quarterly compounding?



Press

Display

g **RESET**

8 **ENTER** 4 **x** **f** **n**

32.00

6000 **f** **PV**

6000.00

10000 **f** **FV**

10000.00

f **i**

1.61

4 **x**

6.44

Quarters.

Present value.

Desired amount.

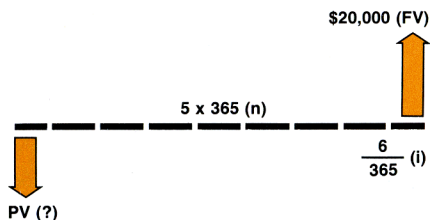
% quarterly interest rate.

% annual interest rate.

Present Value of a Compounded Amount. This calculation finds the present value of a future amount “discounted back” at a given rate for a specified number of periods.

1. Press **g** **RESET**.
2. Input the following in any order:
 - Key in the number of periods, press **f** **n**.
 - Key in the periodic interest rate, press **f** **i**.
 - Key in the future value, press **f** **FV**.
3. Press **f** **PV** to obtain the present value.

Example: In 5 years when your son starts college, you will need \$20,000. You deposit a lump sum in a certificate account that earns 6% compounded daily. How much do you need to deposit today to reach that goal?



Press

9 **RESET**

5 **ENTER** 365 **×** **f** **n**

6 **ENTER** 365 **÷** **f** **i**

20000 **f** **FV**

f **PV**

Display

1825.00

0.02

20000.00

14816.73

Days.

% daily interest rate.

Desired amount.

To be deposited.

Future Value of a Compounded Amount. This calculation finds the future value of an initial amount compounded at a given rate for a specified number of periods.

1. Press **9** **RESET**.
2. Input the following in any order:
 - Key in the number of periods, press **f** **n**.
 - Key in the periodic interest rate, press **f** **i**.
 - Key in the present value, press **f** **PV**.
3. Press **f** **FV** to obtain the future value.

Example 1: Property values in an unattractive area are declining at the rate of 2% per year. If your property is presently valued at \$32,000, what will it be worth in 6 years if this trend continues?

Press

g **RESET**

6 **f** **n**

2 **CHS** **f** **i**

32000 **f** **PV**

f **FV**

Display

6.00

-2.00

32000.00

28346.96

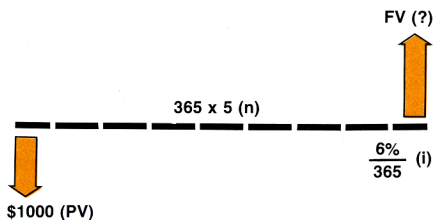
Years.

% depreciation rate.

Present value.

Value in 6 years.

Example 2: The local trading post manager opened up a savings operation 5 years ago, offering 6% compounded daily. Gold miner Yellowstone Sam deposited \$1000 at that time, and now wants to know his present balance and the total accrued interest after all this time.



Press

g **RESET**

5 **ENTER** 365 **x** **f** **n**

6 **ENTER** 365 **÷** **f** **i**

1000 **f** **PV**

f **FV**

RCL **f** **PV** **=**

Display

1825.00

0.02

1000.00

1349.83

349.83

Days.

% daily interest rate.

Original deposit.

Present balance.

Accrued interest.

Interest Rate Conversions

Nominal Rate Converted to Effective Rate. Given a nominal interest rate and the number of compounding periods per year, this keystroke procedure computes the effective annual interest rate.

1. Press **9** **RESET**.
2. Key in the number of compounding periods per year, press **f** **n**.
3. Key in the nominal rate, press **x \div y** **\div** **f** **i**.
4. Key in 100, press **f** **PV**.
5. Press **f** **FV**.
6. Press **RCL** **f** **PV** **=** to obtain effective annual rate.

Example: What is the effective annual rate of interest if the annual nominal rate of 12% is compounded quarterly?

| Press | Display | |
|---|--------------|----------------------------|
| 9 RESET | | |
| 4 f n | 4.00 | Compounding periods. |
| 12 x\divy \div f i | 3.00 | % quarterly interest rate. |
| 100 f PV f FV RCL f PV = | 12.55 | % effective interest rate. |

Add-on Interest Rate Converted to APR. An add-on interest rate determines what portion of the principal will be added on for repayment of a loan. This sum is then divided by the number of months in the loan to determine the monthly payment. For example, a 10% add-on rate for 36 months on \$3000 means add one-tenth of \$3000 for 3 years (300×3). This is usually called the “finance charge.” The total (\$3900) is divided by the number of payments (36) to obtain the monthly payment (\$108.33).

This keystroke procedure converts an add-on interest rate to an annual percentage rate and calculates the monthly payment.

1. Press **9** **RESET**.
2. Key in number of *months* in loan. Press **f** **n** **ENTER** **ENTER** 12 **÷**.
3. Key in add-on rate, press **ENTER** 100 **÷** **×** 1 **+** **÷** **f** **PV** 1 **f** **PMT**.
4. Press **f** **i** to obtain the monthly interest rate.
5. Press **9** **RESET** **f** **i**.
6. Press 12 **×** to obtain the APR.
7. Press **RCL** **f** **n** **f** **n**.
8. Key in the principal and press **f** **PV**.
9. Press **f** **PMT** to obtain the monthly payment.

Example: Calculate the APR and monthly payment of a 5%, \$1000 add-on loan which has a life of 18 months.

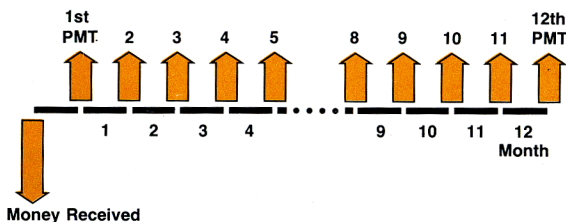
| Press | Display |
|--|-------------------------------|
| 9 RESET | |
| 18 f n ENTER ENTER | 18.00 Months. |
| 12 ÷ | |
| 5 ENTER 100 ÷ × | |
| 1 + ÷ f PV 1 f PMT | |
| f i | 0.77 % monthly interest rate. |
| 9 RESET f i | 0.77 Reuse i. |
| 12 × | 9.27 % APR. |
| RCL f n f n | 18.00 Recall and reuse n. |
| 1000 f PV | 1000.00 Loan principal. |
| f PMT | 59.72 Monthly payments. |

What's Really Happening? A word of explanation about those last keystrokes: pressing **9** **RESET** prepares the calculator to solve for a different variable. Since *n* is unchanged in both parts of the calculation, you can reuse that value by pressing **RCL** **f** **n** **f** **n**. For a detailed explanation, refer to appendix B.

Ordinary Annuities (Payments in Arrears)

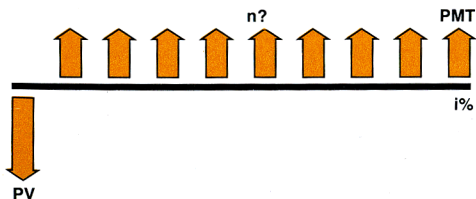
An annuity is a series of equal payments made at regular intervals. The time between annuity payments is called the payment interval or payment period. If your payment is due at the *end* of each payment period, it's called an ordinary annuity or payment in arrears. Examples of ordinary annuities are a car loan (where you drive away now and pay later) or a mortgage (where the payments start one month after you get your loan).

The time/money relationship for an ordinary annuity with monthly payments for a year would look like this:



The following problems all pertain to ordinary annuities, e.g., loans, mortgages, and sinking funds.

Number of Periods (Ordinary Annuity). This procedure calculates the number of periods for an ordinary annuity, when the present value and interest rate are known.



1. Press **g** **RESET**.
2. Input the following in any order:
 - Key in the periodic interest rate, press **f** **i**.
 - Key in the payment, press **f** **PMT**.
 - Key in the present value, press **f** **PV**.
3. Press **f** **n** to obtain the number of periods.

Example 1: A. Hunter borrows \$2000 at 10% interest to go on an African safari. His payments are \$100 a month. How long will it take him to pay off the loan?

Press**g** **RESET**10 **ENTER** 12 \div **f** **i**100 **f** **PMT**2000 **f** **PV****f** **n****Display**

| | |
|------|--------------------------|
| 0.83 | % monthly interest rate. |
|------|--------------------------|

| | |
|--------|------------------|
| 100.00 | Monthly payment. |
|--------|------------------|

| | |
|---------|--------------------|
| 2000.00 | Principal of loan. |
|---------|--------------------|

| | |
|-------|---------|
| 21.97 | Months. |
|-------|---------|

Example 2: Through an insurance fund, you have accumulated \$50,000 for your retirement. How long can you withdraw \$3000 every 6 months (starting 6 months from now) if the fund earns 5% per annum compounded semiannually?

Press**g** **RESET**5 **ENTER** 2 \div **f** **i**3000 **f** **PMT**50000 **f** **PV** **f** **n****Display**

| | |
|------|-----------------------------|
| 2.50 | % semiannual interest rate. |
|------|-----------------------------|

| | |
|---------|--|
| 3000.00 | |
|---------|--|

| | |
|-------|-------------------------|
| 21.83 | Semiannual withdrawals. |
|-------|-------------------------|

Interest Rate or APR (Ordinary Annuity). This routine computes the periodic interest rate for an ordinary annuity, given the number of periods, payment amount, and initial principal.

1. Press **g** **RESET**.
2. Input the following in any order:
 - Key in number of periods, press **f** **n**.
 - Key in payment amount, press **f** **PMT**.
 - Key in present value, press **f** **PV**.
3. Press **f** **i** to obtain the periodic interest rate.

The computed periodic rate is multiplied by the number of periods per year to obtain an annual rate.

Example 1: What is the annual interest rate on a 2-year, \$1775 loan with \$83.65 monthly payments?

Press

Display

g **RESET**

2 **ENTER** 12 **x** **f** **n**

24.00 Number of payments.

83.65 **f** **PMT**

1775 **f** **PV**

1775.00 Amount of loan.

f **i**

1.01 % monthly interest rate.

12 **x**

12.11 % annual interest rate.

Borrowers are sometimes charged fees related to the issuance of a mortgage, which effectively raises the interest rate. Given the basis of the fee charge, the true annual percentage rate may be calculated.

Example 2: A borrower is charged 2 points for the issuance of his mortgage. If the mortgage amount is \$50,000 for 30 years, and the interest rate is 9% per year, with monthly payments, what annual percentage rate is the borrower paying? (1 point is equal to 1% of the mortgage amount.)

First, compute the payment amount which is based on \$50,000.

Press

g **RESET**
 30 **ENTER** 12 **x** **f** **n**
 9 **ENTER** 12 **÷** **f** **i**
 50000 **f** **PV** **f** **PMT**
g **RESET**
f **PMT**
RCL **f** **n** **f** **n**
RCL **f** **PV** 2 **%** **=**
f **PV**
f **i**
 12 **x**

Display

| | |
|-----------------|-----------------------------------|
| 360.00 | Months. |
| 0.75 | % monthly interest rate. |
| 402.31 | Monthly payment. |
| 402.31 | Clear status for APR calculation. |
| 360.00 | Reuse payment. |
| | Recall and reuse time periods. |
| 49000.00 | Effective amount received. |
| 0.77 | % monthly percent-age rate. |
| 9.23 | % APR. |

What's Really Happening? For a mortgage with fees, the borrower is making payments on the original loan amount, which corresponds with the initial calculation of the payment amount. If you borrow \$10,000, but are immediately charged \$500 in fees, you really only receive \$9500. But, your payments are based on \$10,000. With fees, then, you're really paying the same for less money, which generates the need to compute the true APR.

Payment Amount (Ordinary Annuity). This routine calculates the payment amount for an ordinary annuity given the number of periods, the initial principal, and the interest rate.

1. Press **g** **RESET**.

2. Input the following in any order:
 - Key in the number of periods, press **f** **n**.
 - Key in the periodic interest rate, press **f** **i**.
 - Key in the present value, press **f** **PV**.
3. Press **f** **PMT** to obtain the payment amount.

Example: Find the monthly payment amount on a 30-year, \$52,000 mortgage at 9.75% annual interest rate.

| Press | Display |
|---|--------------------------------------|
| g RESET | |
| 30 ENTER 12 × f n | 360.00 Number of payments. |
| 9.75 ENTER 12 ÷ f i | 0.81 % monthly interest rate. |
| 52000 f PV f PMT | 446.76 Monthly payment. |

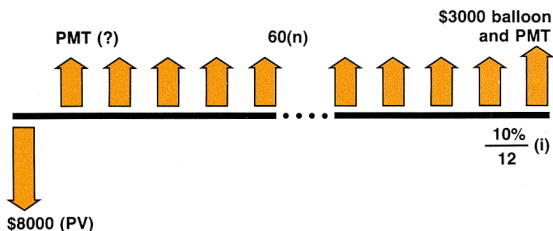
Payment Amount (Ordinary Annuity with Balloon). A common financial occurrence is an annuity that has a large payment at the end, like so:



The last payment—usually considerably larger although it could also be smaller than the others—is called a balloon payment or balloon.

By subtracting the present value of the balloon payment from the loan amount, the problem effectively becomes “What is the monthly payment on a direct reduction loan?”

Example: Yellowstone Sam is heading north, and will invest in an \$8000 dog sled and team. His loan specifies 60 monthly payments at 10% with a balloon payment in the 60th month of \$3000. What will his monthly payments be?

**Press**

g **RESET**

60 **f** **n**

10 **ENTER** 12 **÷** **f** **i**

3000 **f** **FV** **f** **PV**

g **RESET**

f **PV**

RCL **f** **n** **f** **n**

RCL **f** **i** **f** **i**

8000 **RCL** **f** **PV** **-**

f **PV**

f **PMT**

Display

60.00

Months.

0.83

% monthly interest rate.

1823.37

Present value of balloon.

1823.37

Input balloon PV.

60.00

Recall and reuse n.

0.83

Recall and reuse i.

6176.63

Net present value of loan amount less balloon.

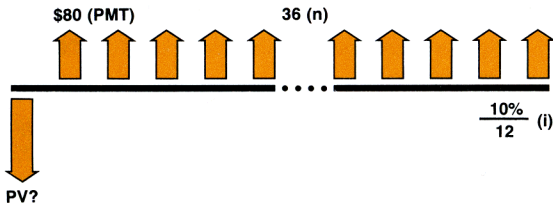
131.24

Monthly payment.

Present Value (Ordinary Annuity). This calculation finds the principal amount of a direct reduction loan when the interest rate, payment amount and number of payments are known.

1. Press **g** **RESET**.
2. Input the following in any order:
 - Key in number of periods, press **f** **n**.
 - Key in the periodic interest rate, press **f** **i**.
 - Key in the payment amount, press **f** **PMT**.
3. Press **f** **PV** to obtain the present value.

Example: Yellowstone Sam decides to purchase a snowmobile. He plans to pay \$80 per month for 3 years, and he's willing to pay 10% annual interest. How much can he afford to pay for the snowmobile?



Press

g **RESET**

36 **f** **n**

80 **f** **PMT**

10 **ENTER** 12 \div **f** **i**

f **PV**

Display

36.00

Number of payments.

80.00

Monthly payment.

0.83

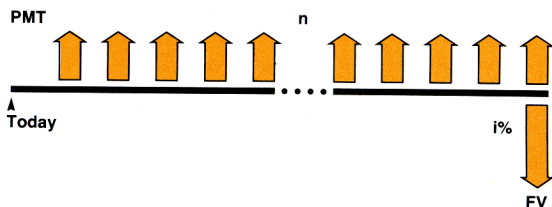
% monthly interest rate.

2479.30

Price he can pay for snowmobile.

Future Value (Ordinary Annuity). With loan calculations, you generally solve for n , i , PMT , or PV . There is another type of ordinary annuity called a “sinking fund,” where you make payments at regular intervals into a fund to discharge a debt (for example, to pay off a bond issue at maturity). With sinking fund calculations, you solve for n , i , PMT , or FV (how much you will have in the fund at a future date).

Sinking fund payments start at the end of the first period, like so:



This is different from opening a savings account with a starting deposit *today*. Savings are annuity due calculations and will be described later in this section.

The following procedure finds the future value of an ordinary annuity:

1. Press **g** **RESET**.
2. Input the following in any order:
 - Key in the number of periods, press **f** **n**.
 - Key in the periodic interest rate, press **f** **i**.
 - Key in the payment amount, press **f** **PMT**.
3. Press **f** **FV** to obtain the future value.

Example: A \$100,000 bond is to be discharged by the sinking fund method. If, starting 6 months from now, you deposit \$3914.75 twice a year into a sinking fund that pays 5% compounded semiannually, will you be able to pay off the bond in 10 years?

Press

g **RESET**
 10 **ENTER** 2 **x** **f** **n**
 5 **ENTER** 2 **÷** **f** **i**
 3914.75 **f** **PMT**
f **FV**

Display

| | |
|------------------|---------------------------------|
| 20.00 | Semiannual periods. |
| 2.50 | % semiannual interest. |
| 3914.75 | Semiannual deposit. |
| 100000.95 | Balance of fund after 10 years. |

Just barely!

Amortization Schedule

This calculation generates an amortization schedule for a direct reduction loan (mortgage) from the first payment to a given payment, when the loan amount, periodic interest rate, and payment amount are known.

1. Key in payment amount, press **STO** 1.
2. Key in periodic interest rate, press **ENTER** **ENTER**
3. Key in loan amount.
4. Press **x $\frac{1}{y}$** **%** to obtain the interest portion of the payment.
5. Press **RCL** 1 **x $\frac{1}{y}$** **=** to obtain the principal portion of the payment.
6. Press **=** to obtain the remaining loan balance.

Repeat steps 4 through 6 for all desired periods.

Example: Generate an amortization schedule for the first two months of a \$30,000 loan, at 7%, with monthly payments of \$200.

| Press | Display | |
|--|-----------------|--------------------------------------|
| 200 STO 1 | 200.00 | Payment. |
| 7 ENTER 12 ÷ ENTER | | |
| ENTER | 0.58 | % monthly interest rate. |
| 30000 x$\frac{1}{y}$ % | 175.00 | Interest portion of first payment. |
| RCL 1 x$\frac{1}{y}$ = | 25.00 | Principal portion of first payment. |
| = | 29975.00 | Remaining balance. |
| x$\frac{1}{y}$ % | 174.85 | Interest portion of second payment. |
| RCL 1 x$\frac{1}{y}$ = | 25.15 | Principal portion of second payment. |
| = | 29949.85 | Remaining balance. |

Accumulated Interest and Remaining Balance. This calculation finds the accumulated interest and remaining balance at any point in the life of a fully amortized mortgage, when the mortgage amount, payment amount, and periodic interest rate are known.

1. Press **g** **RESET** **f** **REG**.
2. Input the following in any order:
 - Key in periodic interest rate, press **f** **i**.
 - Key in payment amount, press **f** **PMT**.
 - Key in amount of loan, press **f** **PV**.
3. Press **f** **n** to obtain total number of periods.
4. Press **STO** 1 **g** **RESET**.
5. Key in the beginning payment number *less one*. Press **□** **f** **n**.
6. Press **RCL** **f** **i** **f** **i**.
7. Press **RCL** **f** **PMT** **f** **PMT**.
8. Press **f** **PV** to obtain the remaining balance at the beginning of the time frame.
9. Press **RCL** 1 to recall the total number of periods.
10. Key in the last payment number of the time frame, press **□** **f** **n**.
11. Press **f** **PV** for the remaining balance at the end of the time frame.
12. Press **□** **RCL** **f** **PMT**. Key in the number of payments made during the time frame, press **□**.
13. Press **xzy** **□** to obtain accumulated interest paid during the time frame.

Example: For tax purposes, a home owner wants to know the accumulated interest paid on his 8.75%, \$45,000 mortgage between payments 22 and 33 inclusive. His payments are \$354; calculate the interest paid.

Press

g **RESET** **f** **REG**

8.75 **ENTER** 12 **÷** **f** **i**

354 **f** **PMT**

45000 **f** **PV** **f** **n**

STO 1 **g** **RESET**

21 **—** **f** **n**

RCL **f** **i** **f** **i**

RCL **f** **PMT** **f** **PMT**

f **PV**

RCL 1

33 **—** **f** **n** **f** **PV**

— **RCL** **f** **PMT** 12 **×**

x²y **—**

Display

0.73

% monthly interest.

360.07

Total number of periods.

360.07

339.07

0.73

Recall and reuse i.

354.00

Recall and reuse PMT.

44415.11

Balance at beginning of 22nd payment.

360.07

44038.57

Remaining balance after 33rd payment.

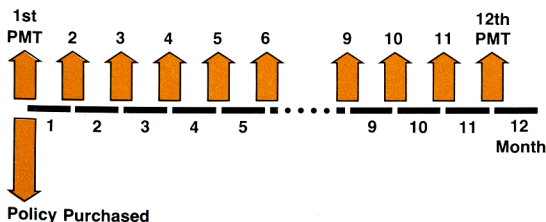
3871.46

Accumulated interest.

Annuity Due (Payments in Advance)

With some annuities—like insurance premiums or a lease—the payment is due at the beginning of the month. This is called an annuity due because the payment falls at the beginning of the payment period. Other terms are payments in advance or anticipated payments.

An annuity due with monthly payments for a year—say, a car insurance policy—looks like this:



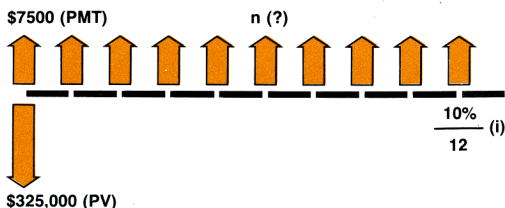
Notice that with an annuity due, you have a payment right away at the beginning of the first interval. (With an ordinary annuity, your payment isn't due until the end of the first period, but you also have a payment at the end of the entire term.)

The following calculations all deal with annuity due problems, e.g., savings, insurance, leases, and rents.

Number of Periods (Annuity Due). This calculation finds the total number of periods required for an initial amount when the payment amount (paid at the beginning of each period) and interest rate are known.

1. Press **9** **RESET**.
2. Key in periodic interest rate, press **f** **i**.
3. Key in payment amount, press **x\divy** **%** **+** **f** **PMT**.
4. Key in initial amount, press **f** **PV**, or key in desired amount and press **f** **FV**.
5. Press **f** **n**.

Example 1: Given an investment possibility of \$325,000 that will immediately produce rental income of \$7500 per month, how long must the investment be held to yield 10% per annum?



Press

9 **RESET**

10 **ENTER** 12 **÷** **f** **i**

Display

0.83

% monthly interest rate.

7500 $\times \div y$ % $+$ f PMT

7562.50

325000 f PV f n

53.43

Months.

Example 2: If you deposit \$50 a month in a savings account that pays 6% interest, how long will it take to reach \$1000?

Press

Display

 g RESET6 ENTER 12 \div f i

0.50

% monthly interest rate.

50 $\times \div y$ % $+$ f PMT1000 f FV

1000.00

Desired amount.

 f n

19.02

Months.

Interest Rate (Annuity Due). This routine calculates the interest rate on an initial amount, given the number of periods and payment amount (paid at the beginning of each period).

1. Press g RESET.
2. Key in *one less* than the total number of periods, press f n.
3. Key in payment amount, press f PMT.
4. Key in initial amount, press $\times \div y$ $-$ f PV.
5. Press f i to obtain the periodic rate. Multiply by the number of payment periods per year to obtain the annual rate or yield.

Example: Equipment worth \$12,000 is leased for 8 years with monthly payments in advance of \$200. The equipment is assumed to have no salvage value at the end of the lease. What yield rate does this represent?

Press

Display

 g RESET8 ENTER 12 \times 1 $-$ f n

95.00

Number of periods less one.

| | | |
|--|---------------|------------------|
| 200 f PMT | 200.00 | Payment amount. |
| 12000 x\divy - f PV | | |
| f i | 1.09 | % monthly yield. |
| 12 x | 13.07 | % annual yield. |

Payment Amount (Annuity Due). This calculation finds the payment amount (paid at the beginning of each period) given the initial amount, number of periods and interest rate.

1. Press **g** **RESET**.
2. Key in the number of payment periods, press **f** **n**.
3. Key in the periodic interest rate, press **f** **i**.
4. Press 1 **x \div y** **%** **+**.
5. Key in initial value, press **x \div y** **\div** **f** **PV**.
6. Press **f** **PMT**.

Example: The owner of a building presently worth \$70,000 intends to lease it for 20 years at the end of which time he assumes the building will be worthless (i.e., has no residual value). How much must the quarterly payments (in advance) be to achieve a 10% annual yield?

Press

Display

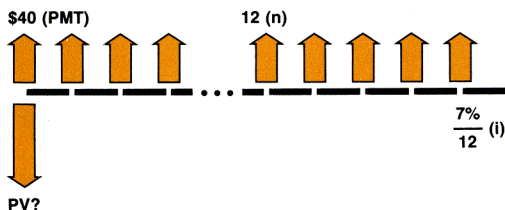
| | | |
|---|----------------|-------------------------------|
| g RESET | | |
| 20 ENTER\uparrow 4 x f n | 80.00 | Number of quarterly payments. |
| 10 ENTER\uparrow 4 \div f i | 2.50 | % quarterly interest. |
| 1 x\divy % + | | |
| 70000 x\divy \div f PV | | |
| f PMT | 1982.27 | Quarterly payment. |

Present Value (Annuity Due). This calculation finds the present value of a series of payments (made at the beginning of each period) given the number of payments, interest rate, and payment amount.

1. Press **g** **RESET**.
2. Key in the number of payment periods, press **f** **n**.

3. Key in the periodic interest rate, press **f** **i**.
4. Key in the payment amount, press **x \div y** **%** **+** **f** **PMT**.
5. Press **f** **PV**.

Example: The owner of a downtown parking lot has achieved full occupancy and a 7% annual yield by renting parking spaces for \$40 per month payable in advance. Several regular customers want to rent their spaces on an annual basis. What annual rent, also payable in advance, will maintain a 7% annual yield rate?



Press

9 **RESET**
 12 **f** **n**
 7 **ENTER** 12 **÷** **f** **i**
 40 **x \div y** **%** **+** **f** **PMT**
f **PV**

Display

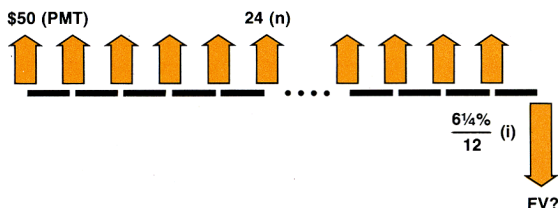
| | |
|---------------|----------------------------|
| 12.00 | Months. |
| 0.58 | % monthly interest rate. |
| 464.98 | Equivalent annual payment. |

Future Value (Annuity Due). This calculation finds the future value of a series of payments in advance, given the number of periods and interest rate.

1. Press **9** **RESET**.
2. Key in the number of periods, press **f** **n**.

3. Key in the periodic interest rate, press **f** **i**.
4. Key in the payment amount, press **x \div y** **%** **+** **f** **PMT**.
5. Press **f** **FV**.

Example: If you can afford to deposit \$50 per month in an account with $6\frac{1}{4}\%$ interest compounded monthly, how much will you have 2 years from now?



Press

9 **RESET**

24 **f** **n**

6.25 **ENTER** 12 **÷** **f** **i**

50 **x \div y** **%** **+** **f** **PMT**

f **FV**

Display

24.00

Number of periods.

0.52

% monthly interest rate.

1281.34

Savings.

Depreciation

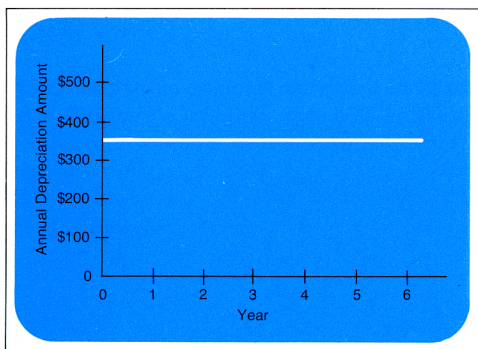
Machines, buildings, delivery trucks, showcases, tools and other tangible assets all decline in value with the passing of time. To provide for the replacement of obsolete or worn-out equipment, you usually set aside a fixed amount of money each year that is equal to the loss in value of that article during the year.

There are three methods of depreciation commonly used: straight-line, sum-of-the-digits, and declining-balance. Let's take a popcorn machine that costs \$2500 brand new and has a salvage value of \$400. We will depreciate the \$2100 over a life expectancy of 6 years, using each of these three methods.

Straight-Line Method—The straight-line method (SL) is simply a matter of dividing the total depreciable amount by the number of

useful years, then subtracting that amount each year from the item's value. The depreciation on the popcorn machine is \$2100, divided by 6 years = \$350 a year.

If you plot this depreciation amount on a graph, it looks like a straight line, hence, the name.



The advantage of the straight-line method is its simplicity—it's easy to figure and it's consistent. Your deduction is always the same.

To calculate the annual depreciation allowance on your HP-27:

1. Calculate and key in depreciable amount (cost less salvage value), press **ENTER↑** **ENTER↑**.
2. Key in asset's useful life (number of years), press **÷** to obtain each year's depreciation.
3. Press **STO** **1** **□** to obtain depreciable value after the first year.
4. Continue pressing **RCL** **1** **□** to obtain the remaining depreciable value for each subsequent year. If book value is needed, add salvage value to depreciable value.

Example: A duplex costing \$41,500 (exclusive of land) is depreciated over 25 years, using the straight-line method. What is the annual depreciation amount and remaining depreciable value for years 1 and 2 if it has no salvage value?

Press41500 **ENTER** **ENTER**25 **÷****STO** 1 **=****RCL** 1 **=****Display****41500.00****1660.00****39840.00****38180.00**

Depreciable value.

Annual depreciation amount.

Remaining depreciable value year 1.

Remaining depreciable value year 2.

In reality, some items depreciate the most during their initial periods of use. For example, the value of your car declines the most in the first two years. Or perhaps your equipment doesn't wear out most in the early years, but you want to increase the initial depreciation for the financial advantages. This is called accelerated depreciation, and you would use one of the two following methods.

Sum-of-the-Years-Digits Method—The sum-of-the-years-digits method (SOYD) is based on the sum of the digits from one to the number of years of the asset's life. For the popcorn machine, the life is 6 years, so:

$$6 + 5 + 4 + 3 + 2 + 1 = 21 \text{ (sum of the years digits)}$$

Theoretically, the first year you use up 6/21 of the asset's life; the second year, 5/21 of the asset's life, etc. So, the first year, you multiply the depreciation amount (\$2100) by that year's use (6/21).

$$\frac{6}{21} \times \$2100 = \$600$$

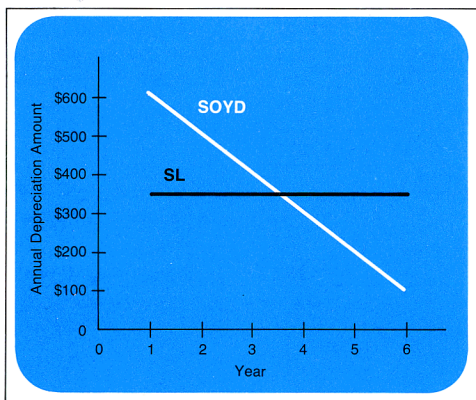
The second year, by that year's use:

$$\frac{5}{21} \times \$2100 = \$500$$

The third year:

$$\frac{4}{21} \times \$2100 = \$400$$

If you plot the depreciation on a graph, you can see the difference between the straight-line method and the sum-of-the-years-digits method.



The keystroke procedure below finds the depreciation and remaining depreciable value using the SOYD method for each year of an asset's depreciable life when its useful life expectancy and cost (less salvage value) are known.

1. Key in the beginning depreciable value, press **ENTER↑**.
2. Key in asset's depreciable life, press **ENTER↑**.
3. Press **ENTER↑** 1 **+** **CHS** **STO** 1 **×** 2 **÷** **÷** **STO** 2 **CL×**.
4. Press 1 **RCL** 1 **+** **STO** 1 **RCL** 2 **×** to obtain the depreciation amount.

5. Press \square to obtain the remaining depreciable value at the end of the year.
6. Repeat steps 4 and 5 for subsequent year's depreciation and remaining depreciable value.

Example: Apartments valued at \$88,000 are depreciated over 25 years using SOYD depreciation. What is the depreciation amount and remaining depreciable value for the first 2 years?

Press**Display**88000 **ENTER****88000.00**

Depreciable value.

25 **ENTER****25.00**

Depreciable life.

ENTER 1 **+** **CHS****STO** 1 **×**2 **÷** **÷****STO** 2 **CLx****0.00**1 **RCL** 1 **+** **STO** 1**RCL** 2 **×****6769.23**Depreciation amount
year 1.**−****81230.77**Remaining depreciable
value, year 1.1 **RCL** 1 **+** **STO** 1**RCL** 2 **×****6498.46**Depreciation amount
year 2.**−****74732.31**Remaining depreciable
value, year 2.

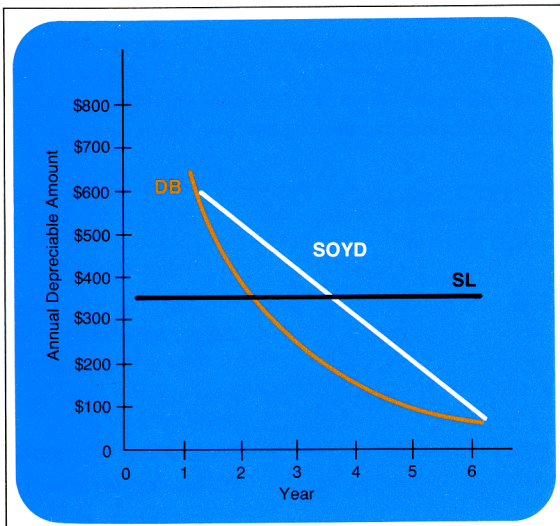
Declining Balance Method—With the declining balance method (sometimes called the fixed-rate method), a constant percentage is applied each year to the remaining balance (book value) to find the depreciation amount. The salvage value is not subtracted initially, but the asset may not be depreciated below this salvage value.

Certain declining balance “factors” are authorized for income tax purposes. A factor of 1.25 simply means 125% declining balance; 2.00 means double-declining balance. To compute the annual

depreciation rate, divide the factor by the asset's estimated life in years. If you use a factor of 1.50 (150%) for the popcorn machine, then you will depreciate it $\frac{150}{6}$ or 25% each year.

| | Depreciation | Balance (Book Value) |
|--|--------------|-------------------------|
| 1 st year 25% of \$2500 = | \$625.00 | \$1875.00 |
| 2 nd year 25% of \$1875 = | \$468.75 | \$1406.25 |
| 3 rd year 25% of \$1406.25 = | \$351.56 | \$1054.69 |
| 4 th year 25% of \$1054.69 = | \$263.67 | \$ 791.02 |
| 5 th year 25% of \$791.02 = | \$197.75 | \$ 593.27 |
| 6 th year \$593.27 - \$400 (salvage value) = | \$193.27 | \$ 400.00 |

You can plot the declining balance depreciation on the graph to compare it with the other two methods.



The following keystroke procedure finds the depreciation and remaining book value for each year of an asset's depreciable life when the declining factor, cost, salvage value, and life expectancy are known:

1. Key in declining factor (1.25 for 125% declining balance, 2.00 for double declining etc.), press **ENTER** 100 **×**.
2. Key in number of years of useful life, press **÷** **STO** 1.
3. Key in cost (do not deduct salvage value).
4. Press **RCL** 1 **%** to obtain first year's depreciation,
5. Press **=** to obtain remaining book value after first year.
6. Repeat steps 4 and 5 to obtain each succeeding year's depreciation and remaining book value until book value is equal to or less than the salvage value. In the period when the remaining book value is less than the salvage value, the previous book value should be reduced by the salvage value to obtain the final year's depreciation.

Example: The Drifter Apartments have a depreciable value of \$86,000. The owner wishes to use 125% declining balance depreciation over 20 years. What is the annual depreciation amount and remaining book value in years 1 and 2?

| Press | Display | |
|--------------------------|----------|--------------------------------------|
| 1.25 ENTER | | |
| 100 × | 125.00 | |
| 20 ÷ STO 1 | 6.25 | |
| 86000 | 86000 | Depreciable value. |
| RCL 1 % | 5375.00 | First year depreciation. |
| = | 80625.00 | Remaining depreciable value, year 1. |
| RCL 1 % | 5039.06 | Second year depreciation. |
| = | 75585.94 | Remaining depreciable value, year 2. |

Discounted Cash Flow Analysis

Discounted cash flow analysis is a way of evaluating investment alternatives in which investments are compared on the same basis—their present dollar worth. Two forms of discounted cash flow analysis are the net present value approach and the discounted or internal rate of return approach. These two functions, **NPV** and **IRR**, are preprogrammed on your HP-27.

Net Present Value. The **g** **NPV** function solves for the net present value according to the formula:

$$\text{NPV} = -\text{INV} + \frac{\text{cf}_1}{(1+i)^1} + \frac{\text{cf}_2}{(1+i)^2} + \dots + \frac{\text{cf}_n}{(1+i)^n}$$

NPV = net present value.

INV = your initial investment or cash outlay.

$\frac{\text{cf}_1}{(1+i)^1}$ = cash flow generated the first year.

$\frac{\text{cf}_2}{(1+i)^2}$ = cash flow generated the second year.

i = the rate of return or desired yield.

INV, the original investment, is negative because it represents a cash outlay. With the net present value approach, you are comparing the sum of the present values of all the future cash flows (all the cf 's) to that initial investment.

At the start, NPV is negative because you've put out a large amount (INV). As the cash returns flow in, NPV will increase. Eventually—hopefully—NPV will turn positive. When $\text{NPV} = 0$, you have reached the break-even point on the investment. If NPV is positive, your investment is profitable.

The following procedure is used to find the net present value when the assumed yield rate (interest rate) and periodic cash flows are known.

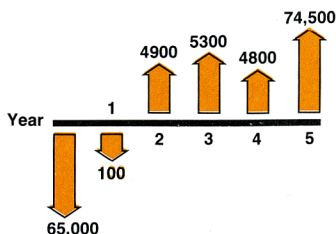
1. Press **g** **RESET**. Press **f** **Σ** .
2. Key in the initial investment and press **f** **PV**.
3. Key in the desired or known yield rate and press **f** **i**.
4. Key in the first cash flow and press **g** **NPV**. The number displayed is the net present value. Repeat this step for all successive cash flows.

As soon as the display shows a positive number, the investment is recovered on a discounted cash flow basis.

The number of cash flows and the sum of their present values are accumulated in registers 8 and 9. Hence, you must clear the registers by pressing **f** **Σ** before solving for NPV.

Example 1: An investor pays \$65,000 for a duplex which he intends to keep 5 years and then sell. The first year he knows he will have to spend a considerable amount for repairs. If he desires a 9% after-tax yield rate and the after-tax cash flows are as follows, will he achieve this yield?

| Year | Cash Flows (\$) |
|------|-----------------|
| 1 | -100 |
| 2 | 4900 |
| 3 | 5300 |
| 4 | 4800 |
| 5 | 74500 |



Press

Display

g **RESET** **f** **Σ**

65000 **f** **PV**

65000.00

Initial investment

9 **f** **i**

9.00

% annual yield.

100 **CHS** **g** **NPV**

-65091.74

First year NPV.

4900 **g** **NPV**

-60967.51

Second year NPV.

5300 **g** **NPV**

-56874.94

Third year NPV.

4800 **g** **NPV**

-53474.50

Fourth year NPV.

74500 **g** **NPV**

-5054.61

Fifth year NPV.

Since the final NPV is negative, the investment does *not* achieve the desired yield.

Internal Rate of Return. The **g** **IRR** function solves for the internal or discounted rate of return for a maximum of 10 uneven cash flows. The same formula applies, only $NPV = 0$.

$$0 = -INV + \frac{cf_1}{(1+i)^1} + \frac{cf_2}{(1+i)^2} + \dots + \frac{cf_{10}}{(1+i)^{10}}$$

Solving for i gives you the rate of return (IRR) on your investment. This is actually quite a complex calculation, but your HP-27 handles it with ease. In fact, the HP-27 is the *only* pocket calculator that's preprogrammed to solve IRR.

Key in the cash flows as follows:

1. Press **g** **RESET**.
2. Key in the amount of the initial investment, and press **f** **PV**.
3. Key in the total number of cash flows and press **f** **n**.
4. Key in the first cash flow (followed by **CHS** if it is negative, i.e., if you had a loss that year). Store that number in register 0 by pressing **STO** 0.

5. Key in the second cash flow and press **STO** 1. Continue keying in cash flows (up to 10) and store them in successive storage registers, up to register 9.
6. Press **9** **IRR**.

If each of the cash flows = 0 or if there are no cash flows, an **Error** message is displayed. If you haven't recovered even part of the initial investment, then logically there is no "return" and you can't solve for IRR.

If all the cash flows are negative, your answer is meaningless.

Example 1: What is the estimated rate of return on a restaurant costing \$200,000 that produces the following cash flows?

| Year | Cash Flow |
|------------------|-----------|
| 1 | \$ -4,000 |
| 2 | \$ 20,000 |
| 3 | \$ 27,000 |
| 4 | \$ 42,000 |
| 5 | \$ 56,000 |
| 6 (You sell it.) | \$230,000 |

Press

Display

9 **RESET**

200000 **f** **PV**

6 **f** **n**

4000 **CHS** **STO** 0

20000 **STO** 1

27000 **STO** 2

42000 **STO** 3

56000 **STO** 4

230000 **STO** 5

9 **IRR**

200000.00

6.00

-4000.00

20000.00

27000.00

42000.00

56000.00

230000.00

12.72

Initial investment.

Number of cash flows.

First cash flow.

Second cash flow.

Third cash flow.

Fourth cash flow.

Fifth cash flow.

Sixth cash flow.

% annual yield or rate of return on investment.

Example 2: In a moment of altruism, Joe Carnegie loaned his nephew, Winston, \$8000 to finish his last two years of college. Winston agreed to pay back the original amount plus \$300 in interest, within five years after graduation.

If Winston repaid the loan as follows, what was benevolent Uncle Joe's rate of return?

| Year | Payment |
|-------------|-----------|
| Junior year | 0. |
| Senior year | 0. |
| Year 1 | \$ 500.00 |
| Year 2 | \$1300.00 |
| Year 3 | \$1000.00 |
| Year 4 | \$1000.00 |
| Year 5 | \$4500.00 |

Press

Display

9 **RESET**

8000 **f** **PV**

8000.00

Initial investment.

7 **f** **n**

7.00

Number of cash flows.

0 **STO** 0

0.00

First cash flow.

0 **STO** 1

0.00

Second cash flow.

500 **STO** 2

500.00

Third cash flow.

1300 **STO** 3

1300.00

Fourth cash flow.

1000 **STO** 4

1000.00

Fifth cash flow.

1000 **STO** 5

1000.00

Sixth cash flow.

4500 **STO** 6

4500.00

Seventh cash flow.

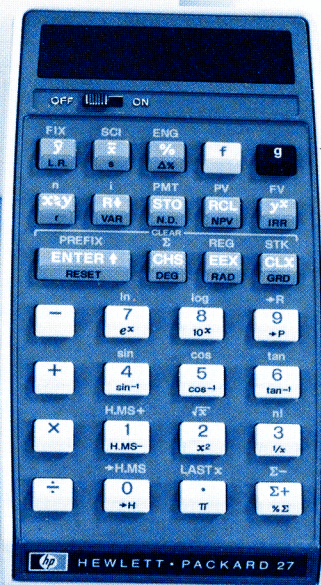
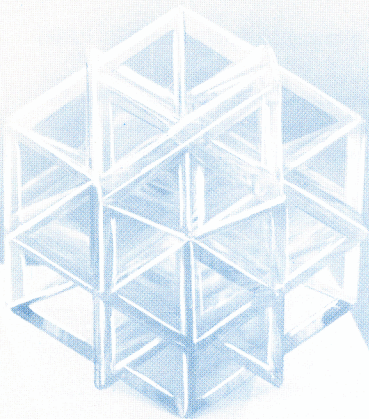
9 **IRR**

0.62

% rate of return.

The following constraints apply to the **IRR** function:

1. The number of cash flows (**n**) must be 1 or more and may not exceed 10.
2. The sequence PV, cf_1 , cf_2 ... cf_n may contain only one sign reversal. That is, if the first several cash flows are negative quantities, they may be followed by positive quantities only. You cannot intersperse positive and negative cash flows.
3. The loss or gain should not be extravagant, and the IRR must be greater than -100% . Extreme values will produce a meaningless answer or a continually blinking display.



Appendix A

Accessories, Maintenance, and Service

Standard Accessories

Your HP-27 comes complete with the following standard accessories: battery pack, ac adapter/recharger, soft carrying case, and the *HP-27 Owner's Handbook*.

CAUTION:

Use of any batteries other than the Hewlett-Packard battery pack may damage your calculator.

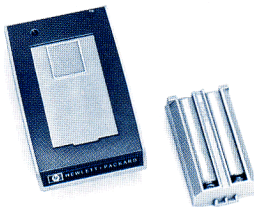
Optional Accessories

Other accessories are shown here. To order additional standard or optional accessories, see your nearest dealer or send the Accessory Order Form, along with a check or money order, to:

Hewlett-Packard Company
19310 Pruneridge Avenue
Cupertino, CA 95014



Security Cradle



Reserve Power Pack

Outside the United States, mail the Accessory Order Form to the Hewlett-Packard Sales Office nearest you.

Calculator Operation

Charging Times

Your calculator contains a rechargeable battery pack. *The batteries must be in the calculator for it to operate.* With the recharger connected to the calculator and to an ac power outlet, the batteries will charge, whether the calculator is ON or OFF. Normal charging times from dead battery to full charge are:

| | |
|----------------|----------|
| Calculator ON | 17 hours |
| Calculator OFF | 6 hours |

Charge the battery before portable use. Shorter charge periods than the above will reduce battery operating time.

Whether the calculator is OFF or ON, the battery pack will never overcharge with the ac adapter/recharger connected to the ac line.

Charging the Battery

The procedure for using the battery charger is as follows:

1. Turn the HP-27 power switch OFF.
2. Insert the recharger plug into the rear connector of the HP-27, and insert the power plug into a live power outlet. The HP-27 may feel warm to the touch while recharging. Don't be concerned; this is normal because part of the charging circuit is inside the calculator.
3. At the end of the charging period, you may continue to use your HP-27 with ac power; or turn the power switch OFF and disconnect the recharger for battery-only operation.

If the battery pack won't hold a charge, it may be defective. If the one-year warranty is in effect, return the pack to Hewlett-Packard according to the shipping instructions. If your warranty has expired, see your nearest dealer or use the Accessory Order Form to purchase a replacement.

Battery Operation

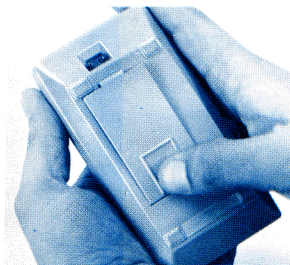
Use only the HP battery pack. A fully-charged pack provides approximately 3 to 5 hours of continuous operation. By turning the power OFF when the calculator is not in use, you can conserve energy and make the HP-27 battery pack last easily through a normal working day.

Low Power Display

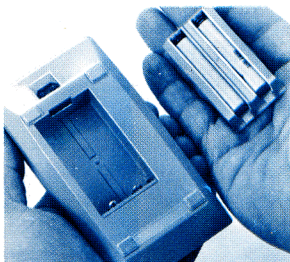
When the batteries get low, several decimal points will appear on the display. This means you have approximately one minute of operating time left. Then you must either charge the battery or insert a fully-charged spare battery pack.

Replacing the Battery Pack

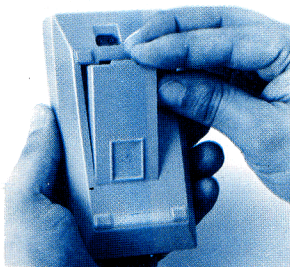
To replace your battery pack, use the following procedure:



1. Turn the HP-27 power switch to OFF, and disconnect the re-charger from the calculator.
2. With your thumb on the square inset on the rear of the calculator, press down and slide the door in the direction of the arrow. The latch will spring open.



3. Remove the battery pack.



4. To insert the new pack, slant the leading edge of the pack into the upper edge of the doorway, then snap the latch down into place.

If you use your HP-27 extensively on batteries, you may want to order the Reserve Power Pack. This optional accessory lets you charge one pack while using the other.

Blank Display

If the display blanks out, turn the calculator OFF, then ON. If 0.00 does not appear on the display, check the following:

1. Make sure the ac adapter/recharger is plugged into an ac outlet. If not, turn the calculator OFF before plugging in the recharger.
2. Examine the battery pack to see if the contacts are dirty.
3. Substitute a fully-charged battery pack, if available, for the one that was in the calculator.

If the display is still blank, your calculator should be serviced. (Refer to *Warranty* in this appendix.)

Temperature Range

Temperature ranges for the calculator are:

| | | |
|------------|---------------|----------------|
| Operating: | 0° to 45°C | 32° to 113°F |
| Charging: | 15° to 40°C | 59° to 104°F |
| Storage: | -40° to +55°C | -40° to +131°F |

Temperatures above or below these specified limits may not cause permanent damage to your calculator, but you probably will get incorrect answers. Also, temperature extremes will damage the batteries.

Serial Number

The serial number of your calculator is located on the back of the case, right above the battery door and below the recessed plug. It's advisable to keep a separate record of your serial number.

Warranty

Full One-Year Warranty

The HP-27 is warranted against defects in materials and workmanship for one (1) year from date of delivery. During the warranty period,

Hewlett-Packard will repair or, at its option, replace at no charge components that prove to be defective, provided the calculator is returned, shipping prepaid, to Hewlett-Packard's Customer Service Facility. (Refer to Shipping Instructions.)

This warranty does not apply if the calculator has been damaged by accident or misuse, or as a result of service or modification by other than an authorized Hewlett-Packard Customer Service Facility. No other express warranty is given by Hewlett-Packard. **HEWLETT-PACKARD SHALL NOT BE LIABLE FOR CONSEQUENTIAL DAMAGES.**

Warranty Transfer

The warranty is recorded by calculator serial number, as well as date of purchase. If you sell your calculator or give it as a gift, the warranty is transferrable and remains in effect for the new owner until the original one-year expiration date. It is not necessary for the owner to notify Hewlett-Packard of the transfer.

Shipping Instructions

Whether the unit is under warranty or not, it is the customer's responsibility to pay shipping charges to the Hewlett-Packard Customer Service Facility. Please return the calculator with all standard accessories and a completed Service Card, describing the problem.

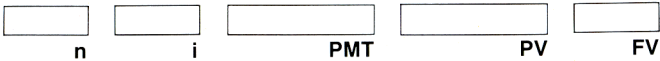
Under normal conditions, your calculator will be repaired and reshipped within five working days of receipt at the Hewlett-Packard Customer Service Facility.

After warranty repairs are completed, the Service Center returns the unit with postage prepaid. On out-of-warranty repairs, the unit is returned C.O.D. (covering shipping costs and the service charge).

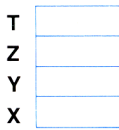
Appendix B

How the HP-27 Registers Work

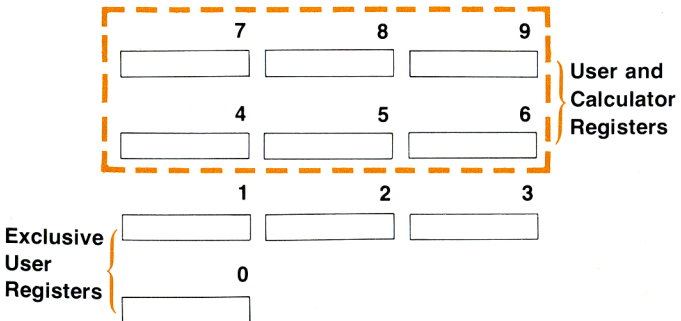
Inside your calculator are five financial registers (associated with the top row keys):



There are also four registers in the operational stack:



And 10 general-purpose storage registers. Four of these (registers 0 thru 3) are dedicated exclusively for your own use. The other six registers (4 thru 9) are shared by the user and the calculator.



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If you press 340 **STO** 5, that number is transferred from the display to register 5:



When you press **RCL** 5, a copy of that number appears on the display but the value is still stored in register 5:

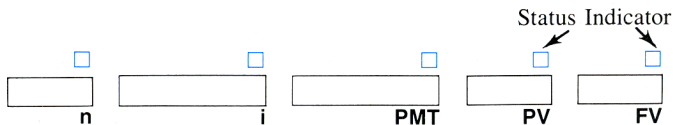
| | |
|---|--------|
| T | 0.00 |
| Z | 0.00 |
| Y | 0.00 |
| X | 340.00 |

← 340 5

When you press **f** **REG**, all 10 storage registers are cleared. If you press **f** **Σ**, just the statistical registers (4 thru 9) are cleared.

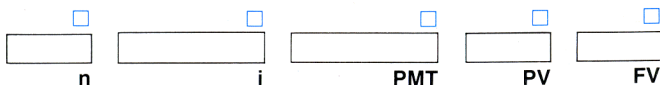
The Financial Registers

Associated with each of the financial registers is a status indicator. (You can't see them but they're there.)



When a status indicator is on (■), the value in the particular register is to be used as data for the next financial calculation. When a status indicator is reset or off (□), the register value cannot be used.

Let's take an example calculating the monthly payment on a 20-year \$27,000 mortgage with 8.5% annual interest. Press **g** **RESET** to reset the status indicators:



Key in 20 (the number of yearly payments):

| | |
|---|------|
| T | 0.00 |
| Z | 0.00 |
| Y | 0.00 |
| X | 20. |

(Display)

Since the n value must be monthly payment periods, convert the yearly figure by pressing **ENTER+** 12 **×**, then input it by pressing **f** **n**. The converted value on the display is copied into the n register and the first status indicator is turned on.

| | |
|---|--------|
| T | 0.00 |
| Z | 0.00 |
| Y | 0.00 |
| X | 240.00 |

(Display)

| | | | | |
|----------------|-------------|-------------|-------------|-------------|
| <div>240</div> | <div></div> | <div></div> | <div></div> | <div></div> |
| n | i | PMT | PV | FV |

The stack lift indicator is set after a financial value has been keyed into a financial register.

Next, key in the monthly interest rate: 8.5 **ENTER+** 12 **÷** **f** **i**. Again, the value on the display is copied into the i register and that status indicator is turned on.

| | |
|---|--------|
| T | 0.00 |
| Z | 0.00 |
| Y | 240.00 |
| X | 0.71 |

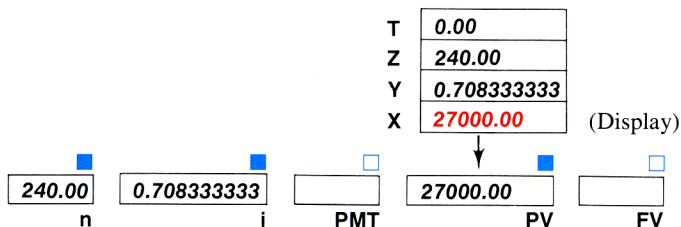
(Display)

| | | | | |
|----------------|-----------------------|-------------|-------------|-------------|
| <div>240</div> | <div>0.70833333</div> | <div></div> | <div></div> | <div></div> |
| n | i | PMT | PV | FV |

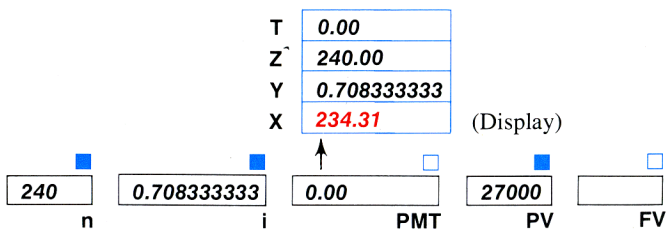
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Remember that although the value on the display is rounded off to two decimal places, your calculator always figures internally to 10 places, so the number placed in **i** is really 0.708333333. This is extremely important when you stop to think that those extra decimal places are used over 240 months!

To solve a top-row financial function, three status indicators must be on. Key in 27000 and press **f** **PV**:



The calculator sees that n, i and PV status indicators are turned on. Once three status indicators are on, pressing one of the other two financial keys will cause the calculator to execute and display the new value. Press **f** **PMT**:



There are two important points to remember: (1) the HP-27 will never allow you to turn on more than three status indicators; and (2) of the three status indicators that are on, at least one of them must be n or i.

Suppose you want to solve for payment again but with a different time value—say, a 25-year loan instead of 20 years. Since only one value changes and the necessary status indicators are already turned

on, you can simply overwrite the new number in the n register. Press 25 **ENTER** 12 **⊗** **f** **n**.

| | | | | | |
|---|-------------|-----------|--|--|--|
| T | 0.708333333 | | | | |
| Z | 0.708333333 | | | | |
| Y | 234.31 | | | | |
| X | 300.00 | (Display) | | | |

| | | | | |
|---------------|--------------------|-------------|--------------|----|
| 300.00 | 0.708333333 | 0.00 | 27000 | |
| n | i | PMT | PV | FV |

Solve for the new payment by pressing **f** **PMT**:

| | | | | | |
|---|-------------|-----------|--|--|--|
| T | 0.708333333 | | | | |
| Z | 0.708333333 | | | | |
| Y | 234.31 | | | | |
| X | 217.41 | (Display) | | | |

| | | | | |
|------------|--------------------|-------------|--------------|----|
| 300 | 0.708333333 | 0.00 | 27000 | |
| n | i | PMT | PV | FV |

Notice that the two calculated answers are stored in the X and Y registers of the stack for your convenience. For example, you can review both payment amounts by pressing **x↔y** to look back at the first answer.

Or, with both answers positioned like that in the stack, you can quickly subtract to calculate the difference between the two payment amounts:

| | | | | | |
|---|-------------|-----------|---|-------------|--------------------------------------|
| T | 0.708333333 | | T | 0.708333333 | |
| Z | 0.708333333 | | Z | 0.708333333 | |
| Y | 234.31 | | Y | 0.708333333 | |
| X | 217.41 | (Display) | X | 16.90 | \$ per month difference in payments. |

The RESET Function

The power of RESET becomes apparent when you do a problem that requires a different status indicator. Before, the prerequisite status indicators were *n*, *i* and *PV* in order to obtain *PMT*.

Suppose you can afford \$225 a month and wish to find the length of the loan. This involves a different set of known values. Now, the prerequisite status indicators are *i*, *PMT* and *PV*, and you want to find *n*.

Because this is a new financial calculation involving different input registers, reset the status indicators by pressing 9 RESET.

| | | |
|---|------------|-----------|
| T | 0.70833333 | |
| Z | 0.70833333 | |
| Y | 0.70833333 | |
| X | 16.90 | (Display) |

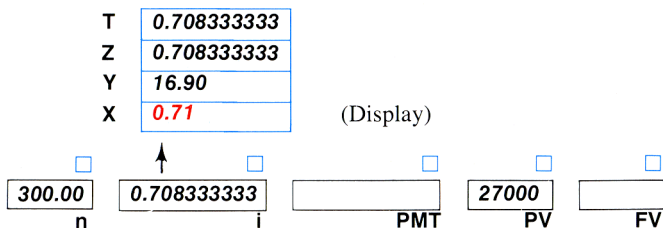
| | | | | |
|--|--|--|---|--|
| 300.00 | 0.70833333 | 0.00 | 27000 | |
| <i>n</i> | <i>i</i> | <i>PMT</i> | <i>PV</i> | <i>FV</i> |

Notice that your previous answer is still on the display, and hence, in the stack. If you want to use this number in a future problem, you can ENTER it up in the stack, store it in one of the 10 manual storage registers, or use it in another financial problem.

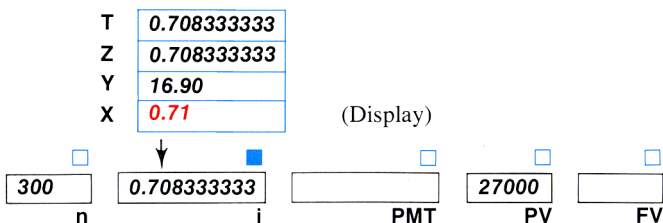
The five status indicators are reset by 9 RESET so the HP-27 is ready for a new problem. The values are still in the financial register—you can use them again or write over them with new values.

The i and PV values are the same as the original problem so we'll use that data over again. But how do you turn on the status indicators?

Press RCL f i to place that number in the display:

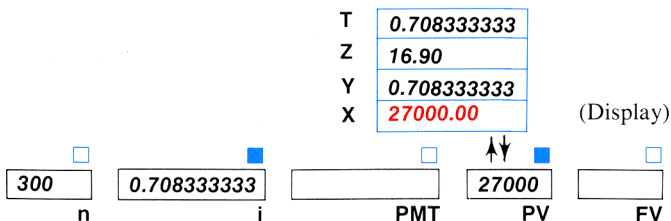


Then press **f** **i** again to transfer the number into the i register and turn on the status indicator:



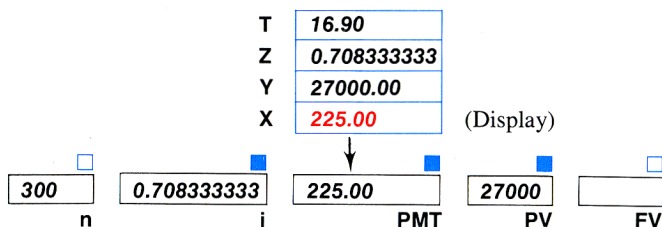
Herein lies one advantage of the **RESET** function: You don't have to calculate i again! (You may be tempted just to key in 0.71 **f** **i**, but multiply that rounded-off figure over 25 years and your answer would be wrong.)

Press **RCL** **f** **PV** **f** **PV** to do the same for the PV value:

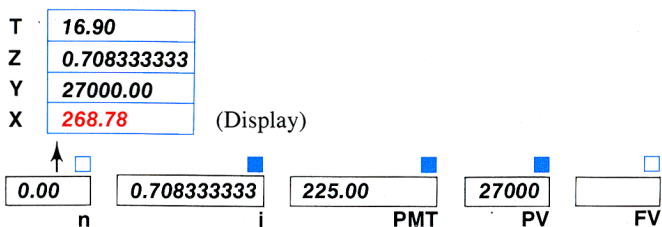


200 How the HP-27 Registers Work

Turn on the third status indicator by inputting the new payment amount, 225 f PMT .



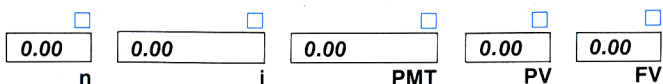
And solve for the length of the loan by pressing f n :



In summary, g RESET resets all five top-row status indicators so that the HP-27 is ready for a new financial problem. It does not erase the values in those registers.

The f REG Function

Pressing f REG clears the five financial registers, clears all 10 storage registers, and clears LAST X. It does not clear the stack.







| | |
|---|-------------|
| T | 16.90 |
| Z | 0.708333333 |
| Y | 27000.00 |
| X | 268.78 |

(Display)

| | | |
|-----------|-----------|-----------|
| 7 0.00 | 8 0.00 | 9 0.00 |
| 4 0.00 | 5 0.00 | 6 0.00 |
| 1 0.00 | 2 0.00 | 3 0.00 |
| 0 0.00 | | |

Any data in the storage registers is eliminated.

The Function

Pressing   clears the shared user-calculator registers 4 thru 9. These are the statistical registers, so you should press   before starting any new statistical problem.

| | | |
|-----------|-----------|-----------|
| 7 0.00 | 8 0.00 | 9 0.00 |
| 4 0.00 | 5 0.00 | 6 0.00 |
| 1 | 2 | 3 |
| 0 | | |

The Statistical Registers

Pressing the $\Sigma+$ key produces several different sums and products of values in the X- and Y-registers and stores this data in registers 4 thru 9:

| Register | Contents |
|----------------------|--|
| R ₄ | Number of entries (n). |
| R ₅ | Summation of x values (Σx). |
| R ₆ | Summation of x ² values (Σx^2). |
| R ₇ | Summation of y values. (Σy). |
| R ₈ | Summation of y ² values (Σy^2). |
| R ₉ | Summation of products of x and y values (Σxy). |
| Display (X-register) | Number of entries (n). |

If you input 50 ENTER 4 $\Sigma+$, the values are stored as follows:

| | | | | |
|---|-------|-------------------------|---|-------|
| T | | | T | 0.00 |
| Z | | | Z | 0.00 |
| Y | 50.00 | $\xrightarrow{\Sigma+}$ | Y | 50.00 |
| X | 4.00 | | X | 1.00 |

| | | |
|--------------|----------------|----------------|
| Σy 7 | Σy^2 8 | Σxy 9 |
| 50.00 | 2500.00 | 200.00 |
| n 4 | Σx 5 | Σx^2 6 |
| 1.00 | 4.00 | 16.00 |
| 1 | 2 | 3 |
| | | |
| 0 | | |
| | | |

If the next data pair were 80 **ENTER** 5 **Σ+**, the contents of registers 4 thru 9 would change as follows:

| | | | | |
|---|-------|---------------|---|-------|
| T | 0.00 | | T | 0.00 |
| Z | 50.00 | | Z | 50.00 |
| Y | 80.00 | → Σ+ → | Y | 80.00 |
| X | 5.00 | | X | 2.00 |

| | | |
|--------|-------------------|-------------------|
| Σy 7 | Σy ² 8 | Σxy 9 |
| 130.00 | 106000.00 | 600.00 |
| n 4 | Σx 5 | Σx ² 6 |
| 2.00 | 9.00 | 41.00 |
| 1 | 2 | 3 |
| | | |
| 0 | | |

Availability of Registers

Some functions utilize or overwrite other registers when solving a calculation. Any data stored in those registers will be lost, so you should be aware of this effect.

All statistical calculations overwrite the X-register or both the X- and Y-registers. The previous X value can be salvaged from **LAST X**.

When solving for **ŷ**, **L.R.** and **r**, the calculator uses the PV and FV registers as “scratchpads.” After execution, PV and FV are cleared. (If an error occurs, PV and FV are not cleared.)

The calculation for iterative **i** uses registers R₇, R₈ and R₉. After execution, these three registers are cleared (unless an error occurred).

The **IRR** function uses the i, PMT and FV registers as “scratchpads.” After execution or if an error occurs, these three financial registers are cleared.

Value Limits

The algorithms in the HP-27 can accommodate a wide range of values; however, the accuracy of your answer also depends on the values you key in (e.g., the “garbage in, garbage out” syndrome.)

A negative number for n will produce an answer, but one that’s meaningless because you used a negative time period.

You cannot have a negative number of entries for statistical problems. That is, you cannot key three inputs into $\Sigma+$, then subtract out four with $\Sigma-$.

The ND function is designed to cope with all realistic statistical applications. All calculations are accurate to $\pm 1 \times 10^{-7}$. However, if large input values are used to calculate extremely small tail areas, the function will produce answers accurate to two significant digits.

Extreme input values should not be used for all financial calculations. For example, do not use negative interest rates close to -100% or extremely small interest rates within $\pm 0.000001\%$.

Iterative solutions for i and IRR are accurate to $\pm 0.0001\%$ for small interest rates and to six significant digits for large interest rates. If the input value limits are not met, your HP-27 will try but may fail to find a solution in one of several ways. For example, you could so choose a combination of investment and cash flows or a combination of PMT , PV and FV such that the problem has no real solution, or it may have more than one solution. Your HP-27 may signal **Error**; it may display one of several possible solutions; it may display what is “almost” a solution; or it may search interminably until it is turned off. In any case, you can easily verify the answer in two ways. You can calculate the NPV for the IRR that is displayed and see if NPV is close to 0, or you can solve for PMT or n using the displayed interest rate and see if it agrees with the original inputs.

Appendix C

Formulas

The following formulas are preprogrammed in your HP-27 calculator.

Statistical Formulas

Mean

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{y} = \frac{\sum y}{n}$$

Standard Deviation

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}}$$

Variance

$$\text{VAR}_x = (s_x)^2$$

$$\text{VAR}_y = (s_y)^2$$

Normal Distribution

$f(x)$ = density function

Q = upper tail area

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-x^2}{2}\right)}$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{\left(\frac{-t^2}{2}\right)} dt$$

Correlation Coefficient

$$r = \frac{\Sigma xy - \frac{1}{n} \Sigma x \Sigma y}{(n - 1) s_x s_y}$$

Linear Regression

$$y = A + Bx$$

where A = y intercept of the line

B = slope of the line

$$A = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2}$$

$$B = \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2}$$

Linear Estimate

$$\hat{y} = A + Bx$$

Financial Formulas**Percentage**

$$\% = \frac{\text{Base} \times \text{Rate}}{100}$$

$$\text{Percent Difference} = \Delta\% = \left(\frac{\text{New Amount} - \text{Base}}{\text{Base}} \right) \times 100$$

Unless otherwise stated:

n = number of time periods

i = periodic interest rate, expressed as a decimal value

PMT = periodic payment

PV = present value or principal

FV = future value

I = interest amount

Compound Amount

$$FV = PV (1 + i)^n$$

Savings (Annuity Due)

$$FV = PMT \left[\frac{(1 + i)^n - 1}{i} \right] (1 + i)$$

Rents, Leases (Annuity Due)

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right] (1 + i)$$

**Direct Reduction Loans
(Ordinary Annuity)**

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

**Direct Reduction Loan with Balloon
Payment (Ordinary Annuity)**

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right] + BAL (1 + i)^{-n}$$

Sinking Fund (Ordinary Annuity)

$$FV = PMT \left[\frac{(1 + i)^n - 1}{i} \right]$$

Discounted Cash Flow Analysis

NPV = Net present value

INV = Initial investment

cf = cash flow

$$NPV = -INV + \frac{cf_1}{(1 + i)^1} + \frac{cf_2}{(1 + i)^2} + \dots + \frac{cf_n}{(1 + i)^n}$$

Depreciation

L = asset's useful life expectancy

SBV = starting book value

SAL = salvage value

DEP_k = depreciation for year k

RBV_k = remaining book value at the end of year k

RDV_k = remaining depreciable value at end of year k

■ Straight-Line Depreciation

$$DEP_k = \frac{SBV - SAL}{L}$$

$$RDV_k = RDV_{k-1} - DEP_k$$

■ Sum-of-the-Digits Depreciation

$$DEP_k = \frac{2(L - k + 1)}{L(L + 1)} \times (SBV - SAL)$$

$$RDV_k = RDV_{k-1} - DEP_k$$

■ Declining Balance Depreciation

F = declining balance factor

$$DEP_k = RBV_{k-1} \times \frac{F}{L}$$

$$RDV_k = RDV_{k-1} - DEP_k$$

Mathematical Formulas**Polar to Rectangular**

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Rectangular to Polar

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right)$$

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
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ETHIOPIA

*EMESCO Ltd.
P.O. Box 2550
Kassate Teshome Bldg.
Omedla Square
Addis Ababa
Tel: 12-13-87
Cable: EMESCO Addis Ababa

GUAM

*Guam Medical Supply, Inc.
Jay Ease Building, Room 210
P.O. Box 8383
Tamuning 96911
Tel: 646-4513

HONG KONG

*Schmidt & Co. (Hong Kong) Ltd.
P.O. Box 297
Connaught Road, Central
Hong Kong
Tel: 240168, 232735
Telex: HX4766
Cable: SCHMIDTCO Hong Kong

INDIA

*Blue Star Ltd.
Sahas
414/2 Vir Savarkar Marg
Prabhadevi
Bombay 400 025
Tel: 45 78 87
Telex: 4093
Cable: FROSTBLUE

Blue Star Ltd.
Bombay 400 020
Tel: 29 50 21

Blue Star Ltd.
Bombay 400 025
Tel: 45 73 01

Blue Star Ltd.
Kanpur 208 001
Tel: 6 88 82

Blue Star Ltd.
Calcutta 700 001
Tel: 23-0131

Blue Star Ltd.
New Delhi 110 024
Tel: 62 32 76

Blue Star Ltd.
Secunderabad 500 003
Tel: 7 63 91, 7 73 93

Blue Star Ltd.
Madras 600 001
Tel: 23954

Blue Star Ltd.
Jamshedpur 831 001
Tel: 7383

Blue Star Ltd.
Bangalore 560 025
Tel: 55668

Blue Star Ltd.
Cochin 682 001
Tel: 32069, 32161, 32282

INDONESIA

*BERCA Indonesia P.T.
P.O. Box 496
1st Floor JL, Cikini Raya 61
Jakarta
Tel: 56038, 40369, 49886
Telex: 2895 Jakarta

IRAN

*Hewlett-Packard Iran
Daftar Machine Building (No. 19)
Roosevelt Avenue, 14th Street
Tehran
Tel: 851082/3/4/5/6
Telex: 212574

IRAQ

Electromac Services
Baghdad
Tel: 95456

*Service

JAPAN

*Yokogawa-Hewlett-Packard Ltd.
 Ohashi Building
 1-59-1 Yoyogi
 Shibuya-ku, Tokyo
 Tel: 03-370-2281/92
 Telex: 232-2024 YHP
 Cable: YHPMARKET TOK 23 724

*Yokogawa-Hewlett-Packard Ltd.
 Nisei Ibaragi Bldg.
 2-2-8, Kasuga
 Ibaragi-shi
 Osaka
 Tel: 0726-23-1641
 Telex: 5332-385 YHP-Osaka

Yokogawa-Hewlett-Packard Ltd.
 Nakamura-Ku, Nagoya City
 Tel: 052-571-5171

Yokogawa-Hewlett-Packard Ltd.
 Yokohama, 221
 Tel: 045-312-1252

Yokogawa-Hewlett-Packard Ltd.
 Mito, 310
 Tel: 0292-25-7470

Yokogawa-Hewlett-Packard Ltd.
 Atsugi, 243
 Tel: 0462-24-0452

KENYA

*Business Machines Kenya Limited
 Olivetti House
 Uhru Highway/Lusaka Road
 P.O. Box 49991 NBI
 Nairobi
 Tel: 556066
 Cable: PRESTO Nairobi

KOREA

*American Trading Company
 Korea, Ltd.
 I.P.O. Box 1103
 Dae Kyung Bldg., 8th Floor
 107 Sejong-Ro
 Chongro Ku, Seoul
 Tel: (4 lines) 73 8924 7
 Cable: AMITRACO Seoul

KUWAIT

*Photo and Cine Equipment
 P.O. Box 270 Safat
 Tel: 422846/423801
 Telex: 2247

LEBANON

Constantin Macridis
 Beirut
 Tel: 366397/8

LIBYA

Kabir Stationery
 Tripoli
 Tel: 35201

H.M. Zeidan and Sons Organization
 Benghazi
 Tel: 94930/94963/93689

MOROCCO

Gerep Ltd.
 Casablanca
 Tel: 258196/279469

MOZAMBIQUE

*A.N. Goncalves, Lta.
 162, 1° Apt. 14 Av. D. Luis
 Caixa Postal 107
 Lourenco Marques
 Tel: 27091, 27114
 Telex: 6-203 NEGON Mo
 Cable: NEGON

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NEW ZEALAND

*Hewlett-Packard (N.Z.) Ltd.
94-96 Dixon Street
P.O. Box 9443
Courtenay Place,
Wellington
Tel: 59-559
Telex: 3898
Cable: HEWPACK Wellington

*Hewlett-Packard (N.Z.) Ltd.
Pakuranga Professional Centre
267 Pakuranga Highway
Box 51092
Pakuranga
Tel: 569-651
Cable: HEWPACK, Auckland

NIGERIA

*The Electronics Instrumentations
Ltd.
N6B/770 Oyo Road
Oluseun House
P.M.B. 5402
Ibadan
Tel: 22325

PAKISTAN

*Mushko & Company Ltd.
38B, Satellite Town
Rawalpindi
Tel: 41924
Cable: REMUS Rawalpindi

Mushko & Company Ltd.
Karachi-3,
Tel: 511027, 512927

PHILLIPINES

*Electronic Specialist & Proponents,
Inc.
Room 417 Federation
Center Bldg.
Muella de Binondo
P.O. Box 2649
Manila
Tel: 48-46-10 & 48-46-25
Cable: Espinc Manila

REUNION ISLANDS

*ZOOM
B.P. 938, 97400 Saint Denis
85 Rue Jean Chatel
Ile de la Reunion
Tel: 21-13-75
Cable: ZOOM

RHODESIA

*Field Technical Sales
45 Kelvin Road North
P.O. Box 3458
Salisbury
Tel: 705231 (5 lines)
Telex: RH 4122

RWANDA

*Buromeca
R.C. Kigali 1228
B.P. 264 Kigali
Rwanda

SAUDI ARABIA

*Modern Electronic Establishment
(M.E.E.)
P.O. Box 1228
Jeddah
Tel: 27798/31173
Telex: 40035

M.E.E.
Riyadh
Tel: 62596/29269

M.E.E.
Al Khobar
Tel: 44678/44813

Riyadh House Establishment
Riyadh
Tel: 21741/27360

*Service

SINGAPORE

- *Hewlett-Packard Singapore
(Pte.) Ltd.
Blk. 2, 6th Floor, Jalan
Bukit Merah
Redhill Industrial Estate
Alexandra P.O. Box 87,
Singapore 3
Tel: 633022
Telex: HPSG RS 21486
Cable: HEWPACK, Singapore

SOUTH AFRICA

- *Hewlett-Packard South Africa
(Pty.), Ltd.
P.O. Box 31716,
Braamfontein Transvaal
Milnerton
30 DeBeer Street
Johannesburg
Street Delivery Zip Code: 2001
P.O. Box Delivery Zip Code: 2017
Tel: 725-2030, 725-2080,
725-2081
Telex: 0226 JH
Cable: HEWPACK Johannesburg

- *Hewlett-Packard South Africa
(Pty.), Ltd.
Breeccastle House
Bree Street
Cape Town
Street Delivery Zip Code: 8001
P.O. Box Delivery Zip Code: 8018
Tel: 2-6941/2/3
Telex: 0006 CT
Cable: HEWPACK Cape Town

- *Hewlett-Packard South Africa
(Pty.), Ltd.
641 Ridge Road, Durban
P.O. Box 37099
Overport, Natal
Street Delivery Zip Code: 4001
P.O. Box Delivery Zip Code: 4067
Tel: 88-6102
Telex: 67954
Cable: HEWPACK

*Service

Hewlett-Packard South Africa
(Pty.), Ltd.
Sandton, Transvaal 2001
Tel: 802-1040/6

SYRIA

Sawah and Company
Damascus
Tel: 16367/19697

Suleiman Hilal el Mlawi
Damascus
Tel: 114663

TAHITI

- *Metagraph
B.P. 1741
Papeete
Tahiti
Tel: 20/320, 29/979
Cable: METAGRAPH PAPEETE
Telex: SOMAC 033 F.P.

TAIWAN

- *Hewlett-Packard Taiwan
39 Chung Hsiao West Road
Section 1
Overpass Insurance Corp. Bldg.
7th Floor
Taipei
Telex: TP824 HEWPACK
Cable: HEWPACK Taipei
Tel: 3819160, 3819161, 3819162

Hewlett-Packard Taiwan
Kaohsiung
Tel: 297319

THAILAND

- *UNIMESA Co., Ltd.
Elsom Research Building
Bangjak Sukumvit Avenue
Bangkok
Tel: 932387, 930338
Cable: UNIMESA Bangkok

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TUNISIA

Societe Samos
Tunis
Tel: 284 355

TURKEY

Melodi Records
Istanbul
Tel: 442636

Talekom
Istanbul
Tel: 494040

UNITED ARAB EMIRATES

*Emitac Limited
P.O. Box 1641
Sharjah
Tel: Sharjah 22779
Dubai 25795
Telex: Sharjah 8033

YEMEN

A. Besse and Co. Yemen Ltd.
Sanaa
Tel: 2182/2342

ZAMBIA

*R.J. Tilbury (Zambia) Ltd.
P.O. Box 2792
Lusaka
Tel: 73793
Cable: ARJAYTEE, Lusaka

**FOR AREAS NOT LISTED,
CONTACT:**

Hewlett-Packard Export
Trade Company
3200 Hillview Avenue
Palo Alto, California 94034
Tel: (415) 493-1501
TWX: 910-373-1260
Telex: 034-8300, 034-8493
Cable: HEWPACK Palo Alto

EUROPE

AUSTRIA

*Hewlett-Packard Ges. m.b.H.
Handelskai 52/53
P.O. Box 7
A-1205 Vienna
Tel: (0222) 35 15 21 to 32
Cable: HEWPACK Vienna
Telex: 75923 hewpack a

BELGIUM

*Hewlett-Packard Benelux
S.A./N.V.
Avenue del Col Vert, 1,
(Groenkraaglaan)
B-1170 Brussels
Tel: (02) 672 22 40
Cable: PALOBEN Brussels
Telex: 23-494 paloben bru

DENMARK

*Hewlett-Packard A/S
Datevej 52
DK-3460 Birkerød
Tel: (01) 81-66-40
Cable: HEWPACK AS
Telex: 16640 hp as

Hewlett-Packard A/S
DK-8600 Silkeborg
Tel: (06) 8271 66

FINLAND

*Hewlett-Packard Oy
Bulevard 26
P.O. Box 185
SF-00120 Helsinki 12
Tel: (90) 13730
Cable: HEWPACKOY Helsinki
Telex: 12-15363 hel

FRANCE

*Hewlett-Packard France
Quartier de Courtaboeuf
Boite Postale No. 6
F-91401 Orsay
Tel: (1) 907 78 25
Cable: HEWPACK Orsay
Telex: 60048

Hewlett-Packard France
F-69130 Ecully
Tel: (78) 33 81 25/83 65 25

Hewlett-Packard France
F-31770 Colomiers
Tel: (61) 78 11 55

Hewlett-Packard France
F-13271 Aéroport de Marignane
Tel: (91) 89 12 36

Hewlett-Packard France
F-35000 Rennes
Tel: (99) 36 33 21

Hewlett-Packard France
F-67000 Strasbourg
Tel: (88) 35 23 20/21

GERMAN FEDERAL REPUBLIC

*Hewlett-Packard GmbH
Vertriebszentrale Frankfurt
Bernerstrasse 117
Postfach 560 140
D-6000 Frankfurt 56
Tel: (0611) 50 04-1
Cable: HEWPACKSA Frankfurt
Telex: 41 32 49 fra

Hewlett-Packard GmbH
D-1000 Berlin 30
Tel: (030) 24 90 86

Hewlett-Packard GmbH
D-7030 Boeblingen, Wurttemberg
Tel: (07031) 66 72 87

Hewlett-Packard GmbH
D-4000 Dusseldorf
Tel: (0211) 63 80 31/5

Hewlett-Packard GmbH
D-2000 Hamburg 1
Tel: (040) 24 13 93

Hewlett-Packard GmbH
D-8012 Ottobrunn
Tel: (089) 601 30 61/7

Hewlett-Packard GmbH
D-3000 Hannover-Kleefeld
Tel: (0511) 55 60 46

Hewlett-Packard GmbH
D-8500 Nuremberg
Tel: (0911) 57 10 66/75

GREECE

*Kostas Karayannis
18 Ermou Str.
Athens 126
Tel: 3230-303
Telex: 315962

*Hewlett-Packard Athens
Kolokotroni Str. 35
Platia Kefallariou/Kifissia
Athens
Tel: 8080337/8080359/
8080429/8018693
Telex: 216588

ICELAND

Skrifstofuvelar H.F.
Reykjavik
Tel: 20560

IRELAND

*Hewlett-Packard Ltd.
King Street Lane
GB-Winnersh, Wokingham
Berks. RG11 5AR.
Tel: Wokingham 784774
Telex: 847178&9

Hewlett-Packard Ltd.
GB-Altrincham, Cheshire
Tel: (061) 928-9021

*Service

ITALY

*Hewlett-Packard Italiana S.p.A.
Via Amerigo Vespucci, 2
I-20124 Milan
Tel: (2) 62 51 (10 lines)
Cable: HEWPACKIT Milan
Telex: 32046

Hewlett-Packard Italiana S.p.A.
I-00143 Roma-Eur
Tel: (6) 5912544/5, 5915947

Hewlett-Packard Italiana S.p.A.
I-10121 Turin
Tel: 53 82 64

Hewlett-Packard Italiana S.p.A.
I-95126 Catania
Tel: (095) 370504

Hewlett-Packard Italiana S.p.A.
I-35100 Padova
Tel: 66 40 62, 66 31 88

Hewlett-Packard Italiana S.p.A.
I-56100 Pisa
Tel: (050) 500022

LUXEMBURG

*Hewlett-Packard Benelux
S.A./N.V.
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B-1170 Brussels
Tel: (02) 672 22 40
Cable: PALOBEN Brussels
Telex: 23-494 paloben bru

NETHERLANDS

*Hewlett-Packard Benelux/N.V.
Weerdestein 117
P.O. Box 7825
NL-Amsterdam, 1011
Tel: (020) 5411522
Cable: PALOBEN Amsterdam
Telex: 13 216 hepa nl

NORWAY

*Hewlett-Packard Norge A/S
Box 149
Nesveien 13
N-1344 Haslum
Tel: (02) 53 83 60
Telex: 16621 hpnas n

POLAND

*Hewlett-Packard
Warsaw Technical Office
Ul, Szpitalna 1/Apartment 50
00-120 Warsaw
Tel: 268031
Telex: 812453

PORTUGAL

Telectra Empresa Técnica de
Equipamentos Electricos
Lisbon
Tel: 686072/3/4

SPAIN

*Hewlett-Packard Española S.A.
Jerez No. 3
E-Madrid 16
Tel: 458 26 00
Telex: 23515 hpe

Hewlett-Packard Española S.A.
E. Seville
Tel: 64 44 54/58

Hewlett-Packard Española S.A.
E-Barcelona, 17
Tel: (3) 2036200-08 & 2044098/9

Hewlett-Packard Española S.A.
E-Bilbao
Tel: 23 83 06/23 82 06

SWEDEN

*Hewlett-Packard Sverige AB
S-431 41 Molndal
Tel: (031) 27 68 00/01

***SWITZERLAND**

Hewlett-Packard (Schweiz) AG
 Zürcherstrasse 20
 P.O. Box 64
 CH-8952 Schlieren-Zürich
 Tel: (01) 98 18 21/24/98 52 40
 Cable: HPAG CH
 Telex: 53933 hpag ch

Hewlett-Packard (Schweiz) AG
 CH-1214 Vernier-Geneva
 Tel: (022) 41 49 50

UNITED KINGDOM

*Hewlett-Packard Ltd.
 King Street Lane
 GB-Winnersh, Wokingham
 Berks. RG11 5 AR.
 Tel: Wokingham 784774
 Tel: 847178&9

Hewlett-Packard Ltd.
 GB-Altrincham, Cheshire
 Tel: (061) 928-9021

Hewlett-Packard Ltd.
 c/o Makro
 GB-Halesowen, Worcs.
 Tel: Birmingham 7860

Hewlett-Packard Ltd.
 GB-Thornton Heath
 CR4 6XL, Surrey
 Tel: (01) 684 0105

Hewlett-Packard Ltd.
 c/o Makro
 GB-New Town, County Durham
 Tel: Washington 464001,
 ext 57/58

USSR

*Hewlett-Packard
 Representative Office USSR
 Hotel Budapest/Room 201
 Petrovskie Linii 2/18
 Moscow
 Tel: 221-79-71

*Service

**EUROPEAN AREAS
 NOT LISTED, CONTACT:**

Hewlett-Packard S.A.
 7, Rue du Bois-du-Lan
 P.O. Box 349
 CH-1217 Meyrin 1
 Geneva, Switzerland
 Tel: (022) 41 54 00
 Cable: HEWPACKSA Geneva
 Telex: 2 24 86

**MEDITERRANEAN AND
 MIDDLE EAST AREAS NOT
 LISTED, CONTACT:**

Hewlett-Packard S.A.
 Mediterranean & Middle
 East Operations
 35, Kolokotroni Str.
 Platia Kefallariou
 GR-Kifissia-Athens
 Tel: 8080337, 8080359,
 8080429, 8018693
 Telex: 21-6588
 Cable: HEWPACKSA Athens

**SOCIALIST COUNTRIES,
 CONTACT:**

Hewlett-Packard Ges.m.b.H.
 Handelskai 52/53
 P.O. Box 7
 A-1205 Vienna
 Tel: (0222) 33 66 06 to 09
 Cable HEWPACK Vienna
 Telex: 75923 hewpak a

Service Card

Refer to the appendix of your Owner's Handbook to diagnose a calculator malfunction. The warranty period for your calculator is one year from date of purchase. Unless **Proof Of Purchase** is enclosed (sales slip or validation) Hewlett-Packard will assume any unit over 12 months old is out of warranty. **Proof Of Purchase** will be returned with your calculator. Should service be required, please return your calculator, charger, batteries and this card protectively packaged to avoid in-transit damage. Such damage is not covered under warranty.

Inside the U.S.A.

Return items safely packaged directly to:

Hewlett-Packard
Corvallis Division Service Dept.
1000 N.E. Circle Blvd. P.O. Box 999
Corvallis, OR 97330

We advise that you insure your calculator and use priority (AIR) mail for distances greater than 300 miles to minimize transit times. All units will be returned by fastest practical means.

Outside the U.S.A.

Where required please fill in the validation below and return your unit to the nearest designated Hewlett-Packard Sales and Service Office. Your warranty will be considered invalid if this completed card is not returned with the calculator.

Model No. _____

Serial No. _____

Date Received _____

Invoice No./Delivery Note No. _____

Sold by:

Calculator Catalog and Buying Guide Request Card

Primary Interest:

☐ *Scientific
Calculators*

☐ *Business
Calculators*

☐ *Both*

Valid in U.S. only



430A

A friend or associate might also want to know about Hewlett-Packard calculators. If you would like us to send him the Hewlett-Packard Calculator **Catalog and Buying Guide**, please mail his name and address on this postage paid Request Card.

Name _____

Title _____

Company _____

Street _____

City _____ State _____ Zip _____

HP-27 Warranty Card

Please fill in and return this card within 10 days of receipt of calculator.

Your Name

Title

Street Address

Date Received

Serial Number

City

State

Zip

① Where Purchased:

☐ 11 Department Store

☐ 20 Direct Mail

☐ 50 From Employer

☐ 12 College Bookstore

☐ 30 HP Salesperson

☐ 99 Other _____

☐ 13 Other Retail Store

☐ 40 Received as gift

② Source of Information Leading to Your Purchase:

☐ 10 Retail Store

☐ 40 Direct Mail Brochure

☐ 80 Professional Association

☐ 20 HP Owner

☐ 50 HP Salesperson

☐ 99 Other _____

☐ 31 Retail Store Advertisement

☐ 60 Previous Use

☐ 32 HP Advertisement

☐ 70 Employer

③ Areas of Application for Your HP-27:

☐ 01 Architecture

☐ 08 Engineering/Technical Management

☐ 15 Plant Operations

☐ 02 Aviation

☐ 09 Financial Management

☐ 16 Pollution Control

| | | |
|--|--|--|
| <input type="checkbox"/> 01 Architecture | <input type="checkbox"/> 08 Engineering/Technical Management | <input type="checkbox"/> 15 Plant Operations |
| <input type="checkbox"/> 02 Aviation | <input type="checkbox"/> 09 Financial Management | <input type="checkbox"/> 16 Pollution Control |
| <input type="checkbox"/> 03 Capital Budgeting | <input type="checkbox"/> 10 Forecasting/Planning | <input type="checkbox"/> 17 Quality Control |
| <input type="checkbox"/> 04 Consulting | <input type="checkbox"/> 11 Investment Analysis | <input type="checkbox"/> 18 Research |
| <input type="checkbox"/> 05 Data Processing | <input type="checkbox"/> 12 Navigation/Guidance | <input type="checkbox"/> 19 Statistical Analysis |
| <input type="checkbox"/> 06 Design/Development | <input type="checkbox"/> 13 Nuclear Science | <input type="checkbox"/> 20 Surveying |
| <input type="checkbox"/> 07 Engineering | <input type="checkbox"/> 14 Physics | <input type="checkbox"/> 99 Other _____ |

④ Category Best Describing Your Industry:

| | | |
|--|---|--|
| <input type="checkbox"/> 01 Aerospace | <input type="checkbox"/> 06 Computers | <input type="checkbox"/> 11 State or Local Gov't |
| <input type="checkbox"/> 02 Armed Services/DOD | <input type="checkbox"/> 07 Education | <input type="checkbox"/> 12 Student |
| <input type="checkbox"/> 03 Chemical Products | <input type="checkbox"/> 08 Federal Gov't | <input type="checkbox"/> 13 Transportation |
| <input type="checkbox"/> 04 Construction | <input type="checkbox"/> 09 Food Production | <input type="checkbox"/> 14 Utilities |
| <input type="checkbox"/> 05 Communications | <input type="checkbox"/> 10 Leasing/Renting | <input type="checkbox"/> 15 Other _____ |

⑤ Number of Employees At Your Location:

| | | | | | |
|----------------------------------|-----------------------------------|------------------------------------|--------------------------------------|---|---|
| <input type="checkbox"/> 01 1-10 | <input type="checkbox"/> 02 11-50 | <input type="checkbox"/> 03 51-100 | <input type="checkbox"/> 04 101-1000 | <input type="checkbox"/> 05 1001-10,000 | <input type="checkbox"/> 06 over 10,000 |
|----------------------------------|-----------------------------------|------------------------------------|--------------------------------------|---|---|

If you are outside the United States:

- Return this card in the enclosed warranty envelope.
- If no envelope, please mail this card to the nearest Hewlett-Packard Sales and Service Office.

Service Information

Must be **completed** and **returned** with your calculator, charger and batteries.

Owner's Name _____

Date _____

Home Phone _____

Work Phone _____

Ship-to address for returning repaired calculator

Street Address _____

City _____

State _____

Zip _____

Describe Problem: _____

Model No. _____

Serial No. _____

Preferred method of payment for out of warranty repairs.

If not specified, unit will be returned C.O.D.

☐ BankAmericard ☐ Master Charge

Card No. _____

Expiration Date _____

Name appearing on credit card _____

☐ Purchase Order, Companies with established Hewlett-Packard credit only. (Include copy of Purchase Order with shipment)

P.O. Number _____

Authorized Signature _____

HEWLETT  PACKARD

Useful Conversion Factors

The following factors are provided to 10 digits of accuracy where possible. Exact values are marked with an asterisk. For more complete information on conversion factors, refer to *Metric Practice Guide E380-74* by the American Society for Testing and Materials (ASTM).

Length

| | |
|--------------------|-----------------------------------|
| 1 inch | = 25.4 millimeters* |
| 1 foot | = 0.304 8 meter* |
| 1 mile (statute)† | = 1.609 344 kilometers* |
| 1 mile (nautical)† | = 1.852 kilometers* |
| 1 mile (statute)† | = 1.150 779 448 miles (nautical)† |

Area

| | |
|----------------|-------------------------------|
| 1 square inch | = 6.451 6 square centimeters* |
| 1 square foot | = 0.092 903 04 square meter* |
| 1 acre | = 43 560 square feet |
| 1 square mile† | = 640 acres |

Volume

| | |
|-------------------|-----------------------------------|
| 1 cubic inch | = 16.387 064 cubic centimeters* |
| 1 cubic foot | = 0.028 316 847 cubic meter |
| 1 ounce (fluid)† | = 29.573 529 56 cubic centimeters |
| 1 ounce (fluid)† | = 0.029 573 530 liter |
| 1 gallon (fluid)† | = 3.785 411 784 liters* |

Mass

| | |
|----------------|----------------------------|
| 1 ounce (mass) | = 28.349 523 12 grams |
| 1 pound (mass) | = 0.453 592 37 kilogram* |
| 1 ton (short) | = 0.907 184 74 metric ton* |

Energy

| | |
|------------------------|------------------------|
| 1 British thermal unit | = 1 055.055 853 joules |
| 1 kilocalorie (mean) | = 4 190.02 joules |
| 1 watt-hour | = 3 600 joules* |

Force

| | |
|-----------------|-------------------------|
| 1 ounce (force) | = 0.278 013 85 newton |
| 1 pound (force) | = 4.448 221 615 newtons |

Power

| | |
|-------------------------|--------------|
| 1 horsepower (electric) | = 746 watts* |
|-------------------------|--------------|

Pressure

| | |
|--------------|-------------------------------|
| 1 atmosphere | = 760 mm Hg at sea level |
| 1 atmosphere | = 14.7 pounds per square inch |
| 1 atmosphere | = 101 325 pascals |

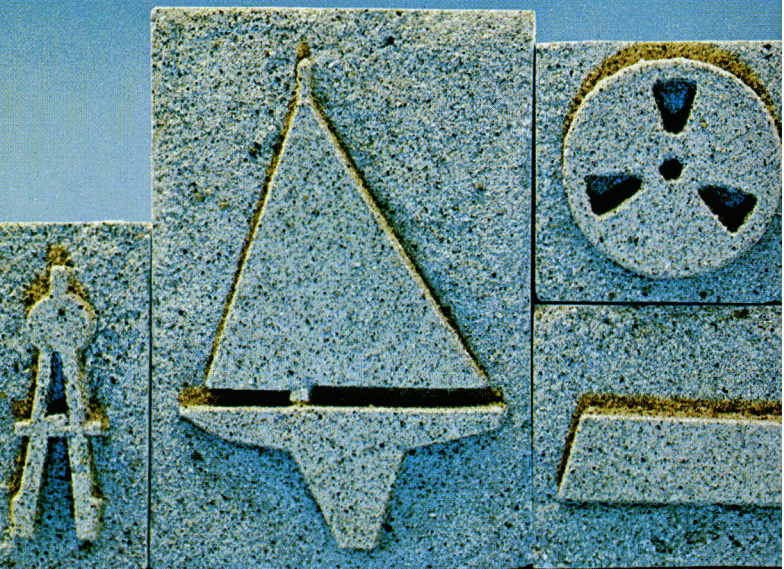
Temperature

| | |
|------------|-----------------------------|
| Fahrenheit | = 1.8 Celsius + 32 |
| Celsius | = 5/9(Fahrenheit - 32) |
| kelvin | = Celsius + 273.15 |
| kelvin | = 5/9 (Fahrenheit + 459.67) |
| kelvin | = 5/9 Rankine |

† U.S. values chosen. * Exact values.



Sales and Service from 172 offices in 65 countries.
19310 Pruneridge Avenue, Cupertino, CA 95014



For additional Sales and Service Information Contact Your
Local Hewlett-Packard Sales Office or Call 408/998-0100
(Ask for Calculator Customer Service)

00027-90001

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www.hpmuseum.org

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