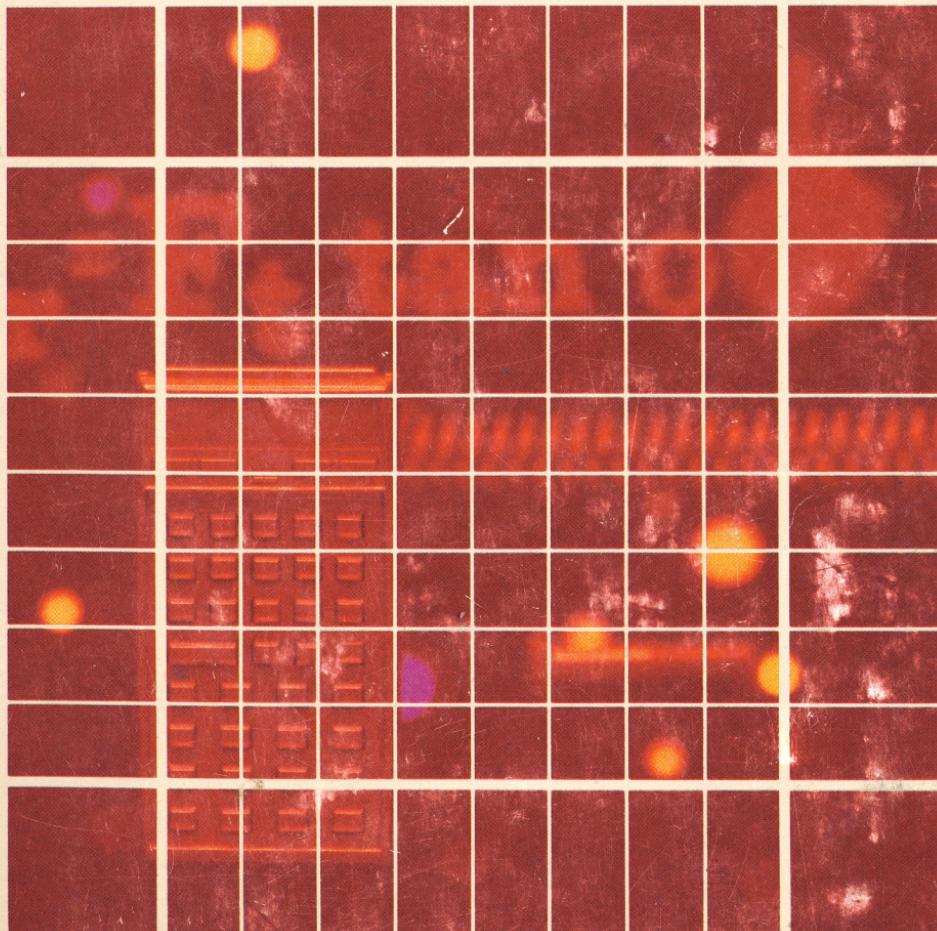


HEWLETT-PACKARD

HP-41C

MACHINE DESIGN  
PAC



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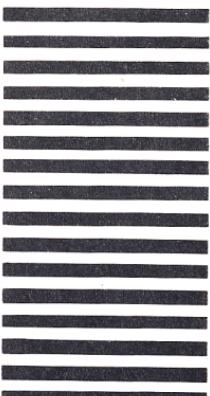
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## INTRODUCTION

The *Machine Design Pac* is a collection of programs for mechanical engineers involved in design or analysis. Topics include cams, gears, linkages, springs, mechanical motions and machine geometries.

Each program in this pac is represented by one program in the Application Module and a section in this manual.

The manual provides a description of each program, a set of instructions for using each program, and one or more example problems, each of which includes a list of the keystrokes required for its solution.

Before plugging in your Application Module, **turn your calculator off**, and be sure you understand the section *Inserting and Removing Application Modules*. Before using a particular program, take a few minutes to read *Format of User Instructions* and *A Word About Program Usage*.

You should first familiarize yourself with a program by running it once or twice while following the complete User Instructions in the manual. Thereafter, the program's prompting should provide the necessary instructions, including which variables are to be input, which keys are to be pressed, and which values will be output.

We hope this pac will assist you in the solution of numerous problems in your discipline. We would appreciate knowing your reactions to the programs, and to this end we have provided a questionnaire inside the front cover of this manual. Would you please take a few minutes to give us your comments on these programs? It is from your comments that we learn how to increase the usefulness of our programs.

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## INSERTING AND REMOVING APPLICATION MODULES

Before you insert an Application Module for the first time, familiarize yourself with the following information.

Up to four Application Modules can be plugged into the ports on the HP-41C. While plugged in, the names of all programs contained in the Module can be displayed by pressing  **CATALOG**.

### CAUTION

Always turn the HP-41C off before inserting or removing any plug-in extension or accessories. Failure to turn the HP-41C off could damage both the calculator and the accessory.

#### To insert Application Modules:

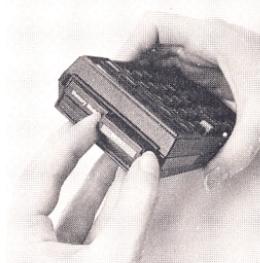
1. Turn the HP-41C off! Failure to turn the calculator off could damage both the Module and the calculator.



2. Remove the port covers. Remember to save the port covers; they should be inserted into the empty ports when no extensions are inserted.



3. Insert the Application Module with the label facing downward as shown, into any port **after** the last Memory Module. For example, if you have a Memory Module inserted in port 1, you can insert an Application Module in any of ports 2, 3, or 4. (The port numbers are shown on the back of the calculator.) **Never insert an Application Module into a lower numbered port than a Memory Module.**



4. If you have additional Application Modules to insert, plug them into any port after the last Memory Module. Be sure to place port covers over unused ports.
5. Turn the calculator on and follow the instructions given in this book for the desired application functions.

#### **To remove Application Modules:**

1. Turn the HP-41C off! Failure to do so could damage both the calculator and the Module.
2. Grasp the desired Module handle and pull it out as shown.



3. Place a port cap into the empty ports.

#### **Mixing Memory Modules and Application Modules**

Any optional accessories (such as the HP 82104A Card Reader, or the HP 82143A Printer) should be treated in the same manner as Application Modules. That is, they can be plugged into any port after the last Memory Module. Also, the HP-41C should be turned off prior to insertion or removal of these extensions.

The HP-41C allows you to leave gaps in the port sequence when mixing Memory and Application Modules. For example, you can plug a Memory Module into port 1 and an Application Module into port 4, leaving ports 2 and 3 empty.

## FORMAT OF USER INSTRUCTIONS

The completed User Instruction Form—which accompanies each program—is your guide to operating the programs in this Pac.

The form is composed of five labeled columns. Reading from left to right, the first column, labeled STEP, gives the instruction step number.

The INSTRUCTIONS column gives instructions and comments concerning the operations to be performed.

The INPUT column specifies the input data, the units of data if applicable, or the appropriate alpha response to a prompted question. Data input keys consist of 0 to 9 and the decimal point (the numeric keys), **EEX** (enter exponent), and **CHS** (change sign).

The FUNCTION column specifies the keys to be pressed after keying in the corresponding input data.

The DISPLAY column specifies prompts, intermediate and final answers, and their units, where applicable.

Above the DISPLAY column is a box which specifies the minimum number of data storage registers necessary to execute the program. Refer to the Owner's Handbook for information on how the SIZE function affects storage configuration.

## A WORD ABOUT PROGRAM USAGE

### Catalog

When an Application Module is plugged into a port of the HP-41C, the contents of the Module can be reviewed by pressing  **CATALOG** 2 (the Extension Catalog). Executing the **CATALOG** function lists the name of each program or function in the Module, as well as functions of any other extention which might be plugged in.

### ALPHA and USER Mode Notation

This manual uses a special notation to signify ALPHA mode. Whenever a statement on the User Instruction Form is printed in gold, the **ALPHA** key must be pressed before the statement can be keyed in. After the statement is input, press **ALPHA** again to return the calculator to its normal operating mode, or to begin program execution. For example, **XEQ SECTION** means press the following keys: **XEQ ALPHA SECTION ALPHA** .

When the calculator is in USER mode, this manual will use the symbols **A - J** and **■ A - ■ E** to refer to the reassigned keys in the top two rows. These key designations will appear on the User Instruction Form and in the keystroke solutions to sample problems.

### Optional HP 82143A Printer

When the optional printer is plugged into the HP-41C, all inputs and results will be printed. The printer should be set in **MANUAL** mode.

## CIRCULAR CAMS

This program computes the parameters necessary for the design of harmonic, cycloidal or parabolic circular cams with roller, point or flat radial follower.

### Equations:

Harmonic: 
$$y = \frac{h}{2} \left[ 1 - \cos \left( \frac{\pi \cdot \Delta}{\beta} \right) \right]$$

Cycloidal: 
$$y = h \left[ \frac{\Delta}{\beta} - \frac{1}{2\pi} \sin \frac{2\pi\Delta}{\beta} \right]$$

Parabolic: 
$$y = h \begin{cases} 2 \left( \frac{\Delta}{\beta} \right)^2 & \text{if } \left( \frac{\Delta}{\beta} \right) \leq 0.5 \\ \left( 1 - 2 \left( 1 - \frac{\Delta}{\beta} \right)^2 \right) & \text{if } \left( \frac{\Delta}{\beta} \right) > 0.5 \end{cases}$$

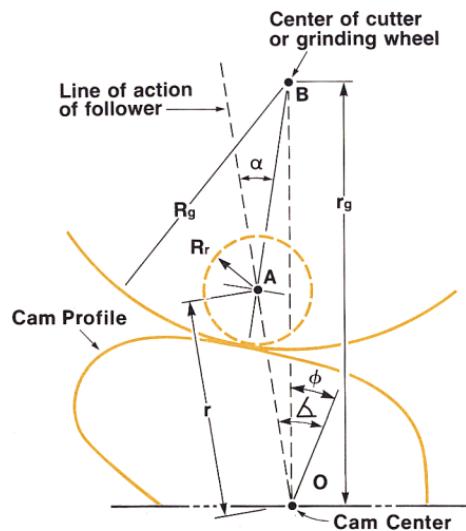
Dwell: 
$$y = 0$$

$$\alpha = \tan^{-1} \left( \frac{180}{\pi r} \frac{dy}{d\Delta} \right)$$

$$r = R_b + y$$

## 8 Circular Cams

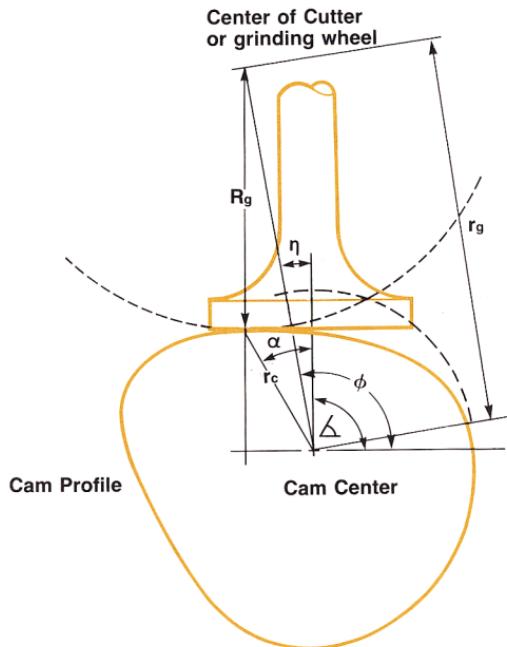
Roller followers:



$$r_g = (r^2 + (R_g - R_r)^2 + 2r(R_g - R_r) \cos \alpha)^{1/2}$$

$$\phi = \sin^{-1} \frac{R_g - R_r}{r_g} + \theta$$

Flat followers:



$$r_c = \left( r^2 + \left( \frac{180}{\pi} \frac{dy}{d\alpha} \right)^2 \right)^{1/2}$$

$$r_g = (R_g^2 + r_c^2 + 2R_g r_c \cos \alpha)^{1/2}$$

$$\phi = \cos^{-1} \left( \frac{r_c + R_g \cos \alpha}{r_g} \right) - \alpha + \alpha$$

where:

$\beta$  is the duration of lift  $h$ .

$h$  is the total cam lift over angle  $\beta$ .

$R_b$  is the base circle radius.

$R_g$  is the grinder radius (set to zero for cam profile).

$R_r$  is the roller radius (set to zero for point follower).

$\alpha$  is the cam angle.

$y$  is the follower lift.

$\frac{dy}{d\alpha}$  is the follower velocity.

$\frac{d^2y}{d\alpha^2}$  is the follower acceleration.

$\alpha$  is the pressure angle.

$\phi$  is the angle from zero to grinder center.

$r_g$  is the center to center distance of grinder and cam.

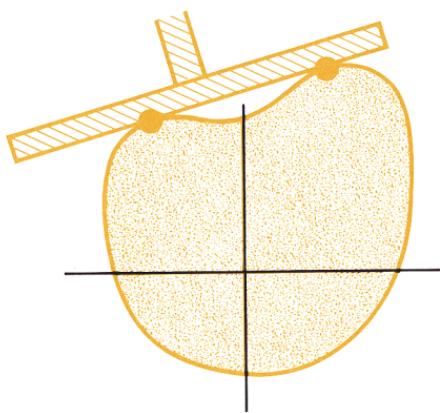
### Reference:

M.F. Spotts, *Design of Machine Elements*, Prentice-Hall 1971.

## 10 Circular Cams

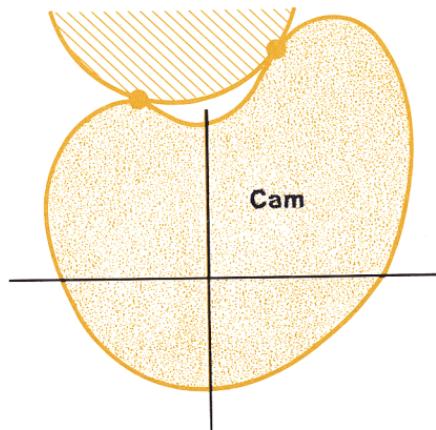
### Remarks:

A flat follower will not properly follow a cam profile with any concave sections, e.g., see figure 1.



**Figure 1**  
**Note two points of contact**

A roller follower will not properly follow a cam profile with concave section whose radius is less than the roller radius, e.g., see figure 2.



**Figure 2**  
**Note two points of contact**



## 12 Circular Cams

### Example:

Using 10 degree increments design a cam with a roller follower which does the following:

SECTION #	LIFT	DURATION	PROFILE
1	-4.5	130°	Parabolic
2	+2.0	100°	Harmonic
3	0	30°	Dwell
4	+2.5	100°	Cycloidal

Grinder radius = 0.500

Base radius = 12.000

Roller radius = 1.000

### Keystrokes (SIZE $\geq$ 020)

XEQ ALPHA DEG ALPHA  
 ENG 3  
 XEQ ALPHA CAM AL.PHA  
 N R/S  
 .5 R/S  
 12 R/S  
 1 R/S  
 PAR R/S  
 130 R/S  
 4.5 CHS R/S  
 13 R/S  
 R/S \*  
 :  
 etc.

### Display

FLAT? Y/N  
 GRNDR=?  
 Rb=?  
 RR=?  
 PROFILE?  
 DUR=?  
 LIFT=?  
 N INT=?  
 $\Delta=0.000E0$   
 $Y=0.000E0$   
 $R=12.00E0$   
 $dY/d\Delta=0.000E0$   
 $d2Y/d\Delta^2=-1.065E-3$   
 PRS  $\Delta=0.000E0$   
 $\Delta G=0.000E0$   
 $RG=11.50E0$   
 $\Delta RC=0.000E0$   
 $RC=11.00E0$   
 $\Delta=10.00E0$   
 $Y=-53.25E-3$   
 $R=11.95E0$   
 $dY/d\Delta=-10.65E-3$   
 $d2Y/d\Delta^2=-1.065E-3$   
 PRS  $\Delta=-2.924E0$   
 $\Delta G=9.872E0$   
 $RG=11.45E0$

$\angle$  RC=9.733E0

RC=10.95E0

:

etc.

$\angle$ =120.0E0

Y=-4.447E0

R=7.553E0

dY/d $\angle$ =-10.65E-3

d2Y/d $\angle$ 2=1.065E-3

PRS  $\angle$ =-4.619E0

$\angle$ G=119.7E0

RG=7.055E0

$\angle$  RC=119.3E0

RC=6.557E0

$\angle$ =130.0E0

Y=-4.500E0

R=7.500E0

dY/d $\angle$ =0.000E0

d2Y/d $\angle$ 2=1.065E-3

PRS  $\angle$ =0.000E0

$\angle$ G=130.0E0

RG=7.000E0

$\angle$  RC=130.0E0

RC=6.500E0

**R/S** \*

HAR **R/S**

100 **R/S**

2 **R/S**

**PROFILE?**

**DUR=?**

**LIFT=?**

**N INT=?**

10 **R/S**

**R/S** \*

:

etc.

$\angle$ =130.0E0

Y=0.000E0

R=7.500E0

dY/d $\angle$ =0.000E0

d2Y/d $\angle$ 2=987.0E-6

PRS  $\angle$ =0.000E0

$\angle$ G=130.0E0

RG=7.000E0

$\angle$  RC=130.0E0

RC=6.500E0

$\angle=140.0E0$   
 $Y=48.94E-3$   
 $R=7.549E0$   
 $dY/d\angle=9.708E-3$   
 $d2Y/d\angle^2=938.7E-6$   
 $PRS \angle=4.214E0$   
 $\angle G=140.3E0$   
 $RG=7.050E0$   
 $\angle RC=140.6E0$   
 $RC=6.552E0$   
 $\vdots$   
etc.

$\angle=220.0E0$   
 $Y=1.951E0$   
 $R=9.451E0$   
 $dY/d\angle=9.708E-3$   
 $d2Y/d\angle^2=-938.7E-6$   
 $PRS \angle=3.368E0$   
 $\angle G=220.2E0$   
 $RG=8.952E0$   
 $\angle RC=220.4E0$   
 $RC=8.453E0$

$\angle=230.0E0$   
 $Y=2.000E0$   
 $R=9.500E0$   
 $dY/d\angle=0.000E0$   
 $d2Y/d\angle^2=-987.0E-6$   
 $PRS \angle=0.000E0$   
 $\angle G=230.0E0$   
 $RG=9.000E0$   
 $\angle RC=230.0E0$   
 $RC=8.500E0$

\*  
DWELL   
30   
0

**PROFILE?**  
**DUR=?**  
**LIFT=?**  
**N INT=?**

3 [R/S]  
[R/S] \*  
:  
etc.

$\angle=230.0E0$   
 $Y=0.000E0$   
 $R=9.500E0$   
 $dY/d\angle=0.000E0$   
 $d2Y/d\angle^2=0.000E0$   
PRS  $\angle=0.000E0$   
 $\angle_G=230.0E0$   
 $RG=9.000E0$   
 $\angle RC=230.0E0$   
 $RC=8.500E0$

$\angle=240.0E0$   
 $Y=0.000E0$   
 $R=9.500E0$   
 $dY/d\angle=0.000E0$   
 $d2Y/d\angle^2=0.000E0$   
PRS  $\angle=0.000E0$   
 $\angle_G=240.0E0$   
 $RG=9.000E0$   
 $\angle RC=240.0E0$   
 $RC=8.500E0$

$\angle=250.0E0$   
 $Y=0.000E0$   
 $R=9.500E0$   
 $dY/d\angle=0.000E0$   
 $d2Y/d\angle^2=0.000E0$   
PRS  $\angle=0.000E0$   
 $\angle_G=250.0E0$   
 $RG=9.000E0$   
 $\angle RC=250.0E0$   
 $RC=8.500E0$

$\angle=260.0E0$   
 $Y=0.000E0$   
 $R=9.500E0$   
 $dY/d\angle=0.000E0$   
 $d2Y/d\angle^2=0.000E0$   
PRS  $\angle=0.000E0$   
 $\angle_G=260.0E0$   
 $RG=9.000E0$   
 $\angle RC=260.0E0$   
 $RC=8.500E0$

**R/S** \*  
 CYC **R/S**  
 100 **R/S**  
 2.5 **R/S**

10 **R/S**

**R/S** \*

:

etc.

**PROFILE?**

**DUR=?**

**LIFT=?**

**N INT=?**

$\alpha=260.0E0$   
 $Y=0.000E0$   
 $R=9.500E0$   
 $dY/d\alpha=0.000E0$   
 $d2Y/d\alpha^2=0.000E0$   
 $PRS \alpha=0.000E0$   
 $\alpha_G=260.0E0$   
 $RG=9.000E0$   
 $\alpha RC=260.0E0$   
 $RC=8.500E0$

$\alpha=270.0E0$   
 $Y=16.13E-3$   
 $R=9.516E0$   
 $dY/d\alpha=4.775E-3$   
 $d2Y/d\alpha^2=923.3E-6$   
 $PRS \alpha=1.647E0$   
 $\alpha_G=270.1E0$   
 $RG=9.016E0$   
 $\alpha RC=270.2E0$   
 $RC=8.517E0$   
 etc.

$\alpha=350.0E0$   
 $Y=2.484E0$   
 $R=11.98E0$   
 $dY/d\alpha=4.775E-3$   
 $d2Y/d\alpha^2=-923.3E-6$   
 $PRS \alpha=1.308E0$   
 $\alpha_G=350.1E0$   
 $RG=11.48E0$   
 $\alpha RC=350.1E0$   
 $RC=10.98E0$

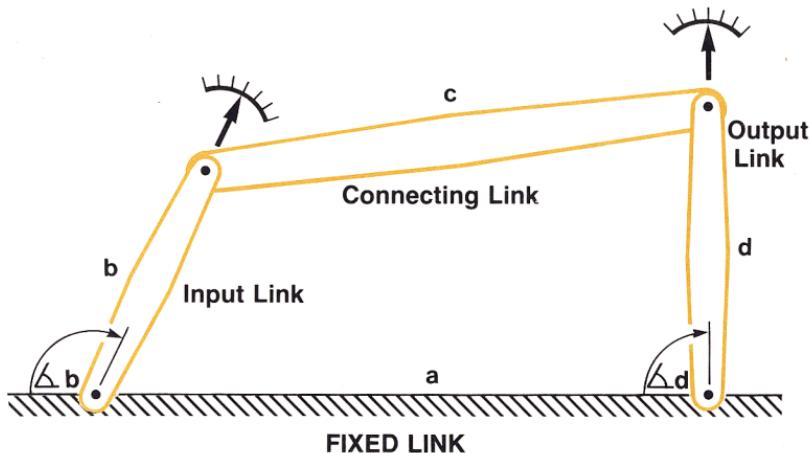
$\alpha=360.0E0$   
 $Y=2.500E0$   
 $R=12.000E0$   
 $dY/d\alpha=0.000E0$   
 $d2Y/d\alpha^2=0.000E0$   
PRS  $\alpha=0.000E0$   
 $\alpha_G=360.0E0$   
 $R_G=11.50E0$   
 $\alpha_{RC}=360.0E0$   
 $R_C=11.00E0$

\* Press **R/S** if you are not using a printer.



## GENERATION OF A FOUR BAR LINKAGE

Given three angular orientations of the input crank and three corresponding angular orientations of the output crank, this program will size the link members to conform to these orientations. Thus, a linkage can be designed to conform exactly at three angular orientations.



### Equations:

$$R_1 \cos \Delta b_1 - R_2 \cos \Delta d_1 + R_3 = \cos(\Delta b_1 - \Delta d_1)$$

$$R_1 \cos \Delta b_2 - R_2 \cos \Delta d_2 + R_3 = \cos(\Delta b_2 - \Delta d_2)$$

$$R_1 \cos \Delta b_3 - R_2 \cos \Delta d_3 + R_3 = \cos(\Delta b_3 - \Delta d_3)$$

$$R_1 = a/d$$

$$R_2 = a/b$$

$$R_3 = \frac{a^2 + b^2 + d^2 - c^2}{2bd}$$

where:

a is the length of the fixed link.

b is the length of the input link.

c is the length of the connecting link.

d is the length of the output link.

$\Delta b$  is the angular orientation of the input link.

$\Delta d$  is the angular orientation of the output link.

## 20 Generation of a Four Bar Linkage

### Remarks:

$\Delta b_1$  may not equal 90 or 270 degrees.

### Reference:

Martin, G.H. *Kinematics and Dynamics of Machines* McGraw-Hill, 1969.

STEP	INSTRUCTIONS	INPUT	FUNCTION	SIZE: 022
				DISPLAY
1	Initialize program.		XEQ GEN4BAR	$\Delta b_1=?$
2	Key in initial angle of input link.	$\Delta b_1$	R/S	$\Delta d_1=?$
3	Key in initial angle of output link.	$\Delta d_1$	R/S	$\Delta b_2=?$
4	Key in intermediate angle of input link.	$\Delta b_2$	R/S	$\Delta d_2=?$
5	Key in intermediate angle of output link.	$\Delta d_2$	R/S	$\Delta b_3=?$
6	Key in final angle of input link.	$\Delta b_3$	R/S	$\Delta d_3=?$
7	Key in final angle of output link.	$\Delta d_3$	R/S	a=?
8	Key in length of fixed link and calculate link lengths.	a	R/S R/S * R/S * R/S * R/S *	a= b= c= d= $\Delta b_1=?$
9	For a new case, go to step 2. Key in only values which change.			
10	If you wish to run <i>Progression of a Four Bar System</i> immediately after this program, start at step 1 of its user instructions but ignore the prompts for a, b, c and d since these values are already stored.			
	*Press R/S if you are not using a printer.			

**Example:**

Design a linkage which will have the following correspondence between input and output links:

#	b	d
1	70	100
2	83.3	116
3	110	140

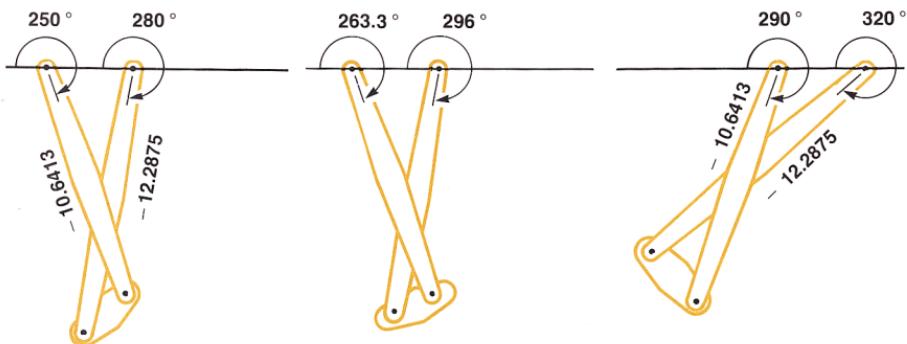
The fixed link is 3.75 units long.

**Keystrokes (SIZE  $\geq$  022)**

XEQ ALPHA DEG ALPHA  
 FIX 4  
 XEQ ALPHA GEN4BAR ALPHA  
 70 R/S  
 100 R/S  
 83.3 R/S  
 116 R/S  
 110 R/S  
 140 R/S  
 3.75 R/S  
 R/S \*  
 R/S \*  
 R/S \*

**Display**

$\Delta b_1 = ?$   
 $\Delta d_1 = ?$   
 $\Delta b_2 = ?$   
 $\Delta d_2 = ?$   
 $\Delta b_3 = ?$   
 $\Delta d_3 = ?$   
 $a = ?$   
 $a = 3.7500$   
 $b = -10.6413^{**}$   
 $c = 2.9170$   
 $d = -12.2875^{**}$



\*Press **R/S** if you are not using a printer.

\*\*The negative signs indicate that the links are opposite to the assumed directions. Thus, all angles are 180 degrees from the originally specified orientations.

## PROGRESSION OF A FOUR BAR SYSTEM

This program calculates angular displacement, velocity and acceleration for the output link of a four bar system (figure 1). (Either the “connecting link” (c) or the “output link” (d) may be selected as the program’s output link.)

Two modes of program operation are provided; manual and automatic progression. Manual mode is useful for analysis of discrete linkage orientations. Automatic mode can be used to simulate link movements through a range of motion. Constant angular velocity of the input link is required for automatic progression.

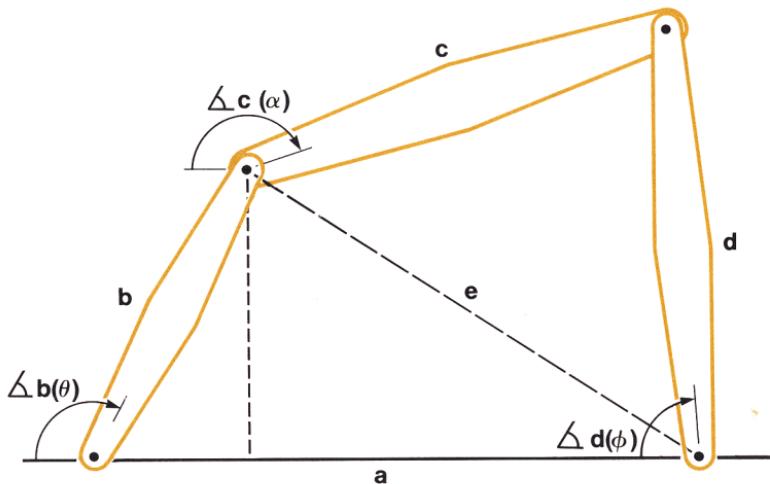


Figure 1 - Four Bar System Showing Positive Angular Conventions

Output Link

$$\phi = \sin^{-1} \left( \frac{b}{e} \sin \theta \right) + \cos^{-1} \left( \frac{d^2 + e^2 - c^2}{2de} \right)$$

Connecting Link

$$\alpha = \sin^{-1} \left( \frac{b}{e} \sin \theta \right) + \cos^{-1} \left( \frac{c^2 + e^2 - d^2}{-2ce} \right)$$

$$e = \sqrt{a^2 + b^2 + 2abc \cos \theta}$$

$$\frac{d\phi}{d\theta} = \frac{R_1 \sin \theta - \sin(\theta - \phi)}{R_2 \sin \phi - \sin(\theta - \phi)}$$

$$R_1 = \frac{a}{d} \quad R_2 = \frac{a}{b}$$

$$\frac{d\alpha}{d\theta} = \frac{S_1 \sin \theta - \sin(\theta - \alpha)}{S_2 \sin \alpha - \sin(\theta - \alpha)}$$

$$S_1 = -\frac{a}{c} \quad S_2 = \frac{a}{b}$$

$$\frac{d^2\phi}{d\theta^2} = \frac{R_1 \cos \theta - R_2 \cos \phi \left( \frac{d\phi}{d\theta} \right)^2 - \left( 1 - \frac{d\phi}{d\theta} \right)^2 \cos(\theta - \phi)}{R_2 \sin \alpha - \sin(\theta - \alpha)}$$

$$\frac{d^2\alpha}{d\theta^2} = \frac{S_1 \cos \theta - S_2 \cos \alpha \left( \frac{d\alpha}{d\theta} \right)^2 - \left( 1 - \frac{d\alpha}{d\theta} \right)^2 \cos(\theta - \alpha)}{S_2 \sin \alpha - \sin(\theta - \alpha)}$$

$$\dot{\phi} = \frac{d\phi}{d\theta} \dot{\theta} \quad \dot{\alpha} = -\frac{d\alpha}{d\theta} \dot{\theta}$$

$$\begin{aligned} \ddot{\phi} &= \frac{d^2\phi}{d\theta^2} = \frac{d^2\phi}{d\theta^2} \left( \frac{d\theta}{dt} \right)^2 + \frac{d^2\theta}{dt^2} \frac{d\phi}{d\theta} \\ &= \dot{\theta}^2 \frac{d^2\theta}{d\theta^2} + \ddot{\theta} \frac{d\theta}{d\theta} \quad \alpha = \dot{\theta}^2 \frac{d^2\alpha}{d\theta^2} + \alpha \frac{d\ddot{\alpha}}{d\theta} \end{aligned}$$

where:

$\angle b$  (or  $\theta$ ) is the orientation of the input link.

$\angle c$  (or  $\alpha$ ) is the orientation of the connecting link.

$\angle d$  (or  $\phi$ ) is the orientation of the output link.

$\angle b$  (or  $d\theta/dt$ ) is the angular velocity of the input link.

$\angle c$  (or  $d\alpha/dt$ ) is the angular velocity of the connecting link.

$\angle d$  (or  $d\phi/dt$ ) is the angular velocity of the output link.

$\angle b$  (or  $d^2\theta/dt^2$ ) is the angular acceleration of the input link.

$\angle c$  (or  $d^2\alpha/dt^2$ ) is the angular acceleration of the connecting link.

$\angle d$  (or  $d^2\phi/dt^2$ ) is the angular acceleration of the output link.

$a$  is the length of the fixed link.

$b$  is the length of the input link.

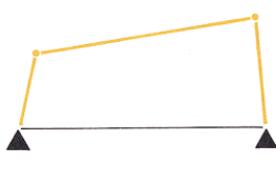
$c$  is the length of the connecting link.

$d$  is the length of the output link.

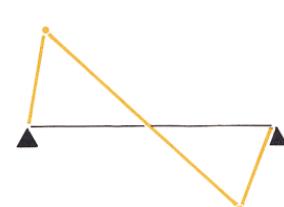
## 24 Progression of a Four Bar System

### Remarks:

Two possible configurations exist for a given set of links:



Configuration A



Configuration B

The program allows either configuration to be selected.

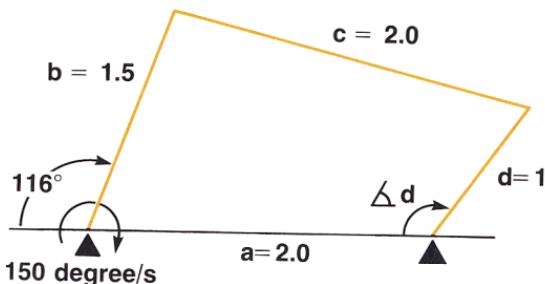
All angular inputs should be consistent with the angular mode of the machine.

STEP	INSTRUCTIONS	INPUT	FUNCTION	SIZE: 020
				DISPLAY
1	Initialize program.		<b>XEQ 4BAR</b>	a=?
2	Key in length of fixed link.	a	<b>R/S</b>	b=?
3	Key in length of input link.	b	<b>R/S</b>	c=?
4	Key in length of connecting link.	c	<b>R/S</b>	d=?
5	Key in length of output link.	d	<b>R/S</b>	TYPE B? Y/N
6	See Remarks. Key in "N" for type A or "Y" for type B.	Y or N	<b>R/S</b>	LINK c? Y/N
7	Key in "Y" for the connecting link or "N" for the output link.	Y or N	<b>R/S</b>	AUTO? Y/N
8	Key in "Y" for automatic link progression or "N" for analysis of individual orientations.	Y or N	<b>R/S</b>	$\Delta b = ?$ or $\Delta b_0 = ?$
9	If you keyed "N" in step 8, go to step 15.			
10	Key in initial angle of link b.	$\Delta b_0$	<b>R/S</b>	$N = ?$
11	Key in number of increments.	N	<b>R/S</b>	$\Delta INC$ ?
12	Key in angular increment.	$\Delta INC$	<b>R/S</b>	$\Delta.b = ?$

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
13	Key in angular velocity of input link.	$\Delta.b$	<b>R/S</b> <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * etc. : :	$\Delta.b =$ $\Delta.c =$ or $\Delta.d =$ $\Delta.c =$ or $\Delta.d =$ $\Delta.c =$ or $\Delta.d =$ $\Delta.b =$ $\Delta.c =$ or $\Delta.d =$ etc. : : $a = ?$
14	For a new case, go to step 2. Key in only values which change.			
15	Key in angle of input link and calculate angle of output link.	$\Delta.b$	<b>R/S</b> <b>R/S</b> *	$\Delta.c =$ or $\Delta.d =$ $\Delta.b = ?$
16	Key in angular velocity of input link and calculate angular velocity of output link.	$\Delta.b$	<b>R/S</b>	$\Delta.c =$ or $\Delta.d =$ $\Delta.b = ?$
17	Key in angular acceleration of input link and calculate acceleration of output link.	$\Delta.b$	<b>R/S</b> * <b>R/S</b> *	$\Delta.c =$ or $\Delta.d =$ $\Delta.b = ?$
18	For another orientation go to step 15. Key in only values which change.			
19	For a new case, go to step 1. Key in only the values which change.			
	*Press <b>R/S</b> if you are not using a printer.			

**Example 1:**

The input link of the four bar linkage below is instantaneously rotating at 150 degrees/second with an angular acceleration of 0.23 degrees/second squared. The input link is at 116°. What are the values of position, velocity, and acceleration of link d? Link c?

**Keystrokes (SIZE≥020)**

**FIX** 4  
**XEQ** **ALPHA** DEG **ALPHA**  
**XEQ** **ALPHA** 4 BAR **ALPHA**  
2 **R/S**  
1.5 **R/S**  
2 **R/S**  
1 **R/S**  
N **R/S**  
N **R/S**  
N **R/S**  
116 **R/S**  
**R/S** \*  
150 **R/S**  
**R/S** \*  
.23 **R/S**  
**R/S** \*  
For link c.  
**XEQ** **ALPHA** 4 BAR **ALPHA**  
**R/S**  
**R/S**  
**R/S**  
**R/S**  
**R/S**  
Y **R/S**

**Display**

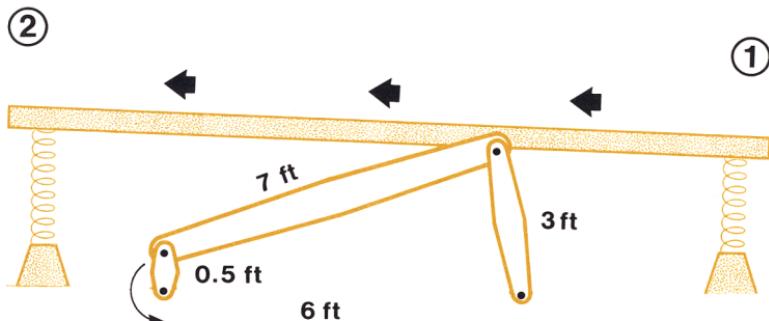
**a=?**  
**b=?**  
**c=?**  
**d=?**  
**TYPE B? Y/N**  
**LINK c? Y/N**  
**AUTO? Y/N**  
**Δ b=?**  
**Δ d=125.7484**  
**Δ.b=?**  
**Δ.d=235.7572**  
**Δ:b=?**  
**Δ:d=227.9894**  
**Δ b=?**  
**a=?**  
**b=?**  
**c=?**  
**d=?**  
**TYPE B? Y/N**  
**LINK c? Y/N**  
**AUTO? Y/N**

R/S  
 R/S  
 R/S \*  
 R/S  
 R/S \*  
 R/S  
 R/S \*

$\Delta b = ?$   
 $\Delta c = 195.5632$   
 $\Delta.b = ?$   
 $\Delta.c = 20.2953$   
 $\Delta:b = ?$   
 $\Delta:c = 204.9014$   
 $\Delta b = ?$

### Example 2:

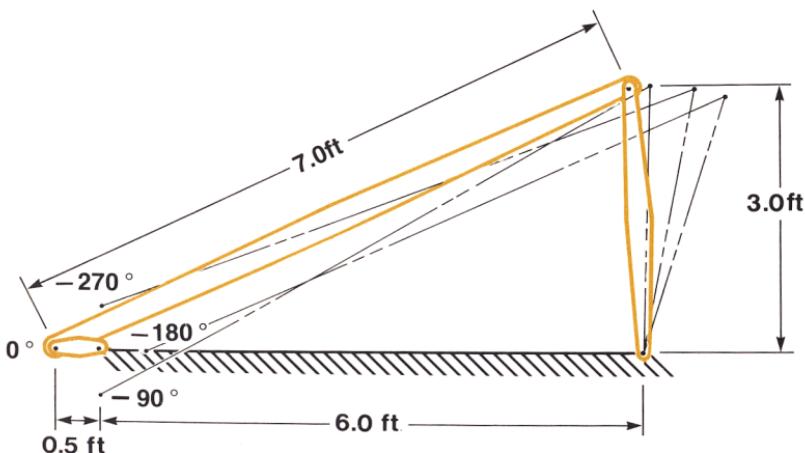
A four bar linkage is to be used to convert rotary motion from an electric motor to the reciprocating motion necessary to activate a shaking conveyor system which moves fruit between two process stations.



$$\Delta.b = -360 \text{ degrees/second}$$

For the geometry shown above, what is the motion of the output link? Start at  $\Delta b = 0^\circ$  and go to  $-330^\circ$  by  $12^\circ, 30^\circ$  increments.

#### Four Bar Shaker Mechanism



Keystrokes (SIZE  $\geq 020$ )

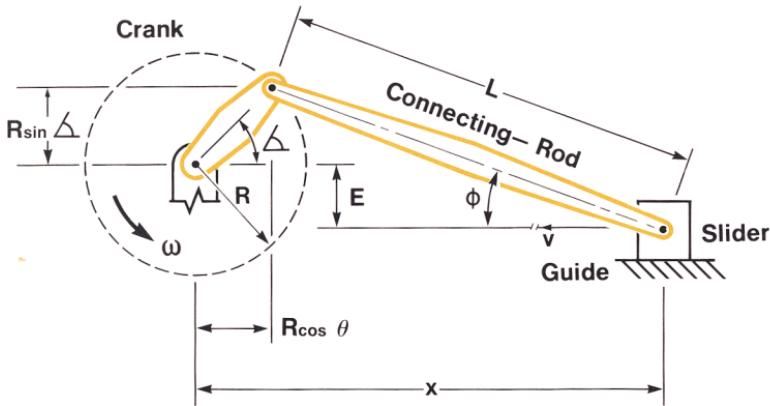
**FIX** 4  
**XEQ** **ALPHA** DEG **ALPHA**  
**XEQ** **ALPHA** 4 BAR **ALPHA**  
6 **R/S**  
.5 **R/S**  
7 **R/S**  
3 **R/S**  
N **R/S**  
N **R/S**  
Y **R/S**  
0 **R/S**  
12 **R/S**  
30 **R/S**  
360 **CHS** **R/S**  
**R/S** \*  
**R/S** \*  
**R/S** \*  
**R/S** \*  
etc.  
:  
**R/S** \*  
**R/S** \*  
**R/S** \*  
**R/S** \*  
**R/S** \*

## Display

**a=?**  
**b=?**  
**c=?**  
**d=?**  
**TYPE B? Y/N**  
**LINK c? Y/N**  
**AUTO? Y/N**  
 $\Delta b_0=?$   
**N=?**  
 $\Delta INC=?$   
 $\Delta.b=?$   
 $\Delta b=0.0000$   
 $\Delta d=86.6926$   
 $\Delta.d=-27.6923$   
 $\Delta:d=339.2909$   
 $\Delta b=330.0000$   
etc.  
:  
 $\Delta b=30.0000$   
 $\Delta d=90.0799$   
 $\Delta.d=-52.2294$   
 $\Delta:d=240.4647$   
**a=?**

## PROGRESSION OF A SLIDER-CRANK

This program calculates the slider displacement, the rod orientation, the slide velocity, the rod velocity, the slider acceleration and the rod acceleration for a slider-crank mechanism rotating at a constant angular velocity.



### Equations:

$$x = R \cos \Delta + L \cos \phi$$

$$\phi = \sin^{-1} \left( \frac{E + R \sin \Delta}{L} \right)$$

$$v = \frac{dx}{dt} = R\omega \left( \frac{-\sin(\Delta + \phi)}{\cos \phi} \right)$$

$$a = \frac{d^2x}{dt^2} = R\omega^2 \left( \frac{-\cos(\Delta + \phi)}{\cos \phi} - \frac{R \cos^2 \Delta}{L \cos^3 \phi} \right)$$

$$\dot{\phi} = \frac{d\phi}{dt} = \omega \frac{R \cos \Delta}{L \cos \phi}$$

$$\ddot{\phi} = \frac{d^2\phi}{dt^2} = \omega^2 \left[ \left( \frac{d\phi}{d\Delta} \right)^2 \tan \phi - \frac{R \sin \Delta}{L \cos \phi} \right]$$

where:

- E is the slider offset.
- L is the connecting rod length.
- R is the crank radius.
- $\omega$  is the crank angular velocity.
- $\Delta$  is the crank angle.
- x is the slider displacement.
- v is the slider velocity.
- a is the slider acceleration.
- $\phi$  is the connecting rod angular displacement.
- $\dot{\phi}$  is the angular velocity of connecting rod.
- $\ddot{\phi}$  is the angular acceleration of connecting rod.

### References:

- H.A. Rothbart, *Mechanical Design and Systems Handbook*, McGraw-Hill, 1964.
- V. M. Faires, *Kinematics*, McGraw-Hill, 1959.

### Remarks:

Angular inputs must be consistent with the angular mode of the machine. Angular outputs are also consistent with the angular mode of the machine. If the calculator is in degree mode  $\phi$  is output in degrees per unit time.

STEP	INSTRUCTIONS	INPUT	FUNCTION	SIZE: 013
				DISPLAY
1	Initialize program.		<b>XEQ CRANK</b>	CRANK=?
2	Key in crank radius.	R	<b>R/S</b>	ROD=?
3	Key in connecting rod length.	L	<b>R/S</b>	ECCEN=?
4	Key in eccentricity.	E	<b>R/S</b>	INI $\Delta$ ?=?
5	Key in initial crank angle.	INI $\Delta$	<b>R/S</b>	$\Delta$ INC=?
6	Key in angular increment.	$\Delta$ INC	<b>R/S</b>	N=?
7	Key in number of increments.	N	<b>R/S</b>	$\Delta$ VEL=?
8	Key in angular velocity of crank and initiate calculation.	$\Delta$ VEL	<b>R/S</b> <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * etc.	$\Delta$ = X= PHI= V= PHI.= a= PHI:= $\Delta$ = etc. CRANK=?
9	For a new case, go back to step 2. Key in only values which change.  *Press <b>R/S</b> if you are not using a printer.			

**Example:**

For a slider crank mechanism with an eccentricity of 1.5 inches, a 2.0 inch crank radius, a 7.0 inch rod and an angular velocity of 4800 RPM, compute the motion for crank angles 0.0 degrees through 330 degrees in 30 degree increments.

**Keystrokes (SIZE  $\geq$  013)**

```

 $\boxed{\text{FIX}}$  2
[XEQ] [ALPHA] DEG [ALPHA]
[XEQ] [ALPHA] CRANK [ALPHA]
2 [R/S]
7 [R/S]
1.5 [R/S]
0 [R/S]
30 [R/S]
12 [R/S]

```

**Display**

```

CRANK=?  

ROD=?  

ECCEN=?  

INI  $\Delta$ ?  

 $\Delta$  INC=?  

N=?  

 $\Delta$  VEL=?  

 $\Delta$  0.00  

X=8.84  

PHI=12.37  

V=-220.55  

PHI.=8,424.26  

a=-660,249.41  

PHI.=271,732.24  

 $\Delta$ =30.00  

:  

etc.  

 $\Delta$ =330.00  

X=8.71  

PHI=4.10  

V=440.31  

PHI.=7,144.40  

a=-564,834.37  

PHI.=2,137,157.26  

CRANK=?

```

\*Press [R/S] if you are not using a printer.

## GEAR FORCES

This program computes three mutually perpendicular forces, resulting from input torque, on spur, helical, bevel or worm gears.

### Helical gear equations:

$$F_t = \frac{T}{R}$$

$$F_{gs} = F_t \tan \phi$$

$$F_{gax} = F_t \tan \Delta$$

$$\tan \phi = \frac{\tan \phi_n}{\cos \Delta}$$

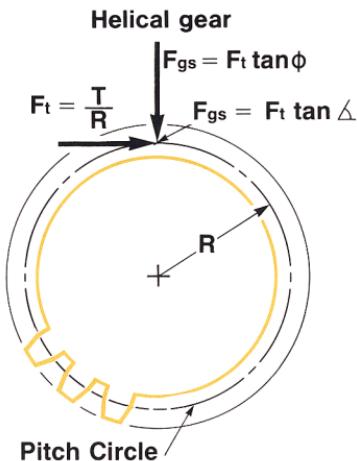


Figure 1 – Helical Gear

where:

$T$  is the input torque.

$R$  is the pitch radius of the input gear.

$F_t$  is the tangential force.

$\Delta$  is the helix angle measured from the axis of the gear (for spur gears  $\Delta = 0$ ).

$\phi_n$  is the pressure angle measured perpendicular to the gear tooth.

$F_{gs}$  is the radial force trying to separate the gears.

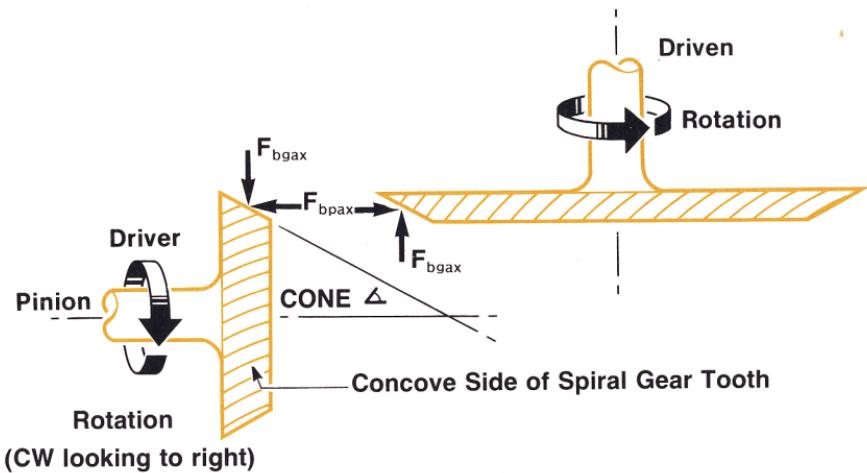
$F_{gax}$  is the force parallel to the gear axis.

## Bevel gear equations:

$$F_t = \frac{T}{R}$$

$$F_{bpax} = F_t \left( \frac{\tan \phi_n \sin (\text{cone } \Delta)}{\cos \Delta} + \tan \Delta \cos (\text{cone } \Delta) \right)$$

$$F_{bgax} = F_t \left( \frac{\tan \phi_n \cos (\text{cone } \Delta)}{\cos \Delta} - \tan \Delta \sin (\text{cone } \Delta) \right)$$



where:

$T$  is the input (pinion) torque.

$R$  is the pitch radius of the pinion gear.

$F_t$  is the tangential force.

$\Delta$  is the pinion spiral angle (zero for straight tooth bevel gears).

$\phi_n$  is the pressure angle measured perpendicular to the gear tooth.

Cone  $\Delta$  is the pitch cone angle of the pinion.

$F_{bpax}$  is the force along the axis of the bevel pinion.

$F_{bgax}$  is the force along the axis of the bevel gear.

## Worm gear equations:

$$F_t = \frac{T}{R}$$

$$F_{gax} = F_t \frac{1 - \frac{f \tan \Delta}{\cos \phi_n}}{\tan \Delta + \frac{F}{\cos \phi_n}}$$

$$F_{ws} = F_t \left( \frac{\sin \phi_n}{\cos \phi_n \sin \Delta + f \cos \Delta} \right)$$

Driver: Worm (Right hand)

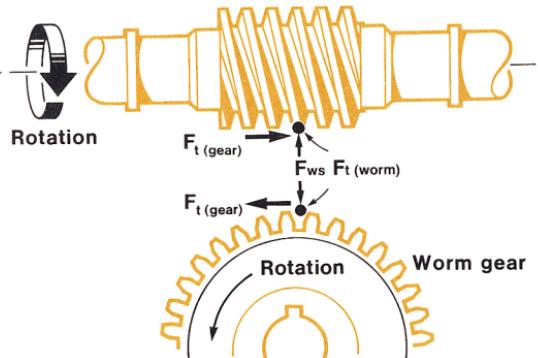


Figure 3  
Worm Gear

where:

$T$  is the input (worm) torque.

$R$  is the pitch radius of the worm.

$F_t$  is the tangential force on the worm.

$\Delta$  is the lead angle of the worm ( $\alpha = \tan^{-1}(L/2\pi r)$ , where  $L$  is the lead of the worm).

$\phi_n$  is the pressure angle measured perpendicular to the worm teeth.

$f$  is the coefficient of friction.

$F_{ws}$  is the separating force between the worm and gear.

$F_{gax}$  is the force parallel to the gear axis.

## Remarks:

For bevel gears, the pinion spiral angle is positive if the concave face of the pinion teeth are facing the direction of rotation (see figure 2). The pinion spiral angle is negative if the convex surface of the pinion teeth face the direction of rotation.

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Initialize program.		[XEQ] GEAR	"SELECT KEY:" HEL WRM BEV
2	For helical gears press [A] ; for worm gears press [C] ; for spiral bevel gears press [E] .		[A] [C] [E]	TRQ=? TRQ=? TRQ=?
3	Key in torque.	T	[R/S]	PCH R=?
4	Key in pitch radius and calculate tangential force.	R	[R/S] [R/S] *	FT= HLX Δ or LED Δ or SPR Δ
5	Go to step 6a for helical gears, step 6b for worm gears or step 6c for spiral bevel gears.			
6a	Key in helix angle.	Δ	[R/S]	NPR Δ=?
7a	Key in normal pressure angle and calculate separating and axial forces.	$\phi_n$	[R/S] [R/S] * [R/S] *	FGS= FGAX= TRQ=?
8a	For a new case go to step 3**.			
6b	Key in lead angle.	Δ	[R/S]	NPR Δ=?
7b	Key in normal pressure angle.	$\phi_n$	[R/S]	FRIC=?
8b	Key in coefficient of friction and calculate separating and axial forces.	f	[R/S] [R/S] * [R/S]	FWS= FGAX= TRQ=?
9b	For a new case go to step 3**.			
6c	Key in spiral angle.	Δ	[R/S]	NPR Δ=?
7c	Key in normal pressure angle.	$\phi_n$	[R/S]	CON Δ=?
8c	Key in cone angle and calculate pinion axial force and gear axial force.	Cone Δ	[R/S] [R/S] * [R/S] *	FbPAX= FbGAX= TRQ=?
9c	For a new case go to step 3**.			
	*Press [R/S] if you are not using a printer.			
	**Key in only values which change.			

**Example 1:**

A helical gear with pitch radius 12 cm sustains a torque of 450,000 dyne-cm. The helix angle is  $30^\circ$  and the normal pressure angle, measured perpendicular to a tooth is  $17.5^\circ$ . Find the tangential, separating, and thrust forces.

**Keystrokes (SIZE  $\geq$  009)**

**XEQ ALPHA DEG ALPHA**  
**FIX 2**  
**XEQ ALPHA GEAR ALPHA**

**A**  
**450000 R/S**  
**12 R/S**  
**R/S \***  
**30 R/S**  
**17.5 R/S**  
**R/S \***  
**R/S \***

**Display**

**“SELECT KEY:”**  
**HEL WRM BEL**  
**TRQ=?**  
**PCH R=?**  
**FT=37,500.00**  
**HLX  $\Delta$ =?**  
**NPR  $\Delta$ =?**  
**FGS=13,652.84**  
**FGAX=21,650.64**  
**TRQ=?**

**Example 2:**

A spiral bevel pinion with mean radius 1.73 inches is subjected to a torque of 745 in-lb. The pinion is cut with a normal pressure angle of  $20^\circ$ , a spiral angle of  $35^\circ$ , with a pitch cone of  $18^\circ$ . Find the forces acting on the pinion. Rotation is in the direction of the concave side of the pinion teeth, so  $\alpha$  is positive  $35^\circ$ .

**Keystrokes (SIZE  $\geq$  009)**

**XEQ ALPHA DEG ALPHA**  
**FIX 2**  
**XEQ ALPHA GEAR ALPHA**

**E**  
**745 R/S**  
**1.73 R/S**  
**R/S \***  
**35 R/S**  
**20 R/S**  
**18 R/S**  
**R/S \***  
**R/S \***

**Display**

**“SELECT KEY:”**  
**HEL WRM BEV**  
**TRQ=?**  
**PCH R=?**  
**FT=430.64**  
**SPR  $\Delta$ =?**  
**NPR  $\Delta$ =?**  
**CON  $\Delta$ =?**  
**FbPAX=345.90**  
**FbGAX=88.80**  
**TRQ=?**

\*Press **R/S** if you are not using a printer.

## STANDARD EXTERNAL INVOLUTE SPUR GEARS

This program calculates various parameters for standard external involute spur gears. Given the diametral pitch, number of teeth, pressure angle, and pin diameter, program will calculate the pitch diameter, tooth thickness, the involute and corresponding flank angle.

The program also calculates various measurements over pins, namely, the theoretical values of the measurement over pins, the radius to the center of the pin, and the measurement over one pin. In addition, given the value of the tooth thinning, the program will return the measurement over pins with tooth thinning.

### Equations:

$$D = \frac{N}{P}$$

$$T = \frac{\pi}{2P}$$

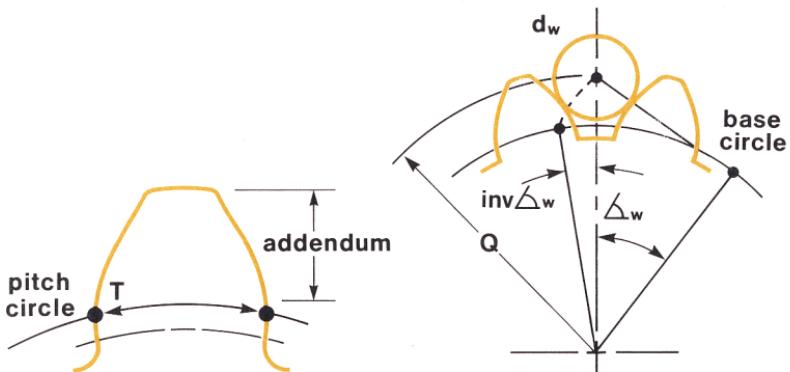
$$\text{inv } \Delta_w \text{ (radians)} = \frac{T}{D} + \tan \Delta - \Delta + \frac{d_w}{D \cos \Delta} - \frac{\pi}{N}$$

$$M = \begin{cases} d_w + 2Q & (N \text{ even}) \\ d_w + 2Q \cos\left(\frac{90}{N}\right) & (N \text{ odd}) \end{cases}$$

$$Q = \frac{D \cos \Delta}{2 \cos \Delta_w}$$

$$R_w = Q + \frac{d_w}{2}$$

$$M_t = M - \text{THIN} \frac{\cos \Delta}{\sin \Delta_w}$$



where:

D is the pitch diameter.

P is the diametral pitch.

N is the number of teeth.

$\Delta$  is the pressure angle.

T is the tooth thickness.

$\Delta_w$  is the flank angle.

M is the measurement over pins.

$R_w$  is the measurement over one pin.

Q is the radius to the pin center.

THIN is the amount of tooth thinning.

$M_t$  is the measurement over pins with tooth thinning.

$d_w$  is the pin diameter.

### Remarks:

Angular inputs and outputs match the mode of the calculator.

### Reference:

Adapted from a program submitted to the HP-65 Users' Library by Mr. John Nemcovich, Los Angeles, CA.

Dudley, D.W., *Gear Handbook*, McGraw-Hill, 1962.

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Initialize program.		<b>XEQ SPUR</b>	N=?
2	Key in number of gear teeth.	N	<b>R/S</b>	P=?
3	Key in diametral pitch and calculate pitch diameter and tooth thickness.	P	<b>R/S</b> <b>R/S</b> * <b>R/S</b> *	D= T= dW=?
4	Key in pin diameter.	$d_w$	<b>R/S</b>	$\Delta$ ?=?
5	Key in pressure angle and calculate involute of flank angle, flank angle, measurement over pins, radius over one pin, and radius to the center of pin.	$\Delta$	<b>R/S</b> <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * <b>R/S</b> *	INV $\Delta$ W= $\Delta$ W= M= RW= Q= THIN=?
6	Key in tooth thinning and calculate measurement over pins with thinning.	THIN	<b>R/S</b> <b>R/S</b> *	MT= THIN=?
7	For a new tooth thinning go back to step 6. For a new case go to step 1. *Press <b>R/S</b> if you are not using a printer.			

**Example:**

A 27-tooth gear with pitch 8 is cut with a  $20^\circ$  pressure angle. The pin diameter is 0.24 inches, and tooth thinning is reckoned at 0.002 inches. Calculate the unknown parameters.

**Keystrokes (SIZE 012)**

**FIX** 4  
**XEQ** **ALPHA** DEG **ALPHA**  
**XEQ** **ALPHA** SPUR **ALPHA**  
 27 **R/S**  
 8 **R/S**  
**R/S** \*  
**R/S** \*  
 .24 **R/S**  
 20 **R/S**  
**R/S** \*  
**R/S** \*  
**R/S** \*  
**R/S** \*  
**R/S** \*  
 .002 **R/S**

**Display**

**N**=?  
**P**=?  
**D**=**3.3750**  
**T**=**0.1963**  
**dW**=?  
**Δ**=?  
**INVΔW**=**1.8565**  
**ΔW**=**25.6215**  
**M**=**3.7514**  
**RW**=**1.8787**  
**Q**=**1.7587**  
**THIN**=?  
**MT**=**3.7470**

\*Press **R/S** if you are not using a printer.

## HELICAL SPRING DESIGN

This program performs one or two point design for helical compression springs, of round wire, with ends square and ground. The design procedure is as follows:

1. The program prompts for design inputs.
2. After you supply these inputs the calculator checks for adequate clearance between coils. If clearance is not adequate the calculator outputs:

NO CLEARANCE

3. The calculator then checks for high stresses and outputs:

HIGH STRESS

if stresses are too high.

4. If only one of the tests above fails the calculator recommends a possible solution to the problem:

SMALLER WIRE

or,

LARGER WIRE

and prompts for a new wire diameter.

If both of the tests above fail no recommendation is given since major changes in the design parameters are required. The calculator restarts the prompt sequence from the beginning.

5. If both of the tests above succeed, the design is at worst uneconomical and the calculator outputs the specifications of the spring. If stresses are extremely low the calculator will recommend smaller wire for a more economical design and prompt for a new wire diameter.

If stresses are moderate the calculator will say:

GOOD

and go back to the beginning of the input sequence. Even with a "GOOD" design, smaller wire, different material, or a different spring diameter may yield a more economical solution.

**Equations:**

$$k = \frac{P_2 - P_1}{L_1 - L_2}$$

$$s_1 = \frac{8 P_1 D W}{\pi d^3}$$

$$s_2 = \frac{8 P_2 D W}{\pi d^3}$$

$$N = \frac{G d^4}{8 D^3 k}$$

$$L_s = (N + 2) d$$

$$L_f = \frac{P_1}{k} + L_1$$

$$s_s = \frac{8 D k (L_f - L_s) W}{\pi d^3}$$

$$W = \frac{4 (D/d) - 1}{4 (D/d) - 4} + \frac{0.615}{(D/d)}$$

$$s_{\max} = \begin{cases} .45 \text{ TS for ferrous materials.} \\ .35 \text{ TS for non-ferrous materials.} \end{cases}$$

$$Y_S = \begin{cases} .65 \text{ TS for ferrous materials.} \\ .55 \text{ TS for non-ferrous materials.} \end{cases}$$

$$TS = b \ln d + a$$

**Design checking logic:**

If  $(L_2 - L_s) < 0.1 (L_f - L_2)$  and  $s_s > s_{\max}$ , the spring lacks sufficient clearance between coils and stresses are too high.

If  $(L_2 - L_s) < 0.1 (L_f - L_2)$  and  $s_s \leq s_{\max}$ , clearance between coils is insufficient.

If  $(L_2 - L_s) \geq 0.1 (L_f - L_2)$  and  $s_s > Y_S$ , stress is too high.

If  $(L_2 - L_s) \geq 0.1 (L_f - L_2)$  and  $s_s \leq Y_S$ , design is satisfactory. If  $s_s \leq 0.3$  TS, stresses are quite conservative. If  $s_s > 0.3$  TS, design is acceptable.

where:

G is the torsional modulus of rigidity.

a and b are tensile strength regression coefficients from table 1 (metric) or table 2 (English).

$P_1$  is the spring load at the most extended operating point (see figure 1).

$L_1$  is spring length, at the most extended operating point.

$P_2$  is spring load at the most compressed operating point.

$L_2$  is the spring length, at the most compressed operating point.

K is the spring constant.

d is the wire diameter.

D is the mean spring diameter.

OD is the outside spring diameter.

ID is the inside spring diameter.

$s_1$  is the Wahl-corrected shear stress at operating point 1.

$s_2$  is the Wahl-corrected shear stress at operating point 2.

N is the number of active coils.

$s_s$  is the Wahl-corrected shear stress when the spring is fully compressed to solid (coils touching).

$L_f$  is the free length of the spring.

$L_s$  is the fully compressed or solid spring length.

W is the Wahl factor which corrects stresses for curvature.

$S_{max}$  is the maximum allowable working stress for the material.

YS is the yield strength of the material.

TS is the tensile strength of the material.

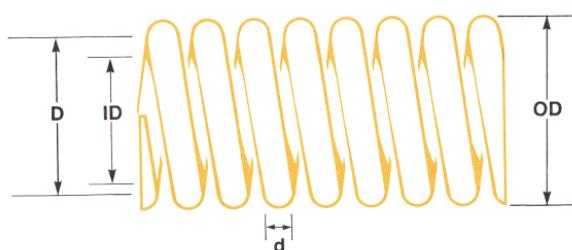
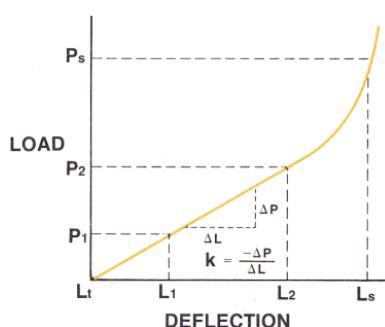


Table 1

**MINIMUM TENSILE STRENGTH REGRESSION COEFFICIENTS  
(Metric Units)**

MATERIAL	MODULUS OF RIGIDITY G,N/(mm) <sup>2</sup>	WIRE DIAMETER RANGE- MILLIMETERS	TENSILE STRENGTH COEF.	
			a,N/(mm) <sup>2</sup>	b,N/(mm) <sup>2</sup>
Music Wire ASTM-A228	$7.93 \times 10^4$	0.41-6.35	2205	-346.1
Alloy Steel ASTM-A232	$7.93 \times 10^4$	0.64-7.62	1921	-249.7
Stainless Steel ASTM-A313	$6.90 \times 10^4$	0.41-1.91 1.91-5.08 5.08-9.40	1851 1950 2221	-209.6 -393.6 -560.4
Oil Tempered ASTM-A229	$7.93 \times 10^4$	0.51-6.86	1827	-304.7
Hard Drawn ASTM-A227	$7.93 \times 10^4$	0.51-3.56 3.56-12.7	1773 1757	-283.4 -270.8
Tempered Valve Spring ASTM-A230	$7.93 \times 10^4$	2.36-5.08	1586	-153.1
Phosphor Bronze ASTM-B159	$4.07 \times 10^4$	0.64-9.40	957	- 63.97

**Table 2**  
**MINIMUM TENSILE STRENGTH REGRESSION COEFFICIENTS**  
**(English Units)**

MATERIAL	MODULUS OF RIGIDITY G,psi	WIRE DIAMETER RANGE- INCHES	TENSILE STRENGTH COEF.	
			a,psi	b,psi
Music Wire ASTM-A228	$11.5 \times 10^6$	0.016-0.25	157400	-50200
Alloy Steel ASTM-A232	$11.5 \times 10^6$	0.025-0.30	161400	-36220
Stainless Steel ASTM-A313	$10.0 \times 10^6$	0.016-0.075 0.075-0.20 0.20 -0.37	170200 98110 59190	-30400 -57090 -81280
Oil Tempered ASTM-A229	$11.5 \times 10^6$	0.020-0.27	122100	-44190
Hard Drawn ASTM-A227	$11.5 \times 10^6$	0.020-0.14 0.14 -0.50	124200 127800	-41110 -39280
Tempered Valve Spring ASTM-A230	$11.5 \times 10^6$	0.093-0.20	158300	-22200
Phosphor Bronze ASTM-B159	$5.9 \times 10^6$	0.025-0.37	108800	-9278

### Reference:

*Design Handbook—Springs, Custom Metal Parts*, Associated Spring Corporation, Bristol, Connecticut, 1970.

### Remarks:

The assumptions implicit to this program are based on engineering practice and experience. Generally, designs found by this program will be conservative, however, caution must be exercised when high or low temperatures, corrosive media or other adverse environmental circumstances exist.

For one point design, specify the free length ( $L_1$ ) and a corresponding zero load ( $P_1$ ), then specify the length ( $L_2$ ) and corresponding load ( $P_2$ ).

Some designs achieved by this program may require coiling the spring wire in such a small radius that the spring material would fail in the manufacturing process. No program check is made for this condition.

STEP	INSTRUCTIONS	INPUT	FUNCTION	SIZE: 019
1	Initialize program. Key in "Y" for ferrous materials or "N" for non-ferrous materials (stainless steel should be considered non-ferrous).		XEQ SPRING	FERROUS?Y/N
2		Y or N	R/S	G=?
3	Refer to tables 1 and 2 for the next three inputs. Use table 1 for forces in newtons and lengths in millimeters or table 2 for forces in pounds and lengths in inches.			
4	Key in modulus of rigidity.	G	R/S	a=?
5	Key in first tensile coefficient.	a	R/S	b=?
6	Key in second tensile coefficient.	b	R/S	P1=?
7	Key in load at point 1.	P1	R/S	L1=?
8	Key in spring length 1.	L1	R/S	P2=?
9	Key in load at point 2.	P2	R/S	L2=?
10	Key in spring length 2.	L2	R/S	OD=?
11a	Key in spring outside diameter.	OD	R/S	d=? (ID=?)**
11b	Key in spring inside diameter.	ID	R/S	d=?
12	Key in diameter of spring wire and start checking procedure.	d	R/S	DIAGNOSTIC
13	If the diagnostic is not "GOOD" or "OK" change inputs. See the program description for more details.			
14	For "GOOD" or "OK, BUT STRESS LOW" cases see outputs.		R/S	*
				GOOD
				K=
				N=
				LF=
				LS=
				ID=
				OD=
				S1=
				S2=
				SS=
				FERROUS? Y/N or, IF "OK", d=?

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
15	<p>For a new case go to step 1. If you wish to see the current value of the variable being prompted for, press <b>■</b>.</p> <p>*Press <b>R/S</b> if you are not using a printer.</p> <p>**If you do not key in OD in step 11a, you will be prompted for ID in step 11b.</p>			

Using Oil Tempered Wire (ASTM-A229), design a spring which supports a load of 270 newtons at a length of 62 millimeters and a load of 470 newtons at 50 millimeters. Wire is available in 0.5 mm increments. Try 4.0 mm wire first. Space available limits the spring diameter to 38.00 mm.

### Variables:

$$P_1 = 270 \text{ N}$$

$$L_1 = 62 \text{ mm}$$

$$P_2 = 470 \text{ N}$$

$$L_2 = 50 \text{ mm}$$

$$d = 4.0 \text{ mm}$$

$$OD = 38 \text{ mm}$$

$$G = 7.93 \times 10^4 \text{ N/mm}^2$$

$$a = 1827 \text{ N/mm}^2 \quad \text{From table 1}$$

$$b = -304.7$$

### Keystrokes (SIZE $\geq$ 019)

**■** **ENG** 3  
**XEQ** **ALPHA** SPRING **ALPHA**  
 Y **R/S**  
 7.93 **EEX** 4 **R/S**  
 1827 **R/S**  
 304.7 **CHS** **R/S**  
 270 **R/S**  
 62 **R/S**  
 470 **R/S**  
 50 **R/S**  
 38 **R/S**  
 4 **R/S**  
**R/S** \*  
**R/S** \*

### Display

**FERROUS? Y/N**  
**G=?**  
**a=?**  
**b=?**  
**P1=?**  
**L1=?**  
**P2=?**  
**L2=?**  
**OD=?**  
**d=?**  
**HIGH STRESS**  
**LARGER WIRE**  
**d=?**

Check to see what the current wire diameter is.



4.000

Try 4.50 mm.

4.5 [R/S]

[R/S] \*

**GOOD****K=16.16E0****N=6.487E0****LF=78.20E0****LS=38.19E0****ID=29.00E0****OD=38.00E0****S1=303.1E0****S2=527.5E0****SS=748.4E0**

### Example 2:

Using music wire (ASTM-A228), design a spring for the loading below;

$$P_1 = 1 \text{ lb}$$

$$L_1 = 1.5 \text{ in}$$

$$P_2 = 10 \text{ lb}$$

$$L_2 = 1.0 \text{ in}$$

$$G = 11.5 \times 10^6 \text{ psi}$$

$$a = 157.4 \times 10^3 \text{ psi}$$

From table 2

$$b = -50.20 \times 10^3 \text{ psi}$$

$$d = 0.035 \text{ or } 0.040$$

$$OD = 0.225 \text{ in}$$

### Keystrokes (SIZE $\geq 019$ )

ENG 3  
[XEQ ALPHA] SPRING ALPHA  
Y [R/S]  
11.5 [EEX] 6 [R/S]  
157.4 [EEX] 3 [R/S]  
50.2 [CHS] [EEX] 3 [R/S]  
1 [R/S]  
1.5 [R/S]  
10 [R/S]  
1 [R/S]  
.225 [R/S]  
.035 [R/S]

### Display

**FERROUS? Y/N****G=?****a=?****b=?****P1=?****L1=?****P2=?****L2=?****OD=?****d=?****HIGH STRESS**

\*Press [R/S] if you are not using a printer.

\* \*.04  \* \***LARGER WIRE** **$d = ?$** **NO CLEARANCE****SMALLER WIRE** **$d = ?$** 

Since neither available wire will work, a change in design constraints is required. After considering the alternatives, it is decided that  $P_2$  can be reduced to 9.0 pounds.

**XEQ** **ALPHA** SPRING **ALPHA**9  \* \*0.035  \* \* \* \* \* \* \* \* \***FERROUS Y/N** **$G = ?$**  **$a = ?$**  **$b = ?$**  **$P1 = ?$**  **$L1 = ?$**  **$P2 = ?$**  **$L2 = ?$**  **$OD = ?$**  **$ID = ?$**  **$d = ?$** **NO CLEARANCE****SMALLER WIRE** **$d = ?$** **GOOD** **$K = 16.00E0$**  **$N = 19.66E0$**  **$LF = 1.563E0$**  **$LS = 758.0E-3$**  **$ID = 155.0E-3$**  **$OD = 225.0E-3$**  **$S1 = 14.47E3$**  **$S2 = 130.3E3$**  **$SS = 186.3E3$**

## FORCED OSCILLATOR WITH ARBITRARY FUNCTION

This program determines the displacement  $x$ , velocity  $\dot{x}$ , and acceleration  $\ddot{x}$  of a damped oscillating mass  $m$  that is being driven by some forcing function. A numerical solution is computed for the nonhomogeneous differential equation

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

where:

$c$  is the damping constant.

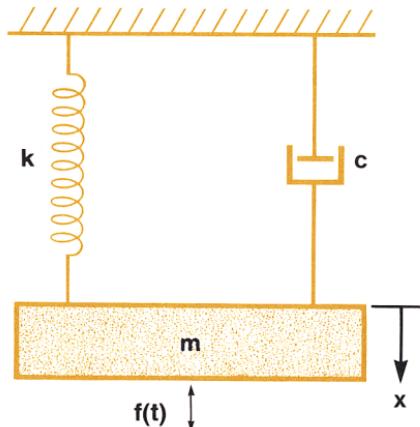
$k$  is the spring constant.

$f(t)$  is the driving force as a function of time.

The damped natural frequency for the system is also calculated if it exists.

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$$

The driving function must be programmed into the HP-41C using an alpha label as the first step. The name of this alpha label is required as a program input. When called, the function will receive time in the  $x$  register, and may access displacement from register 10 and/or velocity from register 11. Registers 16 and up are available to the function for scratch purposes.



### Remarks:

This program uses an improved Euler method to reach a numerical solution. Thus, the smaller the time increment between calculations the more accurate the solution will be. To improve accuracy either decrease the time interval between outputs or increase the number of calculations in the time interval.

SIZE: 016

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Write and key in driving function. (See Examples.)			
2	Initialize program.		<b>XEQ MOTION</b>	M=?
3	Input mass of system.	m	<b>R/S</b>	C=?
4	Input damping coefficient of system.	c	<b>R/S</b>	K=?
5	Input system spring constant and calculate damped natural frequency if one exists).	k	<b>R/S</b> <b>R/S</b> *	OMEGA= T INC=?
6	Key in time increment between outputs.	T INC	<b>R/S</b>	END T=?
7	Key in time at which calculation is to end.	END T	<b>R/S</b>	CAL/T=?
8	Key in number of calculations between outputs (positive integer).	INC/T	<b>R/S</b>	X0=?
9	Key in initial displacement of mass.	X <sub>0</sub>	<b>R/S</b>	X.0=?
10	Key in initial velocity of mass.	• X <sub>0</sub>	<b>R/S</b>	FNC NAME?
11	Key in name of driving function and initiate calculation: Displacement, Velocity, Acceleration	NAME	<b>R/S</b> <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * <b>R/S</b> * • etc.	T= X= X.= X:= T= : etc.
12	For a new case go back to step 1. Key in only values which change. *Press <b>R/S</b> if you are not using a printer.			

**Example 1:**

A mass is being driven by a forcing function of the form

$$f(t) = t^3 + 7t^2 - 14t + 40$$

Constants for the system are  $m = 5$ ,  $c = 2$ , and  $k = 12$ . The mass is held at an initial displacement of 20 at time zero. Determine the position, velocity, and acceleration of the mass at 0.05 second intervals for the first  $\frac{1}{4}$  sec. of the object's motion. Use 1 calculation per time increment.

**Solution:**

t sec	0.05	0.10	0.15	0.20	0.25
x	19.95	19.80	19.56	19.22	18.80
$\dot{x}$	-1.98	-3.92	-5.81	-7.62	-9.36
$\ddot{x}$	-39.22	-38.22	-37.00	-35.58	-33.97

**Keystrokes (SIZE  $\geq 016$ )**

First key function into program memory.

```

PRGM [ ] GTO [ ] [ ]
[ ] LBL ALPHA FT ALPHA
ENTER↑ ENTER↑ ENTER↑
7 [+] [ ] 14 [-] [ ] 40 [+] PRGM
[ ] FIX 2
XEQ ALPHA MOTION ALPHA
5 [R/S]
2 [R/S]
12 [R/S]
[R/S] *
0.05 [R/S]
.25 [R/S]
1 [R/S]
20 [R/S]
0 [R/S]
FT [R/S]
[R/S] *
[R/S] *
[R/S] *

```

**Display**

```

M=? 
C=? 
K=? 
OMEGA=1.54 
T INC=? 
END T=? 
CAL/T=? 
X0=? 
X.0=? 
FNC NAME? 
T=0.00 
X=20.00 
X.=0.00 
X:-40.00

```

**R/S** \*  
etc.  
**R/S** \*  
**R/S** \*  
**R/S** \*  
**R/S** \*

**T=0.05**  
**X=19.95**  
**X.=-1.98**  
**X:=-39.22**  
**X.**  
etc.  
**T=0.25**  
**X=18.80**  
**X.=-9.36**  
**X:=-33.97**

### Example 2:

A mass of 20 g stretches a spring 10 cm. The mass is displaced 4 cm, held, and then released. Find the displacement and velocity at 0.1 second intervals up to 1 second. Use 10 calculations per interval. Let the damping constant equal 50 dyne-s/cm.

$$K = \frac{F}{X} = \frac{mg}{X} = \frac{20 \times 980}{10} = 1960$$

### Keystrokes (SIZE > 016)

### Display

**■ [FIX] 2**

The external force is always zero for this case, but a function must still be supplied.

**■ [GTO] • •**  
**■ [PRGM] ■ [LBL] ■ [ALPHA] ZERO ■ [ALPHA]**  
**■ [CLX] ■ [PRGM]**  
**■ [XEQ] ■ [ALPHA] MOTION ■ [ALPHA]**  
20 **[R/S]**  
50 **[R/S]**  
1960 **[R/S]**  
**[R/S] \***  
.1 **[R/S]**  
1 **[R/S]**

**M=?**  
**C=?**  
**K=?**  
**OMEGA=9.82**  
**T INC=?**  
**END T=?**  
**CAL/T=?**

\*Press **[R/S]** if you are not using a printer.

10 **R/S**  
 4 **R/S**  
 0 **R/S**  
 ZERO **R/S**  
**R/S** \*  
**R/S** \*  
**R/S** \*  
**R/S** \*  
**R/S** \*  
**R/S** \*  
**R/S** \*

etc.

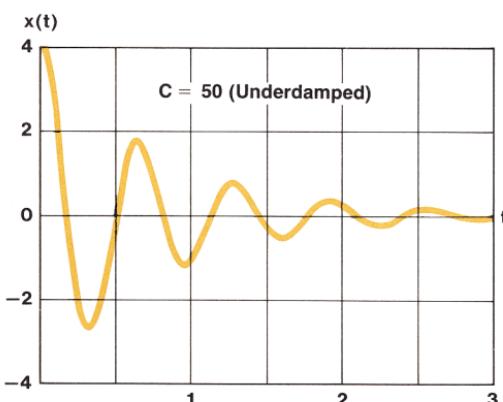
The exact solution to this problem is:

```

X0=?
X.0=?
FNC NAME?
T=0.00
X=4.00
X.=0.00
X:=-392.00
T=0.10
X=2.33
X.=-29.31
X:=-154.93
:
etc.

```

<b>t s</b>	<b>x cm</b>	<b><math>\dot{x}</math> cm/s</b>	<b><math>\ddot{x}</math> cm/s<sup>2</sup></b>
0	4.000	0.000	-392.000
.1	2.334	-29.296	-155.494
.2	-0.827	-28.715	152.880
.3	-2.629	-5.330	270.947
.4	-1.932	17.139	146.511
.5	0.153	20.950	-67.408
.6	1.655	7.187	-180.174
.7	1.503	-9.272	-124.104
.8	0.184	-14.685	18.677
.9	-0.990	-7.173	114.959
1.0	-1.114	4.406	98.133

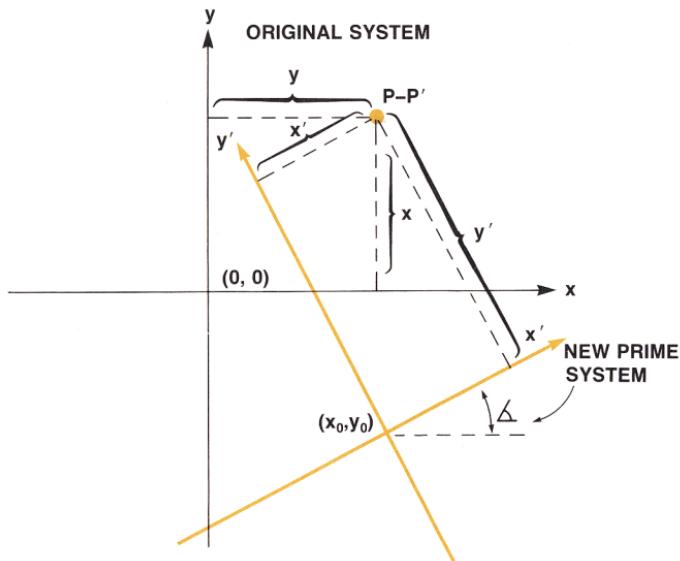


This solution may be more closely approximated by increasing the number of calculations per time interval.

\*Press **R/S** if you are not using a printer.

## COORDINATE TRANSFORMATION

This program provides transformation from a reference coordinate system to a translated rotated system. The new “prime” system is defined by the coordinates of its origin in the original system and a rotation angle relative to the original system. The inverse transformation, from the prime system to the original system, is also provided.



### Equations:

$$x' = (x - x_0) \cos \Delta + (y - y_0) \sin \Delta$$

$$y' = (x - x_0) \sin \Delta + (y - y_0) \cos \Delta$$

where:

$x_0$  is the x coordinate of the origin of the new system.

$y_0$  is the y coordinate of the origin of the new system.

$\Delta$  is the rotation angle of the new system relative to the original system.

(x,y) are the Cartesian coordinates of a point in the original system.

(x',y') are the Cartesian coordinates of a point in the translated-rotated prime system.

### Remarks:

Any angular mode will work.

For pure translation, input zero for  $\Delta$ .

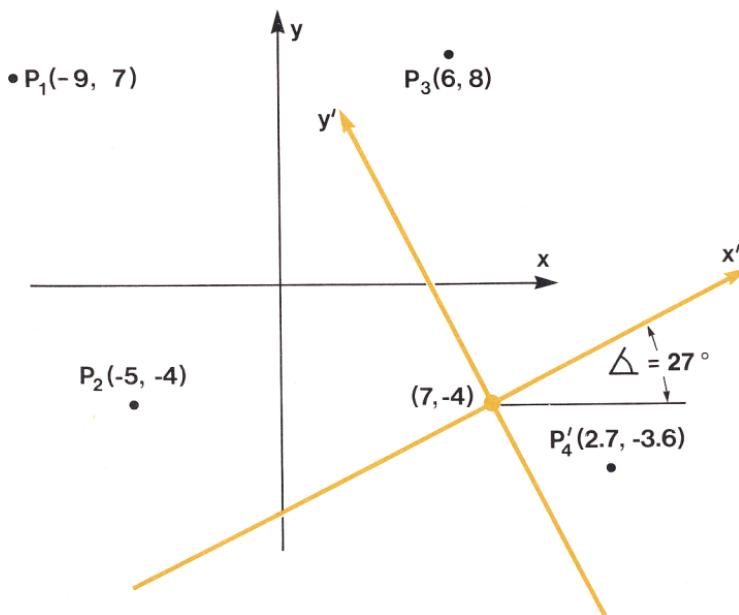
For pure rotation, input zeros for  $x_0$  and  $y_0$ .

STEP	INSTRUCTIONS	INPUT	FUNCTION	SIZE: 006
				DISPLAY
1	Initialize program.		<b>[XEQ] COORD</b>	X0=?
2	Key in x coordinate of translated origin.	$x_0$	<b>[R/S]</b>	Y0=?
3	Key in y coordinate of translated origin.	$y_0$	<b>[R/S]</b>	$\Delta=?$
4	Key in rotation angle of translated axis.	$\Delta$	<b>[R/S]</b>	X=?
5	Optional: Toggle between original and prime transformation by pressing <b>[R/S]</b> .		<b>[R/S]</b>	XP=? or X=?
6	Key in x coordinate.	X	<b>[R/S]</b>	YP=? or Y=?
7	Key in y coordinate.	y	<b>[R/S]</b> <b>[R/S]</b> <b>[R/S]</b> <b>[R/S]</b> *	XP= or X= YP= or Y=? XP=? or X=?
8	For next point go to step 5 or step 6.			

\*Press **[R/S]** if you are not using a printer.

### Example:

The coordinate systems  $(x, y)$  and  $(x', y')$  are shown below:



Convert the points  $P_1$ ,  $P_2$  and  $P_3$  to equivalent coordinates in the  $(x', y')$  system.  
 Convert the point  $P_4'$  to equivalent coordinates in the  $(x, y)$  system.

Keystrokes (SIZE  $\geq 008$ )

XEQ [ALPHA] DEG [ALPHA]  
 [FIX] 2  
 XEQ [ALPHA] COORD [ALPHA]  
 7 [R/S]  
 4 [CHS] [R/S]  
 27 [R/S]  
 9 [CHS] [R/S]  
 7 [R/S]  
 [R/S] \*  
 [R/S] \*  
 5 [CHS] [R/S]  
 4 [CHS] [R/S]  
 [R/S] \*  
 [R/S] \*  
 6 [R/S]  
 8 [R/S]  
 [R/S] \*  
 [R/S] \*

Get prompt for prime system

[R/S]  
 2.7 [R/S]  
 3.6 [CHS] [R/S]  
 [R/S] \*

## Display

$X_0=?$   
 $Y_0=?$   
 $\Delta=?$   
 $X=?$   
 $Y=?$   
 $XP=-9.26$   
 $YP=17.06$   
 $X=?$   
 $Y=?$   
 $XP=-10.69$   
 $YP=5.45$   
 $X=?$   
 $Y=?$   
 $XP=4.56$   
 $YP=11.15$   
 $X=?$

$XP=?$   
 $YP=?$   
 $X=11.04$   
 $Y=-5.98$

\*Press [R/S] if you are not using a printer.

## POINTS ON A CIRCLE

This program calculates coordinates on a circle. Points are at regular angular increments from a given starting point on the circle.

### Equations:

$$x_i = x_0 + R \cos (\angle_0 + i \Delta \angle)$$

$$y_i = y_0 + R \sin (\angle_0 + i \Delta \angle)$$

where:

$(x_i, y_i)$  is the  $i^{\text{th}}$  point.

$\angle_0$  is the initial angle.

$\Delta \angle$  is the angular increment.

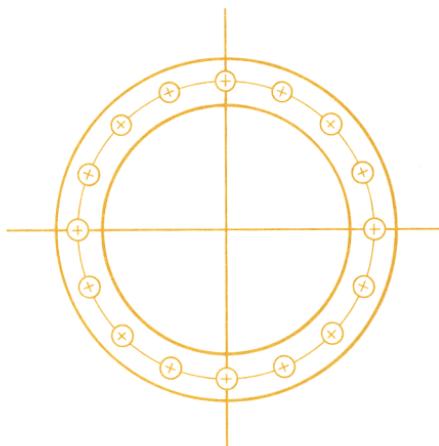
$(x_0, y_0)$  is the circle center.

$R$  is the circle radius.

STEP	INSTRUCTIONS	INPUT	FUNCTION	SIZE: 008
				DISPLAY
1	Initialize program.		[XEQ] POINTS	X0=?
2	Key in x coordinate of circle center.	$x_0$	[R/S]	Y0=?
3	Key in y coordinate of circle center.	$y_0$	[R/S]	R=?
4	Key in radius of circle.	$R$	[R/S]	∠0=?
5	Key in starting angle.	$\angle_0$	[R/S]	DEL ∠=?
6	Key in angular increment and start calculation.	$\Delta \angle$	[R/S] [R/S] * [R/S] * [R/S] * • etc. [R/S] *	∠= X= Y= ∠= X= Y= ⋮ etc. X0=?
7	For a new case go to step 2. Key in only values which change. *Press [R/S] if you are not using a printer.			

**Example 1:**

Two sections of pipe are to be joined by a flange with 16 evenly spaced bolts 9.75 inches from the center of the pipe. Calculate the coordinates of the bolt holes to be drilled. Let  $x_0 = y_0 = \Delta_0 = 0$ .

**Keystrokes (SIZE  $\geq$  008)**

```

 $\boxed{\text{FIX}}$  2
 $\boxed{\text{XEQ}}$   $\boxed{\text{ALPHA}}$  POINTS  $\boxed{\text{ALPHA}}$ 
0  $\boxed{\text{R/S}}$ 
0  $\boxed{\text{R/S}}$ 
9.75  $\boxed{\text{R/S}}$ 
0  $\boxed{\text{R/S}}$ 
360  $\boxed{\text{ENTER}}$  16  $\boxed{+}$   $\boxed{\text{R/S}}$ 
 $\boxed{\text{R/S}}$  *
:
etc.

```

**Display**

$X_0=?$   
 $Y_0=?$   
 $R=?$   
 $\Delta_0=?$   
 $\text{DEL } \Delta=?$   
 $\Delta=0.00$   
 $X=9.75$   
 $Y=0.00$

$\Delta=22.50$   
 $X=9.01$   
 $Y=3.73$

$\Delta=45.00$   
 $X=6.89$   
 $Y=6.89$

$\Delta=67.50$   
 $X=3.73$   
 $Y=9.01$

\*Press  $\boxed{\text{R/S}}$  for each output if you are not using a printer.

$\angle=90.00$  $X=0.00$  $Y=9.75$  $\angle=112.50$  $X=-3.73$  $Y=9.01$  $\angle=135.00$  $X=-6.89$  $Y=6.89$  $\angle=157.50$  $X=-9.01$  $Y=3.73$  $\angle=180.00$  $X=-9.75$  $Y=0.00$  $\angle=202.50$  $X=-9.01$  $Y=-3.73$  $\angle=225.00$  $X=-6.89$  $Y=-6.89$  $\angle=247.50$  $X=-3.73$  $Y=-9.01$  $\angle=270.00$  $X=0.00$  $Y=-9.75$  $\angle=292.50$  $X=3.73$  $Y=-9.01$  $\angle=315.00$  $X=6.89$  $Y=-6.89$  $\angle=337.50$  $X=9.01$  $Y=-3.73$ 

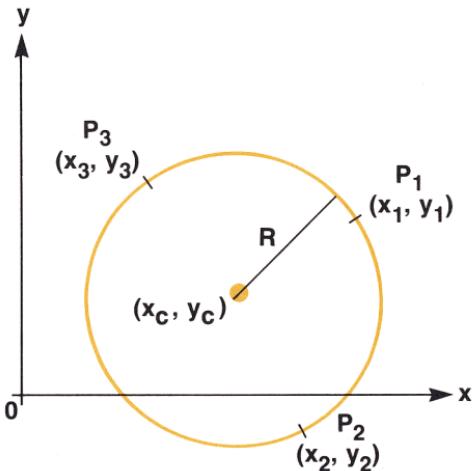
•

**R/S**

\*

## CIRCLE BY THREE POINTS

This program calculates the center ( $x_c$ ,  $y_c$ ) and the radius (R) of a circle given three non-collinear points.



STEP	INSTRUCTIONS	INPUT	FUNCTION	SIZE: 017
				DISPLAY
1	Initialize program.		<b>[XEQ</b> 3POINTS	X1=?
2	Key in point one.	$x_1$	<b>[R/S]</b>	Y1=?
		$y_1$	<b>[R/S]</b>	X2=?
3	Key in point two.	$x_2$	<b>[R/S]</b>	Y2=?
		$y_2$	<b>[R/S]</b>	X3=?
4	Key in point three.	$x_3$	<b>[R/S]</b>	Y3=?
		$y_3$	<b>[R/S]</b>	$x_c=$
			<b>[R/S]</b>	$y_c=$
			<b>[R/S]</b> *	$R=$
			<b>[R/S]</b> *	$X1=?$
5	For a new case go back to step 2. Key in only the values which change. *Press <b>[R/S]</b> if you are not using a printer.			

**Example:**

What circle contains the points (1,1), (3.5, -7.6), and (12, 0.8)?

**Keystrokes (SIZE  $\geq$  017)**

**FIX** 2  
**XEQ** **ALPHA** 3POINTS **ALPHA**  
 1 **R/S**  
 1 **R/S**  
 3.5 **R/S**  
 7.6 **CHS** **R/S**  
 12 **R/S**  
 .8 **R/S**  
**R/S** \*  
**R/S** \*

**Display**

**X1=?**  
**Y1=?**  
**X2=?**  
**Y2=?**  
**X3=?**  
**Y3=?**  
**Xc=6.45**  
**Yc=-2.08**  
**R=6.26**

\*Press **R/S** if you are not using a printer.

## UNIT CONVERSIONS

This program provides unit conversions applicable to mechanical engineering. The program is controlled by unit abbreviations and combinations of unit abbreviations placed in the alpha register.

The table below lists individual unit abbreviations recognized by this program.

HP-41C Abbreviation	Name	Multiplicative Conversion Constant	Homogeneous SI Unit
ANG	angstrom	$1.0 \times 10^{-10}$	m
ATM	atmosphere	$1.01325 \times 10^5$	Pa
BTU	British Thermal Unit (IST)	$1.055056 \times 10^3$	J
C	degree celsius	$1.0 \times 10^0$ ( $\pm 273.15$ )	K
CAL	calorie (IST)	$4.1868 \times 10^0$	J
CM	centimeter	$1 \times 10^{-2}$	m
DYNE	dyne	$1.0 \times 10^{-5}$	N
ERG	erg	$1.0 \times 10^{-7}$	J
F	degree fahrenheit	$5.5555555555 \times 10^{-1}$ ( $\pm 459.67$ )	K
FT	foot	$3.048 \times 10^{-1}$	m
G	gram	$1.0 \times 10^{-3}$	kg
GAL	gallon (U.S.)	$3.785411784 \times 10^{-3}$	m <sup>3</sup>
HP	horsepower	$7.4569987 \times 10^2$ (550 ft·lbf/s)	W
HR	hour (mean solar)	$3.6 \times 10^3$	s
IN	inch	$2.54 \times 10^{-2}$	m
J	joule	$1 \times 10^0$	J
K	kelvin	$1.0 \times 10^0$	K
KG	kilogram	$1.0 \times 10^0$	kg
KM	kilometer	$1 \times 10^3$	m
KPA	kilopascal	$1 \times 10^3$	Pa
KW	kilowatt	$1 \times 10^3$	W
LBF	pound force	$4.448221615 \times 10^0$	N
LBM	pound mass	$4.5359237 \times 10^{-1}$	kg
L	liter	$1.0 \times 10^{-3}$	m <sup>3</sup>
M	meter	$1.0 \times 10^0$	m
MI	mile	$1.609344 \times 10^3$	m
MIC	micron	$1.0 \times 10^{-6}$	m
MIL	1/1000 inch	$2.54 \times 10^{-5}$	m
MIN	minute	$6.0 \times 10^1$	s
ML	1/1000 liter	$1 \times 10^{-6}$	m <sup>3</sup>
MM	1/1000 meter	$1 \times 10^{-3}$	m
N	newton	$1.0 \times 10^0$	N
PA	pascal	$1.0 \times 10^0$	Pa

PDL	poundal	$1.382549544 \times 10^{-1}$	N
PSF	pounds force per square foot	$4.788025833 \times 10^1$	Pa
PSI	pounds force per square inch	$6.8947572 \times 10^3$	Pa
R	degree rankine	$5.555555555 \times 10^{-1}$	K
S	second	$1.0 \times 10^0$	s
SLUG	slug	$1.45939029 \times 10^1$	kg
TON	ton (2000 lb)	$9.0718474 \times 10^2$	kg
W	watt	$1.0 \times 10^0$	W
YD	yard	$9.144 \times 10^{-1}$	m
	null string	$1.0 \times 10^0$	

### Notes On Usage:

The individual abbreviations may be combined into unit strings and equations using the unit control characters \*, /, -, and 1-9. This allows a virtually unlimited set of unit conversions. For instance, the unit string for an acceleration of feet per second squared could be keyed in as:

FT/S\*S

or

FT/S2

Legal options for volumetric flow rate include, but are far from limited to:

FT3/S  
M\*CM\*IN/S  
FT3\*HR/S2

etc.

Only one divide is allowed in a unit string. Thus, all units to the right of the divide sign are included in the denominator.

Two unit strings may be combined in a unit equation using the dash character which stands for "convert to". Thus,

FT3-M3

is read "feet cubed converted to meters cubed". Only one dash is allowed in a unit equation.

If a unit string is not followed by a dash and a second unit string, the conversion defaults to equivalent SI (Standard International) units. For instance, the abbreviated unit equation

MI/HR

and the explicit unit equation

MI/HR-M/S

would both cause conversion from miles per hour to meters per second. This allows shorthand conversions between SI and other units.

Several types of errors are possible in unit conversion syntax. These are:

1. The units keyed in are misspelled, used lower case letters, or were not included in the previous list (e.g., "FEET" for "FT").
2. Unit control characters (\*, /, -, 1-9) were incorrectly used (e.g., FT/S/S).
3. The units specified on the left side of the unit equation were incompatible with those on the right (e.g., an attempt was made to convert from feet to seconds).

Any of these errors will result in the message "DATA ERROR."

STEP	INSTRUCTIONS	INPUT	FUNCTION	SIZE: Any
				DISPLAY
1	Key in unit string.	"String"	ALPHA ALPHA	
2	Key in numeric value to be converted.	Value		
3a	For forward conversion		XEQ FCON	Con Value
3b	For backward conversion		XEQ BCON	Con Value
4	For a new case go to steps 1 or 2. The original value is in LASTX. Stack registers T, Z and Y are not modified.  Note: You will probably find it convenient to assign FCON and BCON to keys.			

Convert 212°F to kelvins. Convert 0.0 kelvins to degrees Fahrenheit.

### Keystrokes

 **FIX** 4  
**ALPHA** F-K **ALPHA**  
212 **XEQ** **ALPHA** FCON **ALPHA**  
0 **XEQ** **ALPHA** BCON **ALPHA**

### Display

  
373.1500  
-459.6700

**Example 2:**

Convert 23 pounds per square inch to atmospheres.

**Keystrokes**

**FIX** 4  
**ALPHA** PSI-ATM **ALPHA**  
 23 **XEQ** **ALPHA** FCON **ALPHA**

**Display**

1.5651

**Example 3:**

Convert 88 feet per second to kilometers per hour. Convert 100 kilometers per hour back to feet per second.

**Keystrokes**

**FIX** 4  
**ALPHA** FT/S-KM/HR **ALPHA**  
 88 **XEQ** **ALPHA** FCON **ALPHA**  
 100 **XEQ** **ALPHA** BCON **ALPHA**

**Display**

95.5606  
 91.1344

**Example 4:**

Convert 10 feet to meters using the default SI conversion.

**Keystrokes**

**FIX** 4  
**ALPHA** FT **ALPHA**  
 10 **XEQ** **ALPHA** FCON **ALPHA**

**Display**

3.0480

**Example 5:**

Convert 20 (btu)(in)/(°F)(ft<sup>3</sup>)(s) to W/(in<sup>2</sup>)(°C).

**Keystrokes**

**FIX** 4  
**ALPHA** BTU\*IN/F\*FT3\*S-W/IN2\*C **ALPHA**  
 20 **XEQ** **ALPHA** FCON **ALPHA**

**Display**

21.9803

APPENDIX A  
**PROGRAM DATA**

Program	# Regs. to Copy	Data Registers	Flags	Display Format	Subprograms Called
Circular Cams	74	00=Data Pointer 01= $\Delta$ 02=GRNDR 03=Rb 04=RR 05=Name 06=Duration 07=Lift 08=N INC 09= $\Delta$ Duration 10=LIFT 11=dY/d $\Delta$ 12=d2Y/d $\Delta$ 2, RG-RR 13= $\pi$ , 180, 200 14= $\Delta$ 15= $\Delta$ /Duration 16= $\alpha$ 17=r	00 Flat Follower 01 Parabolic, 2 <sup>nd</sup> half 21 Stop on AVIEW	Any	INI *? *IN *OUT

Generation of a  
Four Bar Linkage

35	00=cos $\Delta b_1$ , R <sub>1</sub> 01=-cos $\Delta d_1$ , R <sub>2</sub> 02=cos ( $\Delta b_1$ 2 $\Delta d_1$ ), R <sub>3</sub> 03=cos $\Delta b_2$ 04=-cos $\Delta d_2$ 05=cos ( $\Delta b_2$ - $\Delta d_2$ ) 06=cos $\Delta b_3$ 07=-cos $\Delta d_3$ 08=cos ( $\Delta b_3$ - $\Delta d_3$ ) 09=COUNTER 10=USED 11=COUNTER 12=a 13=b 14=c 15=d 16= $\Delta b_1$ 17= $\Delta d_1$ 18= $\Delta b_2$ 19= $\Delta d_2$ 20= $\Delta b_3$ 21= $\Delta d_3$	21 Stop on AVIEW	Any	INI *IN *OUT
----	--	------------------	-----	--------------------

Progression of a Four Bar System	46	00=Data Pointer 01 4 USED 02= $d^2\theta/dt^2$ 03= $d\phi/d\theta$ , ( $d\alpha/d\theta$ ) 04= $\phi_3(\alpha)$ 05= $d_3(-c)$ 06=c (d) 07= $\theta$ 08= $\Delta\theta$ 09= $d\theta/dt$ 10=e 11=N 12=a 13=b 14=c 15=d 16= $\angle 60$ 17=N 18= $\angle$ INC 19= $\angle .60$	21 Stop on AVIEW	Any	INI *IN *OUT +360 *?
Progression of a Slider Crank	35	00=Data Pointer 01=R 02=L 03=E 04=INI $\Delta$ 04= $\Delta$ 06=N	21 Stop on AVIEW	Any	INI *IN +360 *OUT *?

		07= $\Delta$ VEL 08= $\Delta$ 09=PHI 10=COUNTER 11=USED 12= $\Delta$ VEL, Radians		
Gear Forces	42	00=Data Pointer 01=T 02=R 03= $\Delta$ 04=N or $\Delta$ 05=FRIC, CONE $\Delta$ 06= $\cos \Delta$ 07= $F_t$ 08= $F_{bgax}$	00 First Pass 21 Stop on AVIEW	Any KEY INI *IN *OUT
Standard External Involute Spur Gears	36	00=Data Pointer 01=N 02=P 03= $d_\omega$ 04= $\phi$ 05=D 06=INV $\phi$ 07= $\phi_\omega$ 08=t 09= $\pi/180$ , 2g 10=M 11=THIN	00 Even Teeth	Any INI *IN *OUT

Helical Spring Design	59	00=Data Pointer 01=G 02=a 03=b 04=P <sub>1</sub> 05=L <sub>1</sub> 06=P <sub>2</sub> 07=L <sub>2</sub> 08=K 09=D 10=d 11=s <sub>s</sub> 12=L <sub>f</sub> 13=L <sub>s</sub> 14=TS 15=N 16=s <sub>1</sub> 17=s <sub>2</sub>	00 Non-Ferrous 21 Stop on AVIEW	Any	INI *? *IN *OUT
--------------------------	----	---	------------------------------------	-----	--------------------------

Forced Oscillator With Arbitrary Function	36	00=Data Pointer 01=M 02=C 03=K 04=T INC 05=CAL/T 06=X0 07=X.0 08=Function Name 09=Time 10=x 11=dx/dt 12=d <sup>2</sup> x/dt <sup>2</sup> 13=USED 14=END T 15=USED	21 Stop on AVIEW 23 Alpha Input 25 Omega Output 55 Printer?	Any	INI *IN *OUT
Coordinate Transformation	23	00=Data Pointer 01=X0 02=Y0 03=Δ 04=XP, X-X 05=YP, Y-Y 06=cos Δ 07=sin Δ	21 Stop on AVIEW	Any	INI *IN *OUT

Points on a Circle	16	00=Data Pointer 01=X0 02=Y0 03=R 04=Δ 05=Δ Δ 06=360, 2, 400 07=Δ		INI *IN *OUT +360
Circle by Three Points	28	00=Data Pointer, XO 01=X1 02=Y1 03=X2 04=Y2 05=X3 06=Y3 07=X1+X2/2→X1 08=Y1+Y2/2 09=θ <sub>1</sub> , sin θ <sub>1</sub> , 10=(X <sub>3</sub> +X <sub>2</sub> )/2, X <sub>2</sub> 11=(Y <sub>3</sub> +Y <sub>2</sub> )/2, Y <sub>2</sub> 12=θ <sub>2</sub> , sin θ <sub>2</sub> 13=A 14=B 15=cos θ <sub>1</sub> , 16=cos θ <sub>2</sub>	21 Stop on AVIEW Any	INI *IN *OUT





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