MODEL

10

HEWLETT-PACKARD 9810A CALCULATOR STATISTICS BLOCK

OPERATING MANUAL

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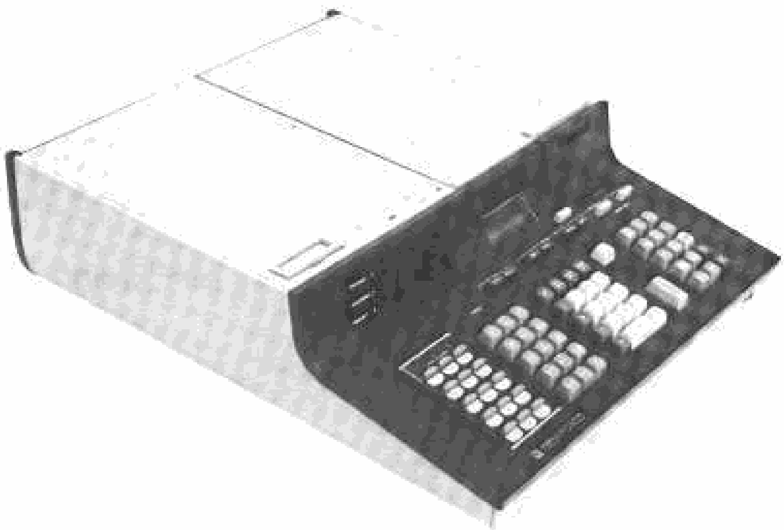
HEWLETT-PACKARD 9810A CALCULATOR
SHOWN WITH STATISTICS BLOCK



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STAT BLOCK DESCRIPTION

The 11214A Statistics Block (hereafter referred to as simply the Stat Block) consists of a statistics oriented Read-Only-Memory (ROM), and an overlay which indicates how the half keys of the left hand key block of the 9810A are defined by the Stat ROM.

The Stat Block provides the user with the power of quick and easy solution of a large number of statistical problems ranging from finding means and variances through sophisticated regression analyses. The number of problem variables permitted ranges from one to five, depending upon the nature of the operation.

The following equipment is supplied with each Stat Block:

Table 1-1. Equipment Supplied with the Stat ROM

PART NO.	QUANTITY	DESCRIPTION
7120-2776	1	Stat Overlay
09810-90026	1	Exerciser Program
09810-90005	2	Operating Manual
09810-90006	1	Pocket Reference Manual
09810-70800	1	Model 10 Stat Pac

STAT BLOCK INSTALLATION

The Stat ROM is designed for use in only ROM Slot 1 of the 9810A Calculator; it may be used concurrently with only those ROM's which are usable in Slots 2 and 3.

CAUTION

THE STAT ROM MUST BE INSTALLED ONLY IN ROM SLOT 1. CALCULATOR POWER SHOULD BE OFF WHEN INSTALLING OR REMOVING ANY ROM, OR THE CALCULATOR MAY BEHAVE IN AN UNPREDICTABLE MANNER, WHICH CAN BE STOPPED ONLY BY MOMENTARILY TURNING THE CALCULATOR OFF.

STAT BLOCK INSTALLATION

Follow this procedure to install the Stat ROM and Overlay in your 9810A Calculator:

1. Switch off the power to the Calculator.
2. Remove any ROM which is already in Slot 1. Do this by pulling the ROM straight up by the wire handle.
3. Remove any Overlay which is already in place over the left hand block of keys. An Overlay is removed by pulling back on the plastic tab (part of the keyboard) holding the Overlay in place, and lifting a corner of the Overlay with a fingernail. An Overlay not in use should be stored on the clips provided on the back of its associated ROM.
4. Install the Stat Overlay by inserting the locking tab of the Overlay into the small slot at the head of the left block of keys, and then pressing the Overlay into place.
5. Position the Stat ROM over Slot 1. See Figure 1-1. The label of the ROM must be 'right-side-up' when viewed from the front of the Calculator. Push the ROM straight down into the Slot until it is firmly seated.
6. Turn the Calculator on.

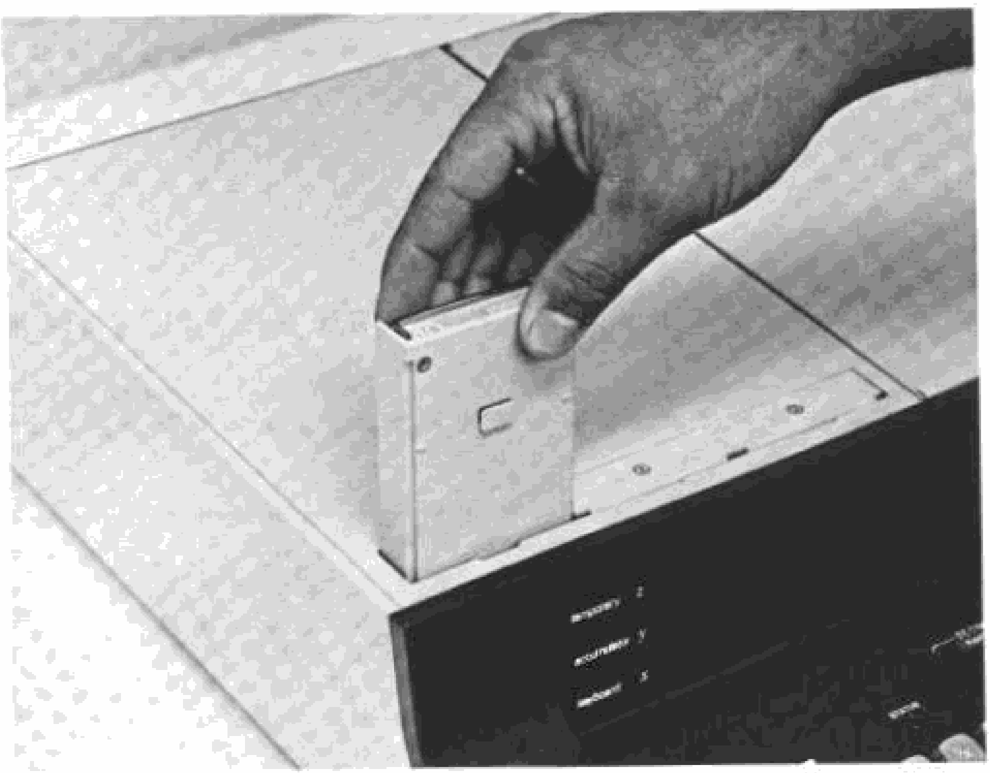


Figure 1-1. Stat Block Installation

ELECTRICAL INSPECTION

The purpose of the exerciser program supplied with the Stat Block is to verify proper operation of the 9810A Calculator - Stat ROM combination. The program may be used in any Calculator configuration; as long as the Stat ROM is properly installed.

To exercise an installed Stat Block:

1. PRESS: [RUN] [STOP] [END] [FIX ( )] [0]
2. PRESS: [LOAD]
3. Insert either side of the Stat exerciser card into the upper slot of the magnetic card reader.
4. After the Calculator's display returns, remove the card and press:

   [END] [CONTINUE]

Proper operation is indicated when the display takes the following form:

temporary z	N.
accumulator y	N.
keyboard x	N.

N must assume the values 1 through 8, and continue to repeat the sequence. The display will remain blank for varying periods of time between each change in value of N. Each of the three indicator lights above the left hand block of keys should light (or blink) at least once during each cycle of the numbers in the display.

If the exerciser program does not operate as described above, please repeat the procedure carefully. If the program still fails to run properly, contact your nearest Hewlett-Packard Sales and Service Office for assistance.

Table 2-2. Computed Quantities and Their Associated Registers

Quantity	Register	Quantity	Register	Quantity	Register
SIGMA		Σxa	011	x_{max}	022
n	000	Σya	012	y_{min}	023
Σx	001	Σza	013	y_{max}	024
Σx^2	002	Σa^2	014	z_{min}	025
Σy	003	Σb	015	z_{max}	026
Σxy	004	Σxb	016	**RANDOM**	
Σy^2	005	Σyb	017	Random Number	027
Σz	006	Σzb	018	**PAIRED** t	
Σxz	007	Σab	019	$\Sigma(x-y)$	001
Σyz	008	Σb^2	020	$\Sigma(x-y)^2$	002
Σz^2	009	**MAX/MIN**		**CHI-SQUARED**	
Σa	010	x_{min}	021	$\Sigma \frac{(x-y)^2}{y}$	001

INTRODUCTION TO THE STAT BLOCK

VARIABLES AND THEIR NAMES

The Stat Block can perform computations on more than one variable. Depending upon the nature of the computation, one through five variables are permitted. At the beginning of a problem, the Operator selects how many variables are desired. The selected variables are called 'active variables'. Selecting the active variables also specifies the registers which correspond to the variables of the problem. This correspondence (between the number of variables in a problem and the registers used to represent the variables) is shown in Table 2-1. This correspondence cannot be altered.

Table 2-1. The Correspondence Between Variables and Registers

If this many variables are selected	...Then these registers represent the variables.
1	X
2	X, Y
3	X, Y, Z
4	X, Y, Z, a
5	X, Y, Z, a, b

We will call the variables by the names of their associated registers. Thus, the names of the five variables are: x, y, z, a, and b.

Many of the operations performed by the Stat Block result in the computation of quantities involving combinations of one or more of the active variables (such as Σx and Σxy). These quantities are stored in the data registers numbered 000 through 027. Sometimes different operations use the same register for different purposes. When this happens those operations cannot be mixed together for use on the same data. Thus, SIGMA, PAIRED t, and CHI-SQUARED are mutually exclusive operations, since they each use reg. 001 for different purposes. Table 2-2 lists the computed quantities and their associated registers. This information is also given in a fold-out at the rear of the manual.

The quantities stored in registers 000 through 027 are referred to by their symbolic names (such as n, Σx, Σxy, etc.) rather than by the number of the register in which they are stored.

USE OF THE NUMBERED DATA REGISTERS

The number of active variables, in conjunction with each Stat Block operation, determines which of registers 000 through 027 are in active use. For example, if the variable setting were two (*x* and *y*), registers 000 through 005, and 021 through 024 would be required for proper Stat Block operation (again, depending upon desired operations), and must not be used for the storage of other data. However, the other registers are free for any use to which you may choose to put them, since (in this example), the Stat Block does not recognize the existence of variables named *z*, *a*, and *b*.

Any of the registers used by the Stat Block may be recalled and their contents learned and used in any desired fashion. However, the contents of those registers must not be altered during such a process; to do so would probably upset any subsequent calculations.

NOTATION

The symbolic names of variables and their associated quantities are written in italics, e.g., x, n, Σxy.

On the other hand, the names of registers are always written as follows: X, Y, Z, a, b, reg. 001. There is one exception to this rule: when the contents of X, Y and Z are shown in an illustration of the display, the registers will be identified by italicized letters (because those are the type of letters on the display window).

ILLEGAL OPERATIONS

The Status Light will come on whenever an illegal operation is attempted. Some illegal operations also halt the execution of a program. The various illegal operations are described, as they occur, in the next four chapters.

BASIC STATISTICAL OPERATIONS

VARIABLES

K = 1, . . . , 5

VARIABLES, followed by one of the digit keys, 1, 2, 3, 4, or 5, specifies the number (and names) of the variables involved in the operations performed by the Stat Block, especially summations performed by SIGMA (Σ).

The number of variables specified is indicated by the three indicator lights above the left hand block of keys. To determine the variable setting indicated by the lights, add the numerical values associated with the lights that are on.

Table 3-1. Selection of Variables

For this many variables	. . . Press . . .	. . . The lights will be . . .	. . . And the names and locations of the variables are:
1	VARIABLES (K) 1	● ○ ○ (1 2 3 — number of variables)	X
2	VARIABLES (K) 2	○ ● ○ (1 2 3 — number of variables)	X, Y
3	VARIABLES (K) 3	○ ○ ● (1 2 3 — number of variables)	X, Y, Z
4	VARIABLES (K) 4	● ○ ● (1 2 3 — number of variables)	X, Y, Z, *a*
5	VARIABLES (K) 5	○ ● ● (1 2 3 — number of variables)	X, Y, Z, *a*, *b*

A desired variable setting may be specified at any time. Specifying a variable setting does not affect the contents of any register. Once specified, a variable setting will remain in effect until it is changed.

If VARIABLES is followed by a key other than a digit of 1 through 5, the Status Light will come on; if the Calculator is running a program, it will halt, displaying the contents of X, Y, and Z.

After the Calculator is turned on, but before VARIABLES is used to specify a particular variable setting, or before SIGMA or MAX/MIN is pressed, none of the indicator lights will be on. If SIGMA or MAX/MIN is pressed without a prior specification of the variable setting, the variable setting is assumed to be three. At that time the indicator lights will indicate three. However, if an attempt is made to perform an operation requiring data obtained from the input variables, before the variable setting is specified (either explicitly, or by default), the Status Light will light.

SUMMATIONS

INITIALIZE SIGMA

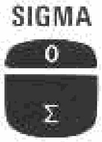
INITIALIZE SIGMA clears (sets to zero) the registers used to accumulate the sums of variables, sums of squares of variables, and the sums of cross products of variables. The number of such accumulator registers in active use during a problem is determined by the variable setting. INITIALIZE SIGMA clears only those accumulator registers which correspond to the current variable setting (one through five). INITIALIZE SIGMA also sets n to zero.

EXAMPLE:

Suppose that the variable setting is two. Then:

PRESSING	RESULTS IN	
	zero → n	(reg. 000)
	zero → Σx	(reg. 001)
	zero → Σx^2	(reg. 002)
	zero → Σy	(reg. 003)
	zero → Σxy	(reg. 004)
	zero → Σy^2	(reg. 005)

SIGMA



SIGMA causes the sums of variables, sums of squares of variables, and sums of cross products of variables, to be updated by the latest values of the active variables. SIGMA also adds one to the value of n.

SIGMA operates only on the variables specified by the variable setting. The new value of each active variable is assumed to be in its associated register: X, Y, Z, *a*, and *b*. SIGMA does not alter the contents of these registers.

EXAMPLE:

Suppose that a series of values of two variables V_1 and V_2 are to be summed. Assign one variable the name x and the name y to the other.

ACTION	RESULTS	REMARKS
PRESS: 		Specifies a variable setting of two, (x and y).
PRESS: 	Clears: n, Σx, Σx^2, Σy, Σxy, and Σy^2.	Removes any residual data from previous problems.
Enter the first value of x into X, and the first value of y into Y.		
PRESS: 	$1 \rightarrow n$ $x \rightarrow \Sigma x$ $x^2 \rightarrow \Sigma x^2$ $y \rightarrow \Sigma y$ $xy \rightarrow \Sigma xy$ $y^2 \rightarrow \Sigma y^2$	Since n and all the active accumulator registers were zero, the effect of the first use of SIGMA is as shown.

SUMMATIONS

Enter the next value of x into X, and the next value of y into Y.

PRESS: [0 / Σ]

$$1 + n \rightarrow n$$
$$x + \Sigma x \rightarrow \Sigma x$$
$$x^2 + \Sigma x^2 \rightarrow \Sigma x^2$$
$$y + \Sigma y \rightarrow \Sigma y$$
$$xy + \Sigma xy \rightarrow \Sigma xy$$
$$y^2 + \Sigma y^2 \rightarrow \Sigma y^2$$

Increments n by one and updates the summations.

Continue entering successive values of the x and y variables, following each set of entries with SIGMA.

Enter the last value of each variable.

PRESS: [0 / Σ]

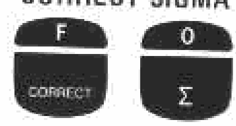
Increments n by one and updates the summations.

At this time n equals the number of paired observations that have been entered.

The accumulated summations are in their respective registers, and may be recalled and manipulated in the same way as numbers in any other registers.

There are four operations that can be performed directly on the summations (on up to three variables) without relocation or manipulation of the summations: MEAN, VARIANCE, REGRESSION, and CORRELATION. Each of these operations can be performed on the data by a single keystroke. See pages 3-5, 3-6, 4-1, and 4-2, respectively.

CORRECT SIGMA



CORRECT SIGMA allows erroneous data to be removed from the summations formed by SIGMA. CORRECT SIGMA is the operational inverse of SIGMA; for any specific values of active variables added into the summations with SIGMA, pressing CORRECT SIGMA, with all active variables equal to their respective values at the time of the error, will exactly undo that use of SIGMA associated with the error. CORRECT SIGMA also reduces the value of n by one.

CORRECT SIGMA operates only on the variables specified by the variable setting. The erroneous value of an active variable is assumed to be in its associated register. Correct values of other active variables which were added into the summations by that same use of SIGMA must also be in their associated registers, or the sums of cross products of variables will not be properly altered.

In the two examples that follow, the variable setting is assumed to be two. However, the procedures illustrated can be used with any variable setting.

CORRECT SIGMA

(continued)

SUMMATIONS

EXAMPLE:

Assume that a series of values of x and y are being summed with SIGMA.

ACTION	REMARKS
Enter the next values of x and y.	Suppose one of those values is incorrect, but that this goes unnoticed.
PRESS: 	The summations are now incorrect.

Suppose at this point you discover the error. Remember that pressing SIGMA did not alter the contents of X and Y. Thus, the erroneous and associated correct value are still in their associated registers.

PRESS: 	The incorrect data has now been removed from the summations.
Enter the correct values of x and y.	
PRESS: 	The error is now completely corrected.

Proceed with the rest of the problem.

EXAMPLE:

Assume that a series of values of x and y are being summed with SIGMA.

ACTION	REMARKS
Enter the next values of x and y.	Suppose that one of those values is incorrect, but that this goes unnoticed.
PRESS: 	The summations are now incorrect.

Suppose that you do not notice the error, and continue with the problem.

Enter the next values of x and y.

PRESS: 

Suppose that at this point the error is discovered.

Reenter the values of x and y associated with the error.	Both x and y must be reentered, or the sum of their cross product will not be adjusted properly.
PRESS: 	The incorrect data has now been removed from the summations.
Enter the correct values of x and y.	
PRESS: 	The error is now completely corrected.

Proceed with the rest of the problem.

MEAN

MEAN computes the arithmetic mean of up to three variables, x, y, and z. If the variable setting is greater than three, the means of the additional variables are not computed; MEAN will operate as if the variable setting were three. MEAN will find the arithmetic mean of x, or of x and y, when the variable setting is one, or two, respectively. The computed mean of each active variable appears in the display register associated with that variable. Display registers associated with nonactive variables are cleared (set to zero).

Table 3-2. Results of MEAN

If the variable setting is . . .	. . . Then the variables are . . .	. . . And MEAN results in:
1	x	zero → Z zero → Y $\bar{x}$ → X
2	x, y	zero → Z $\bar{y}$ → Y $\bar{x}$ → X
3	x, y, z	$\bar{z}$ → Z $\bar{y}$ → Y $\bar{x}$ → X
4, 5	Same as for three variables	

MEAN computes $\bar{x}$ by dividing Σx by n. $\bar{y}$ and $\bar{z}$ are computed in the same way. The quantities needed to compute the mean of a variable are generated by SIGMA as the values of that variable are summed. MEAN does not alter the contents of any of the summation registers.

EXAMPLE:

Suppose that you had two tables of data, and that you wished to find the mean of each one. Since there are two things involved, assume a variable setting of two, and assign the name x to one table, and the name y to the other.

ACTION	REMARKS
PRESS: 	Specifies a variable setting of two.
PRESS: 	Removes any data from previous problems.

Use SIGMA to sum the values of the variables in the tables.

ACTION	REMARKS
PRESS: 	Finds the means; $\bar{x}$ appears in X, $\bar{y}$ in Y, while Z is cleared.

VARIANCE

VARIANCE computes the variances of as many as three variables, x, y, and z. If the variable setting is greater than three, the variances of the additional variables are not computed; VARIANCE will operate as if the variable setting were three. VARIANCE will find the variance of x, or of x and y, when the variable setting is one, or two, respectively. The computed variance of each active variable appears in the display register associated with that variable. Display registers associated with nonactive variables are cleared (set to zero).

Table 3-3. Results of VARIANCE

If the variable setting is . . .	. . Then the variables are . .	. . . And VARIANCE results in:
1	x	zero → Z zero → Y Δ_x^2 → X
2	x, y	zero → Z Δ_y^2 → Y Δ_x^2 → X
3	x, y, z	Δ_z^2 → Z Δ_y^2 → Y Δ_x^2 → X
4, 5	Same as for three variables	

VARIANCE computes Δ_x^2 as shown below; Δ_y^2 and Δ_z^2 are computed in the same way. The quantities needed to compute the variances are generated by SIGMA as the values of the variables are summed.

$$\Delta_x^2 = \frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}$$

VARIANCE does not alter the contents of any of the summation registers.

EXAMPLE:

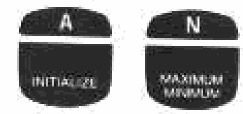
Suppose that you had two tables of data, and that you wished to find the variance of each one. Since there are two things involved, assume a variable setting of two, and assign the name x to one table, and the name y to the other.

ACTION	REMARKS
PRESS: 	Specifies a variable setting of two.
PRESS: 	Removes any data from previous problems.

Use SIGMA to sum the values of the variables in the tables.

ACTION	REMARKS
PRESS: 	Finds the variances; Δ_x^2 appears in X, Δ_y^2 in Y, while Z is cleared.

MAXIMUMS AND MINIMUMS



INITIALIZE MAX/MIN presets the contents of the six registers used for collection of maximum and minimum values of the x, y, and z variables.

The registers used for collection of maximum values are preset to -10^{99}, while registers used for collection of minimum values are preset to $+10^{99}$.

All six registers are preset, regardless of the variable setting.

Table 3-4. Results of INITIALIZE MAX/MIN

Regardless of the variable setting, pressing.	. . . Results in:
	$+10^{99} \rightarrow x_{min}$ $-10^{99} \rightarrow x_{max}$ $+10^{99} \rightarrow y_{min}$ $-10^{99} \rightarrow y_{max}$ $+10^{99} \rightarrow z_{min}$ $-10^{99} \rightarrow z_{max}$

EXAMPLE:

See the combined example on page 3-8.

MAX/MIN is used to collect the maximum and minimum values of as many as three variables, x, y, and z. MAX/MIN does this by comparing the current values of the active variables to the values stored in the max/min registers. If the comparison indicates that the new value of a variable is larger or smaller than its previously associated maximum or minimum, the new value is substituted for the appropriate previous value.

If the variable setting is greater than three, the additional variables are not included in the operation of MAX/MIN; MAX/MIN will operate as if the variable setting were three.

Table 3-5. Results of MAX/MIN

If the variable setting is . . .	. . . MAX/MIN results in:
1	Min $[x, x_{min}] \rightarrow x_{min}$ Max $[x, x_{max}] \rightarrow x_{max}$
2	Min $[y, y_{min}] \rightarrow y_{min}$ Max $[y, y_{max}] \rightarrow y_{max}$
3	Min $[z, z_{min}] \rightarrow z_{min}$ Max $[z, z_{max}] \rightarrow z_{max}$
4, 5	Same as for three variables

[N | MAXIMUM MINIMUM]
(continued)

MAXIMUMS AND MINIMUMS

MAX/MIN does not alter the contents of the X, Y, and Z registers. Also, the registers used to collect the maximum and minimum values of the active variables are separate from the registers used to accumulate the various sums of variables computed by SIGMA. Hence, MAX/MIN may be used in conjunction with either SIGMA, PAIRED t, or CHI-SQUARED, as illustrated by the next example.

CORRECT MAX/MIN is not a valid operation. Its use will not light the Status Light; the operation is simply ignored.

If an error in data entry occurs, and MAX/MIN operates on the incorrect data before the error is discovered, the collected maximums and minimums may or may not be affected. In such an instance, the collected maximums and minimums must be examined in light of the facts to determine what, if any, corrective action should be taken. Such corrective action generally takes the form of manually altering the contents of one of the max/min registers.

EXAMPLE:

Suppose that you had two tables of data and that (probably among other things) you wished to find the maximum and minimum values of each variable. Assign the name x to one variable, and the name y to the other.

ACTION	REMARKS
PRESS: [K] [2]	Specifies a variable setting of two.
PRESS: [A] [N]	Presets the max/min registers. Even though z is not an active variable, z_{min} and z_{max} are also preset.
Optional, press as required: [A] [O | Σ] or [A] [L | χ^2] or [A] [M | t]	
Enter the first values of x and y.	Beginning of data entry.
PRESS: [N]	
Optional, press as required: [O | Σ] or [L | χ^2] or [M | t]	

Continue entering values of x and y, following each set of entries with MAX/MIN, and (optional, do as required) following that with SIGMA, or CHI-SQUARED, or PAIRED t.

After the last values of x and y have been entered and processed, the maximums and minimums of the variables are in their respective max/min registers. The ranges of the variables are easily computed by computing $x_{max} - x_{min}$ and $y_{max} - y_{min}$.

LINEAR REGRESSION

REGRESSION performs a least squares linear regression on two or three variables. The regression is performed using the sums of variables, sums of cross products of variables, and sums of squares of variables, accumulated by SIGMA. The coefficients of the resulting equation are placed in the display. If the variable setting is greater than three, the additional variables are not included in the regression; REGRESSION will operate as if the variable setting were three.

Using REGRESSION while the variable setting is one is an illegal operation; the Status Light will light and the execution of a program will be halted.

Table 4-1. Results of REGRESSION

If the variable setting is . . .	. . . Then the independent variable(s) are	. . . And the equation solved is	. . With a display of:
1	—	No regression is performed	STATUS ●
2	x	$y = a_0 + a_1 x$	zero → Z a_0 → Y a_1 → X
3	x, y	$z = a_0 + a_1 x + a_2 y$	a_0 → Z a_2 → Y a_1 → X
4, 5	Same as for three variables		

The Calculator can be programmed to perform regressions involving four and five variables. The summations needed are provided by SIGMA as the values of the variables entered into the Calculator.

EXAMPLE:

See the combined example of REGRESSION and CORRELATION, on page 4-4.

REGRESSION ANALYSIS

CORRELATION

CORRELATION computes the coefficient of determination (r^2) for a linear regression performed by REGRESSION. The coefficient of determination is a number between 0 and 1, which describes the 'goodness of fit' of the equation determined by the regression. When r^2 is close to 1, the equation closely describes the data; when r^2 is close to 0, the equation is not a good statement of the relationship between the data.

For two variable analyses, the correlation coefficient (r) can be obtained from r^2 by taking the square root of r^2. The proper sign for r can be found by determining the sign of

$$\left(\Sigma xy - \frac{\Sigma x \, \Sigma y}{n}\right).$$

CORRELATION operates on the accumulated data of the active variables in a manner which is similar to REGRESSION. If the variable setting is one, CORRELATION is ignored. If the variable setting is greater than three, the additional variables are not considered in the computations; r^2 is computed as if the variable setting were three.

Table 4-2. Results of CORRELATION

If the variable setting is . . .	. . . Then the independent variable(s) are	. And the coefficient of determination is found for the equation	. . . With a display of:
1	—	No computations are performed	
2	x	$y = a_0 + a_1 x$	zero → Z zero → Y r^2 → X
3	x, y	$z = a_0 + a_1 x + a_2 y$	zero → Z zero → Y r^2 → X
4, 5	Same as for three variables		

CORRELATION does not alter the contents of any of the registers used to accumulate the summations of the active variables.

CORRELATION is independent of REGRESSION. Either operation may be performed prior to the other; the results of each are unaffected by the prior use of the other.

The computations performed by CORRELATION are easily described after defining the quantity $\mu(v_1, v_2)$, where v_1 and v_2 each represent one of the variables x, y, and z.

$$\mu(v_1, v_2) = \Sigma v_1 v_2 - \frac{(\Sigma v_1)(\Sigma v_2)}{n}$$

CORRELATION

Then, if the variable setting is two, (r^2) is defined to be:

$$r^2 = \frac{\mu^2(x, y)}{\mu(x, x)\ \mu(y, y)}$$

If the variable setting is three, then r^2 is:

$$r^2 = \frac{a_1 \mu(x, z) + a_2 \mu(y, z)}{\mu(z, z)}$$

A SAMPLE REGRESSION

Suppose that you have plotted a dependent variable against one independent variable, and wish to perform a linear regression on that data.

Since there are two variables involved, the problem will be worked with a variable setting of two. With a variable setting of two, the independent variable is x and the dependent variable is y. Accordingly, the independent variable of the table of data is named x, while the dependent variable is named y.

INDEPENDENT VARIABLE x	DEPENDENT VARIABLE y
.4	.5
.8	.7
1.0	1.1
1.3	1.1
1.4	1.6
1.8	1.7
1.9	2.1
2.3	2.2
2.6	2.7
2.9	2.8

ACTION	REMARKS
PRESS: [VARIABLES] [2]	Specifies a variable setting of two.
PRESS: [A / illegible] [D / Σ]	Removes any data from previous problems.
Enter .5 into Y and .4 into X.	Beginning of data entry.
PRESS: [D / Σ]	Start of data summation.
Enter .7 into Y and .8 into X.	Entry of next values in table.
PRESS: [D / Σ]	Update summations.

Continue entering data, following each set of entries with SIGMA. Do this until the last set of values has been entered and summed.

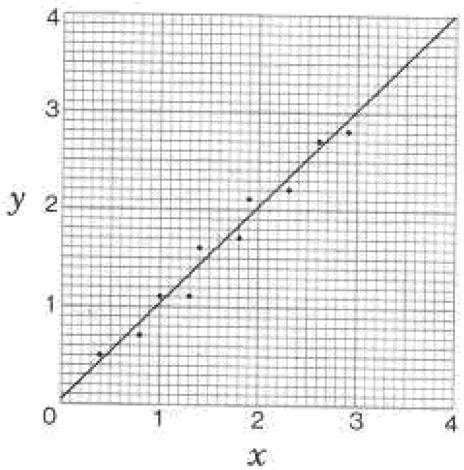
ACTION	REMARKS
Enter 2.8 into Y and 2.9 into X.	Last entry of data.
PRESS: [D / Σ]	Completes the summation of the variables.
PRESS: [E / illegible]	Finds an equation $y = a_0 + a_1 x$ to describe the data. The coefficients are placed in the display:

```
0.00  →  z
 .05  →  y (a0)
 .98  →  x (a1)
```

A SAMPLE REGRESSION

Accordingly, the linear equation that best describes the relationship between x and y is:

$$y = .05 + .98x$$



Complete your analysis of the problem by performing the following operations:

ACTION	REMARKS
PRESS: 	Computes the coefficient of determination; r^2 is placed in the display: 0.00 → z 0.00 → y .97 → x (r^2)
PRESS: 	Computes the means of the variables and places them in the display: 0.00 → z 1.65 → y ($\bar{y}$) 1.64 → x ($\bar{x}$)
PRESS: 	Computes the variances of the variables and places them in the display: 0.00 → z .64 → y (s_y^2) .65 → x (s_x^2)

NOTES

THE PAIRED t STATISTIC

INITIALIZE PAIRED t

INITIALIZE PAIRED t clears (sets to zero) the three registers used to collect the sums of variables needed by PAIRED t. These are registers 000, 001, and 002.

Table 5-1. Results of INITIALIZE PAIRED t

Pressing . . .	. . . Results in:	Reg.
	zero → n	000
	zero → ΣD	001
	zero → ΣD^2	002
Where $D = x - y$		

EXAMPLE:

See the combined example on page 5-3.

PAIRED t

PAIRED t collects the summations needed to compute the paired t statistic from the variables x and y, increments n by one, then computes the statistic and places it and other related information in the display*. The latest values of x and y are lost. PAIRED t is not affected by the variable setting; it always operates on the variables x and y.

PAIRED t **does not** use the summations generated by SIGMA; PAIRED t generates its own sumamtions. In fact, PAIRED t **must not** be used in conjunction with SIGMA.

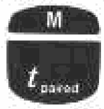
Table 5-2. Results of PAIRED t

Pressing . . .	. . . Results in:
	$\frac{\Sigma(x-y)}{n}$ → Z
	n → Y
	t_{paired} → X

*Provided that more than one pair of x and y values has been entered and operated on. After the first pair of values has been entered and operated on, PAIRED t results in the following display:

0. → z
1. → y (n)
0. → x

PAIRED t



(continued)

THE PAIRED t STATISTIC

The paired t statistic is computed from the formula:

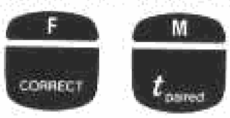
$$t_{\text{paired}} = \frac{\dfrac{\Sigma(x-y)}{n}}{\sqrt{\dfrac{\Sigma(x-y)^2 - \dfrac{[\Sigma(x-y)]^2}{n}}{n(n-1)}}} = \frac{\overline{D}}{s_D}$$

The use of PAIRED t cannot be mixed with the use of SIGMA, or with the use of CHI-SQUARED. This is because all of these operations use (at least) reg. 001 for different purposes.

EXAMPLE:

See the combined example on page 5-3.

CORRECT PAIRED t



CORRECT PAIRED t allows the value of a computed paired t statistic to be corrected to reflect the deletion of erroneous values of previously entered data. CORRECT PAIRED t is the operational inverse of PAIRED t. For any specific values of x and y added into the summations $\Sigma(x-y)$ and $\Sigma(x-y)^2$ by PAIRED t, CORRECT PAIRED t exactly undoes the results of that use of PAIRED t, including the decrementing of n by one.

The paired t statistic is automatically recomputed for the adjusted (deleted value) summations, and is placed in the display registers in the same way as the results of PAIRED t are. At this point the proper values of x and y may be entered and PAIRED t used to compute the correct value of t_{paired}.

There are two things to bear in mind when using CORRECT PAIRED t: first, the values of x and y must be reentered prior to the correction, since PAIRED t alters the contents of X and Y, and second, both x and y must be reentered, even though only one of them may have been incorrect.

Table 5-3. Results of CORRECT PAIRED t

Pressing . . .	. . . With these values in X and Y	. . . Deletes those values from the summations as shown	. . . And results in:
F CORRECT, M t_{paired}	$(x, y)_{\text{wrong}}$	$n - 1 \to n$ $\Sigma(x-y) - (x,y)_{\text{wrong}} \to \Sigma(x-y)$ $\Sigma(x-y)^2 - (x,y)^2_{\text{wrong}} \to \Sigma(x-y)^2$	$\frac{\Sigma(x-y)}{n} \to Z$ $n \to Y$ $t_{\text{paired}} \to X$

THE PAIRED t STATISTIC

EXAMPLE:

Suppose that you wished to compute the paired t statistic for the following data:

x	y
8.8	4.3
7.6	5.2
8.1	3.9
7.9	4.1
7.5	3.9
8.5	3.5
8.3	4.2

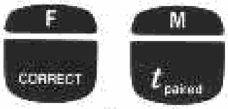
ACTION	REMARKS
PRESS: [A] [M t_{paired}]	Clears the registers used to accumulate the sums generated by PAIRED t.
Enter 4.3 into Y and 8.8 into X.	Beginning of data entry.
PRESS: [M t_{paired}]	Since only one set of values has been entered, t_{paired} is not computed. The display will be: 0.00 → z 1.00 → y (n) 0.00 → x
Enter 5.3 into Y (ERROR) and 7.6 into X.	Assume that the error is not discovered until later.
PRESS: [M t_{paired}]	Computes the summations needed to compute t_{paired}, increments n by one, and places the results in the display: 3.40 → z [(ΣD)/n] 2.00 → y (n) 3.09 → x (t_{paired})

Suppose that at this point you discover the error. To correct it, reenter both values and press CORRECT PAIRED t. Then enter the correct values and press PAIRED t.

ACTION	REMARKS
Enter 5.3 into Y and 7.6 into X.	Reentry of incorrect values.
PRESS: [F CORRECT] [M t_{paired}]	Deletes the erroneous data from the summations, decrements n by one, and places the adjusted results in the display. In this particular case, however, it is the second pair of values that is being deleted, leaving only one pair still in effect. One pair of x and y values is not sufficient to compute t_{paired}, so the display is again: 0.00 → z 1.00 → y (n) 0.00 → x
Enter 5.2 into Y and 7.6 into X.	Entry of correct values.
PRESS: [M t_{paired}]	DISPLAY: 3.45 → z [(ΣD)/n] 2.00 → y (n) 3.29 → x (t_{paired})

(continued)

CORRECT PAIRED t



(continued)

THE PAIRED t STATISTIC

Enter 3.9 into Y and 8.1 into X.

PRESS: 

Continue data entry.

DISPLAY:

3.70 → z $[(\Sigma D)/n]$
3.00 → y (n)
5.64 → x (t_{paired})

Continue making data entries, following each set of entries with PAIRED t, until all of the data has been entered and operated on.

Enter 4.2 into Y and 8.3 into X.

PRESS: 

Last entry of data.

DISPLAY:

3.94 → z $[(\Sigma D)/n]$
7.00 → y (n)
12.72 → x (t_{paired})

Hence, the value of the paired t statistic is 12.72.

THE CHI-SQUARED STATISTIC

INITIALIZE CHI-SQUARED clears (sets to zero) the two registers used to collect the sums of variables needed by CHI-SQUARED. These are registers 000 and 001.

INITIALIZE CHI-SQUARED

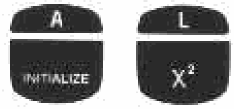


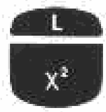
Table 5-4. Results of INITIALIZE CHI-SQUARED

Pressing . . .	. . . Results in:	Reg.
[A INITIALIZE] [L X^2]	zero → n	000
	zero → $\Sigma\frac{(x-y)^2}{y}$	001

EXAMPLE:

See the combined example on page 5-6.

CHI-SQUARED



CHI-SQUARED collects the summations needed to compute the chi-squared statistic from the variables x and y, increments n by one, then computes the statistic and places it and n in the display. The latest values of x and y are lost. CHI-SQUARED is not affected by the variable setting; it always operates on the variables x and y. In the context of normal usage of the chi-squared statistic, x represents the 'observed value', while y represents the 'expected value'.

CHI-SQUARED **does not** use the summations generated by SIGMA; CHI-SQUARED generates its own summations. In fact, CHI-SQUARED **must not** be used in conjunction with SIGMA.

Table 5-5. Results of CHI-SQUARED

Pressing . . .	. . . Results in:
[L X^2]	zero → Z n → Y X^2 → X

The chi-squared statistic is computed from the formula:

$$X^2 = \Sigma\frac{(x-y)^2}{y}$$

The use of CHI-SQUARED cannot be mixed with the use of SIGMA, or with the use of PAIRED t. This is because all of these operations use (at least) reg. 001 for different purposes.

EXAMPLE:

See the combined example on page 5-6.

CORRECT CHI-SQUARED

THE CHI-SQUARED STATISTIC

CORRECT CHI-SQUARED allows the value of a computed chi-squared statistic to be corrected to reflect the deletion of erroneous values of previously entered data. CORRECT CHI-SQUARED is the operational inverse of CHI-SQUARED. For any specific values of x and y added into the summation $\Sigma[(x-y)^2/y]$ by CHI-SQUARED, CORRECT CHI-SQUARED exactly undoes the results of that use of CHI-SQUARED, including the decrementing of n by one.

The adjusted (deleted value) of n and of chi-squared are placed in the display registers. At this point, the proper values of x and y may be entered and CHI-SQUARED used to compute the correct value of chi-squared.

There are two things to bear in mind when using CORRECT CHI-SQUARED: first, the values of x and y must be reentered prior to the correction, since CHI-SQUARED alters the contents of X and Y, and second, both x and y must be reentered, even though only one of them may have been incorrect.

Table 5-6. Results of CORRECT CHI-SQUARED

Pressing . . .	. . . With these values in X and Y	. . . Deletes those values from the summations as shown	. . . And results in:
	$(x, y)_{wrong}$	$n-1 \rightarrow n$ $\Sigma\frac{(x-y)^2}{y} - (x, y)_{wrong} \rightarrow \Sigma\frac{(x-y)^2}{y}$	zero → Z n → Y χ^2 → X

EXAMPLE:

Suppose you wished to compute the chi-squared statistic for the following data:

x OBSERVED VALUE	y EXPECTED VALUE
18	20
32	20
10	20
20	20

ACTION	REMARKS
PRESS: 	Clears the registers used in the computation of χ^2.
Enter 20 into Y and 18 into X.	Beginning of data entry.
PRESS: 	Computes χ^2 and places the result in the display: 0.00 → z 1.00 → y (n) .20 → x (χ^2)
Enter 20 into Y and 33 into X (**ERROR**).	Assume that the error is not discovered until later.

THE CHI-SQUARED STATISTIC

PRESS: [L / x^2]

Computes x^2 and places the result in the display:

0.00 → z
2.00 → y (n)
8.65 → x (x^2)

Suppose that at this point you discover the error. To correct it, reenter both values and press CORRECT CHI-SQUARED. Then enter the correct values and press CHI-SQUARED.

Enter 20 into Y and 33 into X.

Reentry of incorrect values.

PRESS: [F / CORRECT] [L / x^2]

Deletes the erroneous data from the summations, decrements *n* by one, and places the adjusted results in the display:

0.00 → z
1.00 → y (n)
.20 → x (x^2)

Enter 20 into Y and 32 into X.

Entry of correct values.

PRESS: [L / x^2]

DISPLAY:

0.00 → z
2.00 → y (n)
7.40 → x (x^2)

Enter 20 into Y and 10 into X.

Continue data entry

PRESS: [L / x^2]

DISPLAY:

0.00 → z
3.00 → y (n)
12.40 → x (x^2)

Enter 20 into Y and 20 into X.

Last entry of data.

PRESS: [L / x^2]

DISPLAY:

0.00 → z
4.00 → y (n)
12.40 → x (x^2)

Hence, the value of the chi-squared statistic is 12.40.

NOTES

PSEUDO RANDOM NUMBERS

RANDOM is a function which replaces a pseudo random number in reg. 027 with another pseudo random number, which is also placed in X. Y and Z are not disturbed. The process is started by placing a suitable seed in reg. 027 prior to the first use of RANDOM. After that the argument of RANDOM is the previously generated pseudo random number. The numbers are generated uniformly in either of two ranges: −1 to 0, or 0 to +1.

For best results, the seed placed in reg. 027 should have certain properties:

1. be a twelve digit (including guard digits) decimal fraction,
2. contain all ten digits 0 through 9 in an arbitrary order,
3. be of the same sign as the desired sequence of random numbers.

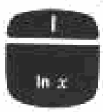
Typical sequences of pseudo random numbers generated by RANDOM are longer than fifty thousand numbers before the sequence begins to repeat itself.

The algorithm used to generate pseudo random numbers is to:

1. multiply the number in reg. 027 by 29,
2. drop the integer portion of the product and use the fractional portion as the next pseudo random number.

TRANSCENDENTAL FUNCTIONS

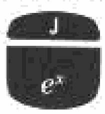
NATURAL LOG



NATURAL LOG replaces the number in X with the natural (base e) logarithm of that number.

Operations to find the natural logarithm of zero, or of a negative number, are illegal, although not fatal. In such cases the Status Light will come on and the number −9.999999999 98 is placed in X. However, the execution of a program will not be interrupted.

EXPONENTIAL FUNCTION



The EXPONENTIAL FUNCTION raises e (2.718...) to the power of the number in X, and places the result in X.

COMMON LOG

COMMON LOG replaces the number in X with the common (base 10) logarithm of that number.

Operations to find the common logarithm of zero, or of a negative number, are illegal, although not fatal. In such cases, the Status Light will come on and the number −4.342944819 98 is placed in X. However, the execution of a program will not be interrupted.

BEFORE AND AFTER - t_{paired}

PAIRED t is useful in solving problems associated with 'paired' experimental models. This type of experiment involves two observations from each subject; often these observations are of 'before' and 'after'.

The formula for t_{paired} is:

$$t_{paired} = \frac{\bar{D}}{\Delta_{\bar{D}}}$$

Where:

1) $D = x - y$, and x and y are a pair of observations
2) $\bar{D}$ is the mean of the various D's or the average difference between paired values
3) $\Delta_{\bar{D}}$ is the standard error of the differences.

A small value of t_{paired} indicates that there is little difference between the 'before' and 'after'. A large value of t_{paired} indicates that there is a difference.

For example, suppose you wanted to determine the effectiveness of hypnotic treatments to reduce or eliminate sleepwalking. Let's say that you have collected data from somnambulists before and after treatment, and obtained the following table of data.

subject numbered:	x(before)*	y(after)*
1	6	2
2	4	3
3	7	5
4	4	4
5	3	3
6	2	2
7	6	1
8	5	2
9	4	2
10	6	3

*Incidences per month

The computed value of t_{paired} for this data is:

$$t_{paired} = 3.59, \text{ with } df = 9$$

This statistic has degrees of freedom equal to the number of subjects less one.

From a table of t values we see that 99 out of 100 t values are less than 3.25. Hence, it is likely that the hypnotic treatments did indeed reduce the incidence of sleepwalking.

SAMPLE PROBLEMS

CONFIDENCE INTERVALS

Suppose you wished to determine the 95% confidence interval for a population mean (μ), given a sample taken from the population:

$$\text{Sample} = \{x_i, i = 1, n\}$$

For a 95% confidence interval, the 'true mean' of the population lies within the interval:

$$\bar{x} - 1.96\, s_{\bar{x}} < \mu < \bar{x} + 1.96\, s_{\bar{x}}$$

The computation of the upper and lower bounds of this interval is simplified by the use of PAIRED t, even though only one variable is involved.

The procedure is to press INITIALIZE PAIRED t, and enter the x_i into X, pressing PAIRED t after each entry. However, the Y register **must** contain zero each time PAIRED t is pressed.

Following the above procedure results in the following quantities being placed in X, Y, and Z after PAIRED t is pressed for the last sample:

$$\bar{x} \rightarrow z$$
$$n \rightarrow y$$
$$\frac{\bar{x}}{s_{\bar{x}}} \rightarrow x$$

Once $\bar{x}$ and $s_{\bar{x}}$ are available, it is an easy task to compute the upper and lower bounds of the confidence interval.

OBSERVED vs EXPECTED - χ^2

CHI-SQUARED is used to identify significant differences between the 'observed results' and 'expected results' of an experiment. The formula used here for χ^2 is:

$$\chi^2 = \Sigma \frac{(x-y)^2}{y}$$

Where:

1) x is the observed value
2) y is the expected value

If the expected and observed values are quite similar, χ^2 will be small; if the values are quite different, χ^2 will be large.

For example, suppose you wished to determine people's preference in beer containers. You could offer beer in paper cups, cans, and glass bottles, and record the results.

If people had no preference, you would expect a third of the subjects to choose each type of container. But, even assuming there were no preference, it is unlikely that exactly one third of the subjects would fall into each category. The chi-squared statistic allows you to determine if the variations in the results are really significant, and as such, actually indicate a preference.

	CHOICES			
	paper cup	can	bottle	total
observed	10	12	38	60
expected	20	20	20	60

We compute χ^2 and find its value to be:

$$\chi^2 = 24.40, \text{ with } df = 2$$

This statistic has degrees of freedom equal to the number of choices less one.

From a χ^2 table we see that 99 out of 100 χ^2 values are less than 9.21. Hence, our value of 24.40 indicates a definite preference for bottled beer.

CURVE FITTING

TWO VARIABLE LINEAR REGRESSION

Recall that two variable regressions are performed by naming the independent variable x and the dependent variable y. The values of the variables are then entered a pair at a time and operated on with SIGMA and a variable setting of two. Then the summations are operated on with REGRESSION and CORRELATION.

For example, suppose you wished to find a formula for determining the time required to cut down a tree (felling time) as a function of the diameter of the tree (diameter at breast).

x DIAMETER AT BREAST (FEET)	y FELLING TIME (MINUTES)
10	.95
12	1.05
15	1.25
17	1.41
20	1.73
22	2.00
25	2.53
27	2.98
30	3.85
32	4.59
35	6.02

For this data, the results of a simple linear regression are:

$$\bar{y} = 2.58$$
$$\bar{x} = 22.27$$

$$s_y^2 = 2.68$$
$$s_x^2 = 68.82$$

$$a_0 = -1.58$$
$$a_1 = .19$$

$$r^2 = .89$$

Thus, the formula:

$$y = -1.58 + .19x$$

accounts for 89% of 'felling time'.

CURVE FITTING

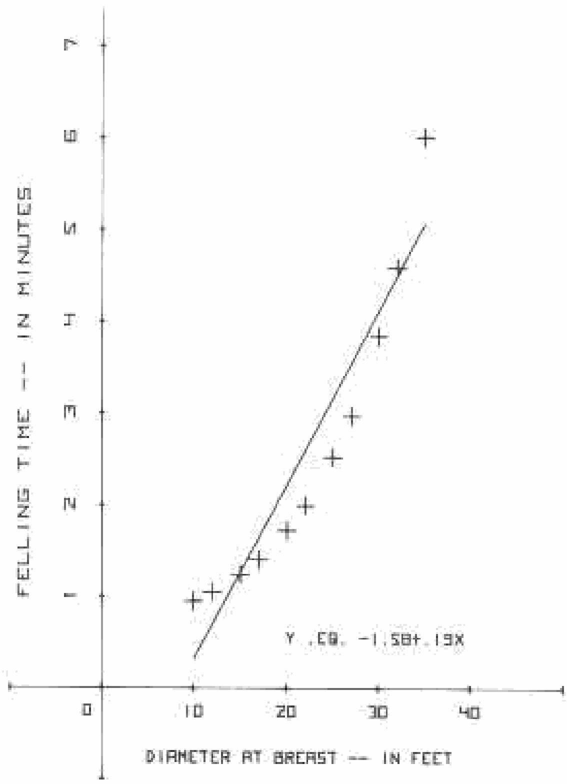


Figure 7-1. A Linear Regression of the Tree Felling Data

TWO VARIABLE POWER REGRESSION

By properly transforming the data prior to a linear regression, and then transforming the coefficients obtained by the regression, you can find the coefficients of the following power curve:

(1) $$y = a_0 x^{a_1}$$

This is easily seen by taking the natural logarithm of both members of (1) to get:

(2) $$\ln y = \ln a_0 + a_1 \ln x$$

Notice the similarity between (2) and the linear model (3).

(2) $$\ln y = \ln a_0 + a_1 \ln x$$
(3) $$y = b_0 + b_1 x$$

TWO VARIABLE POWER REGRESSION (continued)

CURVE FITTING

By entering $\ln y$ for y, and $\ln x$ for x, a linear regression will find the coefficients of (2): $\ln a_0$ and a_1. After the regression, a_0 is recovered by:

(4) $$a_0 = \exp(\ln a_0) = e^{\ln a_0}$$

Enter each pair of entries as follows:

ACTION	REMARKS
PRESS: [ ] [ ] [ ] [ ]	Enter the untransformed y value.
PRESS: [I / illegible]	Transforms the y value.
PRESS: [↑]	Puts the transformed y value in the Y register.
PRESS: [ ] [ ] [ ] [ ]	Enter the untransformed x value.
PRESS: [I / illegible]	Transforms the x value.
PRESS: [0 / Σ]	Sums the transformed data.

After REGRESSION has been used to find $\ln a_0$ (in Y) and a_1 (in X), press the following keys:

ACTION	REMARKS
PRESS: [x⇄y]	Puts $\ln a_0$ in X so it can be operated on with e^x.
PRESS: [I / illegible]	Finds a_0 from $\ln a_0$.
PRESS: [x⇄y]	Restores the final coefficients to the form in which you are used to seeing them.
PRESS: [8 / illegible]	Finds the coefficient of determination (r^2).

The results of a power regression on the tree cutting data are as follows:

$$y = .03x^{1.46} \qquad \text{with } r^2 = .94$$

Thus, the power model accounts for 94% of the 'felling time'.

CURVE FITTING

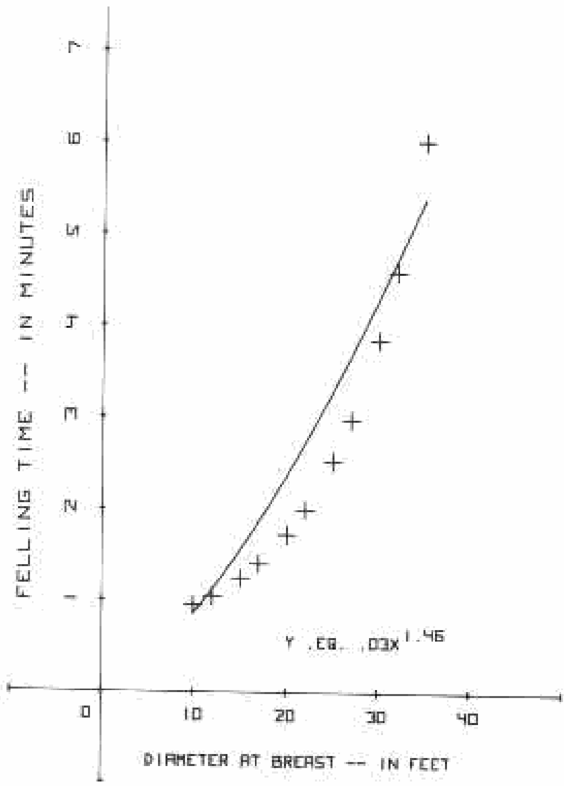


Figure 7-2. A Power Regression of the Tree Felling Data.

TWO VARIABLE EXPONENTIAL REGRESSION

By properly transforming the data prior to a linear regression, and then transforming the coefficients obtained by the regression, you can find the coefficients of the following exponential curve:

(1) $y = a_0 e^{a_1 x}$

This is easily seen by taking the natural logarithm of both members of (1) to get:

(2) $\ln y = \ln a_0 + a_1 x$

Notice the similarity between (2) and the linear model (3).

(2) $\ln y = \ln a_0 + a_1 x$
(3) $y = b_0 + b_1 x$

By entering $\ln y$ for y, and x as simply x, a linear regression will find the coefficients of (2): $\ln a_0$ and a_1. After the regression, a_0 is recovered by:

(4) $a_0 = \exp(\ln a_0) = e^{\ln a_0}$

TWO VARIABLE EXPONENTIAL REGRESSION (continued)

CURVE FITTING

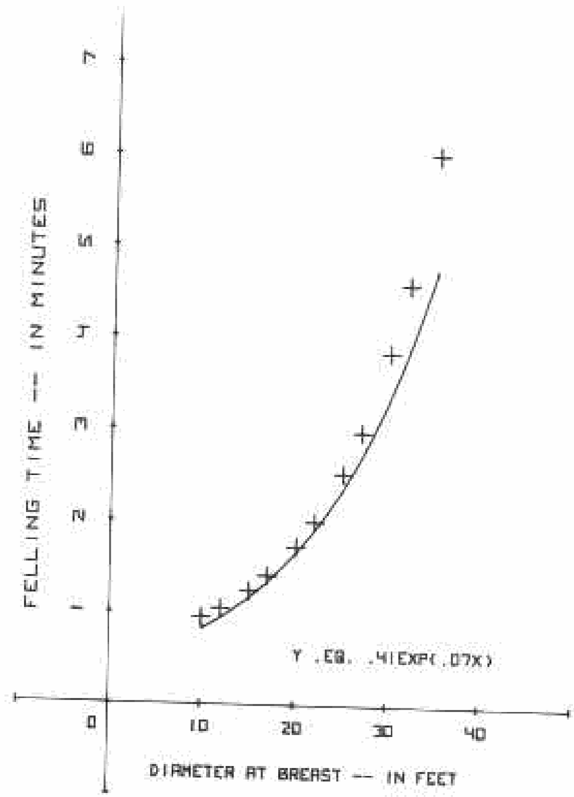


Figure 7-3. An Exponential Regression of the Tree Felling Data.

Enter each pair of entries as follows:

ACTION	REMARKS
PRESS: [ ] [ ] [ ] [ ]	Enter the untransformed y value.
PRESS: [ ]	Transforms the y value.
PRESS: [↑]	Puts the transformed y value in the Y register.
PRESS: [ ] [ ] [ ] [ ]	Enter the x value.
PRESS: [Σ]	Sums the transformed data.

After REGRESSION has been used to find $\ln a_0$ (in Y) and a_1 (in X), press the following keys:

PRESS: [$x \rightleftarrows y$]	Puts $\ln a_0$ in X so that it can be operated on with e^x.

CURVE FITTING

PRESS: [e^x] — Finds a_0 from $\ln a_0$.

PRESS: [$x \rightleftarrows y$] — Restores the final coefficient to the form in which you are used to seeing them.

PRESS: [r^2] — Finds the coefficient of determination (r^2).

The results of an exponential regression on the tree cutting data are as follows:

$$y = .41e^{.07x} \qquad \text{with } r^2 = .99$$

Thus, the exponential model accounts for 99% of the 'felling time'.

THREE VARIABLE LINEAR REGRESSION

Recall that three variable linear regressions are performed by naming the independent variables x and y, and the dependent variable z. The values of the variables are then entered a triplet at a time and operated on with SIGMA and a variable setting of three. Then the summations are operated on with REGRESSION and CORRELATION.

For example, suppose you wished to find a formula relating the price of real estate to the size and elevation of the lot.

PRICE — z in thousands of dollars	ELEVATION — y in thousands of feet	AREA — x in thousands of square feet
6.95	6.7	13
6.20	5.5	14
5.80	5.0	17
5.60	4.9	14
5.75	4.4	18
5.35	4.1	19
5.20	3.5	21

CURVE FITTING

THREE VARIABLE LINEAR REGRESSION (continued)

For this data, the results of a three variable linear regression are:

$$\bar{z} = 5.84$$
$$\bar{y} = 4.87$$
$$\bar{x} = 16.57$$

$$a_0 = .93$$
$$a_2 = .75$$
$$a_1 = .08$$

$$s_z^2 = .35$$
$$s_y^2 = 1.08$$
$$s_x^2 = 8.95$$

$$r^2 = .96$$

Thus, the formula is:

Price = .93 + .08(Area) + .75(Elevation),

and it accounts for 96% of the price of the lots.

PARABOLIC REGRESSION

By the proper substitution of variables, a two variable parabolic regression can be performed using a three variable linear model.

The usual way of writing the three variable equation solved by REGRESSION is:

(1) $$z = a_0 + a_1 x + a_2 y$$

We cannot alter which of the registers are involved in a three-variable regression. However, we can change the names (which are merely symbols) of the variables, if we choose.

CURVE FITTING

Thus, the same equation can be written:

(2) $$v_3 = a_0 + a_1 v_1 + a_2 v_2$$

All that is required here is to keep in mind that v_3 is a dependent variable of v_1 and v_2. Note that there is no restriction on any relationship between v_1 and v_2. During the entry of data, v_3 would be entered where z would have been entered, with v_1 in place of x and v_2 in place of y.

Suppose we let:

v_1 = an independent variable named x

v_2 = the square of that variable; x^2

v_3 = the dependent variable of x and x^2, called y.

So, by the proper substitution, the model for a linear three-variable regression can be made to contain the parabolic two-variable model:

(3) $$y = a_0 + a_1 x + a_2 x^2$$

Notice the similarity among (1), (2) and (3). The only real difference is in the naming of the variables.

(1) $$z = a_0 + a_1 x + a_2 y$$

(2) $$v_3 = a_0 + a_1 v_1 + a_2 v_2$$

(3) $$y = a_0 + a_1 x + a_2 x^2$$

Enter each set of values as follows:

ACTION	REMARKS
PRESS: [ ] [ ] [ ] [ ]	Enter y.
PRESS: [↑]	Put y into Y.
PRESS: [ ] [ ] [ ] [ ]	Enter x.
PRESS: [↑]	Put y into Z.
PRESS: [×]	Generates x^2 in Y; x remains in X.
PRESS: [Σ]	Sums the data.

After all the values have been entered, press MEAN, VARIANCE, REGRESSION, and CORRELATION for a complete analysis of the problem.

PARABOLIC REGRESSION (continued)

CURVE FITTING

For example, consider the table below, and examine the relationship between corn yield (y) and June rainfall (x).

Corn Yield $- y$ bushels per acre	Rainfall $- x$ inches during June
32.0	1.5
43.5	2.6
48.0	3.5
52.5	4.1
49.6	4.6
40.0	6.1
35.6	7.2

The results of the analysis of this data are shown below:

$\bar{y}$ = 43.03 (appears in Z)
$\bar{x}$ = 4.23 (appears in X)

a_0 = 10.07
a_2 = −2.07
a_1 = 18.15

s_y^2 = 57.30 (appears in Z)
s_x^2 = 3.85 (appears in X)

r^2 = .92

APPENDIX

Computed Quantities and Their Associated Registers

Quantity	Register	Quantity	Register	Quantity	Register
SIGMA		Σxa	011	x_{max}	022
n	000	Σya	012	y_{min}	023
Σx	001	Σza	013	y_{max}	024
Σx^2	002	Σa^2	014	z_{min}	025
Σy	003	Σb	015	z_{max}	026
Σxy	004	Σxb	016	**RANDOM**	
Σy^2	005	Σyb	017	Random Number	027
Σz	006	Σzb	018	**PAIRED t**	
Σxz	007	Σab	019	$\Sigma(x-y)$	001
Σyz	008	Σb^2	020	$\Sigma(x-y)^2$	002
Σz^2	009	**MAX/MIN**		**CHI-SQUARED**	
Σa	010	x_{min}	021	$\Sigma \frac{(x-y)^2}{y}$	001

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