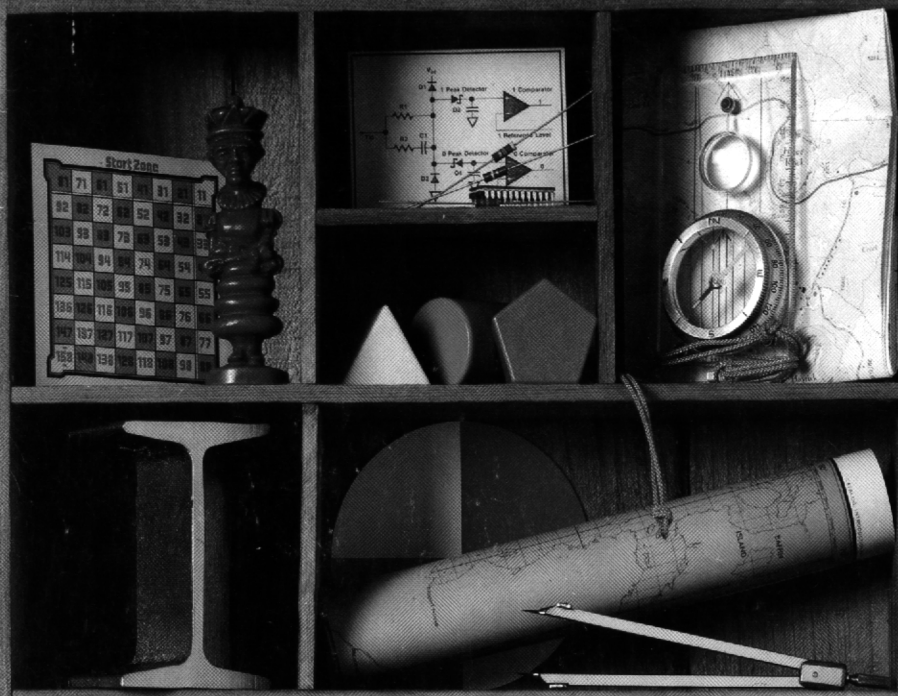

HEWLETT-PACKARD

HP-19C/HP-29C Applications Book

PRINTED IN U.S.A.

INTRODUCTION

Welcome to the world of HP calculators. We know you will be pleased with the quality, versatility, and ease of use of your new HP-19C/HP-29C. This application book is designed to help you get the best from your calculator, whether your interest is in solving specific problems in a particular area or in learning to use the powerful programming capabilities of the HP-19C/HP-29C.

These programs have been chosen from real world problems in a variety of areas; mathematics, statistics, finance, surveying, navigation, science, medicine and games. They demonstrate the many uses of the HP-19C/HP-29C and will give you immediate calculation aids for problems you encounter every day. You will also find them useful as guides to programming techniques and models for writing your own customized software. The comments on each program listing demonstrate the approach used to reach the solution and help you follow the programmer's logic as you become an expert with your own HP-19C/HP-29C.

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A WORD ABOUT PROGRAM USAGE

This Applications Book for the HP-19C/HP-29C provides a diverse selection of programs chosen from a number of areas of interest. Each program includes a brief description, a listing of the program keystrokes, a set of instructions for using the program and one or more example problems, including the actual keystrokes required for the solution.

Explanatory comments have been incorporated in each program listing to aid your understanding of the actual working of each program. Thorough study of the commented listing can help you expand your programming repertoire since interesting techniques can often be found.

The completed User Instruction Form—which accompanies each program—is your guide to operating the programs in this pac.

The form is composed of five labeled columns. Reading from left to right, the first column, labeled STEP, gives the instruction step number.

The INSTRUCTIONS column gives instructions and comments concerning the operations to be performed.

The INPUT-DATA/UNITS column specifies the input data, and the units of the data, if applicable. Data input keys consist of [0] to [9] and decimal point (the numeric keys), [EEX] (enter exponent), and [CHS] (change sign).

The KEYS column specifies the keys to be pressed after keying in the corresponding input data.

The OUTPUT-DATA/UNITS column specifies intermediate and final outputs and their units, wherever applicable.

The following illustrates the User Instruction Form for Quadratic Equation, the first program in this book.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Enter coefficients of quadratic			
	x^2 coefficient	a	[ENTER↑]	
	x coefficient	b	[ENTER↑]	
	constant	c	[GSB] [1]	D
3	If D ≥ 0, roots are real		[R/S]	x_2
			[R/S]	x_1
4	If D < 0, roots are complex of		[R/S]	u (real part)
	form u ± iv		[R/S]	v
				(imaginary part)

Step 1 requires you to key in the program. Switch the HP-19C/HP-29C to PRGM mode, depress f CLEAR PRGM and key in the program steps as listed. The choice of program LABEL 1 is arbitrary and could be changed to fit the user's needs by making corresponding changes in the User's Instructions (and possibly other modifications in the program listing.) Note that some steps on the program listing require keystrokes not explicitly listed for entry in the program, e.g. LBL 1 is keyed in by three keystrokes g LBL 1. (See the Owner's Handbook for a more detailed explanation of keying in programs.)

Step 2 of the User's Instructions asks for the coefficients of the quadratic equation. Switch the calculator to RUN mode. Coefficient a is keyed in and followed by ENTER↑, coefficient b is keyed in and followed by ENTER↑, and coefficient c is keyed in, followed by GSB 1. D is immediately calculated and displayed and program execution stops. Upon depressing R/S the calculator resumes program execution, automatically determining if D is positive or negative and displaying a root of the equation. Depressing R/S again displays the other root.

Display of intermediate or sequential results can be accomplished in several ways; a pause may be used to display a result for approximately 1 second before resumption of program execution, or a R/S command may be used to stop execution and display the result. Execution of the program then resumes after depressing the R/S key. (In these programs we have usually resorted to R/S commands to eliminate the chance of missing important results during the brief pause.) Additionally, with the printer, PRINT X commands may be used to print intermediate results.

If you own the HP-19C with printer you should note that the program listings are written to provide display outputs only and do not include PRINT commands. You will want to take advantage of the printer in recording both intermediate and final results. This can best be done by substituting a PRINT X command for the R/S commands when recording sequential or intermediate results and inserting a PRINT X command at the point in the program where the final result is displayed (usually just prior to a RTN command). Use of the printer in this manner has the advantage of eliminating halts in the program due to R/S commands.

Many of the program comments show points (designated by ***) at which the PRINT X command may be inserted or substituted, if desired. If the length of the program prevents insertion of the printer commands at the various steps the results still may be recorded by manually operating PRx as needed.

For example, in the Quadratic Equation program the R/S instruction at step 15 could have been replaced with a PAUSE instruction if only momentary display of D was desired, or, on the HP-19C, a PRINT X command could be substituted for the R/S command at 15 and inserted after step 34 to provide a printout of the results.

QUADRATIC EQUATION

This program calculates the two roots of a quadratic equation. If the roots are real they are displayed consecutively. If complex, the real part is displayed first, followed by the imaginary part.

Equations:

The roots x_1, x_2 of $ax^2 + bx + c = 0$

are given by
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $D = (b^2 - 4ac)/4a^2$ is positive or zero, the roots are real. In these cases, better accuracy may sometimes be obtained by first calculating the root with the larger absolute value:

If $-\dfrac{b}{2a} \geqslant 0, \quad x_1 = -\dfrac{b}{2a} + \sqrt{D}$

If $-\dfrac{b}{2a} < 0, \quad x_1 = -\dfrac{b}{2a} - \sqrt{D}$

In either case,
$$x_2 = \frac{c}{x_1 a}$$

If $D < 0$, the roots are complex, being

$$u \pm iv = \frac{-b}{2a} \pm \frac{\sqrt{4ac - b^2}}{2a}\, i$$

Remarks:

- The user merely inputs the coefficients in proper order; first a, then b, then c, being careful to observe signs for negative coefficients. The first result displayed is D. If it is positive the roots are real, if negative, they are complex.
- In the case of real roots the program tests for and calculates the larger root first for best accuracy, then displays the roots in reverse order.

Program	Comments		
01 *LBL1			
02 STO0			
03 R↓			
04 X⇄Y			
05 ST÷0	c/a		
06 ÷			
07 2			
08 ÷			
09 CHS	b/2a		
10 STO1			
11 ENT↑			
12 X²			
13 RCL0			
14 -			
15 R/S	*** D		
16 X<0?			
17 GTO0			
18 √X			
19 ST-1			
20 X⇄Y	$-b/2a - \sqrt{D}$		
21 +			
22 RCL1	$-b/2a + \sqrt{D}$		
23 LSTX			
24 X>0?			
25 R↓			
26 R↓	Select		
27 ST÷0			
28 RCL0			
29 GTO3	x_2		
30 *LBL0			
31 ABS			
32 √X			
33 X⇄Y	v		
34 *LBL3			
35 R/S	*** Display		
36 X⇄Y			
37 GTO3			

REGISTERS

0 c/a, x_2	1 x_1	2	3	4	5
6	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Enter coefficients of quadratic			
	x^2 coefficient	a	ENTER↑	
	x coefficient	b	ENTER↑	
	constant	c	GSB 1	D
3	If D ≥ 0, roots are real		R/S	x_2
			R/S	x_1
4	If D < 0, roots are complex of		R/S	u (real part)
	form u ± iv		R/S	v
				(imaginary part)

Example 1:

Find the roots of $x^2 + x - 6 = 0$

Keystrokes:	Outputs:	
1 ENTER↑ 1 ENTER↑		
6 CHS GSB 1 ⟶	6.25	(D)
R/S ⟶	2.00	(x_2)
R/S ⟶	-3.00	(x_1)

Example 2:

Solve the quadratic equation $2x^2 - 3x + 5 = 0$

Keystrokes:	Outputs:	
2 ENTER↑ 3 CHS ENTER↑		
5 GSB 1 ⟶	-1.94	(D)
R/S ⟶	0.75	(u)
R/S ⟶	1.39	(v)

Since D is negative the roots are imaginary and the solutions are of the form $x_{1,2} = 0.75 \pm 1.39i$

Example 3:

A ball is thrown straight up at a velocity of 20 meters per second from a height of 2 meters. At what time, neglecting air resistance, will it reach the ground? The acceleration of gravity is 9.81 meters/second^2. From physics:

$$f(t) = x = \frac{1}{2}gt^2 + V_0t + x_0 = 0 \quad \text{or} \quad \left(-\frac{9.81}{2}\right)t^2 + 20t + 2 = 0$$

Keystrokes:	Outputs:	
9.81 [CHS] [ENTER↑] 2 [÷] 20 [ENTER↑]		
2 [GSB] [1] ——→	4.56	(D)
[R/S] ——→	-0.10	(seconds)
[R/S] ——→	4.18	(*seconds)

*The answer is 4.18 seconds. The root − 0.10 seconds is a legitimate root of the equation but is not relevant to the problem.

BASE CONVERSIONS

This program converts positive numbers to and from base 10 representations. The other base involved may be any integer from 2 to 99, inclusive.

Let x_b be the representation of the number in the original base b. Assume that it is to be converted to the representation x_B in base B. Either b or B must be 10. In general, the bases are stored manually (b in R_1, B in R_2) prior to keying in x_b and pressing GSB 1, which will cause the computation of x_B.

When converting numbers from base 10, b = 10. However, the number stored for b may be either 10 or 100. If the other base B < 10, then store b in R_1 as 10. If, however, B > 10, the value stored for b in R_1 should be 100.

Similarly, when numbers are converted to base 10 representations, B = 10. When b < 10, the value of B stored in R_2 should be 10; when b > 10, a value of 100 should be stored in R_2.

The table below shows examples of the four possible cases:

To convert	**From Base**	**To Base**	**Store in R_1**	**Store in R_2**
	10	2	10	2
	10	16	100	16
	2	10	2	10
	16	10	16	100

A number such as $4B6_{16}$ cannot be represented directly on the display because the display is strictly numeric. Therefore, some convention must be adopted to represent numbers R_a when a > 10. We use the convention of allocating two digit locations for each single character in R_a when a > 10.

For example, $4B6_{16}$ is represented as 041106_{16} by our convention (in hexadecimal system, A = 10, B = 11, C = 12, D = 13, E = 14, F = 15).

When displayed, this number may appear as 41106 or with an exponent

4.1106   04

which is interpreted as $4.B6 \times 16^2$.

The displayed exponent 4 is for base 10 and only serves to locate the decimal point (in the same manner as for decimal numbers).

When base a > 10 (as in the above example), divide the displayed exponent by 2 to get the true exponent of the number. When the displayed exponent is an odd integer, shift the decimal point of the displayed number one place (to the left or right) and adjust its exponent accordingly to make the true exponent an integer.

For example, the displayed number

$$1.112 \quad -03$$

is interpreted as $B.C \times 16^{-2}$ or $0.BC \times 16^{-1}$.

Remarks:

- When the magnitude of the number is very large or very small, this program will take a long time to execute.

- The program will not give error indication for invalid inputs for x_b. For example, 981_8 will be treated the same as 1201_8.

- As the program now stands, the user is forced to make a decision at input time whether the number stored for base 10 is 10 or 100. An alternative approach would be to always store 10, never 100, and have the program decide whether to overwrite the 10 with 100 in some cases. Such an alteration of the program would require about 25 more program steps.

01 *LBL1	$x_b \to x_P$	50 GTO8	
02 STO3		51 *LBL6	
03 RCL1		52 RCL5	
04 STO5		53 RCL0	
05 RCL2		54 Y^X	
06 STO6		55 RCL4	
07 0		56 ×	
08 STO0		57 STO4	*** x_B
09 STO4		58 RTN	
10 EEX		59 *LBL7	
11 1		60 EEX	
12 2		61 4	
13 STO8		62 +	Eliminate round-off error
14 RCL3	Shift right until < 1. R_0 keeps track of no. places shifted (exponent) ...	63 EEX	
15 *LBL9		64 4	
16 1		65 -	
17 X>Y?		66 INT	
18 GTO8		67 RTN	
19 ST+0			
20 CLX			
21 RCL6			
22 ÷			
23 STO3			
24 GTO9	On entry, R_3 contains normalized x_b: $0 < x_b < 1$.		
25 *LBL8			
26 RCL6			
27 RCL3			
28 ×			
29 STO3			
30 GSB7			
31 RCL4			
32 RCL5	Build up x_B.		
33 ×			
34 +			
35 STO4			
36 RCL3			
37 GSB7			
38 RCL3			
39 -			
40 ABS	Do not build mantissa beyond 10^{12}		
41 STO3			
42 1			
43 ST-0			
44 RCL4			
45 RCL8			
46 X≤Y?			
47 GTO6			
48 RCL3			
49 X≠0?			

REGISTERS

0 Used	1 b	2 B	3 x_b	4 Used	5 b
6 B	7	8 10^{12}	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Store bases (one must be 10			
	or 100):			
	• Base to be converted from	b	STO 1	
	• Base to be converted to	B	STO 2	
3	Key in number in base b and			
	calculate number in base B.	x_b	GSB 1	x_B
4	For a new conversion between			
	the same bases, go to step 3;			
	to change either base, go to			
	step 2.			

Example 1:
Convert 0.2937_{10} to base 8 representation. (Since B = 8 < 10, b = 10.)

Keystrokes: **Outputs:**

10 STO 1 8 STO 2
f FIX 9 .2937 GSB 1 ⟶ 0.226277543 (Base 8)

Example 2:
Convert $1.23_{10} \times 10^{-12}$ to base 16. (Since B = 16 > 10, b = 100.)

Keystrokes: **Outputs:**

100 STO 1 16 STO 2
1.23 EEX CHS 12 GSB 1 ⟶ 1.0510030 -20 (Base 16)

This is interpreted as $1.5A3_{16} \times 16^{-10}$.

Example 3:
Convert $7.200067_8 \times 8^{-10}$ to base 10. (Since b = 8 < 10, B = 10.)

Keystrokes: **Outputs:**

8 STO 1 10 STO 2
f SCI 9 7.200067
EEX CHS 10 GSB 1 ⟶ 6.7522840 -09 (Base 10)

Example 4:
Convert $D.2EE4_{16} \times 16^{12}$ to base 10. (Since b = 16 > 10, B = 100.)

Keystrokes: **Outputs:**

16 STO 1 100 STO 2
13.02141404 EEX 24 GSB 1 ⟶ 3.7107314 15 (Base 10)

VECTOR OPERATIONS

This program calculates the basic vector operations of addition, dot (scalar) product, and cross product for three dimensional vectors. It also calculates the angle between two vectors. The program is capable of doing chain calculations whenever the product is a vector (refer to examples).

Equations:

Define a vector $\vec{V}$ in 3 dimensional rectangular coordinate system,

$$\vec{V} = x\vec{i} + y\vec{j} + z\vec{k}$$

then:

Vector addition:

$$\vec{V}_1 + \vec{V}_2 = (x_1 + x_2)\vec{i} + (y_1 + y_2)\vec{j} + (z_1 + z_2)\vec{k}$$

Dot or scalar product:

$$\vec{V}_1 \cdot \vec{V}_2 = x_1 x_2 + y_1 y_2 + z_1 z_2$$

Cross product:

$$\vec{V}_1 \times \vec{V}_2 = (y_1z_2 - z_1y_2)\vec{i} + (z_1x_2 - x_1z_2)\vec{j} + (x_1y_2 - y_1x_2)\vec{k}$$

Angle between vectors:

$$\gamma = \cos^{-1}\frac{\vec{V}_1 \cdot \vec{V}_2}{|\vec{V}_1|\,|\vec{V}_2|}$$

Remarks:

- For two dimensional vectors, simply consider that the k component does not exist, i.e. input 0 for z's.

Step	Key	Comments	Step	Key	Comments
01	*LBL0		50	x	
02	RCL6		51	RCL3	
03	STO3		52	RCL5	
04	R↓		53	GSB8	
05	STO6		54	STO8	
06	R↓		55	R/S	
07	RCL5		56	RCL3	
08	STO2	Input $\vec{V}_1$	57	RCL4	
09	R↓		58	x	
10	STO5		59	RCL1	
11	R↓		60	RCL6	
12	RCL4		61	GSB8	
13	STO1		62	STO9	
14	R↓		63	R/S	
15	STO4		64	RCL1	$\vec{V}_1 \times \vec{V}_2$
16	RTN		65	RCL5	
17	*LBL1		66	x	
18	GSB0		67	RCL2	
19	RCL1		68	RCL4	
20	ST+4		69	*LBL8	
21	RCL4		70	x	
22	R/S		71	-	
23	RCL2	$\vec{V}_1 + \vec{V}_2$	72	STO0	
24	ST+5		73	RCL8	
25	RCL5		74	RCL9	
26	R/S		75	RCL0	
27	RCL3		76	RTN	
28	ST+6		77	*LBL4	
29	RCL6		78	GSB2	
30	RTN		79	0	
31	*LBL2		80	ST.2	
32	GSB0		81	ST.4	
33	RCL1		82	RCL1	
34	RCL4		83	RCL4	
35	x	$\vec{V}_1 \bullet \vec{V}_2$	84	Σ+	
36	RCL2		85	RCL2	
37	RCL5		86	RCL5	
38	x		87	Σ+	
39	+		88	RCL3	
40	RCL3		89	RCL6	
41	RCL6		90	Σ+	γ
42	x		91	RC.2	
43	+		92	RC.4	
44	STO7		93	x	
45	RTN		94	√X	
46	*LBL3		95	ST÷7	
47	GSB0		96	RCL7	
48	RCL2		97	COS⁻¹	
49	RCL6		98	RTN	

REGISTERS

0 $\vec{V}_1 \times \vec{V}_2\ \vec{k}$	1 x_1	2 y_1	3 z_1	4 x_2	5 y_2
6 z_2	7 $\vec{V}_1 \bullet \vec{V}_2$	8 $\vec{V}_1 \times \vec{V}_2\ i$	9 $\vec{V}_1 \times \vec{V}_2\ \vec{j}$	.0 Used	.1 Used
.2 Used	.3 Used	.4 Used	.5 Used	16	17
18	19	20	21	22	23
24	25	26	27	28	29

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input the first vector $\vec{V}_1$	x_1	ENTER↑	x_1
		y_1	ENTER↑	y_1
		z_1	GSB 0	x_1
3	For vector addition, go to step 4			
	For vector dot product, go to			
	step 6.			
	For vector cross product, go			
	to step 8.			
	For the angle between two			
	vectors, go to step 10.			
4	**Vector Addition:**			
	Input the 2[nd] vector $\vec{V}_2$ and			
	calculate $\vec{V}_1 + \vec{V}_2$	x_2	ENTER↑	x_2
		y_2	ENTER↑	y_2
		z_2	GSB 1	$\vec{i}$
			R/S	$\vec{j}$
			R/S	$\vec{k}$
5	For a new case, go to step 2.			
6	**Vector Dot Product:**			
	Input the 2[nd] vector $\vec{V}_2$ and			
	calculate $\vec{V}_1 \cdot \vec{V}_2$	x_2	ENTER↑	x_2
		y_2	ENTER↑	y_2
		z_2	GSB 2	$\vec{V}_1 \cdot \vec{V}_2$
7	For a new case, go to step 2.			
8	**Vector Cross Product:**			
	Input the 2[nd] vector and			
	calculate $\vec{V}_1 \times \vec{V}_2$	x_2	ENTER↑	x_2
		y_2	ENTER↑	y_2
		z_2	GSB 3	$\vec{i}$
			R/S	$\vec{j}$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
			R/S	$\vec{k}$
9	For a new case go to step 2.			
10	**Angle Between Two Vectors:**			
	Input the 2nd vector and			
	calculate γ	x_2	ENTER↑	x_2
		y_2	ENTER↑	y_2
		z_2	GSB 4	γ
11	For a new case go to step 2.			

Example 1: $\vec{V}_1 = (2, 5, 2)$, $\vec{V}_2 = (3, 3, -4)$

Addition: $\vec{V}_1 + \vec{V}_2 = (5, 8, -2)$

Keystrokes:	**Outputs:**	
2 ENTER↑ 5 ENTER↑		
2 GSB 0 ⟶	2.00	
3 ENTER↑ 3 ENTER↑		
4 CHS GSB 1 ⟶	5.00	($\vec{i}$)
R/S ⟶	8.00	($\vec{j}$)
R/S ⟶	-2.00	($\vec{k}$)

Dot product: $\vec{V}_1 \cdot \vec{V}_2 = 13.00$

Keystrokes:	**Outputs:**	
2 ENTER↑ 5 ENTER↑		
2 GSB 0 ⟶	2.00	
3 ENTER↑ 3 ENTER↑		
4 CHS GSB 2 ⟶	13.00	($\vec{V}_1 \cdot \vec{V}_2$)

Cross product: $\vec{V}_1 \times \vec{V}_2 = (-26, 14, -9)$

Keystrokes:	**Outputs:**	
2 ENTER↑ 5 ENTER↑		
2 GSB 0 ⟶	2.00	
3 ENTER↑ 3 ENTER↑		
4 CHS GSB 3 ⟶	-26.00	($\vec{i}$)
R/S ⟶	14.00	($\vec{j}$)
R/S ⟶	-9.00	($\vec{k}$)

Angle:

Keystrokes:	**Outputs:**	
2 ENTER↑ 5 ENTER↑		
2 GSB 0 ⟶	2.00	
3 ENTER↑ 3 ENTER↑		
4 CHS GSB 4 ⟶	67.16°	(γ)

Example 2:

Calculate $(\vec{V}_1 + \vec{V}_2) \cdot \vec{V}_3$ for $\vec{V}_1 = (1.10, 3.00, 4.40)$
$\vec{V}_2 = (1.24, 2.17, 3.03)$, and $\vec{V}_3 = (0.072, 0.231, 0.409)$

Keystrokes:	**Outputs:**	
1.10 ENTER↑ 3 ENTER↑		
4.40 GSB 0 ⟶	1.10	
1.24 ENTER↑ 2.17 ENTER↑		
3.03 GSB 1 ⟶	2.34	$(\vec{V}_1 + \vec{V}_2)$
R/S ⟶	5.17	
R/S ⟶	7.43	
0.072 ENTER↑ 0.231 ENTER↑		
0.409 GSB 2 ⟶	4.40	$((\vec{V}_1 + \vec{V}_2) \cdot \vec{V}_3)$

COMPLEX OPERATIONS

This program allows for chained calculations involving complex variables. The four operations of complex arithmetic (+, −, ×, ÷) are provided, as well as several of the most used functions of a complex variable z($|z|$, z^n, and $z^{1/n}$). Functions and operations may be mixed in the course of a calculation to allow evaluation of expressions like $z_3/(z_1 + z_2)$, $|z_1 + z_2|$, ..., etc., where z_1, z_2 and z_3 are complex numbers of the form x + iy.

Arithmetic Operations

An arithmetic operation needs two numbers to operate on. Both numbers must be input before the operation can be performed. Suppose that $z_1 = 2 + 3i$, $z_2 = 5 - i$, and we wish to find $z_1 - z_2$. This can be calculated by the keystrokes:

2 ENTER↑ 3 GSB 0 5 ENTER↑ 1 CHS GSB 2

The result $z_3 = u + iv$ is found to be $-3 + 4i$. This result is now stored by the program in place of the second complex number z_2. A further calculation $z_3 \times z_4$ could be performed by inputting z_4 and depressing GSB 3 for multiplication. This type of chaining can be continued indefinitely, and functions can be interspersed with arithmetic operations.

Equations:

$$\text{Let } z_j = x_j + iy_j = r_j e^{i\theta_j},\ j = 1, 2$$

$$z = x + iy = r^{i\theta}$$

Where $$r = \sqrt{x^2 + y^2}$$

Let the result in each case be u + iv

$$z_1 + z_2 = u + iv = (x_1 + x_2) + i\,(y_1 + y_2)$$

$$z_1 - z_2 = u + iv = (x_1 - x_2) + i\,(y_1 - y_2)$$

$$z_1 \cdot z_2 = r_1 \cdot r_2 \cdot e^{i(\theta_1 + \theta_2)} = u + iv$$

$$z_1/z_2 = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)} = u + iv$$

$$|z| = r = \sqrt{x^2 + y^2}$$

$$z^n = r^n e^{in\theta} \qquad n = \pm(1, 2, 3, \ldots)$$

$$z^{1/n} = r^{1/n} e^{i\left(\frac{\theta}{n} + \frac{360k}{n}\right)},\ k = 0, 1, \ldots, n-1$$

Step	Key	Comment
01	*LBL0	
02	RCL4	
03	STO2	
04	R↓	
05	STO4	
06	R↓	
07	RCL3	
08	STO1	Input z_1
09	R↓	
10	STO3	
11	0	
12	STO0	
13	RTN	
14	*LBL2	
15	CHS	
16	X⇄Y	$z_1 - z_2$
17	CHS	
18	X⇄Y	
19	*LBL1	
20	GSB0	
21	RCL1	
22	ST+3	$z_1 + z_2$
23	RCL2	
24	ST+4	
25	RCL3	
26	R/S	
27	RCL4	
28	R/S	
29	*LBL3	
30	GSB0	
31	GSB9	
32	ST×5	
33	X⇄Y	$z_1 \times z_2$
34	ST+6	
35	*LBL8	
36	RCL6	
37	RCL5	
38	→R	
39	STO3	
40	R/S	
41	X⇄Y	
42	STO4	
43	RTN	
44	*LBL4	
45	GSB0	
46	GSB9	
47	ST÷5	
48	X⇄Y	z_1/z_2
49	ST-6	

Step	Key	Comment
50	GTO8	
51	*LBL9	
52	RCL2	
53	RCL1	
54	→P	
55	STO5	
56	X⇄Y	
57	STO6	
58	*LBL5	
59	RCL4	
60	RCL3	$\|z_1\|$
61	→P	
62	RTN	
63	*LBL6	
64	STO7	
65	GSB5	z^n
66	RCL7	
67	Y^X	
68	STO5	
69	X⇄Y	
70	RCL7	
71	×	
72	STO6	
73	GTO8	
74	*LBL7	
75	STO7	
76	GSB5	
77	RCL7	
78	1/X	
79	Y^X	
80	X⇄Y	
81	RCL7	
82	÷	
83	3	
84	6	$z^{1/n}$
85	0	
86	RCL0	
87	×	
88	RCL7	
89	÷	
90	+	
91	X⇄Y	
92	→R	
93	R/S	
94	X⇄Y	
95	R/S	
96	ISZ	
97	RCL7	
98	GTO7	

REGISTERS

0 K	1 x_1	2 y_1	3 x_2, Last x	4 y_2, Last y	5 x, ÷, r^n
6 +, −, $n\theta$	7 n	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Key in the first complex number.			
	$z_1 = x_1 + iy_1$	x_1	ENTER↑	
		y_1	GSB 0	0
3	For a function, go to step 7, for			
	arithmetic, go to step 4. A com-			
	plex result is u + iv			
4	**Arithmetic**			
	Key in the second complex			
	number $z_2 = x_2 + iy_2$	x_2	ENTER↑	
		y_2		
5	Select one of the four:			
	• Add (+)		GSB 1	u
			R/S	v
	• Subtract (−)		GSB 2	u
			R/S	v
	• Multiply (×)		GSB 3	u
			R/S	v
	• Divide (÷)		GSB 4	u
			R/S	v
6	The result of the operation has			
	been stored, go to step 7 for a			
	function or to step 4 for further			
	arithmetic.			
7	**Functions**			
	Select one of the 3 functions:			
	• Magnitude ($\|z_1\|$)		GSB 5	$\|z\|$
	• Raise z to integer power (z_1^n)	n	GSB 6	u
			R/S	v
	• Find the roots of ($z^{1/n}$)	n	GSB 7	u
			R/S	v

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
			[R/S]	u_2
			[R/S]	v_2
			⋮	⋮
			[R/S]	u_n
			[R/S]	v_n
8	The result, if complex, has been calculated; go to step 4 for arithmetic or to step 7 for another function.			

Examples:

Keystrokes:	**Outputs:**
1. (3+4i) + (7.4 − 5.6i) = 10.40 − 1.60i	
3 [ENTER↑] 4 [GSB] [0] 7.4 [ENTER↑] 5.6 [CHS] [GSB] [1] ⟶	10.40
[R/S] ⟶	-1.60
2. (3+4i) − (7.4 − 5.6i) = −4.40 + 9.60i	
3 [ENTER↑] 4 [GSB] [0] 7.4 [ENTER↑] 5.6 [CHS] [GSB] [2] ⟶	-4.40
[R/S] ⟶	9.60
3. (3.1+4.6i)(5−12i) = 70.70 −14.20i	
3.1 [ENTER↑] 4.6 [GSB] [0] 5 [ENTER↑] 12 [CHS] [GSB] [3] ⟶	70.70
[R/S] ⟶	-14.20
4. $\frac{3+4i}{7-2i}$ = 0.25 + 0.64i	
3 [ENTER↑] 4 [GSB] [0] 7 [ENTER↑] 2 [CHS] [GSB] [4] ⟶	0.25
[R/S] ⟶	0.64
5. $\frac{1}{2+3i}$ = 0.15 − 0.23i	
1 [ENTER↑] 0 [GSB] [0] 2 [ENTER↑] 3 [GSB] [4] ⟶	0.15
[R/S] ⟶	-0.23
OR:	
2 [ENTER↑] 3 [GSB] [0] 1 [CHS] [GSB] [6] ⟶	0.15
[R/S] ⟶	-0.23

6. $(7-2i)^2 = 45.00 - 28.00i$

7 ENTER↑ 2 CHS GSB 0 2 GSB 6 →	45.00
R/S →	-28.00

7. $\sqrt{7+6i} = \pm (2.85 + 1.05i)$

7 ENTER↑ 6 GSB 0 2 GSB 7 →	2.85
R/S →	1.05
R/S →	-2.85
R/S →	-1.05

8. $\dfrac{23+13i}{(-2+i)+(4-3i)} = 2.50 + 9.00i$

2 CHS ENTER↑ 1 GSB 0 4 ENTER↑ 3 CHS GSB 1 →	2.00
R/S →	-2.00
1 CHS GSB 6 →	0.25
R/S →	0.25
23 ENTER↑ 13 GSB 3 →	2.50
R/S →	9.00

SYSTEM OF LINEAR EQUATIONS WITH 3 UNKNOWNS

This program uses Cramer's rule to solve systems of linear equations with three unknowns.

Equations:

A system of linear equations can be expressed as

$$A\bar{x} = \bar{b}$$

For 3 Unknowns, $$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \bar{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Determinant of the system

$$\text{Det} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$b_i\text{'s are solved by } b_i = \frac{\text{det (i)}}{\text{Det}}$$

Where det (i) is the determinant of the A matrix with the i^{th} column replaced by $\bar{b}$.

Remarks:

If "Error" occurs while running the program, then possibly the determinant is zero. i.e. the system is linearly dependent and this program is not applicable.

Step	Key	Comments
01	*LBL1	
02	0	
03	STO0	
04	RCL6	
05	RCL8	Input a_{ij}'s and calculate Det
06	GSB9	
07	RCL4	
08	RCL9	
09	GSB9	
10	RCL5	
11	RCL7	
12	GSB9	
13	CHS	
14	RCL3	
15	RCL8	
16	GSB9	
17	RCL1	
18	RCL9	
19	GSB9	
20	RCL2	
21	RCL7	
22	*LBL9	
23	DSZ	
24	RCLi	------------
25	×	
26	×	
27	+	
28	ST.0	Input b_1, b_2, b_3 and calculate x_1, x_2, and x_3
29	RTN	
30	*LBL2	
31	ST.3	
32	R↓	
33	ST.2	
34	R↓	
35	ST.1	
36	GSB1	
37	RC.0	
38	ST.5	
39	1	
40	GSB7	
41	R/S	***
42	2	------------
43	GSB7	
44	R/S	***
45	3	
46	GSB7	
47	R/S	***
48	*LBL7	Subroutine to calculate det (i)
49	ST.4	

Step	Key	Comments
50	STO0	
51	GSB8	
52	GSB1	
53	RC.5	
54	÷	
55	ST.0	------------
56	RC.4	
57	STO0	
58	GSB8	Swap register contents
59	RC.0	
60	RTN	
61	*LBL8	
62	RCLi	
63	RC.1	
64	STOi	
65	X⇄Y	
66	ST.1	
67	RCL0	
68	3	
69	+	
70	STO0	
71	RCLi	
72	RC.2	
73	STOi	
74	X⇄Y	
75	ST.2	
76	RCL0	
77	3	
78	+	
79	STO0	
80	RCLi	
81	RC.3	
82	STOi	
83	X⇄Y	
84	ST.3	
85	RTN	

REGISTERS

0 Index	1 a_{11}	2 a_{12}	3 a_{13}	4 a_{21}	5 a_{22}
6 a_{23}	7 a_{31}	8 a_{32}	9 a_{33}	.0 Det	.1 b_1
.2 b_2	.3 b_3	.4 Index	.5 Det	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Store elements of A matrix	a_{11}	STO 1	a_{11}
		a_{12}	STO 2	a_{12}
		a_{13}	STO 3	a_{13}
		a_{21}	STO 4	a_{21}
		a_{22}	STO 5	a_{22}
		a_{23}	STO 6	a_{23}
		a_{31}	STO 7	a_{31}
		a_{32}	STO 8	a_{32}
		a_{33}	STO 9	a_{33}
3	(Optional) to calculate			
	determinant		GSB 1	Det
4	Input $\bar{b}$ to calculate $\bar{x}$	b_1	ENTER↑	b_1
		b_2	ENTER↑	b_2
		b_3	GSB 2	x_1
			R/S	x_2
			R/S	x_3
5	For a new b with the same			
	system, go to step 4.			
6	For a new system, go to step 2.			

Example:

Find x_1, x_2, and x_3 for the following system.

$$\begin{bmatrix} 19 & -4 & -15 \\ -4 & 22 & -10 \\ -15 & -10 & 26 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 40 \\ 0 \\ 0 \end{bmatrix}$$

Keystrokes:	**Outputs:**	
19 STO 1, 4 CHS STO 2,		
15 CHS STO 3, 4 CHS STO 4,		
22 STO 5, 10 CHS STO 6,		
15 CHS STO 7, 10 CHS STO 8,		
26 STO 9		
GSB 1 ⟶	2402.00	(Det)
40 ENTER↑ 0 ENTER↑		
0 GSB 2 ⟶	7.86	(x_1)
R/S ⟶	4.23	(x_2)
R/S ⟶	6.16	(x_3)

ANNUITIES AND COMPOUND AMOUNTS

These programs (1st part and 2nd part) can be used to solve a variety of problems involving money, time and interest. The following variables can be inputs or outputs:

n, which is the number of compounding periods. (For a 30 year loan with monthly payments, n = 12 × 30 = 360.)

i, which is the periodic interest rate expressed as a percent. (For other than annual compounding, divide the annual percentage rate by the number of compounding periods in a year, i.e. 8% annual interest compounded monthly equals 8/12 or 0.667%.)

PMT, which is the periodic payment.

PV, which is the present value of the cash flows or compound amounts.

FV, which is the future value of a compounded amount or a series of cash flows.

BAL, which is the balloon or remaining balance at the end of a series of payments.

Accumulated interest and remaining balance may also be computed with this program.

The program accommodates payments which are made at the end of compounding periods or at the beginning. Payments made at the end of compounding periods (ordinary annuity) are common in direct reduction loans and mortgages while payments at the beginning of compounding periods (annuity due) are common in leasing.

This program uses the convention that cash outlays are input as negative, and cash incomes are input as positive.

1st part: When i is known

The initialization (GSB 0) performs two functions:

1. It sets PMT, PV, and BAL to zero (n and i are not affected).
2. It toggles for the ordinary annuity mode (display = 1), and annuity due mode (display = 0).

Pressing GSB 0 provides a safe, convenient, easy to remember method of preparing the calculator for a new problem. It is not necessary to use GSB 0 between problems containing the same combination of variables. For instance, any number of n, i, PMT, FV problems involving different numbers and/or different combinations of knowns could be done in succession without re-initializing. Only the values which change from problem to problem would have to be keyed in. To change the combination of variables without using

GSB [0], simply input zero for any variable which is no longer applicable. To go from n, i, PMT, PV problems to n, i, PV, FV problems, a zero would be stored (0 STO [3]) in place of PMT. Table I summarizes these procedures.

2nd part: Solving for i
Newton's method is applied to solve problems with unknown i. (Refer to page 81: Newton's Method-Solution to f(x) = 0).

Table I
Possible Solutions Using *Annuities and Compound Amounts*

Allowable Combination of Variables	Applications		Initial Procedure
	Ordinary Annuity	Annuity Due	
n, i, PMT, PV(Input any three and calculate the fourth.)	Direct reduction loan Discounted notes Mortgages	Leases	Use GSB [0] or set BAL to zero
n, i, PMT, PV, BAL. (Input any four and calculate the fifth.)	Direct reduction loan with balloon Discounted notes with balloon	Leases with residual values	None
n, i, PMT, FV (Input any three and calculate the fourth.)	Sinking fund	Periodic savings insurance	Use GSB [0] or set PV to zero
n, i, PV, FV (Input any three and calculate the fourth.)	Compound amount Savings (Annuity mode is not applicable and has no effect)		Use GSB [0] or set PMT to zero.

Equations:

$$PV = \frac{PMT}{i} A\left[1-(1+i)^{-n}\right] + (BAL\ or\ FV)(1+i)^{-n}$$

where

$$A = \begin{cases} 1 & \text{ordinary annuity} \\ (1+i) & \text{annuity due.} \end{cases}$$

Remarks:

- The equation above is solved for i using Newton's method where:

$$i_n = i_{n-1} - \frac{f(i_{n-1})}{f'(i_{n-1})}$$

This is why solutions involving PMT and i take longer than other solutions. It is quite possible to define problems which cannot be solved by this technique. Such problems usually result in an error message but may simply continue to run indefinitely.

- Interest problems with balloon payment of opposite sign to the periodic payments may have more than one mathematically correct answer (or no answer at all). While this program may find one of the answers, it has no way of finding or indicating other possibilities.

1st Part: When i Is Known

```
*LBL0
CLX
STO3
STO4      Toggling
STO5
RC.2      1 for ordinary annuity
X=0?      0 for annuity due
GTO8
0
ST.2
RTN
*LBL8
1
ST.2
RTN
-------------------------
*LBL1
0
STO1
GSB9
RCL5
LSTX      Calculate n
-
RCL4
Pi
+
÷
CHS
LN
RCL7
LN
÷
STO1
-------------------------
RTN       ***
*LBL3
1
STO3
GSB9      Calculate PMT
1/X
RCL4
RC.3
+
x
CHS
STO3
-------------------------
RTN       ***
*LBL4     Calculate PV
1
STO4
GSB9
+
CHS
STO4
RTN       ***
-------------------------
*LBL5
GSB9
RCL4
+
RCL8      Calculate FV (BAL)
÷
CHS
STO5      ***
RTN
-------------------------
*LBL9
1
ST.1
RCL2
%
STO9      Calculate
+
STO7      PMT/i [1 - (1 + i)^-n] R1
RC.2
X=0?
X⇄Y
ST.1
RCL7
RCL1
CHS
Y^x
STO8
RCL5
x
ST.3
1
RCL8
-
ST.0
RCL3
RCL9
÷
STO0
RC.1
x
x
RTN
```

REGISTERS

0 PMT/i	1 n	2 i	3 PMT	4 PV	5 FV (BAL)
6 $n(1+i)^{n-1}$	7 $1+i$	8 $(1+i)^{-n}$	9 i/100	.0 $1-(1+i)^{-n}$	.1 1 or 1 + i
.2 annuity flag	.3 Used	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

2nd Part: Solving For i

Step	Comment	Step	Comment
01 *LBL1	Annuity	50 -	
02 1		51 ÷	Calculate next i
03 GTO3		52 CHS	
04 *LBL2		53 GSB9	
05 0	Annuity due	54 RCL2	
06 *LBL3		55 ÷	
07 ST.2		56 ABS	
08 0	Clear R_2 for sum of	57 RC.5	
09 STO2		58 X≤Y?	
10 RCL5	i terms	59 GTO8	Test increment to i for
11 RCL4		60 RCL2	limit
12 +	$\frac{nPMT + BAL + PV}{n}$	61 RTN	
13 RCL1		62 *LBL9	
14 ÷		63 EEX	
15 RCL3		64 2	
16 +		65 x	Calculate i to % and add
17 RCL4		66 ST+2	to i
18 ÷	guess for i	67 RTN	
19 CHS		68 *LBL0	
20 .		69 1	
21 9		70 ST.1	
22 CHS	If guess is less than –0.9,	71 RCL2	
23 X≤Y?	use –0.9 for guess.	72 %	
24 X⇄Y		73 STO9	
25 GSB9		74 +	1 to R_1 for ordinary
26 X=0?		75 STO7	annuity
27 RTN		76 RC.2	
28 *LBL8		77 X=0?	
29 GSB0	Calculate f(i)	78 X⇄Y	Calculate
30 +		79 ST.1	$\frac{PMT}{i}[1-(1+i)^{-n}] \times R_1$
31 RCL4		80 RCL7	
32 +		81 RCL1	
33 RCL8		82 CHS	
34 RCL1		83 Y^X	
35 RCL7	Calculate f′(i)	84 STO8	
36 ÷		85 RCL5	
37 x		86 x	
38 STO6		87 1	
39 RC.0		88 RCL8	
40 RCL9		89 -	
41 ÷		90 ST.0	
42 -		91 RCL3	
43 RC.1		92 RCL9	
44 x		93 ÷	
45 RCL0		94 STO0	
46 x		95 RC.1	
47 RCL6		96 x	
48 RCL5		97 x	
49 x	f(i)/f′(i)	98 RTN	

REGISTERS

0 PMT/i	1 n	2 i	3 PMT	4 PV	5 FV(BAL)
6 $n(1+i)^{-n-1}$	7 1 + i	8 $(1+i)^{-n}$	9 i/100	.0 $1-(1+i)^{-n}$	.1 1 or 1+i
.2 annuity flag	.3 used	.4	.5 10^{-6}	16	17
18	19	20	21	22	23
24	25	26	27	28	29

1st Part: When i Is Known

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Toggling for ordinary annuity			
	(1.00) and annuity due (0.00)		GSB 0	1.00/0.00
3	Input the known values (i must			
	be known):			
	Number of periods	n	STO 1	n
	Periodic interest rate	i (%)	STO 2	i (%)
	Periodic payment	PMT	STO 3	PMT
	Present value	PV	STO 4	PV
	Future value, balloon or			
	balance	FV (BAL)	STO 5	FV, (BAL)
4	Calculate the unknown value			
	Number of periods		GSB 1	n
	Periodic payment		GSB 3	PMT
	Present value		GSB 4	PV
	Future value, balloon or			
	balance		GSB 5	FV, (BAL)
6	For a new case, go to step 3			
	and change appropriate values.			
	Input zero for any value not			
	applicable in the new case.			

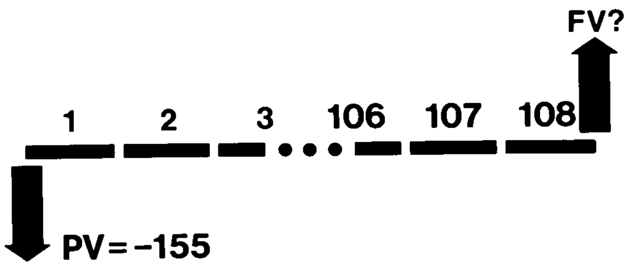
2nd Part: Solving For i

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Input the known values: (0 for			
	not existing values)			
	Number of periods	n	STO 1	n
	Periodic payment	PMT	STO 3	PMT
	Present value	PV	STO 4	PV

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Future value, balloon or			
	balance	FV, (BAL)	STO 5	FV, (BAL)
	and the tolerance for i (say			
	$\epsilon = 10^{-6}$)	ϵ	STO · 5	ϵ
3	Calculate interest rate.			
	For ordinary annuity		GSB 1	i (%)
	For annuity due		GSB 2	i (%)
4	For a new case, go to step 2.			

Example 1:

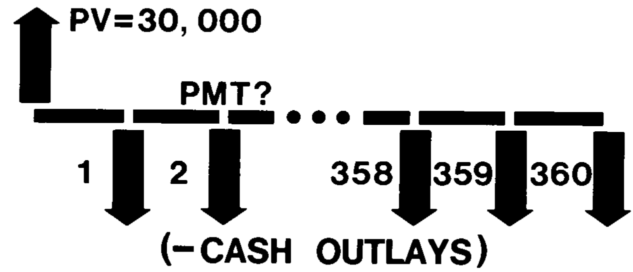
If you place $155 in a savings account paying 5¾% compounded monthly, what sum of money may you withdraw at the end of 9 years?



Keystrokes:	Outputs:	
(Key in the 1st program)		
GSB 0 ⟶	1.00	(ordinary annuity)
9 ENTER↑ 12 × STO 1 ⟶	108.00	(# of month compounding)
5.75 ENTER↑ 12 ÷ STO 2 ⟶	0.48	(% monthly interest rate)
155 CHS STO 4 ⟶	-155.00	(cash outlay)
GSB 5 ⟶	259.74	(FV)
If the interest is changed to 6% what is the sum?		
6 ENTER↑ 12 ÷ STO 2 ⟶	0.50	(% monthly interest rate)
GSB 5 ⟶	265.62	(FV)

Example 2:

You receive $30000 from the bank as a 30 year, 9% mortgage. What monthly payment must you make to the bank to fully amortize the mortgage?

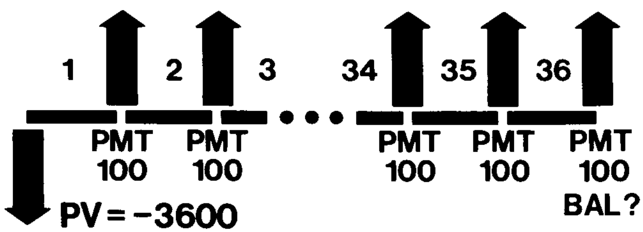


Keystrokes:	**Outputs:**	
(Key in the 1st program)		
GSB 0 ⟶	1.00	
30 ENTER↑ 12 × STO 1 ⟶	360.00	(# monthly payments)
9 ENTER↑ 12 ÷ STO 2 ⟶	0.75	(% monthly interest rate)
30000 STO 4 ⟶	30000.00	(PV)
GSB 3 ⟶	−241.39	(PMT)

Example 3:

Two individuals are constructing a loan with a balloon payment. The loan amount is $3,600 and it is agreed that the annual interest rate will be 10% with 36 monthly payments of $100. What balloon payment amount, to be paid coincident with the 36th payment, is required to fulfill the loan agreement?

(Note the cash flow diagram below is with respect to the loaner. For the loanee, the appropriate diagram will be exactly the opposite.)

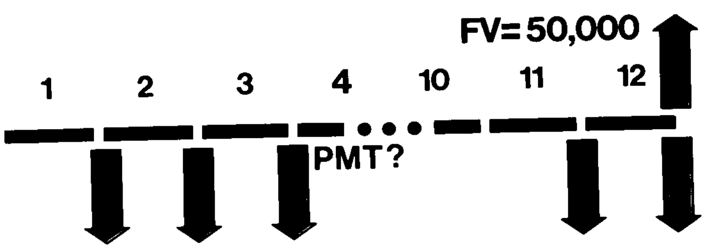


Keystrokes:	**Outputs:**
(Key in the 1st program)	
GSB 0 ——→	1.00
36 STO 1 10 ENTER↑ 12 ÷	
STO 2 100 STO 3 3600	
CHS STO 4 GSB 5 ——→	675.27

(Note that the final payment is \$675.27 + \$100.00 = \$775.27 since the final payment falls at the end of the last period.)

Example 4:

A corporation has determined that a certain piece of equipment costing \$50,000 will be required in 3 years. Assuming a fund paying 7% compounded quarterly is available, what quarterly payment must be made in order to withdraw this cost from the fund if savings are to start at the end of this quarter?



Keystrokes:	**Outputs:**
(Key in the 1st program)	
GSB 0 ——→	1.00
3 ENTER↑ 4 × STO 1 7 ENTER↑	
4 ÷ STO 2 50000 STO 5	
GSB 3 ——→	–3780.69

What single amount, invested immediately, would provide the same effect?

0 STO 3 GSB 4 ——→	–40602.89

Example 5:

This program may also be used to calculate accumulated interest/remaining balance for loans. The accumulated interest between two points in time, n_1 and n_2, is just the total payments made in that period less the principal reduction in that period. The principal reduction is the difference of the remaining balances for the two points in time. The following example demonstrates the concepts above.

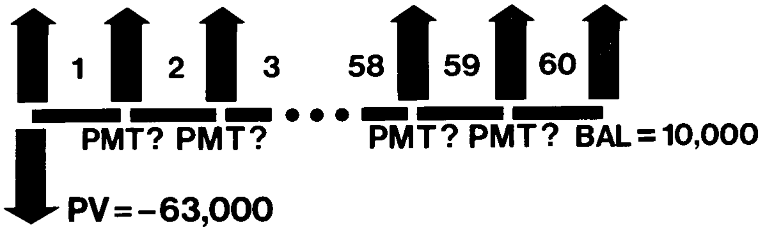
For a 360 month, $50,000 loan at 9½% annual interest, find the remaining balance after the 24th payment and the accrued interest for payments 13-24 (between the 12th and 24th payments!).

First we must calculate the payment on the loan:

Keystrokes:	**Outputs:**	
(Key in the 1st program)		
GSB 0 ⟶	1.00	
360 STO 1 9.5 ENTER↑ 12 ÷		
STO 2 50000 CHS STO 4		
GSB 3 ⟶	420.43	(payment)
The remaining balance is found:		
24 STO 1 GSB 5 ⟶	49352.76	(remaining balance at month 24)
Store this remaining balance and calculate the remaining balance at period 12:		
STO . 4 12 STO 1		
GSB 5 ⟶	49691.68	
The principal reduction between payments 12 and 24 is:		
RCL . 4 − ⟶	338.92	
The accrued interest is 12 payments less the principal reduction:		
RCL 3 12 × ⟶	5045.13	(total paid out)
x⇄y − ⟶	4706.20	(accrued interest)

Example 6:

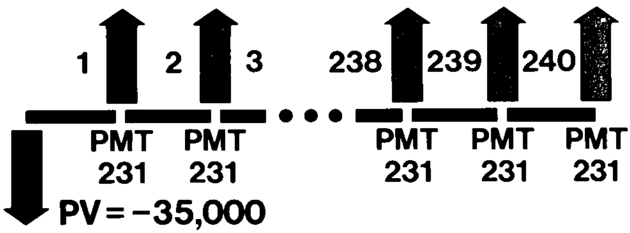
A "third" party leasing firm is considering the purchase of a mini-computer priced at $63,000 and intends to achieve a certain annual yield by leasing the computer to a customer for a 5-year period. Ownership is retained by the leasing firm and at the end of lease they expect to be able to sell the equipment for at least $10,000. If the monthly payment is $1300.16, what is the annual yield? (Since lease payments occur at the start of the periods, this is an annuity due problem).



Keystrokes:	Outputs:	
(Key in the 2nd program)		
5 ENTER↑ 12 × STO 1,		
1300.16 STO 3 63000 CHS		
STO 4 10000 STO 5 EEX		
CHS 6 STO · 5 GSB 2 →	1.08	(% per month)
12 × →	13.00	(% per year)

Example 7:

A fixed term annuity is available which requires a $35,000 initial deposit. In return the depositor will receive monthly payments of $231 for 20 years. What annual interest rate is being applied?



Keystrokes:	Outputs:	
(Key in the 2nd program)		
20 ENTER↑ 12 × STO 1 →	240.00	(# monthly payments)
231 STO 3 →	231.00	(monthly income)
35000 CHS STO 4 →	-35000.00	(initial cash deposit)
0 STO 5 →	0.00	(FV = 0)
EEX CHS 6 STO · 5 →	1. -06	(ε)
GSB 1 →	0.42	(0.42% monthly)
12 × →	5.00	(5% annual interest rate)

Example 8:

Suppose you deposit $100 today in the bank, after 3 years you will have a total of $116.08. If the interest is compounded quarterly, what is the interest rate?

Keystrokes:	**Outputs:**	
(Key in the 2nd program)		
3 ENTER↑ 4 × STO 1 0 STO 3		
100 CHS STO 4 116.08 STO 5		
EEX CHS 6 STO . 5		
GSB 1 ⟶	1.25	(% quarter)
4 × ⟶	5.00	(% annual)

DISCOUNTED CASH FLOW ANALYSIS
NET PRESENT VALUE

Assuming a minimum desired yield (cost of capital, discount rate), this program finds the present value of the future cash flows generated by the investment and subtracts the initial investment from this amount. If the final net present value is a positive value, the investment exceeds the profit objectives assumed. If the final net present value is a negative value, then the investment is not profitable to the extent of the desired yield. If the net present value is zero, the investment meets the profit objectives.

The function associated with the GSB [3] key (#) is designed to accommodate those situations where a series of the cash flows are equal. You enter the number of times these equal periodic cash flows occur with GSB [3], and then the amount only once with GSB [4]. The program automatically assumes 1 for #. If the cash flow occurs only once, there is no need to enter anything for #.

Zero must be entered for all periods with no cash flow. When a cash flow other than the initial investment is an outlay (additional investment, loss, etc.) the value must be entered as a negative number with CHS.

Cash flows are assumed to occur at the end of cash flow periods.

Equation:

$$NPV_k = -INV + \sum_{k=1}^{n} \frac{CF_k}{(1 + i)^k}$$

where:

n = number of cash flows

CF_k = k^{th} cash flow

NPV_k = net present value after k^{th} cash flow

Step	Comment	Step	Comment
01 *LBL1		50 R↓	***
02 CHS		51 RTN	
03 STO1		52 *LBL5	Recall Σn
04 0	-NPV → R_1	53 RCL9	
05 STO9	0 → R_9	54 RTN	
06 1	1 → R_3		
07 STO3			
08 RCL1			
09 CHS			
10 RTN			
11 *LBL2			
12 EEX			
13 2			
14 ÷	$\frac{i}{100}$ → R_2		
15 STO2			
16 LSTX			
17 x			
18 RTN			
19 *LBL3			
20 STO3	# → R_3		
21 RTN			
22 *LBL4			
23 STO4			
24 1			
25 RCL2			
26 +			
27 RCL3			
28 ST+9	Calculate present value of series		
29 Y^x			
30 STO5			
31 RCL1			
32 x			
33 RCL5			
34 1			
35 -			
36 RCL2			
37 ÷			
38 RCL4			
39 x			
40 +			
41 STO1			
42 1			
43 RCL2			
44 +			
45 RCL9			
46 Y^x			
47 ÷			
48 1	Reset n to 1		
49 STO3			

REGISTERS

0	1 NPV	2 i/100	3 #	4 CF	5 $(1 + i)^n$
6	7	8	9 Σn	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Input:			
	Initial investment amount	INV	GSB 1	INV
	Periodic interest (discount)			
	rate	i (%)	GSB 2	i (%)
3	Input the number of equal cash			
	flows if greater than 1.	#	GSB 3	#
4	Input cash flow amounts and			
	calculate net present value	CF	GSB 4	NPV
5	(Optional): Display total number			
	of cash flows entered so far.		GSB 5	n
6	For next cash flow go to step 3.			
7	For a new case go to step 2.			

Example 1:

An investor has an opportunity to purchase a piece of property for $70,000. If the going rate of return on this type of investment is 13.75%, and the after-tax cash flows are forecast as follows, should the investor purchase the property?

Year	Cash Flow ($)	
1	$14,000	
2	11,000	
3	10,000	
4	10,000	
5	10,000	
6	9,100	
7	9,000	
8	9,000	
9	4,500	
10	71,000	(property sold in 10th year)

Keystrokes:	Outputs:	
70000 GSB 1 13.75 GSB 2		
14000 GSB 4 ⟶	–57692.31	(NPV after 1 cash flow)

11000 GSB 4 →	-49190.92	(NPV after 2 cash flows)
3 GSB 3 10000 GSB 4 →	-31172.57	(NPV after 5 cash flows)
9100 GSB 4 →	-26971.76	(NPV after 6 cash flows)
2 GSB 3 9000 GSB 4 →	-20108.39	(NPV after 8 cash flows)
GSB 5 →	8.00	(checking that we've entered 8 periods cash flows so far)
4500 GSB 4 →	-18696.99	(NPV after 9 cash flows)
71000 GSB 4 →	879.93	(NPV after 10 cash flows)

Since the final NPV is positive, the investment exceeds the profit objectives.

Example 2:

The Cooper Company needs a new photocopier and is considering leasing the equipment as an alternative to buying. The end-of-the-year net cash cost of each option is:

PURCHASE

Year	Net Cash Cost
1	$ 533
2	948
3	1,375
4	1,815
5	2,270
Total Net Cash Cost	$6,941

LEASE

Year	Net Cash Cost
1	$1,310
2	1,310
3	1,310
4	1,310
5	1,310
Total Net Cash Cost	$6,550

Looking at total cost, leasing appears to be less. But, purchasing costs less the first two years. Mr. Cooper knows that he can make a 15% return on every dollar he puts in the business; the sooner he can reinvest money, the sooner he earns 15%. Therefore, he decides to consider the **timing of the costs,** discounting the cash flows at 15% to find the present value of the alternatives. Which option should he choose?

Keystrokes:	**Outputs:**
PURCHASE	
0 GSB [1] 15 GSB [2] 533 GSB [4]	
948 GSB [4] 1375 GSB [4]	
1815 GSB [4] 2270 GSB [4] ——→	4250.71
LEASE	
0 GSB [1] 5 GSB [3]	
1310 GSB [4] ——→	4391.32

Leasing has a present value cost of $4391.32, while purchasing has a present value cost of $4250.71. Since these are both expense items, the lowest present value is the most desirable. So, in this case, purchase is the least costly alternative.

CALENDAR FUNCTIONS

For the period March 1, 1900 through February 28, 2100, this program solves for dates and days.

Given a date, the first part calculates an associated day number*. By using this program on two dates, the number of days between those dates may be found.

The second part takes a day number* and finds the corresponding date. The third part calculates the day of the week from a given day number*.

By using the first two parts together, a second date may be calculated from a date and a specified number of days (see example).

A date must be input in mm.ddyyyy format. For instance, June 3, 1975, is keyed in as 6.031975. It is important that the zero between the decimal point and the day of the month be included when the day of the month is less than 10. The day of the week is represented by the digits 0 through 6 where zero is Sunday.

Equations:

To calculate the day number from the date:

$$\text{Julian Day number*} = \text{INT}(365.25\,y') + \text{INT}(30.6001\,m') + d + 1{,}720{,}982$$

where:

$$y' = \begin{cases} \text{year} - 1 & \text{if } m = 1 \text{ or } 2 \\ \text{year} & \text{if } m > 2 \end{cases}$$

$$m' = \begin{cases} \text{month} + 13 & \text{if } m = 1 \text{ or } 2 \\ \text{month} + 1 & \text{if } m > 2 \end{cases}$$

Then days between dates is found by:

$$\text{Days} = \text{Day number}_2 - \text{Day number}_1$$

To calculate the date from a day number:

$$\text{Day \#} = \text{Julian Day Number*} - 1{,}720{,}982$$

$$y' = \text{INT}\left[\frac{\text{Day \#} - 122.1}{365.25}\right]$$

* The Julian Day number is an astronomical convention representing the number of days since January 1, 4713 B.C.

$$m' = \text{INT}\left[\frac{\text{Day \#} - \text{INT}(365.25\,y')}{30.6001}\right]$$

$$\text{Day of the month} = \text{Day \#} - \text{INT}\left[365.25\,y'\right] - \text{INT}\left[30.6001\,m'\right]$$

$$\text{Month} = m = \begin{cases} m' - 13 & \text{if } m' = 14 \text{ or } 15 \\ m' - 1 & \text{if } m' < 14 \end{cases}$$

$$\text{Year} = \begin{cases} y' & \text{if } m > 2 \\ y' + 1 & \text{if } m = 1 \text{ or } 2 \end{cases}$$

To calculate the day of the week:

$$\text{Day of the week} = 7 \times \text{FRAC}\left[(\text{Day \#} + 5)/7\right]$$

Remarks:

- No checking is done to determine if input data represent valid dates.

Step	Key	Comment
01	*LBL1	Break date input into the individual components of mm, dd, yyyy
02	ENT↑	
03	INT	
04	STO7	
05	-	
06	EEX	
07	2	
08	x	
09	ENT↑	
10	INT	
11	STO8	
12	-	
13	EEX	
14	4	
15	x	
16	STO9	
17	RCL7	m + 1
18	1	
19	+	
20	ENT↑	m + 1 → m′ y → y′
21	1/X	
22	.	
23	7	
24	+	
25	CHS	
26	INT	If input to this routine has absolute value 1 or greater, y = y ± 1 m = m ± 2
27	ST+9	
28	RCL4	
29	x	
30	-	
31	RCL2	
32	x	
33	INT	
34	RCL9	Calculate day number
35	RCL1	
36	x	
37	INT	
38	+	
39	RCL8	
40	+	
41	FIX0	
42	RTN	
43	*LBL2	Calculate y′
44	STO6	
45	RCL3	
46	-	
47	RCL1	
48	÷	
49	INT	
50	STO9	Calculate m′
51	RCL1	
52	x	
53	INT	
54	ST-6	
55	RCL6	
56	RCL2	
57	÷	
58	INT	
59	STO7	Calculate day of month
60	RCL6	
61	X⇄Y	
62	RCL2	
63	x	
64	INT	
65	-	
66	STO8	
67	RCL7	Build (m′−1) .dd Part of display
68	1	
69	RCL8	
70	%	
71	-	
72	-	
73	RCL7	Correct m′−1 and y′ to m and y
74	1	
75	4	
76	÷	
77	INT	
78	ST+9	
79	RCL4	
80	x	
81	-	
82	RCL9	Finish building mm.ddyyyy result and display answer
83	EEX	
84	6	
85	÷	
86	+	
87	FIX6	
88	RTN	
89	*LBL3	Calculate day of the week from day #
90	5	
91	+	
92	7	
93	÷	
94	FRC	
95	7	
96	x	
97	RTN	

REGISTERS

0	1 365.25	2 30.6001	3 122.1	4 12	5
6 Day #	7 m	8 d	9 y	.0	.1 Used
.2 Used	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Input constants for calculations:	365.25	STO 1	365.25
		30.6001	STO 2	30.6001
		122.1	STO 3	122.1
		12	STO 4	12
3	For day #, go to step 4. For			
	dates from day #, go to step 7.			
	For day of the week go to step 9.			
4	Input date and calculate day #	date	GSB 1	day #
5	Repeat step 4 for any other date			
6	For # of days between dates			
	calculate day #'s for each and			
	find the difference.	date 1	GSB 1	day #1
			STO . 1	day #1
		date 2	GSB 1	day #2
			RCL . 1	day #1
			−	difference
7	For dates from day#'s, input			
	day # and calculate date	day #	GSB 2	date
8	Repeat step 7 for any other			
	day #			
9	For day of the week from day #,			
	input day # and calculate day			
	of the week.	day #	GSB 3	0 ,..., 6
10	Repeat step 9 for any other			
	day #.			
11	For a new case, go to step 3.			

Example 1:

Senior Lieutenant Yuri Gagarin flew Vostok I into space on April 12, 1961. On July 21, 1969, Neil Armstrong set foot on the moon. How many days had passed between the first manned space flight and the moon landing? On what day of the week did each event take place.

Keystrokes:	Outputs:	
(Key in the program and store constants by: 365.25 STO 1 30.6001 STO 2 122.1 STO 3 12 STO 4)		
4.121961 GSB 1 STO . 1 →	716420.	(day # 1)
7.211969 GSB 1 STO . 2 →	719442.	(day # 2)
RCL . 1 − →	3022.	(days)
RCL . 1 GSB 3 →	3.	(Wednesday)
RCL . 2 GSB 3 →	1.	(Monday)

Example 2:

A short term note is due in 200 days. If the issue date is June 11, 1976, what is the maturity date?*

Keystrokes:	Outputs:	
6.111976 GSB 1 →	721959.	
200 + →	722159.	
GSB 2 →	12.281976	(December 28, 1976)

* First a day number is calculated for the known date, the number of days (200) is added to it, and this new day number is converted to a date.

Some securities use a 30/360 day calendar while this program performs all calculations using the actual number of days. Do not use the program for financial purposes unless you are sure that actual calendar days are correct.

MOON ROCKET LANDER

Imagine for a moment the difficulties involved in landing a rocket on the moon with a strictly limited fuel supply. You're coming down tail-first, freefalling toward a hard rock surface. You'll have to ignite your rockets to slow your descent; but if you burn too much too soon, you'll run out of fuel 100 feet up, and then you'll have nothing to look forward to but cold eternal moon rocks coming faster every second. The object, clearly, is to space your burns just right so that you will alight on the moon's surface with no downward velocity.

The game starts off with the rocket descending at a velocity of 50 feet/second from a height of 500 feet. The velocity and altitude are shown in a combined display as -50.0500, the altitude appearing to the right of the decimal point and the velocity to the left, with a negative sign on the velocity to indicate downward motion. Then the remaining fuel is displayed and the rocket fire count-down begins: "3", "2", "1", "0",. Exactly at zero you may key in a fuel burn. You only have one second, so be ready. A zero burn, which is very common, is accomplished by doing nothing. After a burn the sequence is repeated unless:

1. You have successfully landed—flashing zeros.
2. You have smashed into the lunar surface—flashing crash velocity.

You must take care, however, not to burn more fuel than you have; for if you do, you will free-fall to your doom! The final velocity shown will be your impact velocity (generally rather high). You have 60 units of fuel initially.

Equations:

We don't want to get too specific, because that would spoil the fun of the game; but rest assured that the program is solidly based on some old friends from Newtonian physics:

$$x = x_0 + V_0t + \frac{1}{2}at^2, \quad V = V_0 + at, \quad V^2 = V_0^2 + 2a(x - x_0)$$

where:

x, V, a, and t are distance, velocity, acceleration, and time.

Remarks:

- Only integer values for fuel burn are allowed. [R/S] can be used to stop Moon Rocket Lander at any time.

Program steps	Comments
*LBL1, 5, 0, 0, STO6, 5, 0, CHS, STO7, 6, 0, STO8	Store initial conditions
*LBL0, RCL6, FIX4, EEX, 4	Divide height by 10000 for proper display
÷, RCL7, ABS, +, RCL7, X>0?, GSB4, X⇄Y, CHS	Build VV.Ohhh display, taking negative values into account
PSE, PSE	Display VV.Ohhh
FIX0, RCL8, PSE, 3, PSE, 2, PSE, 1, PSE, 0	Count down for burn
PSE, *LBL9	Accept input
RCL8, X⇄Y, X>Y?	If fuel is gone calculate crash velocity
GTO6, ST-8, 2, x, 5, -, STO9, 2, ÷, RCL6, +, RCL7, +, RCL9, ST+7, R↓, STO6, INT, X>0?	Determine velocity and height
GTO0, RCL7, *LBL7	If no impact go for another burn
PSE, GTO7, *LBL6	Flash crash velocity
RCL8, 2, ., 5, -, ST+6, 2, x, ST+7, RCL6, 1, 0, x, RCL7, X^2, +, √X, CHS, GTO7, *LBL4, X⇄Y, CHS, X⇄Y, RTN	Fuel exhausted: call free-fall crash velocity

REGISTERS

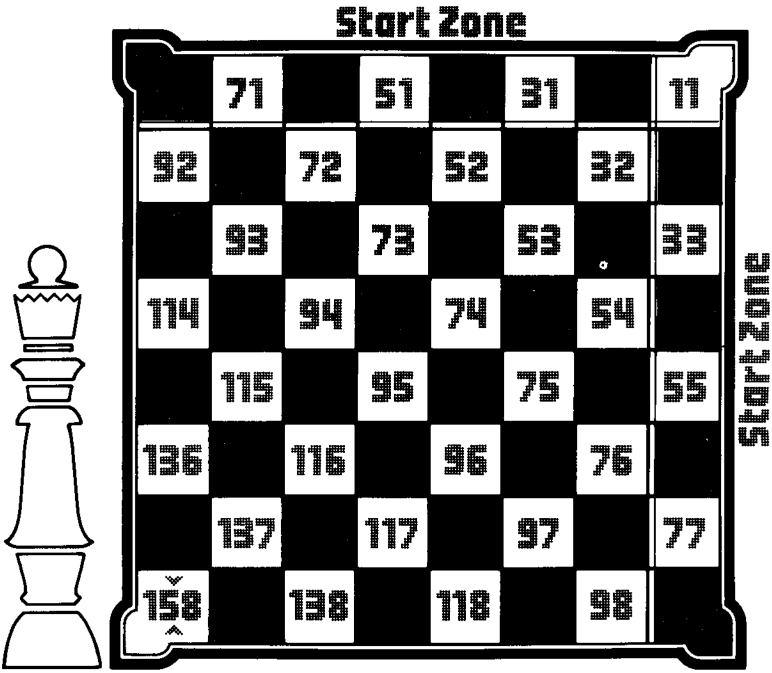
0	1	2	3	4	5
6 X	7 V	8 Fuel	9 Accel.	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Assume manual control.		GSB 1	"V.ALT"
				"FUEL"
				"3"
				"2"
				"1"
				"0"
3	Key in burn upon "0" display:			
	Press and hold R/S until			
	blinking stops.		R/S	
	Enter burn	BURN	R/S	"V. ALT"
				"FUEL"
				"3"
				"2"
				"1"
				"0"
4	Go to step 3 until you land			
	(flashing zeros) or crash			
	(flashing impact velocity).			
5	If you survived last landing			
	attempt, go to step 2 for another			
	try.			

QUEEN BOARD

This game is based on the moves of a chess queen. A queen will be allowed to move only to the left, down, or diagonally to the left. The object of the game is to be the first player to move the queen to the lower left-hand corner of the chess board (square 158), by alternating moves between you and the calculator. You start by placing the queen on any square on the top row or right-hand column. This is your first move. The play then alternates.

The playing board is numbered as follows:



You tell the calculator your moves by keying in the number of the square you start on or move to. Press GSB 1 and the calculator responds with the square it moves to. Square 158 is the winning square.

The program does not check for illegal moves. If you win (by moving to square 158), the program will respond with 168 (the calculator acknowledges the loss by displaying a nonexistent square).

The program is in FIX 0 mode, for integer display.

Reference:

This program is based on an HP-65 Users' Library program by Jacob R. Jacobs.

Some interesting comments on the theory of "Queen Board" may be found in: Gardner, M. "Mathematical Games", Scientific American, vol 236, no 3., p. 134, March 1977.

01 *LBL1		50 RTN	
02 FIX0	Current position R_1	51 *LBL0	
03 STO1		52 1	
04 GSB0		53 5	
05 1		54 8	158 = R_2?
06 X=Y?		55 X=Y?	
07 GTO8	$7 \rightarrow R_0$	56 GTO6	
08 7		57 3	
09 STO0		58 1	127 = R_2?
10 *LBL9		59 -	
11 RCL1		60 X=Y?	
12 RCL0		61 GTO6	
13 EEX		62 1	
14 1		63 -	126 = R_2?
15 x		64 X=Y?	
16 +	$10K + R_1 \rightarrow R_2$	65 GTO6	
17 STO2	Position good?	66 5	
18 GSB0		67 1	
19 1		68 -	75 = R_2?
20 X=Y?	Yes, recall R_2	69 X=Y?	
21 GTO7		70 GTO6	
22 RCL0	$K+R_2 \rightarrow R_2$	71 2	
23 ST+2	Position good?	72 -	73 = R_2?
24 RCL2		73 X=Y?	
25 GSB0	Yes, recall R_2	74 GTO6	
26 1		75 2	
27 X=Y?		76 9	
28 GTO7		77 -	44 = R_2?
29 RCL0		78 X=Y?	
30 EEX		79 GTO9	
31 1		80 3	
32 x	$10K + R_2 \rightarrow R_2$	81 -	41 = R_2?
33 ST+2		82 X=Y?	
34 RCL2	Position good?	83 GTO6	
35 GSB0		84 RTN	
36 1		85 *LBL6	
37 X=Y?	Yes, recall R_2	86 1	
38 GTO7		87 RTN	
39 DSZ			
40 GTO9			
41 RCL1			
42 *LBL8			
43 EEX	Default move		
44 1	$10 + R_1 \rightarrow R_1$		
45 ST+1			
46 RCL1			
47 RTN			
48 *LBL7	Test for good position		

REGISTERS

0 Indirect	1 Used	2 Used	3	4	5
6	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Key in your starting position (first move).	Move	GSB 1	Calc's Move
3	Repeat step 2 until someone wins.			
	Display of 158: calculator wins			
	Display of 168: you win			
4	To begin new game, repeat step 2 with new starting position.			

Example:

Keystrokes: **Outputs:**

55 GSB 1 ⟶ 75.

(You start on 55, and the calculator, after deep and careful thought, moves to 75).

97 GSB 1 ⟶ 127.

(You respond with 97, and the calculator, showing no mercy moves to 127).

148 GSB 1 ⟶ 158.

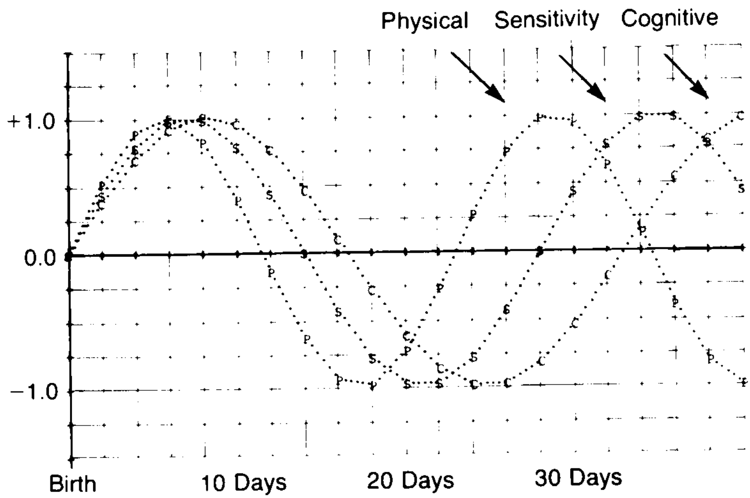
(You try 148, hoping the calculator's batteries run down before it can respond, but no luck—it wins by moving to 158).

BIORHYTHMS

From ancient days philosophers and sages have taught that human happiness lies in the harmonious integration of body, mind, and heart. Now a twentieth-century theory claims to be able to quantitatively gauge the functioning of these three aspects of our selves: the physical, sensitive, and cognitive.

The biorhythm theory is based on the assumption that the human body has inner clocks or metabolic rhythms with constant cycle times. Currently, three cycles starting at birth in a positive direction are postulated. The 23-day or physical cycle relates with physical vitality, endurance and energy. The 28-day cycle or sensitivity cycle relates with sensitivity, intuition and cheerfulness. The 33-day or cognitive cycle relates with mental alertness and judgement.

For each cycle, a day is considered either high, low, or critical. x is the output value for a given cycle. The high ($0 < x \leq 1$) times are regarded as energetic times, you are your most dynamic in the cycle. The low ($-1 \leq x < 0$) times are regarded as the recuperative periods. The critical days ($x = 0$) are regarded as your accident prone days, especially for the physical and sensitivity cycles.



Remarks:

- The birthdate and biodate must be between January 1, 1901, and December 31, 2099.
- The format for input of dates is MM.DDYYYY. For example, June 3, 1976, is keyed in as 6.031976. The program does not check input data. Thus, if an improper format or an invalid date (e.g., February 30) is keyed in, erroneous answers may result.
- This program sets the angular mode to radians (RAD).

Step	Key	Comment	Step	Key	Comment
01	*LBL1		50	STO6	M
02	RAD	Birthdate store	51	-	
03	GSB0		52	EEX	
04	STO9	N_1	53	2	
05	RTN		54	x	
06	GSB0	Biodate	55	ENT↑	D
07	RCL9		56	INT	
08	-		57	STO5	
09	STO8	Store $N_2 - N_1$	58	-	
10	*LBL9		59	EEX	
11	1		60	4	
12	8		61	x	Y
13	STO7		62	STO4	
14	GSB8	23 Day cycle	63	2	
15	GSB8	28 Day cycle	64	RCL6	
16	*LBL8		65	X>Y?	
17	5	# Days	66	GTO6	
18	ST+7		67	1	
19	RCL8		68	ST-4	
20	RCL7		69	1	
21	÷		70	2	
22	FRC		71	ST+6	
23	2		72	*LBL6	
24	x		73	1	
25	Pi		74	ST+6	
26	x		75	RCL6	
27	SIN		76	3	
28	ENT↑		77	0	
29	ABS		78	.	
30	X≠0?		79	6	
31	÷		80	x	
32	LSTX		81	INT	
33	EEX		82	RCL4	
34	7		83	3	
35	+		84	6	
36	EEX		85	5	
37	7		86	.	
38	-		87	2	
39	x	***	88	5	
40	R/S	Bio value	89	x	
41	RTN		90	INT	
42	*LBL7		91	+	
43	1	Next day	92	RCL5	
44	ST+8		93	+	N
45	GSB9		94	RTN	
46	GTO7	Compute N(M, D, Y,)			
47	*LBL0				
48	ENT↑				
49	INT				

REGISTERS

0	1	2	3	4 Y	5 D
6 M	7 23,28,33	8 $N_2 - N_1$	9 N_1	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Key in birthdate	MM.DDYYYY	GSB 1	Day #*
3	Key in biodate and find bio			
	values	MM.DDYYYY	R/S	P
			R/S	S
			R/S	C
4	To find bio values for succeed-			
	ing days.		R/S R/S	P
			R/S	S
			R/S	C
5	For a new birthdate, go to step			
	2; for a new biodate, go to			
	step 3.			
	* See Calendar Functions for			
	explanation of this number.			

Example:

Calculate the bio values for June 29, 1976, for a person born March 27, 1948. Find the values for the two days following also.

Keystrokes:	Outputs:	
3.271948 GSB 1 →	711656	(day #)
6.291976 R/S →	-1.00	(June 29) (P)
R/S →	-0.62	(S)
R/S →	-1.00	(C)
R/S R/S →	-0.98	(June 30) (P)
R/S →	-0.78	(S)
R/S →	-0.97	(C)
R/S →	-0.89	(July 1) (P)
R/S →	-0.90	(S)
R/S →	-0.91	(C)

COUNTDOWN TIMER

This program provides a countdown timer and a calibration routine for measuring elapsed time. When using this program, you should remember that clock circuits of HP calculators are designed for calculator use, not for accurate time keeping. Although the routine may be calibrated quite accurately, highly stable performance should not be expected due to variable conditions about the calculator.

Equations:

$$Ca_{new} = Ca_{old} \frac{\text{HP time}}{\text{Actual Time}}$$

```
01 *LBL1
02  FIX4      Store constant
03  STO2      ------------
04 *LBL9
05     0      Alarm
06   R/S      ------------
07  STO1
08    →H
09  RCL2
10     x      Store time
11  STO0
12  RCL1      ------------
13   R/S      ***
14 *LBL8
15   DSZ
16  GTO8      Loop on counter
17  GTO9      Go to "alarm"
18 *LBL2      ------------
19    →H
20   X⇄Y
21    →H
22   X⇄Y
23     -
24  RCL1      Calibrate constant
25    →H
26     ÷
27   1/X
28  RCL2
29     x
30   R/S
31  GTO1
```

REGISTERS

0 counter	1 time	2 Ca	3	4	5
6	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Input timer constant (try 10000)	Ca	[GSB] [1]	0.0000
3	Input desired time	t(H.MMSS)	[R/S]	t
4	Start timer		[R/S]	0.0000
5	Timer loops for time t. When			
	0.0000 is displayed, time has			
	elapsed. For a new time t, exe-			
	cute step 3 and 4. To calibrate,			
	proceed to step 6.			
6	Input ending time and starting			
	time to calculate new constant	te	[ENTER↑]	
		ts	[GSB] [2]	Ca
	To proceed depress		[R/S]	
	Then go to step 3.			

Example:

Measure elapsed times of 35 seconds and 1 minute 8 seconds.

Keystrokes:	**Outputs:**
10000 [GSB] [1] ⟶	0.0000
0.0035 [R/S] ⟶	0.0035
[R/S] ⟶	0.0000

Timer runs for approximately 32 seconds.

For the second desired time:

Keystrokes:	**Outputs:**
0.0108 [R/S] ⟶	0.0108
[R/S] ⟶	0.0000

Supposing you had noticed the *actual* ending and starting times of the 2nd example were 9:58:03 and 9:57:01, respectively, then calibrate the timer with this information:

Keystrokes:	**Outputs:**
9.5803 ENTER↑ 9.5701	
GSB 2 →	10967.7421
R/S →	0.0000

Now try the calibrated timer for 2 minutes 5 seconds:

0.0205 R/S →	0.0205
R/S →	0.0000

Under the same conditions, the new timer constant 10967.7421 should be used for subsequent use of this program. Your HP calculator will have its own "best" constant for calibration.

BODY SURFACE AREA CALCULATIONS

This program calculates body surface area by either the Dubois or Boyd formula, ... allowing your choice of the preferred method. If cardiac output is known, cardiac index may also be calculated.

The Dubois is undefined, and should not be used, for children with a BSA of less than $0.6m^2$. If the result is less than 0.6, use the Boyd formula instead.

Data inputs are patient's height and weight, in either metric or English units, and if desired, the cardiac output. If the measurements are in English units (inches and pounds) the data are input as negative values and the program automatically converts then to metric units (cm and kilograms).

Equations:

Dubois formula:

$$BSA\ (m^2) = Ht\ (cm)^{0.725} \bullet Wt\ (kg)^{0.425} \bullet 71.84 \bullet 10^{-4}$$

Boyd formula:

$$BSA\ (m^2) = 3.207 \bullet Wt\ (gm)^{(0.7285\ -\ 0.0188\ \log Wt)} \bullet Ht\ (cm)^{0.3} \bullet 10^{-4}$$

Cardiac Index (CI):

$$CI = CO\ (\ell/min)/BSA\ (m^2)$$

Remarks:

- The height and weight may be input in either metric or English units. If English units are used, they must be entered as negative values, by pressing CHS after the number is input. Press GSB 1 to calculate BSA by the Dubois method, or GSB 2 for the Boyd result. The data must be reentered for calculation by the alternate method, if desired.
- Values for BSA calculated by the Dubois method are stored in Register 1 or, if by the Boyd method in Register 2 and may be recalled as needed.
- To calculate cardiac index: select BSA as calculated by the desired method and recall it from storage, then enter cardiac output and press GSB 3.

Step	Key	Comments	Step	Key	Comments
01	*LBL1		50	3	
02	GSB0		51	1	
03	RCL8	Calculate BSA by DuBois method	52	1	
04	.		53	8	
05	7		54	÷	
06	2		55	STO2	***
07	5		56	RTN	
08	Y^X		57	*LBL3	Input CO and calculate CI
09	RCL9		58	X⇄Y	
10	.		59	÷	
11	4		60	RTN	***
12	2		61	*LBL0	
13	5		62	X<0?	Is ht. and wt. input metric or English?
14	Y^X		63	GSB9	
15	×		64	STO9	
16	1		65	R↓	
17	3		66	X<0?	
18	9		67	GSB8	
19	.		68	STO8	
20	2		69	RTN	
21	÷		70	*LBL9	
22	STO1		71	CHS	Convert wt. to metric
23	RTN	***	72	2	
24	*LBL2	Calculate BSA by Boyd method	73	.	
25	GSB0		74	2	
26	RCL8		75	÷	
27	.		76	RTN	
28	3		77	*LBL8	Convert ht. to metric
29	Y^X		78	CHS	
30	RCL9		79	2	
31	EEX		80	.	
32	3		81	5	
33	×		82	4	
34	ENT↑		83	×	
35	LOG		84	RTN	
36	.				
37	0				
38	1				
39	8				
40	8				
41	×				
42	.				
43	7				
44	2				
45	8				
46	5				
47	−				
48	Y^X				
49	×				

REGISTERS

0	1 BSA (DuBois)	2 BSA (Boyd)	3	4	5
6	7	8 Ht.	9 Wt.	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Input patient height (+ cm or −in)	Ht	ENTER↑	
3	Input patient weight (+ kg or −lb)	Wt		
4	Calculate BSA			
	by Dubois formula		GSB 1	BSA (m^2)
	or, by Boyd formula		GSB 2	BSA (m^2)
	Note: Reenter data before per-			
	forming calculation again by			
	alternate method.			
5	Calculate cardiac index. Recall			
	desired BSA from storage	BSA, Dubois	RCL 1	BSA (m^2)
	or,	BSA, Boyd	RCL 2	BSA (m^2)
	Input cardiac output and	CO, ℓ/min	GSB 3	CI, (ℓ/min/m^2)
	calculate cardiac index			

Example:

A patient has the following height and weight.

Ht = 60 in or 152.40 cm
Wt = 100 lbs or 45.45 kg.

Calculate BSA by both the Dubois and Boyd methods. If the cardiac output, (CO) is 5 l/min calculate the cardiac index using the Dubois BSA.

Keystrokes:	Outputs:	
60 CHS ENTER↑		
100 CHS GSB 1 ⟶	1.39	m^2 (Dubois)
152.4 ENTER↑ 45.45 GSB 2 ⟶	1.40	m^2 (Boyd)
RCL 1 5 GSB 3 ⟶	3.59	CI (by Dubois)

PULMONARY FUNCTIONS AND VITAL CAPACITY

The pulmonary function testing package provides calculations of the predicted and percent predicted values for vital capacity (VC), forced expiratory volume after 1 second (FEV_1), maximum expiratory flow rate (MEFR), maximum ventilatory volume after 12 seconds (MVV_{12}), residual volume (RV), total lung capacity (TLC), functional residual capacity (FRC), and forced expiratory flow from 25% to 75% (FEF 25%-75%).

The calculations are performed for either male or female patients, given the patient's height and age.

Equations:

All of the functions (with two exceptions) are calculated from a general equation of the form: (A • Ht(cm)) – (B • AGE(years)) – C, where A, B, and C are constants given in Table 1.

The exceptions are:

- Female TLC: If height is greater than 174 cm (68.5 inches) add 1 cm to height before calculation.
- Female Predicted FEF: (A • Ht(cm)) – (B • AGE(years)) – (0.00005 • AGE^2(years)) – C.

$$25\%\ VC = 0.25\ VC$$

$$75\%\ VC = 0.75\ VC$$

$$\Delta t = t_{75\%} - t_{25\%}$$

$$\text{Measured FEF} = (0.5 \cdot VC)/\Delta t$$

References:

Morris, J.F., Koski, A., Johnson, L.C., American Rev. Resp. Dis., 1971, *103*, 57.

Bates, et. al., Respiratory Function in Disease, W.G. Saunders Co., 1971.

Table 1
Constants For Calculation of Predicted Values

	FEMALE			MALE		
	A	**B**	**C**	**A**	**B**	**C**
Predicted VC	0.045	0.024	2.852	0.058	0.025	4.24
Predicted FEV_1	0.035	0.025	1.932	0.036	0.032	1.26
Predicted MEFR	0.057	0.036	2.532	0.043	0.047	−2.07
Predicted MVV_{12}	0.762	0.81	6.29	0.9	1.51	−27.0
Predicted RV	0.024	−0.012	2.63	0.03	−0.015	3.75
Predicted TLC*	0.078	0.01	7.36	0.094	0.015	9.17
Predicted FRC	0.047	0.00	4.86	0.051	0.00	5.05
Predicted FEF	0.02	0.03	−1.3	0.02	0.04	−2.0

Detailed User Instructions:

Key in the program. Then key in the patient height, in centimeters or inches (if in inches, input as a negative number) and press GSB 1. Then key in patient age in years and press GSB 2. Now any of the predicted values may be calculated by entering the appropriate constants A, B and C from table 1 and pressing GSB 3. The predicted value of the function is displayed. Key in the measured value of the function and press R/S to obtain the percent of predicted value.

The measured forced expiratory flow rate from the 25% and 75% points of a spirogram and predicted and percentage of predicted value are calculated as follows:

Enter A, B, and C from table 1, then press GSB 4. The predicted FEF is displayed. Key in the vital capacity as measured from the spirogram and press R/S. The display will show 25% VC. Read the measured time of this volume from the spirogram, key in this time in seconds and press R/S. The display will now show 75% VC. Determine the time at this volume from the spirogram, key it in and again press R/S. The measured FEF is now displayed. Pressing R/S again results in display of the percentage of predicted value. If desired, the predicted FEF can be recalled by pressing R/S or RCL 0.

*(Note: for female patients over 174 cm in height be sure to add 1 cm to height before calculating TLC, then reenter proper value for height before proceeding with calculations of other functions).

Step	Key	Comment
01	*LBL1	
02	X<0?	
03	GSB0	Store ht.
04	STO4	
05	RTN	
06	*LBL2	
07	STO5	Store age
08	RTN	
09	*LBL3	
10	STO3	
11	R↓	Calculate functions
12	STO2	
13	R↓	
14	STO1	
15	RCL1	
16	RCL4	
17	x	
18	RCL2	
19	RCL5	
20	x	
21	-	
22	RCL3	
23	1	
24	.	
25	3	
26	CHS	
27	X=Y?	
28	GSB9	
29	R↓	
30	-	
31	STO0	
32	RTN	*** Display function
33	X⇄Y	Input meas. value
34	÷	
35	EEX	
36	2	Calculate and display % of predicted
37	x	
38	RTN	
39	*LBL4	
40	GSB3	Calculate predicted FEF
41	R/S	
42	STO6	
43	.	
44	2	
45	5	
46	x	
47	RTN	Input Δt @ .25VC
48	RCL6	
49	.	
50	7	
51	5	
52	x	Input Δt @ .75 VC
53	RTN	Calculate measured FEF
54	X⇄Y	
55	R↓	
56	X⇄Y	
57	-	
58	.	
59	5	
60	RCL6	
61	x	
62	X⇄Y	
63	÷	*** Display Meas. FEF
64	R/S	
65	RCL0	
66	÷	
67	EEX	
68	2	
69	x	*** Display % of predicted FEF.
70	R/S	
71	RCL0	
72	RTN	
73	*LBL0	
74	CHS	Change inches to cm
75	2	
76	.	
77	5	
78	4	
79	x	
80	RTN	
81	*LBL9	
82	R↓	
83	5	Calculate female FEF.
84	EEX	
85	5	
86	CHS	
87	RCL5	
88	X²	
89	x	
90	+	
91	ENT↑	
92	RTN	

REGISTERS

0 Predicted Value	1 A	2 B	3 C	4 ht. (cm)	5 age (Years)
6 Measured VC	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Input patient height in cm, or	Ht, cm.	GSB 1	Ht, cm
	in inches	Ht, in.	CHS GSB	Ht, cm
			1	
3	Input patient age in years	AGE, Yrs.	GSB 2	AGE, Yrs.
4	Calculate predicted values of			
	desired functions.			
	Input A from table I	A	ENTER↑	
	Input B from table I	B	ENTER↑	
	Input C from table I	C	GSB 3	Pred. Value
5	Calculate % of predicted value			
	Input measured value	Meas. Value	R/S	% of Pred.
6	Calculate forced expiratory flow			
	Calculate predicted FEF			
	Input A from table I	A	ENTER↑	
	Input B from table I	B	ENTER↑	
	Input C from table I	C	GSB 4	FEF Pred.
	Input measured VC	VC	R/S	25% VC
	Obtain t @ 25% VC from			
	spirogram and input	$t_{25\%}$ sec.	R/S	75% VC
	Obtain t @ 75% VC from			
	spirogram and input	$t_{75\%}$ sec.	R/S	$FEF_{Meas.}$
	Calculate % predicted FEF		R/S	% $FEF_{Pred.}$
	Recall $FEF_{Pred.}$ if desired		R/S	$FEF_{Pred.}$

Example 1:

Calculate the predicted and percentage of predicted vital capacity, residual volume and forced expiratory flow for a male 6 feet tall, 28 years of age.

Measured values are:

VC = 5.2 ℓ
RV = 2.0 ℓ

Keystrokes: **Outputs:**

Calculate VC: From table 1, A = 0.058, B = 0.025, C = 4.24

Keystrokes	Outputs	
72 CHS GSB 1 →	182.88	(cm)
28 GSB 2 →	28	(years)
.058 ENTER↑ .025 ENTER↑ 4.24		
GSB 3 →	5.67	(ℓ, $VC_{Pred.}$)
5.2 R/S →	91.76	(% Pred.)

Calculate RV: From table 1, A = 0.03, B = -0.015, C = 3.75

Keystrokes	Outputs	
.03 ENTER↑ .015 CHS ENTER↑		
3.75 GSB 3 →	2.16	(ℓ, $RV_{Pred.}$)
2 R/S →	92.75	(% Pred.)

Calculate % of FEF: From table 1, A = 0.02, B = 0.04, C = -2.0

Keystrokes	Outputs	
.02 ENTER↑ .04 ENTER↑ 2.0		
CHS GSB 4 →	4.54	(ℓ, $FEF_{Pred.}$)

Input Measured VC = 5.2 ℓ.

Keystrokes	Outputs	
5.2 R/S →	1.30	(25% VC)

From Spirogram at 25% VC = 1.3 Obtain $t_{25\%}$ = 0.4 sec.

Keystrokes	Outputs	
.4 R/S →	3.90	(75% VC)

From Spirogram at 75% VC = 3.9 Obtain $t_{75\%}$ = 1.0 sec.

Keystrokes	Outputs	
1 R/S →	4.33	(ℓ, FEF)
R/S →	95.50	(% Pred.)
R/S →	4.54	(ℓ, $FEF_{Pred.}$)

Example 2:

Calculate the predicted and percentage of predicted vital capacity for a female patient 5 feet tall, 28 years of age.

Measured VC = 3.0 ℓ
Measured RV = 1.2 ℓ

Keystrokes: **Outputs:**

Calculate VC: From table 1, A = 0.045, B = 0.024, C = 2.852

Keystrokes	Outputs	
60 CHS GSB 1 →	152.40	(cm)
28 GSB 2 →	28.00	(years)
.045 ENTER↑ .024 ENTER↑		
2.852 GSB 3 →	3.33	(ℓ, $VC_{Pred.}$)
3.0 R/S →	89.98	(% Pred.)

Calculate RV: From table 1, A = 0.024, B = −0.012, C = 2.63

.024 ENTER↑ .012 CHS ENTER↑		
2.63 GSB 3 ⟶	1.36	(ℓ, $RV_{Pred.}$)
1.2 R/S ⟶	88.00	(% Pred.)

Calculate % FEF: From table 1, A = 0.02, B = 0.03, C = −1.3

.02 ENTER↑ .03 ENTER↑ 1.3		
CHS GSB 4 ⟶	3.47	(ℓ, $FEF_{Pred.}$)
3.0 R/S ⟶	0.75	(25% VC)

From Spirogram Find t = 0.4 sec @ 25% VC

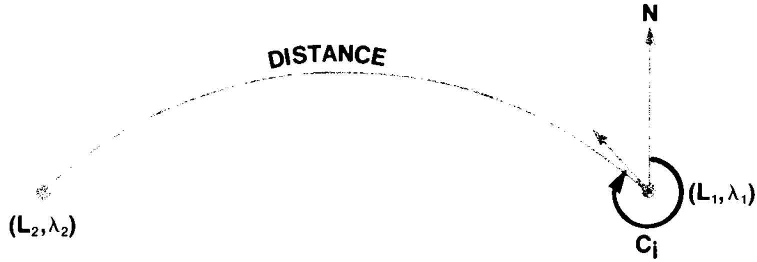
.4 R/S ⟶	2.25	(75% VC)

From Spirogram Find t = 1.0 sec @ 75% VC

1.0 R/S ⟶	2.50	(ℓ, $FEF_{Meas.}$)
R/S ⟶	72.07	(% Pred.)
R/S ⟶	3.47	(ℓ, $FEF_{Pred.}$)

GREAT CIRCLE NAVIGATION

This program calculates the great circle distance between two points and the initial course from the first point. Coordinates are input in degrees-minutes-seconds format. The distance is displayed in nautical miles and the initial course in decimal degrees.



Equations:

$$D = 60 \cos^{-1} \left[\sin L_1 \sin L_2 + \cos L_1 \cos L_2 \cos (\lambda_2 - \lambda_1)\right]$$

$$C = \cos^{-1} \left[\frac{\sin L_2 - \sin L_1 \cos (D/60)}{\sin (D/60) \cos L_1} \right]$$

$$C_i = \begin{cases} C; \sin (\lambda_2 - \lambda_1) < 0 \\ 360 - C; \sin (\lambda_2 - \lambda_1) \geq 0 \end{cases}$$

where:

L_1, λ_1 = coordinates of initial point

L_2, λ_2 = coordinates of final point

D = distance from initial to final point

C_i = initial course from initial to final point

Remarks:

Southern latitudes and eastern longitudes must be entered as negative numbers.

Truncation and round off errors occur when the source and destination are very close together (1 mile or less).

Do not use coordinates located at diametrically opposite sides of the earth.

Do not use latitudes of +90° or -90°.

Do not try to compute initial heading along a line of longitude ($L_1 = L_2$).

This program assumes the calculator is set in DEG mode.

01 *LBL0		50 SIN	
02 →H		51 ÷	
03 STO0	L_1	52 COS^{-1}	C
04 RTN		53 RCL4	
05 →H		54 SIN	
06 STO1	λ_1	55 X<0?	
07 R/S		56 GTO9	
08 →H		57 R↓	
09 STO2	L_2	58 3	
10 R/S		59 6	
11 →H		60 0	
12 STO3	λ_2	61 X⇄Y	
13 R/S		62 -	
14 *LBL1		63 RTN	*** C_i
15 RCL0		64 *LBL9	
16 SIN		65 R↓	
17 RCL2		66 RTN	*** C_i
18 SIN			
19 x			
20 RCL0			
21 COS			
22 RCL2			
23 COS			
24 x			
25 RCL3			
26 RCL1			
27 -			
28 STO4			
29 COS			
30 x			
31 +			
32 STO5			
33 COS^{-1}			
34 STO6			
35 6			
36 0			
37 x			
38 R/S	*** D		
39 RCL2			
40 SIN			
41 RCL0			
42 SIN			
43 RCL5			
44 x			
45 -			
46 RCL0			
47 COS			
48 ÷			
49 RCL6			

REGISTERS

0 L_1	1 λ_1	2 L_2	3 λ_2	4 $\lambda_2 - \lambda_1$	5 COS D/60
6 D/60	7 C_i	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Key in latitude and longitude of origin.	L_1 (D.MS)	GSB 0	L_1 (dec. deg)
		λ_1 (D.MS)	R/S	λ_1 (dec. deg)
3	Key in latitude and longitude of destination.	L_2 (D.MS)	R/S	L_2 (dec. deg)
		λ_2 (D.MS)	R/S	λ_2 (dec. deg)
4	Calculate distance and initial course.		GSB 1	D (n.m.)
			R/S	C_i (dec. deg)

Example 1:

Find the distance and initial course for the great circle from Tokyo (L35°40′N, λ139°45′E) to San Francisco (L37°49′N, λ122°25′E).

Keystrokes:	Outputs:	
35.40 GSB 0 139.45		
CHS R/S ⟶	-139.75	
37.49 R/S 122.25 R/S ⟶	122.42	
GSB 1 ⟶	4460.04	(D, n. m.)
R/S ⟶	54.37	(C_i, dec. deg.)

Example 2:

What is the distance and initial great circle course from L33°53′30″S, λ18°23′10″E to L40°27′10″N, λ73°49′40″W?

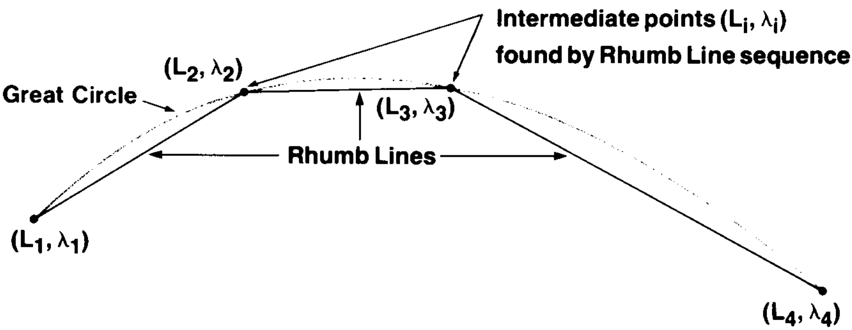
Keystrokes:	Outputs:	
33.533 CHS GSB 0 18.231		
CHS R/S ⟶	-18.39	
40.271 R/S 73.494 R/S ⟶	73.83	
GSB 1 ⟶	6763.09	(D, n. m.)
R/S ⟶	304.48	(C_i, dec. deg.)

RHUMB LINE NAVIGATION

This program is designed to assist in the activity of course planning. You supply the latitude and longitude of the point of origin and the destination. The program calculates the rhumb line course and the distance from origin to the destination.

Since the rhumb line is the constant course path between points on the globe, it forms the basis of short distance navigation. In low and midlatitudes the rhumb line is sufficient for virtually all course and distance calculations which navigators encounter. However, as distance increases or at high latitudes the rhumb line ceases to be an efficient track since it is not the shortest distance between points.

The shortest distance between points on a sphere is the great circle. However, in order to steam great circles, an infinite number of course changes are necessary. Since it is impossible to calculate an infinite number of courses at an infinite number of points, several rhumb lines may be used to approximate a great circle. The more rhumb lines used the closer to the great circle distance the sum of the rhumb line distances will be. The Great Circle Navigation program may be used to calculate intermediate course change points which can be linked by rhumb lines.



Latitudes and longitudes are input in degrees-minutes-seconds. Course is displayed in decimal degrees. Southern latitudes and eastern longitudes are input as negative numbers.

Equations:

$$C = \tan^{-1} \frac{\pi (\lambda_1 - \lambda_2)}{180 (\ln \tan (45 + \frac{1}{2} L_2) - \ln \tan (45 + \frac{1}{2} L_1))}$$

$$D = \begin{cases} 60 (\lambda_2 - \lambda_1) \cos L; \cos C = 0 \\ 60 \dfrac{(L_2 - L_1)}{\cos C}; \text{ otherwise} \end{cases}$$

where:

(L_1, λ_1) = position of initial point

(L_2, λ_2) = position of final point

D = rhumb line distance

C = rhumb line course

Remarks:

No course should pass through either the south or north pole.

Errors in distance calculations may be encountered as cos C approaches zero.

Accuracy deteriorates for very short legs.

This program assumes the calculator is set in DEG mode.

Step	Key	Comment
01	*LBL1	
02	→H	
03	STO3	λ_2
04	R↓	
05	→H	
06	STO2	L_2
07	R↓	
08	→H	λ_1
09	STO1	
10	R↓	
11	→H	L_1
12	STO0	
13	FIX2	$\lambda_1 - \lambda_2$
14	RCL1	
15	RCL3	------------
16	-	
17	STO4	
18	2	Make $-180 \leqslant \lambda_1 - \lambda_2$
19	÷	$\leqslant 180$
20	SIN	
21	SIN^{-1}	
22	9	
23	0	
24	÷	
25	Pi	
26	x	
27	RCL2	
28	GSB9	
29	RCL0	
30	GSB9	
31	-	
32	→P	
33	R↓	C
34	STO5	
35	RCL4	
36	SIN	
37	SIN^{-1}	
38	X<0?	$x < 0$ means east to west,
39	GTO8	
40	RCL5	
41	GTO7	
42	*LBL9	If west to east
43	2	C is answer
44	÷	
45	4	
46	5	
47	+	
48	TAN	
49	LN	

Step	Key	Comment
50	RTN	
51	*LBL8	
52	3	
53	6	E to W 360 - C
54	0	
55	RCL5	
56	ABS	
57	-	
58	*LBL7	
59	ABS	
60	STO6	
61	1	
62	8	
63	0	
64	RCL4	
65	ABS	
66	X≤Y?	is $[\lambda_1 - \lambda_2] > 180°$?
67	GSB6	If so subtract from 360
68	RCL2	
69	COS	
70	x	
71	STO7	
72	RCL2	
73	RCL0	
74	-	
75	RCL5	
76	COS	
77	X≠0?	is C = 90°?
78	÷	
79	ENT↑	
80	X=0?	
81	RCL7	
82	6	
83	0	
84	x	
85	ABS	
86	R/S	*** Distance
87	RCL6	*** Course
88	RTN	------------
89	*LBL6	
90	3	
91	6	If $[\lambda_1 - \lambda_2] > 180°$
92	0	
93	X⇄Y	then $360 - [\lambda_1 - \lambda_2]$
94	-	
95	RTN	

REGISTERS

0 L_1	1 λ_1	2 L_2	3 λ_2	4 $\lambda_1 - \lambda_2$	5 Used
6 C	7 Used	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Key in latitude and longitude of			
	origin	L_1 (D.MS)	ENTER↑	
		λ_1 (D.MS)	ENTER↑	
3	Key in latitude and longitude of			
	destination	L_2 (D.MS)	ENTER↑	
		λ_2 (D.MS)		
4	Calculate distance and course		GSB 1	D (n.m.)
			R/S	C (dec. deg.)
	Note: Southern latitudes and			
	eastern longitudes must be input			
	as negative numbers.			

Example 1:

What is the distance and course from L35°24′12″N, λ125°02′36″W to L41°09′12″N, λ147°22′36″E?

Keystrokes: **Outputs:**

35.2412 ENTER↑ 125.0236
ENTER↑ 41.0912 ENTER↑
147.2236 CHS GSB 1 ⟶ 4135.60 (DIST., n. m.)
R/S ⟶ 274.79 (C, dec. deg.)

Example 2:

What course should be sailed to travel a rhumb line from L2°13′42″S, λ179°07′54″E to L5°27′24″N, λ179°24′36″W? What is the distance?

Keystrokes: **Outputs:**

2.1342 CHS ENTER↑ 179.0754
CHS ENTER↑ 5.2724 ENTER↑
179.2436 GSB 1 ⟶ 469.31 (DIST., n. m.)
R/S ⟶ 10.73 (C, dec. deg.)

SIGHT REDUCTION TABLE

This program calculates the computed altitude, Hc, and azimuth, Zn, of a celestial body given the observer's latitude, L, and the local hour angle, LHA, and declination, (d), of the body. It thus becomes a replacement for the nine volumes of H0 214. Moreover, the user need not bother with the distinctions of same name and contrary name; the program itself resolves all ambiguities of this type.

Equations:

$$Hc = \sin^{-1}\left[\sin d \sin L + \cos d \cos L \cos LHA\right]$$

$$Zn = \begin{cases} Z; & \sin LHA < 0 \\ 360 - Z; & \sin LHA \geq 0 \end{cases}$$

$$Z = \cos^{-1}\left[\frac{\sin d - \sin L \sin Hc}{\cos L \cos Hc}\right]$$

Remarks:

- Southern latitudes and southern declinations must be entered as negative numbers.
- The meridian angle t may be input in place of LHA, but if so, eastern meridian angles must be input as negative numbers.
- The program assumes the calculator is set in DEG mode.

Note:

This program may also be used for star identification by entering observed azimuth in place of local hour angle and observed altitude in place of declination. The outputs are then declination and local hour angle instead of altitude and azimuth. The star may be identified by comparing this computed declination to the list of stars in *The Nautical Almanac*.

01 *LBL1		50 3	
02 →H		51 6	
03 STO0	L	52 0	
04 RTN		53 X⇄Y	
05 →H		54 -	
06 STO1		55 RTN	*** Zn
07 R/S		56 *LBL0	
08 →H	d	57 R↓	
09 STO2		58 RTN	*** Zn
10 RCL0			
11 SIN	LHA		
12 RCL1			
13 SIN			
14 x			
15 RCL0			
16 COS			
17 RCL1			
18 COS			
19 x			
20 RCL2			
21 COS			
22 x			
23 +			
24 STO3			
25 SIN^{-1}			
26 STO4	Hc, dec. deg.		
27 →HMS			
28 FIX4	Hc, D.MS		
29 R/S	***		
30 FIX1			
31 RCL1			
32 SIN			
33 RCL3			
34 RCL0			
35 SIN			
36 x			
37 -			
38 RCL0			
39 COS			
40 ÷			
41 RCL4			
42 COS			
43 ÷			
44 COS^{-1}			
45 RCL2			
46 SIN	Z		
47 X<0?			
48 GTO0			
49 R↓			

REGISTERS

0 L	1 d	2 LHA	3 Sin Hc	4 Hc	5
6	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTION	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Input the following:			
	Observer's latitude	L (D.MS)	GSB 1	
	Declination	d (D.MS)	R/S	
	Local hour angle	L.H.A. (D.MS)		
3	Calculate:			
	Altitude		R/S	Hc (D.MS)
	Azimuth		R/S	Zn (dec. deg.)
	or			
2	Input:			
	Observer's latitude	L (D.MS)	GSB 1	
	Altitude	Hc (D.MS)	R/S	
	Azimuth	Zn (D.MS)		
3	Calculate:			
	Declination		R/S	d (D.MS)
	Local hour angle		R/S	L.H.A. (dec. deg.)

Example 1:

Calculate the altitude and azimuth of the moon if its LHA is 2°39′54″W and its declination 13°51′06″S. The assumed latitude is 33°20′N.

Keystrokes:	Outputs:	
33.20 GSB 1 ——→	33.33	
13.5106 CHS R/S ——→	-13.85	
2.3954 R/S ——→	42.4447	(Hc, D.MS)
R/S ——→	183.5	(Zn, dec. deg.)

Example 2:

Calculate the altitude and azimuth of REGULUS if its LHA is 36°39′18″W and its declination is 12°12′42″N. The assumed latitude is 33°30′N.

Keystrokes:	Outputs:	
33.30 GSB 1 ——→	33.5	
12.1242 R/S ——→	12.2	
36.3918 R/S ——→	50.2425	(Hc, D.MS)
R/S ——→	246.3	(Zn, dec. deg.)

Example 3:

At 6:10 G.M.T. on January 12, 1977 a star peeked through the clouds over Corvallis (L44°34′N, λ123°17′W). An alert observer using a bubble sextant quickly determined its altitude to be 26° and its azimuth 158°. Using *The Nautical Almanac* identify the star.

Keystrokes:	Outputs:	
44.34 [GSB] [1] 26 [R/S]		
158 [R/S] ⟶	-16.3725	(d, D.MS)
[R/S] ⟶	339.4	(L.H.A., dec. deg.)

Obtain G.H.A. by adding latitude to L.H.A.

123.17 [g] [→H] [+] ⟶	462.7	(G.H.A., dec. deg.)

Then convert G.H.A. to S.H.A. by subtracting G.H.A. ARIES (for 6:10 G.M.T., January 12, 1977 G.H.A. ARIES is 203.4 dec. degrees).

203.4 [−] [f] [→H.MS] ⟶	259.2	(S.H.A., D.MS)

From *The Nautical Almanac* we find the star to be SIRIUS (S.H.A. = 258°58.1′, d = S16°41.2′).

NEWTON'S METHOD-SOLUTION TO f(x) = 0

This program uses Newton's method to find a solution for $f(x) = 0$, where $f(x)$ is specified by the user.

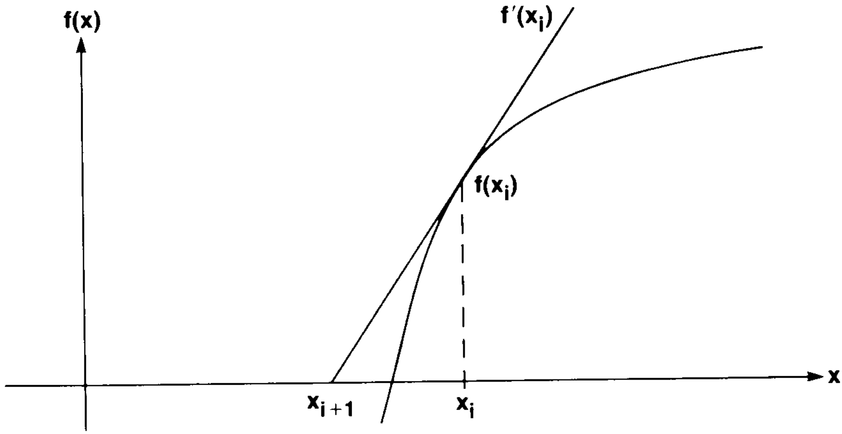
The user must define the function $f(x)$ by keying into program memory the keystrokes required to find $f(x)$, assuming x is in the X-register. 55 program steps are available for defining $f(x)$; the program only uses registers R_0 through R_4, the rest of the registers are available to the user.

The user must provide the program with an initial guess, x_1, for the solution. The closer the initial guess is to the actual solution, the faster the program will converge to an answer. The program will halt when two successive approximations for x, say x_i and x_{i+1}, are within a tolerance ϵ, i.e., when $[x_{i+1} - x_i] < \epsilon$. The value for ϵ must be input by the user. In general a reasonable value for ϵ might be $10^{-6}\ x_1$.

Equations:

The basic formula used by Newton's method to generate the next approximation for the solution is:

$$x_{i+1} = x_i - \frac{f\ (x_i)}{f'\ (x_i)}$$



This program makes a numerical approximation for the derivative $f'\ (x)$ to give the following equation:

$$x_{i+1} = x_i - \delta_i \left[\frac{f(x_i + \delta_i)}{f(x_i)} - 1 \right]^{-1}$$

where:

$$\delta_i = 10^{-5}\ x_i$$

Remarks:

After the routine has finished calculating, the last value of f(x) may be displayed by pressing RCL 4. If this value is not close enough to zero, the program may be run again with a smaller value for ϵ.

Programming Remarks:

This is one of the more complex programs in the book. The main difficulty is that at each iteration both f(x) and f(x + δ) need to be calculated, but the function f is keyed in in only one place in program memory. Large computers handle this problem by the use of a subroutine. This program simulates that technique by a number stored in R_0 known as a flag. The flag is set to 0 to indicate that f(x) is to be calculated, or to 1 if f(x + δ) is to be found. After the calculation of f, a test is made on the flag. If it is 0, the program will branch to an instruction which will store f(x); if it is 1, the program will go on to calculate a derivative based on f(x + δ).

Program	Comments		
01 *LBL1			
02 STO2	Store x, ϵ		
03 X⇄Y			
04 STO1	- - - - - - - - - - - -		
05 *LBL8			
06 CLX			
07 STO0	Set flag to 0 for f(x)		
08 RCL1			
09 GTO0	- - - - - - - - - - - -		
10 *LBL6			
11 R↓			
12 STO4			
13 1			
14 STO0			
15 RCL1	Store f(x) and calculate δ		
16 RCL1			
17 EEX			
18 5			
19 ÷			
20 STO3	- - - - - - - - - - - -		
21 +	User's f(x)		
22 *LBL0	- - - - - - - - - - - -		
23 *LBL7			
24 X=0?			
25 GTO9			
26 RCL0			
27 X=0?			
28 GTO6			
29 R↓			
30 RCL4	Calculate x_{i+1}		
31 ÷			
32 1	x_i and		
33 -			
34 1/X	$[x_{i+1} - x_i] > \epsilon$?		
35 RCL3			
36 x			
37 ST-1			
38 ABS			
39 RCL2			
40 X⇄Y			
41 X>Y?			
42 GTO8	- - - - - - - - - - - -		
43 *LBL9			
44 RCL1			
45 RTN	***		
	Output x_0		

REGISTERS

0 Flag	1 x	2 ϵ	3 δ	4 f(x)	5
6	7	8	9	0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Press GTO 0		GTO 0	
3	Switch to PRGM and key			
	in function f(x)			
4	Switch to RUN			
5	Input initial guess for solution			
	and tolerance to calculate			
	solution.	x_1	ENTER↑	
		ϵ	GSB 1	x_0
6	To recall the last f(x)		RCL 4	f(x)

Example:

Find a root x_0 of the equation $\ln x + 3x - 10.8074 = 0$ in the interval $[1, 5]$. An accuracy of 10^{-4} is acceptable.

Keystrokes: **Outputs:**

GTO 0,

Switch to PRGM

f ln f LAST X 3 ×

+ 10.8074 −

Switch to RUN

1 ENTER↑ EEX CHS 4

GSB 1 ⟶ 3.21 (root)

RCL 4 ⟶ -1.50 -07 (f(3.21))

NUMERICAL INTEGRATION *BY SIMPSON'S FORMULA*

This program will perform numerical integration by Simpson's formula whether a function is known explicitly or only at a finite number of equally spaced points (discrete case).

Discrete Case:

Let $x_0, x_1, \ldots, x_n$ be n equally spaced points ($x_j = x_0 + jh$, $j = 1, 2, \ldots, n$) at which corresponding values $f(x_0), f(x_1), \ldots, f(x_n)$ of the function f(x) are known. The function itself need not be known explicitly. After input of the step size h and the values of $f(x_j)$, $j = 0, 1, \ldots, n$, then the integral

$$\int_{x_0}^{x_n} f(x)\,dx \qquad (1)$$

may be approximated using Simpson's rule:

$$\int_{x_0}^{x_n} f(x)\,dx \approx \frac{h}{3}\Big[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\Big] \qquad (2)$$

In order to apply Simpson's rule, n must be even.

Explicit Functions:

If an explicit formula is known for the function f(x), then the function may be keyed into program memory and numerically integrated by Simpson's rule. The user must specify the endpoints a and b of the interval over which integration is to be performed, and the number of subintervals n into which the interval (a, b) is to be divided. This n must be even; if it is not, Error will be displayed. The program will go on to compute $x_0 = a$, $x_j = x_0 + jh$, $j = 1, 2, \ldots, n-1$, and $x_n = b$ where

$$h = \frac{b - a}{n}$$

The integral $\int_a^b f(x)\,dx$ is approximated by equation (2) above, Simpson's rule.

17 program steps (or more) are available for user's function f(x). Refer to the Instructions for keying in the function f(x).

Remarks:

- Since there are actually 3 routines after LBL 1 for keying in the value of $f(x_j)$, one for $j = 0$, one for j odd, and one for j even, it is important that no other keys be pressed during the entry of the $f(x_j)$, lest the next $f(x_j)$ entered go into the wrong register.
- If n is not even erroneous results will occur.

Program	Comments	Program	Comments
*LBL1		ST+0	
CLRG	Input h	RCL2	
STO4		RCL1	
R/S		STO5	
STO9		-	
*LBL9	Input f_0	RCL3	
RCL3		÷	
R/S		STO4	
STO1		0	
GSB6		STO9	
ENT↑		*LBL8	
+		GSB4	
ST+9	Input f_i for odd i	ST+0	
1		2	
ST+3		ST+9	$x \leftarrow x + h$
RCL3		RCL3	
R/S		RCL9	$R_0 \leftarrow R_0 + 4\ f(x)$
STO1		X=Y?	Exit for $R_9 = n$
GSB6		GTO5	
ST+9		GSB4	
1		GTO8	
ST+3	Input f_i for even i	*LBL4	
GTO9		RCL4	
*LBL2		ST+5	
3		RCL5	Subroutine
RCL9		GSB0	
RCL1		GSB6	
-		RTN	
*LBL7	Calculate	*LBL5	
RCL4		3	
x		RCL0	Exit
X⇄Y	$\int f(x)\,dx$	GTO7	
÷	***	*LBL0	
RTN		RTN	For User's function
*LBL6			
ENT↑			
+			
ST+0	Subroutine		
RTN			
*LBL3			
STO3			
R↓			
STO2			
R↓			
STO1	Input a, b, n		
GSB0	To calculate		
STO0	$\int_a^b f(x)\,dx$		
RCL2			
GSB0			

REGISTERS

0 Used	1 $f(x_i)$, a	2 b	3 n	4 h	5 x
6	7	8	9 Used	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program:			
	i for discrete case only: program step 1 to step 39.			
	ii for explicit functions only: program step 29 to step 83.			
2	For explicit functions, go to step 7, for discrete case, go to step 3.			
3	**Discrete Case:** input h	h	GSB 1	h
4	Repeat this step for j = 0, 1,..., n:			
	Key in the function value at x_j	$f(x_j)$	R/S	j
5	Calculate the integral		GSB 2	the integral
6	For a new case, go to step 3.			
7	**Explicit Function:** To key in your function f(x), first press,		GTO 0	
	then switch to PRGM and key in f(x)			
	Switch back to RUN*			
8	Input a, b, and n to calculate			
	$\int_a^b f(x)dx$	a	ENTER↑	
		b	ENTER↑	
		n	GSB 3	the integral
9	For a new set of a, b, and n, go to step 7.			
	***Note:** Available program steps for f(x) are:			
	• 45 steps when only the EXPLICIT part is keyed in.			
	• 17 steps when both parts are keyed in.			

Example 1:

Given the values below for $f(x_j)$, $j = 0, 1, \ldots, 8$, calculate the approximations to the integral

$$\int_0^2 f(x)\,dx$$

by Simpson's formula.

The value for h is 0.25.

i	0	1	2	3	4	5	6	7	8
x_i	0	.25	.5	.75	1	1.25	1.5	1.75	2
$f(x_i)$	2	2.8	3.8	5.2	7	9.2	12.1	15.6	20

Keystrokes: **Outputs:**

(Key in the program from step 1 to step 39)

0.25 [GSB] [1] ⟶ 0.25
2 [R/S] 2.8 [R/S] 3.8 [R/S] 5.2
[R/S] 7 [R/S] 9.2 [R/S] 12.1
[R/S] 15.6 [R/S] 20 [R/S] ⟶ 8.00
[GSB] [2] ⟶ 16.58 (the integral)

Example 2:

Find the value of

$$\int_0^{2\pi} \frac{dx}{1 - \cos x + 0.25}$$

for n = 16. Note that x is assumed to be in radians. For safety, if you work mostly in degrees, it is good programming practice to set the angular mode to radians at the beginning of the routine, then back to degrees at the end.

Keystrokes: **Outputs:**

(Key in the program from step 29 to step 83)

[GTO] [0],

Switch to PRGM

[g] [RAD] [f] [cos] 1 [x⇄y] [−] .25
[+] [g] [1/x] [g] [DEG],

Switch back to RUN

0 [ENTER↑] [g] [π] 2 [×] 16
[GSB] [3] ⟶ 8.36 (Answer)

IDEAL GAS EQUATION OF STATE

Many gases obey the ideal gas laws quite closely at reasonable temperatures and pressures. This program calculates any one of the four variables when data for the other three and the universal gas constant are entered. Likewise, the value of the universal gas constant can be determined by entering data for the four variables.

Equation:

$$PV = n\,RT$$

where: P is the absolute pressure

V is the volume

n is the number of moles present

R is the Universal Gas Constant

T is the absolute temperature

Table 1

Values of the Universal Gas Constant

Value of R	Units of R	Units of P	Units of V	Units of T
8.314	N - m/g mole-°K	N/m^2	m^3/g mole	°K
83.14	cm^3 - bar/g mole-°K	bar	cm^3/g mole	°K
82.05	cm^3 - atm/g mole-°K	atm	cm^3/g mole	°K
0.08205	ℓ – atm/g mole-°K	atm	ℓ/g mole	°K
0.7302	atm-ft^3/lb mole-°R	atm	ft^3/lb mole	°R
10.73	psi-ft^3/lb mole-°R	psi	ft^3/lb mole	°R
1545	psf-ft^3/lb mole-°R	psf	ft^3/lb mole	°R

Remarks:

- At low temperatures or high pressures the ideal gas law does not represent the behavior of real gases.
- The value of R used must be compatible with the units of P, V, T.
- To ensure proper execution of the program initialize by pressing [GTO] [6] before entering data.

Program	Comments
01 *LBL6	
02 0	
03 RTN	
04 *LBL1	
05 1	
06 GTO0	
07 *LBL2	
08 2	
09 GTO0	
10 *LBL3	Initialize and store data
11 3	
12 GTO0	
13 *LBL4	
14 4	
15 GTO0	
16 *LBL5	
17 5	
18 *LBL0	
19 STO0	
20 R↓	
21 STOi	
22 X≠0?	
23 GTO6	------------
24 1	
25 STOi	
26 RCL1	
27 RCL2	PV
28 ×	
29 RCL3	
30 RCL4	n RT
31 ×	
32 RCL5	
33 ×	
34 GTOi	
35 *LBL1	Calculate P or V
36 *LBL2	
37 X⇄Y	
38 *LBL3	
39 *LBL4	Calculate n, R or T
40 *LBL5	
41 ÷	
42 STOi	***
43 RTN	

REGISTERS

0 Indirect	1 P	2 V	3 n	4 R	5 T
6	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Input four of the following:			
	absolute pressure	P	GSB 1	0.00*
	volume	V	GSB 2	0.00
	number of moles	n	GSB 3	0.00
	universal gas constant	R	GSB 4	0.00
	absolute temperature	T	GSB 5	0.00
3	Calculate one of the following:**			
	absolute pressure	0.00	GSB 1	P
	volume	0.00	GSB 2	V
	number of moles	0.00	GSB 3	n
	universal gas constant	0.00	GSB 4	R
	absolute temperature	0.00	GSB 5	T
4	For a new case, go to step 2 and change appropriate inputs.			
5	If program fails to execute properly press GTO 6 and start again.		GTO 6	
*	Be sure that 0.00 is displayed after each data entry. If not press GTO 6 and reenter all data.			
**	Be sure 0.00 is displayed before GSB is executed to calculate unknown.			

Example 1:

0.63 moles of air are enclosed in 25000 cm^3 of space at 1200°K. What is the pressure in bars? In atmospheres? Assume an ideal gas.

Keystrokes:	**Outputs:**	
25000 GSB 2 0.63 GSB 3		
83.14 GSB 4 1200 GSB 5		
GSB 1 ⟶	2.51	(bars)
82.05 GSB 4 GSB 1 ⟶	2.48	(atm.)

Example 2:

What is the specific volume (ft^3/lb) of a gas at atmospheric pressure and a temperature of 513°R? The molecular weight is 29 lb/lb-mole.

Keystrokes:	**Outputs:**	
513 GSB 5 29 g 1/x		
GSB 3 0.7302 GSB 4 1		
GSB 1 GSB 2 ⟶	12.92	(ft^3/lb)

What is the density?

g 1/x FIX 3 ⟶	0.077	(lb/ft^3)

What is the density at 1.32 atmosphere and 555° R?

1.32 GSB 1 555 GSB 5 GSB 2		
g 1/x ⟶	0.094	(lb/ft^3)

RADIOACTIVE ISOTOPE DECAY

This program is designed to allow calculation of the decay in radioactivity of an isotope over a specified time interval. To use the program, select an isotope and key in its half-life. (Half-life data may be stored for up to 10 different isotopes in available storage registers.) Then key in two of the three variables:

A_0: Initial activity of the isotope.

t: Elapsed time.

A: Present activity.

The program then calculates the missing variable. Thus, for example, you are not restricted to finding the present activity, given time and initial activity; you may also solve for initial activity given time and present activity, or for time given initial activity and present activity.

The continuous memory feature of your calculator allows convenient storage and recall of the half-lifes of up to ten of the isotopes you most commonly use. Prior storage of the half-lifes eliminates having to enter them before each calculation and they are always available.

You may use any units for initial and present activity as long as they are consistent. The elapsed time *must* be input in the units: Days.Hours (DD.HH), where two full decimal places must be allotted to the hours. For instance an elapsed time of 5 days 18 hours would be keyed in and displayed as 5.18; a time of 1 day 6 hours as 1.06; and a time of 12 hours as 0.12.

Equations:

$$A = A_0\left(\frac{1}{2}\right)^{t/\tau_{1/2}}$$

$$t = \frac{\tau_{1/2}\ \ln\,(A/A_0)}{\ln\,(1/2)}$$

where:

A_0 = initial radioactivity

A = present radioactivity

t = time elapsed, in hours

$\tau_{1/2}$ = half-life of radioisotope, in hours

Isotope	Half-Life in Hours ($\tau_{1/2}$)
Cr^{51}	6672
Co^{57}	6480
Co^{60}	46460
I^{125}	1440
I^{131}	193.2
Cs^{137}	262980
H^{3}	107470
C^{14}	5.058×10^7
F^{18}	1.87
P^{32}	343.2
Se^{75}	2880
Sr^{85}	1536
In^{113}	1.73
Xe^{133}	126.5
Hg^{197}	65
Ra^{226}	1.3938×10^7

Remarks:

When recalling previously stored half-life data from the storage registers the program utilizes indirect addressing. Remember that the indirect addresses of storage registers .0 thru .5 are 10 thru 15 respectively.

If half-life of desired isotope has not previously been stored the user may key it in and store it in register 2, for use in the program.

Time is input and displayed in DD.HH format. To prevent "untidy" displays, such as 6.24 instead of 7.00 days, residual hours of 23.5 or greater are presented as 1.00 day.

The variable to be calculated is always input with a value of 0.00.

```
01 *LBL0      Recall selected isotope
02 STO0       half-life
03 RCLi
04 STO2
05 RTN        ------------
06 *LBL1
07 FIX2       Input data
08 STO3
09 R↓         Calculator determines which
10 X=0?       variable is to be calculated
11 STO4
12 GSB9
13 R↓
14 STO5
15 X=0?
16 GTO8
17 RCL4
18 X=0?
19 GTO7
20 GTO6       ------------
21 *LBL9
22 ENT↑       Convert time from DD.HH
23 INT        to hours
24 2
25 4
26 ×
27 X⇄Y
28 FRC
29 EEX
30 2
31 ×
32 +
33 STO4
34 RTN        ------------
35 *LBL8      Calculate A0,
36 RCL3       Initial activity
37 RCL4
38 RCL2
39 ÷
40 .
41 5
42 X⇄Y
43 Yˣ
44 ÷
45 STO5
46 RTN        ------------
47 *LBL7
48 RCL3
49 RCL5       Calculate t, time
```

```
50 ÷
51 STO1
52 LN
53 RCL2
54 ×
55 .
56 5
57 LN
58 ÷
59 2
60 4
61 ÷          ------------
62 ENT↑       Convert from hours to
63 INT        DD.HH
64 X⇄Y
65 FRC
66 2
67 4
68 ×
69 2
70 3
71 .          If t ≥ 23.5 hours,
72 5          round to nearest day
73 X≤Y?
74 GTO5
75 R↓
76 EEX
77 2
78 ÷
79 +
80 RTN        ------------
81 *LBL6      Calculate A,
82 .          Present activity
83 5
84 RCL4
85 RCL2
86 ÷
87 Yˣ
88 RCL5
89 ×
90 STO3
91 RTN
92 *LBL5
93 R↓         Present t ≥ 23.5 hours
94 R↓         as 1 day
95 1
96 +
97 RTN
```

REGISTERS

0 i	1 A/A_0	2 $T_{1/2}$	3 A	4 t	5 A_0
6 *	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

* Registers 6 through .5 are available for isotope half life storage.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
	To store half-lives of commonly used isotopes.			
1′	Store half lifes of desired isotopes in registers 6 through 9 and .0 through .5.*	$\tau_{1/2}$, hrs.	STO 6 ⋮ STO 9 STO • 0 ⋮ STO • 5	
	To calculate variables.			
2	Select desired isotope and initialize by recalling its $\tau_{1/2}$ from storage, using indirect address.	$\tau_{1/2}$ index	GSB 0	
2′	*or,* if isotope half life is not stored, input $\tau_{1/2}$ manually.	$\tau_{1/2}$	STO 2	
3	Key in variables in this format:			
	• Activity at time zero	A_0	ENTER↑	
	• Elapsed time, in days hours format	t, DD.HH	ENTER↑	
	• Present activity	A	GSB 1	unknown
	Important: Input zero for value of unknown variable. Be sure variables are entered in above order.			
4	Other data may be recalled as desired:			
	• Decay factor, A/A_0		RCL 1	
	• Half life, $\tau_{1/2}$		RCL 2	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	• Present activity, A		RCL 3	
	• Elapsed time		RCL 4	
	• Initial activity		RCL 5	
	* Half lifes of up to 10 selected isotopes may be permanently stored in registers 6 through 9 and .0 through .5 having indirect addresses 6 through 15 respectively.			

Example:

An activity of 200 μCi is measured for a standard of Cr^{51} (with half-life 667.20 hours). What is the activity after a week?

Keystrokes:	Outputs:	
667.20 STO 2 →	667.20	($\tau_{1/2}$ for Cr^{51})
200 ENTER↑ →	200.00	(A_o)
7 ENTER↑ →	7.00	(t = 7 days)
0 GSB 1 →	167.97	(A, μC_i)

(OR)

Calculate A_o given A = 167.97 (μC_i) and t = 7.00

0 ENTER↑ →	0	Unknown
7 ENTER↑ →	7.00	(t = 7 days)
167.97 GSB 1 →	200.00	(A_o, μC_i)

ACID-BASE EQUILIBRIUM

This program calculates the hydrogen ion concentration, $[H_3O^+]$, and pH of a solution of a monoprotic weak acid if the ionization constant is known. Likewise, the program will calculate $[OH^-]$ concentration and pOH for solutions of weak bases given the ionization constant of the base. In addition, conversions from concentration to pH or pOH and vice versa and from pH to pOH, $[H_3O^+]$ to $[OH^-]$ etc. are included.

The following equation is used:

$$x^3 + K_a x^2 - (K_w + K_a C_a)\,x - K_w K_a = 0 \qquad (K_b \text{ For bases})$$

where:

$x = [H_3O^+]$ For acid, $[OH^-]$ For base

K_a = Ionization constant of acid $= \dfrac{[H_3O^+]\,[A^-]}{[HA]}$

K_b = Ionization constant of base $= \dfrac{[B^+]\,[OH^-]}{[BOH]}$

K_w = Ionization constant of water $= 10^{-14}$ @ 25° C

C_a or C_b = Concentration (moles/liter) of acid or base

The program uses Newton's method of approximating the solution of a polynomial where one evaluates f(x) successively with approximate values of X. First approximation of x is $x = (K_a C_a + K_w)^{1/2}$. Successive approximations are $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$

The calculation is reiterated until x_{i+1} differs from x_i a small amount (1% or less).

References:

Butler, J.N., "Ionic Equilibrium, A Mathematical Approach", Addison-Wesley, 1964.

Dick, J.G., "Analytical Chemistry", McGraw-Hill, 1973.

This program is based upon a program submitted to the HP Users' Library by Alan J. Rubin.

```
01 *LBL1
02  STO0
03    R↓
04  STO1
05  SCI2
06  RCL0
07     x
08   EEX
09   CHS
10     1
11     4
12     +
13  STO3
14  LSTX
15  RCL1
16     x
17  STO4
18  RCL3
19    √X
20  STO2
```

Calculate and store constants

```
21 *LBL0
22  RCL1
23     +
24  RCL2
25     x
26  RCL3
27     -
28  RCL2
29     x
30  RCL4
31     -
32  RCL2
33     3
34     x
35  RCL1
36     2
37     x
38     +
39  RCL2
40     x
41  RCL3
42     -
43     ÷
44  STO5
45   ABS
46  RCL2
47     9
48     9
49     ÷
```

Iteration sequence

Test approximations

```
50  X>Y?
51  GTO9
52  RCL2
53  RCL5
54     -
55  STO2
56  GTO0
```

Calculate next approximation

Iterate

```
57 *LBL9
58  RCL2
59   R/S
```

*** Result $[H_3O^+]$ or $[OH^-]$

```
60 *LBL2
61   LOG
62   CHS
63  FIX2
64   RTN
```

Convert Conc to pH or pOH or K_a, K_b to pK_a, pK_b ***

```
65 *LBL3
66   CHS
67   10^x
68  SCI2
69   RTN
```

Convert pH or pOH to Conc. or pK_a, pK_b to K_a, K_b ***

```
70 *LBL4
71     1
72     4
73   X≷Y
74     -
75  FIX2
76   RTN
```

Convert pH to pOH or vice versa

***

```
77 *LBL5
78   EEX
79   CHS
80     1
81     4
82   X≷Y
83     ÷
84   RTN
```

Convert $[H_3O^+] \rightarrow [OH^-]$ or $[OH^-] \rightarrow [H_3O^+]$

***

REGISTERS

0 C_a or C_b	1 K	2 X(est)	3 $CK + K_w$	4 $K K_w$	5 $f(x)/f'(x)$
6	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program:			
2	Enter ionization constant:			
	K_a or K_b	K	ENTER↑	K
	or pK_a or pK_b	pK	GSB 3	K
3	Input concentration (moles/liter)			
	of acid or base and calculate			
	conc. of $[H_3O^+]$ if acid (conc. of			
	$[OH^-]$ if base).	Conc.	GSB 1	$[H_3O^+]$ or $[OH^-]$
4	Convert concentration to pH or			
	pOH:			
	$[H_3O^+] \rightarrow$ pH	$[H_3O^+]$		
	or $[OH^-] \rightarrow$ pOH	or $[OH^-]$	R/S	pH or pOH
5	If desired the following con-			
	versions are available:			
	concentration of $[H_3O^+]$ or			
	$[OH^-]$ to pH or pOH	$[H_3O^+]$, $[OH^-]$	GSB 2	pH or pOH
	Ionization const. K_a or K_b to			
	pK_a or pK_b	K_a, K_b	GSB 2	pK_a, pK_b
	pH or pOH to concentration of			
	$[H_3O^+]$ or $[OH^-]$	pH, pOH	GSB 3	$[H_3O^+]$, $[OH^-]$
	pK to ionization constant	pK_a, pK_b	GSB 3	K_a, K_b
	pH to pOH or vice versa	pH, pOH	GSB 4	pOH, pH
	$[H_3O^+]$ to $[OH^-]$ or vice versa	$[H_3O^+]$, $[OH^-]$	GSB 5	$[OH^-]$, $[H_3O^+]$
6	If desired, error of calculation			
	may be reviewed.		RCL 5	f(x)/f′(x)

Example 1:

1. Calculate the pH of a 1.0×10^{-4} molar solution of acetic acid if the ionization constant is 1.75×10^{-5}.

Keystrokes:	**Outputs:**	
1.75 EEX CHS 5 ENTER↑ EEX CHS		
4 GSB 1 ⟶	3.41 -05	($[H_3O^+]$)
R/S ⟶	4.47	(pH)

Example 2:

Calculate the pH of a sample of water containing 0.85 mg of ammonia as the only contaminant. K_b of ammonium hydroxide is 1.8×10^{-5} and the molecular wt. of ammonia is 17.

Keystrokes:	**Outputs:**	
1.8 EEX CHS 5 ENTER↑ 0.85		
EEX CHS 3 ENTER↑ 17		
÷ GSB 1 ⟶	2.25 -05	($[OH^-]$)
R/S ⟶	4.65	(pOH)
GSB 4 ⟶	9.35	(pH)

(Note: After entering ionization constant, calculate molar conc. of $NH_3 = 0.85 \times 10^{-3}/17 = 5 \times 10^{-5}M$).

Example 3:

Water in equilibrium with air contains carbon dioxide which forms a dilute solution of carbonic acid. If distilled water contains 1.35×10^{-5} moles/liter of carbon dioxide, what is the pH?

(The primary ionization constant of carbonic acid is 3.5×10^{-7}, the secondary ionization constant of 4.4×10^{-11} may be neglected).

Keystrokes:	**Outputs:**	
3.5 EEX CHS 7 ENTER↑ 1.35		
EEX CHS 5 GSB 1 ⟶	2.03 -06	($[H_3O^+]$)
R/S ⟶	5.69	(pH)

(Examples 2 and 3 are taken from Kolthoff and Sandell, Textbook of Quantitative Inorganic Analysis, MacMillan, 1948).

CURVE FITTING

This program can be used to fit data to:

1. Straight lines (linear regression); $y = a + bx$.
2. Exponential curves; $y = ae^{bx}$ $(a > 0)$,
3. Logarithmic curves; $y = a + b \ln x$,
4. Power curves; $y = ax^{b}$ $(a > 0)$.

The regression coefficients a and b are found from solving the following equivalent of linear equations.

$$\begin{bmatrix} n & \Sigma X_i \\ \Sigma X_i & \Sigma X_i^2 \end{bmatrix} \begin{bmatrix} A \\ b \end{bmatrix} = \begin{bmatrix} \Sigma Y_i \\ \Sigma Y_i X_i \end{bmatrix}$$

While the relations of the variables are defined as the following:

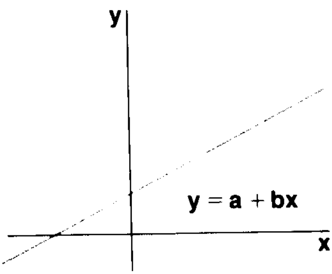
Regression	A	X_i	Y_i	Code
Linear	a	x_i	y_i	5
Exponential	ln a	x_i	$\ln y_i$	6
Logarithmic	a	$\ln x_i$	y_i	7
Power	ln a	$\ln x_i$	$\ln y_i$	8

The coefficient of determination is:

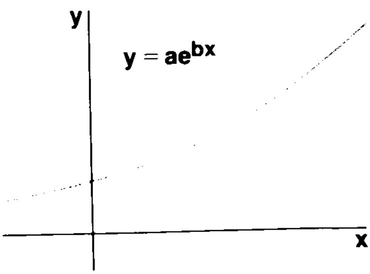
$$r^2 = \frac{A\Sigma Y_i + b\Sigma X_i Y_i - \frac{1}{n}(\Sigma Y_i)^2}{\Sigma (Y_i^2) - \frac{1}{n}(\Sigma Y_i)^2}$$

The type of curve fit must be determined before data input begins, that is, by storing the code number into register 0.

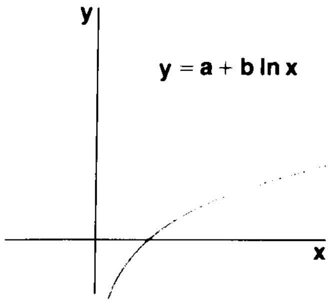
Linear Regression
Code = 5



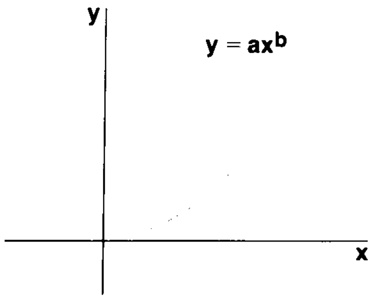
Exponential Curve Fit
Code = 6



Logarithmic Curve Fit
Code = 7



Power Curve Fit
Code = 8



Remarks:

- Negative and zero values of x_i will cause a machine error for logarithmic curve fits. Negative and zero values of y_i will cause a machine error for exponential curve fits. For power curve fits both x_i and y_i must be positive, non-zero values.
- As the differences between x and/or y values become small, the accuracy of the regression coefficients will decrease.

01 *LBL1		50 x	
02 X⇄Y		51 STO7	Determinate
03 GSBi	Input data	52 R↓	
04 Σ+		53 x	
05 RTN		54 RCL7	
06 *LBL7		55 -	
07 LN	Log	56 RTN	
08 RTN		57 *LBL3	
09 *LBL8		58 RCL4	
10 LN		59 RC.3	
11 *LBL6	Power and exp.	60 x	
12 X⇄Y		61 RCL5	
13 LN		62 RC.5	
14 X⇄Y		63 x	
15 RTN		64 +	
16 *LBL2		65 RC.3	
17 RC.0		66 X²	
18 RC.2		67 RC.0	Calculate r^2
19 RC.1		68 ÷	
20 RC.1		69 STO9	
21 GSB9		70 -	
22 STO3		71 RC.4	
23 RC.3		72 RCL9	
24 RC.2		73 -	
25 RC.1		74 ÷	
26 RC.5		75 R/S	
27 GSB9		76 *LBL4	
28 RCL3	Calculate A, b, and a, b	77 GTOi	
29 ÷		78 *LBL8	
30 STO4		79 RCL5	
31 GSBi		80 Y^x	
32 STO6		81 GTO9	
33 R/S		82 *LBL6	
34 RC.0		83 RCL5	
35 RC.5		84 x	
36 RC.1		85 e^x	
37 RC.3		86 *LBL9	
38 GSB9		87 RCL6	Input x to calculate y
39 RCL3		88 x	
40 ÷		89 RTN	
41 STO5		90 *LBL7	
42 RTN		91 LN	
43 *LBL6	Inverse transform	92 *LBL5	
44 *LBL8		93 RCL5	
45 e^x		94 x	
46 *LBL5		95 RCL6	
47 *LBL7		96 +	
48 RTN		97 RTN	
49 *LBL9			

REGISTERS

0 Index	1 x	2 y	3 det	4 A	5 b
6 a	7 Used	8	9 $1/n\ (\Sigma Y)^2$	.0 n	.1 ΣX
.2 ΣX^2	.3 ΣY	.4 ΣY^2	.5 ΣXY	16	17
18	19	20	21	22	23
24	25	26	27	28	29

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Initialize.		[f] [REG]	
3	Store curve fit code (5 or 6 or 7			
	or 8) in register 0	code	[STO] [0]	code
4	(Repeat for i = 1, 2, ..., n.)			
	Input x_i value and y_i value.	x_i	[ENTER↑]	
		y_i	[GSB] [1]	i
5	Calculate regression coeffi-			
	cients		[GSB] [2]	a
			[R/S]	b
6	Calculate r^2.		[GSB] [3]	r^2
7	(Repeat if necessary.) Input x to			
	calculate $\hat{y}$.	x	[GSB] [4]	$\hat{y}$
8	For a new case, go to step 2.			

Example 1:

(Linear, code = 5):

x_i	40.5	38.6	37.9	36.2	35.1	34.6
y_i	104.5	102	100	97.5	95.5	94

Solution:

a = 33.53, b = 1.76

r^2 = 0.99

i.e., y = 33.53 + 1.76 x

For x = 37, $\hat{y}$ = 98.65

Keystrokes: **Outputs:**

[f] [REG] 5 [STO] [0] ⟶ 5.00

40.5 [ENTER↑] 104.5 [GSB] [1]

38.6 [ENTER↑] 102 [GSB] [1]

37.9 [ENTER↑] 100 [GSB] [1]

36.2 [ENTER↑] 97.5 [GSB] [1]

35.1 ENTER↑ 95.5 GSB 1		
34.6 ENTER↑ 94 GSB 1 ⟶	6.00	
GSB 2 ⟶	33.53	(a)
R/S ⟶	1.76	(b)
GSB 3 ⟶	0.99	(r^2)
37 GSB 4 ⟶	98.65	($\hat{y}$)

Example 2:
(Exponential, Code = 6):

x_i	.72	1.31	1.95	2.58	3.14
y_i	2.16	1.61	1.16	.85	0.5

Solution:

$a = 3.45$, $b = -0.58$

$y = 3.45\ e^{-0.58x}$

$r^2 = 0.98$

For $x = 1.5$, $\hat{y} = 1.44$

Example 3:
(Logarithmic, Code = 7):

x_i	3	4	6	10	12
y_i	1.5	9.3	23.4	45.8	60.1

Solution:

$a = -47.02$, $b = 41.39$

$y = -47.02 + 41.39 \ln x$

$r^2 = 0.98$

For $x = 8$, $\hat{y} = 39.06$

For $x = 14.5$, $\hat{y} = 63.67$

Example 4:
(Power, Code = 8):

x_i	10	12	15	17	20	22	25	27	30	32	35
y_i	0.95	1.05	1.25	1.41	1.73	2.00	2.53	2.98	3.85	4.59	6.02

Solution:

$a = .03$, $b = 1.46$

$y = .03x^{1.46}$

$r^2 = 0.94$

For $x = 18$, $\hat{y} = 1.76$

For $x = 23$, $\hat{y} = 2.52$

NORMAL AND INVERSE NORMAL DISTRIBUTION

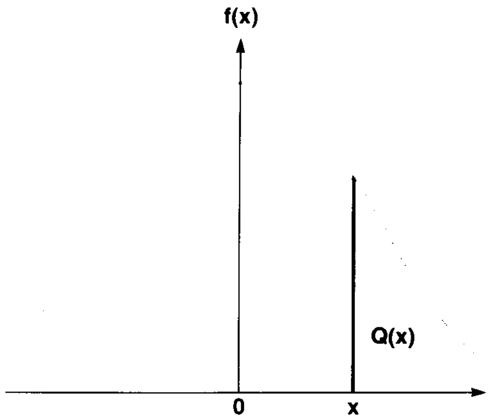
This program evaluates the standard normal density function f(x) and the normal integral Q(x) for given x. If Q is given, x can also be found.

The standard normal distribution has mean 0 and standard deviation 1.

Equations:

1. Standard normal density

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



2. Normal integral

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$$

Polynomial approximation is used to compute Q(x) for given x.

Define $R = f(x)\,(b_1t + b_2\,t^2 + b_3t^3 + b_4t^4 + b_5t^5) + \epsilon(x)$

where:

$$|\epsilon(x)| < 7.5 \times 10^{-8}$$

$$t = \frac{1}{1 + r\,|x|}, \qquad r = 0.2316419$$

$b_1 = .319381530$

$b_2 = -.356563782$

$b_3 = 1.781477937$

$b_4 = -1.821255978$

$b_5 = 1.330274429$

$$\text{Then } Q(x) = \begin{cases} R & \text{if } x \geq 0 \\ 1 - R & \text{if } x < 0 \end{cases}$$

3. Inverse normal

For a given $Q > 0$, x can be found such that

$$Q = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$$

The following rational approximation is used:

$$\text{Define } y = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

where:

$$|\epsilon(Q)| < 4.5 \times 10^{-4}$$

$$t = \begin{cases} \sqrt{\ln \frac{1}{Q^2}} & \text{if } 0 < Q \leq 0.5 \\ \sqrt{\ln \frac{1}{(1-Q)^2}} & \text{if } 0.5 < Q < 1 \end{cases}$$

$c_0 = 2.515517$ $d_1 = 1.432788$

$c_1 = 0.802853$ $d_2 = 0.189269$

$c_2 = 0.010328$ $d_3 = 0.001308$

$$\text{Then } x = \begin{cases} y & \text{if } 0 < Q \leq 0.5 \\ -y & \text{if } 0.5 < Q < 1 \end{cases}$$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1970.

```
01 *LBL1                                   50 ENT↑
02 STO7                                    51 RCL5
03 X²                                      52 x
04 2                                       53 RCL4
05 ÷                                       54 GSB7
06 CHS                                     55 RCL3
07 eˣ                                      56 GSB7
08 Pi                                      57 RTN
09 2             Calculate f(x)            58 *LBL3        -----------
10 x                                       59 STO7
11 √X                                      60 .
12 ÷                                       61 5
13 STO9                                    62 X⇄Y
14 RTN           ------------              63 X>Y?
15 *LBL2                                   64 GSB8
16 GSB1                                    65 X²
17 1                                       66 1/X
18 RCL0                                    67 LN
19 RCL7                                    68 √X
20 ABS                                     69 STO8
21 x                                       70 GSB6
22 +                                       71 1
23 1/X           Calculate Q (x)           72 +
24 GSB6                                    73 STO9
25 RCL2                                    74 CLX          Calculate X
26 GSB7                                    75 RCL2
27 RCL1                                    76 x
28 GSB7                                    77 RCL1
29 RCL9                                    78 GSB7
30 x                                       79 RCL0
31 RCL7                                    80 +
32 X<0?                                    81 RCL9
33 GTO9                                    82 ÷
34 X⇄Y                                     83 -
35 RTN           ------------              84 STO6
36 *LBL9                                   85 RCL7
37 X⇄Y                                     86 .
38 *LBL8                                   87 5
39 1                                       88 X⇄Y
40 -                                       89 X>Y?
41 CHS                                     90 GTO5
42 RTN                                     91 RCL6
43 *LBL7         Subroutines               92 RTN
44 +                                       93 *LBL5
45 x                                       94 RCL6
46 RTN                                     95 CHS
47 *LBL6                                   96 RTN          -----------
48 ENT↑
49 ENT↑
```

REGISTERS

0 r, C_0	1 b_1, C_1	2 b_2, C_2	3 b_3, d_1	4 b_4, d_2	5 b_5, d_3
6 y	7 x, Q	8 t	9 f(x), deno.	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
	i. Normal distribution: from program step 1 to step 57.			
	ii. Inverse normal distribution: from program step 38 to step 96.			
	iii. Both: the entire program.			
2	For normal distribution, go to step 3, for inverse, go to step 7.			
3	Store constants for normal distribution.	r	STO 0	r
		b_1	STO 1	b_1
		b_2	STO 2	b_2
		b_3	STO 3	b_3
		b_4	STO 4	b_4
		b_5	STO 5	b_5
4	Optional: Input x to calculate f(x)	x	GSB 1	f(x)
5	Input x to calculate Q(x)	x	GSB 2	Q(x)
6	For a new x, go to step 4 or step 5.			
7	Store constants for inverse	c_0	STO 0	c_0
		c_1	STO 1	c_1
		c_2	STO 2	c_2
		d_1	STO 3	d_1
		d_2	STO 4	d_2
		d_3	STO 5	d_3
8	Input Q(x) to calculate x.	Q(x)	GSB 3	x
9	For a new Q(x), go to step 8.			

Example 1:

(Normal distribution):

Find f(x) and Q(x) for $x = 1.18$ and $x = -2.28$

Keystrokes:	**Outputs:**	
(Key in the program as shown in the Instructions)		
0.2316419 STO 0 0.31938153		
STO 1 0.356563782 CHS STO		
2 1.781477937 STO 3		
1.821255978 CHS STO 4		
1.330274429 STO 5		
1.18 GSB 1 ⟶	0.20	(f(1.18))
1.18 GSB 2 ⟶	0.12	(Q(1.18))
2.28 CHS GSB 2 ⟶	0.99	(Q(−2.28))
2.28 CHS GSB 1 ⟶	0.03	(f(−2.28))

Example 2:

(Inverse):

Given Q = 0.12 and Q = 0.95, find x's

Keystrokes:	**Outputs:**	
(Key in program as shown in the Instructions)		
2.515517 STO 0 0.802853		
STO 1 0.010328 STO 2		
1.432788 STO 3 0.189269		
STO 4 0.001308 STO 5		
0.12 GSB 3 ⟶	1.18	(x)
0.95 GSB 3 ⟶	−1.65	(x)

FACTORIAL, PERMUTATION AND COMBINATION

$$\text{Factorial} \quad n! = n\,(n-1)\,(n-2) \cdot \cdot \cdot 2 \cdot 1$$

$$\text{Permutation} \quad {}_mP_n = \frac{m!}{(m-n)!} = m(m-1)\ldots(m-n+1)$$

$$\text{Combination} \quad {}_mC_n = \frac{m!}{(m-n)!\,n!} = \frac{m(m-1)\ldots(m-n+1)}{1 \cdot 2 \cdot \ldots \cdot n}$$

where m, n are integers and $0 \leqslant n \leqslant m$.

Remarks:

This program will compute factorials for positive integers between 2 and 69.

$$n! = n\,(n-1)\,(n-2)\,\ldots\,(2)\,(1)$$

For large values of n, the program will take some time to arrive at a result, up to a maximum of about 20 seconds for n = 69.

The program does not check input values and will return incorrect answers for values of $n < 2$ or $n > 69$ or non-integer n.

${}_mP_0 = 1$, ${}_mP_1 = m$, ${}_mP_m = m!$ Therefore n! should be used for large m.

${}_mC_0 = {}_mC_m = 1$

${}_mC_1 = {}_mC_{m-1} = m$

${}_mC_n = {}_mC_{m-n}$

01 *LBL1 02 1 03 ST.0 04 X⇄Y 05 *LBL4 06 Sx.0 07 1 08 - 09 X≠Y? 10 GTO4 11 RC.0 12 RTN	Calculate n! ***	50 GTO8 51 - 52 LSTX 53 X≤Y? 54 GSB9 55 STO1 56 1 57 STO0 58 + 59 STO2 60 CLX 61 X=Y? 62 GTO7 63 *LBL0 64 R↓ 65 ISZ 66 RCL0 67 X>Y? 68 GTO8 69 RCL1 70 X⇄Y 71 + 72 LSTX 73 ÷ 74 RCL2 75 × 76 STO2 77 GTO0	Calculate $_mC_n$
13 *LBL2 14 X>Y? 15 GTO8 16 ENT↑ 17 0 18 X=Y? 19 GTO7 20 CLX 21 1 22 X=Y? 23 GTO6 24 - 25 STO0 26 R↓ 27 STO1 28 *LBL5 29 RCL1 30 1 31 - 32 STO1 33 × 34 DSZ 35 GTO5 36 RTN	Calculate $_mP_n$ ***	78 *LBL9 79 STO2 80 X⇄Y 81 RTN	$_mC_n = {_mC_{m-n}}$
37 *LBL6 38 R↓ 39 R↓ 40 RTN	$_mP_1$ = m	82 *LBL8 83 RCL2 84 RTN	***
41 *LBL7 42 ENT↑ 43 1 44 RTN	*** 1		
45 *LBL8 46 0 47 ÷	Error		
48 *LBL3 49 X>Y?			

REGISTERS

0 n!	1 m,	2 Used	3	4	5
6	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program. **i.** Factorial: from program step 1 to step 12. **ii.** Permutation: from program step 12 to step 47. **iii.** Combination: from program step 41 to step 84.			
2	For factorial, go to step 3, for permutation go to step 5, for combination go to step 7.			
3	Input n to calculate n!	n	GSB 1	n!
4	Repeat step 3 for another n.			
5	Input m and n to calculate $_mP_n$	m	ENTER↑	
		n	GSB 2	$_mP_n$
6	Repeat step 5 for a different set of m and n.			
7	Input m and n to calculate $_mC_n$	m	ENTER↑	
		n	GSB 3	$_mC_n$
8	Repeat step 7 for a different set of m and n.			

Example 1:

(Factorial):

Find n! for n = 5 and n = 10

Keystrokes: **Outputs:**

(Key in the program as shown in the Instructions)

5 GSB 1 ⟶ 120.00 (5!)
10 GSB 1 ⟶ 3628800.00 (10!)

Example 2:

(Permutation):

Find $_{43}P_3$ and $_{73}P_4$.

Keystrokes:	**Outputs:**	
(Key in the program as shown in the Instructions)		
43 ENTER↑ 3 GSB 2 ⟶	74046.00	($_{43}P_3$)
73 ENTER↑ 4 GSB 2 ⟶	26122320.00	($_{73}P_4$)

Example 3:

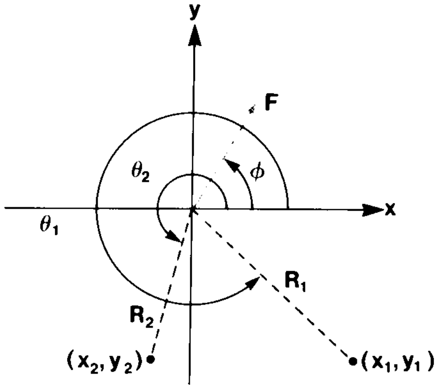
(Combination):

Find $_{73}C_4$ and $_{43}C_3$.

Keystrokes:	**Outputs:**	
(Key in the program as shown in the Instructions)		
73 ENTER↑ 4 GSB 3 ⟶	1088430.00	($_{73}C_4$)
43 ENTER↑ 3 GSB 3 ⟶	12341.00	($_{43}C_3$)

STATIC EQUILIBRIUM AT A POINT

This program calculates the two reaction forces necessary to balance a given two-dimensional force vector. The direction of the reaction forces may be specified as a vector of arbitrary length or by Cartesian coordinates using the point of force application as the origin.



Equations:

$$R_1 \cos \theta_1 + R_2 \cos \theta_2 = F \cos \phi$$
$$R_1 \sin \theta_1 + R_2 \sin \theta_2 = F \sin \phi$$

where:

F is the known force;

ϕ is the direction of the known force;

R_1 is one reaction force;

θ_1 is the direction of R_1;

R_2 is the second reaction force;

θ_2 is the direction of R_2;

The coordinates x_1 and y_1 are referenced from the point where F is applied to the end of the member along which R_1 acts; x_2 and y_2 are the coordinates referenced from the point where F is applied to the end of the member along which R_2 acts.

Remarks:

This program assumes the calculator is set in DEG mode.

Step	Key	Comments
01	*LBL1	Input y_1, x_1
02	→P	
03	X⇄Y	
04	*LBL2	
05	1	Input θ_1, and store $\sin\theta_1$, $\cos\theta_1$
06	→R	
07	STO0	
08	X⇄Y	
09	STO1	
10	RTN	
11	*LBL3	Input y_2, x_2
12	→P	
13	X⇄Y	
14	*LBL4	
15	1	
16	→R	Input θ_2 and store $\sin\theta_2$, $\cos\theta_2$
17	STO2	
18	X⇄Y	
19	STO3	
20	RTN	
21	*LBL5	
22	→R	
23	STO4	
24	X⇄Y	
25	STO5	
26	RCL4	
27	RCL3	
28	x	
29	RCL5	
30	RCL2	
31	x	
32	-	
33	RCL1	Input ϕ and F and calculate reaction forces
34	RCL2	
35	x	
36	RCL0	
37	RCL3	
38	x	
39	-	
40	÷	
41	R/S	***
42	LSTX	
43	STO6	
44	RCL5	
45	RCL0	
46	x	
47	RCL4	
48	RCL1	
49	x	
50	-	
51	RCL6	
52	÷	***
53	RTN	

REGISTERS

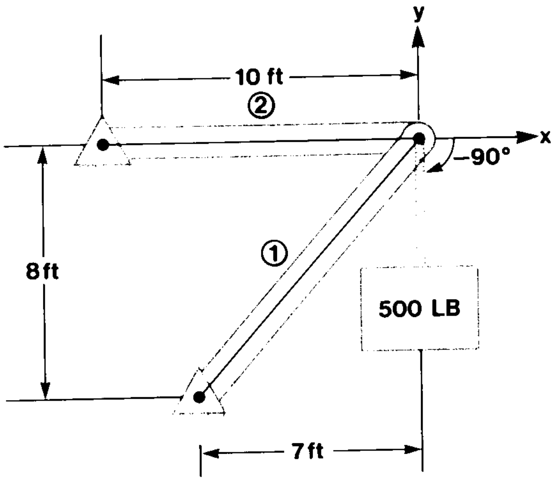
0 $\cos\theta_1$	1 $\sin\theta_1$	2 $\cos\theta_2$	3 $\sin\theta_2$	4 F $\cos\phi$	5 F $\sin\phi$
6 Used	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Define reaction directions as Cartesian coordinates or as vectors of arbitrary magnitude. (Use the point of force application as the origin):			
	Define direction one in rectangular form	y_1	ENTER↑	y_1
		x_1	GSB 1	$\sin \theta_1$
	or in polar form	θ_1	GSB 2	$\sin \theta_1$
	and			
	Define direction two in rectangular form	y_2	ENTER↑	
		x_2	GSB 3	$\sin \theta_2$
	or in polar form	θ_2	GSB 4	$\sin \theta_2$
3	Key in known force: direction *then* magnitude and compute reactions.	ϕ	ENTER↑	
		F	GSB 5	R_1
			R/S	R_2
4	To change force, go to step 3. To change either or both reaction directions, go to step 2.			

Example 1:

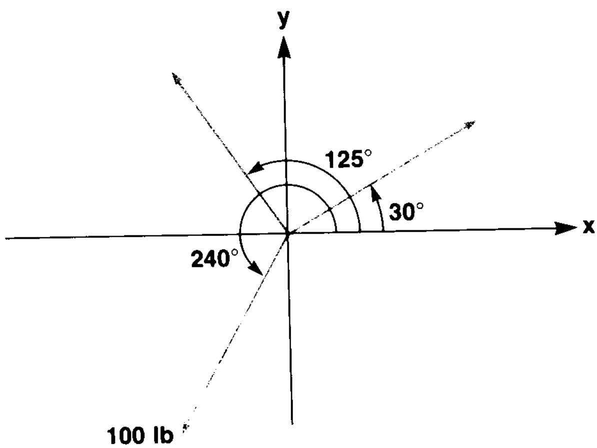
Find the reaction forces in the pin-jointed structure shown below.



Keystrokes:	**Outputs:**	
8 CHS ENTER↑ 7 CHS		
GSB 1 ⟶	-0.75	
0 ENTER↑ 10 CHS GSB 3 ⟶	0.00	
90 CHS ENTER↑ 500 GSB 5 ⟶	-664.38	(R_1)
R/S ⟶	437.50	(R_2)

Example 2:

Find the reaction forces for the diagram below:



Keystrokes:	**Outputs:**	
30 GSB 2 →	0.50	
125 GSB 4 →	0.82	
240 ENTER↑ 100 GSB 5 →	90.98	(R_1)
R/S →	50.19	(R_2)

SECTION PROPERTIES

The properties of arbitrarily shaped sections which are composed of rectangles can be evaluated using this program.

The program calculates the area of the section, the centroid of the area, the moments of inertia about any specified set of axes, the polar moment of inertia about the specified axis, the moments of inertia about an axis translated to the centroid, the moments of inertia of the principal axis, and the rotation angle between the translated axis and the principal axis.

Equations:

$$A_{si} = \Delta x_i \, \Delta y_i$$

$$A = S_{s1} + A_{s2} + A_{s3} + \ldots + A_{sn}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_{oi} A_{si}}{A}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_{oi} A_{si}}{A}$$

$$I_x = \sum_{i=1}^{n} \left(y_{oi}^2 + \frac{\Delta y_i^2}{12} \right) A_{si}$$

$$I_y = \sum_{i=1}^{n} \left(x_{oi}^2 + \frac{\Delta x_i^2}{12} \right) A_{si}$$

$$J = I_x + I_y$$

$$I_{xy} = \sum_{i=1}^{n} x_{oi} \, y_{oi} \, A_{si}$$

$$I_{\bar{x}} = I_x - A\bar{y}^2$$

$$I_{\bar{x}\bar{y}} = I_{xy} - A\bar{x}\bar{y}$$

$$I_{\bar{y}} = I_y - A\bar{x}^2$$

$$\phi = \frac{1}{2} \tan^{-1} \frac{-2 \, I_{\bar{x}\bar{y}}}{I_{\bar{x}} - I_{\bar{y}}}$$

where:

Δx_i is the width of a rectangular element;

Δy_i is the height of a rectangular element;

A_{si} is the area of an element;

A is the total area of the section;

$\bar{x}$ is the x coordinate of the centroid;

$\bar{y}$ is the y coordinate of the centroid;

x_{oi} is the x coordinate of the centroid of an element;

y_{oi} is the y coordinate of the centroid of an element;

I_x is the moment of inertia about the x-axis;

I_y is the moment of inertia about the y-axis;

J is the moment of inertia about the origin;

I_{xy} is the product of inertia;

$I_{\bar{x}}$ is the moment of inertia about the x-axis translated to the centroid;

$I_{\bar{y}}$ is the moment of inertia about the y-axis translated to the centroid;

$I_{\bar{x}\bar{y}}$ is the product of inertia about the translated axis;

ϕ is the angle between the translated axis and the principal axis;

Reference:

Wojciechowski, Felix; "Properties of Plane Cross Sections"; *Machine Design;* P. 105, Jan 22, 1976.

Step	Key	Description	Step	Key	Description
01	*LBL0		50	RCL5	
02	STO4		51	RCL0	
03	R↓		52	÷	Calculate $\bar{x}$, $\bar{y}$, and A
04	STO3		53	RTN	
05	R↓		54	*LBL9	
06	STO2		55	RCL6	
07	X⇄Y	Input Δx, Δy, x_{oi}, and y_{oi} and calculate	56	RCL0	
08	STO1		57	÷	
09	x		58	RTN	
10	ST.5		59	RCL0	
11	ENT↑		60	RTN	
12	ST+0		61	*LBL2	
13	RCL3		62	RCL7	
14	x		63	GSB9	
15	ST+5		64	X²	
16	R↓		65	RCL0	
17	RCL4		66	x	
18	x		67	-	
19	ST+6		68	ST.2	Calculate $I_{\bar{x}}$, $I_{\bar{y}}$, and $I_{\bar{x}\bar{y}}$
20	RCL2		69	R/S	
21	X²		70	RCL8	
22	1		71	GSB1	
23	2		72	X²	
24	÷		73	RCL0	
25	RCL4		74	x	
26	X²		75	-	
27	+		76	ST.3	
28	RC.5		77	R/S	
29	x		78	RCL9	
30	ST+7		79	GSB1	
31	RCL1		80	GSB9	
32	X²		81	x	
33	1		82	RCL0	
34	2		83	x	
35	÷		84	-	
36	RCL3		85	ST.4	
37	X²		86	RTN	
38	+		87	*LBL3	
39	RC.5		88	RC.4	
40	x		89	2	
41	ST+8		90	x	
42	RCL3		91	RC.3	Calculate ϕ
43	RCL4		92	RC.2	
44	x		93	-	
45	RC.5		94	÷	
46	x		95	TAN⁻¹	
47	ST+9		96	2	
48	RTN		97	÷	
49	*LBL1		98	RTN	

REGISTERS

0 ΣA	1 Δx_i	2 Δy_i	3 x_{oi}	4 y_{oi}	5 $\Sigma x_{oi} A_{si}$
6 $\Sigma y_{oi} A_{si}$	7 ΣI_x	8 ΣI_y	9 ΣI_{xy}	.0	.1
.2 $I_{\bar{x}}$	.3 $I_{\bar{y}}$	.4 $I_{\bar{x}\bar{y}}$	.5 A_{si}	16	17
18	19	20	21	22	23
24	25	26	27	28	29

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Initialize		[f] [REG]	
3	Input Δx, Δy, x_{oi}, y_{oi}	Δx	[ENTER↑]	
		Δy	[ENTER↑]	
		x_{oi}	[ENTER↑]	
		y_{oi}	[GSB] [0]	
4	Repeat step 3 for more sections.			
5	To calculate $\bar{x}$, $\bar{y}$, A		[GSB] [1]	$\bar{x}$
			[R/S]	$\bar{y}$
			[R/S]	A
6	Optional: To recall I_x		[RCL] [7]	I_x
	I_y		[RCL] [8]	I_y
	I_{xy}		[RCL] [9]	I_{xy}
7	To calculate $I_{\bar{x}}$		[GSB] [2]	$I_{\bar{x}}$
	$I_{\bar{y}}$		[R/S]	$I_{\bar{y}}$
	$I_{\bar{x}\bar{y}}$		[R/S]	$I_{\bar{x}\bar{y}}$
8	To calculate ϕ		[GSB] [3]	ϕ
9	For a new case, go to step 2.			

Example 1:

Given the rectangle below, find $\bar{x}$, $\bar{y}$, A, I_x, I_y, I_{xy}, $I_{\bar{x}}$, $I_{\bar{y}}$, $I_{\bar{x}\bar{y}}$ and ϕ.

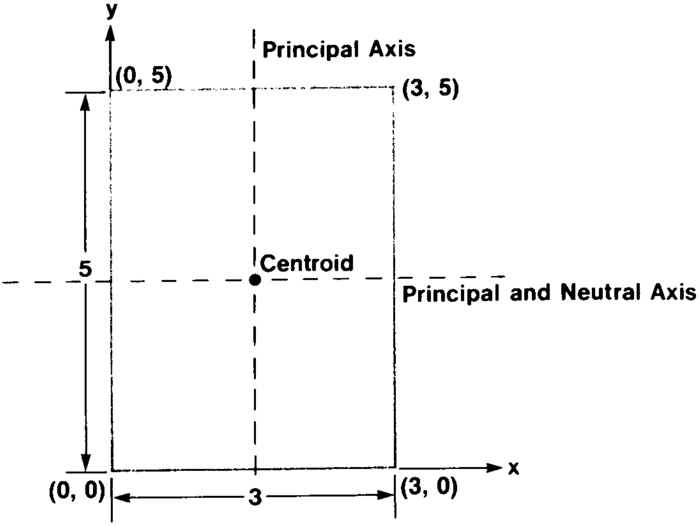


TABLE OF INPUTS

Section	Δx	Δy	x_o	y_o
1	3	5	1.5	2.5

Keystrokes:	Outputs:	
f REG 3 ENTER↑ 5 ENTER↑ 1.5		
ENTER↑ 2.5 GSB 0 →	56.25	
GSB 1 →	1.50	($\bar{x}$)
R/S →	2.50	($\bar{y}$)
R/S →	15.00	(A)
RCL 7 →	125.00	(I_x)
RCL 8 →	45.00	(I_y)
RCL 9 →	56.25	(I_{xy})
GSB 2 →	31.25	($I_{\bar{x}}$)
R/S →	11.25	($I_{\bar{y}}$)
R/S →	0.00	($I_{\bar{x}\bar{y}}$)
GSB 3 →	0.00	(ϕ)

Example 2:

Calculate the section properties for the beam shown below.

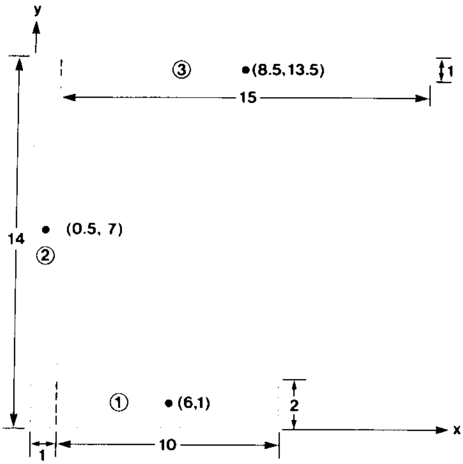


TABLE OF INPUTS

Section	Δx	Δy	x_{oi}	y_{oi}
1	10	2	6	1
2	1	14	0.5	7
3	15	1	8.5	13.5

Keystrokes:	**Outputs:**	
f REG 10 ENTER↑ 2 ENTER↑ 6		
ENTER↑ 1 GSB 0 →	120.00	
1 ENTER↑ 14 ENTER↑ 0.5 ENTER↑		
7 GSB 0 →	49.00	
15 ENTER↑ 1 ENTER↑ 8.5 ENTER↑		
13.5 GSB 0 →	1721.25	
GSB 1 →	5.19	($\bar{x}$)
R/S →	6.54	($\bar{y}$)
R/S →	49.00	(A)
RCL 7 →	3676.33	(I_x)
RCL 8 →	2256.33	(I_y)
RCL 9 →	1890.25	(I_{xy})
GSB 2 →	1580.00	($I_{\bar{x}}$)
R/S →	934.49	($I_{\bar{y}}$)
R/S →	225.61	($I_{\bar{x}\bar{y}}$)
GSB 3 →	−17.48	(ϕ)

FIELD ANGLE OR BEARING TRAVERSE

This program uses angles and/or deflections turned from a reference azimuth and horizontal distances, or quadrant bearings and horizontal distances, to compute the coordinates of successive points in a traverse. For a closed traverse, the area enclosed and closure distance and azimuth are computed.

(Note: Angles left and deflections left must be entered as negative numbers.)

Equations:

$$N_{i+1} = N_1 + HD \cos Az$$

$$E_{i+1} = E_1 + HD \sin Az$$

$$\text{Area} = \sum_{k=1}^{n} LAT_k \left(\tfrac{1}{2} DEP_k + \sum_{j=1}^{k-1} DEP_j \right)$$

where:

$DEP_k = E_{k+1} - E_k$ and $LAT_k = N_{k+1} - N_k$

Step	Key	Comment
01	*LBL1	Store starting point coordinates and 180°
02	FIX4	
03	CLRG	
04	STO1	
05	X≷Y	
06	STO2	
07	1	
08	8	
09	0	
10	STO3	
11	R/S	
12	→H	
13	RCL3	Reference azimuth
14	→H	
15	+	
16	GTO0	
17	*LBL2	
18	→H	Angle input
19	RCL3	
20	→H	
21	+	
22	→HMS	
23	*LBL3	Deflection angle input
24	→H	
25	RCL4	
26	+	
27	*LBL0	
28	1	
29	→R	Compute azimuth
30	→P	
31	*LBL9	
32	X≷Y	
33	X>0?	
34	GTO8	
35	3	
36	6	
37	0	
38	+	
39	*LBL8	
40	STO4	
41	→HMS	
42	R/S	***
43	ST+5	Input horizontal distance
44	RCL4	
45	X≷Y	
46	→R	
47	ST+6	
48	X≷Y	
49	ST+7	Compute next coord. and accumulate area.
50	2	
51	÷	
52	RCL7	
53	-	
54	x	
55	ST+8	
56	RCL6	
57	RCL2	
58	+	
59	R/S	***
60	RCL7	
61	RCL1	
62	+	
63	R/S	***
64	*LBL4	
65	X≷Y	
66	STO9	
67	X≷Y	Convert bearing and quadrant code to azimuth.
68	ENT↑	
69	ENT↑	
70	2	
71	÷	
72	INT	
73	RCL3	
74	x	
75	X≷Y	
76	RCL3	
77	x	
78	COS	
79	RCL9	
80	→H	
81	x	
82	-	
83	GTO0	
84	*LBL5	Area
85	RCL8	
86	ABS	
87	R/S	***
88	RCL7	
89	RCL6	
90	→P	Setup for closure
91	R/S	***
92	GTO9	

REGISTERS

0	1 Beg. E	2 Beg. N	3 180	4 Az	5 Σ HD
6 Lat.	7 Dep.	8 Area	9 Bearing	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

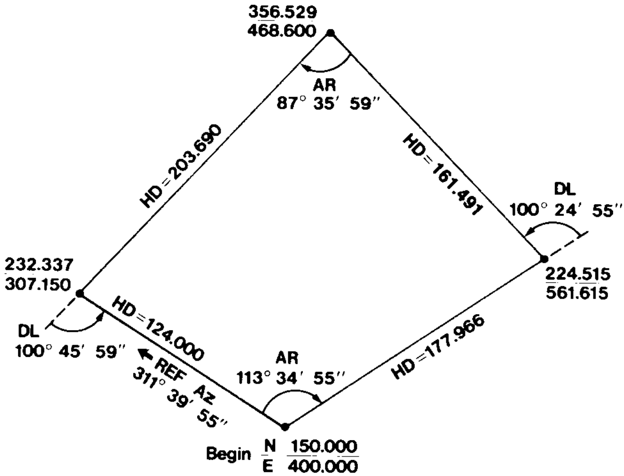
*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Key in beginning coordinates	BEG N	ENTER↑	
		BEG E	GSB 1	180.00
	For Field Angle Traverse			
3	Key in reference azimuth away			
	from beginning point.	REF AZ (D.MS)	R/S	Az (D.MS)
4	Key in field angle:			
	Angle right	ang. right	GSB 2	Az (D.MS)
	or Angle left (−)	-ang. left	GSB 2	Az (D.MS)
	or Deflection right	deflect. right	GSB 3	Az (D.MS)
	or Deflection left (−)	-deflect. left	GSB 3	Az (D.MS)
5	Key in horizontal distance and			
	compute coordinates	HD	R/S	N
			R/S	E
	or			
	For Bearing Traverse			
3′	Key in bearing and quadrant			
	code.	BRG (D.MS)	ENTER↑	
		QD	GSB 4	Az (D.MS)
4′	Key in horizontal distance and			
	compute coordinates.	HD	R/S	N
			R/S	E
	Repeat steps 3, 4, 5, or 3′, 4′ for			
	successive courses.			
6	For closed figure: Compute			
	area, error distance, and error			
	azimuth		GSB 5	Area
			R/S	Error Dist.
			R/S	Error Az (D.MS)

Example 1:

Field Angle Traverse

Traverse the figure below starting at $\frac{\text{N } 150}{\text{E } 400}$.

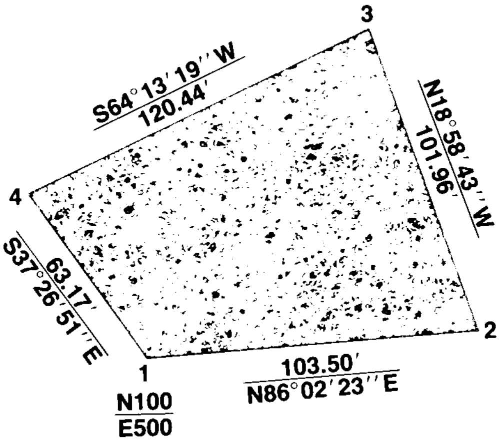


Keystrokes:	**Outputs:**	
150 [ENTER↑] 400 [GSB] [1] →	180.0000	
311.3955 [R/S] →	131.3955	
113.3455 [GSB] [2] →	65.1450	
177.966 [R/S] →	224.5150	(N)
[R/S] →	561.6150	(E)
100.2455 [CHS] [GSB] [3] →	324.4955	
161.491 [R/S] →	356.5285	(N)
[R/S] →	468.6000	(E)
87.3559 [GSB] [2] →	232.2554	
203.690 [R/S] →	232.3372	(N)
[R/S] →	307.1498	(E)
100.4559 [CHS] [GSB] [3] →	131.3955	
124 [R/S] →	149.9048	(N)
[R/S] →	399.7829	(E)
[GSB] [5] →	26558.8204	(Area)
[R/S] →	0.2371	(Error distance)
[R/S] →	246.1844	(Error azimuth)

Example 2:

Bearing Traverse

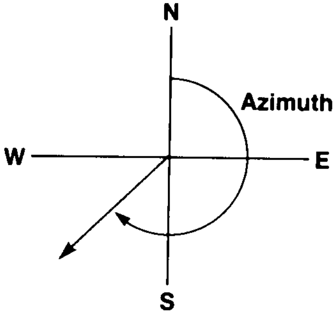
Traverse the figure below starting at $\frac{\text{N } 100}{\text{E } 500}$.

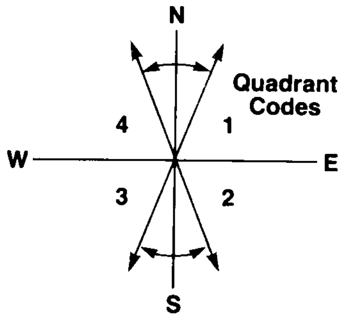


Keystrokes:	Outputs:	
100 ENTER↑ 500 GSB 1 →	180.0000	
86.0223 ENTER↑ 1 GSB 4 →	86.0223	
103.50 R/S →	107.1482	(N)
R/S →	603.2529	(E)
18.5843 ENTER↑ 4 GSB 4 →	341.0117	
101.96 R/S →	203.5657	(N)
R/S →	570.0939	(E)
64.1319 ENTER↑ 3 GSB 4 →	244.1319	
120.44 R/S →	151.1880	(N)
R/S →	461.6395	(E)
37.2651 ENTER↑ 2 GSB 4 →	142.3309	
63.17 R/S →	101.0366	(N)
R/S →	500.0490	(E)
GSB 5 →	8855.4922	(Area)
R/S →	1.0378	(Error distance)
R/S →	2.4219	(Error azimuth)

Remarks:

- If the user does not desire to do Field Angle Traverse, steps 012 through 026 may be eliminated; if he does not desire to do Bearing Traverse, steps 064 through 080 may be eliminated.
- Angles left and deflections left must be entered as negative numbers.
- This program assumes the calculator is set in DEG mode.





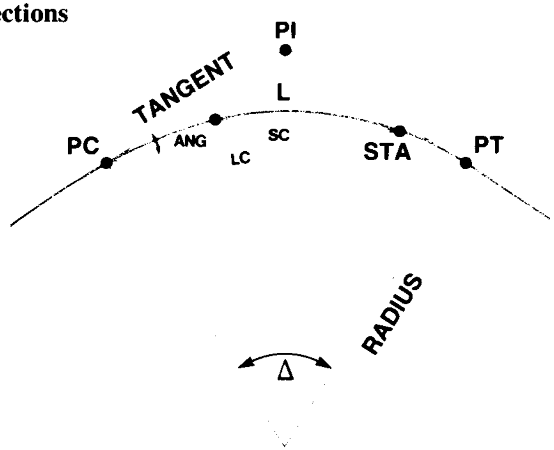
HORIZONTAL CURVE LAYOUT

This program calculates various field data for layout of a horizontal circular curve. The required information on the curve is the PC station and the radius or degree of curve. With this data one computes successively the arc length, deflection angle from tangent to long chord, the long chord from PC to current station, and the short chord from previous station to current station. In addition, the tangent offset and tangent distance are available if desired.

If the central angle is known the program also will compute the total arc length from PC to PT, the station PT and the length of the tangent from PC to PI.

In the program, stations are entered in the form XXXX.XX for station XX+XX.XX. For example: 20 + 10.00 is entered as 2010.00. The degree of curve D, (or central angle subtending an arc of 100 ft.) is entered in degrees with a negative sign, *always*.

PC Deflections



Field data output for PC deflections consist of:

STA-current station

ANG-deflection angle from tangent to long chord

LC-long chord from PC to current station

SC-short chord from previous station to current station

Δ-central angle

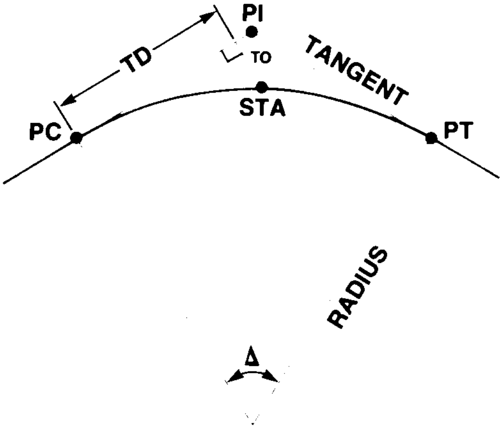
PI-point of intersection of tangents

PC, PT-ends of curve

L-Arc length

R-radius

Tangent Offsets and Distances



Field data output for tangent offsets consists of:

STA-current station

TD-tangent distance

TO-tangent offset

T-distance from PC to PI

Step	Key	Comment
01	*LBL1	
02	CLRG	
03	FIX4	Store R & D
04	X<0?	
05	GSB0	
06	STO1	
07	Pi	
08	x	
09	9	
10	0	
11	÷	
12	STO2	Input PC
13	R↓	
14	STO3	
15	STO4	
16	RTN	
17	*LBL0	
18	CHS	
19	→H	Calculate R from D
20	Pi	
21	x	
22	1/X	
23	1	
24	8	
25	EEX	
26	3	
27	x	
28	RTN	
29	*LBL2	Input station
30	RCL4	
31	ST.2	
32	R↓	
33	STO4	
34	RCL3	
35	-	
36	R/S	*** Calculate L
37	GSB9	
38	x	
39	STO7	
40	→HMS	
41	R/S	***
42	RCL7	Calculate ANG
43	SIN	
44	RCL1	
45	x	
46	2	
47	x	Calculate LC
48	STO5	
49	RCL7	
50	SIN	Calculate TO
51	x	
52	STO8	
53	RCL5	Calculate TD
54	RCL7	
55	COS	
56	x	
57	STO9	dsp LC
58	RCL5	
59	R/S	***
60	RCL4	
61	RC.2	
62	-	
63	GSB9	
64	x	
65	SIN	
66	RCL1	
67	2	
68	x	
69	x	*** Calculate SC
70	RTN	
71	*LBL9	
72	9	
73	0	$\frac{90}{\pi R}$
74	Pi	
75	÷	
76	RCL1	Input Δ
77	÷	
78	RTN	
79	*LBL3	
80	→H	
81	2	
82	÷	
83	STO6	
84	GSB9	
85	÷	Calculate L
86	R/S	***
87	RCL3	
88	+	Calculate PT
89	R/S	***
90	RCL6	
91	TAN	
92	RCL1	
93	x	*** Calculate T
94	R/S	

REGISTERS

0	1 R	2 Ft/Deflect	3 PC	4 STA Current	5 LC
6 Δ/2	7 ANG	8 TO	9 TD	.0	.1
.2 Prev. Sta.	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Input beginning station of curve	PC	ENTER↑	PC
3	Input radius	R	GSB 1	
	or degree of curve (as a negative number)	-D (D.MS)	GSB 1	
3′	Radius or degree of curve are available if desired.		RCL 1	R
			RCL 2	D
4	Input station	STA	GSB 2	L (Arc. length)
			R/S	def. angle
			R/S	long chord
			R/S	short chord
4′	Tangent offset, TO, and tangent distance, TD. are available if desired.		RCL 8	TO
			RCL 9	TD
5	Input central angle	Δ (D.MS)	GSB 3	Arc. length
			R/S	station PT
			R/S	T, length of tan.

Example:

Compute field data for a curve with a central angle of 35° 30′ and degree of curve of 12° 30′. The PC station is 7 + 85.40.

Keystrokes:	Outputs:	
785.40 ENTER↑ 12.30		
CHS GSB 1 ⟶	785.40	(PC)
RCL 1 (if desired) ⟶	458.3662	(R)
For Station 8:		
800 GSB 2 ⟶	14.6000	(L)
R/S ⟶	.5445	(ANG)
R/S ⟶	14.5994	(LC)
R/S ⟶	14.5994	(SC)
RCL 8 (if desired) ⟶	.2325	(TO)
RCL 9 (if desired) ⟶	14.5975	(TD)

For Station 9:

900 GSB 2 →	114.6000	(L)
R/S →	7.0945	(ANG)
R/S →	114.3018	(LC)
R/S →	99.8018	(SC)
RCL 8 →	14.2516	(TO)
RCL 9 →	113.4098	(TD)

For Station 10:

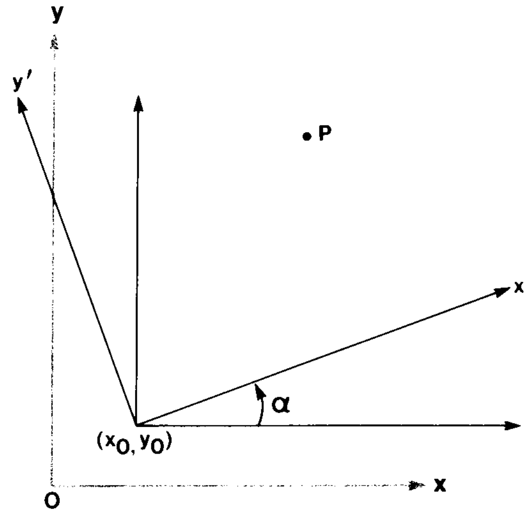
1000 GSB 2 →	214.6000	(L)
R/S →	13.2445	(ANG)
R/S →	212.6454	(LC)
R/S →	99.8018	(SC)
RCL 8 →	49.3252	(TO)
RCL 9 →	206.8455	(TD)
35.30 GSB 3 →	284.0000	(L)
R/S →	1069.4000	(PT)
R/S →	146.7242	(T)

Now calculate field data for PT:

1069.40 GSB 2 →	284.000	(L)
R/S →	17.4500	(ANG)
R/S →	279.4790	(LC)
R/S →	69.3337	(SC)

COORDINATE TRANSLATION AND ROTATION

This program allows for two-dimensional translation and rotation of coordinate axes. Suppose the origin of a coordinate system is translated to a new point, (x_0, y_0), and the x and y axes are rotated through an angle α to give new axes, x' and y'. A point P having coordinates (x, y) with respect to the old system of x and y axes, now has coordinates (x', y') with respect to the new axes. Given α and one pair of coordinates, the program allows you to find the other pair of coordinates.



Equations:

Let Rect (r, θ) denote the operation [→R] when r is in the X-register and θ is in the Y-register. Let Pol (x, y) denote the operation [→P] when x is in X and y is in Y.

Then $(x', y') = \text{Rect}\ (r, \theta - \alpha)$
where $(r, \theta) = \text{Pol}\ (x - x_0, y - y_0)$

and $(x, y) = (x_0, y_0) + \text{Rect}\ (r', \theta' + \alpha)$
where $(r', \theta') = \text{Pol}\ (x', y')$

Remarks:

- The program may be used to solve a problem of translation only, or of rotation only, or of combined translation and rotation. If the problem involves translation alone, a value of $\alpha = 0$ must be input. For rotation alone, the values $x_0 = y_0 = 0$ must be input.
- The program assumes the following sign convention: α should be input as a positive number if the rotation is counterclockwise, and negative if clockwise.
- This program assumes the calculator is set in DEG mode.

01 *LBL0			
02 STO7	$x_0 \uparrow y_0$		
03 R↓	Stored		
04 STO9			
05 R↓			
06 STO8			
07 RTN	- - - - - - - - - - - -		
08 *LBL1			
09 RCL9	Convert x, y to x′, y′		
10 -			
11 X⇄Y			
12 RCL8			
13 -			
14 →P			
15 X⇄Y			
16 RCL7			
17 -			
18 X⇄Y			
19 →R			
20 R/S	***		
21 X⇄Y	***		
22 RTN	- - - - - - - - - - - -		
23 *LBL2			
24 X⇄Y	Convert x′, y′ to x, y		
25 →P			
26 X⇄Y			
27 RCL7			
28 +			
29 X⇄Y			
30 →R			
31 RCL8			
32 +			
33 R/S	***		
34 X⇄Y			
35 RCL9			
36 +	***		
37 RTN			

REGISTERS

0	1	2	3	4	5
6	7 θ	8 x_0	9 y_0	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Initialize:			
	Key in new origin	x_0	ENTER↑	
		y_0	ENTER↑	
	Key in angle of rotation (observe proper sign)	α	GSB [0]	x_0
3	Key in old coordinates and calculate coordinates in new system	x	ENTER↑	
		y	GSB [1]	x′
			[R/S]	y′
4	Key in new coordinates and calculate coordinates in old system	x′	ENTER↑	
		y′	GSB [2]	x
			[R/S]	y

Example 1:

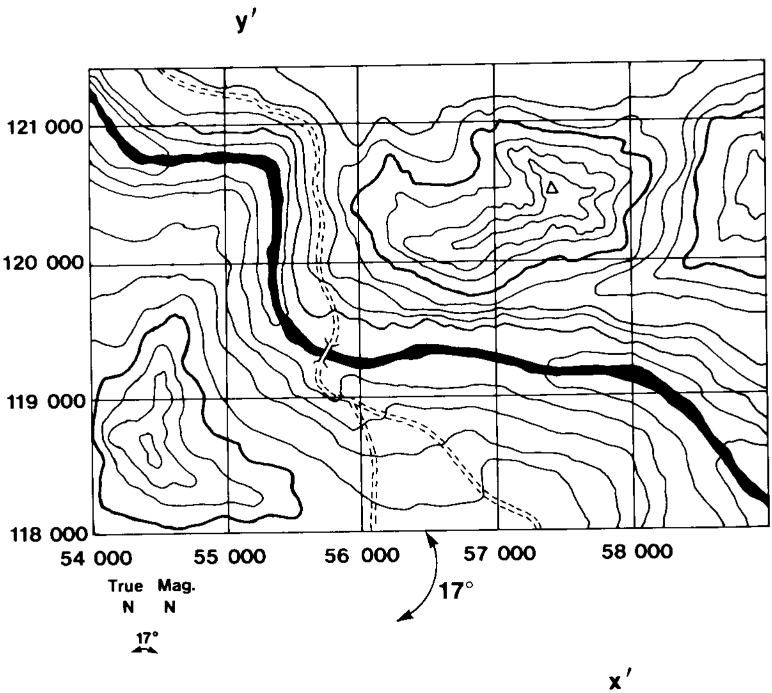
The origin of a coordinate system is translated to the point (–1, 4) and rotated 30° in a positive (counterclockwise) direction. Find the new coordinates of the point whose coordinates in the old system are (1, 3). If the coordinates of a point in the new system are (5, 7), what are its coordinates in the old system?

Keystrokes:	Outputs:	
1 CHS ENTER↑ 4 ENTER↑		
30 GSB [0] ⟶	–1.00	
1 ENTER↑ 3 GSB [1] ⟶	1.23	(x′)
[R/S] ⟶	–1.87	(y′)
5 ENTER↑ 7 GSB [2] ⟶	–0.17	(x)
[R/S] ⟶	12.56	(y)

Example 2:

Backpacker Will B. Bushed's route will take him cross-country from the marked trails of an area. He knows that he will have to check his compass frequently against his map over this terrain, and regrets that the map is in such an inconvenient format for his purposes. In the first place, the grid lines on his map represent distances in feet from an origin about 25 miles away, resulting in such large numbers that the calculations are difficult. Secondly, the map's grid is based on true north while his compass readings are relative to magnetic north, a variation of 17°.

Before he leaves home, Bushed decides to draw a rough version of the map for his own convenience, locating his origin at the grid point (54 000, 118 000) and rotating his axes by 17° in a clockwise direction. As a first step, he wants to find the new coordinates of the bridge and the peak of the hill, whose coordinates in the old system are (55 750, 119 300) and (57 450, 120 500) respectively.



Keystrokes:	**Outputs:**
54000 ENTER↑ 118000 ENTER↑	
17 CHS GSB 0 55750	
ENTER↑ 119300 GSB 1 ⟶	1293.45
R/S ⟶	1754.85

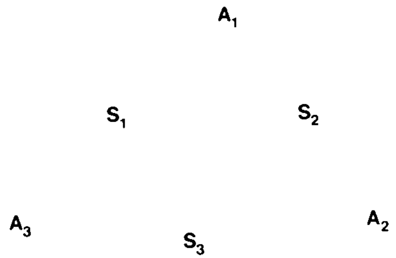
The new coordinates of the bridge are (1293, 1755).

57450 ENTER↑ 120500	
GSB 1 ⟶	2568.32
R/S ⟶	3399.44

The new coordinates of the peak are (2568, 3399).

TRIANGLE SOLUTIONS

This program may be used to find the sides, the angles, and the area of a plane triangle.



In general, the specification of any three of the six parameters of a triangle (3 sides, 3 angles) is sufficient to define the triangle. (The exception is that three angles will not define a triangle.) There are thus five possible cases that this program will handle: two sides and the included angle (SAS), two angles and the included side (ASA), two sides and the adjacent angle (SSA—an ambiguous case), two angles and the adjacent side (AAS), and three sides (SSS).

The results are stored in storage registers 0 through 6 as follows:

AREA	Register 0
SIDE 1	Register 1
ANGLE 1	Register 2
SIDE 2	Register 3
ANGLE 2	Register 4
SIDE 3	Register 5
ANGLE 3	Register 6

Equations:

SAS (S_1, A_1, S_2):

$$S_3 = \sqrt{S_1^2 + S_2^2 - 2\,S_1\,S_2 \cos A_1}$$

$$A_2 = \tan^{-1}\frac{S_1 \sin A_1}{S_2 - S_1 \cos A_1}$$

$$A_3 = \cos^{-1}\left[-\cos(A_1 + A_2)\right]$$

ASA (A_3, S_1, A_1):

$$S_2 = S_1\frac{\sin A_3}{\sin A_2} = S_1\frac{\sin A_3}{\sin(A_1 + A_3)}$$

Now go to SAS.

SSA (S_1, S_2, A_2):

$$A_3 = \sin^{-1}\left(\frac{S_2 \sin A_2}{S_1}\right)$$

$$A_1 = \cos^{-1}\left[-\cos(A_2 + A_3)\right]$$

Now go to SAS.

AAS (A_2, A_1, S_1):

$$S_2 = S_1 \frac{\sin A_3}{\sin A_2} = S_1 \frac{\sin(A_1 + A_2)}{\sin A_2}$$

Now go to SAS.

SSS (S_1, S_2, S_3):

$$A_1 = \cos^{-1}\left(\frac{S_1^2 + S_2^2 - S_3^2}{2 S_1 S_2}\right)$$

Now go to SAS.

$$\text{AREA: } = \tfrac{1}{2} S_1 S_3 \sin A_3.$$

Remarks:

Any angular mode may be used.

Note that the triangle described by the program does not conform to standard triangle notation; i.e., A_1 is not opposite S_1.

Angles must be entered as decimals. The [→H] conversion can be used to convert degrees, minutes, and seconds to decimal degrees.

Accuracy of solution may degenerate for triangles containing extremely small angles.

Step	Key	Comment
01	*LBL1	
02	RCL2	SAS, Calculate A_2, S_3 by law of cosines
03	RCL1	
04	→R	
05	RCL3	
06	X⇄Y	
07	-	
08	→P	
09	STO5	
10	X⇄Y	
11	STO4	
12	RCL2	
13	+	
14	GSB0	
15	STO6	
16	SIN	
17	x	Area
18	RCL1	
19	x	
20	2	
21	÷	
22	STO0	***
23	RTN	
24	*LBL2	
25	RCL1	
26	RCL3	SSS, Find A_1 by law of cosines then go to SAS
27	→P	
28	X²	
29	RCL5	
30	X²	
31	-	
32	RCL1	
33	RCL3	
34	x	
35	2	
36	x	
37	÷	
38	COS⁻¹	
39	STO2	
40	GTO1	
41	*LBL3	
42	RCL6	
43	SIN	
44	RCL2	ASA, Find S_2 then go to SAS
45	RCL6	
46	+	
47	SIN	
48	÷	
49	RCL1	

Step	Key	Comment
50	x	
51	STO3	
52	GTO1	
53	*LBL4	
54	RCL3	SSA, Find A_1, go to SAS (Solution #1)
55	RCL4	
56	SIN	
57	RCL1	
58	÷	
59	x	
60	SIN⁻¹	
61	RCL4	
62	+	
63	GSB0	
64	STO2	
65	GSB1	
66	RCL1	
67	RCL3	Two solution exist if $S_2 > S_1$
68	X≤Y?	
69	RTN	
70	R/S	Solve for Solution #2
71	RCL6	
72	GSB0	
73	STO6	
74	RCL4	
75	+	
76	GSB0	
77	STO2	
78	GTO1	
79	*LBL5	
80	RCL4	
81	RCL2	AAS, Find S_2 go to SAS
82	+	
83	SIN	
84	RCL4	
85	SIN	
86	÷	
87	RCL1	
88	x	
89	STO3	
90	GTO1	
91	*LBL0	
92	COS	
93	CHS	
94	COS⁻¹	
95	RTN	

REGISTERS

0 Area	1 Side 1	2 Angle 1	3 Side 2	4 Angle 2	5 Side 3
6 Angle 3	7	8	9	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

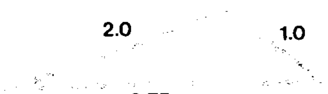
*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Select case and key in data:			
	2A: SAS (2 sides & included angle)			
	Side 1	S_1	STO 1	
	Angle 1	A_1	STO 2	
	Side 2	S_2	STO 3	
			GSB 1	output
	2B: SSS (3 sides)			
	Side 1	S_1	STO 1	
	Side 2	S_2	STO 3	
	Side 3	S_3	STO 5	
			GSB 2	output
	2C: ASA (2 angles & included side)			
	Angle 3	A_3	STO 6	
	Side 1	S_1	STO 1	
	Angle 1	A_1	STO 2	
			GSB 3	output
	2D: SSA (2 sides & adjacent angle) Side 1 (opposite side)	S_1	STO 1	
	Side 2 (adjacent side)	S_2	STO 3	
	Angle 2 (adjacent angle)	A_2	STO 4	
			GSB 4	solution # 1*
			R/S	solution # 2** (If it exists)
	2E: AAS (2 angles & adjacent side)			
	Angle 1 (adjacent angle)	A_1	STO 2	
	Angle 2 (opposite angle)	A_2	STO 4	
	Side 1 (adjacent side)	S_1	STO 1	
			GSB 5	output

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
3	Obtain solution by reviewing registers (use print reg. command if applicable).			
	recall reg. 0		RCL 0	area
	recall reg. 1		RCL 1	S_1
	recall reg. 2		RCL 2	A_1
	recall reg. 3		RCL 3	S_2
	recall reg. 4		RCL 4	A_2
	recall reg. 5		RCL 5	S_3
	recall reg. 6		RCL 6	A_3
	* Review registers at this point for solution #1.			
	** Press R/S, *once only*, for solution #2, (pressing R/S more than once will give erroneous results.)			

Example 1:

Find the angles and the area for the following triangle.



Keystrokes:

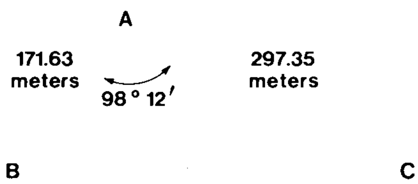
2 STO 1 1 STO 3
2.75 STO 5 GSB 2
(and recall registers) ⟶

Outputs:

Reg 0	0.77	(Area)
Reg 1	2.00	(S_1)
Reg 2	129.84	(A_1)
Reg 3	1.00	(S_2)
Reg 4	33.95	(A_2)
Reg 5	2.75	(S_3)
Reg 6	16.21	(A_3)

Example 2:

A surveyor is to find the area and dimensions of a triangular land parcel. From point A, the distances to B and C are measured with an electronic distance meter. The angle between AB and AC is also measured. Find the area and other dimensions of the triangle.



This is a side-angle-side problem where:

$$S_1 = 171.63,\ A_1 = 98°12' \text{ and } S_2 = 297.35.$$

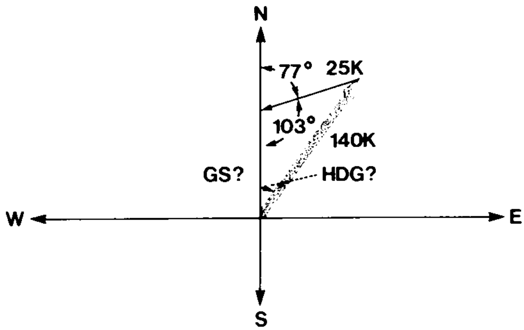
Keystrokes: **Outputs:**

171.63 [STO] [1] 98.12
[g] [→H] [STO] [2] 297.35
[STO] [3] [GSB] [1]

(and recall registers) ⟶	Reg 0	25256.21	(Area (m^2))
	Reg 1	171.63	(AB, m)
	Reg 2	98.20	(ANG. A)
	Reg 3	297.35	(AC, m)
	Reg 4	27.83	(ANG. C)
	Reg 5	363.91	(CB, m)
	Reg 6	53.97	(ANG. B)

Example 3:

A pilot wishes to fly due north. The wind is reported as 25 knots at 77°. Because winds are reported opposite to the direction they blow, this is interpreted as 77 + 180 or 257°. The true airspeed of the aircraft is 140 knots. What heading (HDG) should be flown? What is the ground speed (GS)?



By subtracting the wind direction from 180 (yielding an angle of 103°), the problem reduces to a S_1, S_2, A_2 triangle.

Keystrokes:		Outputs:	
140 STO 1 25 STO 3			
103 STO 4 GSB 4			
(and recall registers) ⟶		25.00	(Side 2)
	Reg 0	1610.64	
	Reg 1	140.00	(TAS)
	Reg 2	66.98	
	Reg 3	25.00	(WIND VEL.)
	Reg 4	103.00	
	Reg 5	132.24	(GS)
	Reg 6	10.02	(HDG)
R/S ⟶	No Operation (No Second Solution)		

Thus, the pilot should fly a heading 10.02° east due north. His ground speed equals 132.24 knots.

Example 4:

Two angles and an adjacent side of a triangle are known. Calculate the area of the triangle, the other two sides and the third angle. The known side is 19.6 ft. and the angle adjacent is 61.06°. The opposite angle is 40.25°.

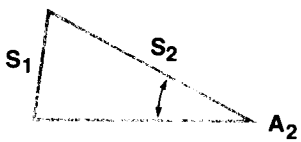
This is an AAS case where S_1 = 19.6 ft., A_1 = 61.06° and A_2 = 40.25°.

Keystrokes:		Outputs:	
19.6 STO 1 61.06 STO 2 40.25 STO 4 GSB 5 (and recall registers) →	Reg 0	255.11	(Area (ft^2))
	Reg 1	19.60	(S_1, ft)
	Reg 2	61.06	(A_1, deg)
	Reg 3	29.75	(S_2, ft)
	Reg 4	40.25	(A_2, deg)
	Reg 5	26.55	(S_3, ft)
	Reg 6	78.69	(A_3, deg)

Example 5:

Given 2 sides and a nonincluded angle solve for the triangle:

Side 1 = 25.6
Side 2 = 32.8
Angle 2 = 42.3°



(Note: Since $S_1 < S_2$ and $A_2 < 90°$, 2 solutions exist.)

Keystrokes:		Outputs:	
25.6 STO 1 32.8 STO 3 42.3 STO 4 GSB 4 (and recall registers) →	Reg 0	410.85	(Area)
	Reg 1	25.60	(S_1)
	Reg 2	78.12	(A_1)
(Solution #1)	Reg 3	32.80	(S_2)
	Reg 4	42.30	(A_2)
	Reg 5	37.22	(S_3)
	Reg 6	59.58	(A_3)
R/S (and recall registers) →	Reg 0	124.68	(Area)
	Reg 1	25.60	(S_1)
	Reg 2	17.28	(A_1)
	Reg 3	32.80	(S_2)
(Solution #2)	Reg 4	42.30	(A_2)
	Reg 5	11.30	(S_3)
	Reg 6	120.42	(A_3)

Example 6:

A triangle has angles of 64°32′ and 35°06′ with the included side 20.96 feet long. Solve for the remainder of the triangle.

Keystrokes:		**Outputs:**	
64.32 [g] [→H] [STO] [6] 20.96 [STO] [1] 35.06 [g] [→H] [STO] [2] [GSB] [3] (and recall registers) ——→	Reg 0	115.66	(Area (ft^2))
	Reg 1	20.96	(S_1, ft)
	Reg 2	35.10	(A_1)
	Reg 3	19.19	(S_2, ft)
	Reg 4	80.37	(A_2)
	Reg 5	12.22	(S_3, ft)
	Reg 6	64.53	(A_3)

CIRCLE DETERMINED BY THREE POINTS

This program calculates the center (x_0, y_0) and radius (r) of a circle given three non-collinear points.

Equations:

Circle determined by three points:

$$y_0 = \frac{K_2 - K_1}{N_2 - N_1} \bullet x_0 = K_2 - N_2 y_0$$

$$r = \sqrt{(x_3 - x_0)^2 + (y_3 - y_0)^2}$$

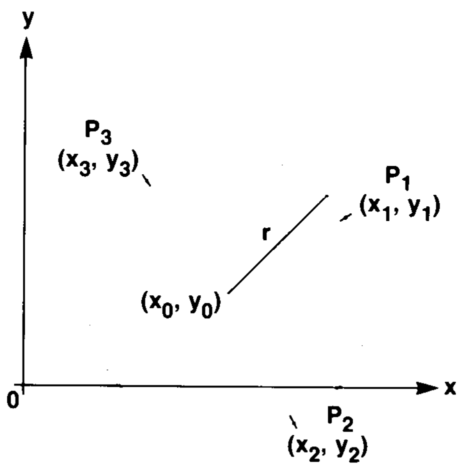
where:

$$K_1 = \frac{(x_2 - x_1)(x_2 + x_1) + (y_2 - y_1)(y_2 + y_1)}{2(x_2 - x_1)}$$

$$K_2 = \frac{(x_3 - x_1)(x_3 + x_1) + (y_3 - y_1)(y_3 + y_1)}{2(x_3 - x_1)}$$

$$N_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$N_2 = \frac{y_3 - y_1}{x_3 - x_1}$$



Remarks:

If $x_1 = x_2$ or $x_1 = x_3$ in the calculation of the center and radius of a circle, then point 1 replaces point 3, point 3 replaces point 2 and point 2 replaces point 1.

01	*LBL1		50	RTN	***
02	RCL3		51	*LBL6	
03	RCL1		52	RCL1	
04	X=Y?		53	RCL2	
05	GTO9		54	→P	
06	RCL5	Check for $x_1 = x_2$ or	55	X^2	
07	X=Y?	$x_1 = x_3$	56	STO0	
08	GTO8		57	RCL3	
09	*LBL5		58	RCL4	
10	GSB6		59	→P	
11	ST.3		60	X^2	
12	X⇄Y		61	RCL0	
13	STO7		62	-	Subroutine to
14	RCL3		63	RCL3	
15	RCL5		64	RCL1	Calculate K_1, K_2,
16	STO3		65	-	
17	X⇄Y		66	2	N_1 and N_2
18	STO5		67	×	
19	RCL4	Calculate y_o, x_o, or r	68	÷	
20	RCL6		69	RCL4	
21	STO4		70	RCL2	
22	X⇄Y		71	-	
23	STO6		72	RCL3	
24	GSB6		73	RCL1	
25	ST.4		74	-	
26	X⇄Y		75	÷	
27	STO8		76	RTN	
28	RCL7		77	*LBL9	----------
29	-		78	RCL5	
30	RC.4		79	X⇄Y	
31	RC.3		80	GTO8	Check for $x_1 = x_2 = x_3$
32	-		81	*LBL8	----------
33	÷		82	RCL1	
34	ST.2		83	RCL3	
35	RC.4		84	RCL5	
36	×		85	STO3	
37	RCL8		86	R↓	
38	-		87	STO1	
39	CHS		88	R↓	
40	ST.1		89	STO5	Swap for $x_1 = x_2$ or
41	R/S	***	90	RCL2	$x_1 = x_3$
42	RC.2		91	RCL4	
43	R/S	***	92	RCL6	
44	RCL4		93	STO4	
45	-		94	R↓	
46	RCL3		95	STO2	
47	RC.1		96	R↓	
48	-		97	STO6	
49	→P		98	GTO5	

REGISTERS

0 Used	1 x_1	2 y_1	3 x_2	4 y_2	5 x_3
6 y_3	7 k_1	8 k_2	9 r	.0	.1 x_o
.2 y_o	.3 N_1	.4 N_2	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Input x_1, y_1; x_2, y_2; x_3, y_3	x_1	STO 1	x_1
		y_1	STO 2	y_1
		x_2	STO 3	x_2
		y_2	STO 4	y_2
		x_3	STO 5	x_3
		y_3	STO 6	y_3
3	Calculate x_0, y_0, and r		GSB 1	x_0
			R/S	y_0
			R/S	r
4	For a new case go to step 2.			

Example:

What circle contains the points (1, 1), (3.5, −7.6) and (12, 0.8)?

Keystrokes:	Outputs:	
1 STO 1 1 STO 2 3.5		
STO 3 7.6 CHS STO 4 12		
STO 5 0.8 STO 6		
GSB 1 ——→	6.45	(x_0)
R/S ——→	−2.08	(y_0)
R/S ——→	6.26	(r)

INTERSECTIONS OF LINES AND LINES, LINES AND CIRCLES, AND CIRCLES AND CIRCLES

This program calculates the point of intersection of two lines, the points of intersection of a coplanar circle and line, or the points of intersection of two coplanar circles.

There are three sub-programs, i.e.,

1. Calculates intersections of lines and lines.

   Lines may be specified by two points (x_1, y_1, and x_2, y_2), or by one point and an angle (θ), where θ is the angle from the positive x-axis to the line.

2. Calculates intersections of circles and lines.

   Lines are specified by two points (x_1, y_1, and x_2, y_2).

   Circles are specified by their center coordinates (x_0, y_0) and the radius (r).

3. Calculates intersections of circles and circles.

   Circles are specified by their center coordinates (x_0, y_0) and the radius (r).

Equations:

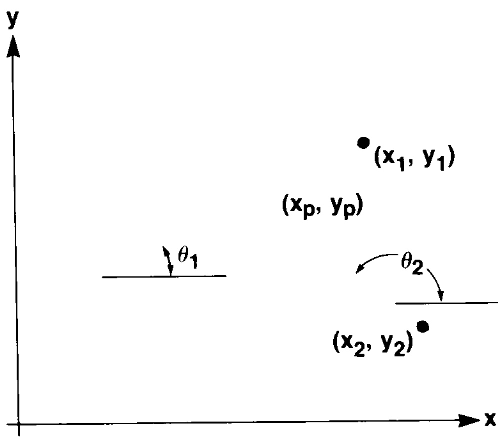
Line-Line Intersection:

$$x_p = \frac{x_1 \tan\theta_1 - x_2 \tan\theta_2 + y_2 - y_1}{\tan\theta_1 - \tan\theta_2}$$

$$y_p = y_1 + (x - x_1)\tan\theta_1$$

$$y_1 = x_1 \tan\theta_1 + C_1$$

$$y_2 = x_2 \tan\theta_2 + C_2$$



Circle-Line Intersections:

$$x_{p1} = x_1 + P_1\cos\theta$$
$$y_{p1} = y_1 + P_1\sin\theta$$
$$x_{p2} = x_1 + P_2\cos\theta$$
$$y_{p2} = y_1 + P_2\sin\theta$$

where:

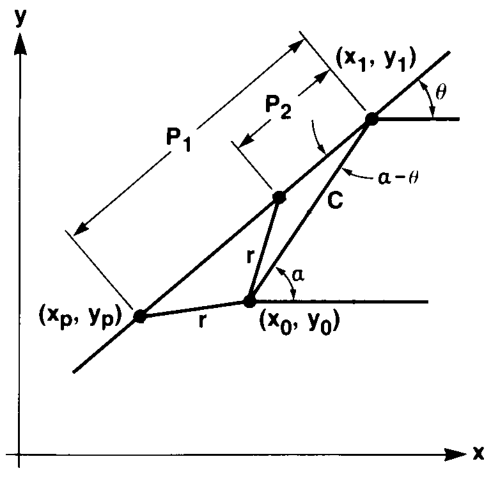
P_1 and P_2 are the roots of

$$P^2 - 2\,D\cos(\theta - \alpha)\,P + D^2 - r^2 = 0$$

$$\theta = \tan^{-1}\left[\frac{y_2 - y_1}{x_2 - x_1}\right]$$

$$\alpha = \tan^{-1}\left[\frac{y_0 - y_1}{x_0 - x_1}\right]$$

$$D = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}$$



Circle-Circle Intersections:

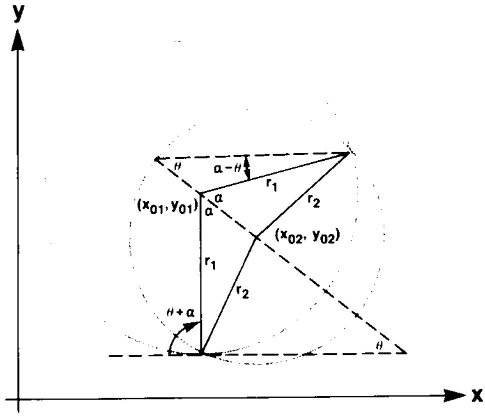
$$x_{p1} = x_{01} + r_1\cos(\theta + \alpha)$$
$$y_{p1} = y_{01} + r_1\sin(\theta + \alpha)$$
$$x_{p2} = x_{01} + r_1\cos(\theta - \alpha)$$

$$y_{p2} = y_{01} + r_1 \sin(\theta - \alpha)$$

$$\theta = \tan^{-1}\left(\frac{y_{02} - y_{01}}{x_{02} - x_{01}}\right)$$

$$\alpha = \cos^{-1}\left[\frac{D^2 + r_1{}^2 - r_2{}^2}{2Dr_1}\right]$$

$$D = \sqrt{(x_{02} - x_{01})^2 + (y_{02} - y_{01})^2}$$



Intersections Part 1: Line-Line

Steps	Description
01 *LBL3 02 STO3 03 R↑ 04 STO2 05 R↓ 06 STO1 07 GTO9	Input one point and angle for the 1st point
08 *LBL4 09 STO7 10 R↓ 11 STO6 12 R↑ 13 STO5 14 GTO8	Input one point and angle for the 2nd point
15 *LBL1 16 STO2 17 X⇄Y 18 STO1 19 GSB6 20 STO3 21 *LBL9 22 RCL2 23 RCL1 24 RCL3 25 TAN 26 × 27 - 28 STO4 29 RTN	Input coordinates of the 1st point
30 *LBL2 31 STO6 32 X⇄Y 33 STO5 34 GSB6 35 STO7 36 *LBL8 37 RCL6 38 RCL5 39 RCL7 40 TAN 41 × 42 - 43 STO8	Input coordinates of the 2nd point.
44 RTN 45 *LBL6 46 R↓ 47 - 48 X⇄Y 49 RCL0 50 - 51 →P 52 R↓ 53 RTN	Subroutine to find the slope and constant.
54 *LBL5 55 RCL7 56 RCL3 57 X=Y? 58 GTO0 59 ABS 60 RC.0 61 X≠Y? 62 GTO6 63 RCL1 64 R/S 65 RCL7 66 TAN 67 × 68 RCL8 69 + 70 RTN	To calculate x_p, y_p
71 *LBL6 72 RCL7 73 ABS 74 RC.0 75 X≠Y? 76 GTO7 77 RCL5 78 R/S 79 GTO9	The 1st line is vertical.
80 *LBL7 81 RCL8 82 RCL4 83 - 84 RCL3 85 TAN 86 RCL7 87 TAN 88 - 89 ÷ 90 R/S 91 *LBL9 92 RCL3 93 TAN 94 × 95 RCL4 96 + 97 RTN	The 2nd line is vertical.

REGISTERS

0 temp x_2	1 x_1'	2 y_1'	3 θ_1	4 c_1	5 x_2'
6 y_2'	7 θ_2	8 c_2	9	.0 90	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

Intersections Part 2: Circle Line

Program	Comments
01 *LBL2	
02 STO0	
03 X⇄Y	
04 STO1	
05 R↓	
06 -	
07 X⇄Y	Input x_1, y_1, x_2, y_2, and calculate
08 RCL0	
09 -	
10 →P	
11 R↓	
12 9	
13 0	
14 X⇄Y	
15 X=Y?	
16 GTO9	
17 TAN	
18 STO3	
19 RCL2	
20 RCL1	
21 RCL3	
22 x	
23 -	
24 STO4	
25 RCL6	
26 -	
27 ST.2	
28 RCL3	
29 x	
30 RCL5	
31 -	
32 STO8	
33 RC.2	
34 X²	
35 RCL5	
36 X²	
37 +	
38 RCL7	
39 X²	
40 -	
41 RCL3	
42 X²	
43 1	
44 +	
45 ST.1	
46 x	
47 CHS	
48 RCL8	
49 X²	
50 +	
51 √X	
52 STO9	***
53 RTN	
54 *LBL5	------------
55 *LBL3	
56 RCL9	
57 RCL8	
58 -	Calculate x_p
59 RC.1	
60 ÷	
61 R/S	***
62 RCL3	
63 x	
64 RCL4	
65 +	
66 *LBL9	***
67 RTN	------------
68 *LBL4	
69 1	
70 CHS	Calculate y_p
71 STx9	
72 GTO5	------------
73 *LBL3	
74 RCL7	
75 X²	
76 RCL1	
77 R/S	***
78 RCL5	
79 -	
80 X²	Calculate x_p and y_p for vertical line.
81 -	
82 √X	
83 ST.3	
84 *LBL8	
85 RCL6	
86 +	
87 RTN	
88 *LBL4	
89 RCL1	
90 R/S	***
91 RC.3	
92 CHS	
93 GTO8	

REGISTERS

0 temp x_2	1 x_1'	2 y_1'	3 $\tan\theta_1$	4 c_1	5 x_0
6 y_0	7 r	8 α	9 β	.0	.1 (1 + m)
.2 $c - y_0$	.3 Used	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

Intersections Part 3: Circle-Circle

Step	Key	Comment
01	*LBL1	
02	STO3	
03	R↓	
04	STO2	Input x_{01}, y_{01}, r_1
05	R↓	
06	STO1	
07	RTN	
08	*LBL2	
09	STO6	
10	R↓	
11	STO5	Input x_{02}, y_{02}, r_2
12	R↓	
13	STO4	
14	RTN	
15	*LBL3	
16	RCL5	
17	RCL2	
18	-	
19	RCL4	
20	RCL1	
21	-	
22	→P	
23	STO8	Calculate x_{p1}, y_{p1}
24	X⇄Y	
25	STO7	
26	RCL8	
27	X²	
28	RCL3	
29	X²	
30	+	
31	RCL6	
32	X²	
33	-	
34	RCL8	
35	2	
36	×	
37	RCL3	
38	×	
39	÷	
40	COS⁻¹	
41	STO9	
42	RCL7	
43	+	
44	GSB9	
45	GTO8	
46	*LBL4	
47	RCL7	
48	RCL9	
49	-	
50	GSB9	
51	*LBL8	
52	RCL0	
53	SIN	Calculate x_{p2}, y_{p2}
54	RCL3	
55	×	
56	RCL2	
57	+	***
58	RTN	
59	*LBL9	
60	STO0	
61	COS	
62	RCL3	
63	×	
64	RCL1	
65	+	
66	R/S	***
67	RTN	

REGISTERS

0 $\theta \pm \alpha$	1 x_{01}	2 y_{01}	3 r_1	4 x_{02}	5 y_{02}
6 r_2	7 θ	8 D	9 α	.0	.1
.2	.3	.4	.5	16	17
18	19	20	21	22	23
24	25	26	27	28	29

*** indicates that "Print X" may be inserted or used to replace "R/S".

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the corresponding program.			
2	For lines and lines, go to step 3.			
	For circles and lines, go to step 9.			
	For circles and circles, go to step 14.			
3	**Lines and Lines**			
4	Initialize	90	STO [.] [0]	90
5	Input the first line:			
	by two points:			
	x_1	x_1	ENTER↑	
	y_1	y_1	ENTER↑	
	x_2	x_2	STO [0]	
	y_2	y_2	GSB [1]	c_1
	or			
	by one point and the angle	x_1	ENTER↑	
		y_1	ENTER↑	
		θ	GSB [3]	c_1
6	Input the 2nd line:			
	by two points:			
	x'_1	x'_1	ENTER↑	
	y'_1	y'_1	ENTER↑	
	x'_2	x'_2	STO [0]	
	y'_2	y'_2	GSB [2]	c_2
	or			
	by one point and the angle	x'_1	ENTER↑	
		y'_1	ENTER↑	
		θ'	GSB [4]	c_2
7	Calculate intersection point		GSB [5]	x_p
			[R/S]	y_p
8	For a new case go to step 5.			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
9	**Circles and Lines**			
10	Input the circle center			
	x_0	x_0	STO 5	x_0
	y_0	y_0	STO 6	y_0
	radius r	r	STO 7	r
11	Input the line by two points			
	x_1	x_1	ENTER↑	
	y_1	y_1	ENTER↑	
	x_2	x_2	STO 0	
	y_2	y_2	GSB 2	c_2
12	Calculate the intersection points			
	x_{p1}		GSB 3	x_{p1}
	y_{p1}		R/S	y_{p1}
	x_{p2}		GSB 4	x_{p2}
	y_{p2}		R/S	y_{p2}
13	For a new case, go to step 9.			
14	**Circles and Circles**			
15	Input circle one			
	x_{01}	x_{01}	ENTER↑	
	y_{01}	y_{01}	ENTER↑	
	r_1	r_1	GSB 1	x_{01}
16	Input circle two			
	x_{02}	x_{02}	ENTER↑	
	y_{02}	y_{02}	ENTER↑	
	r_2	r_2	GSB 2	x_{02}
17	Calculate intersections			
	x_{p1}		GSB 3	x_{p1}
	y_{p1}		R/S	y_{p1}
	x_{p2}		GSB 4	x_{p2}
	y_{p2}		R/S	y_{p2}
18	For a new case, go to step 14.			

Example 1:

Find the intersection of the vertical line specified by two points:

$$P_1 = (0, 0)$$
$$P'_1 = (0, 50)$$

And the oblique line specified by one point and an angle:

$$P_2 = (10, 20)$$
$$\theta = 45°$$

Keystrokes:	Outputs:	
(Key in the first program)		
90 STO · 0 ⟶	90.00	
0 ENTER↑ ENTER↑ STO 0 50		
GSB 1 ⟶	9.9999999 +99	(Neglect)
10 ENTER↑ 20 ENTER↑ 45		
GSB 4 ⟶	10.00	
GSB 5 ⟶	0.00	(x_p)
R/S ⟶	10.00	(y_p)

Example 2:

Find the points of intersection for a circle with center at (0, 0) and radius 50, and the line containing the points (20, 30) and (0, –10).

Keystrokes:	Outputs:	
(Key in the second program)		
0 STO 5 STO 6 50 STO 7 ⟶	50.00	
20 ENTER↑ 30 ENTER↑ 0 STO		
0 10 CHS GSB 2 ⟶	111.36	
GSB 3 ⟶	26.27	(x_{p1})
R/S ⟶	42.54	(y_{p1})
GSB 4 ⟶	–18.27	(x_{p2})
R/S ⟶	–46.54	(y_{p2})

Example 3:

Calculate the points of intersection for circles at (0, 0) radius 50 and (90, 30) radius 70.

Keystrokes:	Outputs:	
(Key in the third program)		
0 ENTER↑ ENTER↑ 50 GSB 1 →	0.00	
90 ENTER↑ 30 ENTER↑ 70		
GSB 2 →	90.00	
GSB 3 →	21.64	(x_{p1})
R/S →	45.07	(y_{p1})
GSB 4 →	44.36	(x_{p2})
R/S →	–23.07	(y_{p2})

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