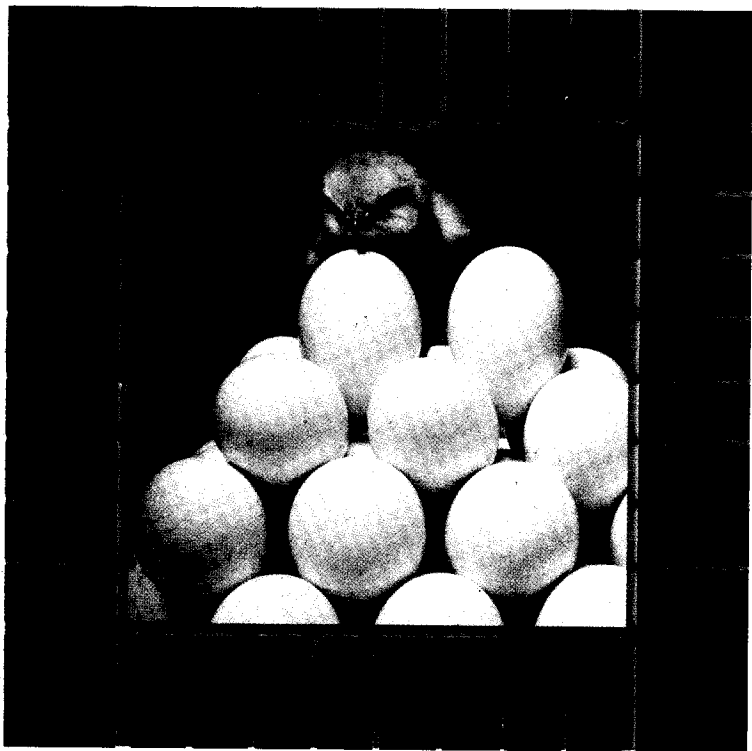


HEWLETT-PACKARD

HP-34C

\$36-

# STATISTICS Applications



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# **Statistics Applications**

**September 1979**

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# Introduction

This Statistics Applications book was written to help you get the most from your HP-34C calculator. The programs were chosen to provide useful calculations for many of the common problems encountered in statistics.

They provide you with immediate problem solving capabilities in your everyday work. You may also find them useful as guides to programming techniques for writing your own customized software.

You will find general information on how to key in and run programs under "A Word About Program Usage" in the Applications book you received with your calculator.

We hope this Statistics Applications book will be a valuable tool in your work and appreciate your comments about it.

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# General Statistics

## Basic Statistics for Two Variables

This program calculates means, standard deviations, covariance, correlation coefficient, coefficients of variation, sums of data points, sum of multiplication of data points, and sums of squares of data points derived from a set of ungrouped data points  $\{(x_i, y_i), i = 1, 2, \dots, n\}$ , or grouped data points  $\{(x_i, y_i, f_i) i = 1, 2, \dots, n\}$ .  $f_i$  denotes the frequency of repetition of  $(x_i, y_i)$ .

$$\text{means } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

$$\text{standard deviations } s_x = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$$

$$\left( \text{or } s_x' = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n}} \right)$$

$$s_y = \sqrt{\frac{\sum y_i^2 - n\bar{y}^2}{n-1}}$$

$$\left( \text{or } s_y' = \sqrt{\frac{\sum y_i^2 - n\bar{y}^2}{n}} \right)$$

$$\text{covariance } s_{xy} = \frac{1}{n-1} \left( \sum x_i y_i - \frac{1}{n} \sum x_i \sum y_i \right)$$

$$\left( \text{or } s_{xy}' = \frac{1}{n} \left[ \sum x_i y_i - \frac{1}{n} \sum x_i \sum y_i \right] \right)$$

$$\text{correlation coefficient } \gamma_{xy} = \frac{s_{xy}}{s_x s_y}$$

Coefficients of variation  $V_x = \frac{s_x}{\bar{x}} \cdot 100$ ,  $V_y = \frac{s_y}{\bar{y}} \cdot 100$

**Notes:**

1.  $n$  is a positive integer and  $n > 1$ .

2. For grouped data  $\bar{x} = \frac{\sum_{i=1}^m f_i x_i}{\sum_{i=1}^m f_i}$ , etc.

| KEY ENTRY               | DISPLAY         |
|-------------------------|-----------------|
| <b>[f] CLEAR [PRGM]</b> | 000-            |
| <b>[h] [LBL] [B]</b>    | 001- 25, 13, 12 |
| <b>[STO] 9</b>          | 002- 23 9       |
| <b>[h] [F?] 0</b>       | 003- 25, 71, 0  |
| <b>[CHS]</b>            | 004- 32         |
| <b>[STO] [+ ] 0</b>     | 005- 23, 51, 0  |
| <b>[g] [R+]</b>         | 006- 15 22      |
| <b>[STO] 8</b>          | 007- 23 8       |
| <b>[g] [R+]</b>         | 008- 15 22      |
| <b>[STO] 7</b>          | 009- 23 7       |
| <b>[f] [R+]</b>         | 010- 14 22      |
| <b>[f] [R+]</b>         | 011- 14 22      |
| <b>[h] [ABS]</b>        | 012- 25 34      |
| <b>[x]</b>              | 013- 61         |
| <b>[x]</b>              | 014- 61         |
| <b>[h] [F?] 0</b>       | 015- 25, 71, 0  |
| <b>[CHS]</b>            | 016- 32         |
| <b>[STO] [+ ] 5</b>     | 017- 23, 51, 5  |

| KEY ENTRY                  | DISPLAY        |
|----------------------------|----------------|
| <b>[RCL] 8</b>             | 018- 24 8      |
| <b>[g] [x<sup>2</sup>]</b> | 019- 15 3      |
| <b>[RCL] 9</b>             | 020- 24 9      |
| <b>[x]</b>                 | 021- 61        |
| <b>[h] [F?] 0</b>          | 022- 25, 71, 0 |
| <b>[CHS]</b>               | 023- 32        |
| <b>[STO] [+ ] 4</b>        | 024- 23, 51, 4 |
| <b>[RCL] 8</b>             | 025- 24 8      |
| <b>[RCL] 9</b>             | 026- 24 9      |
| <b>[x]</b>                 | 027- 61        |
| <b>[h] [F?] 0</b>          | 028- 25, 71, 0 |
| <b>[CHS]</b>               | 029- 32        |
| <b>[STO] [+ ] 3</b>        | 030- 23, 51, 3 |
| <b>[RCL] 7</b>             | 031- 24 7      |
| <b>[g] [x<sup>2</sup>]</b> | 032- 15 3      |
| <b>[RCL] 9</b>             | 033- 24 9      |
| <b>[x]</b>                 | 034- 61        |
| <b>[h] [F?] 0</b>          | 035- 25, 71, 0 |

| KEY ENTRY                    | DISPLAY         |
|------------------------------|-----------------|
| <b>CHS</b>                   | 036- 32         |
| <b>STO</b> <b>+</b> 2        | 037- 23, 51, 2  |
| <b>RCL</b> 7                 | 038- 24 7       |
| <b>RCL</b> 9                 | 039- 24 9       |
| <b>x</b>                     | 040- 61         |
| <b>h</b> <b>F?</b> 0         | 041- 25, 71, 0  |
| <b>CHS</b>                   | 042- 32         |
| <b>STO</b> <b>+</b> 1        | 043- 23, 51, 1  |
| <b>RCL</b> 0                 | 044- 24 0       |
| <b>h</b> <b>CF</b> 0         | 045- 25, 61, 0  |
| <b>R/S</b>                   | 046- 74         |
| <b>h</b> <b>LBL</b> <b>A</b> | 047- 25, 13, 11 |
| <b>x<sub>1</sub>y</b>        | 048- 21         |
| <b>f</b> <b>Σ+</b>           | 049- 14 74      |
| <b>R/S</b>                   | 050- 74         |
| <b>h</b> <b>LBL</b> 1        | 051- 25, 13, 1  |
| <b>h</b> <b>Σ</b>            | 052- 25 8       |
| <b>R/S</b>                   | 053- 74         |
| <b>x<sub>1</sub>y</b>        | 054- 21         |
| <b>R/S</b>                   | 055- 74         |
| <b>h</b> <b>LBL</b> 2        | 056- 25, 13, 2  |
| <b>h</b> <b>s</b>            | 057- 25 9       |
| <b>R/S</b>                   | 058- 74         |
| <b>GTO</b> 9                 | 059- 22 9       |
| <b>h</b> <b>LBL</b> 3        | 060- 25, 13, 3  |
| <b>h</b> <b>s</b>            | 061- 25 9       |
| <b>x<sub>1</sub>y</b>        | 062- 21         |
| <b>R/S</b>                   | 063- 74         |

| KEY ENTRY                     | DISPLAY        |
|-------------------------------|----------------|
| <b>h</b> <b>LBL</b> 9         | 064- 25, 13, 9 |
| <b>RCL</b> 0                  | 065- 24 0      |
| <b>ENTER</b>                  | 066- 31        |
| <b>x<sub>1</sub>y</b>         | 067- 21        |
| 1                             | 068- 1         |
| <b>-</b>                      | 069- 41        |
| <b>+</b>                      | 070- 71        |
| <b>f</b> <b>Σ<sub>x</sub></b> | 071- 14 3      |
| <b>+</b>                      | 072- 71        |
| <b>R/S</b>                    | 073- 74        |
| <b>h</b> <b>LBL</b> 4         | 074- 25, 13, 4 |
| <b>h</b> <b>s</b>             | 075- 25 9      |
| <b>h</b> <b>Σ</b>             | 076- 25 8      |
| <b>x<sub>1</sub>y</b>         | 077- 21        |
| <b>g</b> <b>R+</b>            | 078- 15 22     |
| <b>+</b>                      | 079- 71        |
| <b>EEX</b>                    | 080- 33        |
| 2                             | 081- 2         |
| <b>x</b>                      | 082- 61        |
| <b>R/S</b>                    | 083- 74        |
| <b>h</b> <b>s</b>             | 084- 25 9      |
| <b>h</b> <b>Σ</b>             | 085- 25 8      |
| <b>x<sub>1</sub>y</b>         | 086- 21        |
| <b>f</b> <b>R+</b>            | 087- 14 22     |
| <b>x<sub>1</sub>y</b>         | 088- 21        |
| <b>+</b>                      | 089- 71        |
| <b>EEX</b>                    | 090- 33        |
| 2                             | 091- 2         |

| KEY ENTRY                     | DISPLAY        |
|-------------------------------|----------------|
| $\boxed{x}$                   | 092- 61        |
| $\boxed{R/S}$                 | 093- 74        |
| $\boxed{h} \boxed{LBL} 5$     | 094- 25, 13, 5 |
| $\boxed{h} \boxed{\bar{x}}$   | 095- 25 8      |
| $\boxed{x \div y}$            | 096- 21        |
| $\boxed{STO} 6$               | 097- 23 6      |
| $\boxed{RCL} 5$               | 098- 24 5      |
| $\boxed{RCL} 1$               | 099- 24 1      |
| $\boxed{RCL} 6$               | 100- 24 6      |
| $\boxed{x}$                   | 101- 61        |
| $\boxed{-}$                   | 102- 41        |
| $\boxed{RCL} 0$               | 103- 24 0      |
| 1                             | 104- 1         |
| $\boxed{-}$                   | 105- 41        |
| $\boxed{+}$                   | 106- 71        |
| $\boxed{STO} \boxed{\circ} 0$ | 107- 23 .0     |
| $\boxed{R/S}$                 | 108- 74        |

| KEY ENTRY                     | DISPLAY        |
|-------------------------------|----------------|
| $\boxed{RCL} 0$               | 109- 24 0      |
| $\boxed{ENTER}$               | 110- 31        |
| $\boxed{x \div y}$            | 111- 21        |
| 1                             | 112- 1         |
| $\boxed{-}$                   | 113- 41        |
| $\boxed{+}$                   | 114- 71        |
| $\boxed{+}$                   | 115- 71        |
| $\boxed{R/S}$                 | 116- 74        |
| $\boxed{h} \boxed{LBL} 6$     | 117- 25, 13, 6 |
| $\boxed{h} \boxed{s}$         | 118- 25 9      |
| $\boxed{RCL} \boxed{\circ} 0$ | 119- 24 .0     |
| $\boxed{+}$                   | 120- 71        |
| $\boxed{x}$                   | 121- 61        |
| $\boxed{h} \boxed{1/x}$       | 122- 25 2      |
| $\boxed{R/S}$                 | 123- 74        |
| $\boxed{GTO} 1$               | 124- 22 1      |
|                               |                |

| REGISTERS        |                 |                  | I              |
|------------------|-----------------|------------------|----------------|
| $R_0 n$          | $R_1 \Sigma x$  | $R_2 \Sigma x^2$ | $R_3 \Sigma y$ |
| $R_4 \Sigma y^2$ | $R_5 \Sigma xy$ | $R_6 \bar{y}$    | $R_7 x_i$      |
| $R_8 y_i$        | $R_9 f_i$       | $R_{10} s_{xy}$  | $R_{11}$       |

# 8 Basic Statistics for Two Variables

| STEP | INSTRUCTIONS                                                                       | INPUT DATA/UNITS | KEYS                                                                       | OUTPUT DATA/UNITS  |
|------|------------------------------------------------------------------------------------|------------------|----------------------------------------------------------------------------|--------------------|
| 1    | Key in the program. (For ungrouped data only: line 47–line 124)                    |                  |                                                                            |                    |
| 2    | Initialize.                                                                        |                  | <b>f</b> <b>CLEAR</b> <b>REG</b>                                           |                    |
|      | <b>Ungrouped Data</b>                                                              |                  |                                                                            |                    |
| 3    | Repeat step 3–4 for $i = 1, 2, \dots, n$                                           |                  |                                                                            |                    |
|      | Input: $x_i$                                                                       | $x_i$            | <b>ENTER</b>                                                               |                    |
|      | $y_i$                                                                              | $y_i$            | <b>A</b>                                                                   | $i$                |
| 4    | If you made a mistake in inputting $x_k$ and $y_k$ , then correct by $\rightarrow$ | $x_k$            | <b>ENTER</b>                                                               |                    |
|      |                                                                                    | $y_k$            | <b>x<math>\leftrightarrow</math>y</b> <b>g</b> <b><math>\Sigma</math>-</b> | $k-1$              |
| 5    | Go to step 8 for basic statistic calculations.                                     |                  |                                                                            |                    |
|      | <b>Grouped Data</b>                                                                |                  |                                                                            |                    |
| 6    | Repeat step 6–7 for $i = 1, 2, \dots, n$                                           |                  |                                                                            |                    |
|      | Input: $x_i$                                                                       | $x_i$            | <b>ENTER</b>                                                               |                    |
|      | $y_i$                                                                              | $y_i$            | <b>ENTER</b>                                                               |                    |
|      | $f_i$                                                                              | $f_i$            | <b>B</b>                                                                   | $\Sigma f_i$       |
| 7    | If you made a mistake in inputting $x_k$ and $y_k$ , then correct by $\rightarrow$ | $x_k$            | <b>ENTER</b>                                                               |                    |
|      |                                                                                    | $y_k$            | <b>ENTER</b>                                                               |                    |
|      |                                                                                    | $f_k$            | <b>h</b> <b>SF</b> <b>0</b>                                                |                    |
|      |                                                                                    |                  | <b>B</b>                                                                   | $\Sigma f_i - f_k$ |

| STEP | INSTRUCTIONS                                     | INPUT DATA/UNITS | KEYS         | OUTPUT DATA/UNITS |
|------|--------------------------------------------------|------------------|--------------|-------------------|
| 8    | To calculate basic statistics:                   |                  |              |                   |
|      | $\bar{x}$                                        |                  | <b>GSB</b> 1 | $\bar{x}$         |
|      | $\bar{y}$                                        |                  | <b>R/S</b>   | $\bar{y}$         |
|      | $s_x$                                            |                  | <b>GSB</b> 2 | $s_x$             |
|      | $s'_x$                                           |                  | <b>R/S</b>   | $s'_x$            |
|      | $s_y$                                            |                  | <b>GSB</b> 3 | $s_y$             |
|      | $s'_y$                                           |                  | <b>R/S</b>   | $s'_y$            |
|      | $V_x^*$                                          |                  | <b>GSB</b> 4 | $V_x^*$           |
|      | $V_y^*$                                          |                  | <b>R/S</b>   | $V_y^*$           |
|      | $s_{xy}$                                         |                  | <b>GSB</b> 5 | $s_{xy}$          |
|      | $s'_{xy}$                                        |                  | <b>R/S</b>   | $s'_{xy}$         |
|      | $\gamma_{xy}$                                    |                  | <b>GSB</b> 6 | $\gamma_{xy}$     |
| 9    | To recall sums:                                  |                  |              |                   |
|      | $\Sigma x_i$                                     |                  | <b>RCL</b> 1 | $\Sigma x_i$      |
|      | $\Sigma x_i^2$                                   |                  | <b>RCL</b> 2 | $\Sigma x_i^2$    |
|      | $\Sigma y_i$                                     |                  | <b>RCL</b> 3 | $\Sigma y_i$      |
|      | $\Sigma y_i^2$                                   |                  | <b>RCL</b> 4 | $\Sigma y_i^2$    |
|      | $\Sigma x_i y_i$                                 |                  | <b>RCL</b> 5 | $\Sigma x_i y_i$  |
| 10   | Repeat step 8 if you want the results again.     |                  |              |                   |
| 11   | For a new case, go to step 2.                    |                  |              |                   |
|      |                                                  |                  |              |                   |
|      |                                                  |                  |              |                   |
|      | ***Error 0** will be displayed                   |                  |              |                   |
|      | if $\bar{x}$ or $\bar{y}$ is zero. Press any key |                  |              |                   |
|      | and proceed.                                     |                  |              |                   |

Example 1:

**Example 1:**

For the following set of data, find the means, standard deviations, covariance, correlation coefficient, coefficients of variation, and the sums.

|       |    |    |    |    |    |    |    |
|-------|----|----|----|----|----|----|----|
| $x_i$ | 26 | 30 | 44 | 50 | 62 | 68 | 74 |
| $y_i$ | 92 | 85 | 78 | 81 | 54 | 51 | 40 |

**Keystrokes****Display**

**f** CLEAR **REG**

26 **ENTER** 92 **A** 1.0000

100 **ENTER** 99 **A** 2.0000 (error)

100 **ENTER** 99 **Σ-** 1.0000 (correction)

**xy** **g** **Σ-**

30 **ENTER** 85 **A**

44 **ENTER** 78 **A**

50 **ENTER** 81 **A**

62 **ENTER** 54 **A**

68 **ENTER** 51 **A**

74 **ENTER** 40 **A** 7.0000 ( $n = 7$ )

**GSB** 1 50.5714 ( $\bar{x}$ )

**R/S** 68.7143 ( $\bar{y}$ )

**GSB** 2 18.5010 ( $s_x$ )

**R/S** 17.1286 ( $s'_x$ )

**GSB** 3 19.9976 ( $s_y$ )

**R/S** 18.5142 ( $s'_y$ )

**GSB** 4 36.5838 ( $V_x$ )

**R/S** 29.1026 ( $V_y$ )

**GSB** 5 -354.1429 ( $s_{xy}$ )

**R/S** -303.5510 ( $s'_{xy}$ )

**GSB** 6 -0.9572 ( $\gamma_{xy}$ )

**Example 2:**

Apply the program to the following set of grouped data.

|       |      |      |      |      |      |
|-------|------|------|------|------|------|
| $x_i$ | 4.8  | 5.2  | 3.8  | 4.4  | 4.1  |
| $y_i$ | 15.1 | 11.5 | 14.3 | 13.6 | 12.8 |
| $f_i$ | 1    | 3    | 1    | 6    | 2    |

**Keystrokes****Display**

|                                 |         |                   |
|---------------------------------|---------|-------------------|
| <b>f</b> CLEAR <b>REG</b>       |         |                   |
| 4.8 <b>ENTER</b>                |         |                   |
| 15.1 <b>ENTER</b> 1 <b>B</b>    | 1.0000  |                   |
| 10 <b>ENTER</b> 10 <b>ENTER</b> |         |                   |
| 4 <b>B</b>                      | 5.0000  | (error)           |
| 10 <b>ENTER</b> 10 <b>ENTER</b> |         |                   |
| 4 <b>h</b> <b>SF</b> 0 <b>B</b> | 1.0000  | (correction)      |
| 5.2 <b>ENTER</b>                |         |                   |
| 11.5 <b>ENTER</b> 3 <b>B</b>    |         |                   |
| 3.8 <b>ENTER</b>                |         |                   |
| 14.3 <b>ENTER</b> 1 <b>B</b>    |         |                   |
| 4.4 <b>ENTER</b>                |         |                   |
| 13.6 <b>ENTER</b> 6 <b>B</b>    |         |                   |
| 4.1 <b>ENTER</b>                |         |                   |
| 12.8 <b>ENTER</b> 2 <b>B</b>    | 13.0000 | ( $\sum f_i$ )    |
| <b>GSB</b> 1                    | 4.5231  | ( $\bar{x}$ )     |
| <b>R/S</b>                      | 13.1615 | ( $\bar{y}$ )     |
| <b>GSB</b> 2                    | 0.4494  | ( $s_x$ )         |
| <b>R/S</b>                      | 0.4317  | ( $s_y$ )         |
| <b>GSB</b> 3                    | 1.1087  | ( $s_{xy}$ )      |
| <b>R/S</b>                      | 1.0652  | ( $s_{xy}'$ )     |
| <b>GSB</b> 4                    | 9.9348  | ( $V_x$ )         |
| <b>R/S</b>                      | 8.4238  | ( $V_y$ )         |
| <b>GSB</b> 5                    | -0.3065 | ( $s_{xy}$ )      |
| <b>R/S</b>                      | -0.2830 | ( $s_{xy}'$ )     |
| <b>GSB</b> 6                    | -0.6153 | ( $\gamma_{xy}$ ) |

## Factorial, Permutations and Combinations

This program finds the extended range factorial ( $n$  can be greater than 69), permutations and combinations. Permutations and combinations are functions of the factorial, but this program does not use the factorial key, so that better accuracy and larger range can be obtained.

The equations are:

Factorial 
$$n! = n (n - 1) (n - 2) \cdot \dots \cdot 2 \cdot 1 ; n \geq 1$$

Permutations 
$${}_m P_n = \frac{m!}{(m - n)!} = m(m - 1) \dots (m - n + 1)$$

Combinations 
$${}_m C_n = \frac{m!}{(m - n)! n!} = \frac{m(m - 1) \dots (m - n + 1)}{1 \cdot 2 \cdot \dots \cdot n}$$

where  $m, n$  are integers and  $0 \leq n \leq m$ .

### Notes:

1.  ${}_m P_0 = 1, {}_m P_1 = m, {}_m P_m = m!$
2.  ${}_m C_0 = {}_m C_m = 1$
3.  ${}_m C_1 = {}_m C_{m-1} = m$
4.  ${}_m C_n = {}_m C_{m-n}$
5. In calculating  $n!$ , the accuracy will be reduced for  $n > 69$ , since it is calculated using logarithms.

| KEY ENTRY                             | DISPLAY         |
|---------------------------------------|-----------------|
| $\boxed{f}$ CLEAR $\boxed{PRGM}$      | 000-            |
| $\boxed{h}$ LBL $\boxed{A}$           | 001- 25, 13, 11 |
| $\boxed{f}$ FIX 9                     | 002- 14, 11, 9  |
| $\boxed{STO}$ 1                       | 003- 23 1       |
| $\boxed{STO}$ 3                       | 004- 23 3       |
| 6                                     | 005- 6          |
| 9                                     | 006- 9          |
| $\boxed{x \div y}$                    | 007- 21         |
| $\boxed{f}$ $\boxed{x \leq y}$        | 008- 14 41      |
| $\boxed{GTO}$ 9                       | 009- 22 9       |
| $\boxed{x \div y}$                    | 010- 21         |
| $\boxed{-}$                           | 011- 41         |
| $\boxed{STO}$ $\boxed{f}$ $\boxed{I}$ | 012- 23, 14, 23 |
| $\boxed{h}$ LST $\boxed{x}$           | 013- 25 0       |
| $\boxed{h}$ $\boxed{x!}$              | 014- 25 1       |
| $\boxed{f}$ LOG                       | 015- 14 2       |
| $\boxed{STO}$ 2                       | 016- 23 2       |
| $\boxed{RCL}$ 1                       | 017- 24 1       |
| $\boxed{ENTER*}$                      | 018- 31         |
| $\boxed{f}$ LOG                       | 019- 14 2       |
| $\boxed{STO}$ $\boxed{+}$ 2           | 020- 23, 51, 2  |
| $\boxed{h}$ LBL 7                     | 021- 25, 13, 7  |
| $\boxed{g}$ DSE                       | 022- 15 23      |
| $\boxed{GTO}$ 8                       | 023- 22 8       |
| $\boxed{RCL}$ 2                       | 024- 24 2       |
| $\boxed{h}$ FRAC                      | 025- 25 33      |
| $\boxed{g}$ $\boxed{10^x}$            | 026- 15 2       |
| $\boxed{f}$ FIX 9                     | 027- 14, 11, 9  |
| $\boxed{R/S}$                         | 028- 74         |

| KEY ENTRY                   | DISPLAY         |
|-----------------------------|-----------------|
| $\boxed{RCL}$ 2             | 029- 24 2       |
| $\boxed{f}$ FIX 0           | 030- 14, 11, 0  |
| $\boxed{h}$ INT             | 031- 25 32      |
| $\boxed{h}$ RTN             | 032- 25 12      |
| $\boxed{h}$ LBL 8           | 033- 25, 13, 8  |
| $\boxed{RCL}$ 3             | 034- 24 3       |
| 1                           | 035- 1          |
| $\boxed{-}$                 | 036- 41         |
| $\boxed{STO}$ 3             | 037- 23 3       |
| $\boxed{f}$ LOG             | 038- 14 2       |
| $\boxed{STO}$ $\boxed{+}$ 2 | 039- 23, 51, 2  |
| $\boxed{GTO}$ 7             | 040- 22 7       |
| $\boxed{h}$ LBL 9           | 041- 25, 13, 9  |
| $\boxed{h}$ $\boxed{x!}$    | 042- 25 1       |
| $\boxed{h}$ RTN             | 043- 25 12      |
| $\boxed{h}$ LBL $\boxed{B}$ | 044- 25, 13, 12 |
| $\boxed{f}$ FIX 0           | 045- 14, 11, 0  |
| $\boxed{x \div y}$          | 046- 21         |
| $\boxed{STO}$ 0             | 047- 23 0       |
| $\boxed{x \div y}$          | 048- 21         |
| $\boxed{f}$ $\boxed{x > y}$ | 049- 14 51      |
| $\boxed{GTO}$ 2             | 050- 22 2       |
| $\boxed{f}$ $\boxed{x = y}$ | 051- 14 71      |
| $\boxed{GTO}$ 9             | 052- 22 9       |
| $\boxed{ENTER*}$            | 053- 31         |
| 0                           | 054- 0          |
| $\boxed{f}$ $\boxed{x = y}$ | 055- 14 71      |
| $\boxed{GTO}$ 3             | 056- 22 3       |
| $\boxed{CLX}$               | 057- 34         |

| KEY ENTRY                         | DISPLAY         |
|-----------------------------------|-----------------|
| 1                                 | 058- 1          |
| $\boxed{f} \boxed{x=y}$           | 059- 14 71      |
| $\boxed{GTO} \boxed{5}$           | 060- 22 5       |
| $\boxed{-}$                       | 061- 41         |
| $\boxed{STO} \boxed{f} \boxed{I}$ | 062- 23, 14, 23 |
| $\boxed{g} \boxed{R+}$            | 063- 15 22      |
| $\boxed{STO} \boxed{4}$           | 064- 23 4       |
| $\boxed{h} \boxed{LBL} \boxed{4}$ | 065- 25, 13, 4  |
| $\boxed{RCL} \boxed{4}$           | 066- 24 4       |
| 1                                 | 067- 1          |
| $\boxed{-}$                       | 068- 41         |
| $\boxed{STO} \boxed{4}$           | 069- 23 4       |
| $\boxed{x}$                       | 070- 61         |
| $\boxed{g} \boxed{DSE}$           | 071- 15 23      |
| $\boxed{GTO} \boxed{4}$           | 072- 22 4       |
| $\boxed{h} \boxed{RTN}$           | 073- 25 12      |
| $\boxed{h} \boxed{LBL} \boxed{2}$ | 074- 25, 13, 2  |
| 0                                 | 075- 0          |
| $\boxed{+}$                       | 076- 71         |
| $\boxed{h} \boxed{LBL} \boxed{3}$ | 077- 25, 13, 3  |
| 1                                 | 078- 1          |
| $\boxed{h} \boxed{RTN}$           | 079- 25 12      |
| $\boxed{h} \boxed{LBL} \boxed{1}$ | 080- 25, 13, 1  |
| $\boxed{f} \boxed{FIX} \boxed{0}$ | 081- 14, 11, 0  |
| $\boxed{f} \boxed{x>y}$           | 082- 14 51      |
| $\boxed{GTO} \boxed{2}$           | 083- 22 2       |
| $\boxed{-}$                       | 084- 41         |
| $\boxed{h} \boxed{LST} \boxed{x}$ | 085- 25 0       |
| $\boxed{f} \boxed{x\leq y}$       | 086- 14 41      |

| KEY ENTRY                         | DISPLAY         |
|-----------------------------------|-----------------|
| $\boxed{GSB} \boxed{6}$           | 087- 13 6       |
| $\boxed{STO} \boxed{4}$           | 088- 23 4       |
| 1                                 | 089- 1          |
| $\boxed{STO} \boxed{f} \boxed{I}$ | 090- 23, 14, 23 |
| $\boxed{+}$                       | 091- 51         |
| $\boxed{STO} \boxed{0}$           | 092- 23 0       |
| $\boxed{CLx}$                     | 093- 34         |
| $\boxed{f} \boxed{x=y}$           | 094- 14 71      |
| $\boxed{GTO} \boxed{3}$           | 095- 22 3       |
| $\boxed{h} \boxed{LBL} \boxed{0}$ | 096- 25, 13, 0  |
| $\boxed{g} \boxed{R+}$            | 097- 15 22      |
| 1                                 | 098- 1          |
| $\boxed{RCL} \boxed{f} \boxed{I}$ | 099- 24, 14, 23 |
| $\boxed{+}$                       | 100- 51         |
| $\boxed{STO} \boxed{f} \boxed{I}$ | 101- 23, 14, 23 |
| $\boxed{f} \boxed{x>y}$           | 102- 14 51      |
| $\boxed{GTO} \boxed{5}$           | 103- 22 5       |
| $\boxed{RCL} \boxed{4}$           | 104- 24 4       |
| $\boxed{x\geq y}$                 | 105- 21         |
| $\boxed{+}$                       | 106- 51         |
| $\boxed{h} \boxed{LST} \boxed{x}$ | 107- 25 0       |
| $\boxed{+}$                       | 108- 71         |
| $\boxed{RCL} \boxed{0}$           | 109- 24 0       |
| $\boxed{x}$                       | 110- 61         |
| $\boxed{STO} \boxed{0}$           | 111- 23 0       |
| $\boxed{GTO} \boxed{0}$           | 112- 22 0       |
| $\boxed{h} \boxed{LBL} \boxed{6}$ | 113- 25, 13, 6  |
| $\boxed{STO} \boxed{0}$           | 114- 23 0       |
| $\boxed{x\leq y}$                 | 115- 21         |

| KEY ENTRY                   | DISPLAY       |
|-----------------------------|---------------|
| $\boxed{h}$ $\boxed{RTN}$   | 116- 25 12    |
| $\boxed{h}$ $\boxed{LBL}$ 5 | 117-25, 13, 5 |

| KEY ENTRY       | DISPLAY   |
|-----------------|-----------|
| $\boxed{RCL}$ 0 | 118- 24 0 |
|                 |           |

| REGISTERS  |             |                                | 1 n-69      |
|------------|-------------|--------------------------------|-------------|
| $R_0$ Used | $R_1, m, n$ | $R_2 \text{ Log}(69!) + \dots$ | $R_3 (n-i)$ |
| $R_4 m$    | $R_5 n-1$   | $R_6-R_{12}$ Unused            |             |

| STEP | INSTRUCTIONS                                                                                                                     | INPUT DATA/UNITS | KEYS            | OUTPUT DATA/UNITS |
|------|----------------------------------------------------------------------------------------------------------------------------------|------------------|-----------------|-------------------|
| 1    | Key in the program (For factorial only: key in line 1-line 43. For permutations and combinations only: key in line 41-line 118). |                  |                 |                   |
|      | <b>Factorial (<math>n!</math>)</b>                                                                                               |                  |                 |                   |
| 2    | For $n \leq 69$ .                                                                                                                | $n$              | $\boxed{A}$     | $n!$              |
| 3    | For $n > 69$ .                                                                                                                   | $n$              | $\boxed{A}$     | decimal no.       |
|      |                                                                                                                                  |                  | $\boxed{R/S}$   | exp. of 10        |
|      | <b>Permutations</b>                                                                                                              |                  |                 |                   |
| 4    | Calculate ${}_m P_n$ .                                                                                                           | $m$              | $\boxed{ENTER}$ |                   |
|      |                                                                                                                                  | $n$              | $\boxed{B}$     | ${}_m P_n$        |
|      | <b>Combinations</b>                                                                                                              |                  |                 |                   |
| 5    | Calculate ${}_m C_n$ .                                                                                                           | $m$              | $\boxed{ENTER}$ |                   |
|      |                                                                                                                                  | $n$              | $\boxed{GSB}$ 1 | ${}_m C_n$        |
| 6    | Repeat any of the above steps if desired.                                                                                        |                  |                 |                   |

## 16 Factorial, Permutations and Combinations

### Examples:

1.  $5! = 120$
2.  $69! \approx 1.711224524 \times 10^{98}$
3.  $70! \approx 1.197857069 \times 10^{100}$
4.  $100! \approx 9.332622518 \times 10^{157}$
5.  ${}_{27}P_5 = 9687600.00$
6.  ${}_{73}C_4 = 1088430.00$

### Keystrokes

### Display

|     |                             |                    |                  |
|-----|-----------------------------|--------------------|------------------|
| 5   | <b>A</b>                    | <b>120.0000000</b> | (5!)             |
| 69  | <b>A</b>                    | <b>1.711224 98</b> | (69!)            |
| 70  | <b>A</b>                    | <b>1.197857069</b> | (decimal no.)    |
|     | <b>R/S</b>                  | <b>100.</b>        | ( $10^{100}$ )   |
| 100 | <b>A</b>                    | <b>9.332622518</b> | (decimal no.)    |
|     | <b>R/S</b>                  | <b>157</b>         | ( $10^{157}$ )   |
| 27  | <b>ENTER</b> 5 <b>B</b>     | <b>9,687,600.</b>  | ( ${}_{27}P_5$ ) |
| 73  | <b>ENTER</b> 4 <b>GSB</b> 1 | <b>1,088,430.</b>  | ( ${}_{73}C_4$ ) |

## Moments, Skewness and Kurtosis

For grouped or ungrouped data, moments are used to describe sets of data, skewness is used to measure the lack of symmetry in a distribution, and kurtosis is the relative peakness or flatness of a distribution. For a given set of data

$$\{x_1, x_2, \dots, x_n\}:$$

$$\text{1st moment } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\text{2nd moment } m_2 = \frac{1}{n} \sum x_i^2 - \bar{x}^2$$

$$\text{3rd moment } m_3 = \frac{1}{n} \sum x_i^3 - \frac{3}{n} \bar{x} \sum x_i^2 + 2\bar{x}^3$$

$$\text{4th moment } m_4 = \frac{1}{n} \sum x_i^4 - \frac{4}{n} \bar{x} \sum x_i^3 + \frac{6}{n} \bar{x}^2 \sum x_i^2 - 3\bar{x}^4$$

Moment coefficient of skewness

$$\gamma_1 = \frac{m_3}{m_2^{3/2}}$$

Moment coefficient of kurtosis

$$\gamma_2 = \frac{m_4}{m_2^2}$$

This program also provides the option for calculating those statistics for grouped data (using similar formulas as for ungrouped data):

|           |       |       |     |       |
|-----------|-------|-------|-----|-------|
| data      | $x_1$ | $x_2$ | ... | $x_m$ |
| frequency | $f_1$ | $f_2$ | ... | $f_m$ |

Note that for this case, 1<sup>st</sup> moment

$$\bar{x} = \frac{\sum_{i=1}^m f_i x_i}{\sum_{i=1}^m f_i}$$

### Reference:

Theory and Problems of Statistics, M. R. Spiegel, Schaum's Outline, McGraw-Hill, 1961.

| KEY ENTRY        | DISPLAY         | KEY ENTRY   | DISPLAY        |
|------------------|-----------------|-------------|----------------|
| [f] CLEAR [PRGM] |                 | [h] LST X   | 016- 25 0      |
| [h] LBL [A]      | 001- 25, 13, 11 | [x]         | 017- 61        |
| [ENTER]          | 002- 31         | [STO] [+] 3 | 018- 23, 51, 3 |
| 1                | 003- 1          | [h] LST X   | 019- 25 0      |
| [GSB] 0          | 004- 13 0       | [x]         | 020- 61        |
| [R/S]            | 005- 74         | [STO] [+] 4 | 021- 73, 51, 4 |
| [h] LBL [B]      | 006- 25, 13, 12 | [h] LST X   | 022- 25 0      |
| [GSB] 0          | 007- 13 0       | [x]         | 023- 61        |
| [R/S]            | 008- 74         | [STO] [+] 5 | 024- 73, 51, 5 |
| [h] LBL 0        | 009- 25, 13, 0  | 1           | 025- 1         |
| [h] F? 0         | 010- 25, 71, 0  | [h] F? 0    | 026- 25, 71, 0 |
| [CHS]            | 011- 32         | [CHS]       | 027- 32        |
| [STO] [+] 1      | 012- 23, 51, 1  | [STO] [+] 0 | 028- 23, 51, 0 |
| [x] y            | 013- 21         | [RCL] 0     | 029- 24 0      |
| [x]              | 014- 61         | [h] CF 0    | 030- 25, 61, 0 |
| [STO] [+] 2      | 015- 23, 51, 2  | [h] RTN     | 031- 25 12     |

| KEY ENTRY             | DISPLAY        |
|-----------------------|----------------|
| [h] [LBL] 1           | 032- 25, 13, 1 |
| [RCL] 2               | 033- 24 2      |
| [RCL] 1               | 034- 24 1      |
| [+]                   | 035- 71        |
| [STO] 6               | 036- 23 6      |
| [R/S]                 | 037- 74        |
| [RCL] 3               | 038- 24 3      |
| [RCL] 1               | 039- 24 1      |
| [+]                   | 040- 71        |
| [RCL] 6               | 041- 24 6      |
| [g] [x <sup>2</sup> ] | 042- 15 3      |
| [STO] 8               | 043- 23 8      |
| [-]                   | 044- 41        |
| [STO] 7               | 045- 23 7      |
| [R/S]                 | 046- 74        |
| [RCL] 4               | 047- 24 4      |
| [RCL] 3               | 048- 24 3      |
| [RCL] 6               | 049- 24 6      |
| [x]                   | 050- 61        |
| 3                     | 051- 3         |
| [x]                   | 052- 61        |
| [-]                   | 053- 41        |
| [RCL] 1               | 054- 24 1      |

| KEY ENTRY | DISPLAY   |
|-----------|-----------|
| [+]       | 055- 71   |
| [RCL] 6   | 056- 24 6 |
| [RCL] 8   | 057- 24 8 |
| [x]       | 058- 61   |
| 2         | 059- 2    |
| [x]       | 060- 61   |
| [+]       | 061- 51   |
| [STO] 9   | 062- 23 9 |
| [R/S]     | 063- 74   |
| [RCL] 5   | 064- 24 5 |
| [RCL] 6   | 065- 24 6 |
| [RCL] 4   | 066- 24 4 |
| [x]       | 067- 61   |
| 4         | 068- 4    |
| [x]       | 069- 61   |
| [-]       | 070- 41   |
| [RCL] 8   | 071- 24 8 |
| [RCL] 3   | 072- 24 3 |
| [x]       | 073- 61   |
| 6         | 074- 6    |
| [x]       | 075- 61   |
| [+]       | 076- 51   |
| [RCL] 1   | 077- 24 1 |

| KEY ENTRY                            | DISPLAY        | KEY ENTRY                 | DISPLAY   |
|--------------------------------------|----------------|---------------------------|-----------|
| $\boxed{+}$                          | 078- 71        | 1                         | 089- 1    |
| $\boxed{\text{RCL}} \ 8$             | 079- 24 8      | $\boxed{\cdot}$           | 090- 73   |
| $\boxed{9} \ \boxed{x^2}$            | 080- 15 3      | 5                         | 091- 5    |
| 3                                    | 081- 3         | $\boxed{h} \ \boxed{y^x}$ | 092- 25 3 |
| $\boxed{\times}$                     | 082- 61        | $\boxed{+}$               | 093- 71   |
| $\boxed{-}$                          | 083- 41        | $\boxed{\text{R/S}}$      | 094- 74   |
| $\boxed{\text{STO}} \ 6$             | 084- 23 6      | $\boxed{\text{RCL}} \ 6$  | 095- 24 6 |
| $\boxed{\text{R/S}}$                 | 085- 74        | $\boxed{\text{RCL}} \ 7$  | 096- 24 7 |
| $\boxed{h} \ \boxed{\text{LBL}} \ 2$ | 086- 25, 13, 2 | $\boxed{9} \ \boxed{x^2}$ | 097- 15 3 |
| $\boxed{\text{RCL}} \ 9$             | 087- 24 9      | $\boxed{+}$               | 098- 71   |
| $\boxed{\text{RCL}} \ 7$             | 088- 24 7      | $\boxed{\text{R/S}}$      | 099- 74   |

| REGISTERS              |                        |                        | I                      |
|------------------------|------------------------|------------------------|------------------------|
| $R_0 n$                | $R_1 \Sigma f_i$       | $R_2 \Sigma f_i x_i$   | $R_3 \Sigma f_i x_i^2$ |
| $R_4 \Sigma f_i x_i^3$ | $R_5 \Sigma f_i x_i^4$ | $R_6 x, m_4$           | $R_7 m_2$              |
| $R_8 \bar{x}^2$        | $R_9 m_3$              | $R_{10}-R_{14}$ Unused |                        |

| STEP | INSTRUCTIONS                    | INPUT DATA/UNITS | KEYS                                                   | OUTPUT DATA/UNITS |
|------|---------------------------------|------------------|--------------------------------------------------------|-------------------|
| 1    | Key in the program.             |                  |                                                        |                   |
| 2    | Initialize the program.         |                  | $\boxed{\text{f}} \ \text{CLEAR} \ \boxed{\text{REG}}$ |                   |
|      | <b>Ungrouped Data</b>           |                  |                                                        |                   |
| 3    | Repeat step 3-4 for $i = 1, 2,$ |                  |                                                        |                   |
|      | $\dots, n.$                     |                  |                                                        |                   |
|      | Input $x_i$ .                   | $x_i$            | $\boxed{\text{A}}$                                     | $i$               |
| 4    | If you made a mistake in        |                  |                                                        |                   |

| STEP | INSTRUCTIONS                                                                       | INPUT DATA/UNITS | KEYS              | OUTPUT DATA/UNITS |
|------|------------------------------------------------------------------------------------|------------------|-------------------|-------------------|
|      | inputting $x_k$ , then correct                                                     |                  |                   |                   |
|      | by $\rightarrow$                                                                   | $x_k$            | <b>[h] [SF] 0</b> |                   |
|      |                                                                                    |                  | <b>[A]</b>        | $k-1$             |
| 5    | Go to step 8 for moments calculations.                                             |                  |                   |                   |
|      | <b>Grouped Data</b>                                                                |                  |                   |                   |
| 6    | Repeat step 6-7 for $j = 1, 2, \dots, m$ .                                         |                  |                   |                   |
|      | Input $x_i$                                                                        | $x_i$            | <b>[ENTER+]</b>   |                   |
|      | $f_i$                                                                              | $f_i$            | <b>[B]</b>        | $i$               |
| 7    | If you made a mistake in inputting $x_h$ and $f_h$ , then correct by $\rightarrow$ | $x_h$            | <b>[ENTER+]</b>   |                   |
|      |                                                                                    | $f_h$            | <b>[h] [SF] 0</b> |                   |
|      |                                                                                    |                  | <b>[B]</b>        | $h-1$             |
| 8    | Calculate moments etc: $\bar{x}$                                                   |                  | <b>[GSB] 1</b>    | $\bar{x}$         |
|      | $m_2$                                                                              |                  | <b>[R/S]</b>      | $m_2$             |
|      | $m_3$                                                                              |                  | <b>[R/S]</b>      | $m_3$             |
|      | $m_4$                                                                              |                  | <b>[R/S]</b>      | $m_4$             |
|      | $\gamma_1$                                                                         |                  | <b>[GSB] 2</b>    | $\gamma_1$        |
|      | $\gamma_2$                                                                         |                  | <b>[R/S]</b>      | $\gamma_2$        |
| 9    | Repeat step 8 if you want the results again.                                       |                  |                   |                   |
| 10   | For a new case, go to step 2.                                                      |                  |                   |                   |

## 22 Moments, Skewness, and Kurtosis

### Examples:

#### 1. Ungrouped data

| $i$   | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $x_i$ | 2.1 | 3.5 | 4.2 | 6.5 | 4.1 | 3.6 | 5.3 | 3.7 | 4.9 |

#### Keystrokes

#### Display

|                                  |               |                |
|----------------------------------|---------------|----------------|
| <b>f</b> <b>FIX</b> 4            |               |                |
| <b>f</b> <b>CLEAR</b> <b>REG</b> |               |                |
| 2.1 <b>A</b>                     | <b>1.0000</b> | (i)            |
| 4 <b>A</b>                       | <b>2.0000</b> | (error)        |
| 4 <b>h</b> <b>SF</b> 0 <b>A</b>  | <b>1.0000</b> | (correction)   |
| 3.5 <b>A</b> 4.2 <b>A</b>        |               |                |
| 6.5 <b>A</b> 4.1 <b>A</b>        |               |                |
| 3.6 <b>A</b> 5.3 <b>A</b>        |               |                |
| 3.7 <b>A</b> 4.9 <b>A</b>        | <b>9.0000</b> | (n)            |
| <b>GSB</b> 1                     | <b>4.2111</b> | ( $\bar{x}$ )  |
| <b>R/S</b>                       | <b>1.3899</b> | ( $m_2$ )      |
| <b>R/S</b>                       | <b>0.3864</b> | ( $m_3$ )      |
| <b>R/S</b>                       | <b>5.4894</b> | ( $m_4$ )      |
| <b>GSB</b> 2                     | <b>0.2358</b> | ( $\gamma_1$ ) |
| <b>R/S</b>                       | <b>2.8417</b> | ( $\gamma_2$ ) |

#### 2. Grouped data

| $i$   | 1 | 2 | 3 | 4 | 5 |
|-------|---|---|---|---|---|
| $x_i$ | 3 | 2 | 4 | 6 | 1 |
| $f_i$ | 4 | 5 | 3 | 2 | 1 |

#### Keystrokes

#### Display

|                                  |               |              |
|----------------------------------|---------------|--------------|
| <b>f</b> <b>CLEAR</b> <b>REG</b> |               |              |
| 3 <b>ENTER</b> 4 <b>B</b>        | <b>1.0000</b> | (i)          |
| 5 <b>ENTER</b> 5 <b>B</b>        | <b>2.0000</b> | (error)      |
| 5 <b>ENTER</b> 5                 |               |              |
| <b>h</b> <b>SF</b> 0 <b>B</b>    | <b>1.0000</b> | (correction) |
| 2 <b>ENTER</b> 5 <b>B</b>        |               |              |
| 4 <b>ENTER</b> 3 <b>B</b>        |               |              |
| 6 <b>ENTER</b> 2 <b>B</b>        |               |              |

1 **ENTER** 1 **B****GSB** 1**R/S****R/S****R/S****GSB** 2**R/S****5.0000** $(m)$ **3.1333** $(\bar{x})$ **1.9822** $(m_2)$ **2.1381** $(m_3)$ **11.0479** $(m_4)$ **0.7661** $(\gamma_1)$ **2.8117** $(\gamma_2)$

# Distribution Functions

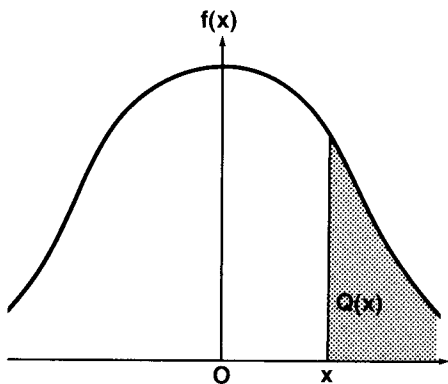
## Normal and Inverse Normal Distribution

This program evaluates the standard normal density function  $f(x)$  and the normal integral  $Q(x)$  for given  $x$ . If  $Q$  is given,  $x$  can also be found. The standard normal distribution has mean 0 and standard deviation 1.

### Equations:

1. Standard normal density

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



## 2. Normal integral

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$$

The integral is calculated by using the integrate key  $\boxed{\int/}$ .

## 3. Inverse normal

For a given  $Q > 0$ ,  $x$  can be found such that

$$Q = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} du$$

The following rational approximation is used:

$$\text{Define } y = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

$$\text{where: } |\epsilon(Q)| < 4.5 \times 10^{-4}$$

$$t = \begin{cases} \sqrt{\ln \frac{1}{Q^2}} & \text{if } 0 < Q \leq 0.5 \\ \sqrt{\ln \frac{1}{(1-Q)^2}} & \text{if } 0.5 < Q < 1 \end{cases}$$

$$c_0 = 2.515517$$

$$c_1 = 0.802853$$

$$c_2 = 0.010328$$

$$d_1 = 1.432788$$

$$d_2 = 0.189269$$

$$d_3 = 0.001308$$

$$\text{Then } x = \begin{cases} y & \text{if } 0 < Q \leq 0.5 \\ -y & \text{if } 0.5 < Q < 1 \end{cases}$$

**Reference:**

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

## Normal Distribution

| KEY ENTRY                             | DISPLAY         |
|---------------------------------------|-----------------|
| <b>f</b> CLEAR <b>PRGM</b>            | 000-            |
| <b>h</b> <b>LBL</b> <b>A</b>          | 001- 25, 13, 11 |
| <b>STO</b> 0                          | 002- 23 0       |
| <b>h</b> <b>LBL</b> 0                 | 003- 25, 13, 0  |
| <b>ENTER</b>                          | 004- 31         |
| <b>x</b>                              | 005- 61         |
| 2                                     | 006- 2          |
| <b>+</b>                              | 007- 71         |
| <b>CHS</b>                            | 008- 32         |
| <b>g</b> <b>e<sup>x</sup></b>         | 009- 15 1       |
| <b>h</b> <b><math>\pi</math></b>      | 010- 25 73      |
| 2                                     | 011- 2          |
| <b>x</b>                              | 012- 61         |
| <b>f</b> <b><math>\sqrt{x}</math></b> | 013- 14 3       |

| KEY ENTRY                             | DISPLAY         |
|---------------------------------------|-----------------|
| <b>+</b>                              | 014- 71         |
| <b>h</b> <b>RTN</b>                   | 015- 25 12      |
| <b>h</b> <b>LBL</b> <b>B</b>          | 016- 25, 13, 12 |
| <b>GSS</b> <b>A</b>                   | 017- 13 11      |
| <b>RCL</b> 0                          | 018- 24 0       |
| <b>CHS</b>                            | 019- 32         |
| <b>RCL</b> 0                          | 020- 24 0       |
| <b>f</b> <b><math>1/x</math></b> 0    | 021- 14, 72, 0  |
| 1                                     | 022- 1          |
| <b>x<math>\leftrightarrow</math>y</b> | 023- 21         |
| <b>-</b>                              | 024- 41         |
| 2                                     | 025- 2          |
| <b>+</b>                              | 026- 71         |
| <b>R/S</b>                            | 027- 74         |

| REGISTERS        |                                       | I |
|------------------|---------------------------------------|---|
| R <sub>0</sub> x | R <sub>1</sub> —R <sub>9</sub> Unused |   |

## Inverse Normal Distribution

| KEY ENTRY                  | DISPLAY        |
|----------------------------|----------------|
| <b>f</b> CLEAR <b>PRGM</b> | 000-           |
| <b>h</b> <b>LBL</b> 1      | 001- 25, 13, 1 |
| <b>g</b> <b>x&lt;0</b>     | 002- 15 41     |
| <b>GTO</b> 0               | 003- 22 0      |
| 1                          | 004- 1         |
| <b>f</b> <b>x&lt;y</b>     | 005- 14 41     |

| KEY ENTRY                             | DISPLAY    |
|---------------------------------------|------------|
| <b>GTO</b> 0                          | 006- 22 0  |
| <b>g</b> <b>R+</b>                    | 007- 15 22 |
| <b>.</b>                              | 008- 73    |
| 5                                     | 009- 5     |
| <b>x<math>\leftrightarrow</math>y</b> | 010- 21    |
| <b>f</b> <b>x&gt;y</b>                | 011- 14 51 |

| KEY ENTRY                    | DISPLAY         |
|------------------------------|-----------------|
| <b>GSB</b> 8                 | 012- 13 8       |
| <b>ENTER</b> *               | 013- 31         |
| <b>x</b>                     | 014- 61         |
| <b>h</b> $\frac{1}{x}$       | 015- 25 2       |
| <b>f</b> <b>LN</b>           | 016- 14 1       |
| <b>f</b> $\sqrt{x}$          | 017- 14 3       |
| <b>STO</b> <b>f</b> <b>I</b> | 018- 23, 14, 23 |
| <b>RCL</b> 2                 | 019- 24 2       |
| <b>x</b>                     | 020- 61         |
| <b>RCL</b> 1                 | 021- 24 1       |
| <b>+</b>                     | 022- 51         |
| <b>RCL</b> <b>f</b> <b>I</b> | 023- 24, 14, 23 |
| <b>x</b>                     | 024- 61         |
| <b>RCL</b> 0                 | 025- 24 0       |
| <b>+</b>                     | 026- 51         |
| <b>RCL</b> <b>f</b> <b>I</b> | 027- 24, 14, 23 |
| <b>RCL</b> 5                 | 028- 24 5       |
| <b>x</b>                     | 029- 61         |
| <b>RCL</b> 4                 | 030- 24 4       |
| <b>+</b>                     | 031- 51         |
| <b>RCL</b> <b>f</b> <b>I</b> | 032- 24, 14, 23 |

| KEY ENTRY                    | DISPLAY         |
|------------------------------|-----------------|
| <b>x</b>                     | 033- 61         |
| <b>RCL</b> 3                 | 034- 24 3       |
| <b>+</b>                     | 035- 51         |
| <b>RCL</b> <b>f</b> <b>I</b> | 036- 24, 14, 23 |
| <b>x</b>                     | 037- 61         |
| 1                            | 038- 1          |
| <b>+</b>                     | 039- 51         |
| <b>+</b>                     | 040- 71         |
| <b>RCL</b> <b>f</b> <b>I</b> | 041- 24, 14, 23 |
| <b>x<sub>2</sub>y</b>        | 042- 21         |
| <b>-</b>                     | 043- 41         |
| <b>h</b> <b>F?</b> 1         | 044- 25, 71, 1  |
| <b>CHS</b>                   | 045- 32         |
| <b>h</b> <b>CF</b> 1         | 046- 25, 61, 1  |
| <b>R/S</b>                   | 047- 74         |
| <b>h</b> <b>LBL</b> 8        | 048- 25, 13, 8  |
| <b>h</b> <b>SF</b> 1         | 049- 25, 51, 1  |
| 1                            | 050- 1          |
| <b>-</b>                     | 051- 41         |
| <b>CHS</b>                   | 052- 32         |
|                              |                 |

| REGISTERS |           |                  | I t       |
|-----------|-----------|------------------|-----------|
| $R_0 c_0$ | $R_1 c_1$ | $R_2 c_2$        | $R_3 d_1$ |
| $R_4 d_2$ | $R_5 d_3$ | $R_6-R_9$ Unused |           |

## 28 Normal and Inverse Normal Distribution

| STEP | INSTRUCTIONS                      | INPUT DATA/UNITS | KEYS         | OUTPUT DATA/UNITS |
|------|-----------------------------------|------------------|--------------|-------------------|
|      | <b>Normal Distribution</b>        |                  |              |                   |
| 1    | Key in the program.               |                  |              |                   |
| 2    | (Optional) Input $x$ to calculate |                  |              |                   |
|      | $f(x)$                            | $x$              | <b>A</b>     | $f(x)$            |
| 3    | Input $x$ to calculate $Q(x)$     | $x$              | <b>B</b>     | $Q(x)$            |
| 4    | For a new $x$ , go to step 2.     |                  |              |                   |
|      |                                   |                  |              |                   |
|      |                                   |                  |              |                   |
|      | <b>Inverse Normal</b>             |                  |              |                   |
|      | <b>Distribution</b>               |                  |              |                   |
| 1    | Key in the program.               |                  |              |                   |
| 2    | Store constants for inverse       |                  |              |                   |
|      | normal distribution.              | $c_0$            | <b>STO</b> 0 |                   |
|      |                                   | $c_1$            | <b>STO</b> 1 |                   |
|      |                                   | $c_2$            | <b>STO</b> 2 |                   |
|      |                                   | $d_1$            | <b>STO</b> 3 |                   |
|      |                                   | $d_2$            | <b>STO</b> 4 |                   |
|      |                                   | $d_3$            | <b>STO</b> 5 |                   |
| 3    | Input $Q$ to calculate $x$ .      | $Q$              | <b>GSB</b> 1 | $x$               |
| 4    | For a new case, go to step 2.     |                  |              |                   |

**Example 1:**

Find  $f(x)$  and  $Q(x)$  for  $x = 1.18$  and  $x = -2.28$ .

**Keystrokes****Display**

(Key in normal distribution program)

**f** **SCI** 2

|      |                     |             |            |          |
|------|---------------------|-------------|------------|----------|
| 1.18 | <b>A</b>            | <b>1.99</b> | <b>-01</b> | $(f(x))$ |
| 1.18 | <b>B</b>            | <b>1.19</b> | <b>-01</b> | $(Q(x))$ |
| 2.28 | <b>CHS</b> <b>B</b> | <b>9.89</b> | <b>-01</b> | $(Q(x))$ |
| 2.28 | <b>CHS</b> <b>A</b> | <b>2.97</b> | <b>-02</b> | $(f(x))$ |

**Example 2:**

Given  $Q = 0.12$  and  $Q = 0.95$ , find  $x$ .

**Keystrokes****Display**

(Key in inverse normal distribution program)

**f** **FIX** 4

|          |              |                |       |
|----------|--------------|----------------|-------|
| 2.515517 | <b>STO</b> 0 |                |       |
| 0.802853 | <b>STO</b> 1 |                |       |
| 0.010328 | <b>STO</b> 2 |                |       |
| 1.432788 | <b>STO</b> 3 |                |       |
| 0.189269 | <b>STO</b> 4 |                |       |
| 0.001308 | <b>STO</b> 5 |                |       |
| 0.12     | <b>GSB</b> 1 | <b>1.1751</b>  | $(x)$ |
| 0.95     | <b>GSB</b> 1 | <b>-1.6452</b> | $(x)$ |

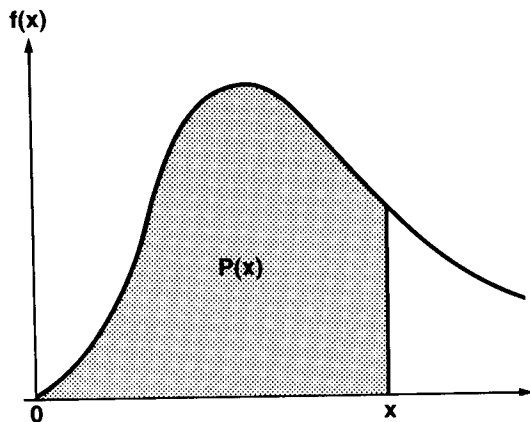
## Chi-Square Distribution

This program evaluates the chi-square density

$$f(x) = \frac{1}{2^{\frac{v}{2}} \Gamma\left(\frac{v}{2}\right)} x^{\frac{v}{2} - 1} e^{-\frac{x}{2}}$$

where:  $x \geq 0$

$v$  is the degrees of freedom.



The integrate key  $\int$  is used to evaluate the cumulative distribution

$$P(x) = \int_0^x f(t) dt$$

**Notes:**

1. Program requires  $v < 141$ . If  $v \geq 141$ , erroneous overflow will result.
2. If both  $x$  and  $v$  are large,  $x^{\frac{v}{2}-1}$  may overflow the machine.
3. If  $v$  is even,

$$\Gamma\left(\frac{v}{2}\right) = \left(\frac{v}{2} - 1\right) !$$

If  $v$  is odd,

$$\Gamma\left(\frac{v}{2}\right) = \left(\frac{v}{2} - 1\right) \left(\frac{v}{2} - 2\right) \dots \left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right)$$

$$4. \quad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

**Reference:**

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

| KEY ENTRY                    | DISPLAY         |
|------------------------------|-----------------|
| <b>f</b> CLEAR <b>PRGM</b>   | 000-            |
| <b>h</b> <b>LBL</b> <b>B</b> | 001- 25, 13, 12 |
| <b>GSB</b> <b>A</b>          | 002- 13 11      |
| 0                            | 003- 0          |
| <b>RCL</b> 2                 | 004- 24 2       |
| <b>f</b> <b>1/x</b> 3        | 005- 14, 72, 3  |
| <b>R/S</b>                   | 006- 74         |
| <b>h</b> <b>LBL</b> <b>A</b> | 007- 25, 13, 11 |
| <b>STO</b> 2                 | 008- 23 2       |
| <b>x↔y</b>                   | 009- 21         |

| KEY ENTRY             | DISPLAY    |
|-----------------------|------------|
| 1                     | 010- 1     |
| <b>STO</b> 3          | 011- 23 3  |
| <b>x↔y</b>            | 012- 21    |
| 2                     | 013- 2     |
| <b>+</b>              | 014- 71    |
| <b>STO</b> 1          | 015- 23 1  |
| <b>h</b> <b>INT</b>   | 016- 25 32 |
| <b>h</b> <b>LST x</b> | 017- 25 0  |
| <b>f</b> <b>x↔y</b>   | 018- 14 61 |
| <b>GTO</b> 1          | 019- 22 1  |

| KEY ENTRY             | DISPLAY        |
|-----------------------|----------------|
| 1                     | 020- 1         |
| $\square$             | 021- 41        |
| $\square$ $\square$   | 022- 25 1      |
| $\square$ 3           | 023- 23 3      |
| $\square$ 2           | 024- 24 2      |
| $\square$ 3           | 025- 22 3      |
| $\square$ $\square$ 1 | 026- 25, 13, 1 |
| $\square$             | 027- 73        |
| 5                     | 028- 5         |
| $\square$ $\square$   | 029- 14 71     |
| $\square$ 2           | 030- 22 2      |
| $\square$             | 031- 21        |
| 1                     | 032- 1         |
| $\square$             | 033- 41        |
| $\square$ $\square$ 3 | 034- 23, 61, 3 |
| $\square$ 1           | 035- 22 1      |
| $\square$ $\square$ 2 | 036- 25, 13, 2 |
| $\square$ $\square$   | 037- 25 73     |
| $\square$ $\square$   | 038- 14 3      |
| $\square$ $\square$ 3 | 039- 23, 61, 3 |

| KEY ENTRY             | DISPLAY        |
|-----------------------|----------------|
| $\square$ 2           | 040- 24 2      |
| $\square$ $\square$ 3 | 041- 25, 13, 3 |
| $\square$             | 042- 31        |
| $\square$             | 043- 31        |
| $\square$ 1           | 044- 24 1      |
| 1                     | 045- 1         |
| $\square$             | 046- 41        |
| $\square$ $\square$   | 047- 25 3      |
| $\square$             | 048- 21        |
| 2                     | 049- 2         |
| $\square$             | 050- 71        |
| $\square$             | 051- 32        |
| $\square$ $\square$   | 052- 15 1      |
| $\square$             | 053- 61        |
| 2                     | 054- 2         |
| $\square$ 1           | 055- 24 1      |
| $\square$ $\square$   | 056- 25 3      |
| $\square$             | 057- 71        |
| $\square$ 3           | 058- 24 3      |
| $\square$             | 059- 71        |

| REGISTERS            |             |         | I                      |
|----------------------|-------------|---------|------------------------|
| $R_0$                | $R_1 \nu/2$ | $R_2 x$ | $R_3 1, \Gamma(\nu/2)$ |
| $R_4$ — $R_9$ Unused |             |         |                        |

| STEP | INSTRUCTIONS                                                            | INPUT DATA/UNITS | KEYS         | OUTPUT DATA/UNITS |
|------|-------------------------------------------------------------------------|------------------|--------------|-------------------|
| 1    | Key in the program.                                                     |                  |              |                   |
| 2    | (Optional) Input degrees of freedom $\nu$ and $x$ to calculate $f(x)$ . | $\nu$            | <b>ENTER</b> |                   |
|      |                                                                         | $x$              | <b>A</b>     | $f(x)$            |
| 3    | Input degrees of freedom $\nu$ and $x$ to calculate $P(x)$ .            | $\nu$            | <b>ENTER</b> |                   |
|      |                                                                         | $x$              | <b>B</b>     | $P(x)$            |
| 4    | For a different case, go to step 2.                                     |                  |              |                   |

**Example 1:**

If degrees of freedom  $\nu = 20$ , find  $f(x)$ ,  $P(x)$  for  $x = 9.6$  and  $x = 15$ .

**Keystrokes****Display**

|          |              |     |          |      |                  |
|----------|--------------|-----|----------|------|------------------|
| <b>f</b> | <b>SCI</b>   | 2   |          |      |                  |
| 20       | <b>ENTER</b> |     |          |      |                  |
| 9.6      | <b>A</b>     |     | 1.53     | -02  | ( $f(9.6)$ )     |
| 20       | <b>ENTER</b> | 9.6 | <b>B</b> | 2.51 | -02 ( $P(9.6)$ ) |
| 20       | <b>ENTER</b> | 15  | <b>A</b> | 5.72 | -02 ( $f(15)$ )  |
| 20       | <b>ENTER</b> | 15  | <b>B</b> | 2.24 | -01 ( $P(15)$ )  |

**Example 2:**

If  $\nu = 3$ , find  $P(x)$  for  $x = 7.82$ .

**Keystrokes****Display**

|          |              |   |      |     |               |
|----------|--------------|---|------|-----|---------------|
| <b>f</b> | <b>SCI</b>   | 2 |      |     |               |
| 3        | <b>ENTER</b> |   |      |     |               |
| 7.82     | <b>B</b>     |   | 9.50 | -01 | ( $P(7.82)$ ) |

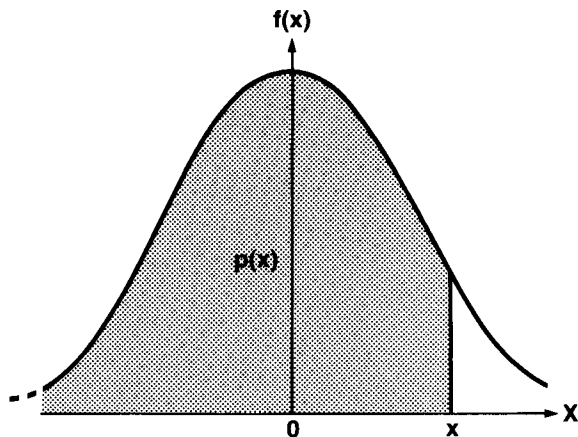
## t Distribution

This program evaluates the cumulative distribution  $P(x, v)$  for a given  $x$  and degrees of freedom  $v$ .

### Equations:

1. Density function

$$f(x) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi v} \Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{x^2}{v}\right)^{-\frac{v+1}{2}}$$



## 2. Cumulative distribution function

$$P(x) = \int_{-\infty}^x f(y) dy$$

The integral is calculated by using the integrate key  $\int \frac{f}{y}$ .

**Remark:**

The program requires  $v < 141$  for  $f(x)$ , otherwise erroneous overflow will result. (The normal distribution is a good approximation for large  $v$ ).

**Reference:**

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

| KEY ENTRY                             | DISPLAY         |
|---------------------------------------|-----------------|
| $\boxed{f}$ CLEAR $\boxed{PRGM}$      | 000-            |
| $\boxed{h}$ $\boxed{LBL}$ $\boxed{A}$ | 001- 25, 13, 11 |
| $\boxed{STO}$ 1                       | 002- 23 1       |
| $\boxed{x \rightarrow y}$             | 003- 21         |
| $\boxed{STO}$ 0                       | 004- 23 0       |
| $\boxed{x \rightarrow y}$             | 005- 21         |
| $\boxed{h}$ $\boxed{LBL}$ 0           | 006- 25, 13, 0  |
| $\boxed{GSB}$ 1                       | 007- 13 1       |
| $\boxed{STO}$ 4                       | 008- 23 4       |
| $\boxed{RCL}$ 0                       | 009- 24 0       |
| $\boxed{GSB}$ 3                       | 010- 13 3       |
| $\boxed{STO}$ 2                       | 011- 23 2       |
| $\boxed{RCL}$ 0                       | 012- 24 0       |
| 1                                     | 013- 1          |
| $\boxed{+}$                           | 014- 51         |
| $\boxed{GSB}$ 3                       | 015- 13 3       |

| KEY ENTRY                      | DISPLAY        |
|--------------------------------|----------------|
| $\boxed{STO}$ 3                | 016- 23 3      |
| $\boxed{RCL}$ 2                | 017- 24 2      |
| $\boxed{+}$                    | 018- 71        |
| $\boxed{h}$ $\boxed{\pi}$      | 019- 25 73     |
| $\boxed{RCL}$ 0                | 020- 24 0      |
| $\boxed{\times}$               | 021- 61        |
| $\boxed{f}$ $\boxed{\sqrt{x}}$ | 022- 14 3      |
| $\boxed{+}$                    | 023- 71        |
| $\boxed{RCL}$ 4                | 024- 24 4      |
| $\boxed{\times}$               | 025- 61        |
| $\boxed{h}$ $\boxed{RTN}$      | 026- 25 12     |
| $\boxed{h}$ $\boxed{LBL}$ 1    | 027- 25, 13, 1 |
| $\boxed{g}$ $\boxed{x^2}$      | 028- 15 3      |
| $\boxed{RCL}$ 0                | 029- 24 0      |
| $\boxed{+}$                    | 030- 71        |
| 1                              | 031- 1         |

| KEY ENTRY           | DISPLAY        |
|---------------------|----------------|
| $+$                 | 032- 51        |
| $\text{RCL } 0$     | 033- 24 0      |
| 1                   | 034- 1         |
| $+$                 | 035- 51        |
| 2                   | 036- 2         |
| $+$                 | 037- 71        |
| $\text{CHS}$        | 038- 32        |
| $h \ y^x$           | 039- 25 3      |
| $h \ \text{RTN}$    | 040- 25 12     |
| $h \ \text{LBL } 3$ | 041- 25, 13, 3 |
| 1                   | 042- 1         |
| $\text{STO } 5$     | 043- 23 5      |
| $x \div y$          | 044- 21        |
| 2                   | 045- 2         |
| $+$                 | 046- 71        |
| $h \ \text{INT}$    | 047- 25 32     |
| $h \ \text{LST } x$ | 048- 25 0      |
| $f \ x \times y$    | 049- 14 61     |
| $\text{GTO } 4$     | 050- 22 4      |
| 1                   | 051- 1         |
| $-$                 | 052- 41        |
| $h \ x!$            | 053- 25 1      |
| $\text{STO } 5$     | 054- 23 5      |
| $h \ \text{RTN}$    | 055- 25 12     |
| $h \ \text{LBL } 4$ | 056- 25, 13, 4 |
| $\cdot$             | 057- 73        |
| 5                   | 058- 5         |

| KEY ENTRY                  | DISPLAY         |
|----------------------------|-----------------|
| $f \ x=y$                  | 059- 14 71      |
| $\text{GTO } 2$            | 060- 22 2       |
| $x \div y$                 | 061- 21         |
| 1                          | 062- 1          |
| $-$                        | 063- 41         |
| $\text{STO } x \ 5$        | 064- 23, 61, 5  |
| $\text{GTO } 4$            | 065- 22 4       |
| $h \ \text{LBL } 2$        | 066- 25, 13, 2  |
| $h \ \pi$                  | 067- 25 73      |
| $f \ \sqrt{x}$             | 068- 14 3       |
| $\text{STO } x \ 5$        | 069- 23, 61, 5  |
| $\text{RCL } 5$            | 070- 24 5       |
| $h \ \text{RTN}$           | 071- 25 12      |
| $h \ \text{LBL } B$        | 072- 25, 13, 12 |
| $\text{GSB } A$            | 073- 13 11      |
| $h \ \text{CF } 0$         | 074- 25, 61, 0  |
| $\text{RCL } 1$            | 075- 24 1       |
| $g \ x=0$                  | 076- 15 71      |
| $\text{GTO } 5$            | 077- 22 5       |
| $g \ x < 0$                | 078- 15 41      |
| $h \ \text{SF } 0$         | 079- 25, 51, 0  |
| $\text{CHS}$               | 080- 32         |
| $\text{RCL } 1$            | 081- 24 1       |
| $f \ \sqrt[3]{x} \ 0$      | 082- 14, 72, 0  |
| $h \ F? \ 0$               | 083- 25, 71, 0  |
| $\text{CHS}$               | 084- 32         |
| $\text{ENTER} \rightarrow$ | 085- 31         |

| KEY ENTRY             | DISPLAY        |
|-----------------------|----------------|
| <b>ENTER</b> +        | 086- 31        |
| <b>ENTER</b> +        | 087- 31        |
| 1                     | 088- 1         |
| <b>x<sub>2</sub>y</b> | 089- 21        |
| <b>-</b>              | 090- 41        |
| 2                     | 091- 2         |
| <b>+</b>              | 092- 71        |
| <b>h</b> <b>F?</b> 0  | 093- 25, 71, 0 |

| KEY ENTRY             | DISPLAY        |
|-----------------------|----------------|
| <b>R/S</b>            | 094- 74        |
| <b>+</b>              | 095- 51        |
| <b>R/S</b>            | 096- 74        |
| <b>h</b> <b>LBL</b> 5 | 097- 25, 13, 5 |
| <b>*</b>              | 098- 73        |
| 5                     | 099- 5         |
| <b>R/S</b>            | 100- 74        |
|                       |                |

| REGISTERS  |            |                         | I                       |
|------------|------------|-------------------------|-------------------------|
| $R_0 \nu$  | $R_1 x$    | $R_2 \Gamma(\nu/2)$     | $R_3 \Gamma[(\nu+1)/2]$ |
| $R_4$ Used | $R_5$ Used | $R_6$ — $R_{14}$ Unused |                         |

| STEP | INSTRUCTIONS                                                                 | INPUT DATA/UNITS | KEYS           | OUTPUT DATA/UNITS |
|------|------------------------------------------------------------------------------|------------------|----------------|-------------------|
| 1    | Key in the program.                                                          |                  |                |                   |
| 2    | (Optional) Input degrees of freedom $\nu$ and $x$ to calculate $f(x, \nu)$ . | $\nu$            | <b>ENTER</b> + |                   |
|      |                                                                              | $x$              | <b>A</b>       | $f(x, \nu)$       |
| 3    | Input degrees of freedom $\nu$ and $x$ to calculate $P(x, \nu)$ .            | $\nu$            | <b>ENTER</b> + |                   |
|      |                                                                              | $x$              | <b>B</b>       | $P(x, \nu)$       |
| 4    | For a different case, go to step 2.                                          |                  |                |                   |

**Example 1:**Find  $f(x, v)$  and  $P(x, v)$  for  $x = 2.2$ ,  $v = 11$ .

| Keystrokes                          | Display                              |
|-------------------------------------|--------------------------------------|
| $\boxed{f}$ $\boxed{SCI}$ 2         |                                      |
| 11 $\boxed{ENTER+}$ 2.2 $\boxed{A}$ | <b>4.37</b> <b>-02</b> ( $f(x, v)$ ) |
| 11 $\boxed{ENTER+}$ 2.2 $\boxed{B}$ | <b>9.75</b> <b>-01</b> ( $P(x, v)$ ) |

**Example 2:**Find  $P(x, v)$  for  $x = -1.75$ ,  $v = 30$ .

| Keystrokes                     | Display                              |
|--------------------------------|--------------------------------------|
| $\boxed{f}$ $\boxed{SCI}$ 2    |                                      |
| 30 $\boxed{ENTER+}$            | <b>30.00</b> ( $v$ )                 |
| 1.75 $\boxed{CHS}$ $\boxed{B}$ | <b>4.51</b> <b>-02</b> ( $P(x, v)$ ) |

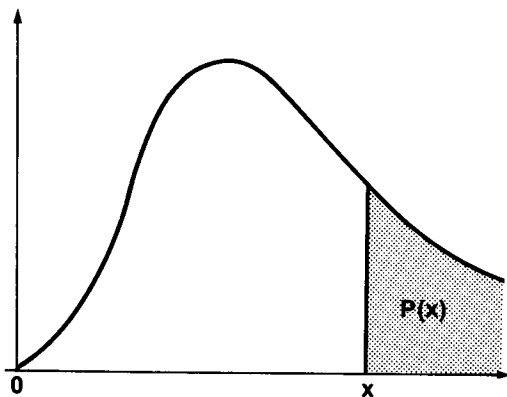
## F Distribution

This program evaluates the integral of the F distribution

### Equations:

1. Density function

$$f(y) = \frac{\Gamma\left(\frac{v_1 + v_2}{2}\right) y^{\frac{v_1}{2} - 1} \left(\frac{v_1}{v_2}\right)^{\frac{v_1}{2}}}{\Gamma\left(\frac{v_1}{2}\right) \Gamma\left(\frac{v_2}{2}\right) \left(1 + \frac{v_1}{v_2} y\right)^{\frac{v_1 + v_2}{2}}}$$



## 2. Cumulative distribution function

$$P(x) = \int_x^{\infty} f(y)dy$$

The integral  $P(x)$  is calculated by using the integrate key  $\int \frac{dx}{x}$ .

**Note:**

Usually  $v_1$  is identified as the degree of freedom for numerator, and  $v_2$  is identified as the degree of freedom for denominator.

| KEY ENTRY                                    | DISPLAY         | KEY ENTRY                 | DISPLAY   |
|----------------------------------------------|-----------------|---------------------------|-----------|
| $\boxed{f}$ CLEAR $\boxed{\text{PRGM}}$      | 000-            | $\boxed{\text{ENTER}}$    | 019- 31   |
| $\boxed{h}$ $\boxed{\text{LBL}}$ $\boxed{A}$ | 001- 25, 13, 11 | $\boxed{\text{ENTER}}$    | 020- 31   |
| $\boxed{\text{STO}}$ 2                       | 002- 23 2       | $\boxed{\text{ENTER}}$    | 021- 31   |
| $\boxed{9}$ $\boxed{R\downarrow}$            | 003- 15 22      | $\boxed{\text{RCL}}$ 0    | 022- 24 0 |
| $\boxed{\text{STO}}$ 1                       | 004- 23 1       | $\boxed{\text{RCL}}$ 1    | 023- 24 1 |
| $\boxed{x \div y}$                           | 005- 21         | $\boxed{+}$               | 024- 71   |
| $\boxed{\text{STO}}$ 0                       | 006- 23 0       | $\boxed{\text{STO}}$ 7    | 025- 23 7 |
| $\boxed{\text{GSB}}$ 3                       | 007- 13 3       | $\boxed{\times}$          | 026- 61   |
| $\boxed{\text{STO}}$ 3                       | 008- 23 3       | 1                         | 027- 1    |
| $\boxed{\text{RCL}}$ 1                       | 009- 24 1       | $\boxed{+}$               | 028- 51   |
| $\boxed{\text{GSB}}$ 3                       | 010- 13 3       | $\boxed{\text{RCL}}$ 0    | 029- 24 0 |
| $\boxed{\text{STO}}$ 4                       | 011- 23 4       | $\boxed{\text{RCL}}$ 1    | 030- 24 1 |
| $\boxed{\text{RCL}}$ 0                       | 012- 24 0       | $\boxed{+}$               | 031- 51   |
| $\boxed{\text{RCL}}$ 1                       | 013- 24 1       | 2                         | 032- 2    |
| $\boxed{+}$                                  | 014- 51         | $\boxed{+}$               | 033- 71   |
| $\boxed{\text{GSB}}$ 3                       | 015- 13 3       | $\boxed{h}$ $\boxed{y^x}$ | 034- 25 3 |
| $\boxed{\text{STO}}$ 5                       | 016- 23 5       | $\boxed{x \div y}$        | 035- 21   |
| $\boxed{\text{RCL}}$ 2                       | 017- 24 2       | $\boxed{\text{RCL}}$ 0    | 036- 24 0 |
| $\boxed{h}$ $\boxed{\text{LBL}}$ 0           | 018- 25, 13, 0  | 2                         | 037- 2    |

| KEY ENTRY   | DISPLAY        |
|-------------|----------------|
| $+$         | 038- 71        |
| 1           | 039- 1         |
| $-$         | 040- 41        |
| $h$ $y^x$   | 041- 25 3      |
| $x \div y$  | 042- 21        |
| $+$         | 043- 71        |
| $RCL$ 7     | 044- 24 7      |
| $RCL$ 0     | 045- 24 0      |
| 2           | 046- 2         |
| $+$         | 047- 71        |
| $h$ $y^x$   | 048- 25 3      |
| $x$         | 049- 61        |
| $RCL$ 5     | 050- 24 5      |
| $x$         | 051- 61        |
| $RCL$ 3     | 052- 24 3      |
| $+$         | 053- 71        |
| $RCL$ 4     | 054- 24 4      |
| $+$         | 055- 71        |
| $h$ $RTN$   | 056- 25 12     |
| $h$ $LBL$ 3 | 057- 25, 13, 3 |
| 1           | 058- 1         |
| $STO$ 6     | 059- 23 6      |
| $x \div y$  | 060- 21        |
| 2           | 061- 2         |
| $+$         | 062- 71        |
| $h$ $INT$   | 063- 25 32     |
| $h$ $LST x$ | 064- 25 0      |

| KEY ENTRY      | DISPLAY         |
|----------------|-----------------|
| $f$ $x \div y$ | 065- 14 61      |
| $GTO$ 1        | 066- 22 1       |
| 1              | 067- 1          |
| $-$            | 068- 41         |
| $h$ $x!$       | 069- 25 1       |
| $STO$ 6        | 070- 23 6       |
| $h$ $RTN$      | 071- 25 12      |
| $h$ $LBL$ 1    | 072- 25, 13, 1  |
| $*$            | 073- 73         |
| 5              | 074- 5          |
| $f$ $x \div y$ | 075- 14 71      |
| $GTO$ 2        | 076- 22 2       |
| $x \div y$     | 077- 21         |
| 1              | 078- 1          |
| $-$            | 079- 41         |
| $STO$ $x$ 6    | 080- 23, 61, 6  |
| $GTO$ 1        | 081- 22 1       |
| $h$ $LBL$ 2    | 082- 25, 13, 2  |
| $h$ $\pi$      | 083- 25 73      |
| $f$ $\sqrt{x}$ | 084- 14 3       |
| $STO$ $x$ 6    | 085- 23, 61, 6  |
| $RCL$ 6        | 086- 24 6       |
| $h$ $RTN$      | 087- 25 12      |
| $h$ $LBL$ $B$  | 088- 25, 13, 12 |
| $GSB$ $A$      | 089- 13, 11     |
| 0              | 090- 0          |
| $RCL$ 2        | 091- 24 2       |

| KEY ENTRY         | DISPLAY        |
|-------------------|----------------|
| $\frac{f}{f-2}$ 0 | 092- 14, 72, 0 |
| 1                 | 093- 1         |
| $\frac{f}{f-2}$   | 094- 41        |

| KEY ENTRY | DISPLAY |
|-----------|---------|
| CHS       | 095- 32 |
| R/S       | 096- 74 |
|           |         |

| REGISTERS             |                                 |            | I                     |
|-----------------------|---------------------------------|------------|-----------------------|
| $R_0 \nu_1$           | $R_1 \nu_2$                     | $R_2 x$    | $R_3 \Gamma(\nu_1/2)$ |
| $R_4 \Gamma(\nu_2/2)$ | $R_5 \Gamma[(\nu_1 + \nu_2)/2]$ | $R_6$ Used | $R_7 \nu_1/\nu_2$     |
| $R_8 - R_5$ Unused    |                                 |            |                       |

| STEP | INSTRUCTIONS                                                       | INPUT DATA/UNITS | KEYS   | OUTPUT DATA/UNITS |
|------|--------------------------------------------------------------------|------------------|--------|-------------------|
| 1    | Key in the program.                                                |                  |        |                   |
| 2    | (Optional) Input $\nu_1$ , $\nu_2$ , and $x$ to calculate $f(x)$ . | $\nu_1$          | ENTER+ |                   |
|      |                                                                    | $\nu_2$          | ENTER+ |                   |
|      |                                                                    | $x$              | A      | $f(x)$            |
| 3    | Input $\nu_1$ , $\nu_2$ and $x$ to calculate $P(x)$ .              | $\nu_1$          | ENTER+ |                   |
|      |                                                                    | $\nu_2$          | ENTER+ |                   |
|      |                                                                    | $x$              | B      | $P(x)$            |
| 4    | For a different case, go to step 2.                                |                  |        |                   |

**Examples:**

$$1. \quad v_1 = 6, v_2 = 7$$

$$P(3.87) = 0.0499$$

$$2. \quad v_1 = 4, v_2 = 20$$

$$P(2.25) = 0.0999$$

**Keystrokes****Display**

**f** **SCI** 2

6 **ENTER** 7 **ENTER**

3.87 **B** 4.99 -02 (P(3.87))

4 **ENTER** 20 **ENTER**

2.25 **B** 9.99 -02 (P(2.25))

# Test Statistics

## t Statistics

### I. Paired t Statistic

Given a set of paired observations from two normal populations with means  $\mu_1, \mu_2$  (unknown)

|       |       |       |     |       |
|-------|-------|-------|-----|-------|
| $x_i$ | $x_1$ | $x_2$ | ... | $x_n$ |
| $y_i$ | $y_1$ | $y_2$ | ... | $y_n$ |

let

$$D_i = x_i - y_i$$

$$\bar{D} = \frac{1}{n} \sum_{i=1}^n D_i$$

$$s_D = \sqrt{\frac{\sum D_i^2 - \frac{1}{n} (\sum D_i)^2}{n - 1}}$$

The test statistic

$$t = \frac{\bar{D}}{s_D} \sqrt{n}$$

which has  $n - 1$  degrees of freedom ( $df$ ) can be used to test the null hypothesis

$$H_0: \mu_1 = \mu_2$$

### II. t Statistic for Two Means

Suppose  $\{x_1, x_2, \dots, x_{n1}\}$  and  $\{y_1, y_2, \dots, y_{n2}\}$  are independent random samples from two normal populations having means  $\mu_1, \mu_2$  (unknown) and the same unknown variance  $\sigma^2$ .

We want to test the null hypothesis

$$H_0: \mu_1 - \mu_2 = d$$

Define

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - d}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{\sum x_i^2 - n_1 \bar{x}^2 + \sum y_i^2 - n_2 \bar{y}^2}{n_1 + n_2 - 2}}}$$

We can use this  $t$  statistic which has the  $t$  distribution with  $n_1 + n_2 - 2$  degrees of freedom ( $df$ ) to test the null hypothesis  $H_0$ .

### References:

1. Statistics in Research, B. Ostle, Iowa State University Press, 1963.
2. Statistical Theory and Methodology in Science and Engineering, K. A. Brownlee, John Wiley & Sons, 1965.

| KEY ENTRY                    | DISPLAY         |
|------------------------------|-----------------|
| <b>f</b> CLEAR <b>PRGM</b>   | 000-            |
| <b>h</b> <b>LBL</b> <b>A</b> | 001- 25, 13, 11 |
| <b>-</b>                     | 002- 41         |
| <b>f</b> <b>Σ+</b>           | 003- 14 74      |
| <b>R/S</b>                   | 004- 74         |

| KEY ENTRY             | DISPLAY        |
|-----------------------|----------------|
| <b>h</b> <b>LBL</b> 1 | 005- 25, 13, 1 |
| <b>h</b> <b>Σ</b>     | 006- 25 8      |
| <b>STO</b> 6          | 007- 23 6      |
| <b>R/S</b>            | 008- 74        |
| <b>h</b> <b>S</b>     | 009- 25 9      |

| KEY ENTRY                                    | DISPLAY         |
|----------------------------------------------|-----------------|
| $\boxed{R/S}$                                | 010- 74         |
| $\boxed{RCL} \ 6$                            | 011- 24 6       |
| $\boxed{RCL} \ 0$                            | 012- 24 0       |
| $\boxed{f} \ \boxed{\sqrt{x}}$               | 013- 14 3       |
| $\boxed{x}$                                  | 014- 61         |
| $\boxed{x \div y}$                           | 015- 21         |
| $\boxed{+}$                                  | 016- 71         |
| $\boxed{R/S}$                                | 017- 74         |
| $\boxed{RCL} \ 0$                            | 018- 24 0       |
| 1                                            | 019- 1          |
| $\boxed{-}$                                  | 020- 41         |
| $\boxed{R/S}$                                | 021- 74         |
| $\boxed{GTO} \ 1$                            | 022- 22 1       |
| $\boxed{h} \ \boxed{LBL} \ \boxed{B}$        | 023- 25, 13, 12 |
| $\boxed{f} \ \boxed{\Sigma+}$                | 024- 14 74      |
| $\boxed{R/S}$                                | 025- 74         |
| $\boxed{RCL} \ 1$                            | 026- 24 1       |
| $\boxed{STO} \ \boxed{\cdot} \ 1$            | 027- 23 .1      |
| $\boxed{RCL} \ 2$                            | 028- 24 2       |
| $\boxed{STO} \ \boxed{\cdot} \ 2$            | 029- 23 .2      |
| $\boxed{RCL} \ 0$                            | 030- 24 0       |
| $\boxed{STO} \ \boxed{\cdot} \ 0$            | 031- 23 .0      |
| 0                                            | 032- 0          |
| $\boxed{f} \ \boxed{CLEAR} \ \boxed{\Sigma}$ | 033- 14 34      |
| $\boxed{R/S}$                                | 034- 74         |
| $\boxed{h} \ \boxed{LBL} \ 2$                | 035- 25, 13, 2  |
| $\boxed{STO} \ 6$                            | 036- 23 6       |

| KEY ENTRY                         | DISPLAY    |
|-----------------------------------|------------|
| $\boxed{RCL} \ \boxed{\cdot} \ 1$ | 037- 24 .1 |
| $\boxed{RCL} \ \boxed{\cdot} \ 2$ | 038- 24 .2 |
| $\boxed{STO} \ 4$                 | 039- 23 4  |
| $\boxed{x \div y}$                | 040- 21    |
| $\boxed{STO} \ 3$                 | 041- 23 3  |
| $\boxed{h} \ \boxed{\bar{x}}$     | 042- 25 8  |
| $\boxed{STO} \ 8$                 | 043- 23 8  |
| $\boxed{x \div y}$                | 044- 21    |
| $\boxed{RCL} \ 0$                 | 045- 24 0  |
| $\boxed{x}$                       | 046- 61    |
| $\boxed{RCL} \ \boxed{\cdot} \ 0$ | 047- 24 .0 |
| $\boxed{+}$                       | 048- 71    |
| $\boxed{STO} \ 7$                 | 049- 23 7  |
| $\boxed{x \div y}$                | 050- 21    |
| $\boxed{-}$                       | 051- 41    |
| $\boxed{RCL} \ 6$                 | 052- 24 6  |
| $\boxed{-}$                       | 053- 41    |
| $\boxed{RCL} \ 4$                 | 054- 24 4  |
| $\boxed{RCL} \ 3$                 | 055- 24 3  |
| $\boxed{RCL} \ 7$                 | 056- 24 7  |
| $\boxed{x}$                       | 057- 61    |
| $\boxed{-}$                       | 058- 41    |
| $\boxed{RCL} \ 2$                 | 059- 24 2  |
| $\boxed{+}$                       | 060- 51    |
| $\boxed{RCL} \ 1$                 | 061- 24 1  |
| $\boxed{RCL} \ 8$                 | 062- 24 8  |
| $\boxed{x}$                       | 063- 61    |

| KEY ENTRY               | DISPLAY    |
|-------------------------|------------|
| $\square$               | 064- 41    |
| $\square$ 0             | 065- 24 0  |
| $\square$ $\cdot$ 0     | 066- 24 .0 |
| $+$                     | 067- 51    |
| 2                       | 068- 2     |
| $-$                     | 069- 41    |
| $\square$ 9             | 070- 23 9  |
| $+$                     | 071- 71    |
| $\square$ $\sqrt{x}$    | 072- 14 3  |
| $\square$ 0             | 073- 24 0  |
| $\square$ $\frac{1}{x}$ | 074- 25 2  |

| KEY ENTRY               | DISPLAY    |
|-------------------------|------------|
| $\square$ $\cdot$ 0     | 075- 24 .0 |
| $\square$ $\frac{1}{x}$ | 076- 25 2  |
| $+$                     | 077- 51    |
| $\square$ $\sqrt{x}$    | 078- 14 3  |
| $\times$                | 079- 61    |
| $+$                     | 080- 71    |
| $\square$               | 081- 74    |
| $\square$ 9             | 082- 24 9  |
| $\square$               | 083- 74    |
| $\square$ 2             | 084- 22 2  |
|                         |            |

| REGISTERS                |                              |                         | I                            |
|--------------------------|------------------------------|-------------------------|------------------------------|
| $R_0 d$                  | $R_1 x$                      | $R_2 y$                 | $R_3 df$                     |
| $R_4 \Sigma x, \Sigma y$ | $R_5 \Sigma x^2, \Sigma y^2$ | $R_6 \Sigma y \Sigma x$ | $R_7 \Sigma y^2, \Sigma x^2$ |
| $R_8 \Sigma xy$          | $R_9 n_1, n_2$               | $R_{10} n_1$            | $R_{11} \Sigma x$            |
| $R_{12} \Sigma x^2$      | $R_{13}-R_{17}$ Unused       |                         |                              |

| STEP | INSTRUCTIONS                                | INPUT DATA/UNITS | KEYS                          | OUTPUT DATA/UNITS |
|------|---------------------------------------------|------------------|-------------------------------|-------------------|
|      | <b>Paired t Statistics</b>                  |                  |                               |                   |
| 1    | Key in the program from line 1-line 22.     |                  |                               |                   |
| 2    | Initialize the program.                     |                  | $\square$ CLEAR $\square$ REG |                   |
| 3    | Repeat steps 3-4 for $i = 1, 2, \dots, n$ . |                  |                               |                   |
|      | Input: $x_i$                                | $x_i$            | $\square$ ENTER $\square$     |                   |
|      | $y_i$                                       | $y_i$            | $\square$ A                   | $i$               |
| 4    | If you made a mistake in                    |                  |                               |                   |

| STEP | INSTRUCTIONS                                                             | INPUT DATA/UNITS | KEYS                            | OUTPUT DATA/UNITS |
|------|--------------------------------------------------------------------------|------------------|---------------------------------|-------------------|
|      | inputting $x_k$ and $y_k$ , then                                         |                  |                                 |                   |
|      | correct by $\rightarrow$                                                 | $x_k$            | <b>ENTER</b>                    |                   |
|      |                                                                          | $y_k$            | <b>- 9 <math>\Sigma</math>-</b> | $k-1$             |
| 5    | To calculate test statistics:                                            |                  |                                 |                   |
|      | $\bar{D}$                                                                |                  | <b>GSB 1</b>                    | $\bar{D}$         |
|      | $S_D$                                                                    |                  | <b>R/S</b>                      | $S_D$             |
|      | $t$                                                                      |                  | <b>R/S</b>                      | $t$               |
|      | $df$                                                                     |                  | <b>R/S</b>                      | $df$              |
| 6    | Repeat step 5 if you want the results again.                             |                  |                                 |                   |
|      | <b>t Statistics for</b>                                                  |                  |                                 |                   |
|      | <b>Two Means</b>                                                         |                  |                                 |                   |
| 1    | Key in the program from line 23–line 84.                                 |                  |                                 |                   |
| 2    | Initialize the program.                                                  |                  | <b>f CLEAR REG</b>              |                   |
| 3    | Repeat step 3–4 for $i = 1, 2, \dots, n_1$ .                             |                  |                                 |                   |
|      | Input $x_i$ .                                                            | $x_i$            | <b>B</b>                        | $i$               |
| 4    | If you made a mistake in inputting $x_k$ , then correct by $\rightarrow$ | $x_k$            | <b>9 <math>\Sigma</math>-</b>   | $k-1$             |
| 5    | Initialize for the 2nd array of data.                                    |                  | <b>R/S</b>                      | 0.0000            |
| 6    | Repeat step 6–7 for $i = 1, 2, \dots, n_2$ .                             |                  |                                 |                   |
|      | Input $y_i$ .                                                            | $y_i$            | <b>B</b>                        | $i$               |
| 7    | If you made a mistake in inputting $y_k$ , then correct by $\rightarrow$ | $y_k$            | <b>9 <math>\Sigma</math>-</b>   | $k-1$             |

| STEP | INSTRUCTIONS                                 | INPUT DATA/UNITS | KEYS         | OUTPUT DATA/UNITS |
|------|----------------------------------------------|------------------|--------------|-------------------|
| 8    | Input $d$ to calculate test statistic: $t$   |                  |              |                   |
|      |                                              | $d$              | <b>GSB</b> 2 | $t$               |
|      | $df$                                         |                  | <b>R/S</b>   | $df$              |
| 9    | Repeat step 8 if you want the results again. |                  |              |                   |
|      |                                              |                  |              |                   |

## Example 1:

|       |    |      |      |      |      |
|-------|----|------|------|------|------|
| $x_i$ | 14 | 17.5 | 17   | 17.5 | 15.4 |
| $y_i$ | 17 | 20.7 | 21.6 | 20.9 | 17.2 |

$$D = -3.2000$$

$$s_D = 1.0000$$

$$t = -7.1554$$

$$df = 4.0000$$

## Keystrokes

## Display

|                                  |         |              |
|----------------------------------|---------|--------------|
| <b>f</b> <b>FIX</b> 4            |         |              |
| <b>f</b> <b>CLEAR</b> <b>REG</b> |         |              |
| 14 <b>ENTER</b> 17 <b>A</b>      | 1.0000  |              |
| 17 <b>ENTER</b> 15 <b>A</b>      | 2.0000  | (error)      |
| 17 <b>ENTER</b> 15 <b>-</b>      |         |              |
| <b>g</b> <b>Σ-</b>               | 1.0000  | (correction) |
| 17.5 <b>ENTER</b> 20.7 <b>A</b>  |         |              |
| 17 <b>ENTER</b> 21.6 <b>A</b>    |         |              |
| 17.5 <b>ENTER</b> 20.9 <b>A</b>  |         |              |
| 15.4 <b>ENTER</b> 17.2 <b>A</b>  | 5.0000  |              |
| <b>GSB</b> 1                     | -3.2000 | $(\bar{D})$  |
| <b>R/S</b>                       | 1.0000  | $(s_D)$      |
| <b>R/S</b>                       | -7.1554 | $(t)$        |
| <b>R/S</b>                       | 4.0000  | $(df)$       |

### Example 2:

|     |    |     |     |     |     |     |     |     |    |    |
|-----|----|-----|-----|-----|-----|-----|-----|-----|----|----|
| $x$ | 79 | 84  | 108 | 114 | 120 | 103 | 122 | 120 |    |    |
| $y$ | 91 | 103 | 90  | 113 | 108 | 87  | 100 | 80  | 99 | 54 |

$$n_1 = 8$$

$$n_2 = 10$$

If  $d = 0$  (i.e.,  $H_0: \mu_1 = \mu_2$ )

then  $t = 1.73$ ,  $df = 16.00$

## Keystrokes

## Display

 CLEAR 

79 **B** 84 **B** 99

**B** 99 **9** **Σ-**

**2.0000**

108 **B** 114 **B** 120

**B** 103 **B** 122

**B** 120 **B**

**8.0000**

R/S

**0.0000**

91 **B** 103 **B** 90

**B** 113 **B** 108

**B** 87 **B** 100 **B**

80 **B** 99 **B** 54 **B**

**10.0000**

0 **GSB** 2

**1.7316**

(1)

R/S

**16.0000**

(df)

## Chi-Square Evaluation

This program calculates the value of the  $\chi^2$  statistic for the goodness of fit test by the equation

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

where  $O_i$  = observed frequency  
 $E_i$  = expected frequency

If the expected values are equal

$$\left( E = E_i = \frac{\sum O_i}{n} \text{ for all } i \right)$$

then

$$\chi^2 = \frac{n \sum O_i^2}{\sum O_i} - \sum O_i$$

### Note:

In order to apply the goodness of fit test to a set of given data, combining some classes may be necessary to make sure that each expected frequency is not too small (say, not less than 5).

### Reference:

Mathematical Statistics, J. E. Freund, Prentice Hall, 1962.

| KEY ENTRY                     | DISPLAY         |
|-------------------------------|-----------------|
| <b>f</b> CLEAR <b>PRGM</b>    | 000-            |
| <b>h</b> <b>LBL</b> <b>A</b>  | 001- 25, 13, 11 |
| <b>ENTER</b>                  | 002- 31         |
| <b>g</b> <b>R+</b>            | 003- 15 22      |
| <b>-</b>                      | 004- 41         |
| <b>g</b> <b>x<sup>2</sup></b> | 005- 15 3       |
| <b>f</b> <b>R+</b>            | 006- 14 22      |
| <b>+</b>                      | 007- 71         |
| <b>h</b> <b>F?</b> 0          | 008- 25, 71, 0  |
| <b>CHS</b>                    | 009- 32         |
| <b>STO</b> <b>+</b> 1         | 010- 23, 51, 1  |
| 1                             | 011- 1          |
| <b>h</b> <b>F?</b> 0          | 012- 25, 71, 0  |
| <b>CHS</b>                    | 013- 32         |
| <b>STO</b> <b>+</b> 0         | 014- 23, 51, 0  |
| <b>RCL</b> 0                  | 015- 24 0       |
| <b>h</b> <b>CF</b> 0          | 016- 25, 61, 0  |
| <b>R/S</b>                    | 017- 74         |
| <b>h</b> <b>LBL</b> 1         | 018- 25, 13, 1  |
| <b>RCL</b> 1                  | 019- 24 1       |

| KEY ENTRY                            | DISPLAY         |
|--------------------------------------|-----------------|
| <b>R/S</b>                           | 020- 74         |
| <b>GTO</b> 1                         | 021- 22 1       |
| <b>h</b> <b>LBL</b> <b>B</b>         | 022- 25, 13, 12 |
| <b>f</b> <b><math>\Sigma</math>+</b> | 023- 14 74      |
| <b>R/S</b>                           | 024- 74         |
| <b>h</b> <b>LBL</b> 2                | 025- 25, 13, 2  |
| <b>RCL</b> 0                         | 026- 24 0       |
| <b>RCL</b> 0                         | 027- 24 0       |
| <b>RCL</b> 2                         | 028- 24 2       |
| <b>x</b>                             | 029- 61         |
| <b>RCL</b> 1                         | 030- 24 1       |
| <b>+</b>                             | 031- 71         |
| <b>RCL</b> 1                         | 032- 24 1       |
| <b>-</b>                             | 033- 41         |
| <b>R/S</b>                           | 034- 74         |
| <b>RCL</b> 1                         | 035- 24 1       |
| <b>RCL</b> 0                         | 036- 24 0       |
| <b>+</b>                             | 037- 71         |
| <b>R/S</b>                           | 038- 74         |
| <b>GTO</b> 2                         | 039- 22 2       |

| REGISTERS            |                                       |                               | I                                     |
|----------------------|---------------------------------------|-------------------------------|---------------------------------------|
| R <sub>0</sub> Index | R <sub>1</sub> $x_2^2$ , $\Sigma O_i$ | R <sub>2</sub> $\Sigma O_i^2$ | R <sub>3</sub> Used                   |
| R <sub>4</sub> Used  | R <sub>5</sub> Used                   | R <sub>6</sub> Unused         | R <sub>7</sub> —R <sub>9</sub> Unused |

| STEP | INSTRUCTIONS                                                                 | INPUT DATA/UNITS | KEYS                          | OUTPUT DATA/UNITS |
|------|------------------------------------------------------------------------------|------------------|-------------------------------|-------------------|
|      | <b>Unequal Expected</b>                                                      |                  |                               |                   |
|      | <b>Frequency</b>                                                             |                  |                               |                   |
| 1    | Key in the program from<br>line 1–line 21.                                   |                  |                               |                   |
| 2    | Initialize the program.                                                      |                  | <b>f</b> CLEAR <b>REG</b>     |                   |
| 3    | Repeat steps 3–4 for $i = 1, 2,$<br>..., $n$ .                               |                  |                               |                   |
|      | Input: $O_i$                                                                 | $O_i$            | <b>ENTER</b> *                |                   |
|      | $E_i$                                                                        | $E_i$            | <b>A</b>                      | $i$               |
| 4    | If you made a mistake in<br>inputting $O_k$ and $E_k$ , then<br>correct by → | $O_k$            | <b>ENTER</b> *                |                   |
|      |                                                                              | $E_k$            | <b>h</b> <b>SF</b> 0 <b>A</b> | $k-1$             |
| 5    | Calculate $\chi_1^2$ .                                                       |                  | <b>GSB</b> 1                  | $\chi_1^2$        |
| 6    | Repeat step 5 if you want<br>the result again.                               |                  |                               |                   |
|      | <b>Equal Expected</b>                                                        |                  |                               |                   |
|      | <b>Frequency</b>                                                             |                  |                               |                   |
| 1    | Key in the program from<br>line 22–line 39.                                  |                  |                               |                   |
| 2    | Initialize the program.                                                      |                  | <b>f</b> CLEAR <b>REG</b>     |                   |
| 3    | Repeat steps 3–4 for $i = 1, 2,$<br>..., $n$ .                               |                  |                               |                   |
|      | Input: $O_i$                                                                 | $O_i$            | <b>B</b>                      | $i$               |
| 4    | If you made a mistake in<br>inputting $O_h$ , then correct<br>by →           | $O_h$            | <b>g</b> <b>Σ-</b>            | $h-1$             |
| 5    | Calculate: $\chi_2^2$                                                        |                  | <b>GSB</b> 2                  | $\chi_2^2$        |
|      | $E$                                                                          |                  | <b>R/S</b>                    | $E$               |

| STEP | INSTRUCTIONS              | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
|------|---------------------------|------------------|------|-------------------|
| 6    | Repeat step 5 if you want |                  |      |                   |
|      | the results again.        |                  |      |                   |

**Example 1:**

Find the value of  $\chi^2$  statistic for the goodness of fit for the following data set:

|       |     |       |       |      |      |      |
|-------|-----|-------|-------|------|------|------|
| $O_i$ | 8   | 50    | 47    | 56   | 5    | 14   |
| $E_i$ | 9.6 | 46.75 | 51.85 | 54.4 | 8.25 | 9.15 |

$$\chi_1^2 = 4.8444 \quad df = 5$$

**Keystrokes****Display**

|                                |        |                |
|--------------------------------|--------|----------------|
| <b>f</b> CLEAR <b>REG</b>      |        |                |
| 8 <b>ENTER</b> 9.6 <b>A</b>    |        |                |
| 50 <b>ENTER</b> 46.75 <b>A</b> |        |                |
| 47 <b>ENTER</b> 51.85 <b>A</b> |        |                |
| 56 <b>ENTER</b> 54.4 <b>A</b>  |        |                |
| 100 <b>ENTER</b> 100 <b>A</b>  | 5.0000 | (error)        |
| 100 <b>ENTER</b> 100           |        |                |
| <b>h</b> <b>SF</b> 0 <b>A</b>  | 4.0000 | (correction)   |
| 5 <b>ENTER</b> 8.25 <b>A</b>   |        |                |
| 14 <b>ENTER</b> 9.15 <b>A</b>  | 6.0000 |                |
| <b>GSB</b> 1                   | 4.8444 | ( $\chi_1^2$ ) |

**Example 2:**

The following table shows the observed frequencies in tossing a die 120 times.  $\chi^2$  can be used to test if the die is fair.

**Note:**

Assume that the expected frequencies are equal.

|                 |    |    |    |    |    |    |
|-----------------|----|----|----|----|----|----|
| number          | 1  | 2  | 3  | 4  | 5  | 6  |
| frequency $O_i$ | 25 | 17 | 15 | 23 | 24 | 16 |

$$\chi^2 = 5.00 \quad df = 5$$

$$E = 20.00$$

| Keystrokes                          | Display |              |
|-------------------------------------|---------|--------------|
| <b>f</b> CLEAR <b>REG</b>           |         |              |
| 25 <b>B</b> 17 <b>B</b> 19 <b>B</b> | 3.0000  | (error)      |
| 19 <b>9</b> <b>Σ-</b>               | 2.0000  | (correction) |
| 15 <b>B</b> 23 <b>B</b> 24 <b>B</b> |         |              |
| 16 <b>B</b>                         | 6.0000  |              |
| <b>GSB</b> 2                        | 5.0000  | ( $\chi^2$ ) |
| <b>R/S</b>                          | 20.0000 | ( $E$ )      |

Since 5.00 is less than 11.07 (5% significance level), no actual differences exist between the observed and expected frequencies.

## Contingency Table

Contingency tables can be used to test the null hypothesis that two variables are independent.

This program calculates the  $\chi^2$  statistic for testing the independence of the two variables. Also Pearson's coefficient of contingency  $C_c$ , which measures the degree of association between the two variables, is calculated.

### I. 2 x k Contingency Table

| $i \backslash j$ | 1        | 2        | ... | k        | Totals |
|------------------|----------|----------|-----|----------|--------|
| 1                | $x_{11}$ | $x_{12}$ | ... | $x_{1k}$ | $R_1$  |
| 2                | $x_{21}$ | $x_{22}$ | ... | $x_{2k}$ | $R_2$  |
| Totals           | $C_1$    | $C_2$    | ... | $C_k$    | $T$    |

### II. 3 x k Contingency Table

| $i \backslash j$ | 1        | 2        | ... | k        | Totals |
|------------------|----------|----------|-----|----------|--------|
| 1                | $x_{11}$ | $x_{12}$ | ... | $x_{1k}$ | $R_1$  |
| 2                | $x_{21}$ | $x_{22}$ | ... | $x_{2k}$ | $R_2$  |
| 3                | $x_{31}$ | $x_{32}$ | ... | $x_{3k}$ | $R_3$  |
| Totals           | $C_1$    | $C_2$    | ... | $C_k$    | $T$    |

Equations:

$$\text{Row sum } R_i = \sum_{j=1}^k x_{ij} \quad \begin{array}{l} i = 1, 2 \text{ (for } 2 \times k) \\ i = 1, 2, 3 \text{ (for } 3 \times k) \end{array}$$

$$\text{Column sum } C_j = \sum_{i=1}^n x_{ij} \quad \begin{array}{l} j = 1, 2, \dots, k \\ n = 2 \text{ (for } 2 \times k) \\ n = 3 \text{ (for } 3 \times k) \end{array}$$

$$\text{Total } T = \sum_{i=1}^n \sum_{j=1}^k x_{ij} \quad \begin{array}{l} n = 2 \text{ (for } 2 \times k) \\ n = 3 \text{ (for } 3 \times k) \end{array}$$

Chi-square statistic

$$\chi^2 = \sum_{i=1}^n \sum_{j=1}^k \frac{(x_{ij} - E_{ij})^2}{E_{ij}} \quad df = (n - 1)(k - 1)$$

$$= T \left( \sum_{i=1}^n \sum_{j=1}^k \frac{x_{ij}^2}{R_i C_j} \right) - T \quad \begin{array}{l} n = 2 \text{ (for } 2 \times k) \\ n = 3 \text{ (for } 3 \times k) \end{array}$$

Contingency coefficient

$$C_c = \sqrt{\frac{\chi^2}{T + \chi^2}}$$

### Reference:

B. Ostle, *Statistics in Research*, Iowa State University Press, 1972.

| KEY ENTRY                    | DISPLAY         |
|------------------------------|-----------------|
| <b>f</b> CLEAR <b>PRGM</b>   | 000-            |
| <b>h</b> <b>LBL</b> 0        | 001- 25, 13, 0  |
| <b>f</b> CLEAR <b>REG</b>    | 002- 14 33      |
| <b>h</b> <b>CF</b> 0         | 003- 25, 61, 0  |
| <b>h</b> <b>CF</b> 1         | 004- 25, 61, 1  |
| 0                            | 005- 0          |
| <b>R/S</b>                   | 006- 74         |
| <b>h</b> <b>LBL</b> <b>B</b> | 007- 25, 13, 12 |
| <b>h</b> <b>SF</b> 0         | 008- 25, 51, 0  |
| <b>h</b> <b>LBL</b> <b>A</b> | 009- 25, 13, 11 |
| <b>h</b> <b>F?</b> 1         | 010- 25, 71, 1  |
| 0                            | 011- 0          |

| KEY ENTRY             | DISPLAY        |
|-----------------------|----------------|
| <b>GSB</b> 9          | 012- 13 9      |
| <b>R/S</b>            | 013- 74        |
| <b>RCL</b> 7          | 014- 24 7      |
| <b>R/S</b>            | 015- 74        |
| <b>h</b> <b>LBL</b> 1 | 016- 25, 13, 1 |
| <b>h</b> <b>F?</b> 1  | 017- 25, 71, 1 |
| 1                     | 018- 1         |
| <b>h</b> <b>F?</b> 1  | 019- 25, 71, 1 |
| <b>STO</b> 3          | 020- 23 3      |
| <b>RCL</b> 4          | 021- 24 4      |
| <b>RCL</b> 1          | 022- 24 1      |
| <b>+</b>              | 023- 71        |

| KEY ENTRY                 | DISPLAY       |
|---------------------------|---------------|
| <b>[STO]</b> 9            | 024- 23 9     |
| <b>[RCL]</b> 5            | 025- 24 5     |
| <b>[RCL]</b> 2            | 026- 24 2     |
| <b>+</b>                  | 027- 71       |
| <b>[STO]</b> <b>+</b> 9   | 028-23, 51, 9 |
| <b>[RCL]</b> 6            | 029- 24 6     |
| <b>[RCL]</b> 3            | 030- 24 3     |
| <b>+</b>                  | 031- 71       |
| <b>[STO]</b> <b>+</b> 9   | 032-23, 51, 9 |
| <b>[RCL]</b> 9            | 033- 24 9     |
| 1                         | 034- 1        |
| <b>-</b>                  | 035- 41       |
| <b>[RCL]</b> 0            | 036- 24 0     |
| <b>[x]</b>                | 037- 61       |
| <b>[R/S]</b>              | 038- 74       |
| <b>[ENTER]</b>            | 039- 31       |
| <b>[ENTER]</b>            | 040- 31       |
| <b>[RCL]</b> 0            | 041- 24 0     |
| <b>+</b>                  | 042- 51       |
| <b>+</b>                  | 043- 71       |
| <b>[f]</b> <b>[x]</b>     | 044- 14 3     |
| <b>[R/S]</b>              | 045- 74       |
| <b>[h]</b> <b>[LBL]</b> 2 | 046-25, 13, 2 |
| <b>[RCL]</b> 1            | 047- 24 1     |
| <b>[R/S]</b>              | 048- 74       |
| <b>[RCL]</b> 2            | 049- 24 2     |
| <b>[R/S]</b>              | 050- 74       |
| <b>[h]</b> <b>[F?]</b> 1  | 051-25, 71, 1 |
| <b>[GTO]</b> 3            | 052- 22 3     |

| KEY ENTRY                         | DISPLAY        |
|-----------------------------------|----------------|
| <b>[RCL]</b> 3                    | 053- 24 3      |
| <b>[R/S]</b>                      | 054- 74        |
| <b>[h]</b> <b>[LBL]</b> 3         | 055-25, 13, 3  |
| <b>[RCL]</b> 0                    | 056- 24 0      |
| <b>[R/S]</b>                      | 057- 74        |
| <b>[GTO]</b> 1                    | 058- 22 1      |
| <b>[h]</b> <b>[LBL]</b> 9         | 059-25, 13, 9  |
| <b>[h]</b> <b>[F?]</b> 0          | 060-25, 71, 0  |
| <b>[CHS]</b>                      | 061- 32        |
| <b>[STO]</b> <b>+</b> 3           | 062-23, 51, 3  |
| <b>[STO]</b> <b>+</b> 0           | 063-23, 51, 0  |
| <b>[STO]</b> 7                    | 064- 23 7      |
| <b>[g]</b> <b>[x<sup>2</sup>]</b> | 065- 15 3      |
| <b>[STO]</b> 8                    | 066- 23 8      |
| <b>[g]</b> <b>[R+]</b>            | 067- 15 22     |
| <b>[h]</b> <b>[F?]</b> 0          | 068-25, 71, 0  |
| <b>[CHS]</b>                      | 069- 32        |
| <b>[STO]</b> <b>+</b> 2           | 070-23, 51, 2  |
| <b>[STO]</b> <b>+</b> 0           | 071-23, 51, 0  |
| <b>[STO]</b> <b>+</b> 7           | 072-23, 51, 7  |
| <b>[g]</b> <b>[x<sup>2</sup>]</b> | 073- 15 3      |
| <b>[STO]</b> 9                    | 074- 23 9      |
| <b>[g]</b> <b>[R+]</b>            | 075- 15 22     |
| <b>[h]</b> <b>[F?]</b> 0          | 076-25, 71, 0  |
| <b>[CHS]</b>                      | 077- 32        |
| <b>[STO]</b> <b>+</b> 1           | 078- 23, 51, 1 |
| <b>[STO]</b> <b>+</b> 0           | 079- 23, 51, 0 |
| <b>[STO]</b> <b>+</b> 7           | 080- 23, 51, 7 |
| <b>[g]</b> <b>[x<sup>2</sup>]</b> | 081- 15 3      |

| KEY ENTRY             | DISPLAY       |
|-----------------------|---------------|
| <b>RCL</b> 7          | 082- 24 7     |
| <b>+</b>              | 083- 71       |
| <b>STO</b> <b>+</b> 4 | 084-23, 51, 4 |
| <b>RCL</b> 9          | 085- 24 9     |
| <b>RCL</b> 7          | 086- 24 7     |
| <b>+</b>              | 087- 71       |
| <b>STO</b> <b>+</b> 5 | 088-23, 51, 5 |
| <b>RCL</b> 8          | 089- 24 8     |
| <b>RCL</b> 7          | 090- 24 7     |

| KEY ENTRY                    | DISPLAY        |
|------------------------------|----------------|
| <b>+</b>                     | 091- 71        |
| <b>STO</b> <b>+</b> 6        | 092-23, 51, 6  |
| <b>RCL</b> <b>f</b> <b>I</b> | 093-24, 14, 23 |
| 1                            | 094- 1         |
| <b>h</b> <b>F?</b> 0         | 095-25, 71, 0  |
| <b>CHS</b>                   | 096- 32        |
| <b>+</b>                     | 097- 51        |
| <b>STO</b> <b>f</b> <b>I</b> | 098-23, 14, 23 |
| <b>h</b> <b>CF</b> 0         | 099-25, 61, 0  |

| REGISTERS                 |                           |                           | I Index   |
|---------------------------|---------------------------|---------------------------|-----------|
| $R_0 T$                   | $R_1 R_1$                 | $R_2 R_2$                 | $R_3 R_3$ |
| $R_4 \sum x_{1j}^2 / c_j$ | $R_5 \sum x_{2j}^2 / c_j$ | $R_6 \sum x_{3j}^2 / c_j$ | $R_7 c_j$ |
| $R_8 x_{3j}^2$            | $R_9 x_{2j}^2$            | $R_{10} - R_{14}$ Unused  |           |

| STEP | INSTRUCTIONS                               | INPUT DATA/UNITS | KEYS                  | OUTPUT DATA/UNITS |
|------|--------------------------------------------|------------------|-----------------------|-------------------|
| 1    | Key in the program.                        |                  |                       |                   |
|      | <b>2xk</b> (go to step 8 for <b>3xk</b> ). |                  |                       |                   |
| 2    | Initialize the program.                    |                  | <b>GSB</b> 0          |                   |
|      |                                            |                  | <b>h</b> <b>SF</b> 1  |                   |
| 3    | Repeat steps 3-6 for $j = 1, 2,$           |                  |                       |                   |
|      | ..., $k$ .                                 |                  |                       |                   |
|      | Input: $x_{1j}$                            | $x_{1j}$         | <b>ENTER</b> <b>+</b> |                   |
|      | $x_{2j}$                                   | $x_{2j}$         | <b>A</b>              | $j$               |
| 4    | (OPTIONAL) Calculate                       |                  |                       |                   |
|      | column sum $C_j$ .                         |                  | <b>R/S</b>            | $C_j$             |

# 60 Contingency Table

| STEP | INSTRUCTIONS                                                                                          | INPUT DATA/UNITS | KEYS          | OUTPUT DATA/UNITS |
|------|-------------------------------------------------------------------------------------------------------|------------------|---------------|-------------------|
| 5    | If you made a mistake in inputting $x_{1h}$ and $x_{2h}$ , then correct by $\rightarrow$              | $x_{1h}$         | <b>ENTER+</b> |                   |
|      |                                                                                                       | $x_{2h}$         | <b>B</b>      | $h-1$             |
| 6    | (OPTIONAL) Calculate column sum $C_h$ .                                                               |                  |               |                   |
|      | (correction)                                                                                          |                  | <b>R/S</b>    | $-C_h$            |
| 7    | Go to step 13 for contingency table calculations.                                                     |                  |               |                   |
|      | <b>3xk</b>                                                                                            |                  |               |                   |
| 8    | Initialize the program.                                                                               |                  | <b>GSB</b> 0  |                   |
| 9    | Repeat steps 9-12 for $j = 1, 2, \dots, k$ .                                                          |                  |               |                   |
|      | Input: $x_{1j}$                                                                                       | $x_{1j}$         | <b>ENTER+</b> |                   |
|      | $x_{2j}$                                                                                              | $x_{2j}$         | <b>ENTER+</b> |                   |
|      | $x_{3j}$                                                                                              | $x_{3j}$         | <b>A</b>      | $j$               |
| 10   | (OPTIONAL) Calculate column sum $C_j$ .                                                               |                  | <b>R/S</b>    | $C_j$             |
| 11   | If you made a mistake in inputting $x_{1h}$ , $x_{2h}$ , and $x_{3h}$ , then correct by $\rightarrow$ | $x_{1h}$         | <b>ENTER+</b> |                   |
|      |                                                                                                       | $x_{2h}$         | <b>ENTER+</b> |                   |
|      |                                                                                                       | $x_{3h}$         | <b>B</b>      | $(h-1)$           |
| 12   | (OPTIONAL) Calculate column sum $C_h$ (correction).                                                   |                  | <b>R/S</b>    | $C_h$             |
| 13   | Calculate: Test Statistics $\chi^2$                                                                   |                  | <b>GSB</b> 1  | $\chi^2$          |
|      | Coefficients $C_c$                                                                                    |                  | <b>R/S</b>    | $C_c$             |
|      | Row Sum 1 $R_1$                                                                                       |                  | <b>GSB</b> 2  | $R_1$             |
|      | Row Sum 2 $R_2$                                                                                       |                  | <b>R/S</b>    | $R_2$             |

| STEP | INSTRUCTION                                      | KEYS | INPUTS<br>DATA/UNITS | OUTPUT<br>DATA/UNITS |
|------|--------------------------------------------------|------|----------------------|----------------------|
|      | Row Sum $3 R_3$ (3xk only)                       |      | <b>R/S</b>           | $R_3$                |
|      | Total $T$                                        |      | <b>GSB</b> 3         | $T$                  |
| 14   | Repeat step 13 if you want<br>the results again. |      |                      |                      |
| 15   | For another case, go to step 2<br>or step 8.     |      |                      |                      |

## Example 1:

|   |                                          |   |   |   |
|---|------------------------------------------|---|---|---|
|   | $\begin{matrix} & j \\ i & \end{matrix}$ | 1 | 2 | 3 |
| A |                                          | 2 | 5 | 4 |
| B |                                          | 3 | 8 | 7 |

## Keystrokes:

**GSB** 0 **h** **SF** 12 **ENTER** 3 **A**5 **ENTER** 8 **A**6 **ENTER** 9 **A****R/S**6 **ENTER** 9 **B****R/S**4 **ENTER** 7 **A****GSB** 1**R/S****GSB** 2**R/S****GSB** 3

## Display:

0.0000

3.0000

15.0000

2.0000

-15.0000

3.0000

0.0221

0.0276

11.0000

18.0000

29.0000

(error)

 $(C_3)$ 

(correction)

 $(-C_3)$  $(x^2)$  $(C_c)$  $(R_1)$  $(R_2)$  $(T)$

**Example 2:**

Find test statistic  $\chi^2$  and coefficient of contingency  $C_c$  for the following set of data

| $i \backslash j$ | 1  | 2  | 3  | 4  |
|------------------|----|----|----|----|
| 1                | 36 | 67 | 49 | 58 |
| 2                | 31 | 60 | 49 | 54 |
| 3                | 58 | 87 | 80 | 68 |

**Keystrokes:****Display:**

|                     |           |              |
|---------------------|-----------|--------------|
| GSB 0               | 0.0000    |              |
| 36 ENTER+ 31 ENTER+ |           |              |
| 58 A                | 1.0000    |              |
| 67 ENTER+ 60 ENTER+ |           |              |
| 87 A                | 2.0000    |              |
| 4 ENTER+ 49 ENTER+  |           |              |
| 80 A                | 3.0000    | (error)      |
| R/S                 | 133.0000  | ( $C_3$ )    |
| 4 ENTER+ 49 ENTER+  |           |              |
| 80 B                | 2.0000    |              |
| R/S                 | -133.0000 | ( $-C_3$ )   |
| 49 ENTER+ 49 ENTER+ |           |              |
| 80 A                | 3.0000    |              |
| 58 ENTER+ 54 ENTER+ |           |              |
| 68 A                | 4.0000    |              |
| GSB 1               | 3.3574    | ( $\chi^2$ ) |
| R/S                 | 0.0692    | ( $C_c$ )    |
| GSB 2               | 210.0000  | ( $R_1$ )    |
| R/S                 | 194.0000  | ( $R_2$ )    |
| R/S                 | 293.0000  | ( $R_3$ )    |
| GSB 3               | 697.0000  | ( $T$ )      |



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