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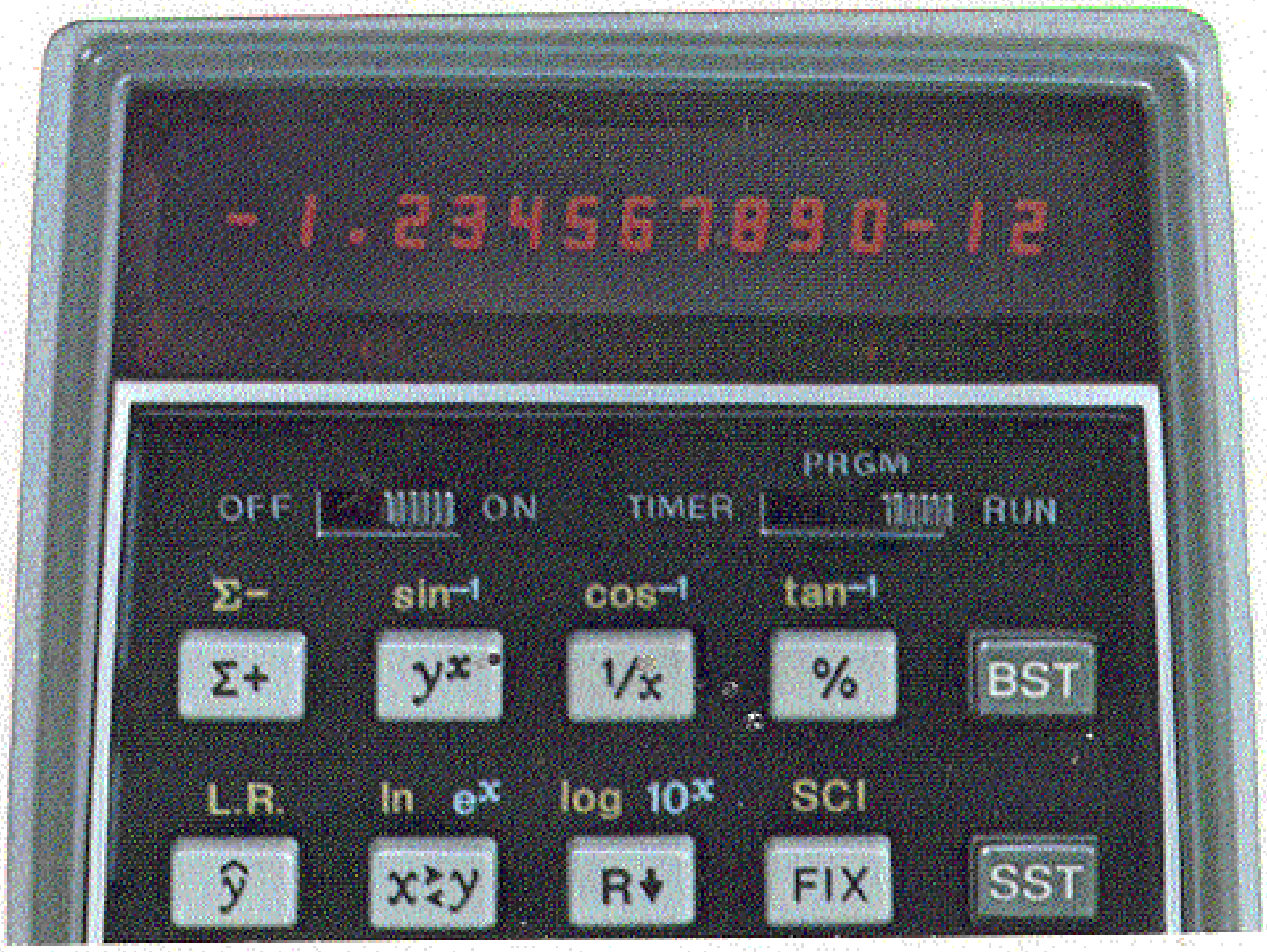
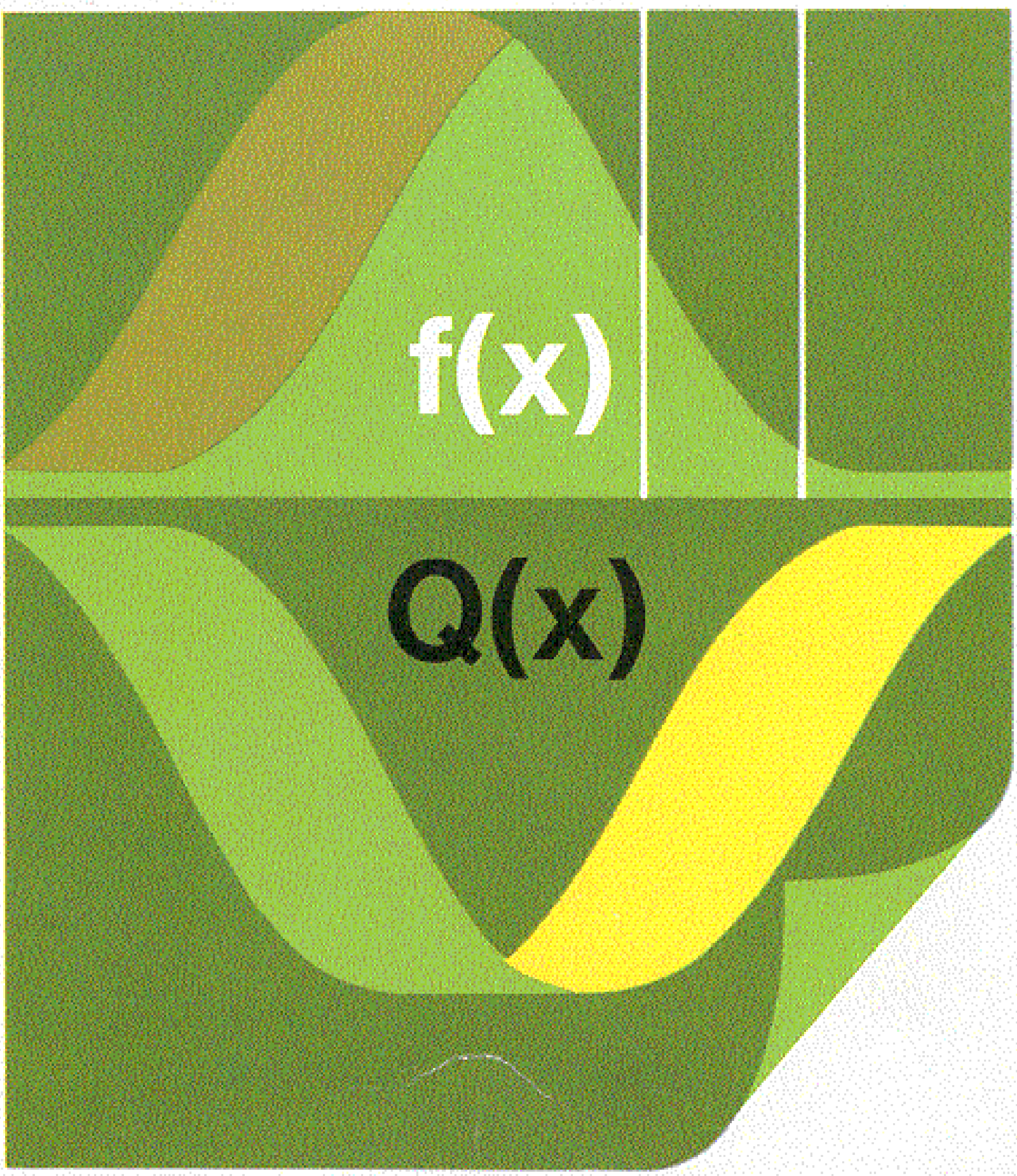
HP-55 statistics programs



a powerful guidebook for the data analyst: 53 programs in such areas as

probability, distribution functions, general statistics, curve fitting, test statistics, and others



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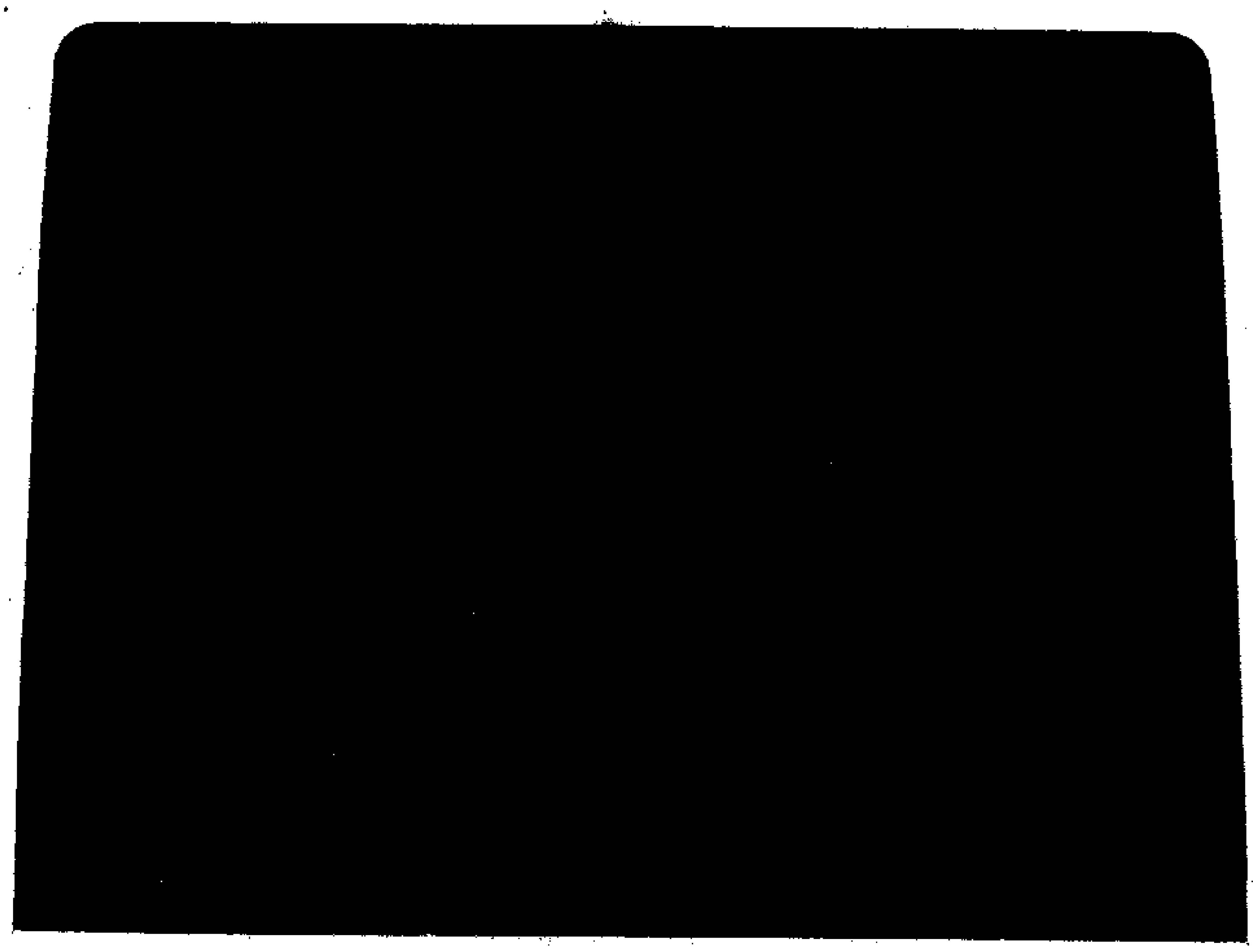

HEWLETT PACKARD

HP-55 statistics programs

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INTRODUCTION

Material in *HP-55 Statistics Programs* has been selected from the areas of probability, general statistics, distribution functions, curve fitting, and test statistics.

Each program includes a general description, formulas used in the program solution, numerical examples, and user instructions. Program listings and register allocations are also given. The body of the book is arranged logically according to subject matter. The back cover contains an index.

We suggest that you first read the material explaining the Format of User Instructions, then use the programs. An understanding of the *HP-55 Owner's Handbook* is also required if, in addition, you wish to track the changes in the storage registers and stack registers on a step-by-step basis.

We hope you find *HP-55 Statistics Programs* a useful tool for your statistical work and welcome your comments, requests, and suggestions–these are our most important source of future user-oriented programs.

CONTENTS

CURVE FITTING

TEST STATISTICS

FORMAT OF USER INSTRUCTIONS

The completed User Instructions form is your guide to operating the programs in this book.

The form is composed of five columns. Reading from left to right, the STEP column gives the instruction step number. A step number with the symbol "prime" (′) placed to its upper right indicates that step is optional or alternate to the step with the same number.

The INSTRUCTIONS column gives instructions and comments concerning the operations to be performed. Steps are executed in sequential order except where the INSTRUCTIONS column directs otherwise.

Normally, the first instruction is "Enter program", which means to store the keystrokes of the program into memory (press BST in RUN mode, switch to PRGM mode, key in the program, then switch back to RUN mode).

Repeated processes, used in most cases for a long string of input/output data, are outlined with a bold border together with a "Perform" instruction.

The INPUT DATA/UNITS column specified the input data to be supplied, and the units of data if applicable.

The KEYS column specifies the keys to be pressed. [↑] is the symbol used to denote the [ENTER↑] key. All other key designations are identical to those appearing on the HP-55. Ignore any blank positions in the KEYS column.

Some programs are sufficiently complex that users have to press additional keys (other than program-control keys) in order to get the answers. Those keys will also be shown in the KEYS column.

The following is an example of User Instructions (for the Behrens-Fisher Statistic program).

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R			0.00
3	Perform 3 for i = 1, 2,..., n_1	x_i	Σ+				i
3′	Delete erroneous data x_k	x_k	f	Σ−			
4	Compute $\bar{x}$ and $s_1/\sqrt{n_1}$		f	$\bar{x}$	STO	0	$\bar{x}$
			g	s	RCL	•	
			0	f	$\sqrt{x}$	÷	$s_1/\sqrt{n_1}$
			STO	1	g	CL•R	0.00
5	Perform 5 for i = 1, 2,..., n_2	y_i	Σ+				j
5′	Delete erroneous data y_h	y_h	f	Σ−			
6	Input D and compute d and θ	D	BST	R/S			d
			R/S				θ
7	For a new case, go to 2						

Step 1: The first step in all programs is to enter the program into the calculator.

Step 2: The initialization step clears the stack and registers $R_{\bullet 0}$ through $R_{\bullet 9}$.

Step 3: This is a loop which accumulates sums for input data x_i's. The first time through the loop the dummy variable i takes the value 1; the second time, i takes the value 2; etc.

Step 3′: Only executed when you want to remove data entered in step 3.

Step 4: User has to press additional keystrokes to compute intermediate results and reinitialize registers. $\bar{x}$ and $s_1/\sqrt{n_1}$ are computed and displayed.

Step 5: This is a loop which accumulates sums for input data y_i's.

Step 5′: Only executed when you want to remove data entered in step 5.

Step 6: D is an input. Answers d and θ are computed.

Step 7: This step gives instructions for starting a new case. In this example, return to step 2.

PERMUTATION

A permutation is an ordered subset of a set of distinct objects. The number of possible permutations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$$_mP_n = \frac{m!}{(m-n)!} = m(m-1)\ldots(m-n+1)$$

where m, n are integers and $0 \leqslant n \leqslant m$.

Notes:

1. $_mP_n$ can also be denoted by P_n^m, P(m,n) or $(m)_n$.
2. $_mP_0 = 1$, $_mP_1 = m$, $_mP_m = m!$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	41	↑
02.	33	STO
03.	00	0
04.	84	R/S
05.	32	g
06.	-35	x=y 35
07.	31	f
08.	-11	x⩽y 11
09.	00	0
10.	81	÷
11.	01	1
12.	32	g
13.	-32	x=y 32
14.	44	CLX
15.	32	g
16.	-38	x=y 38
17.	61	+
18.	51	–
19.	01	1
20.	61	+
21.	71	x
22.	31	f
23.	34	LAST X
24.	34	RCL

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	00	0
26.	01	1
27.	51	–
28.	32	g
29.	-32	x=y 32
30.	23	R↓
31.	-19	GTO 19
32.	23	R↓
33.	23	R↓
34.	-00	GTO 00
35.	31	f
36.	43	n!
37.	-00	GTO 00
38.	01	1
39.	-00	GTO 00
40.	41	↑
41.	31	f
42.	43	n!
43.	22	x⇄y
44.	84	R/S
45.	51	–
46.	31	f
47.	43	n!
48.	81	÷
49.	-00	GTO 00

REGISTERS	
R_0	m
R_1	
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $_{27}P_5 = 9687600.00$
2. $_{73}P_4 = 26122320.00$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input m, n	m	BST	R/S			m
		n	R/S				$_mP_n$
2′	If m ≤ 69, for a faster execution		GTO	4	0		
		m	R/S				m
		n	R/S				$_mP_n$
3	For a new case, go to 2						

COMBINATION

A combination is a selection of one or more of a set of distinct objects without regard to order. The number of possible combinations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$$ {}_mC_n = \frac{m!}{(m-n)!\,n!} = \frac{m(m-1)\ldots(m-n+1)}{1\cdot 2\cdot \ldots \cdot n} $$

where m, n are integers and $0 \leqslant n \leqslant m$.

This program computes ${}_mC_n$ using the following algorithm:

1. If $n \leqslant m - n$

$$ {}_mC_n = \frac{m-n+1}{1} \cdot \frac{m-n+2}{2} \cdot \ldots \cdot \frac{m}{n}\ . $$

2. If $n > m - n$, program computes ${}_mC_{m-n}$.

Notes:

1. ${}_mC_n$, which is also called the binomial coefficient, can be denoted by C_n^m, $C(m,n)$, or $\binom{m}{n}$.

2. ${}_mC_n = {}_mC_{m-n}$

3. ${}_mC_0 = {}_mC_m = 1$

4. ${}_mC_1 = {}_mC_{m-1} = m$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	51	–
02.	31	f
03.	34	LAST X
04.	31	f
05.	–42	x≤y 42
06.	33	STO
07.	00	0
08.	01	1
09.	33	STO
10.	01	1
11.	61	+
12.	33	STO
13.	02	2
14.	44	CLX
15.	32	g
16.	–44	x=y 44
17.	23	R↓
18.	01	1
19.	34	RCL
20.	01	1
21.	61	+
22.	33	STO
23.	01	1
24.	31	f

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	–29	x≤y 29
26.	34	RCL
27.	02	2
28.	–00	GTO 00
29.	34	RCL
30.	00	0
31.	22	x⇄y
32.	61	+
33.	31	f
34.	34	LAST X
35.	81	÷
36.	34	RCL
37.	02	2
38.	71	x
39.	33	STO
40.	02	2
41.	–17	GTO 17
42.	22	x⇄y
43.	–06	GTO 06
44.	01	1
45.	–00	GTO 00
46.		
47.		
48.		
49.		

REGISTERS
R_0 max(n, m–n)
R_1 Used
R_2 Used
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Examples:

1. ${}_{73}C_4 = 1088430.00$

2. ${}_{27}C_5 = 80730.00$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input m, n	m	↑				
		n	BST	R/S			${}_mC_n$
3	For a new case, go to 2						

BAYES' FORMULA

Suppose $E_1, E_2, ..., E_n$ are n mutually exclusive and exhaustive events, and A is an event for which the conditional probabilities, $P[A/E_i]$ of A given E_i, are known. If $P[E_i]$ are given, then the conditional probability $P[E_k/A]$ of any one event E_k given A is

$$P[E_k/A] = \frac{P[E_k]\ P[A/E_k]}{\sum_{i=1}^{n} P[E_i] P[A/E_i]}$$

where k can be 1, 2, ..., or n.

Reference:

E. Parzen, *Modern Probability Theory and its Applications,* John Wiley and Sons, 1960.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	00	0
02.	33	STO
03.	00	0
04.	33	STO
05.	01	1
06.	84	R/S
07.	71	x
08.	33	STO
09.	61	+
10.	00	0
11.	34	RCL
12.	01	1
13.	01	1
14.	61	+
15.	33	STO
16.	01	1
17.	-06	GTO 06
18.	71	x
19.	33	STO
20.	51	–
21.	00	0
22.	34	RCL
23.	01	1
24.	01	1

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	51	–
26.	33	STO
27.	01	1
28.	-06	GTO 06
29.	71	x
30.	34	RCL
31.	00	0
32.	81	÷
33.	-00	GTO 00
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	$\Sigma P[A/E_i] P[E_i]$
R_1	n
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

If $P[E_1] = 0.95$
$P[A/E_1] = 0.005$
$P[E_2] = 0.05$
$P[A/E_2] = 0.995$
then $P[E_1/A] = .09$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for i = 1, 2,..., n	$P[E_i]$	↑				
		$P[A/E_i]$	R/S				i
3′	Delete erroneous data $P[E_m]$,						
	$P[A/E_m]$	$P[E_m]$	↑				
		$P[A/E_m]$	GTO	1	8	R/S	
4	Compute $P[E_k/A]$	$P[E_k]$	↑				
		$P[A/E_k]$	GTO	2	9	R/S	$P[E_k/A]$
5	For a different k, go to 4						
6	For a new case, go to 2						

PROBABILITY OF NO REPETITIONS IN A SAMPLE

Suppose a sample of size n is drawn with replacement from a population containing m different objects. Let P be the probability that there are no repretitions in the sample, then

$$P = \left(1 - \frac{1}{m}\right)\left(1 - \frac{2}{m}\right) \cdots \left(1 - \frac{n-1}{m}\right).$$

Given integers m, n such that $m \geqslant n \geqslant 1$, this program finds the probability P.

Note:

The execution time of the program depends on n; the larger n is, the longer it takes.

Reference:

E. Parzen, *Modern Probability Theory and its Applications,* John Wiley and Sons, 1960.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	00	0
01.	33	STO	26.	–06	GTO 06
02.	02	2	27.	34	RCL
03.	01	1	28.	00	0
04.	33	STO	29.	–00	GTO 00
05.	00	0	30.		
06.	34	RCL	31.		
07.	01	1	32.		
08.	34	RCL	33.		
09.	02	2	34.		
10.	01	1	35.		
11.	51	–	36.		
12.	33	STO	37.		
13.	02	2	38.		
14.	00	0	39.		
15.	32	g	40.		
16.	–27	x=y 27	41.		
17.	23	R↓	42.		
18.	22	x⇄y	43.		
19.	81	÷	44.		
20.	01	1	45.		
21.	22	x⇄y	46.		
22.	51	–	47.		
23.	33	STO	48.		
24.	71	x	49.		

REGISTERS
R_0 Used
R_1 m
R_2 Used
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

In a room containing n persons, what is the probability that no two or more persons have the same birthday for n = 4, 23, 48?

(Note: m = 365)

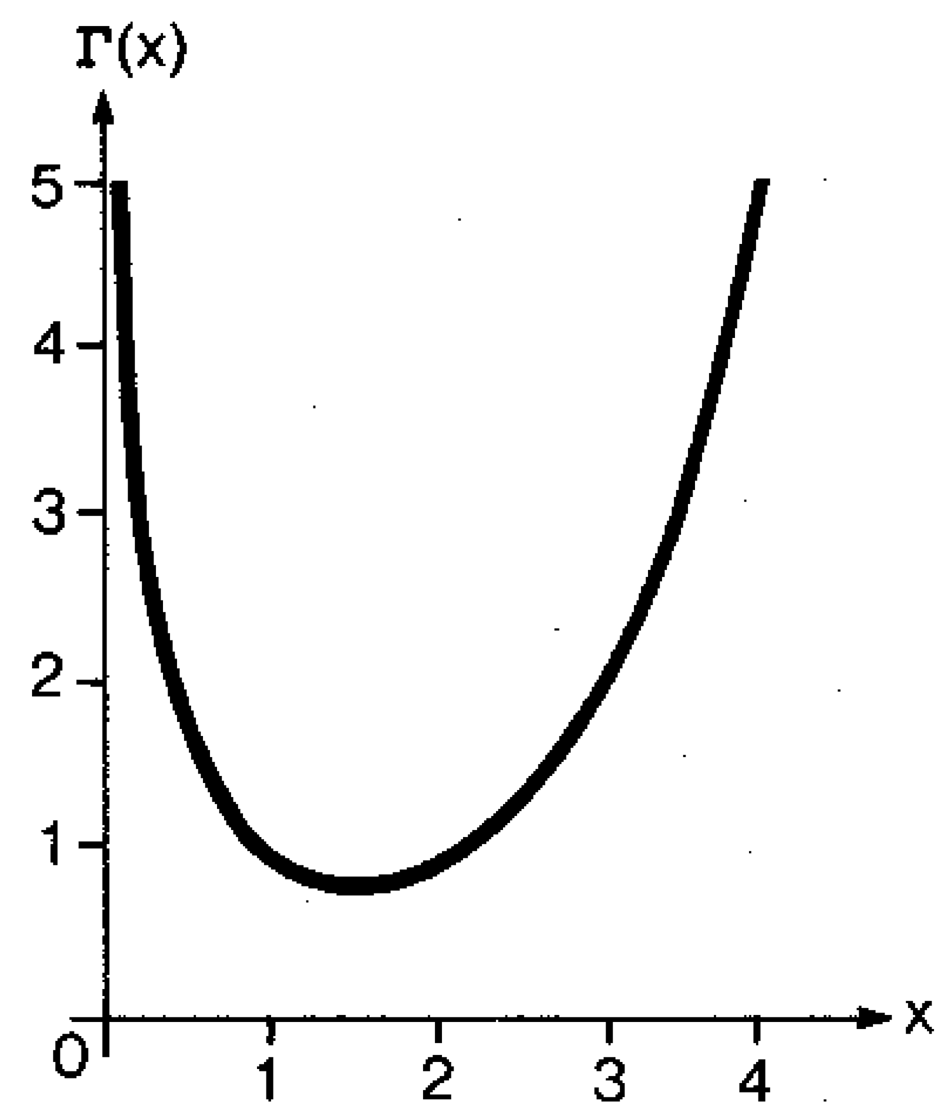
1. n = 4, P = .98
2. n = 23, P = .49
3. n = 48, P = .04

(That is, in a room having 48 persons, the probability that at least two of them will have the same birthday is as high as 1 − .04 = 0.96.)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input m	m	STO	1	BST		
3	Input n	n	R/S				P
4	For different n, go to 3						
5	For a new case, go to 2						

GAMMA FUNCTION

This program approximates the value of the gamma function $\Gamma(x)$ for $1 \leqslant x \leqslant 64$.



$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \, dt$$

$$\cong \sqrt{2\pi/x} \; x^x \; e^{-\left(x - \frac{1}{12x} + \frac{1}{360x^3}\right)}$$

Suppose ϵ is the error, then

$$\frac{\epsilon}{\Gamma(x)} < 2 \times 10^{-7}$$

This approximation is good for large x. In order to increase the accuracy (especially for small values of x), the program computes $\Gamma(x + 5)$, then $\Gamma(x)$ is calculated using the following formula

$$\Gamma(x) = \frac{\Gamma(x+5)}{(x+4)(x+3)(x+2)(x+1)\,x} \, .$$

Note:

This program can be used to find the generalized factorial x! for $0 \leqslant x \leqslant 63$.

$$x! = \Gamma(x + 1)$$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1968.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	05	5
02.	61	+
03.	41	↑
04.	13	$1/x$
05.	41	↑
06.	71	x
07.	41	↑
08.	41	↑
09.	03	3
10.	00	0
11.	81	÷
12.	01	1
13.	51	–
14.	71	x
15.	01	1
16.	02	2
17.	81	÷
18.	22	$x \rightleftarrows y$
19.	31	f
20.	22	ln
21.	51	–
22.	71	x
23.	61	+
24.	42	CHS

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	32	g
26.	22	e^x
27.	22	$x \rightleftarrows y$
28.	41	↑
29.	61	+
30.	31	f
31.	83	π
32.	71	x
33.	31	f
34.	42	$\sqrt{x}$
35.	71	x
36.	33	STO
37.	00	0
38.	44	CLX
39.	05	5
40.	51	–
41.	33	STO
42.	81	÷
43.	00	0
44.	01	1
45.	61	+
46.	31	f
47.	–41	$x \leq y$ 41
48.	34	RCL
49.	00	0

REGISTERS	
R_0	Used
R_1	
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $\Gamma(5.25) = 35.21$
2. $7! = \Gamma(8) = 5040.00$
3. $2.34! = \Gamma(3.34) = 2.80$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST				
3	Input x	x	R/S				$\Gamma(x)$
4	For a new case, go to 3						

INCOMPLETE GAMMA FUNCTION

$$\gamma(a, x) = \int_0^x e^{-t}\, t^{a-1}\, dt$$

$$= x^a e^{-x} \sum_{n=0}^{\infty} \frac{x^n}{a(a+1)\ldots(a+n)}$$

where $a > 0, x > 0$.

This program computes successive partial sums of the above series. The program stops when two consecutive partial sums are equal and displays the last partial sum as the answer.

Note:

When x is too large, computing a new term of the series might cause an overflow. In that case, display shows all 9's and the program stops.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	02	2	R_0 x
01.	33	STO	26.	61	+	R_1 Used
02.	00	0	27.	32	g	R_2 Used
03.	22	x⇄y	28.	–30	x=y 30	R_3
04.	33	STO	29.	–12	GTO 12	R_4
05.	01	1	30.	34	RCL	R_5
06.	12	y^x	31.	00	0	R_6
07.	34	RCL	32.	32	g	R_7
08.	01	1	33.	22	e^x	R_8
09.	81	÷	34.	81	÷	R_9
10.	33	STO	35.	–00	GTO 00	$R_{\bullet 0}$
11.	02	2	36.			$R_{\bullet 1}$
12.	34	RCL	37.			$R_{\bullet 2}$
13.	00	0	38.			$R_{\bullet 3}$
14.	34	RCL	39.			$R_{\bullet 4}$
15.	01	1	40.			$R_{\bullet 5}$
16.	01	1	41.			$R_{\bullet 6}$
17.	61	+	42.			$R_{\bullet 7}$
18.	33	STO	43.			$R_{\bullet 8}$
19.	01	1	44.			$R_{\bullet 9}$
20.	81	÷	45.			
21.	34	RCL	46.			
22.	02	2	47.			
23.	71	x	48.			
24.	33	STO	49.			

Examples:

1. $\gamma(1, 2) = .86$
2. $\gamma(1, 0.1) = .10$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input a, x	a	↑				
		x	BST	R/S			$\gamma(a, x)$
3	For a new case, go to 2						

ERROR FUNCTION AND COMPLEMENTARY ERROR FUNCTION

Error function $\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$

$$= \frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \cdot \ldots \cdot (2n+1)} x^{2n+1}$$

Complementary error function

$$\operatorname{erfc} x = 1 - \operatorname{erf} x$$

where $x > 0$.

This program computes successive partial sums of the series. The program stops when two consecutive partial sums are equal and displays the last partial sum as the answer.

Notes:

1. When x is too large, computing a new term of the series might cause an overflow. In that case, display shows all 9's and the program stops.

2. The execution time of the program depends on x; the larger x is, the longer it takes.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	33	STO
02.	00	0
03.	41	↑
04.	71	x
05.	02	2
06.	71	x
07.	33	STO
08.	01	1
09.	01	1
10.	33	STO
11.	02	2
12.	34	RCL
13.	00	0
14.	34	RCL
15.	01	1
16.	34	RCL
17.	02	2
18.	02	2
19.	61	+
20.	33	STO
21.	02	2
22.	81	÷
23.	34	RCL
24.	00	0
25.	71	x
26.	33	STO
27.	00	0
28.	61	+
29.	32	g
30.	–32	x=y 32
31.	–14	GTO 14
32.	02	2
33.	71	x
34.	31	f
35.	83	π
36.	31	f
37.	42	$\sqrt{x}$
38.	34	RCL
39.	01	1
40.	02	2
41.	81	÷
42.	32	g
43.	22	e^x
44.	71	x
45.	81	÷
46.	84	R/S
47.	01	1
48.	22	x⇄y
49.	51	–

REGISTERS
R_0 Used
R_1 $2x^2$
R_2 Used
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

erf 1.34 = .94

erfc 1.34 = .06

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Compute erf x and erfc x	x	BST	R/S			erf x
			R/S				erfc x
3	For a new case, go to 2						

RANDOM NUMBER GENERATOR

This program calculates uniformly distributed pseudo random numbers u_i in the range

$$0 \leqslant u_i \leqslant 1$$

using the following formula:

$$u_i = \text{Fractional part of } [(\pi + u_{i-1})^5].$$

The user has to specify the starting value u_0 such that

$$0 \leqslant u_0 \leqslant 1.$$

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	33	STO
02.	00	0
03.	84	R/S
04.	31	f
05.	83	π
06.	34	RCL
07.	00	0
08.	61	+
09.	05	5
10.	12	y^x
11.	41	↑
12.	41	↑
13.	43	EEX
14.	09	9
15.	61	+
16.	43	EEX
17.	09	9
18.	51	–
19.	01	1
20.	51	–
21.	51	–
22.	01	1
23.	31	f
24.	–29	x≤y 29

DISPLAY		KEY ENTRY
LINE	CODE	
25.	23	R↓
26.	33	STO
27.	00	0
28.	–03	GTO 03
29.	51	–
30.	–26	GTO 26
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS
R_0 u_i
R_1
R_2
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

The following uniformly distributed pseudo random numbers are generated for $u_0 = 0$: .02, .73, .70, .31, .58, .85, .86, .43, .33, .60, .67, .93, .22, .32, .45, .50, ...

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input u_0	u_0	BST	R/S			u_0
3	Perform 3 for i = 1, 2, 3,...		R/S				u_i
4	For a new case, go to 2						

MEAN, STANDARD DEVIATION, STANDARD ERROR FOR GROUPED DATA

Given a set of data points

$$x_1, x_2, \ldots, x_n$$

with respective frequencies

$$f_1, f_2, \ldots, f_n$$

the program computes the following statistics:

$$\text{mean } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\text{standard deviation } s = \sqrt{\frac{\Sigma f_i x_i^2 - (\Sigma f_i)\bar{x}^2}{\Sigma f_i - 1}}$$

$$\text{standard error } s_{\bar{x}} = \frac{s_x}{\sqrt{\Sigma f_i}} \, .$$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	61	+
03.	00	0
04.	22	x⇄y
05.	71	x
06.	31	f
07.	34	LAST X
08.	22	x⇄y
09.	71	x
10.	31	f
11.	34	LAST X
12.	11	Σ+
13.	-00	GTO 00
14.	42	CHS
15.	34	RCL
16.	83	•
17.	00	0
18.	02	2
19.	51	–
20.	33	STO
21.	83	•
22.	00	0
23.	23	R↓
24.	-01	GTO 01

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	34	RCL
26.	00	0
27.	33	STO
28.	83	•
29.	00	0
30.	34	RCL
31.	83	•
32.	03	3
33.	33	STO
34.	83	•
35.	02	2
36.	31	f
37.	33	$\bar{x}$
38.	84	R/S
39.	32	g
40.	33	s
41.	84	R/S
42.	34	RCL
43.	00	0
44.	31	f
45.	42	$\sqrt{x}$
46.	81	÷
47.	-00	GTO 00
48.		
49.		

REGISTERS	
R_0	Σf_i
R_1	
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	n, Σf_i
$R_{.1}$	$\Sigma f_i x_i$
$R_{.2}$	$\Sigma (f_i x_i)^2$, $\Sigma f_i x_i^2$
$R_{.3}$	$\Sigma f_i x_i^2$
$R_{.4}$	$\Sigma (f_i x_i^2)^2$
$R_{.5}$	$\Sigma f_i^2 x_i^3$
$R_{.6}$	0
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

x_i	2	3.4	7	11	23	3.41
f_i	5	3	4	2	3	1

$\bar{x} = 7.92$

$s = 7.52$

$s_{\bar{x}} = 1.77$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R	STO	0	
			BST				0.00
3	Perform 3 for i = 1, 2,..., n	x_i	↑				
		f_i	R/S				i
3′	Delete erroneous data x_k, f_k	x_k	↑				
		f_k	GTO	1	4	R/S	
4	Compute $\bar{x}$, s and $s_{\bar{x}}$		GTO	2	5	R/S	$\bar{x}$
			R/S				s
			R/S				$s_{\bar{x}}$
5	For a new case, go to 2						

GEOMETRIC MEAN

For a set of n positive numbers $\{a_1, a_2, ..., a_n\}$, the geometric mean is defined by

$$G = (a_1\ a_2\ ...\ a_n)^{\frac{1}{n}}.$$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	01	1
02.	33	STO
03.	01	1
04.	00	0
05.	33	STO
06.	00	0
07.	84	R/S
08.	34	RCL
09.	01	1
10.	71	x
11.	33	STO
12.	01	1
13.	34	RCL
14.	00	0
15.	01	1
16.	61	+
17.	33	STO
18.	00	0
19.	-07	GTO 07
20.	34	RCL
21.	01	1
22.	34	RCL
23.	00	0
24.	13	$^1/x$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	12	y^x
26.	-00	GTO 00
27.	33	STO
28.	81	÷
29.	01	1
30.	34	RCL
31.	00	0
32.	01	1
33.	51	–
34.	33	STO
35.	00	0
36.	-07	GTO 07
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS
R_0 n
R_1 $\Pi\ a_i$
R_2
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

The set of numbers $\{2, 3.4, 3.41, 7, 11, 23\}$ has the geometric mean G = 5.87.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for i = 1, 2,..., n	a_i	R/S				i
3'	Delete erroneous data a_k	a_k	GTO	2	7	R/S	
4	Compute the mean G		GTO	2	0	R/S	G
5	For a new case, go to 2						

HARMONIC MEAN

For a set of n positive numbers $\{a_1, a_2, ..., a_n\}$, the harmonic mean is defined by

$$H = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}} .$$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	00	0
02.	33	STO
03.	00	0
04.	33	STO
05.	01	1
06.	84	R/S
07.	13	$^1/x$
08.	34	RCL
09.	01	1
10.	61	+
11.	33	STO
12.	01	1
13.	34	RCL
14.	00	0
15.	01	1
16.	61	+
17.	33	STO
18.	00	0
19.	-06	GTO 06
20.	34	RCL
21.	00	0
22.	34	RCL
23.	01	1
24.	81	÷

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	-00	GTO 00
26.	13	1/x
27.	33	STO
28.	51	–
29.	01	1
30.	34	RCL
31.	00	0
32.	01	1
33.	51	–
34.	33	STO
35.	00	0
36.	-06	GTO 06
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	n
R_1	$\Sigma 1/a_i$
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

The harmonic mean for the set of numbers $\{2, 3.4, 3.41, 7, 11, 23\}$ is $H = 4.40$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for i = 1, 2,..., n	a_i	R/S				i
3′	Delete erroneous data a_k	a_k	GTO	2	6	R/S	
4	Compute the mean H		GTO	2	0	R/S	H
5	For a new case, go to 2						

GENERALIZED MEAN

For a set of n positive numbers $\{a_1, a_2, ..., a_n\}$, the generalized mean is defined by

$$M(t) = \left(\frac{1}{n} \sum_{k=1}^{n} a_k{}^t \right)^{\frac{1}{t}}$$

where t is any desired number.

Notes:

1. If t = 1, the generalized mean M (1) is the same as the arithmetic mean.
2. If t = –1, the generalized mean M (–1) is the same as the harmonic mean.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	00	0
02.	33	STO
03.	00	0
04.	33	STO
05.	01	1
06.	84	R/S
07.	33	STO
08.	02	2
09.	84	R/S
10.	34	RCL
11.	02	2
12.	12	y^x
13.	34	RCL
14.	01	1
15.	61	+
16.	33	STO
17.	01	1
18.	34	RCL
19.	00	0
20.	01	1
21.	61	+
22.	33	STO
23.	00	0
24.	-09	GTO 09

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	34	RCL
26.	01	1
27.	34	RCL
28.	00	0
29.	81	÷
30.	34	RCL
31.	02	2
32.	13	$1/x$
33.	12	y^x
34.	-00	GTO 00
35.	34	RCL
36.	02	2
37.	12	y^x
38.	33	STO
39.	51	–
40.	01	1
41.	34	RCL
42.	00	0
43.	01	1
44.	51	–
45.	33	STO
46.	00	0
47.	-09	GTO 09
48.		
49.		

REGISTERS
R_0 n
R_1 $\Sigma a_k{}^t$
R_2 t
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

The set of numbers $\{2, 3.4, 3.41, 7, 11, 23\}$ has the generalized mean M (2) = 11.00.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Input t	t	R/S				t
4	Perfrom 4 for i = 1, 2,..., n	a_i	R/S				i
4'	Delete erroneous data a_k	a_k	GTO	3	5	R/S	
5	Compute mean M(t)		GTO	2	5	R/S	M(t)
6	For a new case, go to 2						

MOVING AVERAGE

Given a set of numbers $\{x_1, x_2, x_3, \ldots\}$, this program finds the moving averages of order n (n can be 2, 3, ..., or 9) given by the following sequence of arithmetic means:

$$\frac{x_1 + x_2 + \ldots + x_n}{n}, \frac{x_2 + x_3 + \ldots + x_{n+1}}{n}, \frac{x_3 + x_4 + \ldots + x_{n+2}}{n}, \ldots$$

The numerators are the moving totals of order n.

Note:

The program computes the total and the average of the first n numbers. Then x_{n+1} is added to and x_1 is removed from the total. A new average is computed. Similar procedure goes on until all answers are found. This program is written in such a way that the value that needed to be removed is stored in register R_n (where n is the order). In the following example, the order is 6, hence register R_6 contains the value.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	83	·
03.	06	6
04.	34	RCL
05.	08	8
06.	33	STO
07.	09	9
08.	34	RCL
09.	07	7
10.	33	STO
11.	08	8
12.	34	RCL
13.	06	6
14.	33	STO
15.	07	7
16.	34	RCL
17.	05	5
18.	33	STO
19.	06	6
20.	34	RCL
21.	04	4
22.	33	STO
23.	05	5
24.	34	RCL
25.	03	3
26.	33	STO
27.	04	4
28.	34	RCL
29.	02	2
30.	33	STO
31.	03	3
32.	34	RCL
33.	01	1
34.	33	STO
35.	02	2
36.	34	RCL
37.	00	0
38.	33	STO
39.	01	1
40.	34	RCL
41.	83	·
42.	06	6
43.	33	STO
44.	00	0
45.	11	Σ+
46.	–00	GTO 00
47.		
48.		
49.		

REGISTERS	
R_0	Used
R_1	Used
R_2	Used
R_3	Used
R_4	Used
R_5	Used
R_6	Used
R_7	Used
R_8	Used
R_9	Used
$R_{.0}$	Used
$R_{.1}$	Used
$R_{.2}$	Used
$R_{.3}$	Used
$R_{.4}$	Used
$R_{.5}$	Used
$R_{.6}$	Used
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

For the following set of data {105, 121, 124, 97, 86, 134, 105, 81, 127, 132, 114, 121}, the moving averages of order 6 are 111.17, 111.17, 104.50, 105.00, 110.83, 115.50, 113.33.

The moving totals of order 6 are 667.00, 667.00, 627.00, 630.00, 665.00, 693.00, 680.00.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R	BST		0.00
3	Perform 3 for i = 1, 2,..., n	x_i	R/S				i
4	Compute the moving average						
	of order n		f	$\bar{x}$			average
5	(optional) Compute the moving						
	total of order n		RCL	Σ+			total
6	Input next value	x_k	R/S				n + 1
7	Remove one old value		RCL				
		n*	f	Σ−			n
8	Go to 4						
9	For a new case, go to 2						
	* n can be one of the values						
	2, 3,..., 9.						

COVARIANCE AND CORRELATION COEFFICIENT

For a set of given data points $\{(x_i, y_i), i = 1, 2, ..., n\}$, the covariance and the correlation coefficent are defined as:

$$\text{covariance } s_{xy} = \frac{1}{n-1}\left(\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i\right)$$

$$\text{or } s_{xy}{}' = \frac{1}{n}\left(\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i\right)$$

$$\text{correlation coefficient } r = \frac{s_{xy}}{s_x s_y}$$

where s_x and s_y are standard deviations

$$s_x = \sqrt{\frac{\Sigma x_i^2 - (\Sigma x_i)^2/n}{n-1}}$$

$$s_y = \sqrt{\frac{\Sigma y_i^2 - (\Sigma y_i)^2/n}{n-1}}$$

Note:

$-1 \leqslant r \leqslant 1$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	51	−
01.	32	g	26.	81	÷
02.	44	CL•R	27.	81	÷
03.	84	R/S	28.	-00	GTO 00
04.	31	f	29.		
05.	21	L.R.	30.		
06.	41	↑	31.		
07.	32	g	32.		
08.	33	s	33.		
09.	81	÷	34.		
10.	81	÷	35.		
11.	84	R/S	36.		
12.	41	↑	37.		
13.	41	↑	38.		
14.	32	g	39.		
15.	33	s	40.		
16.	71	x	41.		
17.	71	x	42.		
18.	84	R/S	43.		
19.	34	RCL	44.		
20.	83	•	45.		
21.	00	0	46.		
22.	41	↑	47.		
23.	41	↑	48.		
24.	01	1	49.		

REGISTERS
R_0
R_1
R_2
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$ n
$R_{.1}$ Σx_i
$R_{.2}$ Σx_i^2
$R_{.3}$ Σy_i
$R_{.4}$ Σy_i^2
$R_{.5}$ $\Sigma x_i y_i$
$R_{.6}$ 0
$R_{.7}$ 0
$R_{.8}$ 0
$R_{.9}$ 0

Example:

y_i	92	85	78	81	54	51	40
x_i	26	30	44	50	62	68	74

$r = -.96$

$s_{xy} = -354.14$

$s_{xy}' = -303.55$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2*	Initialize		BST	R/S			0.00
3	Perform 3 for i = 1, 2,..., n	y_i	↑				
		x_i	Σ+				i
3′	Delete erroneous data x_k, y_k	y_k	↑				
		x_k	f	Σ−			
4	Compute correlation coefficient						
	r		R/S				r
5	Compute covariance s_{xy}		R/S				s_{xy}
	(optional) Compute s_{xy}'		R/S				s_{xy}'
6	For a new case, go to 2						
	*Note: If sums are already						
	accumulated in proper registers,						
	skip steps 2, 3 and 3′.						

MOMENTS, SKEWNESS AND KURTOSIS

This program computes the following statistics for a set of given data $\{x_1, x_2, \ldots, x_n\}$:

1st moment $\quad \bar{x} = \dfrac{1}{n} \sum_{i=1}^{n} x_i$

2nd moment $\quad m_2 = \dfrac{1}{n} \Sigma x_i^2 - \bar{x}^2$

3rd moment $\quad m_3 = \dfrac{1}{n} \Sigma x_i^3 - \dfrac{3}{n} \bar{x} \, \Sigma x_i^2 + 2\bar{x}^3$

4th moment $\quad m_4 = \dfrac{1}{n} \Sigma x_i^4 - \dfrac{4}{n} \bar{x} \, \Sigma x_i^3 + \dfrac{6}{n} \bar{x}^2 \, \Sigma x_i^2 - 3\bar{x}^4$

moment coefficient of skewness

$$\gamma_1 = \frac{m_3}{m_2^{3/2}}$$

moment coefficient of kurtosis

$$\gamma_2 = \frac{m_4}{m_2^{\,2}}$$

Reference:

M. R. Spiegel, *Theory and Problems of Statistics*, Schaum's Outline, McGraw-Hill, 1961.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	71	x
02.	03	3
03.	71	x
04.	51	–
05.	34	RCL
06.	01	1
07.	81	÷
08.	34	RCL
09.	00	0
10.	03	3
11.	12	y^x
12.	02	2
13.	71	x
14.	61	+
15.	84	R/S
16.	34	RCL
17.	83	•
18.	02	2
19.	34	RCL
20.	00	0
21.	34	RCL
22.	83	•
23.	05	5
24.	71	x

DISPLAY		KEY ENTRY
LINE	CODE	
25.	04	4
26.	71	x
27.	51	–
28.	34	RCL
29.	83	•
30.	04	4
31.	34	RCL
32.	00	0
33.	32	g
34.	42	x^2
35.	71	x
36.	06	6
37.	71	x
38.	61	+
39.	34	RCL
40.	01	1
41.	81	÷
42.	34	RCL
43.	00	0
44.	04	4
45.	12	y^x
46.	03	3
47.	71	x
48.	51	–
49.	–00	GTO 00

REGISTERS	
R_0	$\bar{x}$
R_1	n
R_2	m_2
R_3	m_3
R_4	m_4
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{\bullet 0}$	n
$R_{\bullet 1}$	Σx_i^2
$R_{\bullet 2}$	Σx_i^4
$R_{\bullet 3}$	Σx_i
$R_{\bullet 4}$	Σx_i^2
$R_{\bullet 5}$	Σx_i^3
$R_{\bullet 6}$	0
$R_{\bullet 7}$	0
$R_{\bullet 8}$	0
$R_{\bullet 9}$	0

Example:

i	1	2	3	4	5	6	7	8	9
x_i	2.1	3.5	4.2	6.5	4.1	3.6	5.3	3.7	4.9

$\bar{x} = 4.21, m_2 = 1.39, m_3 = .39, m_4 = 5.49$

$\gamma_1 = .24, \gamma_2 = 2.84$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL·R	BST		0.00
3	Perform 3 for i = 1, 2,..., n	x_i	↑	↑	x	Σ+	i
3′	Delete erroneous data x_k	x_k	↑	↑	x	f	
			Σ−				
4	Compute the mean $\bar{x}$		f	$\bar{x}$	x⇄y	STO	
			0				$\bar{x}$
5	Compute 2nd moment m_2		RCL	·	1	RCL	
			·	0	STO	1	
			÷	x⇄y	g	x^2	
			−	STO	2		m_2
6	Compute 3rd moment m_3		RCL	·	5	RCL	
			0	RCL	·	1	
			R/S	STO	3		m_3
7	Compute 4th moment m_4		R/S	STO	4		m_4
8	(optional) Compute γ_1, γ_2		RCL	3	RCL	2	
			1	·	5	y^x	
			÷				γ_1
			RCL	4	RCL	2	
			g	x^2	÷		γ_2
9	For a new case, go to 2						

STANDARD ERRORS FOR LINEAR REGRESSION

Suppose $y = a_0 + a_1 x$ is the least squares fit to a set of data points $\{(x_i, y_i), i = 1, 2, ..., n\}$ and $\hat{y}$ is the estimated value on the line for a given x value.

The program computes:

1. Standard error of estimate (of y on x)

$$s_{y \cdot x} = \sqrt{\frac{\Sigma(y_i - \hat{y}_i)^2}{n-2}}$$

$$= \sqrt{\frac{\Sigma y_i^2 - a_0 \Sigma y_i - a_1 \Sigma x_i y_i}{n-2}}$$

2. Standard error of the regression coefficient a_0

$$s_0 = s_{y \cdot x} \sqrt{\frac{\Sigma x_i^2}{n\left[\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right]}}$$

3. Standard error of the regression coefficient a_1

$$s_1 = \frac{s_{y \cdot x}}{\sqrt{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}}$$

Note:

n is a positive integer and $n \neq 1$ or 2.

Reference:

Draper and Smith, *Applied Regression Analysis,* John Wiley and Sons, 1966.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	71	x
02.	51	–
03.	34	RCL
04.	83	•
05.	05	5
06.	34	RCL
07.	01	1
08.	71	x
09.	51	–
10.	34	RCL
11.	83	•
12.	00	0
13.	02	2
14.	51	–
15.	81	÷
16.	31	f
17.	42	$\sqrt{x}$
18.	84	R/S
19.	34	RCL
20.	83	•
21.	02	2
22.	34	RCL
23.	83	•
24.	01	1

DISPLAY		KEY ENTRY
LINE	CODE	
25.	32	g
26.	42	x^2
27.	34	RCL
28.	83	•
29.	00	0
30.	81	÷
31.	51	–
32.	31	f
33.	42	$\sqrt{x}$
34.	81	÷
35.	34	RCL
36.	83	•
37.	02	2
38.	34	RCL
39.	83	•
40.	00	0
41.	81	÷
42.	31	f
43.	42	$\sqrt{x}$
44.	22	$x \rightleftarrows y$
45.	71	x
46.	84	R/S
47.	31	f
48.	34	LAST X
49.	-00	GTO 00

REGISTERS	
R_0	a_0
R_1	a_1
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{\bullet 0}$	n
$R_{\bullet 1}$	Σx_i
$R_{\bullet 2}$	Σx_i^2
$R_{\bullet 3}$	Σy_i
$R_{\bullet 4}$	Σy_i^2
$R_{\bullet 5}$	$\Sigma x_i y_i$
$R_{\bullet 6}$	0
$R_{\bullet 7}$	0
$R_{\bullet 8}$	0
$R_{\bullet 9}$	0

Example:

y_i	92	85	78	81	54	51	40
x_i	26	30	44	50	62	68	74

$a_0 = 121.04$
$a_1 = -1.03$
Regression line is $y = 121.04 - 1.03x$

$s_{y \cdot x} = 6.34$
$s_0 = 7.47$
$s_1 = .14$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2*	Initialize		g	CL•R			0.00
3	Perform 3 for i = 1, 2,..., n	y_i	↑				
		x_i	Σ+				i
3′	Delete erroneous data x_k, y_k	y_k	↑				
		x_k	f	Σ−			
4	Compute a_0, a_1		f	L. R.	STO	0	a_0
			x⇄y	STO	1		a_1
5	Compute standard errors		RCL	•	4	RCL	
			0	RCL	•	3	
			BST	R/S			$s_{y \cdot x}$
			R/S				s_0
			R/S				s_1
6	For a new case, go to 2						
	*Note: If sums are already in						
	proper registers, skip steps 2, 3						
	and 3′.						

PARTIAL CORRELATION COEFFICIENTS

The partial correlation coefficient measures the relationship between any two of the variables when all others are kept constant.

For the case of 3 variables, the partial correlation coefficient between X_1 and X_2 keeping X_3 constant is

$$r_{12\cdot3} = \frac{r_{12} - r_{13}\, r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$

where r_{ij} denotes the correlation coefficient of X_i and X_j.

Similarly, for the case of 4 variables, the partial correlation coefficient between X_1 and X_2 keeping X_3 and X_4 constant is

$$r_{12\cdot34} = \frac{r_{12\cdot4} - r_{13\cdot4}\, r_{23\cdot4}}{\sqrt{(1 - r_{13\cdot4}^2)(1 - r_{23\cdot4}^2)}} = \frac{r_{12\cdot3} - r_{14\cdot3}\, r_{24\cdot3}}{\sqrt{(1 - r_{14\cdot3}^2)(1 - r_{24\cdot3}^2)}} .$$

Any partial correlation coefficient can be computed by means of these formulas (using this program) if correlation coefficients r_{12}, r_{13}, r_{23}, ... are given.

Note:

This program finds $r_{13\cdot2}$, $r_{23\cdot1}$ by similar formulas.

Reference:

S. Wilks, *Mathematical Statistics,* John Wiley and Sons, 1962.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	02	2
03.	32	g
04.	42	x^2
05.	01	1
06.	51	–
07.	22	x⇄y
08.	33	STO
09.	01	1
10.	32	g
11.	42	x^2
12.	01	1
13.	51	–
14.	71	x
15.	31	f
16.	42	$\sqrt{x}$
17.	22	x⇄y
18.	33	STO
19.	00	0
20.	34	RCL
21.	01	1
22.	34	RCL
23.	02	2
24.	71	x
25.	51	–
26.	22	x⇄y
27.	81	÷
28.	84	R/S
29.	34	RCL
30.	01	1
31.	34	RCL
32.	02	2
33.	34	RCL
34.	00	0
35.	–01	GTO 01
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS
R_0 r_{12}, r_{13}, r_{23}
R_1 r_{13}, r_{23}, r_{12}
R_2 r_{23}, r_{12}, r_{13}
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

Suppose $r_{12} = -0.96$, $r_{13} = -0.1$, $r_{23} = 0.12$, then the partial correlation coefficients are

$r_{12\cdot3} = -.96$
$r_{13\cdot2} = .05$
$r_{23\cdot1} = .09.$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input data and compute						
	correlation coefficients	r_{12}	↑				
		r_{13}	↑				
		r_{23}	BST	R/S			$r_{12\cdot3}$
			R/S				$r_{13\cdot2}$
			R/S				$r_{23\cdot1}$
3	For a new case, go to 2						

STANDARDIZED SCORES

Given a set of data $\{x_1, x_2, ..., x_n\}$, this program finds $\{y_1, y_2, ..., y_n\}$ such that

$$y_i = \frac{x_i - \bar{x}}{s}$$

$$\text{for } i = 1, 2, ..., n$$

where $\bar{x}$ and s are sample mean and standard deviation of $\{x_1, x_2, ..., x_n\}$. $\{y_1, y_2, ..., y_n\}$ has mean zero and its standard deviation is 1.

This program can also transform y_i's to z_i's such that $\{z_1, z_2, ..., z_n\}$ has mean μ and standard deviation σ (μ and σ are given).

$$z_i = \sigma y_i + \mu$$

$$\text{for } i = 1, 2, ..., n$$

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.		
01.	34	RCL	26.		
02.	02	2	27.		
03.	51	−	28.		
04.	34	RCL	29.		
05.	03	3	30.		
06.	81	÷	31.		
07.	84	R/S	32.		
08.	34	RCL	33.		
09.	01	1	34.		
10.	71	x	35.		
11.	34	RCL	36.		
12.	00	0	37.		
13.	61	+	38.		
14.	-00	GTO 00	39.		
15.	31	f	40.		
16.	33	$\bar{x}$	41.		
17.	33	STO	42.		
18.	02	2	43.		
19.	32	g	44.		
20.	33	s	45.		
21.	33	STO	46.		
22.	03	3	47.		
23.	-00	GTO 00	48.		
24.			49.		

REGISTERS	
R_0	μ
R_1	σ
R_2	$\bar{x}$
R_3	s
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	n
$R_{.1}$	Σx_i
$R_{.2}$	Σx_i^2
$R_{.3}$	Used
$R_{.4}$	Used
$R_{.5}$	Used
$R_{.6}$	0
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

$\mu = 75, \quad \sigma = 10, \quad s = 10.54$

i	1	2	3	4	5	6	7	8	9	10	11
x_i	57	62	73	48	78	54	59	75	67	81	66
y_i	−.80	−.33	.72	−1.66	1.19	−1.09	−.61	.91	.15	1.48	.05
z_i	66.98	71.72	82.16	58.44	86.90	64.13	68.88	84.06	76.47	89.75	75.52

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R			0.00
3	Input μ, σ if z_i's are desired	μ	STO	0			
		σ	STO	1			
4	Perform 4 for i = 1, 2,..., n	x_i	Σ+				i
4′	Delete erroneous data x_k	x_k	f	Σ−			
5	Compute and store $\bar{x}$, s		GTO	1	5	R/S	s
6	Perform 6 for i = 1, 2,..., n	x_i	BST	R/S			y_i
	(optional) Compute z_i		R/S				z_i
7	For a new case, go to 2						

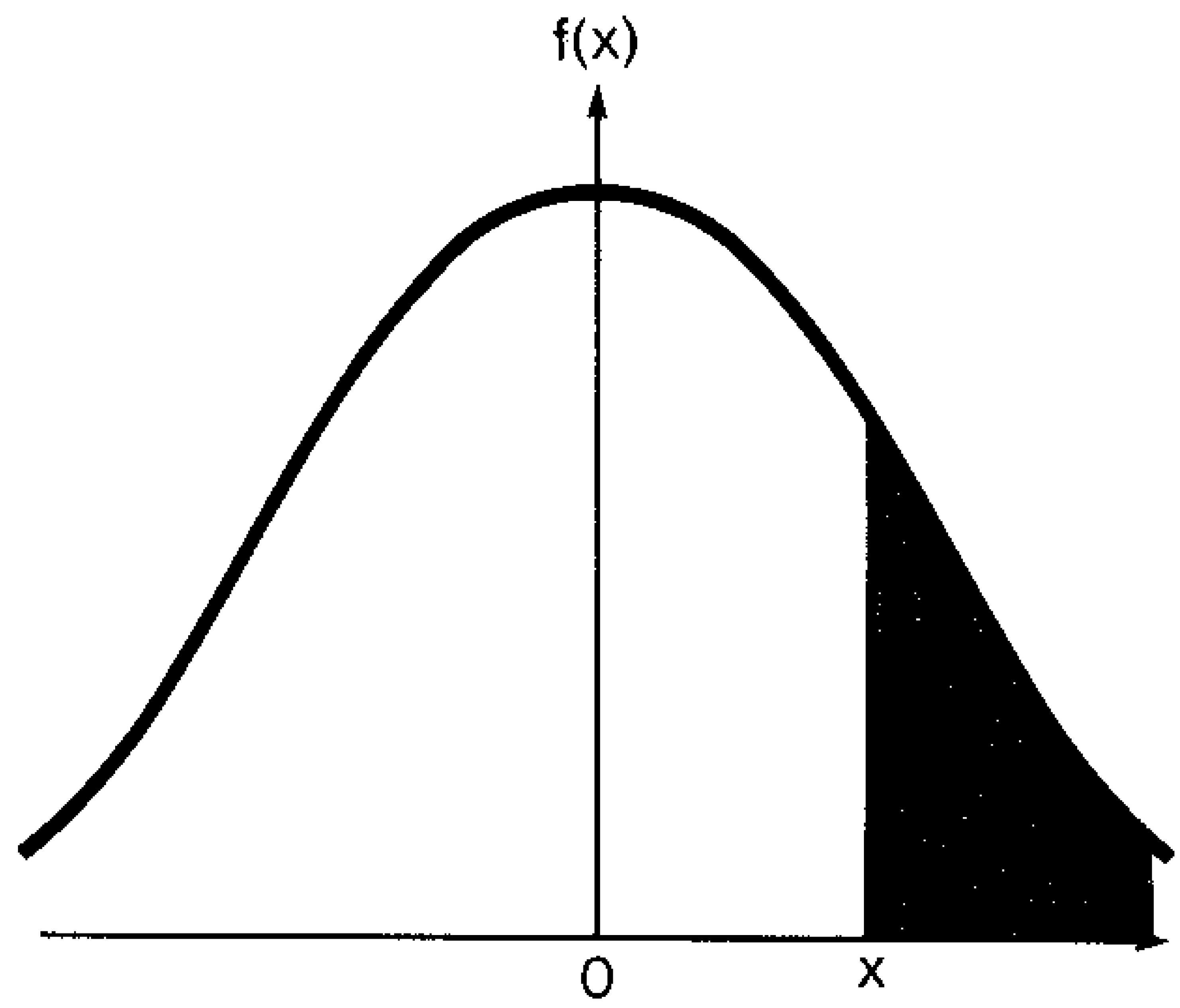
NORMAL DISTRIBUTION

The density function for a standard normal variable is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} .$$

The upper tail area is

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} \, dt .$$



For $x \geqslant 0$, polynomial approximation is used to compute Q(x):

$$Q(x) = f(x)\,(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + \epsilon(x)$$

where $|\epsilon(x)| < 7.5 \times 10^{-8}$

$$t = \frac{1}{1 + rx}, \; r = 0.2316419$$

$b_1 = .31938153,$ $b_2 = -.356563782$

$b_3 = 1.781477937,$ $b_4 = -1.821255978$

$b_5 = 1.330274429$

Note:

The program only works for $x \geqslant 0$. Equations $f(-x) = f(x)$, $Q(-x) = 1-Q(x)$, where $x \geqslant 0$, can be used to find f and Q for negative numbers.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1968.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	41	$\uparrow$
01.	71	x	26.	41	$\uparrow$
02.	02	2	27.	41	$\uparrow$
03.	81	÷	28.	34	RCL
04.	42	CHS	29.	05	5
05.	32	g	30.	71	x
06.	22	e^x	31.	34	RCL
07.	31	f	32.	04	4
08.	83	π	33.	61	+
09.	02	2	34.	71	x
10.	71	x	35.	34	RCL
11.	31	f	36.	03	3
12.	42	$\sqrt{x}$	37.	61	+
13.	81	÷	38.	71	x
14.	33	STO	39.	34	RCL
15.	07	7	40.	02	2
16.	84	R/S	41.	61	+
17.	34	RCL	42.	71	x
18.	00	0	43.	34	RCL
19.	34	RCL	44.	01	1
20.	06	6	45.	61	+
21.	71	x	46.	71	x
22.	01	1	47.	34	RCL
23.	61	+	48.	07	7
24.	13	$1/x$	49.	71	x

REGISTERS
R_0 r
R_1 b_1
R_2 b_2
R_3 b_3
R_4 b_4
R_5 b_5
R_6 x
R_7 f(x)
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Examples:

1. x = 1.18
   f(x) = .20
   Q(x) = .12

2. x = 2.28
   f(x) = .03
   Q(x) = .01

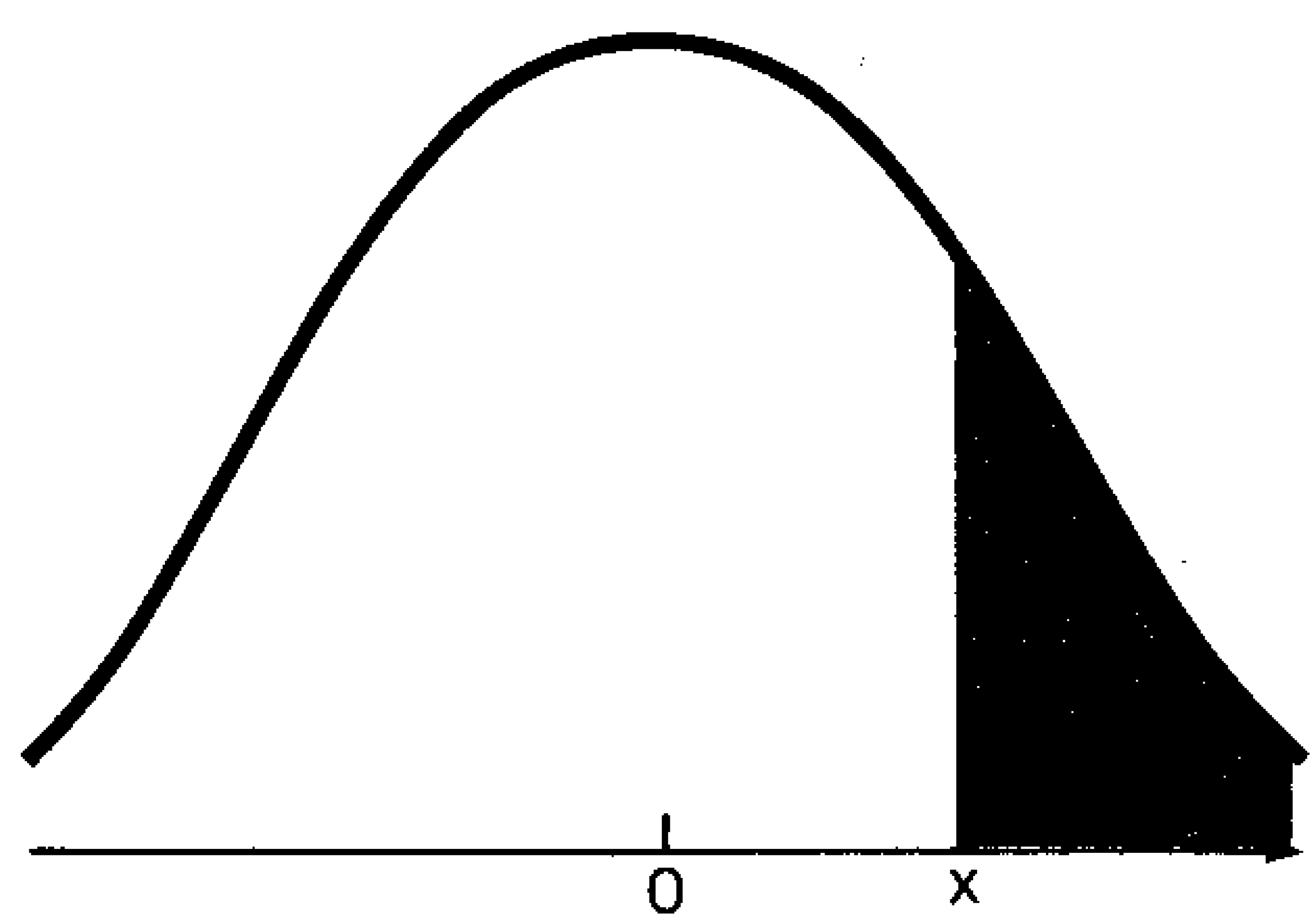
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store constants	r	STO	0			
		b_1	STO	1			
		b_2	STO	2			
		b_3	STO	3			
		b_4	STO	4			
		b_5	STO	5	BST		
3	Input x and compute f(x)	x	$\uparrow$	STO	6	R/S	f(x)
4	Compute Q(x)		R/S				Q(x)
5	For a new case, go to 3						

INVERSE NORMAL INTEGRAL

This program determines the value of x such that

$$Q = \int_x^\infty \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}}\,dt$$

where Q is given and $0 < Q \leq 0.5$.



The following rational approximation is used:

$$x = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

where $|\epsilon(Q)| < 4.5 \times 10^{-4}$

$$t = \sqrt{\ln \frac{1}{Q^2}}$$

$c_0 = 2.515517 \qquad d_1 = 1.432788$

$c_1 = 0.802853 \qquad d_2 = 0.189269$

$c_2 = 0.010328 \qquad d_3 = 0.001308$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1968.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	41	↑
02.	71	x
03.	13	$^1/_x$
04.	31	f
05.	22	ln
06.	31	f
07.	42	$\sqrt{x}$
08.	33	STO
09.	06	6
10.	41	↑
11.	41	↑
12.	41	↑
13.	34	RCL
14.	05	5
15.	71	x
16.	34	RCL
17.	04	4
18.	61	+
19.	71	x
20.	34	RCL
21.	03	3
22.	61	+
23.	71	x
24.	01	1

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	61	+
26.	33	STO
27.	07	7
28.	44	CLX
29.	34	RCL
30.	02	2
31.	71	x
32.	34	RCL
33.	01	1
34.	61	+
35.	71	x
36.	34	RCL
37.	00	0
38.	61	+
39.	34	RCL
40.	07	7
41.	81	÷
42.	51	−
43.	-00	GTO 00
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS
R_0 c_0
R_1 c_1
R_2 c_2
R_3 d_1
R_4 d_2
R_5 d_3
R_6 t
R_7 $1+d_1t+d_2t^2+d_3t^3$
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Examples:

1. Q = 0.12
   x = 1.18

2. Q = 0.05
   x = 1.65

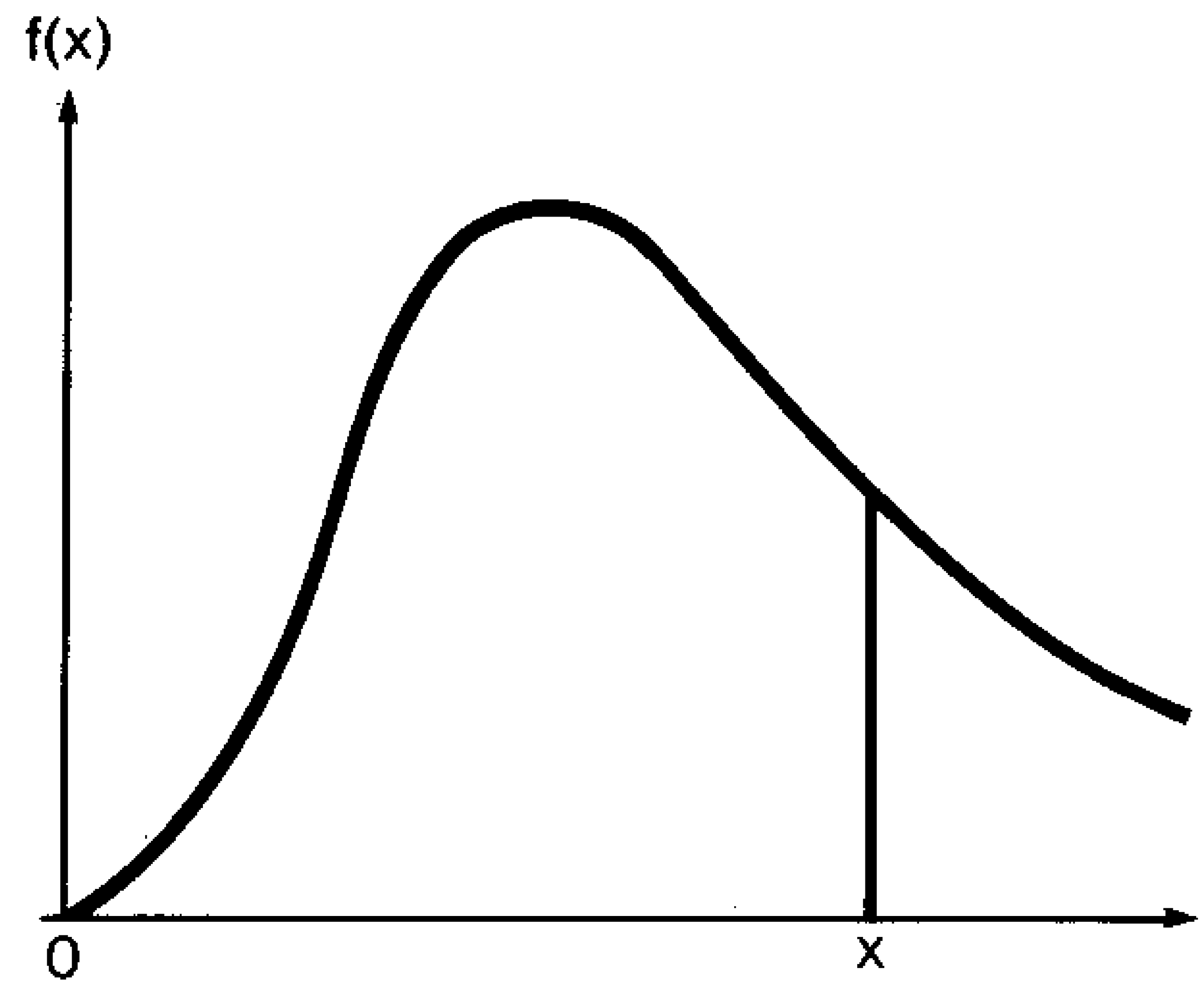
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store constants	c_0	STO	0			
		c_1	STO	1			
		c_2	STO	2			
		d_1	STO	3			
		d_2	STO	4			
		d_3	STO	5	BST		
3	Input Q	Q	R/S				x
4	For a new case, go to 3						

CHI-SQUARE DENSITY FUNCTION

This program evaluates the chi-square density function

$$f(x) = \frac{x^{\frac{\nu}{2}-1}}{2^{\frac{\nu}{2}}\,\Gamma\left(\frac{\nu}{2}\right)\,e^{\frac{x}{2}}}$$

where $x \geqslant 0$ and ν is the degrees of freedom.



Notes:

1. The program requires that $\nu \leqslant 141$. If $\nu > 141$ and ν is even, then the display shows all 9's for $\Gamma(\nu/2)$; if $\nu > 141$ and ν is odd, no warnings are given, but the answers are incorrect.

2. If both x and ν are large, f(x) may overflow the machine.

3. If ν is even,

$$\Gamma\left(\frac{\nu}{2}\right) = \left(\frac{\nu}{2} - 1\right)!$$

If ν is odd,

$$\Gamma\left(\frac{\nu}{2}\right) = \left(\frac{\nu}{2} - 1\right)\left(\frac{\nu}{2} - 2\right)\cdots\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right).$$

4. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

5. f(x) may be used as an input for *Chi-Square Distribution* program to find the cumulative distribution. In that case, record f(x) to as many digits as possible for reentry.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1968.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY	REGISTERS
00.			25.	31	f	R_0 $(\nu/2)-1$
01.	41	↑	26.	42	$\sqrt{x}$	R_1 Used
02.	02	2	27.	71	x	R_2 x
03.	81	÷	28.	84	R/S	R_3
04.	01	1	29.	33	STO	R_4
05.	51	–	30.	02	2	R_5
06.	33	STO	31.	34	RCL	R_6
07.	00	0	32.	00	0	R_7
08.	84	R/S	33.	12	y^x	R_8
09.	83	•	34.	22	x⇄y	R_9
10.	05	5	35.	81	÷	$R_{\bullet 0}$
11.	32	g	36.	02	2	$R_{\bullet 1}$
12.	–20	x=y 20	37.	34	RCL	$R_{\bullet 2}$
13.	23	R↓	38.	00	0	$R_{\bullet 3}$
14.	33	STO	39.	01	1	$R_{\bullet 4}$
15.	71	x	40.	61	+	$R_{\bullet 5}$
16.	01	1	41.	12	y^x	$R_{\bullet 6}$
17.	01	1	42.	81	÷	$R_{\bullet 7}$
18.	51	–	43.	34	RCL	$R_{\bullet 8}$
19.	–09	GTO 09	44.	02	2	$R_{\bullet 9}$
20.	34	RCL	45.	02	2	
21.	01	1	46.	81	÷	
22.	71	x	47.	32	g	
23.	31	f	48.	22	e^x	
24.	83	π	49.	81	÷	

Examples:

1. $\nu = 20$,

$$\Gamma\left(\frac{\nu}{2}\right) = 362880.00$$

f(9.591) = .02

(Press [f] SCI [9] to see 1.527751934–02)

2. $\nu = 3$

$$\Gamma\left(\frac{\nu}{2}\right) = .89$$

f(7.82) = .02

(Press [f] SCI [9] to see 2.235743714–02)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize	1	STO	1	BST		1.00
3	Input ν	ν	R/S				$(\nu/2)-1$
4	If ν is even, go to 6						
5	Compute $\Gamma(\nu/2)$ for odd ν		R/S				$\Gamma(\nu/2)$
	Go to 7						
6	Compute $\Gamma(\nu/2)$ for even ν		f	n!	GTO	2	
			9				$\Gamma(\nu/2)$
7	Input x and compute f(x)	x	R/S				f(x)
8	For a new case, go to 2						

CHI-SQUARE DISTRIBUTION

Given x, ν and f(x), this program uses a series approximation to evaluate the chi-square cumulative distribution

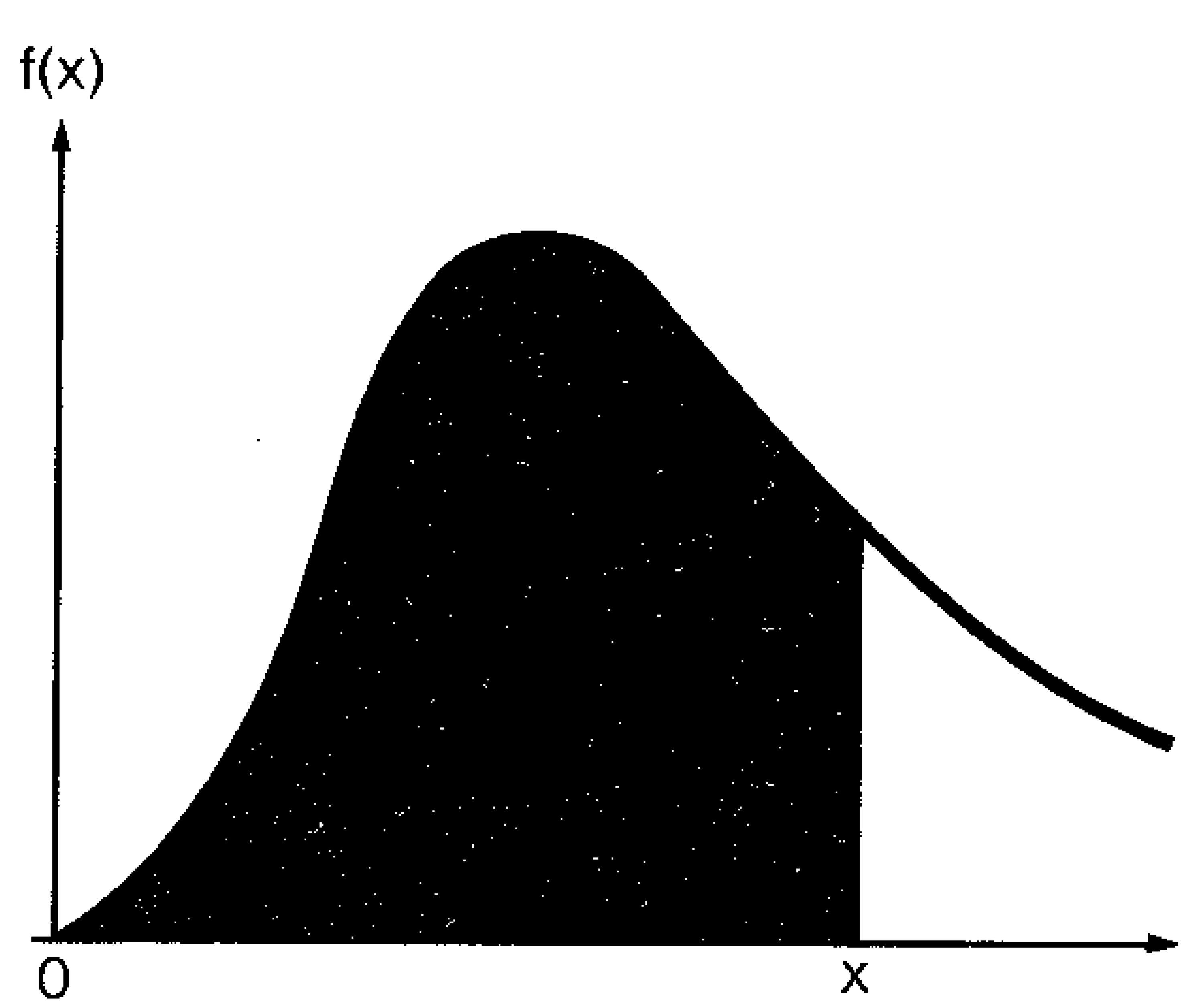
$$P(x) = \int_0^x f(t)\,dt$$

$$= \frac{2x}{\nu} f(x)\left[1 + \sum_{k=1}^{\infty} \frac{x^k}{(\nu+2)(\nu+4)\ldots(\nu+2k)}\right]$$

where $x \geqslant 0$

ν is the degrees of freedom, and density function

$$f(x) = \frac{x^{\frac{\nu}{2}-1}}{2^{\frac{\nu}{2}}\,\Gamma\left(\frac{\nu}{2}\right) e^{\frac{x}{2}}}.$$



The program computes successive partial sums of the series. When two consecutive partial sums are equal, the value is used as the sum of the series.

Note:

f(x) may be computed using *Chi-square Density Function* program.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1968.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	02	2
03.	84	R/S
04.	33	STO
05.	00	0
06.	81	÷
07.	02	2
08.	71	x
09.	71	x
10.	33	STO
11.	01	1
12.	01	1
13.	33	STO
14.	03	3
15.	34	RCL
16.	02	2
17.	34	RCL
18.	00	0
19.	02	2
20.	61	+
21.	33	STO
22.	00	0
23.	81	÷
24.	34	RCL
25.	03	3
26.	71	x
27.	33	STO
28.	03	3
29.	61	+
30.	32	g
31.	-33	x=y 33
32.	-15	GTO 15
33.	34	RCL
34.	01	1
35.	71	x
36.	-00	GTO 00
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS
R_0 ν
R_1 $2xf(x)/\nu$
R_2 x
R_3 Used
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Examples:

1. $f(x) = 1.527751934 \times 10^{-2}$

   $x = 9.591$

   $\nu = 20$

   $P(x) = .03$

Note: For f(x), see *Chi-square Density Function* program.

2. $f(x) = 2.235743714 \times 10^{-2}$

   $x = 7.82$

   $\nu = 3$

   $P(x) = .95$

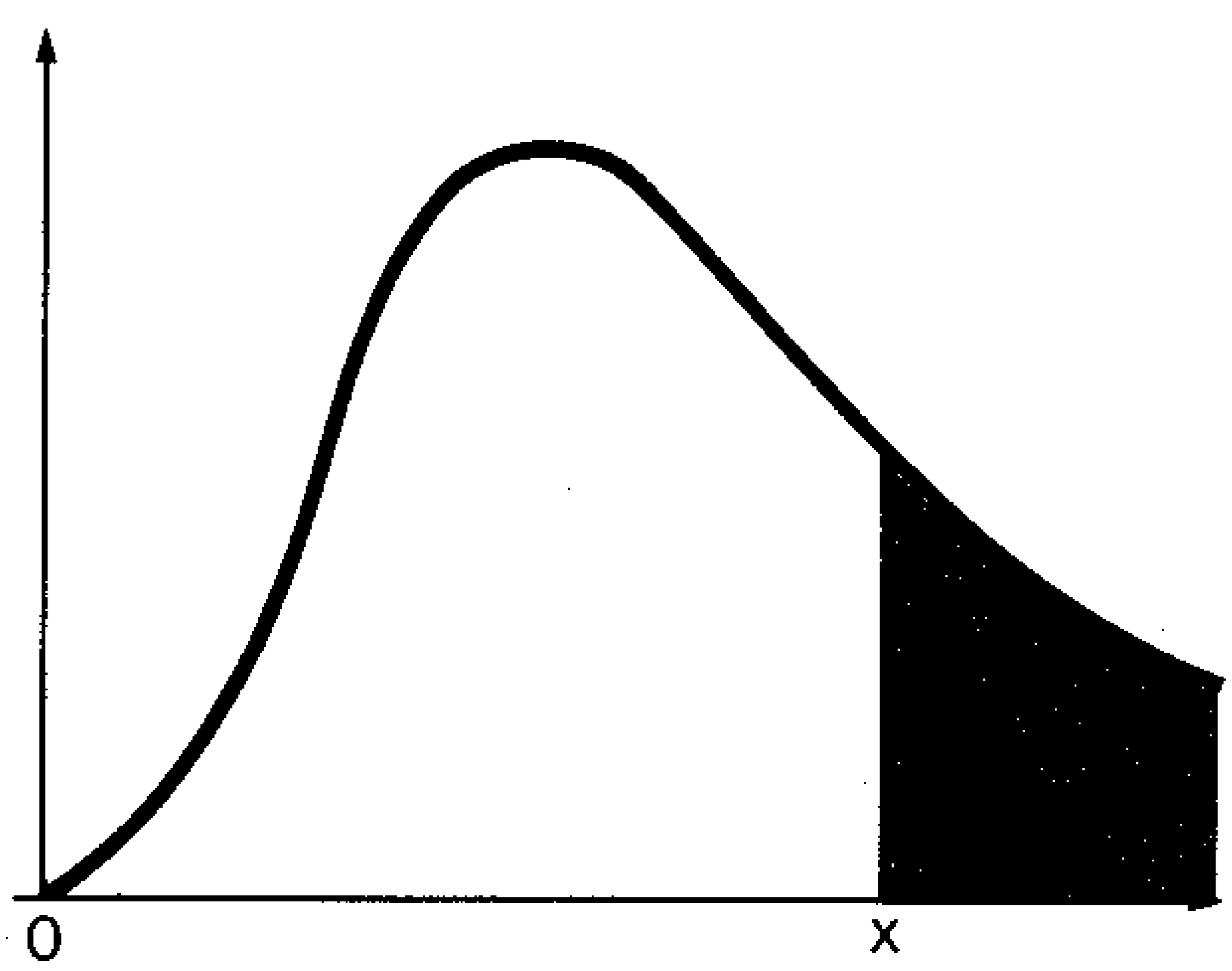
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input f(x), x and ν	f(x)	↑				
		x	BST	R/S			x
		ν	R/S				P(x)
3	For a new case, go to 2						

F DISTRIBUTION

This program evaluates the integral of the F distribution

$$Q(x) = \int_x^\infty \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right) y^{\frac{\nu_1}{2}-1} \left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)\left(1+\frac{\nu_1}{\nu_2}y\right)^{\frac{\nu_1+\nu_2}{2}}} dy$$

for given values of x (> 0), degrees of freedoms ν_1, ν_2, provided either ν_1 or ν_2 is even.



The integral is evaluated by means of the following series:

1. ν_1 even

$$Q(x) = t^{\frac{\nu_2}{2}} \left[1 + \frac{\nu_2}{2}(1-t) + \ldots + \frac{\nu_2(\nu_2+2)\ldots(\nu_2+\nu_1-4)}{2 \cdot 4 \ldots (\nu_1-2)} (1-t)^{\frac{\nu_1-2}{2}} \right]$$

2. ν_2 even

$$Q(x) = 1 - (1-t)^{\frac{\nu_1}{2}} \left[1 + \frac{\nu_1}{2} t + \ldots + \frac{\nu_1(\nu_1+2)\ldots(\nu_2+\nu_1-4)}{2 \cdot 4 \ldots (\nu_2-2)} t^{\frac{\nu_2-2}{2}} \right]$$

where $t = \dfrac{\nu_2}{\nu_2 + \nu_1 x}$.

Note:

If both ν_1, ν_2 are even, the two formulas would generate identical answers. Using the smaller of ν_1, ν_2 could save computation time. For example, if $\nu_1 = 10$, $\nu_2 = 20$, then classify the problem as ν_1 is even to obtain the answer.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1968.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	34	RCL
01.	61	+	26.	00	0
02.	81	÷	27.	71	x
03.	33	STO	28.	34	RCL
04.	00	0	29.	04	4
05.	34	RCL	30.	02	2
06.	02	2	31.	61	+
07.	02	2	32.	33	STO
08.	81	÷	33.	04	4
09.	12	y^x	34.	34	RCL
10.	33	STO	35.	01	1
11.	03	3	36.	32	g
12.	01	1	37.	-44	x=y 44
13.	34	RCL	38.	23	R↓
14.	00	0	39.	81	÷
15.	51	−	40.	33	STO
16.	33	STO	41.	61	+
17.	00	0	42.	05	5
18.	01	1	43.	-19	GTO 19
19.	34	RCL	44.	34	RCL
20.	02	2	45.	05	5
21.	34	RCL	46.	34	RCL
22.	04	4	47.	03	3
23.	61	+	48.	71	x
24.	71	x	49.	-00	GTO 00

REGISTERS
R_0 t, 1−t
R_1 ν_1
R_2 ν_2
R_3 $t^{\nu_2/2}$
R_4 0, 2,...
R_5 Used
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Examples:

1. $\nu_1 = 7, \nu_2 = 6$

   Q (4.21) = .05

2. $\nu_1 = 4, \nu_2 = 20$

   Q (2.25) = .10

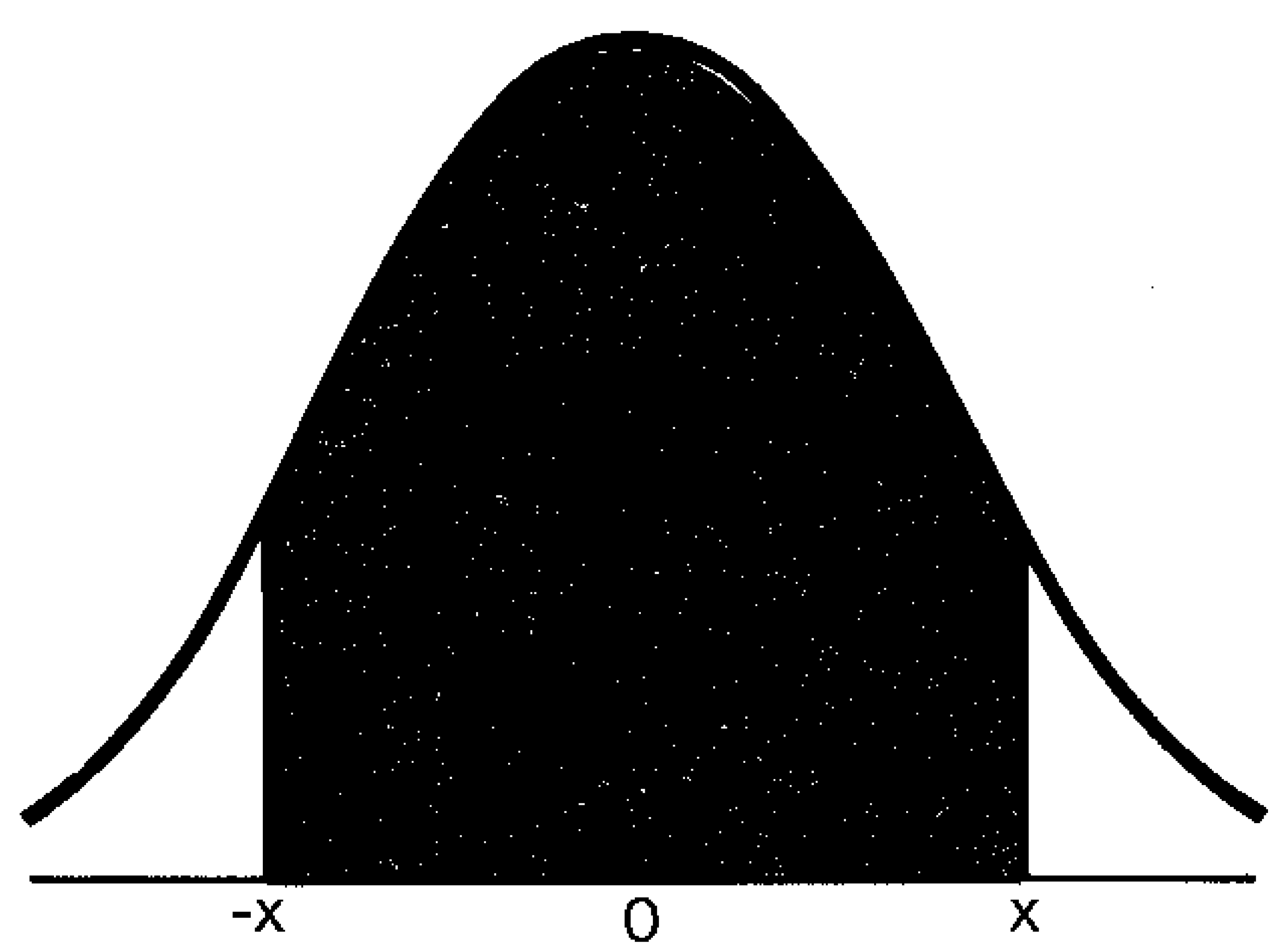
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize	0	STO	4			
		1	STO	5	BST		1.00
3	If ν_2 is even, go to 5						
4	Input ν_1, ν_2 and x	ν_1	STO	1			
		ν_2	STO	2			
		x	RCL	1	x	RCL	
			2	R/S			Q(x)
5	ν_2 even	ν_2	STO	1			
		ν_1	STO	2			
		x	1/x	RCL	1	x	
			RCL	2	R/S		1 – Q(x)
			1	x⇄y	–		Q(x)
6	For a new case, go to 2						

t DISTRIBUTION

This program evaluates the integral for t distribution

$$I(x, \nu) = \int_{-x}^{x} \frac{\Gamma\left(\frac{\nu+1}{2}\right)\left(1 + \frac{y^2}{\nu}\right)^{-\frac{\nu+1}{2}}}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)} dy$$

where $x > 0$,
ν is the degrees of freedom.



Formulas used are:

1. ν even

$$I(x, \nu) = \sin\theta \left\{1 + \frac{1}{2}\cos^2\theta + \frac{1 \cdot 3}{2 \cdot 4}\cos^4\theta + \ldots + \frac{1 \cdot 3 \cdot 5 \ldots (\nu - 3)}{2 \cdot 4 \cdot 6 \ldots (\nu - 2)}\cos^{\nu-2}\theta\right\}$$

2 ν odd

$$I(x,\nu) = \begin{cases} \dfrac{2\theta}{\pi} & \text{if } \nu = 1 \\ \dfrac{2\theta}{\pi} + \dfrac{2}{\pi}\cos\theta\left\{\sin\theta\left[1 + \dfrac{2}{3}\cos^2\theta + \ldots + \dfrac{2\cdot 4 \ldots (\nu-3)}{1\cdot 3 \ldots (\nu-2)}\cos^{\nu-3}\theta\right]\right\} & \text{if } \nu > 1 \end{cases}$$

$$\text{where } \theta = \tan^{-1}\left(\frac{x}{\sqrt{\nu}}\right)$$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1968.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY	REGISTERS
00.			25.	61	+	R_0 1 + ($\cos^2\theta$)/2 + ...
01.	31	f	26.	33	STO	R_1 ν
02.	42	$\sqrt{x}$	27.	03	3	R_2 $\cos^2\theta$
03.	81	÷	28.	34	RCL	R_3 0, 2, 4,... or 1, 3, 5...
04.	32	g	29.	01	1	R_4 θ
05.	14	$\tan^{-1}$	30.	32	g	R_5
06.	33	STO	31.	-41	x=y 41	R_6
07.	04	4	32.	23	R↓	R_7
08.	31	f	33.	81	÷	R_8
09.	13	cos	34.	34	RCL	R_9
10.	32	g	35.	02	2	$R_{.0}$
11.	42	x^2	36.	71	x	$R_{.1}$
12.	33	STO	37.	33	STO	$R_{.2}$
13.	02	2	38.	61	+	$R_{.3}$
14.	01	1	39.	00	0	$R_{.4}$
15.	33	STO	40.	–17	GTO 17	$R_{.5}$
16.	00	0	41.	34	RCL	$R_{.6}$
17.	34	RCL	42.	00	0	$R_{.7}$
18.	03	3	43.	34	RCL	$R_{.8}$
19.	01	1	44.	04	4	$R_{.9}$
20.	61	+	45.	31	f	
21.	71	x	46.	12	sin	
22.	34	RCL	47.	71	x	
23.	03	3	48.	-00	GTO 00	
24.	02	2	49.			

Examples:

1. I (2.201, 11) = .95

2. I (2.75, 30) = .99

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Put machine in RAD mode		f	RAD	BST		
3	If ν is odd, go to 4′						
4	ν is even	0	STO	3			
		x	↑				
		ν	STO	1	R/S		I (x, ν)
4′	If ν = 1, go to 4″	1	STO	3			
		x	↑				
		ν	STO	1	f	$\sqrt{x}$	
			÷	g	$\tan^{-1}$	STO	
			4	GTO	0	8	
			R/S				
			RCL	4	f	cos	
			x	RCL	4	+	
			2	x	f	π	
			÷				I (x, ν)
4″	ν = 1	x	g	$\tan^{-1}$	2	x	
			f	π	÷		I (x, 1)
5	For a new case, go to 3						

BIVARIATE NORMAL DISTRIBUTION

This program evaluates the joint probability density function

$$f(x, y) = \frac{1}{2\pi\ \sigma_1\ \sigma_2 \sqrt{1-\rho^2}}\, e^{-P(x,y)}$$

where

$$P(x, y) = \frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_1)^2}{{\sigma_1}^2} - 2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} + \frac{(y-\mu_2)^2}{{\sigma_2}^2}\right]$$

Notes:

1. $\sigma_1 \neq 0, \sigma_2 \neq 0$
2. The program requires that $\rho^2 < 1$.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	32	g
02.	42	x^2
03.	22	$x \rightleftarrows y$
04.	34	RCL
05.	00	0
06.	51	–
07.	34	RCL
08.	01	1
09.	81	÷
10.	33	STO
11.	06	6
12.	32	g
13.	42	x^2
14.	61	+
15.	34	RCL
16.	06	6
17.	34	RCL
18.	07	7
19.	71	x
20.	34	RCL
21.	04	4
22.	71	x
23.	02	2
24.	71	x

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	51	–
26.	34	RCL
27.	05	5
28.	02	2
29.	71	x
30.	81	÷
31.	42	CHS
32.	32	g
33.	22	e^x
34.	34	RCL
35.	05	5
36.	31	f
37.	42	$\sqrt{x}$
38.	34	RCL
39.	01	1
40.	71	x
41.	34	RCL
42.	03	3
43.	71	x
44.	02	2
45.	71	x
46.	31	f
47.	83	π
48.	71	x
49.	81	÷

REGISTERS
R_0 μ_1
R_1 σ_1
R_2 μ_2
R_3 σ_2
R_4 ρ
R_5 $1-\rho^2$
R_6 $(x-\mu_1)/\sigma_1$
R_7 $(y-\mu_2)/\sigma_2$
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

$\mu_1 = -1, \sigma_1 = 1.5$

$\mu_2 = 1, \ \sigma_2 = 0.5$

$\rho = 0.7$

f(1, 2) = .04

f(−1, 1) = .30

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input μ_1, σ_1, μ_2, σ_2, ρ	μ_1	STO	0			
		σ_1	STO	1			
		μ_2	STO	2			
		σ_2	STO	3	1	↑	
		ρ	STO	4	g	x^2	
			−	STO	5	BST	
3	Input x and y	x	↑				
		y	RCL	2	−	RCL	
			3	÷	STO	7	
			R/S				f(x, y)
4	For different x, y, go to 3						
5	For a new case, go to 2						

LOGARITHMIC NORMAL DISTRIBUTION

If X is a random variable whose logarithm is normally distributed with mean m and variance σ^2, then X has a logarithmic normal distribution with density function

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2\sigma^2}(\ln x - m)^2}$$

where $x > 0$.

This program computes f(x) and the following statistics for given m, σ^2:

$$\text{median} = e^{m}$$

$$\text{mode} = e^{m-\sigma^2}$$

$$\text{mean} = e^{m+(\sigma^2/2)}$$

$$\text{variance} = e^{\sigma^2+2m}\,(e^{\sigma^2} - 1).$$

Note:

The program requires that $\sigma^2 \neq 0$.

Reference:

K. A. Brownlee, *Statistical Theory and Methodology in Science and Engineering,* John Wiley and Sons, 1965.

LINE	CODE	KEY ENTRY
00.		
01.	34	RCL
02.	01	1
03.	34	RCL
04.	00	0
05.	02	2
06.	81	÷
07.	61	+
08.	32	g
09.	22	e^x
10.	84	R/S
11.	32	g
12.	42	x^2
13.	34	RCL
14.	00	0
15.	32	g
16.	22	e^x
17.	01	1
18.	51	–
19.	71	x
20.	84	R/S
21.	31	f
22.	22	ln
23.	34	RCL
24.	01	1

LINE	CODE	KEY ENTRY
25.	51	–
26.	32	g
27.	42	x^2
28.	34	RCL
29.	00	0
30.	81	÷
31.	02	2
32.	81	÷
33.	42	CHS
34.	32	g
35.	22	e^x
36.	31	f
37.	83	π
38.	02	2
39.	71	x
40.	34	RCL
41.	00	0
42.	71	x
43.	31	f
44.	42	$\sqrt{x}$
45.	81	÷
46.	34	RCL
47.	02	2
48.	81	÷
49.	–20	GTO 20

REGISTERS
R_0 σ^2
R_1 m
R_2 x
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example: $\sigma^2 = 1$, m = 1

median = 2.72
mode = 1.00
mean = 4.48
variance = 34.51

f (.1) = .02
f (.6) = .21
f (1) = .24

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store m, σ^2	σ^2	STO	0			
		m	STO	1	BST		
3	Compute median and mode		g	e^x			median
			RCL	1	RCL	0	
			–	g	e^x		mode
4	Compute mean and variance		R/S				mean
			R/S				variance
5	Input x	x	STO	2	R/S		f(x)
6	For a new x, go to 5						

WEIBULL DISTRIBUTION PARAMETER CALCULATION

The Weibull probability density function is given by

$$f(x) = \frac{bx^{(b-1)}}{\theta^b} e^{-\left(\frac{x}{\theta}\right)^b}$$

where $\theta > 0, \quad b > 0, \quad x > 0.$

The cumulative distribution function is

$$F(x) = 1 - e^{-\left(\frac{x}{\theta}\right)^b}$$

For a set of data $\{x_1, \ldots, x_n\}$, the Weibull parameters b and θ are to be calculated for these functions.

A common application is to use Weibull analysis for failure data where all samples are tested to failure. To use the program, list the items in order of increasing time to failure.

The median rank (M. R.) is calculated by

$$\frac{R_i - 0.3}{n + 0.4}$$

where R_i is the rank of failure data x_i. Using this median rank as an approximation of $F(x_i)$, a least squares fit is performed to the linearized form of the cummulative distribution function

$$\ln \ln \left(\frac{1}{1 - F(x)}\right) = b \ln x - b \ln \theta.$$

The solution is similar to the linear regression problem, and estimates of b and θ are obtained.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	01	1
02.	33	STO
03.	00	0
04.	32	g
05.	44	CL·R
06.	84	R/S
07.	33	STO
08.	01	1
09.	84	R/S
10.	31	f
11.	22	ln
12.	34	RCL
13.	00	0
14.	83	·
15.	03	3
16.	51	–
17.	34	RCL
18.	01	1
19.	83	·
20.	04	4
21.	61	+
22.	81	÷
23.	01	1
24.	33	STO

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	61	+
26.	00	0
27.	22	x⇄y
28.	51	–
29.	13	1/x
30.	31	f
31.	22	ln
32.	31	f
33.	22	ln
34.	22	x⇄y
35.	11	Σ+
36.	–09	GTO 09
37.	31	f
38.	21	L. R.
39.	22	x⇄y
40.	84	R/S
41.	81	÷
42.	42	CHS
43.	32	g
44.	22	e^x
45.	–00	GTO 00
46.		
47.		
48.		
49.		

REGISTERS	
R_0	Used
R_1	n
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	n
$R_{.1}$	Used
$R_{.2}$	Used
$R_{.3}$	Used
$R_{.4}$	Used
$R_{.5}$	Used
$R_{.6}$	0
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

x_i: 34, 60, 75, 95, 119, 158 (hours to failure)
(x_i's must be entered in increasing order.)
$n = 6$
$b = 1.95$
$\theta = 104.09$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Input n	n	R/S				
4	Perform 4 for i = 1, 2,..., n	x_i	R/S				i
5	Compute b and θ		GTO	3	7	R/S	b
			R/S				θ
6	For a new case, go to 2						

BINOMIAL DISTRIBUTION

This program evaluates the binomial density function for given p and n:

$$f(x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

where n is a positive integer

$0 < p < 1$ and

$x = 0, 1, 2, \ldots, n.$

The recursive relation

$$f(x + 1) = \frac{p(n - x)}{(x + 1)(1 - p)} f(x)$$

$$(x = 0, 1, 2, \ldots, n - 1)$$

is used to find the cumulative distribution

$$P(x) = \sum_{k=0}^{x} f(k).$$

Notes:

1. $f(0) = P(0)$
2. When x is large, due to round-off error, the computed value for P(x) might be slightly greater than one. In that case, let $P(x) = 1$.
3. The execution time of the program depends on x; the larger x is, the longer it takes.
4. The mean m and the variance σ^2 are given by

$$m = np$$

$$\sigma^2 = np(1 - p).$$

Reference:

E. Parzen, *Modern Probability Theory and its Applications,* John Wiley and Sons, 1960.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	06	6
03.	00	0
04.	33	STO
05.	00	0
06.	34	RCL
07.	03	3
08.	33	STO
09.	04	4
10.	33	STO
11.	05	5
12.	34	RCL
13.	01	1
14.	34	RCL
15.	00	0
16.	51	–
17.	34	RCL
18.	00	0
19.	01	1
20.	61	+
21.	81	÷
22.	34	RCL
23.	02	2
24.	71	x

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	34	RCL
26.	04	4
27.	71	x
28.	33	STO
29.	04	4
30.	33	STO
31.	61	+
32.	05	5
33.	34	RCL
34.	00	0
35.	01	1
36.	61	+
37.	33	STO
38.	00	0
39.	34	RCL
40.	06	6
41.	32	g
42.	–44	x=y 44
43.	–12	GTO 12
44.	34	RCL
45.	04	4
46.	84	R/S
47.	34	RCL
48.	05	5
49.	–00	GTO 00

REGISTERS
R_0 counter
R_1 n
R_2 p, p/(1 – p)
R_3 f(0)
R_4 Used
R_5 Used
R_6 x
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

n = 6, p = 0.49
f(0) = .02
f(4) = .22
P(4) = .90

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input n and p	n	STO	1			
		p	STO	2	STO	4	
			1	–	CHS	RCL	
			1	y^x	STO	3	f(0)
			RCL	2	1	RCL	
			2	–	÷	STO	
			2	BST			
3	For x ⩾ 1	x	R/S				f(x)
			R/S				P(x)
4	For a new x, go to 3						
5	For a new case, go to 2						

POISSON DISTRIBUTION

Density function

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where $\lambda > 0$

and $x = 0, 1, 2, \dots .$

Cumulative distribution is

$$P(x) = \sum_{k=0}^{x} f(k).$$

This program evaluates f(x) and P(x) for a given λ using the recursive relation

$$f(x+1) = \frac{\lambda}{x+1} f(x).$$

Notes:

1. $f(0) = P(0)$
2. When x is large, due to round-off error, the computed value for P(x) might be slightly greater than one. In that case, let $P(x) = 1$.
3. The execution time of the program depends on x; the larger x is, the longer it takes.
4. Mean = variance = λ

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	42	CHS
02.	32	g
03.	22	e^x
04.	33	STO
05.	02	2
06.	84	R/S
07.	33	STO
08.	05	5
09.	00	0
10.	33	STO
11.	00	0
12.	34	RCL
13.	02	2
14.	33	STO
15.	03	3
16.	33	STO
17.	04	4
18.	34	RCL
19.	01	1
20.	34	RCL
21.	00	0
22.	01	1
23.	61	+
24.	81	÷

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	34	RCL
26.	03	3
27.	71	x
28.	33	STO
29.	03	3
30.	33	STO
31.	61	+
32.	04	4
33.	34	RCL
34.	00	0
35.	01	1
36.	61	+
37.	33	STO
38.	00	0
39.	34	RCL
40.	05	5
41.	32	g
42.	–44	x=y 44
43.	–18	GTO 18
44.	34	RCL
45.	03	3
46.	84	R/S
47.	34	RCL
48.	04	4
49.	–06	GTO 06

REGISTERS
R_0 counter
R_1 λ
R_2 f(0)
R_3 Used
R_4 Used
R_5 x
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

$\lambda = 3.2$
$f(0) = .04$
$f(7) = .03$
$P(7) = .98$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input λ	λ	STO	1	BST	R/S	f(0)
3	For x ⩾ 1	x	R/S				f(x)
			R/S				P(x)
4	For a new x, go to 3						
5	For a new case, go to 2						

NEGATIVE BINOMIAL DISTRIBUTION

This program evaluates the negative binomial density function for given p and r:

$$f(x) = \binom{x+r-1}{r-1} p^r (1-p)^x$$

where r is a positive integer

$0 < p < 1$ and

$x = 0, 1, 2, \ldots$.

The recursive relation

$$f(x+1) = \frac{(1-p)(x+r)}{x+1} f(x)$$

is used to find the cumulative distribution

$$P(x) = \sum_{k=0}^{x} f(k).$$

Notes:

1. $f(0) = P(0)$
2. When x is large, due to round-off error, the computed value for P(x) might be slightly greater than one. In that case, let $P(x) = 1$.
3. The execution time of the program depends on x; the larger x is, the longer it takes.
4. The mean m and the variance σ^2 are given by

$$m = \frac{r(1-p)}{p}$$

$$\sigma^2 = \frac{r(1-p)}{p^2} \quad .$$

5. If we interpret p as the probability of success of a given event, then f(x) is the probability that exactly $x + r$ trials will be required to get r successes.

Reference:

E. Parzen, *Modern Probability Theory and its Applications,* John Wiley and Sons, 1960.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	06	6
03.	00	0
04.	33	STO
05.	00	0
06.	34	RCL
07.	03	3
08.	33	STO
09.	04	4
10.	33	STO
11.	05	5
12.	01	1
13.	34	RCL
14.	01	1
15.	51	–
16.	34	RCL
17.	00	0
18.	34	RCL
19.	02	2
20.	61	+
21.	71	x
22.	34	RCL
23.	00	0
24.	01	1

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	61	+
26.	33	STO
27.	00	0
28.	81	÷
29.	34	RCL
30.	04	4
31.	71	x
32.	33	STO
33.	04	4
34.	33	STO
35.	61	+
36.	05	5
37.	34	RCL
38.	00	0
39.	34	RCL
40.	06	6
41.	32	g
42.	–44	x=y 44
43.	–12	GTO 12
44.	34	RCL
45.	04	4
46.	84	R/S
47.	34	RCL
48.	05	5
49.	–00	GTO 00

REGISTERS
R_0 counter
R_1 p
R_2 r
R_3 f(0)
R_4 Used
R_5 Used
R_6 x
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Examples:

$p = .9, r = 4$
$f(0) = .66$
$f(1) = .26$
$P(1) = .92$
$f(2) = .07$
$P(2) = .98$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input p and r	p	STO	1			
		r	STO	2	y^x	STO	
			3	BST			f(0)
3	For x ⩾ 1	x	R/S				f(x)
			R/S				P(x)
4	For a new x, go to 3						
5	For a new case, go to 2						

HYPERGEOMETRIC DISTRIBUTION

This program evaluates the hypergeometric density function for given a, b and n:

$$f(x) = \frac{\binom{a}{x}\binom{b}{n-x}}{\binom{a+b}{n}}$$

where a, b, n are positive intergers

$x \leqslant a$, $n - x \leqslant b$ and

$x = 0, 1, 2, \ldots, n.$

The recursive relation

$$f(x+1) = \frac{(x-a)(x-n)}{(x+1)(b-n+x+1)} f(x)$$

$$(x = 0, 1, 2, \ldots, n-1)$$

is used to find the cumulative distribution

$$P(x) = \sum_{k=0}^{x} f(k).$$

Notes:

1. The program requires that $n \leqslant 69$.
2. $f(0) = P(0)$
3. The execution time of the program depends on x; the larger x is, the longer it takes.
4. When x is large, due to round-off error, the computed value for P(x) might be slightly greater than one. In that case, let $P(x) = 1$.
5. The mean m and the variance σ^2 are given by

$$m = \frac{an}{a+b}$$

$$\sigma^2 = \frac{abn(a+b-n)}{(a+b)^2(a+b-1)} \quad .$$

Reference:

J. E. Freund, *Mathematical Statistics,* Prentice-Hall, 1971.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	33	STO
02.	00	0
03.	34	RCL
04.	01	1
05.	51	–
06.	34	RCL
07.	00	0
08.	34	RCL
09.	03	3
10.	51	–
11.	71	x
12.	34	RCL
13.	00	0
14.	01	1
15.	61	+
16.	81	÷
17.	31	f
18.	34	LAST X
19.	34	RCL
20.	02	2
21.	34	RCL
22.	03	3
23.	51	–
24.	61	+

DISPLAY		KEY ENTRY
LINE	CODE	
25.	81	÷
26.	34	RCL
27.	05	5
28.	71	x
29.	33	STO
30.	05	5
31.	33	STO
32.	61	+
33.	06	6
34.	34	RCL
35.	07	7
36.	01	1
37.	34	RCL
38.	00	0
39.	61	+
40.	33	STO
41.	00	0
42.	32	g
43.	–45	x=y 45
44.	–03	GTO 03
45.	34	RCL
46.	05	5
47.	84	R/S
48.	34	RCL
49.	06	6

REGISTERS
R_0 counter
R_1 a
R_2 b
R_3 n
R_4 f(0)
R_5 Used
R_6 Used
R_7 x
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

Given a = 8, b = 12, n = 6, then
f(0) = .02
f(3) = .32
P(3) = .86
f(5) = .02
P(5) = 1.00

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input a, b, n	a	STO	1			
		b	STO	2			
		n	STO	3	RCL	2	
			f	n!	f	LAST x	
			RCL	3	−	f	
			n!	÷	RCL	1	
			RCL	2	+	f	
			n!	f	LAST x	RCL	
			3	−	f	n!	
			÷	÷	STO	4	f(0)
3	For x ⩾ 1	x	STO	7	RCL	4	
			STO	5	STO	6	
			0	BST	R/S		f(x)
			R/S				P(x)
4	For a new x, go to 3						
5	For a new case, go to 2						

MULTINOMIAL DISTRIBUTION

This program evaluates the joint probability function of k (k can be 2, 3, ..., or 8) random variables having the multinomial distribution

$$f(x_1, x_2, \ldots, x_k) = \frac{n!}{x_1!\, x_2! \ldots x_k!}\, \theta_1^{x_1}\, \theta_2^{x_2} \ldots \theta_k^{x_k}$$

where $\sum_{i=1}^{k} \theta_i = 1, \sum_{i=1}^{k} x_i = n, \quad \theta_i > 0$ and

$$x_i = 0, 1, 2, \ldots, n \quad (i = 1, 2, \ldots, k).$$

The parameters of this distribution are n, $\theta_1, \theta_2, \ldots$ and θ_k.

Note:

The program requires that $n \leqslant 69$.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	31	f
02.	43	n!
03.	34	RCL
04.	01	1
05.	31	f
06.	34	LAST X
07.	12	y^x
08.	22	x⇄y
09.	81	÷
10.	33	STO
11.	71	x
12.	00	0
13.	34	RCL
14.	01	1
15.	33	STO
16.	09	9
17.	34	RCL
18.	02	2
19.	33	STO
20.	01	1
21.	34	RCL
22.	03	3
23.	33	STO
24.	02	2

DISPLAY		KEY ENTRY
LINE	CODE	
25.	34	RCL
26.	04	4
27.	33	STO
28.	03	3
29.	34	RCL
30.	05	5
31.	33	STO
32.	04	4
33.	34	RCL
34.	06	6
35.	33	STO
36.	05	5
37.	34	RCL
38.	07	7
39.	33	STO
40.	06	6
41.	34	RCL
42.	08	8
43.	33	STO
44.	07	7
45.	34	RCL
46.	09	9
47.	33	STO
48.	08	8
49.	-00	GTO 00

REGISTERS	
R_0	Used
R_1	Used
R_2	Used
R_3	Used
R_4	Used
R_5	Used
R_6	Used
R_7	Used
R_8	Used
R_9	Used
$R_{.0}$	n!
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Given $\theta_1 = 0.2, \theta_2 = 0.1, \theta_3 = 0.2, \theta_4 = 0.15, \theta_5 = 0.17, \theta_6 = 0.18$ and $n = 20$,

then $f(1, 2, 3, 4, 5, 5) = 1.274857927\text{–}04$

$f(2, 4, 0, 4, 2, 8) = 1.688980098\text{–}06$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Perform 2 for i = 1, 2,..., k	θ_i	STO				
		i					θ_i
3	If k = 8, go to 6						
4	Set all other θ_i = 1	1					
5	Perform 5 for i = k + 1,..., 8		STO				
		i					1.00
6	Input n	n	f	n!	STO	0	
			STO	·	0	BST	
7	Perform 7 for i = 1, 2,..., k	x_i	R/S				θ_i
8	If k = 8, go to 11						
9	Set all other x_i = 1	1					
10	Perform 10 8 – k times		R/S				1.00
11	Compute $f(x_1,..., x_k)$		RCL	0			$f(x_1,..., x_k)$
12	For new x's		RCL	·	0	STO	
			0				
	Go to 7						
13	For a new case, go to 2						

EXPONENTIAL CURVE FIT

This program computes the least squares fit of n pairs of data points $\{(x_i, y_i), i = 1, 2, ..., n\}$, where $y_i > 0$, for an exponential function of the form

$$y = a\,e^{bx} \qquad (a > 0).$$

The equation is linearized into

$$\ln y = \ln a + bx.$$

The following statistics are computed:

1. Coefficients a, b

$$b = \frac{\Sigma x_i \ln y_i - \frac{1}{n}(\Sigma x_i)(\Sigma \ln y_i)}{\Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2}$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\,\frac{\Sigma x_i}{n}\right]$$

2. Coefficient of determination

$$r^2 = \frac{\left[\Sigma x_i \ln y_i - \frac{1}{n}\Sigma x_i\,\Sigma \ln y_i\right]^2}{\left[\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right]\left[\Sigma(\ln y_i)^2 - \frac{(\Sigma \ln y_i)^2}{n}\right]}$$

3. Estimated value $\hat{y}$ for a given x

$$\hat{y} = a\,e^{bx}$$

Note:

n is a positive integer and $n \neq 1$.

Reference:

K. A. Brownlee, *Statistical Theory and Methodology in Science and Engineering,* John Wiley and Sons, 1965.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	32	g
02.	44	CL•R
03.	84	R/S
04.	31	f
05.	22	ln
06.	22	x⇄y
07.	11	Σ+
08.	-03	GTO 03
09.	31	f
10.	22	ln
11.	22	x⇄y
12.	31	f
13.	11	Σ−
14.	-03	GTO 03
15.	31	f
16.	21	L. R.
17.	32	g
18.	22	e^x
19.	33	STO
20.	00	0
21.	84	R/S
22.	22	x⇄y
23.	33	STO
24.	01	1
25.	84	R/S
26.	32	g
27.	42	x^2
28.	41	↑
29.	41	↑
30.	32	g
31.	33	s
32.	22	x⇄y
33.	81	÷
34.	32	g
35.	42	x^2
36.	71	x
37.	84	R/S
38.	34	RCL
39.	01	1
40.	71	x
41.	32	g
42.	22	e^x
43.	34	RCL
44.	00	0
45.	71	x
46.	-37	GTO 37
47.		
48.		
49.		

REGISTERS	
R_0	a
R_1	b
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	n
$R_{.1}$	Σx_i
$R_{.2}$	Σx_i^2
$R_{.3}$	$\Sigma \ln y_i$
$R_{.4}$	$\Sigma(\ln y_i)^2$
$R_{.5}$	$\Sigma x_i \ln y_i$
$R_{.6}$	0
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

x_i	.72	1.31	1.95	2.58	3.14
y_i	2.16	1.61	1.16	.85	0.5

1. $a = 3.45$, $b = -.58$
   $y = 3.45\, e^{-0.58x}$
2. $r^2 = .98$
3. For $x = 1.5$, $\hat{y} = 1.44$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for i = 1, 2,..., n	x_i	↑				
		y_i	R/S				i
3′	Delete erroneous data x_k, y_k	x_k	↑				
		y_k	GTO	0	9	R/S	
4	Compute a, b and r^2		GTO	1	5	R/S	a
			R/S				b
			R/S				r^2
5	Compute estimated value $\hat{y}$	x	R/S				$\hat{y}$
6	For a new x, go to 5						
7	For a new case, go to 2						

LOGARITHMIC CURVE FIT

This program fits a logarithmic curve

$$y = a + b \ln x$$

to a set of data points

$$\left\{ (x_i, y_i), i = 1, 2, ..., n \right\}$$

where $x_i > 0$.

Program computes:

1. Regression coefficients

$$b = \frac{\Sigma\, y_i \ln x_i - \frac{1}{n} \Sigma \ln x_i \, \Sigma\, y_i}{\Sigma\, (\ln x_i)^2 - \frac{1}{n} (\Sigma \ln x_i)^2}$$

$$a = \frac{1}{n} (\Sigma\, y_i - b\, \Sigma \ln x_i)$$

2. Coefficient of determination

$$r^2 = \frac{\left[\Sigma\, y_i \ln x_i - \frac{1}{n} \Sigma \ln x_i \, \Sigma\, y_i\right]^2}{\left[\Sigma\, (\ln x_i)^2 - \frac{1}{n} (\Sigma \ln x_i)^2\right] \left[\Sigma y_i^2 - \frac{1}{n} (\Sigma\, y_i)^2\right]}$$

3. Estimated value $\hat{y}$ for given x

$$\hat{y} = a + b \ln x$$

Note:

n is a positive integer and $n \neq 1$.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	32	g
02.	44	CL•R
03.	84	R/S
04.	22	$x \rightleftarrows y$
05.	31	f
06.	22	ln
07.	11	$\Sigma+$
08.	-03	GTO 03
09.	22	$x \rightleftarrows y$
10.	31	f
11.	22	ln
12.	31	f
13.	11	$\Sigma-$
14.	-03	GTO 03
15.	31	f
16.	21	L. R.
17.	33	STO
18.	00	0
19.	84	R/S
20.	22	$x \rightleftarrows y$
21.	33	STO
22.	01	1
23.	84	R/S
24.	32	g

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	42	x^2
26.	41	$\uparrow$
27.	41	$\uparrow$
28.	32	g
29.	33	s
30.	22	$x \rightleftarrows y$
31.	81	$\div$
32.	32	g
33.	42	x^2
34.	71	$\times$
35.	84	R/S
36.	31	f
37.	22	ln
38.	34	RCL
39.	01	1
40.	71	$\times$
41.	34	RCL
42.	00	0
43.	61	+
44.	-35	GTO 35
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	a
R_1	b
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	n
$R_{.1}$	$\Sigma \ln x_i$
$R_{.2}$	$\Sigma(\ln x_i)^2$
$R_{.3}$	Σy_i
$R_{.4}$	Σy_i^2
$R_{.5}$	$\Sigma y_i \ln x_i$
$R_{.6}$	0
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

x_i	3	4	6	10	12
y_i	1.5	9.3	23.4	45.8	60.1

1. $a = -47.02,\ b = 41.39$
   $y = -47.02 + 41.39 \ln x$
2. $r^2 = .98$
3. For $x = 8,\ \hat{y} = 39.06$
   For $x = 14.5,\ \hat{y} = 63.67$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for i = 1, 2,..., n	x_i	↑				
		y_i	R/S				i
3′	Delete erroneous data x_k, y_k	x_k	↑				
		y_k	GTO	0	9	R/S	
4	Compute a, b, and r^2		GTO	1	5	R/S	a
			R/S				b
			R/S				r^2
5	Compute estimated value $\hat{y}$	x	R/S				$\hat{y}$
6	For a new x, go to 5						
7	For a new case, go to 2						

POWER CURVE FIT

This program fits a power curve

$$y = ax^b \qquad (a > 0)$$

to a set of data points

$$\{(x_i, y_i), i = 1, 2, ..., n\}$$

where $x_i > 0, y_i > 0$.

By writing this equation as

$$\ln y = b \ln x + \ln a$$

the problem can be solved as a linear regression problem.

Output statistics are:

1. Regression coefficients

$$b = \frac{\Sigma (\ln x_i)(\ln y_i) - \dfrac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}}{\Sigma (\ln x_i)^2 - \dfrac{(\Sigma \ln x_i)^2}{n}}$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\,\frac{\Sigma \ln x_i}{n}\right]$$

2. Coefficient of determination

$$r^2 = \frac{\left[\Sigma (\ln x_i)(\ln y_i) - \dfrac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}\right]^2}{\left[\Sigma (\ln x_i)^2 - \dfrac{(\Sigma \ln x_i)^2}{n}\right]\left[\Sigma (\ln y_i)^2 - \dfrac{(\Sigma \ln y_i)^2}{n}\right]}$$

3. Estimated value $\hat{y}$ for given x

$$\hat{y} = ax^b$$

Note:

n is a positive integer and $n \neq 1$.

Reference:

K. A. Brownlee, *Statistical Theory and Methodology in Science and Engineering,* John Wiley and Sons, 1965.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	32	g
02.	44	CL·R
03.	84	R/S
04.	31	f
05.	22	ln
06.	22	x⇄y
07.	31	f
08.	22	ln
09.	11	Σ+
10.	-03	GTO 03
11.	31	f
12.	22	ln
13.	22	x⇄y
14.	31	f
15.	22	ln
16.	31	f
17.	11	Σ-
18.	-03	GTO 03
19.	31	f
20.	21	L. R.
21.	32	g
22.	22	e^x
23.	33	STO
24.	00	0

DISPLAY		KEY ENTRY
LINE	CODE	
25.	84	R/S
26.	22	x⇄y
27.	33	STO
28.	01	1
29.	84	R/S
30.	32	g
31.	42	x^2
32.	41	↑
33.	41	↑
34.	32	g
35.	33	s
36.	22	x⇄y
37.	81	÷
38.	32	g
39.	42	x^2
40.	71	x
41.	84	R/S
42.	34	RCL
43.	01	1
44.	12	y^x
45.	34	RCL
46.	00	0
47.	71	x
48.	-41	GTO 41
49.		

REGISTERS	
R_0	a
R_1	b
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{\bullet 0}$	n
$R_{\bullet 1}$	$\Sigma \ln x_i$
$R_{\bullet 2}$	$\Sigma (\ln x_i)^2$
$R_{\bullet 3}$	$\Sigma \ln y_i$
$R_{\bullet 4}$	$\Sigma (\ln y_i)^2$
$R_{\bullet 5}$	$\Sigma \ln x_i \ln y_i$
$R_{\bullet 6}$	0
$R_{\bullet 7}$	0
$R_{\bullet 8}$	0
$R_{\bullet 9}$	0

Example:

x_i	10	12	15	17	20	22	25	27	30	32	35
y_i	0.95	1.05	1.25	1.41	1.73	2.00	2.53	2.98	3.85	4.59	6.02

1. a = .03, b = 1.46
   $y = .03x^{1.46}$
2. $r^2 = .94$
3. For x = 18, $\hat{y} = 1.76$
   x = 23, $\hat{y} = 2.52$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for i = 1, 2,..., n	x_i	↑				
		y_i	R/S				
3′	Delete erroneous data x_k, y_k	x_k	↑				
		y_k	GTO	1	1	R/S	
4	Compute a, b, and r^2		GTO	1	9	R/S	a
			R/S				b
			R/S				r^2
5	Compute estimated value $\hat{y}$	x	R/S				$\hat{y}$
6	For a new x, go to 5						
7	For a new case, go to 2						

ANALYSIS OF VARIANCE

The one-way analysis of variance tests the differences between the population means of k treatment groups. Group i (i = 1, 2, ..., k) has n_i observations (treatment group may have equal or unequal number of observations).

$$\text{Sum}_i = \text{sum of observations in treatment group i}$$

$$= \sum_{j=1}^{n_i} x_{ij}$$

$$\text{Total SS} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}^2 - \frac{\left(\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^{k} n_i}$$

$$\text{Treat SS} = \sum_{i=1}^{k} \frac{\left(\sum_{j=1}^{n_i} x_{ij} \right)^2}{n_i} - \frac{\left(\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^{k} n_i}$$

$$\text{Error SS} = \text{Total SS} - \text{Treat SS}$$

$$df_1 = \text{Treat df} = k - 1$$

$$df_2 = \text{Error df} = \sum_{i=1}^{k} n_i - k$$

$$\text{Treat MS} = \frac{\text{Treat SS}}{\text{Treat df}}$$

$$\text{Error MS} = \frac{\text{Error SS}}{\text{Error df}}$$

$$F = \frac{\text{Treat MS}}{\text{Error MS}} \quad \left(\text{with } k - 1 \text{ and } \sum_{i=1}^{k} n_i - k \text{ degrees of freedom} \right)$$

Reference:

J. E. Freund, *Mathematical Statistics*, Prentice-Hall, 1962.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	01	1
01.	33	STO	26.	81	÷
02.	61	+	27.	33	STO
03.	00	0	28.	61	+
04.	32	g	29.	02	2
05.	42	x^2	30.	34	RCL
06.	33	STO	31.	00	0
07.	61	+	32.	33	STO
08.	05	5	33.	07	7
09.	01	1	34.	33	STO
10.	34	RCL	35.	61	+
11.	01	1	36.	06	6
12.	61	+	37.	34	RCL
13.	33	STO	38.	01	1
14.	01	1	39.	33	STO
15.	-00	GTO 00	40.	61	+
16.	01	1	41.	04	4
17.	33	STO	42.	00	0
18.	61	+	43.	33	STO
19.	03	3	44.	00	0
20.	34	RCL	45.	33	STO
21.	00	0	46.	01	1
22.	32	g	47.	34	RCL
23.	42	x^2	48.	07	7
24.	34	RCL	49.	-00	GTO 00

REGISTERS
R_0 Used
R_1 Used
R_2 Used
R_3 Used
R_4 Σn_i
R_5 $\Sigma\Sigma x_{ij}^2$
R_6 $\Sigma\Sigma x_{ij}$
R_7 Sum_i
R_8 0
R_9 0
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

	i \ j	1	2	3	4	5	6
	1	10	8	5	12	14	11
Treatment	2	6	9	8	13		
	3	14	13	10	17	16	

$\text{Sum}_1 = 60.00$
$\text{Sum}_2 = 36.00$
$\text{Sum}_3 = 70.00$
Total SS = 172.93
Treat SS = 66.93
Error SS = 106.00
F = 3.79

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		f	CLR	BST		0.00
3	Perform 3–5 for i = 1, 2,..., k						
4	Perform 4 for j = 1, 2,..., n_i	x_{ij}	R/S				j
5			GTO	1	6	R/S	Sum_i
6	Compute the F statistic		RCL	5	RCL	6	
			g	x^2	RCL	4	
			÷	–			Total SS
			RCL	2	RCL	6	
			g	x^2	RCL	4	
			÷	–			Treat SS
			–				Error SS
			f	LAST x	RCL	3	
			1	–	÷	x⇄y	
			RCL	4	RCL	3	
			–	÷	÷		F
7	For a new case, go to 2						

PAIRED t STATISTIC

Given a set of paired observations from two normal populations with means μ_1, μ_2 (unknown)

x_i	x_1	x_2	$\cdots$	x_n
y_i	y_1	y_2	$\cdots$	y_n

let

$$D_i = x_i - y_i$$

$$\overline{D} = \frac{1}{n} \sum_{i=1}^{n} D_i$$

$$s_D = \sqrt{\frac{\Sigma D_i^2 - \frac{1}{n}(\Sigma D_i)^2}{n-1}}$$

$$s_{\overline{D}} = \frac{s_D}{\sqrt{n}}$$

The test statistic

$$t = \frac{\overline{D}}{s_{\overline{D}}},$$

which has $n - 1$ degrees of freedom (df), can be used to test the null hypothesis

$$H_0: \mu_1 = \mu_2.$$

Reference:

B. Ostle, *Statistics in Research,* Iowa State University Press, 1963.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	32	g
02.	44	CL•R
03.	84	R/S
04.	51	–
05.	11	Σ+
06.	–03	GTO 03
07.	51	–
08.	31	f
09.	11	Σ–
10.	–03	GTO 03
11.	32	g
12.	33	s
13.	34	RCL
14.	83	•
15.	00	0
16.	31	f
17.	42	$\sqrt{x}$
18.	81	÷
19.	33	STO
20.	00	0
21.	31	f
22.	33	$\bar{x}$
23.	34	RCL
24.	00	0

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	81	÷
26.	84	R/S
27.	34	RCL
28.	83	•
29.	00	0
30.	01	1
31.	51	–
32.	–00	GTO 00
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	$s_{\bar{D}}$
R_1	
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	n
$R_{.1}$	ΣD_i
$R_{.2}$	ΣD_i^2
$R_{.3}$	Used
$R_{.4}$	Used
$R_{.5}$	Used
$R_{.6}$	0
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

x_i	14	17.5	17	17.5	15.4
y_i	17	20.7	21.6	20.9	17.2

$t = -7.16$
$df = 4.00$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for i = 1, 2,..., n	x_i	↑				
		y_i	R/S				i
3′	Delete erroneous data x_k, y_k	x_k	↑				
		y_k	GTO	0	7	R/S	
4	Compute t and df		GTO	1	1	R/S	t
			R/S				df
5	For a new case, go to 2						

t STATISTIC FOR TWO MEANS

Suppose $\{x_1, x_2, \ldots, x_{n_1}\}$ and $\{y_1, y_2, \ldots, y_{n_2}\}$ are independent random samples from two normal populations having means μ_1, μ_2 (unknown) and the same unknown variance σ^2.

We want to test the null hypothesis

$$H_0 : \mu_1 - \mu_2 = D$$

where D is a given number.

Define

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - D}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{\Sigma x_i^2 - n_1 \bar{x}^2 + \Sigma y_i^2 - n_2 \bar{y}^2}{n_1 + n_2 - 2}}} .$$

We can use this t statistic, which has the t distribution with $n_1 + n_2 - 2$ degrees of freedom, to test the null hypothesis H_0.

Reference:

K. A. Brownlee, *Statistical Theory and Methodology in Science and Engineering,* John Wiley and Sons, 1965.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	22	$x \rightleftarrows y$
02.	51	−
03.	34	RCL
04.	00	0
05.	13	$^1/x$
06.	34	RCL
07.	83	•
08.	00	0
09.	13	$^1/x$
10.	61	+
11.	31	f
12.	42	$\sqrt{x}$
13.	81	÷
14.	34	RCL
15.	02	2
16.	34	RCL
17.	03	3
18.	32	g
19.	42	x^2
20.	34	RCL
21.	00	0
22.	71	x
23.	51	−
24.	34	RCL

DISPLAY		KEY ENTRY
LINE	CODE	
25.	83	•
26.	02	2
27.	61	+
28.	34	RCL
29.	04	4
30.	32	g
31.	42	x^2
32.	34	RCL
33.	83	•
34.	00	0
35.	71	x
36.	51	−
37.	34	RCL
38.	00	0
39.	34	RCL
40.	83	•
41.	00	0
42.	61	+
43.	02	2
44.	51	−
45.	81	÷
46.	31	f
47.	42	$\sqrt{x}$
48.	81	÷
49.	-00	GTO 00

REGISTERS	
R_0	n_1
R_1	Σx_i
R_2	Σx_i^2
R_3	$\bar{x}$
R_4	$\bar{y}$
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{\bullet 0}$	n_2
$R_{\bullet 1}$	Σy_i
$R_{\bullet 2}$	Σy_i^2
$R_{\bullet 3}$	Used
$R_{\bullet 4}$	Used
$R_{\bullet 5}$	Used
$R_{\bullet 6}$	0
$R_{\bullet 7}$	0
$R_{\bullet 8}$	0
$R_{\bullet 9}$	0

Example:

x: 79, 84, 108, 114, 120, 103, 122, 120

y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54

$n_1 = 8$

$n_2 = 10$

If $D = 0$ (i.e., $H_0 : \mu_1 = \mu_2$)

then $\bar{x} = 106.25$

$\bar{y} = 92.5$

$t = 1.73$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL·R			0.00
3	Perform 3 for $i = 1, 2, \ldots, n_1$	x_i	Σ+				i
3′	Delete erroneous data x_k	x_k	f	Σ−			
4	Store sums and compute $\bar{x}$		RCL	·	0	STO	
			0	RCL	·	1	
			STO	1	RCL	·	
			2	STO	2	f	
			$\bar{x}$	STO	3		$\bar{x}$
5	Initialize for y's		g	CL·R			0.00
6	Perform 6 for $j = 1, 2, \ldots, n_2$	y_j	Σ+				j
6′	Delete erroneous data y_h	y_h	f	Σ−			
7	Compute $\bar{y}$		f	$\bar{x}$	STO	4	$\bar{y}$
8	Input D and compute t	D	RCL	4	+	RCL	
			3	BST	R/S		t
9	For a different D, go to 8						
10	For a new case, go to 2						

ONE SAMPLE TEST STATISTICS FOR THE MEAN

For a normal population (x_1, x_2 ..., x_n) with a known variance σ^2, a test of the null hypothesis

$$H_0: \text{mean } \mu = \mu_0$$

is based on the z statistic (which has a standard normal distribution)

$$z = \frac{\sqrt{n}\ (\bar{x} - \mu_0)}{\sigma}.$$

If the variance σ^2 is unknown, then

$$t = \frac{\sqrt{n}\ (\bar{x} - \mu_0)}{s}$$

is used instead. This t statistic has the t distribution with n – 1 degrees of freedom. $\bar{x}$ and s are the sample mean and standard deviation.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	00	0
03.	84	R/S
04.	31	f
05.	33	$\bar{x}$
06.	34	RCL
07.	00	0
08.	51	–
09.	34	RCL
10.	83	•
11.	00	0
12.	31	f
13.	42	$\sqrt{x}$
14.	71	x
15.	33	STO
16.	01	1
17.	32	g
18.	33	s
19.	34	RCL
20.	01	1
21.	22	$x \rightleftarrows y$
22.	81	÷
23.	84	R/S
24.	34	RCL

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	01	1
26.	22	$x \rightleftarrows y$
27.	81	÷
28.	–00	GTO 00
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	μ_0
R_1	$\sqrt{n}\ (\bar{x} - \mu_0)$
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	n
$R_{.1}$	Σx_i
$R_{.2}$	Σx_i^2
$R_{.3}$	Used
$R_{.4}$	Used
$R_{.5}$	Used
$R_{.6}$	0
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

Suppose $\mu_0 = 2$, for the following set of data

{2.73, 0.45, 2.52, 1.19, 3.51, 2.75, 1.79, 1.83, 1 , 0.87, 1.9 , 1.62, 1.74, 1.92, 1.24, 2.68,}

test statistic t = –.69
and z = –.57 if $\sigma = 1$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL·R			0.00
3	Perform 3 for i = 1, 2,..., n	x_i	$\Sigma+$				i
4	Input μ_0	μ_0	BST	R/S			μ_0
5	Compute t		R/S				t
6	Input σ (if known)	σ	R/S				z
7	For a new case, go to 2						

TEST STATISTICS FOR CORRELATION COEFFICIENT

Under the assumptions of normal correlation analysis, the following t statistic can be used to test the null hypothesis $\rho = 0$,

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

where r is an estimate (based on a sample of size n) of the true correlation coefficient ρ. This t statistic has the t distribution with $n - 2$ degrees of freedom.

To test the null hypothesis $\rho = \rho_0$, the z statistic is used.

$$z = \frac{\sqrt{n-3}}{2} \ln \frac{(1+r)(1-\rho_0)}{(1-r)(1+\rho_0)}$$

where z has approximately the standard normal distribution.

References:

1. Hogg and Craig, *Introduction to Mathematical Statistics,* Macmillan Co., 1970.

2. J. Freund, *Mathematical Statistics,* Prentice-Hall, 1971.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	34	RCL
02.	01	1
03.	02	2
04.	51	–
05.	31	f
06.	42	$\sqrt{x}$
07.	34	RCL
08.	00	0
09.	71	x
10.	01	1
11.	34	RCL
12.	00	0
13.	41	↑
14.	71	x
15.	51	–
16.	31	f
17.	42	$\sqrt{x}$
18.	81	÷
19.	84	R/S
20.	34	RCL
21.	00	0
22.	01	1
23.	61	+
24.	01	1
25.	34	RCL
26.	00	0
27.	51	–
28.	81	÷
29.	01	1
30.	34	RCL
31.	02	2
32.	51	–
33.	71	x
34.	01	1
35.	34	RCL
36.	02	2
37.	61	+
38.	81	÷
39.	31	f
40.	22	ln
41.	34	RCL
42.	01	1
43.	03	3
44.	51	–
45.	31	f
46.	42	$\sqrt{x}$
47.	71	x
48.	02	2
49.	81	÷

REGISTERS	
R_0	r
R_1	n
R_2	ρ_0
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Suppose r = 0.12 and n = 31, then
t = .65 and
z = .64 (for $\rho_0 = 0$).

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input r and n	r	STO	0			
		n	STO	1			
3	(If z is desired) input ρ_0	ρ_0	STO	2			
4	Go to 6 if only z is needed						
5	Compute t		BST	R/S			t
6	Compute z		GTO	2	0	R/S	z
7	For a new case, go to 2						

CHI-SQUARE EVALUATION (EXPECTED VALUES EQUAL)

This program calculates the value of the χ^2 statistic for the goodness of fit test when the expected frequencies are equal.

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} = \frac{n\,\Sigma O_i^2}{\Sigma O_i} - \Sigma O_i$$

where O_i = observed frequency

E = expected frequency = $\dfrac{\Sigma O_i}{n}$.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.		
01.	34	RCL	26.		
02.	83	•	27.		
03.	02	2	28.		
04.	34	RCL	29.		
05.	83	•	30.		
06.	00	0	31.		
07.	71	x	32.		
08.	34	RCL	33.		
09.	83	•	34.		
10.	01	1	35.		
11.	81	÷	36.		
12.	31	f	37.		
13.	34	LAST X	38.		
14.	51	–	39.		
15.	-00	GTO 00	40.		
16.			41.		
17.			42.		
18.			43.		
19.			44.		
20.			45.		
21.			46.		
22.			47.		
23.			48.		
24.			49.		

REGISTERS
R_0
R_1
R_2
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$ n
$R_{.1}$ ΣO_i
$R_{.2}$ ΣO_i^2
$R_{.3}$ Used
$R_{.4}$ Used
$R_{.5}$ Used
$R_{.6}$ 0
$R_{.7}$ 0
$R_{.8}$ 0
$R_{.9}$ 0

Example:

The following table shows the observed frequencies in tossing a die 120 times. Assume that the expected frequencies are equal (E = 20), χ^2 can be used to test if the die is fair.

number	1	2	3	4	5	6
frequency O_i	25	17	15	23	24	16

$\chi^2 = 5.00$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R			0.00
3	Perform 3 for i = 1, 2,..., n	O_i	Σ+				i
3′	Delete erroneous data O_k	O_k	f	Σ–			
4	Compute χ^2		BST	R/S			χ^2
5	For a new case, go to 2						

CHI-SQUARE EVALUATION (EXPECTED VALUES UNEQUAL)

This program calculates the value of the χ^2 statistic for the goodness of fit test by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequency

E_i = expected frequency.

The χ^2 statistic measures the closeness of the agreement between the observed frequencies and expected frequencies.

Note:

In order to apply the goodness of fit test to a set of given data, combining some classes may be necessary to make sure that each expected frequency is not too small (say, not less than 5).

Reference:

J. E. Freund, *Mathematical Statistics,* Prentice-Hall, 1962.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	51	−
01.	00	0	26.	31	f
02.	33	STO	27.	34	LAST X
03.	00	0	28.	22	x⇄y
04.	33	STO	29.	32	g
05.	01	1	30.	42	x^2
06.	84	R/S	31.	22	x⇄y
07.	51	−	32.	81	÷
08.	31	f	33.	33	STO
09.	34	LAST X	34.	51	−
10.	22	x⇄y	35.	01	1
11.	32	g	36.	34	RCL
12.	42	x^2	37.	00	0
13.	22	x⇄y	38.	01	1
14.	81	÷	39.	51	−
15.	33	STO	40.	33	STO
16.	61	+	41.	00	0
17.	01	1	42.	−06	GTO 06
18.	34	RCL	43.		
19.	00	0	44.		
20.	01	1	45.		
21.	61	+	46.		
22.	33	STO	47.		
23.	00	0	48.		
24.	−06	GTO 06	49.		

REGISTERS	
R_0	n
R_1	$\Sigma(O_i - E_i)^2/E_i$
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

1.

O_i	8	50	47	56	5	14
E_i	9.6	46.75	51.85	54.4	8.25	9.15

$\chi^2 = 4.84$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for i = 1, 2,..., n	O_i	↑				
		E_i	R/S				i
3′	Delete erroneous data O_k, E_k	O_k	↑				
		E_k	GTO	2	5	R/S	
4	Recall χ^2 from register R_1		RCL	1			χ^2
5	For a new case, go to 2						

2 x k CONTINGENCY TABLE

Contingency tables can be used to test the null hypothesis that two variables are independent.

	1	2	3	...	k	Totals
A	a_1	a_2	a_3	...	a_k	N_A
B	b_1	b_2	b_3	...	b_k	N_B
Totals	N_1	N_2	N_3	...	N_k	N

Test statistic χ^2 has k – 1 degrees of freedom.

$$\chi^2 = \frac{N}{N_A} \sum_{i=1}^{k} \frac{a_i^2}{N_i} + \frac{N}{N_B} \sum_{i=1}^{k} \frac{b_i^2}{N_i} - N$$

Pearson's coefficient of contingency C measures the degree of association between the two variables.

$$C = \sqrt{\frac{\chi^2}{N + \chi^2}}.$$

Reference:

B. Ostle, *Statistics in Research,* Iowa State University Press, 1963.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	-00	GTO 00
01.	33	STO	26.	34	RCL
02.	03	3	27.	83	•
03.	33	STO	28.	02	2
04.	61	+	29.	34	RCL
05.	01	1	30.	00	0
06.	22	x⇄y	31.	81	÷
07.	33	STO	32.	34	RCL
08.	02	2	33.	83	•
09.	33	STO	34.	04	4
10.	61	+	35.	34	RCL
11.	00	0	36.	01	1
12.	61	+	37.	81	÷
13.	31	f	38.	61	+
14.	42	$\sqrt{x}$	39.	01	1
15.	34	RCL	40.	51	–
16.	03	3	41.	34	RCL
17.	22	x⇄y	42.	00	0
18.	81	÷	43.	34	RCL
19.	34	RCL	44.	01	1
20.	02	2	45.	61	+
21.	31	f	46.	71	x
22.	34	LAST X	47.	-00	GTO 00
23.	81	÷	48.		
24.	11	Σ+	49.		

REGISTERS	
R_0	N_A
R_1	N_B
R_2	a_i
R_3	b_i
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	k
$R_{.1}$	$\Sigma a_i/\sqrt{N_i}$
$R_{.2}$	$\Sigma a_i^2/N_i$
$R_{.3}$	$\Sigma b_i/\sqrt{N_i}$
$R_{.4}$	$\Sigma b_i^2/N_i$
$R_{.5}$	$\Sigma a_i b_i/N_i$
$R_{.6}$	0
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

	1	2	3
A	2	5	4
B	3	8	7

$\chi^2 = .02 \qquad C = .03$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R	STO	0	
			STO	1	BST		0.00
3	Perform 3 for i = 1, 2,..., k	a_i	↑				
		b_i	R/S				i
4	Compute χ^2		GTO	2	6	R/S	χ^2
5	Compute C		↑	↑	RCL	0	
			RCL	1	+	+	
			÷	f	$\sqrt{x}$		C
6	For a new case, go to 2						

2 x 2 CONTINGENCY TABLE WITH YATES CORRECTION

This program calculates χ^2 for a 2 x 2 contingency table containing observed frequencies. Yates correction for continuity is used.

	1	2
Group A	a	b
Group B	c	d

$$\chi^2 = \frac{(a+b+c+d)\,[|ad-bc| - \tfrac{1}{2}(a+b+c+d)]^2}{(a+b)(a+c)(c+d)(b+d)}$$

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	61	+
02.	33	STO
03.	05	5
04.	61	+
05.	61	+
06.	33	STO
07.	04	4
08.	22	$x \rightleftarrows y$
09.	34	RCL
10.	03	3
11.	71	x
12.	34	RCL
13.	01	1
14.	34	RCL
15.	02	2
16.	71	x
17.	51	–
18.	32	g
19.	42	x^2
20.	31	f
21.	42	$\sqrt{x}$
22.	22	$x \rightleftarrows y$
23.	02	2
24.	81	÷

DISPLAY		KEY ENTRY
LINE	CODE	
25.	51	–
26.	32	g
27.	42	x^2
28.	34	RCL
29.	00	0
30.	34	RCL
31.	01	1
32.	61	+
33.	81	÷
34.	34	RCL
35.	00	0
36.	34	RCL
37.	02	2
38.	61	+
39.	81	÷
40.	34	RCL
41.	05	5
42.	81	÷
43.	34	RCL
44.	01	1
45.	34	RCL
46.	03	3
47.	61	+
48.	81	÷
49.	71	x

REGISTERS
R_0 a
R_1 b
R_2 c
R_3 d
R_4 a + b + c + d
R_5 c + d
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

	1	2
A	9	21
B	17	13

$\chi^2 = 3.33$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store data	a	STO	0			
		b	STO	1			
		c	STO	2			
		d	STO	3			
3	Compute χ^2		BST	R/S			χ^2
4	For a new case, go to 2						

BARTLETT'S CHI -SQUARE STATISTIC

$$\chi^2 = \frac{f \ln s^2 - \sum_{i=1}^{k} f_i \ln s_i^2}{1 + \frac{1}{3(k-1)}\left[\left(\sum_{i=1}^{k} \frac{1}{f_i}\right) - \frac{1}{f}\right]}$$

where s_i^2 = sample variance of the i^{th} sample
f_i = degrees of freedom associated with s_i^2
$i = 1, 2, \ldots, k$
k = number of samples

$$s^2 = \frac{\sum_{i=1}^{k} f_i s_i^2}{f}$$

$$f = \sum_{i=1}^{k} f_i .$$

This χ^2 has a chi-square distribution (approximately) with $k - 1$ degrees of freedom, which can be used to test the null hypothesis that $s_1^2, s_2^2, \ldots, s_k^2$ are all estimates of the same population variance σ^2(H_0: Each of $s_1^2, s_2^2, \ldots, s_k^2$ is an estimate of σ^2).

Reference:

A. Hald, *Statistical Theory with Engineering Applications,* John Wiley and Sons, 1960.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	22	ln
01.	33	STO	26.	34	RCL
02.	61	+	27.	03	3
03.	03	3	28.	71	x
04.	13	1/x	29.	34	RCL
05.	33	STO	30.	83	•
06.	61	+	31.	01	1
07.	02	2	32.	51	–
08.	81	÷	33.	34	RCL
09.	34	RCL	34.	02	2
10.	00	0	35.	34	RCL
11.	31	f	36.	03	3
12.	22	ln	37.	13	1/x
13.	34	RCL	38.	51	–
14.	01	1	39.	34	RCL
15.	71	x	40.	83	•
16.	11	Σ+	41.	00	0
17.	-00	GTO 00	42.	01	1
18.	34	RCL	43.	51	–
19.	83	•	44.	03	3
20.	03	3	45.	71	x
21.	34	RCL	46.	81	÷
22.	03	3	47.	01	1
23.	81	÷	48.	61	+
24.	31	f	49.	81	÷

REGISTERS	
R_0	s_i^2
R_1	f_i
R_2	$\Sigma\, 1/f_i$
R_3	Σf_i
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	k
$R_{.1}$	$\Sigma f_i \ln s_i^2$
$R_{.2}$	$\Sigma (f_i \ln s_i^2)^2$
$R_{.3}$	$\Sigma f_i s_i^2$
$R_{.4}$	$\Sigma (f_i s_i^2)^2$
$R_{.5}$	$\Sigma f_i^2 s_i^2 \ln s_i^2$
$R_{.6}$	0
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

i	1	2	3	4	5	6
s_i^2	5.5	5.1	5.2	4.7	4.8	4.3
f_i	10	20	17	18	8	15

$\chi^2 = .25$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R	STO	2	
			STO	3	BST		0.00
3	Perform 3 for i = 1, 2,..., k	s_i^2	STO	0			
		f_i	STO	1	R/S		i
4	Compute χ^2		GTO	1	8	R/S	χ^2
5	For a new case, go to 2						

BEHRENS-FISHER STATISTIC

Suppose $\{x_1, x_2, \ldots, x_{n_1}\}$ and $\{y_1, y_2, \ldots, y_{n_2}\}$ are independent random samples from two normal populations with means μ_1, μ_2 (unknown). If the variances σ_1^2, σ_2^2 can not be assumed equal, then the Behrens-Fisher statistic

$$d = \frac{\bar{x} - \bar{y} - D}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

is used instead of the t statistic to test the null hypothesis

$$H_0 : \mu_1 - \mu_2 = D$$

Critical values of this test are tabulated in the Fisher-Yates Tables for various values of n_1, n_2, α and θ, where α is the level of significance and

$$\theta = \tan^{-1}\left(\frac{s_1}{s_2}\sqrt{\frac{n_2}{n_1}}\right).$$

Notation:

$$\bar{x} = \frac{\Sigma x_i}{n_1} \qquad s_1^2 = \frac{\Sigma x_i^2 - [(\Sigma x_i)^2/n_1]}{n_1 - 1}$$

$$\bar{y} = \frac{\Sigma y_i}{n_2} \qquad s_2^2 = \frac{\Sigma y_i^2 - [(\Sigma y_i)^2/n_2]}{n_2 - 1}$$

Reference:

Fisher and Yates, *Statistical Tables for Biological, Agricultural and Medical Research,* Hafner Publishing Co., 1970.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	23	R↓
01.	41	↑	26.	32	g
02.	41	↑	27.	42	x^2
03.	31	f	28.	34	RCL
04.	33	$\bar{x}$	29.	01	1
05.	22	x⇄y	30.	32	g
06.	23	R↓	31.	42	x^2
07.	61	+	32.	61	+
08.	34	RCL	33.	31	f
09.	00	0	34.	42	$\sqrt{x}$
10.	22	x⇄y	35.	81	÷
11.	51	−	36.	84	R/S
12.	41	↑	37.	34	RCL
13.	41	↑	38.	01	1
14.	32	g	39.	34	RCL
15.	33	s	40.	02	2
16.	34	RCL	41.	81	÷
17.	83	•	42.	32	g
18.	00	0	43.	14	$\tan^{-1}$
19.	31	f	44.	-00	GTO 00
20.	42	$\sqrt{x}$	45.		
21.	81	÷	46.		
22.	33	STO	47.		
23.	02	2	48.		
24.	22	x⇄y	49.		

REGISTERS
R_0 $\bar{x}$
R_1 $s_1/\sqrt{n_1}$
R_2 $s_2/\sqrt{n_2}$
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{\bullet 0}$ Used
$R_{\bullet 1}$ Used
$R_{\bullet 2}$ Used
$R_{\bullet 3}$ Used
$R_{\bullet 4}$ Used
$R_{\bullet 5}$ Used
$R_{\bullet 6}$ 0
$R_{\bullet 7}$ 0
$R_{\bullet 8}$ 0
$R_{\bullet 9}$ 0

Example: x: 79, 84, 108, 114, 120, 103, 122, 120

y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54

$H_0: \mu_1 = \mu_2$ (D = 0), $n_1 = 8$, $n_2 = 10$, $\bar{x} = 106.25$

$s_1/\sqrt{n_1} = 5.88$, d = 1.73, $\theta = 47.88°$ (= .84 radians = 53.20 grads)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R			0.00
3	Perform 3 for i = 1, 2,..., n_1	x_i	Σ+				i
3′	Delete erroneous data x_k	x_k	f	Σ−			
4	Compute $\bar{x}$ and $s_1/\sqrt{n_1}$		f	$\bar{x}$	STO	0	$\bar{x}$
			g	s	RCL	•	
			0	f	$\sqrt{x}$	÷	$s_1/\sqrt{n_1}$
			STO	1	g	CL•R	0.00
5	Perform 5 for i = 1, 2,..., n_2	y_i	Σ+				i
5′	Delete erroneous data y_h	y_h	f	Σ−			
6	Input D and compute d, θ	D	BST	R/S			d
			R/S				θ
7	For a new case, go to 2						

BISERIAL CORRELATION COEFFICIENT

The biserial correlation coefficient r_b is used where one variable Y is quantitatively measured while the other continuous variable X is artificially dichotomized (that is, artificially defined by two groups). It measures the degree of linear association between X and Y.

$$r_b = \frac{n\,(\Sigma' y_i) - n_1\,\Sigma y_i}{na\,\sqrt{n\,\Sigma y_i^2 - (\Sigma y_i)^2}}$$

Suppose X takes the value 0 or 1.

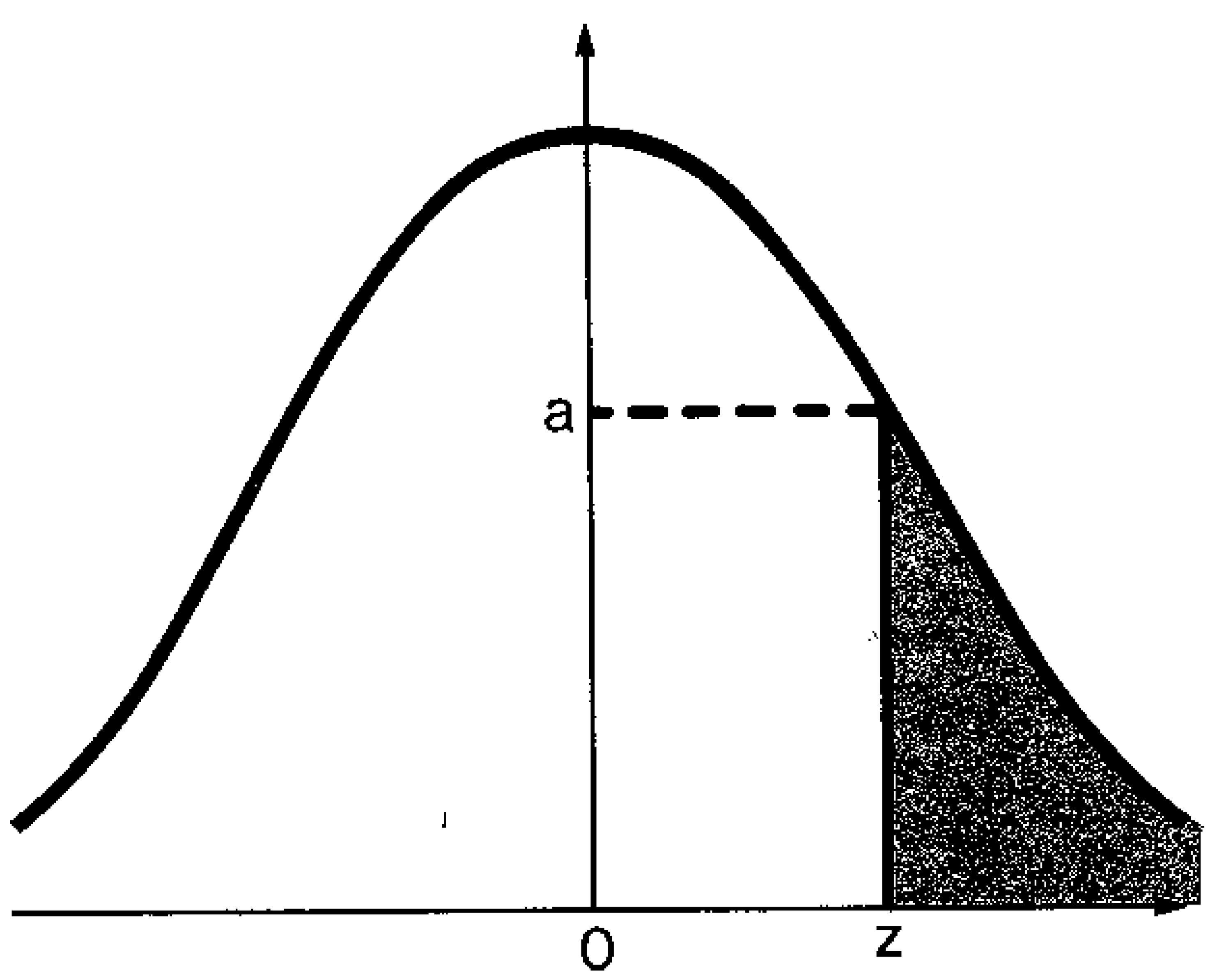
Define n_1 = number of x's such that x = 1

n = total number of data points

$\Sigma' y_i$ = sum of the y's for which x = 1

Σy_i = sum of all y's

a = ordinate of the standard normal curve at point z cutting off a tail of that distribution with area equal to $p = \dfrac{n_1}{n}$.



Note:

Among the necessary assumptions for a meaningful interpretation of r_b are:

1. Y is normally distributed
2. The true distribution of X should be of normal form.

Reference:

B. Ostle, *Statistics in Research,* Iowa State University Press, 1963.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	83	•
01.	34	RCL	26.	02	2
02.	83	•	27.	34	RCL
03.	03	3	28.	83	•
04.	34	RCL	29.	04	4
05.	83	•	30.	61	+
06.	01	1	31.	71	x
07.	61	+	32.	34	RCL
08.	33	STO	33.	02	2
09.	02	2	34.	32	g
10.	31	f	35.	42	x^2
11.	34	LAST X	36.	51	–
12.	34	RCL	37.	31	f
13.	83	•	38.	42	$\sqrt{x}$
14.	00	0	39.	34	RCL
15.	71	x	40.	00	0
16.	22	$x \rightleftarrows y$	41.	34	RCL
17.	34	RCL	42.	83	•
18.	01	1	43.	00	0
19.	71	x	44.	71	x
20.	51	–	45.	71	x
21.	34	RCL	46.	81	÷
22.	83	•	47.	-00	GTO 00
23.	00	0	48.		
24.	34	RCL	49.		

REGISTERS	
R_0	a
R_1	n_1
R_2	Σy_i
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	n
$R_{.1}$	$\Sigma' y_i$
$R_{.2}$	$\Sigma' y_i^2$
$R_{.3}$	$\Sigma y_i - \Sigma' y_i$
$R_{.4}$	$\Sigma y_i^2 - \Sigma' y_i^2$
$R_{.5}$	0
$R_{.6}$	0
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

x_i	0	1	1	0	1	0	0	0	1
y_i	3.1	2.8	5.6	0.3	2.5	2.4	4.8	2.9	7.7

$n_1 = 4$
$n = 9$
$a = 0.40$
$r_b = .59$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL·R	BST		0.00
3	Perform 3 for $x_i = 1$	0	↑				
		y_i	Σ+				
3′	Delete erroneous data y_k	0	↑				
	($x_k = 1$)	y_k	f	Σ−			
4	Perform 4 for $x_i = 0$	y_i	↑				
		0	Σ+				
4′	Delete erroneous data y_h	y_h	↑				
	($x_h = 0$)	0	f	Σ−			
5	Input a and n_1	a	STO	0			
		n_1	STO	1			
6	Compute r_b		R/S				r_b
7	For a new case, go to 2						

SPEARMAN'S RANK CORRELATION COEFFICIENT

Spearman's rank correlation coefficient is defined by

$$r_s = 1 - \frac{6 \sum_{i=1}^{n} D_i^2}{n(n^2 - 1)}$$

where n = number of paired observations (x_i, y_i)

$D_i = \text{rank}(x_i) - \text{rank}(y_i) = R_i - S_i$.

If the X and Y random variables from which these n pairs of observations are derived are independent, then r_s has zero mean and a variance

$$\frac{1}{n-1} \, .$$

A test for the null hypothesis

$$H_0 : X, Y \text{ are independent}$$

is using

$$z = r_s \sqrt{n-1}$$

which is approximately a standardized normal variable (for large n, say $n \geqslant 10$).

If the null hypothesis of independence is not rejected, we can infer that the population correlation coefficient $\rho(x, y) = 0$, but dependence between the variables does not necessarily imply that $\rho(x, y) \neq 0$.

Note:

$-1 \leqslant r_s \leqslant 1$

where $r_s = 1$ indicates complete agreement in order of the ranks and $r_s = -1$ indicates complete agreement in the opposite order of the ranks.

Reference:

J. D. Gibbons, *Nonparametric Statistical Inference,* McGraw Hill, 1971.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	51	–
02.	32	g
03.	42	x^2
04.	33	STO
05.	61	+
06.	01	1
07.	34	RCL
08.	00	0
09.	01	1
10.	61	+
11.	33	STO
12.	00	0
13.	-00	GTO 00
14.	51	–
15.	32	g
16.	42	x^2
17.	33	STO
18.	51	–
19.	01	1
20.	34	RCL
21.	00	0
22.	01	1
23.	51	–
24.	-11	GTO 11

DISPLAY		KEY ENTRY
LINE	CODE	
25.	01	1
26.	34	RCL
27.	01	1
28.	06	6
29.	71	x
30.	34	RCL
31.	00	0
32.	32	g
33.	42	x^2
34.	01	1
35.	51	–
36.	34	RCL
37.	00	0
38.	71	x
39.	81	÷
40.	51	–
41.	84	R/S
42.	34	RCL
43.	00	0
44.	01	1
45.	51	–
46.	31	f
47.	42	$\sqrt{x}$
48.	71	x
49.	-00	GTO 00

REGISTERS
R_0 n
R_1 ΣD_i^2
R_2
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

(Note: Only the ranks R_i's and S_i's are used as the input data.)

Student	x_i Math Grade	y_i Stat Grade	R_i Rank of x_i	S_i Rank of y_i
1	82	81	6	7
2	67	75	14	11
3	91	85	3	4
4	98	90	1	2
5	74	80	11	8
6	52	60	15	15
7	86	94	4	1
8	95	78	2	9
9	79	83	9	6
10	78	76	10	10
11	84	84	5	5
12	80	69	8	13
13	69	72	13	12
14	81	88	7	3
15	73	61	12	14

$r_s = .76$
$z = 2.85$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize	0	STO	0	STO	1	
			BST				0.00
3	Perform 3 for i = 1, 2,..., n	R_i	↑				
		S_i	R/S				i
3′	Delete erroneous data R_k, S_k	R_k	↑				
		S_k	GTO	1	4	R/S	
4	Compute r_s and z		GTO	2	5	R/S	r_s
			R/S				z

DIFFERENCES AMONG PROPORTIONS

Suppose x_1, x_2, ..., x_k are observed values of a set of independent random variables having binomial distributions with parameters n_i and θ_i ($i = 1, 2, ..., k$).

A chi-square statistic given by

$$\chi^2 = \sum_{i=1}^{k} \frac{(x_i - n_i \hat{\theta})^2}{n_i \hat{\theta} (1 - \hat{\theta})}$$

can be used to test the null hypothesis $\theta_1 = \theta_2 = ... = \theta_k$, where

$$\hat{\theta} = \sum_{i=1}^{k} x_i \Big/ \sum_{i=1}^{k} n_i \,.$$

This χ^2 has the chi-square distribution with $k - 1$ degrees of freedom.

Reference:

J. Freund, *Mathematical Statistics,* Prentice-Hall, 1971.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	51	–
02.	33	STO
03.	03	3
04.	33	STO
05.	61	+
06.	01	1
07.	31	f
08.	34	LAST X
09.	33	STO
10.	02	2
11.	33	STO
12.	61	+
13.	00	0
14.	61	+
15.	31	f
16.	42	$\sqrt{x}$
17.	34	RCL
18.	03	3
19.	22	x⇄y
20.	81	÷
21.	34	RCL
22.	02	2
23.	31	f
24.	34	LAST X

DISPLAY		KEY ENTRY
LINE	CODE	
25.	81	÷
26.	11	Σ+
27.	–00	GTO 00
28.	34	RCL
29.	83	•
30.	02	2
31.	34	RCL
32.	00	0
33.	81	÷
34.	34	RCL
35.	83	•
36.	04	4
37.	34	RCL
38.	01	1
39.	81	÷
40.	61	+
41.	01	1
42.	51	–
43.	34	RCL
44.	00	0
45.	34	RCL
46.	01	1
47.	61	+
48.	71	x
49.	–00	GTO 00

REGISTERS
R_0 Σx_i
R_1 $\Sigma(n_i - x_i)$
R_2 x_i
R_3 $n_i - x_i$
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$ k
$R_{.1}$ Used
$R_{.2}$ Used
$R_{.3}$ Used
$R_{.4}$ Used
$R_{.5}$ Used
$R_{.6}$ 0
$R_{.7}$ 0
$R_{.8}$ 0
$R_{.9}$ 0

Example:

	n_i	x_i
Sample 1	400	232
Sample 2	500	260
Sample 3	400	197

$\chi^2 = 6.47$

$\hat{\theta} = .53$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R	STO	0	
			STO	1	BST		0.00
3	Perform 3 for i = 1, 2,..., k	n_i	↑				
		x_i	R/S				i
4	Compute χ^2		GTO	2	8	R/S	χ^2
5	(optional) Compute $\hat{\theta}$		RCL	0	↑	↑	
			RCL	1	+	÷	$\hat{\theta}$
6	For a new case, go to 2						

KENDALL'S COEFFICIENT OF CONCORDANCE

Suppose n individuals are ranked from 1 to n according to some specified characteristic by k observers; the coefficient of concordance W measures the agreement between observers (or concordance between rankings).

$$W = \frac{12 \sum_{i=1}^{n} \left(\sum_{j=1}^{k} R_{ij} \right)^2}{k^2\, n(n^2 - 1)} - \frac{3(n+1)}{n-1}$$

where R_{ij} is the rank assigned to the i^{th} individual by the j^{th} observer.

W varies from 0 (no community of preference) to 1 (perfect agreement). The null hypothesis that the observers have no community of preference may be tested using special tables or, if $n > 7$, by computing

$$\chi^2 = k\,(n-1)\,W$$

which has approximately the chi-square distribution with $n - 1$ degrees of freedom.

Reference:

J. D. Gibbons, *Nonparametric Statistical Inference,* McGraw-Hill, 1971.

Table for small samples:

M. G. Kendall, *Rank Correlation Methods,* Hafner Publishing Co., 1962.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	61	+
03.	02	2
04.	34	RCL
05.	01	1
06.	01	1
07.	61	+
08.	33	STO
09.	01	1
10.	-00	GTO 00
11.	34	RCL
12.	01	1
13.	33	STO
14.	00	0
15.	34	RCL
16.	02	2
17.	32	g
18.	42	x^2
19.	33	STO
20.	61	+
21.	03	3
22.	34	RCL
23.	04	4
24.	01	1
25.	61	+
26.	33	STO
27.	04	4
28.	00	0
29.	33	STO
30.	01	1
31.	33	STO
32.	02	2
33.	34	RCL
34.	04	4
35.	-00	GTO 00
36.	01	1
37.	61	+
38.	81	÷
39.	31	f
40.	34	LAST X
41.	51	−
42.	34	RCL
43.	04	4
44.	01	1
45.	51	−
46.	81	÷
47.	03	3
48.	71	x
49.	-00	GTO 00

REGISTERS	
R_0	k
R_1	j
R_2	ΣR_{ij}
R_3	$\Sigma(\Sigma R_{ij})^2$
R_4	n
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Table for R_{ij} (n = 10, k = 3)

i \ j	1	2	3
1	6	7	3
2	1	4	2
3	9	3	5
4	2	6	1
5	10	8	9
6	3	2	6
7	5	9	8
8	4	1	4
9	8	10	10
10	7	5	7

W = .69
χ^2 = 18.64

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize	0	STO	1	STO	2	
			STO	3	STO	4	
			BST				0.00
3	Perform 3-5 for i = 1, 2,..., n						
4	Perform 4 for j = 1, 2,..., k	R_{ij}	R/S				j
5			GTO	1	1	R/S	i
6	Compute W		RCL	3	4	x	
			RCL	0	g	x^2	
			÷	RCL	4	÷	
			RCL	4	GTO	3	
			6	R/S			W
7	Compute χ^2		RCL	0	x	RCL	
			4	1	–	x	χ^2
8	For a new case, go to 2						

KRUSKAL-WALLIS STATISTIC

Suppose we want to test the null hypothesis that k independent random samples of sizes n_1, n_2, ..., and n_k come from identical continuous populations.

Arrange all values from k samples jointly (as if they were one sample) in an increasing order of magnitude. Let R_{ij} ($i = 1, 2, ..., k, j = 1, 2, ..., n_i$) be the rank of the j^{th} value in the i^{th} sample.

The Kruskal-Wallis statistic H can be used to test the null hypothesis.

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{\left(\sum_{j=1}^{n_i} R_{ij}\right)^2}{n_i} - 3(N+1)$$

where $N = \sum_{i=1}^{k} n_i$.

When all sample sizes are large (> 5), H is distributed approximately as chi-square with k − 1 degrees of freedom. For small samples, the test is based on special tables.

Table for small samples (k = 3):

Alexander and Quade, *On the Kruskal-Wallis Three sample H-statistic,* University of North Carolina, Department of Biostatistics, Inst. Statistics Mimeo Ser. 602, 1968.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	61	+
03.	02	2
04.	34	RCL
05.	01	1
06.	01	1
07.	61	+
08.	33	STO
09.	01	1
10.	-00	GTO 00
11.	34	RCL
12.	01	1
13.	33	STO
14.	61	+
15.	00	0
16.	34	RCL
17.	02	2
18.	32	g
19.	42	x^2
20.	22	x⇄y
21.	81	÷
22.	33	STO
23.	61	+
24.	03	3

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	34	RCL
26.	04	4
27.	01	1
28.	61	+
29.	33	STO
30.	04	4
31.	00	0
32.	33	STO
33.	01	1
34.	33	STO
35.	02	2
36.	34	RCL
37.	04	4
38.	-00	GTO 00
39.	81	÷
40.	34	RCL
41.	00	0
42.	01	1
43.	61	+
44.	81	÷
45.	31	f
46.	34	LAST X
47.	51	−
48.	03	3
49.	71	x

REGISTERS	
R_0	N
R_1	n_i
R_2	ΣR_{ij}
R_3	$\Sigma[(\Sigma R_{ij})^2/n_i]$
R_4	k
R_5	0
R_6	0
R_7	0
R_8	0
R_9	0
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

(Note: Only the ranks R_{ij}'s are used as the input data.)

Sample 1	2.73	0.45	2.52	1.19	3.51	2.75
Ranks R_{1j}	29	5	26	10	33	30

Sample 2	1.79	1.83	1	0.87	1.9	1.62	1.74	1.92
Ranks R_{2j}	11	12	9	7	20	18	19	21

Sample 3	1.24	2.68	0.88	2.5	1.61	1.55	3.03	0.38	0.22
Ranks R_{3j}	14	28	8	25	17	15	32	4	2

Sample 4	0.57	2.54	0.36	1.56	2.39	1.23	−0.1	2.98	2.15	2.25
Ranks R_{4j}	6	27	3	16	24	13	1	31	22	23

N = 33.00
H = 2.29

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		f	CLR	BST		0.00
3	Perform 3-5 for i = 1, 2,..., k						
4	Perform 4 for j = 1, 2,..., n_i	R_{ij}	R/S				j
5			GTO	1	1	R/S	i
6	Compute H		RCL	3	4	x	
			RCL	0			N
			GTO	3	9	R/S	H
7	For a new case, go to 2						

MANN-WHITNEY STATISTIC

This program computes the Mann-Whitney test statistic on two independent samples of equal or unequal sizes. This test is designed for testing the null hypothesis of no difference between two populations.

Mann-Whitney test statistic is defined as

$$U = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - \sum_{i=1}^{n_1} R_i$$

where n_1 and n_2 are the sizes of the two samples. Arrange all values from both samples jointly (as if they were one sample) in an increasing order of magnitude; let R_i ($i = 1, 2, ..., n_1$) be the ranks assigned to the values of the first sample (it is immaterial which sample is referred to as the "first").

When n_1 and n_2 are small, the Mann-Whitney test bases on the exact distribution of U and specially constructed tables. When n_1 and n_2 are both large (say, greater than 8) then

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{n_1 n_2 (n_1 + n_2 + 1)/12}}$$

is approximately a random variable having the standard normal distribution.

Reference:

J. E. Freund, *Mathematical Statistics,* Prentice-Hall, 1962.

Table for small samples:

D. B. Owen, *Handbook of Statistical Tables,* Addison-Wesley, 1962.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	22	x⇄y
01.	33	STO	26.	34	RCL
02.	61	+	27.	02	2
03.	00	0	28.	71	x
04.	34	RCL	29.	02	2
05.	01	1	30.	81	÷
06.	01	1	31.	51	–
07.	61	+	32.	22	x⇄y
08.	33	STO	33.	34	RCL
09.	01	1	34.	02	2
10.	-00	GTO 00	35.	61	+
11.	34	RCL	36.	01	1
12.	02	2	37.	61	+
13.	34	RCL	38.	34	RCL
14.	01	1	39.	01	1
15.	01	1	40.	71	x
16.	61	+	41.	34	RCL
17.	02	2	42.	02	2
18.	81	÷	43.	71	x
19.	61	+	44.	01	1
20.	71	x	45.	02	2
21.	34	RCL	46.	81	÷
22.	00	0	47.	31	f
23.	51	–	48.	42	$\sqrt{x}$
24.	84	R/S	49.	81	÷

REGISTERS
R_0 ΣR_i
R_1 n_1
R_2 n_2
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

(Note: Only the ranks R_i's for the first sample are used as the input data.)

Sample 1	14.9	11.3	13.2	16.6	17	14.1	15.4	13	16.9
Rank R_i	7	1	4	12	14	5	10	3	13

Sample 2	15.2	19.8	14.7	18.3	16.2	21.2	18.9	12.2	15.3	19.4
Rank	8	18	6	15	11	19	16	2	9	17

$n_1 = 9$, $n_2 = 10$, $U = 66.00$, $z = 1.71$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize	0	STO	0	STO	1	
			BST				0.00
3	Store n_2	n_2	STO	2			
4	Perform 4 for $i = 1, 2, \ldots, n_1$	R_i	R/S				i
5	Compute U and z		GTO	1	1	R/S	U
			R/S				z
6	For a new case, go to 2						

MEAN-SQUARE SUCCESSIVE DIFFERENCE

When test and estimation techniques are used, the method of drawing the sample from the population is specified to be random in most cases. If observations are chosen in a sequence $x_1, x_2, \ldots, x_n$, the mean-square successive difference

$$\eta = \sum_{i=1}^{n-1} (x_i - x_{i+1})^2 \Bigg/ \sum_{i=1}^{n} (x_i - \bar{x})^2$$

can be used to test for randomness.

If n is large (say, greater than 20), and the population is normal, then

$$z = \frac{1 - \eta/2}{\sqrt{\dfrac{n-2}{n^2-1}}}$$

has approximately the standard normal distribution. Long trends are associated with large positive values of z and short ocillations with large negative values.

Reference:

Dixon and Massey, *Introduction to Statistical Analysis,* McGraw-Hill, 1969.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	34	RCL
02.	83	•
03.	06	6
04.	22	x⇄y
05.	51	–
06.	31	f
07.	34	LAST X
08.	33	STO
09.	83	•
10.	06	6
11.	11	Σ+
12.	-00	GTO 00
13.	32	g
14.	33	s
15.	32	g
16.	42	x^2
17.	34	RCL
18.	83	•
19.	00	0
20.	01	1
21.	51	–
22.	71	x
23.	34	RCL
24.	83	•

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	04	4
26.	22	x⇄y
27.	81	÷
28.	84	R/S
29.	02	2
30.	81	÷
31.	01	1
32.	22	x⇄y
33.	51	–
34.	34	RCL
35.	83	•
36.	00	0
37.	02	2
38.	51	–
39.	34	RCL
40.	83	•
41.	00	0
42.	32	g
43.	42	x^2
44.	01	1
45.	51	–
46.	81	÷
47.	31	f
48.	42	$\sqrt{x}$
49.	81	÷

REGISTERS	
R_0	
R_1	
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	n
$R_{.1}$	Σx_i
$R_{.2}$	Σx_i^2
$R_{.3}$	$\Sigma(x_i - x_{i+1})$
$R_{.4}$	$\Sigma(x_i - x_{i+1})^2$
$R_{.5}$	Used
$R_{.6}$	x_i
$R_{.7}$	0
$R_{.8}$	0
$R_{.9}$	0

Example:

For the following set of data

$$\{0.53,\ 0.52,\ 0.39,\ 0.49,\ 0.97,\ 0.29,\ 0.65,\ 0.30,\ 0.40,\ 0.06,\ 0.14,\ 0.16,\ 0.68\ 0.22,\ 0.68,\ 0.08,\ 0.52,\ 0.50,\ 0.63,\ 0.20,\ 0.67,\ 0.44,\ 0.64,\ 0.40,\ 0.97,\ 0.03,\ 0.73,\ 0.24,\ 0.57,\ 0.35\}$$

$n = 30$
$\eta = 2.81$
$z = -2.29.$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R	BST		0.00
3	Input x_1	x_1	STO	•	6	Σ+	1.00
4	Perform 4 for i = 2, 3,..., n	x_i	R/S				i
5	Compute η and z		GTO	1	3	R/S	η
			R/S				z
6	For a new case, go to 2						

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