HEWLETT-PACKARD

HP-67/HP-97

Users' Library Solutions

High-Level Math

INTRODUCTION

In an effort to provide continued value to it's customers, Hewlett-Packard is introducing a unique service for the HP fully programmable calculator user. This service is designed to save you time and programming effort. As users are aware, Programmable Calculators are capable of delivering tremendous problem solving potential in terms of power and flexibility, but the real genie in the bottle is program solutions. HP's introduction of the first handheld programmable calculator in 1974 immediately led to a request for program **solutions** — hence the beginning of the HP-65 Users' Library. In order to save HP calculator customers time, users wrote their own programs and sent them to the Library for the benefit of other program users. In a short period of time over 5,000 programs were accepted and made available. This overwhelming response indicated the value of the program library and a Users' Library was then established for the HP-67/97 users.

To extend the value of the Users' Library, Hewlett-Packard is introducing a unique service—a service designed to save you time and money. The Users' Library has collected the best programs in the most popular categories from the HP-67/97 and HP-65 Libraries. These programs have been packaged into a series of low-cost books, resulting in substantial savings for our valued HP-67/97 users.

We feel this new software service will extend the capabilities of our programmable calculators and provide a great benefit to our HP-67/97 users.

A WORD ABOUT PROGRAM USAGE

Each program contained herein is reproduced on the standard forms used by the Users' Library. Magnetic cards are not included. The Program Description I page gives a basic description of the program. The Program Description II page provides a sample problem and the keystrokes used to solve it. The User Instructions page contains a description of the keystrokes used to solve problems in general and the options which are available to the user. The Program Listing I and Program Listing II pages list the program steps necessary to operate the calculator. The comments, listed next to the steps, describe the reason for a step or group of steps. Other pertinent information about data register contents, uses of labels and flags and the initial calculator status mode is also found on these pages. Following the directions in your HP-67 or HP-97 **Owners' Handbook and Programming Guide**, "Loading a Program" (page 134, HP-67; page 119, HP-97), key in the program from the Program Listing I and Program Listing II pages. A number at the top of the Program Listing indicates on which calculator the program was written (HP-67 or HP-97). If the calculator indicated differs from the calculator you will be using, consult Appendix E of your **Owner's Handbook** for the corresponding keycodes and keystrokes converting HP-67 to HP-97 keycodes and vice versa. No program conversion is necessary. The HP-67 and HP-97 are totally compatible, but some differences do occur in the keycodes used to represent some of the functions.

A program loaded into the HP-67 or HP-97 is not permanent—once the calculator is turned off, the program will not be retained. You can, however, permanently save any program by recording it on a blank magnetic card, several of which were provided in the Standard Pac that was shipped with your calculator. Consult your **Owner's Handbook** for full instructions. A few points to remember:

> The Set Status section indicates the status of flags, angular mode, and display setting. After keying in your program, review the status section and set the conditions as indicated before using or permanently recording the program.
>
> REMEMBER! To save the program permanently, **clip** the corners of the magnetic card once you have recorded the program. This simple step will protect the magnetic card and keep the program from being inadvertently erased.

As a part of HP's continuing effort to provide value to our customers, we hope you will enjoy our newest concept.

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Program calculates the eigenvalues of a 3rd order system described by $Ax = \lambda x$.

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Computes Gamma function, Gaussian hypergeometric function, Jacobian polynomial, Legendre polynomial, Gegenbauer polynomial, and first and second Chebychev polynomials. Works for positive or zero subscripts and superscripts, but for large subscripts can be slow. Gamma function for $0 < x < 70.95$.

MISCELLANEOUS SPECIAL FUNCTIONS B 50
Using a recursion relation to compute the Gegenbauer, Laguerre, Hermite, Legendre, and Chebychev (both first and second kind) polynomials. A special key enables one, by inputting alternate starting values, to compute polynomials with non-integer subscripts. Starting values can be computed for non-integer subscripts by using Miscellaneous Special Functions A.

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This program will compute incomplete elliptic integrals of the first or second kind as well as any linear combination of them.

Program Description I

Program Title Eigenvalues for 3rd Order System

Contributor's Name Hewlett-Packard Company

Address 1000 N. E. Circle Boulevard

City Corvallis **State** OR **Zip Code** 97330

Program Description, Equations, Variables This program solves eigenvalues of a 3rd-Order system described by $Ax = \lambda x$, i.e.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Eigenvalues λ_1, λ_2, and λ_3 are solved from

$$\det(\lambda I - A) = 0$$

Roots for the cubic equation is solved by using the exact formula.

Operating Limits and Warnings

Program Description II

Sketch(es)

Sample Problem(s)

(1)

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & -0.05 \end{bmatrix}$$

(2)

$$A_2 = \begin{bmatrix} 13 & -3 & 5 \\ 0 & 4 & 0 \\ -15 & 9 & -7 \end{bmatrix}$$

(3)

$$A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution(s) Load side 1 and 2, Switch to [NORM]

(1) 1 [STO] 1, 0 [STO] 2, ...1 [CHS] [STO] 8, 0.05 [CHS] [STO] 9;
[A] → $\lambda_3 = 1$, $\lambda_1, \lambda_2 = -0.025 \pm 0.9997j$

(2) 13 [STO] 1, 0 [STO] 2, ...0 [STO] 8, 7 [CHS] [STO] 9;
[A] → $\lambda_3 = 8$, $\lambda_1 = 4$, $\lambda_2 = -2$

(3) 1 [STO] 1, 0 [STO] 2, ...
[A] → $\lambda_3 = 1$, $\lambda_1 = \lambda_2 = 0$

Reference(s)

(1) C.G. CULLEN, Matrices and Linear Transformations Addison-Wesley Publishing Co. 1967

(2) HP-97 Standard Pac program 09.

User Instructions

1 $\rightarrow \lambda_3, \lambda_1, \lambda_2$ 2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Load side 1 and side 2				
2	Switch to [NORM]				
3	Input elements of A matrix a_{11}	a_{11}	STO	1	a_{11}
	a_{21}	a_{21}	STO	2	a_{21}
	a_{31}	a_{31}	STO	3	a_{31}
	a_{12}	a_{12}	STO	4	a_{12}
	a_{22}	a_{22}	STO	5	a_{22}
	a_{32}	a_{32}	STO	6	a_{32}
	a_{13}	a_{13}	STO	7	a_{13}
	a_{23}	a_{23}	STO	8	a_{23}
	a_{33}	a_{33}	STO	9	a_{33}
4	Calculate eigenvalues		A		$\lambda_3, \lambda_1, \lambda_2$*
5	For a new case go to step 3				
	*Note: The print out will be of the form				
	x.xxxx (λ_3)				
	x.xxxx T (λ_1 real)				
	x.xxxx Z (λ_1 imagin.)				
	x.xxxx Y (λ_2 real)				
	x.xxxx X (λ_2 imagin.)				

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Calculate coefficients of the 3rd order polynomial
002	SPC	16-11	
003	1	01	
004	0	00	
005	STOI	35 46	
006	RCL8	36 08	
007	RCL6	36 06	
008	RCL5	36 05	
009	RCL9	36 09	
010	GSB9	23 09	
011	RCL1	36 01	
012	x	-35	
013	STOi	35 45	
014	RCL2	36 02	
015	RCL9	36 09	
016	RCL8	36 08	
017	RCL3	36 03	
018	GSB9	23 09	
019	RCL4	36 04	
020	x	-35	
021	ST+i	35-55 45	
022	RCL3	36 03	
023	RCL5	36 05	
024	RCL6	36 06	
025	RCL2	36 02	
026	GSB9	23 09	
027	RCL7	36 07	
028	x	-35	
029	ST+i	35-55 45	
030	ISZI	16 26 46	
031	RCL1	36 01	
032	RCL5	36 05	
033	RCL7	36 07	
034	RCL3	36 03	
035	GSB9	23 09	
036	STOi	35 45	
037	RCL1	36 01	
038	RCL9	36 09	
039	RCL8	36 08	
040	RCL6	36 06	
041	GSB9	23 09	
042	ST+i	35-55 45	
043	RCL5	36 05	
044	RCL9	36 09	
045	RCL4	36 04	
046	RCL2	36 02	
047	GSB9	23 09	
048	ST+i	35-55 45	
049	RCL1	36 01	
050	RCL5	36 05	
051	RCL9	36 09	
052	+	-55	
053	+	-55	
054	CHS	-22	
055	P⇄S	16-51	
056	STO2	35 02	
057	CF0	16 22 00	
058	CF1	16 22 01	
059	RCL0	36 00	Test if constant is zero
060	X=0?	16-43	
061	GTOd	22 16 14	
062	RCL1	36 01	
063	3	03	
064	x	-35	
065	RCL2	36 02	
066	X²	53	
067	-	-45	
068	9	09	
069	÷	-24	
070	STO3	35 03	Solve for the real root λ_3
071	RCL1	36 01	
072	RCL2	36 02	
073	x	-35	
074	9	09	
075	x	-35	
076	RCL0	36 00	
077	2	02	
078	7	07	
079	x	-35	
080	-	-45	
081	RCL2	36 02	
082	3	03	
083	Yˣ	31	
084	2	02	
085	x	-35	
086	-	-45	
087	5	05	
088	4	04	
089	÷	-24	
090	STO4	35 04	
091	X²	53	
092	RCL3	36 03	
093	3	03	
094	Yˣ	31	
095	+	-55	
096	0	00	
097	X≤Y?	16-35	
098	GTO3	22 03	
099	RCL4	36 04	
100	RCL3	36 03	
101	3	03	
102	Yˣ	31	
103	CHS	-22	
104	√X	54	
105	÷	-24	
106	COS⁻¹	16 42	
107	3	03	λ_3 for $Q^3 + R^2 < 0$
108	÷	-24	
109	COS	42	
110	RCL3	36 03	
111	CHS	-22	
112	√X	54	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	a_{11}	a_{21}	a_{31}	a_{21}	a_{22}	a_{32}	a_{13}	a_{23}	a_{33}

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
$\wedge\ s^0$	$\wedge\ s^1$	$\wedge\ s^2$	Q	R	S	T	$\wedge\ {}^-s^0$	$\wedge\ {}^-s^0$	

A	B	C	D	E	I
	$\sqrt{\ }$	$-\sigma$ (or $\frac{-b}{2}$)	ω (or $\frac{\sqrt{\ }}{2}$)	λ_3	Index

97 Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	x	-35	
114	2	02	
115	x	-35	
116	GTO4	22 04	
117	*LBL3	21 03	
118	X⇄Y	-41	
119	√X	54	
120	STO9	35 09	
121	RCL4	36 04	
122	+	-55	
123	GSB5	23 05	S and T
124	RCL4	36 04	
125	RCL9	36 09	
126	-	-45	
127	GSB5	23 05	
128	+	-55	
129	*LBL4	21 04	
130	RCL2	36 02	
131	3	03	
132	÷	-24	λ_3
133	-	-45	
134	STOE	35 15	
135	PRTX	-14	
136	*LBL2	21 02	
137	RCL2	36 02	
138	RCLE	36 15	
139	+	-55	
140	STO8	35 08	
141	RCL0	36 00	
142	RCLE	36 15	
143	÷	-24	
144	CHS	-22	Reduce to 2nd
145	STO7	35 07	order and calcu-
146	*LBL7	21 07	late λ_1, λ_2
147	RCL8	36 08	
148	X^2	53	
149	RCL7	36 07	
150	4	04	
151	x	-35	
152	-	-45	
153	CHS	-22	
154	X>0?	16-44	
155	SF1	16 21 01	
156	X=0?	16-43	
157	SF0	16 21 00	
158	ABS	16 31	
159	√X	54	
160	2	02	
161	÷	-24	
162	STOD	35 14	
163	RCL8	36 08	
164	CHS	-22	
165	2	02	
166	÷	-24	
167	STOC	35 13	
168	F1?	16 23 01	
169	GTO1	22 01	
170	RCLD	36 14	
171	+	-55	
172	0	00	
173	RCLC	36 13	
174	RCLD	36 14	
175	-	-45	
176	0	00	
177	GTO8	22 08	
178	*LBL1	21 01	
179	RCLD	36 14	
180	RCLC	36 13	
181	RCLD	36 14	
182	CHS	-22	
183	*LBL8	21 08	Complex λ_1, λ_2
184	PRST	16-14	
185	P⇄S	16-51	
186	SPC	16-11	
187	RTN	24	
188	*LBL9	21 09	
189	x	-35	
190	R↓	-31	
191	x	-35	Multiply
192	R↑	16-31	
193	-	-45	Subtract
194	RTN	24	
195	*LBL5	21 05	
196	X<0?	16-45	
197	GSB6	23 06	
198	1	01	
199	ENT↑	-21	
200	3	03	$\sqrt[3]{\ }$
201	÷	-24	
202	Y^x	31	
203	F2?	16 23 02	
204	CHS	-22	
205	RTN	24	
206	*LBL6	21 06	
207	SF2	16 21 02	Negative $\sqrt[3]{\ }$
208	CHS	-22	
209	RTN	24	
210	*LBLd	21 16 14	
211	PRTX	-14	
212	RCL1	36 01	For $\lambda_3 = 0$
213	STO7	35 07	
214	RCL2	36 02	
215	STO8	35 08	
216	GTO7	22 07	
217	R/S	51	
220			

LABELS

A $\rightarrow\lambda_3,\lambda_1,\lambda_2$	B	C	D	E
a	b	c	d zero λ_3	e
0	1 Complex	2 2nd order	3 S and T	4 λ_3
5 $\sqrt[3]{\ }$	6 SF2, CHS	7 USED	8 Print	9 x −

FLAGS

0	Double
1	Complex
2	<0
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	4

Program Description I

Program Title Eigenvalues/vectors of 3rd-order Systems W/Distinct Real Eigenvalues.

Contributor's Name Hewlett-Packard Company, APD

Address 19310 Pruneridge Avenue

City Cupertino **State** Ca **Zip Code** 95014

Program Description, Equations, Variables For a system matrix A, the eigenvalues are found from $AX = \lambda X$

(a) Power method

Assume the eigenvalues of A are λ_1, λ_2 and λ_3 where $|\lambda_1| > |\lambda_2| \geq |\lambda_3|$. Now let A operate repeatedly on a vector v, which we express as a linear combination of the eigenvectors $v = c_1v_1 + c_2v_2 + c_3v_3$ then

$$Av = c_1Av_1 + c_2Av_2 + c_3Av_3 = \lambda_1\ (c_1v_1 + c_2\,\frac{\lambda_2}{\lambda_1}\,v_2 + c_3\,\frac{\lambda_3}{\lambda_1}\,v_3)$$

$$A^pv = \lambda_1^p\ [c_1v_1 + c_2\ (\frac{\lambda_2}{\lambda_1})^p v_2 + c_3\ (\frac{\lambda_3}{\lambda_1})^p v_3]$$

$$\text{therefore}\quad \lambda_1 = \lim_{p\to\infty} \frac{(A^{p+1}v)_1}{(A^pv)_1}$$

(b) For deflation method, refer to reference (2).

Operating Limits and Warnings Only works for systems with distinct real eigenvalues with a "good" guess of the initial eigenvector V_1. If a first component of the eigenvectors is zero, then it is necessary to do similarity transformations in order to use this program

Program Description II

Sketch(es)

Sample Problem(s)

(1)

$$A = \begin{bmatrix} -3 & 1 & 0 \\ 2 & -3 & 2 \\ 0 & 1 & -3 \end{bmatrix}, \quad \lambda_1 = -5,\ V_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \quad \lambda_2 = -1,\ V_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \lambda_3 = -3,\ V_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

(2)

$$A = \begin{bmatrix} -3.06 & -1.98 & 4.26 \\ 1.04 & 0.67 & -1.47 \\ -1.76 & -1.14 & 2.44 \end{bmatrix}, \quad \lambda_1 = 0.06,\ V_1 = \begin{bmatrix} 1 \\ -1/2 \\ 1/2 \end{bmatrix}, \quad \lambda_2 = -0.02,\ V_2 = \begin{bmatrix} 1 \\ 4/3 \\ 4/3 \end{bmatrix}, \quad \lambda_3 = 0.01,\ V_3 = \begin{bmatrix} 1 \\ -5/6 \\ 1/3 \end{bmatrix}$$

Solution(s) (1) [NORM] [f] [CL REG] 3 [CHS] [STO] 1, 2 [STO] 2, 0 [STO] 3, [STO] 7, 1 [STO] 4, [STO] 6, 3 [CHS] [STO] 5, [STO] 9, 2 [STO] 8, [A] $\rightarrow a_{ij}$, 1 [STO] [A], [B] $\rightarrow \lambda_1, V_1, \lambda_2, V_2, \lambda_3, V_3$

(2) [NORM] [f] [CL REG] 3.06 [CHS] [STO] 1, 1.04 [STO] 2, 1.76 [CHS] [STO] 3, 1.98 [CHS] [STO] 4, 0.67 [STO] 5, 1.14 [CHS] [STO] 6, 4.26 [STO] 7, 1.47 [CHS] [STO] 8, 2.44 [STO] 9 [A] $\rightarrow a_{ij}$, 1 [STO] [A], [STO] [B], [STO] [C], [B] $\rightarrow \lambda_1, V_1, \lambda_2, V_2, \lambda_3, V_3$

Reference(s)

(1) Charles Cullen, Matrices and Linear Transformations. ADDISON-WESLEY PUB. COMPANY, MARCH 1967

(2) Carl-Erik Froberg, Intro. to Numerical Analysis ADDISON-WESLEY PUB. COMPANY, 1969

User Instructions

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Load side 1 and side 2				
2	Input a_{ij}	a_{11}	STO	1	
		a_{21}	STO	2	
		a_{31}	STO	3	
		a_{12}	STO	4	
		a_{22}	STO	5	
		a_{32}	STO	6	
		a_{13}	STO	7	
		a_{23}	STO	8	
		a_{33}	STO	9	
3	To print a_{ij}		A		a_{ij}'s
4	To calculate eigenvalues and eigenvectors, in-				
	put initial guess of the first eigenvector:				
	v_1^1	v_1^1	STO	A	v_1^1
	v_2^1	v_2^1	STO	B	v_2^1
	v_3^1	v_3^1	STO	C	v_3^1
	and press		B		λ_i's, v^i's
5	For a new case, go to step 2.				

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	
002	0	00	
003	STOI	35 46	
004	*LBL1	21 01	
005	ISZI	16 26 46	
006	RCLi	36 45	
007	PRTX	-14	
008	RCLI	36 46	
009	3	03	Print a_{ij}
010	X=Y?	16-33	
011	SPC	16-11	
012	X⇄Y	-41	
013	6	06	
014	X=Y?	16-33	
015	SPC	16-11	
016	X⇄Y	-41	
017	9	09	
018	X=Y?	16-33	
019	GTO7	22 07	
020	GTO1	22 01	
021	*LBLB	21 12	
022	SPC	16-11	
023	CF0	16 22 00	Calculate λ_1 and v^1
024	*LBL3	21 03	
025	RCL1	36 01	
026	RCLA	36 11	
027	RCL4	36 04	
028	RCLB	36 12	
029	GSB9	23 09	
030	RCL7	36 07	
031	RCLC	36 13	Iteration:
032	×	-35	power method
033	+	-55	
034	STOD	35 14	
035	RCL2	36 02	
036	RCLA	36 11	
037	RCL5	36 05	
038	RCLB	36 12	
039	GSB9	23 09	
040	RCL8	36 08	
041	RCLC	36 13	
042	×	-35	
043	+	-55	
044	STOE	35 15	
045	RCL3	36 03	
046	RCLA	36 11	
047	RCL6	36 06	
048	RCLB	36 12	
049	GSB9	23 09	
050	RCL9	36 09	
051	RCLC	36 13	
052	×	-35	
053	+	-55	
054	STOI	35 46	
055	RCLD	36 14	
056	RCLA	36 11	
057	÷	-24	
058	DSP9	-63 09	
059	RND	16 24	
060	RCL0	36 00	
061	RND	16 24	
062	X=Y?	16-33	
063	GTO0	22 00	
064	RCLD	36 14	
065	RCLA	36 11	
066	÷	-24	
067	STO0	35 00	
068	RCLD	36 14	
069	STOA	35 11	
070	RCLE	36 15	
071	STOB	35 12	
072	RCLI	36 46	
073	STOC	35 13	
074	GTO3	22 03	
075	*LBL0	21 00	
076	DSP4	-63 04	
077	GSB8	23 08	
078	1	01	
079	STOA	35 11	
080	PRTX	-14	
081	RCLE	36 15	
082	RCLD	36 14	
083	÷	-24	
084	STOB	35 12	
085	PRTX	-14	Print λ_1, v^1
086	RCLI	36 46	
087	RCLD	36 14	
088	÷	-24	
089	STOC	35 13	
090	GSB8	23 08	
091	SPC	16-11	
092	RCL5	36 05	
093	RCLB	36 12	
094	RCL4	36 04	
095	×	-35	
096	-	-45	
097	P⇄S	16-51	
098	STO6	35 06	
099	P⇄S	16-51	
100	RCL6	36 06	
101	RCLC	36 13	
102	RCL4	36 04	
103	×	-35	
104	-	-45	Construct B matrix
105	P⇄S	16-51	
106	STO7	35 07	
107	P⇄S	16-51	
108	RCL8	36 08	
109	RCLB	36 12	
110	RCL7	36 07	
111	×	-35	
112	-	-45	

REGISTERS

0 λ_1	1 a_{11}	2 a_{21}	3 a_{31}	4 a_{12}	5 a_{22}	6 a_{32}	7 a_{13}	8 a_{23}	9 a_{33}
S0 λ_3	S1 z_2^3, v_2^3	S2 z_3^3, v_3^3	S3 λ_2	S4 z_2^2, v_2^2	S5 z_3^2, v_3^2	S6 b_{11}	S7 b_{21}	S8 b_{12}	S9 b_{22}

A	B	C	D	E	I
temp, $(x_1^1)_k = 1$	$(\lambda_2^1)_k$	$(x_3^1)_k$	temp Av_1^1, $\frac{-b}{2}$	temp Av_2^1, $\frac{\sqrt{\ }}{2}$	temp Av_3^1

97 Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	P⇄S	16-51	
114	STO8	35 08	
115	P⇄S	16-51	
116	RCL9	36 09	
117	RCLC	36 13	
118	RCL7	36 07	
119	x	-35	
120	-	-45	
121	P⇄S	16-51	
122	STO9	35 09	
123	RCL6	36 06	
124	+	-55	
125	STOI	35 46	
126	X²	53	and solve for the eigenvalues λ_2 and λ_3
127	RCL6	36 06	
128	RCL9	36 09	
129	x	-35	
130	RCL8	36 08	
131	RCL7	36 07	
132	x	-35	
133	-	-45	
134	4	04	
135	x	-35	
136	-	-45	
137	√X	54	
138	2	02	
139	÷	-24	
140	STOD	35 14	
141	RCLI	36 46	
142	2	02	
143	÷	-24	
144	STOE	35 15	
145	RCLD	36 14	
146	-	-45	
147	STO0	35 00	
148	RCLE	36 15	
149	RCLD	36 14	
150	+	-55	
151	STO3	35 03	
152	*LBL2	21 02	
153	RCL0	36 00	
154	GSB8	23 08	
155	RCL6	36 06	
156	-	-45	
157	STO2	35 02	
158	RCL8	36 08	
159	STO1	35 01	
160	X⇄Y	-41	
161	P⇄S	16-51	
162	RCL4	36 04	Solve for v^2
163	RCL7	36 07	
164	R↓	-31	
165	X⇄Y	-41	
166	R↑	16-31	
167	GSB9	23 09	
168	RCL0	36 00	

STEP	KEY ENTRY	KEY CODE	COMMENTS
169	P⇄S	16-51	
170	RCL0	36 00	
171	X⇄Y	-41	
172	-	-45	
173	÷	-24	
174	ST÷1	35-24 01	
175	ST÷2	35-24 02	
176	RCLB	36 12	
177	ST+1	35-55 01	
178	RCLC	36 13	
179	ST+2	35-55 02	
180	RCLA	36 11	
181	PRTX	-14	
182	RCL1	36 01	
183	PRTX	-14	
184	RCL2	36 02	
185	P⇄S	16-51	
186	GSB8	23 08	
187	SPC	16-11	
188	F0?	16 23 00	
189	RTN	24	
190	SF0	16 21 00	
191	P⇄S	16-51	
192	RCL0	36 00	
193	RCL3	36 03	
194	STO0	35 00	
195	X⇄Y	-41	
196	STO3	35 03	
197	RCL1	36 01	
198	RCL2	36 02	
199	RCL4	36 04	
200	RCL5	36 05	
201	STO2	35 02	Calculate v^3
202	R↓	-31	
203	STO1	35 01	
204	R↓	-31	
205	STO5	35 05	
206	R↓	-31	
207	STO4	35 04	
208	GTO2	22 02	
209	*LBL9	21 09	
210	x	-35	
211	R↓	-31	
212	x	-35	x, +
213	R↑	16-31	
214	+	-55	
215	RTN	24	
216	*LBL8	21 08	
217	PRTX	-14	
218	*LBL7	21 07	Print
219	SPC	16-11	
220	RTN	24	
221	R/S	51	

LABELS

A a_{ij}	B λ_i's & v^i	C	D	E
a	b	c	d	e
0 λ_1, v^1	1	2 v^2, v^3	3	4
5	6	7	8	9 x, +

FLAGS

0	v^3
1	
2	
3	

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☒
SCI	☐
ENG	☐
n	4

Program Description I

Program Title Matrix Algebra

Contributor's Name Jeffery J. Straw

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Program Description, Equations, Variables

For two matrices X and Y, each with dimensions less than or equal to three, this program will solve for the product or sum of them, the difference between them, or the result of a scalar multiplication of either. Each matrix is stored upper left justified in a 3x3 array. With the exceptions of the scalar multiplication of Y and the copying of X into Y, all results replace the matrix X and are upper left justified.

Equations used are:

Matrix Product: $X_{ij} = \sum_{n=1}^{3} X_{in} Y_{nj}$, $i = 1,2,3$; $j = 1,2,3$

Matrix Sum or Difference: $X_{ij} = X_{ij} \pm Y_{ij}$, $i = 1,2,3$; $j = 1,2,3$

Scalar Multiplication: $X_{ij} = aX_{ij}$ or $Y_{ij} = bY_{ij}$, $i = 1,2,3$; $j = 1,2,3$

Operating Limits and Warnings The product of two matrices where the number of columns of the first is unequal to the number of rows of the second is undefined, as is the sum or difference of two matrices of unequal dimension. These illegal operations will be undetected by this program, since it assumes each matrix is a submatrix of a 3x3, all undefined elements of which are zero. Correct answers will result for legal operations, but care must be taken that the operations are indeed legal.

Program Description II

Sketch(es)

The matrix $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is stored as $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Sample Problem(s)

Given the following matrices, find P*Q:

$P = \begin{bmatrix} 2 & 7 & 4 \\ 1 & 6 & 3 \end{bmatrix}$ $Q = \begin{bmatrix} 9 & 4 \\ 1 & 4 \\ 2 & 5 \end{bmatrix}$

Solution(s)

```
3 [A]                                                   →  0.00
2 [R/S] 7 [R/S] 4 [R/S] 1 [R/S] 6 [R/S] 3 [R/S]         →  3.00
2 [B]                                                   →  0.00
9 [R/S] 4 [R/S] 1 [R/S] 4 [R/S] 2 [R/S] 5 [R/S]         →  5.00
[C]                                                     →  0.00
[f] [A] → 33.00***     56.00***     0.00***
          21.00***     43.00***     0.00***
           0.00***      0.00***     0.00***
```

P*Q is given in the upper left 2x2 submatrix.

Reference(s)

User Instructions

MATRIX ALGEBRA

prt X	prt Y	X → Y	X-Y → X	aY → Y
# col X	# col Y	X*Y → X	X+Y → X	aX → X

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Load Side 1 and Side 2				
2	Input number of columns in matrix X	n	A		0.00
3	Enter elements of the matrix X by rows	X_{ij}	R/S		X_{ij}
	Repeat step 3 until all elements are entered				
4	To copy X into Y, go to step 10				
	To multiply X by a scalar, go to step 11				
5	Input number of columns in matrix Y	m	B		0.00
6	Enter elements of the matrix Y by rows	Y_{ij}			Y_{ij}
	Repeat step 6 until all elements are entered				
7	For matrix multiplication (X*Y → X)		C		0.00
8	For matrix addition (X+Y → X)		D		0.00
9	For matrix subtraction (X-Y → X)		f	D	0.00
10	To copy X into Y (X → Y)		f	C	0.00
11	To multiply X by a scalar (aX → X)	a	E		a
12	To multiply Y by a scalar (aY → Y)	a	f	E	a
13	To print (display) the matrix X		f	A	X_{ij}***
	(All 9 elements of the 3x3 are given by rows.				
	The result is upper left justified.)				
14	To print (display) the matrix Y		f	B	Y_{ij}***
	(Same note above applies.)				
15	To interchange X and Y		f	P≷S	
16	For further operations, go to the appropriate				
	step, 2-15				
17	To Clear X	x	f	CL REG	x
18	To Clear Y	x	f	P≷S	
			f	CL REG	
			f	P≷S	x

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	
002	CF0	16 22 00	Initialize X to all zeros
003	CLRG	16-53	
004	GTO0	22 00	
005	*LBLB	21 12	
006	SF0	16 21 00	
007	P⇄S	16-51	
008	CLRG	16-53	Initialize Y to 0.
009	P⇄S	16-51	
010	*LBL0	21 00	Test # of columns for proper range (if # = 0, it is treated as 3)
011	ABS	16 31	
012	INT	16 34	
013	STO0	35 00	
014	4	04	
015	X≤Y?	16-35	Error display
016	CLX	-51	
017	1/X	52	
018	*LBL1	21 01	Loop for entering data
019	RCLA	36 11	
020	R/S	51	
021	ISZI	16 26 46	
022	F0?	16 23 00	
023	P⇄S	16-51	
024	STOi	35 45	
025	F0?	16 23 00	
026	P⇄S	16-51	
027	STOA	35 11	
028	RCL0	36 00	This row done? If so, skip the "GTO 1" and reuse the I register
029	RCLI	36 46	
030	X≠Y?	16-32	
031	GTO1	22 01	
032	RCLI	36 46	
033	3	03	
034	.	-62	
035	1	01	
036	÷	-24	
037	INT	16 34	
038	1	01	
039	+	-55	
040	3	03	
041	x	-35	
042	STOI	35 46	
043	3	03	
044	ST+0	35-55 00	
045	GTO1	22 01	
046	*LBLC	21 13	Matrix multiply routine, using registers A,B,C,D,E for data transfer to and from subroutine 2.
047	RCL1	36 01	
048	STOA	35 11	
049	RCL2	36 02	
050	STOB	35 12	
051	RCL3	36 03	
052	STOC	35 13	
053	GSB2	23 02	
054	STO1	35 01	
055	RCLD	36 14	
056	STO2	35 02	

STEP	KEY ENTRY	KEY CODE	COMMENTS
057	RCLE	36 15	
058	STO3	35 03	
059	RCL4	36 04	
060	STOA	35 11	
061	RCL5	36 05	
062	STOB	35 12	
063	RCL6	36 06	
064	STOC	35 13	
065	GSB2	23 02	
066	STO4	35 04	
067	RCLD	36 14	
068	STO5	35 05	
069	RCLE	36 15	
070	STO6	35 06	
071	RCL7	36 07	
072	STOA	35 11	
073	RCL8	36 08	
074	STOB	35 12	
075	RCL9	36 09	
076	STOC	35 13	
077	GSB2	23 02	
078	STO7	35 07	
079	RCLD	36 14	
080	STO8	35 08	
081	RCLE	36 15	
082	STO9	35 09	
083	CLX	-51	
084	RTN	24	
085	*LBL2	21 02	Matrix multiply subroutine
086	P⇄S	16-51	
087	RCL9	36 09	
088	x	-35	
089	RCLB	36 12	
090	RCL6	36 06	
091	x	-35	
092	+	-55	
093	RCLA	36 11	
094	RCL3	36 03	
095	x	-35	
096	+	-55	
097	STOE	35 15	
098	RCLC	36 13	
099	RCL8	36 08	
100	x	-35	
101	RCLB	36 12	
102	RCL5	36 05	
103	x	-35	
104	+	-55	
105	RCLA	36 11	
106	RCL2	36 02	
107	x	-35	
108	+	-55	
109	STOD	35 14	
110	RCLC	36 13	
111	RCL7	36 07	
112	x	-35	

REGISTERS

0	1	2	3	4	5	6	7	8	9
Used	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
Used	Y_{11}	Y_{12}	Y_{13}	Y_{21}	Y_{22}	Y_{23}	Y_{31}	Y_{32}	Y_{33}

A	B	C	D	E	I
Used	Used	Used	Used	Used	Used

97 Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	RCLB	36 12	
114	RCL4	36 04	
115	×	-35	
116	+	-55	
117	RCLA	36 11	
118	RCL1	36 01	
119	×	-35	
120	+	-55	
121	P⇄S	16-51	
122	RTN	24	
123	*LBLb	21 16 12	Loop for displaying (67) or printing (97) the contents of matrix X (LBL a) or matrix Y (LBL b)
124	SF1	16 21 01	
125	GTO3	22 03	
126	*LBLa	21 16 11	
127	CF1	16 22 01	
128	*LBL3	21 03	
129	1	01	
130	STOI	35 46	
131	*LBL4	21 04	
132	F1?	16 23 01	
133	P⇄S	16-51	
134	RCLi	36 45	
135	F1?	16 23 01	
136	P⇄S	16-51	
137	PRTX	-14	
138	ISZI	16 26 46	
139	9	09	
140	RCLI	36 46	
141	X≤Y?	16-35	
142	GTO4	22 04	
143	CLX	-51	
144	RTN	24	
145	*LBLc	21 16 13	X → Y loop
146	9	09	
147	STOI	35 46	
148	*LBL5	21 05	
149	RCLi	36 45	
150	P⇄S	16-51	
151	STOi	35 45	
152	P⇄S	16-51	
153	DSZI	16 25 46	
154	GTO5	22 05	
155	CLX	-51	
156	RTN	24	
157	*LBLd	21 16 14	matrix subtraction or addition routine
158	SF1	16 21 01	
159	GTO6	22 06	
160	*LBLD	21 14	
161	CF1	16 22 01	
162	*LBL6	21 06	
163	9	09	
164	STOI	35 46	
165	*LBL7	21 07	
166	P⇄S	16-51	
167	RCLi	36 45	
168	P⇄S	16-51	
169	F1?	16 23 01	
170	CHS	-22	
171	ST+i	35-55 45	
172	DSZI	16 25 46	
173	GTO7	22 07	
174	CLX	-51	
175	RTN	24	
176	*LBLe	21 16 15	Scalar multiplication routines - aX→X (LBL E) or aY→Y (LBL e)
177	SF2	16 21 02	
178	P⇄S	16-51	
179	*LBLE	21 15	
180	9	09	
181	STOI	35 46	
182	CLX	-51	
183	+	-55	
184	*LBL8	21 08	
185	ST×i	35-35 45	
186	DSZI	16 25 46	
187	GTO8	22 08	
188	F2?	16 23 02	
189	P⇄S	16-51	
190	RTN	24	
191	R/S	51	

LABELS

A # col X	B # col Y	C X*Y → X	D X+Y → X	E aX → X
a prt X	b prt Y	c X → Y	d X-Y → X	e aY → Y
0 OFLO?	1 ent loop	2 subroutine	3 a or b	4 prt loop
5 copy loop	6 d or D	7 ± loop	8 s mult loop	9

FLAGS

0	X or Y
1	used
2	aX or aY
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Program Description I

Program Title 67 - Characteristic Equation of a 4 x 4 Matrix

Contributor's Name Matthew Bishop

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Program Description, Equations, Variables

Given
$$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{pmatrix}$$

The characteristic equation is $\lambda^4+r_1\lambda^3+r_2\lambda^2+r_3\lambda+r_4=0$

where

$r_1=-(a_1+a_6+a_{11}+a_{16})$

$r_2= (a_1+a_{11})a_6 + (a_1+a_{16})a_{11} + (a_1 + a_6)a_{16} - a_8a_{14} - a_{12}a_{15}-a_7a_{10}-a_2a_5-a_3a_4$
$-a_4a_{13}$

$r_3=-\det(A_1)-a_1(a_6a_{11}+a_6a_{16}+a_{11}a_{16}-a_8a_{14}-a_{12}a_{15}-a_7a_{10})+$
$a_2[a_5(a_{11}+a_{16})-a_8a_{13}-a_7a_9]-a_3[-a_9(a_6+a_{16})+a_5a_{10}+a_{12}a_{13}]$
$+a_4[a_{13}(a_6+a_{11})-a_9a_{15}-a_5a_{14}]$

$r_4=a_1\det(A_1)-a_2\det(A_2)+a_3\det(A_3)-a_4\det(A_4)$

where
$$A_1=\begin{pmatrix} a_6 & a_7 & a_8 \\ a_{10} & a_{11} & a_{12} \\ a_{14} & a_{15} & a_{16} \end{pmatrix} \quad A_2=\begin{pmatrix} a_5 & a_7 & a_8 \\ a_9 & a_{11} & a_{12} \\ a_{13} & a_{15} & a_{16} \end{pmatrix} \quad A_3=\begin{pmatrix} a_5 & a_6 & a_8 \\ a_9 & a_{10} & a_{12} \\ a_{13} & a_{14} & a_{16} \end{pmatrix} \text{ and } A_4=\begin{pmatrix} a_5 & a_6 & a_7 \\ a_9 & a_{10} & a_{11} \\ a_{13} & a_{14} & a_{15} \end{pmatrix}$$

NOTE: Trace $(A) = -r_1$ $\det(A) = -r_4$

Operating Limits and Warnings All storage registers are used.

Program Description II

Sketch(es)

Sample Problem(s) Find the characteristic equations of the matrices

1) $$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & -1 \\ 3 & -1 & 0 & 2 \\ -2 & -1 & -1 & 0 \end{pmatrix}$$

Ans. $\lambda^4 - \lambda^3 + 7\lambda + 2 = 0$

2) $$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & -1 \\ 3 & -1 & 0 & 2 \\ -2 & -1 & -1 & 0 \end{pmatrix}$$

Ans. $\lambda^4 - \lambda^3 + 3\lambda^2 + 2\lambda - 5 = 0$

Solution(s) 1) [A] -------------> 1.00 Enter on successive pauses, in this order; 1,0,1,0,1,0,2,-1,3,-1,0,2,-2,-1,-1,0. Each time i+1 will be displayed, except after the last 0 -------------> -1.00,0.00, 7.00. 7.00 flashes, load second card -------------> 2.00

2) [A] -------------> 1.00 Enter on successive pauses in this order; 1,0,0,0,1,0,2,-1,3,-1,0,2,-2,-1,-1,0. Each time i+1 will be displayed, except after the last 0 --------->-1.00,3.00, 2.00. 2.00 flashes, load second card -------------> -5.00.

Reference(s)

User Instructions

CHARACTERISTIC EQUATION OF A 4 x 4 MATRIX
CARD I

1 | 2

Load Data

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1.	Load Card 1			
2.	Initialize		A	1.
3.	Load a_i, $1 \le i \le 15$	a_i		i + 1
4.	Repeat step 3 until 16 is displayed then enter a_{16}.	a_{16}		r_1 r_2 r_3
5.	r_3 will flash for 1-second intervals. During any of these intervals, load side 1 of card 2. The program will then continue.			r_4
6.	For a new matrix go to 1.			
	NOTE: DATA ENTRY ORDER:			
	$A = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{pmatrix}$			
	Also, trace (A) = $-r_1$ det (A) = $-r_4$			

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBL0	21 00	Compute the determinant of a 3 x 3 matrix
002	RCL5	36 05	
003	RCL9	36 09	
004	x	-35	$\begin{pmatrix} R_1 & R_2 & R_3 \\ R_4 & R_5 & R_6 \\ R_7 & R_8 & R_9 \end{pmatrix}$
005	RCL6	36 06	
006	RCL8	36 08	
007	GSB3	23 03	
008	RCL1	36 01	
009	x	-35	
010	RCL6	36 06	
011	RCL7	36 07	
012	x	-35	
013	RCL4	36 04	
014	RCL9	36 09	
015	GSB3	23 03	
016	RCL2	36 02	
017	GSB2	23 02	
018	RCL4	36 04	
019	RCL8	36 08	
020	x	-35	
021	RCL5	36 05	
022	RCL7	36 07	
023	GSB3	23 03	
024	RCL3	36 03	
025	*LBL2	21 02	
026	x	-35	
027	+	-55	
028	RTN	24	
029	*LBL3	21 03	
030	x	-35	
031	-	-45	
032	RTN	24	
033	*LBL4	21 04	Pause for card II
034	MRG	16-62	
035	PSE	16 51	
036	GTO4	22 04	
037	*LBLA	21 11	Initialize
038	9	09	
039	STOI	35 46	
040	*LBL1	21 01	Load input
041	ISZI	16 26 46	
042	2	02	
043	5	05	
044	RCLI	36 46	
045	X≠Y?	16-32	
046	SF2	16 21 02	
047	9	09	
048	-	-45	
049	PSE	16 51	
050	STO:	35 45	
051	F2?	16 23 02	
052	GTO1	22 01	
053	P⇄S	16-51	Compute r_1
054	RCL0	36 00	
055	RCL5	36 05	
056	+	-55	
057	RCLA	36 11	
058	+	-55	
059	RCLI	36 46	
060	+	-55	
061	CHS	-22	
062	PRTX	-14	Display r_1
063	RCL0	36 00	Compute r_2
064	RCLA	36 11	
065	+	-55	
066	RCL5	36 05	
067	x	-35	
068	RCL0	36 00	
069	RCLI	36 46	
070	+	-55	
071	RCLA	36 11	
072	GSB2	23 02	
073	RCL0	36 00	
074	RCL5	36 05	
075	+	-55	
076	RCLI	36 46	
077	GSB2	23 02	
078	RCL7	36 07	
079	RCLD	36 14	
080	GSB3	23 03	
081	RCLB	36 12	
082	RCLE	36 15	
083	GSB3	23 03	
084	RCL6	36 06	
085	RCL9	36 09	
086	GSB3	23 03	
087	RCL1	36 01	
088	RCL4	36 04	
089	GSB3	23 03	
090	RCL2	36 02	
091	RCL8	36 08	
092	GSB3	23 03	
093	RCL3	36 03	
094	RCLC	36 13	
095	GSB3	23 03	
096	PRTX	-14	Display r_2
097	RCL0	36 00	Compute r_3
098	RCL5	36 05	
099	RCL6	36 06	
100	RCL7	36 07	
101	P⇄S	16-51	
102	STO3	35 03	
103	R↓	-31	
104	STO2	35 02	
105	R↓	-31	
106	STO1	35 01	
107	R↓	-31	
108	STO0	35 00	
109	P⇄S	16-51	
110	RCL9	36 09	
111	P⇄S	16-51	
112	STO4	35 04	

REGIS

0	1	2	3	4	5	6	7	8	9
Used	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}

A	B	C	D	E	I
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}

97 Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	RCLA	36 11	
114	STO5	35 05	
115	RCLB	36 12	
116	STO6	35 06	
117	RCLD	36 14	
118	STO7	35 07	
119	RCLE	36 15	
120	STO8	35 08	
121	RCLI	36 46	
122	STO9	35 09	
123	GSB0	23 00	
124	ST×0	35-35 00	
125	P⇄S	16-51	
126	CHS	-22	
127	RCLI	36 46	
128	RCLA	36 11	
129	+	-55	
130	RCLA	36 11	
131	x	-35	
132	RCLA	36 11	
133	RCLI	36 46	
134	GSB2	23 02	
135	RCL7	36 07	
136	RCLD	36 14	
137	GSB3	23 03	
138	RCLB	36 12	
139	RCLE	36 15	
140	GSB3	23 03	
141	RCL6	36 06	
142	RCL9	36 09	
143	GSB3	23 03	
144	RCL0	36 00	
145	GSB3	23 03	
146	RCLA	36 11	
147	RCLI	36 46	
148	+	-55	
149	RCL4	36 04	
150	x	-35	
151	RCL7	36 07	
152	RCLB	36 12	
153	GSB3	23 03	
154	RCL6	36 06	
155	RCL8	36 08	
156	GSB3	23 03	
157	RCL1	36 01	
158	GSB2	23 02	
159	RCL5	36 05	
160	RCLI	36 46	
161	+	-55	
162	RCL8	36 08	
163	x	-35	
164	RCL9	36 09	
165	RCL4	36 04	
166	GSB3	23 03	
167	RCLB	36 12	
168	RCLC	36 13	
169	GSB3	23 03	
170	RCL2	36 02	
171	GSB2	23 02	
172	RCL5	36 05	
173	RCLA	36 11	
174	+	-55	
175	RCLC	36 13	
176	x	-35	
177	RCL8	36 08	
178	RCLE	36 15	
179	GSB3	23 03	
180	RCL4	36 04	
181	RCLD	36 14	
182	GSB3	23 03	
183	RCL3	36 03	
184	GSB2	23 02	
185	PRTX	-14	Display r_3
186	GTO4	22 04	Position memory
187	R/S	51	for new card
190			
200			
210			
220			

LABELS					FLAGS
A Load Data	B	C	D	E	0
a	b	c	d	e	1
0 Used	1 Used	2 x,+	3 x,-	4 Used	2 Used
5	6	7	8	9	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n 2	

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	RCL4	36 04	Compute r_4
002	RCL8	36 08	
003	RCLC	36 13	
004	P⇄S	16-51	
005	STO7	35 07	
006	R↓	-31	
007	STO4	35 04	
008	R↓	-31	
009	STO1	35 01	
010	GSB0	23 00	
011	P⇄S	16-51	
012	RCL1	36 01	
013	x	-35	
014	RCL5	36 05	
015	RCL9	36 09	
016	P⇄S	16-51	
017	RCLD	36 14	
018	STO8	35 08	
019	R↓	-31	
020	STO5	35 05	
021	R↓	-31	
022	STO2	35 02	
023	R↓	-31	
024	ST-0	35-45 00	
025	GSB0	23 00	
026	P⇄S	16-51	
027	RCL2	36 02	
028	x	-35	
029	RCL6	36 06	
030	P⇄S	16-51	
031	STO3	35 03	
032	R↓	-31	
033	ST+0	35-55 00	
034	RCLA	36 11	
035	STO6	35 06	
036	RCLE	36 15	
037	STO9	35 09	
038	GSB0	23 00	
039	P⇄S	16-51	
040	RCL3	36 03	
041	P⇄S	16-51	
042	x	-35	
043	ST-0	35-45 00	
044	RCL0	36 00	
045	PRTX	-14	Display r_4
046	RTN	24	
047	R/S	51	

STEP	KEY ENTRY	KEY CODE	COMMENTS
060			
070			
080			
090			
100			
110			

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

REGISTERS

0	1	2	3	4	5	6	7	8	9
Used	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}

A	B	C	D	E	I
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}

Program Description I

Program Title ONE CARD DETERMINANT AND INVERSE OF A 5×5 MATRIX

Contributor's Name JOHN L. GUSTAFSON

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Program Description, Equations, Variables The matrix A^{-1} is found by computing first the determinant of A, and then the determinant of minor A_{11}, i.e. the matrix (4x4) obtained by deleting the first row and first column of A. If b_1 is the first element of A^{-1}, then

$$b_1 = \frac{\det(A_{11})}{\det(A)}$$

The matrix is then permuted to bring a new element to the a_1 position, and the process is repeated. This continues until an entire column has been found, at which point $\boxed{C}$ may be pressed to find the next column, and so on.

Account must be taken for sign changes in this process, and the fact that the matrix transpose of this "cofactor" matrix is the inverse.

Program execution is quite long, on the order of half an hour to find the entire matrix.

Operating Limits and Warnings Expansion by minors may be inaccurate for an ill-conditioned matrix, like $a_{ij} = \frac{1}{i+j-1}$.

If program is stopped in the middle of execution, data will be in scrambled order in the registers.

Since time between computed values for A^{-1} is long, HP-67 users should probably substitute complete R/S statements for print statements.

Program Description II

Sketch(es)

Data arrangement:

$$\begin{pmatrix} a_1 & a_6 & a_{11} & a_{16} & a_{21} \\ a_2 & a_7 & a_{12} & a_{17} & a_{22} \\ a_3 & a_8 & a_{13} & a_{18} & a_{23} \\ a_4 & a_9 & a_{14} & a_{19} & a_{24} \\ a_5 & a_{10} & a_{15} & a_{20} & a_{25} \end{pmatrix} = \begin{pmatrix} R0 & R5 & S0 & S5 & A \\ R1 & R6 & S1 & S6 & B \\ R2 & R7 & S2 & S7 & C \\ R3 & R8 & S3 & S8 & D \\ R4 & R9 & S4 & S9 & E \end{pmatrix}$$

(Program displays "a" subscript after it is entered.)

Sample Problem(s) Compute the inverse of the tridiagonal matrix

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 2 & 1 \end{pmatrix}$$

(This matrix arises in the problem of fitting a cubic spline to a set of 5 data points.)

Solution(s) 1 [A] 1 [R/S] 0 [R/S] 0 [R/S] 0 [R/S], 2 [R/S] 4 [R/S] 1 [R/S] 0 [R/S] 0 [R/S],
0 [R/S] 1 [R/S] 4 [R/S] 1 [R/S] 0 [R/S], 0 [R/S] 0 [R/S] 1 [R/S] 4 [R/S] 2 [R/S], 0 [R/S] 0 [R/S] 0 [R/S] 1 [R/S] 1 [R/S]
[B] → 12.00 *** (determinant) (several minutes are required.)

	2.17 ***		-1.17 ***		0.33 ***		-0.17 ***		0.17 ***
	-0.58 ***		0.58 ***		-0.17 ***		0.08 ***		-0.06 ***
[C] →	0.17 ***	[C] →	-0.17 ***	[C] →	0.33 ***	[C] →	-0.17 ***	[C] →	0.17 ***
	-0.06 ***		0.08 ***		-0.17 ***		0.58 ***		-0.58 ***
	0.17 ***		-0.17 ***		0.33 ***		-1.17 ***		2.17 ***

(Each column takes about 6 minutes to compute.)

Reference(s)

User Instructions

5x5 Matrix Determinant, Inverse

1 | 2

a_i | $\to \det A$ | $\to A^{-1}$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Load side 1 and side 2.				
2	Input a_1.	a_1	A		1.00
3	Input a_i (repeat for $i=2$ to 25).	a_i	R/S		i
4	Optional: select display mode.		DSP	n	
5	Compute det(A).		B		det A ***
6	Compute first column of A^{-1}.		C		b_1 ***
					b_2 ***
					b_3 ***
					b_4 ***
					b_5 ***
	Compute second column of A^{-1}.		C		b_6 ***
					b_7 ***
					b_8 ***
					b_9 ***
					b_{10} ***
	Compute third column of A^{-1}		C		b_{11} ***
					b_{12} ***
					b_{13} ***
					b_{14} ***
					b_{15} ***
	Compute fourth column of A^{-1}				
			C		b_{16} ***
					b_{17} ***
					b_{18} ***
					b_{19} ***
					b_{20} ***
	Compute last column of A^{-1}		C		b_{21} ***
					b_{22} ***
					b_{23} ***
					b_{24} ***
					b_{25} ***
	HP-67 users may wish to substitute				
	R/S for PRINT at steps 026, 193, 196, 199,				
	202, and 205. When using R/S to output data,				
	it is necessary to push R/S after the last element				
	of a column has been output. Then label C can be called for calculation				

of next column.

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	input a_i, using i as an index register
002	0	00	
003	STOI	35 46	
004	X⇄Y	-41	
005	*LBLa	21 16 11	
006	STOi	35 45	
007	ISZI	16 26 46	
008	RCLI	36 46	
009	R/S	51	
010	GTOa	22 16 11	
011	RTN	24	
012	*LBLB	21 12	
013	0	00	compute determinants of minors times their corresponding a_i, and add to get total determinant.
014	STOI	35 46	
015	GSB0	23 00	
016	GSB5	23 05	
017	GSB0	23 00	
018	GSB5	23 05	
019	GSB0	23 00	
020	GSB5	23 05	
021	GSB0	23 00	
022	GSB5	23 05	
023	GSB0	23 00	
024	GSB5	23 05	
025	CHS	-22	
026	PRTX	-14	
027	SPC	16-11	
028	RTN	24	
029	RTN	24	
030	*LBL0	21 00	Expand 4x4 matrix into its minors to calculate its determinant.
031	0	00	
032	GSB1	23 01	
033	GSB1	23 01	
034	GSB1	23 01	
035	GSB1	23 01	
036	RCLI	36 46	
037	2	02	
038	4	04	
039	STOI	35 46	
040	ENT↑	-21	
041	*LBL4	21 04	
042	R↓	-31	
043	R↓	-31	
044	RCLi	36 45	
045	GSB3	23 03	rotate columns of 4x4
046	GSB3	23 03	
047	GSB3	23 03	
048	GSB3	23 03	
049	X⇄I	16-41	
050	2	02	
051	0	00	
052	+	-55	
053	X⇄I	16-41	
054	STOi	35 45	
055	R↓	-31	
056	DSZI	16 25 46	

STEP	KEY ENTRY	KEY CODE	COMMENTS
057	RCLI	36 46	
058	1	01	
059	9	09	
060	X≠Y?	16-32	
061	GTO4	22 04	
062	R↓	-31	
063	R↓	-31	
064	STOI	35 46	
065	RTN	24	
066	*LBL1	21 01	Find the determinant of the 3x3 minor of the 4x4 minor;
067	P⇄S	16-51	
068	RCL2	36 02	
069	RCL8	36 08	
070	x	-35	
071	RCLE	36 15	
072	x	-35	
073	RCL7	36 07	
074	RCLD	36 14	
075	x	-35	
076	RCL4	36 04	
077	x	-35	
078	+	-55	
079	RCLC	36 13	
080	RCL3	36 03	
081	x	-35	
082	RCL9	36 09	
083	x	-35	
084	+	-55	
085	RCLC	36 13	
086	RCL8	36 08	
087	x	-35	
088	RCL4	36 04	
089	x	-35	
090	-	-45	
091	RCL7	36 07	
092	RCL3	36 03	
093	x	-35	
094	RCLE	36 15	
095	x	-35	
096	-	-45	
097	RCL2	36 02	
098	RCLD	36 14	
099	x	-35	
100	RCL9	36 09	
101	x	-35	
102	-	-45	
103	P⇄S	16-51	
104	RCL6	36 06	multiply by first entry of the 4x4 matrix, accumulate sum
105	x	-35	
106	X⇄Y	-41	
107	-	-45	
108	RCLI	36 46	
109	2	02	
110	4	04	
111	STOI	35 46	
112	ENT↑	-21	

REGISTERS

0	1	2	3	4	5	6	7	8	9
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
S0	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

A	B	C	D	E	I
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	det(A), index

97 Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	*LBL2	21 02	rotate columns of 3x3
114	R↓	-31	
115	R↓	-31	
116	RCLi	36 45	
117	GSB3	23 03	
118	GSB3	23 03	
119	GSB3	23 03	
120	X⇄I	16-41	
121	1	01	
122	5	05	
123	+	-55	
124	X⇄I	16-41	
125	STOi	35 45	
126	DSZI	16 25 46	
127	R↓	-31	
128	RCLI	36 46	
129	2	02	
130	0	00	
131	X≠Y?	16-32	
132	GTO2	22 02	
133	R↓	-31	
134	R↓	-31	
135	STOI	35 46	
136	R↓	-31	
137	RTN	24	
138	*LBL3	21 03	rotation of just one datum
139	X⇄I	16-41	
140	5	05	
141	-	-45	
142	X⇄I	16-41	
143	RCLi	36 45	
144	X⇄Y	-41	
145	STOi	35 45	
146	R↓	-31	
147	RTN	24	
148	*LBLD	21 14	permute matrix rows (5x5)
149	RCLI	36 46	
150	2	02	
151	4	04	
152	STOI	35 46	
153	ENT↑	-21	
154	*LBL6	21 06	
155	R↓	-31	
156	R↓	-31	
157	RCLi	36 45	
158	GSB7	23 07	
159	GSB7	23 07	
160	GSB7	23 07	
161	GSB7	23 07	
162	X⇄I	16-41	
163	4	04	
164	+	-55	
165	X⇄I	16-41	
166	STOi	35 45	
167	R↓	-31	
168	RCLI	36 46	

STEP	KEY ENTRY	KEY CODE	COMMENTS
169	5	05	
170	-	-45	
171	STOI	35 46	
172	1	01	
173	CHS	-22	
174	X≠Y?	16-32	
175	GTO6	22 06	
176	R↓	-31	
177	R↓	-31	
178	STOI	35 46	
179	RTN	24	
180	*LBL7	21 07	rotation of just one datum
181	X⇄I	16-41	
182	1	01	
183	-	-45	
184	X⇄I	16-41	
185	RCLi	36 45	
186	X⇄Y	-41	
187	STOi	35 45	
188	R↓	-31	
189	RTN	24	
190	*LBLC	21 13	compute one column of A^{-1}; rotate to prepare to compute next column
191	GSB0	23 00	
192	÷	-24	
193	PRTX	-14	
194	GSB0	23 00	
195	÷	-24	
196	PRTX	-14	
197	GSB0	23 00	
198	÷	-24	
199	PRTX	-14	
200	GSB0	23 00	
201	÷	-24	
202	PRTX	-14	
203	GSB0	23 00	
204	÷	-24	
205	PRTX	-14	
206	GSBD	23 14	rotate rows
207	SPC	16-11	
208	RTN	24	
209	*LBL5	21 05	multiplies 4x4 determinant by a_{1j} (the element about which the 4x4 is a minor.)
210	X⇄Y	-41	
211	RCLA	36 11	
212	x	-35	
213	+	-55	
214	STOI	35 46	
215	RTN	24	
216	R/S	51	
220			

LABELS

A	B	C	D	E
Store a_i	find det(A)	find A^{-1}	rotate rows	
a loop for A	b	c	d	e
0 used	1 det of 3x3	2 used	3 used	4 used
5 used	6 used	7 used	8	9

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n 2	

Program Description I

Program Title SIMULTANEOUS EQUATIONS IN SIX UNKNOWNS

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Program Description, Equations, Variables

Coefficient Matrix:

$$\begin{Bmatrix} r1 & s1 & t1 & u1 & v1 & w1 & k1 \\ r2 & s2 & t2 & u2 & v2 & w2 & k2 \\ r3 & s3 & t3 & u3 & v3 & w3 & k3 \\ r4 & s4 & t4 & u4 & v4 & w4 & k4 \\ r5 & s5 & t5 & u5 & v5 & w5 & k5 \\ r6 & s6 & t6 & u6 & v6 & w6 & k6 \end{Bmatrix}$$

By Crout's method, let (a_{ij}) be the elements of the given matrix and (A_{ij}) be the elements of the derived matrix. Then

$$A_{ii} = a_{ii} - \sum_{k=1}^{i-1} A_{ik}A_{ki} \qquad \text{(diagonal terms)}$$

$$A_{ij} = a_{ij} - \sum_{k=1}^{j-1} A_{ik}A_{kj} \qquad (i > j\text{, gives the lower half})$$

$$A_{ij} = \left\{a_{ij} - \sum_{k=1}^{i-1} A_{ik}A_{kj}\right\} / A_{ii} \qquad (i < j\text{, gives the upper half})$$

The solution vector is

$$X_i = A_{i,n+1} - \sum_{k=i+1}^{n} A_{ik}X_k \qquad (i = 1, \ldots, n)$$

Operating Limits and Warnings

1. Re-order the equations, such that r1 is not zero.
2. "Error" implies inconsistency.

Program Description II

Sketch(es)

Sample Problem(s) Compute the solution for the following set of equations:

Variables and coefficient matrix:

r	s	t	u	v	w	k
17	34	170	748	3816.5	19669	1781.6
34	170	748	3816.5	19669	105325.625	4864.65
170	748	3816.5	19669	105325.625	573286.75	22810.975
748	3816.5	19669	105325.625	573286.75	3172438.532	90845.9625
3816.5	19669	105325.625	573286.75	3172438.532	17757325.57	412295.4438
19669	105325.625	573286.75	3172438.532	17757325.57	100361561.9	1856770.791

Solution(s)

r = -11.52568830

s = -28.86312210

t = 45.32824695

u = 1.755025950

v = -2.615475715

w = .1994145369

Reference(s)

Nielsen, Kaj L., <u>Methods in Numerical Analysis</u>, page 185, The Macmillan Company, 1956.

Program Description II

Sketch(es)

Sample Problem(s) **Compute the determinant and the inverse of the coefficient matrix in the preceding problem.**

1. In order to compute the determinant of the coefficient matrix, less the k vector, do the following:

1.1 Record the values obtained after calculation for the following:

s2 => record c1 t3 => record c2

u4 => record c3 v5 => record c4

w6 => record c5

1.2 The determinant = r1 x c1 x c2 x c3 x c4 x c5

2. The inverse may be computed by substituting each column of the identity matrix of order 5 for the k vector. The solutions obtained by solving the system with this program represent the respective column vectors of the inverse matrix.

Solution(s) **determinant =**

17 x 102 x 484.5 x 2180.25 x 9447.7507 x 39365.555 = 6.812303266 x 10^{17}

$$A^{-1} = \begin{bmatrix} .23704060 & -.00407230 & -.07938403 & .00976420 & -.00381044 & -.00057157 \\ -.00407230 & .30094782 & -.06239595 & -.06620895 & .02375172 & -.00206822 \\ -.07938403 & -.06239595 & .06240250 & .00928796 & -.00859995 & .00095261 \\ .00976420 & -.06620895 & .00928796 & .01782172 & -.00603319 & .00051864 \\ .00381044 & .02375172 & -.00859995 & -.00603319 & .00264614 & -.00025403 \\ -.00057157 & -.00206822 & .00095261 & .00051864 & -.00025403 & .00002540 \end{bmatrix}$$

Reference(s)

User Instructions

1 6 EQUATIONS CARD 1/2 2

START

RCL (for card 2/2)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Load side 1 and side 2 of card 1/2.				
2.	Place side 1 of card 2/2 into reader slot				
	but do not engage.				
3.	Enter r1:	r1	A		r1
4.	Enter s1, t1, u1, v1, w1, k1,				
	r2, s2, t2, u2, ... thru v5				
	followed by R/S each entry:		R/S		
5.	Immediately upon entry of v5:R/S, engage				
	card 2/2 in the card reader. The card will				
	be read automatically. The calculator will				
	stop with value A_{55} after side 2 of the card				
	has been entered.				A_{55}
6.	Enter w5, k5, r6, s6, t6, ... thru k6				
	followed by R/S each entry:		R/S		
7.	After entry of k6, the calculator will run				
	for approximately 20 seconds and will stop				
	with solution r.				r
8.	To display all solutions:		A		r
			R/S		s
			R/S		t
			R/S		u
			R/S		v
			R/S		w
	Note: If, during step 5, you fail to engage				
	card 2/2 in the card reader, then do the				
	following steps:				
9.	Prepare for Merge:		g	MERGE	
10.	Enter card 2/2, side 1 and side 2.				
11.	Start program:		R/S		A_{55}
12.	Go to Step 6.				

SIX EQUATIONS CARD1/2 **67 Program Listing I** Side 1.

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	f LBL B	31 25 12	
	x	71	
	−	51	
	f LBL C	31 25 13	
	RCL 0	34 00	
	÷	81	
	STO(i)	33 24	
	f DSZ	31 33	
	h RTN	35 22	
010	f LBL 0	31 25 00	RCL 10
	0	00	
	GTO D	22 14	
	f LBL 1	31 25 01	RCL 11
	1	01	
	GTO D	22 14	
	f LBL 2	31 25 02	RCL 12
	2	02	
	GTO D	22 14	
	f LBL 3	31 25 03	RCL 13
020	3	03	
	GTO D	22 14	
	f LBL 4	31 25 04	RCL 14
	4	04	
	GTO D	22 14	
	f LBL 5	31 25 05	RCL 15
	5	05	
	GTO D	22 14	
	f LBL 6	31 25 06	RCL 16
	6	06	
030	GTO D	22 14	
	f LBL 7	31 25 07	RCL 17
	7	07	
	GTO D	22 14	
	f LBL 8	31 25 08	RCL 18
	8	08	
	GTO D	22 14	
	f LBL 9	31 25 09	RCL 19
	9	09	
	f LBL D	31 25 14	
040	1	01	
	0	00	
	+	61	
	h X:I	35 24	
	RCL(i)	34 24	
	h X:Y	35 52	
	h X:I	35 24	
	h R↓	35 53	
	f LBL E	31 25 15	
	x	71	
050	−	51	
	h RTN	35 22	
	g LBLfa	32 25 11	Overlay
	g MERGE	32 41	control.
	h PAUSE	35 72	
	R/S	84	
	h SPACE	35 84	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	f LBL A	31 25 11	START:
	STO 0	33 00	with r1.
	R/S	84	Enter: s1.
060	h X:I	35 24	
	2	02	
	4	04	
	h X:I	35 24	
	f GSB C	31 22 13	
	R/S	84	t1
	f GSB C	31 22 13	
	R/S	84	u1
	f GSB C	31 22 13	
	R/S	84	v1
070	f GSB C	31 22 13	
	R/S	84	w1
	f GSB C	31 22 13	
	R/S	84	k1
	f GSB C	31 22 13	
	R/S	84	r2
	STO 1	33 01	
	R/S	84	s2
	RCL 1	34 01	
	RCL E	34 15	
080	f GSB E	31 22 15	
	STO 0	33 00	
	R/S	84	t2
	RCL 1	34 01	
	RCL D	34 14	
	f GSB B	31 22 12	
	R/S	84	u2
	RCL 1	34 01	
	RCL C	34 13	
	f GSB B	31 22 12	
090	R/S	84	v2
	RCL 1	34 01	
	RCL B	34 12	
	f GSB B	31 22 12	
	R/S	84	w2
	RCL 1	34 01	
	RCL A	34 11	
	f GSB B	31 22 12	
	R/S	84	k2
	RCL 1	34 01	
100	f GSB 9	31 22 09	
	f GSB C	31 22 13	
	R/S	84	r3
	STO 2	33 02	
	R/S	84	s3
	RCL 2	34 02	
	RCL E	34 15	
	f GSB E	31 22 15	
	STO 1	33 01	
	R/S	84	t3
110	RCL 2	34 02	
	RCL D	34 14	
	f GSB E	31 22 15	

REGISTERS

0	1	2	3	4	5	6	7	8	9
s	t	u	v	w	r	used	used	used	used

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
used	used	used	used	used	used	used	used	used	used

A	B	C	D	E	I
used	used	used	used	used	used

67 Program Listing II

Side 2.

STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCL 1	34 01	
	f GSB 8	31 22 08	
	STO 0	33 00	
	R/S	84	u3
	RCL 2	34 02	
	RCL C	34 13	
	f GSB E	31 22 15	
120	RCL 1	34 01	
	f GSB 7	31 22 07	
	f GSB C	31 22 13	
	R/S	84	v3
	RCL 2	34 02	
	RCL B	34 12	
	f GSB E	31 22 15	
	RCL 1	34 01	
	f GSB 6	31 22 06	
	f GSB C	31 22 13	
130	R/S	84	w3
	RCL 2	34 02	
	RCL A	34 11	
	f GSB E	31 22 15	
	RCL 1	34 01	
	f GSB 5	31 22 05	
	f GSB C	31 22 13	
	R/S	84	k3
	RCL 2	34 02	
	f GSB 9	31 22 09	
140	RCL 1	34 01	
	f GSB 4	31 22 04	
	f GSB C	31 22 13	
	R/S	84	r4
	STO 3	33 03	
	R/S	84	s4
	RCL 3	34 03	
	RCL E	34 15	
	f GSB E	31 22 15	
	STO 2	33 02	
150	R/S	84	t4
	RCL 3	34 03	
	RCL D	34 14	
	f GSB E	31 22 15	
	RCL 2	34 02	
	f GSB 8	31 22 08	
	STO 1	33 01	
	R/S	84	u4
	RCL 3	34 03	
	RCL C	34 13	
160	f GSB E	31 22 15	
	RCL 2	34 02	
	f GSB 7	31 22 07	
	RCL 1	34 01	
	f GSB 3	31 22 03	
	STO 0	33 00	
	R/S	84	v4
	RCL 3	34 03	
	RCL B	34 12	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	f GSB E	31 22 15	
170	RCL 2	34 02	
	f GSB 6	31 22 06	
	RCL 1	34 01	
	f GSB 2	31 22 02	
	f GSB C	31 22 13	
	R/S	84	w4
	RCL 3	34 03	
	RCL A	34 11	
	f GSB E	31 22 15	
	RCL 2	34 02	
180	f GSB 5	31 22 05	
	RCL 1	34 01	
	f GSB 1	31 22 01	
	f GSB C	31 22 13	
	R/S	84	k4
	RCL 3	34 03	
	f GSB 9	31 22 09	
	RCL 2	34 02	
	f GSB 4	31 22 04	
	RCL 1	34 01	
190	f GSB 0	31 22 00	
	f GSB C	31 22 13	
	R/S	84	r5
	STO 4	33 04	
	R/S	84	s5
	RCL 4	34 04	
	RCL E	34 15	
	f GSB E	31 22 15	
	STO 3	33 03	
	R/S	84	t5
200	RCL 4	34 04	
	RCL D	34 14	
	f GSB E	31 22 15	
	RCL 3	34 03	
	f GSB 8	31 22 08	
	STO 2	33 02	u5
	R/S	84	
	RCL 4	34 04	
	RCL C	34 13	
	f GSB E	31 22 15	
210	RCL 3	34 03	
	f GSB 7	31 22 07	
	RCL 2	34 02	
	f GSB 3	31 22 03	
	STO 1	33 01	
	R/S	84	v5
	RCL 4	34 04	
	RCL B	34 12	
	f GSB E	31 22 15	
	RCL 3	34 03	
220	f GSB 6	31 22 06	
	RCL 2	34 02	
	f GSB 2	31 22 02	
	RCL 1	34 01	
	GTO f a	22 31 11	

LABELS

A	B	C	D	E
START	used	used	used	used
a used	b	c	d	e
0 used	1 used	2 used	3 used	4 used
5 used	6 used	7 used	8 used	9 used

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

SIX EQUATIONS

67 Program Listing I

Side 1.

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	RCL 9	34 09	
	f GSB E	31 22 15	
	STO 0	33 00	
	R/S	84	w5
	RCL 4	34 04	
	RCL A	34 11	
	f GSB E	31 22 15	
	RCL 3	34 03	
	f GSB 5	31 22 05	
010	RCL 2	34 02	
	f GSB 1	31 22 01	
	RCL 1	34 01	
	RCL 8	34 08	
	f GSB B	31 22 12	
	R/S	84	k5
	RCL 4	34 04	
	f GSB 9	31 22 09	
	RCL 3	34 03	
	f GSB 4	31 22 04	
020	RCL 2	34 02	
	f GSB 0	31 22 00	
	RCL 1	34 01	
	RCL 7	34 07	
	f GSB B	31 22 12	
	STO 0	33 00	
	R/S	84	r6
	STO 5	33 05	
	R/S	84	s6
	RCL 5	34 05	
030	RCL E	34 15	
	f GSB E	31 22 15	
	STO 4	33 04	
	R/S	84	t6
	RCL 5	34 05	
	RCL D	34 14	
	f GSB E	31 22 15	
	RCL 4	34 04	
	f GSB 8	31 22 08	
	STO 3	33 03	
040	R/S	84	u6
	RCL 5	34 05	
	RCL C	34 13	
	f GSB E	31 22 15	
	RCL 4	34 04	
	f GSB 7	31 22 07	
	RCL 3	34 03	
	f GSB 3	31 22 03	
	STO 2	33 02	
	R/S	84	v6
050	RCL 5	34 05	
	RCL B	34 12	
	f GSB E	31 22 15	
	RCL 4	34 04	
	f GSB 6	31 22 06	
	RCL 3	34 03	
	f GSB 2	31 22 02	
	RCL 2	34 02	
	RCL 9	34 09	
	f GSB E	31 22 15	
060	STO 1	33 01	
	R/S	84	w6
	RCL 5	34 05	
	RCL A	34 11	
	f GSB E	31 22 15	
	RCL 4	34 04	
	f GSB 5	31 22 05	
	RCL 3	34 03	
	f GSB 1	31 22 01	
	RCL 2	34 02	
070	RCL 8	34 08	
	f GSB E	31 22 15	
	RCL 1	34 01	
	RCL 6	34 06	
	f GSB E	31 22 15	
	RCL 0	34 00	
	h X:Y	35 52	
	STO 0	33 00	
	R/S	84	k6
	h X:Y	35 52	
080	h R↓	35 53	
	h X:Y	35 52	
	RCL 5	34 05	
	h X:Y	35 52	
	STO 5	33 05	
	h R↓	35 53	
	f GSB 9	31 22 09	
	RCL 4	34 04	
	f GSB 4	31 22 04	
	RCL 3	34 03	
090	f GSB 0	31 22 00	
	RCL 2	34 02	
	RCL 7	34 07	
	f GSB E	31 22 15	
	RCL 1	34 01	
	RCL 5	34 05	
	f GSB B	31 22 12	soln: w
	RCL 5	34 05	
	RCL 4	34 04	
	RCL 6	34 06	
100	f GSB E	31 22 15	
	STO 3	33 03	v
	RCL 7	34 07	
	RCL 4	34 04	
	RCL 8	34 08	
	f GSB E	31 22 15	
	RCL 3	34 03	
	RCL 9	34 09	
	f GSB E	31 22 15	
	STO 2	33 02	u
110	0	00	
	g GSBf e	32 22 15	
	f GSB 1	31 22 01	

REGISTERS

0	1	2	3	4	5	6	7	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

67 Program Listing II

X EQUATIONS — Side 2.

STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCL 3	34 03	
	f GSB 2	31 22 02	
	RCL 2	34 02	
	f GSB 3	31 22 03	
	STO 1	33 01	t
	4	04	
	g GSBfe	32 22 15	
120	f GSB 5	31 22 05	
	RCL 3	34 03	
	f GSB 6	31 22 06	
	RCL 2	34 02	
	f GSB 7	31 22 07	
	RCL 1	34 01	
	f GSB 8	31 22 08	
	STO 0	33 00	s
	9	09	
	g GSBfe	32 22 15	
130	RCL A	34 11	
	f GSB E	31 22 15	
	RCL 3	34 03	
	RCL B	34 12	
	f GSB E	31 22 15	
	RCL 2	34 02	
	RCL C	34 13	
	f GSB E	31 22 15	
	RCL 1	34 01	
	RCL D	34 14	
140	f GSB E	31 22 15	
	RCL 0	34 00	
	RCL E	34 15	
	f GSB E	31 22 15	
	STO 5	33 05	r
	f LBL A	31 25 11	RCL solutions.
	RCL 5	34 05	r
	R/S	84	
	RCL 0	34 00	s
	R/S	84	
150	RCL 1	34 01	t
	R/S	84	
	RCL 2	34 02	u
	R/S	84	
	RCL 3	34 03	v
	R/S	84	
	RCL 4	34 04	w
	R/S	84	
	GTO A	22 11	
	g LBL fe	32 25 15	
160	1	01	
	0	00	
	+	61	
	h STI	35 33	
	RCL(i)	34 24	
	RCL 4	34 04	
	h RTN	35 22	

STEP	KEY ENTRY	KEY CODE	COMMENTS
170			
180			
190			
200			
210			
220			

LABELS

A	B	C	D	E
a	b	c	d	e
0	1	2	3	4
5	6	7	8	9

FLAGS

0
1
2
3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Program Description I

Program Title Roots of Polynomials

Contributor's Name Richard K. Brush
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Program Description, Equations, Variables Let the polynomial be defined by $f(z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n$ where $z = x + jy$.

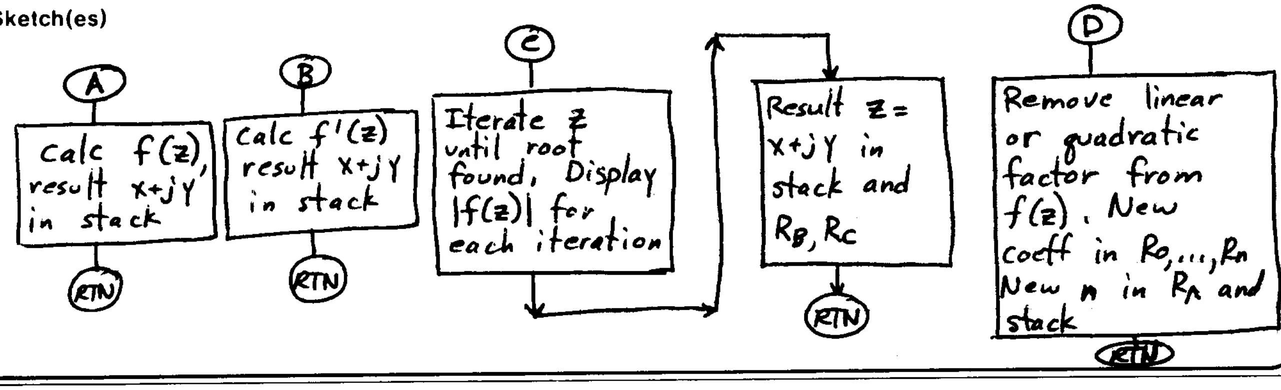
1. Let $z = z_0$ be estimate of root of $f(z)$.
2. Calculate $f(z)$.
3. Calculate $f'(z) = a_1 + 2a_2 z + 3a_3 z^2 + \ldots + n \cdot a_n z^{n-1}$.
4. Calculate $\Delta z = -\dfrac{f(z)}{f'(z)}$.
5. Calculate new estimate of z: $z_{k+1} = z_k + \Delta z$.
6. If $|f(z_{k+1})| > |f(z_k)|$, then set $\Delta z \leftarrow \Delta z/2$, and repeat step 5, until $|f(z_{k+1})| \leq |f(z_k)|$ is satisfied.
7. Repeat 2-6 until $|f(z)| < \epsilon$, where ϵ is supplied by user.
8. If $|\mathrm{Imag}(z)| < \epsilon$ then set $\mathrm{Imag}(z) \leftarrow 0$, and repeat final iteration.
9. Remove the linear factor $(z - z_{root})$ or quadratic factor $(z - z_{root})(z - z_{root}^*)$ by synthetic division.

Operating Limits and Warnings

1. The user must store $n, a_0, a_1, \ldots, a_n$, ϵ, and z_0 in the appropriate registers before program execution.
2. $n \leq 14$
3. $\boxed{h}$ $\boxed{\text{DEG}}$ is used for a "NOP" instruction; hence use DEG mode.
4. $f(z)$ and $f'(z)$ is evaluated using rectangular complex multiplication, which is faster and more accurate than R→P and P→R functions.

Program Description II

Sketch(es)



Sample Problem(s)

1. Let $f(z) = 15 + 11z + 5z^2 + z^3 = (z+3)(z^2+2z+5)$
$f'(z) = 11 + 10z + 3z^2$

Roots are $z_r = -3$, $z_r = -1 \pm j2$.

$f(-2) = 5$, $f'(-2) = 3$

Enter data: $R_A \leftarrow 3$, $R_0 \leftarrow 15$, $R_1 \leftarrow 11$, $R_2 \leftarrow 5$, $R_3 \leftarrow 1$, $R_{59} \leftarrow 1 \times 10^{-8}$.

Solution(s)

Input	Execute	Intermediate Results	Results	
$R_B \leftarrow -2$, $R_C \leftarrow 0$ (z_0)				
	$\boxed{A}$		5	$f(-2)$
	$\boxed{B}$		3	$f'(-2)$
	$\boxed{C}$	5, 7.41, 1.23, .122, 9.04×10^{-4}, 5×10^{-8}, 0.00 $\lvert f(z) \rvert$	-3	z_{root}
$R_B \leftarrow -1$, $R_C \leftarrow 1$ (z_0)	$\boxed{C}$	6.71, 11.5, 5.22, 5.15, .980, .0538, 1.43×10^{-4}, 3×10^{-9}	$-.9999999998$ $+j\,2.000000000$	

Reference(s) Any standard textbook on numerical analysis, e.g. Conte and de Boor, "Elementary Numerical Analysis: An Algorithmic Approach", McGraw-Hill, 1972

Program Description II (Cont.)

Sample Problem No. 2

Find the transfer function $G(s)$ and poles of $G(s)$ (i.e. roots of denominator polynomial) given the magnitude-squared function:

$$M^2(\omega) = |G(j\omega)|^2 = \frac{1}{1 + .5\omega^6 + \omega^{12}}$$

Solution:

From circuit theory, $M^2(\omega) = G(j\omega) \cdot G(-j\omega) = g(-\omega^2)$, where $g(s^2) = G(s) \cdot G(-s)$. Hence, substituting $s^2 = -\omega^2$, gives:

$$G(s) \cdot G(-s) = g(s^2) = \frac{1}{1 - .5s^6 + s^{12}}.$$

The above function has 12 poles, symmetric about the origin, 6 in the right half plane (RHP) and 6 in the left half plane (LHP), belonging to $G(-s)$ and $G(s)$ respectively. Hence we can find 4 complex poles at a time:

Input	Execute	Intermediate Results	Results	Variable(s)
	$\boxed{f}$ $\boxed{CLREG}$			
$R_9 \leftarrow 1 \times 10^{-8}$				ϵ
$R_2 \leftarrow 1$				a_{12}
	$\boxed{f}$ $\boxed{P{\leftrightarrows}S}$			
	$\boxed{f}$ $\boxed{CLREG}$			
$R_0 \leftarrow 1$, $R_6 \leftarrow -.5$				a_0, a_6
$R_A \leftarrow 12$				n
$R_B \leftarrow 1$, $R_C \leftarrow 1$				z_0
	$\boxed{C}$	63.1, 22.0, 7.50, 2.41, .655, .110, 5.13×10^{-3}, 1.25×10^{-5}, 5×10^{-10}		$\lvert f(z) \rvert$
			.6767099819 +j.7362496862	z_{root} (RHP)
	$\boxed{g}$ $\boxed{R{\to}P}$		1.000000000 ∡ 47.41291870°	

Program Description II (Cont.)

Input	Execute	Intermediate Results	Results	Variable(s)
	$\boxed{D}$		10	n
	$\boxed{RCL}$ $\boxed{B}$			
	$\boxed{CHS}$		−.6767099819	Real(Z_{root}), in LHP
	$\boxed{STO}$ $\boxed{B}$			
	$\boxed{D}$		8	n
	$\boxed{C}$	2.92, 1.38, 1.01, * 4.55×10^{9}, 2.12×10^{7}, 1.17×10^{5}, 876, 9.72, .866, 3.02, 1.00, .674, 1.14, .182, .0189, 1.61×10^{-4}, 1.21×10^{-8}, 4.12×10^{-10}		$\lvert f(z) \rvert$
			.2992559410 +j .9541728785	Z_{root} (RHP)
	$\boxed{g}$ $\boxed{R \to P}$		1.000000000 ∠ 72.58708130°	
	$\boxed{D}$		6	n
	$\boxed{RCL}$ $\boxed{B}$			
	$\boxed{CHS}$			Z_{root} (LHP)
	$\boxed{STO}$ $\boxed{B}$			
	$\boxed{D}$		4	n
	$\boxed{C}$	3.45, 1.33, .612, .181 * 1.66, 1.18, .577, .285, .202, .184, .181, 13.7, 2.20, .405, .152, .0545, 5.81×10^{-3}, 5.12×10^{-5}, 4.02×10^{-9}		$\lvert f(z) \rvert$
			−.9759659202 +j .2179231925	Z_{root} (LHP)
	$\boxed{CHS}$			
	$\boxed{g}$ $\boxed{R \to P}$		.9999999976 ∠ 12.58708135°	

* Underlined values of $|f(z)|$ represent iterations in which the modified form of Newton's method is used. (See page 1, Program Description, step 6.)

Program Description II (Cont.)

The poles of $G(s)$ are thus:

$-.67671 \pm j.73625,\ -.29926 \pm j.95417,\ -.97597 \pm j.21792$

or $1.000 \measuredangle (180 \pm 47.41)^\circ,\ 1.000 \measuredangle (180. \pm 72.59)^\circ,\ 1.000 \measuredangle (180 \pm 12.59)^\circ$

To find the coefficients of $G(s)$, remove the RHP poles from $g(s^2)$:

Input	Execute	Results	Variable(s)
	$\boxed{f}$ $\boxed{\text{CLREG}}$		
$R_2 \leftarrow 1$			a_{12}
	$\boxed{f}$ $\boxed{P \lessgtr S}$		
	$\boxed{f}$ $\boxed{\text{CLREG}}$		
$R_0 \leftarrow 1$, $R_6 \leftarrow -.5$			a_0, a_6
$R_A \leftarrow 12$			n
$R_B \leftarrow .6767099819$			
$R_C \leftarrow .7362496862$			Zroot (RHP)
	$\boxed{D}$	10	n
$R_B \leftarrow .2992559410$			
$R_C \leftarrow .9541728785$			Zroot (RHP)
	$\boxed{D}$	8	n
$R_B \leftarrow .9759659202$			
$R_C \leftarrow .2179231925$			Zroot (RHP)
	$\boxed{D}$	6	n
	$\boxed{\text{RCL}}$ $\boxed{0}$	.9999998120	a_0
	$\boxed{\text{RLC}}$ $\boxed{1}$	3.903863530	a_1
	$\boxed{\text{RCL}}$ $\boxed{2}$	7.620075745	a_2
	$\boxed{\text{RCL}}$ $\boxed{3}$	9.388866152	a_3
	$\boxed{\text{RCL}}$ $\boxed{4}$	7.620075835	a_4
	$\boxed{\text{RCL}}$ $\boxed{5}$	3.903863686	a_5
	$\boxed{\text{RCL}}$ $\boxed{6}$	1.000000000	a_6

Program Description II (Cont.)

Hence, $G(s) =$

$$\frac{1}{1 + 3.90386s + 7.62008s^2 + 9.3887s^3 + 7.62008s^4 + 3.90386s^5 + s^6}$$

Reference:

Franklin F. Kuo, "Network Analysis and Synthesis", John Wiley and Sons, 1966, p. 370

User Instructions

Roots of Polynomials

$f(z)$ | $f'(z)$ | Find z_{root} | Reduce

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Store n: $R_A \leftarrow n$	n		
3	Store coefficients $a_0, a_1, \ldots, a_n$ in $R_0, R_1, \ldots, R_n$	$a_0, a_1, \ldots, a_n$		
4	Store ϵ: $R_{S9} \leftarrow \epsilon$	ϵ		
5	Store initial z_0: $R_B \leftarrow Real(z_0)$, $R_C \leftarrow Imag(z_0)$	z_0		
6	*To evaluate polynomial, $f(z)$ (Result in stack)		A	$f(z) = x + jy$
7	*To evaluate derivative, $f'(z)$ (Result in stack)		B	$f'(z) = x + jy$
8	To find root z_{root} of $f(z)$ (z_{root} is displayed in stack, and also stored in R_B, R_C)		C	$\lvert f(z_k) \rvert$ for each iteration; $z_{root} = x + jy$
9	To remove the linear (real root) or quadratic factor (complex root) from $f(z)$ ($n \leftarrow n-1$ for real, $n \leftarrow n-2$ for complex, New coefficients in $R_0, R_1, \ldots, R_n$)		D	n
10	Go back to step 5 for next root(s). Previous z_{root} can be used for new z_0 if desired (in which case go to step 6).			
11	When $n = 1$ or 2, z_{root} may be found manually in closed form.			
	* Optional			

67 Program Listing I

	STEP	KEY ENTRY	KEY CODE	COMMENTS
E	001	f LBL E	31 25 15	[E] Subroutine.
	2	STO D	33 14	Mult (x+jY) by
	3	RCL C	34 13	(R_B+jR_C).
	4	X	71	
	5	h x≷y	35 52	Result is
	6	STO E	33 15	
	7	RCL B	34 12	(x+jY)
	8	X	71	
	9	+	61	
	010	RCL D	34 14	Uses R_D, R_E
	1	RCL B	34 12	for temp
	2	X	71	storage.
	3	RCL C	34 13	
	4	RCL E	34 15	
	5	X	71	
	6	−	51	
	7	h RTN	35 22	RETURN
A	8	f LBL A	31 25 11	[A] Subroutine.
	9	RCL A	34 11	Calc f(z), where
	020	h STI	35 33	$z = R_B + j R_C$
	1	0	00	
	2	ENTER	41	Calls subr. E.
	3	0	00	
(a	4	g LBL a	32 25 11	
	5	RCL (i)	34 24	
	6	+	61	
	7	f GSB E	31 22 15	
	8	f DSZ	31 33	
	9	GTO f a	22 31 11	
	030	RCL (i)	34 24	
	1	+	61	
	2	h RTN	35 22	RETURN
B	3	f LBL B	31 25 12	[B] Subroutine.
	4	RCL A	34 11	Calc f'(z).
	5	h STI	35 33	
	6	0	00	Calls subr E.
	7	ENTER	41	
	8	0	00	
(b	9	g LBL b	32 25 12	
	040	h RCI	35 34	
	1	RCL (i)	34 24	
	2	X	71	
	3	+	61	
	4	f DSZ	31 33	
	5	f GSB E	31 22 15	
	6	f ISZ	31 34	
	7	f DSZ	31 33	
	8	GTO f b	22 31 12	
	9	h RTN	35 22	RETURN
C	050	f LBL C	31 25 13	[C] Subroutine.
	1	h SF 2	35 51 02	1st iter flag.
(c	2	g LBL c	32 25 13	(c) Calc f(z)
	3	f GSB A	31 22 11	
	4	g R→P	32 72	Display |f(z)|
	5	h PAUSE	35 72	
	6	f P≷S	31 42	

	STEP	KEY ENTRY	KEY CODE	COMMENTS
	7	f x=0	31 51	If |f(z)| = 0,
	8	GTO 9	22 09	GoTo 9
	9	RCL 9	34 09	If |f(z)| < ε,
	060	g x>y	32 81	
	1	GTO 8	22 08	GoTo 8
	2	h R↓	35 53	
	3	RCL 7	34 07	If 1st iter,
	4	h F? 2	35 71 02	goto 7
	5	GTO 7	22 07	
	6	g x>y	32 81	If $\|f(z_k)\| <$
	7	GTO 7	22 07	$\|f(z_{k-1})\|$, GoTo 7
	8	2	02	"Back up" Δz:
	9	STO ÷ 5	33 81 05	
	070	STO ÷ 6	33 81 06	$\Delta z \leftarrow \Delta z/2$
	1	RCL B	34 12	
	2	RCL 5	34 05	
	3	−	51	
	4	STO B	33 12	
	5	RCL C	34 13	$z \leftarrow z - \Delta z$
	6	RCL 6	34 06	
	7	−	51	
	8	STO C	33 13	
	9	f P≷S	31 42	Recalc f(z) with
	080	GTO f c	22 31 13	smaller Δz
(7	1	f LBL 7	31 25 07	(7) Normal
	2	h R↓	35 53	iteration:
	3	STO 7	33 07	Save
	4	h x≷y	35 52	$R_{S7}, R_{S8} \leftarrow f(z)_{polar}$
	5	STO 8	33 08	
	6	f P≷S	31 42	
	7	f GSB B	31 22 12	Calc f'(z)
	8	g R→P	32 72	
	9	f P≷S	31 42	Calc $\Delta z = -\frac{f(z)}{f'(z)}$:
	090	h 1/x	35 62	
	1	RCL 7	34 07	
	2	X	71	
	3	CHS	42	$x = -\|\Delta z\|$
	4	h x≷y	35 52	
	5	CHS	42	
	6	RCL 8	34 08	
	7	+	61	$x = \measuredangle \Delta z$
	8	h x≷y	35 52	
	9	f R←P	31 72	
	100	STO 5	33 05	Save
	1	RCL B	34 12	$R_{S5}, R_{S6} \leftarrow \Delta z$
	2	+	61	
	3	STO B	33 12	
	4	h x≷y	35 52	Calc $z \leftarrow z + \Delta z$
	5	STO 6	33 06	
	6	RCL C	34 13	
	7	+	61	
	8	STO C	33 13	
	9	f P≷S	31 42	Repeat iter
	110	GTO f c	22 31 13	
(8	1	f LBL 8	31 25 08	(8) Convergence OK
	2	h F? 2	35 71 02	Do at least 1 iter.

REGISTERS

0	1	2	3	4	5	6	7	8	9
a_0	a_1	a_2	a_3	... etc.	...				
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
					Real (Δz)	Imag (Δz)	|f(z)|	∡ f(z)	ε

A	B	C	D	E	I
n	Real (z)	Imag (z)	Misc.	Misc.	Loop Index

67 Program Listing II

	STEP	KEY ENTRY	KEY CODE	COMMENTS
	3	GTO 7	22 07	---
	4	RCL C	34 13	Test for real root:
	5	f X=0	31 51	
	6	GTO 9	22 09	If Im(z) = 0 or
	7	h ABS	35 64	
	8	RCL 9	34 09	|Im(z)| > ε,
	9	g X≤Y	32 71	GoTo 9, Else
	120	GTO 9	22 09	
	1	0	00	set Im(z) ← 0
	2	STO C	33 13	and go to c.
	3	f P⇄S	31 42	
	4	GTO f c	22 31 13	---
(9	5	f LBL 9	31 25 09	(9) Iteration done.
	6	0	00	Set i ← 0 if real,
	7	h STI	35 33	i ← -1 if cmplx.
	8	RCL C	34 13	
	9	f X≠0	31 61	
	130	f DSZ	31 33	Enter z_{root} in stack
	1	RCL B	34 12	
	2	f P⇄S	31 42	
	3	h RTN	35 22	RETURN
D	4	f LBL D	31 25 14	$\boxed{D}$ Subroutine.
	5	h CF 1	35 61 01	F1 = 0 for real,
	6	RCL C	34 13	= 1 for cmplx root
	7	f X≠0	31 61	
	8	h SF 1	35 51 01	---
	9	RCL A	34 11	Set
	140	h STI	35 33	
	1	0	00	$R_{n+1} \leftarrow 0$
	2	f ISZ	31 34	
	3	STO (i)	33 24	$R_{n+2} \leftarrow 0$
	4	f ISZ	31 34	
	5	STO (i)	33 24	---
	6	0	00	i ← 0
	7	h STI	35 33	
(d	8	g LBL d	32 25 14	(d) For i = 1 to n:
	9	f ISZ	31 34	
	150	h F? 1	35 71 01	
	1	f ISZ	31 34	
	2	RCL (i)	34 24	
	3	f DSZ	31 33	$R_i \leftarrow R_{i+1}$ for real,
	4	h DEG	35 41	
	5	h F? 1	35 71 01	
	6	f DSZ	31 33	
	7	h DEG	35 41	$R_i \leftarrow R_{i+2}$ for cmplx.
	8	STO (i)	33 24	
	9	f ISZ	31 34	
	160	RCL A	34 11	
	1	h RCI	35 34	Test i
	2	g X≤Y	32 71	
	3	GTO f d	22 31 14	---
	4	1	01	n ← n-1 if real,
	5	h F? 1	35 71 01	
	6	2	02	n ← n-2 if cmplx.
	7	RCL A	34 11	
	8	−	51	

	STEP	KEY ENTRY	KEY CODE	COMMENTS
	9	CHS	42	
	170	STO A	33 11	---
	1	RCL B	34 12	
	2	2	02	If cmplx,
	3	×	71	
	4	STO D	33 14	
	5	RCL B	34 12	$R_D \leftarrow 2 \cdot R_B$
	6	g X²	32 54	and
	7	RCL C	34 13	
	8	g X²	32 54	$R_E \leftarrow -(R_B^2 + R_C^2)$
	9	+	61	
	180	CHS	42	
	1	STO E	33 15	
	2	h F? 1	35 71 01	
	3	GTO f e	22 31 15	---
	4	RCL B	34 12	If real,
	5	STO D	33 14	$R_D \leftarrow R_B$ and
	6	0	00	
	7	STO E	33 15	$R_E \leftarrow 0$
(e	8	g LBL e	32 25 15	(e)
	9	RCL A	34 11	
	190	h STI	35 33	---
(6	1	f LBL 6	31 25 06	(6) For i = n-1, ..., 0:
	2	f ISZ	31 34	
	3	RCL (i)	34 24	
	4	RCL E	34 15	
	5	×	71	
	6	f DSZ	31 33	$R_i \leftarrow R_i +$
	7	RCL (i)	34 24	
	8	RCL D	34 14	$R_D \cdot R_{i+1} +$
	9	×	71	
	200	+	61	$R_E \cdot R_{i+2}$
	1	f DSZ	31 33	
	2	h DEG	35 41	
	3	STO+(i)	33 61 24	
	4	h RCI	35 34	Test i
	5	f X>0	31 81	
	6	GTO 6	22 06	---
	7	RCL A	34 11	n in stack
	8	h RTN	35 22	RETURN
	9			
	210			
	220			

LABELS

A calc f(z)	B calc f'(z)	C Find z_{root}	D Reduce Polynomial	E Cmplx mult
a part of A	b part of B	c part of C	d part of D	e part of D
0	1	2	3	4
5	6 part of D	7 part of C	8 part of C	9 part of C

FLAGS

0	
1	Cmplx root
2	1st Iter
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☐	DEG	☒	FIX	☐
1	⊟	⊟	GRAD	☐	SCI	☐
2	⊟	⊟	RAD	☐	ENG	☐
3	☐	☐			n	____

Program Description I

Program Title Miscellaneous Special Functions A

Contributor's Name Matthew A. Bishop
Address 327 Forbes Ave.
City San Rafael **State** CA **Zip Code** 94901

Program Description, Equations, Variables

1) $\Gamma(x+1) = x\Gamma(x)$ if $x \geq 1$ $\quad \Gamma(x) = \dfrac{\Gamma(x+1)}{x}$ if $1 \geq x > 0$.
$\Gamma(x) \simeq 1 + b_1(x-1) + \dots + b_8(x-1)^8$ if $1 \leq x \leq 2$, where
$b_1 = -0.577191652$ $\quad b_3 = -0.897056937$ $\quad b_5 = -0.756704078$
$b_2 = 0.988205891$ $\quad b_4 = 0.918206857$ $\quad b_6 = 0.482199394$
$b_7 = -0.193527818$ $\quad b_8 = 0.035868343$

2) $F(a,b;c;x) = \sum_{n=0}^{\infty} \dfrac{(a)_n (b)_n}{(c)_n} \dfrac{x^n}{n!}$ where $(d)_0 = 1$, $(d)_n = d(d+1)\cdots(d+n-1)$

3) $P_n^{(\alpha,\beta)}(x) = \dfrac{\Gamma(n+1+\alpha)}{\Gamma(n+1)\Gamma(\alpha+1)} F(n+\alpha+\beta+1, -n; 1+2; \frac{1-x}{2})$

4) $P_n(x) = P_n^{(0,0)}(x)$

5) $T_n(x) = F(n, -n; \frac{1}{2}; \frac{1-x}{2})$

6) $C_n^{\lambda}(x) = \dfrac{\Gamma(2\lambda+n)}{\Gamma(n+1)\Gamma(2\lambda)} F(2\lambda+n, -n; \lambda+\frac{1}{2}; \frac{1-x}{2})$

7) $U_n(x) = \dfrac{x T_{n+1}(x) - T_{n+2}(x)}{1-x^2}$

where $\Gamma(x)$ is the Gamma function, $F(a,b;c;x)$ the

Operating Limits and Warnings $\Gamma(x)$: $x > 0$ or Error message results. If $x = 0$, press $\boxed{f}\boxed{P\rightleftarrows S}$.
$F(a,b;c;x)$: If after 400 terms there is no convergence, Error message results. To change this number, switch to W/PRGM, press $\boxed{GTO}\boxed{.}\boxed{0}\boxed{7}\boxed{0}$, then $\boxed{h}\boxed{DEL}$ three times, and enter new number. To disable, choose 0 as the new number.

Program Description I

Program Title Miscellaneous Special Functions A

Contributor's Name Matthew A. Bishop
Address 327 Forbes Ave.
City San Rafael State CA Zip Code 94901

Program Description, Equations, Variables Gaussian Hypergeometric function, $P_n^{(\alpha,\beta)}(x)$ the Jacobi Polynomial, $P_n(x)$ the Legendre Polynomial, $C_n^{\lambda}(x)$ the Gegenbauer Polynomial, and $T_n(x)$ and $U_n(x)$ the Chebychev Polynomials of the first and second kind, respectively.

Subscripts and superscripts must be nonnegative, but they need not be integers. If they are large, the program is slow.

The data card contains the constants for $\Gamma(x)$; they are loaded into registers $S_1 - S_8$. Register S_9 contains 1.

Operating Limits and Warnings $P_n^{(\alpha,\beta)}(x)$: $-1 < \alpha$, β can be any real

$C_n^{\lambda}(x)$: $\lambda > 0$

$U_n(x)$: If $x^2 = 1$, Error message results. Use $U_n(1) = n+1$, $U_n(-1) = (-1)^n(n+1)$

Program Description II

Sketch(es)

Sample Problem(s) Calculate:

1) $\Gamma(0.5) = 1.77$
2) $\Gamma(15) = 14! = 8.71782912 \times 10^{10}$
3) $F(0.3, 0.5; 1; 0.6) = 1.13$
4) $P_6^{(3,4)}(0.56) = -0.58$
5) $P_4(0.4) = -0.11$
6) $T_5(0.93) = -0.31$
7) $C_6^3(0.98) = 373.60$
8) $U_4(0.93) = 2.59$

Typical Errors and recoveries with $\Gamma(x)$:

a) $\Gamma(0)$ = Error. Press $\boxed{f}\boxed{P\rightleftarrows S}$, to reset

b) $\Gamma(-3)$ = Error. Just enter new number. This applies to all $x<0$

c) $\Gamma(90)$ = Error. Press 1 $\boxed{STO}\boxed{9}$ $\boxed{f}$ $\boxed{P\rightleftarrows S}$ to reset. This applies to all $x > 70.95$.

Solution(s) Keystrokes:

1) 0.5 $\boxed{A}$ → 1.77
2) 15 $\boxed{A}$ → $8.71782912 \times 10^{10}$
3) 0.3 $\boxed{\uparrow}$ 0.5 $\boxed{\uparrow}$ 1 $\boxed{\uparrow}$ 0.6 $\boxed{B}$ → 1.13
4) 3 $\boxed{\uparrow}$ 4 $\boxed{\uparrow}$ 0.56 $\boxed{\uparrow}$ 6 $\boxed{C}$ → -0.58
5) 0.4 $\boxed{\uparrow}$ 4 $\boxed{D}$ → -0.11
6) 0.93 $\boxed{\uparrow}$ 5 $\boxed{E}$ → -0.31
7) 3 $\boxed{\uparrow}$ 0.98 $\boxed{\uparrow}$ 6 $\boxed{f}\boxed{A}$ → 373.60
8) 0.93 $\boxed{\uparrow}$ 4 $\boxed{f}\boxed{B}$ → 2.59

Reference(s) Gradshteyn & Ryzhik, Table of Integrals, Series, and Products; 4th Edition, © 1965, Academic Press. (Listed are numbers of equations) 1) [8.331] (Polynomial adapted from HP-65 program); 2) [9.100] (Polynomial adapted from HP-65 program); 3) [8.962-1b]; 4) [8.962-2]; 5) [8.942-1]; 6) [8.932-1a]; 7) [8.941-4]

User Instructions

MISCELLANEOUS SPECIAL FUNCTIONS A

1 $C_n^{\lambda}(x)$ $U_n(x)$ 2

$\Gamma(x)$ $F(a,b;c;x)$ $P_n^{(\alpha,\beta)}(x)$ $P_n(x)$ $T_n(x)$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter both sides of data card.				
2.	Enter both sides of program card.				
3.	Compute:				
	a) The Gamma function $\Gamma(x)$	x	A		$\Gamma(x)$
	b) The Gaussian Hypergeometric	a	↑		
	function $F(a,b;c;x)$	b	↑		
		c	↑		
		x	B		$F(a,b;c;x)$
	c) The Jacobian polynomial $P_n^{(\alpha,\beta)}(x)$	α	↑		
		β	↑		
		x	↑		
		n	C		$P_n^{(\alpha,\beta)}(x)$
	d) The Legendre polynomial $P_n(x)$	x	↑		
		n	D		$P_n(x)$
	e) The Chebychev polynomial of the	x	↑		
	first kind $T_n(x)$	n	E		$T_n(x)$
	f) The Gegenbauer polynomial $C_n^{\lambda}(x)$	λ	↑		
		x	↑		
		n	f	a	$C_n^{\lambda}(x)$
	g) The Chebychev polynomial of the	x	↑		
	second kind $U_n(x)$	n	f	b	$U_n(x)$
4	For a new case, go to 3.				

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STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Compute Γ(x)
002	X<0?	16-45	If x<0, give
003	LN	32	Error message
004	P⇄S	16-51	Initialize
005	ENT↑	-21	
006	1	01	
007	X>Y?	16-34	
008	GTO4	22 04	
009	-	-45	
010	1	01	
011	*LBL8	21 08	If x>2 use
012	X⇄Y	-41	recursion
013	X>Y?	16-34	relation
014	GTO1	22 01	
015	ENT↑	-21	
016	ENT↑	-21	
017	ENT↑	-21	
018	RCL8	36 08	Polynomial
019	×	-35	approximation
020	RCL7	36 07	
021	GSB0	23 00	
022	RCL6	36 06	
023	GSB0	23 00	
024	RCL5	36 05	
025	GSB0	23 00	
026	RCL4	36 04	
027	GSB0	23 00	
028	RCL3	36 03	
029	GSB0	23 00	
030	RCL2	36 02	
031	GSB0	23 00	
032	RCL1	36 01	
033	GSB0	23 00	
034	GSB2	23 02	
035	RCL9	36 09	
036	×	-35	
037	1	01	Initialize
038	STO9	35 09	register 9
039	CLX	-51	
040	+	-55	
041	P⇄S	16-51	Display answer
042	RTN	24	If x>2, use
043	*LBL1	21 01	recursion relation
044	ST×9	35-35 09	
045	X⇄Y	-41	
046	-	-45	
047	1	01	
048	GTO8	22 08	
049	*LBL0	21 00	
050	+	-55	
051	×	-35	
052	RTN	24	If x<1, use
053	*LBL4	21 04	relation to
054	X⇄Y	-41	obtain
055	ENT↑	-21	
056	ST÷9	35-24 09	
057	GSB2	23 02	
058	P⇄S	16-51	
059	GTOA	22 11	
060	*LBLB	21 12	Compute F(a,b;c;x)
061	STO3	35 03	Initialize
062	R↓	-31	
063	STO4	35 04	
064	R↓	-31	
065	STO5	35 05	
066	R↓	-31	
067	STO6	35 06	
068	4	04	
069	0	00	
070	0	00	
071	STOI	35 46	
072	0	00	
073	STOA	35 11	
074	1	01	
075	STOB	35 12	Sum computation
076	*LBL3	21 03	
077	RCL3	36 03	
078	RCLA	36 11	
079	RCL6	36 06	
080	GSB0	23 00	
081	RCLA	36 11	
082	RCL5	36 05	
083	GSB0	23 00	
084	RCL4	36 04	
085	RCLA	36 11	
086	+	-55	
087	÷	-24	
088	RCLA	36 11	
089	GSB2	23 02	
090	STOA	35 11	
091	÷	-24	
092	RCLB	36 12	
093	×	-35	
094	STOB	35 12	
095	+	-55	
096	X=Y?	16-33	
097	RTN	24	
098	DSZI	16 25 46	
099	GTO3	22 03	If more than
100	0	00	400 terms give
101	÷	-24	Error message
102	*LBLC	21 13	Compute $P_n^{(\alpha,\beta)}(x)$
103	STOB	35 12	Initialize
104	R↓	-31	
105	CHS	-22	
106	GSB9	23 09	
107	STOC	35 13	
108	R↓	-31	
109	STOD	35 14	
110	R↓	-31	
111	GSB2	23 02	
112	STOE	35 15	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	USED	USED	USED	USED	USED	USED	USED	USED	

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	USED

A	B	C	D	E	I
USED	USED	USED	USED	USED	COUNTER

97 Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	RCLB	36 12	
114	+	-55	
115	GSBA	23 11	
116	STO7	35 07	
117	RCLE	36 15	
118	GSBA	23 11	Apply Formula
119	ST÷7	35-24 07	
120	RCLB	36 12	
121	1	01	
122	+	-55	
123	GSBA	23 11	
124	ST÷7	35-24 07	
125	RCLB	36 12	
126	RCLD	36 14	
127	+	-55	
128	RCLE	36 15	
129	+	-55	
130	RCLB	36 12	
131	CHS	-22	
132	RCLE	36 15	
133	RCLC	36 13	
134	GSBB	23 12	
135	RCL7	36 07	
136	x	-35	
137	RTN	24	
138	*LBLD	21 14	Compute $P_n(x)$
139	R↓	-31	
140	R↓	-31	
141	CLX	-51	
142	R↓	-31	
143	CLX	-51	
144	R↓	-31	
145	GTOC	22 13	
146	*LBLE	21 15	Compute $T_n(x)$
147	R↓	-31	
148	STO1	35 01	
149	R↑	16-31	
150	CHS	-22	
151	ENT↑	-21	
152	CHS	-22	
153	RCL1	36 01	
154	CHS	-22	
155	GSB9	23 09	
156	.	-62	
157	5	05	
158	X⇄Y	-41	
159	GTOB	22 12	
160	*LBLa	21 16 11	Compute $C_n^\lambda(x)$
161	STO1	35 01	
162	R↓	-31	
163	CHS	-22	
164	GSB9	23 09	
165	STO2	35 02	
166	R↓	-31	
167	2	02	
168	x	-35	

STEP	KEY ENTRY	KEY CODE	COMMENTS
169	STO3	35 03	
170	RCL1	36 01	
171	+	-55	
172	GSBA	23 11	
173	STO7	35 07	
174	RCL3	36 03	
175	GSBA	23 11	
176	ST÷7	35-24 07	
177	RCL1	36 01	
178	GSB2	23 02	
179	GSBA	23 11	
180	ST÷7	35-24 07	
181	RCL3	36 03	
182	RCL1	36 01	
183	+	-55	
184	RCL1	36 01	
185	CHS	-22	
186	RCL3	36 03	
187	GSB9	23 09	
188	RCL2	36 02	
189	GSBB	23 12	
190	RCL7	36 07	
191	x	-35	
192	RTN	24	
193	*LBLb	21 16 12	Computes $U_n(x)$
194	GSB2	23 02	
195	GSBE	23 15	
196	RCL1	36 01	
197	x	-35	
198	STO8	35 08	
199	RCL1	36 01	
200	RCL5	36 05	
201	GSB2	23 02	
202	GSBE	23 15	
203	RCL8	36 08	
204	-	-45	
205	1	01	
206	RCL1	36 01	
207	X²	53	
208	-	-45	
209	÷	-24	
210	CHS	-22	
211	RTN	24	
212	*LBL9	21 09	Computes $\frac{1+x}{2}$
213	GSB2	23 02	
214	2	02	
215	÷	-24	
216	RTN	24	
217	*LBL2	21 02	Computes $1+x$
218	1	01	
219	+	-55	
220	RTN	24	

LABELS

A $\Gamma(x)$	B $F(a,b;c;x)$	C $P_n^{(\alpha,\beta)}(x)$	D $P_n(x)$	E $T_n(x)$
a $C_n^\lambda(x)$	b $U_n(x)$	c	d	e
0 +, ×	1 USED $\Gamma(x)$	2 $1+x$	3 USED	4 USED $\Gamma(x)$
5	6	7	8 USED $\Gamma(x)$	9 $\frac{1+x}{2}$

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☑	DEG	☑	FIX	☑
1	☐	☑	GRAD	☐	SCI	☐
2	☐	☑	RAD	☐	ENG	☐
3	☐	☑			n	2

Program Description I

Program Title Miscellaneous Special Functions B

Contributor's Name Matthew A. Bishop
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Program Description, Equations, Variables Recursion relations and starting values

① $C_{n+1}^{\lambda}(x) = \dfrac{2(\lambda+n)x\,C_n^{\lambda}(x) - (2\lambda+n-1)\,C_{n-1}^{\lambda}(x)}{(n+1)}$ $\quad C_0^{\lambda}(x) = 1 \quad C_1^{\lambda}(x) = 2\lambda x$

② $L_{n+1}^{\alpha}(x) = \dfrac{(2n+\alpha+1-x)\,L_n^{\alpha}(x) - (n+\alpha)\,L_{n-1}^{\alpha}(x)}{(n+1)}$ $\quad L_0^{\alpha}(x) = 1 \quad L_1^{\alpha}(x) = \alpha + 1 - x$

③ $H_{n+1}(x) = 2xH_n(x) - 2n\,H_{n-1}(x)$ $\quad H_0(x) = 1 \quad H_1(x) = 2x$

④ $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$ $\quad T_0(x) = 1 \quad T_1(x) = x$

⑤ $U_{n+1}(x) = 2x\,U_n(x) - U_{n-1}(x)$ $\quad U_0(x) = 1 \quad U_1(x) = 2x$

⑥ $P_n(x) = C_n^{1/2}(x)$

To use this program if n is not an integer, calculate $Z_{n-\mathrm{INT}(n)}$, $Z_{n-\mathrm{INT}(n)+1}$, [where Z is C^{λ}, L^{α}, H, T, U or P and $\mathrm{INT}(n)$ = Largest integer less than n] and use these values as Z_0, Z_1.

To calculate these values, use tables or Miscellaneous Special Functions A

Operating Limits and Warnings If n is negative, an Error message results. If $n \neq \mathrm{INT}(n)$, then unless $n > 2$ the program fails. In this case, use Miscellaneous Special Functions A.

Program Description II

Sketch(es)

Sample Problem(s) ① $C_6^5(2) = 705565.00$

② $L_4^{3.5}(0.3) = 37.93$

③ $H_5(2) = -16.00$

④ $T_3(6) = 846.00$

⑤ $U_3(6) = 1704.00$

⑥ $P_4(0.96) = 0.63$

⑦ $T_{3.3}(0.96) = 0.59$ $[T_{0.3}(0.96) = 0.996,\ T_{1.3}(0.96) = 0.933]$

Solution(s) ① 5 $\boxed{\uparrow}$ 6 $\boxed{\uparrow}$ 2 $\boxed{A}$ $\longrightarrow$ 705565.00

② 3.5 $\boxed{\uparrow}$ 4 $\boxed{\uparrow}$ 0.3 $\boxed{B}$ $\longrightarrow$ 37.93

③ 5 $\boxed{\uparrow}$ 2 $\boxed{C}$ $\longrightarrow$ -16.00

④ 3 $\boxed{\uparrow}$ 6 $\boxed{D}$ $\longrightarrow$ 846.00

⑤ 3 $\boxed{\uparrow}$ 6 $\boxed{E}$ $\longrightarrow$ 1704.00

⑥ 4 $\boxed{\uparrow}$ 0.96 $\boxed{f}$ $\boxed{A}$ $\longrightarrow$ 0.63

⑦ 0.933 $\boxed{\uparrow}$ 0.996 $\boxed{f}$ $\boxed{B}$ $\longrightarrow$ 0.933

3.3 $\boxed{\uparrow}$ 0.96 $\boxed{D}$ $\longrightarrow$ 0.59

Reference(s) Gradshteyn & Ryzhik, Table of Integrals, Series, and Products, 4th edition, ©1965, Academic Press. Equation numbers: ① [8.933-1], [8.937-3], [8.930]; ② [8.971-6], [8.973-1], [8.973-2]; ③ [8.952-2], [8.956-1], [8.956-2]; ④ [8.941-1], [8.943-1], [8.943-2]; ⑤ [8.941-2], [8.943-7], [8.943-8]; ⑥ [8.936-3].

User Instructions

Miscellaneous Special Functions B

1 $P_n(x)$ z_0, z_1 2

$C_n^\lambda(x)$ $L_n^\alpha(x)$ $H_n(x)$ $T_n(x)$ $U_n(x)$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter program				
2.	Compute the Gegenbauer Polynomial $C_n^\lambda(x)$	λ	↑		
		n	↑		
		x	A		$C_n^\lambda(x)$
	or the Laguerre Polynomial $L_n^\alpha(x)$	α	↑		
		n	↑		
		x	B		$L_n^\alpha(x)$
	or the Hermite Polynomial $H_n(x)$	n	↑		
		x	C		$H_n(x)$
	or the Chebychev Polynomial of the first kind $T_n(x)$	n	↑		
		x	D		$T_n(x)$
	or the Chebychev Polynomial of the second kind $U_n(x)$	n	↑		
		x	E		$U_n(x)$
	or the Legendre Polynomial $P_n(x)$	n	↑		
		x	f	A	$P_n(x)$
3.	Change starting points (optional)	z_1	↑		
		z_0	f	B	z_1
4.	For a new case, go to 2 or 3.				

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Compute $C_n^\lambda(x)$
002	CF0	16 22 00	
003	STO0	35 00	
004	R↓	-31	
005	STO1	35 01	
006	R↓	-31	Initialize
007	STO2	35 02	
008	RCL1	36 01	
009	GSB5	23 05	
010	1	01	Compute $C_0^\lambda(x)$
011	F1?	16 23 01	and unless overriden
012	STOA	35 11	store in A
013	RCL1	36 01	
014	X=Y?	16-33	
015	SF0	16 21 00	Compute $C_1^\lambda(x)$ and
016	RCL0	36 00	unless overriden
017	RCL2	36 02	store in B
018	x	-35	
019	2	02	
020	x	-35	
021	F0?	16 23 00	
022	RTN	24	
023	F1?	16 23 01	
024	STOB	35 12	
025	*LBL0	21 00	Use recursion
026	RCLI	36 46	relation
027	RCL2	36 02	
028	+	-55	
029	2	02	
030	x	-35	
031	RCL0	36 00	
032	x	-35	
033	RCLB	36 12	
034	x	-35	
035	RCL2	36 02	
036	2	02	
037	x	-35	
038	RCLI	36 46	
039	+	-55	
040	1	01	
041	-	-45	
042	RCLA	36 11	
043	x	-35	
044	-	-45	
045	RCLI	36 46	
046	1	01	
047	+	-55	
048	÷	-24	
049	GSB6	23 06	
050	ISZI	16 26 46	
051	RCLI	36 46	
052	RCL1	36 01	
053	X>Y?	16-34	
054	GTO0	22 00	
055	RCLB	36 12	Display result
056	SF1	16 21 01	

STEP	KEY ENTRY	KEY CODE	COMMENTS
057	RTN	24	
058	*LBL1	21 01	If n=0 display 1
059	1	01	
060	R/S	51	
061	*LBLB	21 12	Compute $L_n^\alpha(x)$
062	CF0	16 22 00	
063	STO3	35 03	Initialize
064	R↓	-31	
065	STO4	35 04	
066	R↓	-31	
067	STO5	35 05	
068	RCL4	36 04	
069	GSB5	23 05	
070	1	01	Compute $L_0^\alpha(x)$ and
071	F1?	16 23 01	unless overriden
072	STOA	35 11	store in A
073	RCL1	36 01	
074	X=Y?	16-33	
075	SF0	16 21 00	
076	RCL5	36 05	
077	RCL3	36 03	Compute $L_1^\alpha(x)$ and
078	-	-45	unless overriden
079	1	01	store in B
080	+	-55	
081	F0?	16 23 00	
082	RTN	24	
083	F1?	16 23 01	
084	STOB	35 12	
085	*LBL2	21 02	Use recursion
086	RCLI	36 46	relation
087	2	02	
088	x	-35	
089	RCL5	36 05	
090	+	-55	
091	1	01	
092	+	-55	
093	RCL3	36 03	
094	-	-45	
095	RCLB	36 12	
096	x	-35	
097	RCLI	36 46	
098	RCL5	36 05	
099	+	-55	
100	RCLA	36 11	
101	x	-35	
102	-	-45	
103	RCLI	36 46	
104	1	01	
105	+	-55	
106	÷	-24	
107	GSB6	23 06	
108	ISZI	16 26 46	
109	RCLI	36 46	
110	RCL4	36 04	
111	X>Y?	16-34	
112	GTO2	22 02	

REGISTERS

0	1	2	3	4	5	6	7	8	9
$x(C_n^\lambda)$	$n(C_n^\lambda)$	λ	$x(L_n^\alpha)$	$n(L_n^\alpha)$	α	$x(H_n)$	$n(H_n)$		
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
USED	USED				

97 Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	RCLB	36 12	Display result
114	SF1	16 21 01	
115	RTN	24	
116	*LBLC	21 13	Compute $H_n(x)$
117	CF0	16 22 00	
118	STO6	35 06	
119	R↓	-31	Initialize
120	STO7	35 07	
121	GSB5	23 05	
122	R↓	-31	
123	1	01	Compute $H_0(x)$ and
124	F1?	16 23 01	unless overriden
125	STOA	35 11	store in A
126	RCL1	36 01	
127	X=Y?	16-33	
128	SF0	16 21 00	Compute $H_1(x)$ and
129	RCL6	36 06	unless overriden
130	2	02	store in B
131	x	-35	
132	F0?	16 23 00	
133	RTN	24	
134	F1?	16 23 01	
135	STOB	35 12	Use recursion
136	*LBL3	21 03	relation
137	RCLB	36 12	
138	RCL6	36 06	
139	x	-35	
140	RCLA	36 11	
141	RCLI	36 46	
142	x	-35	
143	-	-45	
144	2	02	
145	x	-35	
146	GSB6	23 06	
147	ISZI	16 26 46	
148	RCLI	36 46	
149	RCL7	36 07	
150	X>Y?	16-34	
151	GTO3	22 03	
152	RCLB	36 12	Display answer
153	SF1	16 21 01	
154	RTN	24	
155	*LBLD	21 14	Compute $T_n(x)$
156	CF0	16 22 00	Set indicator for $T_n(x)$
157	*LBL9	21 09	
158	CF2	16 22 02	
159	STO0	35 00	Initialize
160	R↓	-31	
161	X=0?	16-43	
162	GTO1	22 01	
163	1	01	Compute $T_0(x)$ and
164	X=Y?	16-33	unless overriden
165	SF2	16 21 02	store in A
166	F1?	16 23 01	
167	STOA	35 11	
168	-	-45	

STEP	KEY ENTRY	KEY CODE	COMMENTS
169	STOI	35 46	Compute $T_1(x)$ and
170	RCL0	36 00	store in A unless
171	F0?	16 23 00	overriden
172	2	02	
173	F0?	16 23 00	
174	x	-35	
175	F1?	16 23 01	
176	STOB	35 12	
177	F2?	16 23 02	
178	RTN	24	
179	*LBL8	21 08	Compute recursion
180	RCLB	36 12	relation
181	RCL0	36 00	
182	x	-35	
183	2	02	
184	x	-35	
185	RCLA	36 11	
186	-	-45	
187	GSB6	23 06	
188	DSZI	16 25 46	
189	GTO8	22 08	
190	SF1	16 21 01	Display answer
191	RTN	24	
192	*LBLE	21 15	Compute $U_n(x)$
193	SF0	16 21 00	Set indicator for $U_n(x)$
194	GTO9	22 09	
195	*LBLa	21 16 11	Computes $P_n(x)$
196	R↓	-31	
197	R↓	-31	
198	R↓	-31	
199	.	-62	
200	5	05	
201	R↑	16-31	
202	R↑	16-31	
203	GTOA	22 11	
204	*LBLb	21 16 12	Store z_0, z_1
205	STOA	35 11	
206	R↓	-31	
207	STOB	35 12	
208	CF1	16 22 01	Set indicator
209	RTN	24	
210	*LBL5	21 05	If n = 0, Go to 1.
211	X=0?	16-43	
212	GTO1	22 01	
213	FRC	16 44	If $n \neq INT(n)$, store
214	1	01	frac(n) in R_I.
215	+	-55	
216	STOI	35 46	
217	RTN	24	
218	*LBL6	21 06	Load for next pass
219	RCLB	36 12	through recursion
220	STOA	35 11	relation.
221	R↓	-31	
222	STOB	35 12	
223	RTN	24	

LABELS

A $C_n^\lambda(x)$	B $L_n^\alpha(x)$	C $H_n(x)$	D $T_n(x)$	E $U_n(x)$
a $P_n(x)$	b z_0, z_1	c	d	e
0 Recursion $C_n^\lambda(x)$	1 n = 1 ?	2 $L_n^\alpha(x)$	3 $H_n(x)$	4
5 n = 0 ?	6 USED	7	8 $T_n(x)$	9 T_n, U_n

FLAGS

0	is n = 1 ?
1	Alternative starting values
2	is n = 1 (T_n)?
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☑	DEG	☑	FIX	☑
1	☑	☐	GRAD	☐	SCI	☐
2	☐	☑	RAD	☐	ENG	☐
3	☐	☑			n	2

Program Description I

Program Title Incomplete Gamma Function from x to ∞.

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City Carmel **State** CA **Zip Code** 93921

Program Description, Equations, Variables

$$\Gamma(a,x) = \int_x^{\infty} t^{a-1} e^{-t}\,dt = e^{-x} x^{a} \left\{ \frac{1}{x+}\;\frac{1-a}{1+}\;\frac{1}{x+}\;\frac{2-a}{1+}\;\frac{2}{x+}\;\cdots \right\}$$

for $x > 0$, $|a| < \infty$

Take $\Gamma(a,x) = p_k/q_k$ when $p_k/q_k = p_{m-1}/q_{m-1}$ in the HP-67/97 where

$$\left\{ \begin{array}{ll} p_{2m} = p_{2m-1} + d_{2m}\, p_{2m-2}, & p_{2m+1} = x\, p_{2m} + d_{2m+1}\, p_{2m-1} \\ q_{2m} = q_{2m-1} + d_{2m}\, q_{2m-2}, & q_{2m+1} = x\, q_{2m} + d_{2m+1}\, q_{2m-1} \end{array} \right\} m = 1, 2, \ldots$$

and $d_{2m} = m - a$, $d_{2m+1} = m$.

Starting values are $p_0 = 0$, $q_0 = p_1 = 1$, and $q_1 = x$.

p_k/q_k is known as the k^{th} convergent.

Operating Limits and Warnings The continued fraction expansion is presumably exact, but the finite capacity of the HP-67 limits the obtainable accuracy. A partial "accuracy table" is given on the next page. For other values the user is advised to compute $\Gamma(a,x)$ analytically for integer values of a and x in the region of interest. For this purpose the recursion $\Gamma(a,x) = x^{a-1}e^{-x} + (a-1)\Gamma(a-1,x)$ and $\Gamma(1,x) = e^{-x}$ can be used.

Program Description II

In the "accuracy" table below, the relative error $(y_c - y_e)/y_e$ is listed where y_c is the continued fraction solution and y_e is the analytic solution computed on the HP-67. In all of the test cases except one ($\Gamma(7, 200)$) the agreement was exact. For $\Gamma(7, 200)$ the relative error was -1×10^{-10}. The second quantity listed is the number of iterations (contents of register 8) required for convergence. Note: Only the iteration counts for $\Gamma(0, x)$ and $\Gamma(1/2, x)$ have been given since accurate or analytic comparison values are not readily available.

"Accuracy" Table

x	$\Gamma(0,x)$		$\Gamma(1/2,x)$		$\Gamma(1,x)$		$\Gamma(3,x)$		$\Gamma(5,x)$		$\Gamma(7,x)$		$\Gamma(10,x)$	
0.0001	No Solution		No Solution		0	1	0	3	No Solution		No Solution		No Solution	
0.001	No Solution		No Solution		0	1	0	3	No Solution		No Solution		No Solution	
0.01	No Solution		No Solution		0	1	0	3	0	5	No Solution		No Solution	
0.1	No Solution		No Solution		0	1	0	3	0	5	0	7	No Solution	
1.0		42		40	0	1	0	3	0	5	0	7	0	10
2.0		24		25	0	1	0	3	0	5	0	7	0	10
5.0		11		10	0	1	0	3	0	5	0	7	0	10
10.0		8		10	0	1	0	3	0	5	0	7	0	10
50.0		4		4	0	1	0	3	0	4	0	5	0	5
200.0		3		3	0	1	0	3	0	3	-1 -10	3	0	3

$\Gamma(\frac{1}{2}, 5) = 2.774603258 - 03$

$\Gamma(0, 20) = 9.835525289 - 11$

$\Gamma(10, 200) = 7.417614092 - 67$

$\Gamma(3, 0.0001) = 2.000000000 \; 00$

Reference(s) 1. "Handbook of Mathematical Functions", Abramowitz and Stegun, National Bureau of Standards, 1968. §6.5.31 pg 263.

2. The method of convergents is discussed by C.D. Olds, "Continued Fractions", Random House, 1963

User Instructions

Incomplete Gamma Function $\Gamma(a,x)$

a	x			Compute

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Read in program			
2	In any order:			
a	Store a	a	A	
b	Store x	x	B	
3	Compute		E	$\Gamma(a,x)$
4	Repeat steps 2a and/or 2b, then 3 as needed.			

LABELS

A a	B x	C	D	E Compute
a	b	c	d	e
0 ✓	1	2	3	4
5	6	7	8	9 ✓

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS	ON	OFF
0	☐	☑
1	☐	☑
2	☐	☑
3	☐	☑

TRIG	
DEG	☑
GRAD	☐
RAD	☐

DISP	
FIX	☑
SCI	☐
ENG	☐
n	2

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	
002	STO1	35 01	Store a
003	R/S	51	
004	*LBLB	21 12	
005	STO3	35 03	Store x
006	R/S	51	
007	*LBLE	21 15	Compute.
008	RCL3	36 03	
009	STO7	35 07	
010	0	00	Initialize
011	STO4	35 04	
012	STO8	35 08	starting
013	1	01	
014	STO6	35 06	values
015	STO5	35 05	
016	CHS	-22	
017	STO2	35 02	
018	*LBL0	21 00	Iteration return point
019	RCL8	36 08	
020	1	01	
021	+	-55	
022	STO8	35 08	m ← m+1
023	RCL1	36 01	
024	-	-45	d_{2m}
025	ENT↑	-21	
026	ENT↑	-21	
027	RCL4	36 04	
028	×	-35	$d_{2m}p_{2m-2}$
029	RCL5	36 05	
030	+	-55	
031	STO4	35 04	p_{2m}
032	X⇄Y	-41	d_{2m}
033	RCL6	36 06	
034	×	-35	
035	RCL7	36 07	
036	+	-55	
037	STO6	35 06	
038	0	00	q_{2m}
039	X=Y?	16-33	Compute convergent
040	GTO9	22 09	if $q_{2m} \neq 0$.
041	+	-55	
042	÷	-24	
043	*LBL9	21 09	
044	STO2	35 02	Store new convergent
045	RCL8	36 08	d_{2m+1}
046	RCL5	36 05	
047	×	-35	
048	RCL4	36 04	
049	RCL3	36 03	
050	×	-35	
051	+	-55	
052	STO5	35 05	p_{2m+1}
053	RCL8	36 08	
054	RCL7	36 07	
055	×	-35	
056	RCL6	36 06	
057	RCL3	36 03	Gamma
058	×	-35	
059	+	-55	
060	STO7	35 07	q_{2m+1}
061	0	00	
062	X=Y?	16-33	If $q_{2m+1}=0$,
063	GTO0	22 00	iterate again,
064	+	-55	Otherwise compute
065	÷	-24	new convergent
066	RCL2	36 02	Store new convergent
067	X⇄Y	-41	& compare to old
068	STO2	35 02	convergent
069	X≠Y?	16-32	
070	GTO0	22 00	Loop if not equal
071	RCL3	36 03	
072	RCL1	36 01	
073	Yˣ	31	x^a
074	×	-35	
075	RCL3	36 03	
076	CHS	-22	e^{-x}
077	eˣ	33	
078	×	-35	
079	R/S	51	Display result

REGISTERS

0	1	2	3	4	5	6	7	8	9
	a	convergent	x	P_{2m}	P_{2m+1}	q_{2m}	q_{2m+1}	m	

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

Program Description I

Program Title Incomplete Beta Function

Contributor's Name Rex H Shudde
Address 27105 Arriba Way
City Carmel **State** CA **Zip Code** 93921

Program Description, Equations, Variables

$$B_x(a,b) = \int_0^x t^{a-1}(1-t)^{b-1}\,dt = \frac{x^a(1-x)^b}{a}\left\{\frac{1}{1+}\,\frac{d_1}{1+}\,\frac{d_2}{1+}\cdots\right\} \text{ for } 0<x<1,\ a>0,\ b>0$$

Take $B_x(a,b) = p_{2k}/q_{2k}$ when $p_{2k}/q_{2k} = p_{2k-2}/q_{2k-2}$ where
$p_k = d_k p_{k-2} + p_{k-1}$ and $q_k = d_k q_{k-2} + q_{k-1}$ for $k = 1, 2, \ldots$
and

$$d_{2m+1} = -\frac{(a+m)(a+b+m)x}{(a+2m)(a+2m+1)}, \quad m = 0, 1, 2, \ldots$$

$$d_{2m} = \frac{m(b-m)x}{(a+2m-1)(a+2m)}, \quad m = 1, 2, \ldots$$

The starting values are $p_{-1} = 0$, $p_0 = q_0 = q_{-1} = 1$. The reference to this continued fraction states that "best results are obtained for $x < \frac{a-1}{a+b-2}$". The program may be extended by using the relationship $B_x(a,b) = B(a,b) - B_{1-x}(b,a)$ where $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$. The complete Gamma function $\Gamma(a)$ can be computed using other programs in the HP-67/97 library.

Operating Limits and Warnings The continued fraction expansion is exact, but the finite capacity of the HP-67/97 limits obtainable accuracy. A partial "accuracy table" is given on the next page.

Program Description II

In the "accuracy table" below, the relative error $(z_c - z_a)/z_a$ is listed where z_c is the continued fraction solution & z_a is the analytic solution computed on the HP67 (-3-09, for example, means -3×10^{-9}). The second quantity listed is the number of iterations (at about 4.2 seconds/iteration) for convergence. An iteration count is the contents of register 8 after the solution has been obtained. In the table, * denotes values for which the analytic solution suffered severe truncation error; presumably the continued fraction is valid

"Accuracy Table"

x	$B_x(10,3)$		$B_x(10,1)$		$B_x(3,1)$		$B_x(\frac{1}{2},\frac{1}{2})$		$B_x(1,3)$		$B_x(1,10)$		$B_x(3,10)$	
0.0001	-7-10	2	0	2	0	2	0	2	3-09	2	2-08	2	*	2
0.001	5-10	2	0	2	0	2	3-10	3	-3-10	3	-7-10	3	*	3
0.01	0	3	1-10	2	0	2	-1-09	3	-2-10	3	-3-10	4	*6-05	3
0.1	1-10	5	1-10	2	0	2	0	4	1-10	4	2-10	7	3-08	6
0.2	-4-10	4	0	2	0	2	-4-10	5	0	4	-1-09	9	7-09	11
0.5	-7-10	6	0	2	0	2	-1-09	7	7-10	4	0	12	-7-10	11
0.8	-6-9	19	0	2	0	4	9-10	13	-2-9	4	6-05	11	-7-05	15
0.9	2-8	5	0	2	0	2	-8-10	28	-6-08	4	2-03	11	-3-02	13
0.99	1-5	9	0	2	0	2	1-08	53	-5-05	4	No	"	No	"
0.999	2-2	4	0	2	0	2	No	175	-5-02	4	SOLUTION	11	Solution	12
0.9999	No good	5	0	2	0	2	Solution	171	No good	6		"		"

Examples:

$B_{0.9}(3,1) = 2.430000000-01$
$B_{0.9}(10,1) = 3.486784401-02$
$B_{0.5}(3,10) = 1.485928621-03$
$B_{0.5}(\frac{1}{2},\frac{1}{2}) = 1.570796325+00$

The analytic solutions used for checking are:
$B_x(10,3) = x^{10}\left(\frac{1}{10} - \frac{2x}{11} + \frac{x^2}{12}\right)$; $B_x(10,1) = x^{10}/10$;
$B_x(3,1) = x^3/3$; $B_x(\frac{1}{2},\frac{1}{2}) = 2\sin^{-1}\sqrt{x}$;
$B_x(1,3) = [1-(1-x)^3]/3$; $B_x(1,10) = [1-(1-x)^{10}]/10$
$B_x(3,10) = (2!\,9!)/12! - B_{1-x}(10,3)$
Also, $B_x(1,1) = x$ had an error of no more than 8×10^{-10} for all values in the table.

Reference(s) 1. "Handbook of Mathematical Functions", Abramowitz and Stegun, National Bureau of Standards, 1968, §26.5.8 pg 944.

2. The method of convergents is discussed in C.D. Olds, "Continued Fractions", Random House, 1963

User Instructions

Incomplete Beta Function				
Store a	Store b	Store x		Compute $B_x(a,b)$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Read in program card			
2	In any order, store:			
a	Store a	a	A	
b	Store b	b	B	
c	Store x	x	C	
3	Execute program		E	$B_x(a,b)$
4	Repeat any or all of step 2, then step 3 for further values			

LABELS					FLAGS
A a	B b	C x	D	E compute	0
a	b	c	d	e	1
0 ✓	1	2	3	4	2
5	6	7	8	9	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☑	DEG	☑	FIX	☑
1	☐	☑	GRAD	☐	SCI	☐
2	☐	☑	RAD	☐	ENG	☐
3	☐	☑			n	2

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS Beta Fcn
001	*LBLE	21 15	
002	1	01	Initialize
003	STO7	35 07	
004	STO6	35 06	Starting
005	STO4	35 04	
006	0	00	values
007	STO8	35 08	
008	STO5	35 05	
009	*LBL0	21 00	Iteration loop
010	STO9	35 09	Store convergent
011	RCL1	36 01	
012	RCL8	36 08	
013	+	-55	$a+m$
014	ENT↑	-21	
015	ENT↑	-21	
016	RCL2	36 02	
017	+	-55	$a+m+b$
018	x	-35	$(a+m)(a+m+b)$
019	RCL1	36 01	
020	RCL8	36 08	
021	+	-55	
022	RCL8	36 08	
023	+	-55	$(a+2m)$
024	÷	-24	
025	LSTX	16-63	
026	1	01	
027	+	-55	$(a+2m+1)$
028	÷	-24	
029	RCL3	36 03	x
030	x	-35	
031	CHS	-22	d_{2k+1}
032	ENT↑	-21	
033	ENT↑	-21	
034	RCL5	36 05	P_{2k-1}
035	x	-35	
036	RCL4	36 04	P_{2k}
037	+	-55	
038	STO5	35 05	P_{2k+1}
039	CLX	-51	
040	RCL7	36 07	q_{2k-1}
041	x	-35	
042	RCL6	36 06	q_{2k}
043	+	-55	
044	STO7	35 07	q_{2k+1}
045	RCL8	36 08	
046	1	01	
047	+	-55	
048	STO8	35 08	$m \leftarrow m+1$
049	RCL2	36 02	
050	RCL8	36 08	
051	-	-45	
052	x	-35	$m(b-m)$
053	RCL1	36 01	
054	RCL8	36 08	
055	+	-55	
056	RCL8	36 08	
057	+	-55	$a+2m$
058	÷	-24	
059	LSTX	16-63	
060	1	01	
061	-	-45	$a+2m-1$
062	÷	-24	
063	RCL3	36 03	x
064	x	-35	d_{2k}
065	ENT↑	-21	
066	ENT↑	-21	
067	RCL4	36 04	
068	x	-35	
069	RCL5	36 05	
070	+	-55	
071	STO4	35 04	$P_{2k}=d_{2k}P_{2k-2}+P_{2k-1}$
072	X⇄Y	-41	
073	RCL6	36 06	
074	x	-35	
075	RCL7	36 07	
076	+	-55	
077	STO6	35 06	q_{2k}
078	X≠0?	16-42	
079	÷	-24	$P_{2k}/q_{2k} \sim B_x(a,b)$
080	RCL9	36 09	Previous convergent
081	X⇄Y	-41	
082	X≠Y?	16-32	Converged?
083	GTO0	22 00	No
084	RCL3	36 03	
085	RCL1	36 01	
086	Yˣ	31	x^a
087	x	-35	
088	1	01	
089	RCL3	36 03	
090	-	-45	
091	RCL2	36 02	
092	Yˣ	31	$(1-x)^b$
093	x	-35	
094	RCL1	36 01	
095	÷	-24	
096	R/S	51	Display result
097	*LBLA	21 11	
098	STO1	35 01	Store a
099	R/S	51	
100	*LBLB	21 12	
101	STO2	35 02	Store b
102	R/S	51	
103	*LBLC	21 13	
104	STO3	35 03	Store x
105	R/S	51	
110			

REGISTERS

0	1	2	3	4	5	6	7	8	9
	a	b	x	P_{2k}	P_{2k+1}	q_{2k}	q_{2k+1}	m	Convergent
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

Program Description I

Program Title Incomplete Elliptic Integrals

Contributor's Name Stuart D. Augustin
Address Department of Chemistry, Princeton University
City Princeton **State** New Jersey **Zip Code** 08540

Program Description, Equations, Variables This program computes the generalized incomplete integral of the second kind to about 10 significant figures

$$el2(x, k', a, b) = \int_0^x \frac{(a+b\xi^2)\, d\xi}{(1+\xi^2)\sqrt{(1+\xi^2)(1+k'^2\xi^2)}}$$

If $\phi = \tan^{-1} x$ and $k = \sqrt{1-k'^2}$, the standard incomplete integrals of the first and second kind are obtained as special cases

$$F(\phi, k) = el2(x, k', 1, 1)$$

$$E(\phi, k) = el2(x, k', 1, k'^2)$$

Note: $k^2 el2(x, k', a, b) = (b - ak'^2)F(\phi, k) + (a-b)E(\phi, k)$

When $x > 10^{10} \times$ maximum $\{1, 1/|k'|\}$, the results are the complete elliptic integrals. The technique of the arithmetico-geometric mean is used.

Operating Limits and Warnings

There is an error exit if $k' = 0$, but in this case $E(\phi, 1) = \sin\phi$

$$F(\phi, 1) = \ln(\sec\phi + \tan\phi)$$

Program Description II

~~Sketch(es)~~ In the standard trigonometric form

$$el2(x,k',a,b) = \int_0^{\phi} \frac{a+(b-a)\sin^2\alpha}{\sqrt{1-k^2\sin^2\alpha}}\,d\alpha \quad , \quad F(\phi,k) = \int_0^{\phi} \frac{d\alpha}{\sqrt{1-k^2\sin^2\alpha}}$$

$$E(\phi,k) = \int_0^{\phi}\sqrt{1-k^2\sin^2\alpha}\,d\alpha \quad , \quad B(\phi,k) = \int_0^{\phi}\frac{\cos^2\alpha\,d\alpha}{\sqrt{1-k^2\sin^2\alpha}} \quad , \quad D(\phi,k) = \int_0^{\phi}\frac{\sin^2\alpha\,d\alpha}{\sqrt{1-k^2\sin^2\alpha}}$$

where again $x = \tan\phi$, $k^2 = 1 - k'^2$

Sample Problem(s)

1) If $k' = 10^{-11}$ and $x = 10^{10}$, what is $F(x,k')$?

2) If $\phi = 80°$ and $k' = \cos(24°)$ what is $E(x,k')$?

Solution(s)

1) EEX 10 $\boxed{\text{ENTER}\uparrow}$ EEX CHS 11 $\boxed{A}$ $\rightarrow$ 23.71650742

2) 80 $\boxed{g \mid D\rightarrow R}$ $\rightarrow$ 1.396263402 $\boxed{f \mid TAN}$ $\rightarrow$ 5.671281833 ($\tan 80°$)

24 $\boxed{g \mid D\rightarrow R}$ $\rightarrow$ 4.188790205×10^{-1} $\boxed{f \mid COS}$ $\rightarrow$ 9.135454576×10^{-1}

$\boxed{E}$ $\rightarrow$ 1.344059030

Reference(s) Abramowitz and Stegun, Handbook of Mathematical Functions, National Bureau of Standards (AMS 55), 1968.
R. Bulirsch, Numerische Mathematik 7, 78 (1965).

User Instructions

Incomplete Elliptic Integrals

1 ◀ ▶ 2

A	B	C	D	E
$x \uparrow k'$, $F(x,k')$	$x \uparrow k'$, $B(x,k')$	$el2(x,k',a,b)$ $x \uparrow k' \uparrow a \uparrow b$	$x \uparrow k'$, $D(x,k')$	$x \uparrow k'$, $E(x,k')$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Input x, k'	x	E↑	x
		k'		k'
3	Compute $F(x,k')$		A	$F(x,k')$
	or			
3	Compute $B(x,k')$		B	$B(x,k')$
	or			
3	Compute $D(x,k')$		D	$D(x,k')$
	or			
3	Compute $E(x,k')$		E	$E(x,k')$
4	For a new case return to step 2			
4	Input x, k', a, b and compute the generalized form $el2$	x	E↑	x
		k'	E↑	k'
		a	E↑	a
		b	C	$el2(x,k',a,b)$
2'	(Optional) If input is the value of the modular angle α, the relation $k' = \cos\alpha$ can be used (Remember the calculator is in radians mode!)			
	Also If the amplitude ϕ is given, $x = \tan\phi$. Note, however, that if ϕ is near 90°, ϕ is not a good input parameter			
	Complete elliptic integrals can be computed by setting x to 10^{10} times the larger of 1 and $1/\lvert k'\rvert$			

67 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	f LBL A	31 25 11	a, b for F(x, k')
	1	01	
	E↑	41	
	GTO C	22 13	
	f LBL B	31 25 12	a, b for B(x, k')
	1	01	
	E↑	41	
	0	00	
	GTO C	22 13	
010	f LBL D	31 25 14	a, b for D(x, k')
	0	00	
	E↑	41	
	1	01	
	GTO C	22 13	
	f LBL E	31 25 15	a, b for E(x, k')
	g x²	32 54	
	h LST x	35 82	
	h x⇄y	35 52	
	1	01	
020	h x⇄y	35 52	
	f LBL C	31 25 13	Compute el2(x, k', a, b)
	f P⇄S	31 42	
	STO 3	33 03	
	h x⇄y	35 52	
	STO 2	33 02	
	−	51	
	STO 8	33 08	Initialize
	1	01	
	STO 0	33 00	
030	h R↑	35 54	
	f x=0	31 51	
	h RTN	35 22	
	h ABS	35 64	
	÷	81	
	STO 4	33 04	
	STO 7	33 07	
	h R↑	35 54	
	h ABS	35 64	
	STO 1	33 01	Error exit if k' = 0
040	0	00	
	g x=y	32 51	
	÷	81	
	STO 6	33 06	
	h STI	35 33	
	h R↓	35 53	
	g x²	32 54	
	h x⇄y	35 52	
	g x²	32 54	
	+	61	
050	h LST x	35 82	
	1	01	
	+	61	
	STO ÷ 7	33 81 07	
	÷	81	
	f √x	31 54	
	STO 5	33 05	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	f LBL 0	31 25 00	Begin loop
	RCL 7	34 07	
	RCL 0	34 00	
060	RCL 1	34 01	
	x	71	
	STO 9	33 09	
	RCL 6	34 06	
	x	71	
	RCL 5	34 05	
	÷	81	
	STO + 7	33 61 07	
	CL x	44	
	h LST x	35 82	
070	÷	81	Compute A.G.M.
	RCL 6	34 06	
	+	61	
	2	02	
	÷	81	
	STO 6	33 06	
	RCL 9	34 09	
	RCL 5	34 05	
	÷	81	
	STO + 5	33 61 05	
080	RCL 9	34 09	
	CHS	42	
	RCL 4	34 04	
	÷	81	
	h LST x	35 82	
	+	61	
	f x≠0	31 61	
	GTO 1	22 01	
	RCL 9	34 09	branch to avoid dividing by zero
	f √x	31 54	
090	EEX	43	
	CHS	42	
	1	01	
	2	02	
	x	71	
	f LBL 1	31 25 01	
	STO 4	33 04	
	RCL 3	34 03	
	RCL 2	34 02	
	RCL 1	34 01	
100	x	71	
	STO + 3	33 61 03	
	CL x	44	
	RCL 0	34 00	
	÷	81	
	RCL 2	34 02	
	+	61	
	2	02	
	÷	81	
	STO 2	33 02	
110	RCL 0	34 00	
	RCL 1	34 01	
	STO + 0	33 61 00	

REGISTERS

0	1	2	3	4	5	6	7	8	9
S0 used	S1 used	S2 used	S3 used	S4 used	S5 used	S6 used	S7 used	S8 used	S9 used

A	B	C	D	E	I
					used

67 Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
	−	51	
	h ABS	35 64	
	RCL 0	34 00	
	÷	81	
	EEX	43	
	CHS	42	
	5	05	
120	g x>y	32 81	
	GTO 2	22 02	
	h RC I	35 34	
	2	02	
	×	71	
	h ST I	35 33	
	h LST x	35 82	
	RCL 9	34 09	
	f √x	31 54	
	×	71	
130	STO 1	33 01	
	RCL 4	34 04	
	f x<0	31 71	
	f ISZ	31 34	
	GTO 0	22 00	
	f LBL 2	31 25 02	End loop
	RCL 0	34 00	
	RCL 4	34 04	
	f x<0	31 71	
	f ISZ	31 34	
140	÷	81	
	g TAN-1	32 64	
	h RC I	35 34	
	h π	35 73	
	×	71	
	+	61	Compute the
	RCL 2	34 02	elliptic
	×	71	integral
	RCL 0	34 00	
	÷	81	
150	RCL 6	34 06	
	RCL 8	34 08	
	×	71	
	−	51	
	f P⇄S	31 42	
	h RTN	35 22	
160			

STEP	KEY ENTRY	KEY CODE	COMMENTS
170			
180			
190			
200			
210			
220			

LABELS

A F(x, k′)	B B(x, k′)	C el 2	D D(x, k′)	E E(x, k′)
a	b	c	d	e
0 loop	1 branch	2 exit loop	3	4
5	6	7	8	9

FLAGS: 0, 1, 2, 3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☐	FIX	☐
1	☐	☒	GRAD	☐	SCI	☒
2	☐	☒	RAD	☒	ENG	☐
3	☐	☒			n 9	

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