

HEWLETT-PACKARD

# HP-67/HP-97

Math Pac I



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# Introduction

The 19 programs of Math Pac I have been drawn from the fields of number theory, algebra, trigonometry, analytical geometry, calculus, and special functions.

Each program in this pac is represented by one or more magnetic cards and a section in this manual. The manual provides a description of the program with relevant equations, a set of instructions for using the program, and one or more example problems, each of which includes a list of the actual keystrokes required for its solution. Program listings for all the programs in the pac appear at the back of this manual. Explanatory comments have been incorporated in the listings to facilitate your understanding of the actual working of each program. Thorough study of a commented listing can help you to expand your programming repertoire since interesting techniques can often be found in this way.

On the face of each magnetic card are various mnemonic symbols which provide shorthand instructions to the use of the program. You should first familiarize yourself with a program by running it once or twice while following the complete User Instructions in the manual. Thereafter, the mnemonics on the cards themselves should provide the necessary instructions, including what variables are to be input, which user-definable keys are to be pressed, and what values will be output. A full explanation of the mnemonic symbols for magnetic cards may be found in appendix A.

If you have already worked through a few programs in Standard Pac, you will understand how to load a program and how to interpret the User Instructions form. If these procedures are not clear to you, take a few minutes to review the sections, Loading a Program and Format of User Instructions, in your Standard Pac.

Several programs in this pac were based on programs submitted to the HP-65 Users' Library. We wish to acknowledge the following contributors:

John Joseph Herro for *Optimal Scale for a Graph*,

Rene S. Julian for *Rotations in Three-Dimensional Space*,

Stuart D. Augustin for *Bessel Functions*,

Charles R. Ammerman for *Extended Complementary Error Function*.

We hope that Math Pac I will assist you in the solution of numerous problems in your discipline. We would very much appreciate knowing your reactions to the programs in this pac, and to this end we have provided a questionnaire inside the front cover of this manual. Would you please take a few minutes to give us your comments on these programs? It is in the comments we receive from you that we learn how best to increase the usefulness of programs like these.

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## A WORD ABOUT PROGRAM USAGE

This application pac has been designed for both the HP-97 Programmable Printing Calculator and the HP-67 Programmable Pocket Calculator. The most significant difference between the HP-67 and the HP-97 calculators is the printing capability of the HP-97. The two calculators also differ in a few minor ways. The purpose of this section is to discuss the ways that the programs in this pac are affected by the difference in the two machines and to suggest how you can make optimal use of your machine, be it an HP-67 or an HP-97.

Most of the computed results in this pac are output by PRINT statements; most often by the statement PRINTx, and occasionally by the command PRINT STACK. On the HP-97 these results will be output on the printer. On the HP-67 each PRINT command will be interpreted as a PAUSE: the program will halt, display the result for about 5 seconds, then continue execution. The term "PRINT/PAUSE" is used to describe this output condition.

If you own an HP-67, you may want more time to copy down the number displayed by a PRINT/PAUSE. All you need to do is press any key on the keyboard. If the command being executed is PRINTx (eight rapid blinks of the decimal point), pressing a key will cause the program to halt. If the command being executed is PRINT STACK (two slow blinks of the decimal per value), the number in the display will remain there until the depressed key is released; then the next register in the stack will be displayed, and so on. After display of all four registers, the program will halt execution if a key was pressed at any time during the display of the stack contents. In both cases execution of the halted program may be re-initiated by pressing **R/S**.

HP-97 users may also want to keep a permanent record of the values input to a certain program. A convenient way to do this is to set the Print Mode switch to NORMAL before running the program. In this mode all input values and their corresponding user-definable keys will be listed on the printer, thus providing a record of the entire operation of the program.

Several programs in this pac allow you to choose an optional mode which will be referred to on the magnetic card as AUTO. This will apply only to programs that output a long list of results and will allow those results to be output automatically through PRINT/PAUSE commands. If AUTO is not selected, each computed value will be output on the display and the program will halt. The purpose of AUTO mode is to afford maximum convenience to users of both the HP-67 and the HP-97. On the HP-97 it is simplest to have a printed record of each computed result; this can be accomplished just by specifying AUTO. On the HP-67, if every result is to be written down, it may be advantageous *not* to select AUTO, and thus force the program to halt each time a result is found.

Another area that could reflect differences between the HP-67 and the HP-97 is in the keystroke solutions to example problems. It is sometimes necessary in these solutions to include operations that involve prefix keys, namely, **f** on the

HP-97 and **f**, **g**, and **h** on the HP-67. For example, the operation  $10^x$  is performed on the HP-97 as **f**  $10^x$  and on the HP-67 as **g**  $10^x$ . In such cases, the keystroke solution omits the prefix key and indicates only the operation (as here,  $10^x$ ). As you work through the example problems, take care to press the appropriate prefix keys (if any) for your calculator.

Also in keystroke solutions, those values that are output by the command PRINTx will be followed by three asterisks (\*\*\*) .

## FACTORS AND PRIMES



This program will find all prime factors of a positive integer  $n$ , or list all prime numbers between lower and upper bounds specified by the user.

A routine under LBL a is used in determining the factors of an integer  $n$ . This routine selects a trial divisor  $d$  and tests  $d$  as a factor of  $n$ . If  $d$  divides  $n$ , then  $n \leftarrow n/d$  and  $d$  is tested as a factor of the new  $n$ . If  $d$  does not divide  $n$ , a new  $d$  is selected. The process continues until  $d > \sqrt{n}$ , at which point  $n$  is returned as the final factor. The trial divisor  $d$  takes on the values 2,3,5,7; then for  $d > 10$ ,  $d$  takes on those values that satisfy  $(d - 10) \bmod 30 = 1, 3, 7, 9, 13, 19, 21, \text{ or } 27$ . Thus in the first cycle of 30 integers from 11 to 40,  $d$  takes on the values 11, 13, 17, 19, 23, 29, 31, and 37. This technique eliminates from the test those values of  $d$  ( $d > 10$ ) which are divisible by 2, 3, or 5.

To list primes, a lower bound for the search must be specified. The upper bound is an optional input; if omitted, a default value of  $2 \times 10^9$  is assumed. Upper and lower bounds need not be integers. The search for primes also uses LBL a to determine if an integer  $n$  has any factors or is indeed prime. Once an integer  $n$  ( $n \geq 3$ ) has been tested and found to be either prime or non-prime, the next integer tested is  $n+2$ .

### Remarks:

1. The number  $n$  to be factored must be an integer in the range  $0 < n \leq 2 \times 10^9$ . Any other input will result in a program halt with a display of "Error".
2. The upper bound of the search for primes must be greater than or equal to the lower bound, or an Error halt will occur.
3. AUTO mode is available to allow automatic output of all calculated results through PRINT/PAUSE commands. The end of all calculations is signalled by a 0.00 in the display for both modes.
4. Either routine can be quite time-consuming for large integers.

| STEP | INSTRUCTIONS                                            | INPUT<br>DATA/UNITS | KEYS     | OUTPUT<br>DATA/UNITS |
|------|---------------------------------------------------------|---------------------|----------|----------------------|
| 1    | Load side 1 and side 2.                                 |                     |          |                      |
| 2    | To allow automatic output of<br>results, set AUTO mode. |                     | <b>E</b> | 1.00                 |
| 3    | To cancel AUTO mode later.                              |                     | <b>E</b> | 0.00                 |

| STEP | INSTRUCTIONS                                                                               | INPUT DATA/UNITS | KEYS     | OUTPUT DATA/UNITS |
|------|--------------------------------------------------------------------------------------------|------------------|----------|-------------------|
| 4    | To find factors, go to step 5;<br>to find primes, go to step 6.                            |                  |          |                   |
|      | <b>FACTORS</b>                                                                             |                  |          |                   |
| 5    | Key in the integer and find its<br>prime factors (0.00 signals end).                       | n                | <b>A</b> | Factors<br>0.00   |
|      | <b>PRIMES</b>                                                                              |                  |          |                   |
| 6    | Key in the lower bound of the<br>search for primes.                                        | FROM             | <b>B</b> | FROM              |
| 7    | (optional) Key in the upper<br>bound of the search (if omitted,<br>TO = $2 \times 10^9$ ). | TO               | <b>C</b> | TO                |
| 8    | Find all primes between FROM<br>and TO (0.00 signals end of<br>calculation).               |                  | <b>D</b> | Primes<br>0.00    |

**Example 1:**

Find the prime factors of 924. Do not set AUTO mode.

**Keystrokes:**

924 **A** →  
**R/S** →  
**R/S** →  
**R/S** →  
**R/S** →  
**R/S** →

**Outputs:**

2.00  
2.00  
3.00  
7.00  
11.00  
0.00 (end)

Thus  $924 = 2 \times 2 \times 3 \times 7 \times 11$ .

## 01-03

### Example 2:

Find the prime factors of 3623. Do not use AUTO mode.

#### Keystrokes:

3623 **A** →

**R/S** →

3623 is prime.

#### Outputs:

3623.00

0.00 (end)

### Example 3:

Find all prime numbers between 101 and 125. Use AUTO mode.

#### Keystrokes:

101 **B** 125 **C** **E** →

**D** →

#### Outputs:

1.00 (AUTO set)

101.00 \*\*\*

103.00 \*\*\*

107.00 \*\*\*

109.00 \*\*\*

113.00 \*\*\*

0.00 (end)

# GREATEST COMMON DIVISOR, LEAST COMMON MULTIPLE, DECIMAL TO FRACTION



This program contains three different routines: greatest common divisor, least common multiple, and decimal to fraction.

Given integers  $a$  and  $b$ , the first routine finds their greatest common divisor,  $\text{GCD}(a,b)$ . Optional outputs of this routine are the values of the integers  $s$  and  $t$  which satisfy the equation  $\text{GCD}(a,b) = sa + tb$ . The second routine will calculate, for integers  $a$  and  $b$ , their least common multiple,  $\text{LCM}(a,b)$ . This routine is independent of the first, although both share a common subroutine,  $\text{LBL } e$ .

The basic algorithm used in finding  $\text{GCD}(a,b)$  is as follows:

1. If  $b = 0$ ,  $\text{GCD}(a,b) \leftarrow a$  and the program halts.
2. If  $b \neq 0$ ,  $z \leftarrow a \bmod b$ ,  $a \leftarrow b$ , and  $b \leftarrow z$ . Return to 1.

The algorithm is actually extended somewhat to allow calculation of  $s$  and  $t$ . Full details may be found in the reference below.

$\text{LCM}(a,b)$  is found by

$$\text{LCM}(a,b) = \frac{ab}{\text{GCD}(a,b)}$$

The third routine in this program will find rational fractional approximations for decimal values by the method of continued fractions. Each successive approximation is closer to the decimal value than the previous one. For example, if the decimal keyed in is 0.33, the first fractional approximation computed will be  $1/3$ . The program will output first the numerator 1, then the denominator 3, then the 10-digit value of the approximation, 0.333333333, and finally the error in this approximation, displayed in scientific notation. The error is found by subtracting the original value, 0.33, from the value of this approximation. At this step the error is 3.333333300-03.

The program will then go on to compute a closer fractional approximation. In this example, the next approximation would be  $33/100$ . Since this is the exact equivalent of the original decimal value, the program will halt after this step displaying 0.000000000. The last fraction generated can be recalled by pressing **D**.

**Equations:**

The algorithm employed in this routine uses a method of continued fractions, so that the  $n$ th fractional approximation  $f_n$  is computed as

$$f_n = a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{a_4 + \dots + \frac{1}{a_n}}}}$$

Each  $f_i$  is output as a numerator  $N_i$  and a denominator  $D_i$ , which are computed as follows:

$$N_i = a_i N_{i-1} + N_{i-2}$$

$$D_i = a_i D_{i-1} + D_{i-2}$$

where  $N_{-1} = 0$ ,  $D_{-1} = 1$ ,  $N_0 = 1$ , and  $D_0 = 0$  by definition.

The values for the  $a_i$  may be found by the following algorithm:

Let Dec be the original decimal keyed in. Then  $a_1 = \text{INT}(\text{Dec})$ . Let  $x_1 = 1$  and let  $y_1 = \text{FRAC}(\text{Dec})$ . Then

$$a_{i+1} = \text{INT} (x_i/y_i)$$

$$x_{i+1} = y_i$$

$$y_{i+1} = x_i - a_{i+1}y_i$$

**Remarks:**

AUTO mode is available on the Decimal to Fraction routine.

**References:**

(GCD,LCM) D. E. Knuth, *The Art of Computer Programming*, Vol. 2, Addison-Wesley, 1969.

(Decimal to fraction) Charles G. Moore, *An Introduction to Continued Fractions*, National Council of Teachers of Mathematics, 1964.

| STEP | INSTRUCTIONS                                                                                                               | INPUT DATA/UNITS | KEYS  | OUTPUT DATA/UNITS                  |
|------|----------------------------------------------------------------------------------------------------------------------------|------------------|-------|------------------------------------|
| 1    | Load side 1 and side 2.                                                                                                    |                  |       |                                    |
| 2    | For greatest common divisor, go to step 3; for least common multiple, go to step 5; for decimal to fraction, go to step 6. |                  |       |                                    |
|      | <b>GCD</b>                                                                                                                 |                  |       |                                    |
| 3    | Key in integers a and b and find their greatest common divisor.                                                            | a                | ENTER |                                    |
|      |                                                                                                                            | b                | A     | GCD (a,b)                          |
| 4    | (optional) Compute coefficients s and t such that $\text{GCD}(a,b) = sa + tb$ .                                            |                  | R/S   | s                                  |
|      |                                                                                                                            |                  |       | t                                  |
|      | <b>LCM</b>                                                                                                                 |                  |       |                                    |
| 5    | Key in integers a and b and find their least common multiple.                                                              | a                | ENTER |                                    |
|      |                                                                                                                            | b                | B     | LCM (a,b)                          |
|      | <b>DECIMAL→FRACTION</b>                                                                                                    |                  |       |                                    |
| 6    | To allow automatic output of results, set AUTO mode.                                                                       |                  | E     | 1.00                               |
| 7    | To cancel AUTO Mode later.                                                                                                 |                  | E     | 0.00                               |
| 8    | Key in a decimal value and find successive fractional approximations ( $i = 1, 2, \dots$ ).                                | Dec              | C     | Num <sub>i</sub>                   |
|      |                                                                                                                            |                  |       | Den <sub>i</sub>                   |
|      |                                                                                                                            |                  |       | Num <sub>i</sub> /Den <sub>i</sub> |
|      |                                                                                                                            |                  |       | Error <sub>i</sub>                 |

| STEP | INSTRUCTIONS                                                                   | INPUT DATA/UNITS | KEYS     | OUTPUT DATA/UNITS |
|------|--------------------------------------------------------------------------------|------------------|----------|-------------------|
| 9    | To re-output last fractional approximation ( $Error_n$ shown in display only). |                  |          |                   |
|      |                                                                                |                  | <b>D</b> | $Num_n$           |
|      |                                                                                |                  |          | $Den_n$           |
|      |                                                                                |                  |          | $Num_n/Den_n$     |
|      |                                                                                |                  |          | $Error_n$         |

**Example 1:**

Find the greatest common divisor of 406 and 266. Find also the constants s and t.

**Keystrokes:**

406 **ENTER** 266 **A**  $\longrightarrow$   
**R/S**  $\longrightarrow$

**Outputs:**

14.00 \*\*\* (GCD)  
 2.00 \*\*\* (s)  
 -3.00 \*\*\* (t)

That is,  $(2 \times 406) + (-3 \times 266) = 14$ .

**Example 2:**

Find the least common multiple of 406 and 266.

**Keystrokes:**

406 **ENTER** 266 **B**  $\longrightarrow$

**Outputs:**

7714.00 \*\*\* (LCM)

**Example 3:**

A gear designer wants to reduce the angular speed of a rotating shaft by a factor of 0.45647. He will do this by having a *gear* on the driven shaft mesh with a smaller gear, called a *pinion*, on the drive shaft. If  $N_g$  and  $N_p$  are the number of teeth on the gear and pinion respectively, then the reduction in speed is by a factor of  $N_p/N_g$ . Find the best values for  $N_p$  and  $N_g$  subject to the constraint that neither value exceed 100. Do not use AUTO mode.

**Keystrokes:**

.45647 **C**  $\longrightarrow$   
**R/S**  $\longrightarrow$   
**R/S**  $\longrightarrow$   
**R/S**  $\longrightarrow$

**Outputs:**

1. ( $Num_1$ )  
 2. ( $Den_1$ )  
 0.50000000 ( $Frac_1$ )  
 4.353000000-02 ( $Error_1$ )

## 02-05

|     |   |                 |                                      |
|-----|---|-----------------|--------------------------------------|
| R/S | → | 5.              | (Num <sub>2</sub> )                  |
| R/S | → | 11.             | (Den <sub>2</sub> )                  |
| R/S | → | 0.454545455     | (Frac <sub>2</sub> )                 |
| R/S | → | -1.924545500-03 | (Error <sub>2</sub> )                |
|     |   |                 |                                      |
| R/S | → | 21.             | (Num <sub>3</sub> )                  |
| R/S | → | 46.             | (Den <sub>3</sub> )                  |
| R/S | → | 0.456521739     | (Frac <sub>3</sub> )                 |
| R/S | → | 5.173910000-05  | (Error <sub>3</sub> )                |
|     |   |                 |                                      |
| R/S | → | 173.            | (Num <sub>4</sub> > 100,<br>so stop) |

The best values are thus  $N_p = 21$ ,  $N_g = 46$ .

### Example 4:

Generate the series of fractional approximations to  $\pi$ . Use AUTO mode.

#### Keystrokes:

**E** →  
 **$\pi$  C** →

#### Outputs:

1.00 (AUTO set)  
 3. \*\*\*  
 1. \*\*\*  
 3.000000000 \*\*\*  
 -1.415926540-01 \*\*\*  
 22. \*\*\*  
 7. \*\*\*  
 3.142857143 \*\*\*  
 1.264489000-03 \*\*\*  
 333. \*\*\*  
 106. \*\*\*  
 3.141509434 \*\*\*  
 -8.322000000-05 \*\*\*  
 355. \*\*\*  
 113. \*\*\*  
 3.141592920 \*\*\*  
 2.660000000-07 \*\*\*

104348. \*\*\*

33215. \*\*\*

3.141592654 \*\*\*

0.000000000

## BASE CONVERSIONS



This program will convert a positive number in base  $b$ ,  $x_b$ , to its equivalent representation in base  $B$ ,  $x_B$ . The bases  $b$  and  $B$  may take on integer values from 2 to 99, inclusive. Inputs to the program are  $x_b$ ,  $b$ , and  $B$ ; the single output is the value of  $x_B$ . Input of either base,  $b$  or  $B$ , may be omitted if its value is 10 since a default value of 10 is assigned to both  $b$  and  $B$  upon input of  $x_b$  to key **A**. If several conversions are to be done between the same two bases, i.e., there are several values of  $x_b$  for the same  $b$  and  $B$ , then the bases need not be re-input each time. Once the new value of  $x_b$  is keyed in, then pressing **E** will immediately cause the calculation of  $x_B$ , based on the most recent values for  $b$  and  $B$ .

The heart of this program is a routine under LBL **e** which actually converts numbers to and from base 10 representations. If either  $b$  or  $B$  is equal to 10, this routine is executed just once, and then the program halts displaying  $x_B$ . If, on the other hand, neither  $b$  nor  $B$  is 10, then  $x_b$  is first converted to its decimal representation,  $x_{10}$ , and next  $x_{10}$  is converted to  $x_B$ . Thus the routine is here executed twice.

A number such as  $4B6_{16}$  cannot be represented directly on the display because the display is strictly numeric. Therefore, some convention must be adopted to represent numbers  $R_a$  when  $a > 10$ . We use the convention of allocating two digit locations for each single character in  $R_a$  when  $a > 10$ .

For example,  $4B6_{16}$  is represented as  $041106_{16}$  by our convention (in hexadecimal system,  $A = 10$ ,  $B = 11$ ,  $C = 12$ ,  $D = 13$ ,  $E = 14$ ,  $F = 15$ ).

When displayed, this number may appear as 41106 or with an exponent

4.1106          04

which is interpreted as  $4.B6 \times 16^2$ .

The displayed exponent 4 is for base 10 and only serves to locate the decimal point (in the same manner as for decimal numbers).

When base  $a > 10$  (as in the above example), divide the displayed exponent by 2 to get the true exponent of the number. When the displayed exponent is an odd integer, shift the decimal point of the displayed number one place (to the left or right) and adjust its exponent accordingly to make the true exponent an integer.

For example, the displayed number

1.112          -03

is interpreted as  $B.C \times 16^{-2}$  or  $0.BC \times 16^{-1}$ .

**Remarks:**

1. When the magnitude of the number is very large or very small, this program will take a long time to execute.
2. The program will not give any Error indication for invalid inputs for  $x_b$ . For example,  $981_8$  will be treated the same as  $1201_8$ .

| STEP | INSTRUCTIONS                                                                                                             | INPUT DATA/UNITS | KEYS       | OUTPUT DATA/UNITS |
|------|--------------------------------------------------------------------------------------------------------------------------|------------------|------------|-------------------|
| 1    | Load side 1 and side 2.                                                                                                  |                  |            |                   |
| 2    | To cause input values to be output, set Print/Pause mode.                                                                |                  | <b>F A</b> | 1.00              |
| 3    | To cancel Print/Pause mode.                                                                                              |                  | <b>F A</b> | 0.00              |
| 4    | Key in number in first base.                                                                                             | $x_b$            | <b>A</b>   |                   |
| 5    | (optional) Key in first base.<br>(If omitted, default value of b is 10.)                                                 | b                | <b>B</b>   |                   |
| 6    | (optional) Key in second base<br>(If omitted, default value of B is 10.)                                                 | B                | <b>C</b>   |                   |
| 7    | Calculate number in second base.                                                                                         |                  | <b>D</b>   | $x_B$             |
| 8    | To convert another number between the <i>same</i> two bases (from b to B), key in the new $x_b$ and find the new $x_B$ . | $x_b$            | <b>E</b>   | $x_B$             |
| 9    | To change either base, go to step 4.                                                                                     |                  |            |                   |

**Example 1:**

The following octal numbers ( $b = 8$ ) are addresses of a segment of a program in an HP2100 minicomputer: 177700, 177735, 177777. What are the values of these addresses in base 10 ( $B = 10$ )?

**Keystrokes:**177700 **A** 8 **B** **D** →177735 **E** →177777 **E** →**Outputs:**

65472.00 \*\*\*

65501.00 \*\*\*

65535.00 \*\*\*

**Example 2:**

Find the ten-digit binary representation of  $\pi$ . ( $x_b = 3.141592654$ ,  $b = 10$ ,  $B = 2$ )

**Keystrokes:** **$\pi$**  **A** 2 **C** **D** **DSP** **9** →**Outputs:**

11.00100100

**Example 3:**

Convert the following octal numbers ( $b = 8$ ) into hexadecimal ( $B = 16$ ):  
 $7.200067 \times 8^{-10}$ ,  $1.513561778 \times 8^{17}$

**Keystrokes:**7.200067 **EEX** **CHS** 10 **A** 8 **B**16 **C** **D** →**Outputs:**

1.130000031-14 \*\*\*

(1.D003A  $\times 16^{-7}$ )1.513561778 **EEX** 17 **E** →

1.302141404 25 \*\*\*

(13.02141404 24

=D.2EE4  $\times 16^{12}$ )

## OPTIMAL SCALE FOR A GRAPH; PLOTTING



Two separate routines are included in this program. The first finds the optimal scale for a graph, given certain parameters of the graph. The second routine is designed to be of assistance in plotting functions of one variable by generating ordered pairs  $(x, f(x))$  for a range of  $x$ -values specified by the user.

### Optimal scale for a graph

In the first routine the input parameters are the minimum and maximum values on the graph (Min and Max) and the number of major divisions (tics) from top to bottom of the graph. The routine will select a "nice" scale for the graph, meaning that the graph will fill as much of the page as possible, subject to these constraints: (1) the quantity  $\Delta$  represented by one major division will be 1, 2, 4 or 5 times an integral power of 10; (2) the bottom and the top of the graph will be integral multiples of one division; and (3) bottom  $\leq$  Min and top  $\geq$  Max. Outputs of the routine are values for the top and bottom of the graph; the amount of each major division,  $\Delta$ ; and the "efficiency," or percentage of the page filled by the graph. Efficiency is found by  $[(\text{Max} - \text{Min})/(\text{Top} - \text{Bot})] \times 100$ .

### Plotting

In the second routine, the function  $f(x)$  must be specified and loaded into program memory by the user. The user must also input beginning and ending values for  $x$  (Beginx and Endx), and the step size or increment used for  $x$  (Step). Then the routine will output the values of  $(x, f(x))$  for the successive values of  $x$  represented by

$$x_j = \text{Beginx} + j\text{Step} \quad , \quad j=0, 1, 2, \dots, n$$

where  $n$  is such that  $x_{n+1} > \text{Endx}$ . The end of calculations is signalled by a 0.00 in the display.

The AUTO option is provided for output of the ordered pairs  $(x, f(x))$  through Print/Pause commands. If AUTO is not selected, the values will be output one at a time by the use of **R/S**.

Although we have discussed only one  $f(x)$ , there may actually be up to five different functions  $f_i(x)$ ,  $i=1, 2, \dots, 5$ , in program memory at one time. Each function should be under its own label, 1 through 5, and should be followed by

RTN. The function to be evaluated is specified by keying in 1, 2, 3, 4, or 5 and pressing **f** **E**.

92 program steps are available to the user for specifying functions  $f_i(x)$ . This includes all LBL and RTN statements. The functions should assume that upon entry the value of  $x$  will be found in the  $x$ -register. Registers  $R_0$  through  $R_9$  and  $R_{S0}$  through  $R_{S9}$ , as well as the stack, are available to the user. The functions  $f_i(x)$  may use up to two levels of subroutines: note, however, that the only unused labels are 1 through 5.

To specify your functions, you may wish to record them on a blank magnetic card for rapid entry. Alternatively, you may key them into program memory after loading side 1 and side 2 of this card. To link in recorded functions, follow these steps:

1. Load side 1 and side 2 of *Optimal Scale, Plotting*.
2. Press **GTO** **▢** **1** **3** **2**.
3. Press **MERGE**.
4. Load your own magnetic card with the functions  $f_i(x)$  recorded.

To key in a new function:

1. Load side 1 and side 2 of *Optimal Scale, Plotting*.
2. Press **GTO** **▢** **1** **3** **2**.
3. Key in your function(s), beginning each with LBL (1 through 5) and ending each with RTN.

| STEP | INSTRUCTIONS                                                                                          | INPUT DATA/UNITS | KEYS       | OUTPUT DATA/UNITS              |
|------|-------------------------------------------------------------------------------------------------------|------------------|------------|--------------------------------|
| 1    | Load side 1 and side 2.                                                                               |                  |            |                                |
| 2    | For optimal scale of a graph, go to step 3; for plotting, go to step 7.                               |                  |            |                                |
|      | <b>OPTIMAL SCALE FOR A GRAPH</b>                                                                      |                  |            |                                |
| 3    | Key in the minimum value on the graph.                                                                | Min              | <b>A</b>   | Min                            |
| 4    | Key in the maximum value on the graph.                                                                | Max              | <b>B</b>   | Max                            |
| 5    | Key in the number of tics desired and find the graph top, bottom, value of one tic, and % efficiency. | Tics             | <b>C</b>   | Top<br>Bottom<br>$\Delta$<br>% |
| 6    | To change any value, go to the appropriate step, then to step 5.                                      |                  |            |                                |
|      | <b>PLOTTING</b>                                                                                       |                  |            |                                |
| 7    | Load subroutine(s) (either key them in with <b>LBL</b> and <b>RTN</b> , or link from step 132).       |                  |            |                                |
| 8    | Select function under <b>LBL</b> 1, 2, 3, 4 or 5.                                                     | i (1-5)          | <b>f E</b> | i                              |
| 9    | Key in the beginning x-value.                                                                         | Begin x          | <b>f A</b> | Begin x                        |
| 10   | Key in the final or maximum x-value.                                                                  | End x            | <b>f B</b> | End x                          |
| 11   | Key in the step size for x.                                                                           | Step             | <b>f C</b> | Step                           |
| 12   | For automatic output of results, go to step 13; for manual output, go to step 16.                     |                  |            |                                |

| STEP | INSTRUCTIONS                                                                                                                     | INPUT DATA/UNITS | KEYS       | OUTPUT DATA/UNITS |
|------|----------------------------------------------------------------------------------------------------------------------------------|------------------|------------|-------------------|
|      | <b>AUTO mode</b>                                                                                                                 |                  |            |                   |
| 13   | Set AUTO mode to allow auto-matic output of results.                                                                             |                  | <b>E</b>   | 1.00              |
| 14   | To cancel AUTO mode later                                                                                                        |                  | <b>E</b>   | 0.00              |
| 15   | Output successive ordered pairs; program will halt displaying 0.00 when $x > \text{End } x$ .                                    |                  | <b>f D</b> | $x$               |
|      |                                                                                                                                  |                  |            | $f_i(x)$          |
|      | <b>Manual mode</b>                                                                                                               |                  |            |                   |
| 16   | Output first ordered pair.                                                                                                       |                  | <b>f D</b> | $x$               |
|      |                                                                                                                                  |                  | <b>R/S</b> | $f_i(x)$          |
| 17   | For all successive ordered pairs; 0.00 signals end ( $x > \text{End } x$ ).                                                      |                  | <b>R/S</b> | $x$               |
|      |                                                                                                                                  |                  | <b>R/S</b> | $f_i(x)$          |
| 18   | The value for $i$ may be changed at any time. Begin $x$ , End $x$ , and Step need not be re-input if their values are unchanged. |                  |            |                   |

**Example 1:**

Find the best scale to graph a function whose minimum is 20, maximum is 40, with 5 major divisions from top to bottom (figure 1).

**Keystrokes:**

20 **A** 40 **B** 5 **C** →

**Outputs:**

40.00 (Top)  
 20.00 (Bottom)  
 4.00 ( $\Delta$ )  
 100.00 (% Efficiency)

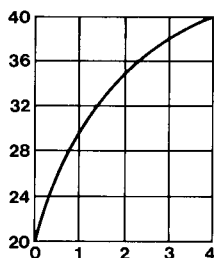


Figure 1

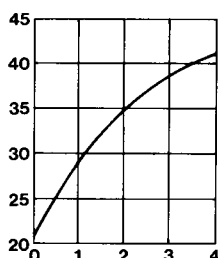


Figure 2

**Example 2:**

Suppose the minimum changes to 21, the maximum to 41, with the number of ticks still 5. Find the new optimal scale (figure 2).

**Keystrokes:**

21 **A** 41 **B** 5 **C** →

**Outputs:**

45.00 (Top)  
 20.00 (Bottom)  
 5.00 ( $\Delta$ )  
 80.00 (% Efficiency)

**Example 3:**

Two different functions are to be plotted in the range from 2.00 to 3.00. The first is  $f_1(x) = e^x$ , and the second is  $f_2(x) = x^x$ . Use a step size of 0.25 for  $f_1(x)$  and 0.2 for  $f_2(x)$ . Load  $f_1(x)$  under LBL 1 and  $f_2(x)$  under LBL 2. Use AUTO mode with  $f_2(x)$ .

**Keystrokes:**

**GTO** **▢** **1** **3** **2**

Switch to PRGM.

**LBL** **1**  **$e^x$**  **RTN** **LBL** **2**

**Outputs:**

ENTER  $\uparrow$   $y^x$  RTN

Switch to RUN.

1  $\downarrow$  E 2  $\downarrow$  A 3  $\downarrow$  B.25  $\downarrow$  C  $\downarrow$  D  $\longrightarrow$ R/S  $\longrightarrow$ R/S  $\longrightarrow$ R/S  $\longrightarrow$ R/S  $\longrightarrow$ R/S  $\longrightarrow$ R/S  $\longrightarrow$ R/S  $\longrightarrow$ R/S  $\longrightarrow$ R/S  $\longrightarrow$ R/S  $\longrightarrow$ 2  $\downarrow$  E .2  $\downarrow$  C E  $\longrightarrow$  $\downarrow$  D  $\longrightarrow$ 

2.00 (x)

7.39 ( $e^x$ )

2.25

9.49

2.50

12.18

2.75

15.64

3.00

20.09

0.00 (end)

1.00 (AUTO set)

2.00 \*\*\* (x)

4.00 \*\*\* ( $x^x$ )

2.20 \*\*\*

5.67 \*\*\*

2.40 \*\*\*

8.18 \*\*\*

2.60 \*\*\*

11.99 \*\*\*

2.80 \*\*\*

17.87 \*\*\*

3.00 \*\*\*

27.00 \*\*\*

0.00 (end)

## COMPLEX OPERATIONS



This program allows for chained calculations involving complex numbers. The four operations of complex arithmetic ( $+$ ,  $-$ ,  $\times$ ,  $\div$ ) are provided, as well as several of the most used functions of a complex variable  $z$  ( $|z|$ ,  $1/z$ ,  $z^n$ ,  $z^{1/n}$ , and  $e^z$ ). Functions and operations may be mixed in the course of a calculation to allow evaluation of expressions like  $z_3/(z_1 + z_2)$ ,  $e^{z_1 z_2}$ ,  $|z_1 + z_2| + |z_2 - z_3|$ , etc., where  $z_1$ ,  $z_2$ ,  $z_3$  are complex numbers of the form  $a + ib$ .

### Keying in a complex number

A complex number is input to the program by keying in its real part, pressing **ENTER**, keying in its imaginary part, and pressing **A**. For example, the complex number  $z_1 = 2 + 3i$  is input as **2** **ENTER** **3** **A**. This number is then stored by the program. There is room in the program to remember up to two complex numbers at a time. A second complex number  $z_2 = 5 - i$  could be input as **5** **ENTER** **1** **CHS** **A**. The program would now contain both the first and the second complex number.

### Functions

The complex functions in this program act on just one number. Thus to perform a function, you need simply to input a complex number  $z$  and then perform the appropriate function. For example, to find the reciprocal of  $(2 + 3i)$ , press **2** **ENTER** **3** **A** **f** **B**. The result is calculated as  $a + ib = 0.15 - 0.23i$ . This result is now stored in place of the original number, and further calculations will operate on this result. All complex functions operate in this same manner, with one exception: since the function  $|z|$  returns a real number, its result is not stored.

### Arithmetic Operations

An arithmetic operation needs two numbers to operate on. Both numbers must be input before the operation can be performed. Suppose that  $z_1 = 2 + 3i$ ,  $z_2 = 5 - i$ , and we wish to find  $z_1 - z_2$ . This can be calculated by the keystrokes **2** **ENTER** **3** **A** **5** **ENTER** **1** **CHS** **A** **C**. The result  $z_3 = a + ib$  is found to be  $-3 + 4i$ . This result is now stored by the program in place of the *second* complex number  $z_2$ . A further calculation  $z_3 \times z_4$  could be performed by inputting  $z_4$  and pressing **D** for multiplication. This type of chaining can be continued indefinitely, and functions can be interspersed with arithmetic operations.

**Equations:**

Let  $z_k = a_k + ib_k = r_k e^{i\theta_k}, k = 1, 2$

$$z = a + ib = re^{i\theta}$$

Let the result in each case be  $u + iv$ .

$$z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$$

$$z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$$

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

$$z_1/z_2 = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

$$|z| = \sqrt{a^2 + b^2}$$

$$1/z = \frac{a}{r^2} - i \frac{b}{r^2}$$

$$z^n = r^n e^{in\theta}$$

$$z^{1/n} = r^{1/n} e^{i\left(\frac{\theta}{n} + \frac{360k}{n}\right)}, k=0,1, \dots, n-1$$

(All  $n$  roots will be output and temporarily stored,  $k = 0, 1, \dots, n-1$ ; at the end of the calculation, the final root will be stored.)

$e^z = e^a (\cos b + i \sin b)$ , where  $b$  is in radians.

**Remarks:**

The logic system for this program may be thought of as a kind of Reverse Polish Notation (RPN) with a stack whose capacity is two complex numbers. Let the bottom register of the complex stack be  $\xi$  and the top register  $\tau$ . These are analogous to the X- and T-registers in the calculator's own four-register stack.\* A complex number  $z_1$  is input to the  $\xi$ -register by the keystrokes  $a_1$  **ENTER**  $b_1$  **A**. Upon input of a second complex number  $z_2$  (as  $a_2$  **ENTER**  $b_2$  **A**),  $z_1$  is moved to  $\tau$  and  $z_2$  is placed in  $\xi$ . The previous contents of  $\tau$  are lost.

\*Each register of the complex stack must actually hold two real numbers: the real and the imaginary part of its complex contents. Thus it takes two of the calculator's registers to represent one register in the complex stack. We will speak of the complex stack registers as though they were each just one register.

Functions operate on the  $\xi$ -register, and the result (except for  $|z|$ ) is left in  $\xi$ ;  $\tau$  is unchanged. Arithmetic operations involve both the  $\xi$ - and  $\tau$ -registers; the result of the operation is left in  $\xi$  and  $\tau$  is unchanged.

| STEP | INSTRUCTIONS                                                                                                  | INPUT DATA/UNITS | KEYS    | OUTPUT DATA/UNITS |
|------|---------------------------------------------------------------------------------------------------------------|------------------|---------|-------------------|
| 1    | Load side 1 and side 2.                                                                                       |                  |         |                   |
| 2    | Key in first complex number                                                                                   |                  |         |                   |
|      | $(a_1 + i b_1).$                                                                                              | $a_1$            | ENTER + |                   |
|      |                                                                                                               | $b_1$            | A       |                   |
| 3    | For a function, go to step 7; for arithmetic, go to step 4. A complex result is $u + iv$ .                    |                  |         |                   |
|      | ARITHMETIC                                                                                                    |                  |         |                   |
| 4    | Key in second complex number                                                                                  |                  |         |                   |
|      | $(a_2 + i b_2).$                                                                                              | $a_2$            | ENTER + |                   |
|      |                                                                                                               | $b_2$            | A       |                   |
| 5    | Select one of four operations:                                                                                |                  |         |                   |
|      | • Add (+)                                                                                                     |                  | B       | $u$               |
|      |                                                                                                               |                  |         | $v$               |
|      | • Subtract (-)                                                                                                |                  | C       | $u$               |
|      |                                                                                                               |                  |         | $v$               |
|      | • Multiply (x)                                                                                                |                  | D       | $u$               |
|      |                                                                                                               |                  |         | $v$               |
|      | • Divide ( $\div$ )                                                                                           |                  | E       | $u$               |
|      |                                                                                                               |                  |         | $v$               |
| 6    | The result of the operation has been stored; go to step 7 for a function or to step 4 for further arithmetic. |                  |         |                   |
|      | FUNCTIONS                                                                                                     |                  |         |                   |
| 7    | Select one of five functions:                                                                                 |                  |         |                   |
|      | • Magnitude ( $ z $ )                                                                                         |                  | f A     | $ z $             |
|      | • Reciprocal ( $1/z$ )                                                                                        |                  | f B     | $u$               |

| STEP | INSTRUCTIONS                                       | INPUT DATA/UNITS | KEYS       | OUTPUT DATA/UNITS |
|------|----------------------------------------------------|------------------|------------|-------------------|
|      |                                                    |                  |            | v                 |
|      | • Raise z to an integer power ( $z^n$ )            | n                | <b>f C</b> | u                 |
|      |                                                    |                  |            | v                 |
|      | • Find the $n^{\text{th}}$ root of z ( $z^{1/n}$ ) |                  |            |                   |
|      | Note: n roots ( $u + iv$ ) will be                 |                  |            |                   |
|      | found.                                             | n                | <b>f D</b> | u                 |
|      |                                                    |                  |            | v                 |
|      | • Raise e to the power z ( $e^z$ )                 |                  | <b>f E</b> | u                 |
|      |                                                    |                  |            | v                 |
| 8    | The result, if complex, has been                   |                  |            |                   |
|      | stored; go to step 4 for arithmetic                |                  |            |                   |
|      | or to step 7 for another function.                 |                  |            |                   |

**Example 1:**

Evaluate the expression

$$\frac{z_1}{z_2 + z_3},$$

where  $z_1 = 23 + 13i$ ,  $z_2 = -2 + i$ ,  $z_3 = 4 - 3i$ . (Suggestion: since the program can remember only two numbers at a time, perform the calculation as

$$z_1 \times [1/(z_2 + z_3)].)$$

**Keystrokes:**

2 **CHS** **ENTER** 1 **A** 4 **ENTER** 3

**CHS** **A** **B**  $\longrightarrow$

**f** **B**  $\longrightarrow$

23 **ENTER** 13 **A** **D**  $\longrightarrow$

**Outputs:**

2.00 \*\*\* real ( $z_2 + z_3$ )  
-2.00 \*\*\* imag ( $z_2 + z_3$ )

0.25 \*\*\*  $1/(z_2 + z_3)$   
0.25 \*\*\*

2.50 \*\*\* ( $z_1/(z_2 + z_3)$ )  
9.00 \*\*\*

**Example 2:**

Find the 3 cube roots of 8.

**Keystrokes:**8 **ENTER** 0 **A** 3 **f** **D** →**Outputs:**

2.00 \*\*\*

0.00 \*\*\*

-1.00 \*\*\*

1.73 \*\*\*

-1.00 \*\*\*

-1.73 \*\*\*

**Example 3:**Evaluate  $e^{z^{-2}}$ , where  $z = (1 + i)$ .**Keystrokes:**1 **ENTER** 1 **A** 2 **f** **C** →**Outputs:**0.00 \*\*\* ( $z^2$ )

2.00 \*\*\*

**f** **B** →0.00 \*\*\* ( $z^{-2}$ )

-0.50 \*\*\*

**f** **E** →0.88 \*\*\* ( $e^{z^{-2}}$ )

-0.48 \*\*\*

## POLYNOMIAL SOLUTIONS



This program will solve polynomial equations with real coefficients of degree 5 and below, provided the high-order coefficient is 1. The equation may be represented as

$$x^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0 = 0, \quad n=2, 3, 4, \text{ or } 5.$$

If the leading coefficient is not 1, it should be made 1 by dividing the entire equation by that coefficient.

The user must store the coefficients of the equation beforehand, beginning with  $a_0$  in  $R_0$  through  $a_{n-1}$  in  $R_{n-1}$ . Zero must be input for those coefficients which are zero. It is not necessary to store the leading coefficient as 1, or any  $a_k$  where  $k > n$ .

After the coefficients have been stored, the user-definable key (**A** through **D**) which represents the order of the polynomial should be pressed. All roots of the equation, real and complex, will then be computed. For example, if coefficients  $a_0$ ,  $a_1$ ,  $a_2$ , and  $a_3$  have been stored in registers  $R_0$  through  $R_3$ , then key **B** should be pressed to compute the four roots of the fourth degree polynomial equation

$$x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0.$$

### Equations:

The routines for third and fifth degree equations use an iterative routine under LBL a to find one real root of the equation. This routine requires that the constant term  $a_0$  not be zero for these equations. (If  $a_0 = 0$ , then zero is a real root and by factoring out  $x$ , the equation may be reduced by one order.) After one root is found, synthetic division is performed to reduce the original equation to a second or fourth degree equation.

To solve a fourth degree equation, it is first necessary to solve the cubic equation

$$y^3 + b_2y^2 + b_1y + b_0 = 0$$

where  $b_2 = -a_2$

$$b_1 = a_3a_1 - 4a_0$$

$$b_0 = a_0(4a_2 - a_3^2) - a_1^2.$$

Let  $y_0$  be the largest real root of the above cubic.

Then the fourth degree equation is reduced to two quadratic equations:

$$x^2 + (A + C)x + (B + D) = 0$$

$$x^2 + (A - C)x + (B - D) = 0$$

where  $A = \frac{a_3}{2}$ ,  $B = \frac{y_0}{2}$

$$D = \sqrt{B^2 - a_0}$$

$$C = \begin{cases} \left( AB - \frac{a_1}{2} \right) / D & \text{if } D \neq 0 \\ \sqrt{A^2 - a_2 + y_0} & \text{if } D = 0 \end{cases}$$

Roots of the fourth degree equation are found by solving the two quadratic equations.

A quadratic equation  $x^2 + a_1x + a_0 = 0$  is solved by the formula  $x_{1,2} =$

$$-\frac{a_1}{2} \pm \sqrt{\frac{a_1^2}{4} - a_0}. \text{ If } D = \frac{a_1^2}{4} - a_0 > 0, \text{ the roots are real; if } D < 0, \text{ the}$$

roots are complex, being  $u \pm iv = -\frac{a_1}{2} \pm i\sqrt{-D}$ .

A real root is output as a single number. Complex roots always occur in pairs of the form  $u \pm iv$ . They are output by loading the stack with  $u$ ,  $v$ ,  $u$ , and  $-v$  in registers T, Z, Y, and X, respectively, and then executing the command Print Stack. If these roots are being output through a Pause (HP-67) rather than a Print (HP-97), some attention may be required to make sure that no roots go unnoticed.

### Remarks:

1. Long execution times ( $\sim 1-2$  minutes) may be expected for equations of degree 3, 4, or 5, as these use an iterative routine once or more.
2. There is one condition in the solution of fourth or fifth degree polynomials that can cause the program to halt displaying Error. It is a very rare condition and you may never encounter it. It will occur when  $b_0 = a_0(4a_2 - a_3^2) - a_1^2 = 0$  in the solution of the cubic to find  $y_0$ . If the calculator halts at line 161 displaying Error, then  $b_0$  has been found to be zero and the following key sequence should be performed to recover from the error: 0 **STO** **7** **RCL** **1** **STO** **0** **RCL** **2** **STO** **1** **D**. After execution of **D**, press **GTO** **044** **R/S**. The program will now continue to execute normally.

| STEP | INSTRUCTIONS                       | INPUT<br>DATA/UNITS | KEYS  | OUTPUT<br>DATA/UNITS |
|------|------------------------------------|---------------------|-------|----------------------|
| 1    | Load side 1 and side 2.            |                     |       |                      |
| 2    | Input coefficients below order     |                     |       |                      |
|      | of polynomial (i.e., for degree n, |                     |       |                      |
|      | input through a <sub>n-1</sub> ).  |                     |       |                      |
|      | Coefficients = 0 must be so        |                     |       |                      |
|      | input.                             | a <sub>0</sub>      | STO 0 |                      |
|      |                                    | a <sub>1</sub>      | STO 1 |                      |
|      |                                    | a <sub>2</sub>      | STO 2 |                      |
|      |                                    | a <sub>3</sub>      | STO 3 |                      |
|      |                                    | a <sub>4</sub>      | STO 4 |                      |
| 3    | Compute roots to polynomial        |                     |       |                      |
|      | of degree                          |                     |       |                      |
|      | • 5                                |                     | A     | Roots 1–5            |
|      | • 4                                |                     | B     | Roots 1–4            |
|      | • 3                                |                     | C     | Roots 1–3            |
|      | • 2                                |                     | D     | Roots 1–2            |
| 4    | A single number will be output     |                     |       |                      |
|      | for a real root; complex pairs of  |                     |       |                      |
|      | roots (u ± iv) will output as      |                     |       |                      |
|      | shown:                             |                     |       | u                    |
|      |                                    |                     |       | v                    |
|      |                                    |                     |       | u                    |
|      |                                    |                     |       | –v                   |
| 5    | For a new equation, return to      |                     |       |                      |
|      | step 2.                            |                     |       |                      |

Example 1:

Solve  $x^5 - x^4 - 101x^3 + 101x^2 + 100x - 100 = 0$ .

Keystrokes:

100 CHS STO 0 100 STO 1  
101 STO 2 101 CHS STO 3

Outputs:

1 **CHS** **STO** **4** **A**  $\longrightarrow$

10.00 \*\*\* (Root 1)  
1.00 \*\*\* (Root 2)  
1.00 \*\*\* (Root 3)  
-1.00 \*\*\* (Root 4)  
-10.00 \*\*\* (Root 5)

### Example 2:

Solve  $4x^4 - 8x^3 - 13x^2 - 10x + 22 = 0$ .

Rewrite the equation as  $x^4 - 2x^3 - \frac{13}{4}x^2 - \frac{10}{4}x + \frac{22}{4} = 0$ .

**Keystrokes:**

**Outputs:**

22 **ENTER** **4** **÷** **STO** **0** 10 **ENTER**

4 **÷** **CHS** **STO** **1** 13 **ENTER** **4** **÷**

**CHS** **STO** **2** 2 **CHS** **STO** **3** **B**  $\longrightarrow$

-1.00 (Roots 1 & 2)  
1.00 are  
-1.00  $-1.00 \pm 1.00i$   
-1.00  
3.12 \*\*\* (Root 3)  
0.88 \*\*\* (Root 4)

### Example 3:

Solve  $x^3 - 4x^2 + 8x - 8 = 0$ .

**Keystrokes:**

**Outputs:**

8 **CHS** **STO** **0** 8 **STO** **1**

4 **CHS** **STO** **2** **C**  $\longrightarrow$

2.00 \*\*\* (Root 1)  
1.00 (Roots 2 & 3)  
1.73 are  
1.00  $1.00 \pm 1.73i$   
-1.73

### Example 4:

Solve  $2x^2 + 5x + 3 = 0$ .

Rewrite the equation as  $x^2 + 2.5x + 1.5 = 0$ .

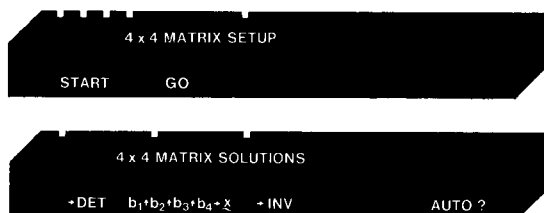
**Keystrokes:**

**Outputs:**

1.5 **STO** **0** 2.5 **STO** **1** **D**  $\longrightarrow$

-1.00 \*\*\* (Root 1)  
-1.50 \*\*\* (Root 2)

## 4x4 MATRIX OPERATIONS



This two-card program allows several of the most important operations involving 4x4 matrices, namely, the calculations of the determinant and inverse of a 4x4 matrix, and the solution of a system of simultaneous equations in 4 unknowns.

The method used in this program is that of Gaussian elimination with partial pivoting. Space does not allow a full treatment of the pertinent equations; however, the Comments section of the program listing shows the operations in detail, step by step.

Basically, the first of these two cards, 4x4 Matrix Setup, allows for input of the matrix A and transforms A into an upper triangular matrix U, assuming A is nonsingular. The multipliers used to accomplish this transformation form a lower triangular matrix, L, which has 1's along its diagonal. If we disregard pivoting, a technique of row interchanges which may improve accuracy and which may introduce one or more permutation matrices, then the relationship among these matrices is  $U = LA$ . At the end of execution of the first card, the original matrix A no longer exists in memory. The initial elements  $a_{ij}$  have been replaced by the elements of U ( $i \leq j$ ) and of L ( $i > j$ ). (The elements of U will still be referred to as  $a_{ij}$ ; those of L will be called  $m_{ij}$  in the program listing comments). The second card, 4x4 Matrix Solutions, uses the transformed matrices U and L to compute the determinant and inverse of A, and to solve systems of simultaneous equations.

### Equations:

Let A =

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

The determinant of A, Det A, is found *after* its transformation to U by the product of the diagonal elements:

$$\text{Det A} = (-1)^k a_{11} a_{22} a_{33} a_{44},$$

where k is the number of row interchanges required by pivoting.

A set of 4 simultaneous equations in 4 unknowns may be written as

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3$$

$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4,$$

where the  $\{x_i\}$  are unknowns and the  $\{b_i\}$  constants.

In matrix notation, this becomes  $A \mathbf{x} = \mathbf{b}$ , where  $\mathbf{x}$  and  $\mathbf{b}$  are the column vectors

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \text{ respectively.}$$

This problem is solved (neglecting pivoting) as  $U\mathbf{x} = L\mathbf{b}$ .

Let C be the inverse of A, i.e., the 4x4 matrix such that  $AC = CA = I$ , where I is the 4x4 matrix such that

$$I_{ij} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}, \quad i, j = 1, 2, 3, 4.$$

C is computed a column at a time in the following way:

let  $\mathbf{c}^{(j)}$  be the  $j^{\text{th}}$  column vector of C, i.e.,

$$\mathbf{c}^{(j)} = \begin{bmatrix} c_{1j} \\ c_{2j} \\ c_{3j} \\ c_{4j} \end{bmatrix}, \quad j = 1, 2, 3, 4.$$

Then  $\mathbf{c}^{(j)}$  is found by the solution of the equation

$$\mathbf{A}\mathbf{c}^{(j)} = \mathbf{I}^{(j)} \quad \text{where } \mathbf{I}^{(j)} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}, i = 1, 2, 3, 4.$$

For example,  $\mathbf{c}^{(1)}$  is found by solution of

$$\mathbf{A} \mathbf{c}^{(1)} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

### Remarks:

1. A halt during the execution of card 1 (Setup) with a display of "Error" indicates that the matrix  $\mathbf{A}$  is singular.
2. If operations are to be carried out on the same matrix over a period of time, it might be convenient to record the elements of the matrix on a magnetic card for rapid input at a later date. Because the program immediately starts operating on the matrix after the last element has been keyed in, the program needs to be modified to halt after the input of  $a_{44}$ . This may be accomplished by the following steps:
  - a. Load side 1 and side 2 of  $4 \times 4$  Matrix Setup.
  - b. Press **GTO**  $\square$  025.
  - c. Switch to PRGM, press **DEL**, **R/S**.
  - d. Switch to RUN and press **A** to start data input.
  - e. After the input of  $a_{44}$ , the program will halt. At this point, the data may be recorded for later use.
  - f. To continue execution, press **B**.

### References:

George E. Forsythe, Michael A. Malcolm, and Cleve B. Moler, *Computer Methods in Mathematical Computation*, Computer Science Department, Stanford University, 1972.

G. Forsythe and C. Moler, *Computer Solution of Linear Algebraic Systems*, Prentice-Hall, 1967.

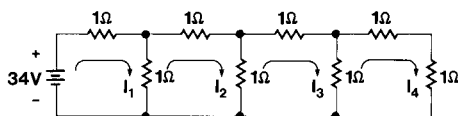
C. Moler, "Matrix Computations with Fortran and Paging," *Comm. ACM*, vol. 15, no. 4, pp. 268-270 (April, 1972).

| STEP | INSTRUCTIONS                                                 | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
|------|--------------------------------------------------------------|------------------|------|-------------------|
| 1    | Load side 1 and side 2 of 4 x 4                              |                  |      |                   |
|      | Matrix Setup.                                                |                  |      |                   |
| 2    | If data has already been stored                              |                  |      |                   |
|      | on magnetic card, go to step 7;                              |                  |      |                   |
|      | to key in data, go to step 3.                                |                  |      |                   |
| 3    | To cause output of elements                                  |                  |      |                   |
|      | $\{a_{ij}\}$ of matrix as they are                           |                  |      |                   |
|      | keyed in, set flag 0.                                        |                  | SF 0 |                   |
| 4    | Prepare to input elements of                                 |                  |      |                   |
|      | matrix in <i>column</i> order ( $a_{11}$ ,                   |                  |      |                   |
|      | $a_{21}$ , $a_{31}$ , $a_{41}$ , $a_{12}$ , $a_{22}$ , etc.) |                  | A    | 1.1               |
| 5    | Display shows i,j; key in element                            |                  |      |                   |
|      | in row i, column j.                                          | $a_{ij}$         | R/S  | next i, j         |
| 6    | Repeat step 5 until all elements                             |                  |      |                   |
|      | of matrix have been keyed in;                                |                  |      |                   |
|      | after $a_{44}$ has been keyed in,                            |                  |      |                   |
|      | program execution will begin                                 |                  |      |                   |
|      | immediately. Go to step 9.                                   |                  |      |                   |
| 7    | If matrix data is already stored on                          |                  |      |                   |
|      | magnetic card, load side 1 and                               |                  |      |                   |
|      | side 2 of data card.                                         |                  |      |                   |
| 8    | Begin program execution.                                     |                  | B    |                   |
| 9    | Load side 1 and side 2 of 4 x 4                              |                  |      |                   |
|      | Matrix Solutions.                                            |                  |      |                   |
| 10   | For automatic output of results,                             |                  |      |                   |
|      | set AUTO mode.                                               |                  | E    | 1.00              |
| 11   | To cancel AUTO mode later                                    |                  | E    | 0.00              |
| 12   | (optional) Compute determinant.                              |                  | A    | Det A             |

| STEP | INSTRUCTIONS                          | INPUT<br>DATA/UNITS | KEYS  | OUTPUT<br>DATA/UNITS |
|------|---------------------------------------|---------------------|-------|----------------------|
| 13   | To solve a system of four             |                     |       |                      |
|      | simultaneous equations, key in        |                     |       |                      |
|      | right-hand side and find $x$ .        | $b_1$               | ENTER |                      |
|      |                                       | $b_2$               | ENTER |                      |
|      |                                       | $b_3$               | ENTER |                      |
|      |                                       | $b_4$               | B     | $x_1$                |
|      |                                       |                     |       | $x_2$                |
|      |                                       |                     |       | $x_3$                |
|      |                                       |                     |       | $x_4$                |
| 14   | Find the inverse of matrix A          |                     |       |                      |
|      | ( $C = A^{-1}$ ), displayed in column |                     |       |                      |
|      | order.                                |                     | C     | $C_{11}$             |
|      |                                       |                     |       | $C_{21}$             |
|      |                                       |                     |       | $C_{31}$             |
|      |                                       |                     |       | $C_{41}$             |
|      |                                       |                     |       |                      |
|      |                                       |                     |       | $C_{12}$             |
|      |                                       |                     |       | $C_{22}$             |
|      |                                       |                     |       | $C_{32}$             |
|      |                                       |                     |       | $C_{42}$             |
|      |                                       |                     |       |                      |
|      |                                       |                     |       | $C_{13}$             |
|      |                                       |                     |       | $C_{23}$             |
|      |                                       |                     |       | $C_{33}$             |
|      |                                       |                     |       | $C_{43}$             |
|      |                                       |                     |       |                      |
|      |                                       |                     |       | $C_{14}$             |
|      |                                       |                     |       | $C_{24}$             |
|      |                                       |                     |       | $C_{34}$             |
|      |                                       |                     |       | $C_{44}$             |
|      |                                       |                     |       | 0.00                 |

**Example 1:**

By applying the technique of loop currents to the circuit below, find the currents  $I_1$ ,  $I_2$ ,  $I_3$ , and  $I_4$ . Do not use AUTO mode.



The equations to be solved are

$$\begin{array}{rrcr}
 2I_1 & -I_2 & & = & 34 \\
 -I_1 & +3I_2 & -I_3 & = & 0 \\
 & -I_2 & +3I_3 & -I_4 & = & 0 \\
 & & -I_3 & +3I_4 & = & 0
 \end{array}$$

In matrix form,

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 34 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Load side 1 and side 2 of 4x4 Matrix Setup. Prepare to store matrix data on a magnetic card.

**Keystrokes:**

**GTO** **◻** **0** **2** **5**

Switch to PRGM

**DEL** **R/S**

Switch to RUN

**A** **2** **R/S** **1** **CHS** **R/S** **0** **R/S** **0** **R/S**

**1** **CHS** **R/S** **3** **R/S** **1** **CHS** **R/S**

**0** **R/S** **0** **R/S** **1** **CHS** **R/S** **3** **R/S**

**1** **CHS** **R/S** **0** **R/S** **0** **R/S** **1** **CHS** **R/S**

**3** **R/S**  $\longrightarrow$

**Outputs:**

4.0

Program halts, displaying 4.0.

W/DATA →

“Crd”

Insert side 1 of a blank magnetic card, see “Crd” and insert side 2.

B → 2.6

Load side 1 and side 2 of 4x4  
Matrix Solutions. → 2.62

34 ENTER 0 ENTER  
 0 ENTER 0 B → 21.00 (I<sub>1</sub>)  
 R/S → 8.00 (I<sub>2</sub>)  
 R/S → 3.00 (I<sub>3</sub>)  
 R/S → 1.00 (I<sub>4</sub>)

**Example 2:**

Find the determinant and inverse of the matrix below. Use AUTO mode.

$$\begin{bmatrix} 7 & 5 & 1 & 3 \\ 5 & 7 & 7 & 7 \\ 3 & 3 & 3 & 5 \\ 1 & 1 & 5 & 1 \end{bmatrix}$$

**Keystrokes:****Outputs:**Load side 1 and side 2 of 4x4  
Matrix Setup

A 7 R/S 5 R/S 3 R/S 1 R/S  
 5 R/S 7 R/S 3 R/S 1 R/S  
 1 R/S 7 R/S 3 R/S 5 R/S  
 3 R/S 7 R/S 5 R/S 1 R/S → 2.5

Load side 1 and side 2 of 4x4  
 Matrix Solutions → 2.46  
 E → 1.00 (AUTO set)  
 A → -256.00 (Det A)  
 DSP 6 C → 0.218750 \*\*\* (c<sub>11</sub>)  
 -0.046875 \*\*\* (c<sub>21</sub>)  
 -0.015625 \*\*\* (c<sub>31</sub>)  
 -0.093750 \*\*\* (c<sub>41</sub>)

-0.281250 \*\*\* ( $c_{12}$ )

0.453125 \*\*\* ( $c_{22}$ )

-0.015625 \*\*\* ( $c_{32}$ )

-0.093750 \*\*\* ( $c_{42}$ )

0.218750 \*\*\* ( $c_{13}$ )

-0.546875 \*\*\* ( $c_{23}$ )

-0.015625 \*\*\* ( $c_{33}$ )

0.406250 \*\*\* ( $c_{43}$ )

0.218750 \*\*\* ( $c_{14}$ )

-0.296875 \*\*\* ( $c_{24}$ )

0.234375 \*\*\* ( $c_{34}$ )

-0.093750 \*\*\* ( $c_{44}$ )

0.000000 (End)

## SOLUTION TO $f(x)=0$ ON AN INTERVAL



This program finds one real root of the equation  $f(x) = 0$  in a finite interval  $[b, c]$ , where  $f(x)$  is a function specified by the user which must be continuous and real on the interval. The program assumes without checking that of the values  $f(b)$  and  $f(c)$ , one will be positive and one negative, i.e.,  $f(b) \times f(c) < 0$ . In this way,  $b$  and  $c$  will bracket the root. An accuracy tolerance  $TOL (\geq 0)$  must also be specified. This number should be the greatest allowable error in the final approximation for the root. That is, the actual root will be no farther away than  $TOL$  from the program's solution for the root.

The function  $f(x)$  should be keyed into program memory under LBL E and should assume that  $x$  will be in the X-register upon entry. 85 program steps, registers  $R_1$  through  $R_7$ ,  $R_{S0}$  through  $R_{S9}$ , and the stack are available for defining  $f(x)$ .

The method used is a combination of bisection (interval-halving) and the secant method. Bisection is often slow but is guaranteed to converge to a root, if one exists in the interval; the secant method is fast but does not always converge. The algorithm employed in this program combines the safety of bisection with some of the speed of the secant method. If the function is known to be well-behaved on the interval in question, then the program in Standard Pac, *Calculus and Roots of  $f(x)$* , may lead to a faster and more convenient solution.

### Remarks:

As each value for  $b$  or  $c$  is input, its function value will be computed and displayed. If you are in doubt about values for  $b$  and  $c$  which will satisfy  $f(b) \times f(c) < 0$ , you may simply keep inputting different values until you strike a good combination. Each new value input overwrites the old.

### References:

George E. Forsythe, Michael A. Malcolm, and Cleve B. Moler, *Computer Methods in Mathematical Computation*, Computer Science Department, Stanford University, 1972.

Richard P. Brent, *Algorithms for Minimization without Derivatives*, Prentice-Hall, 1973.

T. J. Dekker, "Finding a zero by means of successive linear interpolation," in B. Dejon and P. Henrici (editors), *Constructive Aspects of the Fundamental Theorem of Algebra*, Interscience, 1969.

| STEP | INSTRUCTIONS                                                              | INPUT DATA/UNITS | KEYS                | OUTPUT DATA/UNITS |
|------|---------------------------------------------------------------------------|------------------|---------------------|-------------------|
| 1    | Load side 1 and side 2.                                                   |                  |                     |                   |
| 2    | Prepare to key in function.                                               |                  | <b>GTO</b> <b>E</b> |                   |
| 3    | Switch to PRGM. See line 138.                                             |                  |                     |                   |
| 4    | Key in the function $f(x)$ (need not add <b>RTN</b> ).                    |                  |                     |                   |
| 5    | Switch to RUN.                                                            |                  |                     |                   |
| 6    | Key in the end points of the interval (remember $f(b) \times f(c) < 0$ ). | b                | <b>A</b>            | $f(b)$            |
|      |                                                                           | c                | <b>B</b>            | $f(c)$            |
| 7    | Key in the accuracy tolerance.                                            | TOL              | <b>C</b>            | TOL               |
| 8    | Compute a real root.                                                      |                  | <b>D</b>            | root              |
| 9    | To evaluate the function at any point.                                    | x                | <b>E</b>            | $f(x)$            |

**Example 1:**

Find an angle  $\alpha$  between 100 and 101 radians such that  $\sin \alpha = 0.1$ . Hence let  $f(x) = \sin x - 0.1$ . Assume a tolerance of  $10^{-3}$ .

**Keystrokes:****Outputs:**

Load side 1 and side 2.

**GTO** **E**

Switch to PRGM. See line 138.

**RAD** **SIN** .1 **=** **DEG**

Switch to RUN.

|                                    |                |            |
|------------------------------------|----------------|------------|
| 100 <b>A</b> →                     | -0.61          | $(f(100))$ |
| 101 <b>B</b> →                     | 0.35           | $(f(101))$ |
| <b>EEX</b> <b>CHS</b> 3 <b>C</b> → | 1.000000000-03 |            |
| <b>D</b> →                         | 100.63         | (root)     |

**Example 2:**

Find a root of the equation  $\ln x + 3x - 10.8074 = 0$  in the interval  $[1, 5]$ . An accuracy of  $10^{-4}$  is acceptable. Store the constant 10.8074 in  $R_1$ .

**Keystrokes:****Outputs:**

Load side 1 and side 2.

**GTO** **E**

## 08-03

Switch to PRGM. See line 138.

**LN** **LAST X** 3 **×** **+** **RCL** **1** **−**

Switch to RUN.

|                                   |                     |                |        |        |
|-----------------------------------|---------------------|----------------|--------|--------|
| 10.8074                           | <b>STO</b> <b>1</b> | →              | 10.81  |        |
| 1                                 | <b>A</b>            | →              | -7.81  | (f(1)) |
| 5                                 | <b>B</b>            | →              | 5.80   | (f(5)) |
| <b>EEEX</b> <b>CHS</b> 4 <b>C</b> | →                   | 1.000000000-04 |        |        |
| <b>D</b>                          | →                   | 3.21           | (root) |        |

Check the solution by computing its function value.

|          |   |                 |           |
|----------|---|-----------------|-----------|
| <b>E</b> | → | -1.901000000-05 | (f(3.21)) |
|----------|---|-----------------|-----------|

# NUMERICAL INTEGRATION



This program will perform numerical integration whether a function is known explicitly or only at a finite number of equally spaced points (discrete case). The integrals of explicit functions are found using Simpson's rule; discrete case integrals may be approximated by either the trapezoidal rule or Simpson's rule.

## Discrete case

Let  $x_0, x_1, \dots, x_n$  be  $n$  equally spaced points ( $x_j = x_0 + jh, j = 1, 2, \dots, n$ ) at which corresponding values  $f(x_0), f(x_1), \dots, f(x_n)$  of the function  $f(x)$  are known. The function itself need not be known explicitly. After input of the step size  $h$  and the values of  $f(x_j), j = 0, 1, \dots, n$ , then the integral

$$\int_{x_0}^{x_n} f(x) dx$$

may be approximated using

1. The trapezoidal rule:

$$\int_{x_0}^{x_n} f(x) dx \approx \frac{h}{2} \left[ f(x_0) + 2 \sum_{j=1}^{n-1} f(x_j) + f(x_n) \right]$$

2. Simpson's rule:

$$\int_{x_0}^{x_n} f(x) dx \approx \frac{h}{3} \left[ f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right]$$

In order to apply Simpson's rule,  $n$  must be even. If  $n$  is not even, the calculator will halt displaying "Error" if  $\square$  is pressed.

## Explicit functions

If an explicit formula is known for the function  $f(x)$ , then the function may be keyed into program memory and numerically integrated by Simpson's rule. The user must specify the endpoints  $a$  and  $b$  of the interval over which inte-

gration is to be performed, and the number of subintervals  $n$  into which the interval  $(a,b)$  is to be divided. This  $n$  must be even; if it is not, Error will be displayed. The program will go on to compute  $x_0 = a$ ,  $x_j = x_0 + jh$ ,  $j = 1, 2, \dots, n-1$ , and  $x_n = b$  where

$$h = \frac{b-a}{n}.$$

The integral  $\int_a^b f(x) dx$  is approximated by equation (2) above, Simpson's rule.

Up to five different functions  $f_i(x)$ ,  $i = 1, \dots, 5$ , may be loaded into program memory at one time under labels 1 through 5. The function to be integrated is selected by keying in a digit 1, 2, 3, 4, or 5, and pressing **↑ E**. The function under the appropriate label will then be selected. 112 program steps are available for defining the  $f_i(x)$ , as well as registers  $R_1$  through  $R_8$ ,  $R_{S0}$  through  $R_{S9}$ , and the four stack registers. The functions should assume  $x$  is in the X-register upon entry. Two levels of subroutines are allowed in the functions  $f_i(x)$ , but recall that the only labels available are 1 through 5.

Functions  $f_i(x)$  may be keyed into program memory after loading side 1 of *Numerical Integration*, or you may record these functions beforehand on a magnetic card and load them in the following manner:

1. Load side 1 of Numerical Integration.
2. Press **GTO** **□** 112.
3. Press **MERGE**.
4. Load your card with the functions  $f_i(x)$ .

### Remarks:

Note that the function values for the discrete case  $f(x_j)$ ,  $j = 0, 1, \dots, n$ , are keyed into **B**. There are actually three routines in the program which begin with LBL B, one for  $j = 0$ , one for  $j$  odd, and one for  $j$  even. It is important that no other user-definable keys be pressed during the entry of the  $f(x_j)$ , lest the next  $f(x_j)$  entered go into the wrong LBL B.

| STEP | INSTRUCTIONS                                                          | INPUT<br>DATA/UNITS | KEYS     | OUTPUT<br>DATA/UNITS |
|------|-----------------------------------------------------------------------|---------------------|----------|----------------------|
| 1    | Load side 1 of program.                                               |                     |          |                      |
| 2    | For explicit functions, go to step 8; for discrete case, go to step 3 |                     |          |                      |
|      | <b>DISCRETE</b>                                                       |                     |          |                      |
| 3    | Key in the spacing between x-values.                                  | $h$                 | <b>A</b> |                      |

[illegible]

**Example 1:**

Given the values below for  $f(x_j)$ ,  $j = 0, 1, \dots, 8$ , compute the approximations to the integral

$$\int_0^2 f(x) dx$$

by the trapezoidal rule and by Simpson's rule.

The value for  $h$  is 0.25.

|          |   |     |     |     |   |      |      |      |    |
|----------|---|-----|-----|-----|---|------|------|------|----|
| i        | 0 | 1   | 2   | 3   | 4 | 5    | 6    | 7    | 8  |
| $x_i$    | 0 | .25 | .5  | .75 | 1 | 1.25 | 1.5  | 1.75 | 2  |
| $f(x_i)$ | 2 | 2.8 | 3.8 | 5.2 | 7 | 9.2  | 12.1 | 15.6 | 20 |

**Keystrokes:**

.25 **A** 2 **B** 2.8 **B** 3.8 **B**  
 5.2 **B** 7 **B** 9.2 **B** 12.1 **B**  
 15.6 **B** 20 **B** **C**  $\longrightarrow$   
**D**  $\longrightarrow$

**Outputs:**

16.68 \*\*\* (Trapezoidal)  
 16.58 \*\*\* (Simpson's)

**Example 2:**

Find the value of

$$\int_0^{2\pi} \frac{dx}{1 - \cos x + 0.25}$$

for  $n = 10$  and then for  $n = 16$ . Note that  $x$  is assumed to be in radians. For safety, if you work mostly in degrees, it is good programming practice to set the angular mode to radians at the beginning of the routine, then back to degrees at the end. Key the function in under LBL 1.

**Keystrokes:**

**GTO** **□** 112  
 Switch to PRGM.  
**LBL** **1** **RAD** **COS** **1** **x $\div$ y** **-**  
 .25 **+** **1/x** **DEG** **RTN**  
 Switch to RUN.  
 0 **ENTER** **2**  **$\pi$**   **$\times$**  **f** **A** 10 **f** **B**  
 1 **f** **E** **f** **C**  $\longrightarrow$   
 16 **f** **B** **f** **C**  $\longrightarrow$

**Outputs:**

8.22 \*\*\* ( $n=10$ )  
 8.36 \*\*\* ( $n=16$ )

The exact solution is  $\frac{8\pi}{3} = 8.38$ .

## GAUSSIAN QUADRATURE



This program will compute approximations for integrals over finite or infinite intervals by the six-point Gauss-Legendre quadrature method. If  $f(x)$  is the function to be integrated, then either

$$\int_a^{\infty} f(x) \, dx \quad \text{or} \quad \int_a^b f(x) \, dx$$

may be found.

The function  $f(x)$  must be explicitly known and keyed into program memory under LBL E by the user. Upon entry, the value of  $x$  will be in the  $X$ -register. 48 program steps are available for defining  $f(x)$ ; registers  $R_1$  through  $R_9$ ,  $R_{S6}$  through  $R_{S9}$ ,  $R_D$ ,  $R_E$  and the stack are also available to the user.

### Equations:

$$\int_a^b f(x) \, dx \approx \frac{b-a}{2} \sum_{i=1}^6 w_i f\left(\frac{z_i(b-a) + b + a}{2}\right)$$

$$\int_a^{\infty} f(x) \, dx \approx 2 \sum_{i=1}^6 \frac{w_i}{(1 + z_i)^2} f\left(\frac{2}{1 + z_i} + a - 1\right)$$

where

$$z_1 = -z_2 = .2386191861$$

$$z_3 = -z_4 = .6612093865$$

$$z_5 = -z_6 = .9324695142$$

$$w_1 = w_2 = .4679139346$$

$$w_3 = w_4 = .360761573$$

$$w_5 = w_6 = .1713244924$$

### Remarks:

If more program steps are needed to define  $f(x)$ , all of **LBL A** (steps 001–076) may be deleted after executing it (pressing **A**) one time.

### Reference:

Applied Numerical Methods, Carnahan, Luther and Wilks, John Wiley and Sons, 1969.

| STEP | INSTRUCTIONS                         | INPUT DATA/UNITS | KEYS         | OUTPUT DATA/UNITS       |
|------|--------------------------------------|------------------|--------------|-------------------------|
| 1    | Load side 1 and side 2 of program.   |                  |              |                         |
| 2    | Prepare to key in function $f(x)$ .  |                  | <b>GTO E</b> |                         |
| 3    | Switch to PRGM. Line number          |                  |              |                         |
|      | is 177.                              |                  |              |                         |
| 4    | Key in the function $f(x)$ (need     |                  |              |                         |
|      | not add <b>RTN</b> ).                |                  |              |                         |
| 5    | Switch to RUN.                       |                  |              |                         |
| 6    | Initialize.                          |                  | <b>A</b>     |                         |
| 7    | For a finite interval, key in the    |                  |              |                         |
|      | lower and upper bounds of the        |                  |              |                         |
|      | interval and compute the integral.   | a                | <b>ENTER</b> |                         |
|      |                                      | b                | <b>B</b>     | $\int_a^b f(x) dx$      |
| 8    | For an infinite interval, key in the |                  |              |                         |
|      | lower bound of the interval and      |                  |              |                         |
|      | compute the integral.                | a                | <b>C</b>     | $\int_a^\infty f(x) dx$ |

**Example 1:**

Find  $\int_1^{10} \frac{dx}{x}$ .

The function is  $f(x) = \frac{1}{x}$ ; the only key required is **1/x**.

**Keystrokes:**

**GTO E**

Switch to PRGM.

**1/x**

Switch to RUN.

**A 1 ENTER** 10 **B** →

The exact answer is  $\ln 10$ .

**Outputs:**

2.30 \*\*\*

## 10-03

### Example 2:

Find  $\int_0^{\infty} e^{-x} x^{0.8} dx$ .

#### Keystrokes:

**GTO** **E**

Switch to PRGM.

(If Example 1 has been run, delete the key **1/x** .)

**CHS** **e<sup>x</sup>** **LAST X** **CHS** **.8** **y<sup>x</sup>** **×**

Switch to RUN.

**A** (need not be pressed if Example 1 has been run)

0 **C**  $\longrightarrow$

The correct answer is  $\Gamma(1.8) = 0.9314$ .

#### Outputs:

0.92 \*\*\*

# DIFFERENTIAL EQUATIONS



This program solves first- and second-order differential equations by the fourth-order Runge-Kutta method. A first-order equation is of the form  $y' = f(x, y)$ , with initial values  $x_0, y_0$ ; a second-order equation is of the form  $y'' = f(x, y, y')$ , with initial values  $x_0, y_0, y_0'$ .

In either case, the function  $f$  should be keyed into program memory under LBL E, and should assume that  $x$  and  $y$  are in the X- and Y-registers respectively;  $y'$  will be in the Z-register for second-order equations. 56 program steps are available for defining the function, as well as registers  $R_1 - R_8, R_{S0} - R_{S9}$ , and I.

The solution is a numerical solution which calculates  $y_i$  for  $x_i = x_0 + ih$  ( $i = 1, 2, 3, \dots$ ), where  $h$  is an increment specified by the user. The value for  $h$  may be changed at any time during the program's execution. This allows solution of the equation arbitrarily close to a pole ( $y \rightarrow \pm\infty$ ).

The values for  $x_i$  and  $y_i$  may be output in one of two ways. In its normal operation, the program will halt each time a value is calculated for  $x_i$  or  $y_i$ . The user may re-initiate execution by pressing **R/S**. Thus, in its normal use, the program outputs all results by halting and showing the result in the calculator's display. The other way to operate the program is under AUTO mode. In this case, all results are output by a PRINTx command, which means that on an HP-97, the result will appear on the printer, while on the HP-67, the program will pause briefly to display the answer. After that output, the program will automatically go on to calculate the next result.

## Equations:

1<sup>st</sup> -order:

$$y_{i+1} = y_i + \frac{1}{6} (c_1 + 2c_2 + 2c_3 + c_4)$$

where

$$c_1 = hf(x_i, y_i)$$

$$c_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{c_1}{2}\right)$$

$$c_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{c_2}{2}\right)$$

$$c_4 = hf(x_i + h, y_i + c_3)$$

2<sup>nd</sup> -order:

$$y_{i+1} = y_i + h \left[ y'_i + \frac{1}{6} (k_1 + k_2 + k_3) \right]$$

$$y'_{i+1} = y'_i + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_i, y_i, y'_i)$$

$$k_2 = hf \left( x_i + \frac{h}{2}, y_i + \frac{h}{2} y'_i + \frac{h}{8} k_1, y'_i + \frac{k_1}{2} \right)$$

$$k_3 = hf \left( x_i + \frac{h}{2}, y_i + \frac{h}{2} y'_i + \frac{h}{8} k_1, y'_i + \frac{k_2}{2} \right)$$

$$k_4 = hf \left( x_i + h, y_i + h y'_i + \frac{h}{2} k_3, y'_i + k_3 \right)$$

### Remarks:

1. When inputting values for a second-order solution, the values for  $x_0$  and  $y_0$  must be input before the value of  $y'_0$ . All values must be input even if zero.
2. If the program is to be run for different functions, be sure that the first function is no longer in program memory when the second is keyed in. The best way to ensure this is to load the program anew before keying in each function.

| STEP | INSTRUCTIONS                           | INPUT DATA/UNITS | KEYS  | OUTPUT DATA/UNITS |
|------|----------------------------------------|------------------|-------|-------------------|
| 1    | Load side 1 and side 2 of program.     |                  |       |                   |
| 2    | Prepare to load function $f(x, y, y')$ |                  |       |                   |
|      | under LBL E.                           |                  | GTO E |                   |
| 3    | Switch to PRGM.                        |                  |       |                   |
| 4    | Key in the function (need not          |                  |       |                   |
|      | add RTN).                              |                  |       |                   |
| 5    | Switch to RUN.                         |                  |       |                   |
| 6    | Input step size.                       | h                | A     | h/2               |
| 7    | Input initial values for x and y.      | $x_0$            | ENTER |                   |
|      |                                        | $y_0$            | B     | $x_0$             |
| 8    | For a second-order solution,           |                  |       |                   |
|      | input initial value of $y'$ .          | $y'_0$           | C     |                   |

| STEP | INSTRUCTIONS                                                   | INPUT DATA/UNITS | KEYS       | OUTPUT DATA/UNITS |
|------|----------------------------------------------------------------|------------------|------------|-------------------|
| 9    | For AUTO mode go to step 10;<br>for manual use, go to step 13. |                  |            |                   |
|      | <b>AUTO</b>                                                    |                  |            |                   |
| 10   | Select AUTO mode for output<br>by Print/Pause.                 |                  | <b>f E</b> | 1.00              |
| 11   | To cancel AUTO mode later.                                     |                  | <b>f E</b> | 0.00              |
| 12   | Output successive values of<br>x and y.                        |                  | <b>D</b>   | $x_1$             |
|      |                                                                |                  |            | $y_1$             |
|      |                                                                |                  |            | $x_2$             |
|      |                                                                |                  |            | $y_2$             |
|      |                                                                |                  |            | etc.              |
|      | <b>Manual</b>                                                  |                  |            |                   |
| 13   | Output successive values of<br>x and y.                        |                  | <b>D</b>   | $x_1$             |
|      |                                                                |                  | <b>R/S</b> | $y_1$             |
|      |                                                                |                  | <b>R/S</b> | $x_2$             |
|      |                                                                |                  | <b>R/S</b> | $y_2$             |
|      |                                                                |                  |            | etc.              |

**Example 1:**

Solve numerically the first-order differential equation

$$y' = \frac{\sin x + \tan^{-1}(y/x)}{y - \ln(\sqrt{x^2 + y^2})}$$

where  $x_0 = y_0 = 1$ . Let  $h = 0.5$ . The angular mode must be set to radians.

**Keystrokes:**

Load side 1 and side 2 of program

**GTO E**

Switch to PRGM. See line 148.

**RAD STO 1 x↔y STO 2 x↔y**

**→P LN STO 3 R+ RCL 1**

**SIN + RCL 2 RCL 3 - ÷ DEG**

**Outputs:**

Switch to RUN. Do not set Auto mode.

|     |   |      |                   |   |   |   |   |      |                   |
|-----|---|------|-------------------|---|---|---|---|------|-------------------|
| .5  | A | 1    | ENTER             | 1 | B | D | → | 1.50 | (x <sub>1</sub> ) |
| R/S | → | 2.06 | (y <sub>1</sub> ) |   |   |   |   |      |                   |
| R/S | → | 2.00 | (x <sub>2</sub> ) |   |   |   |   |      |                   |
| R/S | → | 2.78 | (y <sub>2</sub> ) |   |   |   |   |      |                   |
| R/S | → | 2.50 | (x <sub>3</sub> ) |   |   |   |   |      |                   |
| R/S | → | 3.28 | (y <sub>3</sub> ) |   |   |   |   |      |                   |

### Example 2:

Solve the second-order equation

$$(1 - x^2) y'' + xy' = x$$

where  $x_0 = y_0 = y_0' = 0$  and  $h = 0.1$ .

Rewrite the equation as

$$y'' = \frac{x(1-y')}{1-x^2}, \quad x \neq 1$$

### Keystrokes:

### Outputs:

Load side 1 and side 2 of program.

GTO E

Switch to PRGM. See line 148.

STO 8 R↓ R↓ 1 - RCL 8 ×

RCL 8 x<sup>2</sup> 1 - ÷

Switch to RUN.

.1 A 0 ENTER 0 B 0 C f E ▶

DSP 4 D →

|            |                   |
|------------|-------------------|
| 1.00       | (AUTO mode)       |
| 0.1000 *** | (x <sub>1</sub> ) |
| 0.0002 *** | (y <sub>1</sub> ) |
| 0.2000 *** | (x <sub>2</sub> ) |
| 0.0013 *** | (y <sub>2</sub> ) |
| 0.3000 *** | (x <sub>3</sub> ) |
| 0.0046 *** | (y <sub>3</sub> ) |
| 0.4000 *** | (x <sub>4</sub> ) |
| 0.0109 *** | (y <sub>4</sub> ) |
| 0.5000 *** | (x <sub>5</sub> ) |
| 0.0217 *** | (y <sub>5</sub> ) |
|            | etc.              |

# INTERPOLATIONS

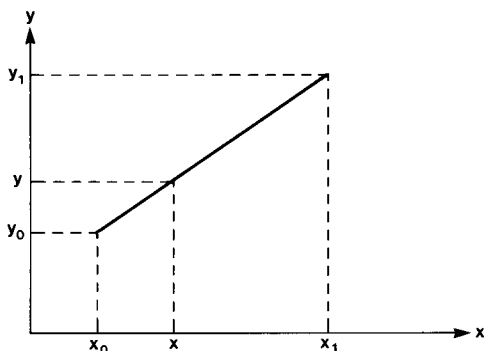


This program allows selection of one of three different interpolation routines: linear, Lagrangian, and finite difference.

## Linear interpolation

If  $y$  is a function of  $x$ , let  $y_0$  and  $y_1$  be known function values corresponding to  $x_0$  and  $x_1$  respectively. Then if  $x_0 < x < x_1$ , the function value of  $x$  can be approximated in a linear fashion by

$$y = \frac{(x_1 - x) y_0 + (x - x_0) y_1}{x_1 - x_0}$$



## Lagrangian interpolation

Given three points,  $(x_0, y_0)$ ,  $(x_1, y_1)$ , and  $(x_2, y_2)$ , the program will evaluate for an argument  $x$  the Lagrangian interpolating polynomial  $P_2(x)$  of degree two which passes through the three points. Let the value of  $P_2(x)$  also be denoted  $y$ .

$$P_2(x) = \sum_{i=0}^2 L_i(x) y_i$$

where

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^2 \frac{(x - x_j)}{(x_i - x_j)}, \quad i=0, 1, 2$$

## Finite difference interpolation

This program interpolates for data points in the region of tabulated data for uniformly spaced abscissas, with spacing  $h$ . The equation used is the backward-interpolation formula of Gauss which uses four pairs of data points and sets up the polynomial for cubic interpolation.

The equation used is:

$$y = y_3 + u\delta y_{-1/2} + \frac{1}{2}u(u+1)\delta^2 y_0 + \frac{1}{3!}u(u+1)(u-1)\delta^3 y_{-1/2}$$

The difference table is:

| u  | x     | y     |             |                    |                           |
|----|-------|-------|-------------|--------------------|---------------------------|
| -2 | $x_1$ | $y_1$ |             |                    |                           |
| -1 | $x_2$ | $y_2$ | $y_2 - y_1$ |                    |                           |
| 0  | $x_3$ | $y_3$ | $y_3 - y_2$ | $y_3 - 2y_2 + y_1$ |                           |
| 1  | $x_4$ | $y_4$ | $y_4 - y_3$ | $y_4 - 2y_3 + y_2$ | $y_4 - 3y_3 + 3y_2 - y_1$ |

where

$$\delta y_{-1/2} = y_3 - y_2$$

$$\delta^2 y_0 = y_4 - 2y_3 + y_2$$

$$\delta^3 y_{-1/2} = y_4 - 3y_3 + 3y_2 - y_1$$

and

$$u = \frac{x - x_3}{h}$$

| STEP | INSTRUCTIONS                                                                                  | INPUT DATA/UNITS | KEYS  | OUTPUT DATA/UNITS |
|------|-----------------------------------------------------------------------------------------------|------------------|-------|-------------------|
| 1    | Load side 1 and side 2 of program.                                                            |                  |       |                   |
| 2    | For linear, go to step 3; for Lagrangian, go to step 7; for finite difference, go to step 12. |                  |       |                   |
|      | <b>LINEAR</b>                                                                                 |                  |       |                   |
| 3    | Input first point.                                                                            | $x_0$            | ENTER |                   |
|      |                                                                                               | $y_0$            | A     | $x_0$             |
| 4    | Input second point.                                                                           | $x_1$            | ENTER |                   |
|      |                                                                                               | $y_1$            | B     | $x_1$             |

| STEP | INSTRUCTIONS                    | INPUT DATA/UNITS | KEYS         | OUTPUT DATA/UNITS |
|------|---------------------------------|------------------|--------------|-------------------|
| 5    | Input an x and find the inter-  |                  |              |                   |
|      | polated y.                      | x                | <b>D</b>     | y                 |
| 6    | Repeat step 5 any number of     |                  |              |                   |
|      | times.                          |                  |              |                   |
|      | <b>LAGRANGIAN</b>               |                  |              |                   |
| 7    | Input first point.              | $x_0$            | <b>ENTER</b> |                   |
|      |                                 | $y_0$            | <b>A</b>     | $x_0$             |
| 8    | Input second point.             | $x_1$            | <b>ENTER</b> |                   |
|      |                                 | $y_1$            | <b>B</b>     | $x_1$             |
| 9    | Input third point.              | $x_2$            | <b>ENTER</b> |                   |
|      |                                 | $y_2$            | <b>C</b>     | $x_2$             |
| 10   | Input an x and find the inter-  |                  |              |                   |
|      | polated y, where $y = P_2(x)$ . | x                | <b>D</b>     | y                 |
| 11   | Repeat step 10 any number of    |                  |              |                   |
|      | times.                          |                  |              |                   |
|      | <b>FINITE DIFFERENCE</b>        |                  |              |                   |
| 12   | Input third abscissa.           | $x_3$            | <b>f A</b>   | $x_3$             |
| 13   | Input abscissa spacing.         | h                | <b>f B</b>   | h                 |
| 14   | Input ordinates 1 through 4.    | $y_1$            | <b>ENTER</b> |                   |
|      |                                 | $y_2$            | <b>ENTER</b> |                   |
|      |                                 | $y_3$            | <b>ENTER</b> |                   |
|      |                                 | $y_4$            | <b>f C</b>   | $\delta^2 y_0$    |
| 15   | Input an x and find the inter-  |                  |              |                   |
|      | polated y.                      | x                | <b>f D</b>   | y                 |
| 16   | Repeat step 15 any number of    |                  |              |                   |
|      | times.                          |                  |              |                   |

**Example 1:**

The points (7.3, 1.9879) and (7.4, 2.0015) are known to lie along a curve which may be approximated by a straight line. Use linear interpolation to find approximations for the function values at 7.33 and 7.37.

**Keystrokes:**

**DSP** **4** 7.3 **ENTER** 1.9879 **A**  
 7.4 **ENTER** 2.0015 **B** 7.33 **D** ►  
 7.37 **D** —————→

**Outputs:**

1.9920 \*\*\*  
 1.9974 \*\*\*

**Example 2:**

The points (1, -5), (3, 1) and (10, 25) lie on a curve which is to be approximated by a second-degree polynomial. Find by Lagrangian interpolation the function values corresponding to  $x = 1.7$  and  $x = 9$ .

**Keystrokes:**

**DSP** **2** 1 **ENTER** 5 **CHS** **A**  
 3 **ENTER** 1 **B** 10 **ENTER** 25 **C**  
 1.7 **D** —————→  
 9 **D** —————→

**Outputs:**

-2.94 \*\*\*  
 21.29 \*\*\*

**Example 3:**

The following table lists four data points with uniformly spaced abscissas (x-values) of spacing 2.

|       |    |   |   |    |
|-------|----|---|---|----|
| i     | 1  | 2 | 3 | 4  |
| $x_i$ | -1 | 1 | 3 | 5  |
| $y_i$ | -1 | 2 | 9 | 30 |

Use finite difference interpolation to approximate the y-values for x-values of -0.5, 2.567, and 4.8.

**Keystrokes:**

3 **f** **A** 2 **f** **B** 1 **CHS**  
**ENTER** 2 **ENTER** 9 **ENTER** 30  
**f** **C** —————→  
 .5 **CHS** **f** **D** —————→  
 2.567 **f** **D** —————→  
 4.8 **f** **D** —————→

**Outputs:**

14.00  
 -0.08 \*\*\*  
 6.64 \*\*\*  
 26.99 \*\*\*

# COORDINATE TRANSFORMATIONS

COORDINATE TRANSFORMATIONS

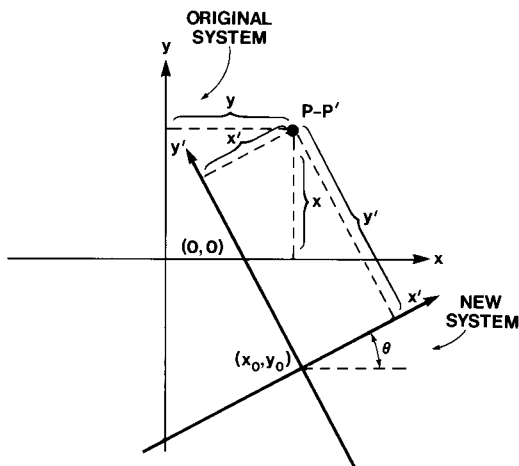
$x_0 + y_0 + \theta$

$x + y = P'$

$x' + y' = P$

This program provides 2-dimensional and 3-dimensional coordinate translation and/or rotation.

For the 2-dimensional case, the coordinates of the origin of the translated system ( $x_0$ ,  $y_0$ ) and the rotation angle ( $\theta$ ) relative to the original system, specify the new coordinate axis. These quantities are input with the **A** key. Subsequently, points specified in the original system ( $x$ ,  $y$ ) may be converted to the translated rotated system ( $x'$ ,  $y'$ ) using the **C** key. Points in the new ( $x'$ ,  $y'$ ) system may be converted to points in the original ( $x$ ,  $y$ ) system using the **E** key.



The 3-dimensional case is analogous to the 2-dimensional case. The only important difference is the specification of the rotation. The rotation axis passes through the translated origin ( $x_0$ ,  $y_0$ ,  $z_0$ ) and is parallel to an arbitrary direction vector ( $a_i$ ,  $b_j$ ,  $c_k$ ). The sign of the rotation angle ( $\theta$ ) is determined by the right-hand rule and the direction of the rotation vector. For instance, the special case of 2-dimensional rotation (rotation in the ( $x$ ,  $y$ ) plane) could be achieved using a direction vector of  $(0, 0, 1)$  and a positive rotation angle for counter-clockwise rotations. The direction vector and angle are input using the **f B** key. The coordinates of the translated origin ( $x_0$ ,  $y_0$ ,  $z_0$ ) are input using **f A**. Conversions from the original system ( $x$ ,  $y$ ,  $z$ ) to the new system ( $x'$ ,  $y'$ ,  $z'$ ) are initiated using **f C** while the inverse conversion is performed with **f E**.

**Equations:**

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

where

$$\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} = \begin{bmatrix} a^2(1-\cos\theta) + \cos\theta & ab(1-\cos\theta) - c\sin\theta & ac(1-\cos\theta) + b\sin\theta \\ ba(1-\cos\theta) + c\sin\theta & b^2(1-\cos\theta) + \cos\theta & bc(1-\cos\theta) - a\sin\theta \\ ca(1-\cos\theta) - b\sin\theta & cb(1-\cos\theta) + a\sin\theta & c^2(1-\cos\theta) + \cos\theta \end{bmatrix}$$

Two dimensional transformations are handled as a special case of three dimensional transformation with (a, b, c) set to (0, 0, 1).

**Remarks:**

1. Degree mode is set when the card is loaded. However, any angular mode will work.
2. For pure translation, input zero for  $\theta$ .
3. For pure rotation, input zeros for  $x_0$ ,  $y_0$ , and  $z_0$ .

**Reference:**

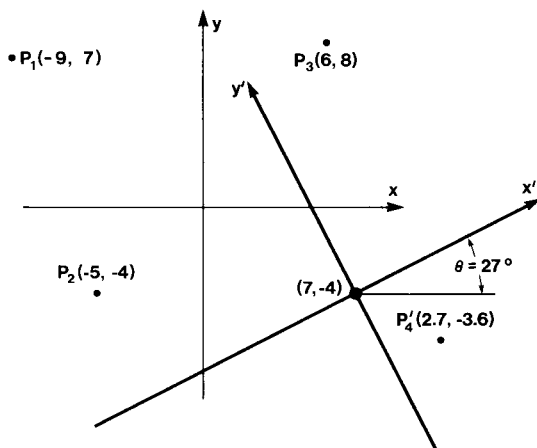
Julian, Rene S., Rotations in Three-Dimensional Space, HP-65 Users' Library Program—01368A

| STEP | INSTRUCTIONS                       | INPUT DATA/UNITS | KEYS  | OUTPUT DATA/UNITS |
|------|------------------------------------|------------------|-------|-------------------|
| 1    | Load side 1 and side 2.            |                  |       |                   |
| 2    | For 2-dimensional transforma-      |                  |       |                   |
|      | tions go to step 3.                |                  |       |                   |
|      | For 3-dimensional transforma-      |                  |       |                   |
|      | tions go to step 6.                |                  |       |                   |
| 3    | Input the origin of the translated |                  |       |                   |
|      | system and the rotation angle.     | $x_0$            | ENTER |                   |
|      |                                    | $y_0$            | ENTER |                   |
|      |                                    | $\theta$         | A     | 1.00              |
| 4    | Transform coordinates from         |                  |       |                   |
|      | the original system to the         |                  |       |                   |
|      | translated-rotated system.         | $x$              | ENTER |                   |
|      |                                    | $y$              | C     | $x'$              |
|      |                                    |                  |       | $y'$              |
|      | or                                 |                  |       |                   |
|      | From the translated-rotated        |                  |       |                   |
|      | system to the original system.     | $x'$             | ENTER |                   |
|      |                                    | $y'$             | E     | $x$               |
|      |                                    |                  |       | $y$               |
| 5    | For a new set of coordinates, go   |                  |       |                   |
|      | to step 4. For a new 2-dimen-      |                  |       |                   |
|      | sional transformation, go to       |                  |       |                   |
|      | step 3.                            |                  |       |                   |

| STEP | INSTRUCTIONS                                                             | INPUT DATA/UNITS | KEYS  | OUTPUT DATA/UNITS    |
|------|--------------------------------------------------------------------------|------------------|-------|----------------------|
| 6    | Input the origin of the translated system.                               | $x_0$            | ENTER |                      |
|      |                                                                          | $y_0$            | ENTER |                      |
|      |                                                                          | $z_0$            | f A   | $x_0$                |
|      | and                                                                      |                  |       |                      |
|      | Input the rotation direction vector and angle.                           | a                | ENTER |                      |
|      |                                                                          | b                | ENTER |                      |
|      |                                                                          | c                | ENTER |                      |
|      |                                                                          | $\theta$         | f B   | $\sqrt{a^2+b^2+c^2}$ |
| 7    | Transform coordinates from original system to translated rotated system. | x                | ENTER |                      |
|      |                                                                          | y                | ENTER |                      |
|      |                                                                          | z                | f C   | $x'$                 |
|      |                                                                          |                  |       | $y'$                 |
|      |                                                                          |                  |       | $z'$                 |
|      | or                                                                       |                  |       |                      |
|      | From the translated-rotated system to the original system.               | $x'$             | ENTER |                      |
|      |                                                                          | $y'$             | ENTER |                      |
|      |                                                                          | $z'$             | f E   | x                    |
|      |                                                                          |                  |       | y                    |
|      |                                                                          |                  |       | z                    |
| 8    | For a new set of coordinates, go to step 7.                              |                  |       |                      |
|      | For a new 3-dimensional transformation go to step 6                      |                  |       |                      |
|      | (either $(x_0, y_0, z_0)$ or $(a, b, c, \theta)$                         |                  |       |                      |
|      | may be changed independently).                                           |                  |       |                      |
|      |                                                                          |                  |       |                      |

**Example 1:**

The coordinate systems  $(x, y)$  and  $(x', y')$  are shown below:



Convert the points  $P_1$ ,  $P_2$  and  $P_3$  to equivalent coordinates in the  $(x', y')$  system.  
Convert the point  $P_4'$  to equivalent coordinates in the  $(x, y)$  system.

**Keystrokes:**

7 **ENTER** 4 **CHS** **ENTER** 27 **A**  $\rightarrow$

9 **CHS** **ENTER** 7 **C**  $\rightarrow$

5 **CHS** **ENTER** 4 **CHS** **C**  $\rightarrow$

6 **ENTER** 8 **C**  $\rightarrow$

2.7 **ENTER** 3.6 **CHS** **E**  $\rightarrow$

**Outputs:**

1.00

-9.26 \*\*\* ( $x'_1$ )

17.06 \*\*\* ( $y'_1$ )

-10.69 \*\*\* ( $x'_2$ )

5.45 \*\*\* ( $y'_2$ )

4.56 \*\*\* ( $x'_3$ )

11.15 \*\*\* ( $y'_3$ )

11.04 \*\*\* ( $x_4$ )

-5.98 \*\*\* ( $y_4$ )

**Example 2:**

A 3-dimensional coordinate system is translated to (2.45, 4.00, 4.25). After translation, a 62.5 degree rotation occurs about the (0, -1, -1) axis. In the original system, a point had the coordinates (3.9, 2.1, 7.0). What are the coordinates of the point in the translated rotated system?

**Keystrokes:****Outputs:**

2.45 **ENTER** 4.00 **ENTER** 4.25  
**f** **A**  $\longrightarrow$   
 0 **ENTER** 1 **CHS** **ENTER** 1 **CHS**  
**ENTER** 62.5 **f** **B**  $\longrightarrow$   
 3.9 **ENTER** 2.1 **ENTER** 7.0  
**f** **C**  $\longrightarrow$

2.45

1.41

3.59 \*\*\* (x')

0.26 \*\*\* (y')

0.59 \*\*\* (z')

In the translated rotated system above, a point has the coordinate (1, 1, 1).  
 What are the corresponding coordinates in the original system?

**Keystrokes:****Outputs:**

1 **ENTER** 1 **ENTER** 1 **f** **E**  $\longrightarrow$

2.91 \*\*\* (x)

4.37 \*\*\* (y)

5.88 \*\*\* (z)

## INTERSECTIONS OF LINES AND LINES, LINES AND CIRCLE AND CIRCLES AND CIRCLES



This program calculates the point of intersection of two coplanar lines, the points of intersection of a coplanar circle and line, or the points of intersection of two coplanar circles.

Lines may be specified by two points ( $x, y$  and  $x', y'$ ), or by one point ( $x, y$ ) and an angle ( $\theta$ ), where  $\theta$  is the angle from the positive  $x$ -axis to the line. Circles are specified by their center coordinates ( $x_0, y_0$ ) and the radius ( $r$ ).

To find the intersection of two lines, input lines specified by two points using **f A** and/or **f B**. Input lines specified by one point and an angle using **A** and/or **B**. Calculate the point of intersection using **f D**. The coordinates of the point of intersection ( $x_p, y_p$ ) will be output. "Error" will be displayed if you input parallel (non-intersecting) lines unless they were also vertical in which case an overflow will be generated.

Calculation of the intersections of a line and a circle requires that the line be input using **A** for point-angle representation or **f A** for point-point representation. The circle is input using **D**. **B**, **f B** and **E** must not be used during circle-line intersection calculations. Once the line parameters and circle parameters have been input, pressing **f D** initiates calculation of one point of intersection and **f E** initiates calculation of the other point. If the line is tangent to the circle, both calculated points will be identical. If the line does not intersect the circle, "Error" will be displayed.

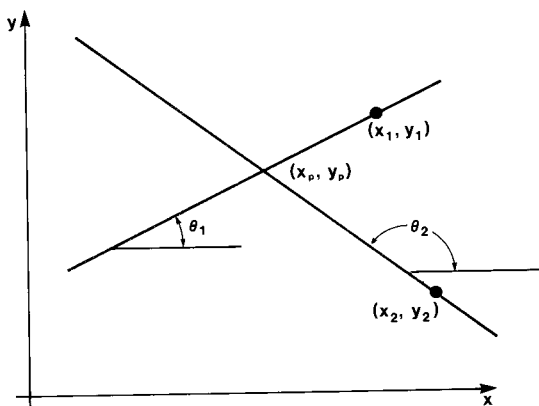
Calculation of the intersections of two circles is accomplished using the **D** and **E** keys to input the circles and **f D** and **f E** to initiate calculation of the points of intersection. If the circles are tangent both calculated points will be identical. If the two circles do not intersect "Error" will be displayed.

### Equations:

Line-Line Intersection:

$$x_p = \frac{x_1 \tan \theta_1 - x_2 \tan \theta_2 + y_2 - y_1}{\tan \theta_1 - \tan \theta_2}$$

$$y_p = y_1 + (x_p - x_1) \tan \theta_1$$



Circle-Line intersections:

$$x_{p1} = x_1 + P_1 \cos \theta$$

$$y_{p1} = y_1 + P_1 \sin \theta$$

$$x_{p2} = x_1 + P_2 \cos \theta$$

$$y_{p2} = y_1 + P_2 \sin \theta$$

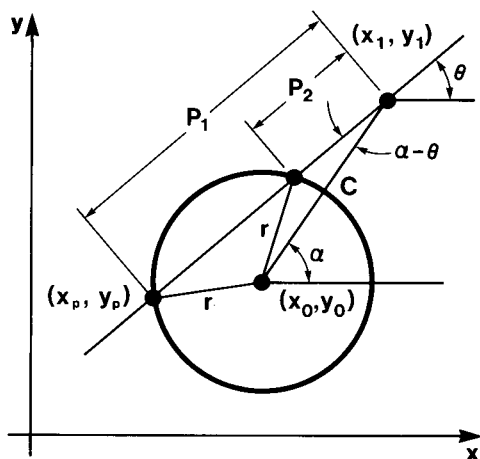
where  $P_1$  and  $P_2$  are the roots of

$$P^2 - 2 D \cos (\theta - \alpha) P + D^2 - r^2 = 0$$

$$\theta = \tan^{-1} \left[ \frac{y_2 - y_1}{x_2 - x_1} \right]$$

$$\alpha = \tan^{-1} \left[ \frac{y_0 - y_1}{x_0 - x_1} \right]$$

$$D = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}$$



Circle-Circle intersections:

$$x_{p1} = x_{01} + r_1 \cos(\theta + \alpha)$$

$$y_{p1} = y_{01} + r_1 \sin(\theta + \alpha)$$

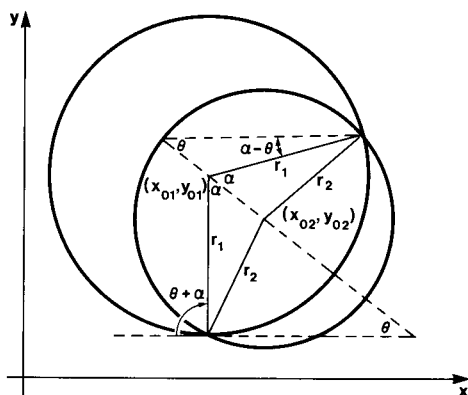
$$x_{p2} = x_{01} + r_1 \cos(\theta - \alpha)$$

$$y_{p2} = y_{01} + r_1 \sin(\theta - \alpha)$$

$$\theta = \tan^{-1} \left( \frac{y_{02} - y_{01}}{x_{02} - x_{01}} \right)$$

$$\alpha = \cos^{-1} \left[ \frac{D^2 + r_1^2 - r_2^2}{2Dr_1} \right]$$

$$D = \sqrt{(x_{02} - x_{01})^2 + (y_{02} - y_{01})^2}$$



**Remarks:**

You may specify any angular mode (degree, radian, grad) after loading the card. When the card is loaded degree mode is set.

| STEP | INSTRUCTIONS                                                                                                                                       | INPUT DATA/UNITS | KEYS  | OUTPUT DATA/UNITS |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------|------------------|-------|-------------------|
| 1    | Load side 1 and side 2<br>(degree mode is set).                                                                                                    |                  |       |                   |
| 2    | For line/line intersections go to<br>step 3. For line/circle inter-<br>sections go to step 6. For<br>circle/circle intersections go<br>to step 10. |                  |       |                   |
|      | <b>LINE/LINE</b>                                                                                                                                   |                  |       |                   |
| 3    | Input two points on each line:                                                                                                                     |                  |       |                   |
|      | First point on line one.                                                                                                                           | $x_1$            | ENTER |                   |
|      |                                                                                                                                                    | $y_1$            | ENTER |                   |
|      | Second point on line one.                                                                                                                          | $x_1'$           | ENTER |                   |
|      |                                                                                                                                                    | $y_1'$           | f A   | $x_1'$            |
|      | First point on line two.                                                                                                                           | $x_2$            | ENTER |                   |
|      |                                                                                                                                                    | $y_2$            | ENTER |                   |
|      | Second point on line two.                                                                                                                          | $x_2'$           | ENTER |                   |
|      |                                                                                                                                                    | $y_2'$           | f B   | $x_2'$            |
|      | or input one point and the angle                                                                                                                   |                  |       |                   |
|      | of each line. Point on line one.                                                                                                                   | $x_1$            | ENTER |                   |
|      |                                                                                                                                                    | $y_1$            | ENTER |                   |
|      | Angle of line one.                                                                                                                                 | $\theta_1$       | A     | $x_1$             |
|      | Point on line two.                                                                                                                                 | $x_2$            | ENTER |                   |
|      |                                                                                                                                                    | $y_2$            | ENTER |                   |
|      | Angle of line two.                                                                                                                                 | $\theta_2$       | B     | $x_2$             |
| 4    | Calculate intersection point.                                                                                                                      |                  | f D   | $x_p, y_p$        |
| 5    | For a new case go to step 3<br>and change either or both lines.                                                                                    |                  |       |                   |

| STEP | INSTRUCTIONS                                                        | INPUT DATA/UNITS | KEYS         | OUTPUT DATA/UNITS |
|------|---------------------------------------------------------------------|------------------|--------------|-------------------|
|      | <b>LINE/CIRCLE</b>                                                  |                  |              |                   |
| 6    | Input two points on line:                                           |                  |              |                   |
|      | First point on line.                                                | $x$              | <b>ENTER</b> |                   |
|      |                                                                     | $y$              | <b>ENTER</b> |                   |
|      | Second point on line.                                               | $x'$             | <b>ENTER</b> |                   |
|      |                                                                     | $y'$             | <b>f A</b>   | $x'$              |
|      | or input one point.                                                 | $x$              | <b>ENTER</b> |                   |
|      |                                                                     | $y$              | <b>ENTER</b> |                   |
|      | and angle of line.                                                  | $\theta$         | <b>A</b>     | $x$               |
| 7    | Input circle center                                                 | $x_0$            | <b>ENTER</b> |                   |
|      |                                                                     | $y_0$            | <b>ENTER</b> |                   |
|      | and radius.                                                         | $r$              | <b>D</b>     | $x_0$             |
| 8    | Calculate one intersection point.                                   |                  | <b>f D</b>   | $x_{p1}, y_{p1}$  |
|      | Calculate the other intersection point.                             |                  | <b>f E</b>   | $x_{p2}, y_{p2}$  |
| 9    | For a new case go to step 6 or 7 and change line or circle or both. |                  |              |                   |
|      | <b>CIRCLE/CIRCLE</b>                                                |                  |              |                   |
| 10   | Input circle one.                                                   | $x_{01}$         | <b>ENTER</b> |                   |
|      |                                                                     | $y_{01}$         | <b>ENTER</b> |                   |
|      |                                                                     | $r_1$            | <b>D</b>     | $x_{01}$          |
|      | Input circle two.                                                   | $x_{02}$         | <b>ENTER</b> |                   |
|      |                                                                     | $y_{02}$         | <b>ENTER</b> |                   |
|      |                                                                     | $r_2$            | <b>E</b>     | $x_{02}$          |
| 11   | Calculate one intersection point.                                   |                  | <b>f D</b>   | $x_{p1}, y_{p1}$  |
|      | Calculate the other intersection point.                             |                  | <b>f E</b>   | $x_{p2}, y_{p2}$  |
| 12   | For a new case go to step 10 and change either or both circles.     |                  |              |                   |

**Example 1:**

Find the intersection of the vertical line specified by two points:

$$P_1 = (0, 0)$$

$$P'_1 = (0, 50)$$

And the oblique line specified by one point and an angle:

$$P_2 = (10, 20)$$

$$\theta = 45^\circ$$

**Keystrokes:**

0 **ENTER** 0 **ENTER** 0 **ENTER**  
50 **f** **A**  $\longrightarrow$

10 **ENTER** 20 **ENTER** 45 **B**  $\longrightarrow$

**f** **D**  $\longrightarrow$

**Outputs:**

0.00

10.00

0.00 \*\*\* ( $x_p$ )

10.00 \*\*\* ( $y_p$ )

**Example 2:**

Calculate the points of intersection for circles at (0, 0) radius 50 and (90, 30) radius 70.

**Keystrokes:**

0 **ENTER** 0 **ENTER** 50 **D**  $\longrightarrow$

90 **ENTER** 30 **ENTER** 70 **E**  $\longrightarrow$

**f** **D**  $\longrightarrow$

**f** **E**  $\longrightarrow$

**Outputs:**

0.00

90.00

21.64 \*\*\* ( $x_{p1}$ )

45.07 \*\*\* ( $y_{p1}$ )

44.36 \*\*\* ( $x_{p2}$ )

-23.07 \*\*\* ( $y_{p2}$ )

**Example 3:**

Find the points of intersection for a circle with center at (0, 0) and radius 50, and the line containing the points (20, 30) and (0, -10).

**Keystrokes:**

0 **ENTER** 0 **ENTER** 50 **D**  $\longrightarrow$

**Outputs:**

0.00

14-07

20 **ENTER** 30 **ENTER** 0 **ENTER**

10 **CHS** **f** **A** →

**f** **D** →

**f** **E** →

0.00

-18.27 \*\*\* ( $x_{p1}$ )

-46.54 \*\*\* ( $y_{p1}$ )

26.27 \*\*\* ( $x_{p2}$ )

42.54 \*\*\* ( $y_{p2}$ )

## CIRCLE COMPUTATIONS



This card combines two separate circle programs. One program calculates the center  $(x_0, y_0)$  and radius  $(r)$  of a circle given three non-collinear points. The other program calculates the coordinates of points on a circle  $(x_i, y_i)$ , given the center and radius of the circle.

To find the center and radius of a circle, simply input three points  $P_1, P_2, P_3$  (represented by  $x$  and  $y$  coordinates) using **f A**, **f B**, and **f C** respectively. After all three points have been input, press **f D** to generate the coordinates of the center  $(x_0, y_0)$  and the radius  $(r)$ .

To find coordinates of points on a circle with a known center and radius, you may choose one of three options:

1. You may key in an angle  $\theta$ , press **B**, and calculate the coordinates of the point on the circle at angle  $\theta$ , where  $\theta$  is measured counterclockwise from a radius parallel to and in the direction of the positive  $x$ -axis.
2. You may manually increment around the circle one point at a time by successively pressing **E**. The outputs are angle  $(\theta)$ , number of the point  $(i)$ , and  $(x, y)$  coordinates of the point.
3. You may automatically increment around the circle by pressing **f E** once. The outputs are the same as those of option two above.

For options two and three above, two input options exist:

1. An initial angle  $(\theta)$  and an incremental angle  $(\Delta\theta)$  are specified and **C** is pressed.
2. An initial angle and the number of increments around a complete circle are specified and **D** is pressed.

## Equations:

Circle determined by three points:

$$y_0 = \frac{K_2 - K_1}{N_2 - N_1}, \quad x_0 = K_2 - N_2 y_0$$

$$r = \sqrt{(x_3 - x_0)^2 + (y_3 - y_0)^2}$$

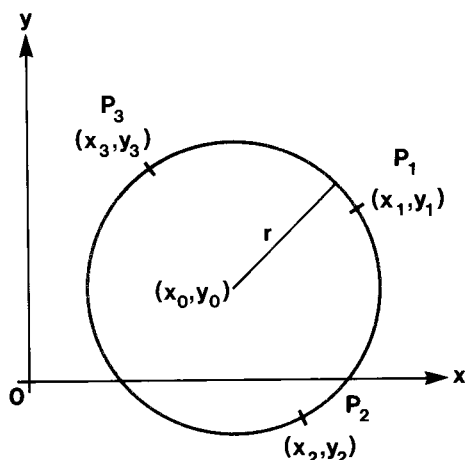
where

$$K_1 = \frac{(x_2 - x_1)(x_2 + x_1) + (y_2 - y_1)(y_2 + y_1)}{2(x_2 - x_1)}$$

$$K_2 = \frac{(x_3 - x_1)(x_3 + x_1) + (y_3 - y_1)(y_3 + y_1)}{2(x_3 - x_1)}$$

$$N_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$N_2 = \frac{y_3 - y_1}{x_3 - x_1}$$



Points on a circle:

$$x_i = x_c + r \cos (\theta_0 + (i - 1) \Delta \theta)$$

$$y_i = y_c + r \sin (\theta_0 + (i - 1) \Delta \theta)$$

$$\Delta \theta = \frac{2 \pi}{n} \quad (\text{for } n \text{ evenly spaced points})$$

$$\theta_i = \theta_0 + (i - 1) \Delta \theta$$

### Remarks:

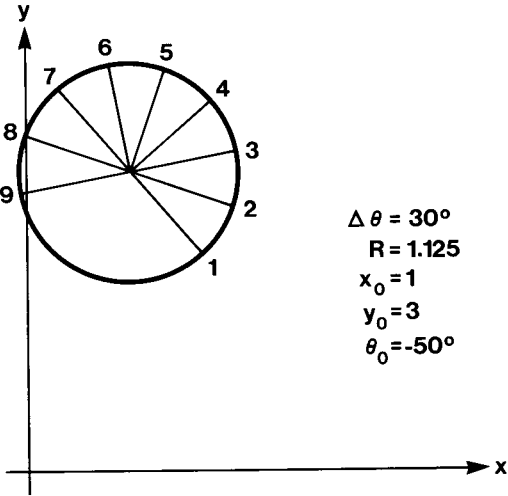
1. If  $x_1 = x_2$  or  $x_1 = x_3$  in the calculation of the center and radius of a circle, then point 1 replaces point 3, point 3 replaces point 2 and point 2 replaces point 1.
2. Degree mode is set when the card is loaded. However the program will also work for radians and grads provided the appropriate mode is set after loading the program.

| STEP | INSTRUCTIONS                                                                                             | INPUT DATA/UNITS | KEYS  | OUTPUT DATA/UNITS |
|------|----------------------------------------------------------------------------------------------------------|------------------|-------|-------------------|
| 1    | Load side 1 and side 2<br>(degrees mode is set).                                                         |                  |       |                   |
| 2    | To determine a circle from three<br>points go to step 3. To generate<br>points on a circle go to step 6. |                  |       |                   |
|      | <b>CIRCLE FROM THREE POINTS</b>                                                                          |                  |       |                   |
| 3    | Input point 1.                                                                                           | $x_1$            | ENTER |                   |
|      |                                                                                                          | $y_1$            | f A   | $x_1$             |
|      | Input point 2.                                                                                           | $x_2$            | ENTER |                   |
|      |                                                                                                          | $y_2$            | f B   | $x_2$             |
|      | Input point 3.                                                                                           | $x_3$            | ENTER |                   |
|      |                                                                                                          | $y_3$            | f C   | $x_3$             |
| 4    | Calculate center coordinates<br>and radius of circle.                                                    |                  | f D   | $x_0, y_0, r$     |
| 5    | For a new case go to step 3 and<br>change any or all of the points.                                      |                  |       |                   |
|      | <b>POINTS ON A CIRCLE</b>                                                                                |                  |       |                   |
| 6    | Input circle center and radius.                                                                          | $x_0$            | ENTER |                   |
|      |                                                                                                          | $y_0$            | ENTER |                   |
|      |                                                                                                          | $r$              | A     | $x_0$             |
| 7    | Optional: input an angle and<br>calculate coordinates.                                                   | $\theta$         | B     | $x, y$            |
| 8    | Input starting angle<br>and                                                                              | $\theta_0$       | ENTER |                   |
|      | angle of increment                                                                                       | $\Delta\theta$   | C     | $\Delta\theta$    |
|      | or                                                                                                       |                  |       |                   |
|      | number of increments.                                                                                    | $n$              | D     | $\Delta\theta$    |
|      |                                                                                                          |                  |       |                   |
|      |                                                                                                          |                  |       |                   |
|      |                                                                                                          |                  |       |                   |
|      |                                                                                                          |                  |       |                   |

| STEP | INSTRUCTIONS                      | INPUT DATA/UNITS | KEYS       | OUTPUT DATA/UNITS       |
|------|-----------------------------------|------------------|------------|-------------------------|
| 9    | Manually increment around         |                  |            |                         |
|      | circle by pressing E for each     |                  |            |                         |
|      | successive increment.             |                  | <b>E</b>   | $\theta_i, i, x_i, y_i$ |
|      | or                                |                  |            |                         |
|      | automatically increment around    |                  |            |                         |
|      | circle.                           |                  | <b>I E</b> | $\theta_i, i, x_i, y_i$ |
| 10   | For a new increment size go       |                  |            |                         |
|      | to step 8. For a new circle go to |                  |            |                         |
|      | step 6.                           |                  |            |                         |

**Example 1:**

Find the coordinates of the points shown on the circle below.



| i | $x_i$ | $y_i$ |
|---|-------|-------|
| 1 | 1.72  | 2.14  |
| 2 | 2.06  | 2.62  |
| 3 | 2.11  | 3.20  |
| 4 | 1.86  | 3.72  |
| 5 | 1.38  | 4.06  |
| 6 | 0.80  | 4.11  |
| 7 | 0.28  | 3.86  |
| 8 | -0.06 | 3.38  |
| 9 | -0.11 | 2.80  |

Keystrokes

Outputs

1 **ENTER** 3 **ENTER** 1.125 **A** ▶

1.00

50 **CHS** **ENTER** 30 **C** →

30.00

**f** **E** →

-50.00 \*\*\* ( $\theta$ )  
1.00 \*\*\* (i)  
1.72 \*\*\* (x)  
2.14 \*\*\* (y)

-20.00 \*\*\*  
2.00 \*\*\*  
2.06 \*\*\*  
2.62 \*\*\*

10.00 \*\*\*  
3.00 \*\*\*  
2.11 \*\*\*  
3.20 \*\*\*

40.00 \*\*\*  
4.00 \*\*\*  
1.86 \*\*\*  
3.72 \*\*\*

70.00 \*\*\*

5.00 \*\*\*

1.38 \*\*\*

4.06 \*\*\*

100.00 \*\*\*

6.00 \*\*\*

0.80 \*\*\*

4.11 \*\*\*

130.00 \*\*\*

7.00 \*\*\*

0.28 \*\*\*

3.86 \*\*\*

160.00 \*\*\*

8.00 \*\*\*

-0.06 \*\*\*

3.38 \*\*\*

190.00 \*\*\*

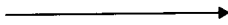
9.00 \*\*\*

-0.11 \*\*\*

2.80 \*\*\*

**R/S** (stops program short of a complete circle.)**Example 2:**

What circle contains the points (1,1), (3.5,-7.6), and (12,0.8)?

**Keystrokes:****Outputs:**1 **ENTER** 1 **f** **A** 3.5 **ENTER** 7.6**CHS** **f** **B** 

3.50

12 **ENTER** .8 **f** **C** →

12.00

**f** **D** →

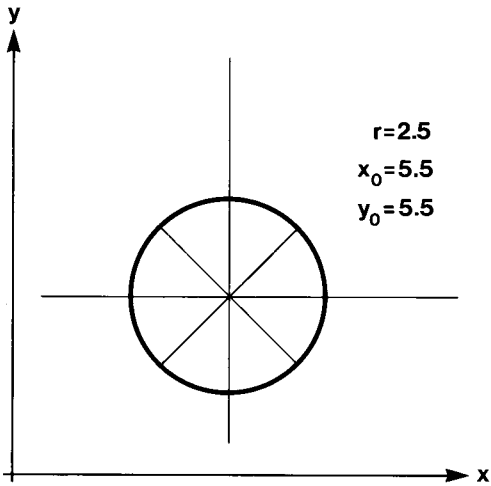
6.45 \*\*\* ( $x_0$ )

-2.08 \*\*\* ( $y_0$ )

6.26 \*\*\* ( $r$ )

**Example 3:**

For the circle below calculate x and y coordinates at 4 equally spaced points, starting at 225°. Use the manual increment feature (**E** key). Also compute x and y at 37°.



**Keystrokes:**

5.5 **ENTER** 5.5 **ENTER** 2.5 **A** ▶

225 **ENTER** 4 **D** →

**E** →

**E** →

**Outputs:**

5.50

90.00 ( $\Delta\theta$ )

225.00 \*\*\*

1.00 \*\*\*

3.73 \*\*\*

3.73 \*\*\*

315.00 \*\*\*

2.00 \*\*\*

7.27 \*\*\*

3.73 \*\*\*

**E** →

405.00 \*\*\*  
3.00 \*\*\*  
7.27 \*\*\*  
7.27 \*\*\*

**E** →

495.00 \*\*\*  
4.00 \*\*\*  
3.73 \*\*\*  
7.27 \*\*\*

37 **B** →

7.50 \*\*\*  
7.00 \*\*\*

# SPHERICAL TRIANGLES

## SPHERICAL TRIANGLES

S+S+S

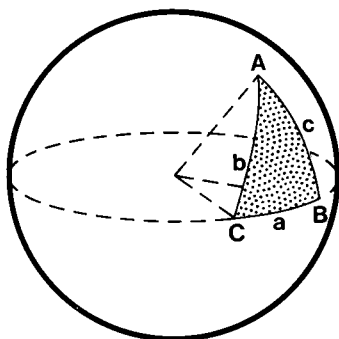
S+A+S

S+S+A

A+A+A

A+S+A

This program will compute solutions to all six cases of spherical triangles, including the two ambiguous cases. In spherical triangles, as opposed to plane triangles, sides and angles have completely reciprocal qualities. Thus a spherical triangle is well defined by the specification of its three angles. Let the angles of the triangle be A, B, C and the sides a, b, c.



### Equations:

The four unambiguous cases are three sides (SSS), three angles (AAA), two sides and the included angle (SAS), and two angles and the included side (ASA). The following equations are used for the four unambiguous cases (laws of cosines):

$$\begin{aligned}\cos a &= \cos b \cos c + \sin b \sin c \cos A \\ \cos A &= -\cos B \cos C + \sin B \sin C \cos a\end{aligned}$$

The two ambiguous cases are two sides and an opposite angle (SSA), and two angles and an opposite side (AAS). The case of SSA is equivalent to specifying a, b, A ( $a \neq b$ ). The solution is found by the following equations:

$$\sin B = \sin b \sin A / \sin a$$

$$\tan \frac{c}{2} = \sin \left( \frac{A+B}{2} \right) \tan \left( \frac{a-b}{2} \right) / \sin \left( \frac{A-B}{2} \right)$$

$$\cos C = \frac{\cos c - \cos a \cos b}{\sin a \sin b}$$

If  $a < b$ , two solutions exist. The alternate solution is found by replacing  $B$  by its supplementary angle  $\cos^{-1}(-\cos B)$ . The program computes both solutions.

The case of AAS is equivalent to specifying  $A, B, a$  ( $A \neq B$ ). The solution is found by the following equations:

$$\sin b = \sin B \sin a / \sin A$$

$$\cot C/2 = \sin \left( \frac{a+b}{2} \right) \tan \left( \frac{A-B}{2} \right) / \sin \left( \frac{a-b}{2} \right)$$

$$\cos c = \frac{\cos C + \cos A \cos B}{\sin A \sin B}$$

If  $A < B$ , two solutions exist. The alternate solution is found by replacing  $b$  by its supplementary angle  $\cos^{-1}(-\cos b)$ .

In the ambiguous cases, two sets of outputs will be given if two solutions exist. Whether one or two solutions exist in these cases, the end of all output for the cases SSA and AAS is signalled by a 0.00 in the display.

For all six cases, the output is similar in format and consists of the output of every parameter of the triangle. The first value output will be the first value input, whether an angle or a side. The second output will be the adjacent value to the first output. Each successive output is adjacent to the one before, thus alternating between sides and angles. For example, if the first value input is a side, the order of the outputs will be first side, first angle, second side, second angle, third side, third angle.

### Remarks:

1. AUTO mode is available to allow automatic output of all results through Print/Pause commands. If AUTO is not selected, each result will be output through a **R/S**.
2. The area of a spherical triangle is determined by the formula  $\text{Area} = r^2 (A + B + C - \pi)$ , where  $r$  is the radius of the sphere and  $A, B, C$  are in radians.
3. The program works in any angular mode. If in DEG mode, decimal degrees must be used. Note that the program sets DEG mode when read in.

| STEP | INSTRUCTIONS                                                                                       | INPUT DATA/UNITS | KEYS         | OUTPUT DATA/UNITS |
|------|----------------------------------------------------------------------------------------------------|------------------|--------------|-------------------|
| 1    | Load side 1 and side 2 of program.                                                                 |                  |              |                   |
| 2    | Select AUTO mode to allow automatic output by Print/Pause.                                         |                  | <b>f E</b>   | 1.00              |
| 3    | To cancel AUTO mode later.                                                                         |                  | <b>f E</b>   | 0.00              |
| 4    | Go to appropriate step for case (SSS, SAS, SSA, AAA, ASA, AAS).                                    |                  |              |                   |
|      | <b>SSS</b>                                                                                         |                  |              |                   |
| 5    | Input three sides.                                                                                 | $S_1$            | <b>ENTER</b> |                   |
|      |                                                                                                    | $S_2$            | <b>ENTER</b> |                   |
|      |                                                                                                    | $S_3$            | <b>A</b>     | OUTPUT            |
|      | <b>SAS</b>                                                                                         |                  |              |                   |
| 6    | Input two sides and included angle.                                                                | $S_1$            | <b>ENTER</b> |                   |
|      |                                                                                                    | $A$              | <b>ENTER</b> |                   |
|      |                                                                                                    | $S_2$            | <b>B</b>     | OUTPUT            |
|      | <b>SSA (ambiguous)</b>                                                                             |                  |              |                   |
| 7    | Input two sides and angle opposite first side (Two sets of outputs will be found if $S_1 < S_2$ ). | $S_1$            | <b>ENTER</b> |                   |
|      |                                                                                                    | $S_2$            | <b>ENTER</b> |                   |
|      |                                                                                                    | $A$              | <b>C</b>     | OUTPUT            |
|      |                                                                                                    |                  |              | 0.00              |
|      | <b>AAA</b>                                                                                         |                  |              |                   |
| 8    | Input three angles.                                                                                | $A_1$            | <b>ENTER</b> |                   |
|      |                                                                                                    | $A_2$            | <b>ENTER</b> |                   |
|      |                                                                                                    | $A_3$            | <b>D</b>     | OUTPUT            |
|      |                                                                                                    |                  |              |                   |
|      |                                                                                                    |                  |              |                   |

| STEP | INSTRUCTIONS                                                                                        | INPUT DATA/UNITS | KEYS         | OUTPUT DATA/UNITS |
|------|-----------------------------------------------------------------------------------------------------|------------------|--------------|-------------------|
|      | <b>ASA</b>                                                                                          |                  |              |                   |
| 9    | Input two angles and included side.                                                                 | $A_1$            | <b>ENTER</b> |                   |
|      |                                                                                                     | S                | <b>ENTER</b> |                   |
|      |                                                                                                     | $A_2$            | <b>E</b>     | OUTPUT            |
|      | <b>AAS</b> (ambiguous)                                                                              |                  |              |                   |
| 10   | Input two angles and side opposite first angle (Two sets of outputs will be found if $A_1 < A_2$ ). | $A_1$            | <b>ENTER</b> |                   |
|      |                                                                                                     | $A_2$            | <b>ENTER</b> |                   |
|      |                                                                                                     | S                | <b>f A</b>   | OUTPUT            |
|      |                                                                                                     |                  |              | 0.00              |
|      | OUTPUT consists of the six parameters of the triangle in the order                                  |                  |              |                   |
|      | First side (angle) input                                                                            |                  |              |                   |
|      | Adjacent angle (side)                                                                               |                  |              |                   |
|      | Adjacent side (angle)                                                                               |                  |              |                   |
|      | Adjacent angle (side)                                                                               |                  |              |                   |
|      | Adjacent side (angle)                                                                               |                  |              |                   |
|      | Adjacent angle (side)                                                                               |                  |              |                   |
|      | If two solutions exist, this schema                                                                 |                  |              |                   |
|      | will be repeated.                                                                                   |                  |              |                   |

**Example 1:**

The three sides of a spherical triangle are 0.20 radians, 0.91 radians, and 0.93 radians. What are the three angles? Do not use AUTO mode.

**Keystrokes:**

**RAD** .2 **ENTER** .91 **ENTER**

.93 **A** → 0.20

**R/S** → 1.59 ( $A_1$ )

**R/S** → 0.91

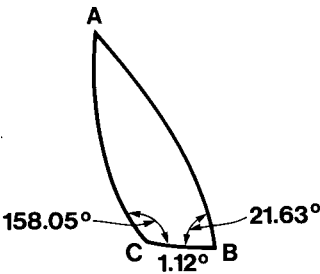
**Outputs:**

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|     |   |      |                   |
|-----|---|------|-------------------|
| R/S | → | 0.25 | (A <sub>2</sub> ) |
| R/S | → | 0.93 |                   |
| R/S | → | 1.40 | (A <sub>3</sub> ) |

Example 2:

Solve the spherical triangle below for the missing parameters. Do not use AUTO mode.



Note that this is an angle-side-angle (ASA) case.

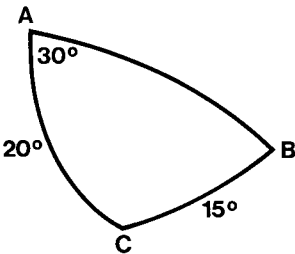
Keystrokes:

Outputs:

|     |        |       |      |       |           |
|-----|--------|-------|------|-------|-----------|
| DEG | 21.63  | ENTER | 1.12 | ENTER |           |
|     | 158.05 | E     | →    |       | 21.63     |
| R/S | →      |       |      |       | 1.12      |
| R/S | →      |       |      |       | 158.05    |
| R/S | →      |       |      |       | 51.90 (b) |
| R/S | →      |       |      |       | 0.52 (A)  |
| R/S | →      |       |      |       | 52.94 (c) |

Example 3:

In the spherical triangle ABC below,  $A = 30^\circ$ ,  $a = 15^\circ$ , and  $b = 20^\circ$ . Find B, C, and c. Use AUTO mode. (Note that as this is a case of SSA, two solutions may exist.)



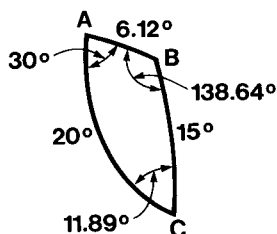
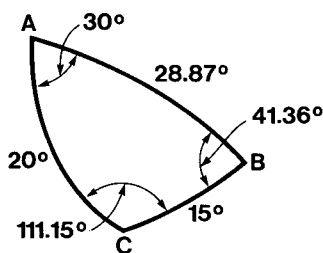
**Keystrokes:**

$\boxed{\text{DEG}}$   $\boxed{f}$   $\boxed{E}$   $\longrightarrow$   
 15  $\boxed{\text{ENTER}}$  20  $\boxed{\text{ENTER}}$  30  $\boxed{C}$   $\longrightarrow$

**Outputs:**

1.00 (AUTO set)  
 15.00 \*\*\*  
 111.15 \*\*\* (C)  
 20.00 \*\*\*  
 30.00 \*\*\*  
 28.87 \*\*\* (c)  
 41.36 \*\*\* (B)  
  
 15.00 \*\*\*  
 11.89 \*\*\* (C)  
 20.00 \*\*\*  
 30.00 \*\*\*  
 6.12 \*\*\* (c)  
 138.64 \*\*\* (B)  
 0.00 (end)

The two possible solutions are pictured below.



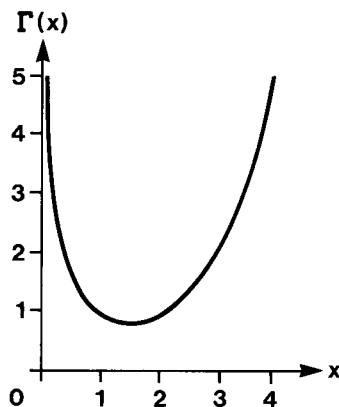
# GAMMA FUNCTION



This program approximates the value of the gamma function,  $\Gamma(x)$ , for  $1 \leq x \leq 70$ .

## Equations:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$$



1.  $\Gamma(x) = (x - 1) \Gamma(x - 1)$
2. For  $1 \leq x \leq 2$ , polynomial approximation can be used.  

$$\Gamma(x) \cong 1 + b_1 (x - 1) + b_2 (x - 1)^2 + \dots + b_8 (x - 1)^8$$

where  $b_1 = -0.577191652$ ,  $b_2 = 0.988205891$   
 $b_3 = -0.897056937$ ,  $b_4 = 0.918206857$   
 $b_5 = -0.756704078$ ,  $b_6 = 0.482199394$   
 $b_7 = -0.193527818$ ,  $b_8 = 0.035868343$

## Remarks:

1. This program can be used to find the generalized factorial  $x!$  for  $0 \leq x \leq 69$ , where  $x! = \Gamma(x+1)$ .
2. When the value keyed in for  $x$  is an integer,  $\Gamma(x)$  is evaluated as the factorial of  $(x-1)$ .
3. If  $x < 1$ , the program will halt and display "Error".

**References:**

Handbook of Mathematical Functions, Abramowitz and Stegun, National Bureau of Standards, 1968.

| STEP | INSTRUCTIONS                       | INPUT DATA/UNITS | KEYS     | OUTPUT DATA/UNITS |
|------|------------------------------------|------------------|----------|-------------------|
| 1    | Load side 1 and side 2 of program. |                  |          |                   |
| 2    | Initialize.                        |                  | <b>A</b> | 0.00              |
| 3    | Key in x and compute $\Gamma(x)$ . | x                | <b>B</b> | $\Gamma(x)$       |
| 4    | Repeat step 3 any number of        |                  |          |                   |
|      | times.                             |                  |          |                   |

**Example:**

Find the gamma function for the following arguments:

5.25, 8, 3.34.

**Keystrokes:**

**Outputs:**

|                        |   |             |
|------------------------|---|-------------|
| <b>A</b> 5.25 <b>B</b> | → | 35.21 ***   |
| 8 <b>B</b>             | → | 5040.00 *** |
| 3.34 <b>B</b>          | → | 2.80 ***    |

# BESSEL FUNCTIONS, ERROR FUNCTION

BESSEL FUNCTIONS, ERROR FUNCTION

n      x \* J<sub>n</sub>(x)      \* J<sub>0</sub> - J<sub>1</sub>      x \* I<sub>n</sub>(x)      x \* erf.erfc

This card combines two separate programs in one. The first routine computes the Bessel functions  $J_n(x)$  and  $I_n(x)$ , where  $n$  is a positive integer and  $x > 0$ . The second of the two routines finds the error function and complementary error function for positive arguments.

## Bessel Functions

The Bessel functions  $J_n(x)$  and  $I_n(x)$  are computed by generating trial values  $T_k$  through the use of recurrence relations. The recurrence is begun at an index  $m$  given by

$$m = 2 \text{ INT} \left[ \frac{6 + \max(n, z) + \frac{9z}{z+2}}{2} \right]$$

where

$$z = \frac{3x}{2}$$

The initial values selected for recurrence are  $T_{m+1} = 10^{-9}$ ,  $T_{m+2} = 0$ .

For the functions  $J_n(x)$ , each term  $T_k$ ,  $0 \leq k \leq m$ , is computed by the relation

$$T_k(x) = \frac{2(k+1)}{x} T_{k+1}(x) - T_{k+2}(x),$$

beginning with  $k = m$ .

$J_n(x)$  is then found by dividing the term  $T_n(x)$  by the normalizing constant

$$K = T_0(x) + 2 \sum_{k=1}^{m/2} T_{2k}(x).$$

After calculating a  $J_n(x)$ , the values of  $J_0(x)$  and  $J_1(x)$  may also be found with very little additional computation.

For the functions  $I_n(x)$ , each  $T_k$  is calculated from the recurrence relation

$$T_k(x) = \frac{2(k+1)}{x} T_{k+1}(x) + T_{k+2}(x),$$

$0 \leq k \leq m$ , beginning with  $k = m$ .

$I_n(x)$  is then found from the equation

$$I_n(x) = e^x \frac{T_n(x)}{T_0(x) + 2 \sum_{k=1}^m T_k(x)}$$

## Error Function

The error function is defined as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

and the complementary error function as

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x).$$

For large values of  $x$  ( $\geq 3$ ), the error function is very close to 1. If  $\operatorname{erfc}(x)$  is computed as  $1 - \operatorname{erf}(x)$ , most of the significant figures of  $\operatorname{erfc}(x)$  will be lost for  $x > 3$ . Hence two different algorithms are employed in this program, one for  $x \leq 3$  and one for  $x > 3$ . For  $x \leq 3$ , the error function is computed by a series sum

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \cdot \dots \cdot (2n+1)} x^{2n+1}$$

and the complementary error function by

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x).$$

For  $x > 3$ , the complementary error function is computed first, by the asymptotic expansion

$$\operatorname{erfc}(x) = \frac{1}{x \sqrt{\pi}} e^{-x^2} \left[ 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot \dots \cdot (2n-1)}{(2x^2)^n} \right]$$

and the error function by

$$\operatorname{erf}(x) = 1 - \operatorname{erfc}(x).$$

The accuracy of the calculation of  $\operatorname{erf}(x)$  and  $\operatorname{erfc}(x)$  from series sums may be controlled by the user's specification of the display setting. If the display is set at DSP 6, for example, the program will halt when two successive terms of the series are equal when rounded to 6 places. Thus if the display is set to DSP N, the result will have N places of significance. Alternatively, the digit N may be keyed into the program on key **f** **E** and the display will be set automatically by the program. For  $x \leq 3$ , it is quite reasonable to specify DSP 9 for maximum accuracy; for  $x > 3$ , the series may not ever converge with DSP 9, and a safer specification would be DSP 6.

### Remarks

1. The range of values  $0 \leq x \leq 10^{-6}$  is out of bounds for the Bessel functions in this program. In this range, however, one may take  $J_0(x) = J_0(0) = I_0(x) = I_0(0) = 1$ , and  $J_n(x) = J_n(0) = I_n(x) = I_n(0) = 0$ ,  $n \neq 0$ .
2. The computation of  $\operatorname{erfc}(x)$  will halt on overflow for  $x \geq 15$ .

### Reference

Handbook of Mathematical Functions, Abramowitz and Stegun, National Bureau of Standards, 1968.

| STEP | INSTRUCTIONS                                                                      | INPUT DATA/UNITS | KEYS       | OUTPUT DATA/UNITS                   |
|------|-----------------------------------------------------------------------------------|------------------|------------|-------------------------------------|
| 1    | Load side 1 and side 2 of program.                                                |                  |            |                                     |
| 2    | For Bessel functions, go to step 3;<br>for error function, go to step 6.          |                  |            |                                     |
|      | <b>BESSEL FUNCTIONS</b>                                                           |                  |            |                                     |
| 3    | To find $J_n(x)$ , go to step 4; to<br>find $I_n(x)$ , go to step 5.              |                  |            |                                     |
| 4    | For $J_n(x)$ :                                                                    |                  |            |                                     |
|      | • Input $n$ ( $n = 0, 1, 2, \dots$ )                                              | $n$              | <b>A</b>   | $n$                                 |
|      | • Input $x$ and find $J_n(x)$                                                     | $x$              | <b>B</b>   | $J_n(x)$                            |
|      | • (optional) Find $J_0(x)$ and $J_1(x)$                                           |                  | <b>C</b>   | $J_0(x)$                            |
|      |                                                                                   |                  | <b>R/S</b> | $J_1(x)$                            |
| 5    | For $I_n(x)$ :                                                                    |                  |            |                                     |
|      | • Input $n$ ( $n = 0, 1, 2, \dots$ )                                              | $n$              | <b>A</b>   | $n$                                 |
|      | • Input $x$ and find $I_n(x)$                                                     | $x$              | <b>D</b>   | $I_n(x)$                            |
|      | <b>ERROR FUNCTION</b>                                                             |                  |            |                                     |
| 6    | Specify places of accuracy<br>desired by setting display or by<br>inputting $N$ . | $N$              | <b>I E</b> | $N$                                 |
| 7    | Key in $x$ and find error function<br>and complementary error<br>function.        | $x$              | <b>E</b>   | $\text{erf}(x)$<br>$\text{erfc}(x)$ |

**Example 1:**

Find  $J_5(9.2)$ ; also find  $J_0(9.2)$  and  $J_1(9.2)$ . Display results to 9 places and compare to table values.

**Keystrokes:**

**DSP** **9** **5** **A** **9.2** **B** →  
**C** →  
**R/S** →

**Outputs:**

-0.100528623 \*\*\*  $J_5(9.2)$   
-0.136748371  $J_0(9.2)$   
0.217408655  $J_1(9.2)$

The actual values from tables are  $J_5(9.2) = -0.10053$ ,  $J_0(9.2) = -0.1367483708$ , and  $J_1(9.2) = 0.2174086550$ .

**Example 2:**Find  $I_3(4.7)$  and  $I_3(5.0)$ .**Keystrokes:**

**DSP** **2** **3** **A** 4.7 **D** →  
 5 **D** →

**Outputs:**

7.42 \*\*\*  $I_3(4.7)$   
 10.33 \*\*\*  $I_3(5.0)$

**Example 3:**

Find erf and erfc of 1.34 to full 9-place accuracy.

**Keystrokes:**

**DSP** **9** 1.34 **E** →

**Outputs:**

0.941913715 \*\*\* erf (1.34)  
 0.058086285 \*\*\* erfc (1.34)

**Example 4:**

Find erf and erfc of 4.55 to 6 places.

**Keystrokes:**

6 **f** **E** 4.55 **E** →

**Outputs:**

1.000000 \*\*\* erf (4.55)  
 1.237404615-10 \*\*\* erfc (4.55)

## HYPERBOLICS



This program computes the hyperbolic functions and their inverses. The stack is preserved during execution of any of the functions on this card. The argument, however, is not saved in the LASTx register.

Note, in the equations below, the appropriate restrictions on the values of the argument in each case.

### Equations:

#### *Hyperbolic functions*

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} \quad (x \neq 0)$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{coth} x = \frac{1}{\tanh x} \quad (x \neq 0)$$

#### *Inverse Hyperbolic Functions*

$$\sinh^{-1} x = \ln [x + (x^2 + 1)^{1/2}]$$

$$\cosh^{-1} x = \ln [x + (x^2 - 1)^{1/2}] \quad x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left[ \frac{1+x}{1-x} \right] \quad x^2 < 1$$

$$\operatorname{csch}^{-1} x = \sinh^{-1} \left[ \frac{1}{x} \right] \quad x \neq 0$$

$$\operatorname{sech}^{-1} x = \cosh^{-1} \left[ \frac{1}{x} \right] \quad 0 < x \leq 1$$

$$\operatorname{coth}^{-1} x = \tanh^{-1} \left[ \frac{1}{x} \right] \quad x^2 > 1$$

| STEP | INSTRUCTIONS                                                                | INPUT DATA/UNITS | KEYS       | OUTPUT DATA/UNITS    |
|------|-----------------------------------------------------------------------------|------------------|------------|----------------------|
| 1    | Load side 1 and side 2 of program.                                          |                  |            |                      |
| 2    | For hyperbolics, go to step 3;<br>for inverse hyperbolics, go to<br>step 4. |                  |            |                      |
|      | <b>HYPERBOLIC FUNCTIONS</b>                                                 |                  |            |                      |
| 3    | Key in argument and compute                                                 |                  |            |                      |
|      | • hyperbolic sine                                                           | x                | <b>B</b>   | sinh x               |
|      | • hyperbolic cosine                                                         | x                | <b>C</b>   | cosh x               |
|      | • hyperbolic tangent                                                        | x                | <b>D</b>   | tanh x               |
|      | • hyperbolic cosecant                                                       | x                | <b>f B</b> | csch x               |
|      | • hyperbolic secant                                                         | x                | <b>f C</b> | sech x               |
|      | • hyperbolic cotangent                                                      | x                | <b>f D</b> | coth x               |
|      | <b>INVERSE HYPERBOLIC<br/>FUNCTIONS</b>                                     |                  |            |                      |
| 4    | Key in argument and compute                                                 |                  |            |                      |
|      | • inverse hyperbolic sine                                                   | x                | <b>A B</b> | sinh <sup>-1</sup> x |
|      | • inverse hyperbolic cosine                                                 | x                | <b>A C</b> | cosh <sup>-1</sup> x |
|      | • inverse hyperbolic tangent                                                | x                | <b>A D</b> | tanh <sup>-1</sup> x |
|      | • inverse hyperbolic cosecant                                               | x                | <b>A</b>   |                      |
|      |                                                                             |                  | <b>f B</b> | csch <sup>-1</sup> x |
|      | • inverse hyperbolic secant                                                 | x                | <b>A</b>   |                      |
|      |                                                                             |                  | <b>f C</b> | sech <sup>-1</sup> x |
|      | • inverse hyperbolic cotangent                                              | x                | <b>A</b>   |                      |
|      |                                                                             |                  | <b>f D</b> | coth <sup>-1</sup> x |

## 19-03

### Example 1:

Evaluate the following hyperbolic functions:

$\sinh 2.5$ ;  $\cosh 3.2$ ;  $\tanh 1.9$ ;  $\operatorname{csch} 4.6$ ;  $\operatorname{sech} -0.25$ ;  $\operatorname{coth} -2.01$ .

#### Keystrokes:

#### Outputs:

|      |                |   |       |              |
|------|----------------|---|-------|--------------|
| 2.5  | <b>B</b>       | → | 6.05  | (sinh 2.5)   |
| 3.2  | <b>C</b>       | → | 12.29 | (cosh 3.2)   |
| 1.9  | <b>D</b>       | → | 0.96  | (tanh 1.9)   |
| 4.6  | <b>f B</b>     | → | 0.02  | (csch 4.6)   |
| .25  | <b>CHS f C</b> | → | 0.97  | (sech -0.25) |
| 2.01 | <b>CHS f D</b> | → | -1.04 | (coth -2.01) |

### Example 2:

Evaluate the following inverse hyperbolic functions:

$\sinh^{-1}(2.4)$ ;  $\cosh^{-1}(90)$ ;  $\tanh^{-1}(-0.65)$ ;  $\operatorname{csch}^{-1}(2)$ ;  $\operatorname{sech}^{-1}(0.4)$ ;  $\operatorname{coth}^{-1}(3.4)$ .

#### Keystrokes:

#### Outputs:

|     |                |   |       |                                    |
|-----|----------------|---|-------|------------------------------------|
| 2.4 | <b>A B</b>     | → | 1.61  | ( $\sinh^{-1} 2.4$ )               |
| 90  | <b>A C</b>     | → | 5.19  | ( $\cosh^{-1} 90$ )                |
| .65 | <b>CHS A D</b> | → | -0.78 | ( $\tanh^{-1} -0.65$ )             |
| 2   | <b>A f B</b>   | → | 0.48  | ( $\operatorname{csch}^{-1} 2$ )   |
| .4  | <b>A f C</b>   | → | 1.57  | ( $\operatorname{sech}^{-1} 0.4$ ) |
| 3.4 | <b>A f D</b>   | → | 0.30  | ( $\operatorname{coth}^{-1} 3.4$ ) |

## PROGRAM LISTINGS

The following listings are included for your reference. A table of keycodes and keystrokes corresponding to the symbols used in the listings can be found in Appendix E of your Owner's Handbook.

| Program                                               | Page   |
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Factors and Primes

|           |                                                             |           |                                                             |                   |    |     |    |     |    |
|-----------|-------------------------------------------------------------|-----------|-------------------------------------------------------------|-------------------|----|-----|----|-----|----|
| 001 *LBLA | Factor integer n.                                           | 057 RCLE  |                                                             |                   |    |     |    |     |    |
| 002 STOB  |                                                             | 058 -     |                                                             |                   |    |     |    |     |    |
| 003 ENT1  |                                                             | 059 X<0?  | If upper < lower, halt on Error.                            |                   |    |     |    |     |    |
| 004 INT   |                                                             | 060 GTOS  |                                                             |                   |    |     |    |     |    |
| 005 X≠Y?  | If non-integer, halt on Error.                              | 061 RCLA  |                                                             |                   |    |     |    |     |    |
| 006 GTOS  |                                                             | 062 INT   |                                                             |                   |    |     |    |     |    |
| 007 0     |                                                             | 063 2     | Handle 2 as special case.                                   |                   |    |     |    |     |    |
| 008 STOD  | Initialize d.                                               | 064 X≠Y   |                                                             |                   |    |     |    |     |    |
| 009 X≠Y   |                                                             | 065 X=Y?  |                                                             |                   |    |     |    |     |    |
| 010 X≠Y?  | If n ≤ 0, halt on Error.                                    | 066 GTOB  |                                                             |                   |    |     |    |     |    |
| 011 GTOS  |                                                             | 067 2     |                                                             |                   |    |     |    |     |    |
| 012 2     |                                                             | 068 ÷     |                                                             |                   |    |     |    |     |    |
| 013 EEK   |                                                             | 069 .     | This routine finds greatest potential prime ≤ user's input. |                   |    |     |    |     |    |
| 014 9     |                                                             | 070 5     |                                                             |                   |    |     |    |     |    |
| 015 X≠Y   |                                                             | 071 +     |                                                             |                   |    |     |    |     |    |
| 016 X>Y?  | If n > 2 x 10 <sup>9</sup> , halt on Error.                 | 072 INT   |                                                             |                   |    |     |    |     |    |
| 017 GTOS  | F1 clear for factors.                                       | 073 2     |                                                             |                   |    |     |    |     |    |
| 018 CF1   | Find factors.                                               | 074 x     |                                                             |                   |    |     |    |     |    |
| 019 GSBa  | -----                                                       | 075 1     |                                                             |                   |    |     |    |     |    |
| 020 RTN   | Lower bound for primes.                                     | 076 -     |                                                             |                   |    |     |    |     |    |
| 021 *LBLB |                                                             | 077 *LBLB |                                                             |                   |    |     |    |     |    |
| 022 STOA  |                                                             | 078 STOE  | Store final prime U.                                        |                   |    |     |    |     |    |
| 023 X<0?  | If negative, halt on Error.                                 | 079 RCLA  |                                                             |                   |    |     |    |     |    |
| 024 GTOS  |                                                             | 080 RTN   | -----                                                       |                   |    |     |    |     |    |
| 025 ENT1  | This routine finds smallest potential prime ≥ user's input. | 081 *LBLB | Routine to list primes.                                     |                   |    |     |    |     |    |
| 026 INT   |                                                             | 082 0     |                                                             |                   |    |     |    |     |    |
| 027 X=Y?  |                                                             | 083 STOD  | Initialize d←0                                              |                   |    |     |    |     |    |
| 028 GTOB  |                                                             | 084 RCLB  |                                                             |                   |    |     |    |     |    |
| 029 1     |                                                             | 085 2     | If L = 2, print 2, add 1 and go.                            |                   |    |     |    |     |    |
| 030 +     |                                                             | 086 X≠Y   |                                                             |                   |    |     |    |     |    |
| 031 *LBLB |                                                             | 087 X=Y?  |                                                             |                   |    |     |    |     |    |
| 032 2     |                                                             | 088 GTOB  |                                                             |                   |    |     |    |     |    |
| 033 X≠Y   | Handle 2 as special case (only even prime).                 | 089 1     | If L = 1, print 1, 2, add 1 and go.                         |                   |    |     |    |     |    |
| 034 X=Y?  |                                                             | 090 X≠Y?  |                                                             |                   |    |     |    |     |    |
| 035 GTOB  |                                                             | 091 GTOI  |                                                             |                   |    |     |    |     |    |
| 036 2     |                                                             | 092 GSBa  | If L ≠ 1 and L ≠ 2, go directly to LBL 1.                   |                   |    |     |    |     |    |
| 037 ÷     |                                                             | 093 +     |                                                             |                   |    |     |    |     |    |
| 038 INT   |                                                             | 094 RCLE  |                                                             |                   |    |     |    |     |    |
| 039 2     |                                                             | 095 X≠Y   |                                                             |                   |    |     |    |     |    |
| 040 x     |                                                             | 096 X>Y?  |                                                             |                   |    |     |    |     |    |
| 041 1     |                                                             | 097 GTOB  |                                                             |                   |    |     |    |     |    |
| 042 +     |                                                             | 098 *LBLB |                                                             |                   |    |     |    |     |    |
| 043 *LBLB |                                                             | 099 GSBa  |                                                             |                   |    |     |    |     |    |
| 044 STOB  | Store beginning prime L.                                    | 100 1     | Output 2.                                                   |                   |    |     |    |     |    |
| 045 STOC  |                                                             | 101 GTOB  |                                                             |                   |    |     |    |     |    |
| 046 2     |                                                             | 102 *LBL1 | -----                                                       |                   |    |     |    |     |    |
| 047 EEK   | 2 x 10 <sup>9</sup> is default upper bound.                 | 103 GSBa  |                                                             |                   |    |     |    |     |    |
| 048 9     |                                                             | 104 RCLB  | Begin main loop.                                            |                   |    |     |    |     |    |
| 049 STOE  |                                                             | 105 RCLC  | Check for factors of current n (R <sub>B</sub> ).           |                   |    |     |    |     |    |
| 050 X≠Y?  | If input ≥ 2 x 10 <sup>9</sup> , halt on Error.             | 106 X=Y?  | If R <sub>B</sub> = R <sub>C</sub> , n is prime.            |                   |    |     |    |     |    |
| 051 GTOS  | Flag 1 set for primes.                                      | 107 GSBa  | Output n.                                                   |                   |    |     |    |     |    |
| 052 SF1   |                                                             | 108 2     |                                                             |                   |    |     |    |     |    |
| 053 RCLA  |                                                             | 109 *LBLB |                                                             |                   |    |     |    |     |    |
| 054 RTN   | -----                                                       | 110 +     |                                                             |                   |    |     |    |     |    |
| 055 *LBLC | Upper bound for primes.                                     | 111 STOC  | Set n to next potential prime.                              |                   |    |     |    |     |    |
| 056 STOA  |                                                             | 112 STOB  |                                                             |                   |    |     |    |     |    |
| REGISTERS |                                                             |           |                                                             |                   |    |     |    |     |    |
| 0         | 1                                                           | 2         | 3                                                           | 4                 | 5  | 6   | 7  | 8   | 9  |
| S0        | S1                                                          | S2        | S3                                                          | S4                | S5 | S6  | S7 | S8  | S9 |
| A Used    |                                                             | B n       |                                                             | C Potential prime |    | D d |    | E U |    |

|     |       |                               |  |  |     |       |                          |
|-----|-------|-------------------------------|--|--|-----|-------|--------------------------|
| 113 | RCLE  |                               |  |  | 169 | RTN   | Loop again for next 30.  |
| 114 | X=Y   |                               |  |  | 170 | GT02  | -----                    |
| 115 | X>Y?  | If n > U, exit.               |  |  | 171 | *LBL3 |                          |
| 116 | GT04  |                               |  |  | 172 | RCLD  | Tests if dln.            |
| 117 | 0     |                               |  |  | 173 | +     |                          |
| 118 | ST0D  | Else loop again.              |  |  | 174 | ST0D  | d ← d + x                |
| 119 | GT01  | -----                         |  |  | 175 | RCLB  | n                        |
| 120 | *LBL4 | Subroutine called from both   |  |  | 176 | X=Y   | n/d                      |
| 121 | 2     | A & D which finds factors     |  |  | 177 | ÷     | d, n/d                   |
| 122 | GSB3  | of n.                         |  |  | 178 | LSTX  | If d > n/d, then d > √n  |
| 123 | X=0?  |                               |  |  | 179 | X>Y?  | Exit.                    |
| 124 | RTN   |                               |  |  | 180 | GT00  |                          |
| 125 | 1     | Check first if n divisible by |  |  | 181 | X=Y   |                          |
| 126 | GSB3  | 2, 3, 5, or 7.                |  |  | 182 | INT   | [n/d] := INT (n/d)       |
| 127 | X=0?  |                               |  |  | 183 | LSTX  | n/d, [n/d]               |
| 128 | RTN   |                               |  |  | 184 | X=Y?  | If non-integer, d does   |
| 129 | 2     |                               |  |  | 185 | RTN   | not divide n.            |
| 130 | GSB3  | LBL 2 check for division by   |  |  | 186 | ST0B  | Else n ← n/d             |
| 131 | X=0?  | integers whose position in a  |  |  | 187 | F1?   |                          |
| 132 | RTN   | cycle of 30 corresponds to    |  |  | 188 | CLX   | If finding primes, exit. |
| 133 | 2     | 11, 13, 17, 19, 23, 29, 31,   |  |  | 189 | F1?   |                          |
| 134 | GSB3  | or 37.                        |  |  | 190 | RTN   |                          |
| 135 | X=0?  |                               |  |  | 191 | RCLD  | If factoring, output d.  |
| 136 | RTN   |                               |  |  | 192 | GSBe  |                          |
| 137 | *LBL2 | -----                         |  |  | 193 | 0     | Check for d a multiple   |
| 138 | 4     |                               |  |  | 194 | GT03  | factor.                  |
| 139 | GSB3  | 11 (=7 +4)                    |  |  | 195 | *LBL0 | -----                    |
| 140 | X=0?  |                               |  |  | 196 | F1?   | Coming here means n      |
| 141 | RTN   |                               |  |  | 197 | CLX   | prime.                   |
| 142 | 2     |                               |  |  | 198 | F1?   | If finding primes, exit. |
| 143 | GSB3  | 13 (=11 +2)                   |  |  | 199 | RTN   |                          |
| 144 | X=0?  |                               |  |  | 200 | RCLB  | Else output last n.      |
| 145 | RTN   |                               |  |  | 201 | GSBe  | -----                    |
| 146 | 4     |                               |  |  | 202 | *LBL4 |                          |
| 147 | GSB3  | 17 (=13 +4)                   |  |  | 203 | CLX   | Exit displaying 0.       |
| 148 | X=0?  |                               |  |  | 204 | F0?   |                          |
| 149 | RTN   |                               |  |  | 205 | SPC   |                          |
| 150 | 2     |                               |  |  | 206 | RTN   |                          |
| 151 | GSB3  | 19 (=17 +2)                   |  |  | 207 | *LBL5 | -----                    |
| 152 | X=0?  |                               |  |  | 208 | F0?   | Auto toggle.             |
| 153 | RTN   |                               |  |  | 209 | GT00  |                          |
| 154 | 4     |                               |  |  | 210 | SF0   |                          |
| 155 | GSB3  | 23 (=19 +4)                   |  |  | 211 | 1     |                          |
| 156 | X=0?  |                               |  |  | 212 | RTN   |                          |
| 157 | RTN   |                               |  |  | 213 | *LBL0 |                          |
| 158 | 6     |                               |  |  | 214 | CF0   |                          |
| 159 | GSB3  | 29 (=23 +6)                   |  |  | 215 | 0     |                          |
| 160 | X=0?  |                               |  |  | 216 | RTN   |                          |
| 161 | RTN   |                               |  |  | 217 | *LBL5 | -----                    |
| 162 | 2     |                               |  |  | 218 | F0?   | Output routine.          |
| 163 | GSB3  | 31 (=29 +2)                   |  |  | 219 | PRTX  | Print if AUTO.           |
| 164 | X=0?  |                               |  |  | 220 | F0?   |                          |
| 165 | RTN   |                               |  |  | 221 | RTN   |                          |
| 166 | E     |                               |  |  | 222 | R/S   |                          |
| 167 | GSB3  | 37 (=31 +6)                   |  |  | 223 | RTN   |                          |
| 168 | X=0?  |                               |  |  | 224 | R/S   | Halt if not.             |

| LABELS                    |               |                |                         |        | FLAGS  | SET STATUS                                                     |                                         |                                         |
|---------------------------|---------------|----------------|-------------------------|--------|--------|----------------------------------------------------------------|-----------------------------------------|-----------------------------------------|
| A                         | B             | C              | D                       | E      | 0      | FLAGS                                                          | TRIG                                    | DISP                                    |
| n → Factors               | Primes from   | Primes to      | → Primes                | AUTO?  | Auto   | ON OFF                                                         |                                         |                                         |
| <sup>a</sup> Factor n     |               |                |                         | Output | Primes | 0 <input type="checkbox"/> <input checked="" type="checkbox"/> | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| <sup>0</sup> Used         | 1 Primes loop | 2 Divisor loop | <sup>3</sup> dln?       | 4 Exit |        | 1 <input type="checkbox"/> <input checked="" type="checkbox"/> | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
| <sup>5</sup> Non-existent | 6             | 7              | <sup>8</sup> L = 1 or 2 | 9      |        | 2 <input type="checkbox"/> <input checked="" type="checkbox"/> | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|                           |               |                |                         |        |        | 3 <input type="checkbox"/> <input checked="" type="checkbox"/> |                                         | n <u>2</u>                              |

GCD, LCM, Decimal to Fraction

|           |                          |           |                             |
|-----------|--------------------------|-----------|-----------------------------|
| 001 *LELA | GCD                      | 057 R4    |                             |
| 002 STOE  | b                        | 058 PRTX  |                             |
| 003 X←Y   |                          | 059 SPC   |                             |
| 004 STOA  | a                        | 060 R4    |                             |
| 005 1     |                          | 061 R4    |                             |
| 006 STOC  |                          | 062 R/S   |                             |
| 007 CLX   |                          | 063 *LBL6 | LCM                         |
| 008 STOD  |                          | 064 STOE  | b                           |
| 009 X=Y?  | If b = 0, list a as GCD. | 065 X←Y   |                             |
| 010 GTOD  |                          | 066 STOA  | a                           |
| 011 STOC  |                          | 067 X     |                             |
| 012 STOE  |                          | 068 STOC  | c = a x b                   |
| 013 1     |                          | 069 X=0?  | If c = 0, LCM = 0           |
| 014 STOD  |                          | 070 GTOD  |                             |
| 015 STOI  |                          | 071 *LBL8 |                             |
| 016 *LBL5 | Main loop.               | 072 GSBc  | Find GCD through iteration. |
| 017 GSBc  |                          | 073 X#0?  |                             |
| 018 X=0?  | If b = 0, list a as GCD. | 074 GTOD  | At end, a = GCD.            |
| 019 GTOD  |                          | 075 RCLC  |                             |
| 020 RCLC  |                          | 076 RCLA  |                             |
| 021 RCLC  | p←s + yq                 | 077 ÷     |                             |
| 022 STOI  | y←s                      | 078 ABS   | LCM =  c + GCD              |
| 023 RCL9  |                          | 079 *LBL3 |                             |
| 024 X     |                          | 080 PRTX  |                             |
| 025 +     |                          | 081 SPC   |                             |
| 026 STOC  |                          | 082 RTN   |                             |
| 027 RCLC  | s←p                      | 083 *LBL6 | Common subroutine which     |
| 028 RCLC  |                          | 084 RCLA  | finds GCD if iterated until |
| 029 STOE  | p←t + xq                 | 085 RCLA  | b = 0.                      |
| 030 RCL9  |                          | 086 RCLB  |                             |
| 031 X     | x←t                      | 087 STOA  |                             |
| 032 +     |                          | 088 ÷     |                             |
| 033 STOD  | t←p                      | 089 INT   | q← -INT (a/b)               |
| 034 GTOD  | Loop again.              | 090 CHS   |                             |
| 035 *LBL0 |                          | 091 STOD  | p←a + bq = a mod b          |
| 036 RCLA  | At end, a is GCD         | 092 RCLC  | a←b                         |
| 037 X=0?  |                          | 093 X     |                             |
| 038 GTOD  | (a < 0)                  | 094 +     |                             |
| 039 CLX   | Load stack with          | 095 STOE  | b←p                         |
| 040 RCLD  |                          | 096 RTN   |                             |
| 041 CHS   | t←Z                      | 097 *LBLC |                             |
| 042 RCLC  |                          | 098 0     | Decimal to fraction.        |
| 043 CHS   |                          | 099 STOA  | Initialize.                 |
| 044 RCLA  | s←Y                      | 100 STOD  |                             |
| 045 CHS   | GCD→X                    | 101 R4    |                             |
| 046 GTOD  |                          | 102 ENT1  |                             |
| 047 *LBL1 |                          | 103 STOE  |                             |
| 048 CLX   | (a > 0)                  | 104 1     |                             |
| 049 RCLD  |                          | 105 STOE  |                             |
| 050 RCLC  |                          | 106 STOC  |                             |
| 051 RCLA  | t                        | 107 SF1   |                             |
| 052 *LBL2 | s                        | 108 *LBL7 |                             |
| 053 PRTX  | GCD                      | 109 FIX   |                             |
| 054 R/S   | Output routine.          | 110 DSP0  |                             |
| 055 R4    |                          | 111 ÷     |                             |
| 056 PRTX  | (optional) Print s, t.   | 112 LSTX  |                             |

| REGISTERS             |                       |    |                     |                     |    |          |         |    |     |
|-----------------------|-----------------------|----|---------------------|---------------------|----|----------|---------|----|-----|
| 0                     | 1                     | 2  | 3                   | 4                   | 5  | 6        | 7       | 8  | 9 q |
| S0                    | S1                    | S2 | S3                  | S4                  | S5 | S6       | S7      | S8 | S9  |
| A                     | B                     |    | C                   | D                   |    | E        | I       |    |     |
| a; Num <sub>i-1</sub> | b; Den <sub>i-1</sub> |    | s; Num <sub>i</sub> | t; Den <sub>i</sub> |    | x; Value | y; Temp |    |     |

|     |       |                                             |     |       |                                             |
|-----|-------|---------------------------------------------|-----|-------|---------------------------------------------|
| 113 | ENT†  |                                             | 169 | GSB5  | Output Num <sub>i</sub> .                   |
| 114 | R↓    |                                             | 170 | RCLD  | Output Den <sub>i</sub> .                   |
| 115 | X≠Y   | a <sub>i</sub>                              | 171 | GSB5  |                                             |
| 116 | INT   |                                             | 172 | +     | Output Num <sub>i</sub> /Den <sub>i</sub> . |
| 117 | STOI  |                                             | 173 | DSP9  |                                             |
| 118 | x     |                                             | 174 | GSB5  |                                             |
| 119 | -     |                                             | 175 | GSB6  |                                             |
| 120 | RCLJ  |                                             | 176 | RCLC  | Halt displaying error.                      |
| 121 | RCLC  |                                             | 177 | -     | -----                                       |
| 122 | x     |                                             | 178 | RTN   | AUTO output.                                |
| 123 | RCLA  |                                             | 179 | *LBL5 |                                             |
| 124 | +     | If Num <sub>i</sub> = 0, clear flag 1.      | 180 | F0?   | If FO set, print and rtn.                   |
| 125 | X=0?  |                                             | 181 | PRTX  |                                             |
| 126 | CF1   |                                             | 182 | F0?   |                                             |
| 127 | RCLC  |                                             | 183 | RTN   | Else halt.                                  |
| 128 | STOA  |                                             | 184 | R/S   |                                             |
| 129 | R↓    |                                             | 185 | RTN   | -----                                       |
| 130 | STOC  |                                             | 186 | *LBL6 | Space if FO set.                            |
| 131 | F1?   | Output Num <sub>i</sub> .                   | 187 | F0?   |                                             |
| 132 | GSB5  |                                             | 188 | SPC   |                                             |
| 133 | CLX   |                                             | 189 | RTN   | -----                                       |
| 134 | RCLJ  |                                             | 190 | *LBL5 | AUTO toggle.                                |
| 135 | RCLD  |                                             | 191 | F0?   |                                             |
| 136 | x     |                                             | 192 | GT00  |                                             |
| 137 | RCLB  |                                             | 193 | SF0   |                                             |
| 138 | +     |                                             | 194 | 1     |                                             |
| 139 | RCLD  |                                             | 195 | RTN   |                                             |
| 140 | STOB  |                                             | 196 | *LBL0 |                                             |
| 141 | R↓    |                                             | 197 | CF0   |                                             |
| 142 | STOD  |                                             | 198 | 0     |                                             |
| 143 | F1?   | Output Den <sub>i</sub> .                   | 199 | FTN   | -----                                       |
| 144 | GSB5  |                                             |     |       |                                             |
| 145 | RCLC  |                                             |     |       |                                             |
| 146 | X≠Y   |                                             |     |       |                                             |
| 147 | +     |                                             |     |       |                                             |
| 148 | DSP9  |                                             |     |       |                                             |
| 149 | F1?   | Output Num <sub>i</sub> /Den <sub>i</sub> . |     |       |                                             |
| 150 | GSB5  |                                             |     |       |                                             |
| 151 | RCLC  | Error = Calc. - orig. value.                |     |       |                                             |
| 152 | -     |                                             |     |       |                                             |
| 153 | X=0?  |                                             |     |       |                                             |
| 154 | GSB6  | If error = 0, halt.                         |     |       |                                             |
| 155 | X=0?  | -----                                       |     |       |                                             |
| 156 | RTN   |                                             |     |       |                                             |
| 157 | SCI   |                                             |     |       |                                             |
| 158 | F1?   | Output error.                               |     |       |                                             |
| 159 | GSB5  |                                             |     |       |                                             |
| 160 | F1?   |                                             |     |       |                                             |
| 161 | GSB6  |                                             |     |       |                                             |
| 162 | R↓    |                                             |     |       |                                             |
| 163 | SF1   | Loop again.                                 |     |       |                                             |
| 164 | GT0?  | -----                                       |     |       |                                             |
| 165 | *LBLD | Recall last fraction.                       |     |       |                                             |
| 166 | FIX   |                                             |     |       |                                             |
| 167 | DSP0  |                                             |     |       |                                             |
| 168 | RCLC  |                                             |     |       |                                             |

| 1b8        |              | KCLL            |              | LABELS     |                      |   | FLAGS                                                        | SET STATUS                              |                                         |  |
|------------|--------------|-----------------|--------------|------------|----------------------|---|--------------------------------------------------------------|-----------------------------------------|-----------------------------------------|--|
| A a†b→GCD  | B a†b→LCM    | C Dec→Frac      | D →Last Frac | E AUTO?    | F AUTO               | 0 | ON OFF                                                       | TRIG                                    | DISP                                    |  |
| a          | b            | c               | d            | e GCD/LCM  | Num <sub>i</sub> ≠ 0 | 1 | <input type="checkbox"/> <input checked="" type="checkbox"/> | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |  |
| 0 Exit GCD | 1 GCD, a > 0 | 2 Output GCD    | 3 Exit LCM   | 4          | 2                    | 1 | <input type="checkbox"/> <input checked="" type="checkbox"/> | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |  |
| 5 AUTO out | 6 Space      | 7 Dec→Frac loop | 8 LCM loop   | 9 GCD loop | 3                    | 2 | <input type="checkbox"/> <input checked="" type="checkbox"/> | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |  |
|            |              |                 |              |            |                      | 3 | <input type="checkbox"/> <input checked="" type="checkbox"/> |                                         | n <u>2</u>                              |  |

## Base Conversions

|                     |                                                  |                     |                                                     |
|---------------------|--------------------------------------------------|---------------------|-----------------------------------------------------|
| 001 *LBLA           | Input $x_b$ (no. in base b to be converted).     | 057 GSB $\epsilon$  | Convert Exit                                        |
| 002 STOE            |                                                  | 058 GTOS            | -----                                               |
| 003 F0?             |                                                  | 059 *LBL2           | Here $b \neq 10$ , $B \neq 10$ .                    |
| 004 SPC             |                                                  | 060 RCLD            | Convert $x_b$ to $x_{10}$ .                         |
| 005 F0?             |                                                  | 061 GSB $\delta$    |                                                     |
| 006 PRTX            |                                                  | 062 STOC            |                                                     |
| 007 1               |                                                  | 063 GSB $\epsilon$  |                                                     |
| 008 0               | Default bases b & B are 10.                      | 064 RCLE            |                                                     |
| 009 STOC            |                                                  | 065 STOE            |                                                     |
| 010 STOD            |                                                  | 066 RCLA            |                                                     |
| 011 EEX             |                                                  | 067 STOC            |                                                     |
| 012 1               |                                                  | 068 GSB $\delta$    | Convert $x_{10}$ to $x_b$ .                         |
| 013 2               |                                                  | 069 STOD            | -----                                               |
| 014 STOE            |                                                  | 070 GSB $\epsilon$  |                                                     |
| 015 R4              |                                                  | 071 *LBL5           |                                                     |
| 016 R4              |                                                  | 072 PRTX            | Exit                                                |
| 017 RTN             |                                                  | 073 F0?             |                                                     |
| 018 *LBL $\epsilon$ | Input base b.                                    | 074 SPC             |                                                     |
| 019 STOD            |                                                  | 075 R/S             |                                                     |
| 020 F0?             |                                                  | 076 *LBL $\delta$   |                                                     |
| 021 PRTX            |                                                  | 077 1               |                                                     |
| 022 F0?             |                                                  | 078 0               | Subroutine tests input in X-register > 10.          |
| 023 SPC             |                                                  | 079 STOD            | If > 10, returns value 100.                         |
| 024 SF2             |                                                  | 080 XZY             | If $\leq 10$ , returns value 10.                    |
| 025 RTN             |                                                  | 081 XZY?            |                                                     |
| 026 *LBLC           | Input base B, to which $x_b$ is to be converted. | 082 EEX             |                                                     |
| 027 STOC            |                                                  | 083 1               |                                                     |
| 028 F2?             |                                                  | 084 STX7            |                                                     |
| 029 GTOD            |                                                  | 085 RCL7            |                                                     |
| 030 F0?             |                                                  | 086 RTN             |                                                     |
| 031 SPC             |                                                  | 087 *LBL $\epsilon$ | Main subroutine; actually converts to/from base 10. |
| 032 *LBL0           |                                                  | 088 0               | -----                                               |
| 033 F0?             |                                                  | 089 STOS            |                                                     |
| 034 PRTX            |                                                  | 090 STOS            |                                                     |
| 035 RTN             |                                                  | 091 RCLE            |                                                     |
| 036 *LBLD           | Find $x_b$ .                                     | 092 *LBL9           | Shift right until < 1.                              |
| 037 RCLC            | Save b and B.                                    | 093 1               |                                                     |
| 038 STOA            |                                                  | 094 XZY?            |                                                     |
| 039 RCLD            |                                                  | 095 GTOD            |                                                     |
| 040 STOI            |                                                  | 096 ST+9            |                                                     |
| 041 1               |                                                  | 097 CLX             | $R_9$ keeps track of no. places shifted (exponent). |
| 042 0               |                                                  | 098 RCLC            |                                                     |
| 043 XZY?            |                                                  | 099 +               |                                                     |
| 044 GTOI            | Is b = 10?                                       | 100 STOE            |                                                     |
| 045 RCLC            | No, try B = 10                                   | 101 GTOS            |                                                     |
| 046 GSB $\delta$    | Yes, test B > 10                                 | 102 *LBL $\epsilon$ | On entry, $R_E$ contains normalized $x_b$ :         |
| 047 STOD            | (Choose 10 or 100).                              | 103 RCLC            | $0 < x_b < 1$ .                                     |
| 048 GSB $\epsilon$  | Convert                                          | 104 RCLE            |                                                     |
| 049 GTOS            | Exit                                             | 105 x               |                                                     |
| 050 *LBL1           | -----                                            | 106 STOE            |                                                     |
| 051 RCLC            | b $\neq 10$                                      | 107 EEX             |                                                     |
| 052 XZY?            | Is B = 10?                                       | 108 4               |                                                     |
| 053 GTOD            | No, branch.                                      | 109 +               |                                                     |
| 054 RCLD            | Yes, test b > 10.                                | 110 EEX             |                                                     |
| 055 GSB $\delta$    | (Choose 10 or 100).                              | 111 4               |                                                     |
| 056 STOC            |                                                  | 112 -               |                                                     |

[illegible]

Optimal Scale for a Graph; Plotting

|           |                                                                          |           |                                                                      |
|-----------|--------------------------------------------------------------------------|-----------|----------------------------------------------------------------------|
| 001 *LBLA | Min                                                                      | 057 RCLB  |                                                                      |
| 002 STOE  |                                                                          | 058 RCL8  |                                                                      |
| 003 RTN   | -----                                                                    | 059 RCL9  | % efficiency =                                                       |
| 004 *LBLB |                                                                          | 060 -     |                                                                      |
| 005 STOD  | Max                                                                      | 061 ÷     |                                                                      |
| 006 RTN   | -----                                                                    | 062 EEX   | $\frac{\text{Max} - \text{Min}}{\text{Top} - \text{Bot}} \times 100$ |
| 007 *LBLC |                                                                          | 063 2     |                                                                      |
| 008 STOC  | #Tics                                                                    | 064 x     |                                                                      |
| 009 1     |                                                                          | 065 RCL8  |                                                                      |
| 010 STO1  |                                                                          | 066 RCL9  | Fill stack and print:                                                |
| 011 RCLD  |                                                                          | 067 RCLA  | Top→T                                                                |
| 012 RCLE  |                                                                          | 068 R↑    | Bot→Z                                                                |
| 013 -     |                                                                          | 069 PRST  | Δ→Y                                                                  |
| 014 STOE  | $R_B \leftarrow \text{Max} - \text{Min}$                                 | 070 RTN   | %→X                                                                  |
| 015 RCLC  |                                                                          | 071 *LBLD |                                                                      |
| 016 ÷     | $n = \text{Floor} (R_B / \#Tics).$                                       | 072 X<0?  | Subroutine to find "floor"                                           |
| 017 LOG   | First guess: $\Delta = 10^n.$                                            | 073 GTD0  | of x, where floor = greatest                                         |
| 018 GSRD  | -----                                                                    | 074 INT   | integer $\leq x.$                                                    |
| 019 10*   | Begin loop.                                                              | 075 RTN   |                                                                      |
| 020 STOA  |                                                                          | 076 *LBL0 |                                                                      |
| 021 *LBL9 |                                                                          | 077 ENT↑  |                                                                      |
| 022 ISZ1  |                                                                          | 078 INT   |                                                                      |
| 023 RCLE  |                                                                          | 079 X=Y?  |                                                                      |
| 024 RCLA  |                                                                          | 080 RTN   |                                                                      |
| 025 =     |                                                                          | 081 1     |                                                                      |
| 026 GSRD  | Trial bottom = $\Delta \times \text{Floor}$                              | 082 -     |                                                                      |
| 027 RCLA  | (Min/Δ).                                                                 | 083 RTN   |                                                                      |
| 028 x     |                                                                          | 084 *LBL0 |                                                                      |
| 029 STO9  |                                                                          | 085 STOE  | Begin x.                                                             |
| 030 RCLA  |                                                                          | 086 RTN   |                                                                      |
| 031 RCLC  |                                                                          | 087 *LBLB |                                                                      |
| 032 x     |                                                                          | 088 STOD  | End x.                                                               |
| 033 +     | $\text{Top} = \text{Bot} + \Delta (\#Tics).$                             | 089 RTN   |                                                                      |
| 034 STOE  |                                                                          | 090 *LBLC |                                                                      |
| 035 RCLD  | If $\text{Max} \leq \text{Top}, \text{exit}.$                            | 091 STOC  |                                                                      |
| 036 X<Y?  |                                                                          | 092 RTN   | Step size.                                                           |
| 037 GT07  |                                                                          | 093 *LBLD |                                                                      |
| 038 RCL1  |                                                                          | 094 RCLE  | Compute $(x, f_i(x)).$                                               |
| 039 4     |                                                                          | 095 STOB  |                                                                      |
| 040 ÷     | Else try new Δ: If 1 mod 4                                               | 096 *LBLB |                                                                      |
| 041 FRC   | = 0, $\Delta \leftarrow 1.25 \Delta$ ; else $\Delta \leftarrow 2\Delta.$ | 097 GSR7  | Output x.                                                            |
| 042 X=0?  |                                                                          | 098 GSR7  |                                                                      |
| 043 GTD0  |                                                                          | 099 GSR7  | Output $f_i(x).$                                                     |
| 044 2     |                                                                          | 100 F0?   |                                                                      |
| 045 GTO6  | This sets Δ to 2, 4, 5, 10, 20,                                          | 101 SPC   | End x.                                                               |
| 046 *LBLB | 40, 50 etc., times initial                                               | 102 RCLD  |                                                                      |
| 047 1     | guess.                                                                   | 103 RCLB  |                                                                      |
| 048 .     |                                                                          | 104 RCLC  |                                                                      |
| 049 2     |                                                                          | 105 +     | $x \leftarrow x + \text{Step}.$                                      |
| 050 5     |                                                                          | 106 X>Y?  | If $x > \text{End } x, \text{exit}.$                                 |
| 051 *LBL6 |                                                                          | 107 GTO8  |                                                                      |
| 052 RCLA  |                                                                          | 108 STOB  |                                                                      |
| 053 x     |                                                                          | 109 GTO8  |                                                                      |
| 054 STOA  | Loop again.                                                              | 110 *LBL0 |                                                                      |
| 055 GTO9  | -----                                                                    | 111 CLX   | Exit.                                                                |
| 056 *LBL7 | Exit routine.                                                            | 112 RTN   |                                                                      |

| REGISTERS |                |               |              |                |        |    |    |       |          |
|-----------|----------------|---------------|--------------|----------------|--------|----|----|-------|----------|
| 0         | 1              | 2             | 3            | 4              | 5      | 6  | 7  | 8 Top | 9 Bottom |
| S0        | S1             | S2            | S3           | S4             | S5     | S6 | S7 | S8    | S9       |
| A Δ       | B Max - Min; x | C #Tics; Step | D Max; End x | E Min; Begin x | I Used |    |    |       |          |

|     |               |                       |  |  |
|-----|---------------|-----------------------|--|--|
| 113 | *LBL7         | Output subroutine     |  |  |
| 114 | F0?           | If AUTO set, print.   |  |  |
| 115 | PRTX          |                       |  |  |
| 116 | F0?           |                       |  |  |
| 117 | RTN           | Otherwise halt.       |  |  |
| 118 | R/S           |                       |  |  |
| 119 | RTN           |                       |  |  |
| 120 | *LBL <i>e</i> | Select LBL <i>i</i> . |  |  |
| 121 | STOI          |                       |  |  |
| 122 | RTN           |                       |  |  |
| 123 | *LBL <i>e</i> |                       |  |  |
| 124 | F0?           | AUTO toggle.          |  |  |
| 125 | GT00          |                       |  |  |
| 126 | SF0           |                       |  |  |
| 127 | 1             |                       |  |  |
| 128 | RTN           |                       |  |  |
| 129 | *LBL0         |                       |  |  |
| 130 | CF0           |                       |  |  |
| 131 | 0             |                       |  |  |
| 132 | RTN           |                       |  |  |

| LABELS               |                    |                    |                                    |                        | FLAGS             | SET STATUS                                                            |                                         |                                         |
|----------------------|--------------------|--------------------|------------------------------------|------------------------|-------------------|-----------------------------------------------------------------------|-----------------------------------------|-----------------------------------------|
| <sup>A</sup> Min     | <sup>B</sup> Max   | <sup>C</sup> #Tics | <sup>D</sup> Floor (x)             | <sup>E</sup> AUTO      | <sup>0</sup> AUTO | FLAGS                                                                 | TRIG                                    | DISP                                    |
| <sup>a</sup> Begin x | <sup>b</sup> End x | <sup>c</sup> Step  | <sup>d</sup> $\rightarrow(x, f_i)$ | <sup>e</sup> <i>i</i>  | 1                 | 0 <input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| <sup>0</sup> Used    | 1                  | 2                  | 3                                  | 4                      | 2                 | 1 <input type="checkbox"/> <input checked="" type="checkbox"/>        | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
| <sup>5</sup>         | <sup>6</sup> Used  | <sup>7</sup> Used  | <sup>8</sup> Plot loop             | <sup>9</sup> Opt. loop | 3                 | 2 <input type="checkbox"/> <input checked="" type="checkbox"/>        | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|                      |                    |                    |                                    |                        |                   | 3 <input type="checkbox"/> <input checked="" type="checkbox"/>        |                                         | n <u>2</u>                              |

## Complex Operations

|                    |                                                             |                    |                                                        |
|--------------------|-------------------------------------------------------------|--------------------|--------------------------------------------------------|
| 001 *LBLA          | Input a <b>b</b>                                            | 057 STOD           |                                                        |
| 002 RCLD           | Last b $\rightarrow$ R <sub>B</sub> (b <sub>1</sub> )       | 058 PRTX           |                                                        |
| 003 STOB           |                                                             | 059 SPC            |                                                        |
| 004 R <sub>1</sub> |                                                             | 060 RTN            |                                                        |
| 005 STOD           | Present b $\rightarrow$ R <sub>D</sub> (b <sub>2</sub> )    | 061 *LBLB          | -----<br>Divide ( $\div$ )                             |
| 006 R <sub>1</sub> |                                                             | 062 RCLB           | r <sub>1</sub> $\theta_1$                              |
| 007 RCLE           | Last a $\rightarrow$ R <sub>C</sub> (a <sub>1</sub> )       | 063 RCLC           |                                                        |
| 008 STOC           |                                                             | 064 +P             |                                                        |
| 009 R <sub>1</sub> | Present a $\rightarrow$ R <sub>E</sub> (a <sub>2</sub> )    | 065 RCLD           |                                                        |
| 010 STOE           |                                                             | 066 RCLE           |                                                        |
| 011 RTN            | -----                                                       | 067 +P             | r <sub>1</sub> $\theta_1$ r <sub>1</sub> $\theta_1$    |
| 012 *LBLB          | Add (+)                                                     | 068 X $\div$ Y     |                                                        |
| 013 RCLC           |                                                             | 069 CHS            | 1/r <sub>2</sub> $-\theta_2$ r <sub>1</sub> $\theta_1$ |
| 014 RCLE           |                                                             | 070 X $\div$ Y     | -----                                                  |
| 015 +              |                                                             | 071 1/X            | z                                                      |
| 016 STOE           | a <sub>2</sub> $\leftarrow$ a <sub>1</sub> + a <sub>2</sub> | 072 GTOD           | r = $\sqrt{a^2 + b^2}$                                 |
| 017 PRTX           |                                                             | 073 *LBLA          | -----                                                  |
| 018 RCLB           |                                                             | 074 RCLD           | 1/z                                                    |
| 019 RCLD           |                                                             | 075 RCLE           |                                                        |
| 020 +              |                                                             | 076 +P             | r $\theta$                                             |
| 021 STOD           | b <sub>2</sub> $\leftarrow$ b <sub>1</sub> + b <sub>2</sub> | 077 PRTX           |                                                        |
| 022 PRTX           |                                                             | 078 SPC            |                                                        |
| 023 SPC            | -----                                                       | 079 RTN            | -----                                                  |
| 024 RTN            | Subtract (-)                                                | 080 *LBLB          | 1/z                                                    |
| 025 *LBLE          |                                                             | 081 RCLD           |                                                        |
| 026 RCLC           |                                                             | 082 RCLE           |                                                        |
| 027 RCLE           |                                                             | 083 +P             |                                                        |
| 028 -              |                                                             | 084 X $\div$ Y     |                                                        |
| 029 STOE           | a <sub>2</sub> $\leftarrow$ a <sub>1</sub> - a <sub>2</sub> | 085 CHS            |                                                        |
| 030 PRTX           |                                                             | 086 X $\div$ Y     |                                                        |
| 031 RCLB           |                                                             | 087 1/X            | 1/r $-\theta$                                          |
| 032 RCLD           |                                                             | 088 GTOD           | -----                                                  |
| 033 -              |                                                             | 089 *LBLC          | z <sup>n</sup>                                         |
| 034 STOD           | b <sub>2</sub> $\leftarrow$ b <sub>1</sub> - b <sub>2</sub> | 090 STOI           | n $\rightarrow$ 1                                      |
| 035 PRTX           |                                                             | 091 RCLD           |                                                        |
| 036 SPC            | -----                                                       | 092 RCLE           |                                                        |
| 037 RTN            | Multiply (x)                                                | 093 +P             | r $\theta$                                             |
| 038 *LBLD          |                                                             | 094 RCLC           |                                                        |
| 039 RCLB           |                                                             | 095 Y <sup>x</sup> |                                                        |
| 040 RCLC           |                                                             | 096 X $\div$ Y     |                                                        |
| 041 +P             |                                                             | 097 RCLC           |                                                        |
| 042 RCLD           | r <sub>1</sub> $\theta_1$                                   | 098 x              |                                                        |
| 043 RCLE           |                                                             | 099 X $\div$ Y     |                                                        |
| 044 +P             |                                                             | 100 GTOD           | r <sup>n</sup> n $\theta$                              |
| 045 *LBL9          | r <sub>2</sub> $\theta_2$ r <sub>1</sub> $\theta_1$         | 101 *LBLD          | -----                                                  |
| 046 X $\div$ Y     |                                                             | 102 STOI           | z <sup>1/n</sup>                                       |
| 047 R $\uparrow$   |                                                             | 103 3              | n $\rightarrow$ 1                                      |
| 048 +              |                                                             | 104 6              |                                                        |
| 049 X $\div$ Y     |                                                             | 105 0              |                                                        |
| 050 R $\uparrow$   |                                                             | 106 X $\div$ Y     |                                                        |
| 051 x              |                                                             | 107 $\div$         |                                                        |
| 052 *LBL8          | r <sub>1</sub> r <sub>2</sub> ( $\theta_1 + \theta_2$ )     | 108 STOA           | 360/n $\rightarrow$ R <sub>A</sub>                     |
| 053 +R             | -----                                                       | 109 RCLD           |                                                        |
| 054 STOE           | Output routine.                                             | 110 RCLE           |                                                        |
| 055 PRTX           |                                                             | 111 +P             | r $\theta$                                             |
| 056 X $\div$ Y     |                                                             | 112 RCLC           |                                                        |

| REGISTERS |                  |                  |                  |                  |    |    |    |    |    |
|-----------|------------------|------------------|------------------|------------------|----|----|----|----|----|
| 0         | 1                | 2                | 3                | 4                | 5  | 6  | 7  | 8  | 9  |
| S0        | S1               | S2               | S3               | S4               | S5 | S6 | S7 | S8 | S9 |
| A 360/n   | B b <sub>1</sub> | C a <sub>1</sub> | D b <sub>2</sub> | E a <sub>2</sub> | F  | G  | H  | I  | n  |



## Polynomial Solutions

|                      |                                    |                      |                                  |                      |               |                 |                    |              |       |
|----------------------|------------------------------------|----------------------|----------------------------------|----------------------|---------------|-----------------|--------------------|--------------|-------|
| 001 *LBLA            | 5 <sup>th</sup> degree.            | 057 2                |                                  |                      |               |                 |                    |              |       |
| 002 4                |                                    | 058 ST+6             | A                                |                      |               |                 |                    |              |       |
| 003 STOE             |                                    | 059 ST+7             | B                                |                      |               |                 |                    |              |       |
| 004 STOI             |                                    | 060 RCL7             |                                  |                      |               |                 |                    |              |       |
| 005 CSB <sub>a</sub> | Find one real root.                | 061 X <sup>2</sup>   |                                  |                      |               |                 |                    |              |       |
| 006 RCL7             |                                    | 062 RCLA             |                                  |                      |               |                 |                    |              |       |
| 007 PRTX             |                                    | 063 -                |                                  |                      |               |                 |                    |              |       |
| 008 RCL4             |                                    | 064 IX               | D                                |                      |               |                 |                    |              |       |
| 009 1.               |                                    | 065 ST09             |                                  |                      |               |                 |                    |              |       |
| 010 CSB <sub>b</sub> | Synthetic division to find         | 066 X=0?             |                                  |                      |               |                 |                    |              |       |
| 011 CSB <sub>b</sub> | new $a_i, i = 3, 2, 1, 0.$         | 067 CT00             |                                  |                      |               |                 |                    |              |       |
| 012 CSB <sub>b</sub> | -----                              | 068 RCL6             |                                  |                      |               |                 |                    |              |       |
| 013 CSB <sub>b</sub> |                                    | 069 RCL7             |                                  |                      |               |                 |                    |              |       |
| 014 *LBLB            | 4 <sup>th</sup> degree (quartic).  | 070 X                |                                  |                      |               |                 |                    |              |       |
| 015 RCL2             |                                    | 071 RCLB             | Compute C for D $\neq 0.$        |                      |               |                 |                    |              |       |
| 016 STOC             |                                    | 072 2                |                                  |                      |               |                 |                    |              |       |
| 017 CHS              |                                    | 073 ÷                |                                  |                      |               |                 |                    |              |       |
| 018 ST02             |                                    | 074 -                |                                  |                      |               |                 |                    |              |       |
| 019 RCL1             | Compute coefficients               | 075 RCL9             |                                  |                      |               |                 |                    |              |       |
| 020 STOB             | $b_2, b_1, b_0$ for cubic equation | 076 ÷                |                                  |                      |               |                 |                    |              |       |
| 021 RCL3             | to find $y_0.$                     | 077 CT01             |                                  |                      |               |                 |                    |              |       |
| 022 STOD             |                                    | 078 *LBL0            | -----                            |                      |               |                 |                    |              |       |
| 023 X                |                                    | 079 RCL6             |                                  |                      |               |                 |                    |              |       |
| 024 RCL0             |                                    | 080 X <sup>2</sup>   | Compute C if D = 0.              |                      |               |                 |                    |              |       |
| 025 STOA             |                                    | 081 RCLC             |                                  |                      |               |                 |                    |              |       |
| 026 4                |                                    | 082 -                |                                  |                      |               |                 |                    |              |       |
| 027 X                |                                    | 083 RCL7             |                                  |                      |               |                 |                    |              |       |
| 028 -                |                                    | 084 2                |                                  |                      |               |                 |                    |              |       |
| 029 STOI             |                                    | 085 X                |                                  |                      |               |                 |                    |              |       |
| 030 RCLC             |                                    | 086 +                |                                  |                      |               |                 |                    |              |       |
| 031 4                |                                    | 087 IX               |                                  |                      |               |                 |                    |              |       |
| 032 X                |                                    | 088 *LBL1            | -----                            |                      |               |                 |                    |              |       |
| 033 RCLD             |                                    | 089 ST08             | C                                |                      |               |                 |                    |              |       |
| 034 X <sup>2</sup>   |                                    | 090 SF0              | Print roots.                     |                      |               |                 |                    |              |       |
| 035 -                |                                    | 091 CSB7             |                                  |                      |               |                 |                    |              |       |
| 036 RCLA             |                                    | 092 *LBL7            | -----                            |                      |               |                 |                    |              |       |
| 037 X                |                                    | 093 RCL6             | First time through, set          |                      |               |                 |                    |              |       |
| 038 RCLB             |                                    | 094 RCL8             | $a_1 = A - C$ and $a_0 = B - D.$ |                      |               |                 |                    |              |       |
| 039 X <sup>2</sup>   |                                    | 095 CHS              |                                  |                      |               |                 |                    |              |       |
| 040 -                |                                    | 096 ST08             |                                  |                      |               |                 |                    |              |       |
| 041 ST00             | If $b_0 = 0$ , Error will occur.   | 097 +                |                                  |                      |               |                 |                    |              |       |
| 042 CF0              |                                    | 098 ST01             |                                  |                      |               |                 |                    |              |       |
| 043 CSBC             | Do not print roots.                | 099 RCL7             |                                  |                      |               |                 |                    |              |       |
| 044 F2?              | Solve cubic.                       | 100 RCL9             | Second time through, set         |                      |               |                 |                    |              |       |
| 045 CT00             | If only one real root, branch.     | 101 CHS              | $a_1 = A + C$ and $a_0 = B + D.$ |                      |               |                 |                    |              |       |
| 046 RCL7             |                                    | 102 ST09             |                                  |                      |               |                 |                    |              |       |
| 047 RCL3             |                                    | 103 +                |                                  |                      |               |                 |                    |              |       |
| 048 X>Y?             |                                    | 104 ST00             |                                  |                      |               |                 |                    |              |       |
| 049 ST07             | Else find largest real root        | 105 CSB0             |                                  |                      |               |                 |                    |              |       |
| 050 RCL7             | from among $R_3, R_4, R_7.$        | 106 RTN              | Solve quadratic $x^2 + a_1 x$    |                      |               |                 |                    |              |       |
| 051 RCL4             |                                    | 107 *LBLC            | $+ a_0 = 0.$                     |                      |               |                 |                    |              |       |
| 052 X>Y?             |                                    | 108 2                | -----                            |                      |               |                 |                    |              |       |
| 053 ST07             |                                    | 109 STOE             | 3 <sup>rd</sup> degree (cubic).  |                      |               |                 |                    |              |       |
| 054 *LBL0            | $y_0 \rightarrow R_7$              | 110 STOI             | Find one real root.              |                      |               |                 |                    |              |       |
| 055 RCLD             | -----                              | 111 CSB <sub>a</sub> |                                  |                      |               |                 |                    |              |       |
| 056 ST06             |                                    | 112 RCL7             |                                  |                      |               |                 |                    |              |       |
| REGISTERS            |                                    |                      |                                  |                      |               |                 |                    |              |       |
| 0 $a_0, b_0$         | 1 $a_1, b_1$                       | 2 $a_2, b_2$         | 3 $a_3, \text{Root}$             | 4 $a_4, \text{Root}$ | 5 $\sqrt{-D}$ | 6 $A, \Delta X$ | 7 $B, \text{Root}$ | 8 $C, \pm 1$ | 9 $D$ |
| S0                   | S1                                 | S2                   | S3                               | S4                   | S5            | S6              | S7                 | S8           | S9    |
| A $a_0$              | B $a_1$                            | C $a_2$              | D $a_3$                          | E 2 or 4             | Counter       |                 |                    |              |       |



|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                    |                    |                                      |                                     |                                     |                    |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|--------------------|--------------------------------------|-------------------------------------|-------------------------------------|--------------------|
| 001 *LBLA<br>002 5<br>003 STOI<br>004 1<br>005 STOB<br>006 STOC<br>007 *LBL9<br>008 RCLB<br>009 RCLC<br>010 1<br>011 0<br>012 ÷<br>013 +<br>014 R/S<br>015 F0?<br>016 PRTX<br>017 STOI<br>018 ISZ1<br>019 4<br>020 RCLB<br>021 X=Y?<br>022 GTOB<br>023 RCLC<br>024 X=Y?<br>025 GTOB<br>026 1<br>027 STOB<br>028 +<br>029 STOC<br>030 GTOS<br>031 *LBL0<br>032 1<br>033 +<br>034 STOB<br>035 GTOS<br>036 *LBLB<br>037 0<br>038 STOB<br>039 1<br>040 STOB<br>041 STOD<br>042 STOE<br>043 RCL5<br>044 AES<br>045 STOC<br>046 2<br>047 RCL6<br>048 GSB0<br>049 3<br>050 RCL7<br>051 GSB0<br>052 4<br>053 RCL8<br>054 GSB0<br>055 1<br>056 RCLB | Input matrix by columns:<br><br>a <sub>11</sub><br>a <sub>21</sub><br>a <sub>31</sub><br>a <sub>41</sub><br>a <sub>12</sub><br>a <sub>22</sub><br>a <sub>32</sub><br>a <sub>42</sub><br>etc.<br><br>R <sub>B</sub> is index i, R <sub>C</sub> is index j<br>for element a <sub>ij</sub> , i, j = 1, 2, 3,<br>4.<br><br>-----<br>Begin execution.<br>Note: pivot is element of<br>greatest absolute value in<br>column.<br><br>n ← k - 1<br><br>Prepare to find pivot,<br>column 1.<br>R <sub>C</sub> ←  a <sub>11</sub>  <br>n ← 2<br>a <sub>21</sub><br><br>n ← 3<br>a <sub>31</sub><br><br>n ← 4<br>a <sub>41</sub> | X=Y?<br>057<br>058 GTOB<br>059 GSB0<br>060 1<br>061 GSB0<br>062 2<br>063 GSB0<br>064 3<br>065 GSB0<br>066 4<br>067 GSB0<br>068 *LBL0<br>069 RCL5<br>070 CHS<br>071 ST÷6<br>072 ST÷7<br>073 ST÷8<br>074 9<br>075 STOI<br>076 GSB0<br>077 GSB0<br>078 GSB0<br>079 2<br>080 STOB<br>081 STOD<br>082 P÷S<br>083 RCL0<br>084 ABS<br>085 STOC<br>086 3<br>087 RCL1<br>088 GSB0<br>089 4<br>090 RCL2<br>091 GSB0<br>092 P÷S<br>093 2<br>094 RCLB<br>095 X=Y?<br>096 GTOB<br>097 1<br>098 0<br>099 x<br>100 GSB0<br>101 2<br>102 GSB0<br>103 3<br>104 GSB0<br>105 4<br>106 GSB0<br>107 *LBL0<br>108 P÷S<br>109 RCL0<br>110 CHS<br>111 ST÷1<br>112 ST÷2 | If n = 1, no row inter-<br>change.<br>Else increment R <sub>0</sub> by n.<br><br>Swap row n with row 1.<br><br>Store multipliers: m <sub>j1</sub> ←<br>-a <sub>ij</sub> /a <sub>11</sub> , j = 2, 3, 4.<br><br>Adjust remaining elts. by<br>multipliers: a <sub>ij</sub> ← a <sub>ij</sub> + m <sub>j1</sub><br>× a <sub>1j</sub> , i, j = 2, 3, 4.<br><br>-----<br>Begin 3 × 3.<br>n ← k - 2.<br>Prepare to find pivot,<br>column 2.<br><br>R <sub>C</sub> ←  a <sub>22</sub>  <br>n ← 3<br>a <sub>32</sub><br><br>n ← 4<br>a <sub>42</sub><br><br>If n = 2, no interchange.<br><br>Otherwise, increment<br>R <sub>0</sub> by 10n.<br><br>Swap row n with row 2 of<br>3 × 3.<br><br>Store multipliers: m <sub>j2</sub> ←<br>-a <sub>ij</sub> /a <sub>22</sub> , j = 3, 4. |                    |                    |                                      |                                     |                                     |                    |
| REGISTERS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                    |                    |                                      |                                     |                                     |                    |
| 0 Pivots                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 4                  | 5 a <sub>11</sub>  | 6 a <sub>21</sub> , m <sub>21</sub>  | 7 a <sub>31</sub> , m <sub>31</sub> | 8 a <sub>41</sub> , m <sub>41</sub> | 9 a <sub>12</sub>  |
| S0 a <sub>22</sub>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | S1 a <sub>32</sub> , m <sub>32</sub>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | S2 a <sub>42</sub> , m <sub>42</sub>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | S3 a <sub>13</sub>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | S4 a <sub>23</sub> | S5 a <sub>33</sub> | S6 a <sub>43</sub> , m <sub>43</sub> | S7 a <sub>14</sub>                  | S8 a <sub>24</sub>                  | S9 a <sub>34</sub> |
| A a <sub>44</sub>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | B i, n                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | C j                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | D k                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | E ±1               | Used               |                                      |                                     |                                     |                    |

|     |       |                                                   |  |     |       |  |                                           |
|-----|-------|---------------------------------------------------|--|-----|-------|--|-------------------------------------------|
| 113 | RCL1  |                                                   |  | 169 | R1    |  |                                           |
| 114 | RCL4  |                                                   |  | 170 | STOC  |  |                                           |
| 115 | x     | Adjust remaining elts. by                         |  | 171 | R1    |  |                                           |
| 116 | ST+5  | multipliers:                                      |  | 172 | STOB  |  | $n \rightarrow R_B$                       |
| 117 | RCL2  | $a_{ij} \leftarrow a_{ij} + m_{12} \times a_{2j}$ |  | 173 | RTN   |  | -----                                     |
| 118 | RCL4  | $i, j = 3, 4.$                                    |  | 174 | *LBL6 |  | Store pivot info in $R_0$ .               |
| 119 | x     |                                                   |  | 175 | ST+0  |  | CHS of $R_E$ for each swap.               |
| 120 | ST+6  |                                                   |  | 176 | RCL6  |  |                                           |
| 121 | RCL1  |                                                   |  | 177 | CHS   |  | Swap $j^{th}$ ( $R_C$ ) elts. in rows     |
| 122 | RCL8  |                                                   |  | 178 | STOE  |  | $m$ ( $R_B$ ) and $k$ ( $R_D$ ).          |
| 123 | x     |                                                   |  | 179 | RTN   |  |                                           |
| 124 | ST+9  |                                                   |  | 180 | *LBL6 |  |                                           |
| 125 | RCL2  |                                                   |  | 181 | STOC  |  |                                           |
| 126 | RCL8  |                                                   |  | 182 | RCLD  |  |                                           |
| 127 | x     |                                                   |  | 183 | RCLC  |  |                                           |
| 128 | RCL4  |                                                   |  | 184 | GSBE  |  | $a_{kj}$                                  |
| 129 | +     |                                                   |  | 185 | RCLB  |  |                                           |
| 130 | STOA  |                                                   |  | 186 | RCLC  |  | $a_{mj} \leftarrow a_{kj}$                |
| 131 | RCL5  | Begin 2 x 2                                       |  | 187 | GSBE  |  |                                           |
| 132 | ABS   | If $ a_{43}  \leq  a_{33} $ , no inter-           |  | 188 | XZ Y  |  | $a_{mj} \leftarrow a_{kj}$                |
| 133 | RCL6  | change.                                           |  | 189 | STOI  |  |                                           |
| 134 | ABS   |                                                   |  | 190 | XZ Y  |  |                                           |
| 135 | XZ Y? |                                                   |  | 191 | RCLD  |  |                                           |
| 136 | STOB  | Otherwise swap.                                   |  | 192 | RCLC  |  |                                           |
| 137 | RCL5  |                                                   |  | 193 | 4     |  |                                           |
| 138 | RCL6  | $a_{43} \leftrightarrow a_{33}$                   |  | 194 | x     |  |                                           |
| 139 | STOE  | $a_{44} \leftrightarrow a_{34}$                   |  | 195 | +     |  |                                           |
| 140 | XZ Y  |                                                   |  | 196 | STOI  |  |                                           |
| 141 | STOE  |                                                   |  | 197 | R1    |  |                                           |
| 142 | RCL9  |                                                   |  | 198 | STOI  |  |                                           |
| 143 | RCL4  |                                                   |  | 199 | RTN   |  |                                           |
| 144 | STOE  |                                                   |  | 200 | *LBL6 |  |                                           |
| 145 | XZ Y  |                                                   |  | 201 | 4     |  | $a_{kj} \leftarrow t$                     |
| 146 | STOA  |                                                   |  | 202 | x     |  | Given $i$ in $Y$ , $j$ in $X$ ,           |
| 147 | .     |                                                   |  | 203 | +     |  | recalls $a_{ij}$ .                        |
| 148 | 4     |                                                   |  | 204 | STOI  |  |                                           |
| 149 | PZS   | Increment $R_0$ by 0.4.                           |  | 205 | CLX   |  |                                           |
| 150 | GSBE  |                                                   |  | 206 | RCL i |  |                                           |
| 151 | PZS   |                                                   |  | 207 | RTN   |  |                                           |
| 152 | *LBL0 |                                                   |  | 208 | *LBLd |  |                                           |
| 153 | RCL5  | Store multiplier:                                 |  | 209 | RCL i |  |                                           |
| 154 | CHS   | $m_{43} \leftarrow -m_{43}/m_{33}$ .              |  | 210 | STOB  |  |                                           |
| 155 | ST+6  |                                                   |  | 211 | ISZ1  |  |                                           |
| 156 | RCL9  |                                                   |  | 212 | RCL6  |  | Used to adjust each                       |
| 157 | RCL6  |                                                   |  | 213 | GSBE  |  | column of 3 x 3 sub-                      |
| 158 | x     |                                                   |  | 214 | RCL7  |  | matrix by multipliers in                  |
| 159 | RCL4  |                                                   |  | 215 | GSBE  |  | $R_6$ , $R_7$ , and $R_8$ .               |
| 160 | +     | $a_{44} \leftarrow a_{44} + m_{43} \times a_{34}$ |  | 216 | RCL8  |  |                                           |
| 161 | STOA  |                                                   |  | 217 | GSBE  |  |                                           |
| 162 | PZS   | DONE                                              |  | 218 | RTN   |  |                                           |
| 163 | RTN   |                                                   |  | 219 | *LBL6 |  |                                           |
| 164 | *LBL6 |                                                   |  | 220 | RCLB  |  |                                           |
| 165 | ABS   | Used to find pivots.                              |  | 221 | x     |  |                                           |
| 166 | RCLC  | $R_6$ contains largest element                    |  | 222 | ST+i  |  | $R_B$ contains $a_{1j}$ , $j = 2, 3, 4$ . |
| 167 | XZ Y? | yet found in column,                              |  | 223 | ISZ1  |  |                                           |
| 168 | RTN   | ignoring sign.                                    |  | 224 | RTN   |  | Goes down column.                         |

| LABELS       |               |                                   |              | FLAGS          |          | SET STATUS                                                     |                                         |                                         |
|--------------|---------------|-----------------------------------|--------------|----------------|----------|----------------------------------------------------------------|-----------------------------------------|-----------------------------------------|
| A Input      | B Execute     | C                                 | D            | E RCL $a_{ij}$ | F Print? | FLAGS                                                          |                                         |                                         |
| a Find pivot | b Store pivot | c $a_{mj} \leftrightarrow a_{kj}$ | d Adjust 3x3 | e Col. Mult.   | 1        | ON OFF                                                         | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| 0 Used       | 1             | 2                                 | 3            | 4              | 2        | 0 <input type="checkbox"/> <input checked="" type="checkbox"/> | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
| 5            | 6             | 7                                 | 8            | 9 Input loop   | 3        | 1 <input type="checkbox"/> <input checked="" type="checkbox"/> | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|              |               |                                   |              |                |          | 2 <input type="checkbox"/> <input checked="" type="checkbox"/> |                                         | n <input type="checkbox"/>              |
|              |               |                                   |              |                |          | 3 <input type="checkbox"/> <input checked="" type="checkbox"/> |                                         |                                         |

## 4 × 4 Matrix Solutions

|             |                                              |             |                                              |             |             |             |             |             |             |
|-------------|----------------------------------------------|-------------|----------------------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 001 #LBLA   | Find determinant.                            | 057 X=0?    | If n = 0, no interchange.                    |             |             |             |             |             |             |
| 002 RCLE    |                                              | 058 GT00    |                                              |             |             |             |             |             |             |
| 003 RCL5    | $R_E = \pm 1$ depending on                   | 059 ST01    | If n $\neq$ 0, swap $b_n$ and $b_2$ .        |             |             |             |             |             |             |
| 004 X       | number of row interchanges.                  | 060 RCL i   |                                              |             |             |             |             |             |             |
| 005 P=S     |                                              | 061 RCL2    |                                              |             |             |             |             |             |             |
| 006 RCL0    |                                              | 062 ST01    |                                              |             |             |             |             |             |             |
| 007 X       |                                              | 063 X=Y     |                                              |             |             |             |             |             |             |
| 008 RCL5    |                                              | 064 ST02    |                                              |             |             |             |             |             |             |
| 009 X       | $ \text{Det}  = a_{11} a_{22} a_{33} a_{44}$ | 065 #LBL0   |                                              |             |             |             |             |             |             |
| 010 P=S     |                                              | 066 P=S     |                                              |             |             |             |             |             |             |
| 011 RCLA    |                                              | 067 RCL2    |                                              |             |             |             |             |             |             |
| 012 X       |                                              | 068 RCL1    |                                              |             |             |             |             |             |             |
| 013 RTN     |                                              | 069 P=S     |                                              |             |             |             |             |             |             |
| 014 #LBL0   | Input vector                                 | 070 RCL2    | $b_3 \leftarrow b_3 + m_{32} b_2$            |             |             |             |             |             |             |
| 015 ST04    | $\underline{b} = (b_1, b_2, b_3, b_4)$       | 071 X       |                                              |             |             |             |             |             |             |
| 016 R4      |                                              | 072 ST+3    |                                              |             |             |             |             |             |             |
| 017 ST03    |                                              | 073 CLX     |                                              |             |             |             |             |             |             |
| 018 R4      |                                              | 074 RCL2    |                                              |             |             |             |             |             |             |
| 019 ST02    |                                              | 075 X       |                                              |             |             |             |             |             |             |
| 020 R4      |                                              | 076 ST+4    | $b_4 \leftarrow b_4 + m_{42} b_2$            |             |             |             |             |             |             |
| 021 ST01    |                                              | 077 RCL0    |                                              |             |             |             |             |             |             |
| 022 #LBL0   |                                              | 078 FRC     | Begin k = 3.                                 |             |             |             |             |             |             |
| 023 RCL0    | Solves $A \underline{x} = \underline{b}$     | 079 RCLD    | Pick off fractional digit                    |             |             |             |             |             |             |
| 024 1       | First find $L \underline{b}$                 | 080 X       | of $R_0$ (= n).                              |             |             |             |             |             |             |
| 025 0       |                                              | 081 X=0?    |                                              |             |             |             |             |             |             |
| 026 ST0D    | Pick off units digit of $R_0$                | 082 GT00    | If n = 0, no interchange.                    |             |             |             |             |             |             |
| 027 ÷       | (= n).                                       | 083 ST01    |                                              |             |             |             |             |             |             |
| 028 FRC     |                                              | 084 RCL i   | If n $\neq$ 0, swap $b_n$ and $b_3$ .        |             |             |             |             |             |             |
| 029 RCLD    |                                              | 085 RCL3    |                                              |             |             |             |             |             |             |
| 030 X       |                                              | 086 ST01    |                                              |             |             |             |             |             |             |
| 031 INT     |                                              | 087 X=Y     | (n can only be 4)                            |             |             |             |             |             |             |
| 032 X=0?    | If n = 0, no interchange.                    | 088 ST03    |                                              |             |             |             |             |             |             |
| 033 GT00    |                                              | 089 #LBL0   |                                              |             |             |             |             |             |             |
| 034 ST01    | If n $\neq$ 0, swap $b_n$ and $b_1$ .        | 090 P=S     |                                              |             |             |             |             |             |             |
| 035 RCL i   |                                              | 091 RCL6    |                                              |             |             |             |             |             |             |
| 036 RCL1    |                                              | 092 P=S     |                                              |             |             |             |             |             |             |
| 037 ST01    |                                              | 093 RCL3    |                                              |             |             |             |             |             |             |
| 038 X=Y     |                                              | 094 X       |                                              |             |             |             |             |             |             |
| 039 ST01    |                                              | 095 ST+4    | $b_4 \leftarrow b_4 + m_{43} b_3$            |             |             |             |             |             |             |
| 040 #LBL0   |                                              | 096 RCLA    |                                              |             |             |             |             |             |             |
| 041 RCL1    |                                              | 097 ST+4    | Now solve $U \underline{x} = \underline{Lb}$ |             |             |             |             |             |             |
| 042 RCL6    |                                              | 098 RCL4    | $b_4 \leftarrow b_4 / a_{44}$                |             |             |             |             |             |             |
| 043 X       |                                              | 099 CHS     | $t \leftarrow -b_4$                          |             |             |             |             |             |             |
| 044 ST+2    | $b_2 \leftarrow b_2 + m_{21} b_1$            | 100 ST0B    |                                              |             |             |             |             |             |             |
| 045 RCL1    |                                              | 101 P=S     |                                              |             |             |             |             |             |             |
| 046 RCL7    |                                              | 102 RCL5    |                                              |             |             |             |             |             |             |
| 047 X       |                                              | 103 ST0C    |                                              |             |             |             |             |             |             |
| 048 ST+3    | $b_3 \leftarrow b_3 + m_{31} b_1$            | 104 RCL9    |                                              |             |             |             |             |             |             |
| 049 RCL1    |                                              | 105 RCL8    | $R_0 \leftarrow a_{33}$                      |             |             |             |             |             |             |
| 050 RCL8    |                                              | 106 RCL7    | $a_{34}$                                     |             |             |             |             |             |             |
| 051 X       |                                              | 107 P=S     | $a_{24}$                                     |             |             |             |             |             |             |
| 052 ST+4    | $b_4 \leftarrow b_4 + m_{41} b_1$            | 108 RCL8    | $a_{14}$                                     |             |             |             |             |             |             |
| 053 RCL0    |                                              | 109 X       |                                              |             |             |             |             |             |             |
| 054 RCLD    | Begin k = 2.                                 | 110 ST+1    | $b_1 \leftarrow b_1 - a_{14} b_4$            |             |             |             |             |             |             |
| 055 ÷       | Pick off tens digit of $R_0$                 | 111 CLX     |                                              |             |             |             |             |             |             |
| 056 INT     | (= n).                                       | 112 RCLB    |                                              |             |             |             |             |             |             |
| REGISTERS   |                                              |             |                                              |             |             |             |             |             |             |
| 0 Pivots    | 1 $b_1$                                      | 2 $b_2$     | 3 $b_3$                                      | 4 $b_4$     | 5 $a_{11}$  | 6 $m_{21}$  | 7 $m_{31}$  | 8 $m_{41}$  | 9 $a_{12}$  |
| S0 $a_{22}$ | S1 $m_{32}$                                  | S2 $m_{42}$ | S3 $a_{13}$                                  | S4 $a_{23}$ | S5 $a_{33}$ | S6 $m_{43}$ | S7 $a_{14}$ | S8 $a_{24}$ | S9 $a_{34}$ |
| A $a_{44}$  | B Used                                       | C Used      | D 10                                         | E $\pm 1$   | F Used      |             |             |             |             |

|     |                  |                                                      |     |                  |  |                                                                                                                                                     |
|-----|------------------|------------------------------------------------------|-----|------------------|--|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| 113 | X                | $b_3 \leftarrow b_3 - a_{24} b_4$                    | 169 | ST02             |  | where $\underline{J}_j$ is the column<br>vector of length 4 with<br>$b_i = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$<br>$i = 1, 2, 3, 4$ |
| 114 | ST+2             |                                                      | 170 | CSB <sub>a</sub> |  |                                                                                                                                                     |
| 115 | CLX              | $b_3 \leftarrow b_3 - a_{34} b_4$                    | 171 | CSB <sub>c</sub> |  |                                                                                                                                                     |
| 116 | RCLB             | -----                                                | 172 | 1                |  |                                                                                                                                                     |
| 117 | X                | $b_3 \leftarrow b_3 - b_3/a_{33}$                    | 173 | ST03             |  | Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                                    |
| 118 | ST+3             |                                                      | 174 | CSB <sub>a</sub> |  |                                                                                                                                                     |
| 119 | RCLC             | $R_B \leftarrow -b_3$                                | 175 | CSB <sub>c</sub> |  |                                                                                                                                                     |
| 120 | ST+3             |                                                      | 176 | 1                |  |                                                                                                                                                     |
| 121 | RCL3             |                                                      | 177 | ST04             |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 122 | CHS              |                                                      | 178 | CSB <sub>a</sub> |  |                                                                                                                                                     |
| 123 | ST0B             |                                                      | 179 | CLX              |  |                                                                                                                                                     |
| 124 | P+S              |                                                      | 180 | RTN              |  |                                                                                                                                                     |
| 125 | RCL0             | $R_c \leftarrow a_{22}$                              | 181 | *LBLc            |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 126 | ST0C             | $a_{23}$                                             | 182 | CLX              |  |                                                                                                                                                     |
| 127 | RCL4             | $a_{13}$                                             | 183 | ST01             |  |                                                                                                                                                     |
| 128 | RCL3             |                                                      | 184 | ST02             |  |                                                                                                                                                     |
| 129 | P+S              |                                                      | 185 | ST03             |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 130 | RCLB             |                                                      | 186 | ST04             |  |                                                                                                                                                     |
| 131 | X                |                                                      | 187 | RTN              |  |                                                                                                                                                     |
| 132 | ST+1             | $b_1 \leftarrow b_1 - a_{13} b_3$                    | 188 | *LBLc            |  |                                                                                                                                                     |
| 133 | CLX              |                                                      | 189 | F0?              |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 134 | RCLB             |                                                      | 190 | ST0B             |  |                                                                                                                                                     |
| 135 | X                |                                                      | 191 | SF0              |  |                                                                                                                                                     |
| 136 | ST+2             | $b_3 \leftarrow b_3 - a_{23} b_3$                    | 192 | 1                |  |                                                                                                                                                     |
| 137 | RCLC             | -----                                                | 193 | RTN              |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 138 | ST+2             | $b_3 \leftarrow b_3/a_{22}$                          | 194 | *LBL0            |  |                                                                                                                                                     |
| 139 | RCL9             |                                                      | 195 | CF0              |  |                                                                                                                                                     |
| 140 | RCL2             |                                                      | 196 | 0                |  |                                                                                                                                                     |
| 141 | CHS              |                                                      | 197 | RTN              |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 142 | X                |                                                      |     |                  |  |                                                                                                                                                     |
| 143 | ST+1             | $b_1 \leftarrow b_1 - a_{12} b_2$                    |     |                  |  |                                                                                                                                                     |
| 144 | RCL5             | -----                                                |     |                  |  |                                                                                                                                                     |
| 145 | ST+1             | $b_1 \leftarrow b_1/a_{11}$                          |     |                  |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 146 | F0?              |                                                      |     |                  |  |                                                                                                                                                     |
| 147 | SPC              |                                                      |     |                  |  |                                                                                                                                                     |
| 148 | RCL1             |                                                      |     |                  |  |                                                                                                                                                     |
| 149 | CSB5             | Output                                               |     |                  |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 150 | RCL2             | $b_1$                                                |     |                  |  |                                                                                                                                                     |
| 151 | CSB5             | $b_2$                                                |     |                  |  |                                                                                                                                                     |
| 152 | RCL3             | $b_3$                                                |     |                  |  |                                                                                                                                                     |
| 153 | CSB5             | $b_4$                                                |     |                  |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 154 | RCL4             | -----                                                |     |                  |  |                                                                                                                                                     |
| 155 | *LBL5            | Output routine.                                      |     |                  |  |                                                                                                                                                     |
| 156 | F0?              | Print for AUTO.                                      |     |                  |  |                                                                                                                                                     |
| 157 | PRTX             |                                                      |     |                  |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 158 | F0?              |                                                      |     |                  |  |                                                                                                                                                     |
| 159 | RTN              |                                                      |     |                  |  |                                                                                                                                                     |
| 160 | R/S              |                                                      |     |                  |  |                                                                                                                                                     |
| 161 | RTN              | Halt otherwise.                                      |     |                  |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 162 | *LBLC            |                                                      |     |                  |  |                                                                                                                                                     |
| 163 | CSBc             | Compute inverse by 4 calls                           |     |                  |  |                                                                                                                                                     |
| 164 | 1                | to LBL a, each one solving                           |     |                  |  |                                                                                                                                                     |
| 165 | ST01             | $A \underline{x} = \underline{J}_j, j = 1, 2, 3, 4,$ |     |                  |  | -----<br>Set all $b_i = 0$ .<br><br>-----<br>AUTO toggle.                                                                                           |
| 166 | CSB <sub>a</sub> |                                                      |     |                  |  |                                                                                                                                                     |
| 167 | CSB <sub>c</sub> |                                                      |     |                  |  |                                                                                                                                                     |
| 168 | 1                |                                                      |     |                  |  |                                                                                                                                                     |

| LABELS   |                  |                  |   |         | FLAGS  | SET STATUS                                                     |                                         |                                         |
|----------|------------------|------------------|---|---------|--------|----------------------------------------------------------------|-----------------------------------------|-----------------------------------------|
| A → Det  | B Input <u>b</u> | C Inverse        | D | E AUTO? | 0 AUTO | FLAGS                                                          | TRIG                                    | DISP                                    |
| a Solve  | b                | c Clear <u>b</u> | d | e       | 1      | ON OFF                                                         |                                         |                                         |
| 0 Used   | 1                | 2                | 3 | 4       | 2      | 0 <input type="checkbox"/> <input checked="" type="checkbox"/> | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
|          |                  |                  |   |         |        | 1 <input type="checkbox"/> <input checked="" type="checkbox"/> | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
|          |                  |                  |   |         |        | 2 <input type="checkbox"/> <input checked="" type="checkbox"/> | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
| 5 Output | 6                | 7                | 8 | 9       | 3      | 3 <input type="checkbox"/> <input checked="" type="checkbox"/> |                                         | n <u>2</u>                              |

Solution to  $f(x) = 0$  on an Interval

|           |                                                   |           |                                                      |
|-----------|---------------------------------------------------|-----------|------------------------------------------------------|
| 001 *LBLA | b                                                 | 057 1     | If $s \geq 1$ , reject $b'$ .                        |
| 002 STOB  |                                                   | 058 X<Y?  |                                                      |
| 003 CSBE  | f(b)                                              | 059 GT01  |                                                      |
| 004 STOB  |                                                   | 060 RCLB  |                                                      |
| 005 RTN   | -----                                             | 061 RCLC  |                                                      |
| 006 *LBLB |                                                   | 062 -     |                                                      |
| 007 STOA  | c                                                 | 063 ABS   |                                                      |
| 008 STOC  |                                                   | 064 4     |                                                      |
| 009 CSBE  |                                                   | 065 ÷     | If $b'$ is closer than                               |
| 010 STOB  | f(c)                                              | 066 RCLD  |                                                      |
| 011 STOB  | -----                                             | 067 RCLC  |                                                      |
| 012 RTN   |                                                   | 068 -     | $\frac{ b-c }{4}$ to c, then reject $b'$ .           |
| 013 *LBLC | TOL                                               | 069 ABS   |                                                      |
| 014 STOE  |                                                   | 070 X<Y?  |                                                      |
| 015 RTN   | -----                                             | 071 GT01  |                                                      |
| 016 *LBLD |                                                   | 072 RCLB  |                                                      |
| 017 RCLB  |                                                   | 073 RCLD  |                                                      |
| 018 X=0?  | If f(b) = 0, exit.                                | 074 RCLB  |                                                      |
| 019 GT05  |                                                   | 075 -     |                                                      |
| 020 RCLB  |                                                   | 076 ABS   | If $ b' - b  \leq \text{TOL1}$ , $b' \leftarrow b +$ |
| 021 RCLC  |                                                   | 077 X>Y?  | $\text{TOL1} \times \text{sgn}(c - b)$ .             |
| 022 -     |                                                   | 078 GT02  |                                                      |
| 023 ABS   |                                                   | 079 X<Y?  |                                                      |
| 024 RCLC  |                                                   | 080 RCLC  |                                                      |
| 025 X>Y?  | If TOL >  b - c , exit.                           | 081 RCLB  |                                                      |
| 026 GT05  |                                                   | 082 -     |                                                      |
| 027 2     |                                                   | 083 ENT1  |                                                      |
| 028 ÷     |                                                   | 084 ABS   |                                                      |
| 029 EEX   |                                                   | 085 ÷     |                                                      |
| 030 CHS   |                                                   | 086 x     |                                                      |
| 031 9     | TOL1 = $10^{-9}b + \frac{1}{2}\text{TOL}$ .       | 087 RCLB  |                                                      |
| 032 RCLB  |                                                   | 088 +     |                                                      |
| 033 x     |                                                   | 089 STOD  |                                                      |
| 034 +     |                                                   | 090 GT02  |                                                      |
| 035 ST01  |                                                   | 091 *LBL1 |                                                      |
| 036 RCLB  |                                                   | 092 RCLB  |                                                      |
| 037 RCLB  | Linear interpolation (secant                      | 093 RCLC  |                                                      |
| 038 RCLB  | method):                                          | 094 +     |                                                      |
| 039 -     |                                                   | 095 2     |                                                      |
| 040 RCLB  | $b' = b - \frac{f(b)}{\frac{f(a) - f(b)}{a - b}}$ | 096 ÷     |                                                      |
| 041 RCLB  |                                                   | 097 STOD  |                                                      |
| 042 -     |                                                   | 098 *LBL2 |                                                      |
| 043 ÷     |                                                   | 099 RCLB  |                                                      |
| 044 ÷     | $b'$ may be next b.                               | 100 STOA  |                                                      |
| 045 RCLB  |                                                   | 101 RCLB  |                                                      |
| 046 X<Y?  |                                                   | 102 STOB  |                                                      |
| 047 -     |                                                   | 103 RCLD  |                                                      |
| 048 STOD  | Test if $b' \in [b, c]$ .                         | 104 STOB  |                                                      |
| 049 RCLB  |                                                   | 105 CSBE  |                                                      |
| 050 -     |                                                   | 106 STOB  |                                                      |
| 051 RCLC  | $s = \frac{b' - b}{c - b}$                        | 107 RCL9  |                                                      |
| 052 RCLB  |                                                   | 108 x     |                                                      |
| 053 -     |                                                   | 109 X<0?  |                                                      |
| 054 ÷     |                                                   | 110 GT03  |                                                      |
| 055 X<0?  | If $s < 0$ , reject $b'$ .                        | 111 RCLB  |                                                      |
| 056 GT01  |                                                   | 112 STOC  |                                                      |

| REGISTERS |     |     |      |       |    |    |    |        |        |
|-----------|-----|-----|------|-------|----|----|----|--------|--------|
| 0 f(a)    | 1   | 2   | 3    | 4     | 5  | 6  | 7  | 8 f(b) | 9 f(c) |
| S0        | S1  | S2  | S3   | S4    | S5 | S6 | S7 | S8     | S9     |
| A a       | B b | C c | D b' | E TOL | F  | G  | H  | I TOL1 | J      |

|     |            |                                    |  |  |
|-----|------------|------------------------------------|--|--|
| 113 | RCL0       | $f(c) \leftarrow f(a)$             |  |  |
| 114 | ST09       | -----                              |  |  |
| 115 | *LBL3      |                                    |  |  |
| 116 | RCL9       |                                    |  |  |
| 117 | ABS        |                                    |  |  |
| 118 | RCL8       |                                    |  |  |
| 119 | ABS        | If $ f(b)  <  f(c) $ , loop again. |  |  |
| 120 | X<Y?       |                                    |  |  |
| 121 | CTOD       |                                    |  |  |
| 122 | RCLB       | Else swap b and c and set          |  |  |
| 123 | RCLC       | $a \leftarrow (\text{new}) c$ .    |  |  |
| 124 | ST0B       |                                    |  |  |
| 125 | X $\neq$ Y |                                    |  |  |
| 126 | ST0C       |                                    |  |  |
| 127 | ST0A       |                                    |  |  |
| 128 | RCLB       |                                    |  |  |
| 129 | RCL9       |                                    |  |  |
| 130 | ST0B       |                                    |  |  |
| 131 | X $\neq$ Y |                                    |  |  |
| 132 | ST09       |                                    |  |  |
| 133 | ST00       |                                    |  |  |
| 134 | CTOD       | Loop again.                        |  |  |
| 135 | *LBL5      | -----                              |  |  |
| 136 | RCLB       | Display root.                      |  |  |
| 137 | RTN        | -----                              |  |  |
| 138 | *LBL6      |                                    |  |  |
| 139 | RTN        | User-defined $f(x)$ .              |  |  |

| LABELS |      |           |                    |             | FLAGS | SET STATUS |  |      |
|--------|------|-----------|--------------------|-------------|-------|------------|--|------|
| A      | B    | C         | D                  | E           |       | FLAGS      |  | DISP |
| b      | c    | TOL       | $\rightarrow$ root | $f(x)$      | 0     |            |  |      |
| a      | b    | c         | d                  | e           | 1     |            |  |      |
| 0      | 1    | Reject b' | 2                  | New a, b, c | 2     |            |  |      |
| 5      | Exit | 7         | 8                  | 9           | 3     |            |  |      |

| ON |                          | OFF |                                     | TRIG |                                     | DISP |                                     |
|----|--------------------------|-----|-------------------------------------|------|-------------------------------------|------|-------------------------------------|
| 0  | <input type="checkbox"/> | 1   | <input checked="" type="checkbox"/> | DEG  | <input checked="" type="checkbox"/> | FIX  | <input checked="" type="checkbox"/> |
| 1  | <input type="checkbox"/> | 2   | <input checked="" type="checkbox"/> | GRAD | <input type="checkbox"/>            | SCI  | <input type="checkbox"/>            |
| 2  | <input type="checkbox"/> | 3   | <input checked="" type="checkbox"/> | RAD  | <input type="checkbox"/>            | ENG  | <input type="checkbox"/>            |
| 3  | <input type="checkbox"/> |     | <input checked="" type="checkbox"/> |      |                                     | n    | 2                                   |

## Numerical Integration

|           |                                                        |           |                                          |
|-----------|--------------------------------------------------------|-----------|------------------------------------------|
| 001 *LBLA | Input h.                                               | 057 *LBL7 |                                          |
| 002 STOD  |                                                        | 058 2     |                                          |
| 003 RTN   | -----                                                  | 059 ÷     | If n is not even, display                |
| 004 *LBLB | Input f(x <sub>0</sub> )                               | 060 FRC   | "Error" by call to E.                    |
| 005 STOD  | R <sub>0</sub> contains TRAP Σ.                        | 061 X≠0?  |                                          |
| 006 STOD  | R <sub>0</sub> contains SIMP Σ.                        | 062 GTOE  | -----                                    |
| 007 0     | n = 0                                                  | 063 RTN   | Compute Simpson area.                    |
| 008 STOC  |                                                        | 064 *LBLC |                                          |
| 009 *LBL9 |                                                        | 065 RCLA  |                                          |
| 010 RTN   | -----                                                  | 066 GSBi  | R <sub>0</sub> ← f(a)                    |
| 011 *LBLB | Input f(x <sub>j</sub> ), j odd.                       | 067 STOD  |                                          |
| 012 STDA  | R <sub>0</sub> ← R <sub>0</sub> + 2 f(x <sub>j</sub> ) | 068 RCLB  |                                          |
| 013 GSB6  |                                                        | 069 GSBi  |                                          |
| 014 ENT1  |                                                        | 070 ST+0  | R <sub>0</sub> ← R <sub>0</sub> + f(b)   |
| 015 +     |                                                        | 071 RCLB  |                                          |
| 016 ST+9  | R <sub>0</sub> ← R <sub>0</sub> + 4 f(x <sub>j</sub> ) | 072 RCLA  |                                          |
| 017 RCLC  |                                                        | 073 STOE  | Set initial x to a.                      |
| 018 1     |                                                        | 074 -     |                                          |
| 019 +     |                                                        | 075 RCLC  |                                          |
| 020 STOC  | n ← n + 1                                              | 076 ÷     |                                          |
| 021 RTN   |                                                        | 077 STOD  | h ← (b - a)/n                            |
| 022 *LBLB | -----                                                  | 078 0     |                                          |
| 023 STDA  | Input f(x <sub>j</sub> ), j even.                      | 079 STOD  | R <sub>0</sub> will count to n.          |
| 024 GSB6  | R <sub>0</sub> ← R <sub>0</sub> + 2 f(x <sub>j</sub> ) | 080 *LBL8 | -----                                    |
| 025 ST+9  |                                                        | 081 RCLD  |                                          |
| 026 RCLC  |                                                        | 082 RCLE  |                                          |
| 027 1     | R <sub>0</sub> ← R <sub>0</sub> + 2 f(x <sub>j</sub> ) | 083 +     |                                          |
| 028 +     |                                                        | 084 STOE  | x ← x + h                                |
| 029 STOC  | n ← n + 1                                              | 085 GSBi  |                                          |
| 030 GTOD  | Exit                                                   | 086 GSB6  |                                          |
| 031 *LBLC | -----                                                  | 087 ST+0  |                                          |
| 032 2     | Compute trapezoidal area.                              | 088 2     | R <sub>0</sub> ← R <sub>0</sub> + 4 f(x) |
| 033 RCL0  |                                                        | 089 ST+9  |                                          |
| 034 GTOD  |                                                        | 090 RCLC  |                                          |
| 035 *LBLD | -----                                                  | 091 RCL9  |                                          |
| 036 RCLC  | Compute Simpson area.                                  | 092 X=Y?  |                                          |
| 037 GSB7  |                                                        | 093 GTOD  | If R <sub>0</sub> = n, exit.             |
| 038 3     | Test n even.                                           | 094 RCLD  |                                          |
| 039 RCL9  |                                                        | 095 RCLE  |                                          |
| 040 *LBL0 |                                                        | 096 +     |                                          |
| 041 RCLA  |                                                        | 097 STOE  |                                          |
| 042 -     | 2 f(x <sub>n</sub> ) were added, so sub-               | 098 GSBi  | x ← x + h                                |
| 043 *LBLd | tract f(x <sub>n</sub> ).                              | 099 GSB6  |                                          |
| 044 RCLD  |                                                        | 100 GTOD  | R <sub>0</sub> ← R <sub>0</sub> + 2 f(x) |
| 045 x     |                                                        | 101 *LBL0 | -----                                    |
| 046 XZY   |                                                        | 102 3     |                                          |
| 047 ÷     |                                                        | 103 RCL0  | Area = $\frac{h}{3} \times R_0$          |
| 048 PRTX  |                                                        | 104 GTOD  |                                          |
| 049 RTN   | Area                                                   | 105 *LBL6 |                                          |
| 050 *LBLc | -----                                                  | 106 ENT1  |                                          |
| 051 STOD  | Input a and b.                                         | 107 +     | Subroutine.                              |
| 052 XZY   | Store b.                                               | 108 ST+0  |                                          |
| 053 STDA  |                                                        | 109 RTN   |                                          |
| 054 RTN   | Store a.                                               | 110 *LBLc | R <sub>0</sub> ← R <sub>0</sub> + 2 f(x) |
| 055 *LBLb | Input n.                                               | 111 STOI  | Input i to select function               |
| 056 STOC  |                                                        | 112 RTN   | 1-5.                                     |

## REGISTERS

|                         |     |     |     |     |    |    |    |              |        |
|-------------------------|-----|-----|-----|-----|----|----|----|--------------|--------|
| 0 Used                  | 1   | 2   | 3   | 4   | 5  | 6  | 7  | 8            | 9 Used |
| S0                      | S1  | S2  | S3  | S4  | S5 | S6 | S7 | S8           | S9     |
| A f(x <sub>j</sub> ), a | B b | C n | D h | E x | F  | G  | H  | I Function i |        |

| LABELS |      |   |          |   |                              | FLAGS |                              | SET STATUS |        |   |   |                             |                                         |      |                                     |     |                                     |  |  |  |  |  |  |
|--------|------|---|----------|---|------------------------------|-------|------------------------------|------------|--------|---|---|-----------------------------|-----------------------------------------|------|-------------------------------------|-----|-------------------------------------|--|--|--|--|--|--|
| A      | h    | B | $f(x_j)$ | C | $\rightarrow \text{TRAP } f$ | D     | $\rightarrow \text{SIMP } f$ | E          | None   | 0 |   |                             | FLAGS                                   |      | TRIG                                |     | DISP                                |  |  |  |  |  |  |
| a      | s1b  | b | n        | c | $\rightarrow f_i$            | d     | Output                       | e          | i      | 1 | 0 | <input type="checkbox"/> ON | <input checked="" type="checkbox"/> OFF | DEG  | <input checked="" type="checkbox"/> | FIX | <input checked="" type="checkbox"/> |  |  |  |  |  |  |
| 0      | Used | 1 |          | 2 |                              | 3     |                              | 4          |        | 2 | 1 | <input type="checkbox"/>    | <input checked="" type="checkbox"/>     | GRAD | <input type="checkbox"/>            | SCI | <input type="checkbox"/>            |  |  |  |  |  |  |
|        |      |   |          |   |                              |       |                              |            |        |   | 2 | <input type="checkbox"/>    | <input checked="" type="checkbox"/>     | RAD  | <input type="checkbox"/>            | ENG | <input type="checkbox"/>            |  |  |  |  |  |  |
| 5      |      | 6 | 2 f(x)   | 7 | Test n                       | 8     | Loop fc                      | 9          | Loop B | 3 | 3 | <input type="checkbox"/>    | <input checked="" type="checkbox"/>     |      |                                     | n   | 2                                   |  |  |  |  |  |  |

## Gaussian Quadrature

|           |  |  |  |  |           |  |  |  |  |
|-----------|--|--|--|--|-----------|--|--|--|--|
| 001 *LBLA |  |  |  |  | 057 5     |  |  |  |  |
| 002 P=S   |  |  |  |  | 058 1     |  |  |  |  |
| 003 .     |  |  |  |  | 059 4     |  |  |  |  |
| 004 2     |  |  |  |  | 060 2     |  |  |  |  |
| 005 3     |  |  |  |  | 061 ST04  |  |  |  |  |
| 006 8     |  |  |  |  | 062 .     |  |  |  |  |
| 007 6     |  |  |  |  | 063 1     |  |  |  |  |
| 008 1     |  |  |  |  | 064 7     |  |  |  |  |
| 009 9     |  |  |  |  | 065 1     |  |  |  |  |
| 010 1     |  |  |  |  | 066 3     |  |  |  |  |
| 011 8     |  |  |  |  | 067 2     |  |  |  |  |
| 012 6     |  |  |  |  | 068 4     |  |  |  |  |
| 013 1     |  |  |  |  | 069 4     |  |  |  |  |
| 014 ST00  |  |  |  |  | 070 9     |  |  |  |  |
| 015 .     |  |  |  |  | 071 2     |  |  |  |  |
| 016 4     |  |  |  |  | 072 4     |  |  |  |  |
| 017 6     |  |  |  |  | 073 ST05  |  |  |  |  |
| 018 7     |  |  |  |  | 074 P=S   |  |  |  |  |
| 019 9     |  |  |  |  | 075 CLX   |  |  |  |  |
| 020 1     |  |  |  |  | 076 RTN   |  |  |  |  |
| 021 3     |  |  |  |  | 077 *LBLB |  |  |  |  |
| 022 9     |  |  |  |  | 078 ST0B  |  |  |  |  |
| 023 3     |  |  |  |  | 079 X=Z   |  |  |  |  |
| 024 4     |  |  |  |  | 080 ST0C  |  |  |  |  |
| 025 6     |  |  |  |  | 081 -     |  |  |  |  |
| 026 ST01  |  |  |  |  | 082 2     |  |  |  |  |
| 027 .     |  |  |  |  | 083 ÷     |  |  |  |  |
| 028 6     |  |  |  |  | 084 ST0A  |  |  |  |  |
| 029 6     |  |  |  |  | 085 RCLC  |  |  |  |  |
| 030 1     |  |  |  |  | 086 RCLB  |  |  |  |  |
| 031 2     |  |  |  |  | 087 +     |  |  |  |  |
| 032 0     |  |  |  |  | 088 2     |  |  |  |  |
| 033 9     |  |  |  |  | 089 ÷     |  |  |  |  |
| 034 3     |  |  |  |  | 090 ST0B  |  |  |  |  |
| 035 8     |  |  |  |  | 091 0     |  |  |  |  |
| 036 6     |  |  |  |  | 092 ST00  |  |  |  |  |
| 037 5     |  |  |  |  | 093 1     |  |  |  |  |
| 038 ST02  |  |  |  |  | 094 0     |  |  |  |  |
| 039 .     |  |  |  |  | 095 ST01  |  |  |  |  |
| 040 3     |  |  |  |  | 096 GSB6  |  |  |  |  |
| 041 6     |  |  |  |  | 097 GSB6  |  |  |  |  |
| 042 0     |  |  |  |  | 098 GSB6  |  |  |  |  |
| 043 7     |  |  |  |  | 099 RCL0  |  |  |  |  |
| 044 6     |  |  |  |  | 100 RCL4  |  |  |  |  |
| 045 1     |  |  |  |  | 101 x     |  |  |  |  |
| 046 5     |  |  |  |  | 102 PRTX  |  |  |  |  |
| 047 7     |  |  |  |  | 103 RTN   |  |  |  |  |
| 048 3     |  |  |  |  | 104 *LBL6 |  |  |  |  |
| 049 ST03  |  |  |  |  | 105 RCL6  |  |  |  |  |
| 050 .     |  |  |  |  | 106 ISZ1  |  |  |  |  |
| 051 9     |  |  |  |  | 107 ST0C  |  |  |  |  |
| 052 3     |  |  |  |  | 108 CHS   |  |  |  |  |
| 053 2     |  |  |  |  | 109 RCL4  |  |  |  |  |
| 054 4     |  |  |  |  | 110 x     |  |  |  |  |
| 055 6     |  |  |  |  | 111 RCLB  |  |  |  |  |
| 056 2     |  |  |  |  | 112 +     |  |  |  |  |

| REGISTERS   |          |             |          |          |          |    |    |         |    |
|-------------|----------|-------------|----------|----------|----------|----|----|---------|----|
| 0 $\Sigma$  | 1        | 2           | 3        | 4        | 5        | 6  | 7  | 8       | 9  |
| S0 $z_1$    | S1 $w_1$ | S2 $z_3$    | S3 $w_3$ | S4 $z_5$ | S5 $w_5$ | S6 | S7 | S8      | S9 |
| A (b - a)/2 |          | B (b + a)/2 |          | C Used   |          | D  |    | E       |    |
|             |          |             |          |          |          |    |    | 10 - 15 |    |

$z_1 = -z_2$

$w_1 = w_2$

$z_3 = -z_4$

$w_3 = w_4$

$z_5 = -z_6$

$w_5 = w_6$

Integral from a to b.

b

a

$R_A \leftarrow (b - a)/2$

$R_B \leftarrow (b + a)/2$

 $R_0$  will accumulate  $\Sigma$ .Set I to point at  $z_1$ .Each call to b computes 2 terms for  $\Sigma$ .

$$\frac{b-a}{2} \Sigma w_i f(x_i)$$

Computes terms of  $\Sigma$ . $z_j$  ( $j = 1, 3, 5$ )I points to  $w_j$ 

$R_0 \leftarrow z_j$

$z_{j+1} = -z_j$

Compute term  $j + 1$ 

$x_{j+1} \leftarrow [z_{j+1}(b-a) + b + a]/2.$

|     |                  |                                         |     |                |                                                             |
|-----|------------------|-----------------------------------------|-----|----------------|-------------------------------------------------------------|
| 113 | CSBE             | $w_j = w_{j+1}$                         | 169 | *LBLB          | Finds one term.                                             |
| 114 | RCL i            | $w_{j+1} f(x_{j+1})$                    | 170 | RCL i          | $\frac{w_j}{(1+z_j)^2} f(x_j)$                              |
| 115 | x                |                                         | 171 | x              |                                                             |
| 116 | ST+0             |                                         | 172 | RCLB           |                                                             |
| 117 | RCLC             |                                         | 173 | X <sup>2</sup> |                                                             |
| 118 | RCLA             | Compute term j.                         | 174 | ÷              |                                                             |
| 119 | x                |                                         | 175 | ST+0           |                                                             |
| 120 | RCLB             |                                         | 176 | RTN            | 48 steps reserved for user-defined function f(x).           |
| 121 | +                |                                         | 177 | *LBLB          |                                                             |
| 122 | CSBE             | $w_j f(x_j)$                            | 178 | RTN            |                                                             |
| 123 | RCL i            |                                         |     |                |                                                             |
| 124 | x                |                                         |     |                |                                                             |
| 125 | ST+0             | I points at $z_{j+2}$ .                 |     |                | Also available are $R_1 - R_9$ , $R_D$ , $R_E$ , and stack. |
| 126 | ISZI             |                                         |     |                |                                                             |
| 127 | RTN              |                                         |     |                |                                                             |
| 128 | *LBLC            | Integral from a to ∞.                   |     |                |                                                             |
| 129 | STOA             | a                                       |     |                |                                                             |
| 130 | 0                |                                         |     |                |                                                             |
| 131 | STO0             | $R_0$ will accumulate $\Sigma$ .        |     |                |                                                             |
| 132 | 1                |                                         |     |                |                                                             |
| 133 | 0                | I points at $z_1$ .                     |     |                |                                                             |
| 134 | STOI             |                                         |     |                |                                                             |
| 135 | CSBc             | Each call to c computes 2               |     |                |                                                             |
| 136 | CSBc             | terms for $\Sigma$ .                    |     |                |                                                             |
| 137 | CSBc             |                                         |     |                |                                                             |
| 138 | RCL0             |                                         |     |                |                                                             |
| 139 | 2                |                                         |     |                |                                                             |
| 140 | x                |                                         |     |                |                                                             |
| 141 | PRTX             | $2 \Sigma \frac{w_i}{(1+z_i)^2} f(x_i)$ |     |                |                                                             |
| 142 | RTN              |                                         |     |                |                                                             |
| 143 | *LBLc            | Computes terms of $\Sigma$ .            |     |                |                                                             |
| 144 | RCL i            | $z_i (i = 1, 3, 5)$ .                   |     |                |                                                             |
| 145 | ISZI             | I points to $w_j$ .                     |     |                |                                                             |
| 146 | STOC             | $R_c + z_j$                             |     |                |                                                             |
| 147 | CHS              | $z_{j+1} = -z_j$                        |     |                |                                                             |
| 148 | CSB9             | $f(x_{j+1})$                            |     |                |                                                             |
| 149 | CSBE             |                                         |     |                |                                                             |
| 150 | CSB8             |                                         |     |                |                                                             |
| 151 | RCLC             |                                         |     |                |                                                             |
| 152 | CSB9             | $z_j$                                   |     |                |                                                             |
| 153 | CSBE             | $f(x_j)$                                |     |                |                                                             |
| 154 | CSB8             |                                         |     |                |                                                             |
| 155 | ISZI             |                                         |     |                |                                                             |
| 156 | RTN              | I points to $z_{j+2}$ .                 |     |                |                                                             |
| 157 | *LBL9            | Computes argument.                      |     |                |                                                             |
| 158 | 1                |                                         |     |                |                                                             |
| 159 | +                | $x_j = \frac{2}{1+z_j} + a - 1$         |     |                |                                                             |
| 160 | STOB             |                                         |     |                |                                                             |
| 161 | 2                |                                         |     |                |                                                             |
| 162 | X <sup>2</sup> Y |                                         |     |                |                                                             |
| 163 | ÷                |                                         |     |                |                                                             |
| 164 | RCLA             |                                         |     |                |                                                             |
| 165 | +                |                                         |     |                |                                                             |
| 166 | 1                |                                         |     |                |                                                             |
| 167 | -                |                                         |     |                |                                                             |
| 168 | RTN              |                                         |     |                |                                                             |

| LABELS  |                                       |                         |             |             | FLAGS | SET STATUS                                                     |                                         |                                         |
|---------|---------------------------------------|-------------------------|-------------|-------------|-------|----------------------------------------------------------------|-----------------------------------------|-----------------------------------------|
| A START | B $a \rightarrow b \rightarrow f_a f$ | C $a \rightarrow f_a f$ | D           | E User f(x) | 0     | FLAGS                                                          | TRIG                                    | DISP                                    |
| a       | Σ terms (B)                           | Σ terms (C)             | d           | e           | 1     | ON OFF                                                         | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| 0       | 1                                     | 2                       | 3           | 4           | 2     | 0 <input type="checkbox"/> <input checked="" type="checkbox"/> | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
| 5       | 6                                     | 7                       | 8 Used in c | 9 Used in c | 3     | 2 <input type="checkbox"/> <input checked="" type="checkbox"/> | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|         |                                       |                         |             |             |       | 3 <input type="checkbox"/> <input checked="" type="checkbox"/> |                                         | n <u>2</u>                              |

## Differential Equations

|           |                                                  |           |                                      |    |             |    |        |    |        |
|-----------|--------------------------------------------------|-----------|--------------------------------------|----|-------------|----|--------|----|--------|
| 001 *LBLA | h/2                                              | 057 RCLC  |                                      |    |             |    |        |    |        |
| 002 2     |                                                  | 058 +     |                                      |    |             |    |        |    |        |
| 003 ÷     | -----                                            | 059 STOC  | $y_{i+1} = y_i + \Delta$             |    |             |    |        |    |        |
| 004 STOA  |                                                  | 060 RCLB  |                                      |    |             |    |        |    |        |
| 005 RTN   | F1 clear for 1 <sup>st</sup> -order.             | 061 RCLD  | $x_{i+1} = x_i + h$                  |    |             |    |        |    |        |
| 006 *LBLB | $y_0$                                            | 062 +     |                                      |    |             |    |        |    |        |
| 007 CF1   |                                                  | 063 STOB  |                                      |    |             |    |        |    |        |
| 008 STOC  | $x_0$                                            | 064 GSB8  |                                      |    |             |    |        |    |        |
| 009 R↓    | -----                                            | 065 RCLC  |                                      |    |             |    |        |    |        |
| 010 STOB  |                                                  | 066 GSB8  |                                      |    |             |    |        |    |        |
| 011 RTN   | F1 set for 2 <sup>nd</sup> -order.               | 067 F8?   | Loop again.                          |    |             |    |        |    |        |
| 012 *LBLC | $y_0$                                            | 068 SPC   | -----                                |    |             |    |        |    |        |
| 013 SF1   |                                                  | 069 GT09  | 2 <sup>nd</sup> -order solution.     |    |             |    |        |    |        |
| 014 STOD  | -----                                            | 070 *LBLD | $y_0'$                               |    |             |    |        |    |        |
| 015 RTN   | Compute solution.                                | 071 RCLD  | $y_0$                                |    |             |    |        |    |        |
| 016 *LBLD | Branch for 2 <sup>nd</sup> -order.               | 072 RCLC  | $x_0$                                |    |             |    |        |    |        |
| 017 F1?   | 1 <sup>st</sup> -order solution.                 | 073 RCLB  |                                      |    |             |    |        |    |        |
| 018 GTOD  |                                                  | 074 GSB8  | $k_1/2$                              |    |             |    |        |    |        |
| 019 *LBL9 |                                                  | 075 STOE  |                                      |    |             |    |        |    |        |
| 020 RCLC  |                                                  | 076 ST09  |                                      |    |             |    |        |    |        |
| 021 RCLB  | $k_1/2$                                          | 077 RCLA  |                                      |    |             |    |        |    |        |
| 022 GSB8  |                                                  | 078 GSB8  | $k_2/2$                              |    |             |    |        |    |        |
| 023 STOE  | $y_i + (k_1/2)$                                  | 079 ST00  |                                      |    |             |    |        |    |        |
| 024 RCLC  |                                                  | 080 ST09  |                                      |    |             |    |        |    |        |
| 025 +     |                                                  | 081 RCLA  |                                      |    |             |    |        |    |        |
| 026 RCLB  |                                                  | 082 RCL9  |                                      |    |             |    |        |    |        |
| 027 RCLA  | $x_i + (h/2)$                                    | 083 RCLD  |                                      |    |             |    |        |    |        |
| 028 +     |                                                  | 084 +     |                                      |    |             |    |        |    |        |
| 029 GSB8  | $k_2/2$                                          | 085 RCLC  | $k_3/2$                              |    |             |    |        |    |        |
| 030 STOB  |                                                  | 086 GSB8  |                                      |    |             |    |        |    |        |
| 031 RCLC  | $y_i + (k_2/2)$                                  | 087 ST+0  |                                      |    |             |    |        |    |        |
| 032 +     |                                                  | 088 ENT↑  |                                      |    |             |    |        |    |        |
| 033 RCLB  |                                                  | 089 +     |                                      |    |             |    |        |    |        |
| 034 RCLA  | $x_i + (h/2)$                                    | 090 ST09  |                                      |    |             |    |        |    |        |
| 035 +     | $k_3/2$                                          | 091 RCLA  |                                      |    |             |    |        |    |        |
| 036 GSB8  |                                                  | 092 ENT↑  |                                      |    |             |    |        |    |        |
| 037 ST+0  |                                                  | 093 +     |                                      |    |             |    |        |    |        |
| 038 ENT↑  |                                                  | 094 GSB8  | $k_4/2$                              |    |             |    |        |    |        |
| 039 +     |                                                  | 095 RCL0  |                                      |    |             |    |        |    |        |
| 040 RCLC  | $y_i + k_3$                                      | 096 ENT↑  |                                      |    |             |    |        |    |        |
| 041 +     |                                                  | 097 +     |                                      |    |             |    |        |    |        |
| 042 RCLB  |                                                  | 098 +     |                                      |    |             |    |        |    |        |
| 043 RCLA  |                                                  | 099 RCLC  |                                      |    |             |    |        |    |        |
| 044 ENT↑  | $h$                                              | 100 +     |                                      |    |             |    |        |    |        |
| 045 +     |                                                  | 101 3     |                                      |    |             |    |        |    |        |
| 046 STOD  | $x_i + h$                                        | 102 ÷     |                                      |    |             |    |        |    |        |
| 047 +     | $k_4/2$                                          | 103 RCLD  |                                      |    |             |    |        |    |        |
| 048 GSB8  |                                                  | 104 +     |                                      |    |             |    |        |    |        |
| 049 RCLC  |                                                  | 105 STOD  | $y_i' + (k_1 + 2k_2 + 2k_3 + k_4)/6$ |    |             |    |        |    |        |
| 050 +     |                                                  | 106 LSTX  |                                      |    |             |    |        |    |        |
| 051 RCL0  |                                                  | 107 RCL0  |                                      |    |             |    |        |    |        |
| 052 ENT↑  |                                                  | 108 RCLC  |                                      |    |             |    |        |    |        |
| 053 +     |                                                  | 109 +     |                                      |    |             |    |        |    |        |
| 054 +     |                                                  | 110 3     |                                      |    |             |    |        |    |        |
| 055 3     | $\Delta = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$ | 111 ÷     | $y_i' + (k_1 + k_2 + k_3)/6$         |    |             |    |        |    |        |
| 056 ÷     |                                                  | 112 +     |                                      |    |             |    |        |    |        |
| REGISTERS |                                                  |           |                                      |    |             |    |        |    |        |
| 0 Used    | 1                                                | 2         | 3                                    | 4  | 5           | 6  | 7      | 8  | 9 Used |
| S0        | S1                                               | S2        | S3                                   | S4 | S5          | S6 | S7     | S8 | S9     |
| A h/2     | B $x_i$                                          |           | C $y_i$                              |    | D $y_i', h$ |    | E Used |    | I      |



## Interpolations

|           |                                  |           |                                                     |
|-----------|----------------------------------|-----------|-----------------------------------------------------|
| 001 #LBLA | F1 set for linear.               | 057 -     |                                                     |
| 002 SF1   | $y_0$                            | 058 STOI  | $(x - x_1)(x - x_2)$                                |
| 003 STOI  | $x_0$                            | 059 x     | $L_0(x)$                                            |
| 004 XZY   |                                  | 060 RCL7  |                                                     |
| 005 STOA  |                                  | 061 x     |                                                     |
| 006 RTN   | -----                            | 062 RCL6  |                                                     |
| 007 #LBLB |                                  | 063 RCLA  |                                                     |
| 008 STOB  | $y_1$                            | 064 -     |                                                     |
| 009 XZY   | $x_1$                            | 065 STOD  | $(x - x_0)(x - x_2)$                                |
| 010 STOB  | -----                            | 066 RCL1  |                                                     |
| 011 RTN   |                                  | 067 x     |                                                     |
| 012 #LBLC | $y_2$                            | 068 RCL8  | $L_1(x)$                                            |
| 013 STOI  |                                  | 069 x     |                                                     |
| 014 XZY   | $x_2$                            | 070 +     |                                                     |
| 015 STOC  | F0 clear first time through.     | 071 RCLD  |                                                     |
| 016 CF0   | F1 clear for Lagrangian.         | 072 RCLC  | $(x - x_0)(x - x_1)$                                |
| 017 CF1   | -----                            | 073 x     |                                                     |
| 018 RTN   |                                  | 074 RCL9  | $L_2(x)$                                            |
| 019 #LBLD |                                  | 075 x     |                                                     |
| 020 STOE  |                                  | 076 +     |                                                     |
| 021 F1?   | If linear, GTO 1.                | 077 PRTX  | $P_2(x)$                                            |
| 022 GTO1  |                                  | 078 RTN   | -----                                               |
| 023 F0?   | If second time through,          | 079 #LBL1 | Linear interpolation.                               |
| 024 GTO0  | GTO 0.                           | 080 RCLB  |                                                     |
| 025 RCLA  | -----                            | 081 RCL6  |                                                     |
| 026 RCLB  | Lagrangian, first time           | 082 -     |                                                     |
| 027 -     | through.                         | 083 RCL7  |                                                     |
| 028 RCLA  |                                  | 084 x     |                                                     |
| 029 RCLC  |                                  | 085 RCL6  |                                                     |
| 030 -     |                                  | 086 RCLA  |                                                     |
| 031 x     |                                  | 087 -     |                                                     |
| 032 ST+7  | $y_0 / [(x_0 - x_1)(x_0 - x_2)]$ | 088 RCL8  |                                                     |
| 033 RCLB  |                                  | 089 x     |                                                     |
| 034 RCLA  |                                  | 090 +     |                                                     |
| 035 -     |                                  | 091 RCLB  | $y = \frac{(x_1 - x)y_1 + (x - x_0)y_0}{x_1 - x_0}$ |
| 036 RCLC  |                                  | 092 RCLA  |                                                     |
| 037 RCLC  |                                  | 093 -     |                                                     |
| 038 -     |                                  | 094 ÷     |                                                     |
| 039 x     |                                  | 095 PRTX  |                                                     |
| 040 ST+8  | $y_1 / [(x_1 - x_0)(x_1 - x_2)]$ | 096 RTN   |                                                     |
| 041 RCLC  |                                  | 097 #LBLA |                                                     |
| 042 RCLA  |                                  | 098 STOA  |                                                     |
| 043 -     |                                  | 099 RTN   |                                                     |
| 044 RCLC  |                                  | 100 #LBLB | Finite difference.                                  |
| 045 RCLB  |                                  | 101 STOB  | $x_3$                                               |
| 046 -     |                                  | 102 RTN   | -----                                               |
| 047 x     |                                  | 103 #LBLC | $h$                                                 |
| 048 ST+9  | $y_2 / [(x_2 - x_0)(x_2 - x_1)]$ | 104 STOI  | -----                                               |
| 049 SF0   |                                  | 105 R4    | $y_4$                                               |
| 050 #LBL0 | F0 set from now on.              | 106 STOE  |                                                     |
| 051 RCL6  | -----                            | 107 R4    | $y_3$                                               |
| 052 RCLB  | Lagrangian                       | 108 STOD  | $y_2$                                               |
| 053 -     |                                  | 109 R+    |                                                     |
| 054 STOE  |                                  | 110 -     | $-3y_3 + 3y_2$                                      |
| 055 RCL6  |                                  | 111 3     |                                                     |
| 056 RCLC  |                                  | 112 x     |                                                     |

## REGISTERS

|            |          |          |                             |                |                |    |      |      |      |
|------------|----------|----------|-----------------------------|----------------|----------------|----|------|------|------|
| 0          | 1        | 2        | 3                           | 4              | 5              | 6  | 7    | 8    | 9    |
|            |          |          |                             |                |                | x  | Used | Used | Used |
| S0         | S1       | S2       | S3                          | S4             | S5             | S6 | S7   | S8   | S9   |
| A          | B        | C        | D                           | E              | F              | G  | H    | I    | J    |
| $x_0, x_3$ | $x_1, h$ | $x_2, u$ | $x - x_0, y_2, \text{Used}$ | $x - x_1, y_3$ | $x - x_2, y_4$ |    |      |      |      |

|     |      |                                             |  |  |
|-----|------|---------------------------------------------|--|--|
| 113 | X←Y  | $-3y_3 + 3y_2 - y_1$                        |  |  |
| 114 | -    | $y_4$                                       |  |  |
| 115 | R↑   | $\delta^3 y_{-1/2}$                         |  |  |
| 116 | +    |                                             |  |  |
| 117 | STO9 |                                             |  |  |
| 118 | RCL5 |                                             |  |  |
| 119 | RCLD |                                             |  |  |
| 120 | -    | $\delta y_{-1/2}$                           |  |  |
| 121 | STO7 |                                             |  |  |
| 122 | RCL1 |                                             |  |  |
| 123 | RCLD |                                             |  |  |
| 124 | +    |                                             |  |  |
| 125 | RCL5 |                                             |  |  |
| 126 | 2    |                                             |  |  |
| 127 | X    |                                             |  |  |
| 128 | -    | $\delta^2 y_0$                              |  |  |
| 129 | STO8 |                                             |  |  |
| 130 | RTN  |                                             |  |  |
| 131 | LBLJ | -----<br>Compute y given x.                 |  |  |
| 132 | RCLA |                                             |  |  |
| 133 | -    |                                             |  |  |
| 134 | RCLB |                                             |  |  |
| 135 | ÷    | $u = (x - x_3)/h$                           |  |  |
| 136 | STOC |                                             |  |  |
| 137 | RCL7 |                                             |  |  |
| 138 | X    |                                             |  |  |
| 139 | RCL5 |                                             |  |  |
| 140 | +    | $y_3 + u \delta y_{-1/2}$                   |  |  |
| 141 | RCLC |                                             |  |  |
| 142 | RCLC |                                             |  |  |
| 143 | 1    |                                             |  |  |
| 144 | +    |                                             |  |  |
| 145 | X    |                                             |  |  |
| 146 | STOD | $u(u+1)$                                    |  |  |
| 147 | 2    |                                             |  |  |
| 148 | ÷    |                                             |  |  |
| 149 | RCL8 |                                             |  |  |
| 150 | X    | $\frac{1}{2} u(u+1) \delta^2 y_0$           |  |  |
| 151 | +    |                                             |  |  |
| 152 | RCLD |                                             |  |  |
| 153 | RCLC |                                             |  |  |
| 154 | 1    |                                             |  |  |
| 155 | -    |                                             |  |  |
| 156 | X    |                                             |  |  |
| 157 | 6    |                                             |  |  |
| 158 | ÷    |                                             |  |  |
| 159 | RCL9 |                                             |  |  |
| 160 | X    | $\frac{1}{6} u(u+1)(u-1) \delta^3 y_{-1/2}$ |  |  |
| 161 | +    |                                             |  |  |
| 162 | PRTX | y                                           |  |  |
| 163 | RTN  | -----                                       |  |  |

| LABELS                 |                      |                                                |                     |   | FLAGS                  | SET STATUS                                                            |                                         |                                         |
|------------------------|----------------------|------------------------------------------------|---------------------|---|------------------------|-----------------------------------------------------------------------|-----------------------------------------|-----------------------------------------|
| A $x_0 \uparrow y_0$   | B $x_1 \uparrow y_1$ | C $x_2 \uparrow y_2$                           | D $x \rightarrow y$ | E | 0 2 <sup>nd</sup> time | FLAGS                                                                 | TRIG                                    | DISP                                    |
| a $x_3$                | b h                  | c $y_1 \uparrow y_2 \uparrow y_3 \uparrow y_4$ | d $x \rightarrow y$ | e | 1 Linear               | 0 <input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| 0 2 <sup>nd</sup> time | 1 Linear             | 2                                              | 3                   | 4 | 2                      | 1 <input type="checkbox"/> <input checked="" type="checkbox"/>        | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
| 5                      | 6                    | 7                                              | 8                   | 9 | 3                      | 2 <input type="checkbox"/> <input checked="" type="checkbox"/>        | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|                        |                      |                                                |                     |   |                        | 3 <input type="checkbox"/> <input checked="" type="checkbox"/>        |                                         | n <u>2</u>                              |

## Coordinate Transformations

|                |                                                               |                |                                                                                  |
|----------------|---------------------------------------------------------------|----------------|----------------------------------------------------------------------------------|
| 001 *LBLA      | Store $\theta$ .                                              | 057 ST01       |                                                                                  |
| 002 ST03       |                                                               | 058 CSB1       |                                                                                  |
| 003 CLX        |                                                               | 059 ST04       |                                                                                  |
| 004 CSB0       | Store 0, $y_0$ , $x_0$ .                                      | 060 1          | Calculate $y$ or $y'$ coefficient.                                               |
| 005 RCL9       | Set rotation axis of 0, 0, 1 in display and recall $\theta$ . | 061 2          |                                                                                  |
| 006 RCL9       | -----                                                         | 062 ST01       |                                                                                  |
| 007 1          |                                                               | 063 CSB1       | Stop if 2 -D.                                                                    |
| 008 RCL3       |                                                               | 064 ST05       |                                                                                  |
| 009 *LBL6      | Store $\theta$ , a, b, and c.                                 | 065 F2?        | Calculate $z$ or $z'$ coefficient.                                               |
| 010 CF0        |                                                               | 066 RTN        |                                                                                  |
| 011 ST03       |                                                               | 067 1          |                                                                                  |
| 012 R+         |                                                               | 068 3          |                                                                                  |
| 013 ST00       |                                                               | 069 ST01       |                                                                                  |
| 014 R+         |                                                               | 070 CSB1       |                                                                                  |
| 015 ST01       |                                                               | 071 ST06       |                                                                                  |
| 016 R+         |                                                               | 072 0          | Stop and display final 3 -D results.                                             |
| 017 ST02       | Calculate $\sqrt{a^2 + b^2 + c^2}$                            | 073 RCL4       |                                                                                  |
| 018 +P         |                                                               | 074 RCL5       |                                                                                  |
| 019 X $\neq$ Y |                                                               | 075 RCL6       |                                                                                  |
| 020 R4         |                                                               | 076 RTN        |                                                                                  |
| 021 +P         |                                                               | 077 *LBLC      | Set matrix done flag. (Program pointer says matrix done.)                        |
| 022 ST=0       | Calculate unit vector components.                             | 078 SF0        |                                                                                  |
| 023 ST=1       |                                                               | 079 GT0C       | Set matrix done flag.                                                            |
| 024 ST=2       | -----                                                         | 080 *LBLC      |                                                                                  |
| 025 RTN        | Store $z_0$ , $y_0$ , $x_0$ .                                 | 081 SF0        | Matrix multiply x and y.                                                         |
| 026 *LBLA      |                                                               | 082 GT0C       |                                                                                  |
| 027 CF0        |                                                               | 083 *LBL1      |                                                                                  |
| 028 ST09       |                                                               | 084 0          |                                                                                  |
| 029 R4         |                                                               | 085 RCL4       |                                                                                  |
| 030 ST08       |                                                               | 086 CSB4       |                                                                                  |
| 031 R4         |                                                               | 087 RCLB       |                                                                                  |
| 032 ST07       |                                                               | 088 CSB4       |                                                                                  |
| 033 RTN        |                                                               | 089 RCLC       |                                                                                  |
| 034 *LBLC      | Set 2 -D flag and input dummy zero.                           | 090 P=S        | Fetch $x_0$ , $y_0$ or $z_0$ depending on which coefficient is being calculated. |
| 035 SF2        | -----                                                         | 091 RCL1       |                                                                                  |
| 036 0          | Store $(x - x_0)$ , $(y - y_0)$ , $(z - z_0)$ .               | 092 P=S        |                                                                                  |
| 037 *LBLC      |                                                               | 093 ST0D       | Matrix multiply z.                                                               |
| 038 RCL9       |                                                               | 094 R4         | If calculating P from P' add appropriate translation distance.                   |
| 039 -          |                                                               | 095 CSB4       |                                                                                  |
| 040 ST0C       |                                                               | 096 RCLD       |                                                                                  |
| 041 CLX        |                                                               | 097 X $\neq$ Y |                                                                                  |
| 042 RCL8       |                                                               | 098 F1?        |                                                                                  |
| 043 -          |                                                               | 099 +          |                                                                                  |
| 044 ST0B       |                                                               | 100 PRTX       | Output result, set stack for 2 -D stop.                                          |
| 045 CLX        |                                                               | 101 RCL4       |                                                                                  |
| 046 RCL7       |                                                               | 102 X $\neq$ Y |                                                                                  |
| 047 -          |                                                               | 103 RTN        | Matrix multiply and counter.                                                     |
| 048 ST0A       |                                                               | 104 *LBL4      |                                                                                  |
| 049 F0?        |                                                               | 105 RCL1       |                                                                                  |
| 050 GT00       | Calculate matrix coefficients if not already done.            | 106 x          |                                                                                  |
| 051 CSB5       | -----                                                         | 107 +          |                                                                                  |
| 052 *LBL0      | Calculate $x$ or $x'$ coefficient.                            | 108 ISZ1       |                                                                                  |
| 053 CF0        |                                                               | 109 ISZ1       |                                                                                  |
| 054 SPC        |                                                               | 110 ISZ1       |                                                                                  |
| 055 1          |                                                               | 111 RTN        |                                                                                  |
| 056 1          |                                                               | 112 *LBL5      |                                                                                  |

| REGISTERS |             |             |                                  |                 |           |           |          |          |          |
|-----------|-------------|-------------|----------------------------------|-----------------|-----------|-----------|----------|----------|----------|
| 0 a       | 1 b         | 2 c         | 3 $\theta$                       | 4 $x(x')$       | 5 $y(y')$ | 6 $z(z')$ | 7 $x_0$  | 8 $y_0$  | 9 $z_0$  |
| S0        | S1 $\ell_1$ | S2 $\ell_2$ | S3 $\ell_3$                      | S4 $m_1$        | S5 $m_2$  | S6 $m_3$  | S7 $n_1$ | S8 $n_2$ | S9 $n_3$ |
| A $x(x')$ | B $y(y')$   | C $z(z')$   | D $\cos \theta, (x_0, y_0, z_0)$ | E $\sin \theta$ | I control |           |          |          |          |

|     |                       |                                                                 |     |                       |                                                       |
|-----|-----------------------|-----------------------------------------------------------------|-----|-----------------------|-------------------------------------------------------|
| 113 | RCL3                  | Calculate $\sin \theta$ , $\cos \theta$ and $1 - \cos \theta$ . | 169 | ST-2                  | $c \sin \theta$                                       |
| 114 | 1                     |                                                                 | 170 | ST+4                  |                                                       |
| 115 | +R                    |                                                                 | 171 | CLX                   |                                                       |
| 116 | STOD                  |                                                                 | 172 | LSTX                  | $b \sin \theta$                                       |
| 117 | CHS                   |                                                                 | 173 | x                     |                                                       |
| 118 | X $\leftrightarrow$ Y |                                                                 | 174 | ST+3                  |                                                       |
| 119 | STOE                  |                                                                 | 175 | ST-7                  |                                                       |
| 120 | CLX                   |                                                                 | 176 | CLX                   |                                                       |
| 121 | 1                     |                                                                 | 177 | LSTX                  |                                                       |
| 122 | +                     |                                                                 | 178 | x                     | $a \sin \theta$                                       |
| 123 | RCL0                  | Recall unit vector components.                                  | 179 | ST+8                  |                                                       |
| 124 | RCL1                  |                                                                 | 180 | ST-6                  |                                                       |
| 125 | RCL2                  |                                                                 | 181 | P $\leftrightarrow$ S | Matrix complete.                                      |
| 126 | R $\downarrow$        | $1 - \cos \theta$                                               | 182 | RTN                   | -----                                                 |
| 127 | P $\leftrightarrow$ S | Store $1 - \cos \theta$ in $R_{S1} - R_{99}$ .                  | 183 | *LBL E                | Set 2D flag and input dummy zero.                     |
| 128 | STO1                  |                                                                 | 184 | SF2                   | -----                                                 |
| 129 | STO2                  |                                                                 | 185 | 0                     | Input x, y, z.                                        |
| 130 | STO3                  |                                                                 | 186 | *LBL E                |                                                       |
| 131 | STO4                  |                                                                 | 187 | STOC                  |                                                       |
| 132 | STO5                  |                                                                 | 188 | R $\downarrow$        |                                                       |
| 133 | STO6                  |                                                                 | 189 | STOB                  |                                                       |
| 134 | STO7                  |                                                                 | 190 | R $\downarrow$        |                                                       |
| 135 | STO8                  |                                                                 | 191 | STOA                  |                                                       |
| 136 | STO9                  |                                                                 | 192 | SF1                   |                                                       |
| 137 | R $\downarrow$        | Multiply by c unit vector component.                            | 193 | F0?                   | Set inverse flag.                                     |
| 138 | ST $\times$ 7         |                                                                 | 194 | GT06                  | Calculate matrix coefficients if not previously done. |
| 139 | ST $\times$ 8         |                                                                 | 195 | GSB5                  | -----                                                 |
| 140 | ST $\times$ 9         |                                                                 | 196 | *LBL 6                | P' $\rightarrow$ P                                    |
| 141 | ST $\times$ 9         |                                                                 | 197 | GSB0                  | -----                                                 |
| 142 | ST $\times$ 6         |                                                                 | 198 | CF1                   |                                                       |
| 143 | ST $\times$ 3         |                                                                 | 199 | RTN                   |                                                       |
| 144 | R $\downarrow$        | Multiply by b unit vector component.                            | 200 | *LBL E                | Set matrix done flag.                                 |
| 145 | ST $\times$ 5         |                                                                 | 201 | SF0                   | -----                                                 |
| 146 | ST $\times$ 5         |                                                                 | 202 | GT0a                  | Set matrix done flag.                                 |
| 147 | ST $\times$ 2         |                                                                 | 203 | *LBL E                | -----                                                 |
| 148 | ST $\times$ 8         |                                                                 | 204 | SF0                   |                                                       |
| 149 | ST $\times$ 4         |                                                                 | 205 | GT0E                  | -----                                                 |
| 150 | ST $\times$ 6         |                                                                 |     |                       |                                                       |
| 151 | R $\downarrow$        | Multiply by a unit vector component.                            |     |                       |                                                       |
| 152 | ST $\times$ 2         |                                                                 |     |                       |                                                       |
| 153 | ST $\times$ 1         |                                                                 |     |                       |                                                       |
| 154 | ST $\times$ 1         |                                                                 |     |                       |                                                       |
| 155 | ST $\times$ 4         |                                                                 |     |                       |                                                       |
| 156 | ST $\times$ 7         |                                                                 |     |                       |                                                       |
| 157 | ST $\times$ 3         |                                                                 |     |                       |                                                       |
| 158 | R $\downarrow$        | c $\rightarrow$ X                                               |     |                       |                                                       |
| 159 | R $\downarrow$        |                                                                 |     |                       |                                                       |
| 160 | RCLD                  | Add $\cos \theta$ .                                             |     |                       |                                                       |
| 161 | ST+1                  |                                                                 |     |                       |                                                       |
| 162 | ST+5                  |                                                                 |     |                       |                                                       |
| 163 | ST+9                  |                                                                 |     |                       |                                                       |
| 164 | CLX                   |                                                                 |     |                       |                                                       |
| 165 | RCL E                 | $\sin \theta$                                                   |     |                       |                                                       |
| 166 | F1?                   | CHS for P' $\rightarrow$ P.                                     |     |                       |                                                       |
| 167 | CHS                   |                                                                 |     |                       |                                                       |
| 168 | x                     |                                                                 |     |                       |                                                       |

| LABELS                                     |                   |                                    |   |                                     | FLAGS                | SET STATUS                                                            |                                         |                                         |
|--------------------------------------------|-------------------|------------------------------------|---|-------------------------------------|----------------------|-----------------------------------------------------------------------|-----------------------------------------|-----------------------------------------|
| A $x_0 \rightarrow y_0 \rightarrow \theta$ | B                 | C $x \rightarrow y \rightarrow P'$ | D | E $x' \rightarrow y' \rightarrow P$ | 0 Matrix done        | FLAGS                                                                 | TRIG                                    | DISP                                    |
| a $x_0 \rightarrow y_0 \rightarrow z_0$    | b atbtct $\theta$ | c xtytz $\rightarrow P'$           | d | e x'ty'tz' $\rightarrow P$          | 1 P' $\rightarrow$ P | 0 <input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| 0 Used                                     | 1 Mult.           | 2                                  | 3 | 4 Mult.                             | 2 2D                 | 1 <input type="checkbox"/> <input checked="" type="checkbox"/>        | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
| 5 Matrix                                   | 6                 | 7                                  | 8 | 9                                   | 3                    | 2 <input type="checkbox"/> <input checked="" type="checkbox"/>        | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|                                            |                   |                                    |   |                                     |                      | 3 <input type="checkbox"/> <input checked="" type="checkbox"/>        |                                         | n <u>2</u>                              |

## Intersections

|                   |                                          |                 |    |                 |    |                   |        |                 |                                                        |
|-------------------|------------------------------------------|-----------------|----|-----------------|----|-------------------|--------|-----------------|--------------------------------------------------------|
| 001 #LBLA         | Input $P_1, P_1'$ .                      | 057 RCL6        |    |                 |    |                   |        |                 |                                                        |
| 002 CSB5          | -----                                    | 058 RCL7        |    |                 |    |                   |        |                 |                                                        |
| 003 #LBLA         | Input $P_1$ and $\theta_1$ .             | 059 LSTX        |    |                 |    |                   |        |                 |                                                        |
| 004 1             |                                          | 060 RTN         |    |                 |    |                   |        |                 | Set flag for alternate point solution.                 |
| 005 STOE          |                                          | 061 #LBLB       |    |                 |    |                   |        |                 | -----                                                  |
| 006 R↓            |                                          | 062 SF1         |    |                 |    |                   |        |                 | Start primary point solution.                          |
| 007 STOD          |                                          | 063 GT00        |    |                 |    |                   |        |                 | -----                                                  |
| 008 R↓            |                                          | 064 #LBLD       |    |                 |    |                   |        |                 | Select type of solution.                               |
| 009 STOC          |                                          | 065 CF1         |    |                 |    |                   |        |                 | -----                                                  |
| 010 R↓            |                                          | 066 #LBL0       |    |                 |    |                   |        |                 | Line-line intersect.                                   |
| 011 STOB          |                                          | 067 CF2         |    |                 |    |                   |        |                 | -----                                                  |
| 012 RTN           | Input $P_2, P_2'$ .                      | 068 SPC         |    |                 |    |                   |        |                 |                                                        |
| 013 #LBLB         | -----                                    | 069 GT01        |    |                 |    |                   |        |                 |                                                        |
| 014 CSB5          | Input $P_2$ and $\theta_2$ .             | 070 #LBL2       |    |                 |    |                   |        |                 |                                                        |
| 015 #LBLB         |                                          | 071 RCLC        |    |                 |    |                   |        |                 |                                                        |
| 016 2             |                                          | 072 1           |    |                 |    |                   |        |                 | Check for input error.                                 |
| 017 STOI          |                                          | 073 X≠Y?        |    |                 |    |                   |        |                 |                                                        |
| 018 R↓            |                                          | 074 GT06        |    |                 |    |                   |        |                 |                                                        |
| 019 STOA          |                                          | 075 RCLD        |    |                 |    |                   |        |                 | If $\theta_1 = \pm 90^\circ$ go to special solution 8. |
| 020 R↓            |                                          | 076 COS         |    |                 |    |                   |        |                 |                                                        |
| 021 STOD          |                                          | 077 X=0?        |    |                 |    |                   |        |                 |                                                        |
| 022 R↓            |                                          | 078 GT08        |    |                 |    |                   |        |                 |                                                        |
| 023 STOB          |                                          | 079 RCLA        |    |                 |    |                   |        |                 |                                                        |
| 024 RTN           |                                          | 080 COS         |    |                 |    |                   |        |                 | If $\theta_2 = \pm 90^\circ$ go to special solution 7. |
| 025 #LBLD         | Input $x_{01}, y_{01}, r_1$ .            | 081 X=0?        |    |                 |    |                   |        |                 |                                                        |
| 026 3             |                                          | 082 GT07        |    |                 |    |                   |        |                 |                                                        |
| 027 STOI          |                                          | 083 RCLB        |    |                 |    |                   |        |                 |                                                        |
| 028 R↓            |                                          | 084 RCLD        |    |                 |    |                   |        |                 |                                                        |
| 029 STOA          |                                          | 085 TAN         |    |                 |    |                   |        |                 | Calculate $x_p$ .                                      |
| 030 R↓            |                                          | 086 STOG        |    |                 |    |                   |        |                 |                                                        |
| 031 STOD          |                                          | 087 X           |    |                 |    |                   |        |                 |                                                        |
| 032 R↓            |                                          | 088 RCL8        |    |                 |    |                   |        |                 |                                                        |
| 033 STOB          |                                          | 089 RCLA        |    |                 |    |                   |        |                 |                                                        |
| 034 RTN           |                                          | 090 TAN         |    |                 |    |                   |        |                 |                                                        |
| 035 #LBLB         |                                          | 091 ST07        |    |                 |    |                   |        |                 |                                                        |
| 036 4             | Input $x_{02}, y_{02}, r_2$ .            | 092 X           |    |                 |    |                   |        |                 |                                                        |
| 037 STOE          |                                          | 093 -           |    |                 |    |                   |        |                 |                                                        |
| 038 R↓            |                                          | 094 RCL9        |    |                 |    |                   |        |                 |                                                        |
| 039 STOD          |                                          | 095 +           |    |                 |    |                   |        |                 |                                                        |
| 040 R↓            |                                          | 096 RCLC        |    |                 |    |                   |        |                 |                                                        |
| 041 STOC          |                                          | 097 -           |    |                 |    |                   |        |                 |                                                        |
| 042 R↓            |                                          | 098 RCL6        |    |                 |    |                   |        |                 |                                                        |
| 043 STOB          |                                          | 099 RCL7        |    |                 |    |                   |        |                 |                                                        |
| 044 RTN           |                                          | 100 -           |    |                 |    |                   |        |                 |                                                        |
| 045 #LBL5         |                                          | 101 +           |    |                 |    |                   |        |                 |                                                        |
| 046 ST07          |                                          | 102 #LBL9       |    |                 |    |                   |        |                 |                                                        |
| 047 X≠Y           | Transform P and P' to P - $\theta$ form. | 103 ENT1        |    |                 |    |                   |        |                 |                                                        |
| 048 ST06          |                                          | 104 ENT1        |    |                 |    |                   |        |                 |                                                        |
| 049 R↓            |                                          | 105 PRYX        |    |                 |    |                   |        |                 | Calculate $y_p$ .                                      |
| 050 -             |                                          | 106 RCLB        |    |                 |    |                   |        |                 |                                                        |
| 051 X≠Y           |                                          | 107 -           |    |                 |    |                   |        |                 |                                                        |
| 052 RT            |                                          | 108 RCL6        |    |                 |    |                   |        |                 |                                                        |
| 053 -             |                                          | 109 X           |    |                 |    |                   |        |                 |                                                        |
| 054 +P            |                                          | 110 RCLC        |    |                 |    |                   |        |                 |                                                        |
| 055 R↓            |                                          | 111 +           |    |                 |    |                   |        |                 |                                                        |
| 056 +             |                                          | 112 PRYX        |    |                 |    |                   |        |                 |                                                        |
| REGISTERS         |                                          |                 |    |                 |    |                   |        |                 |                                                        |
| 0                 | 1                                        | 2               | 3  | 4               | 5  | 6 Used            | 7 Used | 8 $x_2, x_{c1}$ | 9 $y_2, y_{c1}$                                        |
| S0                | S1                                       | S2              | S3 | S4              | S5 | S6                | S7     | S8              | S9                                                     |
| A $\theta_2, r_1$ |                                          | B $x_1, x_{c2}$ |    | C $y_1, y_{c2}$ |    | D $\theta_1, r_2$ |        | E Code          |                                                        |
|                   |                                          |                 |    |                 |    |                   |        |                 |                                                        |

|                                             |                                                      |                           |                                    |  |
|---------------------------------------------|------------------------------------------------------|---------------------------|------------------------------------|--|
| 113 RTN                                     | Calculate $x_p$ for $\theta_1 = \pm 90^\circ$ .      | 169 +                     |                                    |  |
| 114 #LBL7                                   |                                                      | 170 PRTX                  |                                    |  |
| 115 RCLD                                    |                                                      | 171 RTN                   |                                    |  |
| 116 TAN                                     |                                                      | 172 #LBL4                 | Calculate $x_p$ for circle-circle. |  |
| 117 STO6                                    |                                                      | 173 GSB0                  |                                    |  |
| 118 RCL8                                    |                                                      | 174 +                     |                                    |  |
| 119 CT09                                    | Calculate $x_p, y_p$ for $\theta_2 = \pm 90^\circ$ . | 175 RCLD                  |                                    |  |
| 120 #LBL8                                   |                                                      | 176 X $\leftrightarrow$   |                                    |  |
| 121 RCL8                                    |                                                      | 177 -                     |                                    |  |
| 122 RCL8                                    |                                                      | 178 2                     |                                    |  |
| 123 PRTX                                    |                                                      | 179 $\div$                |                                    |  |
| 124 RCL8                                    |                                                      | 180 RCL6                  |                                    |  |
| 125 -                                       |                                                      | 181 $\div$                |                                    |  |
| 126 RCLA                                    |                                                      | 182 RCLA                  |                                    |  |
| 127 TAN                                     |                                                      | 183 $\div$                |                                    |  |
| 128 X                                       |                                                      | 184 COS $\leftrightarrow$ |                                    |  |
| 129 RCL9                                    |                                                      | 185 F1?                   |                                    |  |
| 130 +                                       |                                                      | 186 CHS                   |                                    |  |
| 131 PRTX                                    |                                                      | 187 +                     |                                    |  |
| 132 RTN                                     |                                                      | 188 RCLA                  |                                    |  |
| 133 #LBL3                                   | Circle-line or circle-circle.                        | 189 $\rightarrow R$       |                                    |  |
| 134 RCL6                                    |                                                      | 190 RCL8                  | Calculate $y_p$ for circle-circle. |  |
| 135 X $\leftrightarrow$ I                   |                                                      | 191 +                     |                                    |  |
| 136 CT01                                    |                                                      | 192 PRTX                  |                                    |  |
| 137 #LBL1                                   |                                                      | 193 X $\leftrightarrow$ Y |                                    |  |
| 138 GSB0                                    | Calculate $x_p$ for circle-line.                     | 194 RCL9                  |                                    |  |
| 139 -                                       |                                                      | 195 +                     |                                    |  |
| 140 STO7                                    |                                                      | 196 PRTX                  |                                    |  |
| 141 X $\leftrightarrow$ Y                   |                                                      | 197 RTN                   |                                    |  |
| 142 RCLD                                    |                                                      | 198 #LBL0                 | Calculate $\alpha$ and D.          |  |
| 143 -                                       |                                                      | 199 X $\leftrightarrow$ I |                                    |  |
| 144 COS                                     |                                                      | 200 RCLC                  |                                    |  |
| 145 RCL6                                    |                                                      | 201 RCL9                  |                                    |  |
| 146 X                                       |                                                      | 202 -                     |                                    |  |
| 147 CHS                                     |                                                      | 203 RCLB                  |                                    |  |
| 148 X $\leftrightarrow$ 0 $\leftrightarrow$ |                                                      | 204 RCL8                  |                                    |  |
| 149 SF2                                     |                                                      | 205 -                     |                                    |  |
| 150 ENT $\uparrow$                          |                                                      | 206 $\rightarrow P$       |                                    |  |
| 151 X $\leftrightarrow$                     |                                                      | 207 STO6                  |                                    |  |
| 152 RCL7                                    |                                                      | 208 X $\leftrightarrow$   | Calculate $D^2$ and $r^2$ .        |  |
| 153 -                                       |                                                      | 209 RCLA                  |                                    |  |
| 154 IX                                      |                                                      | 210 X $\leftrightarrow$   |                                    |  |
| 155 F2?                                     |                                                      | 211 RTN                   |                                    |  |
| 156 CHS                                     |                                                      |                           |                                    |  |
| 157 +                                       |                                                      |                           |                                    |  |
| 158 $\div$                                  |                                                      |                           |                                    |  |
| 159 F1?                                     | Calculate $P_2$ .                                    |                           |                                    |  |
| 160 LSTX                                    | Calculate $P_1$ .                                    |                           |                                    |  |
| 161 RCLD                                    | Bring $P_2$ back if flag is set.                     |                           |                                    |  |
| 162 X $\leftrightarrow$ Y                   | Calculate $x_p$ .                                    |                           |                                    |  |
| 163 $\rightarrow R$                         |                                                      |                           |                                    |  |
| 164 RCLB                                    |                                                      |                           |                                    |  |
| 165 +                                       |                                                      |                           |                                    |  |
| 166 PRTX                                    |                                                      |                           |                                    |  |
| 167 X $\leftrightarrow$ Y                   |                                                      |                           |                                    |  |
| 168 RCLC                                    | Calculate $y_p$ .                                    |                           |                                    |  |

| LABELS                                       |                                              |              |                                         |                                         | FLAGS      |                                                     | SET STATUS                    |                              |  |
|----------------------------------------------|----------------------------------------------|--------------|-----------------------------------------|-----------------------------------------|------------|-----------------------------------------------------|-------------------------------|------------------------------|--|
| A $x_1 \uparrow y_1 \uparrow \theta_1$       | B $x_2 \uparrow y_2 \uparrow \theta_2$       | C            | D $x_{01} \uparrow y_{01} \uparrow r_1$ | E $x_{02} \uparrow y_{02} \uparrow r_2$ | 0          | ON OFF                                              |                               |                              |  |
| a $x_1 \uparrow y_1 \uparrow x' \uparrow y'$ | b $x_2 \uparrow y_2 \uparrow x' \uparrow y'$ | c            | d $\rightarrow x_{01}, y_{01}$          | e $\rightarrow x_{02}, y_{02}$          | 1 Prim/sec | <input type="checkbox"/> <input type="checkbox"/>   | DEG <input type="checkbox"/>  | FIX <input type="checkbox"/> |  |
| 0 Used                                       | 1 $x_p$                                      | 2 Line-line  | 3 Used                                  | 4 $x_p$                                 | 2 Quad     | 1 <input type="checkbox"/> <input type="checkbox"/> | GRAD <input type="checkbox"/> | SCI <input type="checkbox"/> |  |
| 5 Line spec.                                 | 6                                            | 7 Vert. line | 8 Vert. line                            | 9 $y_p$                                 | 3          | 2 <input type="checkbox"/> <input type="checkbox"/> | RAD <input type="checkbox"/>  | ENG <input type="checkbox"/> |  |
|                                              |                                              |              |                                         |                                         |            | 3 <input type="checkbox"/> <input type="checkbox"/> |                               | n <u>2</u>                   |  |

## Circles

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| 001 #LBL a | Store $x_1, y_1$ . | 057 x |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | </ |
|------------|--------------------|-------|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|----|

$$y_0 = \frac{k_2 - k_1}{N_2 - N_1}$$

$$x_0 = k_2 - N_2 y_0$$

Output x.

Output y.

Output r and stop with x in Z, y in Y and r in X.

$$P_1$$

$$\swarrow \searrow$$

$$P_3 \rightarrow P_2$$

|           |                                                                   |  |           |                                               |
|-----------|-------------------------------------------------------------------|--|-----------|-----------------------------------------------|
| 113 CLX   |                                                                   |  | 169 PRTX  |                                               |
| 114 RCL6  |                                                                   |  | 170 DSZ1  |                                               |
| 115 ST04  |                                                                   |  | 171 *LBL0 |                                               |
| 116 R4    |                                                                   |  | 172 RCLB  |                                               |
| 117 ST06  |                                                                   |  | 173 RCL1  |                                               |
| 118 R4    |                                                                   |  | 174 -     |                                               |
| 119 ST05  |                                                                   |  | 175 PRTX  |                                               |
| 120 CT01  |                                                                   |  | 176 ST01  |                                               |
| 121 *LBLA | Input $x_0, y_0$ , and $r$ .                                      |  | 177 GSB2  |                                               |
| 122 ST0E  |                                                                   |  | 178 R↑    |                                               |
| 123 R4    |                                                                   |  | 179 CLX   |                                               |
| 124 ST0D  |                                                                   |  | 180 RCL9  |                                               |
| 125 R4    |                                                                   |  | 181 R↑    |                                               |
| 126 ST0C  |                                                                   |  | 182 CLX   |                                               |
| 127 RTN   |                                                                   |  | 183 RCL1  |                                               |
| 128 *LBLC | Calculate $n$ from $\Delta\theta$ .                               |  | 184 R↑    |                                               |
| 129 I     |                                                                   |  | 185 RT↑   |                                               |
| 130 CMS   |                                                                   |  | 186 RTN   |                                               |
| 131 COS↑  |                                                                   |  | 187 *LBLB | Set to begin automatic loop.                  |
| 132 ENT↑  |                                                                   |  | 188 RCLB  |                                               |
| 133 +     |                                                                   |  | 189 ST01  |                                               |
| 134 X↔Y   |                                                                   |  | 190 RCL2  |                                               |
| 135 ÷     |                                                                   |  | 191 RCLA  |                                               |
| 136 LSTX  |                                                                   |  | 192 -     |                                               |
| 137 CT00  |                                                                   |  | 193 ST09  |                                               |
| 138 *LBLD |                                                                   |  | 194 *LBL4 | Automatic loop.                               |
| 139 I     | Calculate $\Delta\theta$ from $n$ .                               |  | 195 GSB2  |                                               |
| 140 CMS   |                                                                   |  | 196 ISZ1  |                                               |
| 141 COS↑  |                                                                   |  | 197 *LBL0 |                                               |
| 142 ENT↑  |                                                                   |  | 198 DSZ1  |                                               |
| 143 +     |                                                                   |  | 199 CT04  |                                               |
| 144 X↔Y   |                                                                   |  | 200 RTN   |                                               |
| 145 ÷     |                                                                   |  | 201 *LBLB | Calculate $x$ and $y$ for given $\theta$ .    |
| 146 LSTX  |                                                                   |  | 202 SPC   |                                               |
| 147 X↔Y   |                                                                   |  | 203 CT00  |                                               |
| 148 *LBL0 |                                                                   |  | 204 *LBL2 |                                               |
| 149 ST0A  | Store $\Delta\theta, n, \theta_0$ and $\theta_0 - \Delta\theta$ . |  | 205 RCL9  | Calculate $x$ and $y$ for contents of $R_9$ . |
| 150 R4    |                                                                   |  | 206 *LBL0 |                                               |
| 151 ABS   |                                                                   |  | 207 RCLC  |                                               |
| 152 .     |                                                                   |  | 208 →R    |                                               |
| 153 S     |                                                                   |  | 209 RCLC  |                                               |
| 154 +     |                                                                   |  | 210 +     |                                               |
| 155 INT   |                                                                   |  | 211 PRTX  |                                               |
| 156 ST0E  |                                                                   |  | 212 X↔Y   |                                               |
| 157 ST01  |                                                                   |  | 213 RCLD  |                                               |
| 158 R4    |                                                                   |  | 214 +     |                                               |
| 159 ST02  |                                                                   |  | 215 PRTX  |                                               |
| 160 ST09  |                                                                   |  | 216 RTN   |                                               |
| 161 RCLA  |                                                                   |  |           |                                               |
| 162 ST-9  |                                                                   |  |           |                                               |
| 163 RTN   |                                                                   |  |           |                                               |
| 164 *LBLB | Calculate and output $\theta, i, x_i$ and $y_i$ .                 |  |           |                                               |
| 165 SPC   |                                                                   |  |           |                                               |
| 166 RCLA  |                                                                   |  |           |                                               |
| 167 ST+9  |                                                                   |  |           |                                               |
| 168 RCL9  |                                                                   |  |           |                                               |

| LABELS                          |                             |                                    |                             |                                 | FLAGS | SET STATUS                 |                                         |                                         |
|---------------------------------|-----------------------------|------------------------------------|-----------------------------|---------------------------------|-------|----------------------------|-----------------------------------------|-----------------------------------------|
| A $x_0 \uparrow y_0 \uparrow r$ | B $\theta \rightarrow x, y$ | C $\theta_0 \uparrow \Delta\theta$ | D $\theta_0 \uparrow n$     | E $\rightarrow \theta, i, x, y$ | 0     | FLAGS                      |                                         |                                         |
|                                 |                             |                                    |                             |                                 |       | ON OFF                     |                                         |                                         |
| A $x_1 \uparrow y_1$            | B $x_2 \uparrow y_2$        | C $x_3 \uparrow y_3$               | D $\rightarrow x_0, y_0, r$ | E AUTO                          | 1     | 0 <input type="checkbox"/> | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| 0 Used                          | 1 $\rightarrow x_0, y_0, r$ | 2 $\rightarrow x_i, y_i$           | 3                           | 4 Loop                          | 2     | 1 <input type="checkbox"/> | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
| 5                               | 6                           | 7                                  | 8                           | 9                               | 3     | 2 <input type="checkbox"/> | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|                                 |                             |                                    |                             |                                 |       | 3 <input type="checkbox"/> |                                         | n <u>2</u>                              |

## Spherical Triangles

|                  |                                    |                   |                                                         |                  |    |                  |    |                   |    |
|------------------|------------------------------------|-------------------|---------------------------------------------------------|------------------|----|------------------|----|-------------------|----|
| 001 #LBLE        | ASA (AcB)                          | 057 F0?           |                                                         |                  |    |                  |    |                   |    |
| 002 SF1          | -----                              | 058 SPC           |                                                         |                  |    |                  |    |                   |    |
| 003 #LBLE        | SAS (aCb)                          | 059 RTN           |                                                         |                  |    |                  |    |                   |    |
| 004 STOB         | Second S (or A)                    | 060 #LBLE         | -----                                                   |                  |    |                  |    |                   |    |
| 005 R4           | Angle (or S)                       | 061 RCLA          | Subroutine to find one                                  |                  |    |                  |    |                   |    |
| 006 STOC         | First S (or A)                     | 062 RCLB          | angle (side).                                           |                  |    |                  |    |                   |    |
| 007 R4           |                                    | 063 STOA          |                                                         |                  |    |                  |    |                   |    |
| 008 STOA         |                                    | 064 RCLC          | Rotate sides (angles)                                   |                  |    |                  |    |                   |    |
| 009 RCLC         |                                    | 065 STOB          |                                                         |                  |    |                  |    |                   |    |
| 010 COS          |                                    | 066 R4            |                                                         |                  |    |                  |    |                   |    |
| 011 RCLA         | $\cos c = \cos a \cos b + \sin a$  | 067 R4            |                                                         |                  |    |                  |    |                   |    |
| 012 SIN          | $\sin b \cos c$                    | 068 STOC          | $\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$ |                  |    |                  |    |                   |    |
| 013 x            |                                    | 069 COS           |                                                         |                  |    |                  |    |                   |    |
| 014 RCLB         |                                    | 070 RCLA          |                                                         |                  |    |                  |    |                   |    |
| 015 SIN          |                                    | 071 COS           |                                                         |                  |    |                  |    |                   |    |
| 016 x            |                                    | 072 RCLB          |                                                         |                  |    |                  |    |                   |    |
| 017 RCLA         | $\cos C = -\cos A \cos B + \sin A$ | 073 COS           | $\cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C}$ |                  |    |                  |    |                   |    |
| 018 COS          | $\sin B \cos c$                    | 074 x             |                                                         |                  |    |                  |    |                   |    |
| 019 RCLB         |                                    | 075 F1?           |                                                         |                  |    |                  |    |                   |    |
| 020 COS          |                                    | 076 CHS           |                                                         |                  |    |                  |    |                   |    |
| 021 x            |                                    | 077 -             |                                                         |                  |    |                  |    |                   |    |
| 022 F1?          |                                    | 078 RCLA          |                                                         |                  |    |                  |    |                   |    |
| 023 CHS          |                                    | 079 SIN           |                                                         |                  |    |                  |    |                   |    |
| 024 +            |                                    | 080 ÷             |                                                         |                  |    |                  |    |                   |    |
| 025 COS-         |                                    | 081 RCLB          |                                                         |                  |    |                  |    |                   |    |
| 026 STOC         | Third S (or A)                     | 082 SIN           |                                                         |                  |    |                  |    |                   |    |
| 027 GTOB         | -----                              | 083 ÷             |                                                         |                  |    |                  |    |                   |    |
| 028 #LBLE        | AAA (ABC)                          | 084 COS-          |                                                         |                  |    |                  |    |                   |    |
| 029 SF1          | -----                              | 085 RTN           |                                                         |                  |    |                  |    |                   |    |
| 030 #LBLE        | SSS (abc)                          | 086 #LBLE         | -----                                                   |                  |    |                  |    |                   |    |
| 031 STOC         |                                    | 087 F0?           | AUTO output.                                            |                  |    |                  |    |                   |    |
| 032 R4           |                                    | 088 PRX           | Print if F0 set.                                        |                  |    |                  |    |                   |    |
| 033 STOB         | Store three sides (or angles).     | 089 F0?           |                                                         |                  |    |                  |    |                   |    |
| 034 R4           |                                    | 090 RTN           |                                                         |                  |    |                  |    |                   |    |
| 035 STOA         |                                    | 091 R/S           |                                                         |                  |    |                  |    |                   |    |
| 036 #LBLE        |                                    | 092 RTN           | Else halt.                                              |                  |    |                  |    |                   |    |
| 037 CSBb         | LBL b finds one angle (or          | 093 #LBLE         | -----                                                   |                  |    |                  |    |                   |    |
| 038 STOE         | side)                              | 094 SF1           | AAS (A,B,a)                                             |                  |    |                  |    |                   |    |
| 039 CSBb         |                                    | 095 #LBLC         | Ambiguous cases.                                        |                  |    |                  |    |                   |    |
| 040 STOI         |                                    | 096 STOE          | -----                                                   |                  |    |                  |    |                   |    |
| 041 CSBb         |                                    | 097 R4            | SSA (a, b, A)                                           |                  |    |                  |    |                   |    |
| 042 STOD         |                                    | 098 STOB          | Angle (or side)                                         |                  |    |                  |    |                   |    |
| 043 CF1          |                                    | 099 SIN           |                                                         |                  |    |                  |    |                   |    |
| 044 #LBLE        | -----                              | 100 X=Y           | Second side (or angle)                                  |                  |    |                  |    |                   |    |
| 045 RCLA         | Output routine.                    | 101 STOA          |                                                         |                  |    |                  |    |                   |    |
| 046 CSBb         | First side (or angle)              | 102 SIN           | First side (or angle)                                   |                  |    |                  |    |                   |    |
| 047 RCLD         | First angle (or side)              | 103 ÷             | $\sin B = \frac{\sin b \sin A}{\sin a}$                 |                  |    |                  |    |                   |    |
| 048 CSBb         |                                    | 104 RCLC          |                                                         |                  |    |                  |    |                   |    |
| 049 RCLB         | Second side (or angle)             | 105 SIN           | $\sin b = \frac{\sin B \sin a}{\sin A}$                 |                  |    |                  |    |                   |    |
| 050 CSBb         |                                    | 106 x             |                                                         |                  |    |                  |    |                   |    |
| 051 RCLE         | Second angle (or side)             | 107 SIN-          |                                                         |                  |    |                  |    |                   |    |
| 052 CSBb         |                                    | 108 CSBb          |                                                         |                  |    |                  |    |                   |    |
| 053 RCLC         | Third side (or angle)              | 109 RCLA          | Find one solution.                                      |                  |    |                  |    |                   |    |
| 054 CSBb         |                                    | 110 RCLB          | If a < b (A < B), have 2                                |                  |    |                  |    |                   |    |
| 055 RCLI         | Third angle (or side)              | 111 X/Y?          | solutions.                                              |                  |    |                  |    |                   |    |
| 056 CSBb         |                                    | 112 GTOB          |                                                         |                  |    |                  |    |                   |    |
| REGISTERS        |                                    |                   |                                                         |                  |    |                  |    |                   |    |
| 0                | 1                                  | 2                 | 3                                                       | 4                | 5  | 6                | 7  | 8                 | 9  |
| S0               | S1                                 | S2                | S3                                                      | S4               | S5 | S6               | S7 | S8                | S9 |
| A First S (or A) |                                    | B Second S (or A) |                                                         | C Third S (or A) |    | D First A (or S) |    | E Second A (or S) |    |
|                  |                                    |                   |                                                         |                  |    |                  |    | F Third A (or S)  |    |

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                    |  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|--|
| <pre> 113 CLX 114 RTN 115 *LBL0 116 RCL1 117 COS 118 CHS 119 COS+ 120 *LBLd 121 STO1 122 RCL2 123 + 124 2 125 ÷ 126 ENT↑ 127 SIN 128 X↔Y 129 RCL1 130 - 131 SIN 132 ÷ 133 RCL4 134 RCLB 135 - 136 2 137 ÷ 138 TAN 139 x 140 FI? 141 1/X 142 TAN+ 143 ENT↑ 144 + 145 STOC 146 COS 147 RCL4 148 COS 149 RCLB 150 COS 151 x 152 FI? 153 CHS 154 - 155 RCL4 156 SIN 157 ÷ 158 RCLB 159 SIN 160 ÷ 161 COS+ 162 STOD 163 GSB9 164 CLX 165 RTN 166 *LBLe 167 F0? 168 CT00 </pre> | <p>Else end.</p> <p>-----</p> <p>For 2<sup>nd</sup> solution, B ← cos<sup>-1</sup><br/>(-cos B)</p> <p>-----</p> <p>Routine finds one solution<br/>given 2 angles and 2 sides.</p> $\tan \frac{C}{2} = \frac{\sin\left(\frac{A+B}{2}\right) \tan\left(\frac{a-b}{2}\right)}{\sin\left(\frac{A-B}{2}\right)}$ $\cot \frac{C}{2} = \frac{\sin\left(\frac{a+b}{2}\right) \tan\left(\frac{A-B}{2}\right)}{\sin\left(\frac{a-b}{2}\right)}$<br>$\cos C = \frac{\cos c - \cos a \cos b}{\sin a \sin b}$<br>$\cos c = \frac{\cos C + \cos A \cos B}{\sin A \sin B}$<br><p>-----</p> <p>AUTO toggle.</p> | <pre> 169 SF0 170 1 171 RTN 172 *LBL0 173 CF0 174 0 175 RTN </pre> |  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|--|

| LABELS |       |       |            | FLAGS    |          | SET STATUS                 |                                       |                                         |                                         |
|--------|-------|-------|------------|----------|----------|----------------------------|---------------------------------------|-----------------------------------------|-----------------------------------------|
| A SSS  | B SAS | C SSA | D AAA      | E ASA    | 0 Auto   | FLAGS                      |                                       | TRIG                                    | DISP                                    |
| a AAS  | b     | c     | d          | e Auto?  | 1 Angles | ON OFF                     |                                       | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| 0 Used | 1     | 2     | 3          |          | 2        | 0 <input type="checkbox"/> | 1 <input checked="" type="checkbox"/> | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
| 5      | 6     | 7     | 8 Auto out | 9 Output | 3        | 2 <input type="checkbox"/> | 3 <input checked="" type="checkbox"/> | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|        |       |       |            |          |          | 3 <input type="checkbox"/> | 4 <input checked="" type="checkbox"/> |                                         | n <u>2</u>                              |

## Gamma Function

|           |                   |                   |                   |                   |                                                                                                                           |                   |                   |                   |      |
|-----------|-------------------|-------------------|-------------------|-------------------|---------------------------------------------------------------------------------------------------------------------------|-------------------|-------------------|-------------------|------|
| 001       | #LBLA             | Load constants.   | 057               | 7                 | b <sub>5</sub>                                                                                                            |                   |                   |                   |      |
| 002       | P=S               |                   | 058               | 8                 |                                                                                                                           |                   |                   |                   |      |
| 003       | .                 |                   | 059               | CHS               |                                                                                                                           |                   |                   |                   |      |
| 004       | 5                 |                   | 060               | STOS              |                                                                                                                           |                   |                   |                   |      |
| 005       | 7                 |                   | 061               | .                 |                                                                                                                           |                   |                   |                   |      |
| 006       | 7                 |                   | 062               | 4                 |                                                                                                                           |                   |                   |                   |      |
| 007       | 1                 |                   | 063               | 8                 |                                                                                                                           |                   |                   |                   |      |
| 008       | 9                 |                   | 064               | 2                 |                                                                                                                           |                   |                   |                   |      |
| 009       | 1                 |                   | 065               | 1                 |                                                                                                                           |                   |                   |                   |      |
| 010       | 6                 |                   | 066               | 9                 |                                                                                                                           |                   |                   |                   |      |
| 011       | 5                 | b <sub>1</sub>    | 067               | 9                 | b <sub>6</sub>                                                                                                            |                   |                   |                   |      |
| 012       | 2                 |                   | 068               | 3                 |                                                                                                                           |                   |                   |                   |      |
| 013       | CHS               |                   | 069               | 9                 |                                                                                                                           |                   |                   |                   |      |
| 014       | STO1              |                   | 070               | 4                 |                                                                                                                           |                   |                   |                   |      |
| 015       | .                 |                   | 071               | STO6              |                                                                                                                           |                   |                   |                   |      |
| 016       | 9                 |                   | 072               | .                 |                                                                                                                           |                   |                   |                   |      |
| 017       | 8                 |                   | 073               | 1                 |                                                                                                                           |                   |                   |                   |      |
| 018       | 8                 |                   | 074               | 9                 |                                                                                                                           |                   |                   |                   |      |
| 019       | 2                 |                   | 075               | 3                 |                                                                                                                           |                   |                   |                   |      |
| 020       | 0                 |                   | 076               | 5                 |                                                                                                                           |                   |                   |                   |      |
| 021       | 5                 | b <sub>2</sub>    | 077               | 2                 | b <sub>7</sub>                                                                                                            |                   |                   |                   |      |
| 022       | 8                 |                   | 078               | 7                 |                                                                                                                           |                   |                   |                   |      |
| 023       | 9                 |                   | 079               | 8                 |                                                                                                                           |                   |                   |                   |      |
| 024       | 1                 |                   | 080               | 1                 |                                                                                                                           |                   |                   |                   |      |
| 025       | STO2              |                   | 081               | 8                 |                                                                                                                           |                   |                   |                   |      |
| 026       | .                 |                   | 082               | CHS               |                                                                                                                           |                   |                   |                   |      |
| 027       | 8                 |                   | 083               | STO7              |                                                                                                                           |                   |                   |                   |      |
| 028       | 9                 |                   | 084               | .                 |                                                                                                                           |                   |                   |                   |      |
| 029       | 7                 |                   | 085               | 0                 |                                                                                                                           |                   |                   |                   |      |
| 030       | 0                 |                   | 086               | 3                 |                                                                                                                           |                   |                   |                   |      |
| 031       | 5                 | b <sub>3</sub>    | 087               | 5                 | b <sub>8</sub>                                                                                                            |                   |                   |                   |      |
| 032       | 6                 |                   | 088               | 8                 |                                                                                                                           |                   |                   |                   |      |
| 033       | 9                 |                   | 089               | 6                 |                                                                                                                           |                   |                   |                   |      |
| 034       | 3                 |                   | 090               | 8                 |                                                                                                                           |                   |                   |                   |      |
| 035       | 7                 |                   | 091               | 3                 |                                                                                                                           |                   |                   |                   |      |
| 036       | CHS               |                   | 092               | 4                 |                                                                                                                           |                   |                   |                   |      |
| 037       | STO3              |                   | 093               | 3                 |                                                                                                                           |                   |                   |                   |      |
| 038       | .                 |                   | 094               | STO8              |                                                                                                                           |                   |                   |                   |      |
| 039       | 9                 |                   | 095               | CLX               |                                                                                                                           |                   |                   |                   |      |
| 040       | 1                 |                   | 096               | P=S               |                                                                                                                           |                   |                   |                   |      |
| 041       | 8                 | b <sub>4</sub>    | 097               | RTN               | -----<br>x → Γ(x)<br><br>(x - 1)<br><br>Error if (x - 1) < 0.<br><br>If (x - 1) integer, GTO b.<br><br><br>Exit when < 1. |                   |                   |                   |      |
| 042       | 2                 |                   | 098               | #LBLB             |                                                                                                                           |                   |                   |                   |      |
| 043       | 0                 |                   | 099               | P=S               |                                                                                                                           |                   |                   |                   |      |
| 044       | 6                 |                   | 100               | 1                 |                                                                                                                           |                   |                   |                   |      |
| 045       | 8                 |                   | 101               | -                 |                                                                                                                           |                   |                   |                   |      |
| 046       | 5                 |                   | 102               | X<0?              |                                                                                                                           |                   |                   |                   |      |
| 047       | 7                 |                   | 103               | GTOE              |                                                                                                                           |                   |                   |                   |      |
| 048       | STO4              |                   | 104               | INT               |                                                                                                                           |                   |                   |                   |      |
| 049       | .                 |                   | 105               | LSTX              |                                                                                                                           |                   |                   |                   |      |
| 050       | 7                 |                   | 106               | X=Y?              |                                                                                                                           |                   |                   |                   |      |
| 051       | 5                 |                   | 107               | GTO6              |                                                                                                                           |                   |                   |                   |      |
| 052       | 6                 |                   | 108               | 1                 |                                                                                                                           |                   |                   |                   |      |
| 053       | 7                 |                   | 109               | STO9              |                                                                                                                           |                   |                   |                   |      |
| 054       | 0                 |                   | 110               | X<Y               |                                                                                                                           |                   |                   |                   |      |
| 055       | 4                 |                   | 111               | #LBL9             |                                                                                                                           |                   |                   |                   |      |
| 056       | 0                 |                   | 112               | X<Y?              |                                                                                                                           |                   |                   |                   |      |
| REGISTERS |                   |                   |                   |                   |                                                                                                                           |                   |                   |                   |      |
| 0         | 1                 | 2                 | 3                 | 4                 | 5                                                                                                                         | 6                 | 7                 | 8                 | 9    |
| S0        | S1 b <sub>1</sub> | S2 b <sub>2</sub> | S3 b <sub>3</sub> | S4 b <sub>4</sub> | S5 b <sub>5</sub>                                                                                                         | S6 b <sub>6</sub> | S7 b <sub>7</sub> | S8 b <sub>8</sub> | S9 Π |
| A         | B                 | C                 | D                 | E                 | F                                                                                                                         | G                 | H                 | I                 | J    |

|     |       |  |  |  |  |  |  |  |  |
|-----|-------|--|--|--|--|--|--|--|--|
| 113 | GT00  |  |  |  |  |  |  |  |  |
| 114 | STX9  |  |  |  |  |  |  |  |  |
| 115 | 1     |  |  |  |  |  |  |  |  |
| 116 | -     |  |  |  |  |  |  |  |  |
| 117 | GT09  |  |  |  |  |  |  |  |  |
| 118 | *LBL0 |  |  |  |  |  |  |  |  |
| 119 | ENT†  |  |  |  |  |  |  |  |  |
| 120 | ENT†  |  |  |  |  |  |  |  |  |
| 121 | ENT†  |  |  |  |  |  |  |  |  |
| 122 | RCL8  |  |  |  |  |  |  |  |  |
| 123 | x     |  |  |  |  |  |  |  |  |
| 124 | RCL7  |  |  |  |  |  |  |  |  |
| 125 | +     |  |  |  |  |  |  |  |  |
| 126 | x     |  |  |  |  |  |  |  |  |
| 127 | RCL6  |  |  |  |  |  |  |  |  |
| 128 | +     |  |  |  |  |  |  |  |  |
| 129 | x     |  |  |  |  |  |  |  |  |
| 130 | RCL5  |  |  |  |  |  |  |  |  |
| 131 | +     |  |  |  |  |  |  |  |  |
| 132 | x     |  |  |  |  |  |  |  |  |
| 133 | RCL4  |  |  |  |  |  |  |  |  |
| 134 | +     |  |  |  |  |  |  |  |  |
| 135 | x     |  |  |  |  |  |  |  |  |
| 136 | RCL3  |  |  |  |  |  |  |  |  |
| 137 | +     |  |  |  |  |  |  |  |  |
| 138 | x     |  |  |  |  |  |  |  |  |
| 139 | RCL2  |  |  |  |  |  |  |  |  |
| 140 | +     |  |  |  |  |  |  |  |  |
| 141 | x     |  |  |  |  |  |  |  |  |
| 142 | RCL1  |  |  |  |  |  |  |  |  |
| 143 | +     |  |  |  |  |  |  |  |  |
| 144 | x     |  |  |  |  |  |  |  |  |
| 145 | 1     |  |  |  |  |  |  |  |  |
| 146 | +     |  |  |  |  |  |  |  |  |
| 147 | RCL9  |  |  |  |  |  |  |  |  |
| 148 | x     |  |  |  |  |  |  |  |  |
| 149 | PRTX  |  |  |  |  |  |  |  |  |
| 150 | P=5   |  |  |  |  |  |  |  |  |
| 151 | RTN   |  |  |  |  |  |  |  |  |
| 152 | *LBL6 |  |  |  |  |  |  |  |  |
| 153 | N!    |  |  |  |  |  |  |  |  |
| 154 | PRTX  |  |  |  |  |  |  |  |  |
| 155 | P=5   |  |  |  |  |  |  |  |  |
| 156 | RTN   |  |  |  |  |  |  |  |  |

| LABELS    |                             |   |   |           | FLAGS | SET STATUS |                                     |                                         |                                         |
|-----------|-----------------------------|---|---|-----------|-------|------------|-------------------------------------|-----------------------------------------|-----------------------------------------|
| A START   | B $x \rightarrow \Gamma(x)$ | C | D | E         | 0     | FLAGS      |                                     | TRIG                                    | DISP                                    |
| 1         | Integers                    |   |   |           | 1     | ON         | OFF                                 | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| 0 Approx. |                             | 2 | 3 | 4         | 2     | 1          | <input checked="" type="checkbox"/> | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
| 5         |                             | 7 | 8 | 9 II loop | 3     | 2          | <input checked="" type="checkbox"/> | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|           |                             |   |   |           |       | 3          | <input checked="" type="checkbox"/> |                                         | n <u>2</u>                              |

## Bessel Functions, Error Function

|                       |                                                   |                       |                                               |
|-----------------------|---------------------------------------------------|-----------------------|-----------------------------------------------|
| 001 *LBLA             | Bessel n                                          | 057 ST09              |                                               |
| 002 ST0A              |                                                   | 058 EEX               |                                               |
| 003 RTN               |                                                   | 059 CHS               |                                               |
| 004 *LBLB             | $J_n(x)$                                          | 060 9                 |                                               |
| 005 CSB <sub>A</sub>  | Initialize                                        | 061 ST0D              |                                               |
| 006 SFB               | F0 set for $J_n$ .                                | 062 RTN               |                                               |
| 007 *LBL9             |                                                   | 063 *LBL <sub>B</sub> |                                               |
| 008 CSB <sub>B</sub>  | Main summing loop.                                | 064 DSZ1              |                                               |
| 009 CF2               | Compute term (even k).                            | 065 SF2               | Compute one term.<br>F2 set except for k = 0. |
| 010 ST+9              |                                                   | 066 RCL1              |                                               |
| 011 CSB <sub>B</sub>  | Accumulate even terms.                            | 067 RCL <sub>A</sub>  |                                               |
| 012 F2?               | Compute term (odd k).                             | 068 X <sub>Y</sub> ?  |                                               |
| 013 GT09              | F2 clear for last term.                           | 069 GT0B              |                                               |
| 014 RCLC              | Loop again.                                       | 070 RCLD              |                                               |
| 015 RCL9              |                                                   | 071 ST0C              |                                               |
| 016 ENT↑              | $R_C = T_n, R_E = T_0$ at end.                    | 072 *LBL0             |                                               |
| 017 +                 |                                                   | 073 R↓                |                                               |
| 018 RCLE              | $J_n(x) = \frac{T_n}{-T_0 + 2 \sum_0^k T_k}$      | 074 RCLE              |                                               |
| 019 -                 |                                                   | 075 F0?               | $T_{k+1}$                                     |
| 020 ÷                 |                                                   | 076 CHS               | CHS for $J_n(-T_{k+1})$                       |
| 021 PRTX              |                                                   | 077 X <sub>Z</sub> Y  |                                               |
| 022 SPC               |                                                   | 078 RCLB              |                                               |
| 023 RTN               |                                                   | 079 x                 | $\frac{2k}{x}$                                |
| 024 *LBL <sub>A</sub> | Initialization for Bessel<br>( $J_n$ and $I_n$ ). | 080 RCLD              | $T_{k+1} + T_k$                               |
| 025 1                 |                                                   | 081 ST0E              |                                               |
| 026 -                 |                                                   | 082 x                 | $T_k \leftarrow \frac{2k}{x} T_k \pm T_{k+1}$ |
| 027 5                 |                                                   | 083 +                 |                                               |
| 028 x                 |                                                   | 084 ST0D              |                                               |
| 029 ST0C              | $R_C + 1.5x$                                      | 085 RTN               |                                               |
| 030 RCL <sub>A</sub>  | n                                                 | 086 *LBLC             |                                               |
| 031 X <sub>Z</sub> Y? |                                                   | 087 RCLE              | Compute $J_0(x), J_1(x)$                      |
| 032 X <sub>Z</sub> Y  |                                                   | 088 RCL9              |                                               |
| 033 6                 | max (n, 1.5x)                                     | 089 ENT↑              |                                               |
| 034 +                 |                                                   | 090 +                 |                                               |
| 035 RCLC              |                                                   | 091 RCLE              |                                               |
| 036 9                 |                                                   | 092 -                 |                                               |
| 037 x                 |                                                   | 093 ÷                 | $J_0(x) = \frac{T_0}{-T_0 + 2 \sum_0^k T_k}$  |
| 038 RCLC              |                                                   | 094 R/S               |                                               |
| 039 2                 |                                                   | 095 RCLD              |                                               |
| 040 +                 |                                                   | 096 CHS               |                                               |
| 041 ÷                 | Compute m.                                        | 097 RCL9              |                                               |
| 042 +                 |                                                   | 098 ENT↑              |                                               |
| 043 2                 |                                                   | 099 +                 |                                               |
| 044 ÷                 |                                                   | 100 RCLE              |                                               |
| 045 INT               |                                                   | 101 -                 |                                               |
| 046 ENT↑              |                                                   | 102 ÷                 |                                               |
| 047 +                 |                                                   | 103 RTN               |                                               |
| 048 2                 |                                                   | 104 *LBLD             |                                               |
| 049 +                 |                                                   | 105 CF0               |                                               |
| 050 ST01              |                                                   | 106 CSB <sub>A</sub>  | $I_n(x)$                                      |
| 051 3                 |                                                   | 107 *LBL8             | F0 clear for $I_n(x)$ .                       |
| 052 RCLC              | $I \leftarrow m + 2$                              | 108 ST+9              | Initialize.                                   |
| 053 ÷                 |                                                   | 109 CSB <sub>B</sub>  |                                               |
| 054 ST0B              |                                                   | 110 F2?               | Accumulate each term.                         |
| 055 0                 |                                                   | 111 GT0B              | Compute next term.                            |
| 056 ST0E              | $R_B + 2/x$                                       | 112 RCLC              | F2 clear for last term.                       |

| REGISTERS     |    |        |    |                 |    |                                    |    |               |                |
|---------------|----|--------|----|-----------------|----|------------------------------------|----|---------------|----------------|
| 0             | 1  | 2      | 3  | 4               | 5  | 6                                  | 7  | 8             | 9 $\Sigma T_k$ |
| S0            | S1 | S2     | S3 | S4              | S5 | S6                                 | S7 | S8            | S9             |
| A n; erf term |    | B 2/x; |    | C 1.5x, $T_n$ . |    | D $T_k; (e^{x^2} \sqrt{\pi})^{-1}$ |    | E $T_{k+1}$ ; |                |
|               |    |        |    |                 |    |                                    |    | I k; places   |                |

|     |                |  |  |     |       |  |  |                                                        |
|-----|----------------|--|--|-----|-------|--|--|--------------------------------------------------------|
| 113 | RCL9           |  |  | 169 | 2     |  |  | erf(x)                                                 |
| 114 | ENT?           |  |  | 170 | x     |  |  | erfc = 1 - erf                                         |
| 115 | +              |  |  | 171 | 1     |  |  |                                                        |
| 116 | RCL6           |  |  | 172 | XZY   |  |  |                                                        |
| 117 | -              |  |  | 173 | -     |  |  |                                                        |
| 118 | ÷              |  |  | 174 | LSTX  |  |  |                                                        |
| 119 | 2              |  |  | 175 | GT02  |  |  |                                                        |
| 120 | RCLB           |  |  | 176 | *LBL3 |  |  | Find erfc, x > 3                                       |
| 121 | ÷              |  |  | 177 | RCLB  |  |  | 1/(2x²) → R <sub>B</sub>                               |
| 122 | e <sup>x</sup> |  |  | 178 | 1/X   |  |  |                                                        |
| 123 | x              |  |  | 179 | ST0B  |  |  | 1/x → R <sub>A</sub>                                   |
| 124 | PRTX           |  |  | 180 | RCLA  |  |  |                                                        |
| 125 | SPC            |  |  | 181 | 1/X   |  |  |                                                        |
| 126 | RTN            |  |  | 182 | ST0A  |  |  |                                                        |
| 127 | *LBL6          |  |  | 183 | *LBL6 |  |  | Loop for erfc                                          |
| 128 | ST0A           |  |  | 184 | RCLB  |  |  | (Stack contains Σ <sub>n-1</sub> )                     |
| 129 | X²             |  |  | 185 | RCLC  |  |  |                                                        |
| 130 | ST0D           |  |  | 186 | 2     |  |  |                                                        |
| 131 | 2              |  |  | 187 | -     |  |  |                                                        |
| 132 | x              |  |  | 188 | ST0C  |  |  | δ <sub>n</sub> ← -δ <sub>n-1</sub> $\frac{2n-1}{2x^2}$ |
| 133 | ST0B           |  |  | 189 | x     |  |  |                                                        |
| 134 | 1              |  |  | 190 | RCLA  |  |  | Σ <sub>n</sub> ← Σ <sub>n-1</sub> + δ <sub>n</sub>     |
| 135 | ST0C           |  |  | 191 | x     |  |  | RND (Σ <sub>n-1</sub> )                                |
| 136 | RCLD           |  |  | 192 | ST0A  |  |  | RND (Σ <sub>n</sub> )                                  |
| 137 | e <sup>x</sup> |  |  | 193 | +     |  |  | If last 2 sums are equal to N places, exit.            |
| 138 | Pi             |  |  | 194 | XZY   |  |  |                                                        |
| 139 | JX             |  |  | 195 | RND   |  |  |                                                        |
| 140 | x              |  |  | 196 | XZY   |  |  |                                                        |
| 141 | ST0D           |  |  | 197 | RND   |  |  |                                                        |
| 142 | 3              |  |  | 198 | X=Y?  |  |  |                                                        |
| 143 | RCLA           |  |  | 199 | GT00  |  |  |                                                        |
| 144 | X?Y?           |  |  | 200 | LSTX  |  |  | Σ <sub>n</sub>                                         |
| 145 | GT03           |  |  | 201 | GT06  |  |  |                                                        |
| 146 | *LBL7          |  |  | 202 | *LBL8 |  |  |                                                        |
| 147 | RCLB           |  |  | 203 | LSTX  |  |  |                                                        |
| 148 | RCLC           |  |  | 204 | RCLD  |  |  |                                                        |
| 149 | 2              |  |  | 205 | ÷     |  |  | erfc(x)                                                |
| 150 | +              |  |  | 206 | 1     |  |  | erf(x)                                                 |
| 151 | ST0C           |  |  | 207 | XZY   |  |  |                                                        |
| 152 | ÷              |  |  | 208 | -     |  |  |                                                        |
| 153 | RCLA           |  |  | 209 | LSTX  |  |  |                                                        |
| 154 | x              |  |  | 210 | XZY   |  |  |                                                        |
| 155 | ST0A           |  |  | 211 | *LBL2 |  |  |                                                        |
| 156 | +              |  |  | 212 | PRTX  |  |  | erf(x)                                                 |
| 157 | XZY            |  |  | 213 | XZY   |  |  | erfc(x)                                                |
| 158 | RND            |  |  | 214 | PRTX  |  |  |                                                        |
| 159 | XZY            |  |  | 215 | XZY   |  |  |                                                        |
| 160 | RND            |  |  | 216 | SPC   |  |  |                                                        |
| 161 | X=Y?           |  |  | 217 | RTN   |  |  |                                                        |
| 162 | GT00           |  |  | 218 | *LBL6 |  |  |                                                        |
| 163 | LSTX           |  |  | 219 | ST01  |  |  | Set DSP to places of accuracy desired for erf (1-9).   |
| 164 | GT07           |  |  | 220 | DSPi  |  |  |                                                        |
| 165 | *LBL8          |  |  | 221 | RTN   |  |  |                                                        |
| 166 | LSTX           |  |  |     |       |  |  |                                                        |
| 167 | RCLD           |  |  |     |       |  |  |                                                        |
| 168 | ÷              |  |  |     |       |  |  |                                                        |

| LABELS       |                          |                                   |                          | FLAGS                 |                  | SET STATUS                                                                   |      |     |
|--------------|--------------------------|-----------------------------------|--------------------------|-----------------------|------------------|------------------------------------------------------------------------------|------|-----|
| A n          | B x → J <sub>n</sub> (x) | C J <sub>0</sub> ; J <sub>1</sub> | D x → I <sub>n</sub> (x) | E x → erf, erfc       | F J <sub>n</sub> | FLAGS                                                                        |      |     |
| 1 Bes. init. | 1 One term               | 2                                 | 3                        | 4 Accuracy            | 5                | ON OFF                                                                       | DEG  | FIX |
| 0 Used       | 2 Print erf              | 3 x > 3 (erfc)                    | 4                        | 5 k = 0 (Bessel)      | 6                | 1 <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> | GRAD | SCI |
| 5            | 6 erfc loop              | 7 erf loop                        | 8 I <sub>n</sub> loop    | 9 J <sub>n</sub> loop | 10               | 2 <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> | RAD  | ENG |
|              |                          |                                   |                          |                       |                  | 3 <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> |      | n 2 |

Hyperbolics

|           |                               |           |                                                 |    |    |    |    |           |         |
|-----------|-------------------------------|-----------|-------------------------------------------------|----|----|----|----|-----------|---------|
| 001 #LBLA | Arc                           | 057 R4    |                                                 |    |    |    |    |           |         |
| 002 SF2   | Set F2 for inverse.           | 058 P2S   |                                                 |    |    |    |    |           |         |
| 003 RTN   | -----                         | 059 RTN   | -----                                           |    |    |    |    |           |         |
| 004 #LBLB | Sinh                          | 060 #LBLD | Tanh                                            |    |    |    |    |           |         |
| 005 P2S   |                               | 061 P2S   |                                                 |    |    |    |    |           |         |
| 006 R↑    |                               | 062 R↑    |                                                 |    |    |    |    |           |         |
| 007 ST00  | Save t                        | 063 ST00  | Save t                                          |    |    |    |    |           |         |
| 008 R4    |                               | 064 R↑    |                                                 |    |    |    |    |           |         |
| 009 F2?   | If inverse, GTO 0.            | 065 ST08  | Save z                                          |    |    |    |    |           |         |
| 010 GT00  |                               | 066 R4    |                                                 |    |    |    |    |           |         |
| 011 e*    |                               | 067 R4    |                                                 |    |    |    |    |           |         |
| 012 ENT↑  |                               | 068 F2?   |                                                 |    |    |    |    |           |         |
| 013 1/X   | Compute sinh x.               | 069 GT04  | If inverse, GTO 4.                              |    |    |    |    |           |         |
| 014 -     |                               | 070 e*    |                                                 |    |    |    |    |           |         |
| 015 2     |                               | 071 ENT↑  |                                                 |    |    |    |    |           |         |
| 016 ÷     |                               | 072 1/X   |                                                 |    |    |    |    |           |         |
| 017 GT01  | Exit.                         | 073 -     |                                                 |    |    |    |    |           |         |
| 018 #LBL0 | -----                         | 074 ST09  | Compute tanh x.                                 |    |    |    |    |           |         |
| 019 ST09  |                               | 075 LSTX  |                                                 |    |    |    |    |           |         |
| 020 X²    | Compute sinh <sup>-1</sup> x. | 076 ENT↑  |                                                 |    |    |    |    |           |         |
| 021 1     |                               | 077 +     |                                                 |    |    |    |    |           |         |
| 022 +     |                               | 078 +     |                                                 |    |    |    |    |           |         |
| 023 JX    |                               | 079 RCL9  |                                                 |    |    |    |    |           |         |
| 024 RCL9  |                               | 080 XZ Y  |                                                 |    |    |    |    |           |         |
| 025 +     |                               | 081 ÷     | Exit.                                           |    |    |    |    |           |         |
| 026 LN    | -----                         | 082 GT05  | -----                                           |    |    |    |    |           |         |
| 027 #LBL1 | Restore t.                    | 083 #LBL4 |                                                 |    |    |    |    |           |         |
| 028 RCL0  |                               | 084 ENT↑  |                                                 |    |    |    |    |           |         |
| 029 R4    |                               | 085 ENT↑  |                                                 |    |    |    |    |           |         |
| 030 P2S   |                               | 086 1     |                                                 |    |    |    |    |           |         |
| 031 RTN   |                               | 087 +     | Compute tanh <sup>-1</sup> x.                   |    |    |    |    |           |         |
| 032 #LBLC |                               | 088 XZ Y  |                                                 |    |    |    |    |           |         |
| 033 P2S   | Cosh                          | 089 CHS   |                                                 |    |    |    |    |           |         |
| 034 R↑    |                               | 090 1     |                                                 |    |    |    |    |           |         |
| 035 ST00  | Save t.                       | 091 +     |                                                 |    |    |    |    |           |         |
| 036 R4    |                               | 092 ÷     |                                                 |    |    |    |    |           |         |
| 037 F2?   | If inverse, GTO 2.            | 093 LN    |                                                 |    |    |    |    |           |         |
| 038 GT02  |                               | 094 2     |                                                 |    |    |    |    |           |         |
| 039 e*    |                               | 095 ÷     |                                                 |    |    |    |    |           |         |
| 040 ENT↑  |                               | 096 #LBL5 |                                                 |    |    |    |    |           |         |
| 041 1/X   |                               | 097 RCL0  |                                                 |    |    |    |    |           |         |
| 042 +     | Compute cosh x.               | 098 RCL8  | Restore t and z.                                |    |    |    |    |           |         |
| 043 2     |                               | 099 R4    |                                                 |    |    |    |    |           |         |
| 044 ÷     |                               | 100 R4    |                                                 |    |    |    |    |           |         |
| 045 GT03  | Exit.                         | 101 P2S   |                                                 |    |    |    |    |           |         |
| 046 #LBL2 | -----                         | 102 RTN   |                                                 |    |    |    |    |           |         |
| 047 ST09  |                               | 103 #LBL6 |                                                 |    |    |    |    |           |         |
| 048 X²    |                               | 104 F2?   | csch                                            |    |    |    |    |           |         |
| 049 1     |                               | 105 GT06  |                                                 |    |    |    |    |           |         |
| 050 -     | Compute cosh <sup>-1</sup> x. | 106 CSBB  |                                                 |    |    |    |    |           |         |
| 051 JX    |                               | 107 1/X   |                                                 |    |    |    |    |           |         |
| 052 RCL9  |                               | 108 RTN   |                                                 |    |    |    |    |           |         |
| 053 +     |                               | 109 #LBL6 |                                                 |    |    |    |    |           |         |
| 054 LN    | -----                         | 110 SF2   | csch x = (sinh x) <sup>-1</sup>                 |    |    |    |    |           |         |
| 055 #LBL3 | Restore t.                    | 111 1/X   | csch <sup>-1</sup> x = sinh <sup>-1</sup> (1/x) |    |    |    |    |           |         |
| 056 RCL0  |                               | 112 GT08  |                                                 |    |    |    |    |           |         |
| REGISTERS |                               |           |                                                 |    |    |    |    |           |         |
| 0         | 1                             | 2         | 3                                               | 4  | 5  | 6  | 7  | 8         | 9       |
| S0 Save t | S1                            | S2        | S3                                              | S4 | S5 | S6 | S7 | S8 Save z | S9 Used |
| A         | B                             | C         | D                                               | E  | F  | G  | H  | I         | J       |

|                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                          |  |  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| 113 #LBL6<br>114 F2°<br>115 GT07<br>116 GSB0<br>117 1/X<br>118 RTN<br>119 #LBL7<br>120 SF2<br>121 1/X<br>122 GT0C<br>123 #LBLd<br>124 F2°<br>125 GT08<br>126 GSB0<br>127 1/X<br>128 RTN<br>129 #LBL8<br>130 SF2<br>131 1/X<br>132 GT0D | Sech<br><br><br>$\text{sech } x = (\cosh x)^{-1}$<br>-----<br><br>$\text{sech}^{-1} x = \cosh^{-1} (1/x)$<br>-----<br>coth<br><br><br>$\text{coth } x = (\tanh x)^{-1}$<br>-----<br><br>$\text{coth}^{-1} x = \tanh^{-1} (1/x)$<br>----- |  |  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|

| LABELS               |                      |                      |                      |                      | FLAGS | SET STATUS                                                     |                                                                |                                         |                                         |
|----------------------|----------------------|----------------------|----------------------|----------------------|-------|----------------------------------------------------------------|----------------------------------------------------------------|-----------------------------------------|-----------------------------------------|
| A Arc                | B Sinh               | C Cosh               | D Tanh               | E                    | 0     | FLAGS                                                          |                                                                | TRIG                                    | DISP                                    |
| a                    | b Csch               | c Sech               | d Coth               | e                    | 1     | ON OFF                                                         |                                                                | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| 0 sinh <sup>-1</sup> | 1 Exit sinh          | 2 cosh <sup>-1</sup> | 3 Exit cosh          | 4 tanh <sup>-1</sup> | 2 Arc | 0 <input type="checkbox"/> <input checked="" type="checkbox"/> | 1 <input type="checkbox"/> <input checked="" type="checkbox"/> | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
| 5 Exit tanh          | 6 csch <sup>-1</sup> | 7 sech <sup>-1</sup> | 8 coth <sup>-1</sup> | 9                    | 3     | 2 <input type="checkbox"/> <input checked="" type="checkbox"/> | 3 <input type="checkbox"/> <input checked="" type="checkbox"/> | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|                      |                      |                      |                      |                      |       |                                                                |                                                                |                                         | n <u>2</u>                              |

# Appendix A

## MAGNETIC CARD

### SYMBOLS AND CONVENTIONS

| SYMBOL OR CONVENTION                                   | INDICATED MEANING                                                                                                                                                                                                                                                                                                                                                  |
|--------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| White mnemonic:<br>x<br>A                              | White mnemonics are associated with the user-definable key they are above when the card is inserted in the calculator's window slot. In this case the value of x could be input by keying it in and pressing A.                                                                                                                                                    |
| Gold mnemonic:<br>y<br>x<br>f E<br>x $\uparrow$ y<br>A | Gold mnemonics are similar to white mnemonics except that the gold f key must be pressed before the user-definable key. In this case y could be input by pressing f E.<br><br>$\uparrow$ is the symbol for ENTER. In this case ENTER is used to separate the input variables x and y. To input both x and y you would key in x, press ENTER, key in y and press A. |
| X<br>A                                                 | The box around the variable x indicates input by pressing STO A.                                                                                                                                                                                                                                                                                                   |
| (x)<br>A                                               | Parentheses indicate an option. In this case, x is not a required input but could be input in special cases.                                                                                                                                                                                                                                                       |
| $\rightarrow$ x<br>A                                   | $\rightarrow$ is the symbol for calculate. This indicates that you may calculate x by pressing key A.                                                                                                                                                                                                                                                              |
| $\rightarrow$ x, y, z<br>A                             | This indicates that x, y, and z are calculated by pressing A once. The values would be printed in x, y, z order.                                                                                                                                                                                                                                                   |
| $\rightarrow$ x; y; z<br>A                             | The semi-colons indicate that after x has been calculated using A, y and z may be calculated by pressing R/S.                                                                                                                                                                                                                                                      |
| $\rightarrow$ "x", y<br>A                              | The quote marks indicate that the x value will be "paused" or held in the display for one second. The pause will be followed by the display of y.                                                                                                                                                                                                                  |
| $\leftrightarrow$ x<br>A                               | The two-way arrow $\leftrightarrow$ indicates that x may be either output or input when the associated user-definable key is pressed. If numeric keys have been pressed between user-definable keys, x is stored. If numeric keys have not been pressed, the program will calculate x.                                                                             |

## SYMBOLS AND CONVENTIONS (Continued)

| SYMBOL OR CONVENTION            | INDICATED MEANING                                                                                                                                                                                                                                                                                                                 |
|---------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>P?</p> <p><b>A</b></p>       | <p>The question mark indicates that this is a mode setting, while the mnemonic indicates the type of mode being set. In this case a print mode is controlled. Mode settings typically have a 1.00 or 0.00 indicator displayed after they are executed. If 1.00 is displayed, the mode is on. If 0.00 is displayed, it is off.</p> |
| <p>START</p> <p><b>A</b></p>    | <p>The word START is an example of a command. The start function should be performed to begin or start a program. It is included when initialization is necessary.</p>                                                                                                                                                            |
| <p>DEL</p> <p><b>A</b></p>      | <p>This special command indicates that the last value or set of values input may be deleted by pressing <b>A</b>.</p>                                                                                                                                                                                                             |
| <p>→ x; ...</p> <p><b>A</b></p> | <p>Three dots (...) indicate that additional output follows. See User Instructions for complete description of variables output.</p>                                                                                                                                                                                              |



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