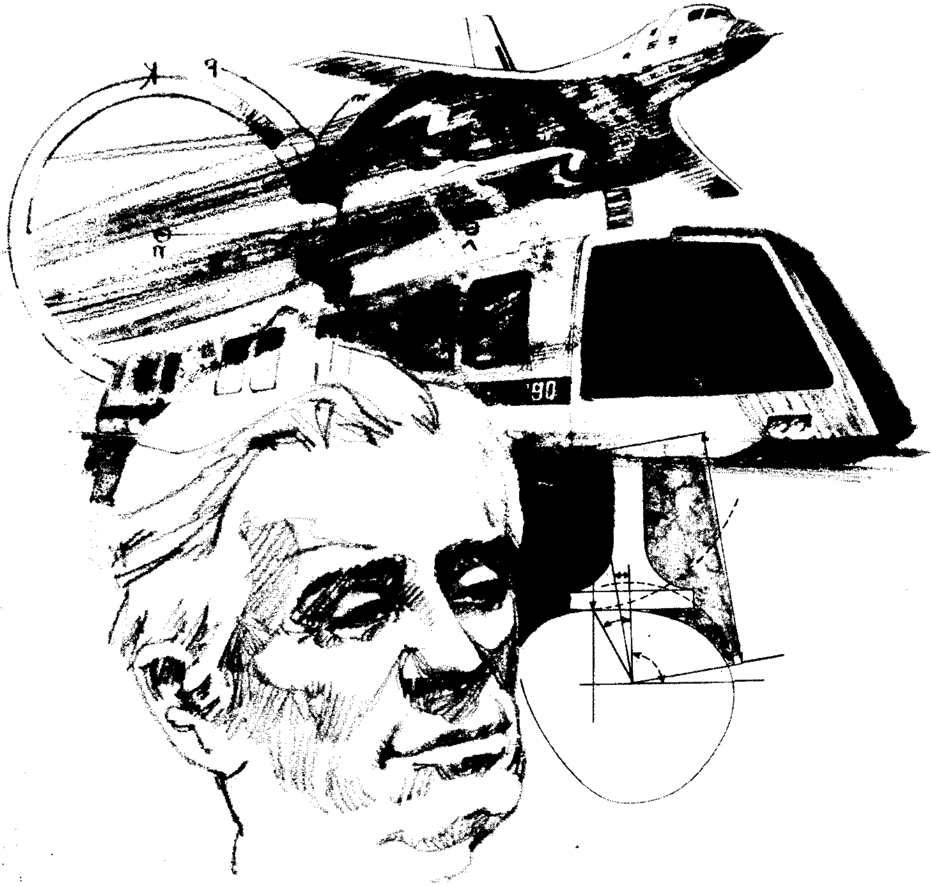
HEWLETT-PACKARD

HP-67 HP-97

M.E. Pac I



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Introduction

The 23 programs of ME Pac I have been drawn from the fields of statics, dynamics, stress analysis, machine design, and thermodynamics.

Each program in this pac is represented by one or more magnetic cards and a section in this manual. The manual provides a description of the program with relevant equations, a set of instructions for using the program, and one or more example problems, each of which includes a list of the actual keystrokes required for its solution. Program listings for all the programs in the pac appear at the back of this manual. Explanatory comments have been incorporated in the listings to facilitate your understanding of the actual working of each program. Thorough study of a commented listing can help you to expand your programming repertoire since interesting techniques can often be found in this way.

On the face of each magnetic card are various mnemonic symbols which provide shorthand instructions to the use of the program. You should first familiarize yourself with a program by running it once or twice while following the complete User Instructions in the manual. Thereafter, the mnemonics on the cards themselves should provide the necessary instructions, including what variables are to be input, which user-definable keys are to be pressed, and what values will be output. A full explanation of the mnemonic symbols for magnetic cards may be found in appendix A.

If you have already worked through a few programs in Standard Pac, you will understand how to load a program and how to interpret the User Instructions form. If these procedures are not clear to you, take a few minutes to review the sections, Loading a Program and Format of User Instructions, in your Standard Pac.

We hope that ME Pac I will assist you in the solution of numerous problems in your discipline. We would very much appreciate knowing your reactions to the programs in this pac, and to this end we have provided a questionnaire inside the front cover of this manual. Would you please take a few minutes to give us your comments on these programs? It is in the comments we receive from you that we learn how best to increase the usefulness of programs like these.

Contents

A WORD ABOUT PROGRAM USAGE

This application pac has been designed for both the HP-97 Programmable Printing Calculator and the HP-67 Programmable Pocket Calculator. The most significant difference between the HP-67 and the HP-97 calculators is the printing capability of the HP-97. The two calculators also differ in a few minor ways. The purpose of this section is to discuss the ways that the programs in this pac are affected by the difference in the two machines and to suggest how you can make optimal use of your machine, be it an HP-67 or an HP-97.

Many of the computed results in this pac are output by PRINT statements; on the HP-97 these results will be output on the printer. On the HP-67 each PRINT command will be interpreted as a PAUSE: the program will halt, display the result for about five seconds, then continue execution. The term "PRINT/PAUSE" is used to describe this output condition.

If you own an HP-67, you may want more time to copy down the number displayed by a PRINT/PAUSE. All you need to do is press any key on the keyboard. If the command being executed is PRINTx (eight rapid blinks of the decimal point), pressing a key will cause the program to halt. Execution of the halted program may be re-initiated by pressing R/S.

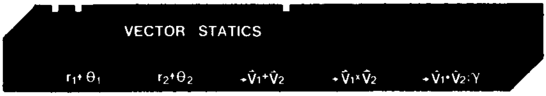
HP-97 users may also want to keep a permanent record of the values input to a certain program. A convenient way to do this is to set the Print Mode switch to NORMAL before running the program. In this mode all input values and their corresponding user-definable keys will be listed on the printer, thus providing a record of the entire operation of the program.

Another area that could reflect differences between the HP-67 and the HP-97 is in the keystroke solutions to example problems. It is sometimes necessary in these solutions to include operations that involve prefix keys, namely, f on the HP-97 and f, g, and h on the HP-67. For example, the operation 10^x is performed on the HP-97 as f 10^x and on the HP-67 as g 10^x. In such cases, the keystroke solution omits the prefix key and indicates only the operation (as here, 10^x). As you work through the example problems, take care to press the appropriate prefix keys (if any) for your calculator.

Also in keystroke solutions, those values that are output by the PRINT command will be followed by three asterisks (***).

Notes

VECTOR STATICS



Part I of this program performs the basic two dimensional vector operations of addition, cross product and dot, scalar, or inner product. In addition, the angle between vectors may be found. Vectors may be input in polar form (r, θ) or rectangular form (x_1, y_1).

Equations:

for addition: $\vec{V}_1 + \vec{V}_2 = (x_1 + x_2)\,\vec{i} + (y_1 + y_2)\,\vec{j}$

for cross products: $\vec{V}_1 \times \vec{V}_2 = (x_1y_2 - x_2y_1)\,\vec{k}$

for dot, scalar, or inner product: $\vec{V}_1 \cdot \vec{V}_2 = x_1x_2 + y_1y_2$

for the angle between vectors: $\gamma = \cos^{-1} \dfrac{\vec{V}_1 \cdot \vec{V}_2}{|\vec{V}_1|\,|\vec{V}_2|}$

where:

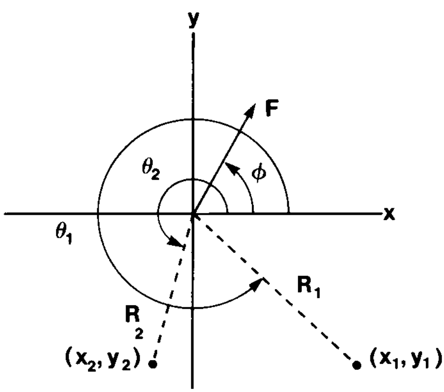
x_1 is the x component of $\vec{V}_1$ $(x_1 = r_1 \cos \theta_1)$;

x_2 is the x component of $\vec{V}_2$ $(x_2 = r_2 \cos \theta_2)$;

y_1 is the y component of $\vec{V}_1$ $(y_1 = r_1 \sin \theta_1)$;

y_2 is the y component of $\vec{V}_2$ $(y_2 = r_2 \sin \theta_2)$;

Part II of this program calculates the two reaction forces necessary to balance a given two-dimensional force vector. The direction of the reaction forces may be specified as a vector of arbitrary length or by Cartesian coordinates using the point of force application as the origin.



Equations:

$$R_1 \cos \theta_1 + R_2 \cos \theta_2 = F \cos \phi$$

$$R_1 \sin \theta_1 + R_2 \sin \theta_2 = F \sin \phi$$

where:

F is the known force;

ϕ is the direction of the known force;

R_1 is one reaction force;

θ_1 is the direction of R_1;

R_2 is the second reaction force;

θ_2 is the direction of R_2.

The coordinates x_1 and y_1 are referenced from the point where F is applied to the end of the member along which R_1 acts; x_2 and y_2 are the coordinates referenced from the point where F is applied to the end of the member along which R_2 acts.

Remarks:

Registers $R_0 - R_3$; $R_{S0} - R_{S9}$ and I are available for user storage.

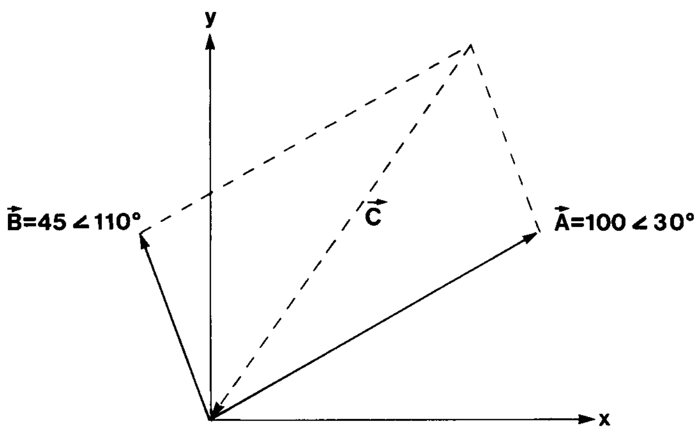
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	To resolve a force in two			
	known directions, go to step 6.			
	For vector addition, cross			
	product, or dot product con-			
	tinue with step 3.			
3	Input $\vec{V}_1$ and $\vec{V}_2$:			
	$\vec{V}_1$ in polar form	r_1	ENTER↑	r_1
		θ_1	A	y_1
	or			
	$\vec{V}_1$ in rectangular form	x_1	ENTER↑	x_1
		y_1	f A	y_1
	and			
	$\vec{V}_2$ in polar form	r_2	ENTER↑	r_2
		θ_2	B	y_2
	or			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	$\vec{V}_2$ in rectangular form.	x_2	ENTER↑	x_2
		y_2	f B	y_2
4	Perform vector operation:			
	add vectors		C	r, θ
	or			
	take cross product		D	$\vec{V}_1 \times \vec{V}_2$
	or			
	take dot (or scalar) product.		E	$\vec{V}_1 \cdot \vec{V}_2$
	(Optionally, calculate angle			
	between vectors after dot			
	product.)		R/S	γ
5	For a new case, go to step 3			
	and change $\vec{V}_1$ and/or $\vec{V}_2$.			
6	Define reaction directions as			
	Cartesian coordinates or as			
	vectors of arbitrary magnitude.			
	(Use the point of force appli-			
	cations as the origin):			
	define direction one in polar			
	form	1	ENTER↑	1.00
		θ_1	A	sin θ_1
	or			
	in rectangular form	x_1	ENTER↑	x_1
		y_1	f A	y_1
	and			
	define direction two in polar			
	form	1	ENTER↑	1.00
		θ_2	B	sin θ_2
	or			
	in rectangular form.	x_2	ENTER↑	x_2
		y_2	f B	y_2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
7	Input known force:			
	magnitude	F	ENTER↑	F
	then direction.	ϕ	f D	F sin ϕ
8	Compute reactions		f E	R_1, R_2
9	To change force, go to step 7.			
	To change either or both			
	directions, go to step 6.			

Example 1:

Forces A and B are shown below. If static equilibrium exists, what is force C.



Keystrokes: **Outputs:**

To obtain $\vec{C}$, add $\vec{A}$ and $\vec{B}$ using negative magnitudes for both.

45 CHS ENTER↑ 110 A 100 CHS

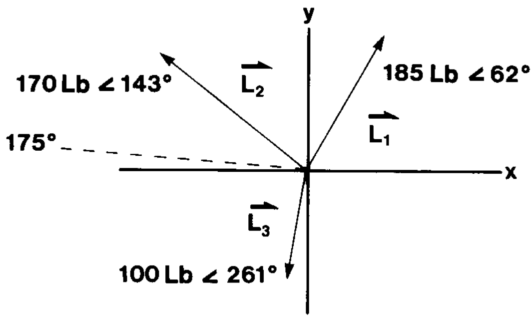
ENTER↑ 30 B C ——→ 116.57 ***

−127.66 ***

$$\vec{C} = 116.57\angle -127.66°$$

Example 2:

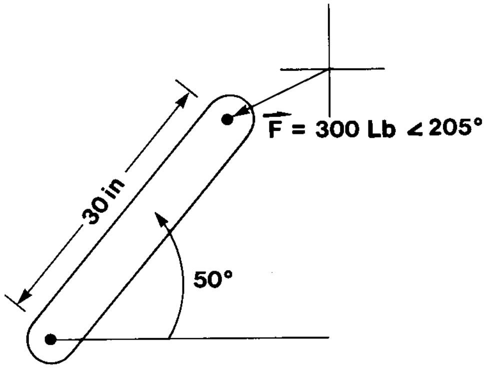
Resolve the following three loads along a 175 degree line.



Keystrokes:	**Outputs:**
First add $\vec{L}_1$ and $\vec{L}_2$.	
185 ENTER↑ 62 A 170 ENTER↑	
143 B C ⟶	270.12 *** (lb)
	100.43 *** (deg)
Define the result as $\vec{V}_1$ and add $\vec{L}_3$.	
A 100 ENTER↑ 261 B C ⟶	178.94 *** (lb)
	111.15 *** (deg)
To resolve the vector, just calculated along the 175° line.	
A 1 ENTER↑ 175 B E ⟶	78.86 *** (lb)
What is the angle between the vector and the line?	
R/S ⟶	63.85 *** (deg)

Example 3:

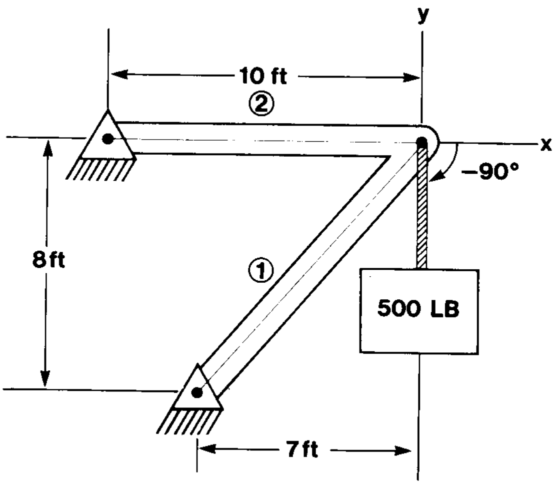
What is the moment at the shaft of the crank pictured below? What is the reaction force transmitted along the member?



Keystrokes:	**Outputs:**	
Moment by cross product ($\vec{V}_1 \times \vec{F}$).		
30 ENTER↑ 50 A 300 ENTER↑		
205 B D ⟶	3803.56	in-lb
Resolution along crank		
1 ENTER↑ 50 A E ⟶	−271.89	lb

Example 4:

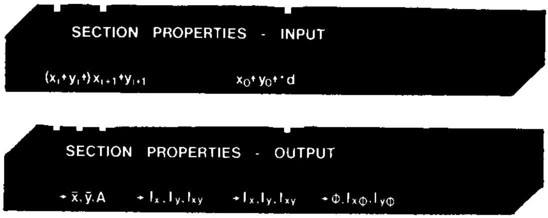
Find the reaction forces in the pin-jointed structure shown below.



Keystrokes:	**Outputs:**
7 CHS ENTER↑ 8 CHS f A ⟶	-8.00
10 CHS ENTER↑ 0 f B ⟶	0.00
500 ENTER↑ 90 CHS f D ⟶	-500.00
f E ⟶	-664.38 *** (R_1)
	437.50 *** (R_2)

Notes

SECTION PROPERTIES



The properties of polygonal sections (see figure 1) may be calculated using this program. The (x, y) coordinates of the vertices of the polygon (which must be located entirely within the first quadrant) are input sequentially for a complete, clockwise path around the polygon. Holes in the cross section, which do not intersect the boundary, may be deleted by following a counter-clockwise path.

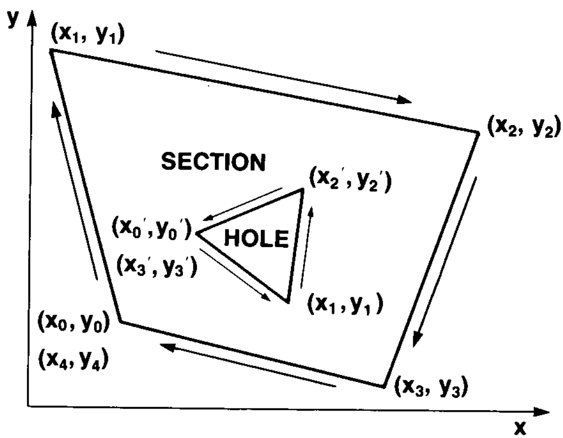


Figure 1 — Polygonal Sections

A special feature allows addition or deletion of circular areas. After the point by point traverse of the section has been completed, circular deletions or additions are specified by the (x,y) coordinates of the circle centers and by the circle diameters. If the diameter is specified as a positive number, the circular areas are added. A negative diameter causes circular areas to be deleted. Example 4 shows an application of this feature.

After all values have been input, the coordinates of the centroid ($\bar{x}$, $\bar{y}$) and the area (A) of the section may be output using card 2, key **A**. The moment of inertia about the x axis (I_x), about the y axis (I_y) and the product of inertia (I_{xy}) are output using **B**. Similar moments, $I_{\bar{x}}$, $I_{\bar{y}}$ and $I_{\overline{xy}}$, about an axis translated to the centroid of the section are calculated when **C** is pressed.

Pressing **D** calculates the moments of inertia, $I_{\bar{x}\phi}$ and $I_{\bar{y}\phi}$, about the principal axis. The rotation angle (ϕ) between the principal axis and the axis which was translated to the centroid is also calculated. The moments of inertia I_x', I_y', the polar moment of inertia J and the product of inertia I_{xy}' may be calculated about any arbitrary axis by specifying its location and rotation with respect to the original axis and pressing **f** **D**.

Equations:

$$A = -\sum_{i=0}^{n} (y_{i+1} - y_i)(x_{i+1} + x_i)/2$$

$$\bar{x} = \frac{-1}{A}\sum_{i=0}^{n} \left[(y_{i+1} - y_i)/8\right]\left[(x_{i+1} + x_i)^2 + (x_{i+1} - x_i)^2/3\right]$$

$$\bar{y} = \frac{1}{A}\sum_{i=0}^{n} \left[(x_{i+1} - x_i)/8\right]\left[(y_{i+1} + y_i)^2 + (y_{i+1} - y_i)^2/3\right]$$

$$I_x = \sum_{i=0}^{n} \left[(x_{i+1} - x_i)(y_{i+1} + y_i)/24\right]\left[(y_{i+1} + y_i)^2 + (y_{i+1} - y_i)^2\right]$$

$$I_y = -\sum_{i=0}^{n} \left[(y_{i+1} - y_i)(x_{i+1} + x_i)/24\right]\left[(x_{i+1} + x_i)^2 + (x_{i+1} - x_i)^2\right]$$

$$I_{xy} = \sum_{i=0}^{n} \frac{1}{(x_{i+1} - x_i)}\left[\frac{1}{8}(y_{i+1} - y_i)^2(x_{i+1} + x_i)(x_{i+1}^2 + x_i^2)\right.$$

$$+ \frac{1}{3}(y_{i+1} - y_i)(x_{i+1}\, y_i - x_i\, y_{i+1})(x_{i+1}^2 + x_{i+1}\, x_i + x_i^2)$$

$$\left. + \frac{1}{4}(x_{i+1}\, y_i - x_i\, y_{i+1})^2(x_{i+1} + x_i)\right]$$

$$I_{\bar{x}} = I_x - A\bar{y}^2$$

$$I_{\bar{y}} = I_y - A\bar{x}^2$$

$$I_{\bar{x}\bar{y}} = I_{xy} - A\bar{x}\bar{y}$$

$$\phi = \frac{1}{2}\tan^{-1}\left(\frac{-2I_{\bar{x}\bar{y}}}{I_{\bar{x}} - I_{\bar{y}}}\right)$$

$$I_x' = I_{\bar{x}}\cos^2\theta + I_{\bar{y}}\sin^2\theta - I_{\bar{x}\bar{y}}\sin 2\theta$$

$$I_y' = I_{\bar{y}}\cos^2\theta + I_{\bar{x}}\sin^2\theta + I_{\bar{x}\bar{y}}\sin 2\theta$$

$$J = I_x' + I_y'$$

$$I_{xy}' = \frac{(I_{\bar{x}} - I_{\bar{y}})}{2}\sin 2\theta + I_{\bar{x}\bar{y}}\cos 2\theta$$

$$A_{circle} = \frac{\pi d^2}{4}$$

$$I_{circle} = \frac{\pi d^4}{64}$$

where:

x_{i+1} is the x coordinate of the current vertex point;
y_{i+1} is the y coordinate of the current vertex point;
x_i is the x coordinate of the previous vertex point;
y_i is the y coordinate of the previous vertex point;
A is the area;
$\bar{x}$ is the x coordinate of the centroid;
$\bar{y}$ is the y coordinate of the centroid;
I_x is the moment of inertia about the x-axis;
I_y is the moment of inertia about the y-axis;
I_{xy} is the product of inertia;
$I_{\bar{x}}$ is the moment of inertia about the x-axis translated to the centroid;
$I_{\bar{y}}$ is the moment of inertia about the y-axis translated to the centroid;
$I_{\bar{x}\bar{y}}$ is the product of inertia about the translated axis;
ϕ is the angle between the translated axis and the principal axis;
$I_{\bar{x}\phi}$ is the moment of inertia about the translated, rotated, principal x-axis;
$I_{\bar{y}\phi}$ is the moment of inertia about the translated, rotated, principal y-axis;
θ is the angle between the original axis and an arbitrary axis.
I_x' is the x moment of inertia about the arbitrary axis;
I_y' is the y moment of inertia about the arbitrary axis;

J is the polar moment of inertia about the arbitrary axis;

I_{xy}' is the product of inertia about the arbitrary axis;

d is the diameter of a circular area.

Reference:

Wojciechowski, Felix; *Properties of Plane Cross Sections; Machine Design;* P. 105, Jan. 22, 1976.

Remarks:

Registers $R_{S0} - R_{S9}$ are available for user storage.

The polygon must be entirely contained in the first quadrant.

Rounding errors will accumulate if the centroid of the section is a large distance from the origin of the coordinate system.

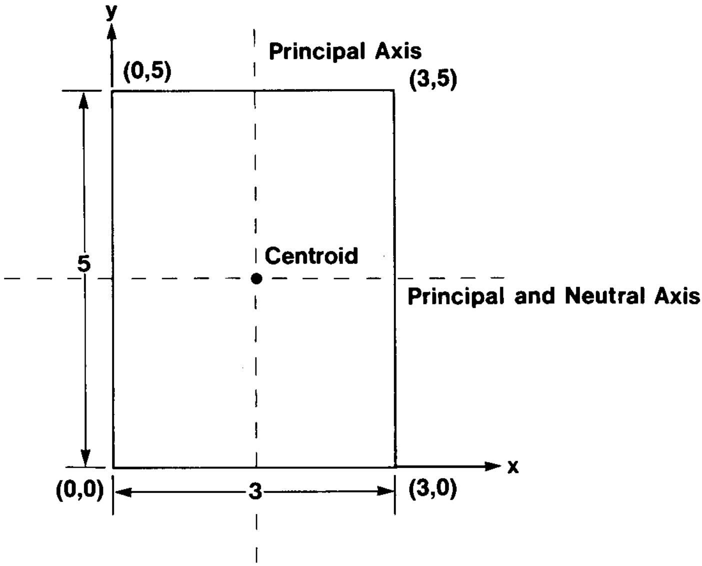
Curved boundaries may be approximated by straight line segments.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2 of			
	card 1.			
2	Initialize.		f A	
3	Key in (x, y) coordinates of			
	first vertex.	x_i	ENTER↑	y_i
		y_i	ENTER↑	y_i
4	Key in (x, y) coordinates of			
	next clockwise vertex.	x_{i+1}	ENTER↑	x_{i+1}
		y_{i+1}	A	y_{i+1}
5	Wait for execution to end, then			
	repeat step 4 for next point.			
	Go to step 6 after you have			
	reinput the starting point.			
6	To delete subsections within			
	the section just traversed,			
	return to step 3, but traverse in			
	a counter-clockwise direction.			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
7	Optional: Add circular areas,	x	ENTER↑	x
		y	ENTER↑	y
		d	C	0.00
	or delete circular areas.	x	ENTER↑	x
		y	ENTER↑	y
		d	CHS C	0.00
8	Load side 1 and side 2 of			
	card 2.			
9	Calculate any or all of the			
	following:			
	Centroid and area;		A	$\bar{x}$, $\bar{y}$, A
	Properties about original			
	axis;		B	I_x, I_y, I_{xy}
	Properties about axis trans-			
	lated to centroid;		C	$I_{\bar{x}}$, $I_{\bar{y}}$, $I_{\overline{xy}}$
	Angular orientation of			
	principal axis and properties			
	about principal axis;		D	ϕ, $I_{\bar{x}\phi}$, $I_{\bar{y}\phi}$
	or			
	Specify arbitrary axis and			
	rotation and calculate			
	properties.	x'	ENTER↑	
		y'	ENTER↑	
		θ	f D	I_x', I_y', J, I_{xy}'
10	To modify the section, go to			
	step 1, but skip step 2. For a			
	new case, go to step 1.			

Example 1:

What is the moment of inertia about the x-axis (I_x) for the rectangular section shown? What is the moment of inertia about the neutral axis through the centroid of the section ($I_{\bar{x}\phi}$)?



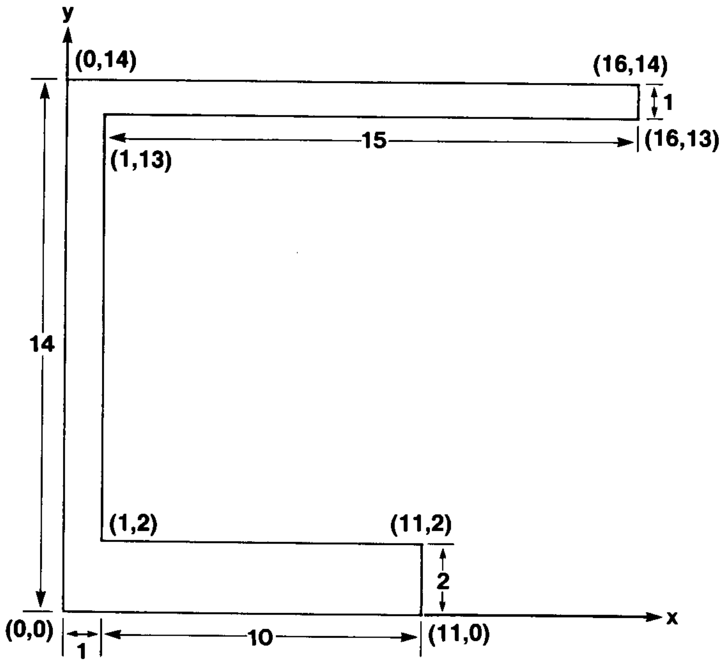
Keystrokes:	**Outputs:**
Load side 1 and side 2 of card 1.	
f A 0 ENTER↑ 0 ENTER↑	
0 ENTER↑ 5 A ⟶	5.00
3 ENTER↑ 5 A ⟶	5.00
3 ENTER↑ 0 A ⟶	0.00
0 ENTER↑ 0 A ⟶	0.00
Load side 1 and side 2 of card 2.	
B ⟶	125.00 *** (I_x)
	45.00 *** (I_y)
	56.25 *** (I_{xy})
D ⟶	0.00 *** (ϕ)
	31.25 *** ($I_{\bar{x}\phi}$)
	11.25 *** ($I_{\bar{y}\phi}$)

Since $\phi = 0$ we would expect $I_{\bar{x}\phi}$ to equal $I_{\bar{x}}$. Press **C** to calculate $I_{\bar{x}}$, $I_{\bar{y}}$ and $I_{\bar{x}\bar{y}}$ and you will see that this prediction is correct. Also, $I_{\bar{x}\bar{y}}$ is zero about the principal axis.

C ⟶	31.25 *** ($I_{\bar{x}}$)
	11.25 *** ($I_{\bar{y}}$)
	0.00 *** ($I_{\bar{x}\bar{y}}$)

Example 2:

Calculate the section properties for the beam shown below.



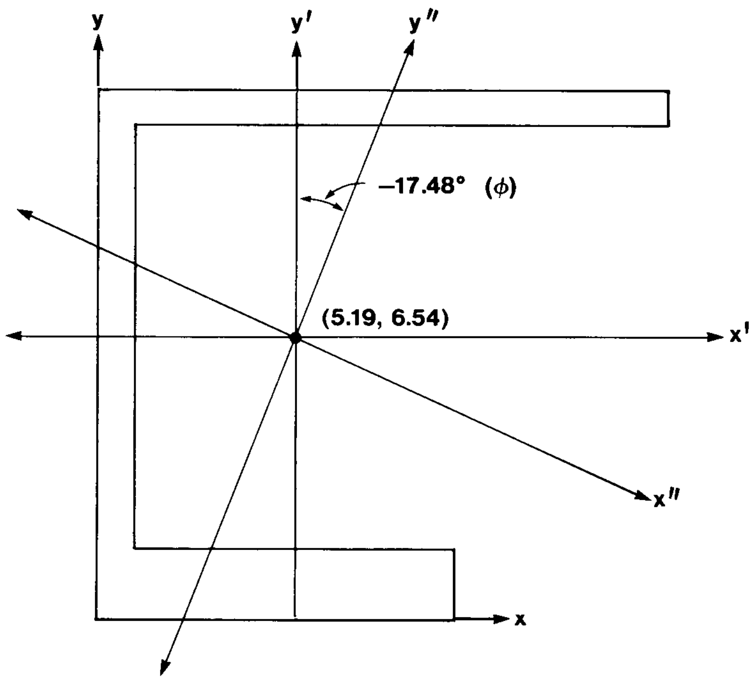
Keystrokes:	**Outputs:**
Load side 1 and side 2 of card 1.	
f **A** 0 **ENTER↑** 0 **ENTER↑**	
0 **ENTER↑** 14 **A** ⟶	14.00
16 **ENTER↑** 14 **A** ⟶	14.00
16 **ENTER↑** 13 **A** ⟶	13.00
1 **ENTER↑** 13 **A** ⟶	13.00
1 **ENTER↑** 2 **A** ⟶	2.00
11 **ENTER↑** 2 **A** ⟶	2.00
11 **ENTER↑** 0 **A** ⟶	0.00

0 ENTER↑ 0 A → 0.00

Load side 1 and side 2 of card 2.

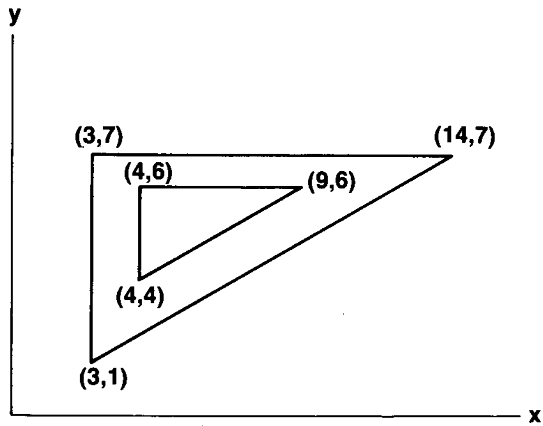
A →	5.19 *** ($\bar{x}$)
	6.54 *** ($\bar{y}$)
	49.00 *** (A)
B →	3676.33 *** (I_x)
	2256.33 *** (I_y)
	1890.25 *** (I_{xy})
C →	1580.00 *** ($I_{\bar{x}}$)
	934.49 *** ($I_{\bar{y}}$)
	225.61 *** ($I_{\bar{x}\bar{y}}$)
D →	-17.48 *** (ϕ)
	1651.04 *** ($I_{\bar{x}\phi}$)
	863.46 *** ($I_{\bar{y}\phi}$)

Below is a figure showing the translated axis and the rotated, principal axis of example 2.



Example 3:

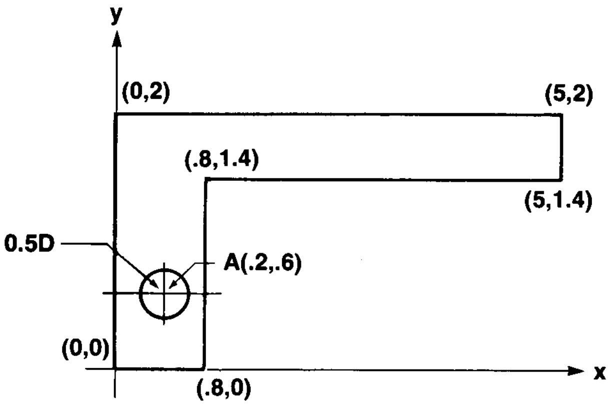
What is the centroid of the section below? The inner triangular boundary denotes an area to be deleted.



Keystrokes:	**Outputs:**
Load side 1 and side 2 of card 1.	
f A 3 ENTER↑ 1 ENTER↑	
3 ENTER↑ 7 A ⟶	7.00
14 ENTER↑ 7 A ⟶	7.00
3 ENTER↑ 1 A ⟶	1.00
Delete inner triangle:	
4 ENTER↑ 4 ENTER↑ 9 ENTER↑	
6 A ⟶	6.00
4 ENTER↑ 6 A ⟶	6.00
4 ENTER↑ 4 A ⟶	4.00
Load side 1 and side 2 of card 2.	
Compute Centroid	
A ⟶	6.85 *** ($\bar{x}$)
	4.94 *** ($\bar{y}$)
	28.00 *** (A)

Example 4:

For the part below, compute the polar moment of inertia about point A. Point A denotes the center of a hole about which the part rotates. The area of the hole must be deleted from the cross section.



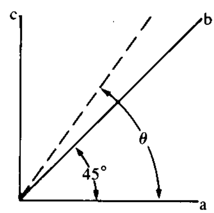
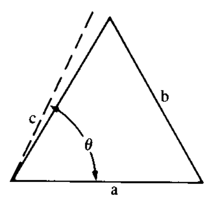
Keystrokes:	**Outputs:**
Load side 1 and side 2 of card 1.	
f A 0 ENTER↑ 0 ENTER↑ 0 ENTER↑	
2 A 5 ENTER↑ 2 A 5 ENTER↑	
1.4 A .8 ENTER↑ 1.4 A .8 ENTER↑	
0 A 0 ENTER↑ 0 A ——→	0.00
Delete the hole.	
.2 ENTER↑ .6 ENTER↑	
.5 CHS C ——→	0.00
Load side 1 and side 2 of card 2. Compute J about point (.2, .6) with θ of zero.	
.2 ENTER↑ .6 ENTER↑	
0 f D ——→	3.91 *** ($I_{x'}$)
	22.22 *** ($I_{y'}$)
	26.13 *** (J)
	7.61 *** ($I_{xy'}$)

STRESS ON AN ELEMENT

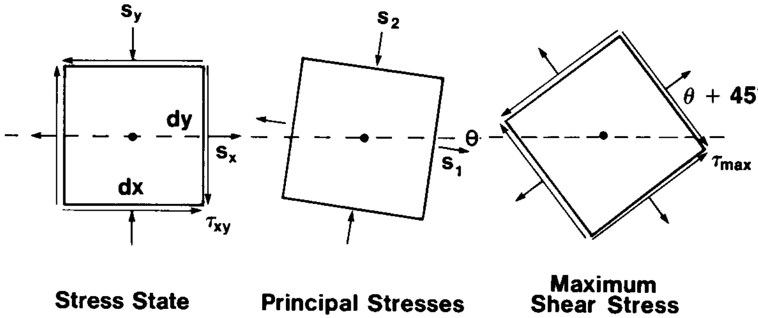
This program reduces data from rosette strain gage measurements and/or performs Mohr circle stress analysis calculations.

Correlations for rectangular and equiangular rosette configurations are included.

Strain Gage Equations:

CONFIGURATION CODE	1	2
TYPE OF ROSETTE	RECTANGULAR	DELTA (EQUIANGULAR)
		
PRINCIPAL STRAINS: ϵ_1, ϵ_2	$\frac{1}{2}\left[\epsilon_a+\epsilon_c \pm \sqrt{2(\epsilon_a-\epsilon_b)^2+2(\epsilon_b-\epsilon_c)^2}\right]$	$\frac{1}{3}\left[\epsilon_a+\epsilon_b+\epsilon_c \pm \sqrt{2(\epsilon_a-\epsilon_b)^2+2(\epsilon_b-\epsilon_c)^2+2(\epsilon_c-\epsilon_a)^2}\right]$
CENTER OF MOHR CIRCLE: $\frac{s_1+s_2}{2}$	$\frac{E(\epsilon_a+\epsilon_c)}{2(1-\nu)}$	$\frac{E(\epsilon_a+\epsilon_b+\epsilon_c)}{3(1-\nu)}$
MAXIMUM SHEAR STRESS: τ_{max}	$\frac{E}{2(1+\nu)}\sqrt{2(\epsilon_a-\epsilon_b)^2+2(\epsilon_b-\epsilon_c)^2}$	$\frac{E}{3(1+\nu)}\sqrt{2(\epsilon_a-\epsilon_b)^2+2(\epsilon_b-\epsilon_c)^2+2(\epsilon_c-\epsilon_a)^2}$
ORIENTATION OF PRINCIPAL STRESSES	$\tan^{-1}\left[\frac{2\epsilon_b-\epsilon_a-\epsilon_c}{\epsilon_a-\epsilon_c}\right]$	$\tan^{-1}\left[\frac{\sqrt{3}(\epsilon_c-\epsilon_b)}{(2\epsilon_a-\epsilon_b-\epsilon_c)}\right]$

The Mohr circle portion of the program converts an arbitrary stress configuration to principal stresses, maximum shear stress and rotation angle. It is then possible to calculate the state of stress for an arbitrary orientation θ'.



Mohr Circle Equations:

$$\tau_{max} = \sqrt{\left(\frac{s_x - s_y}{2}\right)^2 + \tau_{xy}^2}$$

$$s_1 = \frac{s_x + s_y}{2} + \tau_{max}$$

$$s_2 = \frac{s_x + s_y}{2} - \tau_{max}$$

$$\theta = ½ \tan^{-1}\left(\frac{2\tau_{xy}}{s_x - s_y}\right)$$

$$s = \frac{s_1 + s_2}{2} + \tau_{max} \cos 2\theta'$$

$$\tau = \tau_{max} \sin 2\theta'$$

where:

s is the normal stress, and τ is the shear stress.

ϵ_a, ϵ_b, and ϵ_c are the strains measured using rosette gages;

s_x is the stress in the x direction for Mohr circle input;

s_y is the stress in the y direction for Mohr circle input;

τ_{xy} is the shear stress on the element for Mohr circle input;

ϵ_1 and ϵ_2 are the principal strains;

s_1 and s_2 are the principal normal stresses;

τ_{max} is the maximum shear stress;

θ is the counterclockwise angle of rotation from the specified axis to the principal axis. Note that this is opposite to the normal Mohr circle convention.

θ' is an arbitrary rotation angle from the original (x, y) axis;

E is modulus of elasticity.

ν is the Poisson's ratio.

Reference:

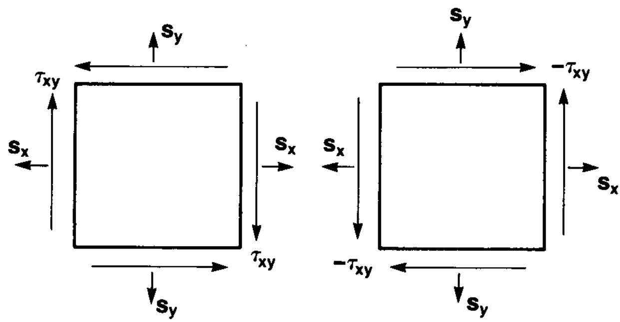
Spotts, M.F., *Design of Machine Elements,* Prentice-Hall, 1971.

Beckwith, T. G., Buck, N. L., *Mechanical Measurements,* Addison-Wesley, 1969

Remarks:

R_0, R_1, R_7, R_8, R_D and R_{S0}–R_{S9} are available for user storage.

Negative stresses and strains indicate compression. Positive and negative shear are represented below:

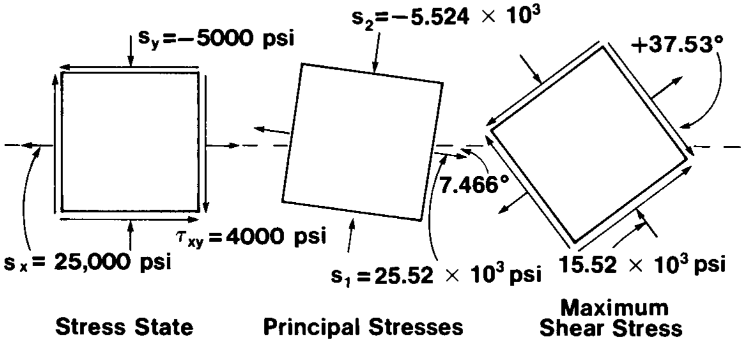


STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	If a stress configuration is			
	known, go to step 8 for Mohr			
	circle evaluation. Continue			
	with step 3 for strain gage			
	data reduction.			
3	Select strain gage			
	configuration:			
	Rectangular		f A	1.000 00
	or Delta.		f B	2.000 00

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
4	Input modulus of elasticity,	E	ENTER↑	E
	then Poisson's ratio.	ν	f D	E
5	Input strains:			
		ϵ_a	ENTER↑	ϵ_a
		ϵ_b	ENTER↑	ϵ_b
		ϵ_c	A	ϵ_a
6	Calculate principal strains			
	and rotation angle.		B	ϵ_1, ϵ_2, θ
7	Skip to step 9 for Mohr circle			
	applications of calculations			
	just completed.			
8	Input stress on element in x			
	direction	s_x	ENTER↑	s_x
	then stress in y direction	s_y	ENTER↑	s_y
	then shear stress.	τ_{xy}	C	0.000 00
9	Calculate principal stresses.		D	s_1, s_2, τ_{max},
				θ
10	Optional: Calculate stress			
	configuration at a specified			
	angle.	θ'	E	s, τ
11	To specify another angle go			
	to step 10. For a new case go			
	to step 2.			

Example 1:

If $s_x = 25000$ psi, $s_y = -5000$ psi, and $\tau_{xy} = 4000$ psi, compute the principal stresses and the maximum shear stress. Compute the normal stresses, where shear stress is maximum ($\theta + 45°$).



Keystrokes:	Outputs:
25000 ENTER↑ 5000 CHS ENTER↑	
4000 C D →	25.52 03 *** (s_1)
	-5.524 03 *** (s_2)
	15.52 03 *** (τ_{max})
	-7.466 00 *** (θ)
45 + →	37.53 00
E →	10.00 03 *** (s)
	15.52 03 *** (τ_1)

Example 2:

A rectangular rosette measures the strains below. What are the principal strains and principal stresses?

$\epsilon_a = 90 \times 10^{-6}$ $\epsilon_b = 137 \times 10^{-6}$ $\epsilon_c = 305 \times 10^{-6}$

$\nu = 0.3$ $E = 30 \times 10^6$ psi

Keystrokes:	Outputs:
f A →	1.000 00
30 EEX 6 ENTER↑ .3 f D →	30.00 06
90 EEX CHS 6 ENTER↑ 137	
EEX CHS 6 ENTER↑ 305 EEX CHS	
6 A →	90.00–06
B →	320.9–06 *** (ϵ_1)
	74.14–06 *** (ϵ_2)
	14.69 00 *** (θ)
D →	11.31 03 *** (s_1)
	5.618 03 *** (s_2)
	2.847 03 *** (τ_{max})
	14.69 00 *** (θ)

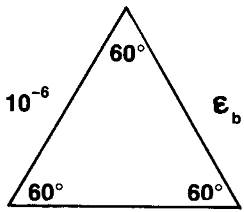
Example 3:

An equiangular rosette measures the strains below. What are the principal strains and stresses?

$$E = 30 \times 10^6 \text{ psi}$$

$$\nu = 0.3$$

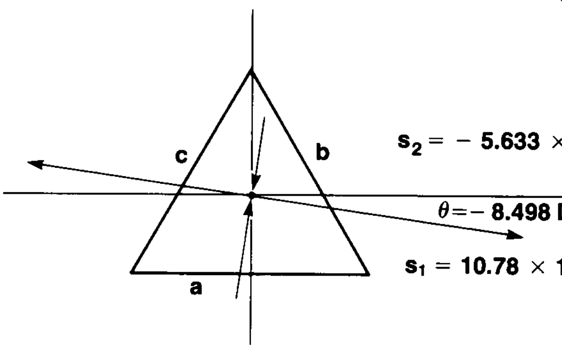
$\varepsilon_c = -200 \times 10^{-6}$



$\varepsilon_b = -20 \times 10^{-6}$

$\varepsilon_a = 400 \times 10^{-6}$

Keystrokes:	Outputs:
[f] [B] →	2.000 00
30 [EEX] 6 [ENTER↑] .3 [f] [D]	
400 [EEX] [CHS] 6 [ENTER↑] 20	
[CHS] [EEX] [CHS] 6 [ENTER↑] 200	
[CHS] [EEX] [CHS] 6 [A] →	400.0–06
[B] →	415.5–06 *** (ϵ_1)
	–295.5–06 *** (ϵ_2)
	–8.498 00 *** (θ)
[D] →	10.78 03 *** (s_1)
	–5.633 03 *** (s_2)
	8.204 03 *** (τ_{max})
	–8.498 00 *** (θ)



$s_2 = -5.633 \times 10^3$ psi

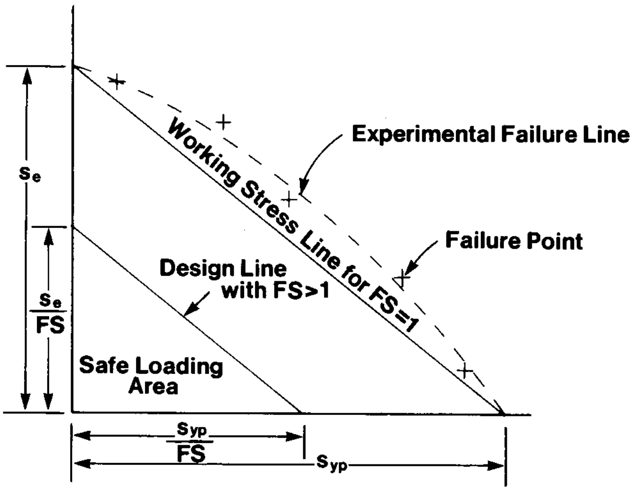
$\theta = -8.498$ Degrees

$s_1 = 10.78 \times 10^3$ psi

SODERBERG'S EQUATION FOR FATIGUE

This program will calculate the seventh variable from the other six values in Soderberg's equation. It is useful in sizing parts for cyclic loading, calculating factors of safety, choosing materials based on size constraints and estimating the fatigue resistance of available parts. Soderberg's equation is graphically represented in figure 1.

Equations:



Working Stress Diagram
Figure 1

$$\frac{s_{yp}}{FS} = \frac{s_{max} + s_{min}}{2} + K\left(\frac{s_{yp}}{s_e}\right)\left(\frac{(s_{max} - s_{min})}{2}\right)$$

$$\frac{s_{max} + s_{min}}{2} = \frac{P_{max} + P_{min}}{2A}$$

$$\frac{s_{max} - s_{min}}{2} = \frac{P_{max} - P_{min}}{2A}$$

where:

s_{yp} is the yield point stress of the material;

s_e is the material endurance stress from reversed bending tests;

K is the stress concentration factor for the part;

FS is the factor of safety (FS $\geqslant$ 1.00)

s_{max} is the maximum stress;

s_{min} is the minimum stress;

P_{max} is the maximum load;

P_{min} is the minimum load;

A is the cross sectional area of the part.

Reference:

Spotts, M. F., *Design of Machine Elements;* Prentice-Hall, Inc., 1971.

Baumeister, T. *Marks Standard Handbook for Mechanical Engineers,* McGraw-Hill Book Company, 1967.

Remarks:

If s_{max} and s_{min} are to be input or calculated instead of P_{max} or P_{min}, simply use 1.00 for the value of area.

R_0–R_7, R_{S0}–R_{S9} and I are available for storage.

This implementation of Soderberg's equation is for ductile materials only.

Values of stress concentration factors and material endurance limits may be found in the referenced sources.

In the presence of corrosive media, or for rough surfaces, fatigue effects may be much more significant than predicted by this program.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Input six of the following seven			
	Yield point stress	s_{yp}	f A	s_{yp}
	Endurance stress	s_e	f B	s_e
	Cross sectional area	A	A	A
	Stress concentration factor	K	B	K
	Maximum load	P_{max}	C	P_{max}
	Minimum load	P_{min}	D	P_{min}
	Factor of safety	FS	E	FS

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
3	Calculate the remaining value:			
	Yield point stress		f A	s_{yp}
	Endurance stress		f B	s_e
	Cross sectional area		A	A
	Stress concentration factor		B	K
	Maximum load		C	P_{max}
	Minimum load		D	P_{min}
	Factor of safety		E	FS
4	Optional: Output values in			
	s_{yp}, s_e, A, K, P_{max}, P_{min},			
	FS order.		f C	OUTPUT
5	For a new case, go to step 2			
	and change appropriate			
	inputs.			

Example 1:

What is the maximum permissible cyclic load for a part if the minimum load is 2000 pounds and the area is 0.5 square inches?

$$s_{yp} = 70000 \text{ psi}$$

$$s_e = 25000 \text{ psi}$$

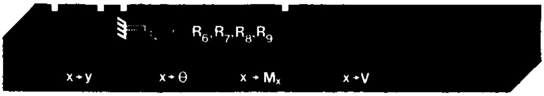
$$K = 1.25$$

$$FS = 2.0$$

Keystrokes:	**Outputs:**	
70000 f A 25000 f B .5 A		
1.25 B 2000 D 2 E C →	8.889 03	(P_{max})
If P_{max} is changed to 10000 pounds what will s_e have to be?		
10000 C f B →	30.43 03	(s_e)

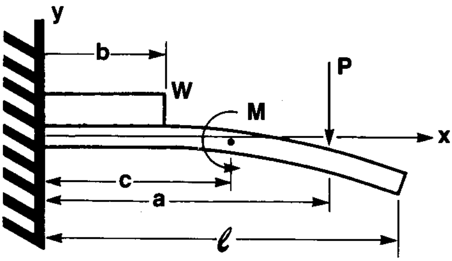
If s_e is changed back to 25000 psi what will the factor of safety be?		
25000 f B E →	1.750 00	(FS)
Output values for review:		
f C →	70.00 03 ***	(s_{yp})
	25.00 03 ***	(s_e)
	500.0 -03 ***	(A)
	1.250 00 ***	(K)
	10.00 03 ***	(P_{max})
	2.000 03 ***	(P_{min})
	1.750 00 ***	(FS)

CANTILEVER BEAMS



This program calculates deflection, slope, moment and shear at any specified point along a rigidly fixed, cantilever beam of uniform cross section. Distributed loads, point loads, applied moments or combinations of all three may be modeled. By using the principle of superposition, complicated beams with multiple point loads, applied moments and combined distributed loads may be analyzed.

Equations:



$$y = y_1 + y_2 + y_3 \qquad \text{(total deflection)}$$

$$y_1 = \frac{PX_1^2}{6EI}(X_1 - 3a) - \frac{Pa^2}{2EI}(x-a)(x>a)^* \qquad \text{(deflection due to point load)}$$

$$y_2 = \frac{-WX_2^2}{6EI}\left[X_2\left(\frac{X_2}{4} - b\right) + 1.5\,b^2\right]$$

$$- \frac{Wb^3}{6EI}(x-b)(x>b) \qquad \text{(distributed load)}$$

$$y_3 = \frac{MX_3^2}{2EI} + \frac{Mc}{EI}(x-c)(x>c) \qquad \text{(applied moment)}$$

$$\theta = \theta_1 + \theta_2 + \theta_3 \qquad \text{(total slope)}$$

$$\theta_1 = \frac{PX_1}{2EI}(X_1 - 2a) \qquad \text{(slope due to point load)}$$

$$\theta_2 = \frac{WX_2}{EI}\left[X_2\left(\frac{X_2}{6} - \frac{b}{2}\right) + \frac{b^2}{2}\right] \qquad \text{(distributed load)}$$

$$\theta_3 = \frac{MX_3}{EI} \quad \text{(applied moment)}$$

$$M_x = M_{x1} + M_{x2} + M_{x3} \quad \text{(total moment)}$$

$$M_{x1} = P(X_1 - a) \quad \text{(moment due to point load)}$$

$$M_{x2} = -W\,(X_2\,(X_2/2 - b) + b^2/2) \quad \text{(distributed load)}$$

$$M_{x3} = M\,(x \leqslant c) \quad \text{(applied moment)}$$

$$V = V_1 + V_2 + V_3 \quad \text{(total shear)}$$

$$V_1 = P\,(x \leqslant a) \quad \text{(shear due to point load)}$$

$$V_2 = W\,(b - X_2) \quad \text{(distributed load)}$$

$$V_3 = 0 \quad \text{(applied moment)}$$

where:

y is the deflection at a distance x from the wall;

θ is the slope (change in y per change in x) at x;

M_x is the moment at x;

V is the shear at x;

I is the moment of inertia of the beam;

E is the modulus of elasticity of the beam;

ℓ is the length of the beam;

P is a concentrated load;

W is a uniformly distributed load with dimensions of force per unit length.

M is an applied moment;

a is the distance from the foundation to the point load;

b is the distance to the end of the distributed load;

c is the distance to the applied moment;

X_1 = x if x $\leqslant$ a or a if x $>$ a;

X_2 = x if x $\leqslant$ b or b if x $>$ b

X_3 = x if x $\leqslant$ c or c if x $>$ c.

*The notation (x $>$ a) is interpreted as 1.00 if x is greater than a and as 0.00 if x is less than or equal to a.

Remarks:

Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.

Registers R_{S0}–R_{S9} are available for user storage.

SIGN CONVENTIONS FOR BEAMS

NAME	VARIABLE	SENSE	SIGN
DEFLECTION	y	↑	+
SLOPE	θ	↑	+
INTERNAL MOMENT	M_x	↺▬↻	+
SHEAR	V	↑▬↓	+
EXTERNAL FORCE OR LOAD	P or W	↓	+
EXTERNAL MOMENT	M	↻	+

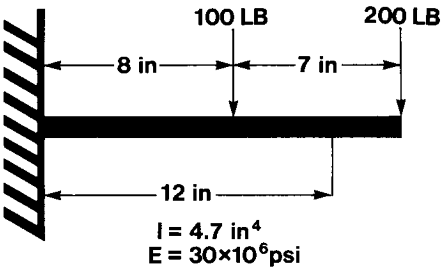
Sums of y, θ, M_x and V may be stored in R_6, R_7, R_8, and R_9, respectively. Note that these registers are indicated on the magnetic card.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Initialize.		f A	0.000 00
3	Input moment of inertia	I	ENTER↑	I
	then modulus of elasticity	E	ENTER↑	E
	then beam length.	l	f B	EI
4	Input load(s):			
	Location of point load	a	ENTER↑	a
	Point load	P	f C	a
	Length of distributed load	b	ENTER↑	b
	Distributed load (force/length)	W	f D	b
	Location of applied moment	c	ENTER↑	c
	Applied moment	M	f E	c
5	Key in x to specify the point			
	of interest and calculate			
	deflection	x	A	y
	or slope	x	B	θ

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	or moment	x	C	M_x
	or shear.	x	D	V
6	For a new calculation with the			
	same loading, go to step 5.			
	For new loads, go to step 4.			
	Be sure to set obsolete			
	loadings to zero. For new			
	beam properties, go to step 3.			
	To restart, go to step 2.			

Example 1:

What is the deflection at x = 12? Neglect the weight of the beam.



Keystrokes: **Outputs:**

f A 4.7 ENTER↑ 30 EEX
6 ENTER↑ 15 f B ⟶ 141.0 06

Compute deflection at 12 inches due to 100 lb weight:
8 ENTER↑ 100 f C 12 A ⟶ –211.8 –06

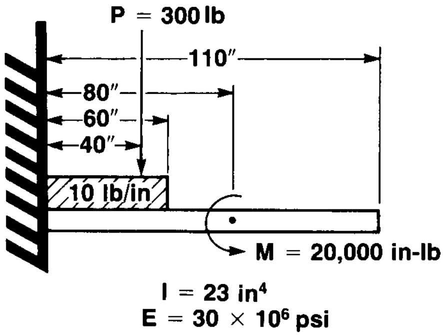
Store deflection due to 100 lb load for addition to deflection due to 200 lb load:
STO 9 ⟶ –211.8–06

Compute deflection at 12 inches due to 200 lb load:
15 ENTER↑ 200 f C 12 A ⟶ –1.123 –03

Compute total deflection:
RCL 9 + ⟶ –1.335 –03

Example 2:

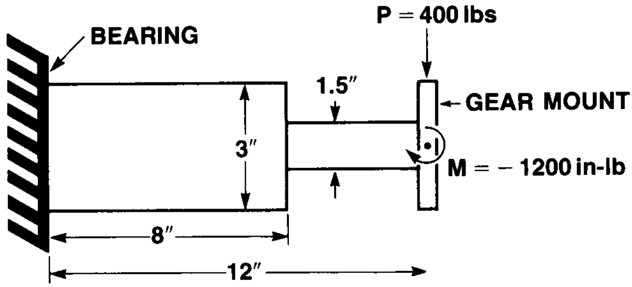
For the beam below, compute deflection, slope, moment and shear at 0, 50, and 90 inches. Neglect the weight of the beam.



Keystrokes:	Outputs:
f A 23 ENTER↑ 30 EEX	
6 ENTER↑ 110 f B 40 ENTER↑	
300 f C 60 ENTER↑ 10 f D	
80 ENTER↑ 20000 f E	
0 A ⟶	0.000 00 (y)
0 B ⟶	0.000 00 (θ)
0 C ⟶	−10.00 03 (M_x)
0 D ⟶	900.00 00 (V)
50 A ⟶	5.211 −03
50 B ⟶	582.1 −06
50 C ⟶	19.50 03
50 D ⟶	100.0 00
90 A ⟶	50.14 −03
90 B ⟶	1.449 −03
90 C ⟶	0.000 00
90 D ⟶	0.000 00

Example 3:

The axle for a gear has the cross sectional shape and properties below. Assuming that the shaft may be modeled as a cantilever, calculate the deflection and slope at the gear mount and the moment and shear at the bearing. Neglect the weight of the axle.



$$E = 30 \times 10^6 \text{ psi}$$
$$I_{0-8} = 3.98 \text{ in}^4$$
$$I_{8-12} = 0.25 \text{ in}^4$$

Keystrokes: **Outputs:**

First compute the deflection and slope from 0 to 8 inches based on larger cross section.

f A 3.98 ENTER↑ 30 EEX
6 ENTER↑ 12 f B 12 ENTER↑
400 f C 12 ENTER↑ 1200
CHS f E 8 A STO 6 ⟶ −1.322 −03 (y_8)
8 B STO 7 ⟶ −294.8 −06 (θ_8)

Compute the deflection at 12 inches assuming no bending occurs from 8 to 12 inches.

4 × RCL 6 + STO 6 ⟶ −2.501 −03 (y_{12})

Compute the moment and shear at the bearing.

0 C ⟶ −6.000 03 (M_0)
0 D ⟶ 400.0 00 (V_0)

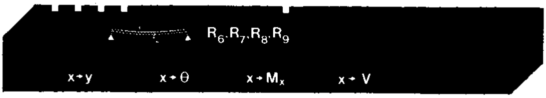
Change to smaller cross section and move origin to shoulder between large and small members.

.25 ENTER↑ 30 EEX 6 ENTER↑
4 f B ⟶ 7.500 06

Add deflection and slope at 12 inches based on smaller cross section to values previously stored for large cross section.

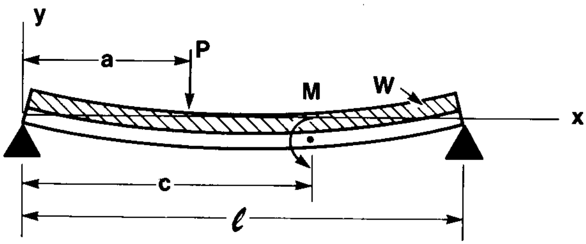
4 A ⟶ −5.831 −03
STO + 6 RCL 6 ⟶ −8.333 −03 (y_{12})
4 B ⟶ −2.773 −03
STO + 7 RCL 7 ⟶ −3.068 −03 (θ_{12})

SIMPLY SUPPORTED BEAMS



This program calculates deflection, slope, moment and shear at any specified point along a simply supported beam of uniform cross section. Distributed loads, point loads, applied moments or combinations of all three may be modeled. By using the principle of superposition, complicated beams with multiple point loads, and multiple applied moments can be analyzed.

Equations:



$$y = y_1 + y_2 + y_3 \qquad \text{(total deflection)}$$

$$y_1 = \frac{P(\ell - a)x}{6EI\,\ell}\left[x^2 + (\ell - a)^2 - \ell^2\right]^* \qquad \text{(deflection due to point load)}$$

$$y_2 = \frac{-Wx}{24EI}\left[\ell^3 + x^2(x - 2\ell)\right] \qquad \text{(distributed load)}$$

$$y_3 = \frac{-Mx}{EI}\left[c - \frac{x^2}{6\ell} - \frac{\ell}{3} - \frac{c^2}{2\ell}\right]^{**} \qquad \text{(applied moment)}$$

$$\theta = \theta_1 + \theta_2 + \theta_3 \qquad \text{(total slope)}$$

$$\theta_1 = \frac{P(\ell - a)}{6EI}\left[3x^2 + (\ell - a)^2 - \ell^2\right]^* \qquad \text{(slope due to point load)}$$

$$\theta_2 = -\frac{W}{24EI}\left[\ell^3 + x^2(4x - 6\ell)\right] \qquad \text{(distributed load)}$$

$$\theta_3 = \frac{-M}{EI}\left[c - \frac{x^2}{2\ell} - \frac{\ell}{3} - \frac{c^2}{2\ell}\right]^{**} \qquad \text{(applied moment)}$$

$$M_x = M_{x1} + M_{x2} + M_{x3} \qquad \text{(total moment)}$$

$$M_{x1} = \frac{P(\ell - a)x^{*}}{\ell} \qquad \text{(moment due to point load)}$$

$$M_{x2} = -\frac{Wx}{2}\left[x - \ell\right] \qquad \text{(distributed load)}$$

$$M_{x3} = \frac{Mx^{**}}{\ell} \qquad \text{(applied moment)}$$

$$V = V_1 + V_2 + V_3 \qquad \text{(total shear)}$$

$$V_1 = \frac{P(\ell - a)^{*}}{\ell} \qquad \text{(shear due to point load)}$$

$$V_2 = W\left(\frac{\ell}{2} - x\right) \qquad \text{(distributed load)}$$

$$V_3 = \frac{M}{\ell} \qquad \text{(applied moment)}$$

where:

y is the deflection at a distance x from the left support;

θ is the slope (change in y per change in x) at x;

M_x is the moment at x;

V is the shear at x;

I is the moment of intertia of the beam;

E is the modulus of elasticity of the beam;

ℓ is the length of the beam;

P is a concentrated load;

W is a uniformly distributed load with dimensions of force per unit length;

M is an applied moment;

a is the distance from the left support to the point load;

c is the distance to the applied moment.

*If x is greater than a, ($\ell - a$) is replaced by $-a$ and x is replaced by ($x - \ell$).

**If x is greater than c, x is replaced by ($x - \ell$) and c is replaced by ($\ell - c$).

Remarks:

Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.

Registers R_{S0}–R_{S9} are available for user storage.

Sums of y, θ, M_x and V may be stored in R_6, R_7, R_8, and R_9, respectively. Note that these registers are indicated on the magnetic card.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Initialize.		f A	0.000 00
3	Input moment of inertia	I	ENTER↑	I
	then modulus of elasticity	E	ENTER↑	E
	then beam length.	*l*	f B	EI
4	Input load(s):			
	Location of point load	a	ENTER↑	a
	Point load	P	f C	a
	Distributed load (force/length)	W	f D	W
	Location of applied moment	c	ENTER↑	c
	Applied moment	M	f E	c
5	Key in x to specify the point of			
	interest and calculate			
	deflection	x	A	y
	or slope	x	B	θ
	or moment	x	C	M_x
	or shear.	x	D	V
6	For a new calculation with the			
	same loading, go to step 5.			
	For new loads, go to step 4. Be			
	sure to set obsolete loadings			
	to zero. For new beam properties,			
	go to step 3. To restart, go to			
	step 2.			

Example 1:

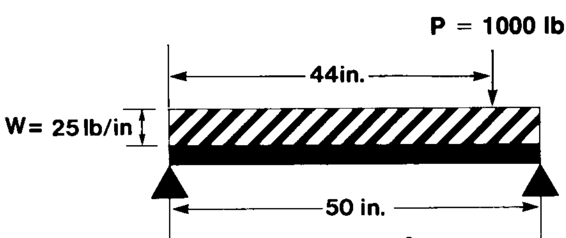
Find the deflection, slope, internal moment and shear at distances of 0, 24 and 60 inches for the beam below. Neglect the weight of the beam.

E = 30×10^6 psi
I = 0.92 in^4

Keystrokes:	Outputs:	
f A .92 ENTER↑ 30 EEX		
6 ENTER↑ 72 f B ⟶	27.60 06	
40 ENTER↑ 10000 f E ⟶	40.00 00	
0 A ⟶	0.000 00	(y_o)
0 B ⟶	-1.771 -03	(θ_o)
0 C ⟶	0.000 00	(M_o)
0 D ⟶	138.9 00	(V_o)
24 A ⟶	-30.92 -03	(y_{24})
24 B ⟶	-322.1 -06	(θ_{24})
24 C ⟶	3.333 03	(M_{24})
24 D ⟶	138.9 00	(V_{24})
60 A ⟶	2.415 -03	(y_{60})
60 B ⟶	40.26 -06	(θ_{60})
60 C ⟶	-1.667 03	(M_{60})
60 D ⟶	138.9 00	(V_{60})

Example 2:

What is the slope of the beam below at x = 38 inches?

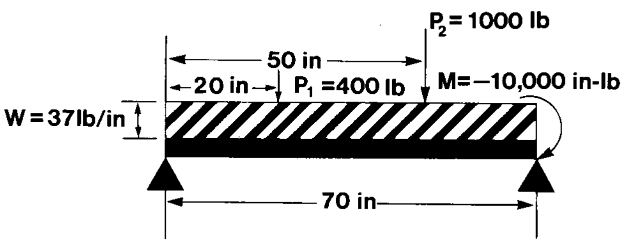


E = 30×10^6 psi
I = 1.30 in^4

Keystrokes:	**Outputs:**	
f A 1.30 ENTER↑ 30 EEX		
6 ENTER↑ 50 f B →	39.00 06	
44 ENTER↑ 1000 f C →	44.00 00	
25 f D →	25.00 00	
38 B →	3.327 -03	(in/in)

Example 3:

What is the total moment at the center of the beam below? (It is not necessary to know the values of E or I to solve the problem. Simply key in 70 and press f B.)



First solve for the effect of the distributed load, P_1, and M.

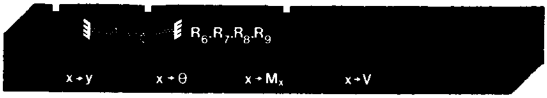
Keystrokes:	**Outputs:**	
f A 70 f B 20 ENTER↑		
400 f C →	20.00 00	
37 f D 70 ENTER↑		
10000 CHS f E →	70.00 00	
70 ENTER↑ 2 ÷ C →	21.66 03	
Store values in R_6.		
STO 6 →	21.66 03	(in-lb)

Now solve for the effect of P_2 and add it to the content of R_6. This is the final answer assuming superposition is valid.

f A 50 ENTER↑ 1000 f C →	50.00 00	
35 C →	10.00 03	(in-lb)
RCL 6 + →	31.66 03	(in-lb)

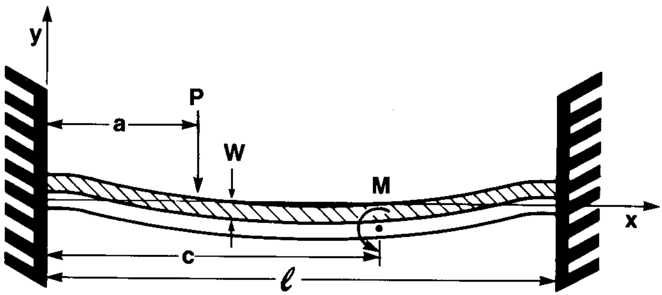
Notes

BEAMS FIXED AT BOTH ENDS



This program calculates deflection, slope, moment and shear at any specified point along a beam of uniform cross section, fixed at both ends. Distributed loads, point loads, applied moments or combinations of all three may be modeled. By using the principle of superposition, complicated beams with multiple point loads, and multiple applied moments can be analyzed.

Equations:



$y = y_1 + y_2 + y_3$ (total deflection)

$$y_1 = \frac{P(\ell - a)^2 x^2}{6EI\ell^3}\left[x(\ell+2a) - 3a\ell\right]^{*} \quad \text{(deflection due to point load)}$$

$$y_2 = \frac{Wx^2}{24EI}\left[x(2\ell-x) - \ell^2\right] \quad \text{(distributed load)}$$

$$y_3 = \frac{M(\ell - c)x^2}{\ell^2 EI}\left[\frac{cx}{\ell} + \frac{\ell - 3c}{2}\right]^{**} \quad \text{(applied moment)}$$

$\theta = \theta_1 + \theta_2 + \theta_3$ (total slope)

$$\theta_1 = \frac{P(\ell - a)^2 x}{2EI\ell^3}\left[x(\ell+2a) - 2a\ell\right]^{*} \quad \text{(slope due to point load)}$$

$$\theta_2 = \frac{Wx}{12EI}\left[x(3\ell - 2x) - \ell^2\right] \quad \text{(distributed load)}$$

$$\theta_3 = \frac{M(\ell - c)x}{\ell^2 EI}\left[\frac{3cx}{\ell} + \ell - 3c\right]^{**} \quad \text{(applied moment)}$$

$$M_x = M_{x1} + M_{x2} + M_{x3} \qquad \text{(total moment)}$$

$$M_{x1} = \frac{P(\ell - a)^2}{\ell^3}\left[x(\ell + 2a) - a\ell\right]^* \qquad \text{(moment due to point load)}$$

$$M_{x2} = \frac{W}{12}\left[6x(\ell - x) - \ell^2\right] \qquad \text{(distributed load)}$$

$$M_{x3} = \frac{M(\ell - c)}{\ell^2}\left[\frac{6cx}{\ell} + \ell - 3c\right]^{**} \qquad \text{(applied moment)}$$

$$V = V_1 + V_2 + V_3 \qquad \text{(total shear)}$$

$$V_1 = \frac{P(\ell - a)^2}{\ell^3}(\ell + 2a) \qquad \text{(shear due to point load)}$$

$$V_2 = \frac{-W}{2}(2x - \ell) \qquad \text{(distributed load)}$$

$$V_3 = \frac{-6M(\ell - c)\,c^{**}}{\ell^3} \qquad \text{(applied moment)}$$

where:

- y is the deflection at a distance x from the left support;
- θ is the slope (change in y per change in x) at x;
- M_x is the moment at x;
- V is the shear at x;
- I is the moment of inertia of the beam;
- E is the modulus of elasticity of the beam;
- ℓ is the length of the beam;
- P is a concentrated load;
- W is a uniformly distributed load with dimensions of force per unit length;
- M is an applied moment;
- a is the distance from the left support to the point load;
- c is the distance to the applied moment.

*If x is greater than a, a is replaced by $(\ell - a)$ and x is replaced by $(\ell - x)$. The signs of θ_1 and V_1 are also changed.

**If x is greater than c, x is replaced by $(\ell - x)$ and c is replaced by $(\ell - c)$. The signs of y_3 and M_{x3} are also changed.

Remarks:

This card differs from other beam cards. The "start" function is not included on LBL f A. You must manually perform the "start" function by storing zero when P, W or M are not included in the problem.

Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.

Registers R_{S0}–R_{S9} are available for user storage.

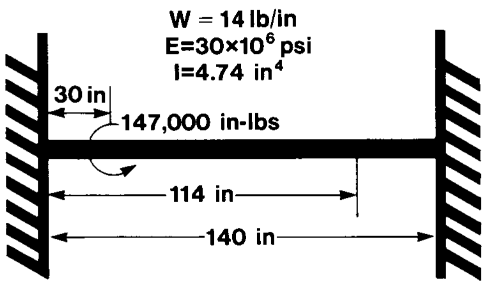
Sums of y, θ, M_x and V may be stored in R_6, R_7, R_8, R_9, respectively. Note that these registers are indicated on the magnetic card.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Input moment of inertia	I	ENTER↑	I
	then modulus of elasticity	E	ENTER↑	E
	then beam length.	l	f B	EI
3	Input load(s):*			
	Location of point load	a	ENTER↑	a
	Point load	P	f C	a
	Distributed load (force/length)	W	f D	W
	Location of applied moment	c	ENTER↑	c
	Applied moment	M	f E	c
4	Key in x to specify the point			
	of interest and calculate			
	deflection	x	A	y
	or slope	x	B	θ
	or moment	x	C	M_x
	or shear.	x	D	V

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
5	For a new calculation with the			
	same loading, go to step 4. For			
	new loads, go to step 3. Be			
	sure to set obsolete loadings to			
	zero. For new beam properties,			
	go to step 2.			
	*Loads must be input, even if			
	zero.			

Example 1:

For the beam below, what are the values of deflection, slope, moment, and shear at an x of 114 inches?

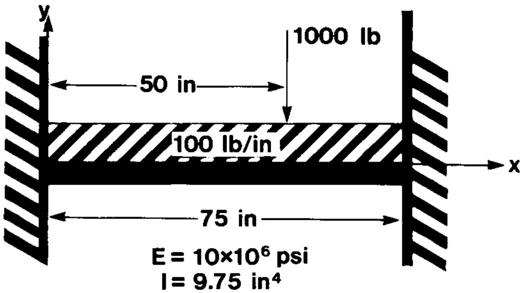


Keystrokes: **Outputs:**

Keystrokes	Outputs	
4.74 ENTER↑ 30 EEX 6 ENTER↑		
140 f B ⟶	142.2 06	
0 f C 30 ENTER↑ 147000 f E		
14 f D ⟶	14.00 00	
114 A ⟶	43.72 −03	(y)
RCL 0 B ⟶	−3.155−03	(θ)
RCL 0 C ⟶	13.05 03	(M_x)
RCL 0 D ⟶	444.7 00	(V)

Example 2:

Find the internal moment at x = 0 for the configuration below.



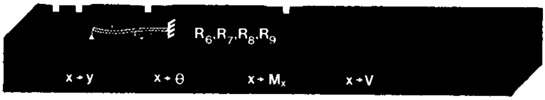
Keystrokes:	Outputs:
9.75 ENTER↑ 10 EEX 6 ENTER↑	
75 f B →	97.50 06
0 f E 100 f D 50 ENTER↑	
1000 f C →	50.00 00
0 C →	–52.43 03 (M_0)

Also, find the deflection at x = 40.

40 A →	–101.0 –03 (Y_{40})

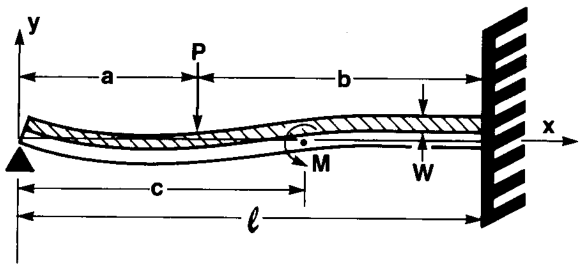
Notes

PROPPED CANTILEVER BEAMS



This program calculates deflection, slope, moment and shear at any specified point along a propped cantilever beam of uniform cross section. Distributed loads, point loads, applied moments or combinations of all three may be modeled. By using the principle of superposition, complicated beams with multiple point loads, and multiple applied moments can be analyzed.

Equations:



$$y = y_1 + y_2 + y_3 \qquad \text{(total deflection)}$$

$$y_1 = \frac{P}{6EI}\left[F(x^3 - 3\ell^2 x) + 3b^2x\right]; \quad x \leqslant a \qquad \text{(deflection due to point load)}$$

$$y_2 = \frac{W}{48EI}(3\ell x^3 - 2x^4 - \ell^3 x) \qquad \text{(distributed load)}$$

$$y_3 = \frac{M}{EI}\, G(x^3 - 3\ell^2 x) + \ell x - cx; \quad x \leqslant c \qquad \text{(applied moment)}$$

$$y_3 = \frac{M}{EI}\, G(x^3 - 3\ell^2 x) + \ell x - ½\,(x^2 + c^2); \quad x > c$$

$$\theta = \theta_1 + \theta_2 + \theta_3 \qquad \text{(total slope)}$$

$$\theta_1 = \frac{P}{6EI}\left[F(3x^2 - 3\ell^2) + 3b^2\right]; \quad x \leqslant a \qquad \text{(slope due to point load)}$$

$$\theta_1 = \frac{P}{6EI}\left[F(3x^2 - 3\ell^2) - 3(x - a)^2\right];\ x > a$$

$$\theta_2 = \frac{W}{48EI}(9\ x^2 - 8x^3 - \ell^3) \qquad \text{(distributed load)}$$

$$\theta_3 = \frac{M}{EI}\left[G(3x^2 - 3\ell^2) + \ell - c\right];\ x \leqslant c \qquad \text{(applied moment)}$$

$$\theta_3 = \frac{M}{EI}\left[G(3x^2 - 3\ell^2) + \ell - x\right];\ x > c$$

$M_x = M_{x1} + M_{x2} + M_{x3}$ (total moment)

$M_{x1} = PFx;\ x \leqslant a$ (moment due to point load)

$M_{x1} = PFx - P(x - b);\ x > a$

$M_{x2} = W\ (3/8x\ \ell - x^2/2)$ (distributed load)

$M_{x3} = 6MGx;\ x \leqslant c$ (applied moment)

$M_{x3} = 6MGx - M;\ x > c$

$V = V_1 + V_2 + V_3$ (total shear)

$V_1 = PF;\ x \leqslant a$ (shear due to point load)

$V_1 = PF - P;\ x > a$

$$V_2 = W\left(\frac{3}{8}\ \ell - x\right) \qquad \text{(distributed load)}$$

$V_3 = 6MG$ (applied moment)

$$F = \left[\frac{3b^2\ \ell - b^3}{2\ell^3}\right]$$

$b = (\ \ell - a)$

$$G = \frac{\ell^2 - c^2}{4\ell^3}$$

where:

- y is the deflection at a distance x from the left support;
- θ is the slope (change in y per change in x) at x;
- M_x is the moment at x;
- V is the shear at x;
- I is the moment of inertia of the beam;
- E is the modulus of elasticity of the beam;
- ℓ is the length of the beam;
- P is a concentrated load;
- W is a uniformly distributed load with dimensions of force per unit length;
- M is an applied moment;
- a is the distance from the left support to the point load;
- c is the distance to the applied moment.

Remarks;

Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.

Registers R_{S0}–R_{S9} and R_B are available for user storage.

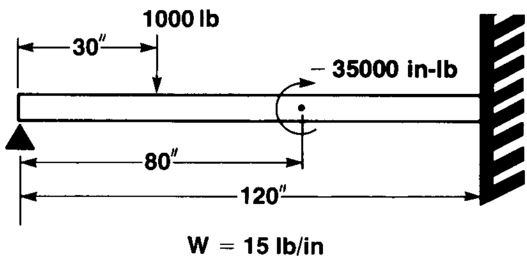
Sums of y, θ, M_X and V may be stored in R_6, R_7, R_8 and R_9, respectively. Note that those registers are indicated on the magnetic card.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Initialize.		f A	0.000 00
3	Input moment of inertia	I	ENTER↑	I
	then modulus of elasticity	E	ENTER↑	E
	then beam length.	l	f B	EI
4	Input load(s):			
	Location of point load	a	ENTER↑	a
	Point load	P	f C	a
	Distributed load (force/length)	W	f D	W
	Location of applied moment	c	ENTER↑	c
	Applied moment.	M	f E	c

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
5	Key in x to specify the point of			
	interest and calculate			
	deflection	x	A	y
	or slope	x	B	θ
	or moment	x	C	M_x
	or shear.	x	D	V
6	For a new calculation with the			
	same loading, go to step 5.			
	For new loads, go to step 4.			
	Be sure to set obsolete			
	loadings to zero. For new			
	beam properties, go to step 3.			
	To restart, go to step 2.			

Example 1:

What are the values of moment and shear at both ends of the beam below? (It is not necessary to know the values of E or I since deflection and slope are not required.)

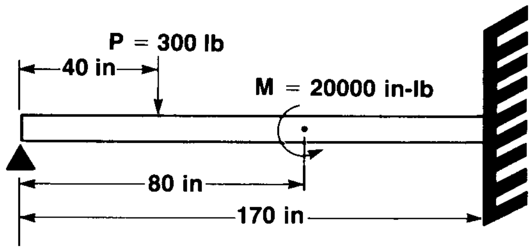


Keystrokes: **Outputs:**

Keystrokes	Outputs	
f A 120 f B 30 ENTER↑		
1000 f C ——→	30.00 00	
80 ENTER↑ 35000 CHS f E		
15 f D ——→	15.00 00	
0 C ——→	0.000 00	(in-lb)
0 D ——→	1.065 03	(lb)
120 C ——→	−35.23 03	(in-lb)
120 D ——→	−1.735 03	(lb)

Example 2:

Calculate the deflection, slope, moment and shear at x = 90 for the beam below.



W = 10 lb/in
$E = 30 \times 10^6$ psi
$I = 23\ in^4$

Keystrokes:	**Outputs:**	
f A 23 ENTER↑ 30 EEX 6 ENTER↑		
170 f B →	690.0 06	
40 ENTER↑ 300 f C 10 f D		
80 ENTER↑ 20000 f E →	80.00 00	
90 A →	-75.73 -03	(in)
90 B →	920.8 -06	(in/in)
90 C →	11.89 03	(in-lb)
90 D →	-229.0 00	(lb)

Notes

HELICAL SPRING DESIGN

HELICAL SPRING DESIGN

→N →s_S →code (→L_f,L_s,D,OD)

This program performs one or two point design for helical compression springs, of round wire, with ends square and ground.

After a tentative spring design has been found, a check can be run to determine whether stresses are acceptable, and whether sufficient clearance between coils is available at the point of highest operating load.

Equations:

$$k = \frac{P_2 - P_1}{L_1 - L_2}$$

$$s_2 = \frac{8\ P_2\ D_H}{\pi d^3}$$

$$D = D_H f_0 - d$$

$$N = \frac{Gd^4}{8\ D^3 k}$$

$$L_s = (N + 2)\ d$$

$$L_f = \frac{P_1}{k} + L_1$$

$$s_S = \frac{8\ D\ k\ (L_f - L_s)\ W}{\pi d^3}$$

$$W = \frac{4\ (D/d) - 1}{4\ (D/d) - 4} + \frac{0.615}{(D/d)}$$

$$s_{max} = \begin{cases} .45\ \text{TS for ferrous materials.} \\ .35\ \text{TS for non-ferrous materials.} \end{cases}$$

$$YS = \begin{cases} .65\ \text{TS for ferrous materials.} \\ .55\ \text{TS for non-ferrous materials.} \end{cases}$$

$$TS = \beta \ln d + \alpha$$

Design checking logic:

If $(L_2 - L_s) < 0.1\,(L_f - L_2)$ and $s_s > s_{max}$, the spring lacks sufficient clearance between coils and stresses are too high; code = 1.

If $(L_2 - L_s) < 0.1\,(L_f - L_2)$ and $s_s \leqslant s_{max}$, clearance between coils is insufficient; code = 2.

If $(L_2 - L_s) \geqslant 0.1\,(L_f - L_2)$ and $s_s > YS$, stress is too high; code = 3.

If $(L_2 - L_s) \geqslant 0.1\,(L_f - L_2)$ and $s_s \leqslant YS$, design is satisfactory. If $s_s \leqslant 0.3\,TS$, stresses are quite conservative and code = 4. If $s_s > 0.3\,TS$, design is acceptable and code = 5.

where:

G is the torsional modulus of rigidity;

α and β are tensile strength regression coefficients from table 1 (metric) or table 2 (English);

P_1 is the spring load at most extended operating point (see figure 1);

L_1 is spring length, at the most extended operating point;

P_2 is spring load at most compressed operating point;

L_2 is the spring length, at the most compressed operating point;

k is the spring constant;

d is the wire diameter;

f_0 is the clearance factor for the spring and the hole (possibly imaginary) in which the spring is designed to work:

$$f_0 = \begin{cases} 0.95 \text{ if } D_H \geqslant 12.70 \text{ mm (0.5 in)} \\ 0.90 \text{ if } D_H < 12.70 \text{ mm (0.5 in);} \end{cases}$$

D_H is the diameter of the hole (possibly imaginary) into which the spring must fit;

s_2 is the uncorrected stress at operating point 2;

N is the number of active coils;

s_s is the Wahl corrected stress when the spring is fully compressed to solid (coils touching);

L_f is the free length of the spring;

L_s is the fully compressed or solid spring length;

D is the mean spring diameter;

OD is the outside spring diameter;

Code is a digit from 1-5, explained in program User Instructions;

W is the Wahl factor which corrects stresses for curvature;

s_{max} is the maximum allowable working stress for the material;

YS is the yield strength of the material;

TS is the tensile strength of the material.

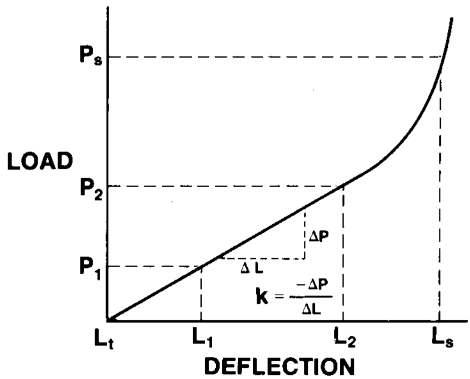


Figure 1-Spring Deflection

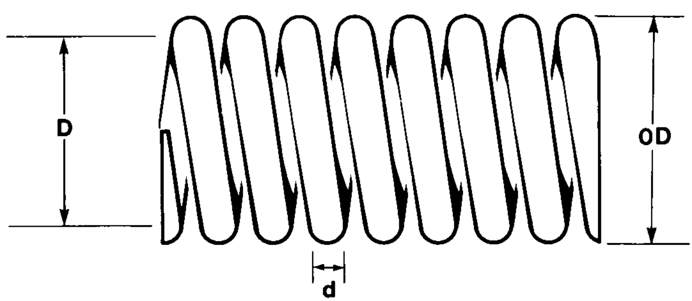


Figure 2-Helical Compression Spring

Table 1
MINIMUM TENSILE STRENGTH REGRESSION COEFFICIENTS
(Metric Units)

MATERIAL	MODULUS OF RIGIDITY G,N/(mm)^2	WIRE DIAMETER RANGE— MILLIMETERS	TENSILE STRENGTH COEF. α,N/(mm)^2	TENSILE STRENGTH COEF. β,N/(mm)^2
Music Wire ASTM-A228	7.93×10^4	0.41–6.35	2205	−346.1
Alloy Steel ASTM-A232	7.93×10^4	0.64–7.62	1921	−249.7
Stainless Steel ASTM-A313	6.90×10^4	0.41–1.91	1851	−209.6
		1.91–5.08	1950	−393.6
		5.08–9.40	2221	−560.4
Oil Tempered ASTM-A229	7.93×10^4	0.51–6.86	1827	−304.7
Hard Drawn ASTM-A227	7.93×10^4	0.51–3.56	1773	−283.4
		3.56–12.7	1757	−270.8
Tempered Valve Spring ASTM-A230	7.93×10^4	2.36–5.08	1586	−153.1
Phosphor Bronze ASTM-B159	4.07×10^4	0.64–9.40	957	− 63.97

Table 2
MINIMUM TENSILE STRENGTH REGRESSION COEFFICIENTS
(English Units)

MATERIAL	MODULUS OF RIGIDITY G,psi	WIRE DIAMETER RANGE—INCHES	TENSILE STRENGTH COEF.	
			α,psi	β,psi
Music Wire ASTM-A228	11.5 × 10^6	0.016–0.25	157400	-50200
Alloy Steel ASTM-A232	11.5 × 10^6	0.025–0.30	161400	-36220
Stainless Steel ASTM-A313	10.0 × 10^6	0.016–0.075	170200	-30400
		0.075–0.20	98110	-57090
		0.20 –0.37	59190	-81280
Oil Tempered ASTM-A229	11.5 × 10^6	0.020–0.27	122100	-44190
Hard Drawn ASTM-A227	11.5 × 10^6	0.020–0.14	124200	-41110
		0.14 –0.50	127800	-39280
Tempered Valve Spring ASTM-A230	11.5 × 10^6	0.093–0.20	158300	-22200
Phosphor Bronze ASTM-B159	5.9 × 10^6	0.025–0.37	108800	-9278

Reference:

Design Handbook–Springs, Custom Metal Parts, Associated Spring Corporation, Bristol, Connecticut, 1970.

Remarks:

Registers R_{s0}–R_{s9} are available for user storage.

The assumptions implicit to this program are based on engineering practice and experience. Generally, designs found by this program will be conservative, however, caution must be exercised when high or low temperatures, corrosive media or other adverse environmental circumstances exist.

For one point design, specify the free length (L_1) and a corresponding zero load (P_1), then specify the length (L_2) and corresponding load (P_2).

Some designs achieved by this program may require coiling the spring wire in such a small radius that the spring material would fail in the manufacturing process. No program check is made for this condition.

If code = 2, then s_2 has no intelligent meaning.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Toggle ferrous (1) or non-			
	ferrous (0) material mode			
	(consider stainless steel			
	non-ferrous).		f A	1.00/0.00
3	Specify material properties			
	from table 1 (Metric) or			
	table 2 (English):			
	Modulus of rigidity	G	ENTER↑	G
	Tensile strength Alpha	α	ENTER↑	α
	Tensile strength Beta	β	f B	G
4	Input load point 1:			
	Force 1	P_1	ENTER↑	P_1
	Corresponding spring			
	length 1	L_1	f C	P_1
5	Input load Point 2 and cal-			
	culate spring constant:			
	Force 2	P_2	ENTER↑	P_2
	Corresponding spring			
	length 2	L_2	f D	k
6	Input wire diameter,	d	ENTER↑	d
	and clearance factor (f_0 = 0.90			
	if spring diameter < 12.70mm			
	(0.5 in); otherwise, f = 0.95).,	f_0	ENTER↑	f_0
	and maximum outside spring			
	diameter.	D_H	f E	s_2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
7	If s_2 is a reasonable value			
	(not extremely high or low for			
	your application), proceed to			
	step 8. Otherwise, you may wish			
	to modify the design specifica-			
	tions in steps 4, 5 or 6.			
8	Compute number of coils.		A	N
9	Compute stress at solid			
	(maximum).		B	s_s
10	Check design.		C	Code
11	If code = 1, the design is			
	over constrained. The specified			
	conditions cannot be met. Try			
	another material, larger D_H,			
	new load points or another type			
	of spring.			
	If code = 2, clearance between			
	coils is not sufficient. Press			
	R↓ to see current wire			
	diameter.		R↓	d
	Key in a smaller wire diameter			
	and calculate a new N. Go			
	back to instruction step 9.	d	R/S	N

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	If code = 3, stress at solid			
	is too high. Press R↓ to			
	see the current wire diameter.		R↓	d
	Key in a larger wire dia-			
	meter and calculate a new N.	d	R/S	N
	Go back to instruction			
	step 9.			
	If code = 4, design is			
	acceptable but smaller wire			
	might also work. Press R↓			
	to see current wire diameter.		R↓	d
	Key in a new smaller wire			
	diameter and calculate N.	d	R/S	N
	Go back to instruction			
	step 9.			
	If code 5, design is			
	acceptable but not necessarily			
	optimal. Try manipulating			
	design parameters to obtain a			
	more economical design.			
12	Display free length, solid			
	length, mean diameter, and			
	outside diameter.		E	L_f, L_s, D, OD
13	Go to steps 2 through 6 for			
	a new case.			

Example 1:

Using Oil Tempered Wire (ASTM-A229), design a spring which supports a load of 270 newtons at a length of 62 millimeters and a load of 470 newtons at 50 millimeters. Wire is available in 0.5 mm increments. Try 4.0 mm wire first. Space available limits the spring diameter to 40.00 mm.

Variables:

$P_1 = 270$ N

$L_1 = 62$ mm

$P_2 = 470$ N

$L_2 = 50$ mm

$d = 4.0$ mm

$D_H = 40.0$ mm

$f_0 = 0.95$ (since $D_H > 12.70$ mm)

$$\left.\begin{aligned} G &= 7.93 \times 10^4 \text{ N/mm}^2 \\ \alpha &= 1827 \text{ N/mm}^2 \\ \beta &= -304.7 \end{aligned}\right\} \text{From table 1}$$

Keystrokes:	**Outputs:**
Select iron wire (press f A until 1.00 is displayed.)	
f A →	1.00 00
7.93 EEX 4 ENTER↑ 1827 ENTER↑	
304.7 CHS f B →	79.30 03
270 ENTER↑ 62 f C 470 ENTER↑	
50 f D →	16.67 00 *** (k)
4 ENTER↑ .95 ENTER↑ 40 f E →	748.0 00 *** (N/mm², s_2)
A →	3.874 00 *** (Coils)
B →	1.446 03 *** (N/mm², s_S)
C →	3.000 00 (Code)

Since Code = 3, select a larger wire. 4.5 mm wire is the next largest, so give it a try.

4.5 R/S →	6.487 00 *** (Coils)
B →	748.4 00 *** (s_S)
C →	5.000 00 (Code)

Since code = 5, design is acceptable. Output free length, solid length, mean diameter and outside diameter.

Keystrokes	Outputs
E ⟶	78.20 00 *** (L_f)
	38.19 00 *** (L_s)
	33.50 00 *** (D)
	38.00 00 *** (OD)

Example 2:

Using music wire (ASTM-A228), design a spring which will work in a 0.25 inch hole, for the loading below:

P_1 = 1 lb $\quad$ L_1 = 1.5 in

P_2 = 10 lb $\quad$ L_2 = 1.0 in

$$\left.\begin{aligned} G &= 11.5 \times 10^6 \text{ psi} \\ \alpha &= 157.4 \times 10^3 \text{ psi} \\ \beta &= -50.20 \times 10^3 \text{ psi} \end{aligned}\right\} \text{From table 2}$$

d = 0.035 or 0.040

f_0 = 0.90 (from User Instructions)

Keystrokes:	**Outputs:**
Since music wire is a ferrous material, press f A until 1.00 is displayed.	
f A ⟶	1.000 00
11.5 EEX 6 ENTER↑ 157.4 EEX	
3 ENTER↑ 50.20 CHS EEX	
3 f B ⟶	11.50 06
1 ENTER↑ 1.5 f C 10 ENTER↑	
1.0 f D ⟶	18.00 00 *** (k)
.035 ENTER↑ .9 ENTER↑	
.25 f E ⟶	148.5 03 *** (s_2, psi)
A ⟶	17.47 00 *** (Coils)
B ⟶	227.7 03 *** (s_s, psi)
C ⟶	3.000 00 (Code)
Try the larger wire.	
.04 R/S ⟶	32.29 00 *** (Coils)
B ⟶	32.66 03 *** (s_s, psi)
C ⟶	2.000 00 (Code)

Since neither available wire will meet these specifications the specifications must be modified. After due consideration, it is decided that P_2 could be lowered to 9 pounds.

9 ENTER↑ 1 f D →	16.00	00 ***	(k)
.04 ENTER↑ .9 ENTER↑			
.25 f E →	89.52	03 ***	(s_2, psi)
A →	36.33	00 ***	(Coils)
B →	4.651	03 ***	(s_s, psi)

Interestingly, and unfortunately, $s_s < s_2$ indicates that this spring cannot be compressed to s_2.

C →	2.000	00	

Sure enough, insufficient clearance. Try the smaller wire.

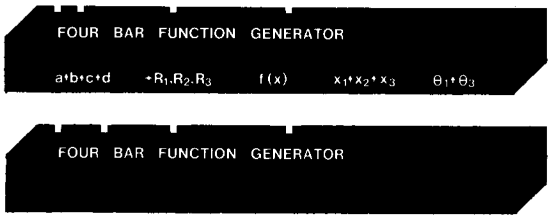
.035 R/S →	21.29	00 ***	(Coils)
B →	169.6	03 ***	(s_s, psi)
C →	5.000	00	(Code)

Since the design checks out, calculate the dimensions:

E →	1.563	00 ***	(L_f)
	815.3	-03 ***	(L_s)
	185.0	-03 ***	(D)
	220.0	-03 ***	(OD)

Notes

FOUR BAR FUNCTION GENERATOR



These cards may be used to design a four bar linkage which will approximate an arbitrary function of one variable. Freudenstein's approach is used in the solution. Cramer's rule is used to solve the 3×3 system of linear equations.

Equations:

Three precision points are used in the solution.

Freudenstein's equations

$$R_1 \cos \theta_1 - R_2 \cos \phi_1 + R_3 = \cos (\theta_1 - \phi_1)$$

$$R_1 \cos \theta_2 - R_2 \cos \phi_2 + R_3 = \cos (\theta_2 - \phi_2)$$

$$R_1 \cos \theta_3 - R_2 \cos \phi_3 + R_3 = \cos (\theta_3 - \phi_3)$$

are solved simultaneously for R_1, R_2 and R_3 which are defined as follows:

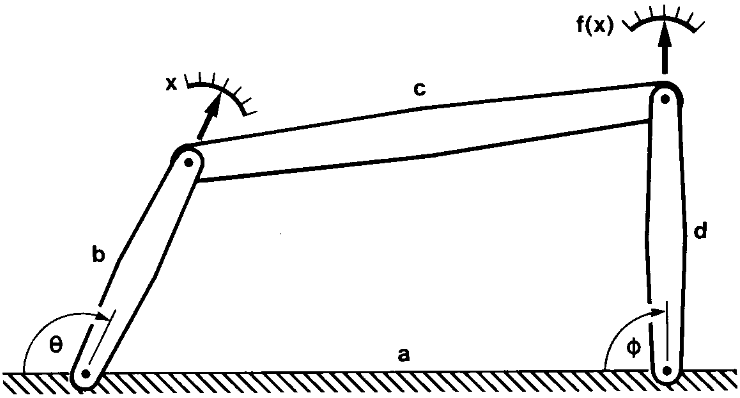
$$R_1 = a/d, \quad R_2 = a/b, \quad R_3 = \frac{a^2 + b^2 + d^2 - c^2}{2bd}$$

where a is the distance between fixed pivots, b is the length of the input link, c is the length of the coupler and d is the length of the output link. θ_1 refers to the angle of the input link at the first precision point, θ_2 the angle at the second point, and θ_3 the angle at the third. ϕ_1 is the angle of the output link at the first precision point, ϕ_2 is the angle at the second point, and ϕ_3 is the angle at the third precision point.

$$\theta_2 = \theta_1 + \frac{x_2 - x_1}{x_3 - x_1} (\theta_3 - \theta_1)$$

$$\phi_2 = \phi_1 + \frac{f(x_2) - f(x_1)}{f(x_3) - f(x_1)} (\phi_3 - \phi_1)$$

x_1, x_2 and x_3 are the precision points or the three points at which the mechanism will yield kinematically exact solutions to the function (f(x)) which is to be generated.



Reference:

Martin, G. H., *Kinematics and Dynamics of Machines* McGraw-Hill, 1969.

Remarks:

f(x) must be stated in 119 or less steps.

$$\left(\cos\,\phi_2 - \frac{\cos\,\phi_1\,\cos\,\theta_2}{\cos\,\theta_1}\right)\left(\frac{\cos\,\theta_3}{\cos\,\theta_1} - 1\right)$$

$$\neq \left(\frac{\cos\,\theta_2}{\cos\,\theta_1} - 1\right)\left(\cos\,\phi_3 - \frac{\cos\,\phi_1\,\cos\,\theta_3}{\cos\,\theta_1}\right)$$

θ_1 may not be equal to 90° or 270°.

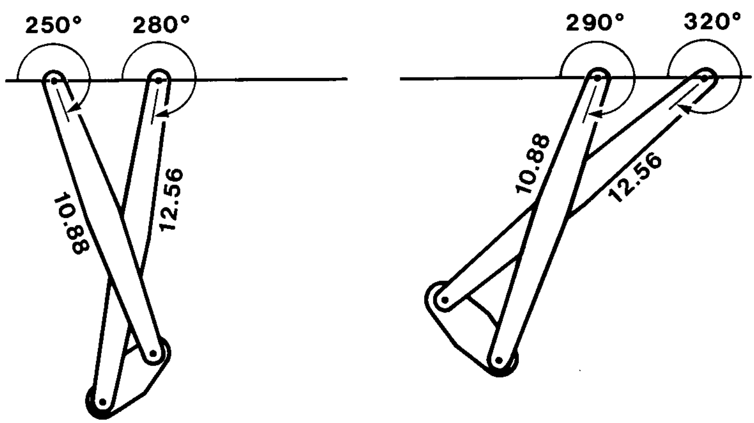
All registers are used.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2 of			
	card 1.			
2	To calculate link ratios, go to			
	step 4.			
3	For function generator, go to			
	step 7.			
4	Input the link lengths.	a	ENTER↑	a
		b	ENTER↑	b
		c	ENTER↑	c
		d	A	a
5	Calculate the link ratios.		B	R_1, R_2, R_3
6	For a new case, go to step 2.			
7	Key the function into memory:			
	i. Go to label C.		GTO C	
	ii. Switch to PRGM mode.			
	iii. Key in the function.*	f(x)		
	iv. Switch to RUN mode.			
	(The argument of the			
	function is in X when the			
	routine is called.)			
8	Input 3 precision points	x_1	ENTER↑	x_1
		x_2	ENTER↑	x_2
		x_3	D	x_1
9	Input starting input angle and			
	final input angle ($\theta_1 \neq 90 \neq 270$)	θ_1	ENTER↑	θ_1
		θ_3	E	θ_2
10	Input starting output angle	ϕ_1	ENTER↑	ϕ_1
	and final output angle.	ϕ_3	f A	ϕ_2
11	Load side 1 and side 2 of			
	card 2.			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
12	Calculate R_1, R_2 and R_3.		f B	R_1, R_2, R_3
13	Input a to calculate b, c,			
	and d.	a	f C	b, c, d
14	For a new case, go to step 1.			
	*119 steps are allowed.			

Example 1:

Suppose the output of a linkage is to be the square root of the input. The input link is to move from 70° to 110° while the output moves from 100° to 140°. Precision points are $x_1 = 3(70°)$, $x_2 = 5$, and $x_3 = 9(110°)$. The distance between foundation pivots is 3.75. What are the remaining link lengths?



Data for input:

$f(x) = \sqrt{x}$

$x_1 = 3,\ x_2 = 5,\ x_3 = 9$

$\theta_1 = 70°,\ \theta_3 = 110°,\ \phi_1 = 100°,\ \phi_3 = 140°$

$a = 3.75$

Keystrokes:	Outputs:
Load side 1 and side 2 of Card 1.	
GTO C	
Switch to PRGM mode.	
$\sqrt{x}$	
Switch to RUN mode.	
3 ENTER↑ 5 ENTER↑ 9 D 70 ENTER↑	
110 E →	83.33 (θ_2)
100 ENTER↑ 140 f A →	115.90 (ϕ_2)
Load side 1 and side 2 of card 2.	
f B →	–0.30 *** (R_1)
	–0.34 *** (R_2)
	1.03 *** (R_3)
3.75 f C →	–10.88 *** (b)
	3.04 *** (c)
	–12.56 *** (d)

Note that should you decide to run the program "PROGRESSION OF FOUR BAR SYSTEM" for the same linkage, then input of a, b, c and d is not necessary since a, b, c and d are already stored in the corresponding registers from this program.

b = –10.88, c = 3.04, d = –12.56 (The negative signs indicate that the links are opposite to the assumed direction i.e., $\theta = 250°$ and $\phi = 280°$).

Example 2:

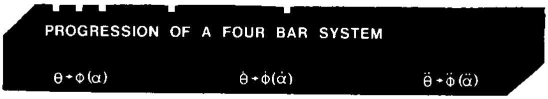
Compute the link ratios for the following link lengths:

a = 1.0
b = 1.371
c = 2.12
d = 1.502

Keystrokes:	Outputs:
Load side 1 and side 2 of Card 1	
DSP 4	
1 ENTER↑ 1.371 ENTER↑	
2.12 ENTER↑ 1.502 A B →	0.6658 *** (R_1)
	0.7294 *** (R_2)
	0.1557 *** (R_3)

Notes

PROGRESSION OF FOUR BAR SYSTEM



This program calculates angular displacement, velocity and acceleration for the output link of a four bar system (figure 1). (Either the "connecting link" (c) or the "output link" (d) may be selected as the program's output link.)

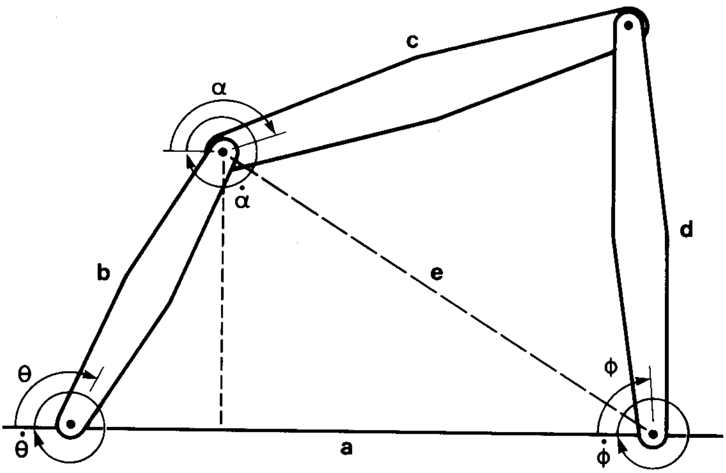


FIGURE 1-FOUR BAR SYSTEM SHOWING POSITIVE ANGULAR CONVENTIONS

Automatic and manual modes of operation are available. In manual mode, the output angle is calculated by keying in the input angle and pressing **A**. The angular output velocity may then be found by keying in the angular input velocity and pressing **C**. After angular velocity is calculated, the output link acceleration is found by keying in the input link acceleration and pressing **E**. In automatic mode, a starting input link angle θ_0, the number of increments n, the angular increment $\Delta\theta$, and the constant input link RPM are input using **f** **E**. The program automatically progresses from θ_0 through n increments of $\Delta\theta$. RPM is output once, followed by groups of four values. The first value, of these four-value groups, is input angle, the second value is output angle, the third value is angular output velocity and the fourth value is angular output acceleration. Example problem 1 demonstrates manual operation while example 2 demonstrates automatic operation.

Equations:

Output Link

$$\phi = \sin^{-1}\left(\frac{b}{e}\sin\theta\right) + \cos^{-1}\left(\frac{d^2 + e^2 - c^2}{2de}\right)$$

Connecting Link

$$\alpha = \sin^{-1}\left(\frac{b}{e}\sin\theta\right) + \cos^{-1}\left(\frac{c^2 + e^2 - d^2}{-2ce}\right)$$

where:

$$e = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

$$\frac{d\phi}{d\theta} = \frac{R_1\sin\theta - \sin(\theta - \phi)}{R_2\sin\phi - \sin(\theta - \phi)}$$

$$R_1 = \frac{a}{d} \qquad R_2 = \frac{a}{b}$$

$$\frac{d\alpha}{d\theta} = \frac{S_1\sin\theta - \sin(\theta - \alpha)}{S_2\sin\alpha - \sin(\theta - \alpha)}$$

$$S_1 = -\frac{a}{c} \qquad S_2 = \frac{a}{b}$$

$$\frac{d^2\phi}{d\theta^2} = \frac{R_1\cos\theta - R_2\cos\phi\left(\frac{d\phi}{d\theta}\right)^2 - \left(1 - \frac{d\phi}{d\theta}\right)^2\cos(\theta - \phi)}{R_2\sin\phi - \sin(\theta - \phi)}$$

$$\frac{d^2\alpha}{d\theta^2} = \frac{S_1\cos\theta - S_2\cos\alpha\left(\frac{d\alpha}{d\theta}\right)^2 - \left(1 - \frac{d\alpha}{d\theta}\right)^2\cos(\theta - \alpha)}{S_2\sin\alpha - \sin(\theta - \alpha)}$$

$$\dot\phi = \frac{d\phi}{d\theta}\dot\theta \qquad \dot\alpha = \frac{d\alpha}{d\theta}\dot\theta$$

$$\ddot\phi = \frac{d^2\phi}{dt^2} = \frac{d^2\phi}{d\theta^2}\left(\frac{d\theta}{dt}\right)^2 + \frac{d^2\theta}{dt^2}\frac{d\phi}{d\theta}$$

$$= \dot\theta^2\frac{d^2\phi}{d\theta^2} + \ddot\theta\frac{d\phi}{d\theta} \qquad \alpha = \dot\theta^2\frac{d^2\alpha}{d\theta^2} + \alpha\frac{d\ddot\alpha}{d\theta}$$

Remarks:

$\dot{\phi}$ has the units of θ, since $\frac{d\phi}{d\theta}$ is dimensionless.

$\frac{d^2\phi}{d\theta^2}$ has units of rad^{-1}. So that the dimensions making up $\ddot{\phi}$ agree, the program assumes $\frac{d^2\theta}{dt^2}$ is given in RPM^2, and $\frac{d^2\phi}{d\theta^2}$ is multiplied by $2\pi \frac{rad}{rev}$:

$$\ddot{\phi}\,\frac{rev}{min^2} = \dot{\theta}^2\,\frac{rev^2}{min^2}\,\frac{d^2\phi}{d\theta^2}\,rad^{-1}\left[\frac{2\pi\ rad}{rev}\right] + \ddot{\theta}\,\frac{rev}{min^2}\,\frac{d\phi}{d\theta}$$

The program could be altered by the appropriate constant change if $\dot{\theta}$ and $\ddot{\theta}$ are in units other than revolutions/time (e.g. for degrees/ time change 2π to $\pi/180$ (radians/degree), or for radians/time, no constant necessary).

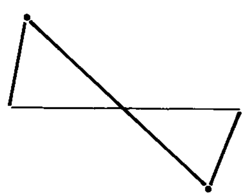
These same remarks apply to $\dot{\alpha}$ and $\ddot{\alpha}$.

An error during calculation of ϕ or α may indicate the linkage may not physically assume the specified position.

The sign of RPM determines the direction of rotation in automatic mode.

Two possible configurations exist for a given set of links:

Configuration A



Configuration B

Configuration A is assumed by the program. To obtain configuration B change step 87 from + to −.

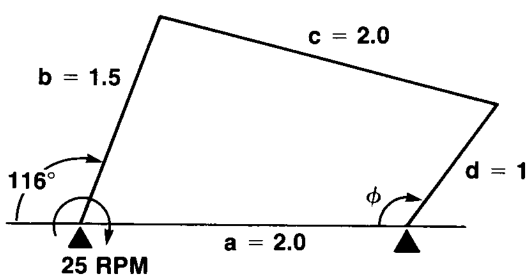
Registers R_{S0}–R_{S9} are available for user storage.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Input link lengths:			
	fixed link	a	ENTER↑	a
	input link	b	ENTER↑	b
	connecting link	c	ENTER↑	c
	output link	d	f A	a
3	If connecting link output			
	values (α, $\dot{\alpha}$, $\ddot{\alpha}$) are desired,			
	rather than output link values			
	(ϕ, $\dot{\phi}$, $\ddot{\phi}$), set connecting link			
	mode by pressing f C. A			
	1.00 appears in the display			
	indicating connecting link			
	mode is on. Pressing f C			
	repeatedly toggles connecting			
	link mode off and on.		f C	1.00/0.00
4	For automatic progression of			
	input link, go to step 9.			
5	Key in input link angle and			
	calculate output angle.	θ	A	ϕ (α)
6	Key in input RPM and calcu-			
	late output RPM.	$\dot{\theta}$(RPM)	C	$\dot{\phi}$ ($\dot{\alpha}$)
7	Key in input link acceleration			
	and calculate output			
	acceleration.	$\ddot{\theta}$(RPM^2)	E	$\ddot{\phi}$ ($\ddot{\alpha}$)
8	For a new input link angle, go			
	to step 5. For the alternate			
	output member (connector, or			
	output link), go to step 3. For			
	a new case, go to step 2.			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
9	Key in starting input link angle	θ_0	ENTER↑	θ_0
	then number of increments	n	ENTER↑	n
	then angular increment	$\Delta\theta$	ENTER↑	$\Delta\theta$
	then RPM (+ or −) and			
	calculate the output θ (α), $\dot{\phi}$ ($\dot{\alpha}$),			
	and $\ddot{\phi}$ ($\ddot{\alpha}$) for constant input			
	RPM between θ_0 and θ_f.	RPM	f E	output
10	For another set of inputs, go			
	to step 9. For the alternate			
	output member (connector or			
	output link) go to step 3. For a			
	new case go to step 2.			

Example 1:

The input link of the four bar linkage below is instantaneously rotating at 25 RPM with an angular acceleration of 2.3 RPM². The input link is at 116°. What are the values of position, velocity, and acceleration of link d? Link c?

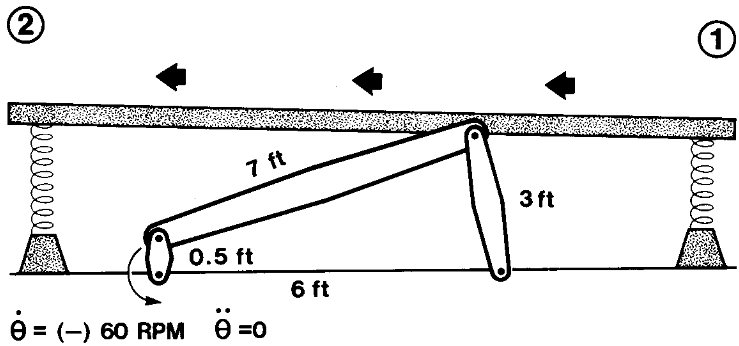


Keystrokes:	**Outputs:**	
2 ENTER↑ 1.5 ENTER↑ 2 ENTER↑		
1 f A →	2.00	
116 A →	125.75	(ϕ)
25 C →	39.29	($\dot{\phi}$)
2.3 E →	2279.89	($\ddot{\phi}$)

f C →	1.00	(connecting link selected)
116 A →	195.56	(α)
25 C →	3.38	($\dot{\alpha}$)
2.3 E →	2049.01	($\ddot{\alpha}$)

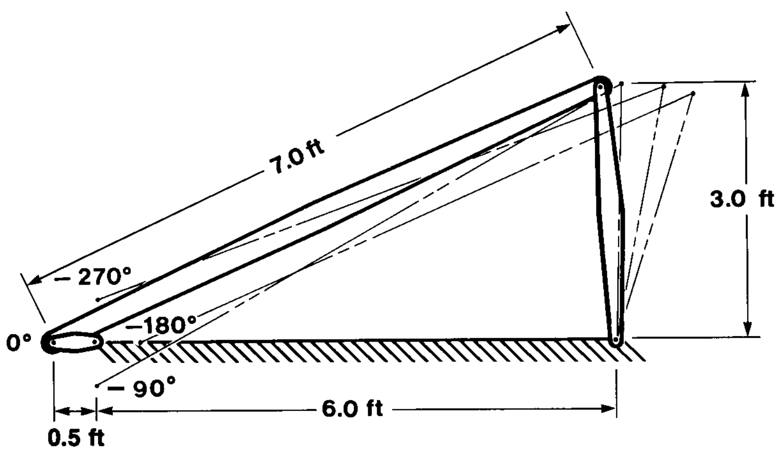
Example 2:

A four bar linkage is to be used to convert rotary motion from an electric motor to the reciprocating motion necessary to activate a shaking conveyor system which moves fruit between two process stations.



For the geometry shown above, what is the motion of the output link? Start at $\theta = 0°$ and go to –330° by 12, 30° increments. Find the corresponding connecting link motion.

Four Bar Shaker Mechanism



Solution:

θ^0	ϕ^0	$\dot{\phi}$(RPM)	$\ddot{\phi}$(RPM²)	α^0	α(RPM)	α(RPM²)
0	86.69	-4.62	3392.91	154.67	-4.62	-92.82
-30	85.63	0.46	3816.90	152.42	-4.20	713.38
-60	87.18	5.70	3615.33	150.67	-2.60	1594.02
-90	91.19	10.12	2592.67	150.01	0.10	2210.83
-120	96.94	12.38	449.28	150.84	3.19	2062.19
-150	102.95	10.93	-2597.20	153.04	5.34	887.00
-180	107.18	5.45	-4998.56	155.83	5.45	-693.75
-210	108.09	-1.86	-5112.85	158.18	3.73	-1628.64
-240	105.56	-7.86	-3351.37	159.45	1.32	-1738.45
-270	100.72	-10.95	-1099.60	159.53	-0.93	-1481.44
-300	95.11	-11.05	887.85	158.59	-2.75	-1133.46
-330	90.08	-8.70	2404.65	156.87	-4.04	-698.86

Keystrokes:	**Outputs:**
6 ENTER↑ .5 ENTER↑ 7 ENTER↑	
3 f A →	6.00
Select output link.	
f C →	0.00 (Press f C again if 1.00 is displayed)
0 ENTER↑ 12 ENTER↑ 30 ENTER↑	
60 CHS f E →	-60.00 *** (RPM)
	0.00 *** (θ_0)
	86.69 *** (ϕ)
	-4.62 *** ($\dot{\phi}$)
	3392.91 *** ($\ddot{\phi}$)
	-30.00 *** (θ_1)
	85.63 *** (ϕ_1)
	0.46 *** ($\dot{\phi}_1$)
	3816.90 *** ($\ddot{\phi}_1$)
	etc.
	⋮
	-330.00 *** (θ_f)
	90.08 *** (ϕ_f)
	-8.70 *** ($\dot{\phi}_f$)
	2404.65 *** ($\ddot{\phi}_f$)

Keys	Output
f C →	1.00
0 ENTER↑ 12 ENTER↑ 30 ENTER↑	
60 CHS f E →	–60.00 *** (RPM)
	0.00 *** (θ_0)
	154.67 *** (α_0)
	–4.62 *** ($\dot{\alpha}_0$)
	–92.82 *** ($\ddot{\alpha}_0$)
	–30.00 *** (θ_1)
	152.42 *** (α_1)
	–4.20 *** ($\dot{\alpha}_1$)
	713.38 *** ($\ddot{\alpha}_1$)
	etc.
	⋮

PROGRESSION OF SLIDER CRANK

In a slider crank mechanism (e.g.,the piston, wrist pin and connecting rod in an internal combustion engine), for given crank radius, connecting rod length, slider offset, crankshaft speed (RPM) and crank position, this program calculates the following: the displacement, velocity, and acceleration of the slider; the connecting rod angle, velocity and acceleration; the maximum and minimum displacements, and the maximum and minimum angular values for ϕ.

Equations:

$$\omega = \frac{\pi N}{30}$$

$$x = R \cos \theta + L \cos \phi$$

$$x_{max} = (R + L) \cos \left[\sin^{-1} \left(\frac{E}{R + L} \right) \right]$$

$$x_{min} = (L - R) \cos \left[\sin^{-1} \left(\frac{E}{L - R} \right) \right]$$

$$\Delta x = x_{max} - x_{min}$$

$$\phi = \sin^{-1} \left(\frac{E + R \sin \theta}{L} \right)$$

$$v = \frac{dx}{dt} = R\omega \left(\frac{-\sin (\theta + \phi)}{\cos \phi} \right)$$

$$a = \frac{d^2x}{dt^2} = R\omega^2 \left(\frac{-\cos (\theta + \phi)}{\cos \phi} - \frac{R \cos^2 \theta}{L \cos^3 \phi} \right)$$

$$\phi_{max} = \sin^{-1} \left(\frac{E + R}{L} \right)$$

$$\phi_{min} = \sin^{-1} \left(\frac{E - R}{L} \right)$$

$$\Delta\phi = \phi_{max} - \phi_{min}$$

$$\dot{\phi} = \frac{d\phi}{dt} = \omega \frac{R \cos \theta}{L \cos \phi}$$

$$\ddot{\phi} = \frac{d^2\phi}{dt^2} = \omega^2 \left[\left(\frac{d\phi}{d\theta} \right)^2 \tan \phi - \frac{R \sin \theta}{L \cos \phi} \right]$$

where:

N is crankshaft speed in RPM;

E is slider offset;

L is connecting rod length;

R is crank radius;

ω is crank angular velocity in radians/sec;

θ is crank angle;

x is slider displacement;

x_{max} is maximum slider displacement;

x_{min} is minimum slider displacement;

Δx is stroke;

v is slider velocity;

a is slider acceleration;

ϕ is connecting rod angular displacement;

ϕ_{max} is maximum connecting rod angular displacement;

ϕ_{min} is minimum connecting rod angular displacement;

$\Delta\phi$ is total angular throw of connecting rod;

$\dot{\phi}$ is angular velocity of connecting rod;

$\ddot{\phi}$ is angular acceleration of connecting rod.

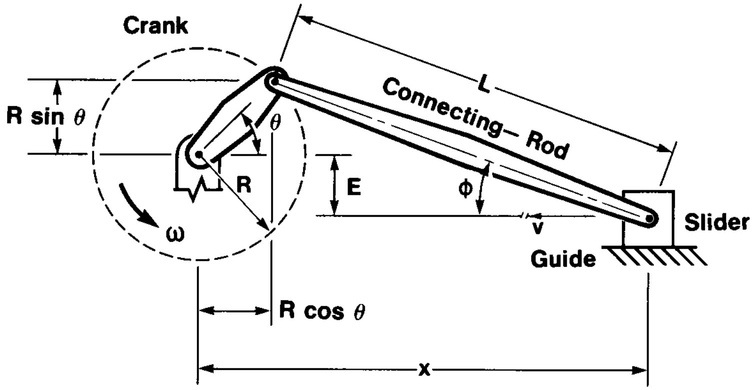
References:

H. A. Rothbart, *Mechanical Design and Systems Handbook,* McGraw-Hill, 1964.

V. M. Faires, *Kinematics,* McGraw-Hill, 1959.

Remarks:

Registers R_{S0}–R_{S9} are available for user storage.



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Input the data for the			
	mechanism.	N	ENTER↑	
		E	ENTER↑	
		L	ENTER↑	
		R	A	ω
3	Calculate maximum dis-			
	placement and minimum			
	displacement of slider.		f A	x_{max}
				x_{min}
4	Calculate maximum and			
	minimum angular displace-			
	ments for connecting rod.		f B	ϕ_{max}
				ϕ_{min}
5	Input crank angle to calculate			
	slider displacement and			
	connecting rod angle.	θ	B	x, ϕ
6	Calculate slider velocity and			
	connecting rod angular			
	velocity.		C	$v, \dot{\phi}$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
7	Calculate slider acceleration			
	and connecting rod angular			
	acceleration.		D	a, $\ddot{\phi}$
8	Repeat steps 5-7 for a			
	different θ.			
9	To calculate x, ϕ, v, $\dot{\phi}$, a, and $\ddot{\phi}$			
	for crank angles between θ_1			
	and θ_2 with n intervals.	θ_1	ENTER↑	
		θ_2	ENTER↑	
		n	E	θ, x, ϕ, v, $\dot{\phi}$, a, $\ddot{\phi}$
10	For a new mechanism, go to			
	step 2.			

Example 1:

For an in-line slider crank mechanism (E = 0), turning at 4800 RPM having a crank radius of 2.0 inches and connecting rod length of 7.0 inches, Find:

(1) x_{max}, x_{min} and ϕ_{max}, ϕ_{min}

(2) x, v, and a of the wrist pin in the slider

(3) ϕ, $\dot{\phi}$, and $\ddot{\phi}$ of the connecting rod

for θ = 0°, 15°, 45°, 90°, 135°, 180°, 225°.

θ°	x(in)	ϕ°	v(in/sec)	$\dot{\phi}$(rad/sec)	a(in/sec^2)	$\ddot{\phi}$(rad/sec^2)
0	9.00	0.00	0.00	143.62	-649701.96	0.00
15	8.91	4.24	-332.20	139.10	-614226.44	-17300.41
45	8.27	11.66	-857.50	103.69	-360454.40	-49902.29
90	6.71	16.60	-1005.31	0.00	150658.43	-75329.22
135	5.44	11.66	-564.22	-103.69	354181.29	-49902.29
180	5.00	0.00	0.00	-143.62	360945.53	0.00
225	5.44	-11.66	564.22	-103.69	354181.29	49902.29

Keystrokes:	**Outputs:**
4800 ENTER↑ 0 ENTER↑	
7 ENTER↑ 2 A →	502.65 *** (ω)
f A →	9.00 *** (x_{max})
	5.00 *** (x_{min})
f B →	16.60 *** (ϕ_{max})
	-16.60 *** (ϕ_{min})
0 B →	9.00 *** (x)
	0.00 *** (ϕ)
C →	0.00 *** (v)
	143.62 *** ($\dot{\phi}$)
D →	-649701.96 *** (a)
	0.00 *** ($\ddot{\phi}$)
15 B →	8.91 *** (x)
	4.24 *** (ϕ)
C →	-332.20 *** (v)
	139.10 *** ($\dot{\phi}$)
D →	-614226.44 *** (a)
	-17300.41 *** ($\ddot{\phi}$)
45 B →	8.27 *** (x)
	11.66 *** (ϕ)
C →	-857.50 *** (v)
	103.69 *** ($\dot{\phi}$)
D →	-360454.40 *** (a)
	-49902.29 *** ($\ddot{\phi}$)
⋮	
225 B →	5.44 ***
	-11.66 ***
C →	564.22 ***
	-103.69 ***
D →	354181.29 ***
	49902.29 ***

Alternatively, the values may be generated automatically.

0 ENTER↑ 225 ENTER↑ 5 E ⟶

0.00	***	(θ)
9.00	***	(x)
0.00	***	(ϕ)
0.00	***	(v)
143.62	***	($\dot{\phi}$)
-649701.96	***	(a)
0.00	***	($\ddot{\phi}$)
45.00	***	
8.27	***	
11.66	***	
-857.58	***	
103.69	***	
-360454.40	***	
-49902.29	***	
90.00	***	
6.71	***	
16.60	***	
-1005.31	***	
0.00	***	
150658.43	***	
-75329.22	***	
⋮		
225.00	***	
5.44	***	
-11.66	***	
564.22	***	
-103.69	***	
354181.29	***	
49902.29	***	

Example 2:

Determine the same values as in example 1 for a slider crank with offset of 1.5 inches (E = 1.5 inches).

Keystrokes:	**Outputs:**
4800 ENTER↑ 1.5 ENTER↑	
7 ENTER↑ 2 A →	502.65 *** (ω)
f A →	8.87 *** (x_{max})
	4.77 *** (x_{min})
f B →	30.00 *** (ϕ_{max})
	-4.10 *** (ϕ_{min})
0 B →	8.84 *** (x)
	12.37 *** (ϕ)
C →	-220.55 *** (v)
	147.03 *** ($\dot{\phi}$)
D →	-660249.41 *** (a)
	4742.62 *** ($\ddot{\phi}$)
15 B →	8.63 *** (x)
	16.75 *** (ϕ)
C →	-552.49 *** (v)
	144.87 *** ($\dot{\phi}$)
D →	-602160.36 *** (a)
	-13194.60 *** ($\ddot{\phi}$)
⋮	
225 B →	5.59 ***
	0.70 ***
C →	719.57 ***
	-101.56 ***
D →	280733.14 ***
	51175.65 ***

$\theta°$	x(in)	$\phi°$	v(in/sec)	$\dot{\phi}$(rad/sec)	a(in/sec^2)	$\ddot{\phi}$(rad/sec^2)
0	8.84	12.37	-220.55	147.03	-660249.41	4742.62
15	8.63	16.75	-552.49	144.87	-602160.36	-13194.60
45	7.78	24.60	-1036.35	111.69	-289750.94	-50429.96
90	6.06	30.00	-1005.31	0.00	291748.80	-83356.80
135	4.95	24.60	-385.37	-111.69	424884.76	-50429.96
180	4.84	12.37	220.55	-147.03	350398.08	4742.62
225	5.59	0.70	719.57	-101.56	280733.14	51175.65

0 ENTER↑ 360 ENTER↑ 8 E ⟶ 0.00 *** (θ)

8.84 *** (x)
12.37 *** (ϕ)

-220.55 *** (v)
147.03 *** ($\dot{\phi}$)

-660249.41 *** (a)
4742.62 *** ($\ddot{\phi}$)

45.00 ***

7.78 ***
24.60 ***

-1036.35 ***
111.69 ***

-289750.94 ***
-50429.96 ***
⋮

360.00 ***

8.84 ***
12.37 ***

-220.55 ***
147.03 ***

-660249.41 ***
4742.62 ***

CIRCULAR CAMS

This program computes the parameters necessary for the design of a harmonic or cycloidal circular cam with a roller, point or flat follower.

Equations:

Harmonic cams:

$$y = \frac{h}{2}\left(1 - \cos\frac{180\theta}{\beta}\right)$$

$$\frac{dy}{d\theta} = \frac{\pi h}{2\beta}\sin\frac{180\theta}{\beta} \qquad \left[\frac{dy}{dt} = \omega\frac{dy}{d\theta}\right]$$

$$\frac{d^2y}{d\theta^2} = \frac{\pi^2 h}{2\beta^2}\cos\frac{180\theta}{\beta} \qquad \left[\frac{d^2y}{dt^2} = \omega^2\frac{d^2y}{d\theta^2}\right]$$

Cycloidal cams:

$$y = h\left[\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right]$$

$$\frac{dy}{d\theta} = \frac{h}{\beta}\left[1 - \cos\frac{2\pi\theta}{\beta}\right] \qquad \left[\frac{dy}{dt} = \omega\frac{dy}{d\theta}\right]$$

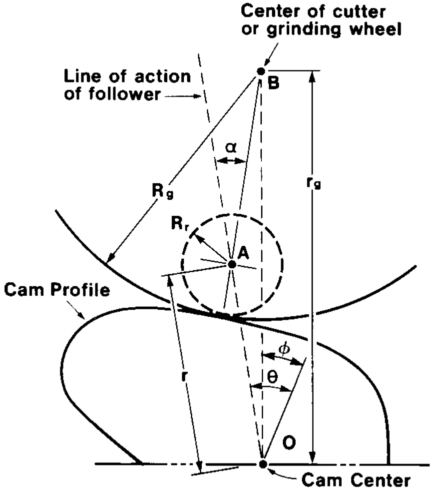
$$\frac{d^2y}{d\theta^2} = \frac{2\pi h}{\beta^2}\sin\frac{2\pi\theta}{\beta} \qquad \left[\frac{d^2y}{dt^2} = \omega^2\frac{d^2y}{d\theta^2}\right]$$

Both cycloidal and harmonic cams:

$$\alpha = \tan^{-1}\left(\frac{180}{\pi r}\frac{dy}{d\theta}\right)$$

$$r = R_b + y$$

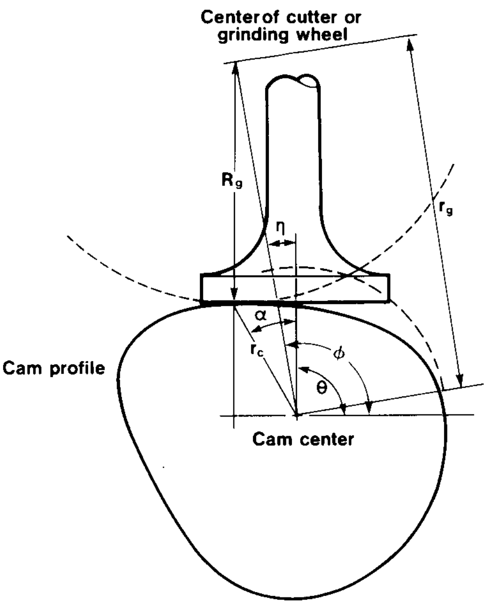
Roller followers:



$$r_g = (r^2 + (R_g - R_r)^2 - 2r(R_g - R_r)\cos\alpha)^{1/2}$$

$$\phi = \sin^{-1}\frac{R_g - R_r}{r_g} + \theta$$

Flat followers:



$$r_c = \left(r^2 + \left(\frac{180}{\pi}\ \frac{dy}{d\theta}\right)^2\right)^{1/2}$$

$$r_g = (R_g^{\ 2} + r_c^{\ 2} + 2R_g\, r_c \cos\alpha)^{1/2}$$

$$\phi = \cos^{-1}\left(\frac{r_c + R_g \cos\alpha}{r_g}\right) - \alpha + \theta$$

where:

β is duration of lift h;

$\Delta\theta$ is angular increment of calculation;

h is total cam lift over angle β;

R_b is base circle radius;

R_g is grinder radius (set to zero for cam profile);

R_r is roller radius (set to zero for point follower);

θ is cam angle;

y is follower lift;

$\dfrac{dy}{d\theta}$ is follower velocity;

$\dfrac{d^2y}{d\theta^2}$ is follower acceleration;

α is pressure angle;

ϕ is angle from zero to grinder center;

r_g is center to center distance of grinder and cam.

Reference:

M.F. Spotts, *Design of Machine Elements,* Prentice-Hall 1971.

Remarks:

A flat follower will not properly follow a cam profile with any concave sections, e.g., see figure 1.

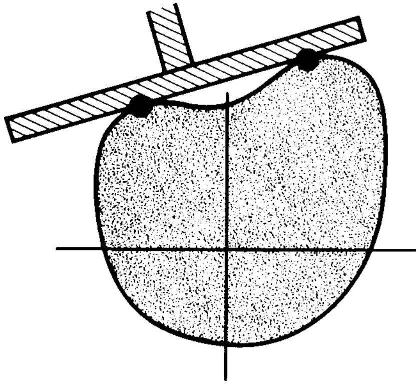


Figure 1
Note two points of contact

A roller follower will not properly follow a cam profile with concave section whose radius is less than the roller radius, e.g., see figure 2.

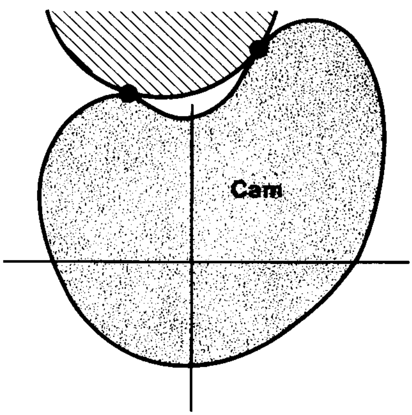


Figure 2
Note two points of contact

When the program is loaded, roller follower and harmonic profile modes are automatically selected.

Profiles other than harmonic and cycloidal may be generated by substituting them instead of label 1 or label 2. Example 3 demonstrates this.

Registers R_8 and R_{S0}–R_{S9} are available for user storage.

For a parabolic profile, substitute the LBL 3 subrotine of ME1-14A for LBL 1 or LBL 2 of ME1-13A.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Select flat or roller follower			
	(1 = flat, 0 = roller. Roller			
	follower is set when card is			
	loaded).		f A	1/0
3	Select cam function type:			
	Harmonic (set when card			
	was loaded)	1	f B	1.000 00
	or cycloidal.	2	f B	2.000 00
4	Input starting angle.	θ_0	ENTER↑	θ_0
5	Input duration of lift.	β	ENTER↑	β
6	Input increment of θ.	$\Delta\theta$	f C	0.000 00
7	Input lift.	h	f D	h
8	Input radius of roller (skip for			
	flat followers).	R_r	ENTER↑	R_r
9	Input radius of grinder (use			
	zero if cam profile is desired).	R_g	ENTER↑	R_g
10	Input base radius.	R_b	f E	$(R_r - R_g)$
11	For automatic output, go to			
	step 15.			
12	Output angle and lift.		B	θ, y
13	Optional: Output other			
	quantities of velocity and			
	acceleration		C	$dy/d\theta, d^2y/d\theta^2$
	and/or pressure angle		D	α
	and/or grinder radius and			
	angle.		E	r_g, ϕ

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
14	For next increment, go to			
	step 12. For a new lift, go to			
	step 4. For a new case, go to			
	step 2.			
15	Automatic output of θ, y, $dy/d\theta$,			
	$d^2y/d\theta^2$, α, r_g, and ϕ with			
	increments of $\Delta\theta$ from θ_0			
	through $\theta_0 + \beta$.		A	θ, y, $dy/d\theta$
16	For next lift, go to step 4. For			
	a new case, go to step 2.			

Example 1:

Design a harmonic cam with a 1.0 inch roller follower, which develops harmonic motion, dropping from a base radius of 12.0 inches to 7.5 inches in 130° of rotation. From 130° to 170°, increase the lift to the original base radius. Using 10° increments, generate the cam profile by letting $R_g = 0$.

θ^0	y (in)	$dy/d\theta$ (in/deg)	$d^2y/d\theta^2$ (in/deg^2)	α^0	r_g(in)	ϕ
0.000 00	0.000 00	0.000 00	−1.314−03	0.000 00	11.00 00	0.000 00
10.00 00	−65.38−03	−13.01−03	−1.276−03	−3.575 00	10.94 00	9.673 00
20.00 00	−257.7−03	−25.27−03	−1.163−03	−7.029 00	10.75 00	19.35 00
30.00 00	−565.9−03	−36.06−03	−983.5−06	−10.24 00	10.45 00	29.03 00
40.00 00	−971.9−03	−44.75−03	−746.4−06	−13.09 00	10.06 00	38.71 00
50.00 00	−1.452 00	−50.84−03	−466.0−06	−15.44 00	9.588 00	48.41 00
60.00 00	−1.979 00	−53.98−03	−158.4−06	−17.15 00	9.070 00	58.14 00
70.00 00	−2.521 00	−53.98−03	158.4−06	−18.07 00	8.534 00	67.92 00
80.00 00	−3.048 00	−50.84−03	466.0−06	−18.02 00	8.007 00	77.79 00
90.00 00	−3.528 00	−44.75−03	746.4−06	−16.84 00	7.520 00	87.79 00
100.0 00	−3.934 00	−36.06−03	983.5−06	−14.37 00	7.101 00	98.00 00
110.0 00	−4.242 00	−25.27−03	1.163 03	−10.57 00	6.777 00	108.4 00
120.0 00	−4.435 00	−13.01−03	1.276−03	−5.628 00	6.571 00	119.1 00
130.0 00	−4.500 00	0.000 00	1.314−03	0.000 00	6.500 00	130.0 00
130.0 00	0.000 00	0.000 00	13.88−03	0.000 00	6.500 00	130.0 00
140.0 00	659.0−03	125.0−03	9.814−03	41.27 00	7.437 00	145.1 00
150.0 00	2.250 00	176.7−03	0.000 00	46.08 00	9.085 00	154.5 00
160.0 00	3.841 00	125.0−03	−9.814−03	32.26 00	10.51 00	162.9 00
170.0 00	4.500 00	0.000 00	−13.88−03	0.000 00	11.00 00	170.0 00

Keystrokes:	**Outputs:**
Select roller follower by pressing f A until zero is displayed.	
f A →	0.000 00
Select harmonic cam:	
1 f B →	1.000 00
0 ENTER↑ 130 ENTER↑	
10 f C →	0.000 00
7.5 ENTER↑ 12 − f D →	−4.500 00
1 ENTER↑ 0 ENTER↑ 12 f E →	1.000 00
A →	0.000 00 *** (θ)
	0.000 00 *** (y)
	0.000 00 *** ($dy/d\theta$)
	−1.314−03 *** ($d^2y/d\theta^2$)
	0.000 00 *** (α)
	11.00 00 *** (r_g)
	0.000 00 *** (ϕ)
	10.00 00 ***
	−65.38−03 ***
	−13.01−03 ***
	−1.276−03 ***
	−3.575 00 ***
	10.94 00 ***
	9.673 00 ***
	⋮
	etc.

For the lift back to the original base radius, input β (170° − 130° = 40°) and $\Delta\theta$. The start of this lift ($\theta_0 = 130°$) is already displayed and does not need to be keyed in again (unless you hit R/S and stopped the calculation prematurely).

Keystrokes:	**Outputs:**
40 ENTER↑ 10 f C →	0.000 00
Key in new lift:	
4.5 f D →	4.500 00
Key in previous roller and grinder radii and new base radius of 7.5:	
1 ENTER↑ 0 ENTER↑ 7.5 f E →	1.000 00
A →	130.0 00 ***

0.000 00 ***

0.000 00 ***
13.88–03 ***

0.000 00 ***

6.500 00 ***
130.0 00 ***

140.0 00 ***
659.0–03 ***

125.0–03 ***
9.814–03 ***

41.27 00 ***

7.437 00 ***
145.1 00
⋮
etc.

Example 2:

Design a cycloidal, flat-faced cam with a lift of 50 millimeters in 40 degrees. The base radius is 500 millimeters and a 200 millimeter cutter is to be used for manufacture. Calculate θ, y, r_g and ϕ at 10 degree increments and calculate dy/dt (dy/dt $= \omega$ dy/dθ) at 20 degrees for a speed of 600 RPM.

Keystrokes:	**Outputs:**
Press f A until 1.000 00 is displayed:	
f A ⟶	1.000 00
Select cycloidal subroutine:	
2 f B ⟶	2.000 00
Input θ_0, β, $\Delta\theta$.	
0 ENTER↑ 40 ENTER↑	
10 f C ⟶	0.000 00
Input h:	
50 f D ⟶	50.00 00
Input R_g and R_b:	
200 ENTER↑ 500 f E ⟶	200.0 00
B ⟶	0.000 00 *** (θ)
	0.000 00 *** (y)

E →	700.0 00 ***	(r_g)
	0.000 00 ***	(ϕ)
B →	10.00 00 ***	
	4.542 00 ***	
E →	708.2 00 ***	
	15.80 00 ***	
B →	20.00 00 ***	
	25.00 00 ***	
E →	739.0 00 ***	
	31.18 00 ***	
Compute dy/dt at 20 degrees:		
C →	2.500 00 ***	($dy/d\theta$)
	0.000 00 ***	($d^2y/d\theta^2$)
x⇄y →	2.500 00	($dy/d\theta$)
600 ENTER↑ 10 ÷ 360 × × →	54.00 03	(dy/dt; mm/sec)
B →	30.00 00 ***	
	45.46 00 ***	
E →	748.9 00 ***	
	35.49 00 ***	
B →	40.00 00 ***	
	50.00 00 ***	
E →	750.0 00 ***	
	40.00 00 ***	

Example 3:

A cam with a flat-faced follower is to convert an angular input to a linear output according to the following equation and its derivatives:

$$y = (\theta/\beta)^2$$

$$y' = 2\ (\theta/\beta)$$

$$y'' = 2$$

Let $\beta = 90°$ and h = 1 inch. Generate the cam profile from 0° to 90° in increments of 15° by setting $R_g = 0$.

$R_b = 3.0$ inches

h = 1.0 inches

The first step is to write a cam function subroutine incorporating the function and the derivatives. The subroutine can access (θ/β) in R_E and must store y′ in R_4, and y′′ in R_3, before returning to the main program with y in the X-register. One such subroutine is shown below:

```
LBL 3
  2
STO 3   (y″calculated and stored)
RCL E
  x
STO 4   (y′calculated and stored)
RCL E
  x²
 RTN    (y calculated)
```

Now, load this sequence into program memory in place of LBL 1 (steps 168-188) or LBL 2 (steps 189-214). After this, the following keystrokes will generate the cam data:

Keystrokes:	**Outputs:**
Select flat-faced follower by pressing f A until 1.000 00 appears:	
f A ——→	1.000 00
Select subroutine 3 (since the new subroutine is LBL 3):	
3 f B ——→	3.000 00
0 ENTER↑ 90 ENTER↑	
15 f C ——→	0.000 00
1 f D ——→	1.000 00
0 ENTER↑ 3.0 f E ——→	0.00 00
A ——→	0.000+00 ***
	0.000+00 ***
	0.000+00 ***
	246.9−06 ***
	0.000+00 ***
	3.000+00 ***
	0.000+00 ***
	15.00+00 ***
	27.78−03 ***
	3.704−03 ***
	246.9−06 ***
	4.009+00 ***
	3.035+00 ***
	19.01+00 ***
	30.00+00 ***

111.1−03 ***

7.407−03 ***
246.9−06 ***
7.768+00 ***

3.140+00 ***
37.77+00 ***

45.00+00 ***
250.0−03 ***

11.11−03 ***
246.9−06 ***

11.08+00 ***

3.312+00 ***
56.08+00 ***

60.00+00 ***
444.4−03 ***

14.81−03 ***
246.9−06 ***

13.84+00 ***

3.547+00 ***
73.84+00 ***

75.00+00 ***
694.4−03 ***

18.52−03 ***
246.9−06 ***

16.02+00 ***

3.844+00 ***
91.02+00 ***

90.00+00 ***
1.000+00 ***

22.22−03 ***
246.9−06 ***

17.66+00 ***

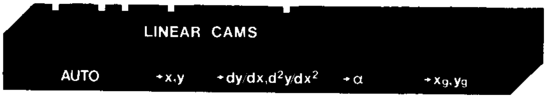
4.198+00 ***
107.7−00 ***

CAM DATA SUMMARY

θ	θ/β	y	r_g	ϕ
0°	0	0	3.000	0
15°	0.167	27.78–03	3.035	19.01
30°	0.333	111.1–03	3.140	37.77
45°	0.500	250.0–03	3.312	56.08
60°	0.667	444.4–03	3.547	73.84
75°	0.833	694.4–03	3.844	91.02
90°	1.000	1.000–00	4.198	107.7

Note that $y = (\theta/\beta)^2$ as specified by the original equation.

LINEAR CAMS



This program computes parameters necessary for the design of harmonic, cycloidal or parabolic profiles for linear cams with roller followers.

Equations:

$$y = hf\,(x/L) + R_b$$

$$x_g = x - (R_g - R_r)\sin\alpha \qquad y_g = y + (R_g - R_r)\cos\alpha$$

$$\alpha = \tan^{-1}\left(\frac{dy}{dx}\right)$$

$$= \tan^{-1}\left(\frac{h}{L}\, f'\,(x/L)\right)$$

$$\frac{dy}{dx} = \frac{h}{L}\, f'\,(x/L)$$

$$\frac{d^2y}{dx^2} = \frac{h}{L^2}\, f''\,(x/L)$$

For harmonic profiles:

$$f\,(x/L) = \left(1 - \cos\left(\frac{180x}{L}\right)\right)$$

For cycloidal profiles:

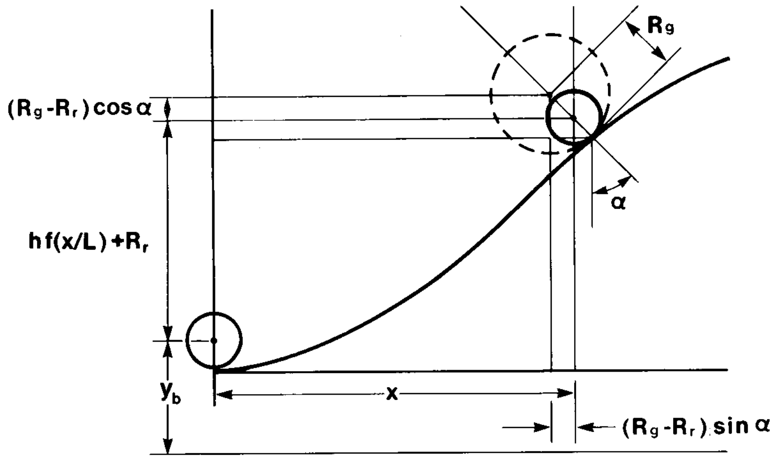
$$f\,(x/L) = \left(\frac{x}{L} - \frac{1}{2\pi}\sin\frac{180x}{L}\right)$$

For parabolic profiles:

$$f\,(x/L) = \begin{cases} 2h\left(\dfrac{x}{L}\right)^2 & \dfrac{x}{L} < .5 \\[2ex] \left[1 - 2\left(1 - \dfrac{x}{L}\right)^2\right] & \dfrac{x}{L} \geqslant .5 \end{cases}$$

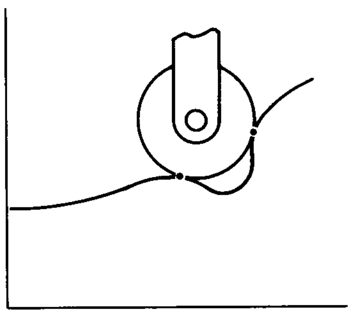
where:

- L is the duration of lift h;
- Δx is the linear increment of calculation;
- h is the total follower lift over length L;
- y_b is the base height from reference datum to roller center;
- R_r is the roller radius (zero for point follower);
- R_g is the grinder radius;
- x is the linear displacement of cam;
- y is the roller center height above datum;
- (x_g, y_g) is the grinder center for displacement x;
- α is the pressure angle.



Remarks:

The roller follower will not properly follow a cam profile with concave sections whose radius is less than the roller radius.



Note two points of contact

When the program is loaded, the harmonic profile mode is assumed. You may change to cycloidal by keying 2 and pressing f A. Parabolic is selected by keying 3 and pressing f A. Keying 1 and pressing f A returns the program to original status.

Arbitrary functions of (x/L) may be substituted in a manner analogous to that of example 3, ME1-13A.

Registers R_{S0}–R_{S9} are available for user storage.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Select cam function type:			
	Harmonic (set when card			
	was loaded)	1	f A	1
	or cycloidal	2	f A	2
	or parabolic	3	f A	3
3	Input starting x.	x_0	ENTER↑	x_0
4	Input duration of lift.	L	ENTER↑	L
5	Input increment of x.	Δx	f B	0.000 00
6	Input lift.	h	f C	h
7	Input height of follower center			
	above reference datum at x_0.	y_b	f D	y_b
8	Input grinder radius.	R_g	ENTER↑	R_g
9	Input roller radius.	R_r	f E	R_g–R_r
10	For automatic output, go to			
	step 14.			
11	Output x, y coordinates of			
	roller.		B	x, y
12	Optional: output other			
	quantities:			
	Follower velocity and			
	acceleration,		C	$dy/dx, d^2y/dx^2$
	and/or pressure angle,		D	α
	and/or grinder coordinates.		E	x_g, y_g

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
13	For next increment, go to			
	step 11. For a new case, go to			
	step 2.			
14	Automatic output of x, y, dy/dx,			
	d^2y/dx^2, α, x_g, and y_g with			
	increments of Δx from x_0			
	through x + L.		A	x,y, dy/dx...
15	For a new case, go to step 2.			

Example:

Design a harmonic, linear profile which has a base follower displacement of 4 cm, and a 3 cm lift over a distance of 7 cm. After the harmonic profile, a cycloidal lift of 2 cm occurs over a distance of 8 cm. Then a parabolic profile returns the follower to its original height (a drop of 5 cm) over a distance of 10 cm.

The follower and the grinder both have a radius of 1 cm. Therefore, the grinder and follower paths are equivalent. Instead of generating redundant grinder data, generate the surface profile by setting $R_g = 0$.

Use 1 cm step size.

Keystrokes:	**Outputs:**
Harmonic segment from 0 cm to 7 cm.	
1 f A ⟶	1.000 00 (harmonic profile)
0 ENTER↑ 7 ENTER↑ 1 f B ⟶	0.000 00
3 f C ⟶	3.000 00
4 f D ⟶	4.000 00
0 ENTER↑ 1 f E ⟶	-1.000 00
A ⟶	0.000 00 *** (x)
	4.000 00 *** (y)
	0.000 00 *** (dy/dx)
	302.1-03 *** (d^2y/dx^2)
	0.000 00 *** (α)
	0.000 00 *** (x_g)

	3.000 00 ***	(y_g)
	1.000 00 ***	
	4.149 00 ***	
	292.1−03 ***	
	272.2−03 ***	
	16.28 00 ***	
	1.280 00 ***	
	3.189 00 ***	
	⋮	
	7.000 00 ***	
	7.000 00 ***	
	0.000 00 ***	
	−302.1−03 ***	
	0.000 00 ***	
	7.000 00 ***	
	6.000 00 ***	
Cycloidal segment from 7 cm to 15 cm.		
2 f A →	2.000 00	
7 ENTER↑ 8 ENTER↑ 1 f B →	0.000 00	
2 f C →	2.000 00	
7 f D →	7.000 00	
A →	7.000 00 ***	(x)
	7.000 00 ***	(y)
	0.000 00 ***	(dy/dx)
	0.000 00 ***	(d^2y/dx^2)
	0.000 00 ***	(α)
	7.000 00 ***	(x_g)
	6.000 00 ***	(y_g)
	8.000 00 ***	
	7.025 00 ***	
	73.22−03 ***	
	138.8−03 ***	
	4.188 00 ***	
	8.073 00 ***	
	6.028 00 ***	

⋮

14.00 00 ***
8.975 00 ***

73.22–03 ***
–138.8–03 ***

4.188 00 ***

14.07 00 ***
7.978 00 ***

15.00 00 ***
9.000 00 ***

0.000 00 ***
0.000 00 ***

0.000 00 ***

15.00 00 ***
8.000 00 ***

Parabolic segment from 15 cm to 25 cm.

3 f A ⟶	3.000 00
15 ENTER↑ 10 ENTER↑ 1 f B	
5 CHS f C 9 f D ⟶	9.000 00
A ⟶	15.00 00 *** (x)
	9.000 00 *** (y)
	0.000 00 *** (dy/dx)
	–200.0–03 *** (d^2y/dx^2)
	0.000 00 *** (α)
	15.00 00 *** (x_g)
	8.000 00 *** (y_g)
	16.00 00 ***
	8.900 00 ***
	–200.00–03 ***
	–200.00–03 ***
	–11.31 00 ***
	15.80 00 ***
	7.919 00 ***

⋮

24.00 00 ***
4.100 00 ***

-200.0-03 ***
200.0-03 ***

-11.31 00 ***

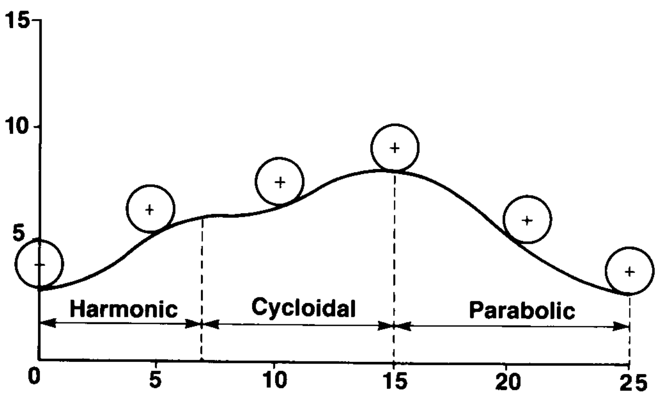
23.80 00 ***
3.119 00 ***

25.00 00 ***
4.000 00 ***

0.000 00 ***
200.0-03 ***

0.000 00 ***

25.00 00 ***
3.000 00 ***



Notes

GEAR FORCES

This program computes three mutually perpendicular forces, resulting from input torque, on helical, bevel or worm gears.

Helical gear equations:

$$F_t = \frac{T}{r}$$

$$F_{gs} = F_t \tan \phi$$

$$F_{gax} = F_t \tan \alpha$$

$$\tan \phi = \frac{\tan \phi_n}{\cos \alpha}$$

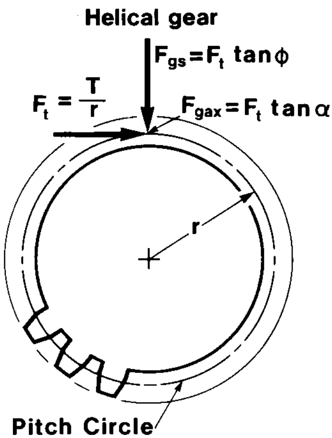


Figure 1-Helical Gear

where:

T is the input torque;

r is the pitch radius of the input gear;

F_t is the tangential force;

α is the helix angle measured from the axis of the gear (for spur gears $\alpha = 0$);

ϕ_n is the pressure angle measured perpendicular to the gear tooth;

ϕ is the pressure angle measured perpendicular to the gear axis;

F_{gs} is the radial force trying to separate the gears;

F_{gax} is the force parallel to the gear axis.

Bevel gear equations:

$$F_t = \frac{T}{r}$$

$$F_{bpax} = F_t \left(\frac{\tan \phi_n \sin (\text{cone}\angle)}{\cos \alpha} + \tan \alpha \cos (\text{cone}\angle) \right)$$

$$F_{bgax} = F_t \left(\frac{\tan \phi_n \cos (\text{cone}\angle)}{\cos \alpha} - \tan \alpha \sin (\text{cone}\angle) \right)$$

$$\tan \phi = \frac{\tan \phi_n}{\cos \alpha}$$

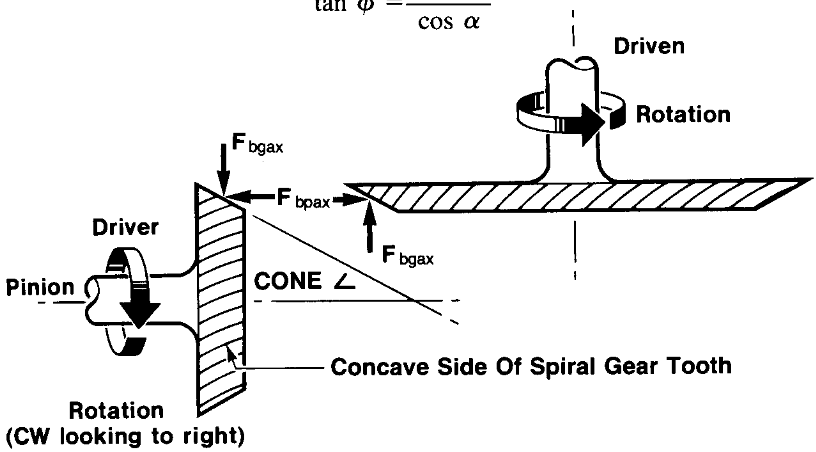


Figure 2—Spiral Bevel Gears

where:

T is the input (pinion) torque;

r is the pitch radius of the pinion gear;

F_t is the tangential force;

α is the pinion spiral angle (zero for straight tooth bevel gears);

ϕ_n is the pressure angle measured perpendicular to the gear tooth;

ϕ is the pressure angle measured perpendicular to the gear axis;

Cone $\angle$ is the pitch cone angle of the pinion;

F_{bpax} is the force along the axis of the bevel pinion;

F_{bgax} is the force along the axis of the bevel gear.

Worm gear equations:

$$F_t = \frac{T}{r}$$

$$F_{ws} = F_t \left(\frac{\sin \phi_n}{\cos \phi_n \sin \alpha + f \cos \alpha} \right)$$

$$F_{gax} = F_t \quad \frac{1 - \dfrac{f \tan \alpha}{\cos \phi_n}}{\tan \alpha + \dfrac{f}{\cos \phi_n}}$$

$$\tan \phi = \frac{\tan \phi_n}{\cos \alpha}$$

Driver: Worm (Right hand)

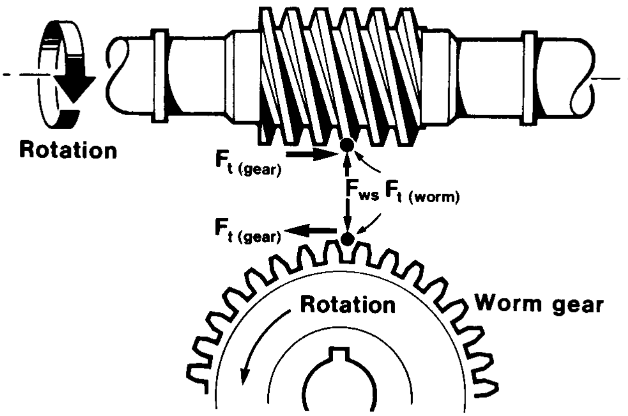


Figure 3
WORM GEAR

where:

T is the input (worm) torque;

n is the pitch radius of the worm;

F_t is the tangential force on the worm;

α is the lead angle of the worm ($\alpha = \tan^{-1}$ (L/2πr), where L is the lead of the worm);

ϕ_n is the pressure angle measured perpendicular to the worm teeth;

ϕ is the pressure angle measured parallel to the worm axis;

f is the coefficient of friction;

F_{ws} is the separating force between the worm and gear;

F_{gax} is the force parallel to the gear axis.

Remarks:

For bevel gears, the spiral angle (α) is positive if the concave face of the pinion teeth are facing the direction of rotation (see figure 2). α is negative if the convex surface of the pinion teeth face the direction of rotation.

Registers R_0–R_3, R_7–R_{S9} and R_c–R_I are available for user storage.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1.			
2	Input torque.	T	ENTER↑	T
3	Input pitch radius and calculate			
	tangential force.	r	A	F_t
4	Input helix angle for helical			
	gears, or spiral angle for spiral			
	bevel gears, or lead angle for			
	worm gears.	α	B	α
5	Input normal pressure angle	ϕ_n	C	ϕ_n
	or			
	input pressure angle.	ϕ	D	ϕ_n
6	For helical gears, go to step 7,			
	for bevel gears, go to step 9,			
	for worm gears, go to step 12.			
7	Calculate separating force			
	and axial force.		E	F_{gs}, F_{gax}
8	For a new case, return to			
	step 2 and modify inputs as			
	necessary.			
9	Input bevel cone angle.	cone∠	f A	cone∠
10	Calculate pinion axial force			
	and gear axial force.		f B	F_{bpax}, F_{bgax}

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
11	For a new case, return to			
	step 2 and modify inputs as			
	necessary.			
12	Input coefficient of friction.	f	f D	f
13	Calculate separating force			
	and gear axial force.		f E	F_{ws}, F_{gax}
14	For a new case, go to step 2			
	and modify inputs as			
	necessary.			

Example 1:

A helical gear with pitch radius 12 cm has a torque applied to it of 450,000 dyne-cm. The helix angle is 30°, and the normal pressure angle, measured perpendicular to a tooth, is 17.5°. Find the tangential, separating, and thrust forces.

Keystrokes:

450000 ENTER↑ 12 A ⟶

30 B 17.5 C E ⟶

Outputs:

37500.00 (F_t)

13652.84 *** (F_{gs})

21650.64 *** (F_{gax})

Example 2:

A spiral pinion with mean radius 1.73 inches is subjected to a torque of 745 in-lb. The pinion is cut with a normal pressure angle of 20°, a spiral angle of 35°, with a pitch cone of 18°. Find the forces acting on the pinion. Rotation is in the direction of the concave side of the pinion teeth, so α is positive 35°.

Keystrokes:

745 ENTER↑ 1.73 A ⟶

35 B 20 C 18 f A f B ⟶

Outputs:

430.64 (F_t)

345.90 *** (F_{bpax})

88.80 *** (F_{bgax})

If the rotation were reversed, leaving all other input values unchanged, what would the forces be?

35 CHS B f B ⟶ -227.65 *** (F_{bpax})
275.16 *** (F_{bgax})

Example 3:

A torque of 512 in-lb is applied to a worm gear having a pitch diameter of 2.92 inches and a lead of 2.20 inches. The normal pressure angle is 20°, and the coefficient of friction is 0.10. Find the lead angle and the forces on the worm and worm gear.

Keystrokes:	Outputs:	
512 ENTER↑ 2.92 ENTER↑		
2 ÷ A ⟶	350.68	(F_t)
Calculate lead angle ($\alpha = \tan^{-1}$ (lead/2πr)).		
2.2 ENTER↑ 2 ÷ π ÷		
2.92 ENTER↑ 2 ÷ ÷ TAN⁻¹ ⟶	13.49	(α)
B 20 C .1 f D f E ⟶	379.10 ***	(F_{ws})
	986.99 ***	(F_{gax})

STANDARD EXTERNAL INVOLUTE SPUR GEARS

This program calculates various parameters for standard external involute spur gears. Given the diametral pitch P, number of teeth N, pressure angle ϕ, and pin diameter d_w, the program will calculate the pitch diameter D, tooth thickness t, and the involute and corresponding flank angle inv ϕ_w and ϕ_w. The flank angle ϕ_w is calculated from the involute by a Newton's method iterative solution for the equation $f(\phi_w) = 0$,

where:

$$f(\phi_w) = \tan \phi_w - \phi_w - \text{inv } \phi_w$$

In this solution, an initial guess is made for ϕ_w:

$$\phi_w^{(0)} = (3 \text{ inv } \phi_w)^{.3}$$

Newton's method then provides refinements of the initial guess by

$$\phi_w^{(n+1)} = \phi_w^{(n)} - \frac{f(\phi_w^{(n)})}{f'(\phi_w^{(n)})}$$

$$= \phi_w^{(n)} - \frac{\tan \phi_w^{(n)} - \phi_w^{(n)} - \text{inv } \phi_w}{\tan^2 \phi_w^{(n)}}$$

The program also calculates various measurements over pins, namely, the theoretical values of the measurement over pins, M; the radius to the center of the pin, l; and the measurement over one pin, R_w. In addition, given the value of the tooth thinning Δt, the program will return the measurement over pins with tooth thinning, M_t.

Equations:

$$D = \frac{N}{P}$$

$$t = \frac{\pi}{2P}$$

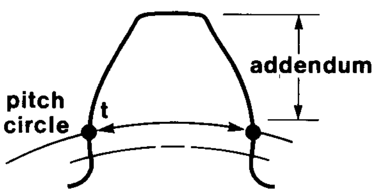
$$\text{inv } \phi_w \text{ (radians)} = \frac{t}{D} + \tan \phi - \frac{\pi \phi}{180} + \frac{d_w}{D \cos \phi} - \frac{\pi}{N}$$

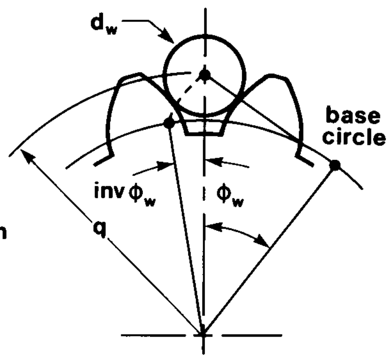
$$M = \begin{cases} d_w + 2q & \text{(N even)} \\ d_w + 2q\cos\left(\dfrac{90}{N}\right) & \text{(N odd)} \end{cases}$$

$$q = \frac{D\cos\phi}{2\cos\phi_w}$$

$$R_w = q + \frac{d_w}{2}$$

$$M_t = M - \Delta t\frac{\cos\phi}{\sin\phi_w}$$





Reference:

Adapted from a program submitted to the HP-65 Users's Library by Mr. John Nemcovich, Los Angeles, CA.

Dudley, D.W., *Gear Handbook,* McGraw-Hill, 1962.

Remarks:

Registers R_0, R_{S0}–R_c and I are available for user storage.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Input diametral pitch P	P	ENTER↑	P
	and number of teeth N to			
	calculate the pitch diameter			
	and tooth thickness t.	N	A	D, t
3	Input pressure angle ϕ	ϕ	ENTER↑	ϕ
	and pin diameter d_w.	d_w	B	ϕ
4	Calculate the involute inv ϕ_w.		C	inv ϕ_w (deg.)
5	Calculate the corresponding			
	flank angle.		D	ϕ_w (deg.)
6	Calculate the measurement			
	over pins (theoretical).		E	M
7	Input tooth thinning and			
	calculate measurement over			
	pins with tooth thinning.	Δt	f A	M_t
8	Calculate radius to the center			
	of pin.		f B	q
9	Calculate measurement over			
	one pin.		f C	R_w
10	To change tooth thinning, go			
	to step 7. To change any other			
	input, go to step 2.			
	Note: If d_w is not known, it may			
	be calculated from the pin			
	constant k and pitch P:	k	ENTER↑	
	$d_w = k/P$	P	÷	d_w
11	To calculate ϕ_w directly from			
	inv ϕ_w: store inv ϕ_w in register 6	inv ϕ_w	STO 6	
	and calculate ϕ_w.		D	ϕ_w (deg.)

Example:

A 27-tooth gear with pitch 8 is cut with a 20° pressure angle. The pin diameter is 0.24 inches, and tooth thinning is reckoned at 0.002 inches. Calculate the unknown parameters.

Keystrokes:	**Outputs:**
8 ENTER↑ 27 A →	3.3750 *** (D)
	0.1963 *** (t)
20 ENTER↑ 0.24 B C →	1.8565 *** (inv ϕ_w)
D →	25.6215 *** (ϕ_w)
E →	3.7514 *** (M)
0.002 f A →	3.7470 *** (M_t)
f B →	1.7587 *** (q)
f C →	1.8787 *** (R_w)

BELT LENGTH

This program computes the belt length around an arbitrary set of pulleys. It may also be used to compute the total length between any connected set of coordinates. The program assumes the coordinates of the first pulley to be (0,0). Optionally the x, y coordinates of the intersections of the belt and pulleys may be output.

(x_i, y_i, R_i) = x, y coordinates and radius of pulley i

R_1 = Radius of first pulley

C.D. = Center to center distance of consecutive pulleys

L = Total length of belt

Equations:

$$L_{12} = \sqrt{C.D._{12}^2 - (R_2 - R_1)^2}$$

$$\text{Arc Length}_2 = R_2\,(\pi - \alpha - \beta - \gamma_2)$$

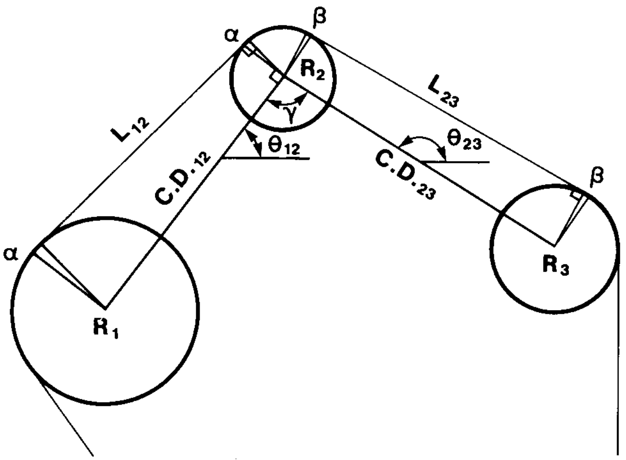
$$\alpha = \tan^{-1}\left(\frac{R_1 - R_2}{L_{12}}\right)$$

$$\beta = \tan^{-1}\left(\frac{R_3 - R_2}{L_{23}}\right)$$

$$\gamma = \theta_{12} - \theta_{23}$$

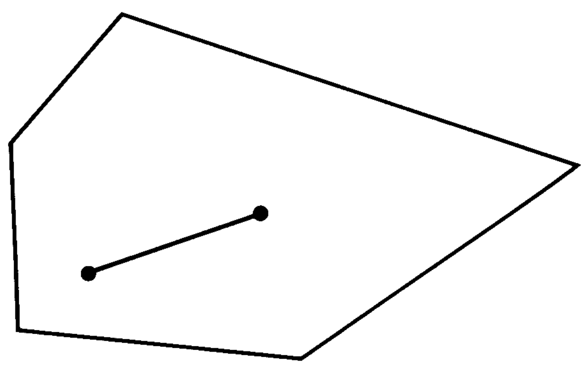
$$\theta_{12} = \tan^{-1}\frac{y_2 - y_1}{x_2 - x_1}$$

$$\theta_{23} = \tan^{-1}\frac{y_3 - y_2}{x_3 - x_2}$$

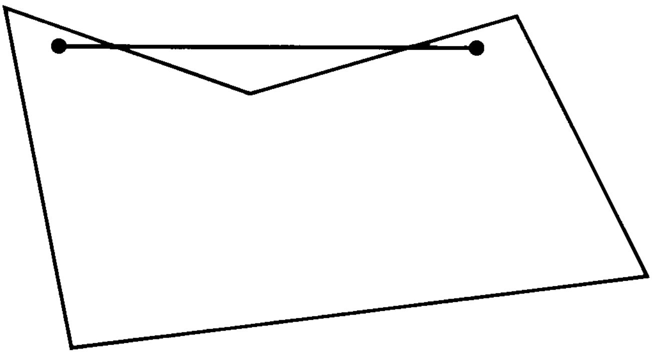


This program generates accurate results for any convex polygon, i.e., a line between any two points within the region bounded by the center-to-center line segments is entirely contained within the region.

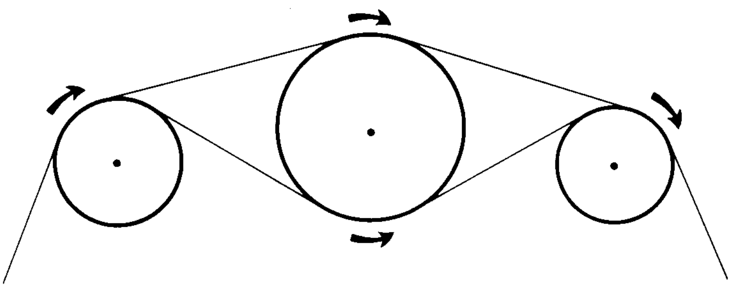
Convex



Concave

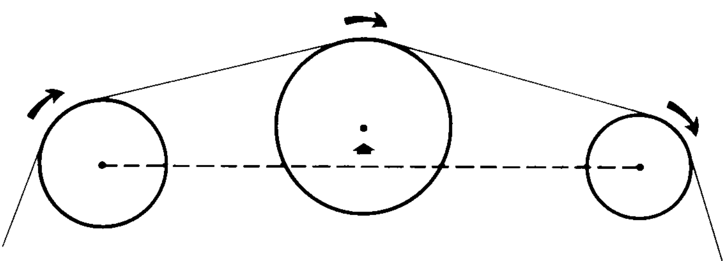


In some cases, there are two physically possible directions for the belt to take:



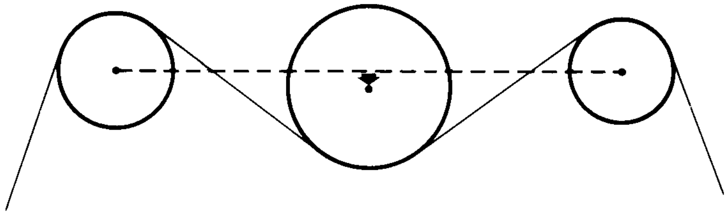
The program chooses the upper side if the middle pulley center lies above the line connecting the previous and following pulleys.

Case 1



The program chooses the lower side if the middle pulley center lies below the line connecting the previous and following pulleys.

Case 2



The program generates inaccurate answers in the second case. Note the figure bounded by the center-to-center line segments for the second case is not convex.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Optional: Toggle for printing			
	belt tangent points for each			
	pulley.*		f A	1.00
3	Input the coordinates (x_1, y_1)			
	and radius of the first pulley.	x_1	ENTER↑	x_1
		y_1	ENTER↑	y_1
		R_1	A	R_1
4	Input the next pulley co-			
	ordinates (x_i, y_i) and radius			
	(R_i).	x_i	ENTER↑	x_i
		y_i	ENTER↑	y_i
		R_i	B	R_i
5	Repeat step 4 for all remaining			
	pulleys.			
6	Calculate the belt length.		C	L
7	For a new case, go to step 2.			
	*Note: Pulley coordinates			
	have to be entered in the			
	clockwise sense.			

Example 1:

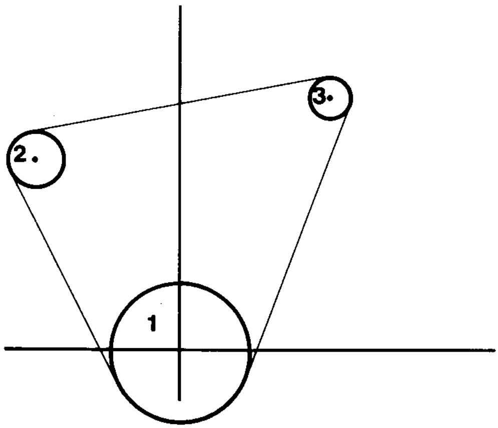
Assume three pulleys are positioned as shown below with the following coordinates and radii:

Pulley 1 (0, 0, 4 inches)

Pulley 2 (–8, 15, 1.5 inches)

Pulley 3 (9, 16, 1 inches).

Find the belt length around the three pulleys.



Keystrokes:	**Outputs:**	
0 ENTER↑ 0 ENTER↑ 4 A ⟶	4.00	(R_1)
8 CHS ENTER↑ 15 ENTER↑ 1.5 B ⟶	1.50	(R_2)
9 ENTER↑ 16 ENTER↑ 1 B ⟶	1.00	(R_3)
C ⟶	66.53	(L)

Example 2:

Find the length of line connecting the points (0, 0), (1.5, 7), (3.2, –6), (0, 0.5), (0, 0). (28.01). Let the radius of each "pulley" be 0.

Keystrokes:	**Outputs:**
0 ENTER↑ 0 ENTER↑ 0 A ⟶	0.00
1.5 ENTER↑ 7 ENTER↑ 0 B ⟶	0.00
3.2 ENTER↑ 6 CHS ENTER↑ 0 B ⟶	0.00

0 ENTER↑ 0.5 ENTER↑ 0 B ——→	0.00	
C ——→	28.01	(L)

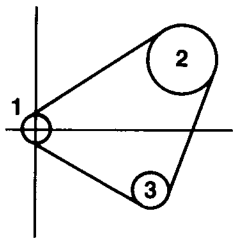
Example 3:

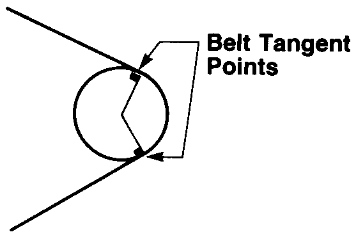
Find the belt length around the following pulley system, also find the belt tangent points on each pulley.

Pulley 1 (0, 0, 2.5)

Pulley 2 (30, 3, 7.5)

Pulley 3 (18, –18, 3.66)





Keystrokes:	**Outputs:**	
f A ——→	1.00	
0 ENTER↑ 0 ENTER↑ 2.5 A ——→	2.5	(R_1)
30 ENTER↑ 3 ENTER↑ 7.5 B ——→	–0.66 ***	(x)
	2.41 ***	(y) 1st pulley
	28.03 ***	(x)
	10.24 ***	(y) 2nd pulley
	7.5	(R_2)
18 ENTER↑ 18 CHS ENTER↑ 3.66		
B ——→	35.84 ***	(x)
	–1.71 ***	(y) 2nd pulley
	20.85 ***	(x)
	–20.30 ***	(y) 3rd pulley
	3.66	(R_3)
C ——→	15.30 ***	(x)
	–20.47 ***	(y) 3rd pulley
	–1.85 ***	(x)
	–1.69 ***	(y) 1st pulley
	109.33	(L)

FREE VIBRATIONS

This program provides an exact solution to the differential equation for a damped oscillator vibrating freely: $m\ddot{x} + c\dot{x} + kx = 0$.

The user inputs the mass m, spring constant k, and damping constant c at **A**. The output will be:

1. ω for an underdamped system, i.e. $c < c_{crit}$. c_{crit} is calculated by pressing **B**.
2. 0 for a critically damped system, i.e. $c = c_{crit}$.
3. -1 for an overdamped system, i.e. $c > c_{crit}$.

The initial conditions are the displacement and velocity at time zero (x_0 and $\dot{x}_0$).

Equations:

$$c_{crit} = 2\sqrt{km}$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$$

$$\ddot{x} = -(c\dot{x} + kx)/m$$

Underdamping $\quad (c^2 - 4km < 0)$

$$x(t) = Re^{-\frac{c}{2m}t}\cos(\omega t - \delta)$$

$$\dot{x}(t) = -R\omega e^{-\frac{c}{2m}t}\sin(\omega t - \delta) - \frac{c}{2m}Re^{-\frac{c}{2m}t}\cos(\omega t - \delta)$$

where:

$$R\cos\delta = x_0$$

$$R\sin\delta = \frac{1}{\omega}\left[\dot{x}_0 + \frac{c}{2m}x_0\right]$$

Critical damping $\quad (c = c_{crit}, \text{ or } c^2 = 4km)$

$$x(t) = (A_{cr} + B_{cr}t)e^{-\frac{c}{2m}t}$$

$$\dot{x}(t) = \left[B_{cr} - \frac{c}{2m}(A_{cr} + B_{cr}t)\right] e^{-\frac{c}{2m}t}$$

where:

$$A_{cr} = x_0$$

$$B_{cr} = \dot{x}_0 + \frac{c}{2m} x_0$$

Overdamping $(c^2 - 4km > 0)$

$$\dot{x}(t) = A_{ov}e^{r_1 t} + B_{ov}e^{r_2 t}$$

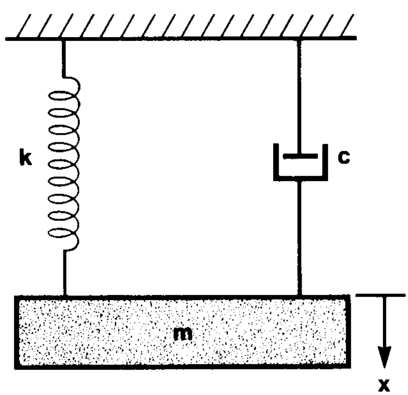
$$x(t) = A_{ov}r_1e^{r_1 t} + B_{ov}r_2e^{r_2 t}$$

where:

$$r_1, r_2 = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$A_{ov} = x_0 - B_{ov}$$

$$B_{ov} = \frac{\dot{x}_0 - r_1 x_0}{r_2 - r_1}$$



Reference:

Boyce, W.E. and DiPrima, R.C., *Elementary Differential Equations,* John Wiley and Sons, 1969.

Remarks:

For overdamping, ω has no meaning and is, in fact, an imaginary number.

For $c = c_{crit}$, $\omega = 0$.

This program sets the angular mode of the calculator to radians. Erroneous answers will occur if degree mode is inadvertently set.

Registers R_{S0}–R_{S9} are available for user storage.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Input the system parameters	m	ENTER↑	
	of mass, damping constant,	c	ENTER↑	
	and spring constant.	k	A	ω or 0 or −1
3	Optional: Calculate c_{crit}		B	c_{crit}
4	Input initial conditions of			
	position	x_0	ENTER↑	x_0
	and velocity.	$\dot{x}_0$	C	$\dot{x}_0$
5	Input t to calculate $x(t)$, $\dot{x}(t)$,			
	and $\ddot{x}(t)$.	t	D	$x(t), \dot{x}(t), \ddot{x}(t)$
6	Repeat step 5 for a different t.			
7	Input t_1 and t_2 and number of			
	intervals (n) to calculate			
	$x(t), \dot{x}(t)$, and $\ddot{x}(t)$ automatically.	t_1	ENTER↑	t_1
		t_2	ENTER↑	t_2
		n	E	$x(t), \dot{x}(t), \ddot{x}(t)$
8	For different initial conditions			
	for the same system, go to			
	step 4.			
9	For a different system, go to			
	step 2.			

Example:

A mass of 20 g stretches a spiral spring 10 cm. The mass is pulled down an additional 4 cm, held, and then released. Find the mass displacement and velocity at 0.1 second intervals up to 1 second for the cases in which (a) c = 50 dyne-sec/cm (b) $c = c_{crit}$ and (c) c = 400 dyne-sec/cm.

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{20g\ (980\ cm/s^2)}{10\ cm} = \frac{20 \times 980}{10}\ \text{dyne/cm}$$

(a) c = 50

Keystrokes:	Outputs:	
20 ENTER↑ 50 ENTER↑ 20 ENTER↑		
980 × 10 ÷ A	9.820	(ω)
B	395.980	(c_{crit})
4 ENTER↑ 0 C	4.000	(x_0)
0 D	4.000 ***	(x)
	1.000000000-09 ***	($\dot{x}$)
	-392.00 ***	($\ddot{x}$)
0.1 D	2.334 ***	(x)
	-29.296 ***	($\dot{x}$)
	-155.494 ***	($\ddot{x}$)
0.2 D	-0.827 ***	(x)
	-28.715 ***	($\dot{x}$)
	152.880 ***	($\ddot{x}$)

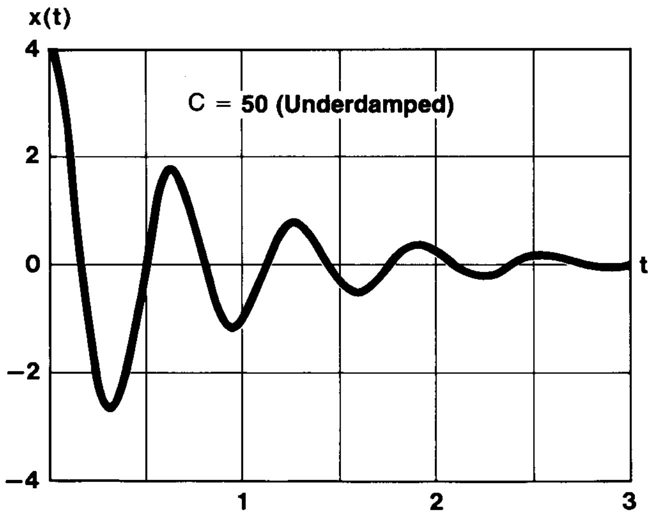
Or, the same results can be achieved automatically.

0 ENTER↑ 1 ENTER↑ 10 E	0.000 ***	(t)
	4.000 ***	(x)
	1.000000000-09 ***	($\dot{x}$)
	-392.000 ***	($\ddot{x}$)
	0.100 ***	
	2.334 ***	
	-29.296 ***	
	-155.494 ***	
	0.200 ***	
	-0.827 ***	
	-28.715 ***	

152.880	***
0.300	***
-2.629	***
-5.330	***
270.947	***
0.400	***
-1.932	***
17.139	***
146.511	***
0.500	***
0.153	***
20.950	***
-67.408	***
0.600	***
1.655	***
7.187	***
-180.174	***
0.700	***
1.503	***
-9.272	***
-124.104	***
0.800	***
0.184	***
-14.685	***
18.677	***
0.900	***
-0.990	***
-7.173	***
114.959	***
1.000	***
-1.114	***
4.406	***
98.133	***

Solution (a) $c = 50$

t s	x cm	$\dot{x}$ cm/s	$\ddot{x}$ cm/s^2
0	4.000	0.00	-392.000
.1	2.334	-29.296	-155.494
.2	-0.827	-28.715	152.880
.3	-2.629	-5.330	270.947
.4	-1.932	17.139	146.511
.5	0.153	20.950	-67.408
.6	1.655	7.187	-180.174
.7	1.503	-9.272	-124.104
.8	0.184	-14.685	18.677
.9	-0.990	-7.173	114.959
1.0	-1.114	4.406	98.133



(b) $c = c_{crit}$

Keystrokes:	Outputs:
20 ENTER↑ 395.98 ENTER↑	
20 ENTER↑ 980 ×	
10 ÷ A →	0.000
4 ENTER↑ 0 C →	4.000 (x_0)
0 D →	4.000 *** (x)
	0.000 *** ($\dot{x}$)
	-392.000 *** ($\ddot{x}$)

0.1 **D** ⟶	2.958 ***
	−14.567 ***
	−1.464 ***
0.2 **D** ⟶	1.646 ***
	−10.826 ***
	53.041 ***

⋮

Or, automatically:

0 **ENTER↑** 1 **ENTER↑** 10 **E** ⟶	0.000 ***
	4.000 ***
	0.000 ***
	−392.000 ***
	0.100 ***
	2.958 ***
	−14.567 ***
	−1.464 ***
	0.200 ***
	1.646 ***
	−10.826 ***
	53.041 ***
	0.300 ***
	0.815 ***
	−6.034 ***
	39.622 ***
	0.400 ***
	0.378 ***
	−2.990 ***
	22.122 ***
	0.500 ***
	0.169 ***
	−1.389 ***
	10.970 ***
	0.600 ***
	0.073 ***
	−0.619 ***
	5.098 ***
	0.700 ***
	0.031 ***

-0.268 ***
2.274 ***

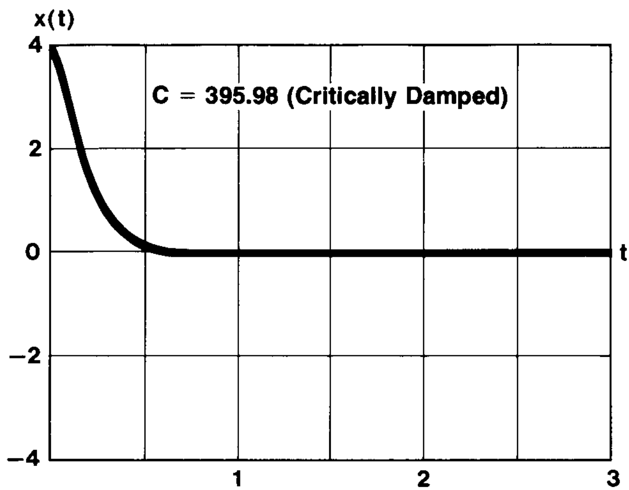
0.800 ***
0.013 ***
-0.114 ***
0.986 ***

0.900 ***
0.005 ***
-0.048 ***
0.419 ***

1.000 ***
0.002 ***
-0.020 ***
0.175 ***

Solution (b) $c = c_{crit}$

t s	x cm	$\dot{x}$ cm/s	$\ddot{x}$ cm/s^2
0	4.000	0.000	-392.000
.1	2.958	-14.567	-1.464
.2	1.646	-10.826	53.041
.3	0.815	-6.034	39.622
.4	0.378	-2.990	22.122
.5	0.169	-1.389	10.970
.6	0.073	-0.619	5.098
.7	0.031	-0.268	2.274
.8	0.013	-0.114	0.986
.9	0.005	-0.048	0.419
1.0	0.002	-0.020	0.175



(c) c = 400

Keystrokes:	Outputs:	
20 ENTER↑ 400 ENTER↑ 20 ENTER↑		
980 × 10 ÷ A →	-1.000 ***	
4 ENTER↑ 0 C →	4.000	(x_0)
0 D →	4.000 ***	(x)
	0.000 ***	$(\dot{x})$
	-392.000 ***	$(\ddot{x})$
0.1 D →	2.963 ***	(x)
	-14.469 ***	$(\dot{x})$
	-0.963 ***	$(\ddot{x})$
0.2 D →	1.660 ***	(x)
	-10.752 ***	$(\dot{x})$
	52.336 ***	$(\ddot{x})$
Or, automatically:		
0 ENTER↑ 1 ENTER↑ 10 E →	0.000 ***	
	4.000 ***	
	0.000 ***	
	-392.000 ***	
	0.100 ***	
	2.963 ***	
	-14.469 ***	
	-0.963 ***	
	0.200 ***	
	1.660 ***	
	-10.752 ***	
	52.336 ***	
	0.300 ***	
	0.833 ***	
	-6.032 ***	
	39.022 ***	
	0.400 ***	
	0.394 ***	
	-3.028 ***	
	21.916 ***	
	0.500 ***	
	0.180 ***	
	-1.433 ***	

11.005 ***

0.600 ***
0.081 ***
–0.656 ***
5.212 ***

0.700 ***
0.035 ***
–0.293 ***
2.384 ***

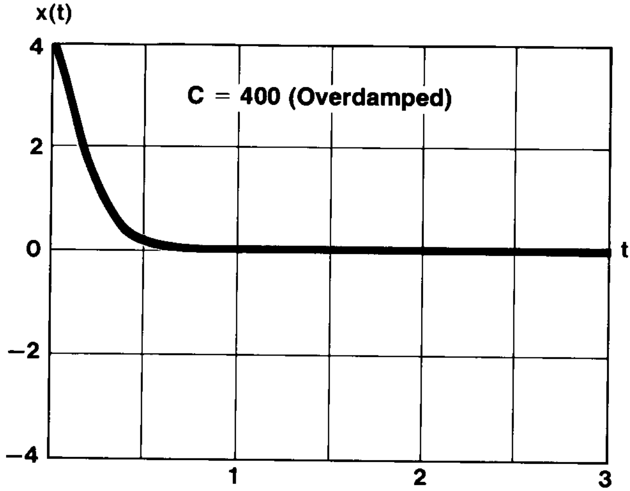
0.800 ***
0.015 ***
–0.129 ***
1.066 ***

0.900 ***
0.007 ***
–0.056 ***
0.470 ***

1.000 ***
0.003 ***
–0.024 ***
0.205 ***

Solution (c) c = 400

t s	x cm	$\dot{x}$ cm/s	$\ddot{x}$ cm/s^2
0	4.000	0.000	–392.000
.1	2.963	–14.469	–0.963
.2	1.660	–10.752	52.336
.3	0.833	–6.032	39.022
.4	0.394	–3.028	21.916
.5	0.180	–1.433	11.005
.6	0.081	–0.656	5.212
.7	0.035	–0.293	2.384
.8	0.015	–0.129	1.066
.9	0.007	–0.056	0.470
1.0	0.003	–0.024	0.205


x(t)
C = 400 (Overdamped)
4
2
0
−2
−4
1
2
3
t

Notes

VIBRATION FORCED BY F_0 cos ωt

This program finds the steady-state solution for an object undergoing damped forced oscillations from a periodic external force of the form $F_0 \cos \omega t$. The differential equation to be solved is

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

The program calculates the following variables: ω_0, ω_n, ζ, ω_{res}, AMP, δ, $x(t)$, $\dot{x}(t)$, and $\ddot{x}(t)$, which are defined as follows:

Equations:

The steady-state solution ($t \rightarrow \infty$) to this equation is

$$x(t) = \frac{F_0}{\Delta} \cos (\omega t - \delta)$$

$$\dot{x}(t) = -\omega \frac{F_0}{\Delta} \sin (\omega t - \delta)$$

where:

$$\Delta = \sqrt{m^2 (\omega_0^2 - \omega^2)^2 + c^2 \omega^2}$$

$$\omega_0 = \sqrt{\frac{k}{m}} = \text{natural frequency or undamped system}$$

$$\omega_n = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \text{damped natural frequency}$$

$$\zeta = \frac{c}{C_{crit}} = \frac{c}{2m\,\omega_0} = \text{damping ratio}$$

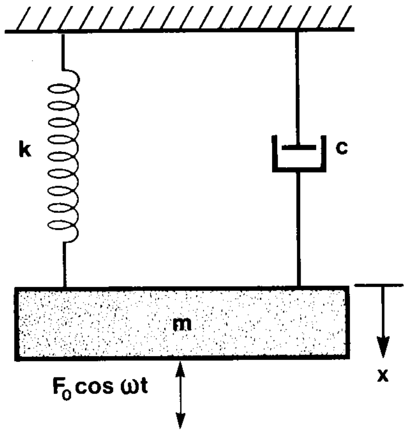
$$\delta = \tan^{-1} \frac{c\omega}{m(\omega_0^2 - \omega^2)}$$

$$AMP = \frac{F_0}{\Delta}$$

ω_{res} is computed from

$$\omega_{res}^2 = \omega_0^2 - \frac{1}{2}\left(\frac{c}{m}\right)^2$$

$$AMP_{max} = \frac{F_0}{\Delta} \text{ (where } \omega = \omega_{res})$$



Reference:

Boyce, W.E. and DiPrima, R.C., *Elementary Differential Equations,* John Wiley and Sons, 1969.

Remarks:

The above solution does not take into account the initial conditions ($x(0)$, $\dot{x}(0)$) of the system, consequently values of $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ calculated by this program are for large values of t. However, should you need values of $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ for the system with initial conditions $x(0)$ and $\dot{x}(0)$, use ME1-18A. Calculate the homogeneous solution $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ and add it to the values (the particular solution) calculated by this program.

This program sets the angular mode of the calculator to radians.

Registers R_{S0}—R_{S9} are available for user storage.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Input the system parameters			
	of mass,	m	ENTER↑	
	damping coefficient	c	ENTER↑	
	and spring constant.	k	A	ω_0, ω_n, ζ
3	Optional: Calculate the			
	resonant frequency ω_{res}.		B	ω_{res}
4	Input the frequency of external			
	excitation	ω	ENTER↑	
	and the external excitation			
	force.	F_0	C	AMP, δ(deg.)
5	Input t to calculate x(t), $\dot{x}(t)$,			
	$\ddot{x}(t)$.	t	D	x, $\dot{x}(t)$ $\ddot{x}(t)$
6	Input t_1 and t_2 and number of			
	intervals to calculate x(t), $\dot{x}(t)$,			
	and $\ddot{x}(t)$ automatically.	t_1	ENTER↑	t_1
		t_2	ENTER↑	t_2
		n	E	n + 1 values of
				x(t), $\dot{x}(t)$,
				$\ddot{x}(t)$ between
				t_1 and t_2
7	For a different external			
	excitation, applied to the same			
	system, go to step 4.			
8	For a different system, go to			
	step 2.			

Example:

A 400-lb. weight is suspended from a spring and stretches it a distance of 2 inches. The damping constant of the system is 0.5 lb-sec/ft. If the weight is driven by a periodic external force whose greatest value is 5 pounds, find (a) the resonant frequency of the system and (b) the amplitude and phase shift of the oscillation that will result if the mass is driven at the resonant frequency. Calculate the position, velocity, and acceleration for t = 6.0 sec. Also calculate the position, velocity, and acceleration for t_1 = 6 sec. and t_2 = 10 sec. with four intervals (n = 4).

$$m = \frac{F}{g} = \frac{400 \text{ lb}}{32.2 \text{ ft/sec}^2} \qquad k = \frac{F}{x} = \frac{400 \text{ lb}}{2 \text{ in}} \frac{12 \text{ in}}{1 \text{ ft}}$$

Keystrokes:	**Outputs:**
400 ENTER↑ 32.2 ÷ .5 ENTER↑	
400 ENTER↑ 2 ÷ 12 × A →	13.900 *** (ω_0)
	13.900 *** (ω_n)
	0.001 *** (ζ)
B →	13.900 (ω_{res})

(To drive the system at the resonant frequency, leave ω_{res} in the display and key in the driving force of 5 pounds).

5 C →	0.719 *** (AMP)
	89.917 *** (δ in deg.)
6 D →	0.712 *** (x)
	-1.464 *** ($\dot{x}$)
	-137.499 *** ($\ddot{x}$)

or automatically:

6 ENTER↑ 10 ENTER↑ 4 E →	6.000 ***
	0.712 ***
	-1.464 ***
	-137.499 ***
	7.000 ***
	0.065 ***
	-9.959 ***
	-12.582 ***
	8.000 ***
	-0.681 ***
	-3.223 ***
	131.577 ***

9.000	***
–0.386	***
8.442	***
74.510	***
10.000	***
0.500	***
7.197	***
–96.508	***

Notes

EQUATIONS OF STATE

This card provides both ideal gas and Redlich-Kwong equations of state. Given four of the five state variables, the fifth is calculated. For the Redlich-Kwong solution, the critical pressure and temperature of the gas must be known. They are not needed for ideal gas solutions.

Values of the Universal Gas Constants

Value of R	Units of R	Units of P	Units of V	Units of T
8.314	N - m/g mole - K	N/m^2	m^3/g mole	K
83.14	cm^3 - bar/g mole - K	bar	cm^3/g mole	K
82.05	cm^3 - atm/g mole - K	atm	cm^3/g mole	K
0.7302	atm - ft^3/lb mole - °R	atm	ft^3/lb mole	°R
10.73	psi - ft^3/lb mole - °R	psi	ft^3/lb mole	°R
1545	psf - ft^3/lb mole - °R	psf	ft^3/lb mole	°R

Critical Temperatures and Pressures

Substance	T_c, K	T_c, °R	P_c, ATM
Ammonia	405.6	730.1	112.5
Argon	151	272	48.0
Carbon dioxide	304.2	547.6	72.9
Carbon monoxide	133	239	34.5
Chlorine	417	751	76.1
Helium	5.3	9.5	2.26
Hydrogen	33.3	59.9	12.8
Nitrogen	126.2	227.2	33.5
Oxygen	154.8	278.6	50.1
Water	647.3	1165.1	218.2
Dichlorodifluoromethane	384.7	692.5	39.6
Dichlorofluoromethane	451.7	813.1	51.0
Ethane	305.5	549.9	48.2
Ethanol	516.3	929.3	63
Methanol	513.2	923.8	78.5
n-Butane	425.2	765.4	37.5
n-Hexane	507.9	914.2	29.9
n-Pentane	469.5	845.1	33.3
n-Octane	568.6	1023.5	24.6
Trichlorofluoromethane	471.2	848.1	43.2

Equations:

Ideal gas:

$$PV = nRT$$

Redlich-Kwong:

$$P = \frac{nRT}{(V - b)} - \frac{a}{T^{1/2}\, V\, (V + b)}$$

$$a = 4.934\ b\ nRT_c^{1.5}$$

$$b = 0.0867 \frac{nRT_c}{P_c}$$

where:

P is the absolute pressure;

V is the volume;

n is the number of moles present;

R is the universal gas constant;

T is the absolute temperature;

T_c is the critical temperature;

P_c is the critical pressure.

Remarks:

P, V, n and T must have units compatible with R.

At low temperatures or high pressures, the ideal gas law does not represent the behavior of real gases.

No equation of state is valid for all substances nor over an infinite range of conditions. The Redlich-Kwong equation gives moderate to good accuracy for a variety of substances over a wide range of conditions. Results should be used with caution and tempered by experience.

Solutions for V, n, R and T, using the Redlich-Kwong equation, require an iterative technique. Newton's method is employed using the ideal gas law to generate the initial guess. Iteration time is generally a function of the amount of deviation from ideal gas behavior. For extreme cases, the routine may fail to converge entirely, resulting in an "error".

Registers R_0, R_1 and R_{S0}—R_{S9} are available for user storage.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Select Redlich-Kwong (1.00) or			
	ideal gas (0.00) using mode			
	toggle.		f A	1.00/0.00
3	If you selected ideal gas in			
	step 2, skip to step 5.			
4	Input critical temperature	T_c	f B	T_c
	and critical pressure.	P_c	f C	P_c
5	Input four of the following:			
	Absolute pressure	P	A	P
	Volume	V	B	V
	Number of moles	n	C	n
	Universal gas constant	R	D	R
	Absolute temperature	T	E	T
6	Calculate remaining value:			
	Absolute pressure		A	P
	Volume		B	V
	Number of moles		C	n
	Universal gas constant		D	R
	Absolute temperature		E	T
7	For a new case, go to steps 2,			
	4, or 5 and change values or			
	mode.			

Example 1:

0.63 g moles of air are enclosed in a 25,000 cm^3 space at 1200 K. What is the pressure in bars? Assume an ideal gas.

Keystrokes: **Outputs:**

Select ideal gas by pressing f A until 0.00 is displayed.

f A f A ⟶ 0.00

25000 B .63 C 83.14 D

1200 E A ⟶ 2.51 (bars)

Example 2:

What is the specific volume (ft^3/lb) of a gas at atmospheric pressure and at a temperature of 513°R? The molecular weight is 29. Assume an ideal gas.

Keystrokes:	**Outputs:**	
f A →	0.00	
513 E 29 1/x C 0.7302		
D 1 A B →	12.92	(ft^3/lb)

What is the density?

1/x →	0.08	(lb/ft^3)

What is the density at 1.32 atmospheres and 555°R?

1.32 A 555 E B 1/x →	0.09	(lb/ft^3)

Example 3:

The specific volume of a gas in a container is 800 cm^3/g mole. The temperature will reach 400 K. What will the pressure be according to the Redlich-Kwong relation?

P_c = 48.2 atm

T_c = 305.5 K

R = 82.05 cm^3 – atm/g mole-K

Keystrokes:	**Outputs:**	
f A →	1.00	
305.5 f B 48.2 f C 82.05		
D 1 C 400 E 800 B A →	36.27	(atm)

Example 4:

6 gram moles of carbon dioxide gas are held at a pressure of 50 atmospheres, and at a temperature of 500 K. What is the volume in cubic centimeters? Use the Redlich-Kwong relation.

T_c = 304.2 K

P_c = 72.9 atm

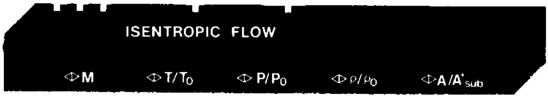
R = 82.05 cm^3 – atm/g mole – K

Keystrokes:	**Outputs:**	
f A →	1.00	
72.9 f C 304.2 f B 82.05		
D 6 C 50 A 500 E B →	4695.86	(cm^3)

How many moles could be contained at this temperature and pressure in 5 liters?

5000 B C →	6.39	(g moles)

ISENTROPIC FLOW FOR IDEAL GASES



This card replaces isentropic flow tables for a specified specific heat ratio k. Inputs and outputs are interchangeable with the exception of k.

The following values are correlated:

M is the Mach number;

T/T_0 is the ratio of flow temperature T to stagnation or zero velocity temperature T_0;

P/P_0 is the ratio of flow pressure P to stagnation pressure P_0;

ρ/ρ_0 is the ratio of flow density ρ to stagnation density ρ_0;

A/A^*_{sub} and A/A^*_{sup} are the ratios of flow area A to the throat area A* in converging—diverging passages. A/A^*_{sub} refers to subsonic flow while A/A^*_{sup} refers to supersonic flow.

Equations:

$$T/T_0 = \frac{2}{2 + (k - 1)\,M^2}$$

$$P/P_0 = (T/T_0)^{k/(k-1)}$$

$$\rho/\rho_0 = (T/T_0)^{1/(k-1)}$$

$$A/A^* = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}\,M^2\right)\right]^{\frac{k+1}{2(k-1)}}$$

In the last equation M^2 is determined using Newton's method. The initial guess used is as follows with a positive exponent for supersonic flow:

$$M_0^2 = (\sqrt{\mathrm{Frac}\,(A/A^*)} + A/A^*)^{\pm 3}$$

Remarks:

After an input of A/A*, the program begins to iterate to find M^2 for future use. This iteration will normally take less than one minute, but may take longer on occasion. For extreme values of k (1.4 is optimum) the routine may fail to converge at all. An "Error" message will eventually halt the routine if it goes out of control.

A/A* values of 1.00 are illegal inputs. Instead, input an M of 1.00.

The calculator uses flag 3 to decide whether to store or calculate a value. If you use the data input keys (setting flag 3) and then wish to calculate a parameter based on a prior input, clear flag 3 before pressing the appropriate user definable keys.

Registers R_0, R_5 and R_{S0}–R_I are available for user storage.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Input specific heat ratio.	k	f A	k
3	Input one of the following:			
	Mach number	M	A	M
	Temperature ratio	T/T_0	B	M
	Pressure ratio	P/P_0	C	M
	Density ratio	ρ/ρ_0	D	M
	Subsonic area ratio	A/A^*_{sub}	E	M
	Supersonic area ratio	A/A^*_{sup}	f E	M
4	Calculate one of the following:			
	Mach number		A	M
	Temperature ratio		B	T/T_0
	Pressure ratio		C	P/P_0
	Density ratio		D	ρ/ρ_0
	Area ratio (subsonic or			
	supersonic)		E	A/A*
4′	Calculate and output all			
	values automatically.		f B	$k, M, T/T_0, P/P_0$
				ρ/ρ_0, A/A*
5	For another calculation based			
	on same input, go to step 4			
	(or 4′). For a new input, go to			
	step 3. For a new specific heat			
	ratio, go to step 2.			

Example 1:

A pilot is flying at Mach 0.93 and reads on air temperature of 15 degrees Celsius (288 K) on a thermometer that reads stagnation temperature T_0. What is the true temperature assuming that k = 1.38?

Keystrokes:	Outputs:	
1.38 [f] [A] →	1.380	
.93 [A] →	0.930	
[B] →	0.859	(T/T_0)
288 [×] →	247.352	(T, K)
273 [−] →	−25.648	(T, °C)

If the same pilot reads a stagnation pressure P_0 of 700 millimeters of mercury, what is the true air pressure?

(Since the data input flag was set when 288 was keyed in, we must either clear it, or input 0.93 again.)

.93 [A] [C] →	0.575	(P/P_0)
700 [×] →	402.843	(mm Hg)

Example 2:

A converging, diverging passage has supersonic flow in the diverging section. At an area ratio A/A* of 1.60, what are the isentropic flow ratios for temperature, pressure and density? What is the Mach number? k = 1.74.

Keystrokes:	Outputs:	
1.74 [f] [A] →	1.740	
1.60 [f] [E] →	2.105	(M)
[B] →	0.379	(T/T_0)
[C] →	0.102	(P/P_0)
[D] →	0.269	(ρ/ρ_0)

or, alternatively, using automatic output.

[f] [B] →	1.740 ***	(k)
	2.105 ***	(M)
	0.379 ***	(T/T_0)
	0.102 ***	(P/P_0)
	0.269 ***	(ρ/ρ_0)
	1.600 ***	(A/A*)

Notes

CONDUIT FLOW

This program solves for the average velocity, or the pressure drop for viscous, incompressible flow in conduits.

Equations:

$$v^2 = \frac{\Delta P/\rho}{2\left(f\frac{L}{D} + \frac{K_T}{4}\right)}$$

For laminar flow (Re < 2300)

$$f = 16/Re$$

For turbulent flow (Re > 2300)

$$\frac{1}{\sqrt{f}} = 1.737 \ln\frac{D}{\epsilon} + 2.28 - 1.737 \ln\left(4.67 \frac{D}{\epsilon \, Re \sqrt{f}} + 1\right)$$

is solved by Newton's method.

$$\frac{1}{\sqrt{f_0}} = 1.737 \ln\frac{D}{\epsilon} + 2.28$$

is used as an initial guess in the iteration.

where:

Re is the Reynolds number, defined as $\rho Dv/\mu$;

D is the pipe diameter;

ϵ is the dimension of irregularities in the conduit surface (see table 2);

f is the Fanning friction factor for conduit flow;

ΔP is the pressure drop along the conduit;

ρ is the density of the fluid;

μ is the viscosity of the fluid;

ν is the kinematic viscosity of the fluid;

L is the conduit length;

v is the average fluid velocity;

K_T is the total of the applicable fitting coefficients in table 1.

Table 1
Fitting Coefficients

Fitting	K
Glove valve, wide open	7.5—10
Angle valve, wide open	3.8
Gate valve, wide open	0.15—0.19
Gate valve, ¾ open	0.85
Gate valve, ½ open	4.4
Gate valve, ¼ open	20
90° elbow	0.4—0.9
Standard 45° elbow	0.35—0.42
Tee, through side outlet	1.5
Tee, straight through	.4
180° bend	1.6
Entrance to circular pipe	0.25—0.50
Sudden expansion	$(1 - A_{up}/A_{dn})^{2*}$
Acceleration from $v = 0$ to $v = v_{entrance}$	1.0

*A_{up} is the upstream area and A_{dn} is the downstream area.

Table 2
Surface Irregularities

Material	ϵ (feet)	ϵ (meters)
Drawn or Smooth Tubing	5.0×10^{-6}	1.5×10^{-6}
Commercial Steel or Wrought Iron	1.5×10^{-4}	4.6×10^{-5}
Asphalted Cast Iron	4.0×10^{-4}	1.2×10^{-4}
Galvanized Iron	5.0×10^{-4}	1.5×10^{-4}
Cast Iron	8.3×10^{-4}	2.5×10^{-4}
Wood Stave	6.0×10^{-4} to 3.0×10^{-3}	1.8×10^{-4} to 9.1×10^{-4}
Concrete	1.0×10^{-3} to 1.0×10^{-2}	3.0×10^{-4} to 3.0×10^{-3}
Riveted Steel	3.0×10^{-3} to 3.0×10^{-2}	9.1×10^{-4} to 9.1×10^{-3}

Reference:

Welty, Wicks, Wilson; *Fundamentals of Momentum, Heat and Mass Transfer*, John Wiley and Sons, Inc., 1969.

Remarks:

The correlation gives meaningless results in the region $2300 < Re < 4000$.

The solution requires an iterative procedure. The time for solution will range from 10 seconds for ΔP, to several minutes for v. The display setting is used to determine when the solution for v is adequately accurate. Time for solution of v is roughly proportional to the number or significant digits in the display setting.

If the conduit is not circular, an equivalent diameter may be calculated using the formula below:

$$D_{eq} = 4 \frac{\text{cross sectional area}}{\text{wetted perimeter}}$$

Unitary consistency must be maintained with the exception of the pressure drop ΔP. If all length units are feet, time is measured in seconds and mass is given in pounds, pressure may be input or output in pounds per square inch, using the f E keys.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2.			
2	Input the following in any order			
	(units must be consistent):			
	Viscosity of fluid	μ	f A	
	or			
	Kinematic viscosity of fluid	ν	f B	ν
	Density	ρ	f C	ρ
	Surface irregularity	ϵ	f D	ϵ
	Length of conduit	L	A	L
	Equivalent diameter of			
	passage	D	B	D
	Total fitting coefficient	K_T	C	$K_T/4$
3	Input one of the following:			
	Fluid velocity	v	D	v
	Pressure drop in compatible			
	units	ΔP	E	ΔP
	or			
	Pressure drop in psi	ΔP(psi)	f E	$144g\Delta P$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
4	Calculate one of the following:			
	Fluid velocity		[D]	v
	Pressure drop in compatible			
	units		[E]	ΔP
	or			
	Pressure drop in psi		[f] [E]	ΔP(psi)
5	Optional: After calculation of			
	ΔP or v, display Reynolds			
	number		[R↓]	Re
	and Fanning friction factor.		[R↓]	f
6	For a new case, go to step 2 or			
	step 3 and change appropriate			
	inputs.			

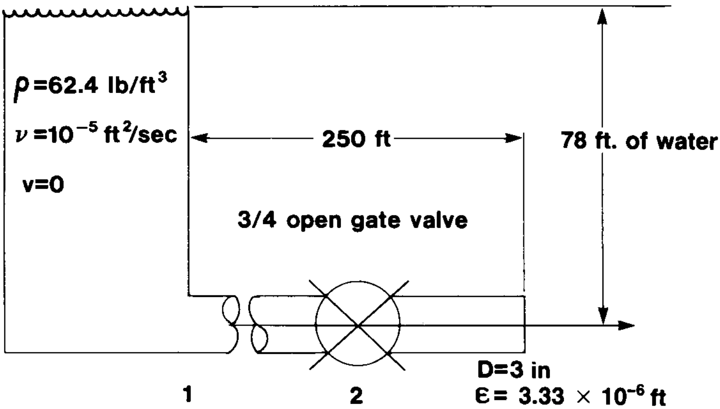
Example 1:

A heat exchanger has 20, 3 meter tube passes (60 m of pipe) with 180 degree bends connecting each pair of tubes (from table 1, $K_T = 10 \times 1.6$). The fluid is water ($\nu = 9.3 \times 10^{-7}$ m²/s, $\rho = 10^3$ kg/m³). The surface roughness is 3×10^{-4} m and the diameter is 2.54×10^{-2} m. If the fluid velocity is 3.05 m/s, what is the pressure loss? What is the Reynolds number? What is the Fanning friction factor?

Keystrokes:	Outputs:	
9.3 [EEX] [CHS] 7 [f] [B] [EEX] 3		
[f] [C] 3 [EEX] [CHS] 4 [f] [D] 60		
[A] 2.54 [EEX] [CHS] 2 [B] 16 [C]		
3.05 [D] [E] ⟶	522. 03	(ΔP, N/m²)
[R↓] ⟶	83.3 03	(Re)
[R↓] ⟶	10.2–03	(f)

Example 2:

For the system shown, what is the volume flow rate?



$$K_T = K_0 + K_1 + K_2$$
$$= 1 + .4 + 0.85 = 2.25$$

Keystrokes: **Outputs:**

First calculate and store ΔP in psi from the given data.

Keystrokes	Outputs	
78 ENTER↑ 62.4 × 144 ÷ f E →	33,8 00	(ΔP, psi)

Now store the other values.

Keystrokes	Outputs	
EEX CHS 5 f B 62.4 f C 3.33 EEX CHS 6 f D 250 A 3 ENTER↑ 12 ÷ B 2.25 C D →	17.8 00	(v, ft/sec)

Calculate volume flow rate (v × Area).

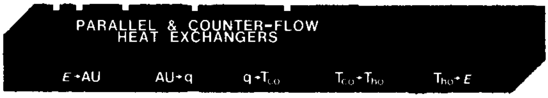
Keystrokes	Outputs	
1.5 ENTER↑ 12 ÷ ENTER↑ × π × × →	873.–03	(ft³/sec)

What will the height of the water be when the velocity is 15 ft/sec?

Keystrokes	Outputs	
15 D f E →	24.7 00	(ΔP, psi)
144 × 62.4 ÷ →	57.0 00	(ft)

Notes

PARALLEL & COUNTER FLOW HEAT EXCHANGERS



This two card set allows analysis of counter-flow, parallel-flow, parallel-counter flow, and cross-flow (both fluids unmixed) heat exchanges.

The program is organized in four segments. The first side of card 1 performs heat balance calculations and acts as controller for the three slave program segments. Slave program segment one, on side 2 of card 1, is applicable to parallel-flow and counter-flow heat exchanges. Counter-flow is selected by pressing **f** **E** until 1.00 appears. Parallel-flow is selected by pressing **f** **E** until 0.00 appears.

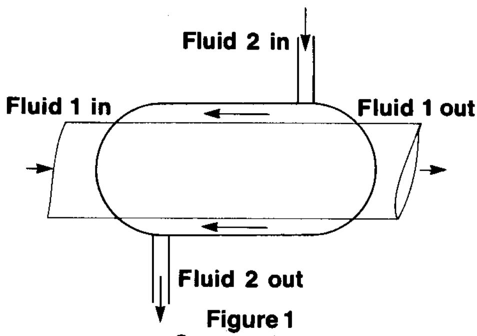


Figure 1
Counter-Flow

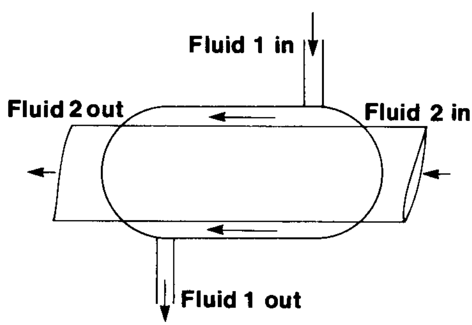


Figure 2
Parallel-Flow

The slave segment for **parallel-counter-flow** configuration (with an even number of tube passes) is on side 1 of card 2.

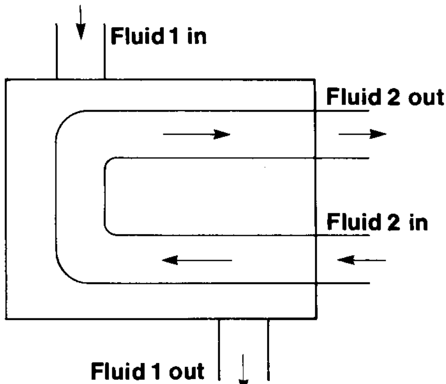


Figure 3 Parallel-Counter-Flow (Even Number Of Tube Passes)

The slave segment for **cross-flow** (with both fluids unmixed) is on side 2 of card 2.

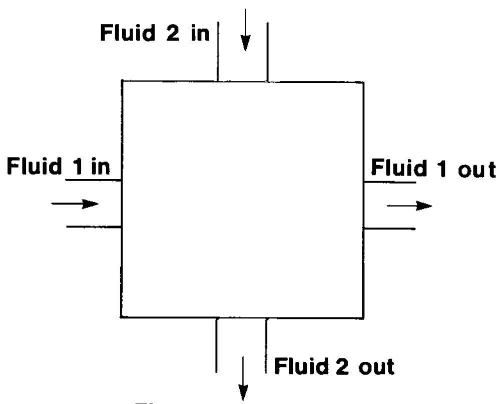


Figure 4 Cross Flow (Both Fluids Unmixed)

Equations:

Heat exchanger effectiveness E is the ratio of actual heat transfer to maximum possible heat transfer.

$$E = \frac{q}{C_{min}\,(T_{hin} - T_{cin})} = \frac{C_h\,(T_{hin} - T_{ho})}{C_{min}\,(T_{hin} - T_{cin})} = \frac{C_c\,(T_{co} - T_{cin})}{C_{min}\,(T_{hin} - T_{cin})}$$

where:

q is the actual heat transfer;

T_{hin} and T_{cin} are the inlet temperatures of the hot and cold fluids, respectively;

T_{ho} and T_{co} are the outlet temperatures of the hot and cold fluids, respectively;

C_h and C_c are the heat capacities of the hot and cold fluids, respectively, e.g., $C_h = m_h \times c_{ph}$, where m_h is the flow rate and c_{ph} is the specific heat capacity of the hot fluid;

C_{min} and C_{max} (which are used later) are the smaller and larger values of C_h and C_c.

Effectiveness can be related to the product of the surface area of an exchanger and the overall transfer coefficient for specific geometries. This product is designated AU. The geometries considered in this pac have the following correlations:

Counter-Flow (See figure 1)

$$E = \frac{1 - e^{-\frac{AU}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)}}{1 - (C_{min}/C_{max})\, e^{-\frac{AU}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)}}$$

For $C_{min}/C_{max} = 1$

$$E = \frac{AU/C_{min}}{1 + AU/C_{min}}$$

Parallel-Flow (See figure 2)

$$E = \frac{1 - e^{-\frac{AU}{C_{min}}(1 + C_{min}/C_{max})}}{1 + C_{min}/C_{max}}$$

For $C_{min}/C_{max} = 0$, C_{min} is set to 1.

Parallel-Counter-Flow; Shell Mixed with an Even Number of Tube Passes (See figure 3)

$$E = \frac{2}{\left(1 + \frac{C_{min}}{C_{max}}\right) + \sqrt{1 + \left(\frac{C_{min}}{C_{max}}\right)^2}\left[\frac{1 + e^{-x}}{1 - e^{-x}}\right]}$$

where:

$$x = \frac{AU}{C_{min}}\sqrt{1 + \left(\frac{C_{min}}{C_{max}}\right)^2}$$

Cross-Flow; Both Fluids Unmixed (See figure 4)

No exact expression exists for this case, but the following is a very good approximation. Note that it cannot be stated explicitly in terms of AU and thus requires an iterative solution.

$$E = 1 - e^{\left(e^{\left(-\frac{AU}{C_{min}}\frac{C_{min}}{C_{max}}y\right)} - 1\right)\left(\frac{C_{max}}{C_{min}}\frac{1}{y}\right)}$$

where:

$$y = \left[\frac{C_{min}}{AU}\right]^{0.22}$$

References:

W.M. Kays and A.L. London, *Compact Heat Exchangers,* National Press, 1955.

Eckert and Drake, *Heat and Mass Transfer,* McGraw-Hill.

Remarks:

Registers R_{S0}-R_{S9}, R_C, R_E, and R_I are available for user storage.

Solution for AU, using the cross-flow slave card takes significantly longer than other solutions because of the iterative technique required.

You should always solve for all values (AU, q, T_{co}, T_{ho} and E). It is quite possible for the heat balance equations to yield meaningless solutions for a particular type of heat exchange. By calculating all results, you are assured that the configuration being used is capable of the performance specified. An error message during calculation of AU or q usually indicates a violation of the second law of thermodynamics.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 of card 1.			
2	Select proper configuration			
	card and side, and load:			
	a. Parallel or counter-flow			
	exchangers→card 1,			
	side 2.			
	b. Parallel-counter-flow			
	(even number of tube			
	passes)→card 2, side 1.			
	c. Cross-flow (both fluids			
	unmixed→card 2, side 2.			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
3	If display says "Crd" press CLx		CLx	0.00
4	If you loaded parallel/			
	counter-flow configurations in			
	step 2, select counter flow			
	(1) or parallel-flow (0) using			
	mode toggle.		f E	1.00/0.00
5	Input the following values			
	Cold fluid inlet temperature	T_{cin}	f A	T_{cin}
	Cold fluid density flow rate	$\dot{m}_c$	ENTER↑	m_c
	then			
	Cold fluid heat capacity	c_{pc}	f B	C_c
	and			
	Hot fluid inlet temperature	T_{hin}	f C	T_{hin}
	Hot fluid density flow rate	$\dot{m}_h$	ENTER↑	m_h
	then			
	Hot fluid heat capacity	c_{ph}	f D	C_h
6	If the remaining known is			
	effectiveness, go to step 7.			
	If area-conductance product,			
	go to step 8. If heat transfer,			
	go to step 9. If cold fluid outlet			
	temperature, go to step 10.			
	If hot fluid outlet temperature,			
	go to step 11.			
7	With effectiveness displayed,			
	calculate area-conductance			
	product.	*E*	A	AU
8	With area-conductance			
	product displayed, calculate			
	heat transfer.	AU	B	q

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
9	With heat transfer displayed,			
	calculate cold fluid outlet			
	temperature.	q	C	T_{co}
10	With cold fluid outlet			
	temperature displayed,			
	calculate hot fluid outlet			
	temperature.	T_{co}	D	T_{ho}
11	With hot fluid outlet tempera-			
	ture displayed, calculate			
	effectiveness.	T_{ho}	E	E
12	Go back to step 6 and com-			
	plete calculation of all outputs.			
13	For a new configuration, go to			
	step 2. It is not necessary to			
	repeat the input process if			
	values remain unchanged.			
14	For new input values, go to			
	step 5 and change appropriate			
	variables.			

Example 1:

Water (c_p = 1 Btu/lb- °F) is used to cool an oil (c_p = .53 Btu/lb- °F) from 200°F to 110°F. The water flow rate is 20,000 pounds per hour while the oil flows at 37,000 pounds per hour. If the water inlet temperature is 55°F and U is 25 Btu/ft²-hr-°F for the heat exchangers being considered, what are the area requirements for counter-flow, parallel-flow, parallel-counter-flow and cross-flow?

Knowns:

c_{pc} = 1.0 Btu/lb-°F

$\dot{m}_c$ = 20,000 lb/hr

c_{ph} = 0.53 Btu/lb-°F

$\dot{m}_h$ = 37,000 lb/hr

T_{cin} = 55°F

$T_{hin} = 200°F$

$T_{ho} = 110°F$

$U = 25$ Btu/ft^2-hr-°F

Keystrokes:	Outputs:	
Load side 1 and side 2 of card 1 and select counter-flow mode.		
55 f A 20000 ENTER↑ 1		
f B 200 f C 37000 ENTER↑		
.53 f D f E →	1.00	(Counter-flow mode on)
110 E →	0.62	(Effectiveness)
Since effectiveness is the same for all configurations, store it for later use.		
STO I →	0.62	
Calculate AU.		
A →	31587.76	(AU)
25 ÷ →	1263.51	(ft^2)
Switch to parallel configuration.		
f E →	0.00	(parallel selected)
RCL I →	0.62	
A →	Error	(Violation of second law)
CL X →	–0.23	
Load parallel-counter flow configuration on side 1 of card 2 and clear display of "Crd."		
CL X →	–0.23	
RCL I A →	Error	(Violation of second law)
CL X →	–0.06	
Load cross-flow configuration on side 2 of card 2 and clear display of "Crd".		
CL X RCL I A →	39383.22	(AU)
25 ÷ →	1575.33	(ft^2)

(Do not alter storage registers if you intend to continue with example 2.)

Example 2:

If a counter flow exchanger with an area of 1000 ft^2 and an overall heat transfer coefficient of 27 Btu/ft^2-hr-°F is available, how close will the outlet temperature of the oil be to 110°F? What will the total heat transfer and outlet water temperature be? All unspecified values remain the same as example 1.

Keystrokes:	**Outputs:**	
Load counter-flow routine on side 2 of card 1 and select counter flow mode.		
CLX f E ⟶	1.00	
Calculate AU product and calculate q.		
27 ENTER↑ 1000 × ⟶	27000.00	(AU)
B ⟶	1656452.69	(q, Btu/hr)
C ⟶	137.82	(T_{co})
D ⟶	115.53	(T_{ho})
E ⟶	0.58	(E)

Notes

PROGRAM LISTINGS

The following listings are included for your reference. A table of keycodes and keystrokes corresponding to the symbols used in the listings can be found in Appendix E of your Owner's Handbook.

Program **Page**

Appendix A
MAGNETIC CARD SYMBOLS AND CONVENTIONS

SYMBOL OR CONVENTION	INDICATED MEANING
White mnemonic: x A	White mnemonics are associated with the user-definable key they are above when the card is inserted in the calculator's window slot. In this case the value of x could be input by keying it in and pressing A.
Gold mnemonic: y x f E	Gold mnemonics are similar to white mnemonics except that the gold f key must be pressed before the user-definable key. In this case y could be input by pressing f E.
x↑y A	↑ is the symbol for ENTER↑. In this case ENTER↑ is used to separate the input variables x and y. To input both x and y you would key in x, press ENTER↑, key in y and press A.
$\boxed{x}$ A	The box around the variable x indicates input by pressing STO A.
(x) A	Parentheses indicate an option. In this case, x is not a required input but could be input in special cases.
→x A	→ is the symbol for calculate. This indicates that you may calculate x by pressing key A.
→x, y, z A	This indicates that x, y, and z are calculated by pressing A once. The values would be printed in x, y, z order.
→x; y; z A	The semi-colons indicate that after x has been calculated using A, y and z may be calculated by pressing R/S.
→"x," y A	The quote marks indicate that the x value will be "paused" or held in the display for one second. The pause will be followed by the display of y.
←→ x A	The two-way arrow ←→ indicates that x may be either output or input when the associated user-definable key is pressed. If numeric keys have been pressed between user-definable keys, x is stored. If numeric keys have not been pressed, the program will calculate x.

P? A	The question mark indicates that this is a mode setting, while the mnemonic indicates the type of mode being set. In this case a print mode is controlled. Mode settings typically have a 1.00 or 0.00 indicator displayed after they are executed. If 1.00 is displayed, the mode is on. If 0.00 is displayed, it is off.
START A	The word START is an example of a command. The start function should be performed to begin or start a program. It is included when initialization is necessary.
DEL A	This special command indicates that the last value or set of values input may be deleted by pressing A.

VECTOR STATICS

TITLE ______

Step	Key	Comment
001	*LBLA	Convert from polar to rectangular.
002	X⇄Y	
003	→R	
004	X⇄Y	
005	*LBLa	Store x, y components of $\vec{V}_1$.
006	STOB	
007	X⇄Y	
008	STOA	
009	X⇄Y	
010	RTN	
011	*LBLB	Convert from polar to rectangular.
012	X⇄Y	
013	→R	
014	X⇄Y	
015	*LBLb	Store x, y components of $\vec{V}_2$.
016	STOD	
017	X⇄Y	
018	STOC	
019	X⇄Y	
020	RTN	
021	*LBLd	Store F cos ϕ and F sin ϕ.
022	X⇄Y	
023	→R	
024	STO8	
025	X⇄Y	
026	STO9	
027	RTN	
028	*LBLC	$\vec{V}_1 + \vec{V}_2$
029	SPC	
030	RCLD	
031	RCLB	
032	+	
033	RCLA	
034	RCLC	
035	+	
036	→P	
037	PRTX	
038	X⇄Y	
039	PRTX	
040	RTN	
041	*LBLD	$\vec{V}_1 \times \vec{V}_2$
042	SPC	
043	RCLA	
044	RCLD	
045	x	
046	RCLC	
047	RCLB	
048	x	
049	-	
050	PRTX	
051	RTN	
052	*LBLE	$\vec{V}_1 \cdot \vec{V}_2$
053	SPC	
054	RCLA	
055	RCLC	
056	x	
7	RCLB	
058	RCLD	
059	x	
060	+	
061	STOE	
062	PRTX	
063	RTN	
064	RCLE	Calculate angle between vectors.
065	RCLA	
066	RCLB	
067	→P	
068	X⇄Y	
069	CLX	
070	RCLC	
071	RCLD	
072	→P	
073	X⇄Y	
074	R↓	
075	x	
076	÷	
077	COS⁻¹	
078	RCLE	
079	X⇄Y	
080	PRTX	
081	RTN	
082	*LBLe	Calculate R_1.
083	SPC	
084	RCLB	
085	RCLA	
086	→P	
087	CLX	
088	1	
089	→R	
090	STO4	
091	X⇄Y	
092	STO5	
093	RCLD	
094	RCLC	
095	→P	
096	CLX	
097	1	
098	→R	
099	STO6	
100	X⇄Y	
101	STO7	
102	R↑	
103	x	
104	R↓	
105	x	
106	R↑	
107	-	
108	STOE	
109	RCL8	
110	RCL7	
111	x	
112	RCL9	

REGISTERS

0	1	2	3	4	5	6	7	8	9
				$\cos\theta_1$	$\sin\theta_1$	$\cos\theta_2$	$\sin\theta_2$	F cos ϕ	F sin ϕ
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
x_1	y_1	x_2	y_2	used	

VECTOR STATICS

TITLE

Step	Key	Comments
001	*LBLA	Convert from polar to rectangular.
002	X⇄Y	
003	→R	
004	X⇄Y	
005	*LBLa	Store x, y components of $\vec{V}_1$.
006	STOB	
007	X⇄Y	
008	STOA	
009	X⇄Y	
010	RTN	
011	*LBLB	Convert from polar to rectangular.
012	X⇄Y	
013	→R	
014	X⇄Y	
015	*LBLb	Store x, y components of $\vec{V}_2$.
016	STOD	
017	X⇄Y	
018	STOC	
019	X⇄Y	
020	RTN	
021	*LBLd	Store F cos ϕ and F sin ϕ.
022	X⇄Y	
023	→R	
024	STO8	
025	X⇄Y	
026	STO9	
027	RTN	
028	*LBLC	$\vec{V}_1 + \vec{V}_2$
029	SPC	
030	RCLD	
031	RCLB	
032	+	
033	RCLA	
034	RCLC	
035	+	
036	→P	
037	PRTX	
038	X⇄Y	
039	PRTX	
040	RTN	
041	*LBLD	$\vec{V}_1 \times \vec{V}_2$
042	SPC	
043	RCLA	
044	RCLD	
045	x	
046	RCLC	
047	RCLB	
048	x	
049	-	
050	PRTX	
051	RTN	
052	*LBLE	$\vec{V}_1 \cdot \vec{V}_2$
053	SPC	
054	RCLA	
055	RCLC	
056	x	
7	RCLB	
058	RCLD	
059	x	
060	+	
061	STOE	
062	PRTX	
063	RTN	
064	RCLE	
065	RCLA	
066	RCLB	
067	→P	Calculate angle between vectors.
068	X⇄Y	
069	CLX	
070	RCLC	
071	RCLD	
072	→P	
073	X⇄Y	
074	R↓	
075	x	
076	÷	
077	COS⁻¹	
078	RCLE	
079	X⇄Y	
080	PRTX	
081	RTN	
082	*LBLe	
083	SPC	
084	RCLB	
085	RCLA	Calculate R_1.
086	→P	
087	CLX	
088	1	
089	→R	
090	STO4	
091	X⇄Y	
092	STO5	
093	RCLD	
094	RCLC	
095	→P	
096	CLX	
097	1	
098	→R	
099	STO6	
100	X⇄Y	
101	STO7	
102	R↑	
103	x	
104	R↓	
105	x	
106	R↑	
107	-	
108	STOE	
109	RCL8	
110	RCL7	
111	x	
112	RCL9	

REGISTERS

0	1	2	3	4	5	6	7	8	9
				$\cos\theta_1$	$\sin\theta_1$	$\cos\theta_2$	$\sin\theta_2$	F $\cos\phi$	F $\sin\phi$
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
x_1	y_1	x_2	y_2	used	

DATE ______________________ AUTHOR ______________________

```
113  RCL6
114    x
115    -
116  RCLE
117    ÷
118  PRTX
119  RCL9
120  RCL4
121    x
122  RCL8
123  RCL5
124    x
125    -
126  RCLE
127    ÷
128  PRTX
129   RTN
```

Calculate R_2.

LABELS					FLAGS
A $r_1 \uparrow \theta_1$	B $r_2 \uparrow \theta_2$	C $\rightarrow \vec{V}_1 + \vec{V}_2$	D $\rightarrow \vec{V}_1 \times \vec{V}_2$	E $\rightarrow V_1 \cdot V_2 ; \gamma$	0
a $x_1 \uparrow y_1$	b $x_2 \uparrow y_2$	c	d $F \uparrow \phi$	e $\rightarrow R_1 ; R_2$	1
0	1	2	3	4	2
5	6	7	8	9	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

SECTION PROPERTIES

TITLE ______________________

Step	Key	Comment
001	*LBLa	Clear registers.
002	CLRG	
003	RTN	
004	*LBLA	Store coordinates.
005	STOD	
006	R↑	
007	STOA	
008	R↑	
009	STOB	
010	R↑	
011	STOC	Sum ΔA.
012	RCLA	
013	+	
014	STO8	
015	RCLD	
016	RCLB	
017	-	
018	STO7	
019	x	
020	2	
021	÷	
022	ST-0	Sum ΔI_y.
023	1	
024	2	
025	÷	
026	RCLC	
027	RCLA	
028	-	
029	STO6	
030	X²	
031	RCL8	
032	GSB4	
033	ST-4	Sum ΔI_x.
034	RCL6	
035	RCLB	
036	RCLD	
037	+	
038	STO9	
039	x	
040	2	
041	4	
042	÷	
043	RCL9	
044	X²	
045	RCL7	
046	GSB4	
047	ST+3	Sum ΔM_x.
048	RCL6	
049	RCL9	
050	RCL7	
051	GSB1	
052	ST+2	Sum ΔM_y.
053	RCL7	
054	RCL8	
055	RCL6	
056	GSB1	
057	ST-1	Sum ΔI_{xy}.
058	RCLC	
059	RCLB	
060	x	
061	RCLA	
062	RCLD	
063	x	
064	-	
065	ENT↑	
066	ENT↑	
067	4	
068	÷	
069	RCL8	
070	x	
071	RCLA	
072	RCLC	
073	x	
074	RCLC	
075	X²	
076	STO9	
077	+	
078	RCLA	
079	X²	
080	ST+9	
081	+	
082	RCL7	
083	x	
084	3	
085	÷	
086	+	
087	x	
088	RCL9	
089	8	
090	÷	
091	RCL8	
092	x	
093	RCL7	
094	X²	
095	x	
096	+	
097	RCL6	
098	X≠0?	
099	÷	
100	ST+5	
101	RCLC	Recall x_i and y_i for next segment.
102	RCLD	
103	RTN	
104	*LBL1	Calculate ΔM_x and ΔM_y.
105	X²	
106	3	
107	÷	
108	X⇌Y	
109	GSB4	
110	8	
111	÷	
112	RTN	

REGISTERS

0	1	2	3	4	5	6	7	8	9
ΣA	ΣM_y	ΣM_x	ΣI_x	ΣI_y	ΣI_{xy}	$(x_{i+1} - x_i)$	$(y_{i+1} - y_i)$	$(x_{i+1} + x_i)$	$(y_{i+1} + y_i)$
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
x_i	y_i	x_{i+1}	y_{i+1}		

DATE ____________________ AUTHOR ____________________

Step	Key	Comments
113	*LBL4	Calculation subroutine.
114	X²	
115	+	
116	x	
117	RTN	
118	*LBLC	Add to sums for circular regions.
119	ENT↑	
120	ABS	
121	x	
122	Pi	
123	x	
124	4	
125	÷	
126	STOA	
127	ST+0	
128	ENT↑	
129	ABS	
130	x	
131	Pi	
132	÷	
133	4	
134	÷	
135	STOB	
136	R↓	
137	STOC	
138	R↓	
139	STOD	
140	R↑	
141	x	
142	RCLA	
143	x	
144	ST+5	
145	RCLB	
146	RCLA	
147	RCLC	
148	X²	
149	x	
150	+	
151	ST+3	
152	RCLB	
153	RCLA	
154	RCLD	
155	X²	
156	x	
157	+	
158	ST+4	
159	RCLA	
160	RCLD	
161	x	
162	ST+1	
163	RCLA	
164	RCLC	
165	x	
166	ST+2	
167	CLX	
168	RTN	

LABELS

A $x_{i+1} \uparrow y_{i+1}$	B	C $x \uparrow y \uparrow \pm d$	D	E
a	b	c	d	e
0	1 Calculate	2	3	4 Calculate
5	6	7	8	9

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

(Card 2)

TITLE ______

Step	Key	Comments
001	*LBLA	Output $\bar{x}$, $\bar{y}$ and A.
002	GSB2	
003	PRTX	
004	X⇄Y	
005	PRTX	
006	RCL0	
007	PRTX	
008	RTN	
009	*LBL2	Calculate $\bar{x}$ and $\bar{y}$.
010	SPC	
011	RCL2	
012	RCL0	
013	÷	
014	STOA	
015	RCL1	
016	RCL0	
017	÷	
018	STOB	
019	RTN	
020	*LBLB	Output Ix, Iy and Ixy.
021	SPC	
022	RCL3	
023	PRTX	
024	RCL4	
025	PRTX	
026	RCL5	
027	PRTX	
028	RTN	
029	*LBLC	Calculate $I\bar{x}$, $I\bar{y}$ and $I\bar{x}\bar{y}$ and ϕ.
030	GSB3	
031	RCLC	
032	PRTX	
033	RCLD	
034	PRTX	
035	RCLE	
036	PRTX	
037	RTN	
038	*LBL3	
039	GSB2	
040	RCL5	
041	RCL0	
042	RCLA	
043	RCLB	
044	x	
045	x	
046	-	
047	STOE	
048	ENT↑	
049	+	
050	RCL4	
051	RCLB	
052	X²	
053	RCL0	
054	x	
055	-	
056	STOD	
057	RCLA	
058	X²	
059	RCL0	
060	x	
061	CHS	
062	RCL3	
063	+	
064	STOC	
065	-	
066	X≠0?	
067	÷	
068	TAN⁻¹	
069	RTN	
070	*LBLD	Calculate $I\bar{x}$, $I\bar{y}$, $I\bar{x}\bar{y}$ and output ϕ.
071	GSB3	
072	STOI	
073	2	
074	÷	
075	PRTX	
076	*LBLE	Rotate to principal axis values.
077	1	
078	→R	
079	X²	
080	STOA	
081	RCLC	
082	x	
083	X⇄Y	
084	X²	
085	STOB	
086	RCLD	
087	x	
088	+	
089	RCLI	
090	SIN	
091	RCLE	
092	x	
093	-	
094	PRTX	
095	LSTX	
096	RCLA	
097	RCLD	
098	x	
099	+	
100	RCLB	
101	RCLC	
102	x	
103	+	
104	PRTX	
105	RTN	
106	*LBLd	Calculate Ix′, Iy′, J and Ixy′.
107	ENT↑	
108	+	
109	STOI	
110	R↓	
111	STOC	
112	R↓	

REGISTERS

0	1	2	3	4	5	6	7	8	9
ΣA	ΣM_y	ΣM_x	ΣI_x	ΣI_y	ΣI_{xy}	$(x_{i+1}-x_i)$	$(y_{i+1}-y_i)$	$(x_{i+1}+x_i)$	$(y_{i+1}+y_i)$
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
$x_i, \bar{y}, \cos^2\phi$	$y_i, \bar{x}, \sin^2\phi$	$x_{i+1}, I\bar{y}$	$y_{i+1}, I\bar{y}$	$I\bar{x}\,\bar{y}$	2ϕ

DATE ______________________ AUTHOR ______________________

```
113  STOD
114  GSB2
115     x
116  RCL6
117     x
118   CHS
119  RCL5
120     +
121  RCLD
122  RCLB
123     -
124  RCLC
125  RCLA
126     -
127     x
128  RCL0
129     x
130     +
131  STOE
132  RCLC
133    X²
134  RCLC
135  RCLA
136     x
137     2
138     x
139     -
140  RCL0
141     x
142  RCL3
143     +
144  STOC
145  RCLD
146    X²
147  RCLD
148  RCLB
149     x
150     2
151     x
152     -
153  RCL0
154     x
155  RCL4
156     +
157  STOD
158  RCLI
159     2
160     ÷
161  GSB6
162     +
163  PRTX
164  RCLC
165  RCLD
166     -
167     2
168     ÷
169  RCLI
170   SIN
171     x
172  RCLE
173  RCLI
174   COS
175     x
176     +
177  PRTX
178   RTN
```

LABELS				
A →$\bar{x}$, $\bar{y}$, ϕ	B →Ix, Iy, Ixy	C →I$\bar{x}$, I$\bar{y}$, I$\bar{x}\bar{y}$	D →I$\bar{x}\phi$ I$\bar{y}\phi$ I$\bar{x}\bar{y}\phi$	E
a	b	c	d →Ix′, Iy′, Ixy′	e
0	1 $\tan^{-1}$	2 $\bar{x}$, $\bar{y}$	3 Ix, Iy, Ixy	4
5	6 Rotate	7	8	9

FLAGS
0
1
2
3

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☒
SCI	☐
ENG	☐
n	2

STRESS ON AN ELEMENT

TITLE ______

Program	Comments
*LBLa	Store code: 1 = rectangular 2 = equiangular
1	
GTO0	
*LBLb	
2	
*LBL0	
STOI	
RTN	
*LBLd	Store ν and E.
STO9	
R↓	
STOE	
RTN	
*LBLA	Store ϵ_a, ϵ_b and ϵ_c.
STOC	
R↓	
STOB	
R↓	
STOA	
RTN	
*LBLB	Calculate ϵ_1 and ϵ_2.
RCLA	
GTOi	
*LBL2	
RCLB	
+	
*LBL1	
RCLC	
+	
STO6	
0	
GTOi	
*LBL2	
RCLC	
RCLA	
-	
*LBL1	
RCLB	
RCLC	
-	
→P	
RCLA	
RCLB	
-	
→P	
2	
√X	
x	
STO5	
2	
GTOi	
*LBL2	
1	
+	
*LBL1	
ST÷5	
ST÷6	
GSB5	Calculate τ_{max} and $\frac{s_1 + s_2}{2}$ from strains.
RCLE	
RCL9	
1	
+	
÷	
ST×5	
RCLE	
1	
RCL9	
-	
÷	
ST×6	
RCLC	
RCLB	
-	
3	
√X	
GTOi	
*LBL1	
2	
RCLB	
x	
RCLA	
-	
RCLC	
-	
RCLA	
RCLC	
GTO4	
*LBL2	
x	
2	
RCLA	
x	
RCLB	
-	
RCLC	
*LBL4	Output θ.
-	
GSB6	
R↓	
PRTX	
RTN	
*LBLC	Calculate τ_{max} and $(s_1 + s_2)/2$ from s_x, s_y and τ_{xy}.
R↑	
R↑	
STO3	
STO6	
R↑	
ST+6	
-	
STO4	
2	
ST÷6	

REGISTERS

0	1	2	3	4	5	6	7	8	9
		2θ	s_x	$s_x - s_y$	τ_{max}	$(s_1 + s_2)/2$			ν
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
ϵ_a	ϵ_b	ϵ_c		E	Control

DATE ______________________ AUTHOR ______________________

Step	Key	Comments
113	÷	
114	R↑	
115	ST02	
116	ST+2	
117	→P	
118	ST05	
119	RCL2	
120	CHS	
121	RCL4	
122	*LBL6	Calculate θ and 2θ.
123	X≠0?	
124	÷	
125	TAN⁻¹	
126	ST02	
127	2	
128	÷	
129	8	
130	RTN	
131	*LBLD	Output s_1, s_2 and τ_{max} and θ.
132	GSB5	
133	RCL5	
134	PRTX	
135	RCL2	
136	2	
137	÷	
138	PRTX	
139	RTN	
140	*LBLE	Calculate s and t from θ'.
141	SPC	
142	ENT↑	
143	+	
144	RCL2	
145	-	
146	RCL5	
147	→R	
148	RCL6	
149	+	
150	PRTX	
151	X⇄Y	
152	PRTX	
153	PTN	
154	*LBL5	Calculate ϵ_1 and ϵ_2 or s_1 and s_2.
155	SPC	
156	RCL6	
157	RCL5	
158	+	
159	PRTX	
160	RCL6	
161	RCL5	
162	-	
163	PRTX	
164	RTN	

LABELS					FLAGS
A $\epsilon_a \uparrow \epsilon_b \uparrow \epsilon_c$	B $\rightarrow \epsilon_1, \epsilon_2, \theta$	C $s_x \uparrow s_y \uparrow \tau_{xy}$	D $\rightarrow s_1, s_2, \tau_{max}, \theta$	E $\theta' \rightarrow s, \tau$	0
a Rectangular	b Equiangular	c	d E↑ν	e	1
0 Store code	1 Rectangular	2 Equiangular	3	4 Output	2
5 Calc	6 θ	7	8	9	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☐
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☒
3	☐	☒			n 3	

SODERBERG'S EQUATION FOR FATIGUE

TITLE

Step	Key	Comments
001	*LBLa	If flag 3 is set, store s_{yp}.
002	STO8	
003	F3?	
004	RTN	
005	GSB1	If flag 3 is not set, calculate s_{yp}.
006	GSB2	
007	RCLB	
008	x	
009	RCL9	
010	÷	
011	CHS	
012	RCLE	
013	1/X	
014	+	
015	÷	
016	STO8	
017	RTN	
018	*LBLb	Store or calculate s_e.
019	STO9	
020	F3?	
021	RTN	
022	GSB2	
023	RCL8	
024	x	
025	RCLB	
026	x	
027	GSB1	
028	CHS	
029	RCL8	
030	RCLE	
031	÷	
032	+	
033	÷	
034	STO9	
035	RTN	
036	*LBLA	Store or calculate A.
037	STOA	
038	F3?	
039	RTN	
040	1	
041	STOA	
042	GSB1	
043	GSB2	
044	RCLB	
045	x	
046	RCL8	
047	x	
048	RCL9	
049	÷	
050	+	
051	RCL8	
052	÷	
053	RCLE	
054	x	
055	STOA	
056	RTN	
057	*LBLB	Store or calculate K.
058	STOB	
059	F3?	
060	RTN	
061	RCL8	
062	RCLE	
063	÷	
064	GSB1	
065	-	
066	GSB2	
067	RCL8	
068	x	
069	RCL9	
070	÷	
071	÷	
072	STOB	
073	RTN	
074	*LBLC	Store or calculate P_{max}.
075	STOC	
076	F3?	
077	RTN	
078	RCLA	
079	ENT↑	
080	+	
081	RCL8	
082	x	
083	RCLE	
084	÷	
085	RCLB	
086	RCL8	
087	x	
088	RCL9	
089	÷	
090	1	
091	-	
092	RCLD	
093	x	
094	+	
095	RCLB	
096	RCL8	
097	x	
098	RCL9	
099	÷	
100	1	
101	+	
102	÷	
103	STOC	
104	RTN	
105	*LBLD	Store or calculate P_{min}.
106	STOD	
107	F3?	
108	RTN	
109	RCLA	
110	ENT↑	
111	+	
112	RCL8	

REGISTERS

0	1	2	3	4	5	6	7	8	9
								s_{yp}	s_e
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
A	K	P_{max}	P_{min}	FS	

DATE ______________ AUTHOR ______________

```
x
RCLE
÷
RCLB
RCL8
x
RCL9
÷
1
+
RCLC
x
-
1
RCLB
RCL8
x
RCL9
÷
-
÷
STOD
RTN
*LBLE        Store or calculate FS.
STOE
F3?
RTN
GSB1
GSB2
RCLB
x
RCL8
x
RCL9
÷
+
RCL9
÷
1/X
STOE
RTN
*LBLc        Output values.
SPC
RCL8
PRTX
RCL9
PRTX
RCLA
PRTX
RCLB
PRTX
RCLC
PRTX
RCLD
PRTX
RCLE
```

```
PRTX
RTN
*LBL2        Q = (Pmax - Pmin)
RCLC
RCLD
CHS
GTO0
*LBL1        Q = (Pmax + Pmin)
RCLC
RCLD
*LBL0        Q/2A
+
RCLA
÷
2
÷
RTN
```

LABELS					FLAGS
A: A	B: K	C: P_{max}	D: P_{min}	E: FS	0
a: s_{yp}	b: s_e	c: PRINT	d	e	1
0: +, A, ÷, 2, ÷	1: s_{ave}	2: s_r	3	4	2
5	6	7	8	9	3: Calc

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☐
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☒
3	☐	☒			n	3

CANTILEVER BEAMS

TITLE

Step	Key	Comment
001	*LBLa	Initialize
002	0	
003	STO3	
004	STO4	
005	STO5	
006	RTN	
007	*LBLb	Store ℓ and EI.
008	STO2	
009	R↓	
010	x	
011	STOE	
012	RTN	
013	*LBLc	Store P and a.
014	STO3	
015	X⇄Y	
016	STOA	
017	RTN	
018	*LBLd	Store W and b.
019	STO4	
020	X⇄Y	
021	STOB	
022	RTN	
023	*LBLe	Store M and c.
024	STO5	
025	X⇄Y	
026	STOC	
027	RTN	
028	*LBLA	y_1'
029	STO0	
030	RCLA	
031	GSB4	
032	LSTX	
033	x	
034	CHS	
035	3	
036	x	
037	F2?	
038	0	
039	RCL1	
040	RCLA	
041	3	
042	x	
043	-	
044	RCL1	
045	x	
046	+	
047	RCL3	
048	x	
049	RCL1	
050	x	
051	RCLB	
052	GSB4	
053	RCLB	
054	3	
055	Y^x	
056	x	
057	F2?	
058	CLX	y_2'
059	STOD	
060	R↓	
061	RCL1	
062	4	
063	÷	
064	RCLB	
065	-	
066	RCL1	
067	x	
068	RCLB	
069	X²	
070	1	
071	.	
072	5	
073	x	
074	+	
075	RCL1	
076	X²	
077	x	
078	RCLD	
079	+	
080	RCL4	
081	x	
082	-	$y' + y_2'$
083	RCLC	
084	GSB4	
085	6	
086	x	
087	RCL1	
088	3	
089	x	
090	X⇄Y	y_3'
091	F2?	
092	CLX	
093	+	
094	RCL5	
095	x	
096	RCL1	
097	x	
098	+	$y_1' + y_2' + y_2'$
099	6	
100	÷	
101	RCLE	
102	÷	y
103	RTN	
104	*LBLB	
105	STO0	
106	RCLA	
107	GSB4	
108	RCL1	θ_1'
109	2	
110	÷	
111	RCLA	
112	-	

REGISTERS

0	1	2	3	4	5	6	7	8	9
x	x'(a)	ℓ	P	W	M				

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
a	b	c		EI	

DATE ______________________ AUTHOR ______________________

Step	Key	Comment
113	RCL3	
114	x	
115	RCL1	
116	x	
117	RCLB	
118	GSB4	
119	R↓	
120	RCL1	
121	6	θ_2'
122	÷	
123	RCLB	
124	2	
125	÷	
126	-	
127	RCL1	
128	x	
129	RCLB	
130	X²	
131	2	
132	÷	
133	+	
134	RCL4	
135	x	
136	RCL1	
137	x	
138	-	
139	RCLC	
140	GSB4	
141	R↓	θ_3'
142	RCL5	
143	RCL1	
144	x	
145	+	
146	RCLE	$\theta_1' + \theta_2' + \theta_3'$
147	÷	
148	RTN	θ
149	*LBLC	
150	STO0	
151	RCLA	
152	GSB4	M_1
153	RCL1	
154	RCLA	
155	-	
156	RCL3	
157	x	
158	RCLB	
159	GSB4	
160	CLX	
161	RCL1	
162	2	M_2
163	÷	
164	RCLB	
165	-	
166	RCL1	
167	x	
168	RCLB	

Step	Key	Comment
169	X²	
170	2	
171	÷	
172	+	
173	RCL4	
174	x	
175	-	$M_1 + M_2$
176	RCLC	
177	GSB4	
178	CLX	M_3
179	RCL5	
180	X⇄Y	
181	F2?	
182	+	
183	RTN	$M_1 + M_2 + M_3$
184	*LBLD	
185	STO0	
186	RCLA	V_1
187	GSB4	
188	0	
189	F2?	
190	RCL3	
191	RCLB	
192	GSB4	
193	CLX	
194	RCL1	
195	RCLB	
196	-	V_2
197	RCL4	
198	x	
199	-	
200	RTN	V
201	*LBL4	
202	CF2	Select smaller of x and a (or b or c) and store as x'.
203	RCL0	
204	STO1	
205	X⇄Y	
206	X≤Y?	
207	STO1	
208	X>Y?	
209	SF2	If x > a set flag.
210	-	
211	RTN	

LABELS					FLAGS
A x→y	B x→θ	C x→M_x	D x→V	E	0
a Start	b I↑E↑ℓ	c a↑P	d b↑W	e c↑M	1
0	1	2	3	4 Store x'	2
5	6	7	8	9	3

SET STATUS				
FLAGS	ON	OFF	TRIG	DISP
0	☐	☒	DEG ☒	FIX ☐
1	☐	☒	GRAD ☐	SCI ☐
2	☐	☒	RAD ☐	ENG ☒
3	☐	☒		n 3

SIMPLY SUPPORTED BEAMS

TITLE

Step	Key	Comments
001	*LBLa	Store zeros for P, W, and M.
002	0	
003	STO3	
004	STO4	
005	STO5	
006	RTN	
007	*LBLb	Store ℓ and EI.
008	STO2	
009	R↓	
010	x	
011	STOE	
012	RTN	
013	*LBLc	Store P and a.
014	STO3	
015	X⇄Y	
016	STOA	
017	RTN	
018	*LBLd	Store W.
019	STO4	
020	RTN	
021	*LBLe	Store M and c.
022	STO5	
023	X⇄Y	
024	STOC	
025	RTN	
026	*LBLB	Set derivative flag.
027	SF0	
028	GTO0	
029	*LBLA	Clear derivative flag.
030	CF0	
031	*LBL0	
032	STO0	
033	RCL2	Compute
034	ENT↑	
035	x	y_2 (EI) or θ_2 (EI).
036	LSTX	
037	x	
038	RCL0	
039	F0?	
040	4	
041	F0?	
042	x	
043	RCL2	
044	2	
045	x	
046	F0?	
047	3	
048	F0?	
049	x	
050	-	
051	RCL0	
052	X²	
053	x	
054	+	
055	RCL4	
056	x	

Step	Key	Comments
057	2	
058	4	
059	÷	
060	RCL0	
061	X⇄Y	
062	x	
063	F0?	
064	LSTX	
065	CHS	
066	GSB1	
067	RCL1	Compute y_1 (EI) or θ_1 (EI).
068	X²	
069	F0?	
070	3	
071	F0?	
072	x	
073	RCLD	
074	X²	
075	+	
076	RCL2	
077	X²	
078	-	
079	x	
080	6	
081	÷	
082	GSB2	
083	RCL1	Compute y_3 (EI) or θ_3 (EI).
084	X²	
085	RCL2	
086	÷	
087	6	
088	÷	
089	F0?	
090	3	
091	F0?	
092	x	
093	RCL2	
094	3	
095	÷	
096	+	
097	RCLD	
098	X²	
099	2	
100	÷	
101	RCL2	
102	÷	
103	+	
104	RCLD	
105	-	
106	x	
107	RCL2	
108	x	
109	RCLI	
110	+	$y = y_1 + y_2 + y_3$
111	RCLE	or
112	÷	$\theta = \theta_1 + \theta_2 + \theta_3$

REGISTERS

0	1	2	3	4	5	6	7	8	9
x	x,(x − ℓ)	ℓ	P	W	M				
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A: a	B:	C: c	D: (ℓ−a), − a; c, ℓ−c	E: EI	I: SUM				

DATE ______________________ AUTHOR ______________________

Step	Key	Comments
113	RTN	
114	*LBLD	Set derivative flag.
115	SF0	
116	GTO0	
117	*LBLC	Clear derivative flag.
118	CF0	
119	*LBL0	Compute M_{x2} or V_2
120	STO0	
121	2	
122	÷	
123	RCL2	
124	F0?	
125	2	
126	F0?	
127	÷	
128	RCL0	
129	-	
130	RCL4	
131	x	
132	x	
133	F0?	
134	LSTX	
135	GSB1	M_{x1} or V_1
136	GSB2	M_{x3} or V_3
137	RCLI	$M = M_{x1} + M_{x2} + M_{x3}$ or $V = V_1 + V_2 + V_3$
138	+	
139	RTN	
140	*LBL1	Store first results.
141	STOI	
142	RCL2	Store ℓ − a and x
143	RCLA	
144	-	
145	STOD	
146	RCL0	If a > x GTO 0
147	STO1	
148	RCLA	
149	X>Y?	
150	GTO0	
151	RCLA	Otherwise store −a for ℓ − a and x − ℓ for x.
152	CHS	
153	STOD	
154	RCL2	
155	ST-1	
156	*LBL0	$\frac{P(\ell - a)}{\ell}$
157	RCL3	
158	RCLD	
159	x	
160	RCL2	
161	÷	
162	F0?	
163	RTN	$\frac{P(\ell - a)x}{\ell}$
164	RCL1	
165	x	
166	RTN	
167	*LBL2	Add first result to second.
168	RCLI	
169	+	
170	STOI	Store x and c.
171	RCL0	
172	STO1	
173	RCLC	
174	STOD	
175	X>Y?	If c > x GTO 0.
176	GTO0	
177	RCL0	Otherwise store x − ℓ for x and ℓ − c for c.
178	RCL2	
179	-	
180	STO1	
181	RCL2	
182	RCLC	
183	-	
184	STOD	
185	*LBL0	$\frac{M}{\ell}$
186	RCL5	
187	RCL2	
188	÷	
189	F0?	
190	RTN	$\frac{M_x}{\ell}$
191	RCL1	
192	x	
193	RTN	

LABELS

A x→y	B x→θ	C x→M_x	D x→V	E RCL x
a Start	b I↑E↑ℓ	c a↑P	d W	e c↑M
0 Used	1 Con	2 Mom	3	4
5	6	7	8	9

FLAGS

0	Derivative
1	
2	
3	

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☐
SCI	☐
ENG	☒
n	3

BEAMS FIXED AT BOTH ENDS

TITLE ______

Step	Key	Comment
001	*LBLd	Store W.
002	STO4	
003	RTN	
004	*LBLb	Store ℓ and EI.
005	STO2	
006	R↓	
007	x	
008	STOE	
009	RTN	
010	*LBLc	Store P and a.
011	STO3	
012	X⇄Y	
013	STOA	
014	RTN	
015	*LBLe	Store M and c.
016	STO5	
017	X⇄Y	
018	STOC	
019	RTN	
020	*LBLB	Set derivative flag and start θ_2 calculation.
021	SF0	
022	STO0	
023	GSB7	
024	RCL2	
025	3	
026	x	
027	RCL0	
028	GSB7	
029	GTO0	
030	*LBLA	Clear derivative flag and start y_2 calculation.
031	CF0	
032	STO0	
033	X²	
034	RCL2	
035	GSB7	
036	RCL0	
037	*LBL0	Complete calculation of θ_2 EI or y_2 EI.
038	SF1	
039	-	
040	RCL0	
041	x	
042	RCL2	
043	X²	
044	-	
045	x	
046	2	
047	4	
048	÷	
049	RCL4	
050	x	
051	RCLA	Calculate y_1 EI or θ_1 EI.
052	GSB1	
053	RCL1	
054	x	
055	RCLD	
056	RCL2	
057	x	
058	3	
059	X⇄Y	
060	x	
061	F0?	
062	LSTX	
063	F0?	
064	-	
065	-	
066	GSB6	
067	6	
068	÷	
069	F0?	
070	3	
071	F0?	
072	x	
073	F0?	
074	GSB3	
075	GSB4	
076	RCLC	Calculate y_3 EI and/or θ_3 EI.
077	GSB1	
078	F0?	
079	3	
080	F0?	
081	x	
082	X⇄Y	
083	F0?	
084	GSB7	
085	+	
086	GSB6	
087	GSB8	
088	RCLE	Calculate y or θ.
089	÷	
090	RTN	
091	*LBLD	Set derivative flag and start V_2 calculation.
092	SF0	
093	STO0	
094	RCL2	
095	X⇄Y	
096	GSB7	
097	GTO0	
098	*LBLC	Clear derivative flag and start M_2 calculation.
099	CF0	
100	STO0	
101	RCL2	
102	X⇄Y	
103	-	
104	RCL0	
105	x	
106	RCL2	
107	X²	
108	6	
109	÷	
110	*LBL0	Complete calculation of V_2 or M_2.
111	SF1	
112	-	

REGISTERS

0	1	2	3	4	5	6	7	8	9
x	x, (ℓ − x)	ℓ	P	W	M				

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
a		c	a, (ℓ−a); c, (ℓ−c)	EI	SUM

DATE ______________________ AUTHOR ______________________

Step	Key	Comments
113	2	
114	÷	
115	RCL4	
116	x	
117	RCLA	
118	GSB1	
119	F0?	
120	GTO0	
121	RCL1	Calculate V_1 or M_1.
122	x	
123	RCLD	
124	RCL2	
125	x	
126	-	
127	*LBL0	
128	x	
129	F0?	
130	GSB3	
131	GSB4	
132	RCLC	
133	GSB1	Calculate V_3 or M_3.
134	F0?	
135	R↓	
136	F0?	
137	CLX	
138	F0?	
139	RCLD	
140	6	
141	x	
142	X⇄Y	
143	GSB7	
144	+	
145	x	
146	*LBL8	
147	F0?	
148	GTO4	Sign change?
149	GSB3	
150	*LBL4	
151	RCLI	Calculate sum.
152	+	
153	RTN	
154	*LBL1	
155	CF2	Store a or c and sum.
156	STOD	
157	R↓	
158	STOI	
159	RCLD	
160	RCL0	
161	STO1	
162	X≤Y?	
163	GTO0	Is x beyond loading point?
164	SF2	Yes – set sign change flag and a = ℓ − a or c = ℓ − c and x = ℓ − x.
165	RCL2	
166	RCLD	
167	-	
168	STOD	
169	RCL2	
170	RCL0	
171	-	
172	STO1	
173	*LBL0	
174	RCL5	$\frac{P(\ell - a)^2}{\ell^3}$ or
175	F1?	
176	RCL3	
177	RCL2	
178	RCLD	
179	-	$\frac{M(\ell - a)}{\ell^3}$
180	F1?	
181	X^2	
182	x	
183	RCL2	
184	3	
185	Y^x	
186	÷	
187	RCL2	
188	RCLD	
189	F1?	
190	GTO0	$\frac{(\ell - 3a)\ell}{2}$
191	3	
192	x	
193	-	
194	RCL2	
195	x	
196	2	
197	÷	
198	RCLD	
199	RCL1	ax
200	x	
201	RTN	
202	*LBL0	
203	GSB7	
204	+	(ℓ + 2a)
205	CF1	
206	RTN	
207	*LBL3	
208	F2?	Sign change.
209	CHS	
210	RTN	
211	*LBL6	Calculation subroutine.
212	x	
213	RCL1	
214	*LBL5	
215	X^2	
216	F0?	
217	√X	
218	x	
219	RTN	
220	*LBL7	
221	ENT↑	2X subroutine
222	+	
223	RTN	

LABELS

A x→y	B x→θ	C x→M_x	D x→V	E
a	b I↑E↑ℓ	c a↑P	d W	e c↑M
0 Used	1 Calc.	2	3 Sign	4 sum
5 Calc.	6 Calc.	7 2x	8	9

FLAGS

0	Derivative
1	P
2	Sign
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☐
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☒
3	☐	☒			n	3

PROPPED CANTILEVER BEAMS

TITLE

Step	Key	Comment
001	*LBLa	
002	0	
003	STO3	Initialize
004	STO5	
005	*LBLd	
006	STO4	Store W.
007	RTN	
008	*LBLb	
009	STO2	Store ℓ and EI.
010	R↓	
011	x	
012	STOE	
013	RTN	
014	*LBLc	
015	STO3	Store P and a.
016	X⇄Y	
017	STOA	
018	RTN	
019	*LBLe	
020	STO5	Store M and c.
021	X⇄Y	
022	STOC	
023	RTN	
024	*LBLB	Store x, clear integral flag, load constants.
025	CF0	
026	STO0	
027	9	
028	ENT↑	
029	8	
030	GTO0	
031	*LBLA	Store x, set integral flag, load constants.
032	SF0	
033	STO0	
034	3	
035	ENT↑	
036	2	
037	*LBL0	
038	RCL0	Calculate 6 y_2 EI or $6\theta_2$ EI.
039	x	
040	RCL2	
041	÷	
042	-	
043	RCL0	
044	X²	
045	x	
046	RCL2	
047	X²	
048	-	
049	RCL2	
050	x	
051	GSB5	
052	RCL0	Calculate $6y_1$ EI or 6 θ_1 EI.
053	X²	
054	F0?	
055	3	
056	F0?	
057	÷	
058	RCL2	
059	X²	
060	-	
061	STOD	
062	x	
063	RCLI	
064	X²	
065	+	
066	GSB4	
067	F2?	
068	GTO0	
069	RCL0	
070	RCLA	
071	-	
072	X²	
073	F0?	
074	LSTX	
075	F0?	
076	x	
077	F0?	
078	3	
079	F0?	
080	÷	
081	-	
082	*LBL0	
083	3	
084	x	
085	RCL3	
086	x	
087	ST+1	EI $(y_1 + y_2)$ EI $(\theta_1 + \theta_2)$
088	6	
089	ST÷1	
090	GSB1	
091	RCLD	
092	x	Calculate y_3 EI or θ_3 EI.
093	3	
094	x	
095	RCL2	
096	+	
097	GSB4	
098	F2?	
099	GTO6	
100	RCL0	
101	GSB4	
102	0	
103	F0?	
104	R↓	
105	F0?	
106	RCLC	
107	X²	
108	+	
109	F0?	
110	2	
111	F0?	
112	÷	

REGISTERS

0	1	2	3	4	5	6	7	8	9
x	sum	ℓ	D	W	M				

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
a		c	$(x^2/3 - \ell^2)$	EI	$(\ell - a)$; c

DATE ______________________ AUTHOR ______________________

Step	Key	Comments
113	GTO0	
114	*LBL6	
115	RCLC	
116	GSB4	
117	*LBL0	
118	-	
119	GSB8	
120	RCLE	$y = y_1 + y_2 + y_3$ or
121	÷	$\theta = \theta_1 + \theta_2 + \theta_3$
122	RTN	
123	*LBLD	Store x, clear integral flag,
124	CF0	multiply by 2.
125	STO0	
126	2	
127	x	
128	GTO0	
129	*LBLC	Store x, set integral flag.
130	SF0	
131	STO0	
132	*LBL0	
133	3	
134	RCL2	Compute M_2 or V_2.
135	x	
136	X⇄Y	
137	4	
138	x	
139	-	
140	GSB5	
141	GSB4	
142	F2?	Compute M_1 or V_1
143	GTO0	
144	1	
145	GSB4	
146	-	
147	F0?	
148	RCLA	
149	F0?	
150	+	
151	*LBL0	
152	RCL3	
153	x	
154	ST+1	
155	GSB1	
156	6	Compute M_3 or V_3
157	x	
158	GSB4	
159	F2?	
160	GTO8	
161	F0?	
162	1	
163	F0?	
164	-	
165	*LBL8	
166	RCL5	
167	x	
168	ST+1	$M_x = M_1 + M_2 + M_3$

Step	Key	Comments
169	RCL1	or
170	RTN	$V = V_1 + V_2 + V_3$
171	*LBL4	Multiply by x if integral
172	F0?	flag is set.
173	RCL0	
174	F0?	
175	x	
176	RTN	Finish y_2, θ_2, M_2 and
177	*LBL5	V_2 calculations.
178	RCL4	
179	x	
180	GSB4	
181	8	
182	÷	
183	STO1	
184	CF1	Store b.
185	RCL2	
186	RCLA	
187	-	
188	STOI	
189	LSTX	
190	GTO0	
191	*LBL1	
192	RCLC	Store c.
193	STOI	
194	*LBL0	Set x ≤ to a or c flag.
195	CF2	
196	RCL0	
197	X≤Y?	
198	SF2	
199	RCLI	Calculate
200	3	
201	x	
202	F1?	$\frac{3a^2\ell - a^3}{2\ell^3}$
203	RCL2	
204	RCL2	
205	x	First pass, then
206	RCLI	
207	X²	
208	-	$\frac{\ell^2 - c^2}{4\ell^3}$ on
209	2	
210	F1?	
211	X²	Second pass.
212	÷	
213	RCL2	
214	3	
215	Yˣ	
216	÷	
217	F1?	
218	RTN	
219	SF1	
220	RCLI	
221	x	
222	RTN	

LABELS

A Start	B l↑E↑ℓ	C a↑P	D W	E c↑M
a x→y	b x→θ	c x→M_x	d x→V	e
0 Used	1 P	2	3	4 x mult
5 W − P	6 Used	7	8 Mult & Sum	9

FLAGS

0	Integral
1	Moment
2	x ≤ d or c
3	

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☐
SCI	☐
ENG	☒
n	3

HELICAL SPRING DESIGN

TITLE

Step	Instruction	Comments
001	*LBLb	
002	STOB	Store β, α, and G.
003	R↓	
004	STOA	
005	R↓	
006	STO9	
007	RTN	
008	*LBLc	
009	STO2	Store load point 1.
010	X⇄Y	
011	STO1	
012	RTN	
013	*LBLd	
014	STO4	Store load point 2 and calculate spring constant k.
015	X⇄Y	
016	STO3	
017	RCL1	
018	-	
019	RCL2	
020	RCL4	
021	-	
022	÷	
023	STO5	
024	SPC	
025	PRTX	
026	RTN	
027	*LBLe	
028	SPC	Calculate uncorrected stress at point 2.
029	x	
030	STO6	
031	CLX	
032	LSTX	
033	RCL3	
034	x	
035	8	
036	x	
037	Pi	
038	÷	
039	X⇄Y	
040	STO8	
041	ST-6	
042	3	
043	Y^x	
044	÷	
045	PRTX	
046	RTN	
047	*LBLA	
048	SPC	Calculate number of coils.
049	RCL9	
050	RCL8	
051	X^2	
052	X^2	
053	x	
054	8	
055	÷	
056	RCL6	
057	X^2	
058	LSTX	
059	x	
060	÷	
061	RCL5	
062	÷	
063	STOC	
064	PRTX	
065	RTN	
066	*LBLB	
067	SPC	Calculate free length.
068	RCL1	
069	RCL5	
070	÷	
071	RCL2	
072	+	
073	STOD	
074	RCLC	
075	2	Calculate solid length.
076	+	
077	RCL6	
078	x	
079	STOE	
080	-	
081	RCL5	Calculate stress at solid.
082	x	
083	RCL6	
084	RCL8	
085	÷	
086	GSB1	
087	x	
088	RCL6	
089	x	
090	RCL8	
091	X^2	
092	÷	
093	RCL8	
094	÷	
095	8	
096	x	
097	Pi	
098	÷	
099	STO0	
100	PRTX	
101	RTN	
102	*LBLE	
103	SPC	Output dimensions.
104	RCLD	
105	PRTX	
106	RCLE	
107	PRTX	
108	RCL6	
109	PRTX	
110	RCL8	
111	+	
112	PRTX	

REGISTERS

0	1	2	3	4	5	6	7	8	9
s_s	P_1	L_1	P_2	L_2	k	OD, D	s_2	d	G
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
α	β	N	L_f	L_s	TS

DATE ______________________ AUTHOR ______________________

Step	Key Entry	Comments
113	RTN	
114	*LBLa	Ferrous/non-ferrous toggle.
115	F0?	
116	GTO0	
117	SF0	
118	0	
119	RTN	
120	*LBL0	
121	1	
122	CF0	
123	RTN	
124	*LBLC	If coil-to-coil clearance is adequate, branch to label zero.
125	RCL4	
126	RCLE	
127	-	
128	RCLD	
129	RCL4	
130	-	
131	.	
132	1	
133	x	
134	X≤Y?	
135	GTO0	
136	GSB2	Check stress with inadequate clearance.
137	R↓	
138	RCL0	
139	X≤Y?	
140	GTO3	
141	1	Change criteria.
142	RTN	
143	*LBL3	Try smaller wire.
144	RCL8	
145	2	
146	RTN	
147	STO8	Store smaller wire size and branch.
148	GTOA	
149	*LBL0	
150	GSB2	Check stress with adequate clearance.
151	RCL0	
152	X≤Y?	
153	GTO3	
154	RCL8	Try larger wire.
155	3	
156	RTN	
157	*LBL5	Store larger wire size and branch.
158	RCL8	
159	ST+6	
160	R↓	
161	STO8	
162	ST-6	
163	GTOA	
164	*LBL3	Check to see if spring is over-designed.
165	RCLI	
166	.	
167	3	
168	x	
169	RCL0	
170	X≤Y?	
171	GTO4	
172	5	Spring is acceptable.
173	RTN	
174	*LBL4	Spring may be over-designed. Try smaller wire.
175	RCL6	
176	4	
177	RTN	
178	GTO5	Smaller wire input.
179	*LBL2	
180	RCL8	Compute wire tensile strength from α and β.
181	LN	
182	RCLB	
183	x	
184	RCLA	
185	+	
186	ENT↑	
187	STOI	
188	.	Convert to s_{max}.
189	3	
190	5	
191	ENT↑	
192	.	
193	1	
194	F0?	
195	CLX	
196	+	
197	x	
198	X⇄Y	Convert to YS.
199	LSTX	
200	.	
201	2	
202	+	
203	x	
204	RTN	
205	*LBL1	Compute Wahl factor.
206	STO8	
207	4	
208	x	
209	1	
210	-	
211	ENT↑	
212	ENT↑	
213	3	
214	-	
215	÷	
216	.	
217	6	
218	1	
219	5	
220	RCL0	
221	÷	
222	+	
223	RTN	

LABELS

A →N	B →s_s	C →code	D	E (→L_f,L_s,D,OD)
a Fe?	b G↑α↑β	c P↑L_1	d P_2↑L_2→k	e d↑f_0↑D_H→S_2
0 Used	1 Wahl	2 TS	3 code 2,4,5	4 code 4
5 Larger d	6	7	8	9

FLAGS

0	Ferrous?
1	
2	
3	

SET STATUS

FLAGS	ON	OFF
0	☒	☐
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☐
SCI	☐
ENG	☒
n	3

FOUR BAR FUNCTION GENERATOR

TITLE

Step	Key	Comments
001	*LBLA	Store d, c, b and a.
002	STO4	
003	R↓	
004	STO3	
005	R↓	
006	STO2	
007	R↓	
008	STO1	
009	RTN	
010	*LBLB	Calculate R_1, R_2 and R_3.
011	RCL1	
012	RCL4	
013	÷	
014	STOA	
015	RCL1	
016	RCL2	
017	÷	
018	STOB	
019	RCL1	
020	X²	
021	RCL2	
022	X²	
023	+	
024	RCL4	
025	X²	
026	+	
027	RCL3	
028	X²	
029	-	
030	RCL2	
031	RCL4	
032	x	
033	2	
034	x	
035	÷	
036	STOC	
037	DSP4	
038	RCLA	
039	SPC	
040	PRTX	
041	RCLB	
042	PRTX	
043	RCLC	
044	PRTX	
045	RTN	
046	*LBLC	f(x) – your function.
047	RTN	
048	*LBLD	Store x_3, x_2, x_1.
049	STOC	
050	R↓	
051	STOB	
052	R↓	
053	STOA	
054	RTN	
055	*LBLE	Store θ_3 and
056	P⇄S	
057	STO3	θ_1 and calculate θ_2.
058	R↓	
059	STO1	
060	R↑	
061	-	
062	RCLA	
063	RCLB	
064	-	
065	x	
066	RCLC	
067	RCLA	
068	-	
069	÷	
070	RCL1	
071	+	
072	STO2	
073	P⇄S	
074	RTN	
075	*LBLa	Store ϕ_3 and ϕ_1 and calculate ϕ_2.
076	P⇄S	
077	STO6	
078	R↓	
079	STO4	
080	R↑	
081	-	
082	CHS	
083	STO8	
084	RCLA	
085	GSBC	
086	STOI	
087	STOE	
088	RCLC	
089	GSBC	
090	RCLI	
091	-	
092	STOI	
093	RCLB	
094	GSBC	
095	RCLE	
096	-	
097	RCLI	
098	÷	
099	RCL8	
100	x	
101	RCL4	
102	+	
103	STO5	
104	P⇄S	
105	RTN	

REGISTERS

0	1	2	3	4	5	6	7	8	9
Used	a, $\cos\theta_1$	b, $\cos\theta_2$	c, $\cos\theta_3$	d, $\cos\phi_1$	$\cos\phi_2$	$\cos\phi_3$	1	1	1

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
Used	θ_1	θ_2	θ_3	ϕ_1	ϕ_2	ϕ_3	R_1	R_2	R_3

A	B	C	D	E	I
R_1, x_1, $\cos(\theta_1-\phi_1)$	R_2, x_2, $\cos(\theta_2-\phi_2)$	R_3, x_3, $\cos(\theta_3-\phi_3)$	Det	Used	Used

DATE ______ AUTHOR ______

LABELS					FLAGS
A $a \uparrow b \uparrow c \uparrow d$	B $\rightarrow R_1, R_2, R_3$	C $f(x)$	D $x_1 \uparrow x_2 \uparrow x_3$	E $\theta_1 \uparrow \theta_3 \rightarrow \theta_2$	0
a $\phi_1 \uparrow \phi_3 \rightarrow \phi_2$	b	c	d	e	1
0	1	2	3	4	2
5	6	7	8	9	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

(Card 2)

TITLE ____________________

Step	Key Entry	Comments
001	*LBLb	Calculate R_1, R_2, and R_3.
002	GSB6	
003	*LBL5	
004	GSB0	
005	STOD	
006	RCLA	
007	STO1	
008	RCLB	
009	STO2	
010	RCLC	
011	STO3	
012	GSB0	
013	RCLD	
014	÷	
015	STOI	
016	PRTX	
017	GSB6	
018	RCLA	
019	STO4	
020	RCLB	
021	STO5	
022	RCLC	
023	STO6	
024	GSB0	
025	RCLD	
026	÷	
027	STOE	
028	PRTX	
029	GSB6	
030	RCLA	
031	STO7	
032	RCLB	
033	STO8	
034	RCLC	
035	STO9	
036	GSB0	
037	RCLD	
038	÷	
039	PRTX	
040	SPC	
041	P⇄S	
042	STO9	
043	RCLE	
044	STO8	
045	RCLI	
046	STO7	
047	RCL9	
048	P⇄S	
049	RTN	
050	*LBLc	Calculate b, c, and d.
051	STOA	
052	P⇄S	
053	STO0	
054	RCL8	
055	÷	
056	STOB	
057	PRTX	
058	X^2	
059	RCL0	
060	RCL7	
061	÷	
062	STOD	
063	X^2	
064	+	
065	RCL0	
066	X^2	
067	+	
068	RCL0	
069	RCL8	
070	÷	
071	RCL0	
072	RCL7	
073	÷	
074	x	
075	2	
076	x	
077	RCL9	
078	x	
079	-	
080	√X	
081	STOC	
082	PRTX	
083	RCLD	
084	P⇄S	
085	PRTX	
086	SPC	
087	RTN	
088	*LBL0	Calculate determinant.
089	RCL5	
090	RCL9	
091	x	
092	RCL8	
093	RCL6	
094	x	
095	-	
096	RCL1	
097	x	
098	RCL4	
099	RCL9	
100	x	
101	RCL7	
102	RCL6	
103	x	
104	-	
105	RCL2	
106	x	
107	-	
108	RCL4	
109	RCL8	
110	x	
111	RCL7	
112	RCL5	

REGISTERS

0 Used	1 $\cos\theta_1$	2 $\cos\theta_2$	3 $\cos\theta_3$	4 $\cos\phi_1$	5 $\cos\phi_2$	6 $\cos\phi_3$	7 1	8 1	9 1
S0 Used	S1 θ_1	S2 θ_2	S3 θ_3	S4 ϕ_1	S5 ϕ_2	S6 ϕ_3	S7 R_1	S8 R_2	S9 R_3

A $\cos(\theta_1 - \phi_1)$, a	B $\cos(\theta_2 - \phi_2)$, b	C $\cos(\theta_3 - \phi_3)$, c	D Det, d	E Used	I Used

DATE ______________________ AUTHOR ______________________

```
113      x
114      -
115   RCL3
116      x
117      +
118    RTN
119  *LBL6
120    P⇄S
121   RCL1
122    COS
123   LSTX
124   RCL4
125      -
126   LSTX
127    COS
128    CHS
129    X⇄Y
130    COS
131   STOA
132     R↓
133    P⇄S
134   STO4
135     R↓
136   STO1
137    P⇄S
138   RCL2
139    COS
140   LSTX
141   RCL5
142      -
143   LSTX
144    COS
145    CHS
146    X⇄Y
147    COS
148   STOB
149     R↓
150    P⇄S
151   STO5
152     R↓
153   STO2
154    P⇄S
155   RCL3
156    COS
157   LSTX
158   RCL6
159      -
160   LSTX
161    COS
162    CHS
163    X⇄Y
164    COS
165   STOC
166     R↓
167    P⇄S
168   STO6
```

Calculate the coefficients for the system of linear equations.

```
169     R↓
170   STO3
171      1
172   STO7
173   STO8
174   STO9
175    RTN
```

LABELS					FLAGS	SET STATUS
A	B	C	D	E	0	FLAGS: ON OFF — 0 ☐ ☒; 1 ☐ ☒; 2 ☐ ☒; 3 ☐ ☒
a	b →R_1,R_2,R_3	c a→b, c, d	d	e	1	TRIG: DEG ☒ GRAD ☐ RAD ☐
0 Determinant	1	2	3	4	2	DISP: FIX ☒ SCI ☐ ENG ☐ n 2
5 R_1,R_2,R_3	6 Coefficients	7	8	9	3	

PROGRESSION OF FOUR-BAR SYSTEM

TITLE

Step	Key	Comment
001	*LBLa	
002	STOD	Store link lengths.
003	R↓	
004	STOC	
005	R↓	
006	STOB	
007	R↓	
008	STOA	
009	RTN	
010	*LBLc	
011	F0?	Toggle connector flag.
012	GTO0	
013	SF0	
014	1	
015	RTN	
016	*LBL0	
017	CF0	
018	0	
019	RTN	
020	*LBLe	
021	CF2	
022	SPC	Store loop parameters.
023	STO9	
024	PRTX	
025	SPC	
026	R↓	
027	STO8	
028	R↓	
029	STOI	
030	R↓	
031	STO7	
032	RCL9	
033	ENT↑	
034	ABS	
035	÷	
036	ST×8	
037	*LBL9	
038	RCL7	Calculate ϕ.
039	PRTX	
040	GSBA	
041	PRTX	
042	RCL9	Calculate $\dot{\phi}$.
043	GSBC	
044	PRTX	
045	0	Calculate $\ddot{\phi}$.
046	GSBE	
047	PRTX	
048	SPC	
049	RCL8	
050	ST+7	Increment θ.
051	DSZI	
052	GTO9	Loop again?
053	R↓	
054	RTN	
055	*LBLA	
056	STO7	
057	RCLC	
058	F0?	
059	CHS	Compute ϕ.
060	RCLD	
061	F0?	
062	X⇄Y	
063	STO5	
064	X⇄Y	
065	STO6	
066	RCL7	
067	RCLB	
068	→R	
069	RCLA	
070	+	
071	→P	
072	STOE	
073	X²	
074	RCL5	
075	X²	
076	+	
077	RCL6	
078	X²	
079	-	
080	RCLE	
081	÷	
082	2	
083	÷	
084	RCL5	
085	÷	
086	COS⁻¹	
087	+	
088	STO4	
089	SIN	Compute $d\phi/d\theta$.
090	RCLA	
091	RCLB	
092	÷	
093	x	
094	RCL7	
095	RCL4	
096	-	
097	SIN	
098	-	
099	LSTX	
100	CHS	
101	RCL7	
102	SIN	
103	RCLA	
104	x	
105	RCL5	
106	÷	
107	+	
108	X⇄Y	
109	÷	
110	STO3	
111	RCL4	
112	RTN	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	Used	RPM²	$d\phi/d\theta$ ($d\alpha/d\theta$)	ϕ (α)	d (−c)	c (d)	θ	$\Delta\theta$	RPM
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
a	b	c	d	e	n

DATE ____________ AUTHOR ____________

Step	Key	Comments
113	*LBLC	
114	STO9	Convert $d\phi/d\theta$ to RPM.
115	RCL3	
116	x	
117	RTN	
118	*LBLE	
119	STO2	Calculate $d^2\phi/d\theta^2$.
120	RCL7	
121	RCL4	
122	-	
123	1	
124	→R	
125	1	
126	RCL3	
127	-	
128	X²	
129	x	
130	RCLA	
131	RCLB	
132	÷	
133	RCL4	
134	X⇄Y	
135	→R	
136	X⇄Y	
137	R↑	
138	-	
139	STO1	
140	CLX	
141	RCL3	
142	X²	
143	x	
144	+	
145	CHS	
146	RCL7	
147	COS	
148	RCLA	
149	x	
150	RCL5	
151	÷	
152	+	
153	RCL1	
154	÷	
155	ENT↑	Convert to RPM².
156	+	
157	Pi	
158	x	
159	RCL9	
160	X²	Add in contribution of input link acceleration.
161	x	
162	RCL2	
163	RCL3	
164	x	
165	+	
166	RTN	

LABELS

A $\theta \to \phi$ (α)	B	C $\dot{\theta} \to \dot{\phi}$ $(\dot{\alpha})$	D	E $\ddot{\theta} \to \ddot{\phi}$ $(\ddot{\alpha})$
a a↑b↑c↑d	b	c connector	d	e θ↑n↑Δθ↑RPM
0 connector	1	2	3	4
5	6	7	8	9 loop

FLAGS

0	α
1	
2	Δ
3	

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG: DEG ☒ GRAD ☐ RAD ☐

DISP: FIX ☒ SCI ☐ ENG ☐ n 2

PROGRESSION OF SLIDER CRANK

TITLE

Step	Key	Comments
001	*LBLB	
002	STO0	Store θ, calculate x and ϕ.
003	*LBL7	
004	SPC	
005	GSB2	
006	COS	
007	STO4	
008	RCLC	
009	x	
010	RCL0	
011	COS	
012	STO3	
013	RCLD	
014	x	
015	+	
016	STO1	
017	PRTX	
018	RCL0	
019	GSB2	
020	PRTX	
021	RTN	
022	*LBLA	Store R, L, E, and N and calculate ω.
023	STOD	
024	R↓	
025	STOC	
026	R↓	
027	STOB	
028	R↓	
029	STOA	
030	RCLA	
031	Pi	
032	x	
033	3	
034	0	
035	÷	
036	STOE	
037	SPC	
038	PRTX	
039	RTN	
040	*LBL0	Calculate x_{max} or x_{min}.
041	RCLB	
042	RCLC	
043	RCLD	
044	F1?	
045	CHS	
046	+	
047	÷	
048	SIN⁻¹	
049	COS	
050	RCLC	
051	RCLD	
052	F1?	
053	CHS	
054	+	
055	x	
056	PRTX	
057	RTN	
058	*LBLa	Output x_{max} and x_{min}.
059	SPC	
060	CF1	
061	GSB0	
062	SF1	
063	STO4	
064	GSB0	
065	RTN	
066	*LBLb	Calculate ϕ_{max} and ϕ_{min}.
067	SPC	
068	1	
069	SIN⁻¹	
070	GSB2	
071	STO4	
072	PRTX	
073	1	
074	CHS	
075	SIN⁻¹	
076	GSB2	
077	PRTX	
078	RTN	
079	*LBL2	Subroutine for ϕ.
080	SIN	
081	RCLD	
082	x	
083	RCLB	
084	+	
085	RCLC	
086	÷	
087	SIN⁻¹	
088	STO2	
089	RTN	
090	*LBLC	Calculate v and $\dot{\phi}$.
091	SPC	
092	RCL0	
093	RCL2	
094	+	
095	SIN	
096	CHS	
097	RCL4	
098	÷	
099	RCLE	
100	x	
101	RCLD	
102	x	
103	PRTX	
104	RCL4	
105	RCLC	
106	x	
107	RCLD	
108	÷	
109	1/X	
110	STO5	
111	RCL3	
112	x	

REGISTERS

0	1	2	3	4	5	6	7	8	9
θ	x	ϕ	$\cos\theta$	$\cos\phi$	$R/(L\cos\phi)$	$(\theta_1-\theta_2)/h$	θ_1	θ_2	n
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
N	E	L	R	ω	Index

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```
113  RCLE
114  x
115  PRTX
116  RTN
117  *LBLD
118  SPC
119  RCL3
120  X²
121  RCLD
122  x
123  RCLC
124  ÷
125  RCL4
126  3
127  Yˣ
128  ÷
129  RCL0
130  RCL2
131  +
132  COS
133  RCL4
134  ÷
135  +
136  CHS
137  RCLD
138  x
139  RCLE
140  X²
141  x
142  PRTX
143  RCL2
144  TAN
145  RCL5
146  RCL3
147  x
148  X²
149  x
150  RCL0
151  SIN
152  RCL5
153  x
154  -
155  RCLE
156  X²
157  x
158  PRTX
159  RTN
160  *LBLE
161  ST09
162  STOI
163  ISZI
164  R↓
165  ST08
166  R↓
167  ST07
168  R↑
169  -
170  CHS
171  RCL9
172  ÷
173  ST06
174  *LBL4
175  SPC
176  SPC
177  RCL7
178  PRTX
179  ST00
180  CF1
181  GSB7
182  GSBC
183  GSBD
184  SF1
185  DSZI
186  GTO3
187  RTN
188  *LBL3
189  RCL6
190  ST+7
191  GTO4
192  RTN
```

Steps 117–159: Calculate a and $\ddot{\phi}$.

Steps 160–192: Automatically output θ, x, ϕ, v, $\dot{\phi}$, a, and $\ddot{\phi}$, for n intervals between θ_1 and θ_2.

LABELS					FLAGS
A N↑E↑L↑R→ω	B θ→x, ϕ	C →v, $\dot{\phi}$	D →a, $\ddot{\phi}$	E θ_1↑θ_2↑n→List	0
a →x_{max},x_{min}	b →ϕ_{max},ϕ_{min}	c	d	e	1 Max – Min
0 x_{max}	1	2 ϕ	3 Used	4 Used	2
5	6	7 Used	8	9	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

TITLE **CIRCULAR CAMS**

Step	Key	Comment
001	*LBLa	Flat or roller toggle.
002	F0?	
003	GTO0	
004	1	
005	SF0	
006	RTN	
007	*LBL0	
008	0	
009	CF0	
010	RTN	
011	*LBLb	Function code store, and clear flag 1.
012	CF1	
013	STOI	
014	RTN	
015	*LBLc	Store increment, duration, angle, π (according to angular mode of calculator) and initialize θ' register.
016	STOC	
017	R↓	
018	STOB	
019	R↓	
020	STO7	
021	1	
022	CHS	
023	COS⁻¹	
024	STO8	
025	0	
026	STO6	
027	RTN	
028	*LBLd	Store lift.
029	STOD	
030	RTN	
031	*LBLe	Store R_b and R_g or $R_r - R_g$ for roller cams.
032	STO9	
033	R↓	
034	STOA	
035	F0?	
036	RTN	
037	-	
038	STOA	
039	RTN	
040	*LBLA	Calculate values automatically.
041	GSBB	
042	GSBC	
043	GSBD	
044	GSBE	
045	RCLB	Check for completion and bring θ into display.
046	RCL6	
047	X≤Y?	
048	GTOA	
049	RCL7	
050	RCLC	
051	-	
052	RTN	
053	*LBLB	Output θ and y.
054	SPC	
055	SPC	
056	RCL7	
057	PRTX	
058	GSB0	
059	RCL5	
060	PRTX	
061	RTN	
062	*LBLC	Output $dy/d\theta$ and $d^2y/d\theta^2$.
063	SPC	
064	RCL4	
065	PRTX	
066	RCL3	
067	PRTX	
068	RTN	
069	*LBLD	Output α.
070	SPC	
071	RCL2	
072	PRTX	
073	RTN	
074	*LBLE	Output r_g and ϕ.
075	SPC	
076	RCL0	
077	PRTX	
078	RCL1	
079	PRTX	
080	RTN	
081	*LBL0	Harmonic if flag 1 is set.
082	1	
083	F1?	
084	STOI	
085	RCL6	$\dot{\theta}/\beta \rightarrow R_E$
086	RCLB	
087	÷	
088	STOE	
089	GSBi	Lift function.
090	RCLD	Calculate lift = h $f(\theta/\beta)$.
091	x	
092	STO5	
093	RCLD	Calculate velocity.
094	RCLB	
095	÷	
096	RCL4	
097	x	
098	STO4	
099	RCLD	Calculate acceleration.
100	RCLB	
101	X²	
102	÷	
103	RCL3	
104	x	
105	STO3	
106	RCL8	Calculate pressure angle.
107	RCL9	
108	RCL5	
109	+	
110	STO1	
111	÷	
112	Pi	

REGISTERS

0	1	2	3	4	5	6	7	8	9
r_g	h	α	y''	y'	y	θ'	θ		R_b
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
R_g or $(R_r - R_g)$	β	Inc	h	θ'/β	Control

DATE ______________________ AUTHOR ______________________

Step	Key	Comments
113	÷	
114	RCL4	
115	x	
116	TAN^{-1}	
117	STO2	If flat follower go to LBL0.
118	F0?	
119	GTO0	
120	RCL1	Calculate grinder radius.
121	→R	
122	RCLA	
123	-	
124	→P	
125	STO0	
126	RCLA	Calculate grinder angular offset.
127	X⇄Y	
128	÷	
129	RCL2	
130	SIN	
131	x	
132	SIN^{-1}	
133	GTO9	
134	*LBL0	Calculate grinder radius.
135	LSTX	
136	RCL1	
137	x	
138	LSTX	
139	→P	
140	STO1	
141	RCL2	
142	RCLA	
143	→R	
144	RCL1	
145	+	
146	→P	
147	STO0	
148	RCLA	Calculate grinder angular offset.
149	RCL2	
150	COS	
151	x	
152	RCL1	
153	+	
154	X⇄Y	
155	÷	
156	COS^{-1}	
157	RCL2	
158	-	
159	CHS	
160	*LBL9	Calculate angle of grinder.
161	RCL7	
162	+	
163	STO1	
164	RCLC	Increment angle.
165	ST+6	
166	ST+7	
167	RTN	
168	*LBL1	Harmonic cams
169	RCL8	f''(θ/β).
170	x	
171	Pi	
172	2	
173	÷	
174	→R	
175	Pi	
176	x	
177	STO3	
178	R↓	f'(θ/β).
179	STO4	f(θ/β).
180	1	
181	RCLE	
182	RCL8	
183	x	
184	COS	
185	-	
186	2	
187	÷	
188	RTN	
189	*LBL2	
190	ENT↑	Cycloidal cam: f'(θ/β).
191	+	
192	RCL8	
193	x	
194	1	
195	→R	
196	CHS	
197	1	
198	+	
199	STO4	
200	R↓	
201	STO1	f''(θ/β).
202	ENT↑	
203	+	
204	Pi	
205	x	
206	STO3	
207	RCLE	
208	RCL1	f(θ/β).
209	2	
210	÷	
211	Pi	
212	÷	
213	-	
214	RTN	

LABELS

					FLAGS
A Auto	B →θ, y	C →y', y''	D →α	E →r_g, φ	0 FLT/RLR
a FLT/RLR	b HAR, CYC	c θ_0↑β↑Δθ	d h	e (R_b↑R_g↑R_b)	1 HAR
0 Used	1 Harmonic	2 Cycloidal	3	4	2
5	6	7	8	9 r_g, φ	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☐
1	☒	☐	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☒
3	☐	☒			n	3

LINEAR CAMS

TITLE ______________________________

Step	Key	Comments
001	*LBLa	Store function code and clear flag 1.
002	CF1	
003	STOI	
004	RTN	
005	*LBLb	Store increment, duration angle, and initialize x′ register.
006	STOC	
007	R↓	
008	STOB	
009	R↓	
010	STO7	
011	0	
012	STO6	
013	RTN	
014	*LBLc	Store lift.
015	STOD	
016	RTN	
017	*LBLd	Store base lift.
018	STO9	
019	RTN	
020	*LBLe	Store $R_g - R_r$.
021	STO8	
022	-	
023	STOA	
024	RTN	
025	*LBLA	Calculate values automatically.
026	GSBB	
027	GSBC	
028	GSBD	
029	GSBE	
030	RCLB	Check for completion and bring x into display.
031	RCL6	
032	X≤Y?	
033	GTOA	
034	RCL7	
035	RCLC	
036	-	
037	RTN	
038	*LBLB	Output x and y.
039	SPC	
040	SPC	
041	RCL7	
042	PRTX	
043	GSB0	
044	RCL5	
045	PRTX	
046	RTN	
047	*LBLC	Output dy/dx and d^2y/dx^2.
048	SPC	
049	RCL4	
050	PRTX	
051	RCL3	
052	PRTX	
053	RTN	
054	*LBLD	Output α.
055	SPC	
056	RCL2	
057	PRTX	
058	RTN	
059	*LBLE	Output x_g and y_g.
060	SPC	
061	RCL0	
062	PRTX	
063	RCL1	
064	PRTX	
065	RTN	
066	*LBL0	Harmonic if flag 1 is set.
067	1	
068	F1?	
069	STOI	
070	RCL6	Store x'/L in R_E.
071	RCLB	
072	÷	
073	STOE	
074	GSBi	Multiply by lift.
075	RCLD	
076	ST×3	
077	ST×4	
078	×	
079	RCL9	y
080	+	
081	STO5	
082	RCLB	Divide by L.
083	ST÷4	
084	ST÷3	
085	ST÷3	
086	RCL4	Calculate α.
087	TAN⁻¹	
088	STO2	$(R_g - R_r)\sin\alpha$ $(R_g - R_r)\cos\alpha$
089	RCLA	
090	→R	
091	RCL5	y_g
092	+	
093	STO1	
094	X⇄Y	x_g
095	CHS	
096	RCL7	
097	+	
098	STO0	
099	RCLC	Increment x.
100	ST+6	
101	ST+7	
102	RTN	
103	*LBL1	Harmonic function.
104	DEG	
105	1	
106	8	
107	0	
108	×	
109	Pi	
110	2	
111	÷	
112	→R	

REGISTERS

0	1	2	3	4	5	6	7	8	9
x_g	y_g	α	y''	y'	y	x'	x	R_r	y_b
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
$R_g - R_r$	L	Δx	h	x'/L	Control

DATE ______________ AUTHOR ______________

```
113   Pi
114   x
115   STO3
116   R↓
117   STO4
118   1
119   RCLE
120   1
121   8
122   0
123   x
124   COS
125   -
126   2
127   ÷
128   RTN
                  ------------
129   *LBL2       Cycloidal function.
130   DEG
131   ENT↑
132   +
133   1
134   8
135   0
136   x
137   1
138   →R
139   CHS
140   1
141   +
142   STO4
143   R↓
144   STO1
145   ENT↑
146   +
147   Pi
148   x
149   STO3
150   RCLE
151   RCL1
152   2
153   ÷
154   Pi
155   ÷
156   -
157   RTN
158   *LBL3
                  ------------
159   .           Parabolic function.
160   5
161   X≤Y?
162   GTO0
                  ------------
163   CLX         x < .5
164   4
165   STO3
166   x
167   STO4
168   RCLE
169   X²
170   2
171   x
172   RTN
                  ------------
173   *LBL0       x ≥ .5
174   CLX
175   4
176   CHS
177   STO3
178   1
179   RCLE
180   -
181   x
182   CHS
183   STO4
184   LSTX
185   X²
186   ENT↑
187   +
188   CHS
189   1
190   +
191   RTN
                  ------------
```

LABELS					FLAGS
A Auto	B →x, y	C y′, y″	D →α	E →x_g, y_g	0
a (1, 2, 3)	b x_0↑L↑Δx	c h	d y_b	e R_g↑R_r	1 Har
0	1 Harmonic	2 Cycloidal	3 Parabolic	4	2
5	6	7	8	9	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☐
1	☒	☐	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☒
3	☐	☒			n	3

GEAR FORCES

TITLE

Step	Key	Comments
001	*LBLA	Calculate F_t.
002	÷	
003	STO6	
004	RTN	
005	*LBLB	Store α.
006	STOA	
007	RTN	
008	*LBLC	Store ϕ_n.
009	STO5	
010	RTN	
011	*LBLD	Convert ϕ to ϕ_n and store.
012	TAN	
013	RCLA	
014	COS	
015	x	
016	TAN⁻¹	
017	STO5	
018	RTN	
019	*LBLE	Calculate F_{gs} and F_{gax}.
020	SPC	
021	RCL6	
022	RCLA	
023	TAN	
024	x	
025	RCL5	
026	TAN	
027	RCLA	
028	COS	
029	÷	
030	RCL6	
031	x	
032	PRTX	
033	X⇄Y	
034	PRTX	
035	RTN	
036	*LBLa	Store bevel gear cone angle.
037	STOB	
038	RTN	
039	*LBLb	Calculate F_{bpax} and F_{bgax}.
040	SPC	
041	RCLB	
042	RCL5	
043	TAN	
044	RCLA	
045	COS	
046	÷	
047	RCL6	
048	x	
049	→R	
050	RCL6	
051	RCLA	
052	TAN	
053	x	
054	RCLB	
055	X⇄Y	
056	→R	
057	R↑	
058	+	
059	R↓	
060	-	
061	R↑	
062	PRTX	
063	X⇄Y	
064	PRTX	
065	RTN	
066	*LBLd	Store coefficient of friction.
067	STO4	
068	RTN	
069	*LBLe	Calculate F_{ws}, F_{gax}.
070	SPC	
071	RCL5	
072	SIN	
073	LSTX	
074	COS	
075	RCLA	
076	SIN	
077	x	
078	RCLA	
079	COS	
080	RCL4	
081	x	
082	+	
083	÷	
084	RCL6	
085	x	
086	PRTX	
087	1	
088	RCLA	
089	TAN	
090	RCL4	
091	x	
092	RCL5	
093	COS	
094	÷	
095	-	
096	RCLA	
097	TAN	
098	RCL4	
099	RCL5	
100	COS	
101	÷	
102	+	
103	÷	
104	RCL6	
105	x	
106	PRTX	
107	RTN	

REGISTERS

0	1	2	3	4	5	6	7	8	9
				f	ϕ_n	F_t			

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
α	cone ∠				

DATE ______________________ AUTHOR ______________________

LABELS					FLAGS
A $T\uparrow r \rightarrow F_t$	B α	C ϕ_n	D ϕ	E $\rightarrow F_{gs}, F_{gax}$	0
a cone ∠	b $\rightarrow F_{bpax}, F_{bgax}$	c	d f	e $\rightarrow F_{ws}, F_{gax}$	1
0	1	2	3	4	2
5	6	7	8	9	3

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☒
SCI	☐
ENG	☐
n	2

STANDARD EXTERNAL INVOLUTE SPUR GEARS

TITLE

Store P and N and calculate D and t.

```
*LBLA
STO1
X⇄Y
STO2
÷
STO5
SPC
PRTX
Pi
2
÷
RCL2
÷
STO8
PRTX
RTN
```

Store d_w and ϕ.

```
*LBLB
STO3
X⇄Y
STO4
RTN
```

Calculate inv ϕ_w.

```
*LBLC
RCL8
RCL5
÷
RCL4
TAN
+
RCL4
Pi
x
1
8
0
÷
-
RCL3
RCL5
÷
RCL4
COS
÷
+
Pi
RCL1
÷
-
STO6
R→D
GTO9
```

Calculate ϕ_w.

```
*LBLD
RAD
RCL6
3
x
.
3
Y^X
STO7
*LBL1
RCL6
RCL7
+
RCL7
TAN
-
LSTX
ENT↑
x
÷
ST+7
RCL7
÷
ABS
EEX
CHS
6
X≤Y?
GTO1
RCL7
SIN
DEG
SIN-1
STO7
GTO9
```

Calculate M.

```
*LBLE
RCL5
2
÷
RCL4
COS
x
RCL7
COS
÷
STOE
2
x
RCL3
RCL1
2
÷
FRC
X=0?
GTO3
R↓
X⇄Y
9
0
RCL1
÷
COS
```

REGISTERS

0	1 N	2 P	3 d_w	4 ϕ	5 D	6 inv ϕ_w	7 ϕ_w	8 t	9 Used
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D M	E q	I

DATE ______ AUTHOR ______

Program	Comments		
113 x 114 + 115 GTO2 116 *LBL3 117 R↓ 118 + 119 *LBL2 120 STOD 121 GTO9			
122 *LBLa 123 RCL4 124 COS 125 x 126 RCL7 127 SIN 128 ÷ 129 CHS 130 RCLD 131 +	Store Δt and calculate M_t.		
132 GTO9 133 *LBLb 134 RCLE	Print q.		
135 GTO9 136 *LBLc 137 RCL3 138 2 139 ÷ 140 RCLE 141 + 142 GTO9 143 RTN	Calculate R_w.		
144 *LBL9 145 SPC 146 PRTX 147 RTN	Space and print.		

LABELS					FLAGS	SET STATUS		
A P↑N→D, t	B ϕ↑d_w	C →inv ϕ_w	D →ϕ_w	E →M	0	FLAGS	TRIG	DISP
a Δt→M_t	b q	c R_w	d	e	1	ON OFF 0 ☐ ☒ 1 ☐ ☒ 2 ☐ ☒ 3 ☐ ☒	DEG ☒ GRAD ☐ RAD ☐	FIX ☒ SCI ☐ ENG ☐ n 4
0	1 ϕ_w	2 Odd N	3 Even N	4	2			
5	6	7	8	9 Print	3			

BELT LENGTH

TITLE

Step	Key	Comment
001	*LBLa	Toggle for printing belt tangent points.
002	0	
003	F2?	
004	RTN	
005	SF2	
006	1	
007	RTN	
008	*LBLA	Input x_1, y_1, R_1
009	CLRG	
010	SF1	
011	STO0	
012	STO1	
013	R↓	
014	STOB	
015	R↓	
016	STOA	
017	1	
018	CHS	
019	COS⁻¹	
020	STOC	
021	RCL0	
022	RTN	
023	*LBLB	Input x_i, y_i, R_i and make calculation.
024	STO4	
025	CLX	
026	RCL3	
027	P⇄S	
028	STO3	
029	P⇄S	
030	X⇄Y	
031	STO3	
032	X⇄Y	
033	-	
034	X⇄Y	
035	RCL2	
036	P⇄S	
037	STO2	
038	P⇄S	
039	X⇄Y	
040	STO2	
041	X⇄Y	
042	-	
043	→P	
044	X²	
045	X⇄Y	
046	X<0?	
047	GSB0	
048	F1?	
049	STO5	
050	F1?	
051	STO6	
052	RCL6	
053	X⇄Y	
054	STO6	
055	GSBE	
056	X⇄Y	
057	RCL1	
058	RCL4	
059	-	
060	X²	
061	-	
062	√X	
063	ST+8	
064	RCL1	
065	RCL4	
066	-	
067	X⇄Y	
068	GSB1	
069	STO7	
070	+	
071	RCL1	
072	×	
073	RCLC	
074	÷	
075	Pi	
076	×	
077	ST+8	
078	F2?	
079	GSBc	
080	RCL4	
081	STO1	
082	CF1	
083	RTN	
084	*LBLE	$\theta_i - \alpha_i$
085	-	
086	1	
087	→R	
088	→P	
089	×	
090	ABS	
091	RCL7	
092	-	
093	RTN	
094	*LBLC	Calculate the total length.
095	RCLA	
096	RCLB	
097	RCL0	
098	GSBB	
099	RCL6	
100	RCL5	
101	GSBE	
102	RCL1	
103	×	
104	RCLC	
105	÷	
106	Pi	
107	×	
108	ST+8	
109	RCL8	
110	RTN	
111	*LBL0	
112	RCLC	

REGISTERS

0	1	2	3	4	5	6	7	8	9
R_1	R_{i-1}	x_i	y_i	R_i	θ_1	θ_i	α	Σ Length	Used

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
		x_{i-1}	y_{i-1}						

A	B	C	D	E	I
x_i	y_i	180°	$\theta_{i-1,\,i} + 90° - \alpha$		

DATE ______ AUTHOR ______

```
113    +        Add 180°
114  LSTX
115    +
116  RTN
                ------------
117  *LBL1      Adjustment for angles.
118   →P
119   R↓
120  COS
121  X<0?
122  GTO9
123  CLX
124  LSTX
125  RTN
                ------------
126  *LBL9      Obtain θ or α.
127  CLX
128  LSTX
129  RCLC
130    +
131    1
132   →R
133   →P
134   R↓
135  RTN
                ------------
136  *LBLc      Calculation for belt tangent
137    1        points.
138  SIN⁻¹
139  RCL6
140    +
141  RCL7
142    -
143  STOD
144  RCL1
145   →R
146  P⇄S
147  RCL2
148    +
149  PRTX
150  X⇄Y
151  RCL3
152    +
153  PRTX
154  SPC
155  P⇄S
156  RCLD
157  RCL4
158   →R
159  RCL2
160    +
161  PRTX
162  X⇄Y
163  RCL3
164    +
165  PRTX
166  SF2
167  SPC
168  SPC
169  RTN
                ------------
```

LABELS					FLAGS	SET STATUS		
A $x_1 \uparrow y_1 \uparrow R_1$	B $x_i \uparrow y_i \uparrow R_i$	C →L	D	E $\theta_i - \alpha_i$	0 last?	FLAGS (ON / OFF)	TRIG	DISP
a points?	b for last i	c points	d	e	1 first	0 ☐ ☒	DEG ☒	FIX ☒
0	1 2π	2	3	4	2 points	1 ☐ ☒	GRAD ☐	SCI ☐
5	6	7	8	9 θ or α	3	2 ☐ ☒	RAD ☐	ENG ☐
						3 ☐ ☒		n 2

FREE VIBRATIONS

TITLE

Step	Key	Comment
001	*LBLA	Store k, c, and m and calculate ω.
002	CLRG	
003	STO4	
004	R↓	
005	STO3	
006	R↓	
007	STO2	
008	RCL4	
009	GSB0	
010	RTN	
011	*LBL0	Calculate $\left[\frac{k}{m}-\left(\frac{c}{2m}\right)^2\right]^{½}$
012	RCL2	
013	÷	
014	STOE	
015	RCL3	
016	RCL2	
017	2	
018	x	
019	÷	
020	STOD	
021	X²	
022	-	
023	RND	
024	X<0?	
025	GTO1	
026	X=0?	
027	RTN	
028	*LBL3	
029	LSTX	
030	√X	
031	STO5	
032	RTN	
033	*LBL1	
034	LSTX	
035	STO5	
036	1	
037	CHS	
038	RTN	
039	*LBLB	Calculate c_{cr}.
040	RCL4	
041	RCL2	
042	x	
043	√X	
044	2	
045	x	
046	RTN	
047	*LBLC	Store initial displacement and velocity.
048	STO1	
049	R↓	
050	STO0	
051	RTN	
052	*LBLD	Calculate displacement, velocity and acceleration at time t.
053	SPC	
054	*LBL4	
055	STO7	
056	RCLD	

Step	Key	Comment
057	x	
058	CHS	
059	e^x	
060	STOC	
061	RCL5	
062	X<0?	
063	GTOc	
064	X=0?	
065	GTOb	
066	*LBLa	For $c > c_r$.
067	RCLD	
068	RCL0	
069	x	
070	RCL1	
071	+	
072	RCL5	
073	÷	
074	RCL0	
075	→P	
076	STOA	
077	R↓	
078	STOB	
079	RCL5	
080	RCL7	
081	x	
082	-	
083	CHS	
084	STO9	
085	COS	
086	RCLC	
087	x	
088	RCLA	
089	x	
090	STO2	
091	PRTX	
092	RCLD	
093	x	
094	RCL9	
095	SIN	
096	RCLC	
097	x	
098	RCL5	
099	x	
100	RCLA	
101	x	
102	+	
103	CHS	
104	PRTX	
105	GTOe	
106	*LBLb	For $c = c_r$.
107	RCLD	
108	RCL0	
109	x	
110	RCL1	
111	+	
112	STOB	

REGISTERS

0	1	2	3	4	5	6	7	8	9
x_0	$\dot{x}_0$	m, x(t)	c, r_1	k, r_2	$\omega, \sqrt{-\omega^2}$	$(t_1 - t_2)/n$	t, t_1	$\dot{x}(t)$	$\omega t - \delta$
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
R, A_{cr}, A_{ov}	δ, B_{cr}, B_{ov}	$e^{-c/2m\ t}$	c/2m	k/m	n

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Step	Key	Comments
113	RCL7	
114	x	
115	RCL0	
116	+	
117	RCLC	
118	x	
119	STO2	
120	PRTX	
121	RCLD	
122	x	
123	CHS	
124	RCLB	
125	RCLC	
126	x	
127	+	
128	PRTX	
129	GTOe	
130	*LBLc	For $c < c_r$.
131	CHS	
132	√X	
133	ENT↑	
134	ENT↑	
135	RCLD	
136	-	
137	STO3	
138	X⇄Y	
139	2	
140	x	
141	-	
142	STO4	
143	RCL1	
144	RCL3	
145	RCL0	
146	x	
147	-	
148	RCL4	
149	RCL3	
150	-	
151	÷	
152	STOE	
153	RCL0	
154	-	
155	CHS	
156	STOA	
157	RCL3	
158	RCL7	
159	x	
160	e^x	
161	RCLA	
162	x	
163	STO2	
164	RCL3	
165	x	
166	STO8	
167	RCL4	
168	RCL7	
169	x	
170	e^x	
171	RCLB	
172	x	
173	ST+2	
174	RCL4	
175	x	
176	ST+8	
177	RCL2	
178	PRTX	
179	RCL8	
180	PRTX	
181	*LBLe	Calculate acceleration.
182	RCLD	
183	2	
184	x	
185	x	
186	RCL2	
187	RCLE	
188	x	
189	+	
190	CHS	
191	PRTX	
192	RTN	
193	*LBLE	Calculate displacement, velocity and acceleration automatically.
194	STOI	
195	R↓	
196	X⇄Y	
197	STO7	
198	-	
199	RCLI	
200	÷	
201	STO6	
202	ISZI	
203	*LBL9	
204	RCL7	
205	SPC	
206	PRTX	
207	GSB4	
208	DSZI	
209	GTO8	
210	RTN	
211	*LBL8	
212	RCL6	
213	ST+7	
214	GTO9	
215	RTN	

LABELS

A m↑c↑k→ω	B →c_{cr}	C x_0↑$\dot{x}_0$	D t→x, $\dot{x}$, $\ddot{x}$	E t_1↑t_2↑n→List
a $c > c_{cr}$	b $c = c_{cr}$	c $c < c_{cr}$	d $\ddot{x}$	e Used
0 ω	1 $c > c_{cr}$	2 $c = c_{cr}$	3 $c < c_{cr}$	4 Used
5	6	7	8 Used	9 t→x

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☐	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☒	ENG	☐
3	☐	☒			n	3

VIBRATIONS FORCED BY $F_0COS\omega t$

TITLE

Step	Key Entry	Comments
001	*LBLA	Store k, c, and m and calculate ω_0, ω_n and ζ.
002	SPC	
003	CLRG	
004	SF0	
005	STO4	
006	R↓	
007	STO3	
008	R↓	
009	STO2	
010	RCL4	
011	GSB0	
012	PRTX	
013	RCL3	
014	RCL4	
015	RCL2	
016	x	
017	√X	
018	2	
019	x	
020	÷	
021	PRTX	
022	RTN	
023	*LBL0	Calculate ω_0 and ω_n.
024	RCL2	
025	÷	
026	STOE	
027	√X	
028	PRTX	
029	RCLE	
030	RCL3	
031	RCL2	
032	2	
033	x	
034	÷	
035	STOD	
036	X²	
037	-	
038	RND	
039	X<0?	
040	GTO1	
041	X=0?	
042	RTN	
043	*LBL3	
044	LSTX	
045	√X	
046	STO5	
047	RTN	
048	*LBL1	
049	LSTX	
050	STO5	
051	1	
052	CHS	
053	RTN	
054	*LBL4	
055	X²	
056	-	
057	RCL2	
058	x	
059	RCL9	
060	RCL3	
061	x	
062	X⇄Y	
063	→P	
064	STOC	
065	RCL8	
066	X⇄Y	
067	÷	
068	PRTX	
069	R↓	
070	STOB	
071	RTN	
072	*LBLC	Store F_0 and ω and calculate AMP and δ.
073	SPC	
074	STO8	
075	R↓	
076	STO9	
077	R↑	
078	RCLE	
079	RCL9	
080	GSB4	
081	R→D	
082	PRTX	
083	RTN	
084	*LBLD	Input t and calculate x(t), $\dot{x}$(t) and $\ddot{x}$(t).
085	SPC	
086	*LBL6	
087	STO6	
088	RCL9	
089	x	
090	RCLB	
091	-	
092	COS	
093	LSTX	
094	SIN	
095	X⇄Y	
096	RCL8	
097	x	
098	RCLC	
099	÷	
100	PRTX	
101	RCL4	
102	x	
103	STO0	
104	X⇄Y	
105	RCL8	
106	x	
107	RCLC	
108	÷	
109	RCL9	
110	x	
111	CHS	
112	PRTX	

REGISTERS

0	1	2	3	4	5	6	7	8	9
k x(t)	c $\dot{x}$(t)	m	c	k	$\omega_n, \sqrt{-\omega_n^2}$	t, t_1	ω_{res}^2	F_0	ω
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
$(t_1 - t_2)/n$	δ	Δ	c/2m	k/m	0.00

DATE ______________________ AUTHOR ______________________

```
113  RCL3
114  x
115  STO1
116  CHS
117  RCL0
118  -
119  RCL9
120  RCLE
121  x
122  COS
123  RCLB
124  x
125  +
126  RCL2
127  ÷
128  PRTX
129  RTN
```

Calculate ω_{res}.

```
130  *LBLB
131  CF0
132  RCLE
133  RCLD
134  X²
135  2
136  x
137  -
138  X>0?
139  GTO9
140  ABS
141  √X
142  STO7
143  RTN
144  *LBL9
145  √X
146  STO7
147  DSP9
148  DSP3
149  RTN
```

Calculate x (t), $\dot{x}$(t) and $\ddot{x}$(t) and output the values automatically.

```
150  *LBLE
151  STOI
152  R↓
153  X⇄Y
154  STO6
155  -
156  RCLI
157  ÷
158  STOA
159  ISZI
160  *LBL7
161  SPC
162  RCL6
163  PRTX
164  GSB6
165  DSZI
166  GTO8
167  RTN
168  *LBL8
169  RCLA
170  ST+6
171  GTO7
172  RTN
```

LABELS					FLAGS
A m↑c↑k	B $\rightarrow \omega_{res}$	C $\omega \uparrow F_0 \rightarrow$ AMP, δ	D $t \rightarrow x, \dot{x}, \ddot{x}$	E $t_1 \uparrow t_2 \uparrow n \rightarrow \ldots$	0
a	b	c	d	e	1
0 ω_0, ω_n	1 $c > c_{cr}$	2	3 $c < c_{cr}$	4 AMP, δ	2
5	6 Used	7 $t \rightarrow x_1$	8 $t + \Delta t$	9 ω_{res}	3

SET STATUS				
FLAGS	ON	OFF	TRIG	DISP
0	☐	☒	DEG ☐	FIX ☒
1	☐	☒	GRAD ☐	SCI ☐
2	☐	☒	RAD ☒	ENG ☐
3	☐	☒		n 3

EQUATIONS OF STATE

TITLE

Step	Key	Comments
001	*LBLa	Redlich-Kwong; ideal gas toggle.
002	F0?	
003	GTO0	
004	0	
005	SF0	
006	RTN	
007	*LBL0	
008	1	
009	CF0	
010	RTN	
011	*LBLb	Store T_c.
012	CF3	
013	STOC	
014	RTN	
015	*LBLc	Store P_c.
016	CF3	
017	STOD	
018	RTN	
019	*LBLA	P code.
020	5	
021	GTO0	
022	*LBLB	V code.
023	6	
024	GTO0	
025	*LBLC	n code.
026	7	
027	GTO0	
028	*LBLD	R code.
029	8	
030	GTO0	
031	*LBLE	T code.
032	9	
033	*LBL0	Store input.
034	CF1	
035	STOI	
036	R↓	
037	STO(i)	
038	F3?	
039	RTN	
040	1	Dummy 1.00 for unknown and GTO ideal gas.
041	STO(i)	
042	GTO(i)	
043	*LBL5	Ideal gas solution for P and V.
044	*LBL6	
045	RCL7	
046	RCL8	
047	x	
048	RCL9	
049	x	
050	RCL5	
051	RCL6	
052	x	
053	÷	
054	STO(i)	
055	GTO0	
056	*LBL7	

Step	Key	Comments
057	*LBL8	Ideal gas solution for n, R and T.
058	SF1	
059	*LBL9	
060	RCL5	
061	RCL6	
062	x	
063	RCL7	
064	÷	
065	RCL8	
066	÷	
067	RCL9	
068	÷	
069	STO(i)	
070	*LBL0	Stop if ideal gas is desired.
071	F0?	
072	RTN	
073	GSB1	Calculate P by Redlich-Kwong.
074	GTO0	
075	*LBL2	
076	F1?	
077	GSB1	
078	*LBL0	
079	RCLE	
080	RCL9	
081	x	
082	RCL6	
083	RCLB	
084	-	
085	STO4	
086	÷	
087	RCLA	
088	RCL9	
089	√X	
090	÷	
091	STO2	
092	RCL6	
093	÷	
094	LSTX	
095	RCLB	
096	+	
097	STO3	
098	÷	
099	-	
100	RCL5	Calculate f(P).
101	-	
102	GSB(i)	Calculate f′(P).
103	÷	
104	ST-(i)	Loop again?
105	RCL(i)	
106	÷	
107	ABS	
108	EEX	
109	CHS	
110	4	
111	X≤Y?	
112	GTO2	

REGISTERS

0	1	2	3	4	5	6	7	8	9
		$a/T^{1/2}$	(V + b)	(V − b)	P	V	n	R	T

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
a	b	T_c	P_c	nR	Control

DATE ______________________ AUTHOR ______________________

Step	Key	Comments
113	RCL i	Display result of iteration.
114	RTN	
115	*LBL6	$\frac{\partial P}{\partial V}$
116	RCL6	
117	ENT↑	
118	+	
119	RCLB	
120	+	
121	RCL2	
122	x	
123	RCL3	
124	RCL6	
125	x	
126	X²	
127	÷	
128	RCLE	
129	RCL9	
130	x	
131	RCL4	
132	X²	
133	÷	
134	-	
135	RTN	
136	*LBL9	$\frac{\partial P}{\partial T}$
137	RCLE	
138	RCL4	
139	÷	
140	RCL2	
141	2	
142	÷	
143	RCL9	
144	÷	
145	RCL6	
146	÷	
147	RCL3	
148	÷	
149	+	
150	RTN	
151	*LBL7	$\frac{\partial P}{\partial n}$ or $\frac{\partial P}{\partial R}$
152	*LBL8	
153	RCL9	
154	RCL6	
155	x	
156	RCL4	
157	X²	
158	÷	
159	RCL6	
160	ENT↑	
161	+	
162	RCLB	
163	+	
164	RCLE	
165	÷	
166	RCL6	
167	÷	
168	RCL3	
169	X²	
170	÷	
171	RCL2	
172	x	
173	-	
174	RCLE	
175	x	
176	RCL i	
177	÷	
178	RTN	
179	*LBL5	Display P.
180	LSTX	
181	+	
182	STO5	
183	R/S	
184	*LBL1	Calculate a and b.
185	RCL7	
186	RCL8	
187	x	
188	STOE	
189	.	
190	0	
191	8	
192	6	
193	7	
194	RCLD	
195	÷	
196	X⇄Y	
197	RCLC	
198	x	
199	x	
200	STOB	
201	LSTX	
202	x	
203	RCLC	
204	√X	
205	x	
206	4	
207	.	
208	9	
209	3	
210	4	
211	x	
212	STOA	
213	RTN	

LABELS

A ↔P	B ↔V	C ↔n	D ↔R	E ↔T
a R – K?	b T_c	c P_c	d	e
0 Used	1 a, b	2 Iter	3	4
5 P	6 V	7 n	8 R	9 T

FLAGS

0	R – K
1	a, b
2	
3	calc

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

ISENTROPIC FLOW FOR IDEAL GASES

TITLE

Step	Key	Comment
001	*LBLa	Store k, k − 1, 1/(k − 1).
002	STO2	
003	1	
004	-	
005	STO3	
006	1/X	
007	STO4	
008	RCL2	
009	RTN	
010	*LBLA	Output M.
011	F3?	
012	GTO0	
013	RCL1	
014	√X	
015	RTN	
016	*LBL0	Store M^2.
017	X²	
018	STO1	
019	√X	
020	RTN	
021	*LBLB	Output T/T_0.
022	F3?	
023	GTO0	
024	2	
025	RCL1	
026	RCL3	
027	x	
028	2	
029	+	
030	÷	
031	RTN	
032	*LBL0	Convert T/T_0 to M^2.
033	2	
034	X⇄Y	
035	÷	
036	2	
037	-	
038	RCL3	
039	÷	
040	STO1	
041	√X	
042	RTN	
043	*LBLC	Output P/P_0.
044	F3?	
045	GTO0	
046	GSBB	
047	RCL2	
048	RCL3	
049	÷	
050	Yˣ	
051	RTN	
052	*LBL0	Convert P/P_0 to T/T_0 and GTO B.
053	RCL3	
054	RCL2	
055	÷	
056	Yˣ	
057	SF3	
058	GTOB	
059	*LBLD	Output ρ/ρ_0.
060	F3?	
061	GTO0	
062	GSBB	
063	RCL4	
064	Yˣ	
065	RTN	
066	*LBL0	Convert ρ/ρ_0 to T/T_0 and GTO B.
067	SF3	
068	RCL3	
069	Yˣ	
070	GTOB	
071	*LBLE	Set −3 in display for subsonic guess.
072	3	
073	CHS	
074	X⇄Y	
075	F3?	
076	GTO1	
077	GTO3	
078	*LBL1	Make guess of M^2.
079	ENT↑	
080	STO6	
081	FRC	
082	√X	
083	+	
084	X⇄Y	
085	Yˣ	
086	STO1	
087	*LBL2	Iterate by Newton's method to find M^2 Corresponding to A/A^*.
088	RCL6	
089	GSB3	
090	÷	
091	1	
092	-	
093	.	
094	5	
095	RCL8	
096	÷	
097	.	
098	5	
099	RCL1	
100	÷	
101	-	
102	÷	
103	ST+1	
104	RCL1	
105	÷	
106	ABS	
107	EEX	
108	CHS	
109	4	
110	X≤Y?	
111	GTO2	
112	RCL1	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	M^2	k	k − 1	1/k − 1		A/A^*	(k−1)/k+1	Used	Used

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

DATE ____________ AUTHOR ____________

Step	Key	Comments
113	√X	
114	RTN	
115	*LBLe	Set +3 in display for supersonic guess.
116	3	
117	X⇄Y	
118	F3?	
119	GTO1	
120	*LBL3	Convert M^2 to A/A*.
121	2	
122	RCL2	
123	1	
124	+	
125	÷	
126	RCL3	
127	LSTX	
128	÷	
129	STO7	
130	RCL1	
131	x	
132	+	
133	STO8	
134	RCL7	
135	2	
136	x	
137	1/X	
138	Y^X	
139	RCL1	
140	√X	
141	÷	
142	RTN	
143	*LBLb	Output values.
144	SPC	
145	CF3	
146	RCL2	
147	PRTX	
148	SPC	
149	GSBA	
150	PRTX	
151	GSBB	
152	PRTX	
153	GSBC	
154	PRTX	
155	GSBD	
156	PRTX	
157	GSBE	
158	PRTX	
159	RTN	

LABELS

A M→M	B T/T_0→M	C P/P_0→M	D ρ/ρ_0→M	E A/A^*_{sub}→M
a k	b →k,M,T/T_0…	c	d	e A/A^*_{sup}→M
0 Used	1 M^2 guess	2 M^2 iter	3 A/A*	4
5	6	7	8	9

FLAGS

0	
1	
2	
3	DATA?

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	3

CONDUIT FLOW

TITLE ______

Step	Key	Comments
001	*LBLa	Set divide by ρ flag.
002	SF2	
003	GTO0	
004	*LBLb	Clear divide by ρ flag.
005	CF2	
006	*LBL6	
007	STO9	Store μ or ν.
008	GTO0	
009	*LBLc	Store ρ.
010	STOA	
011	GTO0	
012	*LBLd	Store ϵ.
013	STOE	
014	GTO0	
015	*LBLA	Store L.
016	STO3	
017	GTO0	
018	*LBLB	Store D.
019	STOD	
020	GTO0	
021	*LBLC	Store $K_T/4$.
022	4	
023	÷	
024	STO8	
025	*LBL0	Clear data input flag.
026	CF3	
027	RTN	
028	*LBLe	Convert input psi to lb/ft-sec² and store.
029	4	
030	6	
031	3	
032	2	
033	×	
034	STO4	
035	LSTX	
036	÷	
037	F3?	Convert lb/ft-sec² to psi and display.
038	RTN	
039	GSBE	
040	4	
041	6	
042	3	
043	2	
044	÷	
045	RTN	Store pressure.
046	*LBLE	
047	STO4	
048	F3?	Compute pressure drop.
049	RTN	
050	CF0	
051	GSB9	
052	RCL2	
053	X^2	
054	×	
055	RCLA	
056	×	

Step	Key	Comments
057	STO4	Set f and Re in display.
058	RCL5	
059	1/X	
060	X^2	
061	RCL1	
062	RCL4	Store velocity.
063	RTN	
064	*LBLD	
065	STO2	
066	F3?	Guess v.
067	RTN	
068	SF0	Iterate to find v.
069	GSB9	
070	*LBL3	
071	RND	
072	STO0	
073	GSB6	
074	RND	
075	RCL0	
076	X⇄Y	
077	X≠Y?	Set f, Re and v in display.
078	GTO3	
079	RCL5	
080	1/X	
081	X^2	
082	RCL1	
083	RCL2	
084	RTN	Calculate constants.
085	*LBL9	
086	RCLA	
087	F2?	
088	ST÷9	
089	RCLD	
090	RCLE	
091	÷	
092	STO6	
093	LN	
094	1	
095	.	
096	7	
097	3	
098	7	
099	STO7	
100	×	
101	2	
102	.	
103	2	
104	8	
105	+	
106	STOC	
107	STO5	
108	F0?	Is flow turbulent?
109	GTO7	
110	*LBL8	
111	1	
112	6	

REGISTERS

0	1	2	3	4	5	6	7	8	9
v	Re	v	L	ΔP	$1/\sqrt{f}$	D/ε	1.737	$K_T/4$	ν, μ
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
ρ	Used	$1/\sqrt{f_0}$	D	ε	

DATE ______________ AUTHOR ______________

```
113  RCL2
114  RCLD
115  x
116  RCL9
117  ÷
118  STO1
119  2
120  3
121  0
122  0
123  X≤Y?
124  GTO2
125  R↓
126  ÷
127  √X
128  1/X
129  STO5
130  GTO7
131  *LBL2
132  RCLC
133  RCL5
134  -
135  4
136  .
137  6
138  7
139  RCL6
140  x
141  RCL1
142  ÷
143  RCL5
144  x
145  1
146  +
147  STOB
148  LN
149  RCL7
150  x
151  -
152  RCLB
153  1/X
154  CHS
155  1
156  +
157  RCL7
158  x
159  RCL5
160  ÷
161  1
162  +
163  ÷
164  ST+5
165  RCL5
166  ÷
167  ABS
168  EEX
```

Iterate to find $\frac{1}{\sqrt{f}}$ for turbulent flow.

```
169  CHS
170  3
171  X≤Y?
172  GTO2
173  *LBL7
174  RCL5
175  1/X
176  X²
177  RCL3
178  x
179  RCLD
180  ÷
181  RCL6
182  +
183  2
184  x
185  RCL4
186  RCLA
187  ÷
188  X⇄Y
189  F0?
190  GTO0
191  RTN
192  *LBL0
193  ÷
194  √X
195  STO2
196  RTN
```

Start calculation of ΔP or v.

Finish v calculation.

LABELS					FLAGS
A L	B D	C K_T	D v	E ΔP	0 v calc.
a μ	b ν	c ρ	d ϵ	e ΔP (psi)	1
0 Used	1	2 iter-$1/\sqrt{f}$	3 iter-v	4	2 ρ divide
5	6	7 calc.	8 turb?	9 →f	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☐
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☒
3	☐	☒			n	2

HEAT EXCHANGERS (Card 1)

TITLE ______________________

Step	Key	Comments
001	*LBLa	Store T_{cin}.
002	STO2	
003	RTN	
004	*LBLb	Store C_c.
005	x	
006	STO3	
007	RTN	
008	*LBLc	Store T_{hin}.
009	STO1	
010	RTN	
011	*LBLd	Store C_h.
012	x	
013	STO4	
014	RTN	
015	*LBLe	Clear flag 1 for counter flow, set for parallel flow.
016	F1?	
017	GTO0	
018	0	
019	SF1	
020	RTN	
021	*LBL0	
022	1	
023	CF1	
024	RTN	
025	*LBLA	Calculate AU from *E*.
026	STO5	
027	GSB1	
028	GSB0	
029	STO8	
030	RTN	
031	*LBLB	Calculate q from AU.
032	STO8	
033	GSB1	
034	GSB2	
035	RCL7	
036	x	
037	RCL1	
038	RCL2	
039	-	
040	x	
041	STO6	
042	RTN	
043	*LBLC	Calculate T_{co} from q.
044	STO6	
045	RCL3	
046	÷	
047	RCL2	
048	+	
049	RTN	
050	*LBLD	Calculate T_{ho} from T_{co}.
051	RCL2	
052	-	
053	RCL3	
054	x	
055	RCL4	
056	÷	
057	RCL1	
058	-	
059	CHS	
060	RTN	
061	*LBLE	Calculate *E* from T_{co}.
062	STO5	
063	GSB1	
064	RCL4	
065	RCL7	
066	÷	
067	RCL1	
068	RCL5	
069	-	
070	x	
071	RCL1	
072	RCL2	
073	-	
074	÷	
075	STO5	
076	RTN	
077	*LBL0	Calculate AU for $C_{min}/C_{max} = 0$.
078	X≠0?	
079	GTO0	
080	1	
081	RCL5	
082	-	
083	LN	
084	CHS	
085	RTN	
086	*LBL2	Calculate *E* for $C_{min}/C_{max} = 0$.
087	X≠0?	
088	GTO2	
089	1	
090	RCL8	
091	CHS	
092	e^x	
093	-	
094	RTN	
095	*LBL1	Store C_{min} and C_{min}/C_{max}.
096	RCL3	
097	RCL4	
098	X>Y?	
099	X⇄Y	
100	STO7	
101	X⇄Y	
102	÷	
103	STO0	
104	RTN	

REGISTERS

0	1	2	3	4	5	6	7	8	9
C_{min}/C_{max}	T_{hin}	T_{cin}	C_c	C_h	*E*	q	C_{min}	AU	C_{max}

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

(Parallel-Flow and Counter-Flow)

DATE ______________ AUTHOR ______________

STEP	KEY ENTRY	COMMENTS
113	*LBL0	Counter-flow AU calculations.
114	F1?	
115	GTO8	
116	RCL5	
117	1/X	
118	-	
119	1	
120	LSTX	
121	-	
122	÷	
123	LN	
124	1	
125	RCL0	
126	-	
127	X=0?	
128	GTO7	
129	÷	
130	RCL7	
131	x	
132	RTN	
133	*LBL7	Counter-flow for C_{min}/C_{max} = 1.
134	RCL5	
135	1	
136	RCL5	
137	-	
138	÷	
139	RCL7	
140	x	
141	RTN	
142	*LBL8	Parallel-flow AU calculation.
143	RCL0	
144	1	
145	+	
146	RCL5	
147	x	
148	CHS	
149	1	
150	+	
151	LN	
152	CHS	
153	1	
154	RCL0	
155	+	
156	÷	
157	RCL7	
158	x	
159	RTN	
160	*LBL2	Counter flow *E* calculation.
161	F1?	
162	GTO8	
163	1	
164	-	
165	RCL8	
166	RCL7	
167	÷	
168	x	
169	e^x	
170	1	
171	X⇄Y	
172	-	
173	LSTX	
174	RCL0	
175	x	
176	1	
177	X⇄Y	
178	-	
179	X=0?	
180	GTO9	
181	÷	
182	RTN	
183	*LBL9	Counter-flow *E* calculation where C_{min}/C_{max} = 1.
184	RCL8	
185	RCL7	
186	÷	
187	ENT↑	
188	ENT↑	
189	1	
190	+	
191	÷	
192	RTN	
193	*LBL8	Parallel-flow *E* calculation.
194	1	
195	+	
196	RCL8	
197	RCL7	
198	÷	
199	x	
200	CHS	
201	e^x	
202	CHS	
203	1	
204	+	
205	1	
206	RCL0	
207	+	
208	÷	
209	RTN	

LABELS

A $E \to$ AU	B AU→q	C q→T_{co}	D $T_{co} \to T_{ho}$	E $T_{ho} \to E$
a T_{cin}	b $\dot{m}_c c_{pc}$	c T_{hin}	d $\dot{m}_h c_{ph}$	e CNT?
0 $E \to$ AU	1 C_{min}	2 AU→E	3	4
5	6	7 C_{min}/C_{max} = 1	8 parallel	9 C_{min}/C_{max} = 1

FLAGS

0	
1	CNT?
2	
3	

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☒	☐
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☒
SCI	☐
ENG	☐
n	2

(Card 2)
(Side 1: Parallel-Counter-Flow)

TITLE

Calculate AU for parallel-counter-flow.

```
113  *LBL0
114  GSB6
115     2
116     x
117  RCL6
118     2
119  RCL5
120     ÷
121     +
122  RCL9
123     -
124     ÷
125   CHS
126     1
127     +
128    LN
129  RCL6
130     ÷
131   CHS
132  RCL7
133     ÷
134  LSTX
135  ENT↑
136     x
137     x
138   RTN
```

Calculate E for parallel-c counter-flow.

```
139  *LBL2
140  GSB6
141  RCL8
142  RCL7
143     ÷
144  RCL6
145     x
146   CHS
147    eˣ
148     1
149   X⇄Y
150     +
151     1
152  LSTX
153     -
154     ÷
155  RCL6
156     x
157  RCL9
158     +
159     2
160   X⇄Y
161     ÷
162   RTN
163  *LBL6
164  RCL0
165     1
166     +
167  STO9
168  RCL0
169    X²
170     1
171     +
172    √X
173  STO6
174   RTN
```

REGISTERS

0	1	2	3	4	5	6	7	8	9
C_{min}/C_{max}	T_{hin}	T_{cin}	C_c	C_h	E	Used	C_{min}	AU_i	Used
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
AU_{i-1}	$F(AU_i)$		$F(AU_{i-1})$		

(Side 2: Cross-Flow)

DATE ______________________ AUTHOR ______________________

Step	Key	Comments
113	*LBL0	Calculate AU for cross-flow.
114	FIX	
115	0	
116	STOA	
117	1	
118	RCL5	
119	CHS	
120	STOD	
121	+	
122	LN	
123	CHS	
124	STO6	
125	*LBL6	
126	RCL8	
127	GSB2	
128	RCL5	
129	-	
130	STOB	
131	RCLA	
132	RCL6	
133	STOA	
134	-	
135	RCLD	
136	RCLB	
137	STOD	
138	-	
139	÷	
140	x	
141	ST-6	
142	RCL6	
143	÷	
144	RND	
145	X≠0?	
146	GTO6	
147	RCL6	
148	RTN	
149	*LBL2	Calculate E for cross-flow.
150	RCL8	
151	RCL7	
152	÷	
153	ENT↑	
154	ENT↑	
155	.	
156	2	
157	2	
158	Y^x	
159	RCL8	
160	÷	
161	÷	
162	LSTX	
163	X⇄Y	
164	CHS	
165	e^x	
166	1	
167	-	
168	x	
169	e^x	
170	CHS	
171	1	
172	+	
173	RTN	

LABELS

A	B	C	D	E
a	b	c	d	e
0 E→AU	1	2 AU→E	3	4
5	6 iterate	7	8	9

FLAGS

0
1
2
3

SET STATUS

FLAGS (ON OFF)	TRIG	DISP
0 ☐ ☐	DEG ☐	FIX ☐
1 ☐ ☐	GRAD ☐	SCI ☐
2 ☐ ☐	RAD ☐	ENG ☐
3 ☐ ☐		n____

HEWLETT hp PACKARD

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A B ● D E

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