

HEWLETT-PACKARD

HP-67/HP-97

Stat Pac I



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Introduction

The 21 programs of Stat Pac I have been drawn from the fields of general statistics and related areas.

Each program in this pac is represented by one or more magnetic cards and a section in this manual. The manual provides a description of the program with relevant equations, a set of instructions for using the program, and one or more example problems, each of which includes a list of the actual keystrokes required for its solution. Program listings for all the programs in the pac appear at the back of this manual. Explanatory comments have been incorporated in the listings to facilitate your understanding of the actual working of each program. Thorough study of a commented listing can help you to expand your programming repertoire, since interesting techniques can often be found in this way.

On the face of each magnetic card are various mnemonic symbols which provide shorthand instructions to the use of the program. You should first familiarize yourself with a program by running it once or twice while following the complete User Instructions in the manual. Thereafter, the mnemonics on the cards themselves should provide the necessary instructions, including what variables are to be input, which user-definable keys are to be pressed, and what values will be output. A full explanation of the mnemonic symbols for magnetic cards may be found in appendix A.

If you have already worked through a few programs in the Standard Pac, you will understand how to load a program and how to interpret the User Instructions form. If these procedures are not clear to you, take a few minutes to review the sections, "Loading a Program" and "Format of User Instructions," in your Standard Pac.

We hope that Stat Pac I will assist you in the solution of numerous problems in your discipline. We would very much appreciate knowing your reactions to the programs in this pac, and to this end we have provided a questionnaire inside the front cover of this manual. Would you please take a few minutes to give us your comments on these programs? It is in the comments we receive from you that we learn how best to increase the usefulness of programs like these.

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A WORD ABOUT PROGRAM USAGE

This application pac has been designed for both the HP-97 Programmable Printing Calculator and the HP-67 Programmable Pocket Calculator. The most significant difference between the HP-67 and the HP-97 calculators is the printing capability of the HP-97. The two calculators also differ in a few minor ways. The purpose of this section is to discuss the ways that the programs in this pac are affected by the differences in the two machines, and to suggest how you can make optimal use of your machine, be it an HP-67 or an HP-97.

Some of the computed results in this pac are output by PRINTx statements. On the HP-97, these results will be output on the printer. On the HP-67, each PRINT command will be interpreted as a PAUSE: the program will halt, display the result for about five seconds, then continue execution. The term "PRINT/PAUSE" is used to describe this output condition.

If you own an HP-67, you may want more time to copy down the number displayed by a PRINT/PAUSE. All you need to do is press down any key on the keyboard. If the command being executed is PRINTx (eight rapid blinks of the decimal point), pressing down a key will cause the program to halt. If the command being executed is PRINT STACK (two slow blinks of the decimal point), the number in the display will remain there until the depressed key is released; then the next register in the stack will be displayed, and so on. After display of all four registers, the program will halt execution if a key was pressed at any time during the display of the stack contents. In both cases, execution of the halted program may be re-initiated by pressing **R/S**.

HP-97 users may also want to keep a permanent record of the values input to a certain program. A convenient way to do this is to set the Print Mode switch to NORMAL before running the program. In this mode, all input values and their corresponding user-definable keys will be listed on the printer, thus providing a record of the entire operation of the program.

Another area that could reflect differences between the HP-67 and the HP-97 is in the keystroke solutions to example problems. It is sometimes necessary in these solutions to include operations that involve prefix keys, namely, **f** on the HP-97 and **f**, **g**, and **h** on the HP-67. For example, the operation **10^x** is performed on the HP-97 as **f** **10^x** and on the HP-67 as **g** **10^x**. In such cases, the keystroke solution omits the prefix key and indicates only the operation (as

here, 10^x). As you work through the example problems, take care to press the appropriate prefix keys (if any) for your calculator.

Also in keystroke solutions, those values which are output by the command PRINTx will be followed by three asterisks (***)

BASIC STATISTICS FOR TWO VARIABLES

BASIC STATISTICS FOR TWO VARIABLES

START $x_1 + y_1 (N_1 +)$ $x_k + y_k (N_k -)$ $x_1 + \dots + (N_1 +)$ $x_k + \dots (N_k -)$

This program calculates means, standard deviations, covariance, correlation coefficient, coefficients of variation, sums of data points, sum of multiplication of data points, and sums of squares of data points derived from a set of ungrouped data points $\{(x_i, y_i), i = 1, 2, \dots, n\}$, or grouped data points $\{(x_i, y_i, f_i), i = 1, 2, \dots, n\}$. f_i denotes the frequency of repetition of (x_i, y_i) .

$$\text{means } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

$$\text{standard deviations } s_x = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$$

$$\left(\text{or } s_x' = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n}} \right)$$

$$s_y = \sqrt{\frac{\sum y_i^2 - n\bar{y}^2}{n-1}}$$

$$\left(\text{or } s_y' = \sqrt{\frac{\sum y_i^2 - n\bar{y}^2}{n}} \right)$$

covariance

$$s_{xy} = \frac{1}{n-1} \left(\sum x_i y_i - \frac{1}{n} \sum x_i \sum y_i \right)$$

$$\left(\text{or } s_{xy}' = \frac{1}{n} \left[\sum x_i y_i - \frac{1}{n} \sum x_i \sum y_i \right] \right)$$

$$\text{correlation coefficient } r_{xy} = \frac{s_{xy}}{s_x s_y}$$

$$\text{Coefficients of variation } V_x = \frac{s_x}{\bar{x}} \cdot 100, \quad V_y = \frac{s_y}{\bar{y}} \cdot 100$$

Note: n is a positive integer and $n > 1$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		f A	1.00
4	For grouped data points, go to 8			
5	For ungrouped data points, do 6~7			
	for $i=1, 2, \dots, n$			
6	Input x_i	x_i	ENTER	
	y_i	y_i	B	i
7	If you made a mistake in inputting x_k and y_k , then correct by $\rightarrow$	x_k	ENTER	
		y_k	C	$i-1$
8	For grouped data points do 9~10			
	for $i=1, 2, \dots, n$			
9	Input x_i	x_i	ENTER	
	y_i	y_i	ENTER	
	f_i	f_i	D	Σf_i
10	If you made a mistake in inputting x_k , y_k and f_k , then correct by $\rightarrow$	x_k	ENTER	
		y_k	ENTER	
		f_k	E	$\Sigma f_i - f_k$
11	Calculate means: $\bar{x}$		f B	$\bar{x}$
	$\bar{y}$		R/S	$\bar{y}$
12	Calculate coefficients of variation: V_x		f B	V_x
	V_y		R/S	V_y

13	Calculate standard deviations:			
	S_x		f C	S_x
	S_y		R/S	S_y
	S_x'		f C	S_x'
	S_y'		R/S	S_y'
14	Calculate: covariance			
	S_{xy}		f D	S_{xy}
	S_{xy}'		R/S	S_{xy}'
15	Calculate correlation			
	coefficient γ_{xy}		f D	γ_{xy}
16	Calculate sums: Σx_i		f E	Σx_i
	Σy_i		R/S	Σy_i
	$\Sigma x_i y_i$		R/S	$\Sigma x_i y_i$
17	Calculate sums of squares			
	Σx_i^2		f E	Σx_i^2
	Σy_i^2		R/S	Σy_i^2
	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example 1:

For the following set of data, find the means, standard deviations, covariance, correlation coefficient, coefficients of variation, and the sums.

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Keystrokes:

A 0.00

f **A** $\longrightarrow$ 1.00

26 **ENTER** 92 **B** $\longrightarrow$ 26.00 *** (x_1)
92.00 *** (y_1)
1.00 ***

Outputs:

0.00

1.00

26.00 *** (x_1)
92.00 *** (y_1)
1.00 ***

100	ENTER B	→	100.00 ***	(x_2)
			100.00 ***	(y_2) (error)
			2.00 ***	
100	ENTER C	→	100.00 ***	(x_2)
			100.00 ***	(y_2) (correction)
			1.00 ***	
30	ENTER B	→	30.00 ***	(x_2)
			85.00 ***	(y_2)
			2.00 ***	
44	ENTER B	→	44.00 ***	(x_3)
			78.00 ***	(y_3)
			3.00 ***	
50	ENTER B	→	50.00 ***	(x_4)
			81.00 ***	(y_4)
			4.00 ***	
62	ENTER B	→	62.00 ***	(x_5)
			54.00 ***	(y_5)
			5.00 ***	
68	ENTER B	→	68.00 ***	(x_6)
			51.00 ***	(y_6)
			6.00 ***	
74	ENTER B	→	74.00 ***	(x_7)
			40.00 ***	(y_7)
			7.00 ***	
f	B	→	50.57 ***	($\bar{x}$)
R/S		→	68.71 ***	($\bar{y}$)
f	B	→	36.58 ***	(V_x)
R/S		→	29.10 ***	(V_y)
f	C	→	18.50 ***	(s_x)
R/S		→	20.00 ***	(s_y)
f	C	→	17.13 ***	(s'_x)
R/S		→	18.51 ***	(s'_y)

f D	→	-354.14 *** (s_{xy})
R/S	→	-303.55 *** (s_{xy}')
f D	→	-0.96 *** (γ_{xy})
f E	→	354.00 *** (Σx_i)
R/S	→	481.00 *** (Σy_i)
R/S	→	22200.00 *** ($\Sigma x_i y_i$)
f E	→	19956.00 *** (Σx_i^2)
R/S	→	35451.00 *** (Σy_i^2)

Example 2:

Apply the program to the following set of grouped data.

x_i	4.8	5.2	3.8	4.4	4.1
y_i	15.1	11.5	14.3	13.6	12.8
f_i	1	3	1	6	2

Keystrokes:**Outputs:**

A	→	0.00	
f A	→	1.00	
4.8 ENTER 15.1 ENTER 1 D	→	4.80 ***	(x_1)
		15.10 ***	(y_1)
		1.00 ***	(f_1)
		1.00 ***	
5.2 ENTER 11.5 ENTER 3 D	→	5.20 ***	(x_2)
		11.50 ***	(y_2)
		3.00 ***	(f_2)
		4.00 ***	(Σf_i)
10 ENTER 10 ENTER 4 D	→	10.00 ***	
		10.00 ***	(error)
		4.00 ***	
		8.00 ***	
10 ENTER 10 ENTER 4 E	→	10.00 ***	
		10.00 ***	(correction)
		4.00 ***	
		4.00 ***	
3.8 ENTER 14.3 ENTER 1 D	→	3.80 ***	(x_3)
		14.30 ***	(y_3)
		1.00 ***	(f_3)
		5.00 ***	(Σf_i)

4.4 **ENTER** 13.6 **ENTER** 6 **D** $\longrightarrow$ 4.40 *** (x_4)
 13.60 *** (y_4)
 6.00 *** (f_4)
 11.00 *** (Σf_i)

4.1 **ENTER** 12.8 **ENTER** 2 **D** $\longrightarrow$ 4.10 *** (x_5)
 12.80 *** (y_5)
 2.00 *** (f_5)
 13.00 *** (Σf_i)

f B $\longrightarrow$ 4.52 *** ($\bar{x}$)
R/S $\longrightarrow$ 13.16 *** ($\bar{y}$)

f B $\longrightarrow$ 9.93 *** (V_x)
R/S $\longrightarrow$ 8.42 *** (V_y)

f C $\longrightarrow$ 0.45 *** (s_x)
R/S $\longrightarrow$ 1.11 *** (s_y)

f C $\longrightarrow$ 0.43 *** (s'_x)
R/S $\longrightarrow$ 1.07 *** (s'_y)

f D $\longrightarrow$ -0.31 *** (s_{xy})
R/S $\longrightarrow$ -0.28 *** (s'_{xy})
f D $\longrightarrow$ -0.62 *** (γ_{xy})

f E $\longrightarrow$ 58.80 *** (Σx_i)
R/S $\longrightarrow$ 171.10 *** (Σy_i)
R/S $\longrightarrow$ 770.22 *** ($\Sigma x_i y_i$)

f E $\longrightarrow$ 268.38 *** (Σx_i^2)
R/S $\longrightarrow$ 2266.69 *** (Σy_i^2)

FACTORIAL, PERMUTATION AND COMBINATION

FACTORIAL, PERMUTATION AND COMBINATION

START

P?

$n \rightarrow n!$

$m \rightarrow n \rightarrow {}_m P_n$

$m \rightarrow n \rightarrow {}_m C_n$

This program finds the extended range factorial (n can be greater than 69), permutation and combination. Permutation and combination are functions of the factorial, but this program will not use the factorial key, so that better accuracy and larger range can be obtained.

The equations are:

Factorial $n! = n (n-1) (n-2) \cdot \cdot \cdot 2 \cdot 1$

Permutation ${}_m P_n = \frac{m!}{(m-n)!} = m(m-1) \dots (m-n+1)$

Combination ${}_m C_n = \frac{m!}{(m-n)! n!} = \frac{m(m-1) \dots (m-n+1)}{1 \cdot 2 \cdot \dots \cdot n}$

where m, n are integers and $0 \leq n \leq m$.

Notes: 1. ${}_m P_0 = 1, {}_m P_1 = m, {}_m P_m = m!$
therefore $n!$ should be used for large m .

2. ${}_m C_0 = {}_m C_m = 1$

3. ${}_m C_1 = {}_m C_{m-1} = m$

4. ${}_m C_n = {}_m C_{m-n}$

5. In calculating $n!$, the accuracy will be reduced for $n > 69$, since it is calculated by taking Log., ie
$$n! = \log^{-1} [\log (n) + \log [(n-1)!]]$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Go to 5 or 6 or 7			
5	Calculate $n!$			
	(i) $n \leq 69$	n	C	$n!$
	(ii) $n > 69^{**}$	n	C	n
			R/S	exp. of 10
			R/S	decimal No.
6	Calculate ${}_mP_n$	m	ENTER	
		n	D	${}_mP_n$
7	Calculate ${}_mC_n$	m	ENTER	
		n	E	${}_mC_n$
	For a new case, go to 2			
	*Note: to clear print mode			
	press $\rightarrow$		CLF	
			0	
	**In Print Mode, the 3-number			
	result will be printed out			
	automatically.			

Examples:

1. $5! = 120$
2. $69! = 1.711224524 \times 10^{98}$
3. $70! = 1.197857069 \times 10^{100}$
4. $100! = 9.332622518 \times 10^{157}$
5. ${}_{27}P_5 = 9687600.00$
6. ${}_{73}C_4 = 1088430.00$

Keystrokes:**Output:**

A	→	0.00	
B	→	1.00	
5 C	→	5.00 ***	
		120.00 ***	(5!)
69 C	→	69.00 ***	
		1.711224524 +98 ***	(69!)
70 C	→	70.00***	
		100.00 ***	(10 ¹⁰⁰)
		1.197857069 ***	(decimal no.)
100 C	→	100.00 ***	
		157.00 ***	(10 ¹⁵⁷)
		9.332622518 ***	(decimal no.)
27 ENTER 5 D	→	27.00 ***	
		5.00 ***	
		9687600.00 ***	(₂₇ P ₅)
73 ENTER 4 E	→	73.00 ***	
		4.00 ***	
		1088430.00 ***	(₇₃ C ₄)

MOMENTS, SKEWNESS AND KURTOSIS (FOR GROUPED OR UNGROUPED DATA)

MOMENTS, SKEWNESS AND KURTOSIS

START $x_i (\Sigma +)$ $x_i (\Sigma -)$ $y_i \cdot f_i (\Sigma +)$ $y_i \cdot f_i (\Sigma -)$

For grouped or ungrouped data, moments are used to describe sets of data, skewness is used to measure the lack of symmetry in a distribution, and kurtosis is the relative peakness or flatness of a distribution. For a given set of data

$$\{x_1, x_2, \dots, x_n\}:$$

$$1^{st} \text{ moment} \quad \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$2^{nd} \text{ moment} \quad m_2 = \frac{1}{n} \sum x_i^2 - \bar{x}^2$$

$$3^{rd} \text{ moment} \quad m_3 = \frac{1}{n} \sum x_i^3 - \frac{3}{n} \bar{x} \sum x_i^2 + 2\bar{x}^3$$

$$4^{th} \text{ moment} \quad m_4 = \frac{1}{n} \sum x_i^4 - \frac{4}{n} \bar{x} \sum x_i^3 + \frac{6}{n} \bar{x}^2 \sum x_i^2 - 3\bar{x}^4$$

Moment coefficient of skewness

$$\gamma_1 = \frac{m_3}{m_2^{3/2}}$$

Moment coefficient of kurtosis

$$\gamma_2 = \frac{m_4}{m_2^2}$$

This program also provides the option for calculating those statistics for grouped data (using similar formulas as for ungrouped data):

data	y_1	y_2	...	y_m
frequency	f_1	f_2	...	f_m

Note that for this case, 1st moment

$$\bar{x} = \frac{\sum_{i=1}^m f_i x_i}{\sum_{i=1}^m f_i}$$

Reference: Theory and Problems of Statistics, M. R. Spiegel, Schaum's Outline, McGraw-Hill, 1961

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		f A	1.00
4	For grouped data, go to 12,			
5	Do 6~7 for $i = 1, 2, \dots, n$ for ungrouped data			
6	Input x_i	x_i	B	i
7	If you made a mistake in inputting x_k , then correct by $\rightarrow$	x_k	C	$i-1$
8	Calculate: $\bar{x}$		f B	$\bar{x}$
9	Calculate: m_2		f C	m_2
	m_3		R/S	m_3
	m_4		R/S	m_4
10	Calculate: γ_1		f D	γ_1
	γ_2		R/S	γ_2
11	For a new case, go to 2			
12	Do 13~14 for $j = 1, 2, \dots, m$ for grouped data			
13	Input and	y_j f_j	ENTER D	j
14	If you made a mistake in inputting y_h and f_h , then correct by	y_h f_h	ENTER E	$j-1$
15	Go to 8			
	*Note: to clear print mode press $\rightarrow$		CLF 0	

Examples:

1. Ungrouped data

i	1	2	3	4	5	6	7	8	9
x_i	2.1	3.5	4.2	6.5	4.1	3.6	5.3	3.7	4.9

$\bar{x} = 4.21, m_2 = 1.39, m_3 = 0.39, m_4 = 5.49$

$\gamma_1 = 0.24, \gamma_2 = 2.84$

Keystrokes:

Outputs:

A

f A

2.1 B

4 B

4 C

3.5 B

4.2 B

6.5 B

4.1 B

3.6 B

5.3 B

0.00

1.00

2.10 ***

4.00 ***

4.00 ***

3.50 ***

4.20 ***

6.50 ***

4.10 ***

3.60 ***

5.30 ***

1.00 ***

2.00 ***

1.00 ***

2.00 ***

3.00 ***

4.00 ***

5.00 ***

6.00 ***

7.00 ***

(x₁)

(x₂) (error)

(x₂) (correction)

(x₂)

(x₃)

(x₄)

(x₅)

(x₆)

(x₇)

2	ENTER 5 D	→	2.00 *** 5.00 *** 2.00 ***	(y_2) (f_2)
5	ENTER 5 D	→	5.00 *** 5.00 *** 3.00 ***	(y_3) (f_3) (error)
5	ENTER 5 E	→	5.00 *** 5.00 *** 2.00 ***	(y_3) (f_3) (correction)
4	ENTER 3 D	→	4.00 *** 3.00 *** 3.00 ***	(y_3) (f_3)
6	ENTER 2 D	→	6.00 *** 2.00 *** 4.00 ***	(y_4) (f_4)
1	ENTER 1 D	→	1.00 *** 1.00 *** 5.00 ***	(y_5) (f_5)
f	B	→	3.13 ***	($\bar{x}$)
f	C	→	1.98 ***	(m_2)
R/S		→	2.14 ***	(m_3)
R/S		→	11.05 ***	(m_4)
f	D	→	0.77 ***	(γ_1)
R/S		→	2.81 ***	(γ_2)

RANDOM NUMBER GENERATOR



Random numbers are useful in a wide variety of applications, e.g., simulation, sampling, computer programming, numerical analysis and games. This program calculates (1) uniformly distributed numbers, (2) random integers, (3) normally distributed numbers, (4) exponentially distributed numbers, (5) mean, standard deviation and counter of the numbers generated.

This program calculates:

1. Uniformly distributed pseudo random numbers u_i in the range $a < u_i < b$:
The multiplicative linear congruential method is used.

$$u_{i+1} = f_{i+1} (b - a) + a$$

where $i = 0, 1, 2, \dots$ and

$$f_{i+1} = \text{fractional part of } (997 f_i)$$

$$f_0 = 0.5284163.$$

The period has length 500000, i.e., 500000 different numbers can be generated before repeating. The least significant digits (the righthand digits) of u_i are not as random as the most significant digits (the left-hand digits). Thus random digits, if needed, should be taken from the most significant end of the numbers. This generator passes the chi-square frequency test for uniformity, serial test and run tests for randomness.

If a different sequence of numbers is desired, a different starting value f_0 ($0 < f_0 < 1$) can be used. Some program steps (the starting value stored under **LBL** **0**) must be changed accordingly. Note that if $10^7 \times f_0$ is not divisible by 2 or 5, then the period of the generator has length 500000. All the tests mentioned above should be applied to the new generator before using it.

2. Pseudo random integers d_i such that $1 \leq d_i \leq k$:

Suppose u_i ($i = 1, 2, \dots$) is a sequence of uniformly distributed pseudo random numbers between 0 and 1.

$$d_i = 1 + \text{integer part of } (ku_i)$$

3. Normally distributed pseudo random numbers n_i if the mean m and the standard deviation σ are given:

Suppose u_i ($i = 1, 2, \dots$) is a sequence of uniformly distributed pseudo random numbers between 0 and 1.

Let

$$v_1 = (2u_1 - 1) \quad , \quad v_2 = (2u_{i+1} - 1) \\ S = v_1^2 + v_2^2 \quad (i = 1, 2, \dots)$$

If $S \geq 1$, discard the two uniform numbers u_i, u_{i+1} and generate the next two numbers in the sequence until $S < 1$. Then compute the normally distributed numbers according to the following equations

$$n_i = \sigma v_1 \sqrt{\frac{-2 \ln S}{S}} + m \\ n_{i+1} = \sigma v_2 \sqrt{\frac{-2 \ln S}{S}} + m$$

4. Exponentially distributed pseudo random numbers e_i with mean μ :

Suppose u_i ($i = 1, 2, \dots$) is a sequence of uniformly distributed pseudo random numbers between 0 and 1.

$$e_i = -\mu \ln u_i$$

5. The mean $\bar{x}$, standard deviation s and counter n of the random numbers computed:

$$\bar{x} = \sum_{i=1}^n x_i / n \\ s = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$$

where x_i can be u_i, d_i, n_i or e_i .

Reference:

Donald E. Knuth, *The Art of Computer Programming*, Vol. 2, Addison-Wesley, 1971.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	For random integers, go to 6			
	For normal numbers, go to 9			
	For exponential numbers, go to 12			
3	Input interval range for uniform numbers	a	ENTER	
		b	f A	b
4	Perform 4 for i=1, 2, . . .		A	u_i
5	For $\bar{x}$ and s, go to 14			
6	Input maximum integer desired	k	f B	k
7	Perform 7 for i=1, 2, . . .		B	d_i
8	For $\bar{x}$ and s, go to 14			
9	For normal numbers, input mean	m	ENTER	
	and standard deviation	σ	f C	σ
10	Perform 10 for i=1, 2, . . .		C	n_i
11	For $\bar{x}$ and s, go to 14			
12	Input mean for exponential numbers	μ	f D	μ
13	Perform 13 for i=1, 2, . . .		D	e_i
14	Optional: Calculate the mean		E	$\bar{x}$
	the standard deviation		R/S	s
	the counter		R/S	n
15	To continue the calculation, go back to 4, 7, 10, or 13			
16	For a new case, go to 2			

Example 1:

Generate a sequence of uniform pseudo random numbers between 0 and 1.

Keystrokes:**Outputs:**

0 **ENTER** 1 **f** **A** → 0.00 *** (a)
 1.00 *** (b)

A → 0.83 ***
A → 0.56 ***
A → 0.27 ***
A → 0.04 ***
A → 0.20 ***
A → 0.75 ***
A → 0.83 ***
A → 0.95 ***

E → 0.55 *** (mean)
R/S → 0.34 *** (s)
R/S → 8.00 *** (counter)

A → 0.68 ***
A → 0.63 ***
A → 0.22 ***
 etc.

Example 2:

Use the random number generator to simulate the successive tosses of a die.

Keystrokes:**Outputs:**

6 **f** **B** → 6.00 *** (k)

B → 5.00 ***
B → 4.00 ***
B → 2.00 ***
B → 1.00 ***
B → 2.00 ***
B → 5.00 ***
 etc.

Example 3:

A professor decides to assign grades randomly and without bias to the students. The grades should have a normal distribution with average grade being 75 and standard deviation being 10. How can the random number generator be used for this purpose?

Keystrokes:75 **ENTER** 10 **f** **C****Outputs:**

75.00 *** (m)
10.00 *** (σ)

C	→	87.42 ***
C	→	77.17 ***
C	→	67.44 ***
C	→	81.23 ***
C	→	89.91 ***
C	→	85.32 ***
		etc.

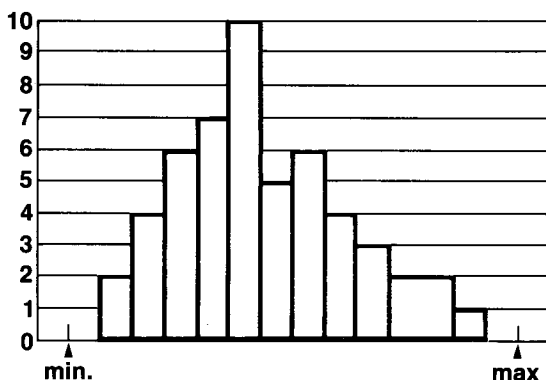
Example 4:

Suppose a radioactive substance emits alpha particles at a rate such that on the average, one particle is emitted every 5 seconds. Note that the amount of time between two successive emissions has the exponential distribution with mean 5. Generate a sequence of random numbers so that each of them can be used as the amount of time between two emissions.

Keystrokes:5 **f** **D****Outputs:**5.00 *** (μ)

D	→	0.93 ***
D	→	2.92 ***
D	→	6.49 ***
D	→	15.93 ***
D	→	8.14 ***
D	→	1.44 ***
		etc.

HISTOGRAM



A histogram or bar chart can provide a meaningful way of representing tabular data or the output of an algorithm. By viewing a histogram, trends and biases can be spotted easily.

This program sorts input data into 24 intervals or bins of equal width between specified upper and lower limits.

One is added to the bin whose number is calculated. This procedure is repeated for all x values in the data set. After all data has been input, pressing **f** **B**, **R/S**, **R/S** will cause the printout of the total number of inputs, the mean of the inputs, and the standard deviation of the inputs. Pressing **f** **A** gives the number of inputs in each bin and a representation of the histogram. The bins are arranged in maximum x value to minimum x value order.

The 24 intervals are stored three at a time in registers $R_1 \sim R_8$.

Incorrect values may be deleted at any time by keying them in and pressing **E**. However, if the value is out of bounds, then "Error" will be displayed. Press **CLX** and continue.

To start the program you must specify the minimum expected value $x_{\min}$ and the maximum expected value $x_{\max}$.

Equations:

For the histogram:

$$\text{mean} = \frac{\sum x}{n}$$

$$\text{standard deviation} = \sqrt{\frac{\sum x^2 - n \bar{x}^2}{n-1}}$$

$$y_i = 1 + \text{INT} \left[24 \frac{x_i - x_{\min}}{x_{\max} - x_{\min}} \right]$$

where

y_i = interval number

x_i = input data

$x_{\min}$ = lower limit of histogram

$x_{\max}$ = upper limit of histogram

INT = integer part of

Remark:

Because each interval is represented by only three digits, overflow from one interval to the next lower interval will occur for most of the intervals if there are more than 999 counts in the interval.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input minimum value	$x_{\min}$	ENTER	
	maximum value	$x_{\max}$	C	$x_{\min}, x_{\max}$
5	Do 6~7 for $i=1, 2, \dots, n$			
6	Input x_i	x_i	D	i
7	If you made a mistake in			
	inputting x_k , then correct by $\rightarrow$	x_k	E	$i-1$
8	List histogram		f A	LIST

9	Print n, $\bar{x}$, s		f B	n
			R/S	$\bar{x}$
			R/S	s
10	For a new case, go to 2			
	*Note: to clear print mode			
	press→		CLF	
			0	

Example 1:

Compute a histogram of the following data with $x_{\min}=0$, $x_{\max}=24$. (18.1, 14.3, 8.4, 0.7, 20.2, 14, 17.2, 24, 8.8, 5.7, 13.2, 22.1, 15.7, 18.9, 23).

Keystrokes:

Outputs:

A	→	0.00	
B	→	1.00	
0 ENTER 24 C	→	0.00 ***	($x_{\min}$)
		24.00 ***	($x_{\max}$)
18.1 D	→	18.10 ***	
		1. ***	
14.3 D	→	14.30 ***	
		2. ***	
8.4 D	→	8.40 ***	
		3. ***	
0.7 D	→	0.70 ***	
		4. ***	
9.9 D	→	9.90 ***	(error)
		5. ***	
9.9 E	→	9.90 ***	(correction)
		4. ***	
20.2 D	→	20.20 ***	
		5. ***	
14.0 D	→	14.00 ***	
		6. ***	
17.2 D	→	17.20 ***	
		7. ***	

24	D	→	24.00 ***	
			8. ***	
8.8	D	→	8.80 ***	
			9. ***	
5.7	D	→	5.70 ***	
			10. ***	
13.2	D	→	13.20 ***	
			11. ***	
22.1	D	→	22.10 ***	
			12. ***	
15.7	D	→	15.70 ***	
			13. ***	
18.9	D	→	18.90 ***	
			14. ***	
23	D	→	23.00 ***	
			15. ***	
f	B	→	15.00 ***	(n)
R/S		→	14.95 ***	($\bar{x}$)
R/S		→	6.71 ***	(s)
f	A	→	0.00 ***	
			1.00 ***	
			1. ***	
			1.00 ***	
			2.00 ***	
			0. ***	
			2.00 ***	
			3.00 ***	
			0. ***	
			3.00 ***	
			4.00 ***	
			0. ***	
			4.00 ***	
			5.00 ***	
			0. ***	

5.00 ***
6.00 ***
1. ***
6.00 ***
7.00 ***
0. ***
7.00 ***
8.00 ***
0. ***
8.00 ***
9.00 ***
2. ***
9.00 ***
10.00 ***
0. ***
10.00 ***
11.00 ***
0. ***
11.00 ***
12.00 ***
0. ***
12.00 ***
13.00 ***
0. ***
13.00 ***
14.00 ***
1. ***
14.00 ***
15.00 ***
2. ***
15.00 ***
16.00 ***
1. ***
16.00 ***
17.00 ***
0. ***

17.00 ***

18.00 ***

1. ***

18.00 ***

19.00 ***

2. ***

19.00 ***

20.00 ***

0. ***

20.00 ***

21.00 ***

1. ***

21.00 ***

22.00 ***

0. ***

22.00 ***

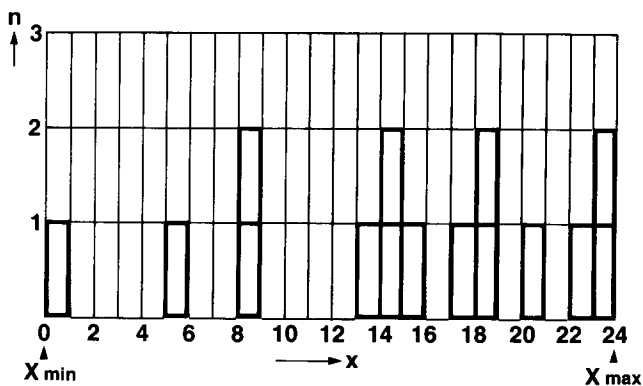
23.00 ***

1. ***

23.00 ***

24.00 ***

2. ***



ANALYSIS OF VARIANCE (ONE WAY)

ANALYSIS OF VARIANCE (ONE WAY)

START

P?

 $x_{ij} (\Sigma +)$ $x_{..m} (\Sigma -)$ $\bar{x}_i, s_i, \text{Sum}_i$

The one-way analysis of variance is used to test if observed differences among k sample means can be attributed to chance or whether they are indicative of actual differences among the corresponding population means. Suppose the i^{th} sample has n_i observations (samples may have equal or unequal number of observations). The null hypothesis we want to test is that the k population means are all equal. This program generates the complete ANOVA table.

1. Mean of observations in the i^{th} sample ($i=1, 2, \dots, k$)

$$\bar{x}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij}$$

2. Standard deviation of observations in the i^{th} sample

$$s_i = \left[\left(\sum_{j=1}^{n_i} x_{ij}^2 - n_i \bar{x}_i^2 \right) / (n_i - 1) \right]^{1/2}$$

3. Sum of observations in the i^{th} sample

$$\text{Sum}_i = \sum_{j=1}^{n_i} x_{ij}$$

4. Total sum of squares

$$\text{TSS} = \sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij}^2 - \frac{\left(\sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^k n_i}$$

5. Treatment sum of squares

$$\text{TrSS} = \sum_{i=1}^k \frac{\left(\sum_{j=1}^{n_i} x_{ij} \right)^2}{n_i} - \frac{\left(\sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^k n_i}$$

6. Error sum of squares

$$\text{ESS} = \text{TSS} - \text{TrSS}$$

7. Treatment degrees of freedom

$$\text{df}_1 = k - 1$$

8. Error degrees of freedom

$$\text{df}_2 = \sum_{i=1}^k n_i - k$$

9. Total degrees of freedom

$$\text{df}_3 = \text{df}_1 + \text{df}_2 = \sum_{i=1}^k n_i - 1$$

10. Treatment mean square

$$\text{TrMS} = \frac{\text{TrSS}}{\text{df}_1}$$

11. Error mean square

$$\text{EMS} = \frac{\text{ESS}}{\text{df}_2}$$

12. The F ratio

$$F = \frac{\text{TrMS}}{\text{EMS}} \quad (\text{with degrees of freedom } \text{df}_1, \text{df}_2)$$

Reference:

J. E. Freund, *Mathematical Statistics*, Prentice Hall, 1962.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Do 4 ~ 8 for i=1, 2, ..., k			
5	Do 6 for j=1, 2, ..., n _i			
6	Input x _{ij}	x _{ij}	C	j
7	If you made a mistake in inputting x _{im} , then correct by	x _{im}	D	j-1
8	Calculate: mean $\bar{x}_i$		E	$\bar{x}_i$
	standard deviation s _i		R/S	s _i
	sum Sum _i		R/S	Sum _i
9	Calculate: total sum of squares		f A	TSS
	treatment sum of squares		R/S	TrSS
	error sum of squares		R/S	ESS
10	Calculate degrees of freedom			
	df ₁		f B	df ₁
	df ₂		R/S	df ₂
	df ₃		R/S	df ₃
11	Calculate:			
	treatment mean square		f C	TrMS
	error mean square		R/S	EMS
	F ratio		R/S	F
12	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example:

The following are the scores obtained in an achievement test by random samples of students from four different schools:

$i \backslash j$	1	2	3	4	5	6	7
School 1	88	99	96	68	85		
School 2	78	62	98	83	61	88	
School 3	80	61	74	92	78	54	77
School 4	71	65	90	46			

Calculate the ANOVA table and test the null hypothesis that the differences among the sample means can be attributed to chance. Use significance level $\alpha = 0.01$.

Keystrokes:**Outputs:**

A	→	0.00	
B	→	1.00	
88 C	→	88.00 ***	
		1.00 ***	
99 C	→	99.00 ***	
		2.00 ***	
96 C	→	96.00 ***	
		3.00 ***	
68 C	→	68.00 ***	
		4.00 ***	
85 C	→	85.00 ***	
		5.00 ***	
E	→	87.20 ***	$(\bar{x}_1)$
R/S	→	12.15 ***	(s_1)
R/S	→	436.00 ***	(Sum_1)
78 C	→	78.00 ***	
		1.00 ***	

06-05

62 **C** → 62.00 ***
2.00 ***

98 **C** → 98.00 ***
3.00 ***

83 **C** → 83.00 ***
4.00 ***

61 **C** → 61.00 ***
5.00 ***

88 **C** → 88.00 ***
6.00 ***

E → 78.33 *** ($\bar{x}_2$)

R/S → 14.62 *** (s_2)

R/S → 470.00 *** (Sum_2)

80 **C** → 80.00 ***
1.00 ***

61 **C** → 61.00 ***
2.00 ***

74 **C** → 74.00 ***
3.00 ***

92 **C** → 92.00 ***
4.00 ***

78 **C** → 78.00 ***
5.00 ***

54 **C** → 54.00 ***
6.00 ***

77 **C** → 77.00 ***
7.00 ***

E → 73.71 *** ($\bar{x}_3$)

R/S → 12.61 *** (s_3)

R/S → 516.00 *** (Sum_3)

71	C	→	71.00 ***	
			1.00 ***	
66	C	→	66.00 ***	(error)
			2.00 ***	
66	D	→	66.00 ***	(correction)
			1.00 ***	
65	C	→	65.00 ***	
			2.00 ***	
90	C	→	90.00 ***	
			3.00 ***	
46	C	→	46.00 ***	
			4.00 ***	
E		→	68.00 ***	($\bar{x}_4$)
R/S		→	18.13 ***	(s_4)
R/S		→	272.00 ***	(Sum ₄)
f	A	→	4530.00 ***	(TSS)
R/S		→	930.44 ***	(TrSS)
R/S		→	3599.56 ***	(ESS)
f	B	→	3.00 ***	(df ₁)
R/S		→	18.00 ***	(df ₂)
R/S		→	21.00 ***	(df ₃)
f	C	→	310.15 ***	(TrMS)
R/S		→	199.98 ***	(EMS)
R/S		→	1.55 ***	(F)

ANOVA Table

	SS	df	MS	F
Treatments	930.44	3	310.15	1.55
Error	3599.56	18	199.98	
Total	4530.00	21		

Since $F = 1.55$ does not exceed $F_{.01,3,18} = 5.09$, the null hypothesis can not be rejected. We conclude that the means of the scores for the four schools are not significantly different.

TWO WAY ANALYSIS OF VARIANCE (NO REPLICATIONS)



The analysis of variance is the analysis of the total variability of a set of data (measured by their total sum of squares) into components which can be attributed to different sources of variation.

The two way analysis of variance tests the row effects and the column effects independently. This program will generate the ANOVA table for the case such that (1) each cell only has one observation and (2) the row and column effects do not interact.

Equations:

1. Sums

$$\text{Row RS}_i = \sum_j x_{ij} \quad i = 1, 2, \dots, r$$

$$\text{Column CS}_j = \sum_i x_{ij} \quad j = 1, 2, \dots, c$$

2. Sums of squares

$$\text{Total TSS} = \sum \sum x_{ij}^2 - (\sum \sum x_{ij})^2 / rc$$

$$\text{Row RSS} = \sum_i \left(\sum_j x_{ij} \right)^2 / c - (\sum \sum x_{ij})^2 / rc$$

$$\text{Column CSS} = \sum_j \left(\sum_i x_{ij} \right)^2 / r - (\sum \sum x_{ij})^2 / rc$$

$$\text{Error ESS} = \text{TSS} - \text{RSS} - \text{CSS}$$

3. Degrees of freedom

$$\text{Row df}_1 = r - 1$$

$$\text{Column df}_2 = c - 1$$

$$\text{Error df}_3 = (r - 1)(c - 1)$$

4. F ratios

$$\text{Row } F_1 = \frac{\text{RSS}}{\text{df}_1} \bigg/ \frac{\text{ESS}}{\text{df}_3}$$

$$\text{Column } F_2 = \frac{\text{CSS}}{\text{df}_2} \bigg/ \frac{\text{ESS}}{\text{df}_3}$$

Reference:

Dixon and Massey, *Introduction to Statistical Analysis*, McGraw-Hill, 1969.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input: number of rows r	r	ENTER	
	number of columns c	c	C	c
5	Do 5~9 for $i=1, 2, \dots, r$			
	Do 6 for $j=1, 2, \dots, c$			
6	Input x_{ij}	x_{ij}	D	j
7	If you made a mistake in inputting x_{im} , then correct by	x_{im}	E	$j-1$
8	Calculate row sums RS_i		f A	RS_i
9	Re-initialize for columns		f B	0.00
10	Do 11~14 for $j=1, 2, \dots, c$			
11	Do 12 for $i=1, 2, \dots, r$			
12	Input x_{ij}	x_{ij}	D	i
13	If you made a mistake in inputting x_{hj} , then correct by	x_{hj}	E	$i-1$
14	Calculate column sums CS_j		f C	CS_j
15	Calculate F ratios: Row F_1		f D	F_1
	Column F_2		R/S	F_2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
16	Calculate degrees of freedom:			
	row df_1		[F D]	df_1
	column df_2		[R/S]	df_2
	error df_3		[R/S]	df_3
17	Calculate sums of squares:			
	row RSS		[F E]	RSS
	column CSS		[R/S]	CSS
	error ESS		[R/S]	ESS
	total TSS		[R/S]	TSS
18	For a new case, go to 2			
	*Note: to clear print mode			
	press →		[CLF]	
			[0]	

Example:

Apply this program to analyze the following set of data.

		Column			
Row	i \ j	1	2	3	4
	1	7	6	8	7
	2	2	4	4	4
	3	4	6	5	3

Keystrokes:

A	_____→	0.00	
B	_____→	1.00	
3 [ENTER] 4 C	_____→	3.00 ***	(r)
		4.00 ***	(c)
7 D	_____→	7.00 ***	
		1.00 ***	
6 D	_____→	6.00 ***	
		2.00 ***	
8 D	_____→	8.00 ***	
		3.00 ***	

7	D	→	7.00 ***	
			4.00 ***	
f	A	→	28.00 ***	(RS ₁)
2	D	→	2.00 ***	
			1.00 ***	
4	D	→	4.00 ***	
			2.00 ***	
4	D	→	4.00 ***	
			3.00 ***	
4	D	→	4.00 ***	
			4.00 ***	
f	A	→	14.00 ***	(RS ₂)
4	D	→	4.00 ***	
			1.00 ***	
7	D	→	7.00 ***	(error)
			2.00 ***	
7	E	→	7.00 ***	(correction)
			1.00 ***	
6	D	→	6.00 ***	
			2.00 ***	
5	D	→	5.00 ***	
			3.00 ***	
3	D	→	3.00 ***	
			4.00 ***	
f	A	→	18.00 ***	(RS ₃)
f	B	→	0.00 ***	
7	D	→	7.00 ***	
			1.00 ***	
2	D	→	2.00 ***	
			2.00 ***	
4	D	→	4.00 ***	
			3.00 ***	
f	C	→	13.00 ***	(CS ₁)
6	D	→	6.00 ***	
			1.00 ***	
4	D	→	4.00 ***	
			2.00 ***	

6	D	→	6.00 ***	
			3.00 ***	
f	C	→	16.00 ***	(CS ₂)
8	D	→	8.00 ***	
			1.00 ***	
4	D	→	4.00 ***	
			2.00 ***	
5	D	→	5.00 ***	
			3.00 ***	
f	C	→	17.00 ***	(CS ₃)
7	D	→	7.00 ***	
			1.00 ***	
4	D	→	4.00 ***	
			2.00 ***	
3	D	→	3.00 ***	
			3.00 ***	
f	C	→	14.00 ***	(CS ₄)
f	D	→	11.70 ***	(F ₁)
R/S		→	1.00 ***	(F ₂)
f	D	→	2.00 ***	(df ₁)
R/S		→	3.00 ***	(df ₂)
R/S		→	6.00 ***	(df ₃)
f	E	→	26.00 ***	(RSS)
R/S		→	3.33 ***	(CSS)
R/S		→	6.67 ***	(ESS)
R/S		→	36.00 ***	(TSS)

ANOVA

	SS	df	F ratio
Row	26.00	2	11.70
Column	3.33	3	1.00
Error	6.67	6	
Total	36.00		

ANALYSIS OF COVARIANCE (ONE WAY)

ANALYSIS OF COVARIANCE (ONE WAY)

START New i $x_{ij} + y_{ij} (Z+)$ $x_{ij} + y_{ij} (Z-)$ $+ Sx : Sy$

ANALYSIS OF COVARIANCE (ONE WAY)

The one way analysis of covariance program tests the effect of one variable separately from the effect of a second variable, if the second variable represents an actual measurement for each individual (rather than a category).

Suppose (x_{ij}, y_{ij}) represents the j^{th} observation from the i^{th} population ($i = 1, 2, \dots, k, j = 1, 2, \dots, n_i$). Note that samples may have equal or unequal number of observations. The analysis of covariance tests for a difference in means of residuals. The residuals are the differences of the observations and a regression quantity based on the associated second variable. The analysis of covariance procedure is based on the separations of the sums of squares and the sums of products into several portions. This program will generate the complete ANOCOV table.

Equations:

1. Sums and sums of squares

$$Sx_i = \sum_j x_{ij} \quad (i = 1, 2, \dots, k)$$

$$TSSx = \sum_i Sx_i^2 - \frac{(\sum_i Sx_i)^2}{\sum_i n_i}$$

$$ASSx = \sum_i \frac{\left(\sum_j x_{ij} \right)^2}{n_i} - \frac{(\sum_i Sx_i)^2}{\sum_i n_i}$$

$$WSSx = TSSx - ASSx$$

2. Degrees of freedom

$$df_1 = k - 1$$

$$df_2 = \sum_i n_i - k$$

3. Mean squares and F statistic

$$AMS_x = \frac{ASS_x}{df_1}$$

$$WMS_x = \frac{WSS_x}{df_2}$$

$$F_x = \frac{AMS_x}{WMS_x} \text{ with degrees of freedom } df_1, df_2$$

By changing x_{ij} to y_{ij} , similar formulas for y_{ij} can be obtained.

4. Sums of products

$$TSP = \sum_i \sum_j x_{ij} y_{ij} - \frac{(\sum_i \sum_j x_{ij})(\sum_i \sum_j y_{ij})}{\sum_i n_i}$$

$$ASP = \sum_i \frac{\left(\sum_j x_{ij}\right)\left(\sum_j y_{ij}\right)}{n_i} - \frac{(\sum_i \sum_j x_{ij})(\sum_i \sum_j y_{ij})}{\sum_i n_i}$$

$$WSP = TSP - ASP$$

5. Residual sums of squares

$$TSS\hat{y} = TSS_y - \frac{(TSP)^2}{TSS_x}$$

$$WSS\hat{y} = WSS_y - \frac{(WSP)^2}{WSS_x}$$

$$ASS\hat{y} = TSS\hat{y} - WSS\hat{y}$$

6. Residual degrees of freedom

$$df_3 = k - 1$$
$$df_4 = \sum_i n_i - k - 1$$

7. Residual mean squares and F statistic

$$AMS\hat{y} = \frac{ASS\hat{y}}{df_3}$$
$$WMS\hat{y} = \frac{WSS\hat{y}}{df_4}$$
$$F = \frac{AMS\hat{y}}{WMS\hat{y}} \text{ with degrees of freedom } df_3, df_4$$

ANOCOV Table

	degrees of freedom	SSx	SP	SSy	degrees of freedom	Residuals		
						SS $\hat{y}$	MS $\hat{y}$	F statistic
Among means	df ₁	ASSx	ASP	ASSy	df ₃	ASS $\hat{y}$	AMS $\hat{y}$	F
Within groups	df ₂	WSSx	WSP	WSSy	df ₄	WSS $\hat{y}$	WMS $\hat{y}$	
Total		TSSx	TSP	TSSy		TSS $\hat{y}$		

Remarks:

- 1. F_x can be used to test if the X means are equal (ANOVA for X).
- 2. F_y can be used to test if the Y means (not making use of the X values) are equal (ANOVA for unadjusted Y).

Reference:

Dixon and Massey, *Introduction to Statistical Analysis*, McGraw-Hill, 1969.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2 of card 1			
2	Initialize		A	0.00
3	To set print mode*		f E	1.00
4	Do 5~9 for $i=1, 2, \dots, k$			
5	Initialize for new i		B	i
6	Do 6 for $j=1, 2, \dots, n_j$			
7	Input x_{ij} and y_{ij}	x_{ij}	ENTER	
		y_{ij}	C	j
8	If you made a mistake in inputting x_{im} and y_{im} , then correct by	x_{im}	ENTER	
		y_{im}	D	$j-1$
9	Calculate the i^{th} sums Sx_i Sy_i		E R/S	Sx_i Sy_i
10	Calculate: the total sum of squares for x Among means sum of squares for x Within groups sum of squares for x		f A R/S R/S	$TSSx$ $ASSx$ $WSSx$
11	Calculate: the total sum of squares for y Among means sum of squares for y Within groups sum of squares for y		f A R/S R/S	$TSSy$ $ASSy$ $WSSy$
12	Calculate: F_x F_y degrees of freedom df_1 df_2		f B R/S R/S R/S	F_x F_y df_1 df_2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
13	Load side 1 of card 2			
14	Calculate: the total sum of products (SP)		f C	TSP
	among means SP		R/S	ASP
	within group SP		R/S	WSP
15	Calculate: TSSŷ		f D	TSSŷ
	WSSŷ		R/S	WSSŷ
	ASSŷ		R/S	ASSŷ
16	Calculate: residual mean squares		f E	AMSŷ
	WMSŷ		R/S	WMSŷ
	the F statistic		R/S	F
	the degrees of freedom df ₃		R/S	df ₃
	df ₄		R/S	df ₄
17	For a new case, go to 1			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example:

		j			
		1	2	3	4
i	x	3	2	1	2
	1 y	10	8	8	11
	x	4	3	3	5
	2 y	12	12	10	13
	x	1	2	3	1
	3 y	6	5	8	7

(k = 3, n₁ = n₂ = n₃ = 4)

Keystrokes:

Load side 1 and side 2 of card 1

Keystrokes:	Outputs:	
A →	0.00	
f E →	1.00	
B →	1.00 ***	
3 ENTER 10 C →	3.00 *** 10.00 *** 1.00 ***	
2 ENTER 8 C →	2.00 *** 8.00 *** 2.00 ***	
5 ENTER 5 C →	5.00 *** 5.00 *** 3.00 ***	(error)
5 ENTER 5 D →	5.00 *** 5.00 *** 2.00 ***	(correction)
1 ENTER 8 C →	1.00 *** 8.00 *** 3.00 ***	
2 ENTER 11 C →	2.00 *** 11.00 *** 4.00 ***	
E →	8.00 ***	(S _{x1})
R/S →	37.00 ***	(S _{y1})
B →	2.00 ***	
4 ENTER 12 C →	4.00 *** 12.00 *** 1.00 ***	
3 ENTER 12 C →	3.00 *** 12.00 *** 2.00 ***	
3 ENTER 10 C →	3.00 *** 10.00 *** 3.00 ***	

5	ENTER	13	C	→	5.00 ***	
					13.00 ***	
					4.00 ***	
E				→	15.00 ***	(Sx ₂)
R/S				→	47.00 ***	(Sy ₂)
B				→	3.00 ***	
1	ENTER	6	C	→	1.00 ***	
					6.00 ***	
					1.00 ***	
2	ENTER	5	C	→	2.00 ***	
					5.00 ***	
					2.00 ***	
3	ENTER	8	C	→	3.00 ***	
					8.00 ***	
					3.00 ***	
1	ENTER	7	C	→	1.00 ***	
					7.00 ***	
					4.00 ***	
E				→	7.00 ***	(Sx ₃)
R/S				→	26.00 ***	(Sy ₃)
f	A			→	17.00 ***	(TSSx)
R/S				→	9.50 ***	(ASSx)
R/S				→	7.50 ***	(WSSx)
f	A			→	71.67 ***	(TSSy)
R/S				→	55.17 ***	(ASSy)
R/S				→	16.50 ***	(WSSy)
f	B			→	5.70 ***	(Fx)
R/S				→	15.05 ***	(Fy)
R/S				→	2.00 ***	(df ₁)
R/S				→	9.00 ***	(df ₂)

Load side 1 of card 2

f C → 27.00 *** (TSP)

R/S → 20.75 *** (ASP)

R/S → 6.25 *** (WSP)

f D → 28.78 *** (TSS $\hat{y}$)R/S → 11.29 *** (WSS $\hat{y}$)R/S → 17.49 *** (ASS $\hat{y}$)f E → 8.75 *** (AMS $\hat{y}$)R/S → 1.41 *** (WMS $\hat{y}$)

R/S → 6.20 *** (F)

R/S → 2.00 *** (df₃)R/S → 8.00 *** (df₄)

ANOCOV Table

Residuals

	df	SSx	SP	SSy	df	SS $\hat{y}$	MS $\hat{y}$	F
Among means	2	9.50	20.75	55.17	2	17.49	8.75	6.20
Within groups	9	7.50	6.25	16.50	8	11.29	1.41	
Total		17.00	27.00	71.67		28.78		

NORMAL AND INVERSE NORMAL DISTRIBUTION

NORMAL & INVERSE NORMAL DISTRIBUTION

START

NORMAL & INVERSE NORMAL DISTRIBUTION

P?

$x \leftarrow f(x)$

$x \leftarrow Q(x)$

$Q(x) \leftarrow x$

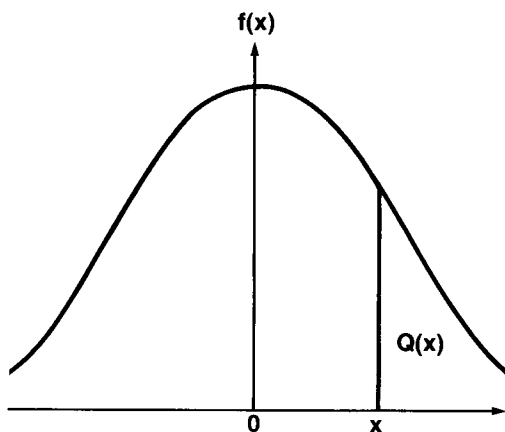
This program evaluates the standard normal density function $f(x)$ and the normal integral $Q(x)$ for given x . If Q is given, x can also be found.

The standard normal distribution has mean 0 and standard deviation 1.

Equations:

1. Standard normal density

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



2. Normal integral

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$$

Polynomial approximation is used to compute $Q(x)$ for given x .

Define $R = f(x) (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + \epsilon(x)$

where $|\epsilon(x)| < 7.5 \times 10^{-8}$

$$t = \frac{1}{1+r|x|}, \quad r = 0.2316419$$

$$b_1 = .319381530, \quad b_2 = -.356563782$$

$$b_3 = 1.781477937, \quad b_4 = -1.821255978$$

$$b_5 = 1.330274429$$

$$\text{Then } Q(x) = \begin{cases} R & \text{if } x \geq 0 \\ 1-R & \text{if } x < 0 \end{cases}$$

3. Inverse normal

For a given $Q > 0$, x can be found such that

$$Q = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt.$$

The following rational approximation is used:

$$\text{Define } y = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

where $|\epsilon(Q)| < 4.5 \times 10^{-4}$

$$t = \begin{cases} \sqrt{\ln \frac{1}{Q^2}} & \text{if } 0 < Q \leq 0.5 \\ \sqrt{\ln \frac{1}{(1-Q)^2}} & \text{if } 0.5 < Q < 1 \end{cases}$$

$c_0 = 2.515517$
 $c_1 = 0.802853$
 $c_2 = 0.010328$

$d_1 = 1.432788$
 $d_2 = 0.189269$
 $d_3 = 0.001308$

Then
$$x = \begin{cases} y & \text{if } 0 < Q \leq 0.5 \\ -y & \text{if } 0.5 < Q < 1 \end{cases}$$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2 of card 1			
2	Initialize		A	0.00
3	Load side 1 and side 2 of card 2			
4	To set print mode* Optional: Step 5		B	1.00
5	Input x to compute f(x)	x	C	f(x)
6	Input x to compute Q(x) For a new case of x, go to 5 or 6	x	D	Q(x)
7	Input Q(x) to compute x For a new case of Q(x), go to 7	Q(x)	E	x
	*Note: to clear print mode press →		O	
			STO	
			A	
			STO	
			B	

Example 1:

Find $f(x)$ and $Q(x)$ for $x = 1.18$ and $x = -2.28$.

Keystrokes:**Outputs:**

Load side 1 and side 2 of card 1

A → 0.00

Load side 1 and side 2 of card 2

B → 1.00

1.18 **C** → 1.18 ***
0.20 *** (f(1.18))

1.18 **D** → 1.18 ***
0.12 *** (Q(1.18))

2.28 **CHS D** → -2.28 ***
0.99 *** (Q(-2.28))

2.28 **CHS C** → -2.28 ***
0.03 *** (f(-2.28))

Example 2:

Given $Q = 0.12$ and $Q = 0.95$, find x .

(If you have run through Example 1, then you can proceed; otherwise you have to load programs as described in Example 1).

Keystrokes:**Outputs:**

0.12 **E** → 0.12 ***
1.18 *** (x)

0.95 **E** → 0.95 ***
-1.65 *** (x)

CHI-SQUARE DISTRIBUTION

CHI-SQUARE DISTRIBUTION

START

P?

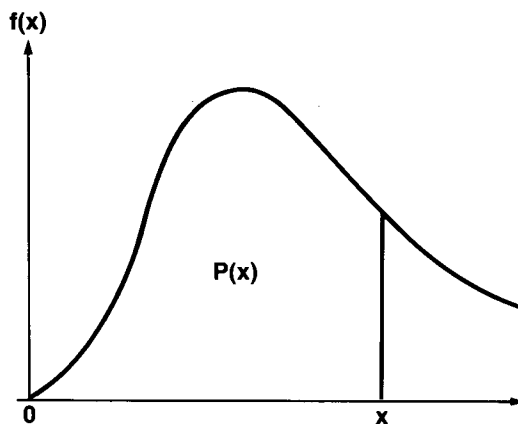
 $\nu \rightarrow \Gamma(\cdot)$ $x \rightarrow f(x)$ $x \rightarrow P(x)$

This program evaluates the chi-square density

$$f(x) = \frac{1}{2^{\frac{\nu}{2}} \Gamma\left(\frac{\nu}{2}\right)} x^{\frac{\nu}{2}-1} e^{-\frac{x}{2}}$$

where $x \geq 0$

ν is the degrees of freedom.



Series approximation is used to evaluate the cumulative distribution

$$P(x) = \int_0^x f(t) dt$$

$$= \left(\frac{x}{2}\right)^{\frac{\nu}{2}} \frac{e^{-\frac{x}{2}}}{\Gamma\left(\frac{\nu+2}{2}\right)} \left[1 + \sum_{k=1}^{\infty} \frac{x^k}{(\nu+2)(\nu+4)\dots(\nu+2k)} \right]$$

The program computes successive partial sums of the above series. When two consecutive partial sums are equal, the value is used as the sum of the series.

Notes: 1. Program requires $\nu < 141$. If $\nu > 141$, erroneous overflow will result.

2. If both x and ν are large, $f(x)$ may overflow the machine.

3. If ν is even,

$$\Gamma\left(\frac{\nu}{2}\right) = \left(\frac{\nu}{2} - 1\right)!$$

If ν is odd,

$$\Gamma\left(\frac{\nu}{2}\right) = \left(\frac{\nu}{2} - 1\right)\left(\frac{\nu}{2} - 2\right) \dots \left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right)$$

$$4. \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input degrees of freedom ν	ν	C	$\Gamma(\nu/2)$
	Optional: Step 5			
5	Input x to compute $f(x)$	x	D	$f(x)$
6	Input x to compute $P(x)$	x	E	$P(x)$
	(i) For a new case with the same ν go to 5 or 6			
	(ii) For a new case with different ν , go to 2			
	*Note: to clear print mode			
	press →		CLF	
			O	

Example 1:

If degrees of freedom $\nu = 20$, find $f(x)$, $P(x)$ for $x = 9.6$ and $x = 15$.

Keystrokes:**Outputs:**

A	→	0.00	
B	→	1.00	
20 C	→	20.00 ***	
		362880.00 ***	$(\Gamma(20/2))$
9.6 D	→	9.60 ***	
		0.02 ***	$(f(9.6))$
9.6 E	→	9.60 ***	
		0.03 ***	$(P(9.6))$
15 E	→	15.00 ***	
		0.22 ***	$(P(15))$
15 D	→	15.00 ***	
		0.06 ***	$(f(15))$

Example 2:

If $\nu = 3$, find $f(x)$ and $P(x)$ for $x = 7.82$.

Keystrokes:**Outputs:**

A	→	0.00	
B	→	1.00	
3 C	→	3.00 ***	
		0.89 ***	$(\Gamma(3/2))$
7.82 D	→	7.82 ***	
		0.02 ***	$(f(7.82))$
7.82 E	→	7.82 ***	
		0.95 ***	$(P(7.82))$

t DISTRIBUTION

t DISTRIBUTION

START

P?

v

x=f(x)

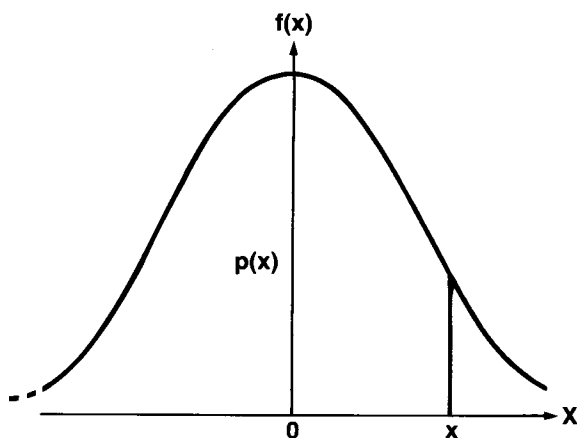
x=P(x)

This program evaluates the t density function $f(x)$ and the cumulative distribution $P(x)$ for a given x and degrees of freedom ν .

Equations:

1. Density function

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu} \Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$



2. Cumulative distribution function

$$P(x) = \int_{-\infty}^x f(y) dy$$

$$\text{Let } \theta = \tan^{-1} \left(\frac{|x|}{\sqrt{\nu}} \right)$$

(a) ν even

$$\text{Let } R = \sin \theta \left\{ 1 + \frac{1}{2} \cos^2 \theta + \frac{1 \cdot 3}{2 \cdot 4} \cos^4 \theta + \dots \right. \\ \left. + \frac{1 \cdot 3 \cdot 5 \dots (\nu-3)}{2 \cdot 4 \cdot 6 \dots (\nu-2)} \cos^{\nu-2} \theta \right\}$$

(b) ν odd

$$\text{Let } R = \begin{cases} \frac{2\theta}{\pi} & \text{if } \nu = 1 \\ \frac{2\theta}{\pi} + \frac{2}{\pi} \cos \theta \left\{ \sin \theta \left[1 + \frac{2}{3} \cos^2 \theta + \dots \right. \right. \\ \left. \left. \frac{2 \cdot 4 \dots (\nu-3)}{1 \cdot 3 \dots (\nu-2)} \cos^{\nu-3} \theta \right] \right\} & \text{if } \nu > 1 \end{cases}$$

$$\text{Then } P(x) = \begin{cases} \frac{1+R}{2} & \text{if } x > 0 \\ \frac{1-R}{2} & \text{if } x \leq 0 \end{cases}$$

Remark:

The program requires $\nu < 141$ for $f(x)$, otherwise erroneous overflow will result.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input degrees of freedom ν	ν	C	ν
	Optional: Step 5			
5	Input x to compute f(x)	x	D	f(x)
6	Input x to compute P(x)	x	E	P(x)
7	(i) For a new case with the same ν go to 5 or 6			
	(ii) For a new case with a different ν , go to 2			
	*Note: to clear print mode press →		CLF	
			0	

Example 1:

Find f(x) and P(x) for $x = 2.2$, $\nu = 11$.

Keystrokes:

Outputs:

A	→	0.00	
B	→	1.00	
11 C	→	11.00 ***	(ν)
2.2 E	→	2.20 ***	(x)
		0.97***	(P(2.2))
2.2 D	→	2.20 ***	(x)
		0.04 ***	(f(2.2))

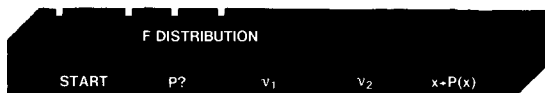
Example 2:

Find $f(x)$ and $P(x)$ for $x = -1.75$, $\nu = 30$.

Keystrokes:**Outputs:**

A	→	0.00	
B	→	1.00	
30 C	→	30.00 ***	(ν)
1.75 CHS D	→	-1.75 ***	(x)
		0.09 ***	($f(-1.75)$)
1.75 CHS E	→	-1.75 ***	(x)
		0.05 ***	($P(-1.75)$)

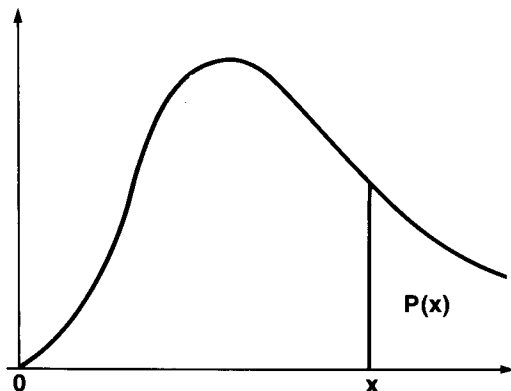
F DISTRIBUTION



This program evaluates the integral of the F distribution

$$P(x) = \int_x^\infty \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right) y^{\frac{\nu_1}{2} - 1} \left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}}}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right) \left(1 + \frac{\nu_1}{\nu_2} y\right)^{\frac{\nu_1 + \nu_2}{2}}} dy$$

for given values of x (>0), degrees of freedom ν_1 , ν_2 , provided either ν_1 or ν_2 is even.



The integral is evaluated by means of the following series:

(1) ν_1 even

$$P(x) = t^{\frac{\nu_2}{2}} \left[1 + \frac{\nu_2}{2}(1-t) + \dots + \frac{\nu_2(\nu_2+2) \dots (\nu_2+\nu_1-4)}{2 \cdot 4 \dots (\nu_1-2)} (1-t)^{\frac{\nu_1-2}{2}} \right]$$

(2) ν_2 even

$$P(x) = 1 - (1-t)^{\frac{\nu_1}{2}} \left[1 + \frac{\nu_1}{2}t + \dots + \frac{\nu_1(\nu_1+2) \dots (\nu_2+\nu_1-4)}{2 \cdot 4 \dots (\nu_2-2)} t^{\frac{\nu_2-2}{2}} \right]$$

$$\text{where } t = \frac{\nu_2}{\nu_2 + \nu_1 x}$$

Note: Usually ν_1 is identified as the degree of freedom for numerator, and ν_2 is identified as the degree of freedom for denominator.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input ν_1	ν_1	C	ν_1
5	Input ν_2	ν_2	D	ν_2
6	Input x to calculate P(x)	x	E	P(x)
7	For a new case go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Examples:

1. $\nu_1 = 7, \nu_2 = 6$

$P(4.21) = 0.05$

2. $\nu_1 = 4, \nu_2 = 20$

$P(2.25) = 0.10$

Keystrokes:

Outputs:

A	→	0.00	
B	→	1.00	
7 C	→	7.00 ***	(ν_1)
6 D	→	6.00 ***	(ν_2)
4.21 E	→	4.21 ***	(x)
		0.05 ***	(P(x))
4 C	→	4.00 ***	(ν_1)
20 D	→	20.00 ***	(ν_2)
2.25 E	→	2.25 ***	(x)
		0.10 ***	(P(x))

MULTIPLE LINEAR REGRESSION

MULTIPLE LINEAR REGRESSION

START

P?

$x_1, \dots, (z_1, \dots)$

$x_2, \dots, (z_2, \dots)$

$\dots$

For a set of data points $\{(x_i, y_i, z_i), i = 1, 2, \dots, n\}$ this program fits a linear equation of the form

$$z = a + bx + cy$$

by the least squares method.

Regression coefficients a, b, c can be found by solving the normal equations:

$$\left. \begin{aligned} \sum z_i &= an + b \sum x_i + c \sum y_i \\ \sum x_i z_i &= a \sum x_i + b \sum x_i^2 + c \sum x_i y_i \\ \sum y_i z_i &= a \sum y_i + b \sum x_i y_i + c \sum y_i^2 \end{aligned} \right\} \quad i = 1, 2, \dots, n$$

$$c = \frac{A - B}{[n \sum x_i^2 - (\sum x_i)^2][n \sum y_i^2 - (\sum y_i)^2] - [n \sum x_i y_i - (\sum x_i)(\sum y_i)]^2}$$

$$\text{where } A = [n \sum x_i^2 - (\sum x_i)^2] [n \sum y_i z_i - (\sum y_i)(\sum z_i)]$$

$$B = [n \sum x_i y_i - (\sum x_i)(\sum y_i)] [n \sum x_i z_i - (\sum x_i)(\sum z_i)]$$

$$b = \frac{[n \sum x_i z_i - (\sum x_i)(\sum z_i)] - c [n \sum x_i y_i - (\sum x_i)(\sum y_i)]}{n \sum x_i^2 - (\sum x_i)^2}$$

$$a = \frac{1}{n} (\sum z_i - c \sum y_i - b \sum x_i)$$

$$R^2 = \frac{a \sum z_i + b \sum x_i z_i + c \sum y_i z_i - \frac{1}{n} (\sum z_i)^2}{(\sum z_i^2) - \frac{(\sum z_i)^2}{n}}$$

Reference: Introduction to the Theory of Statistics, Mood and Graybill, McGraw-Hill, 1963

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Do 5~6 for $i = 1, 2, \dots, n$			
5	Input x_i	x_i	ENTER +	
	y_i	y_i	ENTER +	
	z_i	z_i	C	i
6	If you made a mistake in inputting x_k , y_k , and z_k , then correct by $\rightarrow$	x_k	ENTER +	
		y_k	ENTER +	
		z_k	D	$i-1$
7	Calculate coefficients: a		E	a
	b		R/S	b
	c		R/S	c
8	Calculate the square of multiple correlation coefficient R^2		f A	R^2
9	Calculate estimated z from regression, input: x	x	ENTER +	
	y	y	f B	$\hat{z}$
10	Repeat step 9 for different (x, y)'s			
11	Recall sums: Σx_i		f C	Σx_i
	Σy_i		R/S	Σy_i
	Σz_i		R/S	Σz_i
12	Recall sums of squares: Σx_i^2		f D	Σx_i^2
	Σy_i^2		R/S	Σy_i^2
	Σz_i^2		R/S	Σz_i^2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
13	Recall sums of cross products: $\Sigma x_i y_i$		1 E	$\Sigma x_i y_i$
	$\Sigma x_i z_i$		R/S	$\Sigma x_i z_i$
	$\Sigma y_i z_i$		R/S	$\Sigma y_i z_i$
14	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example:

A set of data points are given as the following

$\begin{matrix} & i \\ & \diagdown \end{matrix}$	1	2	3	4
x_i	1.5	0.45	1.8	2.8
y_i	0.7	2.3	1.6	4.5
z_i	2.1	4.0	4.1	9.4

Find the regression line, coefficients a, b, c, R^2 , $\hat{z}$, sums, sums of squares, and sums of products.

Keystrokes:

A →

B →

1.5 **ENTER** 0.7 **ENTER** 2.1 **C** →

9 **ENTER** 9 **ENTER** 9 **C** →

0.00

1.00

1.50 ***
0.70 ***
2.10 ***
1.00 ***

9.00 ***
9.00 ***
9.00 ***
2.00 ***

(error)

9 **ENTER** 9 **ENTER** 9 **D** $\longrightarrow$ 9.00 ***
 9.00 ***
 9.00 ***
 1.00 ***
 (correction)

0.45 **ENTER** 2.3 **ENTER** 4 **C** $\longrightarrow$ 0.45 ***
 2.30 ***
 4.00 ***
 2.00 ***

1.8 **ENTER** 1.6 **ENTER** 4.1 **C** $\longrightarrow$ 1.80 ***
 1.60 ***
 4.10 ***
 3.00 ***

2.8 **ENTER** 4.5 **ENTER** 9.4 **C** $\longrightarrow$ 2.80 ***
 4.50 ***
 9.40 ***
 4.00 ***

E $\longrightarrow$ -0.10 *** (a)

R/S $\longrightarrow$ 0.79 *** (b)

R/S $\longrightarrow$ 1.63 *** (c)

f **A** $\longrightarrow$ 1.00 *** (R^2)

DSP **9** **PRINT X** $\longrightarrow$ 0.998411259 ***

DSP **2**

2 **ENTER** 3 **f** **B** $\longrightarrow$ 2.00 ***

3.00 ***

6.37 *** ($\hat{z}$)

f **C** $\longrightarrow$ 6.55 *** (Σx_i)

R/S $\longrightarrow$ 9.10 *** (Σy_i)

R/S $\longrightarrow$ 19.60 *** (Σz_i)

f **D** $\longrightarrow$ 13.53 *** (Σx_i^2)

R/S $\longrightarrow$ 28.59 *** (Σy_i^2)

R/S $\longrightarrow$ 125.58 *** (Σz_i^2)

f **E** $\longrightarrow$ 17.57 *** ($\Sigma x_i y_i$)

R/S $\longrightarrow$ 38.65 *** ($\Sigma x_i z_i$)

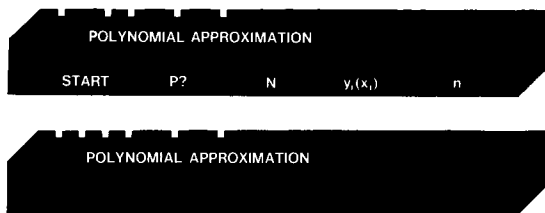
R/S $\longrightarrow$ 59.53 *** ($\Sigma y_i z_i$)

Regression line is

$$z = -0.10 + 0.79x + 1.63y$$

For $x = 2$ and $y = 3$, $\hat{z} = 6.37$

POLYNOMIAL APPROXIMATION



Suppose $x_0, x_1, \dots, x_N$ are equally spaced points ($x_0 < x_N$) at which the corresponding values $f(x_0), f(x_1), \dots, f(x_N)$ of a function $f(x)$ are known.

This program approximates in the least squares sense the function $f(x)$ by a polynomial of degree m , where $2 \leq m \leq 4$. The special Chebyshev polynomials for discrete intervals are used.

Equations:

Let $f_n(x)$ be the orthogonal polynomials ($x = 0, 1, 2, \dots, N$) such that

$$f_0(x) = 1$$

$$f_1(x) = 1 - \frac{2x}{N} \text{ and}$$

$$(n+1)(N-n)f_{n+1}(x) = (2n+1)(N-2x)f_n(x) - n(N+n+1)f_{n-1}(x)$$

where

$$n = 1, 2, \dots, m-1.$$

Then let

$$(f_n, f_n) = \frac{(N+n+1)!(N-n)!}{(2n+1)(N!)^2}$$

$$(f, f_n) = \sum_{j=0}^n f_n(j) f(x_j)$$

and

$$a_n = \frac{(f, f_n)}{(f_n, f_n)}.$$

This program computes all values of (f, f_n) for $n = 0, 1, 2, 3, 4$. If the degree $m = 4$, all terms are used; if $m = 3$, (f, f_4) is replaced by zero in later calculations; and if $m = 2$, (f, f_4) and (f, f_3) are both replaced by zero.

Let $g_n(u)$ be the symmetrical form of the orthogonal polynomial in the domain $-1 < u < 1$ such that

$$g_0(u) = 1 \quad g_1(u) = u$$

and

$$g_{n+1}(u) = \frac{(2 + 1) N}{(n + 1) (N - n)} u g_n(u) - \frac{n(N + n + 1)}{(n + 1) (N - n)} g_{n-1}(u)$$

where

$$n = 1, 2, \dots, m - 1.$$

The program computes the coefficients of the polynomial

$$\sum_{n=0}^N a_n g_n(u) = b_0 + b_1 u + b_2 u^2 + b_3 u^3 + b_4 u^4. \quad (1)$$

Then $g_n(u)$ is shifted to a proper interval between x_0 and x_N by letting

$$u = \beta + \alpha x$$

where

$$\alpha = \frac{-2}{x_N - x_0}$$

$$\beta = \frac{x_N + x_0}{x_N - x_0}$$

The transformation is done in two steps. First, let $z = u - \beta$, thus (1) becomes

$$c_0 + c_1 z + c_2 z^2 + c_3 z^3 + c_4 z^4 \quad (2)$$

where

$$\begin{aligned} c_0 &= b_0 + b_1 \beta + b_2 \beta^2 + b_3 \beta^3 + b_4 \beta^4 \\ c_1 &= b_1 + 2b_2 \beta + 3b_3 \beta^2 + 4b_4 \beta^3 \\ c_2 &= b_2 + 3b_3 \beta + 6b_4 \beta^2 \\ c_3 &= b_3 + 4 b_4 \beta \\ c_4 &= b_4. \end{aligned}$$

Then set $z = \alpha x$ and (2) becomes

$$d_0 + d_1 x + d_2 x^2 + d_3 x^3 + d_4 x^4 \tag{3}$$

where

$$d_i = \alpha^i c_i \ (i = 0, 1, 2, 3, 4).$$

(3) is the polynomial approximation for the function $f(x)$.

Note: $N \geq 4$ has to be satisfied in order to make the program work.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2 of card 1			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input N**	N	C	N
5	Do 6 for i = 0,1,2, . . . , N			
6	Input $y_i(x_i)$	$y_i(x_i)$	D	i
7	Input n for nth order fit	n	E	0.00
8	Load side 1 and side 2 of card 2			
9	To continue execution of			
	program		F A	1.00

14-05

4.48 **D** → 4.48***
3.00 ***

5.75 **D** → 5.75 ***
4.00 ***

7.39 **D** → 7.39 ***
5.00 ***

9.49 **D** → 9.49 ***
6.00 ***

12.18 **D** → 12.18 ***
7.00 ***

15.64 **D** → 15.64 ***
8.00 ***

20.09 **D** → 20.09 ***
9.00 ***

3 **E** → 3.00 *** (n)

Load side 1 and side 2 of Card 2

f **A** → 1.00
3 **ENTER** 1 **f** **B** → 3.00 *** (x_N)
1.00 *** (x_0)

f **C** → -1.79 *** (d_0)

R/S → 7.03 *** (d_1)

R/S → -3.85 *** (d_2)

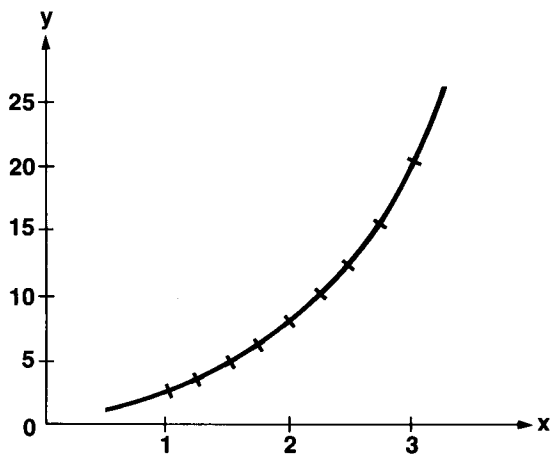
R/S → 1.31 *** (d_3)

R/S → 0.00 *** (d_4)

2 **f** **D** → 2.00 ***
7.35 *** ($\hat{y}$)

3	f	D	→	3.00 ***	
				20.06 ***	($\hat{y}$)
1	f	D	→	1.00 ***	
				2.69 ***	($\hat{y}$)

The polynomial is $-1.79 + 7.03 x - 3.85 x^2 + 1.31 x^3$.



t STATISTICS

t STATISTICS

START

P?

 $x_i + y_i (\Sigma +)$ $x_k + y_k (\Sigma -)$ $+ \bar{D}, S_D, \dots$

I. Paired t Statistic

Given a set of paired observations from two normal populations with means μ_1, μ_2 (unknown)

x_i	x_1	x_2	...	x_n
y_i	y_1	y_2	...	y_n

let

$$D_i = x_i - y_i$$

$$\bar{D} = \frac{1}{n} \sum_{i=1}^n D_i$$

$$s_D = \sqrt{\frac{\sum D_i^2 - \frac{1}{n} (\sum D_i)^2}{n - 1}}$$

$$s_{\bar{D}} = \frac{s_D}{\sqrt{n}}$$

The test statistic

$$t = \frac{\bar{D}}{s_{\bar{D}}}$$

which has $n - 1$ degrees of freedom (df) can be used to test the null hypothesis

$$H_0: \mu_1 = \mu_2$$

Reference:

Statistics in Research, B. Ostle, Iowa State University Press, 1963.

II. t Statistic For Two Means

Suppose $\{x_1, x_2, \dots, x_{n1}\}$ and $\{y_1, y_2, \dots, y_{n2}\}$ are independent random samples from two normal populations having means μ_1, μ_2 (unknown) and the same unknown variance σ^2 .

We want to test the null hypothesis

$$H_0: \mu_1 - \mu_2 = d$$

Define

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - d}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{\sum x_i^2 - n_1 \bar{x}^2 + \sum y_i^2 - n_2 \bar{y}^2}{n_1 + n_2 - 2}}}$$

We can use this t statistic which has the t distribution with $n_1 + n_2 - 2$ degrees of freedom (df) to test the null hypothesis H_0 .

Note: $n_2, \sum y_i, \sum y_i^2, n_1, \sum x_i, \sum x_i^2$ are in registers R_1 through R_6 .

Reference:

Statistical Theory and Methodology in Science and Engineering, K. A. Brownlee, John Wiley & Sons, 1965.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	For paired t statistic, go to 6			
5	For t statistic for two means,			
	go to 11			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Paired t statistic:			
6	Do 7~8 for $i = 1, 2, \dots, n$			
7	Input: x_i	x_i	ENTER	
	y_i	y_i	C	i
8	If you made a mistake in inputting x_k, y_k then correct by	x_k	ENTER	
		y_k	D	$i-1$
9	Calculate: $\bar{D}$		E	$\bar{D}$
	s_D		R/S	s_D
	test statistic:		R/S	t
	degrees of freedom		R/S	df
10	For a new case, go to 2			
	t statistic for two means:			
11	Do 12 ~ 13 for $i = 1, 2, \dots, n_1$			
12	Input x_i	x_i	F A	i
13	If you made a mistake in inputting x_k , then correct by $\rightarrow$	x_k	F B	$i-1$
14	Null hypothesis test	d	F C	d
15	Do 16~17 for $j = 1, 2, \dots, n_2$	y_j	F A	j
16	If you made a mistake in inputting y_k , then correct by $\rightarrow$	y_k	F B	$j-1$
17	Calculate: t		F D	t
	df		R/S	df
18	For a different value of d	d	F C	d
	Calculate: t		F D	t
	df		R/S	df
19	For a new case, go to 2			
	*Note: to clear print mode			
	press $\rightarrow$		CLF	
			0	

Example 1:

x_1	14	17.5	17	17.5	15.4
y_1	17	20.7	21.6	20.9	17.2

$$\bar{D} = -3.20$$

$$s_D = 1.00$$

$$t = -7.16$$

$$df = 4.00$$

Keystrokes:**A****B**14 **ENTER** 17 **C**17 **ENTER** 15 **C**17 **ENTER** 15 **D**17.5 **ENTER** 20.7 **C**17 **ENTER** 21.6 **C**17.5 **ENTER** 20.9 **C**15.4 **ENTER** 17.2 **C****Outputs:**

0.00

1.00

14.00 ***

17.00 ***

1.00 ***

17.00 ***

15.00 ***

2.00 ***

(error)

17.00 ***

15.00 ***

1.00 ***

(correction)

17.50 ***

20.70 ***

2.00 ***

17.00 ***

21.60 ***

3.00 ***

17.50 ***

20.90 ***

4.00 ***

15.40 ***

17.20 ***

5.00 ***

E	→	-3.20 ***	($\bar{D}$)
R/S	→	1.00 ***	(s_D)
R/S	→	-7.16 ***	(t)
R/S	→	4.00 ***	(df)

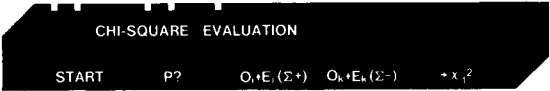
Example 2:

x: 79, 84, 108, 114, 120, 103, 122, 120
y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54
 $n_1 = 8$
 $n_2 = 10$
If $d = 0$ (i.e., $H_0: \mu_1 = \mu_2$)
then $t = 1.73$, $df = 16.00$

Keystrokes:		Outputs:	
A	→	0.00	
B	→	1.00	
79 f A	→	79.00 ***	
		1.00 ***	
84 f A	→	84.00 ***	
		2.00 ***	
99 f A	→	99.00 ***	
		3.00 ***	(error)
99 f B	→	99.00 ***	
		2.00 ***	(correction)
108 f A	→	108.00 ***	
		3.00 ***	
114 f A	→	114.00 ***	
		4.00 ***	
120 f A	→	120.00 ***	
		5.00 ***	
103 f A	→	103.00 ***	
		6.00 ***	
122 f A	→	122.00 ***	
		7.00 ***	

120	f	A	→	120.00 *** 8.00 ***	
0	f	C	→	0.00 ***	(d)
91	f	A	→	91.00 *** 1.00 ***	
103	f	A	→	103.00 *** 2.00 ***	
90	f	A	→	90.00 *** 3.00 ***	
113	f	A	→	113.00 *** 4.00 ***	
108	f	A	→	108.00 *** 5.00 ***	
87	f	A	→	87.00 *** 6.00 ***	
100	f	A	→	100.00 *** 7.00 ***	
80	f	A	→	80.00 *** 8.00 ***	
99	f	A	→	99.00 *** 9.00 ***	
54	f	A	→	54.00 *** 10.00 ***	
f	D		→	1.73 ***	(t)
R/S			→	16.00 ***	(df)

CHI-SQUARE EVALUATION



This program calculates the value of the χ^2 statistic for the goodness of fit test by the equation

$$\chi^2_1 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequency
 E_i = expected frequency

If the expected values are equal

$$\left(E = E_i = \frac{\sum O_i}{n} \text{ for all } i \right)$$

then

$$\chi^2_2 = \frac{n \sum O_i^2}{\sum O_i} - \sum O_i$$

Note: In order to apply the goodness of fit test to a set of given data, combining some classes may be necessary to make sure that each expected frequency is not too small (say, not less than 5).

Reference: Mathematical Statistics, J. E. Freund, Prentice Hall, 1962

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	For equal expected values, go to 10			
5	Do 6~7 for i = 1,2, , n			
6	Input: O _i	O _i	ENTER	
	E _i	E _i	C	i

16-03

50 **ENTER** 46.75 **C** → 50.00 ***
 46.75 ***
 2.00 ***

47 **ENTER** 51.85 **C** → 47.00 ***
 51.85 ***
 3.00 ***

56 **ENTER** 54.4 **C** → 56.00 ***
 54.40 ***
 4.00 ***

5 **ENTER** 8.25 **C** → 5.00 ***
 8.25 ***
 5.00 ***

100 **ENTER** 100 **C** → 100.00 *** (error)
 100.00 ***
 6.00 ***

100 **ENTER** 100 **D** → 100.00 *** (correction)
 100.00 ***
 5.00 ***

14 **ENTER** 9.15 **C** → 14.00 ***
 9.15 ***
 6.00 ***

E → 4.84 *** (χ_1^2)

Example 2:

The following table shows the observed frequencies in tossing a die 120 times. χ^2 can be used to test if the die is fair.

Note: Assume that the expected frequencies are equal.

number	1	2	3	4	5	6
frequency O_i	25	17	15	23	24	16

$$\chi_2^2 = 5.00$$

$$E = 20.00$$

Keystrokes:

Outputs:

A	→	0.00	
B	→	1.00	
25 f A	→	25.00 ***	
		1.00 ***	
17 f A	→	17.00 ***	
		2.00 ***	
19 f A	→	19.00 ***	(error)
		3.00 ***	
19 f B	→	19.00 ***	(correction)
		2.00 ***	
15 f A	→	15.00 ***	
		3.00 ***	
23 f A	→	23.00 ***	
		4.00 ***	
24 f A	→	24.00 ***	
		5.00 ***	
16 f A	→	16.00 ***	
		6.00 ***	
f C	→	5.00 ***	(χ^2_2)
f D	→	20.00 ***	(E)

CONTINGENCY TABLE

CONTINGENCY TABLE				
START	2 x k	$x_{11}, \dots, (C_1)$	$x_{12}, \dots, (C_2)$	$\dots, x_{2k}, C_k$
				$\dots, R_1; R_2; T$

Contingency tables can be used to test the null hypothesis that two variables are independent.

I. 2 x k CONTINGENCY TABLE

i \ j	1	2	...	k	Totals
1	x_{11}	x_{12}	...	x_{1k}	R_1
2	x_{21}	x_{22}	...	x_{2k}	R_2
Totals	C_1	C_2	...	C_k	T

Test statistic

$$\chi^2 = \frac{T}{R_1} \sum_{i=1}^k \frac{x_{1i}^2}{C_i} + \frac{T}{R_2} \sum_{i=1}^k \frac{x_{2i}^2}{C_i} - T$$

Degrees of freedom df = k - 1

Pearson's coefficient of contingency C_c measures the degree of association between the two variables

$$C_c = \sqrt{\frac{\chi^2}{T + \chi^2}}$$

II. 3 x k CONTINGENCY TABLE

i \ j	1	2	...	k	Totals
1	x_{11}	x_{12}	...	x_{1k}	R_1
2	x_{21}	x_{22}	...	x_{2k}	R_2
3	x_{31}	x_{32}	...	x_{3k}	R_3
Totals	C_1	C_2	...	C_k	T

This program computes the χ^2 statistic (with $2(k - 1)$ degrees of freedom) for testing the independence of the two variables. Also Pearson's coefficient of contingency C_c , which measures the degree of association between the two variables, is calculated.

Equations:

$$\text{Row sum } R_i = \sum_{j=1}^k x_{ij} \quad i = 1, 2, 3$$

$$\text{Column sum } C_j = \sum_{i=1}^3 x_{ij} \quad j = 1, 2, \dots, k$$

$$\text{Total } T = \sum_{i=1}^3 \sum_{j=1}^k x_{ij}$$

Chi-square statistic

$$\begin{aligned} \chi^2 &= \sum_{i=1}^3 \sum_{j=1}^k \frac{(x_{ij} - E_{ij})^2}{E_{ij}} \\ &= T \left(\sum_{i=1}^3 \sum_{j=1}^k \frac{x_{ij}^2}{R_i C_j} \right) - T \end{aligned}$$

Where the expected frequency

$$E_{ij} = \frac{R_i C_j}{T}$$

Contingency coefficient

$$C_c = \sqrt{\frac{\chi^2}{T + \chi^2}}$$

Reference:

B. Ostle, *Statistics in Research*, Iowa State University Press, 1972.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		f A	1.00
4	For 2 x k go to 5			
	For 3 x k go to 11			
5	Do 6~7 for $j = 1, 2, \dots, k$			
	for $2 \times k$			
6	Input: x_{1j}	x_{1j}	ENTER*	
	x_{2j}	x_{2j}	B	j
	(Optional) Calculate column sum C_j		R/S	C_j
7	If you made a mistake in inputting x_{1h} and x_{2h} , then correct by $\rightarrow$	x_{1h}	ENTER*	
		x_{2h}	C	j-1
	(Optional) Calculate column sum C_h (correction)		R/S	$-C_h$
8	Calculate: χ^2		D	χ^2
	C_c		R/S	C_c
9	Calculate: row sums R_1		E	R_1
	R_2		R/S	R_2
	total T		R/S	T
10	For a new case go to 2			
11	Do 12~13 for $j = 1, 2, \dots, k$			
	for 3 X k			
12	Input x_{1j}	x_{1j}	ENTER*	
	x_{2j}	x_{2j}	ENTER*	
	x_{3j}	x_{3j}	f B	j
	(Optional) Calculate column sum C_j		R/S	C_j

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
13	If you made a mistake in			
	inputting X_{1h} , X_{2h} and X_{3h} ,			
	then correct by $\rightarrow$	X_{1h}	ENTER	
		X_{2h}	ENTER	
		X_{3h}	f C	j-1
	(Optional) Calculate column			
	sum C_h (correction)		R/S	$-C_h$
14	Calculate: χ^2		f D	χ^2
	C_c		R/S	C_c
15	Calculate: row sums R_1		f E	R_1
	R_2		R/S	R_2
	R_3		R/S	R_3
	total T		R/S	T
16	For a new case go to 2			
	*Note: to clear print mode			
	press $\rightarrow$		CLF	
			0	

Example 1:

A random sample of 250 men and 250 women were polled as to their desires concerning the ownership of television sets. The data in the following table resulted. Apply the program to analyze the result of the poll, i.e., can the hypothesis that the desire to own a television set is independent of sex be rejected?

Results of Sample Poll on Television Ownership

Classification	Men	Women	Total
Want television	80	120	200
Don't want television	170	130	300
Total	250	250	500

Keystrokes:

A →
f A →
80 **ENTER** 170 **B** →

Outputs:

0.00
1.00
80.00 ***
170.00 ***
1.00 ***

120 **ENTER** 130 **B** →

120.00 ***
130.00 ***
2.00 ***

D → 13.33 *** (χ^2)

$$\chi^2 = 13.33 > \chi^2_{.99(1)} = 6.63$$

Thus, the hypothesis that desire to own a television set is independent of sex is rejected.

Example 2:

Find test statistic χ^2 and coefficient of contingency C_c for the following set of data.

	1	2	3
A	2	5	4
B	3	8	7

Keystrokes:

A →
f A →
2 **ENTER** 3 **B** →

Outputs:

0.00
1.00
2.00 ***
3.00 ***
1.00 ***

R/S → 5.00 *** (C_1)

5 **ENTER** 8 **B** →

5.00 ***
8.00 ***
2.00 ***

Outputs:

R/S → 125.00 *** (C₁)

R/S → 214.00 *** (C₂)

R/S $\longrightarrow$ 133.00 *** (C₃)

R/S → 178.00 *** (C₄)

R/S $\longrightarrow$ -133.00 *** (-C₃)

58 ENTER 54 ENTER 68 f B → 58.00 ***
 54.00 ***
 68.00 ***
 4.00 ***

R/S → 180.00 *** (C₄)

f D → 3.36 *** (χ^2)

R/S → 0.07 *** (C_c)

f E → 210.00 *** (R₁)

R/S → 194.00 *** (R₂)

R/S → 293.00 *** (R₃)

R/S → 697.00 *** (T)

SPEARMAN'S RANK CORRELATION COEFFICIENT

SPEARMAN'S RANK CORRELATION COEFFICIENT

START

P?

 $R_i + S_i$ (0-) $R_k + S_k$ (0-) $\rightarrow r_s, z$

Spearman's rank correlation coefficient is a measure of rank correlation under the following circumstance: n individuals are ranked from 1 to n according to some specified characteristic by 2 observers, and we wish to know if the 2 rankings are substantially in agreement with one another.

Spearman's rank correlation coefficient is defined by

$$r_s = 1 - \frac{6 \sum_{i=1}^n D_i^2}{n(n^2 - 1)}$$

where n = number of paired observations (x_i, y_i)
 D_i = rank (x_i) - rank (y_i) = $R_i - S_i$

If the X and Y random variables from which these n pairs of observations are derived are independent, then r_s has zero mean and a variance equals to

$$\frac{1}{n - 1}$$

A test for the null hypothesis

H_0 : X, Y are independent

is using

$$z = r_s \sqrt{n - 1}$$

which is approximately a standardized normal variable (for large n , say $n \geq 10$).

If the null hypothesis of independence is not rejected, we can infer that the population correlation coefficient $\rho(x, y) = 0$, but dependence between the variables does not necessarily imply that $\rho(x, y) \neq 0$.

Note: $-1 \leq r_s \leq 1$

$r_s = 1$ indicates complete agreement in order of the ranks and $r_s =$

-1 indicates complete agreement in the opposite order of the ranks.

Reference: Nonparametric Statistical Inference, J. D. Gibbons, McGraw Hill, 1971

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Do 5~6 for $i=1, 2, \dots, n$			
5	Input R_i	R_i	ENTER	
	S_i	S_i	C	i
6	If you made a mistake in inputting R_k and S_k , then correct by $\rightarrow$	R_k	ENTER	
		S_k	D	$i-1$
7	Calculate: r_s		E	r_s
	z		R/S	z
8	For a new case, go to 2			
	*Note: to clear print mode			
	press $\rightarrow$		CLF	
			0	

Example:

The following data set is the result of two tests in a class; find r_s and z .

Student	x_i Math Grade	y_i Stat Grade	R_i Rank of x_i	S_i Rank of y_i
1	82	81	6	7
2	67	75	14	11
3	91	85	3	4
4	98	90	1	2
5	74	80	11	8
6	52	60	15	15
7	86	94	4	1
8	95	78	2	9
9	79	83	9	6
10	78	76	10	10
11	84	84	5	5
12	80	69	8	13
13	69	72	13	12
14	81	88	7	3
15	73	61	12	14

Keystrokes:

A
B
6 ENTER 7 C

14 ENTER 11 C

3 ENTER 4 C

1 ENTER 2 C

Outputs:

0.00
1.00
6.00 ***
7.00 ***
1.00 ***

14.00 ***
11.00 ***
2.00 ***

3.00 ***
4.00 ***
3.00 ***

1.00 ***
2.00 ***
4.00 ***

11	ENTER 8 C	→	11.00 *** 8.00 *** 5.00 ***	
5	ENTER 5 C	→	5.00 *** 5.00 *** 6.00 ***	(errors)
5	ENTER 5 D	→	5.00 *** 5.00 *** 5.00 ***	(correction)
15	ENTER 15 C	→	15.00 *** 15.00 *** 6.00 ***	
4	ENTER 1 C	→	4.00 *** 1.00 *** 7.00 ***	
2	ENTER 9 C	→	2.00 *** 9.00 *** 8.00 ***	
9	ENTER 6 C	→	9.00 *** 6.00 *** 9.00 ***	
10	ENTER 10 C	→	10.00 *** 10.00 *** 10.00 ***	
5	ENTER 5 C	→	5.00 *** 5.00 *** 11.00 ***	
8	ENTER 13 C	→	8.00 *** 13.00 *** 12.00 ***	

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13 **ENTER** 12 **C** → 13.00 ***
12.00 ***
13.00 ***

7 **ENTER** 3 **C** → 7.00 ***
3.00 ***
14.00 ***

12 **ENTER** 14 **C** → 12.00 ***
14.00 ***
15.00 ***

E → 0.76 *** (r_s)
R/S → 2.85 *** (z)

$\bar{x}$ AND R CONTROL CHARTS

In quality control, a chart is used to decide periodically whether a process is in statistical control. The use of such a chart facilitates the detection and elimination of assignable causes of process variation, thereby reducing rejects and rework, improving product quality, and lowering inspection cost. The $\bar{x}$ chart and R chart are two of the most frequently encountered, they deal with measurement data.

Suppose x_{ij} represents the j^{th} data point from the i^{th} sample, $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. This program computes (1) the sample mean $\bar{x}$ and the sample range R_i , (2) the over-all mean $\bar{\bar{x}}$ and the average range $\bar{R}$, (3) the upper control limit $U_{\bar{x}}$ and the lower control limit $L_{\bar{x}}$ for $\bar{x}$, and (4) the upper control limit U_R and the lower control limit L_R for R.

Equations:

1.

$$\bar{x}_i = \sum_{j=1}^n x_{ij}/n$$

$$R_i = x_{\max} - x_{\min}$$

where $x_{\max}$ is the maximum of the x values and $x_{\min}$ is the minimum of the x values in the i^{th} sample.

2.

$$\bar{\bar{x}} = \sum_{i=1}^m \bar{x}_i/m$$

$$\bar{R} = \sum_{i=1}^m R_i/m$$

3.

$$L_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$

$$U_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$$

where A_2 is the factor for the $\bar{x}$ chart, which can be found in the following table.

4.

$$L_R = D_3\bar{R}$$

$$U_R = D_4\bar{R}$$

D_3 and D_4 are factors for the R chart, which can be found in the table.

Factors for determining from R the 3-sigma control limits for $\bar{x}$ and R charts.

Number of observations in subgroup n	Factor for $\bar{x}$ chart A_2	Factors for R chart	
		Lower limit D_3	Upper limit D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.39	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59

All factors are based on the normal distribution.

The table is reproduced from *Statistical Quality Control*, by Grant and Leavenworth, 1972, with permission of McGraw-Hill Book Company.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Do 5~ 9for $i=1, 2, \dots, m$			
5	Do 6~7 for $j=1, 2, \dots, n$			
6	Input x_{ij}	x_{ij}	C	j
7	If you made a mistake in inputting x_{ik} , then correct by ** $\rightarrow$	x_{ik}	D	$j-1$
8	Calculate: $x_{\max}$		E	$x_{\max}$
	$x_{\min}$		E	$x_{\min}$
9	Calculate: the mean $\bar{x}_i$		f A	$\bar{x}_i$
	the range R_i		f A	R_i
10	Calculate: $\bar{\bar{x}}$		f B	$\bar{\bar{x}}$
	$\bar{R}$		f B	$\bar{R}$
11	Calculate the $\bar{x}$ limits:			
	the lower	A_2	f C	$L_{\bar{x}}$
	the upper		f C	$U_{\bar{x}}$
12	Calculate L_R	D_3	f D	L_R
13	Calculate U_R	D_4	f E	U_R
14	For a new case, go to 2			
	*Note: to clear print mode press $\rightarrow$		0	
			STO	
			E	
	**Note: If there are two or more x_{ik} 's entered incorrectly (one follows the other), then do not try to correct them, go to step 2.			

Example:

For the following set of data, find the lower and upper control limits for $\bar{x}$ and R.

		i \ j	1	2	3	4	5
Sample	1		10.04	10.00	10.02	10.01	10.02
	2		10.00	10.01	10.03	10.02	10.01
	3		10.02	10.02	10.02	10.04	10.01

(Note: $n = 5, A_2 = 0.58, D_3 = 0, D_4 = 2.11$)

Keystrokes:**Outputs:**

A	→	0.00	
B	→	1.00	
10.04 C	→	10.04 ***	
		1.00 ***	
10 C	→	10.00 ***	
		2.00 ***	
10.02 C	→	10.02 ***	
		3.00 ***	
11.11 C	→	11.11 ***	(error)
		4.00 ***	
11.11 D	→	11.11 ***	(correction)
		3.00 ***	
10.01 C	→	10.01 ***	
		4.00 ***	
10.02 C	→	10.02 ***	
		5.00 ***	
E	→	10.04 ***	($x_1 \text{ max}$)
E	→	10.00 ***	($x_1 \text{ min}$)
f A	→	10.02 ***	($\bar{x}_1$)
f A	→	0.04 ***	(R_1)
10 C	→	10.00 ***	
		1.00 ***	
10.01 C	→	10.01 ***	
		2.00 ***	
10.03 C	→	10.03 ***	
		3.00 ***	
10.02 C	→	10.02 ***	
		4.00 ***	

10.01	C	→	10.01 ***	
			5.00 ***	
E		→	10.03 ***	($x_2 \max$)
E		→	10.00 ***	($x_2 \min$)
A		→	10.01 ***	($\bar{x}_2$)
A		→	0.03 ***	(R_2)
10.02	C	→	10.02 ***	
			1.00 ***	
10.02	C	→	10.02 ***	
			2.00 ***	
10.04	C	→	10.04 ***	(error)
			3.00 ***	
10.04	D	→	10.04 ***	(correction)
			2.00 ***	
10.02	C	→	10.02 ***	
			3.00 ***	
10.04	C	→	10.04 ***	
			4.00 ***	
10.01	C	→	10.01 ***	
			5.00 ***	
E		→	10.04 ***	($x_3 \max$)
E		→	10.01 ***	($x_3 \min$)
f A		→	10.02 ***	($\bar{x}_3$)
f A		→	0.03 ***	(R_3)
f B		→	10.02 ***	($\bar{x}$)
f B		→	0.03 ***	($\bar{R}$)
0.58 f C		→	10.00 ***	($L_{\bar{x}}$)
f C		→	10.04 ***	($U_{\bar{x}}$)
0 f D		→	0.00 ***	(L_R)
2.11 f E		→	0.07 ***	(U_R)

Reference:

Grant and Leavenworth, *Statistical Quality Control*, McGraw-Hill, 1972

OPERATING CHARACTERISTIC CURVES

OPERATING CHARACTERISTIC CURVES

START

P?

fin:

N

n+c

p+P₀

This program evaluates the probability P_a of acceptance for a single sampling plan with finite or infinite lot size.

Equations:

1. Finite lot size

The hypergeometric distribution is used to evaluate the probability P_a . The lot size N , sample size n and the acceptance number c (maximum allowable number of defectives in the sample) should be given. The probability P_a , which is the ordinate of the type A operating characteristic curve, can be computed for the different values of the fraction defective p in the lot.

$$P_a = \sum_{x=0}^c f(x)$$

$$f(x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$$

where $f(x)$ is the hypergeometric density function, M is the number of defectives in a lot which is calculated as the integer part of Np .

The recursive relation

$$f(x+1) = \frac{(x-M)(x-n)}{(x+1)(N-M-n+x+1)} f(x)$$

$$(x = 0, 1, 2, \dots, n-1)$$

is used to find the probability

$$P_a = \sum_{x=0}^c f(x)$$

with starting value

$$f(0) = \frac{\binom{N-M}{n}}{\binom{N}{n}}$$

The binomial coefficient $\binom{N}{n}$ is computed by the formula

$$\binom{N}{n} = \frac{N(N-1) \dots (N-n+1)}{1 \cdot 2 \cdot \dots \cdot n}$$

2. Infinite lot size

The binomial distribution is used to evaluate the probability P_a . The sample size n and the acceptance number c should be given. The probability P_a , which is the ordinate of the type B operating characteristic curve, can be computed for different values of the fraction defective p .

$$P_a = \sum_{x=0}^c f(x)$$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where $0 \leq p < 1$.

The recursive relation

$$f(x+1) = \frac{p(n-x)}{(x+1)(1-p)} f(x)$$

$$(x = 0, 1, 2, \dots, n-1)$$

is used to find the probability

$$P_a = \sum_{x=0}^c f(x)$$

with starting value

$$f(0) = (1-p)^n$$

Remarks:

1. The program requires that $0 \leq p < 1$.
2. For the type A curve (finite lot size), if $c = 0$, $P_a = f(0)$.

- 3. For certain combinations of N , n , and c (usually when they are large), an overflow condition will occur. In that case, the program halts and the display shows all 9's.
- 4. If the lot size is finite (type A), the execution time mainly depends on the sample size n and the acceptance number c ; the larger they are, the longer it takes.
- 5. The type A OC curve for finite lot sizes is really a set of discrete points, since defectives can occur only as whole numbers. For very large lot sizes, these points come very close together, giving a practically continuous curve.

Type B OC curves can be considered as suitable approximations to type A OC curves, provided the sample size n is small compared with the lot size N (in general, if $n/N \leq 0.1$).

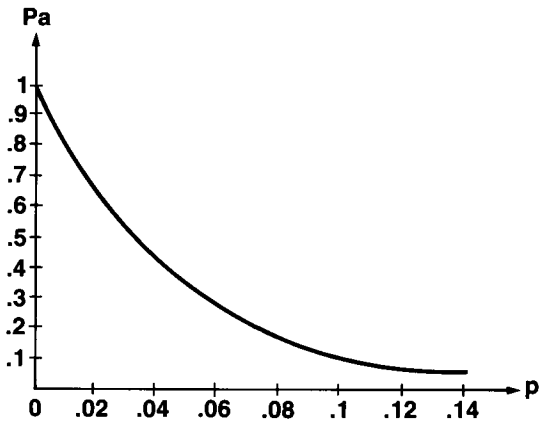
- 6. The lot size N has a relatively small effect on the type A OC curve as long as n/N is not large. The absolute sample size n is a much more controlling factor in determining the type A OC curve.
- 7. The acceptance number c affects drastically the probability of acceptance for the type B OC curve for any given fraction defective p .

References:

- 1. Dodge and Romig, *Sampling Inspection Tables*, John Wiley and Sons, 1959.
- 2. Grant and Leavenworth, *Statistical Quality Control*, McGraw-Hill, 1972.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	For infinite lot size (type B), go to 11			
5	For finite lot size (type A), do 6~9			
6	Input lot size	N	C	N
7	Input: sample size	n	ENTER	n
	acceptance number	c	D	c
8	Calculate probability P_a	p	E	P_a

0.08	E	→	0.08 *** 0.17 ***
0.1	E	→	0.10 *** 0.11 ***
0.12	E	→	0.12 *** 0.07 ***
0.14	E	→	0.14 *** 0.04 ***



Example 2:

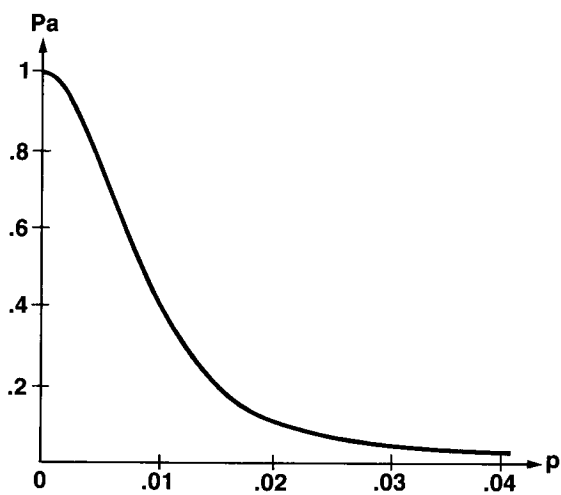
Find the type B OC curve for the sampling plan with $n = 200$, $c = 1$ (compute P_a for $p = 0, 0.01, 0.02, 0.03$ and 0.04).

Keystrokes:

Outputs:

A	→	0.00	
B	→	1.00	
200	ENTER	1	f D
		200.00 ***	(n)
		1.00 ***	(c)
0	f E	→	0.00 *** 1.00 ***

0.01	f	E	→	0.01 ***	0.40 ***
0.02	f	E	→	0.02 ***	0.09 ***
0.03	f	E	→	0.03 ***	0.02 ***
0.04	f	E	→	0.04 ***	2.656338303-03 ***



SINGLE- AND MULTI-SERVER QUEUES

SINGLE- AND MULTI-SERVER QUEUES

$U \leftarrow P \leftarrow P_0$

$-P_0 \leftarrow P_0$

$-L_q \leftarrow L$

$-T_q \leftarrow T$

$t \leftarrow P(t)$

I. Infinite Customers

Suppose there are n ($n \geq 1$) identical stations available to service calls from an infinite number of customers. Let λ be the arrival rate of customers (Poisson input), μ be the service rate of each server (exponential service), and let the service discipline be first-come, first-served. Assume all customers wait in a single line and are directed to whichever station is available. Assume further that, no customers are lost from the queue.

This program computes the following values for given n , λ and μ .

Equations:

1. The intensity factor

$$\rho = \frac{\lambda}{\mu}$$

(ρ must be less than n)

2. The probability that all servers are idle

$$P_0 = \left[\sum_{k=0}^{n-1} \frac{\rho^k}{k!} + \frac{\rho^n}{n! \left(1 - \frac{\rho}{n} \right)} \right]^{-1}$$

3. The probability that all servers are busy

$$P_b = \frac{\rho^n P_0}{n! \left(1 - \frac{\rho}{n} \right)}$$

4. The average number of customers in the queue

$$L_q = \frac{\rho P_b}{n - \rho}$$

5. The average number of customers in the system (waiting or being served)

$$L = L_q + \rho$$

6. The average waiting time in the queue

$$T_q = \frac{L_q}{\lambda}$$

7. The average flow time through the system

$$T = \frac{L}{\lambda}$$

8. The probability of waiting longer than time t

$$P(t) = P_b e^{-(n\mu - \lambda)t}$$

Remarks:

1. n must be an integer greater than or equal to 1.
2. $\rho < n$, otherwise the queue increases without bound.
3. λ and μ are rates, that is, numbers per unit time.

II. Finite Customers

Suppose there are n ($n \geq 1$) identical stations available to service calls. This program handles the case in which demand arises from a finite rather than an infinite population of customers.

Let the number of customers m be fixed; let a be the mean time between service calls; and s be the mean time to serve one customer. Given m , n , s and a , this program computes the following values.

Equations:

1. The average number of customers in the system (waiting or being served)

$$L = \frac{\sum_{k=0}^m k Q_k}{\sum_{k=0}^m Q_k}$$

where

$$Q_0 = 1$$

$$(m - k + 1) \rho Q_{k-1} = \begin{cases} kQ_k & \text{if } 1 \leq k \leq n \\ nQ_k & \text{if } n < k \leq m \end{cases}$$

and

$$\rho = \frac{s}{a}$$

2. The average flow time through the system

$$T = aL$$

3. The average number of customers in the queue

$$L_q = m \left[(\rho + 1) \left(\frac{L}{m} - 1 \right) + 1 \right]$$

4. The average waiting time in the queue

$$T_q = aL_q$$

5. The over-all efficiency factor of the system

$$F = - (\rho + 1) \left(\frac{L}{m} - 1 \right)$$

Remarks:

1. For large values of m and/or small values of ρ , the calculation of Q_k in the routine under **[F] [C]** may underflow. To avoid this, the program tests to see if $Q_k < 10^{-90}$. If it does, the program will halt its recursive solution for Q_k and go directly to the calculation of L . This should not affect the calculated value of L .
2. For certain combinations of m , n , s and a , an overflow condition will occur. In that case, the program halts and the display shows all 9's.
3. The execution time for L depends on m ; the larger m is, the longer it takes. A rough estimate of the time for this routine (**[F] [C]**) is given by $m/30$ minutes.

4. Suppose instead of knowing s and a , the service rate μ of each server and the arrival rate λ are given. Then the following formulas can be used to compute s and a in order to run this program.

$$s = \frac{1}{\mu}$$

$$a = \frac{1}{\lambda}$$

Note that

$$\rho = \frac{\lambda}{\mu}$$

References:

1. H. M. Wagner, *Principles of Operations Research with Applications to Managerial Decisions*, Prentice-Hall, 1969.
2. James Martin, *Systems Analysis for Data Transmission*, Prentice-Hall, 1972.
3. Hillier and Lieberman, *Introduction to Operations Research*, Holden-Day, 1970.
4. Peck and Hazelwood, *Finite Queuing Tables*, John Wiley and Sons, 1958.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	For finite customers go to 11			
3	Do 4 ~ 9 for infinite customers			
4	Input μ	μ	ENTER	
	λ	λ	ENTER	
	n	n	A	ρ
5	Calculate P_o		B	P_o
	P_b		B	P_b
6	Calculate: L_q		C	L_q
	L		C	L

7	Calculate: T_q		D	T_q
	T		D	T
8	Input t to calculate $P(t)$	t	E	$P(t)$
9	For a different t , go to 8			
10	For a new case, go to 2			
11	Do 12 ~ 16 for finite customers			
12	Input: number of customers	m	ENTER	
	number of servers	n	f A	m
13	Input: service time	s	ENTER	
	arrival time	a	f B	ρ
14	Calculate: customers in system		f C	L
	time through system		f C	T
15	Calculate: queue length		f D	L_q
	waiting time in queue		f D	T_q
16	Calculate efficiency factor F		f E	F
17	For a new case, go to 2			

Example 1:

Bank customers arrive at a bank on the average of 1.2 customers per minute. They join a common queue for 3 tellers, each teller serves at a rate of 30 customers per hour. Find ρ , P_0 , P_b , L_q , L , T_q , T and the probability $P(2)$ that a customer will have to wait for more than 2 minutes.

(Note: Service rate $\mu = \frac{30}{60} = 0.5$ customers per minute)
Arrival rate $\lambda = 1.2$ customers per minute

Keystrokes:

.5 **ENTER** 1.2 **ENTER** 3 **A**

Outputs:

0.5 *** (μ)
1.20 *** (λ)
3.00 *** (n)
2.40 *** (ρ)

B	→	0.06 ***	(P_0)
B	→	0.65 ***	(P_b)
C	→	2.59 ***	(L_q)
C	→	4.99 ***	(L)
D	→	2.16 ***	(T_q)
D	→	4.16 ***	(T)
2 E	→	2.00 ***	(t)
		0.36 ***	($P(t)$)

Example 2:

A laundromat has 12 washers which require an average of 4 hours of service after every 60 hours of operation. If there is only one service person in the laundromat, find ρ , L , T , L_q , T_q and F .

Keystrokes:**Outputs:**

12 ENTER 1 f A	→	12.00 ***	(m)
		1.00 ***	(n)
4 ENTER 60 f B	→	4.00 ***	(s)
		60.00 ***	(a)
		0.07 ***	(ρ)
f C	→	1.64 ***	(L)
f C	→	98.66 ***	(T)
f D	→	0.95 ***	(L_q)
f D	→	57.24 ***	(T_q)
f E	→	0.92 ***	(F)

PROGRAM LISTINGS

The following listings are included for your reference. A table of keycodes and keystrokes corresponding to the symbols used in the listings can be found in Appendix E of your Owners Handbook.

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Basic Statistics for Two Variables

001 *LBLA		057 ST09							
002 CLRG	Initialize	058 P=S							
003 CFB		059 CSB9							
004 CF1		060 CSB8							
005 CF2		061 RTN							
006 P=S		062 *LBL5	Correction for x_k, y_k, f_k						
007 CLPG		063 SF1							
008 P=S		064 CSB0							
009 0		065 CF1							
010 RTN		066 RTN							
011 *LBL3	Set flag 0 for print.	067 *LBL3							
012 SF0		068 F10							
013 1		069 GT04							
014 RTN		070 Z+							
015 *LBL0	Correction for x_k, y_k	071 RTN							
016 F00		072 *LBL4							
017 CSB0		073 Z-							
018 SF1		074 RTN							
019 XZY		075 *LBL6	$\bar{x}, \bar{y}$						
020 Z-		076 X							
021 CSB9		077 CSB9							
022 CSB8		078 R/S							
023 CF1		079 XZY							
024 RTN		080 CSB9							
025 *LBL2	Input x_i, y_i	081 CSB8							
026 F00		082 RTN							
027 CSB0		083 *LBL6							
028 XZY		084 -							
029 Z+		085 ST08							
030 CSB9		086 XZY							
031 CSB8		087 P=S							
032 RTN		088 ST08							
033 *LBL0	Input x_i, y_i, f_i	089 P=S	V_x, V_y						
034 ST00		090 S							
035 F10		091 EEV							
036 CNE		092 0							
037 ST+9		093							
038 R1		094 XZ1							
039 ST05		095 LSTN							
040 R1		096 X							
041 ST06		097 XZY							
042 R1		098 RCL0							
043 F00		099 +							
044 CSB0		100 CSB9							
045 R1		101 R/S							
046 ABS		102 XZY							
047 CSB9		103 P=S							
048 ST01		104 RCL0							
049 *LBL2		105 P=S							
050 RCL0		106 +							
051 RCL6		107 CSB9							
052 CSB3		108 CSB8							
053 DSZ1		109 RTN							
054 GT00		110 *LBL0							
055 RCL9		111 S							
056 P=S		112 CSB9							
REGISTERS									
0 $\bar{x}$	1	2	3	4	5	6	7	8	9 Σf_i
S0 $\bar{y}$	S1	S2	S3	S4 Σx_i	S5 Σx_i^2	S6 Σy_i	S7 Σy_i^2	S8 $\Sigma x_i y_i$	S9 n
A x_i	B y_i	C f_i	D	E	F	G	H	I	J

113	R/S			169	CSB9		
114	XZY	x_k, y_k		170	RTN		
115	CSB9			171	*LBLd		
116	CSB8			172	S		
117	RTN			173	RCLC		
118	*LBLc			174	+		
119	S			175	x		
120	*LBLI			176	1/X	$7x_k$	
121	P/S			177	CSB9		
122	RCL9	x_k', y_k'		178	CSB8		
123	P/S			179	RTN		
124	ENT?			180	*LBLa		
125	XZY			181	RCLZ		
126	I			182	CSB9	$\Sigma x_i, \Sigma y_i$	
127	-			183	R/S		
128	+			184	XZY	$\Sigma x_i y_i$	
129	JX			185	CSB9		
130	+			186	R/S		
131	CSB9			187	P/S		
132	F2?			188	RCL8		
133	CSB8			189	P/S		
134	CF2			190	CSB9		
135	R/S			191	CSB8		
136	LSTX			192	RTN		
137	S			193	*LBLa		
138	XZY			194	P/S		
139	SF2			195	RCL7		
140	CT01			196	RCL5	$\Sigma x_i^2, \Sigma y_i^2$	
141	RTN			197	P/S		
142	*LBLd			198	CSB9		
143	R			199	R/S		
144	XZY			200	XZY		
145	P/S			201	CSB9		
146	ST00			202	CSB8		
147	RCL8			203	RTN		
148	RCL4			204	*LBL8		
149	RCL8			205	XZY		
150	x	x_{xy}, y_{xy}		206	PRTX	Print x_i, y_i .	
151	-			207	XZY		
152	RCL9			208	PRTX		
153	I			209	RTN		
154	-			210	*LBL9		
155	+			211	F0"	Subroutine for print.	
156	P/S			212	PRTX		
157	ST0E			213	RTN		
158	CSB9			214	*LBL8		
159	R/S			215	F0"		
160	P/S			216	SPC	Subroutine for space.	
161	RCL9			217	RTN		
162	P/S						
163	ENT?						
164	XZY						
165	I						
166	-						
167	+						
168	+						

LABELS					FLAGS	SET STATUS		
A Start	B $x_i^2 y_i (\Sigma+)$	C $x_k^2 y_k (\Sigma-)$	D x_i, y_i, f_i	E x_k, y_k, f_k	0 Print	ON OFF		
A Print	D Used	C Used	D Used	E $\Sigma x_i, \dots$	1 $\Sigma- (\text{Correct.})$	0 ☐ ☐	DEG ☐	FIX ☐
B Print x_i, y_i	1 Used	2 Used	3 Used	4 Used	2 S_y	1 ☐ ☐	GRAD ☐	SCI ☐
B	B	7	8 Space	9 Print	3	2 ☐ ☐	RAD ☐	ENG ☐
						3 ☐ ☐		n <u> </u>

Factorial, Permutation and Combination

001 *LBL4		057 LSTX	
002 CLRG	Initialize	058 XZY0	
003 CF0		059 GSB6	
004 0		060 ST07	
005 RTN		061 1	
006 *LBL6		062 ST01	
007 SFB	Set flag 0 for print.	063 +	
008 1		064 ST06	
009 RTN		065 CLN	
010 *LBLD		066 X=Y0	
011 XZY		067 GT03	
012 GSB6	Input m, n for mP_n	068 *LBL0	
013 XZY		069 RJ	
014 GSB6		070 1	
015 XZY0		071 RCL1	
016 GT02		072 +	
017 ENT1		073 ST01	
018 0		074 XZY0	
019 X=Y0		075 GT05	
020 GT03		076 RCL7	
021 CLN		077 XZY	
022 1		078 +	
023 X=Y0		079 LSTX	
024 GT04		080 +	
025 -		081 RCL6	
026 ST01		082 x	
027 RJ		083 ST06	
028 ST07		084 GT00	
029 *LBL1		085 *LBL4	
030 RCL7		086 RJ	
031 1		087 RJ	
032 -		088 GSB6	$mP_1 = m$
033 ST07		089 GSB6	
034 -		090 RTN	
035 DSZ1		091 *LBL6	
036 GT01		092 ST06	
037 GSB6		093 XZY	
038 GSB6		094 RTN	
039 RTN		095 *LBL5	
040 *LBL2		096 RCL6	
041 0		097 GSB6	
042 +		098 GSB6	
043 *LBL3	Error	099 RTN	
044 ENT1		100 *LBLC	
045 1		101 GSB6	
046 GSB6		102 ST01	
047 GSB6		103 ST03	
048 RTN		104 6	
049 *LBL5		105 9	
050 XZY		106 XZY	
051 GSB6	Input m, n for mC_n	107 XZY0	
052 XZY		108 GT09	
053 GSB6		109 XZY	
054 XZY0		110 -	
055 GT02		111 ST01	
056 -		112 LSTX	
REGISTERS			
0	1 m, n	2 Log(99) +	3 (n - i)
4	5	6 Used	7 m
8 n - 1	9		
S0	S1	S2	S3
S4	S5	S6	S7
S8	S9		
A	B	C	D
E	m - 68		

[illegible]

Moments, Skewness and Kurtosis
(For Grouped or Ungrouped Data)

001 #LBL4	Initialize	057 F10	Correction for y_h, f_h .						
002 CLRC		058 CMS							
003 P2S		059 ST+1							
004 CLRC		060 X2Y							
005 P2S		061 x							
006 CF1		062 ST+2							
007 CF0		063 LSTX							
008 0		064 x							
009 RTN		065 ST+3							
010 #LBL4		066 LSTX							
011 SF0	Set flag 0 for print.	067 x	$\bar{x}$						
012 1		068 ST+4							
013 RTN		069 LSTX							
014 #LBLB		070 x							
015 STOK		071 ST+5							
016 CSB9		072 RCLC							
017 Z+		073 1							
018 #LBL1		074 F10							
019 P2S		075 CMS							
020 RCL4		076 +							
021 RCL5	Input x_i for ungrouped data.	077 STOK	m_2						
022 RCL9		078 CSB9							
023 P2S		079 CSB8							
024 STOK		080 RTN							
025 R4		081 #LBLC							
026 STOK		082 SF1							
027 R4		083 CSB8							
028 STOK		084 CF1							
029 RCL4		085 RTN							
030 3		086 #LBL1							
031 Y*	Correction for x_k .	087 RCL2	m_3						
032 F10		088 RCL1							
033 CMS		089 +							
034 ST+4		090 STOK							
035 RCL4		091 CSB9							
036 4		092 CSB8							
037 Y*		093 RTN							
038 F10		094 #LBLC							
039 CMS		095 RCL3							
040 ST+5		096 RCL1							
041 RCL1	Input y_i, f_i for grouped data.	097 +	m_3						
042 CSB9		098 RCL6							
043 CSB8		099 X*							
044 RTN		100 STOK							
045 #LBLC		101 -							
046 CSB9		102 STOK							
047 SF1		103 CSB9							
048 Z-		104 R2S							
049 CSB1		105 RCL4							
050 CF1		106 RCL3							
051 RTN	REGISTERS	107 RCL6							
052 #LBLD		108 x							
053 X2Y		109 3							
054 CSB9		110 x							
055 X2Y		111 -							
056 CSB9		112 RCL1							
REGISTERS									
0	1 n or Σf_i	2 Σx_i or $\Sigma f_i y_i$	3 Σx_i^2 or $\Sigma f_i y_i^2$	4 Σx_i^3 or $\Sigma f_i y_i^3$	5 Σx_i^4 or $\Sigma f_i y_i^4$	6 $\bar{x}, m_4$	7 m_2	8 $\bar{x}^2$	9 m_3
S0	S1	S2	S3	S4 Σx_i	S5 Σx_i^2	S6 Σy_i	S7 Σy_i^2	S8 Σxy	S9 n
A x_i	B		C n		D		E		I

113	+			169	F0?		
114	RCL6			170	SPC		
115	RCL8			171	RTN		Subroutine for space.
116	x						
117	2						
118	x						
119	+						
120	ST09						
121	CSB9						
122	R/S						
123	RCL5						
124	RCL6						
125	RCL4						
126	x	m_4					
127	4						
128	x						
129	-						
130	RCL8						
131	RCL3						
132	x						
133	6						
134	x						
135	+						
136	RCL1						
137	÷						
138	RCL8						
139	X ²						
140	3						
141	x						
142	-						
143	ST06						
144	CSB9						
145	CSB8						
146	RTN						
147	*LBL4						
148	RCL9	γ_1					
149	RCL7						
150	1						
151	.						
152	5						
153	Y ^X						
154	+						
155	CSB9						
156	R/S						
157	RCL6						
158	RCL7						
159	X ²	γ_2					
160	+						
161	CSB9						
162	CSB8						
163	RTN						
164	*LBL9	Subroutine for print.					
165	F0?						
166	PRTX						
167	RTN						
168	*LBL8						

LABELS					FLAGS	SET STATUS		
A Start	B x_i ($\Sigma+$)	C x_{ik} ($\Sigma-$)	D $y_i, t f_i$ ($\Sigma+$)	E $y_h, t f_h$ ($\Sigma-$)	0 Print			
Print?	$b \rightarrow \bar{x}$	$c \rightarrow m_2, m_3, m_4$	$d \rightarrow \gamma_1, \gamma_2$	e	1 Correction	ON OFF		
0	1 Used	2	3	4	2	0 ☐ ☒	DEG ☒	FIX ☒
						1 ☐ ☒	GRAD ☐	SCI ☐
						2 ☐ ☒	RAD ☐	ENG ☐
						3 ☐ ☒		n <u>2</u>

Random Number Generator

001 #LBL4	Input a, b, clear secondary registers.	057 #LBL5	Normal number generator.						
002 P2S		058 #LRLC							
003 CLRG		059 CSB7							
004 P2S		060 ST07							
005 GT0C		061 CSB7							
006 #LBL4		062 2							
007 CSB7		063 x							
008 RCLC		064 1							
009 RCLD		065 -							
010 -		066 ST02							
011 x	u _i	067 RCL7							
012 RCLD		068 2							
013 +		069 x							
014 #LBL2		070 1							
015 PRTX		071 -							
016 ST09		072 ST01							
017 2+		073 +F							
018 RCLA		074 X*							
019 +		075 1							
020 ST0A		076 X&Y*							
021 RCL9	077 GT05								
022 X*	078 R4								
023 RCLB	079 ENT?								
024 +	080 LN								
025 ST0B	081 2								
026 1	082 x								
027 RCL1	083 CHS								
028 +	084 X2Y								
029 ST01	085 +								
030 RCL9	086 IX								
031 PRTN	Input k.	087 ST08							
032 #LBL6		088 RCL1							
033 ST0C		089 CSB6							
034 CSB8		090 RTN							
035 RCLD		091 #LRLC							
036 #LBL3		092 RCL8							
037 PRTX		093 RCL2							
038 SPC		094 #LBL6							
039 RTN		095 x							
040 #LRLB		096 RCLC							
041 CSB7	Random digit generator.	097 x							
042 RCLD		098 RCLD							
043 x		099 +							
044 INT		100 GT02							
045 1		101 #LBL4							
046 +		102 GT06							
047 GT02		103 #LRLD							
048 #LRLC		104 CSB7							
049 ST0C		105 LN							
050 X2Y		106 CHS							
051 ST0D	Input m, σ.	107 RCLD							
052 CSB8		108 x							
053 RCLD		109 GT02							
054 CSB4		110 #LRLC							
055 RCLC		111 SPC							
056 GT03		112 x							
REGISTERS									
0	1 V ₁	2 V ₂	3	4	5	6	7 u _i	8 $\sqrt{-2 \ln S/S}$	9
S0	S1	S2	S3	S4 Σx _i	S5 Σx _i ²	S6	S7	S8	S9 n
A Used	B Used	C b or σ		D a or k or m or μ		E f _{i+1}	I index n		

113	CSB4				
114	R/S				
115	S	s			
116	CSB4				
117	R/S				
118	P=S				
119	RCL9				
120	P=S				
121	CSB4	n			
122	SPC				
123	RTN				
124	*LBL4	Subroutine to print.			
125	PRTX				
126	RTN				
127	*LBL0				
128	.				
129	5				
130	2				
131	8				
132	4	u ₀			
133	1				
134	6				
135	3				
136	STOE				
137	0				
138	STOA				
139	STOB				
140	STOI				
141	SPC				
142	RTN				
143	*LBL7				
144	RCLC				
145	9				
146	9				
147	7	Random number generator.			
148	x				
149	FRC				
150	STOE				
151	RTN				

LABELS					FLAGS	SET STATUS		
A → u _i	B → d _i	C → n _i	D → e _i	E → x; s; n	0	FLAGS	TRIG	DISP
a a/b →	b k →	c m/σ →	d μ →	e	1	ON OFF	DEG ☒	FIX ☒
0 u ₀ to R _E	1 Used	2	3 Print, space	4 Print	2	1 ☐ ☒	GRAD ☐	SCI ☐
5 Used	6 n _i , n _i +1	7 997 x u _i	8	9	3	2 ☐ ☒	RAD ☐	ENG ☐
						3 ☐ ☒		n <u>2</u>

Histogram

001	*LBL*	Initialize	05*	RCLD		
002	CLRG		058	XZY		
003	P=S		059	X=Y*		
004	CLRG		060	GSB2		
005	P=S		061	RCLC		
006	CF0		062	-		
007	CF1		063	RCLA		
008	0		064	1-X		
009	RTN		065	X		
010	*LBLC		066	INT		
011	R1		067	1		
012	GSB8	Input $x_{\min}$, $x_{\max}$.	068	XZY		
013	STOC		069	+		
014	R1		070	LSTX		
015	STOD		071	3		
016	GSB8		072	+		
017	GSB7		073	INT		
018	GSB7		074	1		
019	XZY		075	+		
020	-		076	STOI		
021	2		077	1		
022	4		078	-		
023	STOE		079	3		
024	+		080	-		
025	STOA		081	-		
026	RTN		082	CTOI		
027	*LBL*		083	*LBL9		
028	0		084	RCL9		
029	STOI		085	DSP0		
030	XZY		086	GSB8		
031	GSB8	Correction	087	GSB7		
032	2-		088	DSP2		
033	SF1		089	CF1		
034	GSB8		090	R/S		
035	CF1		091	RTN		
036	RTN		092	*LBL1		
037	*LBLC		093	GSB4		
038	STOB		094	F10		
039	0		095	CHS		
040	STOI	Input x_i .	096	ST+1		
041	R1		097	CTO9		
042	GSB8		098	*LBL4		
043	RCLC		099	3		
044	XZY		100	CHS		
045	CTOB		101	Y		
046	R1		102	10*		
047	RCLD		103	RTN		
048	XZY		104	*LBL*		
049	XZY		105	SPC		
050	CTOB		106	0		
051	0		107	STOI		
052	XZY		108	RCLC		
053	2+		109	STOB		
054	*LBL*		110	*LBL5		
055	STO9		111	IS21		
056	RCL0		112	2		
REGISTERS						
0 x_i	1 1, 2, 3	2 4, 5, 6	3 7, 8, 9	4 10, 11, 12	5 13, 14, 15	6 16, 17, 18
7 19, 20, 21	8 22, 23, 24	9 n				
S0	S1	S2	S3	S4 Σx_i	S5 Σx_i^2	S6 Σy_i
S7 Σy_i^2	S8 $\Sigma x_i y_i$	S9 n				
A $(x_{\max} - x_{\min})/24$	B $x_{\min}$, last range	C $x_{\min}$	D $x_{\max}$	E 24	F Counter 1-8	

Listing

113	ST09		169	RCL9	
114	GSBe		170	-	
115	RCLi		171	x	
116	EEX		172	+	
117	3		173	PRTX	
118	x		174	ST08	
119	INT		175	RTN	
120	DSP0		176	*LBL8	Set flag 0 for print.
121	PRTX		177	SF0	
122	SPC		178	1	
123	DSP2		179	RTN	
124	1		180	*LBL6	
125	ST09		181	GSB7	For n, $\bar{x}$, s
126	GSBe		182	GSB7	
127	RCLi		183	PZS	
128	EEX		184	RCL9	
129	3		185	PZS	
130	x		186	GSB8	
131	FRC		187	R/S	
132	EEX		188	R	
133	3		189	GSB8	
134	x		190	R/S	
135	INT		191	S	
136	DSP0		192	GSB8	
137	PRTX	Listing	193	GSB7	
138	SPC		194	RTN	
139	DSP2		195	*LBL8	Subroutine for print.
140	0		196	F0?	
141	ST09		197	PRTX	
142	GSBe		198	RTN	
143	RCLi		199	*LBL7	Subroutine for space.
144	EEX		200	F0?	
145	6		201	SPC	
146	x		202	RTN	
147	FRC		203	*LBL2	
148	EEX		204	RCLA	Correction for input equals to $x_{\max}$.
149	3		205	2	
150	x		206	$\div$	
151	INT		207	-	
152	DSP0		208	RTN	
153	PRTX				
154	SPC				
155	DSP2				
156	RCLi				
157	8				
158	X $\div$ Y?				
159	CT05				
160	RTN				
161	*LBL6				
162	RCLB				
163	PRTX				
164	RCLC				
165	RCLA				
166	RCLi				
167	3				
168	x				

LABELS					FLAGS	SET STATUS		
A Start	B Print	C $x_{\max}$, $x_{\min}$	D Input	E Correct	F Print	FLAGS	TRIG	DISP
1 List	2 n, $\bar{x}$, s	3 y	4 10^x	5 Used	1 Correction	0 ☐ ON ☒ OFF	DEG ☒	FIX ☒
0 Error	1 Sorting	2 Cor. for $x_{\max}$	3	4	2	1 ☐ ☒	GRAD ☐	SCI ☐
5 Listing	6	7 Space	8 Print	9 Print index	3	2 ☐ ☒	RAD ☐	ENG ☐
						3 ☐ ☒		n <u>2</u>

Analysis of Variance (One Way)

001 *LBLA		057 RTN							
002 CLRG	Initialize	058 *LBL9	Clear registers for new i.						
003 P=S		059 P=S							
004 CLRG		060 CLRG							
005 P=S		061 P=S							
006 CF0		062 RTN							
007 CF1		063 *LBLD	Correction						
008 CF2		064 CSB3							
009 0		065 -							
010 RTN		066 CSB8							
011 *LBLC	Input x_{ij} .	067 RTN							
012 F2?		068 *LBLB	Set flag 0 for print.						
013 CSB9		069 SF0							
014 CSB3		070 1							
015 Z+		071 RTN							
016 *LBLB		072 *LBLA							
017 P=S		073 RCL4							
018 RCL4		074 RCL7							
019 RCL5		075 X?							
020 P=S		076 RCL6							
021 ST0B		077 +	TSS						
022 R1		078 ST0B							
023 ST0A		079 -							
024 R1		080 ST0B							
025 CSB3		081 CSB3	TrSS						
026 CSB8		082 R/S							
027 RTN		083 RCL5							
028 *LBLB		084 RCL8							
029 1		085 -							
030 ST+9		086 ST01							
031 SF2		087 CSB3							
032 RCL4		088 R/S							
033 ST+7		089 RCL0	ESS						
034 RCLB		090 RCL1							
035 ST+4		091 -							
036 P=S		092 ST02							
037 RCL9	$\bar{x}_i$	093 CSB3							
038 P=S		094 CSB8							
039 ST+6		095 RTN							
040 RCL4		096 *LBLB	Subroutine for space.						
041 X?		097 F0?							
042 P=S		098 SPC							
043 RCL9		099 RTN							
044 P=S		100 *LBL3	Subroutine to print.						
045 +		101 F0?							
046 ST+5		102 PRX							
047 -		103 RTN							
048 CSB3		104 *LBLA							
049 R/S		105 RCL9							
050 S		106 1							
051 CSB3	s_i	107 -	df_1						
052 R/S		108 ST03							
053 RCL4		109 CSB3							
054 CSB3		110 R/S							
055 CSB8	Sum _i	111 RCL6	df_2						
056 CSB8		112 RCL9							
REGISTERS									
0 TSS	1 T,SS	2 ESS	3 df_1	4 $\Sigma \Sigma x_{ij}^2$	5 $\Sigma (\Sigma x_{ij})^2 / n_i$	6 Σn_i	7 $\Sigma \Sigma x_{ij}$	8 Σx_{ij}	9 k
S0	S1	S2	S3	S4 Σx_i	S5 Σx_i^2	S6 Σy_i	S7 Σy_i^2	S8 $\Sigma x_i y_i$	S9 n
A $\Sigma x_{ij}, df_2$		B $\Sigma x_{ij}^2, F$		C	D	E	I		

113	-			
114	STOA			
115	CSB3			
116	R/S			
117	RCLA			
118	RCL3	df_3		
119	+			
120	CSB3			
121	CSB0			
122	RTN			
123	*LBLc			
124	RCL1	T, MS		
125	RCL3			
126	÷			
127	CSB3			
128	R/S			
129	RCL2	EMS		
130	RCLA			
131	÷			
132	CSB3			
133	R/S			
134	÷			
135	CSB3			
136	CSB0	F		
137	STOB			
138	RTN			

LABELS					FLAGS	SET STATUS			
A Start	B Print?	C x_{ij} ($\Sigma+$)→	D x_{im} ($\Sigma-$)→	E $\bar{x}, s_i, \text{sum}_i$	0 Print	FLAGS		TRIG	DISP
a TSS, ...	b $df_1, \dots$	c $T, MS \dots$	d	e	1 Correction	ON	OFF	DEG ☒	FIX ☒
0 Space	1	2	3 Print	4	2 New data	1	☐	GRAD ☐	SCI ☐
5	6	7	8 $\Sigma x_i, \Sigma x_i^2$	9 CLREG	3	2	☐	RAD ☐	ENG ☐
						3	☐		n <u>2</u>

Two Way Analysis of Variance

001 *LBLA	Initialize	057 RCL6	
002 CLRG		058 x	
003 CF8		059 ÷	
004 0		060 STO7	
005 RTN		061 CMS	
006 *LBLC	Input r, c.	062 RCL2	
007 X=Y		063 +	
008 ST05		064 STO1	
009 GSB9		065 RCL3	
010 R1		066 RCL6	
011 ST06		067 ÷	
012 GSB9		068 RCL7	
013 GSB8		069 -	
014 RTN		070 STO2	
015 *LBLD	Input x _{ij} .	071 RCL4	
016 ST+7		072 RCL5	
017 GSB9		073 ÷	
018 X²		074 RCL7	
019 ST+2		075 -	
020 RCL4		076 STO3	
021 1		077 RCL2	
022 +		078 +	
023 ST04		079 CMS	
024 GSB9		080 RCL1	
025 RTN		081 +	
026 *LBL6		082 STO4	
027 RCL7		083 RCL5	
028 ST+1		084 1	
029 X²	Calculate RS _i .	085 -	
030 ST+3		086 STO5	
031 *LBL8		087 RCL6	
032 RCL7		088 1	
033 0		089 -	
034 ST08		090 STO6	
035 ST07		091 x	
036 X²+Y		092 STO7	
037 GSB9		093 ÷	
038 GSB8		094 ST08	
039 RTN		095 RCL2	
040 *LBL1		096 RCL5	
041 RCL4	Re-initialize for column.	097 ÷	
042 ST08		098 RCL8	
043 0		099 ÷	
044 ST02		100 GSB8	
045 ST08		101 GSB9	
046 GSB8		102 R/S	
047 RTN		103 RCL3	
048 *LBLC		104 RCL6	
049 RCL7	CS _i	105 ÷	
050 X²		106 RCL8	
051 ST+4		107 ÷	
052 ST08		108 GSB9	
053 *LBL4		109 GSB8	
054 RCL1	F ₁ , F ₂	110 RTN	
055 X²		111 *LBL4	
056 RCL5		112 RCL5	

REGISTERS								
0	1 $\sum \sum x_{ij}$, TSS	2 $\sum \sum x_{ij}^2$, RSS	3 Used	4 Used	5 r, r - 1	6 c, c - 1	7 Used	8 Used
S0	S1	S2	S3	S4	S5	S6	S7	S8
								S9
A i, j	B r	C	D	E	F	G	H	I

[illegible]

Analysis of Covariance (One Way) (Card 1)

001 #LBLA		057 RCLA	
002 CLRG	Initialize	058 +	
003 P=S		059 STOA	
004 CLRG		060 RCL5	
005 P=S		061 RCLB	
006 CFB		062 +	
007 0		063 STOB	
008 STOB		064 RCL6	
009 RTN		065 RCLC	
010 #LBLB		066 +	
011 P=S		067 STOC	
012 0		068 RCL7	
013 STOA	For new i.	069 RCLD	
014 STOB		070 +	
015 STOB		071 STOD	
016 STOB		072 RCL8	
017 STOB		073 RCLC	
018 STOB		074 +	
019 P=S		075 STOE	
020 ISZ1		076 P=S	
021 RCL1		077 RCL9	Σn_i
022 CSB0		078 ST+0	
023 CSB1		079 P=S	
024 RTN		080 ST+0	
025 #LBL9	Subroutine to print x_{ij} , y_{ij} .	081 RCL4	$\Sigma \frac{(\Sigma x_{ij})^2}{n_i}$
026 R4		082 P=S	
027 PRTH		083 CSB0	
028 R4		084 P=S	
029 PRTH		085 RCL6	$\Sigma \frac{(\Sigma y_{ij})^2}{n_i}$
030 RTN		086 CSB0	
031 #LBL6	Set flag 0 for print.	087 RCL4	
032 SF0		088 RCL6	
033 1		089 +	
034 STOB		090 RCL9	
035 RTN		091 +	
036 #LBLD	Correction for $x_{im} \cdot y_{im}$.	092 P=S	
037 F00		093 ST+5	
038 CSB9		094 P=S	
039 X=V		095 RCL6	
040 Z=		096 RCL4	
041 STOB		097 P=S	
042 CSB0		098 CSB0	Σx_i
043 CSB1		099 R=S	
044 RTN		100 R4	
045 #LBLC	Input for x_{ij} , y_{ij} .	101 CSB0	
046 F00		102 CSB1	Σy_i
047 CSB9		103 R=S	
048 X=V		104 #LBLB	
049 Z=		105 X=	
050 STOB		106 RCL9	
051 CSB0		107 +	
052 CSB1		108 ST+2	
053 RTN		109 RTN	
054 #LBLB		110 #LBLA	TSS_x
055 P=S		111 RCLB	
056 RCL4		112 RCLA	

REGISTERS									
0 Σn_i	1 TSS_x	2 Used	3 WSS_x	4 TSP_i	5 Used	6 WSP	7 WSS_0	8 1 or 0	9 j
50 Σn_i	51 TSS_y	52 Used	53 WSS_y	54 Σx_{ij}	55 Σx_{ij}^2	56 Σy_{ij}	57 Σy_{ij}^2	58 $\Sigma x_{ij} y_{ij}$	59 $n_i \neq j$
A $\Sigma \Sigma x_{ij}$	B $\Sigma \Sigma x_{ij}^2$	C $\Sigma \Sigma y_{ij}$	D $\Sigma \Sigma y_{ij}^2$	E $\Sigma \Sigma x_{ij} y_{ij}$	I i = 1, 2, ... k				

113	CSB7		165	-	
114	R/S	ASS _x	170	÷	
115	RCL2		171	÷	
116	RCLA		172	CSB0	
117	CSB6		173	R/S	
118	R/S	WSS _x	174	P=S	
119	-		175	RCL2	
120	STO3		176	RCL1	
121	CSB0		177	1	
122	CSB1		178	-	
123	R/S		179	÷	
124	*LBL4	TSS _y	180	RCL3	
125	RCL0		181	P=S	
126	RCLC		182	RCL0	
127	P=S		183	RCL1	
128	CSB7		184	-	
129	P=S		185	÷	
130	R/S	ASS _y	186	÷	F _y
131	P=S		187	CSB0	
132	RCL2		188	R/S	
133	RCLC		189	RCL1	
134	CSB6		190	1	
135	P=S		191	-	df ₁
136	R/S		192	CSB0	
137	-		193	R/S	
138	P=S	WSS _y	194	RCL0	
139	STO3		195	RCL1	
140	P=S		196	-	df ₂
141	CSB0		197	CSB0	
142	CSB1		198	CSB1	
143	R/S		199	RTN	
144	*LBL7	For (TSS) _x or _y	200	*LBL0	Subroutine for print.
145	X ²		201	F0?	
146	RCL0		202	PRTX	
147	÷		203	RTN	
148	-		204	*LBL1	Subroutine for space.
149	STO1		205	F0?	
150	CSB0		206	SPC	
151	RTN		207	RTN	
152	*LBL6	For (ASS) _x or _y			
153	X ²				
154	RCL0				
155	÷				
156	-				
157	STO2				
158	CSB0				
159	RTN				
160	*LBL4				
161	RCL2				
162	RCL1				
163	1				
164	-				
165	÷				
166	RCL3				
167	RCL0	F _x			
168	RCL1				

LABELS					FLAGS	SET STATUS		
A Start	B New i	C x _{ij} † y _{ij} (Σ+)	D x _m † y _m (Σ-)	E s _{xi} † y _i	0 Print			
8 TSS _x ...	9 F _x ...	c	d	9 Print	1			
0 Print	1 Space	2	3	4	2			
5	6 ASS _x	7 TSS _x	8 Σ x _{ij} /n _i	9 Print x _{ij} , y _{ij}	3			

FLAGS		TRIG		DISP
0	ON OFF	DEG	☒	FIX ☒
1	☐ ☒	GRAD	☐	SCI ☐
2	☐ ☒	RAD	☐	ENG ☐
3	☐ ☒			n <u>2</u>

(Card 2)

001 *LBLC		057 +	
002 CSB2		058 CSB0	AMS _Q
003 RCLC		059 R/S	
004 RCLA		060 RCL7	
005 RCLC		061 RCL0	
006 X		062 RCL1	WMS _Q
007 RCL0		063 -	
008 +		064 1	
009 -	TSP	065 -	
010 ST04		066 +	
011 CSB0		067 CSB0	
012 R/S		068 R/S	F
013 RCL5		069 +	
014 RCLA		070 CSB0	
015 RCLC		071 R/S	
016 X		072 RCL1	
017 RCL0		073 1	df ₃
018 +	ASP	074 -	
019 -		075 CSB0	
020 CSB0		076 R/S	
021 R/S		077 RCL0	
022 -	WSP	078 RCL1	
023 ST06		079 -	
024 CSB0		080 1	df ₄
025 CSB1		081 -	
026 RTN		082 CSB0	
027 *LBL4		083 CSB1	
028 P/S		084 RTN	Subroutine for print.
029 RCL1		085 *LBL0	
030 P/S		086 F0?	
031 RCL4		087 PRTX	
032 X?		088 RTN	
033 RCL1		089 *LBL1	Subroutine for space.
034 +	TSS _Q	090 F0?	
035 -		091 SPC	
036 CSB0		092 RTN	
037 R/S		093 *LBL2	
038 P/S		094 CF0	Subroutine for setting flag 0 for print.
039 RCL3		095 RCL0	
040 P/S		096 1	
041 RCL6		097 X=Y?	
042 X?		098 SF0	
043 RCL3		099 RTN	
044 +			
045 -	WSS _Q		
046 ST07			
047 CSB0			
048 R/S			
049 -			
050 CSB0	ASS _Q		
051 CSB1			
052 R/S			
053 *LBLC			
054 RCL1			
055 1			
056 -			

REGISTERS

0 Σn_i	1 TSS _x	2 Used	3 WSS _x	4 TSP _i	5 Used	6 WSP	7 WSS _Q	8 1 or 0	9 j
S0 Σn_i	S1 TSS _y	S2 Used	S3 WSS _y	S4 Σx_{ij}	S5 Σx_{ij}^2	S6 Σv_{ij}	S7 Σv_{ij}^2	S8 $\Sigma x_{ij}v_{ij}$	S9 $n_{i \neq j}$
A $\Sigma \Sigma x_{ij}$	B $\Sigma \Sigma x_{ij}^2$	C $\Sigma \Sigma v_{ij}$	D $\Sigma \Sigma v_{ij}^2$	E $\Sigma \Sigma x_{ij}v_{ij}$	i = 1, 2, ... k				

LABELS					FLAGS					SET STATUS																								
A	B	C	D	E	0 Print																													
a	b	c TSP; ...	d TSSq; ...	e AMSq; ...	1					<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th colspan="2">FLAGS</th> <th>TRIG</th> <th>DISP</th> </tr> <tr> <td>0</td> <td>☐ ON ☒ OFF</td> <td>DEG ☒</td> <td>FIX ☒</td> </tr> <tr> <td>1</td> <td>☐ ☒</td> <td>GRAD ☐</td> <td>SCI ☐</td> </tr> <tr> <td>2</td> <td>☐ ☒</td> <td>RAD ☐</td> <td>ENG ☐</td> </tr> <tr> <td>3</td> <td>☐ ☒</td> <td></td> <td>n <u>2</u></td> </tr> </table>					FLAGS		TRIG	DISP	0	☐ ON ☒ OFF	DEG ☒	FIX ☒	1	☐ ☒	GRAD ☐	SCI ☐	2	☐ ☒	RAD ☐	ENG ☐	3	☐ ☒		n <u>2</u>
FLAGS		TRIG	DISP																															
0	☐ ON ☒ OFF	DEG ☒	FIX ☒																															
1	☐ ☒	GRAD ☐	SCI ☐																															
2	☐ ☒	RAD ☐	ENG ☐																															
3	☐ ☒		n <u>2</u>																															
0 Print	1 Space	2 Used	3	4	2																													
5	6	7	8	9	3																													

Normal and Inverse Normal Distribution (Card 1)

001 #LBLA		057 2							
002 .		058 CHS							
003 2		059 ST07							
004 3		060 .							
005 1		061 3							
006 6		062 1							
007 4		063 9							
008 1		064 3							
009 9		065 8							
010 ST03		066 1							
011 1		067 5							
012 .		068 3							
013 3		069 ST08							
014 3		070 P=S							
015 0		071 2							
016 2		072 .							
017 7		073 5							
018 4		074 1							
019 4		075 5							
020 2	Store constants for normal distribution.	076 5							
021 9		077 1							
022 ST04		078 7							
023 1		079 ST01							
024 .		080 .							
025 8		081 8							
026 2		082 0							
027 1		083 2							
028 2		084 8							
029 5		085 5							
030 5		086 3							
031 9		087 ST02							
032 7		088 .							
033 8		089 0							
034 CHS		090 1							
035 ST05		091 0							
036 1		092 3							
037 .		093 2							
038 7		094 8							
039 8		095 ST03							
040 1		096 1							
041 4		097 .							
042 7		098 4							
043 7		099 3							
044 9		100 2							
045 3		101 7							
046 7		102 8							
047 ST06		103 8							
048 .		104 ST04							
049 2		105 .							
050 5		106 1							
051 6		107 8							
052 5		108 9							
053 6		109 2							
054 3		110 6							
055 7		111 9							
056 8		112 ST05							
REGISTERS									
0	1	2	3 r	4 b ₁	5 b _a	6 b ₃	7 b ₂	8 b ₁	9
S0	S1 C ₀	S2 C ₁	S3 C ₂	S4 d ₁	S5 d ₂	S6 d ₃	S7	S8	S9
A	B	C	D	E	F	G	H	I	J

113 .
 114 0
 115 0
 116 1
 117 3
 118 0
 119 0
 120 ST06
 121 P=5
 122 0
 123 ST0A
 124 ST0B
 125 RTN

LABELS					FLAGS	SET STATUS		
A Start	B	C	D	E	0	FLAGS	TRIG	DISP
a	b	c	d	e	1	ON OFF 0 ☐ ☒	DEG ☒	FIX ☒
0	1	2	3	4	2	1 ☐ ☒	GRAD ☐	SCI ☐
5	6	7	8	9	3	2 ☐ ☒	RAD ☐	ENG ☐
						3 ☐ ☒		n <u>2</u>

(Card 2)

001 *LBLB		057 x	
002 1	Store 1 in R_A for print.	058 RCL2	
003 STOA		059 x	
004 RTN		060 F0?	
005 *LBLE		061 CSB9	
006 CSB9		062 F0?	
007 STOI		063 CSB6	
008 ENT+		064 RTN	
009 x		065 *LBL1	
010 2		066 CF0	
011 +		067 RCL1	
012 CHS	068 CHS		
013 e*	069 STOI		
014 P1	070 CSB:		
015 2	071 1		
016 x	072 XZ?		
017 -	073 -		
018 +	074 STOI		
019 STOI	075 CSB9		
020 CSB9	076 CSB6		
021 CSB6	077 RCL9		
022 CSB3	078 RTN		
023 RCL2	079 *LBLE		
024 RTN	080 CSB9		
025 *LBLE	081 X(0?)		
026 CSB9	082 GT00		
027 STOI	083 1		
028 CSB5	084 XZ?		
029 CSB:	085 GT00		
030 RCL1	086 R1		
031 X(0?)	087 .		
032 GTOI	088 5		
033 SF0	089 XZ?		
034 *LBLE	090 X)Y?		
035 1	091 CSB8		
036 RCL1	092 ENT+		
037 RCL2	093 x		
038 x	094 1/X		
039 +	095 LN		
040 1/X	096 TX		
041 ENT+	097 PZ5		
042 ENT+	098 STOI		
043 ENT+	099 RCL3		
044 RCL4	100 x		
045 x	101 RCL2		
046 RCL5	102 +		
047 +	103 RCL7		
048 =	104 x		
049 RCL6	105 RCL1		
050 +	106 +		
051 x	107 RCL7		
052 RCL7	108 RCL6		
053 +	109 x		
054 x	110 RCL5		
055 RCL8	111 +		
056 +	112 RCL7		
Input Q(x) and calculate x.			

REGISTERS									
0	1 x	2 f(x)	3 r	4 b_5	5 b_4	6 b_3	7 b_2	8 b_1	9
S0	S1 C_0	S2 C_1	S3 C_2	S4 d_1	S5 d_2	S6 d_3	S7 t	S8	S9
A 1 or 0		B 1 or 0		C		D		E	

113	x			169	RTN		
114	RCL4			170	*LBL2		Restore 1 to R _A .
115	+			171	1		
116	RCL7			172	STDA		
117	x			173	RTN		
118	1						
119	+						
120	÷						
121	RCL7						
122	XZY						
123	-						
124	PSS						
125	F1?						
126	CHS						
127	GSB9						
128	GSB6						
129	CF1						
130	RTN						
131	*LBL8						
132	SF1						
133	1						
134	-						
135	CHS						
136	RTN						
137	*LBL9						
138	RCLA						
139	X)0?						
140	GSB7						
141	R4						
142	RTN						
143	*LBL7						
144	R4						
145	PRTX						
146	R↑						
147	RTN						
148	*LBL6						
149	RCLA						
150	X)0?						
151	SPC						
152	R4						
153	RTN						
154	*LBL5						
155	RCLA						
156	X)0?						
157	GSB4						
158	R4						
159	RTN						
160	*LBL4						
161	STOB						
162	CLX						
163	STDA						
164	RTN						
165	*LBL3						
166	RCLB						
167	X)0?						
168	GSB2						

LABELS					FLAGS	SET STATUS		
A	Print?	C x→f(x)	D x→Q(x)	E Q(x)→x	0 x positive	FLAGS	TRIG	DISP
a	b	c Q(x)	d	e	1 Q(x) > .5	0 ON OFF	DEG ☒	FIX ☒
0 Error	1 x < 0	2 1→R _A	3 R _B > 0?	4 0→R _A	2	1 ☒	GRAD ☐	SCI ☐
5 R _A →R _B	6 Space	7 Print	8	9 Print	3	2 ☐	RAD ☐	ENG ☐
						3 ☐		n <u>2</u>

Chi-Square Distribution

001 #LBLA		057 Y*	
002 CLRG		058 RCL2	
003 CF0	Initialize	059 2	
004 CF1		060 ÷	
005 0		061 CHS	
006 RTN		062 e*	
007 #LBLB	Set flag for print.	063 x	
008 SF0		064 2	
009 1		065 RCL1	
010 RTN		066 Y*	
011 #LBLC		067 ÷	
012 CSB9	Input ν.	068 RCL3	
013 1		069 +	
014 ST03		070 ST05	
015 XZY		071 FI?	
016 2	Calculate $\Gamma\left(\frac{\nu}{2}\right)$	072 CSB9	
017 ÷		073 FI?	
018 ST01		074 CSB8	
019 INT		075 CF1	
020 LSTX		076 RTN	
021 X#Y*		077 #LBLB	
022 GT01		078 CSB8	
023 1		079 RCL2	
024 -		080 RCL1	
025 N!		081 ÷	
026 CSB9		082 STX5	
027 CSB8		083 2	
028 ST03		084 RCL1	
029 R/S		085 x	
030 #LBL1		086 ST06	
031 .		087 1	
032 5		088 ST04	
033 X=Y*		089 #LBL3	
034 GT02		090 RCL2	
035 XZY		091 RCL6	
036 1		092 2	
037 -		093 +	
038 STX2		094 ST06	
039 GT01		095 ÷	
040 #LBL2		096 RCL4	
041 P!		097 ~	
042 JX		098 ST04	
043 RCL3		099 +	
044 \		100 X#Y*	
045 ST03		101 GT03	
046 CSB9		102 RCL5	
047 CSB8		103 x	
048 R/S		104 CSB9	
049 #LBLD		105 CSB8	
050 SF1		106 RTN	
051 #LBL6	Input x, calculate f(x).	107 #LBL9	
052 CSB9		108 F0?	
053 ST02		109 PRTX	
054 RCL1		110 RTN	
055 1		111 #LBL8	
056		112 F0?	
REGISTERS			
0	1 ν/2	2 x	3 1, Γ(ν/2)
4 Used	5 f(x)	6 Used	7
8	9		
S0	S1	S2	S3
S4	S5	S6	S7
S8	S9		
A	B	C	D
E	F	G	H
I	J	K	L

t Distribution

001 *LBLA		057 XZY	
002 CLRC		058 2	
003 CF0		059 ÷	
004 CF1	Initialize	060 ST01	
005 0		061 INT	
006 RTN		062 LSTX	
007 *LBLB	Set flag 0 for print.	063 X#Y?	
008 SFO		064 GT01	
009 1		065 1	
010 RTN		066 -	Compute $\Gamma(p/2)$.
011 *LBLC		067 M?	
012 ST00	Input ν .	068 ST03	
013 CSB7		069 RTN	
014 CSB5		070 *LBL1	
015 RTN		071 -	
016 *LBLD	Input x , compute $f(x)$.	072 5	
017 CSB7		073 X#Y?	
018 ST0A		074 GT02	
019 RCL0		075 XZY	
020 CSB4		076 1	
021 ST0B		077 -	
022 RCL0		078 STX3	
023 1		079 GT01	
024 +		080 *LBL2	Compute $R(x)$.
025 CSB4		081 P?	
026 ST0C		082 JX	
027 RCLA		083 RCL3	
028 RCLC		084 x	
029 RCLB		085 ST03	
030 ÷		086 RTN	
031 P?		087 *LBL5	
032 RCL0		088 ST0A	
033 x		089 CSB7	
034 JX		090 ABS	
035 ÷		091 RCL0	
036 1		092 RAD	
037 RCLA		093 JX	
038 X?		094 ÷	
039 RCL0		095 TAN-	
040 ÷		096 ST02	
041 +		097 RCL0	
042 RCL0		098 2	
043 1		099 ÷	
044 +		100 INT	
045 2		101 LSTX	
046 ÷		102 X#Y?	
047 CHS		103 GT04	
048 Y?		104 0	
049 x		105 ST05	
050 ST0D		106 *LBL6	
051 CSB7		107 RCL2	
052 CSB5		108 COS	
053 RTN		109 X?	
054 *LBL4		110 ST03	For even ν .
055 1		111 RCL2	
056 ST03		112 SIN	

REGISTERS									
0 ν	1	2 θ	3 Used	4 $\sin \theta$	5 Used	6 Used	7 $2\theta/\pi$, R	8 Used	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A x	B $\Gamma(p/2)$	C $\Gamma((p+1)/2)$	D	E	F	G	H	I	J

113	ST04			169	PI		
114	RCL0			170	+		
115	- 2			171	RCL7		
116	X=Y?			172	+		
117	GT00			173	RCL4		
118	+			174	GT06		
119	- 1			175	RTN		
120	-			176	#LBL9		
121	ST01			177	RCL7		
122	- 1			178	RCL4		
123	ST06			179	GT06		
124	#LBL3			180	RTN		
125	RCL3			181	#LBL8		
126	x			182	RCL4		
127	RCL5			183	RCL4		
128	- 1			184	GT06		
129	+			185	RTN		
130	x			186	#LBL6		
131	LSTX			187	X)0?		
132	- 1			188	GT00		
133	+			189	X=Y		
134	ST05			190	- 1		
135	+			191	-		
136	ST+6			192	CMS		
137	DS21			193	- 2		
138	GT03			194	+		
139	RCL6			195	GSB7		
140	RCL4			196	GSB5		
141	x			197	RTN		
142	F1?			198	#LBL0		
143	GT0e			199	X=Y		
144	RCL4			200	- 1		
145	GT06			201	+		
146	#LBLe			202	- 2		
147	RTN			203	+		
148	#LBL4			204	GSB7		
149	RCL2			205	GSB5		
150	- 2			206	RTN		
151	x			207	#LBL7		
152	PI			208	F0?		
153	+			209	PRTX		
154	ST07			210	RTN		
155	RCL0			211	#LBL5		
156	- 1			212	F0?		
157	ST05			213	SPC		
158	ST-0			214	RTN		
159	X=Y?						
160	GT09						
161	SF1						
162	GSB6						
163	CF1						
164	RCL2						
165	COS						
166	x						
167	- 2						
168	x						

LABELS					FLAGS	SET STATUS		
A Start	B Print	C $\nu \rightarrow$	D $x \rightarrow f(x)$	E $x \rightarrow P(x)$	0 Print	FLAGS	TRIG	DISP
1 $\nu/2$ integer	2 even ν	c	d	e Used	1 odd ν	ON OFF		
0 $P(x)$, $x > 0$	1 $\nu/2$ no integer	2 $\sqrt{\pi} x$	3 even ν	4 odd ν	2	0 ☐ ON ☒ OFF	DEG ☒	FIX ☒
5 Space	6 $P(x)$, $x \leq 0$	7 Print	8 For even ν	9 For odd ν	3	1 ☐ ON ☒ OFF	GRAD ☐	SCI ☐
						2 ☐ ON ☒ OFF	RAD ☐	ENG ☐
						3 ☐ ON ☒ OFF		n <u>2</u>

F Distribution

001 *LBLA		057 RCL4	
002 CLR6		058 GSB9	
003 CF0	Initialize	059 RTN	
004 CF1		060 *LBL5	
005 0		061 1	
006 RTN		062 ST05	
007 *LBLB	Set flag 0 for print.	063 RCL3	Even ν_1 .
008 SF0		064 -	
009 1		065 ST03	
010 RTN		066 RCL2	
011 *LBLC	Input ν_1 .	067 2	
012 ST01		068 +	
013 GSB9		069 x	
014 RTN		070 ST+5	
015 *LBLD		071 DSZ1	
016 ST02	Input ν_2 .	072 GT03	
017 GSB9		073 GT02	
018 RTN		074 *LBL3	
019 *LBLE		075 RCL2	
020 GSB9	Input x.	076 2	
021 ST06		077 +	
022 ENT1		078 ST02	
023 RCL1		079 RCL7	
024 x		080 2	
025 RCL2		081 +	
026 +		082 ST07	
027 RCL2		083 +	
028 X ² Y		084 RCL3	
029 +		085 x	
030 ST03		086 x	
031 RCL1		087 ST+5	
032 2		088 DSZ1	
033 +	If ν_1 even then go to LBL d else go to LBL e.	089 GT03	
034 FRC		090 *LBL2	Even ν_1 .
035 0		091 RCL5	
036 X=Y?		092 RCL4	
037 GT0d		093 x	
038 GT0e		094 F1?	
039 RTN		095 GSB9	
040 *LBLd		096 F1?	
041 RCL3		097 GSB8	
042 RCL2		098 RTN	
043 2		099 *LBL e	
044 ST07		100 CF1	
045 +	For even ν_1 .	101 RCL1	
046 Y*		102 RCL2	
047 ST04		103 ST01	For odd ν_1 .
048 RCL1		104 X ² Y	
049 2		105 ST02	
050 -		106 1	
051 2		107 RCL3	
052 +		108 -	
053 ST01		109 ST03	
054 0		110 GSBd	
055 X#Y?		111 SF1	
056 GT05		112 1	
REGISTERS			
0	1 ν_1 or ν_2	2 ν_2 or ν_1	3 1, 1 - t
			4 $t^{1/2}$ or $t^{1/2}$
			5 Used
			6 x
			7 Used
			8 Used
			9 Used
S0	S1	S2	S3
			S4
			S5
			S6
			S7
			S8
			S9
A	B	C	D
			E
			I Used

Multiple Linear Regression

001 *LBLA		057 X ²	
002 CLRC		058 CSB2	
003 CF0	Initialize	059 ST+1	
004 CF1		060 RTN	
005 0		061 *LBLB	
006 RTN		062 RCL0	
007 *LBLC		063 RCL4	
008 STOC		064 x	
009 R4		065 RCL7	
010 STOB		066 X ²	
011 R4		067 -	
012 STOA		068 STOD	
013 CSB7		069 RCL0	
014 7		070 RCL3	
015 STOI		071 x	
016 R4	Input x _i , y _i , z _i	072 RCL8	
017 CSB1		073 RCL9	
018 8		074 x	
019 STOI		075 -	
020 RCLB	Compute $\Sigma x_i, \Sigma y_i, \Sigma z_i;$	076 x	
021 CSB9	$\Sigma x_i^2, \Sigma y_i^2, \Sigma z_i^2; \Sigma x_i y_i,$	077 STOC	
022 CSB1	$\Sigma y_i z_i, \Sigma z_i x_i$	078 RCL0	
023 9		079 RCL1	
024 STOI		080 x	
025 RCLC		081 RCL7	
026 CSB9		082 RCL8	
027 CSB1		083 x	
028 RCLA		084 -	
029 RCLB		085 STOA	
030 >		086 RCL0	
031 CSB2		087 RCL2	
032 ST+1		088 x	
033 RCLA		089 RCL7	
034 RCLC		090 RCL9	
035 >		091 x	
036 CSB2		092 -	
037 ST+2		093 STOB	
038 RCLB		094 x	
039 RCLC		095 RCLC	
040 -		096 X ² Y	
041 CSB2		097 -	
042 ST+3		098 RCLD	
043 1		099 RCL0	
044 CSB2		100 RCL5	
045 ST+0		101 x	
046 RCL0		102 RCL8	
047 CSB9		103 X ²	
048 RTN		104 -	
049 *LBL1		105 y	
050 CSB2	Subroutine for $\Sigma x_i, \Sigma x_i^2 \dots$	106 RCLA	
051 ST+1		107 X ²	
052 RCL1		108 -	
053 3		109 ÷	
054 -		110 STOC	
055 STOI		111 RCLB	
056 R4		112 RCLA	

REGISTERS										
0	n	1 $\Sigma x_i y_i$	2 $\Sigma x_i z_i$	3 $\Sigma y_i z_i$	4 Σx_i^2	5 Σy_i^2	6 Σz_i^2	7 Σx_i	8 Σy_i	9 Σz_i
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9	
A	Used, a	B	Used, b	C	z _i , B in b, c	D	$[n \Sigma x_i^2 - (\Sigma x_i)^2]$	E	$n \Sigma x_i^2 - (\Sigma x_{ij})^2$	I

Polynomial Approximation (Card 1)

001 *LBLA	Initialize	057 ST+6	Calculate $f_n(i) f(x_i)$.
002 CLR6		058 GT00	
003 CFB		059 *LBLA	
004 CF1		060 RCL8	
005 0		061 ENT1	
006 RTN		062 +	
007 *LBLC	Input N.	063 1	
008 F00		064 +	
009 PRTX		065 x	
010 ST07		066 RCL7	
011 F00		067 RCL6	
012 SPC		068 -	
013 RTN		069 x	
014 *LBLD	Input y_i .	070 X=Y	
015 F10		071 RCL7	
016 GT07		072 RCL8	
017 SF1		073 +	
018 F00		074 1	
019 PRTX		075 +	
020 ST01		076 x	
021 ST02		077 RCL8	
022 ST03		078 x	
023 ST04		079 -	
024 ST05		080 RCL7	
025 2		081 RCL8	
026 ST06		082 -	
027 *LBL0		083 ÷	
028 1		084 RCL8	
029 ST08		085 1	
030 RCL6		086 +	
031 2		087 ST08	
032 +		088 ÷	
033 F00		089 RCL9	
034 PRTX		090 X=Y	
035 F00		091 ST09	
036 SPC		092 RTN	
037 F10		093 *LBLA	Input order of polynomial.
038 *LBL7		094 F00	
039 F00		095 PRTX	
040 PRTX		096 ENT1	
041 ST+1		097 2	
042 1		098 X=Y	
043 RCL6		099 GT02	
044 RCL7		100 CLX	
045 +		101 3	
046 -		102 X=Y	
047 1		103 GT03	
048 ST+2		104 CLX	
049 ST09		105 4	
050 GSB0		106 X=Y	
051 ST+3		107 GT04	
052 GSB0		108 CLX	
053 ST+4		109 0	
054 GSB0		110 ÷	
055 ST+5		111 *LBL2	
056 2		112 CLX	

REGISTERS									
0	1 (f, t ₀), a ₀	2 (f, t ₁), a ₁	3 (f, t ₂), a ₂	4 (f, t ₃), a ₃	5 (f, t ₄), a ₄	6 2j	7 N	8 n	9 f _n (i) f(x _i)
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A 1 for print	B	C	D	E	F	G	H	I	J

113	ST04			169	X $\div$ Y		
114	#LBL3			170	X		
115	CLX			171	ST09		
116	ST05			172	LSTX		
117	#LBL4			173	RCL6		
118	RCL1			174	X		
119	RCL7			175	ST07		
120	1			176	CLX		
121	ST08			177	R/S		
122	+			178	#LBLd	(f, fi)	
123	$\div$			179	RCL7		
124	ST01			180	RCL8		
125	GSBd			181	1		
126	ST $\div$ 2			182	+		
127	GSBd			183	+		
128	ST $\div$ 3			184	N!		
129	GSBd			185	RCL7		
130	ST $\div$ 4			186	RCL8		
131	GSBd			187	-		
132	ST $\div$ 5			188	N!		
133	RCL7			189	X		
134	RCL7			190	RCL8		
135	RCL7			191	RCL8		
136	1			192	1		
137	-			193	+		
138	2			194	ST06		
139	X			195	+		
140	ST06			196	$\div$		
141	$\div$			197	RCL7		
142	3			198	N!		
143	X			199	ENT $\uparrow$		
144	ST08			200	X		
145	Rd			201	+		
146	2			202	RTN		
147	+			203	#LBLB	Set flag 0 for print.	
148	RCL6			204	SF0		
149	$\div$			205	1		
150	ST06			206	ST0A		
151	RCL3			207	RTN		
152	X						
153	ST-1						
154	RCL8						
155	RCL3						
156	X						
157	ST03						
158	RCL7						
159	RCL7						
160	RCL7						
161	2						
162	-						
163	$\div$						
164	5						
165	X						
166	3						
167	+						
168	RCL8						

LABELS					FLAGS	SET STATUS		
A Start	B Print?	C N $\rightarrow$	D $V_i \rightarrow$	E n $\rightarrow$	0 Print	FLAGS	TRIG	DISP
a	b	c	d	e (f_n, f_n)	1 $V_n, i > 1$	ON OFF		
0 Print i	1	2 Used	3 Used	4 Used	2	0 ☐ ON ☐ OFF	DEG ☐	FIX ☐
5	6	7 $i > 1$	8	9 $f_n(i) f(x_i)$	3	1 ☐ ON ☐ OFF	GRAD ☐	SCI ☐
						2 ☐ ON ☐ OFF	RAD ☐	ENG ☐
						3 ☐ ON ☐ OFF		n <u>2</u>

(Card 2)

001 RCL4		057 RCL7	
002 R+		058 XZY	
003 R+		059 x	
004 3		060 ST+8	
005 +		061 LSTX	
006 ENT+		062 RCL9	
007 ENT+		063 x	
008 5		064 RCL5	
009 -		065 x	
010 ÷		066 ST05	
011 2		067 LSTX	
012 x		068 STx8	
013 3		069 RCL6	
014 ÷		070 x	
015 RCL7	Continue	071 ST+1	
016 +		072 RCL8	
017 ST07		073 ST-3	
018 RCL4		074 *LBL8	
019 x		075 RCLA	
020 ST-2		076 X)0?	
021 LSTX		077 SFO	
022 RCL9		078 1	
023 x		079 R/S	
024 ST04		080 *LBL6	
025 R+		081 XZY	
026 R+		082 CSB9	
027 4		083 XZY	
028 +		084 CSB8	
029 ENT+		085 -	
030 ENT+		086 ST08	
031 7		087 LSTX	
032 -		088 2	
033 ÷		089 x	
034 3		090 +	
035 x		091 RCL8	
036 4		092 ÷	
037 ÷		093 ST06	
038 RCL6		094 2	
039 XZY		095 RCL8	
040 x		096 +	
041 ST06		097 ST08	
042 LSTX		098 RCL1	
043 RCL8		099 RCL6	
044 x		100 RCL2	
045 ST08		101 x	
046 R+		102 +	
047 ENT+		103 RCL6	
048 ENT+		104 ENT+	
049 ENT+		105 x	
050 3		106 RCL3	
051 -		107 x	
052 ÷		108 +	
053 7		109 ST01	
054 x		110 RCL2	
055 4		111 RCL6	
056 +		112 RCL3	

Input x_N, x_0 .

REGISTERS

0	1, b ₀ , c ₀ , d ₀	2, b ₁ , c ₁ , d ₁	3, b ₂ , c ₂ , d ₂	4, b ₃ , c ₃ , d ₃	5, b ₄ , c ₄ , d ₄	6 Used	7 Used	8 Used	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A 1 for print	B	C	D	E	F	G	H	I	J

113	x			169	RCL1		
114	2			170	CSB9		Output $d_0, d_1, d_2, d_3,$
115	x			171	R/S		$d_4.$
116	+			172	RCL2		
117	ST02			173	CSB9		
118	RCL4			174	R/S		
119	RCL6			175	RCL3		
120	x			176	CSB9		
121	3			177	R/S		
122	x			178	RCL4		
123	ST+3			179	CSB9		
124	RCL6			180	R/S		
125	x			181	RCL5		
126	ST+2			182	CSB9		
127	3			183	R/S		
128	+			184	RTN		
129	RCL6			185	LBLd		
130	x			186	CSB9		Input x, calculate $\hat{y}$.
131	ST+1			187	ST07		
132	RCL5			188	RCL2		
133	RCL6			189	x		
134	x			190	RCL7		
135	4			191	x ²		
136	x			192	RCL3		
137	ST+4			193	x		
138	RCL6			194	+		
139	x			195	RCL7		
140	1			196	3		
141	.			197	y ⁿ		
142	5			198	RCL4		
143	x			199	x		
144	ST+3			200	+		
145	LSTX			201	RCL7		
146	+			202	x ²		
147	RCL6			203	x ²		
148	x			204	RCL5		
149	ST+2			205	x		
150	4			206	+		
151	+			207	RCL1		
152	RCL6			208	+		
153	x			209	CSB8		
154	ST+1			210	RTN		
155	RCL8			211	LBL9		Subroutine for space
156	CHS			212	F0?		and print.
157	ENT†			213	SPC		
158	ENT†			214	LBL8		
159	ENT†			215	F0?		
160	ST×2			216	PRTX		
161	x			217	RTN		
162	ST×3						
163	x						
164	ST×4						
165	x						
166	ST×5						
167	R/S						
168	LBLc						

LABELS					FLAGS	SET STATUS		
A	B Set flag 0	C	D	E	0 Print	FLAGS		
a (continue)	b $x_1 \rightarrow x_0$	c $\rightarrow d_0, d_1,$	d $x \rightarrow \hat{y}$	e	1	ON OFF	TRNG	DISP
0	1	2	3	4	2	0 ☐ ☐	DEG ☐	FIX ☐
0	1	2	3	4	2	1 ☐ ☐	GRAD ☐	SCI ☐
0	1	2	3	4	2	2 ☐ ☐	RAD ☐	ENG ☐
0	1	2	3	4	2	3 ☐ ☐		n <u>2</u>
0	1	2	3	4	2			

t Statistics

001 *LBLA	Initialize	057 ST-2	Set flag 0 for print.						
002 CPO		058 X ²							
003 0		059 ST-3							
004 ST01		060 RCL1							
005 ST02		061 1							
006 ST03		062 -							
007 RTN		063 ST01							
008 *LBLC		064 GSB0							
009 F0 ⁰		065 RTN							
010 GSB9		066 *LBLB		Print x _i , y _i .					
011 -	067 SF0								
012 ST+2	068 1								
013 X ²	069 RTN								
014 ST+3	070 *LBL9								
015 RCL1	071 X ² Y								
016 1	072 SPC								
017 +	073 PRTX								
018 ST01	074 X ² Y								
019 GSB0	075 PRTX	Input x _i or y _i for t statistic for two means.							
020 RTN	076 RTN								
021 *LBL E	077 *LBL6								
022 RCL2	078 GSB0								
023 RCL1	079 ST+2								
024 ÷	080 X ²								
025 GSB1	081 ST+3								
026 GSB0	082 RCL1								
027 R/S	083 1								
028 RCL3	084 +								
029 RCL2	085 ST01	d							
030 X ²	086 GSB0								
031 RCL1	087 GSB1								
032 ÷	088 RTN								
033 -	089 *LBLC								
034 RCL1	090 ST07								
035 1	091 RCL1								
036 -	092 ST04								
037 ÷	093 RCL2								
038 IX	094 ST05								
039 GSB0	095 RCL3								
040 R/S	096 ST06	REGISTERS							
041 RCL1	097 0								
042 IX	098 ST01								
043 ÷	099 ST02								
044 +	100 ST03								
045 GSB0	101 RCL7								
046 R/S	102 GSB0								
047 RCL1	103 GSB1								
048 1	104 RTN								
049 -	105 *LBLd								
050 GSB0	106 RCL6	0 Used							
051 GSB1	107 RCL5								
052 RTN	108 X ²								
053 *LBLD	109 RCL4								
054 F0 ⁰	110 ÷								
055 GSB9	111 -								
056 -	112 RCL3								
REGISTERS									
0 Used	1 n ₁ , n ₂	2 Used	3 Used	4 n ₁	5 Σx_1	6 Σx_1^2	7 d	8 n ₁ + n ₂ - 2	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	F	G	H	I	J

113	+		169	#LBL1	
114	RCL2		170	F0?	
115	X ²		171	SPC	
116	RCL1		172	RTN	
117	÷				
118	-				
119	RCL1				
120	RCL4				
121	+				
122	2	t_2, df_2			
123	-				
124	ST08				
125	÷				
126	JX				
127	1				
128	RCL1				
129	÷				
130	1				
131	RCL4				
132	÷				
133	+				
134	JX				
135	x				
136	RCL5				
137	RCL4				
138	÷				
139	RCL2				
140	RCL1				
141	÷				
142	-				
143	RCL7				
144	-				
145	X ² Y				
146	÷				
147	CSB0				
148	R/S				
149	RCL8				
150	CSB0				
151	CSB1				
152	RTN				
153	#LBL6				
154	CSB0				
155	ST-2				
156	X ²	For correction of x_k or y_k .			
157	ST-3				
158	RCL1				
159	1				
160	-				
161	ST01				
162	CSB0				
163	CSB1				
164	RTN				
165	#LBL0				
166	F0?				
167	PRTX				
168	RTN				

Subroutine for space.

For correction of x_k or y_k .

Subroutine for print.

LABELS					FLAGS	SET STATUS		
A Start	B Print	C x_1, y_1	D x_k, y_k	E $\rightarrow D, S_D \dots$	0 Print	FLAGS		TRIG
^a x_1 or y_1	^b x_k or y_k	^c $d \rightarrow$	^d $t_2; df_2$	^e	1	0 ☐ ON	0 ☐ OFF	DEG ☐
0 Print	1 Space	2	3	4	2	1 ☐ ON	1 ☐ OFF	GRAD ☐
5	6	7	8	9 Print x_1, y_1	3	2 ☐ ON	2 ☐ OFF	RAD ☐
						3 ☐ ON	3 ☐ OFF	ENG ☐

n 2

Chi-Square Evaluation

001 #LBLA		057 RCL3																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																			
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113 RTN					
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LABELS					FLAGS	SET STATUS		
A Start	B Print	C $O_i \uparrow E_i \rightarrow$	D $O_k \uparrow E_k$	E X_i^2	0 Print	FLAGS		DISP
a $O_j (\Sigma+)$	b $O_h (\Sigma-)$	c X_2^2	d E	e	1 Equal E_i	ON OFF	TRIG	FIX ☒
0	1 X_2^2	2	3	4	2	0 ☐ ☒	DEG ☒	SCI ☐
5	6	7 Space	8 Print	9 Print	3	1 ☐ ☒	GRAD ☐	ENG ☐
						2 ☐ ☒	RAD ☐	n <u>2</u>
						3 ☐ ☒		

Contingency Table

001	#LBLA		057	#LBLA					
002	CLRC	Initialize	058	SF0					Set flag for print.
003	CF0		059	1					
004	CF1		060	RTN					Print x_{ij} , x_{2j} .
005	0		061	#LBL9					
006	RTN		062	X2Y					
007	#LBLB	Input x_{ij} , x_{2j} for $2 \times k$.	063	PRTX					
008	F0?		064	X2Y					
009	CSB9		065	PRTX					
010	0		066	RTN					
011	STOC		067	#LBLb					$3 \times k$
012	R4		068	F0?					
013	CSB0		069	CSB8					Input x_{ij} , x_{2j} , x_{3j} .
014	RTN		070	STOC					
015	#LBLD		071	F1?					
016	RCL0		072	CHS					
017	RCL4		073	ST+3					
018	x	$2 \times k$	074	STOB					
019	RCL1	X^2	075	STO?					
020	+		076	ENT?					
021	RCL0		077	x					
022	RCL5		078	STOB					
023	RCL2		079	R4					
024	+		080	#LBL0					Input x_{ij} , x_{2j} , for $3 \times k$ and $2 \times k$.
025	x		081	STOB					
026	+		082	F1?					
027	RCL0		083	CHS					
028	-		084	ST+2					
029	STOC		085	ST+0					
030	CSB7		086	ST+7					
031	R/S		087	ENT?					
032	RCLD	$2 \times k$	088	x					
033	RCL0	C_c	089	STOB					
034	RCLD		090	R4					
035	+		091	STOA					
036	+		092	F1?					
037	TX		093	CHS					
038	CSB7		094	ST+1					
039	CSB6		095	ST+0					
040	RTN		096	ST+7					
041	#LBL E	$2 \times k$	097	ENT?					
042	RCL1	R_1, R_2, T	098	x					
043	CSB7		099	RCL7					
044	R/S		100	+					
045	RCL2		101	ST+4					
046	CSB7		102	RCL9					
047	R/S		103	RCL7					
048	+		104	+					
049	CSB7		105	ST+5					
050	CSB6		106	RCL8					
051	RTN		107	RCL7					
052	#LBLC	$2 \times k$ correction.	108	+					
053	SF1		109	ST+6					
054	CSBB		110	1					
055	CF1		111	F1?					
056	RTN		112	CHS					
REGISTERS									
0 T	1 R ₁	2 R ₂	3 R ₃	4 $\Sigma^2 x_{1j}/C_j$	5 $\Sigma^2 x_{2j}/C_j$	6 $\Sigma^2 x_{3j}/C_j$	7 C _j	8 x_{3j}^2	9 x_{2j}^2
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A x_{1j}	B x_{2j}	C x_{3j}	D X^2	E k	F	G	H	I	J

113	RCL E			169	RCL 0				
114	+			170	CSB 7				
115	STOE			171	RTN				3 x k correction.
116	GSB 7			172	#LBL c				
117	GSB 6			173	SF 1				
118	CF 1			174	GSB 6				
119	R/S			175	CF 1				
120	RCL 7			176	RTN				Print x_{1j}, x_{2j}, x_{3j} .
121	GSB 7			177	#LBL 8				
122	GSB 6			178	R 1				
123	RTN			179	R 1				
124	#LBL 4			180	PRTX				
125	RCL 1			181	R 1				
126	RCL 2			182	PRTX				
127	RCL 3			183	R 1				
128	+			184	PRTX				
129	+			185	RTN				
130	STOE	3 x k		186	#LBL 7				Subroutine for print.
131	RCL 4	x^2		187	F0?				
132	RCL 1			188	PRTX				
133	+			189	RTN				
134	STOE			190	#LBL 6				Subroutine for space.
135	RCL 5			191	F0?				
136	RCL 2			192	SPC				
137	+			193	RTN				
138	ST+9								
139	RCL 6								
140	RCL 3								
141	+								
142	ST+9								
143	RCL 9								
144	1								
145	-								
146	RCL 0								
147	x								
148	GSB 7								
149	R/S								
150	ENT 1	3 x k							
151	ENT 1	C_c							
152	RCL 0								
153	+								
154	+								
155	JX								
156	GSB 7								
157	GSB 6								
158	R/S								
159	#LBL e								
160	RCL 1	3 x k							
161	GSB 7	R_1, R_2, R_3, T							
162	R/S								
163	RCL 2								
164	GSB 7								
165	R/S								
166	RCL 3								
167	GSB 7								
168	R/S								

LABELS					FLAGS		SET STATUS		
A Start	B x_{1j}, x_{2j}	C Correction	D $x^2; C_c$	E $R_1; R_2; T$	0 Print	FLAGS	TRIG	DISP	
Print	Input 3 x k	Correction	$x^2; C_c$	$R_1; R_2$	1 Correction	ON OFF			
Used	1	2	3	4	2	0 ☐ ☒	DEG ☒	FIX ☒	
	Space	Print	Print	Print	3	1 ☐ ☒	GRAD ☐	SCI ☐	
						2 ☐ ☒	RAD ☐	ENG ☐	
						3 ☐ ☒		n <u>2</u>	

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LABELS					FLAGS	SET STATUS		
A Start	B Print	C ($\Sigma+$)	D ($\Sigma-$)	E $\rightarrow r_s, z$	0 Print	FLAGS	TRIG	DISP
a	b	c	d	e	1 ($\Sigma-$)	ON OFF 0 ☐ ☒	DEG ☒	FIX ☒
0	1	2	3	4	2	1 ☐ ☒	GRAD ☐	SCI ☐
5	6	7	8	9	3	2 ☐ ☒	RAD ☐	ENG ☐
						3 ☐ ☒		n <u>2</u>

$\bar{x}$ and R Control Chart

001 *LBLA		057 R/S	
002 CLRG	Initialize	058 *LBLB	
003 CF8		059 RCL5	
004 CF1		060 CSB9	
005 0		061 RTN	
006 RTN	Store 1 in R _E for print.	062 *LBLA	
007 *LBLB		063 CF1	
008 1		064 RCL6	
009 STOE		065 1	
010 RTN		066 +	
011 *LBLC		067 STOE	$\bar{x}_i$
012 STOE		068 RCL2	
013 RCL4		069 RCL1	
014 STOA		070 ÷	
015 RCL5		071 CSB9	
016 STOE		072 ST+7	
017 RCL0		073 R/S	
018 CSB9		074 *LBLA	
019 F10		075 RCL4	
020 CTO1		076 RCL5	
021 0		077 -	
022 STO1	Input x _{ij} .	078 ST+8	R _i
023 STO2		079 CSB9	
024 STO3		080 CSB7	
025 XZY		081 R/S	
026 STO4		082 *LBLB	
027 STO5		083 RCL7	
028 SF1		084 RCL6	
029 *LBL1		085 ÷	$\bar{x}$
030 RCL4		086 CSB9	
031 XZ1		087 RTN	
032 X1Y0		088 *LBLB	
033 STO4		089 RCL8	
034 RCL5		090 RCL6	
035 XZY		091 +	
036 X4Y0		092 STO3	R
037 STO5		093 CSB9	
038 F00		094 CSB7	
039 CMS		095 R/S	
040 ST+2		096 *LBLC	
041 XZ		097 RCL3	
042 F00		098 x	
043 CMS		099 RCL7	
044 ST+3		100 RCL6	L $\bar{x}$
045 RCL1		101 ÷	
046 1		102 XZY	
047 F00		103 -	
048 CMS		104 CSB9	
049 +		105 R/S	
050 STO1		106 *LBLC	
051 CSB9		107 LSTX	
052 RTN		108 2	U $\bar{x}$
053 *LBLB		109 x	
054 CSB7		110 +	
055 RCL4		111 CSB9	
056 CSB9	x _{max} , x _{min}	112 CSB7	

REGISTERS

0 x _{ij}	1 n	2 Σx _{ij}	3 Σx _{ij} ² , R	4 x _{max}	5 x _{min}	6 m	7 Σx _i	8 ΣR _i	9 Used
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A Last x _{max}	B Last x _{min}	C	D	E 1 for print	F				

Operating Characteristic Curves

001	*LBL4		057	=	
002	CF0	Initialize	058	LSTX	
003	CLR6		059	RCL1	
004	0		060	RCL4	
005	RTN		061	-	
006	*LBL6	Set flag 0 for print.	062	RCL2	
007	SF0		063	-	
008	1		064	+	
009	RTN		065	+	
010	*LBL0	Store N.	066	RCL6	
011	GSB3		067	X	
012	ST01		068	ST06	
013	RTN		069	ST+7	
014	*LBL0		070	RCL3	
015	ST03	Store n, c for finite lot size.	071	1	
016	X=		072	RCL8	
017	ST02		073	+	
018	GSB3		074	ST08	
019	X=		075	X=	
020	GSB3		076	GT00	
021	RTN		077	1	
022	*LBL6		078	RCL7	
023	GSB4		079	X=	
024	GSB3		080	X=	
025	RCL1		081	GSB3	
026	X		082	R=	
027	INT		083	*LBL4	
028	ST04		084	-	
029	RCL1		085	LSTX	
030	RCL2	Finite lot size.	086	X=	
031	GSB4		087	GSB9	
032	RCL1		088	ST05	
033	RCL4		089	1	
034	-		090	ST07	
035	RCL2		091	+	
036	GSB4		092	ST06	
037	R1		093	CLX	
038	=		094	X=	
039	ST05		095	GT08	
040	ST06		096	*LBL1	
041	ST07		097	R+	
042	RCL3		098	1	
043	0		099	RCL7	
044	ST08		100	+	
045	X=		101	ST07	
046	GSB5		102	X=	
047	*LBL6		103	GT07	
048	RCL4		104	RCL5	
049	-		105	X=	
050	RCL8		106	+	
051	RCL2		107	LSTX	
052	-		108	=	
053	X		109	RCL6	
054	RCL8		110	X	
055	1		111	ST06	
056	+		112	GT01	

REGISTERS									
0	1 N, n	2 n, p	3 c, f(0)	4 M	5 f(0)	6 , c	7 , counter	8 Used	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	F	G	H	I	J

113	RTN			169	RCL8		
114	*LBL7			170	X		
115	RCL6			171	RCL4		
116	RTN			172	X		
117	*LBL8			173	STO4		
118	1			174	ST+5		
119	GSB3			175	RCL7		
120	RTN			176	1		
121	*LBL9			177	+		
122	STO6			178	STO7		
123	X $\leftrightarrow$ Y			179	RCL6		
124	RTN			180	X $\leftrightarrow$ Y $\leftrightarrow$		
125	*LBL4			181	CTO2		
126	STO6			182	1		
127	X $\leftrightarrow$ Y			183	RCL5		
128	STO1			184	X $\leftrightarrow$ Y $\leftrightarrow$		
129	GSB3			185	X $\leftrightarrow$ Y		
130	X $\leftrightarrow$ Y			186	GSB3		
131	GSB3			187	R/S		
132	RTN			188	*LBL6		
133	*LBL4			189	RCL3		
134	STO2			190	GSB3		
135	GSB4			191	R/S		
136	GSB3			192	RTN		
137	STO2			193	*LBL5		
138	RCL2			194	RCL5		
139	1			195	GSB3		
140	-			196	R/S		
141	CHS			197	RTN		
142	÷			198	*LBL3		
143	STO8			199	F0?		
144	LSTX			200	PRTX		
145	RCL1			201	RTN		
146	Y $\leftrightarrow$ X			202	*LBL4		
147	STO3			203	F0?		
148	RCL6			204	SPC		
149	0			205	RTN		
150	STO7						
151	X $\leftrightarrow$ Y $\leftrightarrow$						
152	GSB6						
153	CLX						
154	RCL1						
155	X $\leftrightarrow$ Y						
156	X $\leftrightarrow$ Y $\leftrightarrow$						
157	CTO6						
158	RCL3						
159	STO4						
160	STO5						
161	*LBL2						
162	RCL1						
163	RCL7						
164	-						
165	RCL7						
166	1						
167	+						
168	÷						

LABELS					FLAGS	SET STATUS		
A Start	B Print	C N	D nfc→	E p→P _a	0 Print	FLAGS	TRIG	DISP
a f(x)	b	c	d nfc→	e p→P _a	1	ON OFF	DEG ☒	FIX ☒
0 P _a	1 f(x+1)	2 Used	3 Print	4 Space	2	1 ☐	GRAD ☐	SCI ☐
5 c, f(0)	6 f(0)	7 c	8 1	9 STO 6	3	2 ☐	RAD ☐	ENG ☐
						3 ☐		n <u>2</u>

Subroutine for print.

Subroutine for space.

Single- and Multi-Server Queues

001 #LBLA		057 RCL1							
002 CSB9		058 RCL3							
003 ST01		059 -							
004 ST01		060 ÷							
005 R4	ρ	061 ST04						Lq, L	
006 ST02		062 SPC							
007 XZY		063 PRTX							
008 ST05		064 R/S							
009 ÷		065 #LBLE							
010 ST03		066 RCL3							
011 PRTX		067 +							
012 R/S		068 ST06							
013 #LBLE		069 PRTX							
014 1		070 R/S							
015 ST04		071 #LBLD							
016 0		072 RCL4							
017 #LBL1		073 RCL2						Tq, T	
018 RCL4		074 ÷							
019 +		075 SPC							
020 LSTX		076 PRTX							
021 RCL3		077 R/S							
022 x		078 #LBLD							
023 RCL1		079 RCL6							
024 RCL1		080 RCL2							
025 -		081 ÷							
026 1		082 PRTX							
027 +		083 R/S							
028 ÷		084 #LBLE						P(t)	
029 ST04	P_0, P_D	085 SPC							
030 R4		086 PRTX							
031 DSZ1		087 RCL1							
032 ST01		088 RCL5							
033 1		089 x							
034 RCL3		090 RCL2							
035 RCL1		091 -							
036 ÷		092 x							
037 -		093 CWS							
038 RCL4		094 e ^x							
039 XZY		095 RCL1							
040 ÷		096 x							
041 ST01		097 PRTX							
042 +		098 SPC							
043 1/x		099 R/S							
044 SPC		100 #LBL9						Print u, λ , n.	
045 PRTX		101 R4							
046 R/S		102 R4							
047 #LBLE		103 SPC							
048 RCL1		104 PRTX							
049 x		105 R1							
050 ST01		106 PRTX							
051 PRTX		107 R1							
052 R/S		108 PRTX							
053 #LBLE		109 RTN							
054 RCL1		110 #LBLA						m, n	
055 RCL3		111 CSB8							
056 x		112 ST02							
REGISTERS									
0	1 n, m	2 λ , n	3 ρ	4 Lq, k	5 μ , O_k , L	6 L, ΣO_k	7 $\Sigma k O_k$, -F	8 a	9 Used
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	F	G	H	I	Used, P_D

113 R4		169 ST05	
114 ST01		170 SPC	
115 R/S		171 PRTX	
116 *LBL6	P	172 R/S	T
117 GSB8		173 *LBLc	
118 ST08		174 RCL8	
119 +		175 x	
120 ST03		176 PRTX	
121 PRTX		177 R/S	Lq
122 R/S		178 *LBLd	
123 *LBLc		179 RCL5	
124 CLX		180 RCL1	
125 ST07		181 +	
126 1		182 1	
127 ST04		183 -	
128 ST05		184 RCL3	
129 ST06		185 1	
130 *LBL3		186 +	
131 RCL2		187 x	
132 RCL4		188 ST07	
133 X>Y?		189 1	
134 X<Y		190 +	
135 RCL3		191 RCL1	
136 X<Y		192 x	
137 +		193 SPC	
138 RCL1		194 PRTX	
139 RCL4		195 R/S	
140 -		196 *LBLd	Tq
141 1	L	197 RCL8	
142 +		198 x	
143 x		199 PRTX	
144 RCL5		200 R/S	
145 x		201 *LBLe	
146 ST05		202 RCL7	
147 EEX		203 CHS	F
148 CHS		204 SPC	
149 9		205 PRTX	
150 0		206 SPC	
151 X>Y?		207 R/S	
152 GT02		208 *LBL8	Print m, n, s, a.
153 R4		209 R4	
154 ST+6		210 SPC	
155 RCL4		211 PRTX	
156 x		212 R1	
157 ST+7		213 PRTX	
158 RCL1		214 RTN	
159 RCL4			
160 1			
161 +			
162 ST04			
163 X<Y?			
164 GT03			
165 *LBL2			
166 RCL7			
167 RCL6			
168 +			

LABELS					FLAGS	SET STATUS		
A ...→p	B →P ₀ ..P _b	C →L _q ..→L	D →T _q ..→T	E t→P(t)	0	FLAGS		
p mfn→	D Sta→p	C →L _q ..→T	D →L _q ..→T _q	e →F	1	TRIG		
					2	DISP		
0	1 P ₀ ..P _b	2 L	3 K	4	3	ON OFF	DEG ☐	FIX ☐
						0 ☐ ☐	GRAD ☐	SCI ☐
						1 ☐ ☐	RAD ☐	ENG ☐
5	6	7	8 Print	9 Print		2 ☐ ☐		n—2
						3 ☐ ☐		

Appendix A

MAGNETIC CARD

SYMBOLS AND CONVENTIONS

SYMBOL OR CONVENTION	INDICATED MEANING
White mnemonic: x A	White mnemonics are associated with the user-definable key they are above when the card is inserted in the calculator's window slot. In this case the value of x could be input by keying it in and pressing A .
Gold mnemonic: y x f E	Gold mnemonics are similar to white mnemonics except that the gold f key must be pressed before the user-definable key. In this case y could be input by pressing f E .
x ↑ y A	↑ is the symbol for ENTER↑ . In this case ENTER↑ is used to separate the input variables x and y. To input both x and y you would key in x, press ENTER↑ , key in y and press A .
X A	The box around the variable x indicates input by pressing STO A .
(x) A	Parentheses indicate an option. In this case, x is not a required input but could be input in special cases.
→ x A	→ is the symbol for calculate. This indicates that you may calculate x by pressing key A .
→ x, y, z A	This indicates that x, y, and z are calculated by pressing A once. The values would be printed in x, y, z order.
→ x; y; z A	The semi-colons indicate that after x has been calculated using A , y and z may be calculated by pressing R/S .
→ "x", y A	The quote marks indicate that the x value will be "paused" or held in the display for one second. The pause will be followed by the display of y.
↔ x A	The two-way arrow ↔ indicates that x may be either output or input when the associated user-definable key is pressed. If numeric keys have been pressed between user-definable keys, x is stored. If numeric keys have not been pressed, the program will calculate x.

SYMBOLS AND CONVENTIONS (Continued)

SYMBOL OR CONVENTION	INDICATED MEANING
P? A	The question mark indicates that this is a mode setting, while the mnemonic indicates the type of mode being set. In this case a print mode is controlled. Mode settings typically have a 1.00 or 0.00 indicator displayed after they are executed. If 1.00 is displayed, the mode is on. If 0.00 is displayed, it is off.
START A	The word START is an example of a command. The start function should be performed to begin or start a program. It is included when initialization is necessary.
DEL A	This special command indicates that the last value or set of values input may be deleted by pressing A .
→X; ... A	Three dots (...) indicate that additional output follows. See User Instructions for complete description of variables output.



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A ● C D E

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