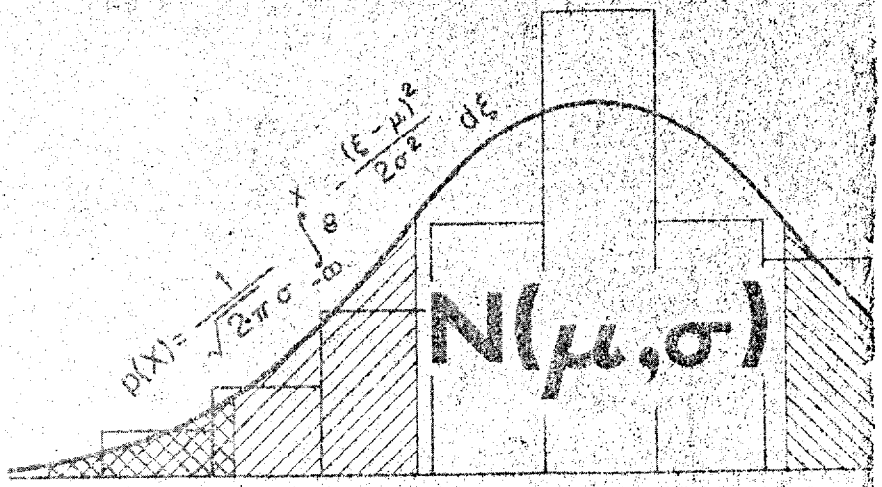
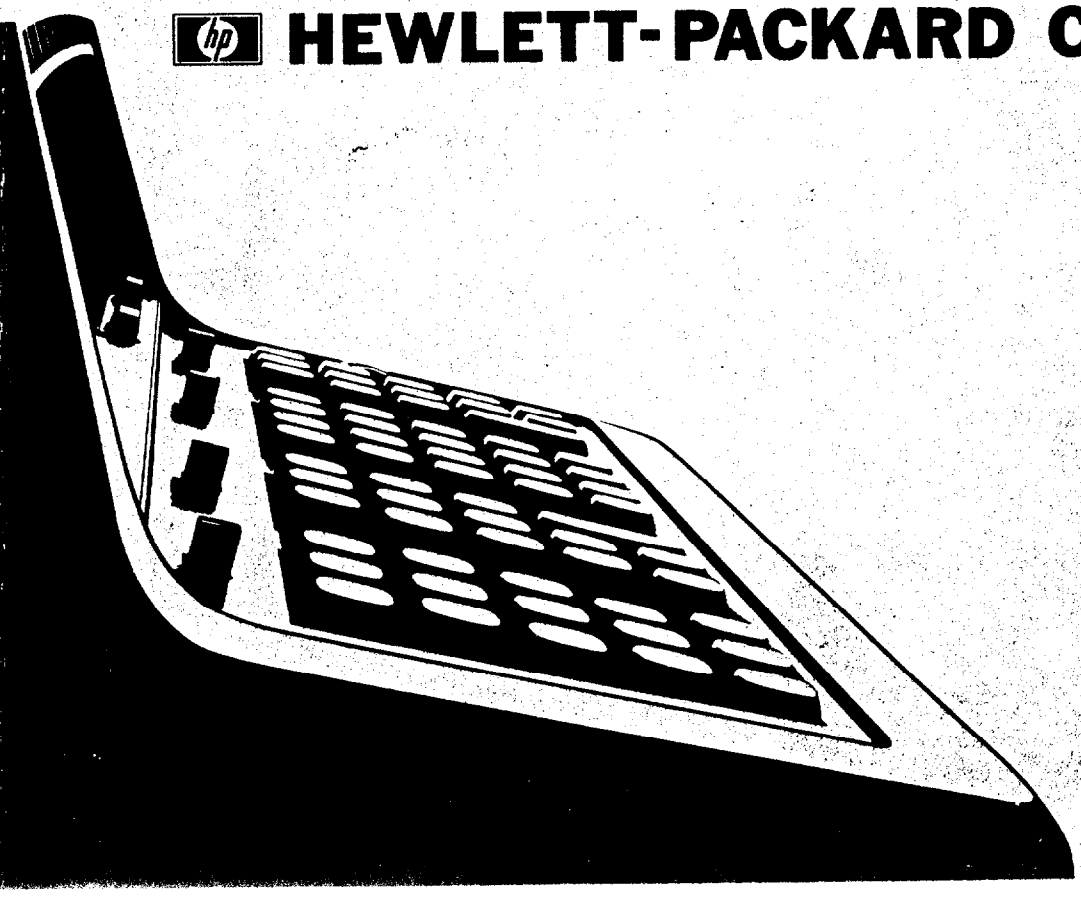


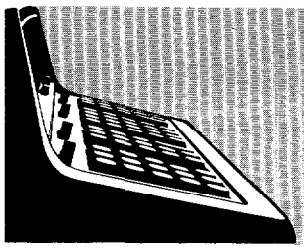
HEWLETT-PACKARD CALCULATOR

STAT-PAC

Vol. I



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INTRODUCTION

This package of statistical programs for the Hewlett-Packard 9100A/B calculator has been designed to assist you in your statistical problem solving. Many of the programs contained in this Volume are actual customer supplied programs or customer suggested programs. A future package, Volume II will be generated from additional programs submitted by you, our customer.

We hope that you will find this package valuable in your everyday statistical computations. Your comments and suggestions for future statistically related calculator applications are welcome. If you develop a calculator program which you would like to have included in STAT-PAC VOLUME II, please forward it to us for review. Please write your programs in a format containing: Program Description, Reference, User Instructions, Program Steps, and a typical Example illustrating how you employ the program. All programs placed in Volume II will, of course, be included with credit to the author and company affiliation.

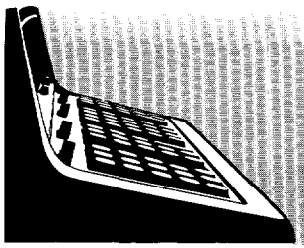
We wholeheartedly hope that this collection of statistical programs will significantly reduce your computational "chores" and increase the 9100 Calculator's usefulness to you.

Sincerely,

HEWLETT-PACKARD COMPANY

Dave Cole
Applications Engineer
P.O. Box 301
Loveland, Colorado 80537

DC/bd

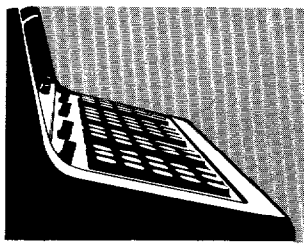


CONTENTS

This package is divided into ten sections, these being:

I — General Statistics
II — Distribution Functions
III — Test Statistics
IV — Curve Fitting
V — Harmonic Analysis
VI — Sampling Theory
VII — Analysis of Variance
VIII — Operations Research
IX — Reliability and Quality Control
X — Plotter Programs

The programs included in each section are designed to be both functional and informational in that many are general enough to be applied directly to any statistical data reduction problem, whereas others are quite specific in application. The specialized programs such as Buffon Needle Experiment, Fourier Series - Sampled X(t), Gompertz Curve Fit, Weibull Distribution Via Failed and Suspended Sets, Control Ellipses Plot, and others demonstrate the variety and range of problems solvable on the Hewlett-Packard 9100 Calculator.



PROGRAM LISTING

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I-3	Cumulative Mean ($\bar{X}_c$) and Cumulative Standard Deviation (s_c)	
I-4	Two Variable Sums of Squares (With Corrector Option)	
I-5	Mean, Variance, Moment Coefficient of Skewness, Moment Coefficient of Kurtosis	
I-6	Covariance and Coefficient of Correlation	
I-7	Permutations	
I-8	Combinations	
I-9	Histogram Generation (Without Plot)	9100B ONLY
I-10	Arithmetic, Geometric, Harmonic, Means	
I-11	Random Number Generator (Uniform)	
I-12	Random Number Generator (Normal)	
I-13	Normal Probability Paper Plot Points	
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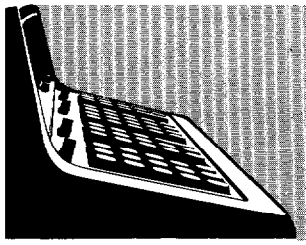
IX-1	Weibull Distribution Parameter Calculation For Failure Data	9100B ONLY
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X-18	General Point Plot	
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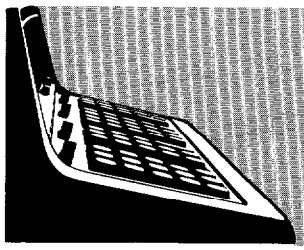


STAT-PAC -- PROGRAM LIBRARY DUPLICATION

The table below lists those programs which are found published in the 9100A/B Libraries or in the PLOTTER PACKET, 09100-76999. These programs, in most cases, have been modified for printer accommodation and are included in the STAT-PAC in order to arrange all your statistics programs into a single compact packet. These programs will be denoted by a ▲ following their STAT-PAC numbers.

TITLE	STAT-PAC No.	
Mean and Standard Deviation of Grouped Data	I-2	9100B Library
Histogram Generation (Without Plot	I-9	9100B Library
Random Number Generator (Uniform)	I-11	9100B Library
Normal Probability Integral	II-1	9100B Library
Poisson Distribution	II-4	9100B Library
χ^2 – Chi Square Distribution	II-5	9100B Library
F – Distribution	II-9	9100B Library
t Statistic for Means of Two Samples	III-1	9100A Library
χ^2 – Chi Square Evaluation Expected Values Equal ($E_i = E$)	III-5	9100B Library
χ^2 2 x 2 Contingency Table	III-6	9100A Library
χ^2 Chi Square Evaluation Expected Values Unequal ($E_i \neq E_j$)	III-7	9100B Library
Linear Regression and Correlation Coefficient	IV-1	9100B Library
Multiple Linear Regression 3 Variable	IV-3	9100B Library
Least Squares Fit – Power Curve	IV-6	9100B Library
Least Squares Fit – Exponential	IV-7	9100B Library
The Least Square Parabola	IV-8	9100B Library
Program For Simultaneous Solution of Four Equations in Four Unknowns With Printer	IV-13	9100B Library
One Way Analysis of Variance m x n	VII-1	9100B Library
Two Way Analysis of Variance	VII-2	9100B Library
Two Way Analysis of Variance With Replicates	VII-3	9100B Library
Weibull Distribution Parameter Calculation For Failure Data	IX-1	9100B Library
Histogram Generation (With Plot)	X-1	9100B Library
Normal Curve Overlay	X-2	9100B Library
Cumulative Probability Functions	X-4	9100B Library

TITLE	STAT-PAC No.	
Linear Regression And Plot	X-6	Plotter Packet
Power Curve Regression And Plot	X-8	Plotter Packet
Exponential Regression And Plot	X-10	Plotter Packet



9120A PRINTER

Each program has built-in printer commands eliminating your need to change the programs to accommodate a printer. These print commands have been incorporated both at input and output points in the programs. The USER INSTRUCTIONS do not assume that a printer is attached to the calculator, and contain PRESS: CONTINUE's between various output displays. If the printer is used, the USER will find that the calculator is automatically advanced through the output displays printing the results. However, if no printer is used, the operator advances through the output displays by use of the CONTINUE key as indicated in the instructions. Similarly, if no printer is used, the operator must press a final CONTINUE after the last display to place the calculator "in-ready" for another case.

To convert the STAT-PAC programs for use without a printer, the PRINT program commands should be replaced by STOP's and CONTINUE's. To assist in these changes, the program step pages have been annotated to locate the program steps requiring alteration. These annotations are:

s : change step to STOP

All other PRINT commands should be replaced by CONTINUE's

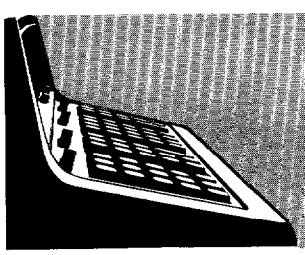
9125A PLOTTER PROGRAMS

Section X of the STAT-PAC contains 20 programs for the calculator/printer/plotter combination. These programs, such as Curve Fitting, Histogramming, Distribution Function Plotting, and Point Plotting, make optimum use of the plotter. The plotting section of the STAT-PAC contains general purpose plotting programs for:

a. Plotting data points prior to an analysis

b. Plotting general functions such as families of parametric distributions

It also contains very specific programs for plotting various distribution functions; normal, log normal, χ^2 (Chi-Squared), and others relating to regression analysis. Where possible, plot programs have been included for applications from the various ten sections.



STAT-PAC USAGE

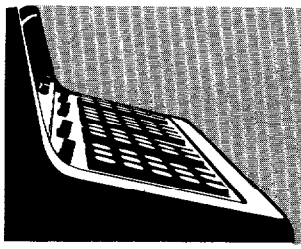
Each program in the STAT-PAC is documented with:

1. Program Description, Mathematical Formulas, References
2. USER INSTRUCTIONS
3. EXAMPLE Problem
4. Program STEPS

To ensure proper usage, the user should review the program description and mathematical formulation to verify its acceptance for his problem. Since many of the programs are customer programs, and since a variety of references were employed, a mathematical verification is justified.

Next, the program should be keyed into the calculator using the program steps. If the 9120A printer is not available, the PRINT commands (steps) should be replaced by the appropriate STOP or CONTINUE command.

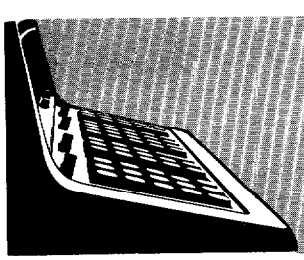
Finally, the program EXAMPLE should be run to verify proper interpretation of the USER INSTRUCTIONS and error-free program step entry.



FLAG USAGE

Programs which generate statistics for sets of data $\{X_i\}$, $\{X_i, Y_i\}$, $\{X_i, Y_i, Z_i\}$, etc., use the FLAG to signal that the last entry $\{X_n\}$, $\{X_n, Y_n\}$, ..., has been entered. The USER INSTRUCTIONS for this operation are given below:

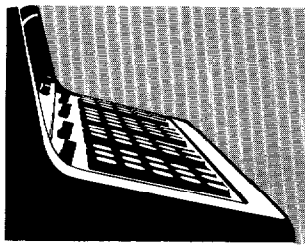
USER INSTRUCTIONS	USER RESPONSE
DISPLAY 0 ——— Z i ——— Y 0 ——— X	
ENTER DATA: X_i → X PRESS: CONTINUE } "entered"	Here the user places the input data into the prescribed display register. The data is "entered" via the "CONTINUE".
NO ← Has all data been entered? → YES	
PRESS: SET FLAG PRESS: CONTINUE	The FLAG is set after all data has been "entered".



ERROR CORRECTING OPTIONS

Several of the STAT-PAC programs contain "error correcting options". These options allow the user to correct for data input errors without re-entering the entire data set. The USER INSTRUCTIONS are seen below:

USER INSTRUCTIONS	USER RESPONSE
DISPLAY 0 ——— Z i ——— Y 0 ——— X ENTER DATA: X_i ⟶ X PRESS: CONTINUE	Enter the elements of the desired data set $\{X_n\}$.
NO ← Has all data been correctly "entered"? → YES	Here the user reviews his input data to verify that each element of $\{X_n\}$ has been "entered". The set of data actually "entered" may contain errors, however, the "entered" set $\{X_n'\}$ contains the desired set $\{X_n\}$.
NO ← Has all the data been "entered" free of errors? → YES Employ Correction Option PRESS: SET FLAG PRESS: CONTINUE	



KEY MNEMONICS

The program step pages employ the following mnemonics:

KEY	MNEMONIC	CODE
0	0	00
1	1	01
2	2	02
3	3	03
4	4	04
5	5	05
6	6	06
7	7	07
8	8	10
9	9	11
e	e	12
a	a	13
b	b	14
f	f	15
c	c	16
d	d	17
CLEAR	CLR	20
.	.	21
ROLL ↑	RUP	22
x→( )	XTO	23
y⇄( )	YE	24
↓	DN	25
ENTER EXP	EEX	26

KEY	MNEMONIC	CODE
↑	UP	27
$x \rightleftarrows y$	XEY	30
ROLL ↓	RDN	31
CHG SIGN	CHS	32
+	+	33
−	-	34
÷	DIV	35
×	X	36
CLEAR x	CLX	37
$y \rightarrow ( )$	YTO	40
STOP	STP	41
FMT	FMT	42
IF FLAG	IFG	43
GO TO ( )	GTO	44
PRINT	PNT	45
END	END	46
CONT	CNT	47
IF $x=y$	X=Y	50
IF $x<y$	X<Y	52
IF $x>y$	X>Y	53
SET FLAG	SFL	54
$\lvert y \rvert$	|Y|	55
π	π	56
PAUSE	PSE	57
ACC +	AC+	60
RCL	RCL	61

KEY	MNEMONIC	CODE
TO POLAR	POL	62
ACC —	AC-	63
int x	INT	64
ln x	LN	65
TO RECT	RCT	66
$x \leftarrow ( )$	XFR	67
hyper ▾	HYP	67
sin x	SIN	70
tan x	TAN	71
arc ▾	ARC	72
cos x	COS	73
e^x	EXP	74
log x	LOG	75
$\sqrt{x}$	$\sqrt{}$	76
▲SUB▼	SUB	77
RETURN	RTN	77

STAT-PAC NOMENCLATURE

The following list defines the mathematical notations used in the package.

NOTATION	DEFINITION
X_i, Y_i, x_i, y_i	subscripted sample points
$\bar{X}$, $\bar{Y}$, M	sample means
s_x, S, σ_x	sample standard deviations
s_{xy} , cov (x, y)	Covariance of X and Y
r_{xy}	Coefficient of Correlation between X and Y
pe_x	probable errors
n, N, n_x, n_y	number of data points
f_i	frequency of a data group
a_3	moment coefficient of skewness
a_4	moment coefficient of kurtosis
m_3	3rd moment about the mean
m_4	4th moment about the mean
${}_nP_r$, P_r^n	permutations of n things taken r at a time
${}_nC_r$, C_r^n , $\binom{n}{r}$	combinations of n things taken r at a time
A	arithmetic mean
G	geometric mean
H	harmonic mean
RN	random number
r.c.f.	relative cumulative frequency
p(X), f(X)	probability density function
P(X), f(X), P	probability distribution function
B(r, n, p)	binomial distribution

NOTATION	DEFINITION
γ, n, V, df	degrees of freedom
$\Gamma(\alpha)$	Gamma function
F(X)	cumulative distribution function
ln X	natural logarithm of X
log X	logarithm (base 10) of X
H_0	null hypothesis
H_A	alternative hypothesis
t	t statistic
α	level of significance
χ^2	chi-squared statistic
O_i	observed frequency
E	expected frequency
m, a, B	slope of linear regression line
b, c	intercept of linear regression line
X_{rms}, Y_{rms}	root mean square of X, Y
a_0, a_1, a_2, a_3	coefficients of polynomial regression
a_0, a_1, a_2, a_3	coefficients of multiple linear regression
$R_{x,\,yz}$, $R_{y,\,xz}$, $R_{z,\,xy}$	coefficients of linear multiple correlation
ω	radian frequency
Δt	time increment between sampled data
a_i	cosine coefficients in a Fourier Series
b_i	sine coefficients in a Fourier Series
F	F statistic
N!	N factorial
A	average (expected) value
R	range of a data set
$\hat{\ }$	estimate of some parameter (i.e., $\hat{s}_x$)
exp (X)	e^X
var (X)	variance of $\{X_i\}$

SECTION I GENERAL STATISTICS

			Page Number
I-1	Mean, Standard Deviation and Probable Error		1
I-2	Mean and Standard Deviation of Grouped Data		4
I-3	Cumulative Mean (X_c) and Cumulative Standard Deviation (s_c)		7
I-4	Two Variable Sums of Squares (With Corrector Option)		10
I-5	Mean, Variance, Moment Coefficient of Skewness, Moment Coefficient of Kurtosis		14
I-6	Covariance and Coefficient of Correlation		17
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MEAN, STANDARD DEVIATION AND PROBABLE ERROR

This program calculates the mean, $\bar{X}$; standard deviation, s_x; probable error, pe_x; the standard deviation from the mean, $s_{\bar{x}}$; and the probable error of the mean, $pe_{\bar{x}}$.

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$s_x = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1}} = \sqrt{\frac{\Sigma (X_i)^2 - n(\bar{X})^2}{n - 1}}$$

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}}$$

$$pe_x = .6745 s_x \qquad pe_{\bar{x}} = .6745 s_{\bar{x}}$$

References: 1. Reference Data for Radio Engineers, International Telephone and Telegraph Corporation, 1956.

2. Treatment of Experimental Data, Worthing and Geffner, John Wiley And Sons, 1943.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

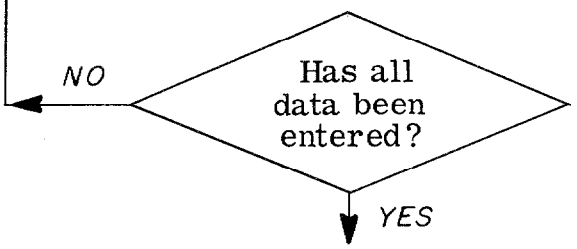
DISPLAY

0	Z
i	Y
0	X

(i indicates point to be entered)

ENTER DATA: X_i → X

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

$s_{\bar{X}}$	Z
$\bar{X}$	Y
n	X

$s_{\bar{X}}$ = Standard deviation
$\bar{X}$ = Mean
n = Number of data points

PRESS: CONTINUE

DISPLAY

$pe_{\bar{X}}$	Z
pe_X	Y
$s_{\bar{X}}$	X

To run another case

EXAMPLE

Sample Data

1
2
3
4
5

Results:

1.581	Z	s_X
3.0	Y	$\bar{X}$
5	X	n

.477	Z	$pe_{\bar{X}}$
1.066	Y	pe_X
.707	X	$s_{\bar{X}}$

00	CLR	20		40	PNT	45		
01	1	01		41	XEY	30		
02	XTO	23		42	DN	25		
03	d	17		43	√	76		
04	XEY	30		44	DIV	35		
05	STP	41	ENTRY	45	YTO	40		
06	IFG	43		46	c	16		
07	1	01		47	.	21		
08	a	13		48	6	06		
09	PNT	45		49	7	07		
0a	PNT	45		4a	4	04		
0b	UP	27		4b	5	05		
0c	X	36		4c	X	36		
0d	AC+	60		4d	RUP	22		
10	0	00		50	X	36		
11	UP	27		51	c	16		
12	d	17		52	PNT	45	S	
13	UP	27		53	PNT	45		
14	1	01		54	GTO	44		
15	+	33		55	0	00		
16	YTO	40		56	0	00		
17	d	17		57	END	46		
18	0	00						
19	GTO	44						
1a	0	00						
1b	5	05						
1c	d	17						
1d	UP	27						
20	1	01						
21	-	34						
22	YTO	40						
23	d	17						
24	UP	27						
25	RCL	61						
26	RUP	22						
27	DIV	35						
28	YTO	40						
29	f	15						
2a	X	36						
2b	f	15						
2c	X	36						
2d	DN	25						
30	-	34						
31	d	17						
32	UP	27						
33	1	01						
34	-	34						
35	f	15						
36	RDN	31						
37	DIV	35						
38	d	17						
39	RDN	31						
3a	√	76						
3b	XEY	30						
3c	RUP	22						
3d	PNT	45	S					

$s_{\bar{x}}$

n

ΣX^2

ΣX

MEAN AND STANDARD DEVIATION OF GROUPED DATA

This program calculates the mean, $\bar{X}$, and standard deviation, S, of a set of data points X_1, X_2, $\cdots$, X_K with frequencies f_1, f_2, $\cdots$, f_K respectively. The equations used are

$$\bar{X} = \frac{\sum_{i=1}^{K} f_i X_i}{\sum_{i=1}^{K} f_i}$$

$$S = \sqrt{\frac{\sum_{i=1}^{K} f_i (X_i - \bar{X})^2}{\sum_{i=1}^{K} f_i - 1}}$$

Reference: Introduction to the Theory of Statistics
by Mood and Graybill

McGraw - Hill 1963

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

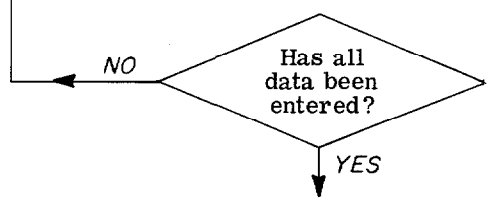
0	Z
i	Y
0	X

(i indicates pair of points to be entered)

ENTER DATA:

f_i	Y
X_i	X

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

S	Z
$\overline{X}$	Y
n	X

S = Standard Deviation

$\overline{X}$ = Mean

n = Number of Data Points

EXAMPLES

SAMPLE DATA

X_i	f_i
1	3
2	3
3	1
4	2
5	1

$s_x = 1.4337$

$\overline{X} = 2.5$

n = 10

SAMPLE DATA

X_i	f_i
41	3
38	5
37	2
39	18
40	22

$s_x = .9010$

$\overline{X} = 39.38$

n = 50

00	CLR	20		40	+	33	
01	XTO	23		41	DN	25	
02	c	16		42	√	76	
03	1	01		43	UP	27	
04	XTO	23		44	f	15	
05	d	17		45	UP	27	
06	XEY	30		46	c	16	
07	STP	41	ENTRY	47	PNT	45	S
08	IFG	43		48	PNT	45	
09	2	02		49	GTO	44	
0a	8	10		4a	0	00	
0b	PNT	45		4b	0	00	
0c	PNT	45		4c	END	46	
0d	RUP	22					
10	c	16					
11	RUP	22					
12	+	33					
13	YTO	40					
14	c	16					
15	RUP	22					
16	X	36					
17	XEY	30					
18	X	36					
19	AC+	60					
1a	0	00					
1b	UP	27					
1c	d	17					
1d	UP	27					
20	1	01					
21	+	33					
22	YTO	40					
23	d	17					
24	0	00					
25	GTO	44					
26	0	00					
27	7	07					
28	YE	24					
29	f	15					
2a	UP	27					
2b	DN	25					
2c	c	16					
2d	DIV	35					
30	RDN	31					
31	X	36					
32	XTO	23					
33	f	15					
34	1	01					
35	RUP	22					
36	-	34					
37	DN	25					
38	DIV	35					
39	YE	24					
3a	e	12					
3b	CHS	32					
3c	DIV	35					
3d	e	12					

	Σf_i
	n
	$\Sigma f_i X_i^2$
	$\Sigma f_i X_i$

CUMULATIVE MEAN ($\bar{X}_c$) AND CUMULATIVE STANDARD DEVIATION (s_c)

This program calculates the cumulative mean ($\bar{X}_c$) and the cumulative standard deviation (s_c) from summary information, namely N, ΣX and $\Sigma(X)^2$, thus saving time where N_0 and N_1 are large, or where the individual values are not available from storage. The program employs the following formulas, where the zero (0) subscript refers to previous data and the one (1) subscript refers to current data.

$$\text{Cumulative Mean} = \bar{X}_c = \frac{\Sigma X_0 + \Sigma X_1}{N_0 + N_1} = \frac{N_0\bar{X}_0 + N_1\bar{X}_1}{N_0 + N_1}$$

Cumulative Standard Deviation = s_c

$$s_c = \left[\frac{(N_0 + N_1)\left(\Sigma(X_0^2) + \Sigma(X_1^2)\right) - (\Sigma X_0 + \Sigma X_1)^2}{(N_0 + N_1)(N_0 + N_1 - 1)} \right]^{\frac{1}{2}}$$

$$s_c^2 = (s_c)^2$$

$$N_c = N_1 + N_0$$

"This work was supported by the U.S. Atomic Energy Commission."

This program was written by Mr. L. E. Snodgrass of Sandia Corporation, Albuquerque, New Mexico.

USER INSTRUCTIONS

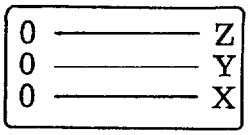
DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY



ENTER DATA:

$\Sigma X_0^2 \longrightarrow Y$
$\Sigma X_1^2 \longrightarrow X$

PRESS: CONTINUE

ENTER DATA:

$\Sigma X_0 \longrightarrow Y$
$\Sigma X_1 \longrightarrow X$

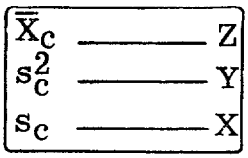
PRESS: CONTINUE

ENTER DATA:

$N_0 \longrightarrow Y$
$N_1 \longrightarrow X$

PRESS: CONTINUE

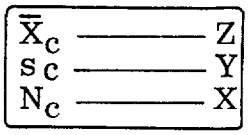
DISPLAY PRINT



$\bar{X}_c$
s_c^2

PRESS: CONTINUE

DISPLAY PRINT



s_c
N_c

To run another case

EXAMPLE

Sample Calculations

Original Experiment

Data	N_0	ΣX_0	ΣX_0^2	s_0	$\bar{X}_0$
2.5 2.1 2.5 2.1 2.1	5	11.3	25.73	.21909	2.260

Current Experiment

Data	N_1	ΣX_1	ΣX_1^2	s_1	$\bar{X}_1$
3.0 3.1 3.2 4.0	4	13.3	44.85	.45735	3.325

Cumulative Results

N_c	s_c	s_c^2	$\bar{X}_c$
9	.64614	.41750	2.733

00	CLR	20		40	RDN	31	
01	STP	41	ENTRY	41	PNT	45	
02	PNT	45		42	PNT	45	
03	PNT	45		43	END	46	
04	+	33					
05	YTO	40					
06	d	17					
07	DN	25					
08	1	01					
09	STP	41	ENTRY				
0a	PNT	45					
0b	PNT	45					
0c	+	33					
0d	YTO	40					
10	e	12					
11	RDN	31					
12	UP	27					
13	X	36					
14	YTO	40					
15	c	16					
16	0	00					
17	UP	27					
18	UP	27					
19	2	02					
1a	STP	41	ENTRY				
1b	PNT	45					
1c	PNT	45					
1d	+	33					
20	YTO	40					
21	f	15					
22	1	01					
23	-	34					
24	YTO	40					
25	a	13					
26	f	15					
27	X	36					
28	YTO	40					
29	b	14					
2a	RCL	61					
2b	DIV	35					
2c	UP	27					
2d	d	17					
30	X	36					
31	c	16					
32	-	34					
33	b	14					
34	DIV	35					$N_0 + N_1 - 1$
35	DN	25					$(N_0 + N_1 - 1)(N_0 + N_1)$
36	UP	27					$(\Sigma X_1 + \Sigma X_0)^2$
37	√	76					$\Sigma X_1^2 + \Sigma X_0^2$
38	CNT	47					
39	RDN	31					$\Sigma X_1 + \Sigma X_0$
3a	PNT	45	S				$N_1 + N_0$
3b	PNT	45					
3c	f	15					
3d	XEY	30					

TWO VARIABLE SUMS OF SQUARES (WITH CORRECTION OPTION)

This program computes the sums of squares for a table of points (X_i, Y_i). The quantities computed are:

ΣX = Sum of X Values
ΣY = Sum of Y Values
ΣX^2 = Sum of Squared X Values
ΣY^2 = Sum of Squared Y Values
ΣXY = Sum of XY Products
n = Number of Data Pairs

This program has an option for correcting erroneously entered data. The program prints the computed quantities and also leaves them in the calculator memory for further use.

To correct for erroneously entered data the error data is re-entered (with the error) and the sums are corrected automatically.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
i	Y
0	X

ENTER DATA:

Y_i	Y
X_i	X

PRESS: CONTINUE

NO — Has all data been correctly entered? — YES

NO — Were any data input errors made? — YES

PRESS: GO TO (5)(0)

PRESS: CONTINUE

DISPLAY

0	Z
j	Y
0	X

*

RE-ENTER DATA:

Y_j	Y
X_j	X

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

NO — Have all error sets been re-entered? — YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

n	Z
ΣY	Y
ΣX	X

PRESS: CONTINUE

DISPLAY

ΣXY	Z
ΣY^2	Y
ΣX^2	X

To run another case

*j represents an index on erroneously entered data sets, independent of i.

EXAMPLES

1)

i	X	Y
1	1	1
2	3	2
3	4	4
4	6	4
5	8	5
6	9	7
7	11	8
8	14	9

EXAMPLES (Con't)

Results:

n	=	8
ΣY	=	40
ΣX	=	56
ΣXY	=	364
ΣY^2	=	256
ΣX^2	=	524

2)

Same as before but the first data set is entered erroneously as 1, 2

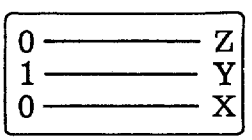
i	X	Y	
1	1	<2>	(error)
2	1	1	(entered
3	3	2	correct
4	4	4	set)
5	6	4	
6	8	5	
7	9	7	
8	11	8	
9	14	9	

After the entry of (14, 9)

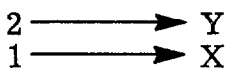
PRESS: GO TO (5)(0)

PRESS: CONTINUE

DISPLAY



RE-ENTER THE ERROR



PRESS: CONTINUE

PRESS: SET FLAG

PRESS: CONTINUE

EXAMPLES (Con't)

Results:

Same as Example 1

Step	Key	Code	T	U	V	Step	Key	Code	T	U	V	Step	Key	Code	T	U	V
00	CLR	20				40	f	15				80	a	13			
01	XTO	23				41	PNT	45	S			81	YE	24			
02	d	17				42	PNT	45				82	9	11			
03	XTO	23				43	b	14				83	+	33			
04	c	16				44	UP	27				84	YTO	40			
05	XTO	23				45	c	16				85	9	11			
06	b	14				46	UP	27				86	+	33			
07	XTO	23				47	d	17				87	0	00			
08	a	13				48	PNT	45	S			88	UP	27			
09	1	01				49	PNT	45				89	RDN	31			
0a	XEY	30				4a	PNT	45				8a	GTO	44			
0b	STP	41	ENTRY			4b	GTO	44				8b	5	05			
0c	IFG	43				4c	8	10				8c	7	07			
0d	3	03				4d	d	17				8d	END	46			
10	a	13				50	0	00									
11	PNT	45				51	UP	27									
12	PNT	45				52	1	01									
13	AC+	60				53	UP	27									
14	UP	27				54	0	00									
15	X	36				55	XTO	23									
16	XEY	30				56	9	11									
17	YE	24				57	STP	41	ENTRY								
18	d	17				58	IFG	43									
19	+	33				59	3	03									
1a	YEX	24				5a	a	13									
1b	d	17				5b	PNT	45									
1c	DN	25				5c	PNT	45									
1d	X	36				5d	AC-	63									
20	b	14				60	UP	27									
21	+	33				61	X	36									
22	YTO	40				62	XEY	30									
23	b	14				63	YE	24									
24	DN	25				64	d	17									
25	DN	25				65	-	34									
26	X	36				66	YE	24									
27	c	16				67	d	17									
28	+	33				68	DN	25									
29	YTO	40				69	X	36									
2a	c	16				6a	b	14									
2b	a	13				6b	XEY	30									
2c	UP	27				6c	-	34									
2d	1	01				6d	YTO	40									
30	+	33				70	b	14									
31	YTO	40				71	DN	25									
32	a	13				72	DN	25									
33	+	33				73	X	36									
34	0	00				74	c	16									
35	UP	27				75	XEY	30									
36	RDN	31				76	-	34									
37	GTO	44				77	YTO	40									
38	0	00				78	c	16									
39	b	14				79	a	13									
3a	a	13				7a	UP	27									
3b	UP	27				7b	1	01									
3c	e	12				7c	-	34									
3d	UP	27				7d	YTO	40									

MEAN, VARIANCE, MOMENT COEFFICIENT OF SKEWNESS AND MOMENT COEFFICIENT OF KURTOSIS

This program computes the mean, variance, coefficients of skewness and kurtosis, and standard deviation of a table of data. The equations used are:

Mean = $$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

Variance = $$S^2 = \frac{1}{n-1} \left[\sum_{i=1}^{n} X_i^2 - n (\bar{X})^2 \right] = \frac{SS}{n-1}$$

m_3 = 3rd moment = $$\frac{1}{n} \sum_{i=1}^{n} X_i^3 - \frac{3}{n} \bar{X} \sum_{i=1}^{n} X_i^2 + 2 (\bar{X})^3$$

m_4 = 4th moment = $$\frac{1}{n} \sum_{i=1}^{n} X_i^4 - \frac{4}{n} \bar{X} \sum_{i=1}^{n} X_i^3 + \frac{6}{n} (\bar{X})^2 \sum_{i=1}^{n} X_i^2 - 3 (\bar{X})^4$$

a_3 = moment coefficient of skewness = $$\frac{m_3}{S^3}$$

a_4 = moment coefficient of kurtosis = $$\frac{m_4}{S^4}$$

The user may wish to replace the (n - 1) divisor in the variance equation by n. If so, place a zero (0) in step (4)(3).

Reference: Theory and Problems of Statistics: M.R. Spiegel McGraw-Hill, 1961.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

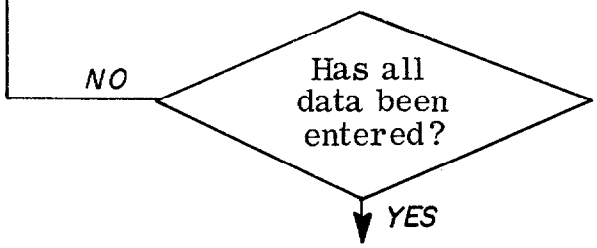
PRESS: CONTINUE

DISPLAY

0	Z
i	Y
0	X

ENTER DATA: X_i → X

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

SS	Z
S^2	Y
$\bar{X}$	X

PRESS: CONTINUE

DISPLAY

1	Z
a_3	Y
S	X

PRESS: CONTINUE

DISPLAY

2	Z
a_4	Y
n	X

To run another case

EXAMPLE

i	X_i
1	2.1
2	3.5
3	4.2
4	6.5
5	4.1
6	3.6
7	5.3
8	3.7
9	4.9

Results:

SS	=	12.509
S^2	=	1.564
$\bar{X}$	=	4.211
a_3	=	.198
S	=	1.250
a_4	=	.068
n	=	9.000

00	CLR	20		40	-	34		80	e	12	
01	XTO	23		41	RUP	22		81	X	36	
02	c	16		42	XEY	30		82	DN	25	
03	XTO	23		43	1	01		83	+	33	
04	d	17		44	-	34		84	d	17	
05	XTO	23		45	DN	25		85	DIV	35	
06	b	14		46	DIV	35		86	f	15	
07	1	01		47	f	15		87	UP	27	
08	XEY	30		48	PNT	45	S	88	X	36	
09	STP	41	ENTRY	49	PNT	45		89	X	36	
0a	IFG	43		4a	YTO	40		8a	X	36	
0b	3	03		4b	a	13		8b	3	03	
0c	2	02		4c	c	16		8c	X	36	
0d	PNT	45		4d	UP	27		8d	DN	25	
10	PNT	45		50	f	15		90	-	34	
11	UP	27		51	UP	27		91	d	17	
12	X	36		52	3	03		92	DIV	35	
13	AC+	60		53	X	36		93	DIV	35	
14	X	36		54	e	12		94	d	17	
15	UP	27		55	X	36		95	UP	27	
16	DN	25		56	DN	25		96	2	02	
17	X	36		57	-	34		97	RDN	31	
18	b	14		58	d	17		98	PNT	45	
19	+	33		59	DIV	35		99	PNT	45	
1a	YTO	40		5a	f	15		9a	END	46	
1b	b	14		5b	UP	27					
1c	DN	25		5c	X	36					
1d	c	16		5d	X	36					
20	+	33		60	2	02					
21	YTO	40		61	X	36					
22	c	16		62	DN	25					
23	d	17		63	+	33					
24	UP	27		64	a	13					
25	1	01		65	DIV	35					
26	+	33		66	$\sqrt{}$	76					
27	YTO	40		67	DIV	35					
28	d	17		68	UP	27					
29	+	33		69	1	01					
2a	0	00		6a	RDN	31					
2b	UP	27		6b	PNT	45	S				
2c	RDN	31		6c	PNT	45					
2d	GTO	44		6d	b	14					
30	0	00		70	UP	27					
31	9	11		71	f	15					
32	RCL	61		72	UP	27					
33	UP	27		73	4	04					
34	d	17		74	X	36					S^2 (m_2)
35	DIV	35		75	c	16					ΣX^4
36	YTO	40		76	X	36					ΣX^3
37	f	15		77	DN	25					n
38	DN	25		78	-	34					ΣX^2
39	UP	27		79	f	15					ΣX_i $\bar{X}$
3a	X	36		7a	UP	27					
3b	d	17		7b	X	36					
3c	X	36		7c	6	06					
3d	RDN	31		7d	X	36					

This program determines the covariance and coefficient of correlation derived from a table of data containing X and Y observations as given below:

X	Y
X_1	Y_1
X_2	Y_2
.	.
.	.
.	.
X_n	Y_n

$$\text{Covariance} = S_{XY} = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})$$

$$= \frac{1}{n-1} \left\{ \Sigma X_i Y_i - \frac{1}{n} \Sigma X_i \Sigma Y_i \right\}$$

$$\text{Coefficient of Correlation} = r_{XY} = \frac{S_{XY}}{S_X S_Y}$$

This program computes

$$\overline{X} = \frac{1}{n} \Sigma X_i$$

$$\overline{Y} = \frac{1}{n} \Sigma Y_i$$

$$S_X = \frac{1}{n-1} \left\{ \Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n} \right\}$$

$$S_Y = \frac{1}{n-1} \left\{ \Sigma Y_i^2 - \frac{(\Sigma Y_i)^2}{n} \right\}$$

S_{XY}

and

r_{XY}

Reference: Statistical Theory with Engineering Applications, A. Hald, John Wiley & Sons, 1960.

USER INSTRUCTIONS

PRESS: [X] [Y] [Z] on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
i	Y
0	X

ENTER DATA:

$Y_i \longrightarrow Y$

$X_i \longrightarrow X$

PRESS: CONTINUE

NO ← Has all data been entered? → YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

$\overline{Y}$	Z
$\overline{X}$	Y
n	X

PRESS: CONTINUE

DISPLAY

$S_Y{}^2$	Z
$S_X{}^2$	Y
S_{XY}	X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

S_Y	Z
S_X	Y
r_{XY}	X

To run another case

EXAMPLE

X	Y
1	1
3	2
4	4
6	4
8	5
9	7
11	8
14	9

Results:

5.	$\overline{Y}$
7.	$\overline{X}$
8.	n
8.00	$S_Y{}^2$
18.86	$S_X{}^2$
12.00	S_{XY}
2.83	S_Y
4.34	S_X
.977	r_{XY}

00	CLR	20		40	f	15		80	UP	27	
01	XTO	23		41	UP	27		81	a	13	
02	d	17		42	X	36		82	DIV	35	
03	XTO	23		43	a	13		83	PNT	45	S
04	c	16		44	DIV	35		84	PNT	45	
05	XTO	23		45	DN	25		85	c	16	
06	b	14		46	-	34		86	UP	27	
07	XTO	23		47	a	13		87	d	17	
08	a	13		48	UP	27		88	UP	27	
09	1	01		49	1	01		89	b	14	
0a	XEY	30		4a	-	34		8a	PNT	45	S
0b	STP	41	ENTRY	4b	DN	25		8b	PNT	45	
0c	IFG	43		4c	DIV	35		8c	RUP	22	
0d	3	03		4d	YTO	40		8d	√	76	
10	c	16		50	d	17		90	RUP	22	
11	PNT	45		51	c	16		91	√	76	
12	PNT	45		52	UP	27		92	X	36	
13	AC+	60		53	e	12		93	DN	25	
14	ARC	72		54	UP	27		94	DIV	35	
15	UP	27		55	X	36		95	c	16	
16	X	36		56	a	13		96	√	76	
17	XEY	30		57	DIV	35		97	UP	27	
18	YE	24		58	DN	25		98	d	17	
19	d	17		59	-	34		99	√	76	
1a	+	33		5a	a	13		9a	RUP	22	
1b	YE	24		5b	UP	27		9b	PNT	45	
1c	d	17		5c	1	01		9c	PNT	45	
1d	DN	25		5d	-	34		9d	END	46	
20	X	36		60	DN	25					
21	b	14		61	DIV	35					
22	+	33		62	YTO	40					
23	YTO	40		63	c	16					
24	b	14		64	b	14					
25	RUP	22		65	UP	27					
26	UP	27		66	f	15					
27	X	36		67	UP	27					
28	c	16		68	e	12					
29	+	33		69	X	36					
2a	YTO	40		6a	CNT	47					
2b	c	16		6b	a	13					
2c	a	13		6c	DIV	35					
2d	UP	27		6d	DN	25					
30	1	01		70	-	34					
31	+	33		71	a	13					
32	YTO	40		72	UP	27					
33	a	13		73	1	01					
34	+	33		74	-	34					n
35	0	00		75	DN	25					$\Sigma XY/SXY$
36	XEY	30		76	DIV	35					$\Sigma Y^2/S_Y^2$
37	UP	27		77	YTO	40					$\Sigma X^2/S_X^2$
38	0	00		78	b	14					ΣY
39	GTO	44		79	e	12					ΣX
3a	0	00		7a	UP	27					
3b	b	14		7b	a	13					
3c	d	17		7c	DIV	35					
3d	UP	27		7d	f	15					

This program evaluates the formula for the permutations of n objects taken r at a time:

$$ {}_nP_r = \frac{n!}{(n-r)!} = P_r^n $$

$$ = n(n-1)(n-2) \ldots (n-r+1) $$

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

n ⟶ Y
r ⟶ X

PRESS: CONTINUE

DISPLAY

P^n_r ——— Z
n ——— Y
r ——— X

To run another case

EXAMPLE

1. ${}_5P_5 = 5! = 120$

2. ${}_8P_4 = 1680$

3. How many different numbers can be formed by using six out of the nine digits, 1, 2, 3, ... 9 ?

$${}_9P_6 = 60480.$$

00	CLR	20
01	STP	41
02	UP	27
03	DN	25
04	XTO	23
05	c	16
06	-	34
07	CLX	37
08	X>Y	53
09	DN	25
0a	STP	41
0b	RDN	31
0c	AC+	60
0d	UP	27

ENTRY

10	1	01
11	RDN	31
12	X=Y	50
13	2	02
14	0	00
15	DN	25
16	X	36
17	UP	27
18	1	01
19	-	34
1a	f	15
1b	GTO	44
1c	1	01
1d	2	02

20	RCL	61
21	c	16
22	PNT	45
23	PNT	45
24	END	46

r

n

n - r

This program evaluates the formula for the combination of n objects taken r at a time:

$$ {}_nC_r = \frac{n!}{(n-r)!\,r!} = C_r^n $$

$$ = \frac{n(n-1)(n-2)\ \ldots\ (n-r+1)}{r!} $$

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA:

n	→ Y
r	→ X

PRESS: CONTINUE

DISPLAY

C_r^n	Z
n^r	Y
r	X

To run another case

EXAMPLE

1. ${}_8C_4 = 70.$

2. In how many ways can a group of 600 people be selected from 602 people? Use the identity.

$${}_{602}C_{600} = {}_{602}C_2 = 180901.$$

$${}_nC_r = {}_nC_{n-r}$$

```
00   CLR   20
01   STP   41     ENTRY
02   XTO   23
03     d   17
04     1   01
05   AC+   60
06   RDN   31
07     f   15
08   XEY   30
09     d   17
0a   X<Y   52
0b     1   01
0c     a   13
0d   RCL   61

10   DIV   35
11    DN   25
12     X   36
13     1   01
14    UP   27
15   CHS   32
16   AC-   63
17   GTO   44
18     0   00
19     7   07
1a     e   12
1b    YE   24
1c     d   17
1d   XEY   30

20     +   33
21   PNT   45
22   PNT   45
23   END   46
```

r

n, n-1, . . . , n-r

1, 2, 3

HISTOGRAM GENERATION (WITHOUT PLOT)

This program generates a histogram table of ten windows given a data set of positive numbers. In addition, it determines the mean (M_x) and the variance (σ_x^2) of the raw data, and the mean (M_h) and variance (σ_h^2) of the normalized histogram data. Since the raw data is normalized by the program to values $0 \leq h \leq 10$, the new mean and variance are given by

$$M_h = \frac{M_x}{W}$$

$$\sigma_h^2 = \frac{\sigma_x^2}{W^2}$$

where W is the histogram window width (normalization factor). The window width W is chosen such that all normalized data entries X/W will lie between 0 and 10. Thus, if the data ranges from 0 → 200, a W of 20 would be proper.

This program uses Indirect Addressing. The (+) registers are used for storage whereas the (−) registers are used for program steps.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

ENTER PROGRAM: (Starting Address is (-0)(0))

PRESS: GO TO (-) (0) (0)

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: W → X

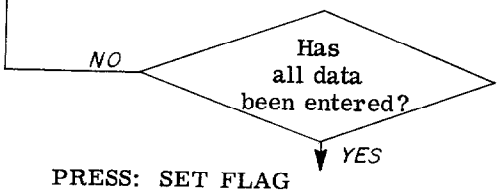
PRESS: CONTINUE

→ DISPLAY

N	Z
N	Y
1	X

ENTER DATA: X → X

PRESS: CONTINUE



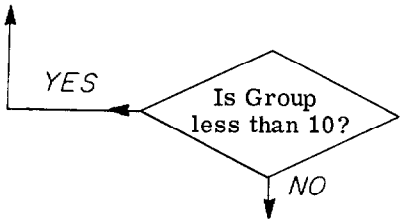
PRESS: SET FLAG

PRESS: CONTINUE

→ DISPLAY

%	Z	(% of data points in Group)
K_A	Y	(Number of data points in Group)
Group	X	(Group #)

PRESS: CONTINUE



PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

σ_x^2	Z
M_x	Y
0	X

PRESS: CONTINUE

DISPLAY

σ_h^2	Z
M_h	Y
1	X

PRESS: GO TO

PRESS: —

PRESS: 0

PRESS: 0

} To consider another set of data.

EXAMPLE

The data set is:

104, 92, 83, 78, 58, 135, 146, 24, 74, 85, 81, 128, 140, 113, 79, 78, 53, 42, 34, 85, 96, 110, 133, 158, 171, 108, 84, 90, 73, 11, 51, 118, 68, 139, 92, 109, 89, 124, 91, 116.

The data varies between 0 and 200 so W is chosen to be 20.

Result

Group	K_A	%
1	1	2.5
2	2	5
3	4	10
4	6	15
5	11	27.5
6	7	17.5
7	5	12.5
8	3	7.5
9	1	2.5
10	0	0

N = 40

σ_x^2 = 1252.644 σ_h^2 = 3.132

M_x = 93.575 M_h = 4.679

Step	Key	Code		Step	Key	Code		Step	Key	Code	
00	CLR	20	Minus Page	40	6	06		80	YTO	40	
01	XTO	23		41	0	00		81	c	16	
02	0	00		42	0	00		82	b	14	
03	XTO	23		43	0	00		83	DIV	35	
04	1	01		44	0	00		84	EEX	26	
05	XTO	23		45	0	00		85	2	02	
06	2	02		46	0	00		86	X	36	
07	XTO	23		47	0	00		87	1	01	
08	3	03		48	0	00		88	RUP	22	
09	XTO	23		49	1	01		89	+	33	
0a	4	04		4a	9	11		8a	XFR	67	
0b	XTO	23		4b	0	00		8b	c	16	
0c	5	05		4c	0	00		8c	XEY	30	
0d	XTO	23		4d	0	00		8d	PNT	45	S
10	6	06		50	IFG	43		90	PNT	45	
11	XTO	23		51	1	01		91	UP	27	
12	7	07		52	d	17		92	1	01	
13	XTO	23		53	CLX	37		93	0	00	
14	8	10		54	1	01		94	X>Y	53	
15	XTO	23		55	+	33		95	6	06	
16	9	11		56	SFL	54		96	0	00	
17	STP	41	ENTRY	57	GTO	44		97	RCL	61	
18	XTO	23		58	3	03		98	UP	27	
19	a	13		59	d	17		99	b	14	
1a	1	01		5a	YTO	40		9a	DIV	35	
1b	CHS	32		5b	b	14		9b	YTO	40	
1c	RDN	31		5c	CLX	37		9c	f	15	
1d	DN	25		5d	UP	27		9d	X	36	
20	1	01		60	EEX	26		a0	f	15	
21	+	33		61	9	11		a1	X	36	
22	UP	27		62	+	33		a2	DN	25	
23	DN	25		63	YTO	40		a3	-	34	
24	STP	41	ENTRY	64	-	34		a4	b	14	
25	IFG	43		65	7	07		a5	DIV	35	
26	5	05		66	-	34		a6	f	15	
27	a	13		67	0	00		a7	UP	27	
28	PNT	45		68	UP	27		a8	0	00	
29	PNT	45		69	0	00		a9	PNT	45	
2a	UP	27		6a	0	00		aa	PNT	45	S
2b	X	36		6b	0	00		ab	a	13	
2c	AC+	60		6c	0	00		ac	DIV	35	
2d	XEY	30		6d	YE	24		ad	XEY	30	
30	a	13		70	0	00					
31	DIV	35		71	0	00					
32	DN	25		72	0	00					
33	INT	64		73	0	00					
34	UP	27		74	0	00					
35	EEX	26		75	0	00					
36	9	11		76	0	00					
37	+	33		77	0	00					
38	YTO	40		78	0	00					
39	-	34		79	1	01					
3a	4	04		7a	9	11					
3b	CLX	37		7b	0	00					
3c	CLX	37		7c	0	00					
3d	YE	24		7d	0	00					

User Keys			
0	K_0	8	K_8
1	K_1	9	K_9
2	K_2	E	W
3	K_3	F	N
4	K_4	G	K_A
5	K_5	H	
6	K_6	I	ΣX^2
7	K_7	J	

b0	RDN	31
b1	DIV	35
b2	DIV	35
b3	DN	25
b4	XEY	30
b5	UP	27
b6	1	01
b7	PNT	45
b8	PNT	45
b9	GTO	44
ba	-	34
bb	0	00
bc	0	00

Minus
Page

S

ARITHMETIC, GEOMETRIC, HARMONIC MEANS

This program calculates the arithmetic, geometric, and harmonic means of a set of n data values. The program evaluates the following formulas:

$$A = \frac{1}{n} \sum_{i=1}^{n} X_i \quad \text{(arithmetic mean)}$$

$$G = \sqrt[n]{X_1 \cdot X_2 \cdots X_n} \quad \text{(geometric mean)}$$

$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{X_i}}$$

Reference: Theory and Problems of Statistics, Murray R. Spiegel, McGraw-Hill, 1961.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

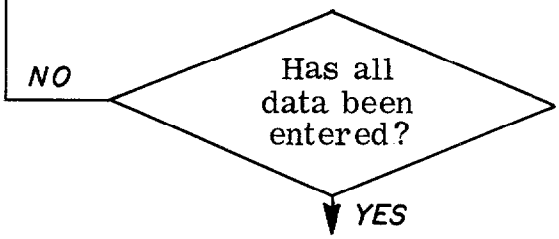
PRESS: CONTINUE

DISPLAY

```
i ——— Z
i ——— Y
i ——— X
```

ENTER DATA: X_i ⟶ X

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

```
H ——— Z
G ——— Y
A ——— X
```

To run another case

EXAMPLE

Data: 2, 4, 8

Results:

```
3.42857 —— Z
4.00000 —— Y
4.66667 —— X
```

Harmonic mean
Geometric mean
Arithmetic mean

```
00  CLR  20
01   1   01
02  XTO  23
03   d   17
04  YTO  40
05   c   16
06  UP   27
07  UP   27
08  STP  41      ENTRY
09  IFG  43
0a   2   02
0b   a   13
0c  PNT  45
0d  PNT  45

10  UP   27
11   1   01
12  AC+  60
13   d   17
14  XEY  30
15   X   36
16  YTO  40
17   d   17
18  UP   27
19   1   01
1a  XEY  30
1b  DIV  35
1c   c   16
1d   +   33

20  YTO  40
21   c   16
22   f   15
23  UP   27
24   1   01
25   +   33
26  DN   25
27  GTO  44
28   0   00
29   6   06
2a   d   17
2b  LN   65
2c  UP   27
2d   f   15

30  DIV  35
31  DN   25
32  EXP  74
33  XTO  23
34   b   14
35   e   12
36  UP   27
37   f   15
38  DIV  35
39  YTO  40
3a   a   13
3b  UP   27
3c   c   16
3d  DIV  35

40   b   14
41  UP   27
42   a   13
43  RUP  22
44  PNT  45
45  RUP  22
46  PNT  45
47  RUP  22
48  PNT  45
49  PNT  45
4a  PNT  45      S
4b  GTO  44
4c   0   00
4d   0   00

50  END  46
```

A

G

$\Sigma\frac{1}{X_i}$

πX_i

ΣX_i

n

RANDOM NUMBER GENERATOR (UNIFORM)

This program calculates uniformly distributed random numbers (RN) in the range $0 \leq RN_i \leq 1$ using the formula given below:

$$RN_i = \left[\pi + RN_{(i-1)}\right]^8 - \text{Int.} \left\{ \left[\pi + RN_{(i-1)}\right]^8 \right\}$$

RN_i is the current random number and $RN_{(i-1)}$ is the last calculated random number. More than 10,000 random numbers may be generated before values are repeated.

USER INSTRUCTIONS

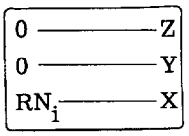
DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM:

PRESS: CONTINUE

DISPLAY



TO RESET PROBLEM

PRESS: END

EXAMPLE

$RN_1 = .5310$

$RN_2 = .6918$

$RN_3 = .3712$

$RN_4 = .6890$

00	CLR	20
01	UP	27
02	π	56
03	+	33
04	DN	25
05	UP	27
06	X	36
07	DN	25
08	UP	27
09	X	36
0a	DN	25
0b	UP	27
0c	X	36
0d	DN	25
10	UP	27
11	INT	64
12	-	34
13	DN	25
14	PNT	45
15	GTO	44
16	0	00
17	1	01
18	END	46

S

RANDOM NUMBER GENERATION (NORMAL)

This program generates a sequence of Normally distributed random numbers with mean and variance $\bar{n}$ and σ_n^2. The generation technique involves transforming Uniform random variables to normal variables by the formulas:

$$N_i = (-2 \ln \mu_i)^{\frac{1}{2}} \cos(2\pi \cdot \mu_{i+1})$$

$$N_{i+1} = (-2 \ln \mu_i)^{\frac{1}{2}} \sin(2\pi \cdot \mu_{i+1})$$

where (μ_i, μ_{i+1}) are independent uniform random variables, $0 \leq \mu_i \leq 1$.

The N_i thus generated are normally distributed with mean zero and unity variance.

The set $\{N_i\}$ are modified to generate a more general set of mean ($\bar{n}$) and variance (σ_n^2) by

$$n_i = \sigma_n N_i + \bar{n}$$

$$n_{i+1} = \sigma_n N_{i+1} + \bar{n}$$

The set $\{n_i\}$ is now distributed normally with mean $\bar{n}$ and variance σ_n^2.

Reference: Handbook of Mathematical Functions, U.S. Dept of Commerce, Applied Mathematics Series, 1964, page 953.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

SET: [ ] RADIANS

SET: Decimal Wheel as required

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA:

σ_n^2	Y
$\bar{n}$	X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA:

μ_2	Y	*
μ_1	X	

PRESS: CONTINUE

DISPLAY

n_{i+1}	Z
n_{i+1}	Y
n_i	X

The program will automatically continue to generate and print a series of $\{n_i\}$ until the operator presses STOP.

*μ_1 and μ_2 are two initializing uniform random variables such that:

USER INSTRUCTIONS (Con't)

$$\mu_1 \neq \mu_2$$
$$0 \leq \mu_1 \leq 1$$
$$0 \leq \mu_2 \leq 1$$

EXAMPLE

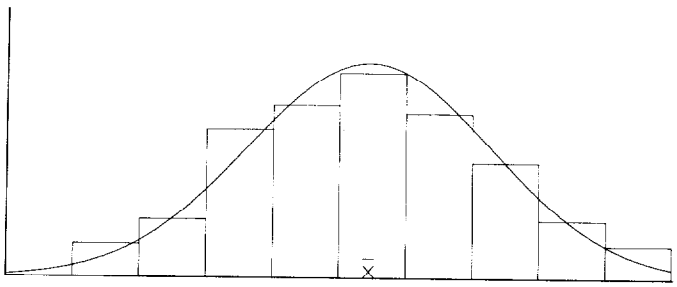
$\sigma_n^2 = 1$
$\bar{n} = 3$

$\mu_2 = .75$
$\mu_1 = .25$

Results:

The histogram seen below represents the first 120 samples generated by this program. The histogram was generated by program STAT-PAC X-1. The superimposed normal curve was generated by program STAT-PAC X-2. The computed mean and variance are:

$\bar{n}_{computed} = 2.99$
$\sigma^2_{n\ computed} = 1.01$
$w = .55$



Loc	Cat	Key		Loc	Cat	Key
00	CLR	20		40	UP	27
01	1	01		41	X	36
02	STP	41	ENTRY	42	DN	25
03	PNT	45		43	UP	27
04	PNT	45		44	X	36
05	XTO	23		45	DN	25
06	d	17		46	UP	27
07	DN	25		47	INT	64
08	√	76		48	-	34
09	XTO	23		49	YE	24
0a	c	16		4a	e	12
0b	2	02		4b	YE	24
0c	STP	41	ENTRY	4c	f	15
0d	PNT	45		4d	IFG	43
10	PNT	45		50	3	03
11	AC+	60		51	6	06
12	LN	65		52	e	12
13	UP	27		53	UP	27
14	2	02		54	f	15
15	X	36		55	GTO	44
16	DN	25		56	1	01
17	CHS	32		57	2	02
18	√	76		58	END	46
19	RUP	22				
1a	UP	27				
1b	π	56				
1c	X	36				
1d	2	02				
20	X	36				
21	DN	25				
22	UP	27				
23	COS	73				
24	RUP	22				
25	X	36				
26	RUP	22				
27	SIN	70				
28	X	36				
29	c	16				
2a	X	36				
2b	RUP	22				
2c	X	36				
2d	d	17				
30	+	33				
31	RUP	22				
32	+	33				
33	DN	25				
34	PNT	45	S			
35	SFL	54				
36	f	15				
37	UP	27				
38	π	56				
39	+	33				
3a	DN	25				
3b	UP	27				
3c	X	36				
3d	DN	25				

NORMAL PROBABILITY PAPER PLOT POINTS

This program determines the Relative Cumulative Frequencies, in percent, from a set of grouped data. The percentages can be plotted on Normal Probability graph paper for determining the normalcy of the data.

Formulas used:

$$r.c.f.(i) = \frac{100}{N} \sum_{j=1}^{i} f_j$$

$$N = \sum_{i=1}^{n} f_i$$

n = Number of Groups

f_i = Frequency of the i^{th} group

The input data consists of a Cumulative Frequency table of grouped data as illustrated below.

Group	Group Description	Frequency
1	Below Boundary 1	f_1
2	Boundary 1 to Boundary 2	f_2
.	.	.
.	.	.
.	.	.
n	Above Boundary (n-1)	f_n

Reference: Introduction to Statistical Inference, J.C.R. Li, Edward Brothers, Inc., 1957.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

```
0 ——— Z
0 ——— Y
0 ——— X
```

ENTER DATA: N → X

PRESS: CONTINUE

DISPLAY

```
0 ——— Z
i ——— Y
0 ——— X
```

ENTER DATA: Upper Boundary and Frequency

UB_i → Y
f_i → X

PRESS: CONTINUE

DISPLAY

```
i ——— Z
r.c.f.% — Y
r.c.f.% — X
```

*

After the n^{th} group is entered, N is printed again.

To run another case

*If printer is used, the print blocks will be:

i
UB_i
f_i
$r.c.f._i$ %

EXAMPLE

Group	Grouped description	Frequency
1	Below 20	6
2	20 to 30	12
3	30 to 40	25
4	40 to 50	39
5	50 to 60	48
6	60 to 70	47
7	70 to 80	36
8	80 to 90	22
9	90 to 100	10
10	100 to 110	5

N = 250

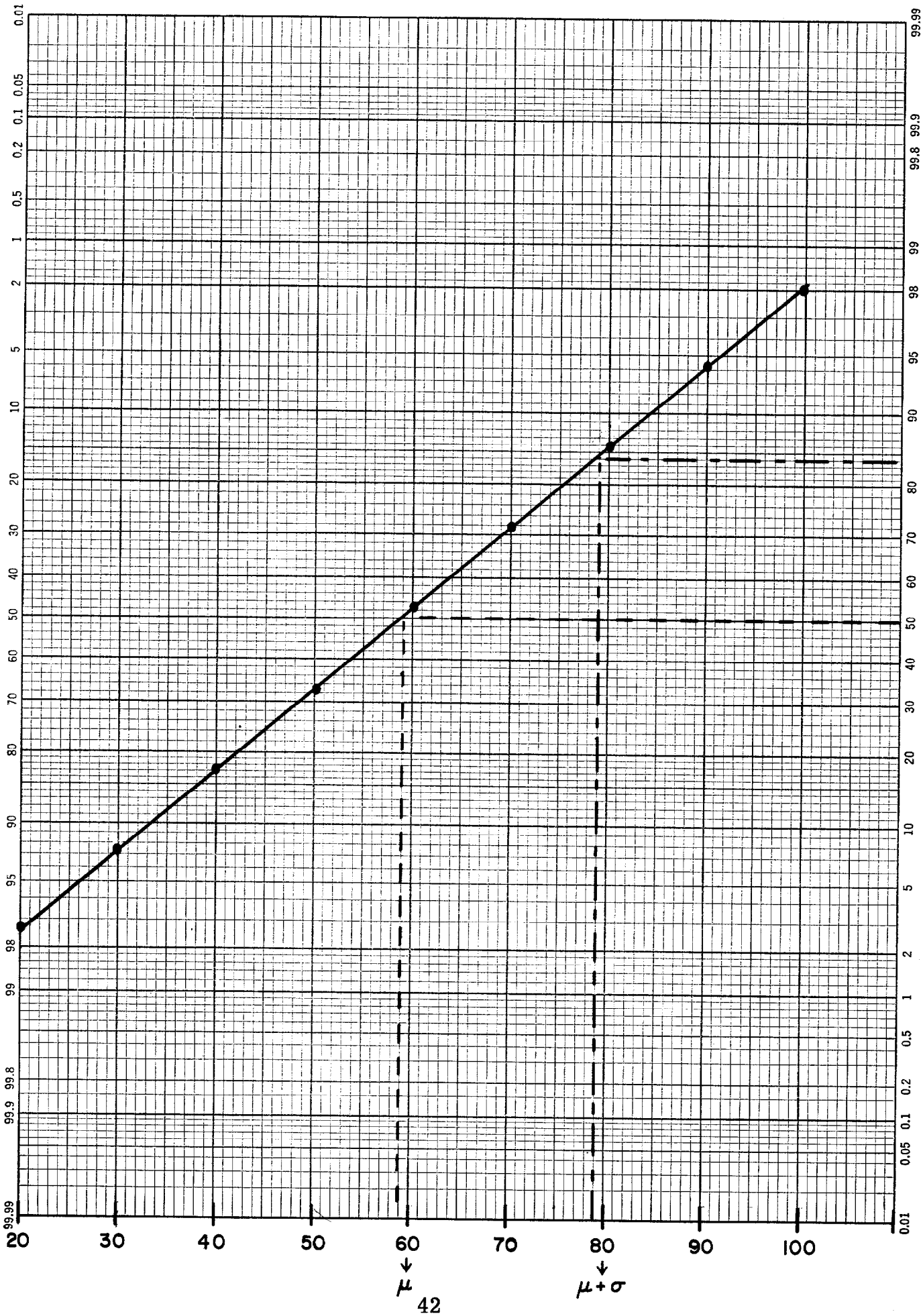
Results:

i	= 1
UB_i	= 20
f_i	= 6
$r.c.f._i$%	= 2.40
	= 2
	= 30
	= 12
	= 7.20
	= 3
	= 40
	= 25
	= 17.20
	= 4
	= 50
	= 39
	= 32.80
	= 5
	= 60
	= 48
	= 52.00

EXAMPLE (Con't)

i	= 6
UB_i	= 70
f_i	= 47
$r.c.f._i\%$	= 70.80
	= 7
	= 80
	= 36
	= 85.20
	= 8
	= 90
	= 22
	= 94.00
	= 9
	=100
	= 10
	= 98.00
	= 10
	=110
	= 5
	=100.00

The $r.c.f._i\%$ are plotted on the following graph.


0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9
99.99
20
30
40
50
60
70
80
90
100
μ
μ+σ

Step	Key	Code
00	CLR	20
01	XTO	23
02	d	17
03	STP	41
04	XTO	23
05	e	12
06	PNT	45
07	PNT	45
08	PNT	45
09	1	01
0a	XEY	30
0b	STP	41
0c	RUP	22
0d	PNT	45
10	RUP	22
11	PNT	45
12	RUP	22
13	PNT	45
14	UP	27
15	f	15
16	+	33
17	YTO	40
18	f	15
19	e	12
1a	DIV	35
1b	EEX	26
1c	2	02
1d	X	36
20	d	17
21	UP	27
22	1	01
23	+	33
24	DN	25
25	XTO	23
26	d	17
27	RDN	31
28	PNT	45
29	PNT	45
2a	PNT	45
2b	DN	25
2c	f	15
2d	UP	27
30	e	12
31	X=Y	50
32	3	03
33	c	16
34	0	00
35	RDN	31
36	1	01
37	+	33
38	0	00
39	GTO	44
3a	0	00
3b	b	14
3c	e	12
3d	UP	27

ENTRY

ENTRY

S

S

Step	Key	Code
40	UP	27
41	PNT	45
42	PNT	45
43	PNT	45
44	PNT	45
45	PNT	45
46	GTO	44
47	0	00
48	0	00
49	END	46

group #

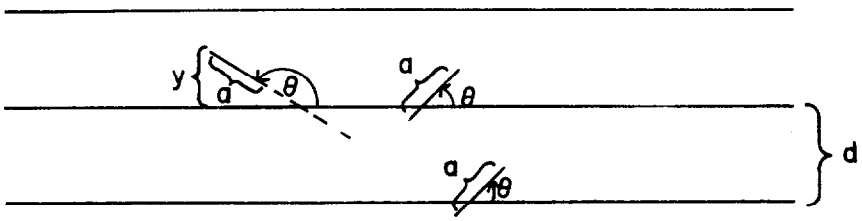
N

c.f.

(C) HEWLETT-PACKARD

BUFFON NEEDLE EXPERIMENT

This program simulates the well known Buffon Needle Experiment described in the reference. The problem is to determine the probability that a needle of length a will intersect a line on a grid of lines separated by distance d. The geometry of the experiment is seen below:



The needle of length $a \leq d$ is randomly tossed on the grid. By considering the angle θ between the needle and a line of the grid, and the distance Y between the upper tip of the needle and the line it intersects or the nearest line below it, the possible relations can be derived:

$$0 \leq \theta \leq \pi$$

$$0 \leq Y \leq d$$

The needle intersects a line when:

$$0 \leq Y \leq a \sin \theta .$$

It can be shown that the probability of an intersection is theoretically given by:

$$P_T = \frac{2a}{\pi d} .$$

This formula involving π has intrigued mathematicians since it allows one to experimentally determine π .

This program randomly chooses a θ from a uniform distribution such that:

$$0 \leq \theta \leq \pi$$

then independently choose a Y from a uniform distribution such that:

$$0 \leq Y \leq 1$$

The grid distance d is defined to be 1.

Thus $P_T = \frac{2a}{\pi}$.

The value of Y is then compared against

$$Y \leq a \sin \theta.$$

If Y is less than or equal to a sin θ , an intersection has occurred. The number of failing throws is then used to evaluate the experimental probability:

$$P_E = \frac{\#\ \text{successful}}{\text{successful} + \text{failed}}$$

The program displays:

trials

P_E

P_T

and proceeds to toss another needle.

Reference: Theory of Probability, M. E. Munroe, McGraw-Hill, 1951.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

SET: [ ] RADIANS

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA:

R_Y	→ Z	*
R_θ	→ Y	
a	→ X	

PRESS: CONTINUE

DISPLAY

Trials	Z
P_E	Y
P_T	X

The program continues indefinitely.

*R_Y is an initializing value for the routine which generates the random values of Y. R_Y should be $0 \leq R_Y \leq 1$.

R_θ is a similar initializer for generating R_θ. R_θ should not equal R_Y. R_θ should be $0 \leq R_\theta \leq \pi$.

EXAMPLE

$R_Y = .75$
$R_\theta = .5$
$a = .5$

Results after 2100 trials,

$P_E = .312$
$P_T = .318$

Step	Key	Code
00	CLR	20
01	STP	41
02	PNT	45
03	PNT	45
04	XTO	23
05	d	17
06	RUP	22
07	XTO	23
08	a	13
09	2	02
0a	X	36
0b	π	56
0c	DIV	35
0d	YTO	40
0e	-	34
0f	YTO	40
10	a	13
11	DN	25
12	X<Y	52
13	5	05
14	6	06
15	X=Y	50
16	5	05
17	6	06
18	e	12
19	UP	27
1a	1	01
1b	+	33
1c	c	16
1d	π	56
1e	RUP	22
1f	+	33
20	DN	25
21	UP	27
22	X	36
23	DN	25
24	UP	27
25	X	36
26	DN	25
27	UP	27
28	X	36
29	DN	25
2a	UP	27
2b	X	36
2c	DN	25
2d	UP	27
2e	INT	64
2f	-	34
30	YTO	40
31	b	14
32	π	56
33	X	36
34	DN	25
35	SIN	70
36	UP	27
37	d	17
38	X	36
39	a	13
3a	UP	27
3b	π	56
3c	+	33
3d	DN	25
3e	UP	27
3f	INT	64
40	-	34
41	YTO	40
42	a	13
43	DN	25
44	X<Y	52
45	5	05
46	6	06
47	X=Y	50
48	5	05
49	6	06
4a	e	12
4b	UP	27
4c	1	01
4d	+	33
4e	YTO	40
4f	e	12
50	f	15
51	GTO	44
52	6	06
53	0	00
54	f	15
55	UP	27
56	1	01
57	+	33
58	YTO	40
59	f	15
5a	e	12
5b	XEY	30
5c	+	33
5d	UP	27
5e	DN	25
5f	XEY	30
60	DIV	35
61	c	16
62	PSE	57
63	b	14
64	RDN	31
65	GTO	44
66	1	01
67	1	01
68	END	46

ENTRY

MEDIAN FINDER

This program determines the median of a set of n numbers (X_i). The median is defined as:

> M = The middle value, or the arithmetic mean of the two middle values, of a set of numbers which have been arranged in order of magnitude.

Thus the median of the set { 8, 5, 4, 8, 10, 4, 3, 6, 8 } is 6 because when arranged in order { 3, 4, 4, 5, 6, 8, 8, 8, 10 } 6 is the middle value.

This program determines the median M of a set of data unordered by magnitude.

The "median finding" portion of this program was written by Mr. E. J. Schmidt, ITT Federal Electric Corporation, Paramus, N. J. Various input-output modifications were made for user convenience.

Reference: Theory and Problems of Statistics, Murray Spiegel, McGraw-Hill Book Company, 1961, page 47.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $M_{guess} \longrightarrow X$

(M_{guess} can be any member of $\{X_i\}$ except $X_{guess} \geq X_{max}$, or $X_{guess} \leq X_{min}$)

PRESS: CONTINUE

DISPLAY

0	Z
i	Y
0	X

ENTER DATA: $X_i \longrightarrow X$

PRESS: CONTINUE

Has all data been entered?

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

M	Z
M	Y
M	X

USER INSTRUCTIONS (Con't)

or

DISPLAY

0	Z
1	Y
0	X

*

To run another case

*If this display appears the value of M_{guess} was too close to X_{min} or X_{max}. A new value of M_{guess} was calculated by the program and the program is ready for re-entry of $\{X_i\}$. Return to the point of X_i entry, and repeat USER INSTRUCTIONS.

EXAMPLE

$\{X_i\}$ = { 8, 5, 4, 8, 10, 4, 3, 6, 8 }

Let M_{guess} = 7

Result: M = 6

Let M_{guess} = 9

Note that the DISPLAY:

0	Z
1	Y
0	X

appears indicating that M_{guess} was too near X_{max} or X_{min}. But, by re-entering the data set:

Result: M = 6

00	CHS	32		Plus Page	40	DN	25		80	CNT	47
01	X=Y	50			41	GTO	44		81	CNT	47
02	5	05			42	0	00		82	CNT	47
03	4	04			43	a	13		83	CNT	47
04	4	04			44	c	16		84	CNT	47
05	CHS	32			45	GTO	44		85	CNT	47
06	X=Y	50			46	0	00		86	CNT	47
07	5	05			47	a	13		87	CNT	47
08	8	10			48	c	16		88	CNT	47
09	a	13			49	UP	27		89	CNT	47
0a	UP	27			4a	b	14		8a	CNT	47
0b	UP	27			4b	+	33		8b	CNT	47
0c	PNT	45	s		4c	2	02		8c	CNT	47
0d	PNT	45			4d	DIV	35		8d	CNT	47
10	GTO	44			50	DN	25		90	CNT	47
11	-	34			51	GTO	44		91	CNT	47
12	0	00			52	0	00		92	CNT	47
13	0	00			53	a	13		93	CNT	47
14	f	15			54	b	14		94	CNT	47
15	GTO	44			55	GTO	44		95	CNT	47
16	0	00			56	0	00		96	CNT	47
17	a	13			57	a	13		97	CNT	47
18	f	15			58	b	14		98	CNT	47
19	UP	27			59	UP	27		99	CNT	47
1a	e	12			5a	a	13		9a	CNT	47
1b	+	33			5b	+	33		9b	CNT	47
1c	2	02			5c	2	02		9c	CNT	47
1d	DIV	35			5d	DIV	35		9d	CNT	47
20	DN	25			60	DN	25		a0	CNT	47
21	GTO	44			61	GTO	44		a1	CNT	47
22	0	00			62	0	00		a2	CNT	47
23	a	13			63	a	13		a3	CNT	47
24	e	12			64	CNT	47		a4	CNT	47
25	GTO	44			65	CNT	47		a5	CNT	47
26	0	00			66	CNT	47		a6	CNT	47
27	a	13			67	CNT	47		a7	CNT	47
28	e	12			68	CNT	47		a8	CNT	47
29	UP	27			69	CNT	47		a9	CNT	47
2a	d	17			6a	CNT	47		aa	CNT	47
2b	+	33			6b	CNT	47		ab	CNT	47
2c	2	02			6c	CNT	47		ac	CNT	47
2d	DIV	35			6d	CNT	47		ad	CNT	47
30	DN	25			70	CNT	47				
31	GTO	44			71	CNT	47				
32	0	00			72	CNT	47				
33	a	13			73	CNT	47				
34	d	17			74	CNT	47				
35	GTO	44			75	CNT	47				
36	0	00			76	CNT	47				
37	a	13			77	CNT	47				
38	d	17			78	CNT	47				
39	UP	27			79	CNT	47				
3a	c	16			7a	CNT	47				
3b	+	33			7b	CNT	47				
3c	2	02			7c	CNT	47				
3d	DIV	35			7d	CNT	47				

b0	CNT	47		10	XTO	23			50	8	10
b1	CNT	47		11	-	34			51	RDN	31
b2	CNT	47		12	f	15			52	YE	24
b3	CNT	47		13	EEX	26			53	8	10
b4	CNT	47		14	9	11			54	1	01
b5	CNT	47		15	XTO	23			55	+	33
b6	CNT	47		16	d	17			56	YTO	40
b7	CNT	47		17	0	00			57	8	10
b8	CNT	47		18	STP	41		ENTRY	58	d	17
b9	CNT	47		19	IFG	43			59	RUP	22
ba	CNT	47		1a	8	10			5a	XEY	30
bb	CNT	47		1b	1	01			5b	X>Y	53
bc	CNT	47		1c	PNT	45			5c	7	07
bd	CNT	47		1d	PNT	45			5d	6	06
c0	CNT	47		20	YTO	40			60	e	12
c1	CNT	47		21	-	34			61	X>Y	53
c2	CNT	47		22	f	15			62	7	07
c3	CNT	47		23	YE	24			63	8	10
c4	CNT	47		24	9	11			64	f	15
c5	CNT	47		25	YTO	40			65	X>Y	53
c6	CNT	47		26	9	11			66	7	07
c7	CNT	47		27	X>Y	53			67	a	13
c8	CNT	47		28	5	05			68	GTO	44
c9	CNT	47		29	1	01			69	4	04
ca	CNT	47		2a	RDN	31			6a	3	03
cb	CNT	47		2b	YE	24			6b	YE	24
cc	CNT	47		2c	7	07			6c	c	16
cd	CNT	47		2d	1	01			6d	YE	24
d0	CNT	47		30	+	33			70	b	14
d1	CNT	47		31	YTO	40			71	YEX	24
d2	CNT	47		32	7	07			72	a	13
d3	CNT	47		33	c	16			73	GTO	44
d4	CNT	47		34	RUP	22			74	4	04
d5	CNT	47		35	XEY	30			75	3	03
d6	CNT	47		36	X<Y	52			76	YE	24
d7	CNT	47		37	6	06			77	d	17
d8	CNT	47		38	b	14			78	YE	24
d9	CNT	47		39	b	14			79	e	12
da	CNT	47		3a	X<Y	52			7a	YE	24
db	CNT	47		3b	6	06			7b	f	15
dc	CNT	47		3c	d	17			7c	GTO	44
dd	CNT	47		3d	a	13			7d	4	04
00	CLR	20	Minus Page	40	X<Y	52					
01	STP	41		41	7	07					
02	PNT	45	ENTRY	42	1	01					
03	PNT	45		43	0	00					
04	XTO	23		44	UP	27					
05	9	11		45	1	01					
06	1	01		46	UP	27					
07	XEY	30		47	XFR	67					
08	XTO	23		48	-	34					
09	c	16		49	f	15					
0a	XTO	23		4a	+	33					
0b	7	07		4b	0	00					
0c	XTO	23		4c	GTO	44					
0d	8	10		4d	1	01					

80	3	03	c0	ARC	72
81	XFR	67	c1	1	01
82	7	07	c2	X=Y	50
83	YE	24	c3	+	33
84	8	10	c4	3	03
85	-	34	c5	4	04
86	5	05	c6	ARC	72
87	CHS	32	c7	0	00
88	X<Y	52	c8	X=Y	50
89	9	11	c9	+	33
8a	9	11	ca	3	03
8b	YEX	24	cb	8	10
8c	a	13	cc	ARC	72
8d	1	01	cd	1	01

90	-	34	d0	CHS	32
91	YTO	40	d1	X=Y	50
92	9	11	d2	+	33
93	0	00	d3	4	04
94	UP	27	d4	4	04
95	RUP	22	d5	ARC	72
96	GTO	44	d6	2	02
97	0	00	d7	CHS	32
98	4	04	d8	X=Y	50
99	CHS	32	d9	+	33
9a	X>Y	53	da	4	04
9b	a	13	db	8	10
9c	6	06	dc	ARC	72
9d	YE	24	dd	3	03

a0	f	15
a1	1	01
a2	+	33
a3	GTO	44
a4	9	11
a5	1	01
a6	X=Y	50
a7	+	33
a8	1	01
a9	4	04
aa	ARC	72
ab	4	04
ac	X=Y	50
ad	+	33

b0	1	01
b1	8	10
b2	ARC	72
b3	3	03
b4	X=Y	50
b5	+	33
b6	2	02
b7	4	04
b8	ARC	72
b9	2	02
ba	X=Y	50
bb	+	33
bc	2	02
bd	8	10

UNIVERSAL FOUR OPERATIONS PROGRAM

This program will perform a given arithmetic operation on a list of numbers. It will add, subtract, multiply or divide each number in the list by a given constant designated by the operator. The user enters the constant and the type of operation in a single entry. The program saves time in operations such as unit conversions, and it is not necessary to write a special program for each type of operation.

Although printout of each result is included, the program may be used without a printer by changing the PRINT commands in steps (2)(4), (3)(6), (4)(6) and (5)(6) to CONTINUE commands.

This program was written by Monsieur Claude Cardot, of Laboratoires De Marcoussis, Centre De Recherches De La Compangnie Général D'Electricité, Marcoussis, France.

USER INSTRUCTIONS

PRESS: Z on the 9120A

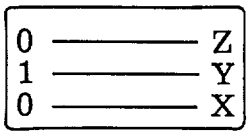
PRESS: END

ENTER PROGRAM

PRESS: END

PRESS: CONTINUE

DISPLAY

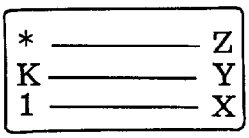


ENTER DATA: K (operating number) → X

PRESS: +, -, x, or ÷ to designate desired operation

PRESS: CONTINUE

DISPLAY

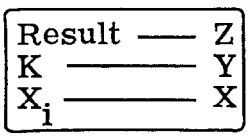


*Denotes Insignificant Display

ENTER DATA: X_i → X

PRESS: CONTINUE

DISPLAY



ENTER DATA: Next X_i → X

PRESS: CONTINUE

For new operating quantity K and/or new type of operation.

EXAMPLES

1. K = π → X

Operation: ÷

X_i	RESULT
5.770	1.837
3.438	1.094
7.123	2.267
4.999	1.591
6.283	2.000

2. K = sin 65° → X

Operation: x (Multiply)

X_i	RESULT
231.4	209.72
579.6	525.30
788.9	714.99
187.3	169.75
422.7	383.10

00	CLR	20	40	STP	41
01	1	01	41	XEY	30
02	RUP	22	42	UP	27
03	STP	41	43	DN	25
04	AC+	60	44	-	34
05	-	34	45	RUP	22
06	1	01	46	PNT	45
07	X=Y	50	47	STP	41
08	2	02	48	GTO	44
09	c	16	49	4	04
0a	RCL	61	4a	1	01
0b	+	33	4b	RCL	61
0c	1	01	4c	RUP	22
0d	X=Y	50	4d	1	01
10	3	03	50	STP	41
11	b	14	51	XEY	30
12	RCL	61	52	UP	27
13	UP	27	53	DN	25
14	1	01	54	DIV	35
15	XEY	30	55	RUP	22
16	DIV	35	56	PNT	45
17	DN	25	57	STP	41
18	X=Y	50	58	GTO	44
19	4	04	59	5	05
1a	b	14	5a	1	01
1b	RCL	61	5b	END	46
1c	RUP	22			
1d	1	01			
20	STP	41			
21	X	36			
22	XEY	30			
23	RDN	31			
24	PNT	45			
25	STP	41			
26	RUP	22			
27	f	15			
28	RDN	31			
29	GTO	44			
2a	2	02			
2b	1	01			
2c	RCL	61			
2d	RUP	22			
30	1	01			
31	STP	41			
32	+	33			
33	RUP	22			
34	f	15			
35	XEY	30			
36	PNT	45			
37	STP	41			
38	GTO	44			
39	3	03			
3a	2	02			
3b	RCL	61			
3c	RUP	22			
3d	1	01			

SECTION II DISTRIBUTION FUNCTIONS

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NORMAL PROBABILITY INTEGRAL

This program computes the integral of the standardized normal distribution

$$P(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{1}{2}z^2} dz \qquad (X \leq 7)$$

The following equations are used

$$P(X) = \frac{\text{erf}\left(\frac{X}{\sqrt{2}}\right)}{2} + \frac{1}{2}$$

where

$$\text{erf}(X) = \frac{2}{\sqrt{\pi}} e^{-X^2} \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \cdots (2n+1)} X^{2n+1}$$

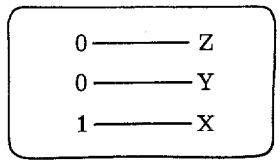
Reference: Handbook of Mathematical Functions
by Abramowitz and Stegun

National Bureau of Standards 1964

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A
PRESS: END
ENTER PROGRAM
PRESS: CONTINUE

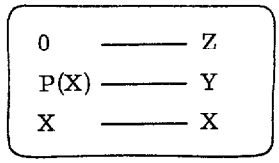
→DISPLAY



ENTER DATA: X ⟶ X

PRESS: CONTINUE

DISPLAY



└— To run another case

EXAMPLES

X = .3	P(X) = .618
X = -.3	P(X) = .382
X = 0	P(X) = .5
X = 3	P(X) = .999

00	CLR	20		40	RDN	31	
01	1	01		41	DIV	35	
02	STP	41	ENTRY	42	RDN	31	
03	CNT	47		43	YE	24	
04	X=Y	50		44	a	13	
05	0	00		45	+	33	
06	a	13		46	YE	24	
07	GTO	44		47	a	13	
08	1	01		48	YE	24	
09	2	02		49	b	14	
0a	.	21		4a	X>Y	53	
0b	5	05		4b	3	03	
0c	XEY	30		4c	a	13	
0d	GTO	44		4d	a	13	
10	6	06		50	UP	27	
11	0	00		51	2	02	
12	XTO	23		52	DIV	35	
13	c	16		53	.	21	
14	X<Y	52		54	5	05	
15	SFL	54		55	+	33	
16	CHS	32		56	1	01	
17	UP	27		57	IFG	43	
18	2	02		58	XEY	30	
19	√	76		59	-	34	
1a	DIV	35		5a	CLX	37	
1b	DN	25		5b	RDN	31	
1c	UP	27		5c	XEY	30	
1d	X	36		5d	c	16	
20	UP	27		60	PNT	45	S
21	DN	25		61	PNT	45	
22	RUP	22		62	GTO	44	
23	EXP	74		63	0	00	
24	DIV	35		64	0	00	
25	2	02		65	END	46	
26	X	36					
27	π	56					
28	√	76					
29	DIV	35					
2a	1	01					
2b	YTO	40					
2c	a	13					
2d	RUP	22					
30	DIV	35					
31	YTO	40					
32	b	14					
33	2	02					
34	DIV	35					
35	EEX	26					
36	1	01					
37	0	00					
38	CHS	32					
39	RUP	22					
3a	YE	24					
3b	b	14					
3c	RDN	31					
3d	+	33					

This program determines the value of X such that

$$\int_{-\infty}^{X} \frac{e^{-\frac{\lambda^2}{2}}}{\sqrt{2\pi}} \, d\lambda = P$$

where P is given. This program performs the inverse of Program STAT-PAC II-1 Normal Probability Integral. P represents an area under the normal curve where X is the upper limit of integration.

The following equations are used:

$$P(X) = \frac{\operatorname{erf}\left(\frac{X}{\sqrt{2}}\right)}{2} + \frac{1}{2}$$

$$\operatorname{erf}(X) = \frac{2}{\sqrt{\pi}} e^{-X^2} \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \cdot \ldots \cdot (2n+1)} X^{2n+1}$$

Reference: Handbook of Mathematical Functions, Abramowitz and Stegun, National Bureau of Standards, 1964.

USER INSTRUCTIONS

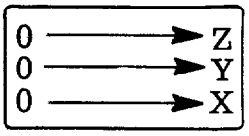
PRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

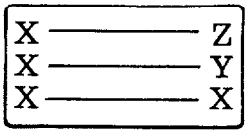
DISPLAY



ENTER DATA: P → X

PRESS: CONTINUE

DISPLAY



To run another case

To observe the program converging on the proper value of X, place a PAUSE in step (6)(7).

EXAMPLES

1. P = .5000, result, X = 0.0

2. P = .841344746, result, X = 1.0

3. P = .999999713, result, X = 5.0

4. P = .022750132, result, X = -2.0

00	CLR	20	40	YE	24	80	1	01
01	STP	41	41	e	12	81	0	00
02	PNT	45	42	RDN	31	82	UP	27
03	XTO	23	43	+	33	83	CHS	32
04	b	14	44	RDN	31	84	.	21
05	UP	27	45	DIV	35	85	1	01
06	1	01	46	RDN	31	86	X	36
07	XTO	23	47	YE	24	87	YTO	40
08	c	16	48	f	15	88	c	16
09	CHS	32	49	+	33	89	GTO	44
0a	6	06	4a	YE	24	8a	1	01
0b	.	21	4b	f	15	8b	0	00
0c	5	05	4c	YE	24	8c	UP	27
0d	YTO	40	4d	e	12	8d	c	16
10	d	17	50	X>Y	53	90	X<Y	52
11	UP	27	51	4	04	91	8	10
12	c	16	52	0	00	92	2	02
13	+	33	53	f	15	93	GTO	44
14	YTO	40	54	UP	27	94	1	01
15	d	17	55	2	02	95	0	00
16	CLX	37	56	DIV	35	96	d	17
17	XEY	30	57	.	21	97	UP	27
18	X<Y	52	58	5	05	98	UP	27
19	SFL	54	59	+	33	99	PNT	45
1a	CHS	32	5a	1	01	9a	PNT	45
1b	UP	27	5b	IFG	43	9b	END	46
1c	2	02	5c	XEY	30			
1d	√	76	5d	-	34			
20	DIV	35	60	CLX	37			
21	DN	25	61	RDN	31			
22	UP	27	62	XEY	30			
23	X	36	63	b	14			
24	UP	27	64	UP	27			
25	DN	25	65	DN	25			
26	RUP	22	66	-	34			
27	EXP	74	67	CNT	47			
28	DIV	35	68	UP	27			
29	2	02	69	DN	25			
2a	X	36	6a	|Y|	55			
2b	π	56	6b	EEX	26			
2c	√	76	6c	CHS	32			
2d	DIV	35	6d	9	11			
30	1	01	70	X>Y	53			
31	YTO	40	71	9	11			
32	f	15	72	6	06			
33	RUP	22	73	DN	25			
34	DIV	35	74	0	00			
35	YTO	40	75	X>Y	53			
36	e	12	76	8	10			
37	2	02	77	c	16			
38	DIV	35	78	UP	27			
39	EEX	26	79	c	16			
3a	1	01	7a	X>Y	53			
3b	0	00	7b	8	10			
3c	CHS	32	7c	2	02			
3d	RUP	22	7d	GTO	44			

ENTRY

P(X)

BINOMIAL DISTRIBUTION

This program evaluates the Binomial (Bernoulli) distribution

$$B(r, n, p) = {}_nC_r \ (p)^r \ (1-p)^{n-r}$$

where:

p = probability of a success

r = number of successes

n = number of trials

Reference: Probabilistic Reliability, Martin Shooman, McGraw-Hill, 1968.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: Probability

p	→ Z
n	→ Y
r	→ X

PRESS: CONTINUE

DISPLAY

B(n, r, p)	Z
n	Y
r	X

└ To run another case

EXAMPLE

What is the probability of twice obtaining obtaining a 3 on six rolls of a die?

$$p = \frac{1}{6}$$

$$n = 6$$

$$r = 2$$

$$B(2,\ 6,\ \tfrac{1}{6}) = .2009388$$

```
00  CLR  20
01  STP  41     ENTRY
02  XTO  23
03    d  17
04  YTO  40
05    c  16
06    1  01
07  AC+  60
08  RDN  31
09  YTO  40
0a    b  14
0b    f  15
0c  XEY  30
0d    d  17

10  X<Y  52
11    1  01
12    d  17
13  RCL  61
14  DIV  35
15   DN  25
16    X  36
17    1  01
18   UP  27
19  CHS  32
1a  AC-  63
1b  GTO  44
1c    0  00
1d    b  14

20   LN  65
21  XEY  30
22    d  17
23    X  36
24   DN  25
25  EXP  74
26    X  36
27    1  01
28   UP  27
29    b  14
2a    -  34
2b    e  12
2c  XEY  30
2d   LN  65

30    X  36
31   DN  25
32  EXP  74
33    X  36
34    c  16
35   UP  27
36    d  17
37  PNT  45
38  PNT  45
39  END  46
```

p

n

r

n

1

POISSON DISTRIBUTION

The Poisson density function is defined by $f(n;\lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$ where λ may be estimated by

$\hat{\lambda}$ = the expected value of X_i $(n = 0, 1, 2, \ldots)$

$$\sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} = e^{-\lambda} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^{-\lambda} e^{\lambda} = 1$$

The Poisson density function is a discrete density which is used to evaluate such things as component failure probabilities. It is also used to approximate the Binomial Distribution when the number of events (N) is large and the probability that an event will happen (p) is small i.e., a general rule of thumb is for $p \leq .1$ and $\hat{\lambda} = Np \leq 5$.

Reference: Mathematical Statistics
by John E. Freund

1962 Prentice-Hall

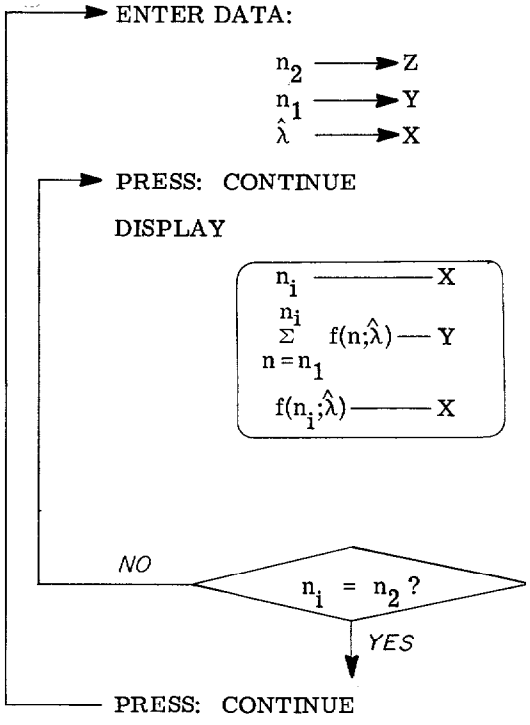
USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE



EXAMPLE

A Poisson distribution is given by

$$f(n;.72) = \frac{(.72)^n e^{-.72}}{n!}$$

Find: a) $f(0;.72)$

$\hat{\lambda} = .72,\ n_1 = 0,\ n_2 = 0$

$f(0;.72) = .48675226$

b) $f(3;.72)$

$\hat{\lambda} = .72,\ n_1 = 3,\ n_2 = 3$

$f(3;.72) = .03027988$

If 3% of the electric bulbs manufactured by a company are defective find the probability that in a sample of 100 bulbs

a) Between 1 and 3 bulbs will be defective.

Expected no. of bulbs that are defective $= \hat{\lambda} = (.03)(100) = 3$

$$\sum_{n=1}^{3} f(n;3) = .59744482$$

where $\hat{\lambda} = 3,\ n_1 = 1,\ n_2 = 3$

b) Less than or equal to 2 bulbs are defective

$$\sum_{n=0}^{2} f(n;3) = .42319008$$

where $\hat{\lambda} = 3,\ n_1 = 0,\ n_2 = 2$

00	CLR	20	ENTRY	40	STP	41
01	STP	41		41	PNT	45
02	PNT	45		42	PNT	45
03	XTO	23		43	CLX	37
04	c	16		44	XTO	23
05	YTO	40		45	f	15
06	b	14		46	d	17
07	RUP	22		47	RUP	22
08	XTO	23		48	X=Y	50
09	d	17		49	0	00
0a	XEY	30		4a	0	00
0b	CHS	32		4b	UP	27
0c	EXP	74		4c	1	01
0d	UP	27		4d	+	33
10	c	16		50	YTO	40
11	LN	65		51	b	14
12	UP	27		52	c	16
13	b	14		53	GTO	44
14	X	36		54	0	00
15	XEY	30		55	b	14
16	EXP	74		56	END	46
17	RUP	22				
18	X	36				
19	YTO	40				
1a	a	13				
1b	b	14				
1c	UP	27				
1d	UP	27				
20	1	01				
21	-	34				
22	X>Y	53				
23	2	02				
24	b	14				
25	RDN	31				
26	X	36				
27	RUP	22				
28	GTO	44				
29	2	02				
2a	1	01				
2b	a	13				
2c	RUP	22				
2d	UP	27				
30	0	00				
31	XEY	30				
32	X=Y	50				
33	ARC	72				
34	1	01				
35	XEY	30				
36	DN	25				
37	DIV	35				
38	XEY	30				
39	UP	27				
3a	AC+	60				
3b	b	14				
3c	RDN	31				
3d	RCL	61				

χ^2 CHI SQUARE DISTRIBUTION

This program evaluates the Chi Square Distribution Integral for a given value of χ^2 and ν degrees of freedom; i.e., the program evaluates

$$P(\chi^2,\nu)= \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})} \int_0^{\chi^2} T^{\nu/2-1} e^{-T/2}\, dT \qquad 0 \le \chi^2$$

The series approximation used to evaluate the integral is

$$P(\chi^2,\nu)=(\tfrac{1}{2}\chi^2)^{\nu/2} \frac{e^{-\chi^2/2}}{\Gamma(\frac{\nu+2}{2})} \left\{1+\sum_{r=1}^{\infty} \frac{\chi^{2r}}{(\nu+2)(\nu+4)\cdots(\nu+2r)}\right\}$$

Reference: Handbook of Mathematical Functions
by Abramowitz and Stegun

National Bureau of Standards 1964

ER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

ENTER DATA:

χ^2 ⟶ Y
ν ⟶ X

PRESS: CONTINUE

DISPLAY

P(χ^2, ν) — Z
χ^2 — Y
ν — X

EXAMPLES

P(7.88, 1) = .995

P(10.6, 2) = .995

P(12.8, 17) = .251

GENERAL FORM

$P(X^2, \nu)$

00	CLR	20		40	DIV	35	80	7	07
01	STP	41	ENTRY	41	GTO	44	81	4	04
02	CNT	47		42	3	03	82	e	12
03	XTO	23		43	6	06	83	+	33
04	d	17		44	4	04	84	DN	25
05	YTO	40		45	LN	65	85	X	36
06	c	16		46	X	36	86	c	16
07	2	02		47	DN	25	87	UP	27
08	DIV	35		48	EXP	74	88	d	17
09	YTO	40		49	X	36	89	PNT	45
0a	f	15		4a	d	17	8a	PNT	45
0b	DN	25		4b	UP	27	8b	END	46
0c	LN	65		4c	2	02			
0d	UP	27		4d	X	36			
10	d	17		50	DN	25			
11	UP	27		51	DIV	35			
12	1	01		52	UP	27			
13	+	33		53	4	04			
14	2	02		54	-	34			
15	DIV	35		55	X<Y	52			
16	YTO	40		56	5	05			
17	e	12		57	0	00			
18	DN	25		58	2	02			
19	X	36		59	X>Y	53			
1a	DN	25		5a	6	06			
1b	EXP	74		5b	1	01			
1c	UP	27		5c	DN	25			
1d	f	15		5d	DIV	35			
20	CHS	32		60	UP	27			
21	EXP	74		61	c	16			
22	X	36		62	|Y|	55			
23	e	12		63	DIV	35			
24	UP	27		64	π	56			
25	INT	64		65	DIV	35			
26	X=Y	50		66	DN	25			
27	4	04		67	√	76			
28	4	04		68	X	36			
29	2	02		69	UP	27			
2a	LN	65		6a	d	17			
2b	X	36		6b	XTO	23			
2c	DN	25		6c	f	15			
2d	EXP	74		6d	0	00			
30	X	36		70	XTO	23			
31	c	16		71	e	12			
32	√	76		72	1	01			
33	DIV	35		73	XEY	30			
34	d	17		74	2	02			
35	DIV	35		75	AC+	60			
36	UP	27		76	c	16			
37	2	02		77	X	36			
38	-	34		78	f	15			
39	0	00		79	DIV	35			
3a	X=Y	50		7a	EEX	26			
3b	6	06		7b	9	11			
3c	a	13		7c	CHS	32			
3d	DN	25		7d	X<Y	52			

HYPERGEOMETRIC DISTRIBUTION

This program computes various functions associated with the hypergeometric distribution which is given by:

$$f(X) = \frac{\binom{k}{X}\binom{N-k}{n-X}}{\binom{N}{n}} \qquad X = 0, 1, 2, \ldots, [n, k]$$

where [n, k] is the smaller of n and k.

- N - is the number of items (successes and failures) in a finite population.
- n - is the number of items drawn in sample without replacement from the population.
- k - is the number of failures in the finite population.
- X - is the number of failures in the sample.

f(X) represents the probability of exactly X successes and n - X failures in the sample of n items.

The function

$$F(X) = \sum_{r=0}^{X} f(r)$$

yields the probability of X or fewer failures in the sample of n items.

The mean and variance of the distribution are given by

$$\mu = \frac{nk}{N}$$

$$\sigma^2 = \frac{k(N-k)n(N-n)}{N^2(N-1)}$$

This program determines f(X), F(X), μ, σ, and σ^2. It uses program STAT-PAC I-8, COMBINATIONS, as a subroutine.

Reference: Handbook of Tables for Probability and Statistics, Chemical Rubber Co. 1966.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B.

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA:

N →	Z
n →	Y
k →	X

PRESS: CONTINUE

DISPLAY

F(X)	Z	(X = 0, 1,
f(X)	Y	2, ...)
X	X	

When X exceeds [n, k] the final display appears:

DISPLAY

σ^2	Z
σ	Y
μ	X

To run another case

EXAMPLE

N = 5

n = 3

k = 2

.1000	Z	
.1000	Y	X = 0
0.	X	
.7000	Z	
.6000	Y	X = 1
1.	X	
1.0000	Z	
.3000	Y	X = 2
2.	X	
.3600	Z	
.6000	Y	(μ, σ, σ^2)
1.2000	X	

00	CLR	20	Plus Page	40	UP	27		80	CNT	47	
01	XTO	23		41	a	13		81	CNT	47	
02	c	16		42	GTO	44		82	CNT	47	
03	XTO	23		43	SUB	77		83	CNT	47	
04	7	07		44	-	34		84	CNT	47	
05	1	01		45	0	00		85	CNT	47	
06	STP	41	ENTRY	46	0	00		86	CNT	47	
07	PNT	45		47	XFR	67		87	CNT	47	
08	PNT	45		48	8	10		88	CNT	47	
09	XTO	23		49	XEY	30		89	CNT	47	
0a	b	14		4a	DIV	35		8a	CNT	47	
0b	YTO	40		4b	XFR	67		8b	CNT	47	
0c	a	13		4c	7	07		8c	CNT	47	
0d	RDN	31		4d	XEY	30		8d	CNT	47	
10	YTO	40		50	+	33		90	CNT	47	
11	9	11		51	YTO	40		91	CNT	47	
12	UP	27		52	7	07		92	CNT	47	
13	c	16		53	UP	27		93	CNT	47	
14	X>Y	53		54	c	16		94	CNT	47	
15	6	06		55	PNT	45	S	95	CNT	47	
16	2	02		56	PNT	45		96	CNT	47	
17	b	14		57	UP	27		97	CNT	47	
18	UP	27		58	1	01		98	CNT	47	
19	c	16		59	+	33		99	CNT	47	
1a	X>Y	53		5a	YTO	40		9a	CNT	47	
1b	6	06		5b	c	16		9b	CNT	47	
1c	2	02		5c	a	13		9c	CNT	47	
1d	GTO	44		5d	GTO	44		9d	CNT	47	
20	SUB	77		60	1	01		a0	CNT	47	
21	-	34		61	2	02		a1	CNT	47	
22	0	00		62	XFR	67		a2	CNT	47	
23	0	00		63	9	11		a3	CNT	47	
24	YTO	40		64	UP	27		a4	CNT	47	
25	8	10		65	b	14		a5	CNT	47	
26	XFR	67		66	-	34		a6	CNT	47	
27	9	11		67	X	36		a7	CNT	47	
28	UP	27		68	a	13		a8	CNT	47	
29	b	14		69	X	36		a9	CNT	47	
2a	-	34		6a	GTO	44		aa	CNT	47	
2b	a	13		6b	-	34		ab	CNT	47	
2c	UP	27		6c	2	02		ac	CNT	47	
2d	c	16		6d	3	03		ad	CNT	47	
30	-	34		70	CNT	47					Intermediate Results
31	DN	25		71	CNT	47					
32	GTO	44		72	CNT	47					N
33	SUB	77		73	CNT	47					
34	-	34		74	CNT	47					n
35	0	00		75	CNT	47					
36	0	00		76	CNT	47					K
37	XFR	67		77	CNT	47					
38	8	10		78	CNT	47					X
39	X	36		79	CNT	47					
3a	YTO	40		7a	CNT	47					
3b	8	10		7b	CNT	47					
3c	XFR	67		7c	CNT	47					
3d	9	11		7d	CNT	47					F(X)

```
b0  CNT  47      Plus
b1  CNT  47      Page
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

Step	Op	Code
00	UP	27
01	0	00
02	XTO	23
03	f	15
04	XTO	23
05	e	12
06	RDN	31
07	XTO	23
08	d	17
09	1	01
0a	AC+	60
0b	RDN	31
0c	f	15
0d	XEY	30
10	d	17
11	X<Y	52
12	2	02
13	1	01
14	RCL	61
15	DIV	35
16	DN	25
17	X	36
18	1	01
19	UP	27
1a	CHS	32
1b	AC-	63
1c	GTO	44
1d	0	00
20	c	16
21	DN	25
22	RTN	77
23	UP	27
24	XFR	67
25	9	11
26	XEY	30
27	-	34
28	DN	25
29	X	36
2a	XFR	67
2b	9	11
2c	DIV	35
2d	DIV	35
30	UP	27
31	1	01
32	-	34
33	DN	25
34	DIV	35
35	b	14
36	UP	27
37	a	13
38	X	36
39	XFR	67
3a	9	11
3b	DIV	35
3c	DN	25
3d	XEY	30

Minus
Page

Step	Op	Code
40	√	76
41	XEY	30
42	PNT	45
43	PNT	45
44	PNT	45
45	PNT	45
46	PNT	45
47	PNT	45
48	GTO	44
49	+	33
4a	0	00
4b	0	00
4c	END	46

S

This program evaluates the t-distribution integral for a given value of t and γ degrees of freedom; i.e., the program evaluates:

$$P(t, \gamma) = \int_{-\infty}^{t} \frac{\Gamma\left(\frac{\gamma+1}{2}\right)\left(1+\frac{X^2}{\gamma}\right)^{-\frac{\gamma+1}{2}}}{\sqrt{\gamma\pi}\ \Gamma\left(\frac{\gamma}{2}\right)}\ dx$$

The series used to evaluate the integral are:

γ EVEN

$$P(t,\gamma) = \sin\theta\ \left\{1+\frac{1}{2}\cos^2\theta+\frac{1\cdot3}{2\cdot4}\cos^4\theta+\ldots+\frac{1\cdot3\ \ldots\ (\gamma-3)}{2\cdot4\ \ldots\ (\gamma-2)}\cos^{\gamma-2}\theta\right\}$$

γ ODD

$$P(t,\gamma) = \frac{2\theta}{\pi} + \frac{2\cos\theta}{\pi}\left[\sin\theta\left\{1+\frac{2}{3}\cos^3\theta+\ldots+\frac{2\cdot4\ \ldots\ (\gamma-3)}{1\cdot3\ \ldots\ (\gamma-2)}\cos^{\gamma-2}\theta\right\}\right]$$

where:

$$\theta = \tan^{-1}\frac{t}{\sqrt{\gamma}}$$

Reference: Handbook of Mathematical Functions, Abramowitz & Stegun, National Bureau of Standards (1964).

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

SET: | | RADIANS |

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA:

(t statistic)	t	→ Y
(degrees of freedom)	γ	→ X

PRESS: CONTINUE

DISPLAY

$P(t,\gamma)$	Z
γ	Y
t	X

To run another case

EXAMPLE

General Form:

$$P(t,\gamma)$$

A) $P(2.201, 11) = 0.975$

B) $P(2.5 , 23) = 0.99$

C) $P(0.26 , 10) = 0.60$

00	CLR	20		40	DN	25	
01	STP	41	ENTRY	41	X	36	
02	XTO	23		42	1	01	
03	c	16		43	+	33	
04	YTO	40		44	UP	27	
05	d	17		45	AC-	63	
06	UP	27		46	AC-	63	
07	RDN	31		47	f	15	
08	√	76		48	X>Y	53	
09	DIV	35		49	3	03	
0a	DN	25		4a	a	13	
0b	ARC	72		4b	DN	25	
0c	TAN	71		4c	a	13	
0d	XTO	23		4d	SIN	70	
10	a	13		50	X	36	
11	COS	73		51	IFG	43	
12	UP	27		52	6	06	
13	X	36		53	3	03	
14	YTO	40		54	UP	27	
15	b	14		55	DN	25	
16	CLX	37		56	a	13	
17	RDN	31		57	COS	73	
18	1	01		58	X	36	
19	X=Y	50		59	a	13	
1a	5	05		5a	+	33	
1b	5	05		5b	2	02	
1c	SFL	54		5c	UP	27	
1d	XEY	30		5d	π	56	
20	UP	27		60	DIV	35	
21	2	02		61	DN	25	
22	X=Y	50		62	X	36	
23	4	04		63	CLX	37	
24	b	14		64	UP	27	
25	IFG	43		65	DN	25	
26	UP	27		66	1	01	
27	DN	25		67	-	34	
28	DIV	35		68	|Y|	55	
29	DN	25		69	2	02	
2a	UP	27		6a	DIV	35	
2b	INT	64		6b	CLX	37	
2c	X=Y	50		6c	RDN	31	
2d	SFL	54		6d	+	33	
30	.	21		70	c	16	
31	DN	25		71	UP	27	
32	3	03		72	d	17	
33	-	34		73	PNT	45	
34	UP	27		74	CNT	47	
35	DN	25		75	END	46	
36	1	01					
37	+	33					
38	RDN	31					
39	AC+	60					
3a	RCL	61					
3b	DIV	35					
3c	b	14					
3d	X	36					

θ

Y

t

BETA DISTRIBUTION

This program determines the Beta Distribution which is given by:

$$f(X) = \frac{\Gamma(\alpha + \beta + 2)\ X^{\alpha}\ (1 - X)^{\beta}}{\Gamma(\alpha + 1)\ \Gamma(\beta + 1)}$$

where

$0 < X < 1$

$-1 < \beta$

$-1 < \alpha$

The program also calculates the mean (μ) and variance ($\sigma 2$) of the distribution. These are given by:

$$\mu = \frac{\alpha + 1}{\alpha + \beta + 2}$$

$$\sigma^2 = \frac{(\alpha + 1)(\beta + 1)}{(\alpha + \beta + 2)^2 (\alpha + \beta + 3)}$$

The program uses program 09100-70024, GAMMA FUNCTION, as a subroutine thus restricting α and β to values less than 68.

Reference: Handbook of Tables for Probability and Statistics, Chemical Rubber Company, 1966.

USER INSTRUCTIONS

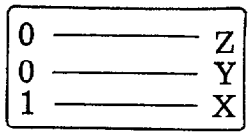
PRESS: X Y Z on the 9120A

PRESS: END

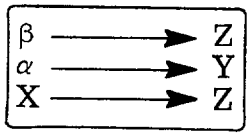
ENTER PROGRAM: Side A followed by Side B.

PRESS: CONTINUE

DISPLAY

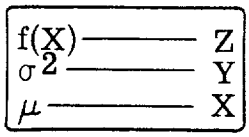


ENTER DATA:



PRESS: CONTINUE

DISPLAY



To run another case

EXAMPLE

$\beta = .5$ $f(X) = 1.2732$

$\alpha = .5$ $\sigma^2 = .0625$

$X = .5$ $\mu = .5$

Error Cases

If $\beta < -1$

DISPLAY



If $\alpha < -1$

DISPLAY



If $X < 0$

DISPLAY



If $X > 1$

DISPLAY



If an error display occurs:

PRESS: END

PRESS: CONTINUE

ENTER DATA:

			Plus Page ENTRY								
00	CLR	20		40	UP	27		80	6	06	
01	1	01		41	a	13		81	1	01	
02	STP	41		42	+	33		82	-	34	
03	PNT	45		43	b	14		83	YTO	40	
04	PNT	45		44	+	33		84	d	17	
05	XTO	23		45	GTO	44		85	UP	27	
06	c	16		46	SUB	77		86	DN	25	
07	YTO	40		47	8	10		87	INT	64	
08	b	14		48	1	01		88	-	34	
09	DN	25		49	XFR	67		89	CLX	37	
0a	YTO	40		4a	9	11		8a	GTO	44	
0b	a	13		4b	X	36		8b	-	34	
0c	UP	27		4c	YTO	40		8c	0	00	
0d	1	01		4d	9	11		8d	0	00	
10	CHS	32		50	1	01		90	CNT	47	
11	X>Y	53		51	UP	27		91	CNT	47	
12	UP	27		52	b	14		92	CNT	47	
13	STP	41		53	+	33		93	CNT	47	
14	XEY	30		54	CLX	37		94	CNT	47	
15	DN	25		55	XTO	23		95	CNT	47	
16	X>Y	53		56	e	12		96	CNT	47	
17	UP	27		57	XTO	23		97	CNT	47	
18	STP	41		58	f	15		98	CNT	47	
19	c	16		59	GTO	44		99	CNT	47	
1a	UP	27		5a	SUB	77		9a	CNT	47	
1b	0	00		5b	8	10		9b	CNT	47	
1c	X>Y	53		5c	1	01		9c	CNT	47	
1d	UP	27		5d	XFR	67		9d	CNT	47	
20	STP	41		60	9	11		a0	CNT	47	
21	1	01		61	XEY	30		a1	CNT	47	
22	X<Y	52		62	DIV	35		a2	CNT	47	
23	UP	27		63	YTO	40		a3	CNT	47	
24	STP	41		64	9	11		a4	CNT	47	
25	DN	25		65	1	01		a5	CNT	47	
26	LN	65		66	UP	27		a6	CNT	47	
27	UP	27		67	a	13		a7	CNT	47	
28	b	14		68	+	33		a8	CNT	47	
29	X	36		69	CLX	37		a9	CNT	47	
2a	DN	25		6a	XTO	23		aa	CNT	47	
2b	EXP	74		6b	e	12		ab	CNT	47	
2c	UP	27		6c	XTO	23		ac	CNT	47	
2d	1	01		6d	f	15		ad	CNT	47	
30	UP	27		70	GTO	44					
31	c	16		71	SUB	77					
32	-	34		72	8	10					
33	DN	25		73	1	01					
34	LN	65		74	XFR	67					
35	UP	27		75	9	11					
36	a	13		76	XEY	30					
37	X	36		77	DIV	35					
38	DN	25		78	YTO	40					
39	EXP	74		79	-	34					
3a	X	36		7a	f	15					
3b	YTO	40		7b	GTO	44					
3c	9	11		7c	-	34					
3d	2	02		7d	6	06					

$(1-X)^{\beta}X^{\alpha}$

β

α

X

$\Gamma(\lambda)$ + | - | σ^2 | f(X)

```
b0   CNT   47        Plus
b1   CNT   47        Page
b2   CNT   47
b3   CNT   47
b4   CNT   47
b5   CNT   47
b6   CNT   47
b7   CNT   47
b8   CNT   47
b9   CNT   47
ba   CNT   47
bb   CNT   47
bc   CNT   47
bd   CNT   47

c0   CNT   47
c1   CNT   47
c2   CNT   47
c3   CNT   47
c4   CNT   47
c5   CNT   47
c6   CNT   47
c7   CNT   47
c8   CNT   47
c9   CNT   47
ca   CNT   47
cb   CNT   47
cc   CNT   47
cd   CNT   47

d0   CNT   47
d1   CNT   47
d2   CNT   47
d3   CNT   47
d4   CNT   47
d5   CNT   47
d6   CNT   47
d7   CNT   47
d8   CNT   47
d9   CNT   47
da   CNT   47
db   CNT   47
dc   CNT   47
dd   END   46
```

00	X>Y	53	Minus Page	40	+	33		80	UP	27	
01	CLX	37		41	DN	25		81	1	01	
02	1	01		42	EXP	74		82	+	33	
03	+	33		43	UP	27		83	DN	25	
04	5	05		44	f	15		84	X	36	
05	+	33		45	UP	27		85	YTO	40	
06	DN	25		46	d	17		86	-	34	
07	UP	27		47	X>Y	53		87	e	12	
08	X	36		48	5	05		88	UP	27	
09	AC+	60		49	2	02		89	b	14	
0a	2	02		4a	X<Y	52		8a	UP	27	
0b	XEY	30		4b	5	05		8b	a	13	
0c	DIV	35		4c	9	11		8c	+	33	
0d	3	03		4d	GTO	44		8d	2	02	
10	-	34		50	6	06		90	+	33	
11	e	12		51	3	03		91	DN	25	
12	DIV	35		52	1	01		92	DIV	35	
13	2	02		53	+	33		93	XFR	67	
14	DIV	35		54	DN	25		94	-	34	
15	+	33		55	X	36		95	f	15	
16	e	12		56	GTO	44		96	UP	27	
17	DIV	35		57	4	04		97	XFR	67	
18	7	07		58	5	05		98	-	34	
19	DIV	35		59	DN	25		99	e	12	
1a	1	01		5a	DIV	35		9a	RUP	22	
1b	-	34		5b	UP	27		9b	PNT	45	S
1c	e	12		5c	1	01		9c	PNT	45	
1d	DIV	35		5d	-	34		9d	END	46	
20	6	06		60	GTO	44					
21	DIV	35		61	4	04					
22	5	05		62	6	06					
23	+	33		63	DN	25					
24	6	06		64	CNT	47					
25	0	00		65	RTN	77					
26	DIV	35		66	b	14					
27	f	15		67	UP	27					
28	DIV	35		68	a	13					
29	-	34		69	+	33					
2a	UP	27		6a	2	02					
2b	.	21		6b	+	33					
2c	5	05		6c	DN	25					
2d	+	33		6d	UP	27					
30	f	15		70	X	36					
31	LN	65		71	UP	27					
32	X	36		72	1	01					
33	DN	25		73	+	33					
34	+	33		74	DN	25					
35	π	56		75	X	36					
36	UP	27		76	b	14					
37	+	33		77	UP	27					
38	DN	25		78	1	01					
39	LN	65		79	+	33					
3a	UP	27		7a	DN	25					
3b	2	02		7b	XEY	30					
3c	DIV	35		7c	DIV	35					
3d	DN	25		7d	a	13					

F-DISTRIBUTION

This program evaluates the integral of the F distribution density function

$$Q = \int_F^{\infty} \frac{\Gamma\left(\frac{V_1 + V_2}{2}\right)}{\Gamma\left(\frac{V_1}{2}\right)\Gamma\left(\frac{V_2}{2}\right)} \frac{X^{V_{1/2}-1}}{\left(1 + \frac{V_1}{V_2} X\right)^{\frac{V_1+V_2}{2}}} \left(\frac{V_1}{V_2}\right)^{V_{1/2}} dx$$

for given values of F, V_1, V_2 .

The integral is evaluated by means of the following series:

V_2 EVEN

$$Q\ (F / V_1, V_2) = 1 - (1-X)^{V_{1/2}} \left[1 + \frac{V_1 X}{2} + \ldots + \frac{V_1\ (V_1+2)\ldots(V_2+V_1-4)}{2\cdot 4\ldots(V_2-2)} X^{\frac{V_2-2}{2}} \right]$$

V_2 ODD

$$Q\ (F / V_1, V_2) = X^{V_{2/2}} \left[1 + \frac{V_2}{2}(1-X) + \ldots + \frac{V_2(V_2+2)\ldots(V_2+V_1-4)}{2\cdot 4\ldots(V_1-2)} (1-X)^{\frac{V_1-2}{2}} \right]$$

V_1 and V_2 both odd

$$Q(F/V_1, V_2) = 1 - A + B$$

$$A = \begin{cases} \frac{2}{\pi}\left\{\theta + \sin\theta\cos\theta\left[1 + 2/3\cos^2\theta + \ldots + \frac{2\cdot 4\ldots(V_2-3)}{3\cdot 5\ldots(V_2-2)}\cos^{V_2-2}\theta\right]\right\} & V_2 > 1 \\ \frac{2\theta}{\pi} & V_2 = 1 \end{cases}$$

$$\theta = \text{Arc Tan}\sqrt{\frac{F}{V_2}}$$

$$B = \begin{cases} 2\frac{\left(\frac{V_2-1}{2}\right)!}{\sqrt{\pi}\left(\frac{V_2-2}{2}\right)!}\sin\theta_1 \cdot \cos^{V2}\theta_1\left\{1 + \frac{V_2+1}{3}\sin^2\theta_1 + \ldots + \frac{(V_2+1)(V_2+3)\ldots(V_2+V_2-4)}{(3)(5)\ldots(V_1-2)} \cdot \sin^{V_2-3}\theta_1\right\} & V_1 > 1 \\ 0 & V_1 = 1 \end{cases}$$

$$\theta_1 = \text{Arc Tan}\sqrt{\frac{V_1 F}{V_2}}$$

Reference: Handbook of Mathematical Functions, Abramowitz and Stegun, National Bureau of Standards (1964)

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

SET: ☐ RADIANS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ PRESS: END

PRESS: CONTINUE

ENTER DATA: F ratio → Z
V_1 for numerator → Y
V_2 for denominator → X

PRESS: CONTINUE

DISPLAY

$Q(F/V_1,V_2)$ — Z
0 ——— Y
0 ——— X

To calculate significance level for new data.

EXAMPLE

General Form $Q(F/V_1, V_2)$

$Q(4.21/7, 6) = 0.05$
$Q(11.4/4, 5) = 0.01$
$Q(3.79/7, 7) = .05$

			Plus Page ENTRY
00	CLR	20	
01	STP	41	
02	PNT	45	
03	PNT	45	
04	XTO	23	
05	f	15	
06	YTO	40	
07	e	12	
08	RDN	31	
09	YTO	40	
0a	b	14	
0b	X	36	
0c	DN	25	
0d	+	33	
10	DN	25	
11	DIV	35	
12	f	15	
13	UP	27	
14	1	01	
15	XTO	23	
16	c	16	
17	2	02	
18	DIV	35	
19	DN	25	
1a	UP	27	
1b	INT	64	
1c	X=Y	50	
1d	3	03	
20	6	06	
21	e	12	
22	XEY	30	
23	2	02	
24	DIV	35	
25	DN	25	
26	UP	27	
27	INT	64	
28	X<Y	52	
29	8	10	
2a	1	01	
2b	SFL	54	
2c	RCL	61	
2d	AC-	63	
30	XEY	30	
31	AC+	60	
32	1	01	
33	RUP	22	
34	-	34	
35	UP	27	
36	DN	25	
37	YTO	40	
38	d	17	
39	e	12	
3a	UP	27	
3b	4	04	
3c	-	34	
3d	f	15	
40	+	33	
41	UP	27	
42	2	02	
43	-	34	
44	CLX	37	
45	X=Y	50	
46	5	05	
47	d	17	
48	YTO	40	
49	f	15	
4a	DN	25	
4b	DIV	35	
4c	d	17	
4d	X	36	
50	c	16	
51	X	36	
52	1	01	
53	+	33	
54	YTO	40	
55	c	16	
56	DN	25	
57	2	02	
58	-	34	
59	f	15	
5a	GTO	44	
5b	4	04	
5c	1	01	
5d	c	16	
60	UP	27	
61	1	01	
62	UP	27	
63	d	17	
64	CHS	32	
65	+	33	
66	DN	25	
67	LN	65	
68	UP	27	
69	e	12	
6a	X	36	
6b	2	02	
6c	DIV	35	
6d	DN	25	
70	EXP	74	
71	X	36	
72	IFG	43	
73	7	07	
74	8	10	
75	1	01	
76	XEY	30	
77	-	34	
78	CLX	37	
79	UP	27	
7a	PNT	45	S
7b	PNT	45	
7c	GTO	44	
7d	0	00	
80	0	00	
81	f	15	
82	UP	27	
83	2	02	
84	DIV	35	
85	DN	25	
86	INT	64	
87	UP	27	
88	UP	27	
89	2	02	
8a	X	36	
8b	LN	65	
8c	RUP	22	
8d	X	36	
90	DN	25	
91	EXP	74	
92	XTO	23	
93	c	16	
94	1	01	
95	-	34	
96	UP	27	
97	DN	25	
98	2	02	
99	-	34	
9a	RDN	31	
9b	X	36	
9c	|Y|	55	
9d	RUP	22	
a0	X<Y	52	
a1	9	11	
a2	9	11	
a3	DN	25	
a4	π	56	
a5	X	36	
a6	c	16	
a7	DIV	35	
a8	2	02	
a9	DIV	35	
aa	GTO	44	
ab	-	34	
ac	0	00	
ad	0	00	

F

V_1

V_2

b0	CNT	47	Plus Page
b1	CNT	47	
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	
c0	CNT	47	
c1	CNT	47	
c2	CNT	47	
c3	CNT	47	
c4	CNT	47	
c5	CNT	47	
c6	CNT	47	
c7	CNT	47	
c8	CNT	47	
c9	CNT	47	
ca	CNT	47	
cb	CNT	47	
cc	CNT	47	
cd	CNT	47	
d0	CNT	47	
d1	CNT	47	
d2	CNT	47	
d3	CNT	47	
d4	CNT	47	
d5	CNT	47	
d6	CNT	47	
d7	CNT	47	
d8	CNT	47	
d9	CNT	47	
da	CNT	47	
db	CNT	47	
dc	CNT	47	
dd	CNT	47	

00	YTO	40	Minus Page
01	c	16	
02	YE	24	
03	b	14	
04	f	15	
05	DIV	35	
06	e	12	
07	X	36	
08	RDN	31	
09	√	76	
0a	ARC	72	
0b	TAN	71	
0c	UP	27	
0d	SIN	70	

10	YTO	40
11	-	34
12	f	15
13	UP	27
14	X	36
15	YTO	40
16	d	17
17	RCL	61
18	1	01
19	X=Y	50
1a	7	07
1b	4	04
1c	XTO	23
1d	b	14

20	f	15
21	+	33
22	4	04
23	-	34
24	e	12
25	UP	27
26	2	02
27	-	34
28	X>Y	53
29	4	04
2a	6	06
2b	YTO	40
2c	-	34
2d	e	12

30	DN	25
31	DIV	35
32	d	17
33	X	36
34	b	14
35	X	36
36	1	01
37	+	33
38	YTO	40
39	b	14
3a	DN	25
3b	2	02
3c	-	34
3d	UP	27

40	YE	24
41	-	34
42	e	12
43	GTO	44
44	2	02
45	7	07
46	IFG	43
47	9	11
48	6	06
49	RUP	22
4a	XEY	30
4b	3	03
4c	X>Y	53
4d	5	05

50	d	17
51	2	02
52	DIV	35
53	UP	27
54	DN	25
55	1	01
56	-	34
57	RDN	31
58	X	36
59	RUP	22
5a	X<Y	52
5b	5	05
5c	6	06
5d	RDN	31

60	c	16
61	DIV	35
62	1	01
63	UP	27
64	d	17
65	-	34
66	√	76
67	RUP	22
68	X	36
69	RUP	22
6a	√	76
6b	LN	65
6c	XEY	30
6d	f	15

70	X	36
71	RDN	31
72	EXP	74
73	X	36
74	b	14
75	X	36
76	YTO	40
77	c	16
78	YE	24
79	-	34
7a	f	15
7b	YTO	40
7c	e	12
7d	DN	25

80	COS	73
81	UP	27
82	X	36
83	YTO	40
84	d	17
85	f	15
86	UP	27
87	1	01
88	YTO	40
89	-	34
8a	e	12
8b	XTO	23
8c	b	14
8d	SFL	54

90	-	34
91	X<Y	52
92	3	03
93	b	14
94	CLX	37
95	XTO	23
96	b	14
97	UP	27
98	d	17
99	√	76
9a	X	36
9b	e	12
9c	SIN	70
9d	X	36

a0	e	12	
a1	+	33	
a2	2	02	
a3	X	36	
a4	π	56	
a5	DIV	35	
a6	c	16	
a7	XEY	30	
a8	-	34	
a9	1	01	
aa	+	33	
ab	CLX	37	
ac	UP	27	
ad	PNT	45	S

b0	PNT	45
b1	GTO	44
b2	+	33
b3	0	00
b4	0	00
b5	END	46

Minus
Page

This program computes various functions associated with the negative binomial distribution which is given by:

$$f(X) = \binom{X + r - 1}{r - 1} p^r q^X \quad ,$$

where $X = 0, 1, 2, 3, \ldots$, and $q = 1 - p$.

Definitions:

p - is the probability of success of a given event.

q - is the probability of failure of a given event.

f(X)- is the probability that exactly X + r trials will be required to produce r successes.

The cumulative distribution F(X) is given by

$$F(X) = \sum_{\lambda = 0}^{X} f(\lambda)$$

The mean and variance of the distribution are given by:

$$\mu = \frac{r}{p}$$

$$\sigma^2 = \frac{rq}{p^2}$$

This program computes μ, σ^2, f(X), and F(X).

Reference: Handbook of Tables for Probability and Statistics, Chemical Rubber Company, 1966.

USER INSTRUCTIONS

PRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM:

PRESS: CONTINUE

DISPLAY

0	Z
1	Y
0	X

ENTER DATA:

r	Y
p	X

PRESS: CONTINUE

DISPLAY

2	Z
σ^2	Y
μ	X

PRESS: CONTINUE

DISPLAY

F(X)	Z
f(X)	Y
X	X

Repeat until F(X) is sufficiently near unity.

To run another case:

PRESS: END

EXAMPLE

r = 4

p = .9

Results:

2	
.4938	σ^2
4.4444	μ

.6561	F(X)
.6561	f(X)
0.	X

.9185	F(X)
.2624	f(X)
1.	X

.9841	F(X)
.0656	f(X)
2.	X

.9973	F(X)
.0131	f(X)
3.	X

.

.

.

00	CLR	20		40	-	34		
01	XTO	23		41	a	13		
02	c	16		42	UP	27		
03	XTO	23		43	1	01		
04	9	11		44	-	34		
05	1	01		45	0	00		
06	XEY	30		46	XTO	23		
07	STP	41	ENTRY	47	e	12		
08	PNT	45		48	XTO	23		
09	PNT	45		49	f	15		
0a	XTO	23		4a	RDN	31		
0b	b	14		4b	XTO	23		
0c	YTO	40		4c	d	17		
0d	a	13		4d	1	01		
10	DIV	35		50	AC+	60		
11	AC+	60		51	RDN	31		
12	DIV	35		52	f	15		
13	UP	27		53	XEY	30		
14	1	01		54	d	17		
15	XEY	30		55	X<Y	52		
16	-	34		56	6	06		
17	DN	25		57	5	05		
18	X	36		58	RCL	61		
19	2	02		59	DIV	35		
1a	XEY	30		5a	DN	25		
1b	UP	27		5b	X	36		
1c	e	12		5c	1	01		
1d	PNT	45	s	5d	UP	27		
20	PNT	45		60	CHS	32		
21	a	13		61	AC-	63		
22	UP	27		62	GTO	44		
23	b	14		63	5	05		
24	LN	65		64	2	02		
25	X	36		65	YEX	24		
26	DN	25		66	8	10		
27	EXP	74		67	DN	25		
28	UP	27		68	X	36		
29	1	01		69	DN	25		
2a	UP	27		6a	YEX	24		
2b	b	14		6b	9	11		
2c	-	34		6c	+	33		
2d	DN	25		6d	YTO	40		
30	LN	65		70	9	11		$p^r q^x$
31	UP	27		71	UP	27		
32	c	16		72	c	16		F(X)
33	X	36		73	PNT	45	s	
34	DN	25		74	PNT	45		r
35	EXP	74		75	UP	27		
36	X	36		76	1	01		p
37	YTO	40		77	+	33		
38	8	10		78	YTO	40		X
39	c	16		79	c	16		
3a	UP	27		7a	GTO	44		
3b	a	13		7b	2	02		
3c	+	33		7c	1	01		
3d	1	01		7d	END	46		

GAMMA DISTRIBUTION

This program determines the Gamma Distribution given by:

$$f(X) = \frac{X^{\alpha} \; e^{-\frac{X}{\beta}}}{\Gamma(\alpha + 1)\, \beta^{\alpha + 1}} \qquad (0 < X < \infty)$$

where α and β are parameters with limitations:

$$\alpha > -1$$

and

$$\beta > 0$$

This program incorporates program 09100-70024, GAMMA FUNCTION, and is restricted to values of $\alpha < 68.98$.

Also evaluated are the mean (μ) and variance (σ^2) of the distribution

$$\mu = \beta(\alpha + 1)$$

$$\sigma^2 = \beta^2(\alpha + 1)$$

Reference: Handbook of Tables for Probability and Statistics , Chemical Rubber Company, 1966.

USER INSTRUCTIONS

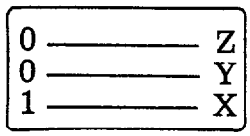
PRESS: X Y Z on the 9120A

PRESS: END

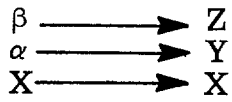
ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

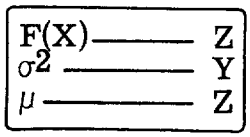
DISPLAY



ENTER DATA:



PRESS: CONTINUE



To run another case

EXAMPLE

Parameters:

$\beta = 2$
$\alpha = .5$
$X = 3$

Results:

$f(X) = .15418$
$\sigma^2 = 6$
$\mu = 3$

The program has three input data checks, these being:

1. is $\alpha > -1$?
2. is $\beta > 0$?
3. is $X > 0$?

If any of these checks indicate bad data, the calculator STOPS indicating the type of error by DISPLAY.

Error 1. ($\beta < 0$)

DISPLAY



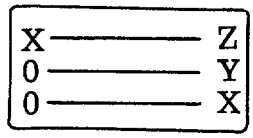
Error 2. ($\alpha < -1$)

DISPLAY



Error 3. ($X < 0$)

DISPLAY



00	CLR	20	Plus Page ENTRY	40	UP	27		80	UP	27	
01	1	01		41	YTO	40		81	+	33	
02	STP	41		42	d	17		82	DN	25	
03	PNT	45		43	UP	27		83	LN	65	
04	PNT	45		44	DN	25		84	UP	27	
05	XTO	23		45	INT	64		85	2	02	
06	c	16		46	-	34		86	DIV	35	
07	YTO	40		47	CLX	37		87	DN	25	
08	b	14		48	X>Y	53		88	+	33	
09	DN	25		49	CLX	37		89	DN	25	
0a	YTO	40		4a	1	01		8a	GTO	44	
0b	a	13		4b	+	33		8b	-	34	
0c	UP	27		4c	5	05		8c	0	00	
0d	1	01		4d	+	33		8d	0	00	
10	CHS	32		50	DN	25		90	CNT	47	
11	X>Y	53		51	UP	27		91	CNT	47	
12	UP	27		52	X	36		92	CNT	47	
13	STP	41		53	AC+	60		93	CNT	47	
14	DN	25		54	2	02		94	CNT	47	
15	0	00		55	XEY	30		95	CNT	47	
16	X>Y	53		56	DIV	35		96	CNT	47	
17	UP	27		57	3	03		97	CNT	47	
18	STP	41		58	-	34		98	CNT	47	
19	UP	27		59	e	12		99	CNT	47	
1a	c	16		5a	DIV	35		9a	CNT	47	
1b	XEY	30		5b	2	02		9b	CNT	47	
1c	X>Y	53		5c	DIV	35		9c	CNT	47	
1d	UP	27		5d	+	33		9d	CNT	47	
20	STP	41		60	e	12		a0	CNT	47	
21	a	13		61	DIV	35		a1	CNT	47	
22	DIV	35		62	7	07		a2	CNT	47	
23	DN	25		63	DIV	35		a3	CNT	47	
24	CHS	32		64	1	01		a4	CNT	47	
25	EXP	74		65	-	34		a5	CNT	47	
26	UP	27		66	e	12		a6	CNT	47	
27	c	16		67	DIV	35		a7	CNT	47	
28	LN	65		68	6	06		a8	CNT	47	
29	UP	27		69	DIV	35		a9	CNT	47	
2a	b	14		6a	5	05		aa	CNT	47	
2b	X	36		6b	+	33		ab	CNT	47	
2c	DN	25		6c	6	06		ac	CNT	47	
2d	EXP	74		6d	0	00		ad	CNT	47	
30	X	36		70	DIV	35					
31	b	14		71	f	15					
32	UP	27		72	DIV	35					$X^{\alpha} e^{-\frac{X}{\beta}} \beta^{\alpha+1}$
33	1	01		73	-	34					
34	+	33		74	UP	27					β
35	a	13		75	.	21					α
36	LN	65		76	5	05					X
37	X	36		77	+	33					
38	DN	25		78	f	15					
39	EXP	74		79	LN	65					$\Gamma(\phi)$
3a	DIV	35		7a	X	36					
3b	YTO	40		7b	DN	25					
3c	9	11		7c	+	33					
3d	b	14		7d	π	56					

b0	CNT	47	Plus Page
b1	CNT	47	
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	
c0	CNT	47	
c1	CNT	47	
c2	CNT	47	
c3	CNT	47	
c4	CNT	47	
c5	CNT	47	
c6	CNT	47	
c7	CNT	47	
c8	CNT	47	
c9	CNT	47	
ca	CNT	47	
cb	CNT	47	
cc	CNT	47	
cd	CNT	47	
d0	CNT	47	
d1	CNT	47	
d2	CNT	47	
d3	CNT	47	
d4	CNT	47	
d5	CNT	47	
d6	CNT	47	
d7	CNT	47	
d8	CNT	47	
d9	CNT	47	
da	CNT	47	
db	CNT	47	
dc	CNT	47	
dd	CNT	47	

Minus
Page

00	EXP	74
01	UP	27
02	f	15
03	UP	27
04	d	17
05	X>Y	53
06	1	01
07	0	00
08	X<Y	52
09	1	01
0a	7	07
0b	GTO	44
0c	2	02
0d	1	01
10	1	01
11	+	33
12	DN	25
13	X	36
14	GTO	44
15	1	01
16	3	03
17	DN	25
18	DIV	35
19	UP	27
1a	1	01
1b	-	34
1c	GTO	44
1d	0	00
20	4	04
21	DN	25
22	XFR	67
23	9	11
24	XEY	30
25	DIV	35
26	b	14
27	UP	27
28	1	01
29	+	33
2a	a	13
2b	X	36
2c	XEY	30
2d	X	36
30	PNT	45
31	PNT	45
32	END	46

LOG NORMAL DISTRIBUTION

This program computes the value of the log-normal distribution for a given set of parameters, a, μ, σ, over a specified range of X.

Definitions:

$$f(X) = \frac{1}{(X - a)\,\sigma\sqrt{2\pi}} \quad \exp \left\{ -\left[\frac{(\ln [X - a] - \mu)^2}{2\sigma^2} \right] \right\}$$

where

$$X > a$$

The program determines f(X) over a range

$$X_i \leq X \leq X_f$$

in increments of ΔX.

$$X_i = X_{initial}$$
$$X_f = X_{final}$$

Reference: Probability and Statistics for Engineers, Miller and Freund, Prentice Hall, 1965, page 77.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA:

a	Z
σ	Y
μ	X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA:

ΔX	Z
X_f	Y
X_i	X

PRESS: CONTINUE

DISPLAY

f(X)	Z
X	Y
ΔX	Z

When $X = X_f$

EXAMPLE

$a = 0$ $\quad \Delta X = .1$
$\sigma = 1$ $\quad X_f = 1$
$\mu = 1$ $\quad X_i = .1$

Results:

X	f(X)
.1	.01708
.2	.06627
.3	.11722
.4	.15902
.5	.19030
.6	.21237
.7	.22706
.8	.23602
.9	.24063
1.0	.24197

00	CLR	20		40	-	34	
01	1	01		41	DN	25	
02	STP	41	ENTRY	42	DIV	35	
03	PNT	45		43	π	56	
04	PNT	45		44	√	76	
05	AC+	60		45	DIV	35	
06	DN	25		46	2	02	
07	YTO	40		47	√	76	
08	d	17		48	DIV	35	
09	0	00		49	c	16	
0a	UP	27		4a	UP	27	
0b	UP	27		4b	a	13	
0c	2	02		4c	PNT	45	s
0d	STP	41	ENTRY	4d	PNT	45	
10	PNT	45		50	a	13	
11	PNT	45		51	UP	27	
12	XTO	23		52	c	16	
13	c	16		53	+	33	
14	YTO	40		54	YTO	40	
15	b	14		55	c	16	
16	DN	25		56	b	14	
17	YTO	40		57	X>Y	53	
18	a	13		58	1	01	
19	c	16		59	b	14	
1a	UP	27		5a	X=Y	50	
1b	d	17		5b	1	01	
1c	-	34		5c	b	14	
1d	0	00		5d	GTO	44	
20	X>Y	53		60	0	00	
21	5	05		61	0	00	
22	0	00		62	END	46	
23	X=Y	50					
24	5	05					
25	0	00					
26	f	15					
27	XEY	30					
28	LN	65					
29	XEY	30					
2a	-	34					
2b	DN	25					
2c	UP	27					
2d	X	36					
30	2	02					
31	DIV	35					
32	e	12					
33	DIV	35					
34	DIV	35					
35	DN	25					
36	CHS	32					
37	EXP	74					
38	UP	27					
39	e	12					
3a	DIV	35					
3b	c	16					
3c	UP	27					
3d	d	17					

Δx

x_f

x_i

a

σ

μ

ERROR FUNCTION

This program computes Erf (X) as defined below:

$$\text{Erf}(X) = \frac{2}{\sqrt{\pi}} \int_0^X e^{-t^2} dt$$

The program applies the approximating expansion given in the reference.

$$\text{Erf}(X) = 1 - \frac{1}{[\,1 + a_1X + a_2X^2 + a_3X^3 + a_4X^4 + a_5X^5 + a_6X^6\,]^{16}}$$

where

$$\begin{aligned} a_1 &= .0705230784 \\ a_2 &= .0422820123 \\ a_3 &= .0092705272 \\ a_4 &= .0001520143 \\ a_5 &= .0002765672 \\ a_6 &= .0000430638 \end{aligned}$$

The absolute error resulting from this approximating function is less than $3 \cdot 10^{-7}$.

Reference: Approximations for Digital Computers, C. Hastings, Jr. Princeton University Press, 1955.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: X → X

PRESS: CONTINUE

DISPLAY

Erf(X)	Z
Erf(X)	Y
Erf(X)	X

To run another case

EXAMPLE

1)
X = 2.3

Erf(2.3) = .99885680

2)

Since $\text{Erf}\left(\frac{X}{\sqrt{2}}\right) = 2\int_0^X f(t)\,dt$

where f(t) is the normal probability density.

The Erf program can be used to determine normal probability integrals.

Thus

$$2\int_0^1 f(t)dt = \text{Erf}\left(\frac{1}{\sqrt{2}}\right)$$

$$= .68268936$$

Thus

$$\int_0^1 f(t)dt = .34134468$$

00	CLR	20		40	+	33		80	DN	25	
01	1	01		41	RUP	22		81	+	33	
02	STP	41	ENTRY	42	XEY	30		82	1	01	
03	PNT	45		43	f	15		83	+	33	
04	PNT	45		44	X	36		84	DN	25	
05	XTO	23		45	.	21		85	LN	65	
06	f	15		46	0	00		86	UP	27	
07	UP	27		47	0	00		87	1	01	
08	.	21		48	0	00		88	6	06	
09	0	00		49	1	01		89	X	36	
0a	7	07		4a	5	05		8a	DN	25	
0b	0	00		4b	2	02		8b	EXP	74	
0c	5	05		4c	0	00		8c	XEY	30	
0d	2	02		4d	1	01		8d	1	01	
10	3	03		50	4	04		90	XEY	30	
11	0	00		51	3	03		91	DIV	35	
12	7	07		52	XEY	30		92	1	01	
13	8	10		53	X	36		93	XEY	30	
14	4	04		54	RDN	31		94	-	34	
15	XEY	30		55	+	33		95	UP	27	
16	X	36		56	RUP	22		96	DN	25	
17	UP	27		57	XEY	30		97	DN	25	
18	X	36		58	f	15		98	PNT	45	
19	.	21		59	X	36		99	PNT	45	
1a	0	00		5a	.	21		9a	END	46	
1b	4	04		5b	0	00					
1c	2	02		5c	0	00					
1d	2	02		5d	0	00					
20	8	10		60	2	02					
21	2	02		61	7	07					
22	0	00		62	6	06					
23	1	01		63	5	05					
24	2	02		64	6	06					
25	3	03		65	7	07					
26	XEY	30		66	2	02					
27	X	36		67	XEY	30					
28	RDN	31		68	X	36					
29	+	33		69	RDN	31					
2a	RUP	22		6a	+	33					
2b	XEY	30		6b	RUP	22					
2c	f	15		6c	XEY	30					
2d	X	36		6d	f	15					
30	.	21		70	X	36					
31	0	00		71	.	21					
32	0	00		72	0	00					
33	9	11		73	0	00					
34	2	02		74	0	00					
35	7	07		75	0	00					
36	0	00		76	4	04					
37	5	05		77	3	03					
38	2	02		78	0	00					
39	7	07		79	6	06					
3a	2	02		7a	3	03					
3b	XEY	30		7b	8	10					
3c	X	36		7c	XEY	30					
3d	RDN	31		7d	X	36					

x

JOINT NORMAL PROBABILITY DENSITY FUNCTION

Two random variables X and Y are said to have a Bivariate Normal Distribution if their joint density function is given by

$$f(X,Y) = \frac{1}{2\pi s_X s_Y \sqrt{1 - r^2}} \quad e^{-P(X,Y)}$$

where

$$P(X, Y) = \frac{1}{2(1 - r^2)} \left\{ \frac{(X - \bar{X})^2}{s_X^2} - 2r \frac{(X - \bar{X})(Y - \bar{Y})}{s_X s_Y} + \frac{(Y - \bar{Y})^2}{s_Y^2} \right\}$$

r = Correlation Coefficient Between X and Y

$\bar{X}$ = Sample Mean of X

$\bar{Y}$ = Sample Mean of Y

s_X = Sample Standard Deviation of X

s_Y = Sample Standard Deviation of Y

X and Y are random variables

This program evaluates $f(X, Y)$ for given values of r, $\bar{X}$, $\bar{Y}$, s_X, s_Y, X, and Y.

Reference: Introduction to Mathematical Statistics, R. V. Hogg, A. T. Craig, 1959, page 218, Macmillan.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A
PRESS: END
ENTER PROGRAM
PRESS: CONTINUE
DISPLAY

0	Z
1	Y
0	X

ENTER DATA:

$\overline{Y}$	Y
$\overline{X}$	X

PRESS: CONTINUE
DISPLAY

0	Z
2	Y
0	X

ENTER DATA:

s_Y	Y
s_X	X

PRESS: CONTINUE
DISPLAY

0	Z
3	Y
0	X

ENTER DATA:

r	Z
Y	Y
X	X

PRESS: CONTINUE
DISPLAY

f(X, Y)	Z
f(X, Y)	Y
f(X, Y)	X

To run another case

EXAMPLE

$\overline{X}$	=	-1
$\overline{Y}$	=	1
s_X	=	1.5
s_Y	=	.5
r	=	.7
Y	=	2
X	=	1

Results:

f(X, Y) = .0400399

00	CLR	20		40	-	34		
01	1	01		41	c	16		
02	XEY	30		42	UP	27		
03	STP	41	ENTRY	43	X	36		
04	PNT	45		44	a	13		
05	PNT	45		45	DIV	35		
06	XTO	23		46	DIV	35		
07	d	17		47	DN	25		
08	YTO	40		48	+	33		
09	c	16		49	e	12		
0a	CLR	20		4a	UP	27		
0b	2	02		4b	X	36		
0c	XEY	30		4c	1	01		
0d	STP	41	ENTRY	4d	XEY	30		
10	PNT	45		50	-	34		
11	PNT	45		51	2	02		
12	XTO	23		52	XEY	30		
13	b	14		53	X	36		
14	YTO	40		54	RDN	31		
15	a	13		55	CHS	32		
16	CLR	20		56	DIV	35		
17	3	03		57	DN	25		
18	XEY	30		58	EXP	74		
19	STP	41	ENTRY	59	XEY	30		
1a	PNT	45		5a	$\sqrt{}$	76		
1b	PNT	45		5b	DIV	35		
1c	YEX	24		5c	2	02		
1d	d	17		5d	DIV	35		
20	XEY	30		60	π	56		
21	-	34		61	DIV	35		
22	YEX	24		62	b	14		
23	d	17		63	DIV	35		
24	c	16		64	a	13		
25	-	34		65	DIV	35		
26	YTO	40		66	DN	25		
27	c	16		67	UP	27		
28	d	17		68	UP	27		
29	X	36		69	PNT	45	s	
2a	DN	25		6a	PNT	45		
2b	YTO	40		6b	PNT	45		
2c	e	12		6c	PNT	45		
2d	X	36		6d	GTO	44		
30	2	02		70	0	00		
31	X	36		71	0	00		
32	b	14		72	END	46		
33	DIV	35						
34	a	13						
35	DIV	35						
36	d	17						
37	UP	27						
38	X	36						
39	b	14						
3a	DIV	35						
3b	DIV	35						
3c	DN	25						
3d	XEY	30						

S_Y

S_X

$\overline{Y}$, $Y - \overline{Y}$

$\overline{X}$, Y, $(X - \overline{X})$

r

CUMULATIVE BINOMIAL DISTRIBUTION

This program determines the cumulative binomial distribution given by:

$$P\{X \leq r\} = \sum_{x=r}^{n} B(r, n, p)$$

where:

$$B(r, n, p) = {}_nC_r \, p^r \, (1-p)^{n-r}$$

p = Probability of a Success
r = Number of Successes
n = Number of Trials

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

```
0 ———— Z
0 ———— Y
0 ———— X
```

ENTER DATA:

```
p ————→ Z
n ————→ Y
r ————→ X
```

PRESS: CONTINUE

DISPLAY

```
P(X≤r) —— Z
n ———— Y
r ———— X
```

PRESS: CONTINUE

DISPLAY

```
1-p ———— Z
p ———— Y
0 ———— X
```

To run another case

EXAMPLE

If the probability of a successful trial is .25 (p = .25) and if 8 trials are to be performed, (n = 8), what is the probability of at least 5 successes, (r = 5) .

$P(X \leq 5) = .0273$

00	CLR	20		40	DN	25	
01	STP	41	ENTRY	41	EXP	74	
02	XTO	23		42	X	36	
03	a	13		43	b	14	
04	YTO	40		44	UP	27	
05	b	14		45	a	13	
06	RUP	22		46	PNT	45	s
07	XTO	23		47	PNT	45	
08	c	16		48	c	16	
09	1	01		49	UP	27	
0a	RDN	31		4a	1	01	
0b	X=Y	50		4b	XEY	30	
0c	3	03		4c	-	34	
0d	9	11		4d	UP	27	
10	XEY	30		50	0	00	
11	CLX	37		51	PNT	45	s
12	X=Y	50		52	PNT	45	
13	4	04		53	PNT	45	
14	0	00		54	PNT	45	
15	DN	25		55	PNT	45	
16	c	16		56	GTO	44	
17	-	34		57	0	00	
18	DIV	35		58	0	00	
19	YTO	40		59	END	46	
1a	d	17					
1b	b	14					
1c	XEY	30					
1d	a	13					
20	-	34					
21	RUP	22					
22	+	33					
23	RDN	31					
24	XEY	30					
25	AC+	60					
26	RCL	61					
27	DIV	35					
28	DN	25					
29	X	36					
2a	d	17					
2b	X	36					
2c	1	01					
2d	+	33					
30	UP	27					
31	CHS	32					
32	AC+	60					
33	f	15					
34	XEY	30					
35	CLX	37					
36	X<Y	52					
37	2	02					
38	6	06					
39	c	16					
3a	LN	65					
3b	XEY	30					
3c	b	14					
3d	X	36					

STORAGE

r

n

p

SECTION III TEST STATISTICS

		Part Number
III-1	t Statistic for Means of Two Samples	1
III-2	Paired Observation Test	4
III-3	t Statistic – Testing a Mean $\bar{X}$ Equal to μ_0	8
III-4	χ^2 Statistic – Testing a Population Variance Equal to a Value σ_0^2	11
III-5	χ^2 – Chi Square Evaluation Expected Values Equal ($E_i = E$)	14
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t STATISTIC FOR MEANS OF TWO SAMPLES

This program calculates the value of the t statistic from the equation

$$t = \left| \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{1}{n_x} + \frac{1}{n_y}} \sqrt{\frac{\sum(X_i - \bar{X})^2 + \sum(Y_i - \bar{Y})^2}{n_x + n_y - 2}}} \right|$$

where $X_1, X_2, \cdots X_{n_x}$ and $Y_1, Y_2, \cdots Y_{n_y}$ are samples from normal populations with means μ_X, and μ_Y, both with same variance σ^2. The number of degrees of freedom $n = (n_x + n_y - 2)$. The sample means are calculated by the equations.

$$\bar{X} = \frac{\sum X_i}{n_x} \qquad \bar{Y} = \frac{\sum Y_i}{n_y}$$

The sample estimates of the standard deviations are also displayed. These are calculated by the equations

$$S_x = \sqrt{\frac{\sum(X_i - \bar{X})^2}{n_x - 1}} \qquad S_y = \sqrt{\frac{\sum(Y_i - \bar{Y})^2}{n_y - 1}}$$

This program may be used to test the null hypothesis $\mu_X = \mu_Y$ where we don't assume knowledge of σ^2.

Reference: Statistical Theory and Methodology in Science and Engineering
by K.A. Brownlee
Second Edition
John Wiley and Sons, Inc. 1965

USER INSTRUCTIONS

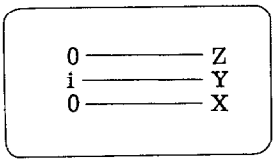
DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

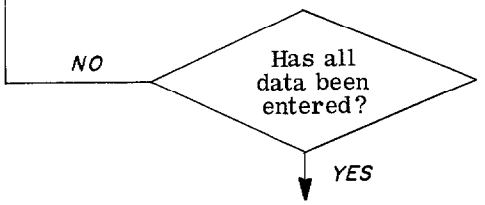
DISPLAY



(i indicates point to be entered)

ENTER DATA: $X_i \longrightarrow X$

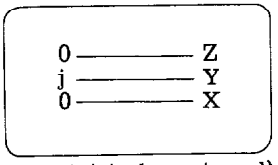
PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

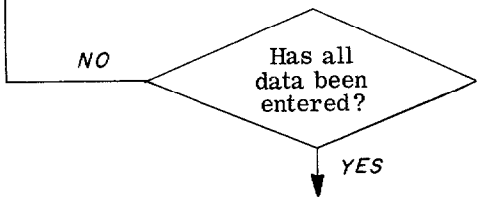
DISPLAY



(j indicates point to be entered)

ENTER DATA: $Y_j \longrightarrow X$

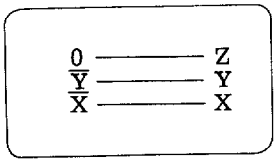
PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

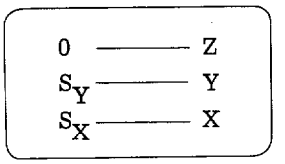
DISPLAY



PRESS: CONTINUE

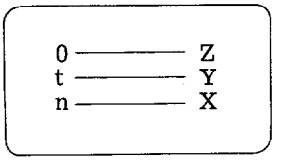
USER INSTRUCTIONS (Con't)

DISPLAY



PRESS: CONTINUE

DISPLAY



PRESS: CONTINUE To restart a new problem

EXAMPLE

X 79, 84, 108, 114, 120, 103, 122, 120

Y 91, 103, 90, 113, 108, 87, 100, 80, 99, 54

$\overline{Y}$ = 92.50
$\overline{X}$ = 106.25

S_Y = 16.82
S_X = 16.64

t = 1.73
n = 16.00

00	CLR	20		40	GTO	44			80	+	33			
01	1	01		41	0	00			81	DN	25			
02	XTO	23		42	4	04			82	XEY	30			
03	a	13		43	0	00			83	DIV	35			
04	XEY	30		44	UP	27			84	b	14			
05	STP	41	ENTRY	45	e	12			85	UP	27			
06	IFG	43		46	UP	27			86	e	12			
07	1	01		47	b	14			87	-	34			
08	c	16		48	PNT	45	S		88	a	13			
09	PNT	45		49	PNT	45			89	√	76			
0a	PNT	45		4a	c	16			8a	RUP	22			
0b	YTO	40		4b	UP	27			8b	√	76			
0c	d	17		4c	a	13			8c	X	36			
0d	UP	27		4d	UP	27			8d	DN	25			
10	X	36		50	1	01			90	DIV	35			
11	AC+	60		51	-	34			91	|Y|	55			
12	d	17		52	DN	25			92	d	17			
13	UP	27		53	DIV	35			93	UP	27			
14	1	01		54	d	17			94	0	00			
15	+	33		55	UP	27			95	RDN	31			
16	0	00		56	1	01			96	PNT	45	S		
17	UP	27		57	-	34			97	PNT	45			
18	RDN	31		58	f	15			98	PNT	45			
19	GTO	44		59	XEY	30			99	PNT	45			
1a	0	00		5a	DIV	35			9a	GTO	44			
1b	5	05		5b	RUP	22			9b	0	00			
1c	d	17		5c	√	76			9c	0	00			
1d	RDN	31		5d	RUP	22			9d	END	46			
20	RCL	61		60	√	76								
21	RUP	22		61	UP	27								
22	DIV	35		62	0	00								
23	YTO	40		63	RDN	31								
24	e	12		64	XEY	30								
25	X	36		65	PNT	45	S							
26	e	12		66	PNT	45								
27	X	36		67	1	01								
28	DN	25		68	UP	27								
29	-	34		69	a	13								
2a	YTO	40		6a	DIV	35								
2b	f	15		6b	1	01								
2c	a	13		6c	UP	27								
2d	UP	27		6d	d	17								
30	1	01		70	DIV	35								
31	X<Y	52		71	RDN	31								
32	4	04		72	+	33								
33	3	03		73	YE	24								
34	RCL	61		74	a	13								$1, n_1$
35	XTO	23		75	2	02								
36	c	16		76	-	34								
37	YTO	40		77	DN	25								
38	b	14		78	+	33								
39	d	17		79	YTO	40								n
3a	XTO	23		7a	d	17								
3b	a	13		7b	c	16								
3c	CLR	20		7c	UP	27								
3d	1	01		7d	f	15								

PAIRED OBSERVATION TEST

Given a table of paired observations as illustrated below,

sample	X_i	Y_i
1	X_1	Y_1
2	X_2	Y_2
3	X_3	Y_3
.	.	.
.	.	.
.	.	.
n	X_n	Y_n

this program determines the statistics associated with the difference, D, defined by

$$D_i = X_i - Y_i$$

The following statistics are determined;

$$\bar{D} = \frac{1}{n} \sum_{i=1}^{n} D_i$$

$$S_D = \sqrt{\frac{\Sigma D_i^2 - \frac{(\Sigma D_i)^2}{n}}{n - 1}}$$

$$S_{\bar{D}} = \frac{S_D}{\sqrt{n}}$$

$$t \text{ (test statistic)} = \frac{\bar{D}}{S_{\bar{D}}}$$

$$\gamma \text{(degrees of freedom)} = n - 1$$

Application

H_o : = $\mu_x = \mu_y$
H_A : = $\mu_x \neq \mu_y$
α = confidence level (.05)
$t = \frac{\bar{D}}{S_{\bar{D}}}$

Conclusion:

If $t > t_{(\frac{\alpha}{2}, n-1)}$, we reject the hypothesis H_o .

Reference: Statistics in Research, by Ostle, Iowa State University Press, 1966.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

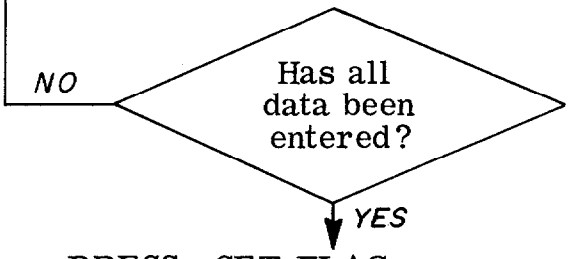
0	Z
i	Y
0	X

ENTER DATA:

$Y_i \longrightarrow Y$

$X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

$S_{\bar{D}}$	Z
S_D	Y
$\bar{D}$	X

PRESS: CONTINUE

DISPLAY

t	Z
γ	Y
n	X

To run another case

EXAMPLE

Sample	X	Y
1	14.	17.
2	17.5	20.7
3	17.	21.6
4	17.5	20.9
5	15.4	17.2

Results

$S_{\bar{D}} = .4472$

$S_D = 1.$

$\bar{D} = -3.2$

$t = -7.1554$

$\gamma = 4$

$n = 5$

00	CLR	20		40	XTO	23	
01	1	01		41	b	14	
02	XTO	23		42	UP	27	
03	d	17		43	d	17	
04	XEY	30		44	√	76	
05	STP	41	ENTRY	45	DIV	35	
06	IFG	43		46	b	14	
07	2	02		47	UP	27	
08	1	01		48	c	16	
09	PNT	45		49	PNT	45	S
0a	PNT	45		4a	PNT	45	
0b	XEY	30		4b	XEY	30	
0c	-	34		4c	DN	25	
0d	DN	25		4d	XEY	30	
10	UP	27		50	DIV	35	
11	X	36		51	d	17	
12	AC+	60		52	UP	27	
13	d	17		53	1	01	
14	UP	27		54	-	34	
15	1	01		55	d	17	
16	+	33		56	PNT	45	
17	YTO	40		57	PNT	45	
18	d	17		58	END	46	
19	0	00					
1a	UP	27					
1b	RDN	31					
1c	GTO	44					
1d	0	00					
20	5	05					
21	d	17					
22	UP	27					
23	1	01					
24	-	34					
25	YTO	40					
26	d	17					
27	f	15					
28	XEY	30					
29	DIV	35					
2a	YTO	40					
2b	c	16					
2c	f	15					
2d	UP	27					
30	X	36					
31	d	17					
32	DIV	35					
33	e	12					
34	XEY	30					
35	-	34					
36	d	17					S_D
37	UP	27					$\overline{D}$
38	1	01					n
39	-	34					
3a	DN	25					ΣD_i^2
3b	DIV	35					
3c	DN	25					ΣD_i
3d	√	76					

t STATISTIC - TEST A MEAN $\overline{X}$ EQUAL TO μ_o

This program calculates the test statistic t used in testing a sample mean $\overline{X}$ equal to a value μ_o.

Hypothesis

$H_o: \quad \mu = \mu_o$

$H_A: \quad \mu \neq \mu_o$

test statistic

$$t = \frac{\overline{X} - \mu_o}{S_{\overline{X}}}$$

where

$\overline{X}$ = Sample Mean

S_X = Sample Standard Deviation

$S_{\overline{X}} = \frac{S_X}{\sqrt{n}}$

α is the level of significance

and

n is the number of observations in the sample

Conclusion:

Reject H_o if $t > \left| t_{(n-1,\ \alpha/2)} \right|$

Reference: Statistics in Research, by Ostle, Iowa State University Press, 1966.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY



ENTER DATA:



PRESS: CONTINUE

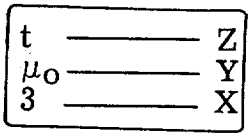
DISPLAY



ENTER DATA: μ_o → X

PRESS: CONTINUE

DISPLAY



To run another case

EXAMPLE

$\bar{X} = 175$

$S_X = 25$

$n = 15$

Test to see if $\mu = \mu_o$ where

$\mu_o = 188$ and $\alpha = .05$

$H_o : \mu = 188$

$H_A : \mu \neq 188$

$\alpha = .05$

Results: $t = -2.01395$

Conclusion: Since $t_{(14,\ .025)} = 2.14$,

the test statistic t (-2.01395) is not greater than $|t_{(n-1,\ \alpha/2)}|$ (2.14) and the hypothesis H_o is accepted.

00	CLR	20
01	1	01
02	STP	41
03	PNT	45
04	PNT	45
05	RUP	22
06	√	76
07	XEY	30
08	RDN	31
09	DIV	35
0a	DN	25
0b	AC+	60
0c	0	00
0d	UP	27

ENTRY

10	UP	27
11	2	02
12	STP	41
13	XTO	23
14	d	17
15	RDN	31
16	RCL	61
17	RDN	31
18	XEY	30
19	-	34
1a	DN	25
1b	XEY	30
1c	DIV	35
1d	d	17

20	UP	27
21	3	03
22	PNT	45
23	PNT	45
24	END	46

μ_0

$\bar{x}$

$s_{\bar{x}}$

χ^2 STATISTIC TESTING A POPULATION VARIANCE EQUAL TO A VALUE σ_0^2

This program determines the test statistic necessary to test a variance against a given value. The equations used are:

$$X^2 = \frac{(n-1)\,s^2}{\sigma_o^2} = \chi^2 \quad \text{(Chi-Squared)}$$

where

n = Number of data observations in sample
s^2 = Sample variance
σ_o^2 = Given value to be tested against

Hypothesis:

H_o: $\sigma^2 = \sigma_o^2$
H_A: a. $\sigma^2 < \sigma_o^2$
b. $\sigma^2 > \sigma_0^2$

Level of significance : α

Critical regions: $X^2 < X^2_{n-1,\ 1-\alpha/2}$, $X^2 > X^2_{n-1,\alpha/2}$

Reference: Introduction to Statistical Inference, Jerome C. R. Li, Edwards Brothers, Inc., 1959.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA:

n	Z
s^2	Y
σ_o^2	X

PRESS: CONTINUE

DISPLAY

X^2	Z
X^2	Y
X^2	X

To run another case

EXAMPLE

A sample of 10 observations is drawn from a population. The observations;

4.8, 3.2, 3.6, 4.8, 6.1, 5.6, 4.7, 5.3, 5.1, and 7.5 ,

have a sample variance

$s^2 = 1.52622$

Question: Is the population variance equal to 4 ?

$(\sigma_o^2 = 4)$

Hypothesis:

$H_o: \sigma^2 = 4$

$H_A: \sigma^2 < 4 \quad (X^2 < X^2_{n-1,\, \alpha/2})$

$\sigma^2 > 4 \quad (X^2 > X^2_{n-1,\, \alpha/2})$

Level of significance $\alpha = 5\%$

Results: $X^2 = 3.43400$

Conclusion: $X^2_{9,\ 9.5} = 2.70039$,

$X^2_{9,\ .05} = 19.0228$

The computed X^2 lies inside the critical region so the H_o is accepted.

00	CLR	20
01	1	01
02	STP	41
03	PNT	45
04	PNT	45
05	DIV	35
06	1	01
07	XEY	30
08	RDN	31
09	-	34
0a	DN	25
0b	X	36
0c	DN	25
0d	UP	27

ENTRY

10	UP	27
11	PNT	45
12	END	46

χ^2 CHI SQUARE EVALUATION
EXPECTED VALUES EQUAL ($E_i = E$)

This program calculates the value of χ^2 by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E)^2}{E}$$

where

O_i — observed frequency

E — expected frequency of O_i is $E = \frac{\sum_{i=1}^{n} O_i}{n}$

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

USER INSTRUCTIONS

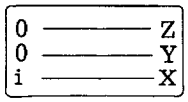
DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

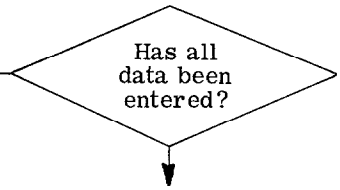
DISPLAY



(i indicates point to be entered)

ENTER DATA: O_i ⟶ X

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY



To run another case

EXAMPLES

The table shows the observed and expected frequencies in tossing a die 120 times. Calculate χ^2 for testing if the die is fair.

FACE	i	1	2	3	4	5	6
O_i Observed Frequency		25	17	15	23	24	16
E_i Expected Frequency		20	20	20	20	20	20

$E = 20$

$\chi^2 = 5.0$

$n = 6$

00	CLR	20		40	0	00	
01	1	01		41	END	46	
02	XTO	23					
03	d	17					
04	STP	41	ENTRY				
05	IFG	43					
06	1	01					
07	b	14					
08	PNT	45					
09	PNT	45					
0a	UP	27					
0b	X	36					
0c	AC+	60					
0d	0	00					
10	UP	27					
11	d	17					
12	UP	27					
13	1	01					
14	+	33					
15	YTO	40					
16	d	17					
17	DN	25					
18	GTO	44					
19	0	00					
1a	4	04					
1b	YE	24					
1c	d	17					
1d	1	01					
20	-	34					
21	YTO	40					
22	d	17					
23	RCL	61					
24	UP	27					
25	d	17					
26	DIV	35					
27	YTO	40					
28	f	15					
29	X	36					
2a	f	15					
2b	X	36					
2c	RDN	31					
2d	-	34					
30	f	15					
31	DIV	35					
32	d	17					
33	RUP	22					
34	PNT	45					
35	RUP	22					
36	PNT	45					
37	RUP	22					
38	PNT	45	S				
39	PNT	45					
3a	PNT	45					
3b	PNT	45					
3c	GTO	44					
3d	0	00					

χ^2 2 x 2 CONTINGENCY TABLE

This program calculates χ^2 for a 2 x 2 contingency table containing observed frequencies. The table is shown below.

OBSERVED RESULTS

	I	II	TOTALS
GROUP A	a	b	a + b
GROUP B	c	d	c + d

(Note there is only one degree of freedom.) The equation used to evaluate χ^2 is

$$\chi^2 = \frac{(ad-bc)^2(a+b+c+d)}{(a+b)(c+d)(a+c)(b+d)}$$

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY



DATA:

a → Y
b → X

PRESS: CONTINUE

DISPLAY



ENTER DATA:

c → Y
d → X

PRESS: CONTINUE

DISPLAY



EXAMPLES

FREQUENCIES OBSERVED

	I	II
GROUP A	75 (a)	25 (b)
GROUP B	65 (c)	35 (d)

$\chi^2 = 2.381$

```
00  CLR  20
01   1   01
02  STP  41     ENTRY
03  PNT  45
04  AC+  60
05  XTO  23
06   b   14
07  YTO  40
08   a   13
09  DN   25
0a  DN   25
0b   2   02
0c  STP  41     ENTRY
0d  PNT  45

10  AC+  60
11  XTO  23
12   d   17
13  YTO  40
14   c   16
15  RCL  61
16  XEY  30
17  AC+  60
18   X   36
19   a   13
1a  UP   27
1b   b   14
1c   +   33
1d  DN   25

20   X   36
21   c   16
22  UP   27
23   d   17
24   +   33
25  DN   25
26   X   36
27  YE   24
28   a   13
29   d   17
2a   X   36
2b   b   14
2c  UP   27
2d   c   16

30   X   36
31  DN   25
32   -   34
33  DN   25
34  UP   27
35   X   36
36   e   12
37   X   36
38   a   13
39  DIV  35
3a   0   00
3b  UP   27
3c  RUP  22
3d  PNT  45

40  PNT  45
41  END  46
```

χ^2 CHI SQUARE EVALUATION
EXPECTED VALUES UNEQUAL ($E_i \neq E_j$)

This program calculates the value of χ^2 by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i — observed frequency

E_i — expected frequency of O_i

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

USER INSTRUCTIONS

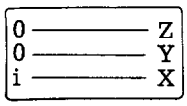
DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

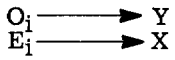
PRESS: CONTINUE

DISPLAY

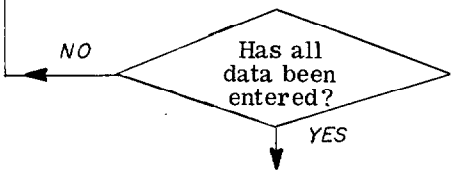


(i indicates points to be entered)

ENTER DATA:



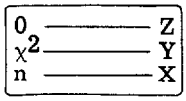
PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY



To run another case

EXAMPLES

The table shows the observed and expected frequencies of some numbers. Calculate χ^2.

i	1	2	3	4	5	6
O_i - Observed Frequency	8	50	47	56	5	14
E_i - Expected Frequency	9.6	46.75	51.85	54.4	8.25	9.15

$\chi^2 = 4.844$

$n = 6$

00	CLR	20	
01	1	01	
02	AC+	60	
03	f	15	
04	STP	41	ENTRY
05	IFG	43	
06	1	01	
07	b	14	
08	PNT	45	
09	PNT	45	
0a	-	34	
0b	XEY	30	
0c	UP	27	
0d	X	36	
10	DN	25	
11	XEY	30	
12	DIV	35	
13	1	01	
14	AC+	60	
15	0	00	
16	UP	27	
17	UP	27	
18	GTO	44	
19	0	00	
1a	3	03	
1b	RCL	61	
1c	UP	27	
1d	1	01	
20	-	34	
21	0	00	
22	RDN	31	
23	PNT	45	S
24	PNT	45	
25	PNT	45	
26	PNT	45	
27	GTO	44	
28	0	00	
29	0	00	
2a	END	46	

χ^2 - 2 x K CONTINGENCY TABLE

This program calculates χ^2 for a 2 x K contingency table containing observed frequencies. The table is shown below:

	1	2	3	4	5	TOTALS
A	a_1	a_2	a_3	...	a_K	N_A
B	b_1	b_2	b_3	...	b_K	N_B
TOTALS	N_1	N_2	N_3	...	N_K	N

The number of degrees of freedom $\gamma = K - 1$

The equation used to evaluate χ^2 is:

$$\chi^2 = \frac{N}{N_A}\left[\frac{a_1^2}{N_1} + \frac{a_2^2}{N_2} + \ldots + \frac{a_K^2}{N_K}\right] + \frac{N}{N_B}\left[\frac{b_1^2}{N_1} + \frac{b_2^2}{N_2} + \ldots + \frac{b_K^2}{N_K}\right] - N$$

Reference: Mathematical Statistics, John E. Freund, Prentice-Hall, 1962.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

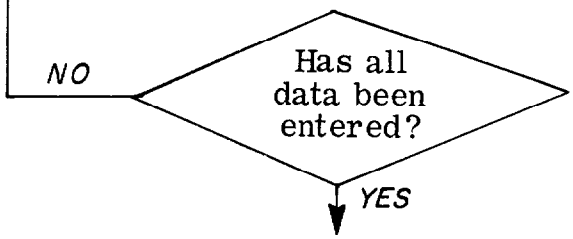
DISPLAY

0 ——— Z
0 ——— Y
i ——— X

ENTER DATA:

a_i ⟶ Y
b_i ⟶ X

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 ——— Z
χ^2 ——— Y
γ ——— X

EXAMPLE

(2 x 3) Table

	I	II	III
A	2	5	4
B	3	8	7

$\chi^2 = .022$

$\gamma = 2$

00	CLR	20		40	+	33	
01	XTO	23		41	YTO	40	
02	d	17		42	a	13	
03	XTO	23		43	d	17	
04	c	16		44	X	36	
05	1	01		45	e	12	
06	XTO	23		46	DIV	35	
07	b	14		47	c	16	
08	STP	41	ENTRY	48	UP	27	
09	IFG	43		49	f	15	
0a	3	03		4a	DIV	35	
0b	d	17		4b	a	13	
0c	PNT	45		4c	X	36	
0d	PNT	45		4d	-	34	
10	AC+	60		50	DN	25	
11	UP	27		51	+	33	
12	DN	25		52	b	14	
13	+	33		53	UP	27	
14	YTO	40		54	2	02	
15	a	13		55	-	34	
16	XEY	30		56	CLX	37	
17	DN	25		57	RDN	31	
18	RDN	31		58	PNT	45	S
19	X	36		59	PNT	45	
1a	a	13		5a	PNT	45	
1b	DIV	35		5b	PNT	45	
1c	XEY	30		5c	GTO	44	
1d	YEX	24		5d	0	00	
20	d	17		60	0	00	
21	+	33		61	END	46	
22	YTO	40					
23	d	17					
24	DN	25					
25	RDN	31					
26	X	36					
27	a	13					
28	DIV	35					
29	XEY	30					
2a	YE	24					
2b	c	16					
2c	+	33					
2d	YTO	40					
30	c	16					
31	b	14					
32	UP	27					
33	1	01					
34	+	33					
35	YTO	40					
36	b	14					
37	CLX	37					
38	UP	27					
39	RUP	22					
3a	GTO	44					
3b	0	00					
3c	8	10					
3d	RCL	61					

t STATISTIC FOR TESTING CORRELATION COEFFICIENT

This program calculates the test statistic necessary in testing the hypothesis that correlation coefficient ρ is equal to zero. The equations used are:

$$t_{n-2} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

where:

n = number of data points

r = estimate of the correlation coefficient

Application:

H_0 : $\rho = 0$

H_A : $\rho \neq 0$

α : Confidence level

t : $\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$

Conclusion:

If $t > t_{(1-\frac{\alpha}{2},\ n-2)}$, we reject the hypothesis H_0.

Reference: Statistics in Research, B. Ostle, Iowa State Press, 1966.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA

r	→ Y
n	→ X

PRESS: CONTINUE

DISPLAY

t	Z
r	Y
n	X

To run another case

EXAMPLE

Given the data and results from the second example of Program STAT-PAC IV-1 LINEAR REGRESSION AND CORRELATION COEFFICIENT, determine if $\rho = 0$.

$n = 5$
$r = .9$

Result:

$t_3 = 3.57624$

Choosing $\alpha = .05$, $t_{.975, 3} = 3.18$.

Thus the test statistic $t > t_{(1-\frac{\alpha}{2}, n-2)}$

$(3.57 > 3.18)$

and we reject H_0.

Step	Key	Code
00	CLR	20
01	1	01
02	STP	41
03	AC+	60
04	UP	27
05	2	02
06	-	34
07	DN	25
08	√	76
09	X	36
0a	RUP	22
0b	XEY	30
0c	DN	25
0d	UP	27

ENTRY

10	X	36
10	1	01
12	XEY	30
13	-	34
14	DN	25
15	√	76
16	DIV	35
17	UP	27
18	RCL	61
19	PNT	45
1a	PNT	45
1b	END	46

r

n

BARTLETT'S χ^2 STATISTIC FOR VARIANCE HOMOGENEITY

This program computes a χ^2 statistic defined as:

$$\chi^2 = \frac{f \ln S^2 - \sum_{i=1}^{K} f_i \ln S_i^2}{1 + \frac{1}{3(K-1)} \left[\left(\sum_{i=1}^{K} \frac{1}{f_i} \right) - \frac{1}{\Sigma f_i} \right]} \qquad (1)$$

where:

S_i^2 = A Sample Variance

f_i = The Degrees of Freedom Associated With S_i^2

K = Number of Sample Variances

$$S^2 = \frac{\sum_{i=1}^{K} f_i S_i^2}{f}$$

$$f = \sum_{i=1}^{K} f_i$$

This χ^2 statistic has an approximate χ^2 distribution with K - 1 degrees of freedom.

This statistic may be used to test the hypothesis that the K sample variances $S_1^2, S_2^2, \ldots, S_K^2$ are all estimates of the same population variance σ^2.

$H_o = S_1^2, S_2^2, \ldots, S_K^2$ are each estimates of σ^2.

$H_A = S_1^2, S_2^2, \ldots, S_K^2$ are not all estimates of σ^2.

Statistic: χ^2 as defined in (1)

Conclusion: If the computed χ^2 exceeds χ_α^2, the hypothesis H_o is rejected.

Reference: Statistical Theory with Engineering Applications, A. Hald, John Wiley & Sons, 1960.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
i	Y
0	X

ENTER DATA:

S_i^2 → Y

f_i → X

PRESS: CONTINUE

NO ← Has all the data been entered? → YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	Z
χ^2	Y
K-1	X

(d.f.)

To run another case

EXAMPLE

The variances in the table represent variances in gas mileage measured during road testing. f_i is obtained from the number of cars in each sample road test. The samples represent slight changes in topology and local climatic conditions.

i	f_i	S_i^2
1	20	5.5
2	15	4.3
3	15	4.9
4	18	5.7
5	20	5.1

Results

$\chi^2 = .3804$

d.f. = 4

Application

H_o: The 5 samples variances each represent the population variance.

H_A: The 5 samples variances do not all represent the population variance

α : .05

Conclusion:

$\chi^2_{.05} = 9.4877$,

$\chi^2_{computed}$ does not exceed $\chi^2_{.05}$.

The hypothesis H_o is not rejected.

00	CLR	20		40	d	17	
01	XTO	23		41	UP	27	
02	d	17		42	c	12	
03	XTO	23		43	DIV	35	
04	c	16		44	DN	25	
05	XTO	23		45	LN	65	
06	b	14		46	UP	27	
07	1	01		47	e	12	
08	XEY	30		48	X	36	
09	STP	41	ENTRY	49	b	14	
0a	IFG	43		4a	-	34	
0b	4	04		4b	YTO	40	
0c	0	00		4c	a	13	
0d	PNT	45		4d	c	16	
10	PNT	45		50	UP	27	
11	UP	27		51	1	01	
12	1	01		52	UP	27	
13	AC+	60		53	e	12	
14	XEY	30		54	DIV	35	
15	DIV	35		55	DN	25	
16	c	16		56	-	34	
17	XEY	30		57	3	03	
18	+	33		58	DIV	35	
19	YTO	40		59	f	15	
1a	c	16		5a	UP	27	
1b	XEY	30		5b	1	01	
1c	1	01		5c	-	34	
1d	XEY	30		5d	DN	25	
20	DIV	35		60	DIV	35	
21	DN	25		61	1	01	
22	XEY	30		62	+	33	
23	X	36		63	a	13	
24	RDN	31		64	XEY	30	
25	YE	24		65	DIV	35	
26	d	17		66	f	15	
27	+	33		67	UP	27	
28	YTO	40		68	1	01	
29	d	17		69	-	34	
2a	RUP	22		6a	0	00	
2b	DIV	35		6b	RDN	31	
2c	LN	65		6c	PNT	45	S
2d	X	36		6d	PNT	45	
30	b	14		70	GTO	44	
31	+	33		71	0	00	
32	YTO	40		72	0	00	
33	b	14		73	END	46	
34	0	00					
35	RDN	31					
36	f	15					
37	XEY	30					
38	1	01					
39	+	33					
3a	0	00					
3b	GTO	44					
3c	0	00					
3d	9	11					

Numerator

$\Sigma f_i \ln S_i$

Σf_i^1

$\Sigma f_i \ S_i^2$

Σf_i

n

CALCULATED Y_c, RESIDUAL, AND ACCUMULATED RESIDUAL OF LINEAR REGRESSION

This program calculates the quantities defined below for the linear regression model

$$Y = mX + b$$

given m, b and n the number of data points;

Y_c = calculated $Y = mX_i + b$

e_i = residual at $X_i = Y_i - Y_c$

accumulated residual = $\sum_{i=1}^{n} e_i$

This program is to be used with programs STAT-PAC III-12 and III-13.

This program was written by Mr. Guy D. Nell, Clifford Shaw and Associates, Investment Advisors, Dallas, Texas.

Reference: Statistics – An Introductory Analysis, Taro Yamane, Harper & Row, 1964.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

DISPLAY

0	Z
0	Y
0	X

ENTER DATA:

m	Y
b	X

PRESS: CONTINUE

DISPLAY

0	Z
i	Y
0	X

ENTER DATA:

Y_i	Y
X_i	X

PRESS: CONTINUE

DISPLAY

Y_c	Z
e_i	Y
i	X

NO

Has all data been entered?

YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

Σe_i	Z
n	Y
0	X

USER INSTRUCTIONS (Con't)

To run another case

EXAMPLE

m = .7789
b = 8.0737

	Given		Results	
i	X	Y	Y_c	e_i
1	1	9.1	8.85	-.25
2	.5	8.2	8.46	.26
3	2	9.5	9.63	.13
4	1	9.0	8.85	-.15
			Σe_i = -.00015	
			n = 4	

00	CLR	20	
01	STP	41	ENTRY
02	PNT	45	
03	PNT	45	
04	XTO	23	
05	a	13	
06	YTO	40	
07	b	14	
08	CLR	20	
09	1	01	
0a	XEY	30	
0b	STP	41	ENTRY
0c	IFG	43	
0d	2	02	
10	d	17	
11	PNT	45	
12	PNT	45	
13	UP	27	
14	b	14	
15	X	36	
16	a	13	
17	+	33	
18	RUP	22	
19	XEY	30	
1a	DN	25	
1b	-	34	
1c	1	01	
1d	AC+	60	
20	f	15	
21	PNT	45	S
22	PNT	45	
23	UP	27	
24	0	00	
25	RDN	31	
26	XEY	30	
27	1	01	
28	+	33	
29	0	00	
2a	GTO	44	
2b	0	00	
2c	b	14	
2d	RCL	61	
30	UP	27	
31	0	00	
32	PNT	45	S
33	PNT	45	
34	PNT	45	
35	GTO	44	
36	0	00	
37	0	00	
38	END	46	

b

M

This program computes the Durbin-Watson statistic d, defined as:

$$d = \frac{\sum_{i=1}^{n-1} (e_{i+1} - e_i)^2}{\sum_{i=1}^{n} e_i^2}$$

where the data values $\{e_i\}$ are the residuals calculated by program STAT-PAC III-11.

This test statistic is used to test for any serial correlation of the ε_i term associated with the linear regression model

$$Y_i = mX_i + b + \varepsilon_i$$

The error term ε_i is in general a random regression disturbance term which is independent of X and Y. In many business and economic cases, the ε_i may indeed be dependent thus reducing the usefulness of the above linear model.

This program was written by Mr. Guy D. Nell, Clifford Shaw & Associates, Investment Advisors, Dallas, Texas.

Reference: Statistics – An Introductory Analysis, Taro Yamane, Harper & Row, 1964, page 809.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
i	Y
0	X

ENTER DATA: e_i → X

PRESS: CONTINUE

NO — Has all data been entered? — YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	Z
d	Y
n	X

To run another case

EXAMPLE

Using the e_i (residuals) from the data of program STAT-PAC III-11, determine the Durbin-Watson statistic d.

i	e_i
1	-.2474
2	.26315
3	.1315
4	-.1474

Results:

d = 2.0993

This statistic is now used to test for serial correlation as described on page 811 of the reference.

00	CLR	20		40	DIV	35	
01	1	01		41	b	14	
02	XTO	23		42	UP	27	
03	b	14		43	1	01	
04	XEY	30		44	-	34	
05	STP	41	ENTRY	45	0	00	
06	PNT	45		46	RDN	31	
07	PNT	45		47	PNT	45	S
08	XEY	30		48	PNT	45	
09	RDN	31		49	GTO	44	
0a	XTO	23		4a	0	00	
0b	a	13		4b	0	00	
0c	UP	27		4c	END	46	
0d	X	36					

10	DN	25	
11	AC+	60	
12	1	01	
13	UP	27	
14	b	14	
15	+	33	
16	YTO	40	
17	b	14	
18	DN	25	
19	RUP	22	
1a	STP	41	ENTRY
1b	IFG	43	
1c	3	03	
1d	d	17	

20	PNT	45	
21	PNT	45	
22	UP	27	
23	X	36	
24	UP	27	
25	YE	24	
26	a	13	
27	XEY	30	
28	-	34	
29	DN	25	
2a	UP	27	
2b	X	36	
2c	DN	25	
2d	XEY	30	

30	AC+	60	
31	0	00	
32	UP	27	
33	1	01	
34	UP	27	
35	b	14	
36	+	33	
37	YTO	40	
38	b	14	
39	0	00	
3a	GTO	44	
3b	1	01	
3c	a	13	
3d	RCL	61	

e_t

n

$\Sigma(e_{t+1} - e_t)^2$

Σe_t^2

STATISTIC FOR SERIAL CORRELATION COEFFICIENT (ORDER 1)

This program determines the serial correlation coefficient of order 1, given by:

$$r_1 = \frac{\sum_{i=2}^{n} \chi_i \chi_{i-1}}{\sum_{i=1}^{n} \chi_i^2}$$

where the data set $\{\chi_i\}$ is any time ordered set of data.

This statistic is used to test the statistical independence of a series of successive observations.

The circular correlation coefficient can be obtained by entering one extra data value, χ_{n+1}, equal to χ_1.

This program was written by Mr. Guy D. Nell, Clifford Shaw & Associates, Investment Advisors, Dallas, Texas.

Reference: Statistics – An Introductory Analysis, Taro Yamane, Harper & Row, 1964, page 866.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

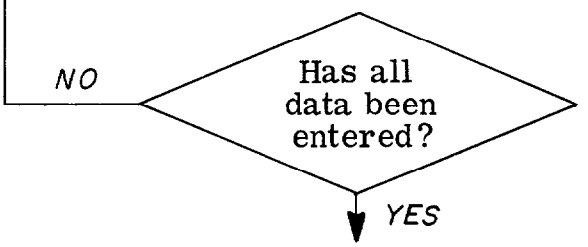
DISPLAY

0	Z
i	Y
0	X

ENTER DATA:

X_i → X

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	Z
n	Y
r_1	X

To run another case

EXAMPLE

1) Determine the serial correlation coefficient of the residuals computed by program STAT-PAC III-11 .

i	$X_i = e_i$
1	-.2474
2	.26315
3	.1315
4	-.1474

Result: $r_1 = -.3376$

2) Determine the circular correlation coefficient of the residuals of example 1 .

i	$X_i = e_i$
1	-.2474
2	.26315
3	.1315
4	-.1474
5	-.2474

Result: $r_{circular} = -.0792$

```
00  CLR  20
01   1   01
02  XTO  23
03   b   14
04  XEY  30
05  STP  41      ENTRY
06  PNT  45
07  PNT  45
08  XEY  30
09  RDN  31
0a  XTO  23
0b   a   13
0c   1   01
0d  UP   27

10   b   14
11   +   33
12  YTO  40
13   b   14
14   0   00
15  UP   27
16  RDN  31
17  STP  41      ENTRY
18  IFG  43
19   3   03
1a   6   06
1b  PNT  45
1c  PNT  45
1d  UP   27

20  YE   24
21   a   13
22  XEY  30
23   X   36
24  UP   27
25   X   36
26  DN   25
27  AC+  60
28   0   00
29  UP   27
2a   1   01
2b  UP   27
2c   b   14
2d   +   33

30   0   00
31  YTO  40
32   b   14
33  GTO  44
34   1   01
35   7   07
36   1   01
37   -   34
38  UP   27
39  RCL  61
3a  DIV  35
3b   0   00
3c  RDN  31
3d  PNT  45

40  PNT  45
41  END  46
```

e_{t-1}

n

$\Sigma e_{t-1} e_t$

Σe_{t-1}^2

SPEARMAN'S RANK CORRELATION COEFFICIENT

The statistic to measure the degree of association between two variables X and Y, when the distributions of X and Y are unknown, is called a distribution - free statistic. Spearman's rank correlation coefficient was developed to measure the association between X and Y and depends only upon the ranks (or order) of the X and Y observations. This statistic is given by:

$$r_s = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}$$

where

n = number of paired observations (X_i, Y_i)

$d = (X_i - Y_i)$

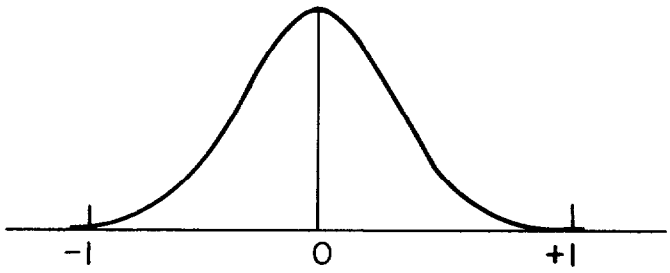
The variance of the Spearman's coefficient is:

$$\sigma_r^2 = \frac{1}{n - 1}$$

The test statistic is defined by:

$$Z = \frac{r_s}{\sigma_r}$$

For $n > 20$, the statistic r_s is approximately normal bounded by ± 1. Its distribution is in fact a truncated normal distribution as seen below.



Use of r_s statistic

We want to test the dependence between X and Y. To do this, we test the null hypothesis that the population rank correlation coefficient ρ_s is 0. The Z statistic is a standardized normal variable with unit variance.

The statistic r_s can also be applied subjectively since it possesses properties similar to the regression coefficient r. That is

$$-1 \leq r_s \leq 1$$

where (+1) indicates complete agreement in order of the ranks and (-1) indicates complete agreement in the opposite order of the ranks.

Reference: Statistics – An Introductory Analysis, Taro Yamane, Harper & Row, 1967, page 467.

USER INSTRUCTIONS

EXAMPLE

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

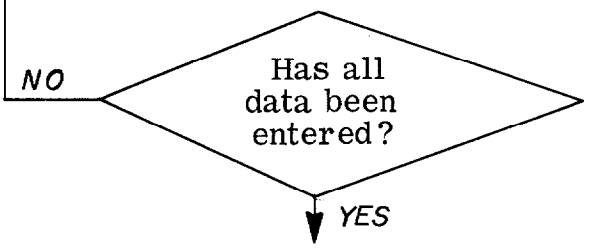
DISPLAY

0 ——— Z
i ——— Y
0 ——— X

ENTER DATA:

Y_i ⟶ Y
X_i ⟶ X

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

r_s ——— Z
$\bar{Z}$ ——— Y
σ_r ——— X

To run another case

See next page.

EXAMPLE

The following table lists the average grade attained in a student's senior year and the number of extra curricular activity events the student participated in. The data was taken at Panther High School, El Paso, Texas.

Student	X Average Senior Year Grade	Y Number of Activities	Rank X	Rank Y
1	82	18	9	5
2	67	11	18	11
3	91	25	4	1
4	98	17	1	6
5	74	10	15	12
6	52	9	20	13
7	86	13	6	9
8	81	14	10	8
9	75	15	14	7
10	83	23	8	2
11	89	21	5	3
12	92	8	3	14
13	95	20	2	4
14	79	2	12	19
15	73	6	16	16
16	78	5	13	17
17	65	0	19	20
18	84	12	7	10
19	69	7	17	15
20	80	4	11	18

Results:

r_s = .6150
Z = 2.6809
σ_r = .2294

Conclusion: Since Z represents a standard normal variate, a Z value of 2.68 means that r_s is 2.68 standard deviations from 0 in the sampling distribution of r_s. Thus, the null hypothesis that ρ_s is equal to zero is rejected.

00	CLR	20
01	1	01
02	XEY	30
03	STP	41
04	IFG	43
05	1	01
06	a	13
07	PNT	45
08	PNT	45
09	-	34
0a	DN	25
0b	UP	27
0c	X	36
0d	1	01

ENTRY

10	AC+	60
11	UP	27
12	f	15
13	+	33
14	0	00
15	UP	27
16	RDN	31
17	GTO	44
18	0	00
19	3	03
1a	f	15
1b	UP	27
1c	X	36
1d	1	01

20	-	34
21	f	15
22	X	36
23	e	12
24	UP	27
25	6	06
26	X	36
27	DN	25
28	XEY	30
29	DIV	35
2a	1	01
2b	XEY	30
2c	-	34
2d	f	15

30	UP	27	
31	1	01	
32	-	34	
33	XEY	30	
34	DIV	35	
35	DN	25	
36	√	76	
37	DIV	35	
38	PNT	45	s
39	PNT	45	
3a	GTO	44	
3b	0	00	
3c	0	00	
3d	END	46	

Σd^2

n

F – TEST FOR THE RATIO OF TWO SAMPLE VARIANCES (SNEDECOR)

This program determines the probability that an experimental value of $F(v_1, v_2)$ is greater than or equal to $F_T(v_1, v_2)$, F_T meaning F theoretical. This probability α is given by:

$$\alpha = P_r\,(F \geq F_T)$$

$$\alpha = \frac{\Gamma\left(\frac{v_1 + v_2}{2}\right)}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)}\; v_1^{\frac{v_1}{2}}\; v_2^{\frac{v_2}{2}} \int_{F_T}^{\infty} \frac{F^{\frac{v_1}{2}-1}\,dF}{(v_2 + v_1 F)^{\frac{(v_1+v_2)}{2}}}$$

where $F(v_1, v_2) = \frac{s_1^2}{s_2^2}$ = ratio of two sample variances

This integral is evaluated by the formulas:

1. for $\frac{v_1 F}{v_2} < 1$,

$$a = \arctan\sqrt{\frac{v_1 F}{v_2}} \leq \frac{\pi}{4}$$

$$\alpha = 1 - \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} \frac{\cos^{2m} a \sin^{2n} a}{n} \left[1 + \sum_{j=0}^{\infty} \frac{(m+n)\ldots(m+n+j-1)\sin^{2j} a}{(n+1)\ldots(n+j)}\right]$$

2. for $\frac{v_1 F}{v_2} > 1$,

$$a = \arctan\sqrt{\frac{v_2}{v_1 F}} < \frac{\pi}{4}$$

$$\alpha = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} \frac{\cos^{2n} a \sin^{2m} a}{m} \left[1 + \sum_{j=0}^{\infty} \frac{(m+n)\ldots(m+n+j-1)\sin^{2j} a}{(m+1)\ldots(m+j)}\right]$$

In both cases:

$$m = \frac{v_1}{2}, \quad n = \frac{v_2}{2}$$

This program determines α when F, v_1, and v_2 are given.

This program was written by professors Marcel Gauthier and Jean-Claude Warmoes, of École Polytéchnique, Université de Montréal, Canada.

Reference: Le Contrôle Statistique des Fabrications, René Cavé, Éditions Eyrolles.

USER INSTRUCTIONS

SET: [ ] RADIANS

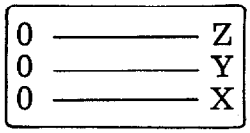
DEPRESS: X Y Z on the 9120A

PRESS: END

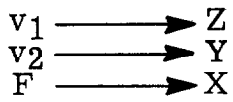
ENTER PROGRAM 1

PRESS: CONTINUE

DISPLAY



ENTER DATA:



PRESS: CONTINUE

DISPLAY

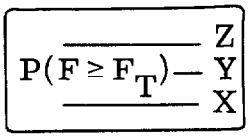


PRESS: END

ENTER PROGRAM 2

PRESS: CONTINUE

DISPLAY



To run another case:

Repeat USER INSTRUCTIONS.

EXAMPLE

v_1	v_2	F_{exp}	Result
12	3	5.2	.10
1	1	1	.50
60	120	1.66	.01
9	30	2.57	.025
4	60	5.31	.001

00	CLR	20	ENTRY	40	DN	25		80	+	33	
01	STP	41		41	c	16		81	DN	25	
02	PNT	45		42	+	33		82	+	33	
03	PNT	45		43	1	01		83	YTO	40	
04	XTO	23		44	-	34		84	b	14	
05	a	13		45	DN	25		85	IFG	43	
06	2	02		46	XEY	30		86	8	10	
07	DIV	35		47	DIV	35		87	b	14	
08	YTO	40		48	b	14		88	9	11	
09	c	16		49	X	36		89	XTO	23	
0a	DN	25		4a	a	13		8a	a	13	
0b	2	02		4b	X	36		8b	1	01	
0c	DIV	35		4c	YTO	40		8c	UP	27	
0d	YTO	40		4d	b	14		8d	UP	27	
10	d	17		50	e	12		90	END	46	
11	c	16		51	+	33					
12	DIV	35		52	YTO	40					
13	a	13		53	e	12					
14	X	36		54	b	14					
15	1	01		55	XEY	30					
16	X>Y	53		56	DIV	35					
17	2	02		57	1	01					
18	6	06		58	EEX	26					
19	d	17		59	CHS	32					
1a	UP	27		5a	4	04					
1b	YE	24		5b	X>Y	53					
1c	c	16		5c	6	06					
1d	YTO	40		5d	5	05					
20	d	17		60	1	01					
21	DN	25		61	UP	27					
22	1	01		62	GTO	44					
23	XEY	30		63	3	03					
24	DIV	35		64	7	07					
25	SFL	54		65	e	12					
26	DN	25		66	LN	65					
27	√	76		67	XTO	23					
28	ARC	72		68	b	14					
29	TAN	71		69	a	13					
2a	SIN	70		6a	LN	65					
2b	UP	27		6b	UP	27					
2c	X	36		6c	d	17					
2d	YTO	40		6d	X	36					
30	a	13		70	LN	65					
31	1	01		71	-	34					
32	XTO	23		72	a	13					
33	e	12		73	√	76					
34	XTO	23		74	ARC	72					
35	b	14		75	SIN	70					
36	UP	27		76	COS	73					
37	f	15		77	LN	65					
38	+	33		78	UP	27					
39	YTO	40		79	c	16					
3a	f	15		7a	X	36					
3b	d	17		7b	2	02					
3c	+	33		7c	X	36					
3d	UP	27		7d	b	14					

00	CLR	20	40	+	33
01	9	11	41	YTO	40
02	UP	27	42	b	14
03	a	13	43	0	00
04	X<Y	52	44	XTO	23
05	SFL	54	45	f	15
06	CNT	47	46	c	16
07	d	17	47	UP	27
08	UP	27	48	GTO	44
09	UP	27	49	0	00
0a	1	01	4a	a	13
0b	X=Y	50	4b	+	33
0c	2	02	4c	YTO	40
0d	b	14	4d	e	12
10	X>Y	53	50	f	15
11	2	02	51	CHS	32
12	3	03	52	YE	24
13	-	34	53	b	14
14	DN	25	54	+	33
15	UP	27	55	YTO	40
16	UP	27	56	b	14
17	LN	65	57	0	00
18	YE	24	58	XTO	23
19	f	15	59	f	15
1a	+	33	5a	c	16
1b	YTO	40	5b	UP	27
1c	f	15	5c	d	17
1d	DN	25	5d	+	33
20	GTO	44	60	GTO	44
21	0	00	61	0	00
22	a	13	62	a	13
23	π	56	63	f	15
24	$\sqrt{}$	76	64	YE	24
25	LN	65	65	b	14
26	UP	27	66	+	33
27	f	15	67	DN	25
28	+	33	68	EXP	74
29	YTO	40	69	IFG	43
2a	f	15	6a	7	07
2b	e	12	6b	3	03
2c	UP	27	6c	UP	27
2d	1	01	6d	1	01
30	X<Y	52	70	XEY	30
31	6	06	71	-	34
32	3	03	72	DN	25
33	X=Y	50	73	UP	27
34	4	04	74	UP	27
35	b	14	75	PNT	45
36	+	33	76	PNT	45
37	YTO	40	77	PNT	45
38	e	12	78	PNT	45
39	UP	27	79	PNT	45
3a	f	15	7a	PNT	45
3b	CHS	32	7b	END	46
3c	UP	27			
3d	b	14			

r — DISTRIBUTION

Given n pairs of values (x_i, y_i). Program STAT-PAC IV-1 allows us to determine the line of best fit $Y = mX + b$ as well as the correlation coefficient r_{exp} (r_{exp} = r experimental).

$$r_{exp} = \frac{\text{cov}(x, y)}{\sqrt{\text{var } x \text{ var } y}} \qquad \text{on } v = (n - 2) \text{ df}$$

The present program evaluates

$$\alpha = \text{Probability} \quad \left[|r_{exp}| \geqslant r_{\alpha} \right]$$

by using the relationships:

$$t_{(v = n - 2)} = \sqrt{\frac{n - 2}{1 - r^2}} \cdot r$$

$$t^2 (v) = \frac{r^2}{1 - r^2} \quad (n - 2) = F(1, v)$$

where r_{α} is a given theoretical or expected value.

Consequently the series approximation for the F distribution (STAT-PAC III-15) can be used here.

This program was written by professors Marcel Gauthier and Jean-Claude Warmoes of École Polytéchnique, Université de Montréal, Canada.

Reference: Le Contrôle Statistique des Fabrications, Rene Cave, Editions Eyrolles.

USER INSTRUCTIONS

SET: [ ] RADIANS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM 1

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
0 —— X

ENTER DATA:

$v \longrightarrow Y$ (n - 2)
$r_{exp} \longrightarrow X$

PRESS: CONTINUE

DISPLAY

1 —— Z
1 —— Y
1 —— X

PRESS: END

ENTER PROGRAM 2

PRESS: CONTINUE

DISPLAY

$P_r[|r_{exp}| \geq r_\alpha]$ —— Y

To run another case:
Repeat USER INSTRUCTIONS.

EXAMPLE

Determine $P_r[|r_{exp}| \geq r_\alpha]$

v	r_{exp}	Result
1	.9877	.10
10	.5760	.05
20	.4921	.02
45	.3721	.01
100	.3211	.001

00	CLR	20		40	e	12		80	√	76	
01	STP	41	ENTRY	41	XTO	23		81	ARC	72	
02	PNT	45		42	b	14		82	SIN	70	
03	PNT	45		43	UP	27		83	COS	73	
04	UP	27		44	f	15		84	LN	65	
05	X	36		45	+	33		85	UP	27	
06	1	01		46	YTO	40		86	c	16	
07	XEY	30		47	f	15		87	X	36	
08	-	34		48	d	17		88	2	02	
09	XEY	30		49	+	33		89	X	36	
0a	DIV	35		4a	UP	27		8a	b	14	
0b	DN	25		4b	DN	25		8b	+	33	
0c	X	36		4c	c	16		8c	DN	25	
0d	1	01		4d	+	33		8d	+	33	
10	RDN	31		50	1	01		90	YTO	40	
11	XTO	23		51	-	34		91	b	14	
12	a	13		52	DN	25		92	IFG	43	
13	2	02		53	XEY	30		93	9	11	
14	DIV	35		54	DIV	35		94	8	10	
15	YTO	40		55	b	14		95	9	11	
16	c	16		56	X	36		96	XTO	23	
17	DN	25		57	a	13		97	a	13	
18	2	02		58	X	36		98	1	01	
19	DIV	35		59	YTO	40		99	UP	27	
1a	YTO	40		5a	b	14		9a	UP	27	
1b	d	17		5b	e	12		9b	END	46	
1c	c	16		5c	+	33					
1d	DIV	35		5d	YTO	40					
20	a	13		60	e	12					
21	X	36		61	b	14					
22	1	01		62	XEY	30					
23	IFG	43		63	DIV	35					
24	3	03		64	1	01					
25	3	03		65	EEX	26					
26	d	17		66	CHS	32					
27	UP	27		67	4	04					
28	YEX	24		68	X>Y	53					
29	c	16		69	7	07					
2a	YTO	40		6a	2	02					
2b	d	17		6b	1	01					
2c	DN	25		6c	UP	27					
2d	1	01		6d	GTO	44					
30	XEY	30		70	4	04					
31	DIV	35		71	2	02					
32	SFL	54		72	e	12					
33	DN	25		73	LN	65					
34	√	76		74	XTO	23					
35	ARC	72		75	b	14					
36	TAN	71		76	a	13					
37	SIN	70		77	LN	65					
38	UP	27		78	UP	27					
39	X	36		79	d	17					
3a	YTO	40		7a	X	36					
3b	a	13		7b	LN	65					
3c	1	01		7c	-	34					
3d	XTO	23		7d	a	13					

00	CLR	20	40	+	33
01	9	11	41	YTO	40
02	UP	27	42	b	14
03	a	13	43	0	00
04	X<Y	52	44	XTO	23
05	SFL	54	45	f	15
06	CNT	47	46	c	16
07	d	17	47	UP	27
08	UP	27	48	GTO	44
09	UP	27	49	0	00
0a	1	01	4a	a	13
0b	X=Y	50	4b	+	33
0c	2	02	4c	YTO	40
0d	b	14	4d	e	12
10	X>Y	53	50	f	15
11	2	02	51	CHS	32
12	3	03	52	YEX	24
13	-	34	53	b	14
14	DN	25	54	+	33
15	UP	27	55	YTO	40
16	UP	27	56	b	14
17	LN	65	57	0	00
18	YEX	24	58	XTO	23
19	f	15	59	f	15
1a	+	33	5a	c	16
1b	YTO	40	5b	UP	27
1c	f	15	5c	d	17
1d	DN	25	5d	+	33
20	GTO	44	60	GTO	44
21	0	00	61	0	00
22	a	13	62	a	13
23	π	56	63	f	15
24	√	76	64	YEX	24
25	LN	65	65	b	14
26	UP	27	66	+	33
27	f	15	67	DN	25
28	+	33	68	EXP	74
29	YTO	40	69	IFG	43
2a	f	15	6a	7	07
2b	e	12	6b	3	03
2c	UP	27	6c	UP	27
2d	1	01	6d	1	01
30	X<Y	52	70	XEY	30
31	6	06	71	-	34
32	3	03	72	DN	25
33	X=Y	50	73	UP	27
34	4	04	74	UP	27
35	b	14	75	PNT	45
36	+	33	76	PNT	45
37	YTO	40	77	PNT	45
38	e	12	78	PNT	45
39	UP	27	79	PNT	45
3a	f	15	7a	PNT	45
3b	CHS	32	7b	END	46
3c	UP	27			
3d	b	14			

SECTION IV CURVE FITTING

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LINEAR REGRESSION AND CORRELATION COEFFICIENT

This program calculates the equation of the straight line of best fit of a set of data points. The best fit is determined by minimizing the sum of the squares of the deviations of the data points from the line.

The program calculates m and b for the equation:

$$Y = mX + b$$

The program also calculates a correlation coefficient r, an indication of goodness of fit. Note: $-1 \le r \le 1$ where the sign corresponds to the slope m. If $r = 0$, there is no correlation, and if $r = \pm 1$, there is perfect correlation or a perfect fit.

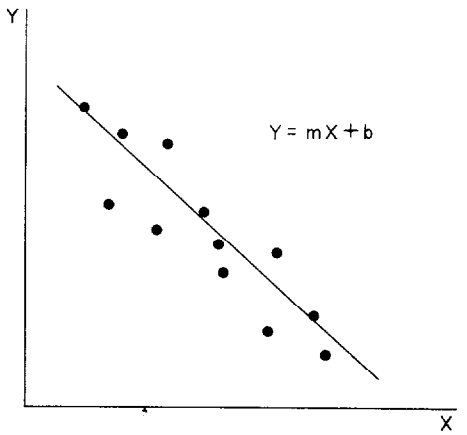
The defining equations are:

$$m = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_i - \overline{X})^2}$$

$$b = \overline{Y} - m\overline{X}$$

$$\text{where } \overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n} \text{ and } \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$r = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \overline{X})^2 \sum_{i=1}^{n} (Y_i - \overline{Y})^2}} = \frac{n\sum_{i=1}^{n} X_i Y_i - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{\sqrt{\left[n\sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2\right]\left[n\sum_{i=1}^{n} Y_i^2 - (\sum_{i=1}^{n} Y_i)^2\right]}}$$



Reference: Mathematical Statistics
by John E. Freund
Prentice-Hall, 1962

USER INSTRUCTIONS

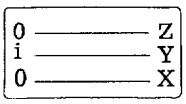
DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

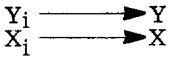
PRESS: CONTINUE

DISPLAY

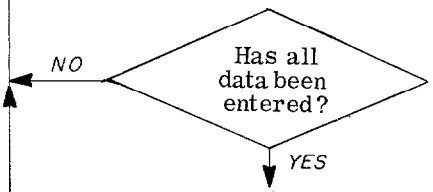


(i indicates pair of points to be entered)

ENTER DATA:



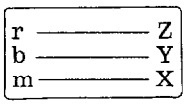
PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY



EXAMPLES

X	Y
26	92
30	85
44	78
50	81
62	54
68	51
74	40

r = -.96
b = 121.04
m = -1.03

Y = -1.03X + 121.04

X	Y
0	1
1	3
2	2
3	4
4	5

r = .9
b = 1.2
m = .9

Y = .9X + 1.2

00	CLR	20		40	DN	25		80	X	36
01	XTO	23		41	XTO	23		81	f	15
02	d	17		42	a	13		82	XEY	30
03	XTO	23		43	e	12		83	-	34
04	c	16		44	UP	27		84	e	12
05	XTO	23		45	a	13		85	PNT	45
06	b	14		46	DIV	35		86	PNT	45
07	1	01		47	YE	24		87	END	46
08	XTO	23		48	f	15				
09	a	13		49	DIV	35				
0a	XEY	30		4a	YTO	40				
0b	STP	41	ENTRY	4b	e	12				
0c	IFG	43		4c	X	36				
0d	3	03		4d	e	12				
10	a	13		50	X	36				
11	PNT	45		51	d	17				
12	PNT	45		52	XEY	30				
13	AC+	60		53	-	34				
14	UP	27		54	YTO	40				
15	X	36		55	d	17				
16	XEY	30		56	c	16				
17	YE	24		57	UP	27				
18	d	17		58	f	15				
19	+	33		59	UP	27				
1a	YEX	24		5a	X	36				
1b	d	17		5b	a	13				
1c	DN	25		5c	X	36				
1d	X	36		5d	DN	25				
20	b	14		60	-	34				
21	+	33		61	YTO	40				
22	YTO	40		62	c	16				
23	b	14		63	b	14				
24	RUP	22		64	UP	27				
25	UP	27		65	f	15				
26	X	36		66	UP	27				
27	c	16		67	e	12				
28	+	33		68	X	36				
29	YTO	40		69	a	13				
2a	c	16		6a	X	36				
2b	a	13		6b	DN	25				
2c	UP	27		6c	-	34				
2d	1	01		6d	d	17				
30	+	33		70	UP	27				
31	YTO	40		71	DN	25				
32	a	13		72	$\sqrt{}$	76				
33	0	00		73	DIV	35				
34	RDN	31		74	c	16				
35	XEY	30		75	$\sqrt{}$	76				
36	0	00		76	DIV	35				
37	GTO	44		77	d	17				
38	0	00		78	RUP	22				
39	b	14		79	XEY	30				
3a	a	13		7a	DIV	35				
3b	UP	27		7b	YE	24				
3c	1	01		7c	e	12				
3d	-	34		7d	e	12				

LINEAR REGRESSION WITH MEANS, RMS VALUES, VARIANCES, AND CORRELATION COEFFICIENT

This program fits a curve of the form $Y = mX + b$, solving for the slope (m) and intercept (b), given the input data (X_1, Y_1), (X_2, Y_2), . . . (X_n, Y_n) for any number of sets (n).

The means ($\overline{Y}$ and $\overline{X}$), rms values (Yrms and Xrms), standard deviations (σY and σX), and correlation coefficient (r) are also found.

The following equations are used:

1) $\overline{Y} = \frac{\Sigma Y}{n}$; $\overline{X} = \frac{\Sigma X}{n}$ (means).

2) $N = \overline{XY} - \overline{X}\,\overline{Y}$ (intermediate value).

3) $Yrms = \sqrt{\overline{Y^2}} = \sqrt{\frac{\Sigma Y^2}{n}}$; $Xrms = \sqrt{\overline{X^2}} = \sqrt{\frac{\Sigma X^2}{n}}$

4) $\sigma Y = \sqrt{\overline{Y^2} - (\overline{Y})^2}$; $\sigma X = \sqrt{\overline{X^2} - (\overline{X})^2}$ (standard deviations).

5) $m = \frac{N}{(\sigma X)^2}$ (slope).

6) $b = \overline{Y} - m\overline{X}$ (intercept).

7) $r = \frac{m\sigma X}{\sigma Y}$ (correlation coefficient).

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	——	Z
n	——	Y
0	——	X

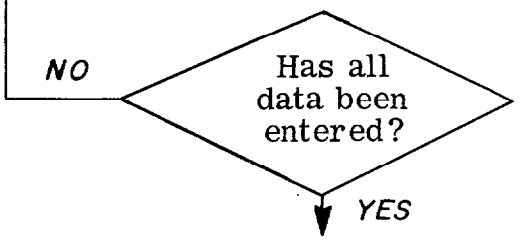
n indicates the data set (X_n, Y_n) to be entered.

ENTER DATA:

$Y_n \longrightarrow Z$

$X_n \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

n	——	Z
$\overline{Y}$	——	Y
$\overline{X}$	——	X

PRESS: CONTINUE

DISPLAY

n	——	Z
Yrms	——	Y
Xrms	——	X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

n	——	Z
σY	——	Y
σX	——	X

PRESS: CONTINUE

DISPLAY

r	——	Z
b	——	Y
m	——	X

To run another case

EXAMPLE

Data: (n = 5)

X	Y
1	3
0	1
-.5	0
2	5
-2	-3

Solution:

$\overline{Y} = 1.2$

$\overline{X} = .1$

Yrms = 2.966

Xrms = 1.360

$\sigma Y = 2.713$

$\sigma X = 1.356$

m = 2
b = 1
r = 1

00	CLR	20		40	c	16		80	c	16			
01	XTO	23		41	UP	27		81	UP	27			
02	a	13		42	f	15		82	X	36			
03	XTO	23		43	DIV	35		83	d	17			
04	b	14		44	YTO	40		84	XEY	30			
05	XTO	23		45	c	16		85	-	34			
06	c	16		46	UP	27		86	f	15			
07	XTO	23		47	a	13		87	RDN	31			
08	d	17		48	XEY	30		88	√	76			
09	1	01		49	DIV	35		89	XEY	30			
0a	AC+	60		4a	YTO	40		8a	PNT	45	s		
0b	f	15		4b	a	13		8b	PNT	45			
0c	XEY	30		4c	RDN	31		8c	UP	27			
0d	CLX	37		4d	PNT	45	s	8d	e	12			
10	STP	41	ENTRY	50	PNT	45		90	X	36			
11	IFG	43		51	X	36		91	DN	25			
12	3	03		52	e	12		92	XEY	30			
13	b	14		53	RUP	22		93	DIV	35			
14	PNT	45		54	DIV	35		94	a	13			
15	PNT	45		55	RDN	31		95	UP	27			
16	UP	27		56	XEY	30		96	e	12			
17	YEX	24		57	-	34		97	X	36			
18	a	13		58	YTO	40		98	c	16			
19	+	33		59	e	12		99	XEY	30			
1a	YE	24		5a	d	17		9a	-	34			
1b	a	13		5b	RUP	22		9b	e	12			
1c	X	36		5c	DIV	35		9c	PNT	45			
1d	RDN	31		5d	YTO	40		9d	END	46			
20	YE	24		60	d	17							
21	b	14		61	DN	25							
22	+	33		62	√	76							
23	YE	24		63	UP	27							
24	b	14		64	b	14							
25	DN	25		65	UP	27							
26	UP	27		66	f	15							
27	YE	24		67	DIV	35							
28	c	16		68	YTO	40							
29	+	33		69	b	14							
2a	YE	24		6a	RDN	31							
2b	c	16		6b	√	76							
2c	X	36		6c	PNT	45							
2d	RDN	31		6d	PNT	45	s						
30	YE	24		70	b	14							
31	d	17		71	UP	27							
32	+	33		72	a	13							
33	YE	24		73	UP	27							
34	d	17		74	X	36							
35	CLX	37		75	DN	25							
36	RDN	31		76	-	34							
37	X	36		77	e	12							
38	GTO	44		78	XEY	30							
39	0	00		79	DIV	35							
3a	9	11		7a	YTO	40							
3b	UP	27		7b	e	12							
3c	1	01		7c	√	76							
3d	AC-	63		7d	UP	27							

MULTIPLE LINEAR REGRESSION
THREE VARIABLE

This program fits any number of data points (X_i, Y_i, Z_i) to a linear, two variable equation of the form:

$$Z = a_o + a_1 X + a_2 Y$$

where X and Y are the independent variables.

Development:

The constants a_o, a_1, and a_2 of the equation may be found by solving simultaneously the following normal equations which represent the least square plane (approximating plane) formed by the data points.

$$\sum Z = a_o n + a_1 \sum X + a_2 \sum Y$$

$$\sum XZ = a_o \sum X + a_1 \sum X^2 + a_2 \sum XY$$

$$\sum YZ = a_o \sum Y + a_1 \sum XY + a_2 \sum Y^2$$

In the program the constant a_2 is found from solving the equations by matrix algebra. Therefore,

$$a_2 = \frac{n(\sum X^2 \sum YZ - \sum XZ \sum XY) - \sum X(\sum X \sum YZ - \sum Y \sum XZ) + \sum Z(\sum X \sum XY - \sum Y \sum X^2)}{D}$$

Where D (the determinant) = $\begin{vmatrix} n & \sum X & \sum Y \\ \sum X & \sum X^2 & \sum XY \\ \sum Y & \sum XY & \sum Y^2 \end{vmatrix}$

After finding a_2, the solution is reduced to two equations in two unknowns which are:

$$M = a_o n + a_1 \sum X$$

$$N = a_o \sum X + a_1 \sum X^2$$

where $M = (\sum Z - a_2 \sum Y)$ $\quad$ $N = (\sum XZ - a_2 \sum XY)$

These two equations are then solved for a_o and a_1.

Reference:

Introduction to the Theory of Statistics
Mood and Graybill
McGraw-Hill, 1963

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A
PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

i	Z
0	Y
0	X

ENTER DATA: $Z_i \rightarrow Z$, $Y_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE

NO ← Has last set of points been entered ? → YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

a_2	Z
a_1	Y
a_0	X

To reset problem

EXAMPLES

(A) Equation of the form:

$$Z = a_o + a_1 X + a_2 Y$$

Input data:

X	Y	Z
1	0	3
0	1	4
1	1	6
3	4	19
2	2	11

Solution:

$$Z = 1 + 2X + 3Y$$

(B) Equation of the form:

$$Z = a_o + a_1 (\log X) + a_2 (\log Y)$$

Note to enter data ;
ENTER: $Z_i \rightarrow X$
PRESS: ↑
ENTER: $Y_i \rightarrow X$
PRESS: log X
PRESS: ↑
ENTER: $X_i \rightarrow X$
PRESS: log X

Input data:

X	Y	Z
1	2	4.6505
1	1	4.5
4	3.63	5.9841
10	5	6.8495
8	16	6.9082
13	7	7.1504
3	10	5.9542

Solution:

$$Z = 4.5 + 2 (\log X) + .4999 (\log Y)$$

00	CLR	20	Plus Page	40	UP	27	80	CNT 47
01	XTO	23		41	X	36	81	CNT 47
02	a	13		42	XEY	30	82	CNT 47
03	XTO	23		43	YE	24	83	CNT 47
04	b	14		44	8	10	84	CNT 47
05	XTO	23		45	+	33	85	CNT 47
06	c	16		46	YE	24	86	CNT 47
07	XTO	23		47	8	10	87	CNT 47
08	9	11		48	XFR	67	88	CNT 47
09	XTO	23		49	-	34	89	CNT 47
0a	-	34		4a	d	17	8a	CNT 47
0b	f	15		4b	X	36	8b	CNT 47
0c	XTO	23		4c	UP	27	8c	CNT 47
0d	-	34		4d	X	36	8d	CNT 47
10	e	12		50	RUP	22	90	CNT 47
11	XTO	23		51	YE	24	91	CNT 47
12	8	10		52	9	11	92	CNT 47
13	1	01		53	+	33	93	CNT 47
14	XTO	23		54	YE	24	94	CNT 47
15	d	17		55	9	11	95	CNT 47
16	RDN	31		56	b	14	96	CNT 47
17	STP	41	ENTRY	57	+	33	97	CNT 47
18	IFG	43		58	YTO	40	98	CNT 47
19	7	07		59	b	14	99	CNT 47
1a	1	01		5a	RUP	22	9a	CNT 47
1b	PNT	45		5b	YE	24	9b	CNT 47
1c	PNT	45		5c	-	34	9c	CNT 47
1d	YTO	40		5d	f	15	9d	CNT 47
20	-	34		60	+	33	a0	CNT 47
21	d	17		61	YTO	40	a1	CNT 47
22	X	36		62	-	34	a2	CNT 47
23	XEY	30		63	f	15	a3	CNT 47
24	YE	24		64	1	01	a4	CNT 47
25	a	13		65	UP	27	a5	CNT 47
26	+	33		66	d	17	a6	CNT 47
27	YE	24		67	+	33	a7	CNT 47
28	a	13		68	YTO	40	a8	CNT 47
29	RDN	31		69	d	17	a9	CNT 47
2a	AC+	60		6a	CLX	37	aa	CNT 47
2b	UP	27		6b	UP	27	ab	CNT 47
2c	X	36		6c	GTO	44	ac	CNT 47
2d	XEY	30		6d	1	01	ad	CNT 47
30	YE	24		70	7	07		ΣZ^2
31	-	34		71	YE	24		
32	e	12		72	-	34		ΣYZ
33	+	33		73	f	15		
34	YE	24		74	d	17		ΣXY
35	-	34		75	UP	27		
36	e	12		76	1	01		ΣY
37	DN	25		77	-	34		
38	X	36		78	YTO	40		ΣXZ
39	c	16		79	d	17		
3a	+	33		7a	GTO	44		n
3b	YTO	40		7b	-	34		
3c	c	16		7c	0	00		ΣZ
3d	RUP	22		7d	0	00		ΣX

b0	CNT	47
b1	CNT	47
b2	CNT	47
b3	CNT	47
b4	CNT	47
b5	CNT	47
b6	CNT	47
b7	CNT	47
b8	CNT	47
b9	CNT	47
ba	CNT	47
bb	CNT	47
bc	CNT	47
bd	CNT	47

c0	CNT	47
c1	CNT	47
c2	CNT	47
c3	CNT	47
c4	CNT	47
c5	CNT	47
c6	CNT	47
c7	CNT	47
c8	CNT	47
c9	CNT	47
ca	CNT	47
cb	CNT	47
cc	CNT	47
cd	CNT	47

d0	CNT	47
d1	CNT	47
d2	CNT	47
d3	CNT	47
d4	CNT	47
d5	CNT	47
d6	CNT	47
d7	CNT	47
d8	CNT	47
d9	CNT	47
da	CNT	47
db	CNT	47
dc	CNT	47
dd	CNT	47

			Minus Page								
00	DN	25		40	X	36		80	RUP	22	
01	f	15		41	RUP	22		81	X	36	
02	X	36		42	XEY	30		82	c	16	
03	CHS	32		43	d	17		83	XEY	30	
04	X	36		44	X	36		84	-	34	
05	RUP	22		45	XFR	67		85	YTO	40	
06	YE	24		46	-	34		86	c	16	
07	-	34		47	e	12		87	f	15	
08	e	12		48	X	36		88	UP	27	
09	YTO	40		49	RDN	31		89	d	17	
0a	-	34		4a	+	33		8a	YTO	40	
0b	e	12		4b	RUP	22		8b	d	17	
0c	X	36		4c	XEY	30		8c	YE	24	
0d	d	17		4d	b	14		8d	-	34	
10	X	36		50	X	36		90	e	12	
11	RDN	31		51	e	12		91	YTO	40	
12	+	33		52	X	36		92	-	34	
13	a	13		53	RDN	31		93	f	15	
14	UP	27		54	-	34		94	XTO	23	
15	X	36		55	f	15		95	b	14	
16	d	17		56	RUP	22		96	CLR	20	
17	X	36		57	X	36		97	YE	24	
18	DN	25		58	a	13		98	-	34	
19	-	34		59	X	36		99	f	15	
1a	UP	27		5a	RDN	31		9a	c	16	
1b	YE	24		5b	+	33		9b	UP	27	
1c	-	34		5c	d	17		9c	d	17	
1d	e	12		5d	RUP	22		9d	DIV	35	
20	YTO	40		60	X	36		a0	RUP	22	
21	-	34		61	c	16		a1	XEY	30	
22	e	12		62	X	36		a2	DIV	35	
23	b	14		63	RDN	31		a3	DN	25	
24	X	36		64	-	34		a4	IFG	43	
25	X	36		65	f	15		a5	b	14	
26	DN	25		66	RUP	22		a6	4	04	
27	-	34		67	X	36		a7	AC+	60	
28	b	14		68	b	14		a8	d	17	
29	UP	27		69	X	36		a9	YE	24	
2a	f	15		6a	DN	25		aa	b	14	
2b	X	36		6b	+	33		ab	UP	27	
2c	a	13		6c	XFR	67		ac	a	13	
2d	X	36		6d	-	34		ad	RUP	22	
30	2	02		70	f	15					
31	X	36		71	DIV	35					
32	DN	25		72	YTO	40					
33	+	33		73	9	11					
34	YTO	40		74	UP	27					
35	-	34		75	DN	25					
36	f	15		76	b	14					
37	YE	24		77	X	36					
38	9	11		78	e	12					
39	UP	27		79	XEY	30					Y
3a	DN	25		7a	-	34					ΣX^2
3b	f	15		7b	a	13					ΣY^2
3c	X	36		7c	YTO	40					
3d	CHS	32		7d	a	13					

b0	SFL	54
b1	GTO	44
b2	9	11
b3	d	17
b4	AC-	63
b5	XEY	30
b6	YE	24
b7	e	12
b8	f	15
b9	DIV	35
ba	e	12
bb	XEY	30
bc	X	36
bd	RDN	31
c0	-	34
c1	RUP	22
c2	YE	24
c3	9	11
c4	RUP	22
c5	PNT	45
c6	PNT	45
c7	END	46

MULTIPLE LINEAR REGRESSION
FOUR VARIABLE

$$X_1 = a_1 + a_2X_2 + a_3X_3 + a_4X_4 \qquad (1)$$

This program determines the coefficients of the normal equations required for determining the least squares regression equation (1). An option is available for correcting erroneously inputted data.

Given a set of data points (X_{1i}, X_{2i}, X_{3i}, X_{4i}) the normal equations are defined as:

$$a_1N + a_2\Sigma X_2 + a_3\Sigma X_3 + a_4\Sigma X_4 = \Sigma X_1$$

$$a_1\Sigma X_2 + a_2\Sigma X_2^2 + a_3\Sigma X_2X_3 + a_4\Sigma X_2X_4 = \Sigma X_1X_2$$

$$a_1\Sigma X_3 + a_2\Sigma X_2X_3 + a_3\Sigma X_3^2 + a_4\Sigma X_3X_4 = \Sigma X_1X_3$$

$$a_1\Sigma X_4 + a_2\Sigma X_2X_4 + a_3\Sigma X_3X_4 + a_4\Sigma X_4^2 = \Sigma X_1X_4$$

The quantities, N, ΣX_2, ΣX_3, ΣX_4, ΣX_1, ΣX_2^2, ΣX_3^2, ΣX_4^2, ΣX_1X_2, ΣX_1X_3, ΣX_1X_4, ΣX_2X_3, ΣX_2X_4, ΣX_3X_4 are computed and outputted in a manner convenient for immediate application of program STAT-PAC IV-13, Simultaneous Solution of Four Equations in Four Unknowns.

Reference: Theory and Problems of Statistics, Murray Spiegel, Shaum's Outline Series, McGraw-Hill Book Company, 1961, page 273.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 ——— Z
i ——— Y
0 ——— X

ENTER DATA:

X_2 ⟶ Y
X_1 ⟶ X

PRESS: CONTINUE

DISPLAY

0 ——— Z
i ——— Y
0 ——— X

ENTER DATA:

X_4 ⟶ Y
X_3 ⟶ X

PRESS: CONTINUE

NO ← Have all sets (X_1, X_2, X_3, X_4) been entered correctly? *

YES

Has the data been entered free of errors?

YES

NO → ENTER: Updater Program

PRESS: SET FLAG

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

The coefficients are now printed out ready for entry into program STAT-PAC IV-13. These are printed as follows:

N ΣX_2 ΣX_3 ΣX_4 ΣX_1	Equation 1
ΣX_2 ΣX_2^2 ΣX_2X_3 ΣX_2X_4 ΣX_1X_2	Equation 2
ΣX_3 ΣX_2X_3 ΣX_3^2 ΣX_3X_4 ΣX_1X_3	Equation 3
ΣX_4 ΣX_2X_4 ΣX_3X_4 ΣX_4^2 ΣX_1X_4	Equation 4

To run another case: Repeat USER INSTRUCTIONS.

*If an error has been made in entering any of the sets (X_1, X_2, X_3, X_4), re-enter the correct sets and run the UPDATER program STAT-PAC IV-5, Normal Equation Coefficient Updater. DO NOT alter any of the storage registers as you load the UPDATER program.

EXAMPLE

$$X_1 = a_1 + a_2X_2 + a_3X_3 + a_4X_4$$

X_1	X_2	X_3	X_4
4	1	1	1
7.5	1.5	2	3
8	-1	3	5
13	2	4	6

Results:

N	=	4.0	Equation 1	ΣX_3	=	10.0	Equation 3
ΣX_2	=	3.5		ΣX_2X_3	=	9.0	
ΣX_3	=	10.0		ΣX_3^2	=	30.0	
ΣX_4	=	15.0		ΣX_3X_4	=	46.0	
ΣX_1	=	32.5		ΣX_1X_3	=	95.0	
ΣX_2	=	3.5	Equation 2	ΣX_4	=	15.0	Equation 4
ΣX_2^2	=	8.25		ΣX_2X_4	=	12.5	
ΣX_2X_3	=	9.0		ΣX_3X_4	=	46.0	
ΣX_2X_4	=	12.5		ΣX_4^2	=	71.0	
ΣX_1X_2	=	33.25		ΣX_1X_4	=	144.5	

Upon inputting these coefficients to program STAT-PAC IV-13, the regression coefficients a_1, a_2, a_3, and a_4 are determined.

$$a_1 = a_2 = a_3 = a_4 = 1$$

00	CLR	20	Plus	40	CNT	47		80	CNT	47	
01	XTO	23	Page	41	CNT	47		81	CNT	47	
02	d	17		42	CNT	47		82	CNT	47	
03	XTO	23		43	CNT	47		83	CNT	47	
04	c	16		44	CNT	47		84	CNT	47	
05	XTO	23		45	CNT	47		85	CNT	47	
06	b	14		46	CNT	47		86	CNT	47	
07	XTO	23		47	CNT	47		87	CNT	47	
08	a	13		48	CNT	47		88	CNT	47	
09	XTO	23		49	CNT	47		89	CNT	47	
0a	9	11		4a	CNT	47		8a	CNT	47	
0b	XTO	23		4b	CNT	47		8b	CNT	47	
0c	8	10		4c	CNT	47		8c	CNT	47	
0d	XTO	23		4d	CNT	47		8d	CNT	47	
10	7	07		50	CNT	47		90	CNT	47	
11	XTO	23		51	CNT	47		91	CNT	47	
12	6	06		52	CNT	47		92	CNT	47	
13	XTO	23		53	CNT	47		93	CNT	47	
14	5	05		54	CNT	47		94	CNT	47	
15	XTO	23		55	CNT	47		95	CNT	47	
16	4	04		56	CNT	47		96	CNT	47	
17	XTO	23		57	CNT	47		97	CNT	47	
18	3	03		58	CNT	47		98	CNT	47	
19	ARC	72		59	CNT	47		99	CNT	47	
1a	GTO	44		5a	CNT	47		9a	CNT	47	
1b	-	34		5b	CNT	47		9b	CNT	47	
1c	0	00		5c	CNT	47		9c	CNT	47	
1d	0	00		5d	CNT	47		9d	CNT	47	
20	CNT	47		60	CNT	47		a0	CNT	47	
21	CNT	47		61	CNT	47		a1	CNT	47	
22	CNT	47		62	CNT	47		a2	CNT	47	
23	CNT	47		63	CNT	47		a3	CNT	47	
24	CNT	47		64	CNT	47		a4	CNT	47	
25	CNT	47		65	CNT	47		a5	CNT	47	
26	CNT	47		66	CNT	47		a6	CNT	47	
27	CNT	47		67	CNT	47		a7	CNT	47	
28	CNT	47		68	CNT	47		a8	CNT	47	
29	CNT	47		69	CNT	47		a9	CNT	47	
2a	CNT	47		6a	CNT	47		aa	CNT	47	
2b	CNT	47		6b	CNT	47		ab	CNT	47	
2c	CNT	47		6c	CNT	47		ac	CNT	47	
2d	CNT	47		6d	CNT	47		ad	CNT	47	
30	CNT	47		70	CNT	47					
31	CNT	47		71	CNT	47					
32	CNT	47		72	CNT	47					
23	CNT	47		73	CNT	47					
34	CNT	47		74	CNT	47					
35	CNT	47		75	CNT	47					
36	CNT	47		76	CNT	47					
37	CNT	47		77	CNT	47					
38	CNT	47		78	CNT	47					
39	CNT	47		79	CNT	47					
3a	CNT	47		7a	CNT	47					
3b	CNT	47		7b	CNT	47					
3c	CNT	47		7c	CNT	47					
3d	CNT	47		7d	CNT	47					

X_1	$\Sigma X_4^2\ (X_3)$
X_2	ΣX_3^2
$\Sigma X_3X_4\ (X_3)$	ΣX_2^2
ΣX_2X_4	ΣX_4
ΣX_2X_3	ΣX_3
ΣX_1X_4	ΣX_2
ΣX_1X_3	ΣX_1
ΣX_1X_2	N

			Plus Page
b0	CNT	47	
b1	CNT	47	
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	
c0	CNT	47	
c1	CNT	47	
c2	CNT	47	
c3	CNT	47	
c4	CNT	47	
c5	CNT	47	
c6	CNT	47	
c7	CNT	47	
c8	CNT	47	
c9	CNT	47	
ca	CNT	47	
cb	CNT	47	
cc	CNT	47	
cd	CNT	47	
d0	CNT	47	
d1	CNT	47	
d2	CNT	47	
d3	CNT	47	
d4	CNT	47	
d5	CNT	47	
d6	CNT	47	
d7	CNT	47	
d8	CNT	47	
d9	CNT	47	
da	CNT	47	
db	CNT	47	
dc	CNT	47	
dd	CNT	47	

00	XTO	23	Minus Page	40	YE	24		80	+	33	
01	2	02		41	b	14		81	YTO	40	
02	f	15		42	+	33		82	4	04	
03	UP	27		43	YTO	40		83	YE	24	
04	1	01		44	b	14		84	0	00	
05	+	33		45	XEY	30		85	XFR	67	
06	0	00		46	DN	25		86	-	34	
07	STP	41	ENTRY	47	UP	27		87	f	15	
08	IFG	43		48	X	36		88	X	36	
09	a	13		49	RDN	31		89	RDN	31	
0a	2	02		4a	YE	24		8a	YE	24	
0b	PNT	45		4b	8	10		8b	5	05	
0c	XTO	23		4c	+	33		8c	+	33	
0d	0	00		4d	YE	24		8d	YTO	40	
10	YTO	40		50	8	10		90	5	05	
11	1	01		51	DN	25		91	XFR	67	
12	UP	27		52	X	36		92	1	01	
13	1	01		53	XEY	30		93	RUP	22	
14	AC+	60		54	YE	24		94	X	36	
15	DN	25		55	2	02		95	DN	25	
16	RDN	31		56	+	33		96	YE	24	
17	YE	24		57	YE	24		97	3	03	
18	d	17		58	2	02		98	+	33	
19	+	33		59	DN	25		99	YTO	40	
1a	YTO	40		5a	UP	27		9a	3	03	
1b	d	17		5b	X	36		9b	0	00	
1c	XEY	30		5c	RDN	31		9c	UP	27	
1d	DN	25		5d	YE	24		9d	GTO	44	
20	X	36		60	9	11		a0	0	00	
21	RDN	31		61	+	33		a1	2	02	
22	YE	24		62	YTO	40		a2	f	15	
23	7	07		63	9	11		a3	UP	27	
24	+	33		64	RUP	22		a4	d	17	
25	YTO	40		65	YE	24		a5	UP	27	
26	7	07		66	c	16		a6	c	16	
27	RUP	22		67	+	33		a7	PNT	45	s
28	UP	27		68	YTO	40		a8	b	14	
29	X	36		69	c	16		a9	UP	27	
2a	DN	25		6a	RDN	31		aa	e	12	
2b	YE	24		6b	XFR	67		ab	RUP	22	
2c	a	13		6c	0	00		ac	PNT	45	
2d	+	33		6d	RUP	22		ad	a	13	s
30	YTO	40		70	X	36					
31	a	13		71	RDN	31					
32	0	00		72	YE	24					
33	UP	27		73	6	06					
34	f	15		74	+	33					
35	UP	27		75	YTO	40					
36	0	00		76	6	06					
37	STP	41	ENTRY	77	XFR	67					
38	PNT	45		78	1	01					
39	PNT	45		79	RUP	22					
3a	YTO	40		7a	X	36					
3b	-	34		7b	RDN	31					
3c	f	15		7c	YE	24					
3d	RDN	31		7d	4	04					

X_4

```
b0   UP    27        Minus
b1   XFR   67        Page
b2    4    04
b3   UP    27
b4   XFR   67
b5    3    03
b6   PNT   45    S
b7   XFR   67
b8    7    07
b9   UP    27
ba    c    16
bb   RUP   22
bc   PNT   45    S
bd   XFR   67

c0    9    11
c1   UP    27
c2   XFR   67
c3    2    02
c4   UP    27
c5   XFR   67
c6    6    06
c7   PNT   45    S
c8    b    14
c9   UP    27
ca   XFR   67
cb    3    03
cc   RUP   22
cd   PNT   45

d0   XFR   67
d1    8    10
d2   UP    27
d3   XFR   67
d4    5    05
d5   UP    27
d6    0    00
d7   PNT   45    S
d8   PNT   45
d9   PNT   45
da    0    00
db   UP    27
dc   UP    27
dd   STP   41
```

NORMAL EQUATION COEFFICIENT UPDATER (CORRECTOR)

This program is to be used to correct the summations computed by program STAT-PAC IV-4. To correct for erroneous data entered into the above program, simply repeat the same erroneous entries into this UPDATER program.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM:

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY



ENTER DATA: (Erroneous Data Set)

$X_2 \longrightarrow Y$
$X_1 \longrightarrow X$

PRESS: CONTINUE

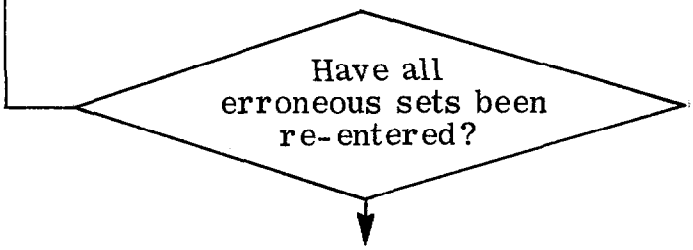
DISPLAY



ENTER DATA:

$X_4 \longrightarrow Y$
$X_3 \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

The coefficients are now printed out ready for entry into program STAT-PAC IV-13. These are printed out as follows:

See program STAT-PAC IV-4.

00	d	17	Minus Page	40	YTO	40		80	YE	24	
01	UP	27		41	b	14		81	0	00	
02	UP	27		42	XEY	30		82	XFR	67	
03	f	15		43	DN	25		83	-	34	
04	CNT	47		44	UP	27		84	f	15	
05	STP	41	ENTRY	45	X	36		85	X	36	
06	IFG	43		46	RDN	31		86	RDN	31	
07	9	11		47	YE	24		87	YE	24	
08	d	17		48	8	10		88	5	05	
09	PNT	45		49	-	34		89	-	34	
0a	XTO	23		4a	YE	24		8a	YTO	40	
0b	0	00		4b	8	10		8b	5	05	
0c	YTO	40		4c	DN	25		8c	XFR	67	
0d	1	01		4d	X	36		8d	1	01	
10	UP	27		50	XEY	30		90	RUP	22	
11	1	01		51	YE	24		91	X	36	
12	AC-	63		52	2	02		92	DN	25	
13	DN	25		53	-	34		93	YE	24	
14	RDN	31		54	YE	24		94	3	03	
15	YE	24		55	2	02		95	-	34	
16	d	17		56	DN	25		96	YTO	40	
17	-	34		57	UP	27		97	3	03	
18	YTO	40		58	X	36		98	0	00	
19	d	17		59	RDN	31		99	UP	27	
1a	XEY	30		5a	YE	24		9a	GTO	44	
1b	DN	25		5b	9	11		9b	0	00	
1c	X	36		5c	-	34		9c	0	00	
1d	RDN	31		5d	YTO	40		9d	f	15	
20	YE	24		60	9	11		a0	UP	27	
21	7	07		61	RUP	22		a1	d	17	
22	-	34		62	YE	24		a2	UP	27	
23	YTO	40		63	c	16		a3	c	16	
24	7	07		64	-	34		a4	PNT	45	S
25	RUP	22		65	YTO	40		a5	b	14	
26	UP	27		66	c	16		a6	UP	27	
27	X	36		67	RDN	31		a7	e	12	
28	DN	25		68	XFR	67		a8	RUP	22	
29	YE	24		69	0	00		a9	PNT	45	S
2a	a	13		6a	RUP	22		aa	a	13	
2b	-	34		6b	X	36		ab	UP	27	
2c	YTO	40		6c	RDN	31		ac	XFR	67	
2d	a	13		6d	YE	24		ad	4	04	
30	0	00		70	6	06		X_1			ΣX_4^2
31	UP	27		71	-	34					
32	UP	27		72	YTO	40		X_2			ΣX_3^2
33	f	15		73	6	06					
34	STP	41	ENTRY	74	XFR	67		ΣX_3X_4			ΣX_2^2
35	PNT	45		75	1	01					
36	PNT	45		76	RUP	22		ΣX_2X_4			ΣX_4
37	YTO	40		77	X	36					
38	-	34		78	RDN	31		ΣX_2X_3			ΣX_3
39	f	15		79	YE	24					
3a	RDN	31		7a	4	04		ΣX_1X_4			ΣX_2
3b	YE	24		7b	-	34					
3c	b	14		7c	YTO	40		ΣX_1X_3			ΣX_1
3d	-	34		7d	4	04					
								ΣX_1X_2			N

b0	UP	27	
b1	XFR	67	
b2	3	03	s
b3	PNT	45	
b4	XFR	67	
b5	7	07	
b6	UP	27	
b7	c	16	
b8	RUP	22	
b9	PNT	45	s
ba	XFR	67	
bb	9	11	
bc	UP	27	
bd	XFR	67	
c0	2	02	
c1	UP	27	
c2	XFR	67	
c3	6	06	
c4	PNT	45	s
c5	b	14	
c6	UP	27	
c7	XFR	67	
c8	3	03	
c9	RUP	22	
ca	PNT	45	s
cb	XFR	67	
cc	8	10	
cd	UP	27	
d0	XFR	67	
d1	5	05	
d2	UP	27	
d3	0	00	
d4	PNT	45	s
d5	PNT	45	
d6	PNT	45	
d7	0	00	
d8	UP	27	
d9	UP	27	
da	STP	41	

LEAST SQUARES FIT - POWER CURVE

This program computes the least squares fit and correlation coefficient of N pairs of data points for a power curve of the form:

$$Y = aX^b \qquad (a > 0)$$

The equation is linearized into $\ln Y = b \ln X + \ln a$

where

$$b = \frac{N\sum(\ln X \ln Y) - \sum \ln X \sum \ln Y}{N\sum(\ln X)^2 - (\sum \ln X)^2}$$

and

$$r = \frac{N\sum \ln X \ln Y - (\sum \ln X)(\sum \ln Y)}{\sqrt{\left[N\sum(\ln X)^2 - (\sum \ln X)^2\right]\left[N\sum(\ln Y)^2 - (\sum \ln Y)^2\right]}}$$

$$\ln a = \frac{\sum \ln Y}{N} - \frac{\sum \ln X}{N} \; b$$

Note: $X_i > 0$ and $Y_i > 0$, $i = 1, \cdots, N$

Reference: Statistical Theory and Methodology in Science and Engineering
by K.A. Brownlee

John Wiley and Sons 1965

USER INSTRUCTIONS

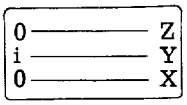
DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

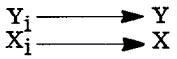
PRESS: CONTINUE

DISPLAY



(i indicates pair of points to be entered)

ENTER DATA:



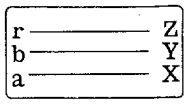
PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY



EXAMPLES

$Y = aX^b$

X	Y
1.0001	25.58
3.16	14.55
10	9.26
31.6	5.63
100	3.48
316	2.12
1000	1.7

r = -0.9964

b = -0.4022

a = 23.5871

$Y = 23.5871X^{-.4022}$

X	Y
1	3
2	4.2574
3	5.2248
4	6.0417
5	6.7624

r = 1.000

b = .505

a = 3.000

$Y = 3X^{.505}$

00	CLR	20		40	a	13	80	YE	24
01	XTO	23		41	UP	27	81	e	12
02	d	17		42	1	01	82	e	12
03	XTO	23		43	-	34	83	X	36
04	c	16		44	YTO	40	84	f	15
05	XTO	23		45	a	13	85	XEY	30
06	b	14		46	e	12	86	-	34
07	1	01		47	UP	27	87	e	12
08	XTO	23		48	a	13	88	XEY	30
09	a	13		49	DIV	35	89	EXP	74
0a	XEY	30		4a	YE	24	8a	PNT	45
0b	STP	41	ENTRY	4b	f	15	8b	PNT	45
0c	IFG	43		4c	DIV	35	8c	END	46
0d	4	04		4d	YTO	40			
10	0	00		50	e	12			
11	PNT	45		51	X	36			
12	PNT	45		52	e	12			
13	LN	65		53	X	36			
14	XEY	30		54	d	17			
15	LN	65		55	XEY	30			
16	XEY	30		56	-	34			
17	AC+	60		57	YTO	40			
18	UP	27		58	d	17			
19	X	36		59	c	16			
1a	XEY	30		5a	UP	27			
1b	YE	24		5b	f	15			
1c	d	17		5c	UP	27			
1d	+	33		5d	X	36			
20	YE	24		60	a	13			
21	d	17		61	X	36			
22	DN	25		62	DN	25			
23	X	36		63	-	34			
24	b	14		64	YTO	40			
25	+	33		65	c	16			
26	YTO	40		66	b	14			
27	b	14		67	UP	27			
28	RUP	22		68	f	15			
29	UP	27		69	UP	27			
2a	X	36		6a	e	12			
2b	c	16		6b	X	36			
2c	+	33		6c	a	13			
2d	YTO	40		6d	X	36			
30	c	16		70	DN	25			
31	a	13		71	-	34			
32	UP	27		72	d	17			
33	1	01		73	UP	27			
34	+	33		74	DN	25			
35	YTO	40		75	$\sqrt{}$	76			
36	a	13		76	DIV	35			
37	CLX	37		77	c	16			
38	RDN	31		78	$\sqrt{}$	76			
39	XEY	30		79	DIV	35			
3a	0	00		7a	d	17			
3b	GTO	44		7b	RUP	22			
3c	0	00		7c	XEY	30			
3d	b	14		7d	DIV	35			

LEAST SQUARES FIT - EXPONENTIAL

This program computes the least squares fit and a correlation coefficient of n pairs of data points for an exponential function of the form:

$$y = ae^{bx} \qquad (a > 0)$$

The equation is linearized into

$$\ln y = \ln a + bx$$

or

$$Y = A + bx$$

Using a linear regression method,

$$b = \frac{n\sum xY - \sum x \sum Y}{n\sum x^2 - (\sum x)^2}$$

$$A = \frac{\sum Y - b\sum x}{n}$$

$$a = e^A$$

the correlation coefficient is given by

$$r = \frac{n\sum xY - \sum x \sum Y}{\sqrt{\left[n\sum x^2 - (\sum x)^2\right]\left[n\sum Y^2 - (\sum Y)^2\right]}}$$

Note: $Y_i > 0 \quad i = 1, \cdots, n$

Reference: Statistical Theory and Methodology in Science and Engineering
by K.A. Brownlee

John Wiley and Sons 1965

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

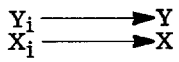
PRESS: CONTINUE

DISPLAY

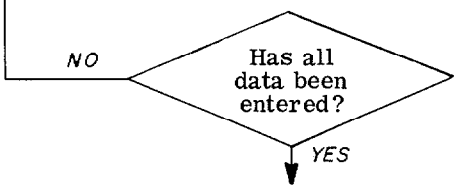


(i indicates pair of points to be entered)

ENTER DATA:



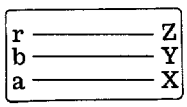
PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY



EXAMPLES

GENERAL FORM: $Y = ae^{bX}$

X	Y
.5	7.12
1.2	11.67
3.1	44.75
7.4	935.64

r = 1.000
b = .707
a = 4.998
$Y = 4.998e^{.707X}$

X	Y
.72	2.16
1.31	1.61
1.95	1.16
2.58	.85

r = -1.000
b = -.503
a = 3.103
$Y = 3.103e^{-.503X}$

00	CLR	20		40	d	17		80	PNT	45
01	XTO	23		41	c	16		81	PNT	45
02	a	13		42	X	36		82	END	46
03	XTO	23		43	UP	27				
04	b	14		44	RCL	61				
05	XTO	23		45	X	36				
06	c	16		46	DN	25				
07	1	01		47	-	34				
08	XEY	30		48	YTO	40				
09	YTO	40		49	c	16				
0a	d	17		4a	d	17				
0b	STP	41	ENTRY	4b	UP	27				
0c	IFG	43		4c	a	13				
0d	3	03		4d	X	36				
10	9	11		50	e	12				
11	PNT	45		51	UP	27				
12	PNT	45		52	X	36				
13	XEY	30		53	DN	25				
14	LN	65		54	-	34				
15	AC+	60		55	YTO	40				
16	UP	27		56	a	13				
17	X	36		57	d	17				
18	XEY	30		58	UP	27				
19	YE	24		59	b	14				
1a	b	14		5a	X	36				
1b	+	33		5b	f	15				
1c	YE	24		5c	UP	27				
1d	b	14		5d	X	36				
20	DN	25		60	DN	25				
21	X	36		61	-	34				
22	c	16		62	a	13				
23	+	33		63	X	36				
24	YTO	40		64	DN	25				
25	c	16		65	√	76				
26	RUP	22		66	UP	27				
27	UP	27		67	c	16				
28	X	36		68	XEY	30				
29	a	13		69	DIV	35				
2a	+	33		6a	YTO	40				
2b	YTO	40		6b	b	14				
2c	a	13		6c	c	16				
2d	CLX	37		6d	UP	27				
30	UP	27		70	a	13				
31	d	17		71	DIV	35				
32	UP	27		72	e	12				
33	1	01		73	UP	27				
34	+	33		74	DN	25				
35	0	00		75	X	36				
36	GTO	44		76	f	15				
37	0	00		77	XEY	30				
38	9	11		78	-	34				
39	d	17		79	d	17				
3a	UP	27		7a	DIV	35				
3b	1	01		7b	b	14				
3c	-	34		7c	RDN	31				
3d	YTO	40		7d	EXP	74				

THE LEAST SQUARE PARABOLA

Development:

The least square parabola approximating the set of points $(X_1, Y_1) \ldots\ldots (X_i, Y_i)$ has the equation:

$$Y = a_o + a_1 x + a_2 x^2$$

where the constants a_o, a_1, and a_2 are determined by solving simultaneously the following normal equations:

$$\Sigma Y = a_o n + a_1 \Sigma X + a_2 \Sigma X^2$$
$$\Sigma XY = a_o \Sigma X + a_1 \Sigma X^2 + a_2 \Sigma X^3$$
$$\Sigma X^2 Y = a_o \Sigma X^2 + a_1 \Sigma X^3 + a_2 \Sigma X^4$$

In the program the constant a_2 is found by matrix algebra; the determinate (D) involved in the solution is:

$$D = \begin{vmatrix} n & \Sigma X & \Sigma X^2 \\ \Sigma X & \Sigma X^2 & \Sigma X^3 \\ \Sigma X^2 & \Sigma X^3 & \Sigma X^4 \end{vmatrix}$$

The equation for a_2 is therefore:

$$a_2 = \frac{\left(n(\Sigma X^2 \Sigma X^2 Y - \Sigma X^3 \Sigma XY) - \Sigma X(\Sigma X \Sigma X^2 Y - \Sigma X^2 \Sigma XY) + \Sigma Y \left[\Sigma X \Sigma X^3 - (\Sigma X^2)^2\right]\right)}{D}$$

After finding a_2 the solution is reduced to two equations in two unknowns which are:

$$N = a_o n + a_1 \Sigma X$$
$$M = a_o \Sigma X + a_1 \Sigma X^2$$

where $M = \Sigma XY - a_2 \Sigma X^3$ and $N = \Sigma Y - a_2 \Sigma X^2$

These equations are then solved for a_o and a_1.

NOTE: Curves with the following equations may also be fitted with this program:

$$Y = a_o + a_1 X \quad (1)$$
$$\log Y = a_o + a_1 X \quad (2)$$
$$\log Y = a_o + a_1 X + a_2 X^2 \quad (3)$$
$$Y = a_o + a_1 \log X \quad (4)$$
$$\log Y = a_o + a_1 \log X \quad (5)$$
$$\log Y = a_o + a_1 (\log X) + a_2 (\log X)^2 \quad (6)$$

An equation of the form of (6) is solved in the examples.

The general form, representing all of these equations, which can be fitted is:

$$f(Y) = a_o + a_1 f(X) + a_2 f^2(X)$$

Reference:

Publisher -- McGraw-Hill
Authors -- Alexander M. Mood & Franklin A. Graybill
Introduction into the Theory of Statistics -- 2nd Edition (1961)

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: (Side A followed by Side B)

PRESS: END

PRESS: CONTINUE

DISPLAY

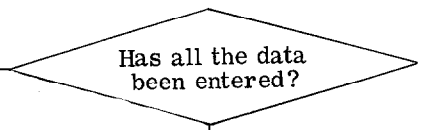


(i indicates pair of points to be entered)

ENTER DATA:



PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY



To calculate coefficients for new data:

PRESS: END

EXAMPLES

(A) Equation of the form: $Y = a_0 + a_1 X + a_2 X^2$

Data:

X	Y
3	29
0	2
5	67
2	16
1.5	11
4	46
1	7

Solution: $Y = 2 + 3X + 2X^2$

(B) Equation of the form:
$\log Y = a_0 + a_1 \log X + a_2 (\log X)^2$

Note: Data to be entered is $\log Y_i$, $\log X_i$; therefore to enter data sets:

ENTER: $Y_i \rightarrow X$
PRESS: log X
PRESS: ↑
ENTER: $X_i \rightarrow Y$
PRESS: log X

Data:

X	Y
1	2.7183
2	35.1595
3	245.3746
4	1188.7946
5	4530.5750

Solution: $\log Y = .43 + 3.0 \log X + 2.30 (\log X)^2$

Step	Key	Code		Step	Key	Code		Step	Key	Code	
00	CLR	20	Plus Page	40	a	13		80	X	36	
01	XTO	23		41	RUP	22		81	RDN	31	
02	c	16		42	X	36		82	-	34	
03	XTO	23		43	XEY	30		83	f	15	
04	b	14		44	YE	24		84	RUP	22	
05	XTO	23		45	-	34		85	X	36	
06	a	13		46	e	12		86	2	02	
07	XTO	23		47	+	33		87	X	36	
08	-	34		48	YTO	40		88	a	13	
09	f	15		49	-	34		89	X	36	
0a	XTO	23		4a	e	12		8a	DN	25	
0b	-	34		4b	1	01		8b	+	33	
0c	e	12		4c	UP	27		8c	YE	24	
0d	1	01		4d	d	17		8d	-	34	
10	XTO	23		50	+	33		90	f	15	
11	d	17		51	YTO	40		91	UP	27	
12	RDN	31		52	d	17		92	DN	25	
13	RDN	31		53	CLX	37		93	b	14	
14	STP	41	ENTRY	54	UP	27		94	X	36	
15	IFG	43		55	GTO	44		95	d	17	
16	5	05		56	1	01		96	X	36	
17	8	10		57	3	03		97	f	15	
18	PNT	45		58	XFR	67		98	RUP	22	
19	PNT	45		59	-	34		99	XEY	30	
1a	AC+	60		5a	e	12		9a	GTO	44	
1b	X	36		5b	UP	27		9b	-	34	
1c	XEY	30		5c	f	15		9c	0	00	
1d	YE	24		5d	UP	27		9d	0	00	
20	c	16		60	X	36		a0	CNT	47	
21	+	33		61	b	14		a1	CNT	47	
22	YE	24		62	RUP	22		a2	CNT	47	
23	c	16		63	X	36		a3	CNT	47	
24	UP	27		64	RUP	22		a4	CNT	47	
25	DN	25		65	CHS	32		a5	CNT	47	
26	X	36		66	X	36		a6	CNT	47	
27	DN	25		67	YE	24		a7	CNT	47	
28	YE	24		68	d	17		a8	CNT	47	
29	-	34		69	1	01		a9	CNT	47	
2a	f	15		6a	-	34		aa	CNT	47	
2b	+	33		6b	YE	24		ab	CNT	47	
2c	YE	24		6c	d	17		ac	CNT	47	
2d	-	34		6d	d	17		ad	CNT	47	
30	f	15		70	RUP	22					
31	DN	25		71	X	36					
32	X	36		72	DN	25					
33	XEY	30		73	+	33					
34	YE	24		74	a	13					ΣX^3
35	b	14		75	UP	27					ΣX^2
36	+	33		76	X	36					ΣXY
37	YE	24		77	d	17					n
38	b	14		78	X	36					ΣX
39	X	36		79	DN	25					ΣY
3a	a	13		7a	-	34					
3b	XEY	30		7b	b	14					
3c	+	33		7c	UP	27					
3d	YTO	40		7d	X	36					

```
b0   CNT   47      Plus
b1   CNT   47      Page
b2   CNT   47
b3   CNT   47
b4   CNT   47
b5   CNT   47
b6   CNT   47
b7   CNT   47
b8   CNT   47
b9   CNT   47
ba   CNT   47
bb   CNT   47
bc   CNT   47
bd   CNT   47

c0   CNT   47
c1   CNT   47
c2   CNT   47
c3   CNT   47
c4   CNT   47
c5   CNT   47
c6   CNT   47
c7   CNT   47
c8   CNT   47
c9   CNT   47
ca   CNT   47
cb   CNT   47
cc   CNT   47
cd   CNT   47

d0   CNT   47
d1   CNT   47
d2   CNT   47
d3   CNT   47
d4   CNT   47
d5   CNT   47
d6   CNT   47
d7   CNT   47
d8   CNT   47
d9   CNT   47
da   CNT   47
db   CNT   47
dc   CNT   47
dd   CNT   47
```

00	X	36	Minus Page	40	YTO	40
01	X	36		41	a	13
02	RDN	31		42	CLR	20
03	-	34		43	XFR	67
04	e	12		44	-	34
05	RUP	22		45	e	12
06	X	36		46	UP	27
07	a	13		47	c	16
08	X	36		48	UP	27
09	RDN	31		49	d	17
0a	+	33		4a	DIV	35
0b	d	17		4b	RUP	22
0c	RUP	22		4c	XEY	30
0d	X	36		4d	DIV	35
10	c	16		50	DN	25
11	X	36		51	IFG	43
12	RDN	31		52	6	06
13	-	34		53	2	02
14	f	15		54	AC+	60
15	XTO	23		55	b	14
16	-	34		56	YE	24
17	e	12		57	-	34
18	RUP	22		58	e	12
19	X	36		59	UP	27
1a	b	14		5a	a	13
1b	X	36		5b	RUP	22
1c	RDN	31		5c	SFL	54
1d	+	33		5d	GTO	44
20	DN	25		60	4	04
21	RDN	31		61	a	13
22	X	36		62	AC-	63
23	e	12		63	XEY	30
24	X	36		64	YE	24
25	RDN	31		65	e	12
26	-	34		66	f	15
27	RUP	22		67	DIV	35
28	YE	24		68	e	12
29	-	34		69	XEY	30
2a	f	15		6a	X	36
2b	RDN	31		6b	RDN	31
2c	DIV	35		6c	-	34
2d	b	14		6d	RUP	22
30	XEY	30		70	YE	24
31	X	36		71	-	34
32	RDN	31		72	f	15
33	-	34		73	RUP	22
34	YE	24		74	PNT	45
35	c	16		75	PNT	45
36	a	13		76	PNT	45
37	RUP	22		77	PNT	45
38	XTO	23		78	PNT	45
39	-	34		79	PNT	45
3a	f	15		7a	END	46
3b	X	36				
3c	RDN	31				
3d	-	34				

ΣX^4

ΣX^2Y

CUBIC REGRESSION (LEAST SQUARES)

This program determines the best fit least squares cubic for a set of N points (X_i, Y_i). The cubic is given by:

$$y = a_0 + a_1X + a_2X^2 + a_3X^3 \quad .$$

The normal equation required for determining the coefficients are:

$$\Sigma Y = a_0N + a_1\Sigma X + a_2 \Sigma X^2 + a_3\Sigma X^3$$

$$\Sigma XY = a_0\Sigma X + a_1\Sigma X^2 + a_2\Sigma X^3 + a_3\Sigma X^4$$

$$\Sigma X^2Y = a_0\Sigma X^2 + a_1\Sigma X^3 + a_2\Sigma X^4 + a_3\Sigma X^5$$

$$\Sigma X^3Y = a_0\Sigma X^3 + a_1\Sigma X^4 + a_2\Sigma X^5 + a_3\Sigma X^6$$

This program determines the summations (Σ) which are then input to program STAT-PAC IV-13, Simultaneous Solution of Four Linear Equations in Four Unknowns, (With Printer), for determination of a_0, a_1, a_2 and a_3 .

The data points and the cubic regression curve may be plotted by the use of program STAT-PAC X-11.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM :

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY

0	Z
i	Y
0	X

ENTER DATA:

Y_i	Y
X_i	X

PRESS: CONTINUE

NO ← Has all data been entered? → YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

N	Z
ΣX	Y
ΣX^2	X

PRESS: CONTINUE

DISPLAY

ΣX^3	Z
ΣY	Y
ΣX	X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

ΣX^2	Z
ΣX^3	Y
ΣX^4	X

PRESS: CONTINUE

DISPLAY

ΣXY	Z
ΣX^2	Y
ΣX^3	X

PRESS: CONTINUE

DISPLAY

ΣX^4	Z
ΣX^5	Y
ΣX^2Y	X

PRESS: CONTINUE

DISPLAY

ΣX^3	Z
ΣX^4	Y
ΣX^5	X

PRESS: CONTINUE

DISPLAY

ΣX^6	Z
ΣX^3Y	Y
0	X

To run another case

EXAMPLE

X	Y
-2	-23
-1	2
0	5
1	4
2	17

Results

N	=	5
ΣX	=	0
ΣX^2	=	10
ΣX^3	=	0
ΣY	=	5
ΣX	=	0
ΣX^2	=	10
ΣX^3	=	0
ΣX^4	=	34
ΣXY	=	82
ΣX^2	=	10
ΣX^3	=	0
ΣX^4	=	34
ΣX^5	=	0
ΣX^2Y	=	-18
ΣX^3	=	0
ΣX^4	=	34
ΣX^5	=	0
ΣX^6	=	130
ΣX^3Y	=	322

EXAMPLE (Con't)

Upon inputting these values in to program STAT-PAC IV-13, the coefficients are found to be:

a_0	=	5
a_1	=	-2
a_2	=	-2
a_3	=	3

00	CLR	20	Minus Page	40	XEY	30		80	UP	27					
01	XTO	23		41	YEX	24		81	f	15					
02	d	17		42	-	34		82	UP	27					
03	XTO	23		43	f	15		83	c	16					
04	c	16		44	+	33		84	PNT	45	S				
05	XTO	23		45	YEX	24		85	b	14					
06	b	14		46	-	34		86	UP	27					
07	XTO	23		47	f	15		87	e	12					
08	a	13		48	XEY	30		88	UP	27					
09	XTO	23		49	X	36		89	f	15					
0a	9	11		4a	XEY	30		8a	PNT	45	S				
0b	XTO	23		4b	YEX	24		8b	c	16					
0c	8	10		4c	-	34		8c	UP	27					
0d	XTO	23		4d	e	12		8d	b	14					
10	7	07		50	+	33		90	UP	27					
11	XTO	23		51	YEX	24		91	a	13					
12	-	34		52	-	34		92	PNT	45	S				
13	f	15		53	e	12		93	XFR	67					
14	XTO	23		54	DN	25		94	9	11					
15	-	34		55	X	36		95	UP	27					
16	e	12		56	XEY	30		96	c	16					
17	1	01		57	YEX	24		97	UP	27					
18	XEY	30		58	9	11		98	b	14					
19	STP	41	ENTRY	59	+	33		99	PNT	45	S				
1a	IFG	43		5a	YEX	24		9a	a	13					
1b	7	07		5b	9	11		9b	UP	27					
1c	d	17		5c	XEY	30		9c	XFR	67					
1d	PNT	45		5d	X	36		9d	-	34					
20	PNT	45		60	XEY	30		a0	f	15					
21	AC+	60		61	YEX	24		a1	UP	27					
22	UP	27		62	8	10		a2	XFR	67					
23	X	36		63	+	33		a3	8	10					
24	XEY	30		64	YEX	24		a4	PNT	45	S				
25	YEX	24		65	8	10		a5	b	14					
26	c	16		66	XEY	30		a6	UP	27					
27	+	33		67	X	36		a7	a	13					
28	YEX	24		68	DN	25		a8	UP	27					
29	c	16		69	YEX	24		a9	XFR	67					
2a	XEY	30		6a	7	07		aa	-	34					
2b	X	36		6b	+	33		ab	f	15					
2c	XEY	30		6c	YTO	40		ac	PNT	45	S				
2d	YEX	24		6d	7	07		ad	XFR	67					
									+					+	
30	b	14		70	0	00								ΣX^2Y	
31	+	33		71	UP	27									
32	YEX	24		72	d	17								ΣXY	
33	b	14		73	UP	27									
34	XEY	30		74	1	01								ΣX^4	
35	X	36		75	+	33									
36	XEY	30		76	YTO	40								ΣX^3	
37	YEX	24		77	d	17									
38	a	13		78	+	33								ΣX^2	
39	+	33		79	0	00									
3a	YEX	24		7a	GTO	44								n	
3b	a	13		7b	1	01									
3c	XEY	30		7c	9	11								ΣY	
3d	X	36		7d	d	17			ΣX^3Y					ΣX	

b0	-	34	Minus
b1	e	12	Page
b2	UP	27	
b3	XFR	67	
b4	7	07	
b5	UP	27	
b6	0	00	
b7	PNT	45	S
b8	PNT	45	
b9	PNT	45	
ba	PNT	45	
bb	PNT	45	
bc	PNT	45	
bd	GTO	44	
c0	0	00	
c1	0	00	
c2	END	46	

$\simeq x^6$

$\simeq x^5$

LINEAR LEAST SQUARES: Y = A + BX
(WITH ERROR STATISTICS)

This program computes the linear regression coefficients A and B, the root mean square error (ERMS), the error in each coefficient, s_A, s_B, and the "t" statistic for each coefficient t_A, t_B. Given a set of data points X_i, Y_i, i = 1, n, the following formulas are employed:

$$A = \frac{\begin{vmatrix} \Sigma Y & \Sigma X \\ \Sigma XY & \Sigma X^2 \end{vmatrix}}{\begin{vmatrix} n & \Sigma X \\ \Sigma X & \Sigma X^2 \end{vmatrix}}$$

$$B = \frac{\Sigma Y - nA}{\Sigma X}$$

$$\Sigma (\text{Residuals})^2 = \Sigma Y^2 - 2A\Sigma Y + 2AB\Sigma X - 2B\Sigma XY + nA^2 + B^2\Sigma X^2$$

$$\text{ERMS} = \sqrt{\frac{\Sigma(\text{Residuals})^2}{n - 2}}$$

$$s_B = \frac{(\text{ERMS})^2}{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}$$

$$s_A = \Sigma X^2 \cdot \frac{s_B}{n}$$

$$t_A = \frac{A}{s_A}$$

$$t_B = \frac{B}{s_B}$$

This program was written by Mr. Patrick Ward of the U. S. Army Ballistic Research Laboratories, Aberdeen Proving Grounds, Maryland.

Reference: Introduction to the Theory of Statistics, Mood and Graybill, 2nd Edition, McGraw-Hill, page 351.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM 1:

PRESS: CONTINUE

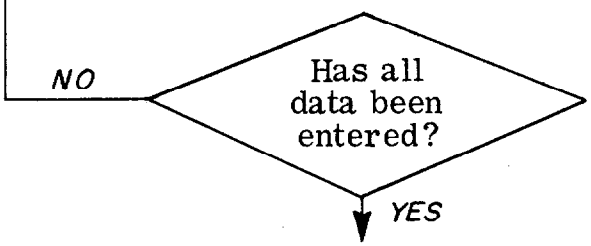
DISPLAY

0	Z
i	Y
0	X

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

1	Z
B	Y
A	X

PRESS: END

ENTER PROGRAM 2: (do not alter the DISPLAY)

PRESS: CONTINUE

DISPLAY

ERMS	Z
s_B	Y
s_A	X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

0	Z
t_B	Y
t_A	X

To run another case: repeat USER INSTRUCTIONS

EXAMPLE

Y_1	=	3
X_1	=	0
Y_2	=	0
X_2	=	1
Y_3	=	-13
X_3	=	2
Y_4	=	-36
X_4	=	3
Y_5	=	-69
X_5	=	4

Results:	B	= -18
	A	= 13
	ERMS	= 10.80123
	s_B	= 3.41565
	s_A	= 8.36660
	t_B	= -5.26986
	t_A	= 1.55380

Program 1

00	CLR	20	
01	XTO	23	
02	d	17	
03	XTO	23	
04	9	11	
05	XTO	23	
06	8	10	
07	XTO	23	
08	c	16	
09	1	01	
0a	XEY	30	
0b	STP	41	ENTRY
0c	IFG	43	
0d	4	04	
10	0	00	
11	PNT	45	
12	PNT	45	
13	AC+	60	
14	UP	27	
15	X	36	
16	XEY	30	
17	YE	24	
18	d	17	
19	+	33	
1a	YEX	24	
1b	d	17	
1c	DN	25	
1d	X	36	
20	XEY	30	
21	YE	24	
22	9	11	
23	+	33	
24	YEX	24	
25	9	11	
26	DN	25	
27	DN	25	
28	X	36	
29	XEY	30	
2a	YEX	24	
2b	8	10	
2c	+	33	
2d	YEX	24	
30	8	10	
31	c	16	
32	UP	27	
33	1	01	
34	+	33	
35	YTO	40	
36	c	16	
37	+	33	
38	0	00	
39	UP	27	
3a	RDN	31	
3b	GTO	44	
3c	0	00	
3d	b	14	
40	1	01	
41	-	34	
42	d	17	
43	X	36	
44	f	15	
45	UP	27	
46	X	36	
47	DN	25	
48	-	34	
49	YTO	40	
4a	a	13	
4b	f	15	
4c	YE	24	
4d	9	11	
50	UP	27	
51	DN	25	
52	YEX	24	
53	9	11	
54	RUP	22	
55	X	36	
56	e	12	
57	UP	27	
58	d	17	
59	X	36	
5a	DN	25	
5b	XEY	30	
5c	-	34	
5d	a	13	
60	DIV	35	
61	YTO	40	
62	a	13	
63	c	16	
64	X	36	
65	e	12	
66	XEY	30	
67	-	34	
68	f	15	
69	DIV	35	
6a	YTO	40	
6b	b	14	
6c	a	13	
6d	UP	27	
70	1	01	
71	RDN	31	
72	PNT	45	
73	PNT	45	
74	END	46	

Program 2

00	UP	27	
01	e	12	
02	X	36	
03	a	13	
04	RUP	22	
05	X	36	
06	f	15	
07	X	36	
08	DN	25	
09	-	34	
0a	b	14	
0b	UP	27	
0c	YE	24	
0d	9	11	
10	X	36	
11	DN	25	
12	+	33	
13	2	02	
14	CHS	32	
15	X	36	
16	a	13	
17	UP	27	
18	X	36	
19	c	16	
1a	X	36	
1b	DN	25	
1c	+	33	
1d	b	14	
20	UP	27	
21	X	36	
22	d	17	
23	X	36	
24	XEY	30	
25	YE	24	
26	8	10	
27	+	33	
28	DN	25	
29	+	33	
2a	c	16	
2b	UP	27	
2c	2	02	
2d	-	34	

	ΣY^2
	ΣXY
	a
	b
	n
	ΣX^2
	ΣY
	ΣX

30	DN	25	
31	DIV	35	
32	YTO	40	
33	e	12	
34	c	16	
35	UP	27	
36	d	17	
37	X	36	
38	f	15	
39	UP	27	
3a	X	36	
3b	DN	25	
3c	-	34	
3d	d	17	
40	UP	27	
41	e	12	
42	X	36	
43	DN	25	
44	XEY	30	
45	DIV	35	
46	UP	27	
47	c	16	
48	DIV	35	
49	e	12	
4a	XEY	30	
4b	DIV	35	
4c	e	12	
4d	√	76	
50	RDN	31	
51	√	76	
52	XEY	30	
53	√	76	
54	PNT	45	S
55	PNT	45	
56	UP	27	
57	a	13	
58	XEY	30	
59	DIV	35	
5a	b	14	
5b	RUP	22	
5c	DIV	35	
5d	0	00	
60	RDN	31	
61	XEY	30	
62	PNT	45	
63	PNT	45	
64	END	46	

MULTIPLE LINEAR REGRESSION (3 VARIABLE) STATISTICS FAMILY

This program consists of 3 parts computing statistics associated with the regression model:

$$Z = a_0 + a_1X + a_2Y \qquad (X_1 = a_0 + a_1X_2 + a_2X_3)$$

PART a —— determines the correlation coefficients between X, Y, and Z using the equations:

$$r_{XY} = r_{23} = \frac{\Sigma XY - \frac{\Sigma X \Sigma Y}{n}}{nS_X S_Y}$$

$$r_{XZ} = r_{12} = \frac{\Sigma XZ - \frac{\Sigma X \Sigma Z}{n}}{n\, S_X S_Z}$$

$$r_{YZ} = r_{13} = \frac{\Sigma YZ - \frac{\Sigma Y \Sigma Z}{n}}{n\, S_Y S_Z}$$

$$S_X^2 = \frac{1}{n}\left\{\Sigma X^2 - \frac{(\Sigma X)^2}{n}\right\}$$

$$S_Y^2 = \frac{1}{n}\left\{\Sigma Y^2 - \frac{(\Sigma Y)^2}{n}\right\}$$

$$S_Z^2 = \frac{1}{n}\left\{\Sigma Z^2 - \frac{(\Sigma Z)^2}{n}\right\}$$

This program must be run first in the sequence.

PART b —— computes the coefficients of linear multiple correlation given by:

$$R_{Z,XY} = R_{1.23} = \left[\frac{r_{13}^2 + r_{12}^2 - 2r_{13}r_{12}r_{23}}{1 - r_{23}^2}\right]^{\frac{1}{2}}$$

$$R_{X,YZ} = R_{2.13} = \left[\frac{r_{12}^2 + r_{23}^2 - 2r_{13}r_{12}r_{23}}{1 - r_{13}^2}\right]^{\frac{1}{2}}$$

$$R_{Y,XZ} = R_{3.12} = \left[\frac{r_{13}^2 + r_{23}^2 - 2r_{13}r_{12}r_{23}}{1 - r_{12}^2}\right]^{\frac{1}{2}}$$

A coefficient of linear multiple correlation will have a value between 0 and 1. Zero indicates no linear relationship between the variables whereas 1 indicates a perfect dependence.

PART c —— determines the constants a_0, a_1, and a_2 from the equations defined by program STAT-PAC IV-3. PART c can be run immediately following PART a if the coefficients of linear multiple correlation are not desired.

Reference: Theory and Problems of Statistics, M.R. Spiegel, McGraw-Hill, 1961, page 271.

USER INSTRUCTIONS

PART a

DEPRESS: [X] [Y] [Z] on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

i	Z
0	Y
0	X

ENTER DATA:

$Z_i \longrightarrow Z$
$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE

NO ← Has all data been entered? → YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

S_Z	Z
S_Y	Y
S_X	X

PRESS: CONTINUE

DISPLAY

r_{YZ}	Z
r_{XZ}	Y
r_{XY}	X

USER INSTRUCTIONS (Con't)

To rerun PART a

PRESS: END

EXAMPLE

i	X	Y	Z
1	1	0	3
2	0	1	4
3	1	1	6
4	3	4	19
5	2	2	11

results:

$S_Z = 5.8856$
$S_Y = 1.3565$
$S_X = 1.0198$

$r_{YZ} = .9820$
$r_{XZ} = .9263$
$r_{XY} = .8386$

USER INSTRUCTIONS

PART b

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM:

PRESS: CONTINUE

DISPLAY

$R_{X,YZ}$ —— Z
$R_{Y,XZ}$ —— Y
$R_{Z,XY}$ —— X

EXAMPLE (Con't)

$R_{X,YZ} = 1.0000$
$R_{Y,XZ} = 1.0000$
$R_{Z,XY} = 1.0000$

USER INSTRUCTIONS

PART c

DEPRESS: [X] [Y] [Z] on the 9120A

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY



EXAMPLE (Con't)

a_2 = 3.0000

a_1 = 2.0000

a_0 = 1.0000

00	CLR	20	Plus Page	40	UP	27		80	CNT	47	
01	XTO	23		41	X	36		81	CNT	47	
02	a	13		42	XEY	30		82	CNT	47	
03	XTO	23		43	YE	24		83	CNT	47	
04	b	14		44	8	10		84	CNT	47	
05	XTO	23		45	+	33		85	CNT	47	
06	c	16		46	YE	24		86	CNT	47	
07	XTO	23		47	8	10		87	CNT	47	
08	9	11		48	XFR	67		88	CNT	47	
09	XTO	23		49	-	34		89	CNT	47	
0a	-	34		4a	d	17		8a	CNT	47	
0b	f	15		4b	X	36		8b	CNT	47	
0c	XTO	23		4c	UP	27		8c	CNT	47	
0d	-	34		4d	X	36		8d	CNT	47	
10	e	12		50	RUP	22		90	CNT	47	
11	XTO	23		51	YE	24		91	CNT	47	
12	8	10		52	9	11		92	CNT	47	
13	1	01		53	+	33		93	CNT	47	
14	XTO	23		54	YE	24		94	CNT	47	
15	d	17		55	9	11		95	CNT	47	
16	RDN	31		56	b	14		96	CNT	47	
17	STP	41	ENTRY	57	+	33		97	CNT	47	
18	IFG	43		58	YTO	40		98	CNT	47	
19	7	07		59	b	14		99	CNT	47	
1a	1	01		5a	RUP	22		9a	CNT	47	
1b	PNT	45		5b	YE	24		9b	CNT	47	
1c	PNT	45		5c	-	34		9c	CNT	47	
1d	YTO	40		5d	f	15		9d	CNT	47	
20	-	34		60	+	33		a0	CNT	47	
21	d	17		61	YTO	40		a1	CNT	47	
22	X	36		62	-	34		a2	CNT	47	
23	XEY	30		63	f	15		a3	CNT	47	
24	YE	24		64	1	01		a4	CNT	47	
25	a	13		65	UP	27		a5	CNT	47	
26	+	33		66	d	17		a6	CNT	47	
27	YE	24		67	+	33		a7	CNT	47	
28	a	13		68	YTO	40		a8	CNT	47	
29	RDN	31		69	d	17		a9	CNT	47	
2a	AC+	60		6a	CLX	37		aa	CNT	47	
2b	UP	27		6b	UP	27		ab	CNT	47	
2c	X	36		6c	GTO	44		ac	CNT	47	
2d	XEY	30		6d	1	01		ad	CNT	47	
30	YE	24		70	7	07					
31	-	34		71	d	17					
32	e	12		72	UP	27					
33	+	33		73	1	01					
34	YE	24		74	-	34					
35	-	34		75	YTO	40					
36	e	12		76	d	17					
37	DN	25		77	XFR	67					
38	X	36		78	-	34					
39	c	16		79	e	12					
3a	+	33		7a	GTO	44					
3b	YTO	40		7b	-	34					
3c	c	16		7c	0	00					
3d	RUP	22		7d	0	00					

E	ΣZ^2
D	ΣYZ
C	ΣXY
B	ΣY
A	ΣXZ
9	n
8	ΣZ
7	ΣX

```
b0  CNT  47      Plus
b1  CNT  47      Page
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

Y

ΣX^2

ΣY^2

			Minus Page
00	UP	27	
01	f	15	
02	GTO	44	
03	SUB	77	
04	8	10	
05	b	14	
06	YTO	40	
07	-	34	
08	d	17	
09	XFR	67	
0a	8	10	
0b	UP	27	
0c	e	12	
0d	GTO	44	
10	SUB	77	
11	8	10	
12	b	14	
13	YTO	40	
14	-	34	
15	c	16	
16	XFR	67	
17	-	34	
18	f	15	
19	UP	27	
1a	b	14	
1b	GTO	44	
1c	SUB	77	
1d	8	10	
20	b	14	
21	YTO	40	
22	-	34	
23	b	14	
24	XFR	67	
25	-	34	
26	c	16	
27	XEY	30	
28	UP	27	
29	XFR	67	
2a	-	34	
2b	d	17	
2c	PNT	45	
2d	PNT	45	
30	a	13	
31	UP	27	
32	f	15	
33	UP	27	
34	b	14	
35	X	36	
36	d	17	
37	DIV	35	
38	DN	25	
39	-	34	
3a	d	17	
3b	DIV	35	
3c	XFR	67	
3d	-	34	
40	d	17	
41	DIV	35	
42	XFR	67	
43	-	34	
44	b	14	
45	DIV	35	
46	YTO	40	
47	-	34	
48	a	13	
49	c	16	
4a	UP	27	
4b	f	15	
4c	UP	27	
4d	e	12	
50	X	36	
51	d	17	
52	DIV	35	
53	DN	25	
54	-	34	
55	d	17	
56	DIV	35	
57	XFR	67	
58	-	34	
59	d	17	
5a	DIV	35	
5b	XFR	67	
5c	-	34	
5d	c	16	
60	DIV	35	
61	YTO	40	
62	-	34	
63	d	17	
64	XFR	67	
65	9	11	
66	UP	27	
67	e	12	
68	UP	27	
69	b	14	
6a	X	36	
6b	d	17	
6c	DIV	35	
6d	DN	25	
70	-	34	
71	d	17	
72	DIV	35	
73	XFR	67	
74	-	34	
75	c	16	
76	DIV	35	
77	XFR	67	
78	-	34	
79	b	14	
7a	DIV	35	
7b	XFR	67	
7c	-	34	
7d	d	17	
80	YTO	40	
81	-	34	
82	c	16	
83	UP	27	
84	XFR	67	
85	-	34	
86	a	13	
87	PNT	45	
88	PNT	45	
89	STP	41	
8a	STP	41	
8b	UP	27	
8c	X	36	
8d	d	17	
90	DIV	35	
91	DN	25	
92	-	34	
93	d	17	
94	UP	27	
95	0	00	
96	-	34	
97	DN	25	
98	DIV	35	
99	DN	25	
9a	√	76	
9b	UP	27	
9c	RTN	77	
9d	END	46	

00	GTO	44	Minus Page	40	YE	24
01	SUB	77		41	-	34
02	2	02		42	a	13
03	c	16		43	YTO	40
04	GTO	44		44	d	17
05	SUB	77		45	RTN	77
06	4	04		46	f	15
07	6	06		47	UP	27
08	YTO	40		48	X	36
09	-	34		49	e	12
0a	9	11		4a	UP	27
0b	GTO	44		4b	X	36
0c	SUB	77		4c	DN	25
0d	4	04		4d	+	33
10	6	06		50	e	12
11	YTO	40		51	UP	27
12	-	34		52	f	15
13	8	10		53	X	36
14	GTO	44		54	d	17
15	SUB	77		55	X	36
16	4	04		56	2	02
17	6	06		57	X	36
18	XFR	67		58	DN	25
19	-	34		59	-	34
1a	8	10		5a	d	17
1b	UP	27		5b	UP	27
1c	XFR	67		5c	X	36
1d	-	34		5d	1	01
20	9	11		60	XEY	30
21	PNT	45	S	61	-	34
22	PNT	45		62	DN	25
23	GTO	44		63	DIV	35
24	SUB	77		64	DN	25
25	2	02		65	√	76
26	c	16		66	UP	27
27	0	00		67	f	15
28	UP	27		68	UP	27
29	UP	27		69	YE	24
2a	STP	41		6a	d	17
2b	STP	41		6b	YE	24
2c	f	15		6c	e	12
2d	UP	27		6d	YTO	40
30	YE	24		70	f	15
31	-	34		71	DN	25
32	d	17		72	RTN	77
33	YTO	40		73	END	46
34	f	15				
35	e	12				
36	UP	27				
37	YE	24				
38	-	34				
39	c	16				
3a	YTO	40				
3b	e	12				
3c	d	17				
3d	UP	27				

Step	Key	Code
00	YE	24
01	-	34
02	f	15
03	UP	27
04	DN	25
05	f	15
06	X	36
07	CHS	32
08	X	36
09	RUP	22
0a	YE	24
0b	-	34
0c	e	12
0d	YTO	40
10	-	34
11	e	12
12	X	36
13	d	17
14	X	36
15	RDN	31
16	+	33
17	a	13
18	UP	27
19	X	36
1a	d	17
1b	X	36
1c	DN	25
1d	-	34
20	UP	27
21	YE	24
22	-	34
23	e	12
24	YTO	40
25	-	34
26	e	12
27	b	14
28	X	36
29	X	36
2a	DN	25
2b	-	34
2c	b	14
2d	UP	27
30	f	15
31	X	36
32	a	13
33	X	36
34	2	02
35	X	36
36	DN	25
37	+	33
38	YTO	40
39	-	34
3a	f	15
3b	YE	24
3c	9	11
3d	UP	27

Minus Page

Step	Key	Code
40	DN	25
41	f	15
42	X	36
43	CHS	32
44	X	36
45	RUP	22
46	XEY	30
47	d	17
48	X	36
49	XFR	67
4a	-	34
4b	e	12
4c	X	36
4d	RDN	31
50	+	33
51	RUP	22
52	XEY	30
53	b	14
54	X	36
55	e	12
56	X	36
57	RDN	31
58	-	34
59	f	15
5a	RUP	22
5b	X	36
5c	a	13
5d	X	36
60	RDN	31
61	+	33
62	d	17
63	RUP	22
64	X	36
65	c	16
66	X	36
67	RDN	31
68	-	34
69	f	15
6a	RUP	22
6b	X	36
6c	b	14
6d	X	36
70	DN	25
71	+	33
72	XFR	67
73	-	34
74	f	15
75	DIV	35
76	YTO	40
77	9	11
78	UP	27
79	DN	25
7a	b	14
7b	X	36
7c	e	12
7d	XEY	30

Step	Key	Code
80	-	34
81	a	13
82	YTO	40
83	a	13
84	RUP	22
85	X	36
86	c	16
87	XEY	30
88	-	34
89	YTO	40
8a	c	16
8b	f	15
8c	UP	27
8d	d	17
90	YTO	40
91	d	17
92	YE	24
93	-	34
94	e	12
95	YTO	40
96	-	34
97	f	15
98	XTO	23
99	b	14
9a	CLR	20
9b	YE	24
9c	-	34
9d	f	15
a0	c	16
a1	UP	27
a2	d	17
a3	DIV	35
a4	RUP	22
a5	XEY	30
a6	DIV	35
a7	DN	25
a8	IFG	43
a9	b	14
aa	8	10
ab	AC+	60
ac	d	17
ad	YE	24

			Minus Page
b0	b	14	
b1	UP	27	
b2	a	13	
b3	RUP	22	
b4	SFL	54	
b5	GTO	44	
b6	a	13	
b7	3	03	
b8	AC-	63	
b9	XEY	30	
ba	YE	24	
bb	e	12	
bc	f	15	
bd	DIV	35	
c0	e	12	
c1	XEY	30	
c2	X	36	
c3	RDN	31	
c4	-	34	
c5	RUP	22	
c6	YE	24	
c7	9	11	
c8	RUP	22	
c9	PNT	45	
ca	PNT	45	
cb	END	46	

GOMPERTZ CURVE FIT

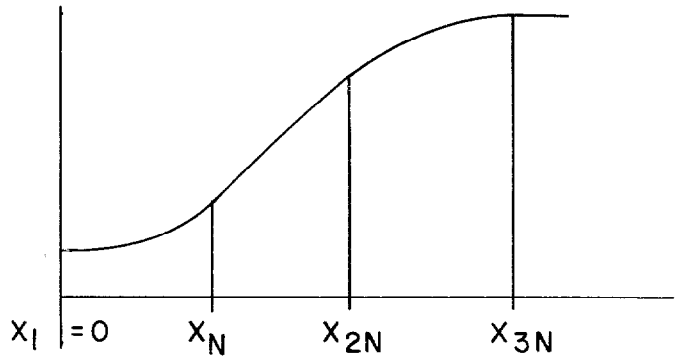
This program fits a GOMPERTZ curve to observed data. The GOMPERTZ curve is given by the equation:

$$Y_c = Ka^{(b^X)}$$

or

$$\log Y_c = \log K + b^X \log a \quad .$$

The usual geometric form of the curve is given below:



3N Observations:

Notice that the curve is divided into 3 sections, each having N observations.

The b, log K, and log a are determined by computing the quantities:

$$\Sigma_1 = \sum_{i=1}^{N} \log Y_i$$

$$\Sigma_2 = \sum_{i=N+1}^{2N} \log Y_i$$

$$\Sigma_3 = \sum_{i=2N+1}^{3N} \log Y_i$$

$$b = \left(\frac{\Sigma_3 - \Sigma_2}{\Sigma_2 - \Sigma_1}\right)^{\frac{1}{N}}$$

$$\log K = \frac{1}{N} \left[\frac{\Sigma_1 \Sigma_3 - (\Sigma_2)^2}{\Sigma_1 + \Sigma_3 - 2\Sigma_2} \right]$$

$$\log a = (\Sigma_2 - \Sigma_1) \frac{(b - 1)}{(b^N - 1)^2}$$

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM :

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA: N ⟶ X

PRESS: CONTINUE

→ DISPLAY

– ——— Z
– ——— Y
i ——— X

ENTER DATA:

Group 1i —— Z
Group 2i —— Y
Group 3i —— X

PRESS: CONTINUE

NO ← Does i = N? → YES

DISPLAY

log a ——— Z
b ——— Y
log K ——— X

→ PRESS: CONTINUE

DISPLAY

Y_c ——— Z
X ——— Y
N ——— X

(repeat for X = 1, 2, . . .)

USER INSTRUCTIONS (Con't)

To run another case:

PRESS: END

EXAMPLE

The following table shows the kilowatt requirements of a group of atomic power plants.

Year	X	July Kilowatts	
1954	0	226	
1955	1	244	
1956	2	265	Group 1
1957	3	287	
1958	4	290	
1959	5	317	
1960	6	316	
1961	7	362	Group 2
1962	8	378	
1963	9	417	
1964	10	442	
1965	11	461	
1966	12	526	Group 3
1967	13	566	
1968	14	649	

N = 5

	Group 11	226
	Group 21	317
	Group 31	442
	Group 12	244
	Group 22	316
	Group 32	461
	⋮	265
		362
		526
		287
		378
		566
		290
		417
		649

Input data

EXAMPLE (Con't)

Results:

log a	=	.49712
b	=	1.04514
log K	=	1.87293
Y_c	=	234.44858
X	=	0.00000
N	=	5.00000
	=	246.88231
	=	1.00000
	=	5.00000
	=	260.58266
	=	2.00000
	=	5.00000
	=	275.71470
	=	3.00000
	=	5.00000
	=	292.46981
	=	4.00000
	=	5.00000
	=	311.07049
	=	5.00000
	=	5.00000
	=	331.77638
	=	6.00000
	=	5.00000
	=	354.89147
	=	7.00000
	=	5.00000

.
.
.

00	CLR	20	
01	XTO	23	
02	d	17	
03	STP	41	ENTRY
04	XTO	23	
05	a	13	
06	1	01	
07	XTO	23	
08	c	16	
09	STP	41	ENTRY
0a	PNT	45	
0b	PNT	45	
0c	LN	65	
0d	XEY	30	
10	LN	65	
11	AC+	60	
12	d	17	
13	RUP	22	
14	LN	65	
15	+	33	
16	YTO	40	
17	d	17	
18	c	16	
19	UP	27	
1a	a	13	
1b	X=Y	50	
1c	2	02	
1d	6	06	
20	1	01	
21	+	33	
22	DN	25	
23	GTO	44	
24	0	00	
25	7	07	
26	d	17	
27	UP	27	
28	e	12	
29	X	36	
2a	f	15	
2b	UP	27	
2c	X	36	
2d	DN	25	
30	-	34	
31	a	13	
32	DIV	35	
33	f	15	
34	UP	27	
35	+	33	
36	e	12	
37	XEY	30	
38	-	34	
39	d	17	
3a	+	33	
3b	DN	25	
3c	DIV	35	
3d	YTO	40	

40	b	14	
41	e	12	
42	UP	27	
43	f	15	
44	-	34	
45	UP	27	
46	d	17	
47	-	34	
48	DN	25	
49	DIV	35	
4a	RDN	31	
4b	LN	65	
4c	XEY	30	
4d	a	13	
50	DIV	35	
51	DN	25	
52	EXP	74	
53	XTO	23	
54	c	16	
55	LN	65	
56	UP	27	
57	a	13	
58	X	36	
59	DN	25	
5a	EXP	74	
5b	UP	27	
5c	1	01	
5d	-	34	
60	DN	25	
61	UP	27	
62	X	36	
63	DN	25	
64	DIV	35	
65	c	16	
66	UP	27	
67	1	01	
68	-	34	
69	DN	25	
6a	X	36	
6b	YTO	40	
6c	d	17	
6d	DN	25	
70	EXP	74	
71	LOG	75	
72	UP	27	
73	CLX	37	
74	XTO	23	
75	e	12	
76	c	16	
77	UP	27	
78	b	14	
79	EXP	74	
7a	LOG	75	
7b	PNT	45	S
7c	PNT	45	
7d	DN	25	

80	LN	65	
81	UP	27	
82	e	12	
83	X	36	
84	DN	25	
85	EXP	74	
86	UP	27	
87	d	17	
88	X	36	
89	b	14	
8a	+	33	
8b	DN	25	
8c	EXP	74	
8d	UP	27	
90	e	12	
91	UP	27	
92	a	13	
93	PNT	45	S
94	PNT	45	
95	1	01	
96	XEY	30	
97	AC+	60	
98	c	16	
99	GTO	44	
9a	8	10	
9b	0	00	
9c	END	46	

N

ln K

i

ΣGrp 1

ΣGrp 3

ΣGrp 2

PROGRAM FOR SIMULTANEOUS SOLUTION OF FOUR EQUATIONS IN FOUR UNKNOWNS WITH PRINTER

This program in its original form was written by Dr. Stefan J. Medwadowski, a Consulting Structural Engineer in San Francisco. One of his hobbies is programming the -hp- 9100A.

4x4 SYSTEM OF LINEAR ALGEBRAIC EQUATIONS

Given a system of linear algebraic equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 &= p_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 &= p_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 &= p_3 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 &= p_4 \end{aligned}$$

or a matrix notation:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

i.e., $[\, a_{ij} \,] \{ x_i \} = \{ p_i \}$ with i, j = 1, 2, 3, 4

Such systems occur frequently in the solution of boundary value problems of structural mechanics, such as those which arise in the theory of thin elastic shells or plates.

It is assumed that the solution of the system exists; i.e., that the determinant of the a_{ij} coefficient matrix does not vanish. The coefficients a_{ij} are assumed real.

NOTE: None of the determinates of the leading submatrices may be zero, or

$$\begin{vmatrix} a_{11} \end{vmatrix} \neq 0$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

Should one or more of these conditions exist (and should therefore the illegal operation light come on), it may be removed by re-arranging the sequence of the equations within the system. It is always possible to do this as a consequence of the postulated existence of a unique solution.

Method of Solution: Cholewski's Method

Reference: Salvadori and Baron, Numerical Method in Engineering, Prentice-Hall, 1952.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

ENTER DATA: a_{ij} (or P_i) ⟶ X

NOTE: Data is entered row by row in the following order with a CONTINUE following each entry:

$a_{11}, a_{12}, a_{13}, a_{14}$, P_1

$a_{21}, a_{22}, a_{23}, a_{24}$, P_2

$a_{31}, a_{32}, a_{33}, a_{34}$, P_3

$a_{41}, a_{42}, a_{43}, a_{44}$, P_4

DISPLAY

*	Z
*	Y
x_1	X

PRESS: CONTINUE

DISPLAY

*	Z
*	Y
x_2	X

PRESS: CONTINUE

DISPLAY

*	Z
*	Y
x_3	X

PRESS: CONTINUE

DISPLAY

*	Z
*	Y
x_4	X

To rerun program:

Repeat USER INSTRUCTIONS

*Denotes Insignificant Display

EXAMPLE

$$2x_1 + x_2 + 3x_3 + x_4 = 7$$
$$x_1 + 4x_2 + x_3 = 10$$
$$3x_1 - 5x_3 - 2x_4 = 10$$
$$8x_1 - x_2 + 4x_3 + 2x_4 = 22$$

$x_1 = 3$ $\quad$ $x_3 = -1$

$x_2 = 2$ $\quad$ $x_4 = 2$

00	CLR	20	Plus Page	40	YE	24		80	X	36	
01	STP	41	ENTRY	41	d	17		81	b	14	
02	PNT	45		42	b	14		82	RUP	22	
03	RDN	31		43	RUP	22		83	X	36	
04	STP	41	ENTRY	44	X	36		84	RDN	31	
05	PNT	45		45	RDN	31		85	+	33	
06	RUP	22		46	-	34		86	STP	41	ENTRY
07	DIV	35		47	c	16		87	PNT	45	
08	YTO	40		48	DIV	35		88	XEY	30	
09	f	15		49	YE	24		89	-	34	
0a	RDN	31		4a	a	13		8a	a	13	
0b	STP	41	ENTRY	4b	YTO	40		8b	DIV	35	
0c	PNT	45		4c	-	34		8c	YE	24	
0d	RUP	22		4d	f	15		8d	-	34	
10	DIV	35		50	DN	25		90	e	12	
11	YTO	40		51	X	36		91	YTO	40	
12	e	12		52	STP	41	ENTRY	92	+	33	
13	RDN	31		53	PNT	45		93	1	01	
14	STP	41	ENTRY	54	XEY	30		94	c	16	
15	PNT	45		55	-	34		95	X	36	
16	RUP	22		56	c	16		96	RUP	22	
17	DIV	35		57	DIV	35		97	XEY	30	
18	YTO	40		58	STP	41	ENTRY	98	CNT	47	
19	b	14		59	PNT	45		99	CNT	47	
1a	RDN	31		5a	RUP	22		9a	GTO	44	
1b	STP	41	ENTRY	5b	f	15		9b	-	34	
1c	PNT	45		5c	XEY	30		9c	0	00	
1d	RUP	22		5d	X	36		9d	0	00	
20	DIV	35		60	RDN	31		a0	CNT	47	
21	YTO	40		61	YTO	40		a1	CNT	47	
22	a	13		62	-	34		a2	CNT	47	
23	STP	41	ENTRY	63	e	12		a3	CNT	47	
24	PNT	45		64	XEY	30		a4	CNT	47	
25	UP	27		65	STP	41	ENTRY	a5	CNT	47	
26	UP	27		66	PNT	45		a6	CNT	47	
27	f	15		67	XEY	30		a7	CNT	47	
28	X	36		68	-	34		a8	CNT	47	
29	STP	41	ENTRY	69	YTO	40		a9	CNT	47	
2a	PNT	45		6a	c	16		aa	CNT	47	
2b	XEY	30		6b	d	17		ab	CNT	47	
2c	-	34		6c	X	36		ac	CNT	47	
2d	YTO	40		6d	e	12		ad	CNT	47	
30	c	16		70	RUP	22					
31	DN	25		71	X	36					
32	e	12		72	RDN	31					
33	X	36		73	+	33					
34	STP	41	ENTRY	74	STP	41	ENTRY				
35	PNT	45		75	PNT	45					
36	XEY	30		76	XEY	30					
37	-	34		77	-	34					
38	STP	41	ENTRY	78	YE	24					
39	PNT	45		79	a	13					
3a	XTO	23		7a	YTO	40					
3b	d	17		7b	-	34					
3c	c	16		7c	d	17					
3d	DIV	35		7d	c	16					

```
 b0  CNT   47        Plus
 b1  CNT   47        Page
 b2  CNT   47
 b3  CNT   47
 b4  CNT   47
 b5  CNT   47
 b6  CNT   47
 b7  CNT   47
 b8  CNT   47
 b9  CNT   47
 ba  CNT   47
 bb  CNT   47
 bc  CNT   47
 bd  CNT   47

 c0  CNT   47
 c1  CNT   47
 c2  CNT   47
 c3  CNT   47
 c4  CNT   47
 c5  CNT   47
 c6  CNT   47
 c7  CNT   47
 c8  CNT   47
 c9  CNT   47
 ca  CNT   47
 cb  CNT   47
 cc  CNT   47
 cd  CNT   47

 d0  CNT   47
 d1  CNT   47
 d2  CNT   47
 d3  CNT   47
 d4  CNT   47
 d5  CNT   47
 d6  CNT   47
 d7  CNT   47
 d8  CNT   47
 d9  CNT   47
 da  CNT   47
 db  CNT   47
 dc  CNT   47
 dd  CNT   47
```

00	XFR	67	Minus Page	40	+	33		80	-	34				
01	-	34		41	b	14		81	RUP	22				
02	f	15		42	RUP	22		82	YE	24				
03	XEY	30		43	X	36		83	-	34				
04	X	36		44	RDN	31		84	d	17				
05	RDN	31		45	+	33		85	c	16				
06	+	33		46	STP	41	ENTRY	86	X	36				
07	STP	41	ENTRY	47	PNT	45		87	RDN	31				
08	PNT	45		48	XEY	30		88	-	34				
09	XEY	30		49	-	34		89	YTO	40				
0a	-	34		4a	YE	24		8a	d	17				
0b	a	13		4b	-	34		8b	f	15				
0c	DIV	35		4c	f	15		8c	X	36				
0d	YTO	40		4d	STP	41	ENTRY	8d	a	13				
10	0	00		50	PNT	45		90	UP	27				
11	STP	41	ENTRY	51	RDN	31		91	e	12				
12	PNT	45		52	X	36		92	X	36				
13	UP	27		53	RDN	31		93	DN	25				
14	UP	27		54	-	34		94	+	33				
15	f	15		55	XEY	30		95	c	16				
16	X	36		56	YE	24		96	UP	27				
17	STP	41	ENTRY	57	0	00		97	b	14				
18	PNT	45		58	RDN	31		98	X	36				
19	XEY	30		59	YE	24		99	DN	25				
1a	-	34		5a	-	34		9a	+	33				
1b	YTO	40		5b	e	12		9b	DN	25				
1c	c	16		5c	X	36		9c	YE	24				
1d	d	17		5d	RDN	31		9d	-	34				
20	X	36		60	-	34		a0	e	12				
21	e	12		61	RDN	31		a1	-	34				
22	RUP	22		62	YE	24		a2	d	17				
23	X	36		63	+	33		a3	UP	27				
24	RDN	31		64	1	01		a4	XFR	67				
25	+	33		65	RUP	22		a5	-	34				
26	STP	41	ENTRY	66	c	16		a6	c	16				
27	PNT	45		67	RUP	22		a7	PNT	45				
28	XEY	30		68	X	36		a8	PNT	45				
29	-	34		69	RDN	31		a9	DN	25				
2a	DN	25		6a	-	34		aa	XEY	30				
2b	UP	27		6b	XFR	67		ab	PNT	45	S			
2c	YE	24		6c	-	34		ac	DN	25				
2d	-	34		6d	f	15		ad	PNT	45	S			
30	e	12		70	DIV	35								
31	YTO	40		71	a	13								
32	a	13		72	YTO	40								
33	X	36		73	c	16								
34	YE	24		74	X	36								
35	-	34		75	XEY	30								
36	d	17		76	YE	24								
37	c	16		77	1	01								
38	XEY	30		78	-	34								
39	X	36		79	YTO	40								
3a	XEY	30		7a	a	13								
3b	YE	24		7b	d	17								
3c	-	34		7c	X	36								
3d	d	17		7d	RDN	31								

```
b0    a     13
b1   PNT    45   S   Minus
b2    c     16       Page
b3   PNT    45   S
b4   PNT    45
b5   XFR    67
b6    -     34
b7    c     16
b8   PNT    45
b9   PNT    45
ba   PNT    45
bb   CLR    20
bc   STP    41
bd   STP    41

c0   CHS    32
c1   CHS    32
c2   CHS    32
c3   CHS    32
c4   CHS    32
c5   CHS    32
c6   CHS    32
c7   CHS    32
c8   CHS    32
c9   CHS    32
ca   YTO    40
cb   YTO    40
cc   YTO    40
cd   YTO    40

d0   END    46
```

LEAST SQUARES REGRESSION
$Y = c_0 X^a + c_1 X^b$

This program calculates the coefficients of the equation $Y = c_0 X^a + c_1 X^b$ of least squares fit of a set of data points. The exponents a and b are any real numbers specified by the user.

Defining Equations are:

$$c_1 = \frac{\Sigma X^{2a} \Sigma X^b Y - \Sigma X^a Y \Sigma X^{a+b}}{\Sigma X^{2b} \Sigma X^{2a} - [\Sigma X^{a+b}]^2}$$

$$c_0 = \frac{[\Sigma X^a Y - b_0 \Sigma X^{a+b}]}{\Sigma X^{2a}}$$

where all $X_i > 0$.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER EXPONENTS:

a ⟶ Y
b ⟶ X

PRESS: CONTINUE

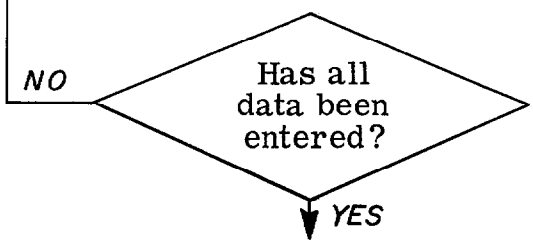
DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

Y_i ⟶ Y
X_i ⟶ X

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

▲ ——— Z
c_1 ——— Y
c_0 ——— X

▲Denotes Insignificant Display

USER INSTRUCTIONS (Con't)

To run another case, Repeat USER INSTRUCTIONS.

EXAMPLE

X	Y
1	9
4	-44
9	-699
16	-4056

$a = \frac{1}{2}$

$b = 3$

$Y = c_0 X^{\frac{1}{2}} + c_1 X^3$

Results:

$c_1 = -1$

$c_0 = 10$

00	CLR	20		40	8	10		80	0	00	
01	STP	41	ENTRY	41	X	36		81	0	00	
02	PNT	45		42	DN	25		82	0	00	
03	PNT	45		43	√	76		83	0	00	
04	XTO	23		44	YEX	24		84	0	00	
05	b	14		45	9	11		85	0	00	
06	YTO	40		46	+	33		86	0	00	
07	a	13		47	YEX	24		87	0	00	
08	CLR	20		48	9	11		88	0	00	
09	STP	41	ENTRY	49	GTO	44		89	0	00	
0a	IFG	43		4a	0	00		8a	0	00	
0b	4	04		4b	9	11		8b	0	00	
0c	c	16		4c	YEX	24		8c	0	00	
0d	PNT	45		4d	9	11		8d	0	00	
10	PNT	45		50	YTO	40		90	0	00	
11	LN	65		51	9	11		91	0	00	
12	XTO	23		52	e	12		92	0	00	
13	8	10		53	X	36		93	0	00	
14	UP	27		54	d	17		94	0	00	
15	a	13		55	UP	27		95	0	00	
16	X	36		56	f	15		96	0	00	
17	DN	25		57	X	36		97	0	00	
18	EXP	74		58	DN	25		98	0	00	
19	X	36		59	-	34		99	0	00	
1a	UP	27		5a	UP	27		9a	0	00	
1b	X	36		5b	YEX	24		9b	0	00	
1c	RDN	31		5c	9	11		9c	0	00	
1d	AC+	60		5d	YTO	40		9d	0	00	
20	RDN	31		60	9	11		a0	0	00	
21	XEY	30		61	DN	25		a1	0	00	
22	DIV	35		62	UP	27		a2	0	00	
23	UP	27		63	X	36		a3	0	00	
24	X	36		64	YEX	24		a4	0	00	
25	YEX	24		65	c	16		a5	0	00	
26	8	10		66	f	15		a6	0	00	
27	b	14		67	X	36		a7	0	00	
28	X	36		68	DN	25		a8	0	00	
29	DN	25		69	YEX	24		a9	0	00	
2a	EXP	74		6a	c	16		aa	0	00	
2b	X	36		6b	-	34		ab	0	00	
2c	UP	27		6c	DN	25		ac	0	00	
2d	X	36		6d	DIV	35		ad	0	00	
30	DN	25		70	UP	27					
31	YEX	24		71	DN	25					
32	c	16		72	DN	25					
33	+	33		73	YEX	24					
34	YEX	24		74	9	11					
35	c	16		75	X	36					
36	XEY	30		76	e	12					
37	YEX	24		77	XEY	30					
38	d	17		78	-	34					
39	+	33		79	f	15					
3a	YEX	24		7a	DIV	35					
3b	d	17		7b	RDN	31					
3c	DN	25		7c	PNT	45					
3d	YEX	24		7d	STP	41					

```
b0      0       00
b1      0       00
b2      0       00
b3      0       00
b4      0       00
b5      0       00
b6      0       00
b7      0       00
b8      0       00
b9      0       00
ba      0       00
bb      0       00
bc      0       00
bd      0       00

c0      0       00
c1      0       00
c2      0       00
c3      0       00
c4      0       00
c5      0       00
c6      0       00
c7      0       00
c8      0       00
c9      0       00
ca      0       00
cb      0       00
cc      0       00
cd      0       00

d0      0       00
d1      0       00
d2      0       00
d3      0       00
d4      0       00
d5      0       00
d6      0       00
d7      0       00
d8      0       00
d9      0       00
da      0       00
db      0       00
dc      0       00
dd      END     46
```

SECTION V HARMONIC ANALYSIS

			Page Number
V-1	Sine Wave Curve Fit		1
V-2	Fourier Analysis (12 Ordinate Scheme)	9100B ONLY	5
V-3	Fourier Series [Sampled X(t)]	9100B ONLY	13

This program accepts discrete samples of a curve of the form:

$$Y(t) = c_0 + c_1 \sin [\omega_1 t + \phi]$$

where ω_1 and t are known and determines c_0, c_1, and ϕ. The function Y(t) is obtained for N discrete values of t at Δt increments for one complete cycle of Y(t).

The quantities c_0, c_1, and ϕ are obtained from the relations:

$$c_0 = \frac{1}{N} \Sigma Y_n \quad ,$$

$$A = \frac{2}{N} \sum_{k=1}^{N} Y_k \cos \frac{2\pi k}{N}$$

$$B = \frac{2}{N} \sum_{k=1}^{N} Y_k \sin \frac{2\pi k}{N}$$

$$c_1 = \sqrt{A^2 + B^2}$$

$$\phi = \arctan \frac{A}{B}$$

Restrictions: N should be chosen such that at least 8 samples are obtained for the complete sine wave cycle.

Reference: Measurement and Analysis of Random Data, J.S. Bendat, A.G. Piersol, John Wiley & Sons, 1966, page 287.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

SET: | | RADIANS |

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA: N ⟶ X

PRESS: CONTINUE

DISPLAY

0 ——— Z
n ——— Y
0 ——— X

ENTER DATA: Y_n ⟶ X

After the Nth data value Y_N is entered:

DISPLAY

ϕ ——— Z
c_1 ——— Y
c_0 ——— X

To run another case

EXAMPLE

$y(t) = c_0 + c_1 \sin [\omega_1 t + \phi]$

$\omega_1 = \frac{360^o}{24}$ or $\frac{2\pi}{24}$ radians/hr.

$\omega_1 = .262$ radians/hr.

$\Delta t = 1$ hr.

t(hr.)	y(t)
1	1.953
2	1.998
3	1.977
4	1.888
5	1.740
6	1.540
7	1.303
8	1.046
9	.787
10	.541
11	.327
12	.159
13	.047
14	.001
15	.023
16	.111
17	.260
18	.460
19	.696
20	.953
21	1.213
22	1.459
23	1.673
24	1.841

N = 24 Data Set

$y_1 \longrightarrow y_{24}$

N = 12 Data Set

$y_2, y_4, \cdots y_{24}$

N = 8 Data Set

$y_3, y_6, \cdots y_{24}$

EXAMPLE (Con't)

Results for all these sets

$$\begin{aligned} \phi &= 1 \\ c_1 &= 1 \\ c_0 &= 1 \end{aligned}$$

$y(t) = 1 + 1 \sin [.262t + 1]$

00	CLR	20		40	6	06	
01	XTO	23		41	0	00	
02	a	13		42	UP	27	
03	XTO	23		43	GTO	44	
04	b	14		44	0	00	
05	STP	41	ENTRY	45	8	10	
06	XTO	23		46	f	15	
07	d	17		47	UP	27	
08	1	01		48	d	17	
09	UP	27		49	DIV	35	
0a	e	12		4a	YTO	40	
0b	+	33		4b	9	11	
0c	0	00		4c	b	14	
0d	STP	41	ENTRY	4d	UP	27	
10	PNT	45		50	a	13	
11	PNT	45		51	POL	62	
12	UP	27		52	UP	27	
13	1	01		53	2	02	
14	XEY	30		54	X	36	
15	AC+	60		55	d	17	
16	XEY	30		56	DIV	35	
17	e	12		57	XEY	30	
18	UP	27		58	YEX	24	
19	d	17		59	9	11	
1a	DIV	35		5a	YTO	40	
1b	π	56		5b	9	11	
1c	X	36		5c	XEY	30	
1d	2	02		5d	RUP	22	
20	X	36		60	PNT	45	
21	DN	25		61	RUP	22	
22	XTO	23		62	PNT	45	
23	c	16		63	RUP	22	
24	COS	73		64	PNT	45	S
25	XEY	30		65	PNT	45	
26	X	36		66	PNT	45	
27	RDN	31		67	PNT	45	
28	YEX	24		68	PNT	45	
29	b	14		69	PNT	45	
2a	+	33		6a	GTO	44	
2b	YTO	40		6b	0	00	
2c	b	14		6c	0	00	
2d	DN	25		6d	END	46	
30	c	16					
31	SIN	70					
32	X	36					
33	DN	25					
34	YEX	24					
35	a	13					
36	+	33					
37	YTO	40					
38	a	13					
39	d	17					
3a	UP	27					
3b	e	12					
3c	X=Y	50					
3d	4	04					

C_0

$\Sigma Y_n \sin \pi n/N$

$\Sigma Y_n \cos 2\pi n/N$

$N_{2\pi n/N}$

n

ΣY_n

FOURIER ANALYSIS
(12 ORDINATE SCHEME)

This program computes the first four sine and cosine terms of the Fourier Series obtained from 12 observed ordinates. The 12 ordinates, $\mu_0, \mu_1, \mu_2, \ldots \mu_{11}$ represent the value of some function f(X) evaluated at $X = 0, \frac{\pi}{6}, \frac{2\pi}{6}, \ldots \frac{11\pi}{6}$ respectively.

This program computes a_0, a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4

where

$$f(X) = a_0 + a_1\cos X + a_2\cos 2X + a_3\cos 3X + a_4\cos 4X + \ldots$$
$$+ b_1\sin X + b_2\sin 2X + b_3\sin 3X + b_4\sin 4X + \ldots$$

The following formulas are applied:

$$V_1 = \mu_1 + \mu_{11} \qquad W_1 = \mu_1 - \mu_{11}$$
$$V_2 = \mu_2 + \mu_{10} \qquad W_2 = \mu_2 - \mu_{10}$$
$$V_3 = \mu_3 + \mu_9 \qquad W_3 = \mu_3 - \mu_9$$
$$V_4 = \mu_4 + \mu_8 \qquad W_4 = \mu_4 - \mu_8$$
$$V_5 = \mu_5 + \mu_7 \qquad W_5 = \mu_5 - \mu_7$$

$$a_0 = \frac{1}{12}\left[\mu_0 + V_1 + V_2 + V_3 + V_4 + V_5 + \mu_6\right]$$

$$a_1 = \frac{1}{6}\left[\mu_0 + \frac{\sqrt{3}}{2}V_1 + \frac{1}{2}V_2 - \frac{1}{2}V_4 - \frac{\sqrt{3}}{2}V_5 - \mu_6\right]$$

$$a_2 = \frac{1}{6}\left[\mu_0 + \frac{1}{2}V_1 - \frac{1}{2}V_2 - V_3 - \frac{1}{2}V_4 + \frac{1}{2}V_5 + \mu_6\right]$$

$$a_3 = \frac{1}{6}\left[\mu_0 - V_2 + V_4 - \mu_6\right]$$

$$a_4 = \frac{1}{6}\left[\mu_0 - \frac{1}{2}V_1 - \frac{1}{2}V_2 + V_3 - \frac{1}{2}V_4 - \frac{1}{2}V_5 + \mu_6\right]$$

$$b_1 = \frac{1}{6}\left[\frac{1}{2}W_1 + \frac{\sqrt{3}}{2}W_2 + W_3 + \frac{\sqrt{3}}{2}W_4 + \frac{1}{2}W_5\right]$$

$$b_2 = \frac{1}{6}\left[\frac{\sqrt{3}}{2}W_1 + \frac{\sqrt{3}}{2}W_2 - \frac{\sqrt{3}}{2}W_4 - \frac{\sqrt{3}}{2}W_5\right]$$

$$b_3 = \frac{1}{6}\left[W_1 - W_3 + W_5\right]$$

$$b_4 = \frac{1}{6}\left[\frac{\sqrt{3}}{2}W_1 - \frac{\sqrt{3}}{2}W_2 + \frac{\sqrt{3}}{2}W_4 - \frac{\sqrt{3}}{2}W_5\right]$$

Reference: The Calculus of Observations, Sir Edmund Whittaker and G. Robinson, Dover Publications, 1967, page 267.

USER INSTRUCTIONS

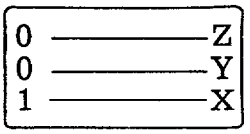
DEPRESS: X Y Z on the 9120A

PRESS: END

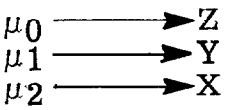
ENTER PROGRAM 1: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

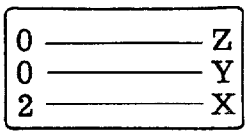


ENTER DATA:

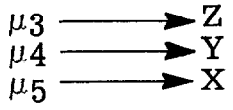


PRESS: CONTINUE

DISPLAY

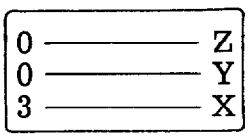


ENTER DATA:

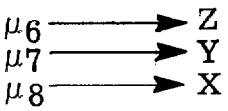


PRESS: CONTINUE

DISPLAY



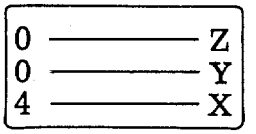
ENTER DATA:



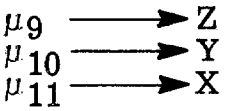
PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

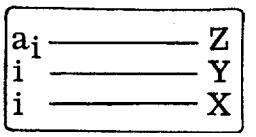


ENTER DATA:



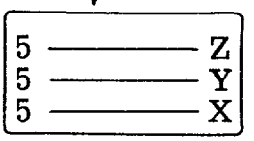
PRESS: CONTINUE

DISPLAY



When the display is:

DISPLAY



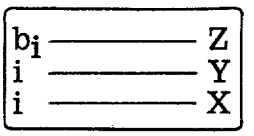
PRESS: GO TO (-)(0)(0)

ENTER PROGRAM 2

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY



To run another case, repeat USER INSTRUCTIONS

EXAMPLE

$\mu_0 = 0$
$\mu_1 = .262$
$\mu_2 = .524$
$\mu_3 = .786$
$\mu_4 = 1.047$
$\mu_5 = 1.309$
$\mu_6 = 0$
$\mu_7 = -1.309$
$\mu_8 = -1.047$
$\mu_9 = -.786$
$\mu_{10} = -.524$
$\mu_{11} = -.262$

Results:

$a_0 = 0$
$a_1 = 0$
$a_2 = 0$
$a_3 = 0$
$a_4 = 0$

$b_1 = .977$
$b_2 = -.453$
$b_3 = .262$
$b_4 = -.151$

Or

$f(X) = .977 \sin X - .453 \sin 2X + .262 \sin 3X - .151 \sin 4X \ldots$

The μ_i and resulting Fourier series are plotted in the example of program STAT-PAC X-18.

00	CLR	20	Plus Page	40	AC+	60		80	CNT	47	
01	1	01		41	YTO	40		81	CNT	47	
02	STP	41	ENTRY	42	5	05		82	CNT	47	
03	PNT	45		43	DN	25		83	CNT	47	
04	AC+	60		44	4	04		84	CNT	47	
05	XTO	23		45	STP	41	ENTRY	85	CNT	47	
06	d	17		46	PNT	45		86	CNT	47	
07	YTO	40		47	PNT	45		87	CNT	47	
08	c	16		48	AC+	60		88	CNT	47	
09	0	00		49	ARC	72		89	CNT	47	
0a	RDN	31		4a	GTO	44		8a	CNT	47	
0b	0	00		4b	-	34		8b	CNT	47	
0c	AC+	60		4c	0	00		8c	CNT	47	
0d	YTO	40		4d	0	00		8d	CNT	47	
10	b	14		50	CNT	47		90	CNT	47	
11	DN	25		51	CNT	47		91	CNT	47	
12	2	02		52	CNT	47		92	CNT	47	
13	STP	41	ENTRY	53	CNT	47		93	CNT	47	
14	AC+	60		54	CNT	47		94	CNT	47	
15	PNT	45		55	CNT	47		95	CNT	47	
16	XTO	23		56	CNT	47		96	CNT	47	
17	a	13		57	CNT	47		97	CNT	47	
18	YTO	40		58	CNT	47		98	CNT	47	
19	9	11		59	CNT	47		99	CNT	47	
1a	0	00		5a	CNT	47		9a	CNT	47	
1b	RDN	31		5b	CNT	47		9b	CNT	47	
1c	0	00		5c	CNT	47		9c	CNT	47	
1d	AC+	60		5d	CNT	47		9d	CNT	47	
20	YTO	40		60	CNT	47		a0	CNT	47	
21	8	10		61	CNT	47		a1	CNT	47	
22	DN	25		62	CNT	47		a2	CNT	47	
23	3	03		63	CNT	47		a3	CNT	47	
24	STP	41	ENTRY	64	CNT	47		a4	CNT	47	
25	PNT	45		65	CNT	47		a5	CNT	47	
26	AC+	60		66	CNT	47		a6	CNT	47	
27	YE	24		67	CNT	47		a7	CNT	47	
28	9	11		68	CNT	47		a8	CNT	47	
29	+	33		69	CNT	47		a9	CNT	47	
2a	YTO	40		6a	CNT	47		aa	CNT	47	
2b	7	07		6b	CNT	47		ab	CNT	47	
2c	-	34		6c	CNT	47		ac	CNT	47	
2d	-	34		6d	CNT	47		ad	CNT	47	
30	YE	24		70	CNT	47					
31	9	11		71	CNT	47					
32	a	13		72	CNT	47					
33	XEY	30		73	CNT	47					
34	+	33		74	CNT	47					
35	YTO	40		75	CNT	47					
36	a	13		76	CNT	47					
37	-	34		77	CNT	47					
38	-	34		78	CNT	47					
39	YTO	40		79	CNT	47					
3a	6	06		7a	CNT	47					
3b	0	00		7b	CNT	47					
3c	RDN	31		7c	CNT	47					
3d	0	00		7d	CNT	47					

```
b0  CNT  47      Plus
b1  CNT  47      Page
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

Step	Key	Code		Step	Key	Code		Step	Key	Code	
00	YE	24	Minus Page	40	X	36		80	7	07	
01	c	16		41	2	02		81	UP	27	
02	+	33		42	DIV	35		82	2	02	
03	YTO	40		43	b	14		83	DIV	35	
04	-	34		44	+	33		84	DN	25	
05	f	15		45	d	17		85	-	34	
06	-	34		46	UP	27		86	a	13	
07	-	34		47	2	02		87	UP	27	
08	YE	24		48	DIV	35		88	2	02	
09	c	16		49	DN	25		89	DIV	35	
0a	d	17		4a	+	33		8a	DN	25	
0b	XEY	30		4b	XFR	67		8b	+	33	
0c	+	33		4c	7	07		8c	6	06	
0d	YTO	40		4d	UP	27		8d	DIV	35	
10	d	17		50	2	02		90	2	02	
11	-	34		51	DIV	35		91	UP	27	
12	-	34		52	DN	25		92	PNT	45	
13	YTO	40		53	-	34		93	PNT	45	S
14	-	34		54	a	13		94	b	14	
15	e	12		55	UP	27		95	UP	27	
16	DN	25		56	3	03		96	d	17	
17	0	00		57	√	76		97	-	34	
18	AC+	60		58	X	36		98	XFR	67	
19	UP	27		59	2	02		99	7	07	
1a	RCL	61		5a	DIV	35		9a	+	33	
1b	+	33		5b	DN	25		9b	XFR	67	
1c	YTO	40		5c	-	34		9c	5	05	
1d	f	15		5d	XFR	67		9d	-	34	
20	DN	25		60	5	05		a0	6	06	
21	XFR	67		61	-	34		a1	DIV	35	
22	8	10		62	6	06		a2	3	03	
23	XEY	30		63	DIV	35		a3	UP	27	
24	+	33		64	1	01		a4	PNT	45	
25	YTO	40		65	UP	27		a5	PNT	45	
26	e	12		66	PNT	45	S	a6	b	14	S
27	-	34		67	PNT	45		a7	UP	27	
28	-	34		68	XFR	67		a8	f	15	
29	YTO	40		69	5	05		a9	UP	27	
2a	8	10		6a	UP	27		aa	2	02	
2b	f	15		6b	b	14		ab	DIV	35	
2c	UP	27		6c	+	33		ac	DN	25	
2d	1	01		6d	f	15		ad	-	34	
30	2	02		70	UP	27					
31	DIV	35		71	2	02					
32	0	00		72	DIV	35					
33	UP	27		73	DN	25					
34	PNT	45	S	74	+	33					
35	PNT	45		75	d	17					
36	XFR	67		76	UP	27					
37	-	34		77	2	02					
38	f	15		78	DIV	35					
39	XTO	23		79	DN	25					
3a	f	15		7a	-	34					
3b	UP	27		7b	e	12					
3c	3	03		7c	-	34					
3d	√	76		7d	XFR	67					

```
b0    d     17
b1    UP    27
b2    2     02
b3    DIV   35
b4    DN    25
b5    -     34
b6    e     12
b7    +     33
b8    XFR   67
b9    7     07
ba    UP    27
bb    2     02
bc    DIV   35
bd    DN    25

c0    -     34
c1    a     13
c2    UP    27
c3    2     02
c4    DIV   35
c5    DN    25
c6    -     34
c7    XFR   67
c8    5     05
c9    +     33
ca    6     06
cb    DIV   35
cc    4     04
cd    UP    27

d0    PNT   45    S
d1    PNT   45
d2    XFR   67
d3    6     06
d4    XTO   23
d5    e     12
d6    XFR   67
d7    8     10
d8    XTO   23
d9    d     17
da    5     05
db    UP    27
dc    UP    27
dd    END   46
```

00	XFR	67	Minus Page	40	UP	27		80	END	46	
01	-	34		41	a	13					
02	e	12		42	X	36					
03	XTO	23		43	DN	25					
04	f	15		44	-	34					
05	XFR	67		45	e	12					
06	9	11		46	UP	27					
07	XTO	23		47	a	13					
08	b	14		48	X	36					
09	3	03		49	DN	25					
0a	√	76		4a	-	34					
0b	UP	27		4b	6	06					
0c	2	02		4c	DIV	35					
0d	DIV	35		4d	2	02					
10	YTO	40		50	UP	27					
11	a	13		51	PNT	45	S				
12	c	16		52	PNT	45					
13	UP	27		53	c	16					
14	2	02		54	UP	27					
15	DIV	35		55	d	17					
16	f	15		56	-	34					
17	UP	27		57	e	12					
18	a	13		58	+	33					
19	X	36		59	6	06					
1a	DN	25		5a	DIV	35					
1b	+	33		5b	3	03					
1c	d	17		5c	UP	27					
1d	+	33		5d	PNT	45	S				
20	b	14		60	PNT	45					
21	UP	27		61	c	16					
22	a	13		62	UP	27					
23	X	36		63	a	13					
24	DN	25		64	X	36					
25	+	33		65	UP	27					
26	e	12		66	f	15					
27	UP	27		67	X	36					
28	2	02		68	DN	25					
29	DIV	35		69	-	34					
2a	DN	25		6a	b	14					
2b	+	33		6b	UP	27					
2c	6	06		6c	a	13					
2d	DIV	35		6d	X	36					
30	1	01		70	DN	25					
31	UP	27		71	+	33					
32	PNT	45	S	72	e	12					
33	PNT	45		73	UP	27					
34	c	16		74	a	13					
35	UP	27		75	X	36					
36	a	13		76	DN	25					
37	X	36		77	-	34					
38	UP	27		78	6	06					
39	f	15		79	DIV	35					
3a	X	36		7a	4	04					
3b	DN	25		7b	UP	27					
3c	+	33		7c	PNT	45					
3d	b	14		7d	PNT	45	S				

W_1 W_3 W_5 W_2

FOURIER SERIES [SAMPLED X(t)]

This program calculates the first 15 Fourier sine or cosine coefficients associated with a discrete (sampled) time function X(t).

Definitions:

$X(t) = X(k\Delta t)$, $k = 1, 2, \ldots , N$

Δt = sampling interval chosen such that the highest frequency present in X(t) is

$$f_{max} \leq \frac{1}{2\Delta t}$$

$$X_k = A_0 + \sum_{q=1}^{N/2} A_q \cos\left(\frac{2\pi qk}{N}\right) + \sum_{q=1}^{N/2-1} B_q \sin\left(\frac{2\pi qk}{N}\right) \qquad (1)$$

$$A_q = \frac{2}{N} \sum_{k=1}^{N} X_k \cos\frac{2\pi qk}{N} \quad , q = 1, 2, \ldots , \frac{N}{2} - 1 \qquad (2)$$

$$B_q = \frac{2}{N} \sum_{k=1}^{N} X_k \sin\frac{2\pi qk}{N}, \quad q = 1, 2, \ldots , \frac{N}{2} - 1 \qquad (3)$$

This program computes either A_q, eq. (2) or B_q, eq. (3) for $q = 1, 2, \ldots 15$.

Assumptions:

$N\Delta t = T$ (period of X(t)).

N is even

At points where X(t) is discontinuous, the midpoint value should be used for the input value.

Reference: Measurement and Analysis of Random Data, J.S. Bendat, A.G. Piersol, John Wiley & Sons, 1966, page 287.

USER INSTRUCTIONS

SET: [ ] RADIANS

DEPRESS: X Y Z on the 9120A

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM

PRESS: GO TO (-)(4)(b)

SET: PROGRAM

PRESS: (sin X) or (cos X) for appropriate series.

SET: [ ] RUN

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

→ ENTER DATA: N ⟶ X

→ PRESS: CONTINUE

DISPLAY

N ——— Z
k ——— Y
0 ——— X

ENTER DATA: X_k ⟶ X

After X_N has been entered, the results are displayed and printed.

DISPLAYS

B_1 ——— Z
B_2 ——— Y
B_3 ——— X

CONT:

B_4 ——— Z
B_5 ——— Y
B_6 ——— X

CONT:

USER INSTRUCTIONS (Con't)

B_7 ——— Z
B_8 ——— Y
B_9 ——— X

CONT:

B_{10} ——— Z
B_{11} ——— Y
B_{12} ——— X

CONT:

B_{13} ——— Z
B_{14} ——— Y
B_{15} ——— X

To run another case:

EXAMPLE

Determine the sine terms for the following function: [sin x → (-)(4)(b)].

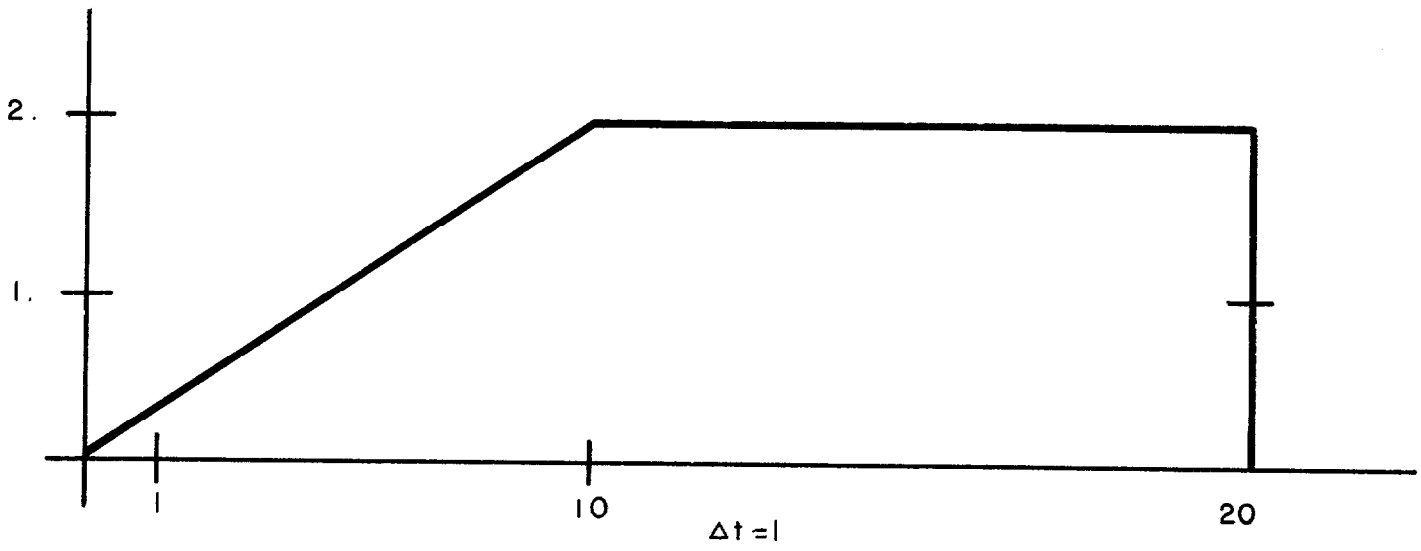
N = 20

k Δt	X(t)
1	.2
2	.4
3	.6
4	.8
5	1.0
6	1.2
7	1.4
8	1.6
9	1.8
10	2.0
11	2.0
12	2.0
13	2.0
14	2.0
15	2.0
16	2.0
17	2.0
18	2.0
19	2.0
20	1.0

EXAMPLE (Con't)

Results:

B_1	=	-.63138
B_2	=	-.30777
B_3	=	-.19626
B_4	=	-.13764
B_5	=	-.10000
B_6	=	-.07265
B_7	=	-.05095
B_8	=	-.03249
B_9	=	-.01584
B_{10}	=	0.00000
B_{11}	=	.01584
B_{12}	=	.03249
B_{13}	=	.05095
B_{14}	=	.07265
B_{15}	=	.10000



3

00	CLR	20	Minus Page	40	1	01		80	-	34	
01	XTO	23		41	RUP	22		81	f	15	
02	1	01		42	YTO	40		82	XEY	30	
03	XTO	23		43	0	00		83	2	02	
04	2	02		44	XFR	67		84	DIV	35	
05	XTO	23		45	-	34		85	XFR	67	
06	3	03		46	f	15		86	-	34	
07	XTO	23		47	X	36		87	e	12	
08	4	04		48	π	56		88	X	36	
09	XTO	23		49	X	36		89	X>Y	53	
0a	5	05		4a	DN	25		8a	2	02	
0b	XTO	23		4b	SIN	70		8b	6	06	
0c	6	06		4c	X	36		8c	XFR	67	
0d	XTO	23		4d	f	15		8d	-	34	
10	7	07		50	+	33		90	d	17	
11	XTO	23		51	YE	24		91	UP	27	
12	8	10		52	1	01		92	UP	27	
13	XTO	23		53	YE	24		93	PNT	45	
14	9	11		54	2	02		94	PNT	45	
15	XTO	23		55	YE	24		95	RCL	61	
16	a	13		56	3	03		96	XEY	30	
17	XTO	23		57	YE	24		97	UP	27	
18	b	14		58	4	04		98	d	17	
19	XTO	23		59	YE	24		99	PNT	45	
1a	c	16		5a	5	05		9a	c	16	
1b	XTO	23		5b	YE	24		9b	UP	27	
1c	d	17		5c	6	06		9c	b	14	
1d	XTO	23		5d	YE	24		9d	UP	27	
20	-	34		60	7	07		a0	a	13	
21	f	15		61	YE	24		a1	PNT	45	
22	STP	41	ENTRY	62	8	10		a2	XFR	67	
23	XTO	23		63	YE	24		a3	9	11	
24	-	34		64	9	11		a4	UP	27	
25	e	12		65	YE	24		a5	XFR	67	
26	UP	27		66	a	13		a6	8	10	
27	UP	27		67	YE	24		a7	UP	27	
28	2	02		68	b	14		a8	XFR	67	
29	XEY	30		69	YE	24		a9	7	07	
2a	DIV	35		6a	c	16		aa	PNT	45	
2b	XFR	67		6b	YE	24		ab	XFR	67	
2c	-	34		6c	d	17		ac	6	06	
2d	f	15		6d	YE	24		ad	UP	27	
30	XEY	30		70	e	12			q		B_8
31	+	33		71	YE	24					
32	YTO	40		72	f	15			B_{15}		B_7
33	-	34		73	XFR	67			B_{14}		B_6
34	f	15		74	0	00			B_{13}		B_5
35	DIV	35		75	XEY	30			B_{12}		B_4
36	0	00		76	1	01			B_{11}		B_3
37	STP	41	ENTRY	77	+	33			B_{10}		B_2
38	PNT	45		78	1	01			B_9		B_1
39	PNT	45		79	6	06					
3a	RUP	22		7a	X>Y	53					
3b	DIV	35		7b	4	04					
3c	2	02		7c	2	02					
3d	X	36		7d	XFR	67					

b0	XFR	67
b1	5	05
b2	UP	27
b3	XFR	67
b4	4	04
b5	PNT	45
b6	XFR	67
b7	3	03
b8	UP	27
b9	XFR	67
ba	2	02
bb	UP	27
bc	XFR	67
bd	1	01

Minus
Page

c0	PNT	45
c1	PNT	45
c2	XFR	67
c3	-	34
c4	d	17
c5	UP	27
c6	UP	27
c7	PNT	45
c8	PNT	45
c9	PNT	45
ca	PNT	45
cb	GTO	44
cc	0	00
cd	0	00

d0	CHS	32
d1	CHS	32
d2	CHS	32
d3	CHS	32
d4	CHS	32
d5	CHS	32
d6	CHS	32
d7	CHS	32
d8	CHS	32
d9	CHS	32
da	YTO	40
db	YTO	40
dc	YTO	40
dd	YTO	40

$$(O)\ K \cdot \frac{2}{N}$$

SECTION VI SAMPLING THEORY

		Page Number
VI-1	Sample Size n For Continuous Data	1
VI-2	Sample Size n In Sampling For Proportions	5
VI-3	Neyman Allocation In Stratified Random Sampling	9

SAMPLE SIZE n FOR CONTINUOUS DATA

This program calculates the necessary sample size (n) required to achieve a desired precision (d), assuming a small risk α that the precision may not be reached. The equations required are:

$$n_o = \frac{t^2 s^2}{d^2}$$

$$n = \frac{n_o}{1 + \frac{n_o}{N}}$$

where

- t = t statistic associated with α ($\alpha = .05$, $t = 1.96$)
- d = Chosen margin of error
- s^2 = Sample variance
- N = Population size in sampling units

The quantity n represents the sample size required to ensure:

$$P_r (|\bar{y} - \bar{Y}| \geq d) = \alpha$$

where

- $\bar{y}$ = Sample unit mean
- $\bar{Y}$ = Population unit mean

Reference: Sampling Techniques, Cochran, Wiley, 1965, page 75.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA:

d	Z
s^2	Y
t	X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: N → X

PRESS: CONTINUE

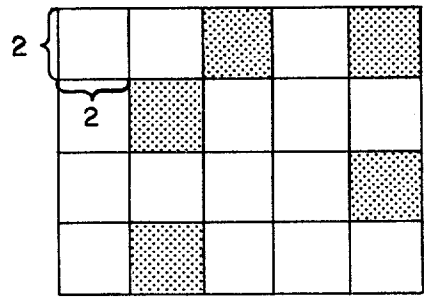
DISPLAY

n	Z
N	Y
0	X

To run another case

EXAMPLE

Ottumwa National Forest



In the spring of 1969, a complete deer population head count was made of the Ottumwa National Forest. This forest is 8 miles by 10 miles. The sampling unit was a square with 2 mile sides. The population unit mean was 40 deer. ($\bar{Y}$ = 40). The population unit variance was determined to be 25. (s^2 = 25)

In order to determine the deer population following the 1969 fall harvesting, how many units (2 mile squares) must be sampled in spring 1970 within a unit margin of error of 10% $\bar{Y}$? (d = 4)

t	=	1.96
d	=	4
s^2	=	25
N	=	4 x 5 = 20

Results:

n	=	4.6

Thus 5 squares must be sampled to obtain a new deer population.

STAT-PAC VI-1

Step	Key	Code	
00	CLR	20	
01	1	01	
02	STP	41	ENTRY
03	PNT	45	
04	PNT	45	
05	X	36	
06	X	36	
07	XEY	30	
08	RUP	22	
09	UP	27	
0a	X	36	
0b	DN	25	
0c	DIV	35	
0d	YTO	40	
10	d	17	
11	CLR	20	
12	2	02	
13	STP	41	
14	XTO	23	
15	e	12	
16	YEX	24	
17	d	17	
18	UP	27	
19	DN	25	
1a	DIV	35	
1b	1	01	
1c	+	33	
1d	DN	25	
20	DIV	35	
21	UP	27	
22	RCL	61	
23	PNT	45	
24	PNT	45	
25	END	46	

n_0

N

O

SAMPLE SIZE (n) IN SAMPLING FOR PROPORTIONS

This program calculates the necessary sample size (n) to achieve a desired precision d, assuming a small risk α that the desired precision may not be reached. $[P_r (|p - P| \geq d) = \alpha]$

The equations used are:

$$n_o = \frac{t^2(p - p^2)}{d^2} = \frac{t^2 pq}{d^2}$$

$$n = \frac{n_o}{1 + \frac{n_o}{N}}$$

where,

t = t statistic associated with α (α = .05, t = 1.96)
d = Desired precision
p = Sample proportion
N = Population size
q = 1 - p
P = Population proportion

Reference: Sampling Techniques, Cochran, Wiley, 1965, page 75.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

```
0 ———— Z
0 ———— Y
1 ———— X
```

ENTER DATA:

```
d ————→ Z
p ————→ Y
t ————→ X
```

PRESS: CONTINUE

DISPLAY

```
0 ———— Z
0 ———— Y
2 ———— X
```

ENTER DATA: N ————→ X

PRESS: CONTINUE

DISPLAY

```
n ———— Z
N ———— Y
0 ———— X
```

To run another case

EXAMPLE

Last year a small sample of TV watching habits was taken in Livingston, Montana. This sample indicated that 70% of the Livingston TV sets were used for more than 4 hours per day.

How large a sample is necessary to accurately determine what proportion of TV owners are using their sets over 4 hours per day?

N = (total number of sets)= 1500

p = .7 (estimated proportion)

d = 0.1 (desired precision)

α = .05 (t = 1.96)

n = 77.

00	CLR	20
01	1	01
02	STP	41
03	PNT	45
04	PNT	45
05	XTO	23
06	f	15
07	DN	25
08	UP	27
09	X	36
0a	XEY	30
0b	-	34
0c	DN	25
0d	RDN	31

ENTRY

10	X	36
11	DN	25
12	DIV	35
13	f	15
14	UP	27
15	X	36
16	DN	25
17	X	36
18	YTO	40
19	d	17
1a	CLR	20
1b	2	02
1c	STP	41
1d	XTO	23

ENTRY

20	e	12
21	YEX	24
22	d	17
23	UP	27
24	DN	25
25	DIV	35
26	1	01
27	+	33
28	DN	25
29	DIV	35
2a	e	12
2b	UP	27
2c	0	00
2d	PNT	45

30	PNT	45
31	END	46

n_0

N

t

NEYMAN ALLOCATION IN STRATIFIED RANDOM SAMPLING

This program calculates the optimum allocation of a fixed number of samples between the N strata.

The equation used

$$n_h = n \frac{N_h s_h}{\Sigma N_h s_h}$$

n = Number of Items to be Allocated

N_h = The Number of Items in Strata h

s_h = Standard Deviation of Strata h

n_h = Number of Items Allocated in Strata h

N = Number of Strata ($N \leq 6$)

Reference: Sampling Techniques by Cochran, Wiley, 1965.

USER INSTRUCTIONS

SET: [ ] FIXED POINT

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM A

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

N ———→ Y
n ———→ X

PRESS: CONTINUE

DISPLAY

0 ——— Z
h ——— Y
0 ——— X

ENTER DATA:

s_h ———→ Y
N_h ———→ X

PRESS: CONTINUE

DISPLAY

1 ——— Z
1 ——— Y
1 ——— X

(When this display appears, all data has been entered.)

PRESS: END

ENTER PROGRAM B

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

DISPLAY

h ——— Z
h ——— Y
n_h ——— X

To run a new case, return to the beginning of the USER INSTRUCTIONS.

EXAMPLE

See next page.

EXAMPLE

The following table shows the stratification of all farms in Milliken County, and the average sugar beet tonnage/acre per farm in each stratum for the year 1968.

Stratum (h)	Farm Size (acres)	Average Tonnage/Acre	Standard Deviation s_h	Number for farms N_h
1	0 - 50	9	4.1	21
2	50 - 100	13	6.3	62
3	100 - 150	16	8.2	104
4	150 - 200	17.5	10.1	181
5	200 - 250	20	9.0	75
6	250 ——	19.6	5.8	14

Great Mountain Sugar would like to determine if its 1969 "Increased Tonnage Incentives" plan has been effective by taking samples of 100 Milliken County farms. How should they distribute the 100 samples?

$N = 6$

$n = 100$

N_h	s_h
$N_1 = 21,$	$s_1 = 4.1$
$N_2 = 62,$	$s_2 = 6.3$
$N_3 = 104,$	$s_3 = 8.2$
$N_4 = 181,$	$s_4 = 10.1$
$N_5 = 75,$	$s_5 = 9.0$
$N_6 = 14,$	$s_6 = 5.8$

Results:

$n_1 = 2$

$n_2 = 10$

$n_3 = 22$

$n_4 = 47$

$n_5 = 17$

$n_6 = 2$

Step	Key	Code		Step	Key	Code		Step	Key	Code	
00	CLR	20		40	0	00		80	0	00	
01	STP	41	ENTRY	41	RDN	31		81	0	00	
02	PNT	45		42	5	05		82	0	00	
03	PNT	45		43	XEY	30		83	0	00	
04	AC+	60		44	e	12		84	0	00	
05	DN	25		45	X<Y	52		85	0	00	
06	1	01		46	6	06		86	0	00	
07	XEY	30		47	2	02		87	0	00	
08	STP	41	ENTRY	48	0	00		88	0	00	
09	PNT	45		49	STP	41	ENTRY	89	0	00	
0a	PNT	45		4a	PNT	45		8a	0	00	
0b	X	36		4b	PNT	45		8b	0	00	
0c	YTO	40		4c	X	36		8c	0	00	
0d	d	17		4d	YTO	40		8d	0	00	
10	0	00		50	9	11		90	0	00	
11	RDN	31		51	0	00		91	0	00	
12	DN	25		52	RDN	31		92	0	00	
13	2	02		53	6	06		93	0	00	
14	XEY	30		54	XEY	30		94	0	00	
15	STP	41	ENTRY	55	e	12		95	0	00	
16	PNT	45		56	X<Y	52		96	0	00	
17	PNT	45		57	6	06		97	0	00	
18	X	36		58	2	02		98	0	00	
19	YTO	40		59	0	00		99	0	00	
1a	c	16		5a	STP	41	ENTRY	9a	0	00	
1b	0	00		5b	PNT	45		9b	0	00	
1c	RDN	31		5c	PNT	45		9c	0	00	
1d	DN	25		5d	X	36		9d	0	00	
20	3	03		60	YTO	40		a0	0	00	
21	XEY	30		61	8	10		a1	0	00	
22	e	12		62	YEX	24		a2	0	00	
23	X<Y	52		63	8	10		a3	0	00	
24	6	06		64	YTO	40		a4	0	00	
25	2	02		65	8	10		a5	0	00	
26	0	00		66	d	17		a6	0	00	
27	STP	41	ENTRY	67	+	33		a7	0	00	
28	PNT	45		68	DN	25		a8	0	00	
29	PNT	45		69	YEX	24		a9	0	00	
2a	X	36		6a	9	11		aa	0	00	
2b	YTO	40		6b	YTO	40		ab	0	00	
2c	b	14		6c	9	11		ac	0	00	
2d	0	00		6d	+	33		ad	0	00	
30	RDN	31		70	c	16					
31	4	04		71	+	33					
32	XEY	30		72	b	14					
33	e	12		73	+	33					
34	X<Y	52		74	a	13					
35	6	06		75	+	33					
36	2	02		76	f	15					
37	0	00		77	DIV	35					
38	STP	41	ENTRY	78	YTO	40					
39	PNT	45		79	f	15					
3a	PNT	45		7a	1	01					
3b	X	36		7b	UP	27					
3c	YTO	40		7c	UP	27					
3d	a	13		7d	STP	41					

6	N_6S_6
5	N_5S_5
4	N_4S_4
3	N_3S_3
2	N_2S_2
1	N_1S_1
0	N
.	$n / \frac{\Sigma}{n}$

b0	0	00		10	DIV	35		50	PNT	45	s
b1	0	00		11	DN	25		51	PNT	45	
b2	0	00		12	PNT	45	s	52	PNT	45	
b3	0	00		13	PNT	45		53	PNT	45	
b4	0	00		14	e	12		54	PNT	45	
b5	0	00		15	X=Y	50		55	PNT	45	
b6	0	00		16	5	05		56	CLR	20	
b7	0	00		17	6	06		57	END	46	
b8	0	00		18	3	03					
b9	0	00		19	UP	27					
ba	0	00		1a	b	14					
bb	0	00		1b	UP	27					
bc	0	00		1c	f	15					
bd	0	00		1d	DIV	35					
c0	0	00		20	DN	25					
c1	0	00		21	PNT	45	s				
c2	0	00		22	PNT	45					
c3	0	00		23	e	12					
c4	0	00		24	X=Y	50					
c5	0	00		25	5	05					
c6	0	00		26	6	06					
c7	0	00		27	4	04					
c8	0	00		28	UP	27					
c9	0	00		29	a	13					
ca	0	00		2a	UP	27					
cb	0	00		2b	f	15					
cc	0	00		2c	DIV	35					
cd	0	00		2d	DN	25					
d0	0	00		30	PNT	45	s				
d1	0	00		31	PNT	45					
d2	0	00		32	e	12					
d3	0	00		33	X=Y	50					
d4	0	00		34	5	05					
d5	0	00		35	6	06					
d6	0	00		36	5	05					
d7	0	00		37	YEX	24					
d8	0	00		38	9	11					
d9	0	00		39	XEY	30					
da	0	00		3a	UP	27					
db	0	00		3b	f	15					
dc	0	00		3c	DIV	35					
dd	END	46		3d	DN	25					
00	1	01	Program B	40	PNT	45	s				
01	UP	27		41	PNT	45					
02	d	17		42	e	12					
03	UP	27		43	X=Y	50					
04	f	15		44	5	05					
05	DIV	35		45	6	06					
06	DN	25		46	6	06					
07	PNT	45	s	47	YEX	24					
08	PNT	45		48	8	10					
09	2	02		49	XEY	30					
0a	UP	27		4a	UP	27					
0b	c	16		4b	f	15					
0c	UP	27		4c	DIV	35					
0d	f	15		4d	DN	25					

SECTION VII ANALYSIS OF VARIANCE

			Page Number
VII-1	One Way Analysis of Variance m x n		1
VII-2	Two Way Analysis of Variance	9100B ONLY	5
VII-3	Two Way Analysis of Variance with Replicates	9100B ONLY	11
VII-4	Analysis of Variance F Test For Column Means		19

ONE WAY ANALYSIS OF VARIANCE m x n

This program separates the total variance in a table of data into a portion due to chance and a portion due to differences between and population means underlying each column of sample data. It then calculates the variance ratio.

$$F = \frac{nm(m-1) \sum_{j=1}^{n} (X_j - \bar{X})}{(n-1) \sum_{i=1}^{m} \sum_{j=1}^{n} (X_{ij} - \bar{X}_j)}$$

with $\nu_1 = n - 1$ degrees of freedom

$\nu_2 = n(m-1)$ degrees of freedom

where

$$\bar{X} = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij}$$

$$\bar{X}_j = \frac{1}{n} \sum_{j=1}^{n} X_{ij}$$

The equation used by the program is:

$$F = \frac{nm(m-1) \left\{ \sum_{j=1}^{n} \left[\sum_{i=1}^{m} X_{ij} \right]^2 - \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 \right\}}{(n-1) \left\{ \sum_{j=1}^{n} \sum_{i=1}^{m} X^2_{ij} - \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 - \frac{1}{m} \sum_{j=1}^{n} \left[\sum_{i=1}^{m} X_{ij} \right]^2 + \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 \right\}}$$

Reference: Freund, John E., Mathematical Statistics, Prentice Hall (1962)

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

→ DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA: n Columns → Y, m Rows → X

PRESS: CONTINUE

→ DISPLAY

0 ——— Z
j ——— Y jth Column
i ——— X ith Row

Enter data Column by Column

ENTER DATA: X_{ij} → X

PRESS: CONTINUE

NO ← Last data entered? → YES

DISPLAY

F ——— Z F=variance ratio
ν_1 ——— Y ν_1=degrees of freedom in numerator
ν_2 ——— X ν_2=degrees of freedom in denominator

← PRESS: CONTINUE for new case

EXAMPLE

General form

i Rows \ j Columns	1	2	.	.	.	.	n
1	X_{11}						X_{1n}
2							
.							
.		X_{ij}					
.							
m	X_{m1}						X_{mn}

Rows \ Columns			
	172	203	161
	185	172	149
	165	187	183
	194	183	156
	212	179	144

$F = 5.01$

$\nu_1 = 2$

$\nu_2 = 12$

00	CLR	20		40	-	34	80	X	36
01	XTO	23		41	XTO	23	81	f	15
02	9	11		42	a	13	82	XEY	30
03	XTO	23		43	CLX	37	83	CNT	47
04	b	14		44	XTO	23	84	RUP	22
05	STP	41	ENTRY	45	b	14	85	PNT	45
06	XEY	30		46	X<Y	52	86	RUP	22
07	PNT	45		47	1	01	87	PNT	45
08	XEY	30		48	8	10	88	RUP	22
09	PNT	45		49	d	17	89	PNT	45
0a	PNT	45		4a	UP	27	8a	PNT	45
0b	YTO	40		4b	1	01	8b	PNT	45
0c	c	16		4c	-	34	8c	PNT	45
0d	UP	27		4d	YTO	40	8d	END	46
10	1	01		50	d	17			
11	+	33		51	RCL	61			
12	YTO	40		52	UP	27			
13	d	17		53	X	36			
14	XEY	30		54	c	16			
15	CLX	37		55	DIV	35			
16	RDN	31		56	d	17			
17	XTO	23		57	DIV	35			
18	a	13		58	DN	25			
19	STP	41	ENTRY	59	-	34			
1a	PNT	45		5a	YTO	40			
1b	PNT	45		5b	e	12			
1c	YEX	24		5c	RDN	31			
1d	b	14		5d	YEX	24			
20	+	33		60	9	11			
21	YEX	24		61	d	17			
22	b	14		62	DIV	35			
23	UP	27		63	DN	25			
24	X	36		64	XEY	30			
25	AC+	60		65	-	34			
26	DN	25		66	e	12			
27	a	13		67	XEY	30			
28	UP	27		68	-	34			
29	1	01		69	XEY	30			
2a	+	33		6a	DIV	35			
2b	d	17		6b	c	16			
2c	X>Y	53		6c	X	36			
2d	1	01		6d	UP	27			
30	5	05		70	1	01			
31	DN	25		71	-	34			
32	b	14		72	YTO	40			
33	UP	27		73	f	15			
34	X	36		74	DN	25			
35	DN	25		75	DIV	35			
36	YEX	24		76	d	17			
37	9	11		77	UP	27			
38	+	33		78	1	01			
39	YEX	24		79	-	34			
3a	9	11		7a	DN	25			
3b	CLX	37		7b	X	36			
3c	RDN	31		7c	UP	27			
3d	1	01		7d	c	16			

TWO WAY ANALYSIS OF VARIANCE (m x 4)

This program analyzes the total statistical variance in a table of data by separating the total variance into two parts, the variance among rows of data, and the variance between columns of data, and comparing each to the variance due to random influence. In a table of four columns and m rows it calculates the variance ratio between columns.

$$F_c = \frac{m \sum_{j=1}^{4} (\bar{X}_j - \bar{X})^2 \Big/ 3}{\sum_{j=1}^{4} \sum_{i=1}^{m} (X_{ij} - \bar{X}_j - \bar{X}_i + \bar{X})^2 \Big/ (m-1)(3)}$$

with $V_1 = 3$ degrees of freedom

and $V_2 = 3(m-1)$ degrees of freedom

and the variance ratio between rows:

$$F_r = \frac{4 \sum_{i=1}^{m} (\bar{X}_i - \bar{X})^2 \Big/ (m-1)}{\sum_{j=1}^{4} \sum_{i=1}^{m} (X_{ij} - \bar{X}_j - \bar{X}_i + \bar{X})^2 \Big/ 3(m-1)}$$

with $V_1 = m - 1$ degrees of freedom
$V_2 = 3(m-1)$ degrees of freedom

where:

$$\bar{X}_j = \frac{1}{m} \sum_{i=1}^{m} X_{ij} \qquad \bar{X}_i = \frac{1}{4} \sum_{j=1}^{4} X_{ij}$$

$$\bar{X} = \frac{1}{4m} \sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}$$

V_1 = degrees of freedom in numerator
V_2 = degrees of freedom in denominator

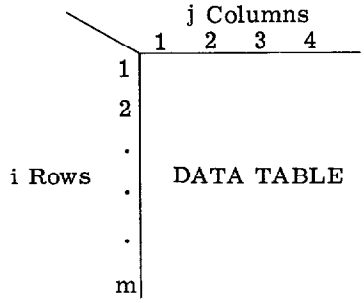
The equations used by the program are:

$$F_r = \frac{4(m-1)\left\{\frac{1}{4}\sum_{i=1}^{m}\left[\sum_{j=1}^{4} X_{ij}\right]^2 - \frac{1}{4m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2\right\}}{3\left\{\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}^2 - \frac{1}{4m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{m}\sum_{j=1}^{4}\left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4}\sum_{i=1}^{m}\left[\sum_{j=1}^{4} X_{ij}\right]^2 + \frac{1}{2m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2\right\}}$$

$$F_c = \frac{3m\left\{\frac{1}{m}\sum_{j=1}^{4}\left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2\right\}}{(m-1)\left\{\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}^2 - \frac{1}{4m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{m}\sum_{j=1}^{4}\left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4}\sum_{i=1}^{m}\left[\sum_{j=1}^{4} X_{ij}\right]^2 + \frac{1}{2m}\left[\sum_{j=1}^{4}\sum_{i=1}^{m} X_{ij}\right]^2\right\}}$$

Reference: Freund, John E., Mathematical Statistics, Prentice Hall (1962)

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: m → X number of rows in data table

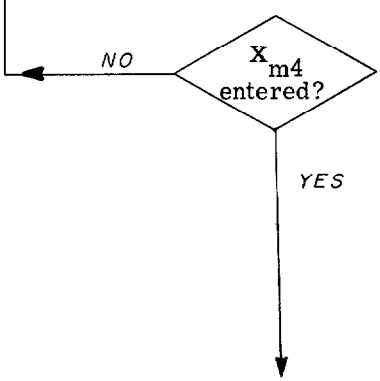
PRESS: CONTINUE

DISPLAY

0	Z
i	Y
j	X

ENTER DATA: X_{ij} → X j = column, i = row
(data entered row by row)

PRESS: CONTINUE



2

USER INSTRUCTIONS (Con't)

DISPLAY

F_c	Z	F ratio between columns
V_1	Y	V_1 degrees of freedom in numerator
V_2	X	V_2 degrees of freedom in denominator

PRESS: CONTINUE

DISPLAY

F_r	Z	F ratio between rows
V_1	Y	V_1 degrees of freedom in numerator
V_2	X	V_2 degrees of freedom in denominator

To re-run program:

PRESS: END

EXAMPLE

General form

j Columns

i Rows	1	2	3	4
1	X_{11}	. . .		X_{14}
2	.			
.	.	X_{ij}		
.	.			
m	X_{m1}	. . .		X_{m4}

Rows	Columns			
	58.2	49.1	60.1	75.8
	56.2	54.1	70.9	58.2
	65.3	51.6	39.2	48.7

$F_c = 0.43$ $V_1 = 3$ $V_2 = 6$

$F_r = 0.92$ $V_1 = 2$ $V_2 = 6$

00	CLR	20		40	YE	24		80	CNT	47	
01	XTO	23	Plus Page	41	c	16		81	CNT	47	
02	a	13		42	+	33		82	CNT	47	
03	XTO	23		43	YE	24		83	CNT	47	
04	b	14		44	c	16		84	CNT	47	
05	XTO	23		45	UP	27		85	CNT	47	
06	c	16		46	X	36		86	CNT	47	
07	XTO	23		47	AC+	60		87	CNT	47	
08	9	11		48	CLX	37		88	CNT	47	
09	XTO	23		49	RDN	31		89	CNT	47	
0a	8	10		4a	4	04		8a	CNT	47	
0b	STP	41	ENTRY	4b	STP	41	ENTRY	8b	CNT	47	
0c	RUP	22		4c	PNT	45		8c	CNT	47	
0d	YTO	40		4d	PNT	45		8d	CNT	47	
10	d	17		50	YE	24		90	CNT	47	
11	1	01		51	7	07		91	CNT	47	
12	STP	41	ENTRY	52	+	33		92	CNT	47	
13	PNT	45		53	YE	24		93	CNT	47	
14	XTO	23		54	7	07		94	CNT	47	
15	7	07		55	YE	24		95	CNT	47	
16	YE	24		56	8	10		96	CNT	47	
17	a	13		57	+	33		97	CNT	47	
18	+	33		58	YE	24		98	CNT	47	
19	YE	24		59	8	10		99	CNT	47	
1a	a	13		5a	UP	27		9a	CNT	47	
1b	UP	27		5b	X	36		9b	CNT	47	
1c	X	36		5c	AC+	60		9c	CNT	47	
1d	AC+	60		5d	YE	24		9d	CNT	47	
20	CLX	37		60	7	07		a0	CNT	47	
21	RDN	31		61	DN	25		a1	CNT	47	
22	2	02		62	YE	24		a2	CNT	47	
23	STP	41	ENTRY	63	9	11		a3	CNT	47	
24	PNT	45		64	UP	27		a4	CNT	47	
25	YE	24		65	X	36		a5	CNT	47	
26	7	07		66	CLX	37		a6	CNT	47	
27	+	33		67	RDN	31		a7	CNT	47	
28	YE	24		68	+	33		a8	CNT	47	
29	7	07		69	GTO	44		a9	CNT	47	
2a	YE	24		6a	-	34		aa	CNT	47	
2b	b	14		6b	0	00		ab	CNT	47	
2c	+	33		6c	0	00		ac	CNT	47	
2d	YE	24		6d	CNT	47		ad	CNT	47	
30	b	14		70	CNT	47					
31	UP	27		71	CNT	47					
32	X	36		72	CNT	47					
33	AC+	60		73	CNT	47					
34	CLX	37		74	CNT	47					
35	RDN	31		75	CNT	47					
36	3	03		76	CNT	47					
37	STP	41	ENTRY	77	CNT	47					
38	PNT	45		78	CNT	47					
39	YE	24		79	CNT	47					
3a	7	07		7a	CNT	47					
3b	+	33		7b	CNT	47					
3c	YE	24		7c	CNT	47					
3d	7	07		7d	CNT	47					

ΣX_{ij}

M

ΣX_{ij}

b0	CNT	47	Plus Page
b1	CNT	47	
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	
c0	CNT	47	
c1	CNT	47	
c2	CNT	47	
c3	CNT	47	
c4	CNT	47	
c5	CNT	47	
c6	CNT	47	
c7	CNT	47	
c8	CNT	47	
c9	CNT	47	
ca	CNT	47	
cb	CNT	47	
cc	CNT	47	
cd	CNT	47	
d0	CNT	47	
d1	CNT	47	
d2	CNT	47	
d3	CNT	47	
d4	CNT	47	
d5	CNT	47	
d6	CNT	47	
d7	CNT	47	
d8	CNT	47	
d9	CNT	47	
da	CNT	47	
db	CNT	47	
dc	CNT	47	
dd	CNT	47	

			Minus Page				
00	YE	24		40	XEY	30	
01	9	11		41	RUP	22	
02	1	01		42	-	34	
03	-	34		43	YTO	40	
04	CLX	37		44	b	14	
05	X<Y	52		45	DN	25	
06	6	06		46	DIV	35	
07	8	10		47	d	17	
08	RCL	61		48	UP	27	
09	AC-	63		49	1	01	
0a	UP	27		4a	-	34	
0b	X	36		4b	DN	25	
0c	4	04		4c	XTO	23	
0d	DIV	35		4d	d	17	
10	d	17		50	X	36	
11	DIV	35		51	UP	27	
12	DN	25		52	3	03	
13	-	34		53	X	36	
14	AC+	60		54	XEY	30	
15	a	13		55	PNT	45	s
16	UP	27		56	PNT	45	
17	X	36		57	a	13	
18	b	14		58	UP	27	
19	UP	27		59	b	14	
1a	X	36		5a	DIV	35	
1b	DN	25		5b	3	03	
1c	+	33		5c	X	36	
1d	c	16		5d	UP	27	
20	UP	27		60	d	17	
21	X	36		61	X	36	
22	DN	25		62	XEY	30	
23	+	33		63	PNT	45	s
24	UP	27		64	PNT	45	
25	YE	24		65	GTO	44	
26	8	10		66	6	06	
27	DN	25		67	c	16	
28	UP	27		68	GTO	44	
29	X	36		69	+	33	
2a	DN	25		6a	1	01	
2b	+	33		6b	1	01	
2c	d	17		6c	END	46	
2d	DIV	35					
30	f	15					
31	-	34					
32	UP	27					
33	YE	24					
34	9	11					
35	4	04					
36	DIV	35					
37	f	15					
38	-	34					
39	YTO	40					
3a	a	13					
3b	e	12					
3c	RUP	22					
3d	-	34					

TWO WAY ANALYSIS OF VARIANCE WITH REPLICATES (THREE WAY)

This program analyzes the total statistical variance in a table of data by separating the total variance into three parts, the variance among rows, the variance between columns, and the variance due to interactions.

Computational Equations

$$SST = \sum_{i}^{k}\sum_{j}^{n}\sum_{r}^{m} X_{ijr}^2 - \frac{1}{knm}\left[\sum_{i}^{k}\sum_{j}^{n}\sum_{r}^{m} X_{ijr}\right]^2$$

$$SSA = \frac{1}{nm}\sum_{i}^{k}\left[\sum_{j}^{n}\sum_{r}^{m} X_{ijr}\right]^2 - \frac{1}{knm}\left[\sum_{i}^{k}\sum_{j}^{n}\sum_{r}^{m} X_{ijr}\right]^2$$

$$SSB = \frac{1}{km}\sum_{j}^{n}\left[\sum_{r}^{m}\sum_{i}^{k} X_{ijr}\right]^2 - \frac{1}{knm}\left[\sum_{i}^{k}\sum_{j}^{n}\sum_{r}^{m} X_{ijr}\right]^2$$

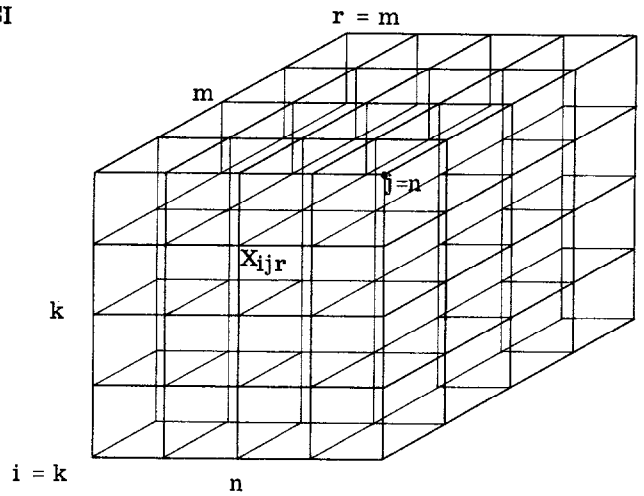
$$SSI = \frac{1}{m}\sum_{i}^{k}\sum_{j}^{n}\left[\sum_{r}^{m} X_{ijr}\right]^2 - \frac{1}{nm}\sum_{i}^{k}\left[\sum_{j}^{n}\sum_{r}^{m} X_{ijr}\right]^2 - \frac{1}{km}\sum_{j}^{n}\left[\sum_{i}^{k}\sum_{r}^{m} X_{ijr}\right]^2 + \frac{1}{kmn}\left[\sum_{i}^{k}\sum_{j}^{n}\sum_{r}^{m} X_{ijr}\right]^2$$

$$F_R = \frac{SSA}{(k-1)} \quad \frac{kn(m-1)}{SSE}$$

$$F_C = \frac{SSB}{(n-1)} \quad \frac{kn(m-1)}{SSE}$$

$$F_I = \frac{SSI}{(k-1)(n-1)} \quad \frac{kn(m-1)}{SSE}$$

$$SSE = SST - SSA - SSB - SSI$$



(n ≤ 8)

Reference: Mathematical Statistics, John E. Freund, Prentice Hall, 1963

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A
PRESS: END

ENTER PROGRAM 1: Side A followed by Side B

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: rows columns repetitions
k → Z, n → Y, m → X

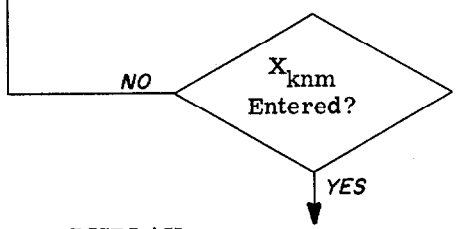
PRESS: CONTINUE

DISPLAY

i	Z	row
j	Y	column
r	X	repetition

ENTER DATA: X_{ijr} → X

PRESS: CONTINUE



DISPLAY

I	Z	Intermediate results, DO NOT ALTER.
I	Y	
I	X	

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM 2:

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

F_{row}	Z
V_1	Y
V_2	X

PRESS: CONTINUE

DISPLAY

F_{column}	Z
V_1	Y
V_2	X

PRESS: CONTINUE

DISPLAY

$F_{interaction}$	Z
V_1	Y
V_2	X

EXAMPLE

n = 3 columns

k = 4 rows	58.2	56.2	65.3	
	52.6	41.2	60.8	
	49.1	54.1	51.6	
	42.8	50.5	48.4	
	60.1	70.9	39.2	m = 2
	58.3	73.2	40.7	repetition
	75.8	58.2	48.7	
	71.5	51.0	41.4	

Results:

$F_{row} = 4.42$ $F_{column} = 9.39$
$V_1 = 3$ $V_1 = 2$
$V_2 = 12$ $V_2 = 12$

$F_{interaction} = 14.93$
$V_1 = 6$
$V_2 = 12$

Step	Key	Code
00	CLR	20
01	STP	41
02	PNT	45
03	PNT	45
04	XTO	23
05	d	17
06	YTO	40
07	c	16
08	DN	25
09	YTO	40
0a	b	14
0b	CLR	20
0c	XTO	23
0d	a	13

Plus Page
ENTRY

Step	Key	Code
40	CNT	47
41	CNT	47
42	CNT	47
43	CNT	47
44	CNT	47
45	CNT	47
46	CNT	47
47	CNT	47
48	CNT	47
49	CNT	47
4a	CNT	47
4b	CNT	47
4c	CNT	47
4d	CNT	47

Step	Key	Code
80	CNT	47
81	CNT	47
82	CNT	47
83	CNT	47
84	CNT	47
85	CNT	47
86	CNT	47
87	CNT	47
88	CNT	47
89	CNT	47
8a	CNT	47
8b	CNT	47
8c	CNT	47
8d	CNT	47

10	XTO	23
11	9	11
12	XTO	23
13	8	10
14	XTO	23
15	7	07
16	XTO	23
17	6	06
18	XTO	23
19	5	05
1a	XTO	23
1b	4	04
1c	XTO	23
1d	3	03

50	CNT	47
51	CNT	47
52	CNT	47
53	CNT	47
54	CNT	47
55	CNT	47
56	CNT	47
57	CNT	47
58	CNT	47
59	CNT	47
5a	CNT	47
5b	CNT	47
5c	CNT	47
5d	CNT	47

90	CNT	47
91	CNT	47
92	CNT	47
93	CNT	47
94	CNT	47
95	CNT	47
96	CNT	47
97	CNT	47
98	CNT	47
99	CNT	47
9a	CNT	47
9b	CNT	47
9c	CNT	47
9d	CNT	47

20	XTO	23
21	-	34
22	f	15
23	XTO	23
24	-	34
25	e	12
26	XTO	23
27	1	01
28	XTO	23
29	0	00
2a	GTO	44
2b	-	34
2c	0	00
2d	0	00

60	CNT	47
61	CNT	47
62	CNT	47
63	CNT	47
64	CNT	47
65	CNT	47
66	CNT	47
67	CNT	47
68	CNT	47
69	CNT	47
6a	CNT	47
6b	CNT	47
6c	CNT	47
6d	CNT	47

a0	CNT	47
a1	CNT	47
a2	CNT	47
a3	CNT	47
a4	CNT	47
a5	CNT	47
a6	CNT	47
a7	CNT	47
a8	CNT	47
a9	CNT	47
aa	CNT	47
ab	CNT	47
ac	CNT	47
ad	CNT	47

30	CNT	47
31	CNT	47
32	CNT	47
33	CNT	47
34	CNT	47
35	CNT	47
36	CNT	47
37	CNT	47
38	CNT	47
39	CNT	47
3a	CNT	47
3b	CNT	47
3c	CNT	47
3d	CNT	47

70	CNT	47
71	CNT	47
72	CNT	47
73	CNT	47
74	CNT	47
75	CNT	47
76	CNT	47
77	CNT	47
78	CNT	47
79	CNT	47
7a	CNT	47
7b	CNT	47
7c	CNT	47
7d	CNT	47

Storage

0	8
1	9
2	a
3	b
4	c
5	d
6	e
7	f

Step	Key	Code
b0	CNT	47
b1	CNT	47
b2	CNT	47
b3	CNT	47
b4	CNT	47
b5	CNT	47
b6	CNT	47
b7	CNT	47
b8	CNT	47
b9	CNT	47
ba	CNT	47
bb	CNT	47
bc	CNT	47
bd	CNT	47
c0	CNT	47
c1	CNT	47
c2	CNT	47
c3	CNT	47
c4	CNT	47
c5	CNT	47
c6	CNT	47
c7	CNT	47
c8	CNT	47
c9	CNT	47
ca	CNT	47
cb	CNT	47
cc	CNT	47
cd	CNT	47
d0	CNT	47
d1	CNT	47
d2	CNT	47
d3	CNT	47
d4	CNT	47
d5	CNT	47
d6	CNT	47
d7	CNT	47
d8	CNT	47
d9	CNT	47
da	CNT	47
db	CNT	47
dc	CNT	47
dd	CNT	47

Plus
Page

Step	Key	Code
00	XTO	23
01	2	02
02	1	01
03	+	33
04	YTO	40
05	9	11
06	GTO	44
07	SUB	77
08	9	11
09	c	16
0a	YE	24
0b	5	05
0c	+	33
0d	YE	24
10	5	05
11	GTO	44
12	SUB	77
13	9	11
14	c	16
15	YE	24
16	4	04
17	+	33
18	YE	24
19	4	04
1a	GTO	44
1b	SUB	77
1c	9	11
1d	c	16
20	YE	24
21	3	03
22	+	33
23	YE	24
24	3	03
25	GTO	44
26	SUB	77
27	9	11
28	c	16
29	YE	24
2a	2	02
2b	+	33
2c	YE	24
2d	2	02
30	GTO	44
31	SUB	77
32	9	11
33	c	16
34	YE	24
35	8	10
36	+	33
37	YTO	40
38	8	10
39	GTO	44
3a	SUB	77
3b	9	11
3c	c	16
3d	YE	24

Minus
Page

Step	Key	Code
40	-	34
41	e	12
42	+	33
43	YE	24
44	-	34
45	e	12
46	GTO	44
47	SUB	77
48	9	11
49	c	16
4a	YE	24
4b	-	34
4c	f	15
4d	+	33
50	YE	24
51	-	34
52	f	15
53	XFR	67
54	1	01
55	UP	27
56	X	36
57	XFR	67
58	0	00
59	+	33
5a	YTO	40
5b	0	00
5c	CLX	37
5d	XTO	23
60	1	01
61	RDN	31
62	YE	24
63	9	11
64	b	14
65	X>Y	53
66	0	00
67	2	02
68	XFR	67
69	5	05
6a	UP	27
6b	X	36
6c	XFR	67
6d	4	04
70	UP	27
71	X	36
72	DN	25
73	+	33
74	XFR	67
75	3	03
76	UP	27
77	X	36
78	DN	25
79	+	33
7a	XFR	67
7b	2	02
7c	UP	27
7d	X	36

Step	Key	Code
80	DN	25
81	+	33
82	XFR	67
83	8	10
84	UP	27
85	X	36
86	DN	25
87	+	33
88	XFR	67
89	-	34
8a	e	12
8b	UP	27
8c	X	36
8d	DN	25
90	+	33
91	XFR	67
92	-	34
93	f	15
94	UP	27
95	X	36
96	DN	25
97	+	33
98	DN	25
99	UP	27
9a	UP	27
9b	STP	41
9c	DN	25
9d	c	16
a0	X=Y	50
a1	5	05
a2	3	03
a3	1	01
a4	+	33
a5	a	13
a6	UP	27
a7	d	17
a8	X=Y	50
a9	c	16
aa	5	05
ab	1	01
ac	+	33
ad	YTO	40

b0	a	13
b1	XFR	67
b2	9	11
b3	RDN	31
b4	STP	41
b5	PNT	45
b6	PNT	45
b7	UP	27
b8	X	36
b9	AC+	60
ba	YE	24
bb	7	07
bc	+	33
bd	YE	24

Minus
Page

ENTRY

c0	7	07
c1	DN	25
c2	GTO	44
c3	a	13
c4	5	05
c5	DN	25
c6	XFR	67
c7	7	07
c8	YE	24
c9	1	01
ca	+	33
cb	YE	24
cc	1	01
cd	UP	27

d0	X	36
d1	RDN	31
d2	YE	24
d3	6	06
d4	+	33
d5	YE	24
d6	6	06
d7	CLX	37
d8	XTO	23
d9	7	07
da	XTO	23
db	a	13
dc	RUP	22
dd	RTN	77

00	YTO	40	Minus Page	40	-	34		80	DN	25					
01	1	01		41	YTO	40		81	DIV	35					
02	f	15		42	3	03		82	UP	27					
03	UP	27		43	RCL	61		83	d	17					
04	X	36		44	XFR	67		84	PNT	45	S				
05	d	17		45	5	05		85	PNT	45					
06	DIV	35		46	-	34		86	CNT	47					
07	c	16		47	XFR	67		87	XFR	67					
08	DIV	35		48	4	04		88	2	02					
09	b	14		49	-	34		89	UP	27					
0a	DIV	35		4a	XFR	67		8a	XFR	67					
0b	e	12		4b	3	03		8b	3	03					
0c	XEY	30		4c	-	34		8c	X	36					
0d	-	34		4d	d	17		8d	b	14					
10	YTO	40		50	UP	27		90	UP	27					
11	e	12		51	1	01		91	c	16					
12	XTO	23		52	-	34		92	X	36					
13	f	15		53	c	16		93	DN	25					
14	XFR	67		54	X	36		94	DIV	35					
15	0	00		55	b	14		95	UP	27					
16	UP	27		56	X	36		96	d	17					
17	c	16		57	DN	25		97	PNT	45	S				
18	DIV	35		58	XEY	30		98	PNT	45					
19	d	17		59	YTO	40		99	PNT	45					
1a	DIV	35		5a	d	17		9a	PNT	45					
1b	f	15		5b	DIV	35		9b	PNT	45					
1c	-	34		5c	YTO	40		9c	PNT	45					
1d	YTO	40		5d	2	02		9d	END	46					
20	5	05		60	XFR	67									
21	XFR	67		61	5	05									
22	1	01		62	X	36									
23	UP	27		63	b	14									
24	b	14		64	UP	27									
25	DIV	35		65	1	01									
26	d	17		66	-	34									
27	DIV	35		67	YTO	40									
28	f	15		68	b	14									
29	-	34		69	DN	25									
2a	YTO	40		6a	DIV	35									
2b	4	04		6b	UP	27									
2c	XFR	67		6c	d	17									
2d	6	06		6d	PNT	45	S								
30	UP	27		70	PNT	45									
31	d	17		71	CNT	47									
32	DIV	35		72	XFR	67									
33	XFR	67		73	2	02									
34	5	05		74	UP	27									
35	-	34		75	XFR	67									
36	XFR	67		76	4	04									
37	1	01		77	X	36									
38	UP	27		78	c	16									
39	d	17		79	UP	27									
3a	DIV	35		7a	1	01									
3b	b	14		7b	-	34									
3c	DIV	35		7c	YTO	40									
3d	DN	25		7d	c	16									

ANALYSIS OF VARIANCE F TEST FOR MEANS

This program operates on 2 or more columns of data containing equal numbers of items, classified according to a single criterion:

$$\begin{array}{c|l} & \overset{K}{\overline{\qquad\qquad\qquad\qquad}} \\ R & \begin{array}{l} X_{11}\ X_{12}\ X_{13}\ \cdot\ \cdot\ \cdot\ X_{1K} \\ X_{21}\ X_{22}\ \cdot\ \cdot\ \cdot \\ X_{31}\ X_{32}\ \cdot\ \cdot\ \cdot \\ \cdot\quad\cdot\quad\cdot \\ \cdot\quad\cdot\quad\cdot \\ \cdot\quad\cdot\quad\cdot \\ X_{R1} \end{array} \end{array}$$

The program generates an Analysis of Variance Table containing the sums of squares among the column means and within each group. This essentially amounts to breaking down the total variation in the data to its component parts, i.e., among and within groups. It subsequently computes the corresponding mean squares and F ratio for testing the hypothesis that the means of the columns are not significantly different.

ANOV Table

	Sum of Squares	Degrees of Freedom	Mean Squares
Among Means	$Sam = \sum_{i=1}^{K} \left[\frac{\left(\sum_{j=1}^{R} X_{ij}\right)^2}{R}\right] - \frac{\left[\sum_{i=1}^{K}\sum_{j=1}^{R} X_{ij}\right]^2}{N}$	K - 1 df_1	Sam/K-1
Within Groups	$Swg = \sum_{i=1}^{K}\sum_{j=1}^{R} (X_{ij}^2) - \sum_{i=1}^{K} \left[\frac{\left(\sum_{j=1}^{R} X_{ij}\right)^2}{R}\right]$	N - K df_2	Swg/N-K
Total	$St = \sum_{i=1}^{K}\sum_{j=1}^{R} (X_{ij})^2 - \frac{\left[\sum_{i=1}^{K}\sum_{j=1}^{R} X_{ij}\right]^2}{N}$	N - 1	

$$F = \frac{\dfrac{\sum_{i=1}^{K}\left[\dfrac{\left(\sum_{j=1}^{R} X_{ij}\right)^2}{R}\right] - \dfrac{\left[\sum_{i=1}^{K}\sum_{j=1}^{R} X_{ij}\right]^2}{N}}{K-1}}{\dfrac{\sum_{i=1}^{K}\sum_{j=1}^{R}(X_{ij})^2 - \sum_{i=1}^{K}\left[\dfrac{\left(\sum_{j=1}^{R} X_{ij}\right)^2}{R}\right]}{N-K}} = \frac{\frac{Sam}{K-1}}{\frac{Swg}{N-K}}$$

Where:

i = Index of Rows
j = Index of Columns
K = No. of Columns
R = No. of Rows
N = Total Number of Observations

For abbreviations in the program:

Sam = A - B
Swg = C - A

The calculated F is to be compared with the value found in a table of F percentiles at $\nu_1 = K - 1$, $\nu_2 = N - K$. The hypothesis is rejected if $F_{calc.} > F_{table}$. The latter will depend on the level of confidence used in the table (usually 95%).

Data is entered column by column beginning with column 1. Thus data is entered as $X_{11}, X_{21}, X_{31}, \ldots, X_{R1}; X_{12}, X_{22}, \ldots, X_{R2}$; etc.

This program was written by Dr. Thomas F. Brodasky of the UPJOHN COMPANY, Kalamazoo, Michigan.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: X_{ij} → X *

PRESS: CONTINUE

NO ← Has the last entry of the j^{th} column been entered? → YES

PRESS: SET FLAG

*After all columns are entered;

PRESS: GO TO (4)(9)

PRESS: CONTINUE

DISPLAY

Sam	Z
Swg	Y
St	X

PRESS: CONTINUE

DISPLAY

Sam/K-1	Z
Swg/N-K	Y
F	X

To run another case:

PRESS: END

EXAMPLE

		K = 1	K = 2	K = 3
R	1	2.10	2.20	3.00
	2	3.00	2.30	2.20
	3	2.60	2.90	2.80
	4	2.50	2.60	2.80
	5	2.80	2.10	2.40

$df_1 = K - 1 = 2$

$df_2 = N - K = 12$

Answer:

Sam = 0.1373

Swg = 1.3200

St = 1.4573

Sam/K-1 = 0.0687

Swg/N-K = 0.1100

F_{calc} = 0.6242

F_{table} = 3.89 (@ 95% confidence)

00	CLR	20		40	1	01		80	UP	27	
01	XTO	23		41	YEX	24		81	RUP	22	
02	9	11		42	9	11		82	DIV	35	
03	XTO	23		43	+	33		83	XEY	30	
04	a	13		44	YEX	24		84	PNT	45	
05	XTO	23		45	9	11		85	PNT	45	
06	b	14		46	GTO	44		86	PNT	45	
07	XTO	23		47	0	00		87	PNT	45	
08	c	16		48	b	14		88	END	46	
09	XTO	23		49	d	17					
0a	d	17		4a	UP	27					
0b	CLR	20		4b	X	36					
0c	STP	41	ENTRY	4c	b	14					
0d	IFG	43		4d	DIV	35					
10	2	02		50	YTO	40					
11	7	07		51	f	15					
12	PNT	45		52	c	16					
13	PNT	45		53	XEY	30					
14	UP	27		54	-	34					
15	X	36		55	YTO	40					
16	UP	27		56	d	17					
17	RDN	31		57	a	13					
18	a	13		58	UP	27					
19	+	33		59	c	16					
1a	YTO	40		5a	-	34					
1b	a	13		5b	RUP	22					
1c	1	01		5c	a	13					
1d	RUP	22		5d	XEY	30					
20	AC+	60		60	f	15					
21	CLX	37		61	-	34					
22	UP	27		62	d	17					
23	UP	27		63	RDN	31					
24	GTO	44		64	PNT	45	S				
25	0	00		65	PNT	45					
26	c	16		66	1	01					
27	e	12		67	YEX	24					
28	UP	27		68	9	11					
29	b	14		69	YTO	40					
2a	+	33		6a	d	17					
2b	YTO	40		6b	-	34					
2c	b	14		6c	RDN	31					
2d	f	15		6d	DIV	35					
30	UP	27		70	YTO	40					
31	X	36		71	e	12					
32	e	12		72	a	13					
33	DIV	35		73	UP	27					
34	c	16		74	c	16					
35	+	33		75	-	34					
36	YTO	40		76	UP	27					
37	c	16		77	b	14					
38	f	15		78	XEY	30					
39	UP	27		79	d	17					
3a	d	17		7a	-	34					
3b	+	33		7b	RDN	31					
3c	YTO	40		7c	DIV	35					
3d	d	17		7d	e	12					

SECTION VIII OPERATIONS RESEARCH

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The program determines P_n (the probability of a waiting line of length n) and $\overline{n}$ (the mean value of a waiting line) for a single-station Queueing problem.

The equations used are:

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

$$\overline{n} = \frac{\frac{\lambda}{\mu}}{\left(1 - \frac{\lambda}{\mu}\right)}$$

where

λ = Mean arrival rate
μ = Mean service rate
$\frac{\lambda}{\mu} < 1$
n = Number of units in the waiting line.

Reference: Introduction to Operations Research, Churchman, Ackoff, and Arnoff, Wiley, 1964.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA:

n	Z
λ	Y
μ	X

PRESS: CONTINUE

DISPLAY

P_n	Z
$\bar{n}$	Y
2	X

To run another case :

PRESS: CONTINUE

EXAMPLE

If the mean arrival rate is 10 units/day, and the mean service rate is 20 units/day, what is the probability of 4 units in the waiting line?

n	=	4
λ	=	10
μ	=	20

Results:

P_4	=	.03125
$\bar{n}$	=	1

00	CLR	20	
01	1	01	
02	STP	41	ENTRY
03	PNT	45	
04	PNT	45	
05	DIV	35	
06	YTO	40	
07	e	12	
08	1	01	
09	XEY	30	
0a	-	34	
0b	YTO	40	
0c	f	15	
0d	e	12	
10	RUP	22	
11	XEY	30	
12	LN	65	
13	X	36	
14	XEY	30	
15	EXP	74	
16	RUP	22	
17	X	36	
18	UP	27	
19	RCL	61	
1a	DIV	35	
1b	2	02	
1c	PNT	45	
1d	PNT	45	
20	END	46	

$\frac{\lambda}{\mu}$

$(1 - \frac{\lambda}{\mu})$

QUEUEING WITH FINITE CUSTOMERS

This program computes the "average number in system, $\bar{n}$" and the "queue length, L_Q" when the following parameters are known:

N = Number of Available Customers
M = Number of Service Channels
MTTR = Mean Service Channels
MTBF = Mean Time Between Calls for Service Per Customer

The recursive equation applied is:

$$\frac{X}{1-X}\,(N-n+1)\,P_{n-1} = \begin{cases} nP_n & \text{when } 0 \le n \le M \\ MP_n & \text{when } M < n \le N \end{cases}$$

where

$$X = \frac{MTTR}{MTTR + MTBF}$$

and

P_n = Probability of n customers in the system.

The above recursive formula is iteratively applied to solve

$$\bar{n} = \frac{\Sigma n P_n}{\Sigma P_n} \quad \text{(average number in system)}$$

$$F = \frac{\frac{\bar{n}}{N} - 1}{X - 1} \quad \text{(efficiency factor)}$$

and $L_Q = N(1 - F)$. (queue length)

This program varies M (number of service channels) and iteratively solves for $\bar{n}$, F, and L_Q.

For certain highly unlikely combinations of N, MTTR and MTBF, an overflow condition will occur. However the program is protected against this eventuality and it re-cycles to the initial DISPLAY when an illegal combination is attempted.

Computing times for each channel value (M) can range between 30 - 60 seconds.

This program was written by Mr. E.J. Schmidt of ITT Federal Electric, Paramus, N.J.

Reference: Finite Queueing Tables, Peck and Hazelwood, Wiley, 1958.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA:

N	Z
MTTR	Y
MTBF	X

PRESS: CONTINUE

DISPLAY

M	Z
$\bar{n}$	Y
L_Q	X

To run another case:

EXAMPLE

Given a "system" with 625 units (customers) each having a 30,000 hour MTBF, and maintenance conditions such that the MTTR is 50 hours, find the average number in system ($\bar{n}$) and the queue length L_Q for varying M (number of channels).

Input data:

N = 625 (number of units - customers - available)

MTTR = 50 (mean time to restore)

MTBR = 30,000 (mean time between failures, at 'failure it "calls' for service")

Results:

```
 1
31.72550
30.73671
 2
 1.41706
  .37776
 3
 1.09101
  .05116
 4
 1.04723
  .00731
```

00	CLR	20		40	DN	25		80	c	16	
01	XTO	23		41	YEX	24		81	X	36	
02	9	11		42	9	11		82	b	14	
03	1	01		43	+	33		83	RUP	22	
04	STP	41	ENTRY	44	YTO	40		84	a	13	
05	PNT	45		45	9	11		85	RUP	22	
06	PNT	45		46	EEX	26		86	PNT	45	S
07	DIV	35		47	9	11		87	PNT	45	
08	YTO	40		48	8	10		88	STP	41	
09	d	17		49	X<Y	52		89	c	16	
0a	DN	25		4a	0	00		8a	RDN	31	
0b	0	00		4b	0	00		8b	GTO	44	
0c	YTO	40		4c	DN	25		8c	1	01	
0d	c	16		4d	a	13		8d	1	01	
10	RUP	22		50	+	33		90	END	46	
11	1	01		51	YTO	40					
12	+	33		52	a	13					
13	YTO	40		53	DN	25					
14	b	14		54	GTO	44					
15	RUP	22		55	2	02					
16	+	33		56	2	02					
17	YTO	40		57	CLR	20					
18	f	15		58	YEX	24					
19	EEX	26		59	9	11					
1a	9	11		5a	a	13					
1b	3	03		5b	DIV	35					
1c	CHS	32		5c	YTO	40					
1d	XTO	23		5d	a	13					
20	a	13		60	c	16					
21	UP	27		61	DIV	35					
22	1	01		62	1	01					
23	UP	27		63	-	34					
24	CHS	32		64	UP	27					
25	AC+	60		65	d	17					
26	RCL	61		66	XEY	30					
27	c	16		67	+	33					
28	X<Y	52		68	RDN	31					
29	5	05		69	CHS	32					
2a	7	07		6a	X	36					
2b	DN	25		6b	.	21					
2c	d	17		6c	9	11					
2d	X	36		6d	X<Y	52					
30	f	15		70	7	07					
31	X	36		71	c	16					
32	e	12		72	XEY	30					
33	UP	27		73	DIV	35					
34	b	14		74	b	14					
35	X<Y	52		75	X	36					
36	XEY	30		76	c	16					
37	CNT	47		77	XEY	30					
38	DN	25		78	INT	64					
39	DIV	35		79	GTO	44					
3a	UP	27		7a	1	01					
3b	DN	25		7b	0	00					
3c	e	12		7c	RDN	31					
3d	X	36		7d	-	34					

10^{-95}

N

$\frac{MTTR}{MTBF}$

QUEUEING WITH INFINITE CUSTOMERS

This program computes the "queue length" of a system when the following system parameters are known:

K = Number of Service Stations

MTTR = Mean Service Time

MTBF = "Ensemble" Mean Time Between Calls for Service From Total Source of Customers

The equation applied is:

$$L_Q = \frac{1}{(K - R)\left[\dfrac{\sum_{n=0}^{K-1} \frac{1}{n!} R^n}{\frac{K}{K-R}\frac{1}{K!}R^K} + 1\right]}$$

where,

$$R = \frac{MTTR}{MTBF}$$

L_Q is defined to be "average length of non-empty queues".

This program varies K (number of service stations) and determines L_Q for each K. The user can optionally output

A_{VS} = Average Time In System
A_{VW} = Average Time In Waiting

This program was written by Mr. E.J. Schmidt of ITT Federal Electric, Paramus, N.J.

Reference: Finite Queueing Models, Peck and Hazelwood, Wiley, 1958.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

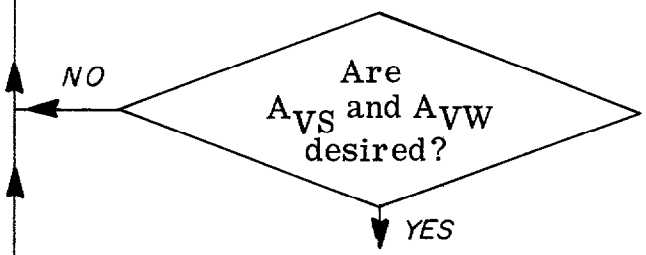
ENTER DATA:

MTTR → Y
MTBF → X

PRESS: CONTINUE

DISPLAY

0	Z
K	Y
L_Q	X



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

K	Z
A_{VS}	Y
A_{VW}	X

To run another case

PRESS: END

EXAMPLE

Consider the problem posed by program STAT-PAC VIII-2. To convert the finite model for an infinite model, determine the ensemble

$$\text{MTBF} = \frac{\text{unit MTBF}}{N}$$

For the example,

Unit MTBF = 30,000
N = 625

Thus

MTTR = 50
ensemble MTBF = 48

Results:

K	=	2.
L_Q	=	.37225 *
	=	3.
	=	.05130
	=	4.
	=	.00790
	=	5.
	=	.00115
	=	6.
	=	.00015

.
.
.

*PRESS: SET FLAG – CONTINUE to obtain:

A_{VS} = 68.61227

A_{VW} = 18.61227

00	CLR	20		40	7	07		80	XEY	30	
01	1	01		41	d	17		81	X	36	
02	STP	41	ENTRY	42	UP	27		82	XEY	30	
03	PNT	45		43	c	16		83	+	33	
04	PNT	45		44	+	33		84	PNT	45	
05	YTO	40		45	YTO	40		85	PNT	45	
06	9	11		46	d	17		86	STP	41	
07	DIV	35		47	RCL	61		87	DN	25	
08	YTO	40		48	a	13		88	GTO	44	
09	f	15		49	X>Y	53		89	0	00	
0a	DN	25		4a	5	05		8a	d	17	
0b	INT	64		4b	1	01		8b	END	46	
0c	UP	27		4c	XTO	23					
0d	1	01		4d	e	12					
10	+	33		50	SFL	54					
11	YTO	40		51	1	01					
12	a	13		52	GTO	44					
13	XTO	23		53	1	01					
14	d	17		54	7	07					
15	XTO	23		55	d	17					
16	e	12		56	RUP	22					
17	XTO	23		57	DIV	35					
18	b	14		58	a	13					
19	XTO	23		59	DIV	35					
1a	c	16		5a	UP	27					
1b	f	15		5b	f	15					
1c	UP	27		5c	-	34					
1d	b	14		5d	1	01					
20	DIV	35		60	RDN	31					
21	c	16		61	X	36					
22	X	36		62	RDN	31					
23	YTO	40		63	XEY	30					
24	c	16		64	+	33					
25	b	14		65	RDN	31					
26	UP	27		66	X	36					
27	e	12		67	DN	25					
28	X=Y	50		68	DIV	35					
29	3	03		69	a	13					
2a	4	04		6a	XEY	30					
2b	1	01		6b	UP	27					
2c	+	33		6c	CLX	37					
2d	YTO	40		6d	RDN	31					
30	b	14		70	PNT	45					
31	GTO	44		71	PNT	45					
32	1	01		72	STP	41					
33	b	14		73	IFG	43					
34	IFG	43		74	7	07					
35	5	05		75	9	11					
36	5	05		76	GTO	44					
37	1	01		77	0	00					
38	UP	27		78	d	17					
39	CLX	37		79	UP	27					
3a	AC+	60		7a	YEX	24					
3b	a	13		7b	9	11					
3c	X=Y	50		7c	YTO	40					
3d	4	04		7d	9	11					

SECTION IX RELIABILITY AND QUALITY CONTROL

			Page Number
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IX-2	Weibull Distribution Via Failed and Suspended Sets	9100B ONLY	7
IX-3	General Statistics For Quality Control		17
IX-4	Repeatability		21

WEIBULL DISTRIBUTION PARAMETER CALCULATION FOR FAILURE DATA

The Weibull probability density function is given by

$$f(X) = \frac{bX^{(b-1)}}{\theta^b} e^{-(\frac{X}{\theta})^b}$$

and the cumulative distribution function is given by

$$F(X) = 1 - e^{-(\frac{X}{\theta})^b}$$

For a set of data, the Weibull parameters b and θ are to be calculated for these functions.

A common application is to use Weibull analysis for failure data where all samples are tested to failure. To use the program, list the items in order of increasing time to failure. The number of items and times to failure are entered. The parameters b , θ , and r are displayed. r is a correlation coefficient indicating goodness of fit. The time required for 10% (B_{10}) to fail is displayed and times to other failure percentages ($B_{\%}$) may be requested.

The Median Rank (M.R.) is calculated by the equation

$$M.R. = \frac{j-.3}{N+4}$$

where j = failure order number

N = number of samples tested

This is an approximation of F(X).

The cumulative distribution function is linearized into the form

$$bLnX - bLn\theta = LnLn\left(\frac{1}{1-F(X)}\right)$$

A least squares fit is performed which calculates the slope, intercept, and correlation coefficient. The solution is similar to the linear regression program 09100-70803. Thus estimates of b and θ are obtained.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

ENTER DATA: N → X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
j	X

(j indicates point to be entered)

ENTER DATA: t_j → X (Data must be ordered)

NO ← Last Entry? → YES

PRESS: CONTINUE

DISPLAY

r	Z
θ	Y
b	X

PRESS: CONTINUE

DISPLAY

0	Z
B%	Y
%	X

(first time will be 10%)

ENTER DATA: % → X

TO RESTART A NEW PROBLEM

PRESS: END

EXAMPLES

TEST DATA

Hours to failure (must be ordered)

34

60

75

95

119

158

N = 6 (number of samples)

r = .999

θ = 104.091

b = 1.953

B_{10} = 32.887

B_{90} = 159.539

00	CLR	20	Plus Page
01	XTO	23	
02	d	17	
03	XTO	23	
04	c	16	
05	XTO	23	
06	b	14	
07	STP	41	ENTRY
08	PNT	45	
09	PNT	45	
0a	XTO	23	
0b	8	10	
0c	UP	27	
0d	.	21	

40	X	36
41	b	14
42	+	33
43	YTO	40
44	b	14
45	RUP	22
46	UP	27
47	X	36
48	c	16
49	+	33
4a	YTO	40
4b	c	16
4c	a	13
4d	UP	27

80	CNT	47
81	CNT	47
82	CNT	47
83	CNT	47
84	CNT	47
85	CNT	47
86	CNT	47
87	CNT	47
88	CNT	47
89	CNT	47
8a	CNT	47
8b	CNT	47
8c	CNT	47
8d	CNT	47

10	4	04	
11	+	33	
12	DN	25	
13	XTO	23	
14	9	11	
15	1	01	
16	XTO	23	
17	a	13	
18	STP	41	ENTRY
19	PNT	45	
1a	PNT	45	
1b	LN	65	
1c	UP	27	
1d	a	13	

50	1	01
51	+	33
52	XFR	67
53	8	10
54	XEY	30
55	UP	27
56	0	00
57	RDN	31
58	X>Y	53
59	6	06
5a	4	04
5b	XTO	23
5c	a	13
5d	XEY	30

90	CNT	47
91	CNT	47
92	CNT	47
93	CNT	47
94	CNT	47
95	CNT	47
96	CNT	47
97	CNT	47
98	CNT	47
99	CNT	47
9a	CNT	47
9b	CNT	47
9c	CNT	47
9d	CNT	47

20	UP	27
21	.	21
22	3	03
23	-	34
24	XFR	67
25	9	11
26	DIV	35
27	1	01
28	XEY	30
29	-	34
2a	1	01
2b	XEY	30
2c	DIV	35
2d	DN	25

60	DN	25
61	GTO	44
62	1	01
63	8	10
64	e	12
65	UP	27
66	a	13
67	DIV	35
68	YE	24
69	f	15
6a	DIV	35
6b	YTO	40
6c	e	12
6d	X	36

a0	CNT	47
a1	CNT	47
a2	CNT	47
a3	CNT	47
a4	CNT	47
a5	CNT	47
a6	CNT	47
a7	CNT	47
a8	CNT	47
a9	CNT	47
aa	CNT	47
ab	CNT	47
ac	CNT	47
ad	CNT	47

30	LN	65
31	LN	65
32	XEY	30
33	AC+	60
34	UP	27
35	X	36
36	XEY	30
37	YE	24
38	d	17
39	+	33
3a	YE	24
3b	d	17
3c	CNT	47
3d	DN	25

70	e	12
71	X	36
72	d	17
73	XEY	30
74	-	34
75	YTO	40
76	d	17
77	c	16
78	UP	27
79	f	15
7a	GTO	44
7b	-	34
7c	0	00
7d	0	00

	N
	N+4

```
b0   CNT   47      Plus
b1   CNT   47      Page
b2   CNT   47
b3   CNT   47
b4   CNT   47
b5   CNT   47
b6   CNT   47
b7   CNT   47
b8   CNT   47
b9   CNT   47
ba   CNT   47
bb   CNT   47
bc   CNT   47
bd   CNT   47

c0   CNT   47
c1   CNT   47
c2   CNT   47
c3   CNT   47
c4   CNT   47
c5   CNT   47
c6   CNT   47
c7   CNT   47
c8   CNT   47
c9   CNT   47
ca   CNT   47
cb   CNT   47
cc   CNT   47
cd   CNT   47

d0   CNT   47
d1   CNT   47
d2   CNT   47
d3   CNT   47
d4   CNT   47
d5   CNT   47
d6   CNT   47
d7   CNT   47
d8   CNT   47
d9   CNT   47
da   CNT   47
db   CNT   47
dc   CNT   47
dd   CNT   47
```

00	UP	27	Minus Page	40	YTO	40
01	X	36		41	f	15
02	a	13		42	RUP	22
03	X	36		43	LN	65
04	DN	25		44	XTO	23
05	-	34		45	e	12
06	YTO	40		46	1	01
07	c	16		47	0	00
08	b	14		48	XTO	23
09	UP	27		49	d	17
0a	f	15		4a	UP	27
0b	UP	27		4b	EEX	26
0c	e	12		4c	2	02
0d	X	36		4d	DIV	35
10	a	13		50	1	01
11	X	36		51	-	34
12	DN	25		52	XEY	30
13	-	34		53	CHS	32
14	d	17		54	DIV	35
15	UP	27		55	DN	25
16	DN	25		56	LN	65
17	√	76		57	LN	65
18	DIV	35		58	UP	27
19	c	16		59	f	15
1a	√	76		5a	X	36
1b	DIV	35		5b	e	12
1c	d	17		5c	+	33
1d	RUP	22		5d	0	00
20	XEY	30		60	RDN	31
21	DIV	35		61	EXP	74
22	YE	24		62	XEY	30
23	e	12		63	d	17
24	e	12		64	CNT	47
25	X	36		65	RUP	22
26	f	15		66	PNT	45
27	XEY	30		67	RUP	22
28	-	34		68	PNT	45
29	e	12		69	RUP	22
2a	CHS	32		6a	PNT	45
2b	DIV	35		6b	PNT	45
2c	XEY	30		6c	STP	41
2d	EXP	74		6d	GTO	44
30	XEY	30		70	4	04
31	CHS	32		71	8	10
32	CNT	47	S	72	END	46
33	RUP	22				
34	PNT	45				
35	RUP	22				
36	PNT	45				
37	RUP	22				
38	PNT	45				
39	PNT	45				
3a	UP	27				
3b	1	01				
3c	XEY	30				
3d	DIV	35				

WEIBULL ANALYSIS VIA FAILED AND SUSPENDED SETS

This program determines the Weibull parameters b, and θ , from a grouping of failure data into sets of failed items and suspended items.

A Weibull analysis is obtained by proper choice of a representative point from each failed set. This representative point is the latest failure in the set of failed items.

The Weibull distribution function is given by:

$$f(X) = \frac{bX^{(b-1)}}{\theta^b} \quad e^{-(\frac{X}{\theta})^b}$$

The program outputs:

r , Goodness of Fit
b , Weibull Slope
M, Median Life
Q_5, 5%, Rank at any point X
R, Median rank at any point X
Q_{95}, 95%, Rank at any point X
θ, Time for 63.2% to fail
B_{10}, Time required for 10% to fail

The formulas applied are:

$$\lambda_K = \lambda_{K-1} + T_K \left(\frac{N + 1 - \lambda_{K-1}}{1 + N - \sum_{i=0}^{K-1} S_i - \sum_{i=1}^{K-1} T_i} \right)$$

where:

λ_K = The mean order number of K^{th} representative point
S_i = Size of the (i + 1)th suspended set
T_i = Size of the i^{th} failed set
N = Total number of failed and suspended sets

The latest failure in T_K has a median rank given by

$$R_K = \frac{\lambda_K - .3}{N + .4}$$

The 5% and 95% ranks at point whose mean order number is λ is

$$Q_5 = \frac{\lambda}{N+1} + t_5 \sigma$$

$$Q_{95} = \frac{\lambda}{N+1} + t_{95} \sigma$$

t_5 and t_{95} are given by

$$t_5 = -1.645 + .2885\ \alpha_3 + .0565\ \alpha_3^2 \qquad \text{for } \alpha_3 < 0$$

$$t_5 = -1.645 + .2871\ \alpha_3 + .041\ \alpha_3^2 \qquad \text{for } \alpha_3 > 0$$

$$t_{95} = 1.645 + .2871\ \alpha_3 - .041\ \alpha_3^2 \qquad \text{for } \alpha_3 < 0$$

$$t_{95} = 1.645 + .2885\ \alpha_3 - .0565\ \alpha_3^2 \qquad \text{for } \alpha_3 > 0$$

Note that t_5 is always less than zero and t_{95} is always greater than zero.

$$\alpha_3 = \frac{2(1-2Q)}{\sigma\sqrt{N+3}} = \text{the skewness of the rank}$$

$$\sigma = \sqrt{\frac{Q(1-Q)}{N+2}}$$

The cumulative distribution

$$F(X) = 1 - e^{-(\frac{X}{\theta})^b}$$

is linearized into the form

$$b \ln X - b \ln \theta = \ln \ln \left(\frac{1}{1 - F(X)} \right)$$

The X_i used for this linearizing are the latest failures in each failed set.

The program can optionally output the time required for Y% to fail.

Program Assumptions:

$R < 1$

$.0155\ \alpha_3^2 < 1$

The sets S_i are ordered from shortest life to longest life.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM 1: Side A followed by Side B

PRESS: CONTINUE

PRESS: (X→)(-)(e)
(X→)(-)(f)

ENTER DATA:

N → Y
F → X

(F is the number of failed sets)

PRESS: CONTINUE

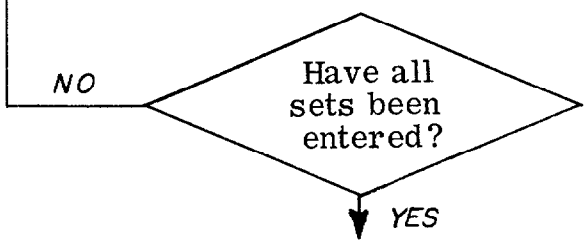
ENTER DATA:

S_i → Z (Size of the (i+1) suspended set)
T_i → Y (Size of the i^{th} failed set)
X_i → X (Latest failure in failed set)

PRESS: CONTINUE

DISPLAY

Q5 ——— Z
R ——— Y
Q95 ——— X



DISPLAY

1 ——— Z
1 ——— Y
1 ——— X

USER INSTRUCTIONS (Con't)

(Y = 0 when all sets have been entered)

PRESS: END

ENTER PROGRAM 2: Side A

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM 2: Side B

ENTER DATA:

N → Y
F → X

PRESS: CONTINUE

DISPLAY

M ——— Z
b ——— Y
r ——— X

PRESS: CONTINUE

DISPLAY

N ——— Z
θ ——— Y
0 ——— X

PRESS: CONTINUE

DISPLAY

0 ——— Z
B% ——— Y
% ——— X

If another B% is desired enter the % →X.

To run another case, repeat USER INSTRUCTIONS.

EXAMPLE (Con't)

Input Data

N = 18
F = 4

S_i	T_i	X_i
2.	3.	1740.
1.	2.	2700.
2.	1.	3510.
4.	3.	4304.
9	9	

Results:

Q_S	R	Q_{95}
.05654	.16592	.33118
.14569	.29675	.48201
.20883	.37670	.56538
.69452	.85640	.95876

M = 3362.62
b = 2.329
r = .91171
N = 18
θ = 3935.60

$B_{10\%}$ = 1497.78
$B_{50\%}$ = 3362.62

00	CLR	20	Plus Page	40	DN	25	80	0	00	
01	XTO	23		41	DIV	35	81	0	00	
02	d	17		42	c	16	82	0	00	
03	STP	41	ENTRY	43	AC+	60	83	0	00	
04	RUP	22		44	RCL	61	84	0	00	
05	1	01		45	.	21	85	0	00	
06	RUP	22		46	3	03	86	0	00	
07	+	33		47	-	34	87	0	00	
08	YTO	40		48	a	13	88	0	00	
09	a	13		49	INT	64	89	0	00	
0a	RUP	22		4a	GTO	44	8a	0	00	
0b	XTO	23		4b	-	34	8b	0	00	
0c	b	14		4c	0	00	8c	0	00	
0d	STP	41	ENTRY	4d	0	00	8d	0	00	
10	PNT	45		50	0	00	90	0	00	
11	LN	65		51	0	00	91	0	00	
12	YE	24		52	0	00	92	0	00	
13	-	34		53	0	00	93	0	00	
14	e	12		54	0	00	94	0	00	
15	+	33		55	0	00	95	0	00	
16	YE	24		56	0	00	96	0	00	
17	-	34		57	0	00	97	0	00	
18	e	12		58	0	00	98	0	00	
19	YTO	40		59	0	00	99	0	00	
1a	c	16		5a	0	00	9a	0	00	
1b	XEY	30		5b	0	00	9b	0	00	
1c	DN	25		5c	0	00	9c	0	00	
1d	UP	27		5d	0	00	9d	0	00	
20	X	36		60	0	00	a0	0	00	
21	XTO	23		61	0	00	a1	0	00	
22	5	05		62	0	00	a2	0	00	
23	XFR	67		63	0	00	a3	0	00	
24	-	34		64	0	00	a4	0	00	
25	f	15		65	0	00	a5	0	00	
26	+	33		66	0	00	a6	0	00	
27	YTO	40		67	0	00	a7	0	00	
28	-	34		68	0	00	a8	0	00	
29	f	15		69	0	00	a9	0	00	
2a	c	16		6a	0	00	aa	0	00	
2b	XEY	30		6b	0	00	ab	0	00	
2c	d	17		6c	0	00	ac	0	00	
2d	RUP	22		6d	0	00	ad	0	00	
30	+	33		70	0	00				
31	YTO	40		71	0	00				
32	d	17		72	0	00				
33	RCL	61		73	0	00				
34	a	13		74	0	00				
35	INT	64		75	0	00				
36	-	34		76	0	00				
37	RDN	31		77	0	00				
38	X	36		78	0	00				
39	f	15		79	0	00				
3a	RUP	22		7a	0	00				
3b	-	34		7b	0	00				
3c	d	17		7c	0	00				
3d	+	33		7d	0	00				

$\alpha_3, t_5 + t_{95}$

$(W+1) \cdot 155\alpha^2$

$R + F_L$

θ

σ ΣS

$.576\alpha_3$ ΣA

ΣT

			Plus Page
b0	0	00	
b1	0	00	
b2	0	00	
b3	0	00	
b4	0	00	
b5	0	00	
b6	0	00	
b7	0	00	
b8	0	00	
b9	0	00	
ba	0	00	
bb	0	00	
bc	0	00	
bd	0	00	
c0	0	00	
c1	0	00	
c2	0	00	
c3	0	00	
c4	0	00	
c5	0	00	
c6	0	00	
c7	0	00	
c8	0	00	
c9	0	00	
ca	0	00	
cb	0	00	
cc	0	00	
cd	0	00	
d0	0	00	
d1	0	00	
d2	0	00	
d3	0	00	
d4	0	00	
d5	0	00	
d6	0	00	
d7	0	00	
d8	0	00	
d9	0	00	
da	0	00	
db	0	00	
dc	0	00	
dd	0	00	

00	UP	27	Minus Page	40	√	76		80	RUP	22	
01	.	21		41	XTO	23		81	X	36	
02	6	06		42	5	05		82	UP	27	
03	-	34		43	1	01		83	.	21	
04	DN	25		44	+	33		84	0	00	
05	DIV	35		45	RUP	22		85	5	05	
06	b	14		46	XTO	23		86	6	06	
07	XEY	30		47	c	16		87	5	05	
08	+	33		48	-	34		88	XEY	30	
09	YTO	40		49	-	34		89	X	36	
0a	b	14		4a	2	02		8a	X	36	
0b	UP	27		4b	X	36		8b	1	01	
0c	1	01		4c	XFR	67		8c	.	21	
0d	-	34		4d	5	05		8d	6	06	
10	XEY	30		50	RUP	22		90	4	04	
11	CHS	32		51	X	36		91	5	05	
12	DIV	35		52	DN	25		92	-	34	
13	DN	25		53	DIV	35		93	DN	25	
14	LN	65		54	YTO	40		94	+	33	
15	LN	65		55	0	00		95	XFR	67	
16	UP	27		56	.	21		96	6	06	
17	UP	27		57	5	05		97	YE	24	
18	X	36		58	7	07		98	0	00	
19	YE	24		59	5	05		99	X>Y	53	
1a	8	10		5a	6	06		9a	a	13	
1b	+	33		5b	XEY	30		9b	d	17	
1c	YE	24		5c	X	36		9c	YE	24	
1d	8	10		5d	UP	27		9d	0	00	
20	DN	25		60	X	36		a0	RUP	22	
21	YE	24		61	.	21		a1	-	34	
22	9	11		62	0	00		a2	-	34	
23	+	33		63	1	01		a3	DN	25	
24	YE	24		64	5	05		a4	+	33	
25	9	11		65	5	05		a5	a	13	
26	XFR	67		66	X	36		a6	UP	27	
27	5	05		67	0	00		a7	INT	64	
28	X	36		68	RUP	22		a8	-	34	
29	XFR	67		69	XTO	23		a9	DN	25	
2a	7	07		6a	6	06		aa	-	34	
2b	+	33		6b	X>Y	53		ab	YE	24	
2c	YTO	40		6c	d	17		ac	0	00	
2d	7	07		6d	9	11		ad	XFR	67	
30	RCL	61		70	RUP	22					
31	a	13		71	+	33					
32	INT	64		72	YE	24					
33	DIV	35		73	0	00					
34	UP	27		74	RUP	22					
35	1	01		75	a	13					
36	+	33		76	+	33					
37	RUP	22		77	.	21					
38	-	34		78	2	02					
39	X	36		79	8	10					
3a	RDN	31		7a	8	10					
3b	XEY	30		7b	5	05					
3c	DIV	35		7c	YTO	40					
3d	XEY	30		7d	a	13					

b0	0	00	Minus
b1	-	34	Page
b2	RUP	22	
b3	XFR	67	
b4	5	05	
b5	X	36	
b6	RUP	22	
b7	X	36	
b8	c	16	
b9	+	33	
ba	RUP	22	
bb	+	33	
bc	b	14	
bd	YE	24	
c0	b	14	
c1	INT	64	
c2	-	34	
c3	XEY	30	
c4	YE	24	
c5	b	14	
c6	RUP	22	
c7	PNT	45	S
c8	PNT	45	
c9	b	14	
ca	UP	27	
cb	1	01	
cc	-	34	
cd	YTO	40	
d0	b	14	
d1	X>Y	53	
d2	UP	27	
d3	UP	27	
d4	STP	41	
d5	GTO	44	
d6	+	33	
d7	1	01	
d8	0	00	
d9	RUP	22	
da	-	34	
db	GTO	44	
dc	7	07	
dd	2	02	

00	XTO	23	Plus Page
01	a	13	
02	YTO	40	
03	e	12	
04	XFR	67	
05	-	34	
06	e	12	
07	UP	27	
08	X	36	
09	UP	27	
0a	XFR	67	
0b	8	10	
0c	X	36	
0d	a	13	
10	YE	24	
11	7	07	
12	X	36	
13	XEY	30	
14	YE	24	
15	7	07	
16	XEY	30	
17	-	34	
18	YE	24	
19	7	07	
1a	XFR	67	
1b	-	34	
1c	f	15	
1d	X	36	
20	DN	25	
21	XEY	30	
22	-	34	
23	XFR	67	
24	7	07	
25	XEY	30	
26	DIV	35	
27	YTO	40	
28	b	14	
29	XTO	23	
2a	d	17	
2b	XFR	67	
2c	-	34	
2d	e	12	
30	X	36	
31	XFR	67	
32	8	10	
33	-	34	
34	a	13	
35	DIV	35	
36	DN	25	
37	EXP	74	
38	UP	27	
39	.	21	
3a	6	06	
3b	9	11	
3c	3	03	
3d	1	01	
40	5	05	
41	X	36	
42	DN	25	
43	LN	65	
44	UP	27	
45	b	14	
46	DIV	35	
47	DN	25	
48	EXP	74	
49	XTO	23	
4a	c	16	
4b	XFR	67	
4c	8	10	
4d	UP	27	
50	X	36	
51	XFR	67	
52	9	11	
53	UP	27	
54	a	13	
55	X	36	
56	DN	25	
57	XEY	30	
58	-	34	
59	d	17	
5a	X	36	
5b	DN	25	
5c	√	76	
5d	YE	24	
60	7	07	
61	DIV	35	
62	c	16	
63	UP	27	
64	b	14	
65	RUP	22	
66	PNT	45	S
67	e	12	
68	UP	27	
69	GTO	44	
6a	-	34	
6b	0	00	
6c	0	00	
6d	END	46	

00	c	16	Minus Page	40	PNT	45	
01	LN	65		41	STP	41	
02	UP	27		42	GTO	44	
03	b	14		43	2	02	
04	X	36		44	3	03	
05	2	02		45	END	46	
06	LN	65					
07	LN	65					
08	-	34					
09	b	14					
0a	DIV	35					
0b	DN	25					
0c	EXP	74					
0d	UP	27					
10	0	00					
11	PNT	45	s				
12	PNT	45					
13	YTO	40					
14	a	13					
15	1	01					
16	UP	27					
17	b	14					
18	DIV	35					
19	YTO	40					
1a	d	17					
1b	a	13					
1c	LN	65					
1d	XTO	23					
20	e	12					
21	1	01					
22	0	00					
23	XTO	23					
24	f	15					
25	UP	27					
26	EEX	26					
27	2	02					
28	DIV	35					
29	1	01					
2a	-	34					
2b	XEY	30					
2c	CHS	32					
2d	DIV	35					
30	DN	25					
31	LN	65					
32	LN	65					
33	UP	27					
34	d	17					θ
35	X	36					
36	e	12					b
37	+	33					
38	0	00					Med. life
39	RDN	31					$\frac{1}{b}$
3a	EXP	74					
3b	XEY	30					ln 0
3c	f	15					
3d	PNT	45	s				%

GENERAL STATISTICS FOR QUALITY CONTROL

This program written by J.S. Madachy of Monsanto Research Corp., Miamisburg, Ohio, determines various statistics useful in quality control. Given a sample of n observations the following statistics are computed:

$\bar{X}$	=	Sample Mean
s_X^2	=	Sample Variance
s_X	=	Sample Standard Deviation
$\bar{X} + 3s_X$	=	Upper 3 Sigma Limit
$\bar{X} - 3s_X$	=	Lower 3 Sigma Limit
$X_{max} - \bar{X}$	=	Maximum Positive Excursion From Mean
$\bar{X} - X_{min}$	=	Maximum Negative Excursion From Mean
X_{max}	=	Maximum Observation
X_{min}	=	Minimum Observation
R	=	Range ($X_{max} - X_{min}$)

The sample mean and variance are given by:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$s_X^2 = \frac{\sum_{i=1}^{n} X_i^2 - \frac{(\Sigma X_i)^2}{n}}{n - 1}$$

This program has been designed for use with the 9120A Printer but can be used without the printer by placing STOP statements in program locations containing PRINT statements as shown. Replace other PRINT statements with CONTINUE. Results appear in the Y register.

USER INSTRUCTIONS

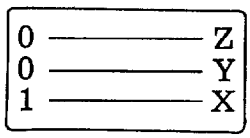
DEPRESS: Y on the 9120A

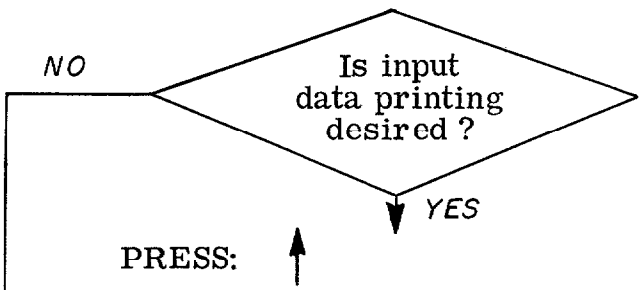
PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY



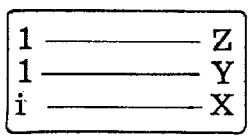


PRESS:

ENTER DATA: $X_1 \longrightarrow X$

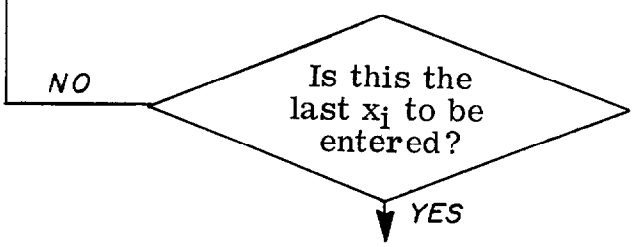
PRESS: CONTINUE

DISPLAY



(i is input data number: i = 2, 3, . . . n)

ENTER DATA: $X_i \longrightarrow X$



PRESS: SET FLAG

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

The general statistics will now be printed in the following order:

n	# of Data Entries
$\overline{X}$	Mean Value of Data Set
s_x^2	Variance of Data Set
s_x	Standard Deviation of Data Set

$\overline{X} + 3s_x$

$\overline{X} - 3s_x$

Max $X_i - \overline{X}$

$\overline{X}$ - Min X_i

Max X_i

Min X_i

Range

EXAMPLE

The following measurements represent the observed diameters of ball bearing produced by machine A.

Input data:

1.157
1.152
1.150
1.141
1.135
1.116
1.133
1.128
1.146
1.123
1.121
1.139

n	=	12
$\overline{X}$	=	1.13675
s_x^2	=	.00017
s_x	=	.01310

EXAMPLE (Con't)

$\overline{X} + 3s_x = 1.17606$
$\overline{X} - 3s_x = 1.09744$

Max $X_i - \overline{X} = .02025$
$\overline{X}$ - Min $X_i = .02075$

Max $X_i = 1.157$
Min $X_i = 1.116$

Range = .041

00	CLR	20		40	GTO	44		80	PNT	45	
01	1	01		41	1	01		81	PNT	45	
02	XTO	23		42	b	14		82	PNT	45	
03	d	17		43	PNT	45	S	83	PNT	45	
04	STP	41	ENTRY	44	-	34		84	PNT	45	
05	YTO	40		45	f	15		85	GTO	44	
06	c	16		46	UP	27		86	0	00	
07	RDN	31		47	d	17		87	0	00	
08	X>Y	53		48	DIV	35		88	RDN	31	
09	8	10		49	PNT	45		89	IFG	43	
0a	8	10		4a	YTO	40		8a	9	11	
0b	RUP	22		4b	c	16		8b	6	06	
0c	XTO	23		4c	f	15		8c	PNT	45	
0d	a	13		4d	X	36		8d	RUP	22	
10	XTO	23		50	e	12		90	X=Y	50	
11	b	14		51	XEY	30		91	2	02	
12	UP	27		52	-	34		92	3	03	
13	X	36		53	DN	25		93	GTO	44	
14	AC+	60		54	XEY	30		94	0	00	
15	d	17		55	DIV	35		95	b	14	
16	UP	27		56	PNT	45	S	96	PNT	45	
17	UP	27		57	DN	25		97	PNT	45	
18	2	02		58	√	76		98	PNT	45	
19	XTO	23		59	UP	27		99	SFL	54	
1a	d	17		5a	PNT	45	S	9a	GTO	44	
1b	STP	41	ENTRY	5b	PNT	45		9b	2	02	
1c	RDN	31		5c	+	33		9c	5	05	
1d	c	16		5d	+	33		9d	END	46	
20	X=Y	50		60	c	16					
21	8	10		61	UP	27					
22	8	10		62	DN	25					
23	RUP	22		63	+	33					
24	UP	27		64	PNT	45	S				
25	a	13		65	RUP	22					
26	X>Y	53		66	-	34					
27	YTO	40		67	PNT	45	S				
28	a	13		68	PNT	45					
29	b	14		69	+	33					
2a	X<Y	52		6a	a	13					
2b	YTO	40		6b	-	34					
2c	b	14		6c	b	14					
2d	DN	25		6d	UP	27					
30	UP	27		70	c	16					
31	X	36		71	-	34					
32	AC+	60		72	PNT	45	S				
33	d	17		73	DN	25					
34	UP	27		74	PNT	45	S				
35	1	01		75	PNT	45					
36	IFG	43		76	b	14					
37	4	04		77	UP	27					
38	3	03		78	a	13					
39	+	33		79	PNT	45	S				
3a	YTO	40		7a	UP	27					
3b	d	17		7b	PNT	45					
3c	UP	27		7c	DN	25					
3d	RUP	22		7d	-	34					

REPEATABILITY

This program computes parameters associated with the repeatability of a measuring (or experimental) process. The statistics are given by:

sample = Collection of n_1 repeat readings on a single piece

X = A reading on a piece

n_1 = Number of repeat readings on a piece

$\bar{X}_i$ = Average Value of the i^{th} Sample

s_{ei}^2 = Variance (error) of the i^{th} Sample

n = Number of pieces tested

$\bar{\bar{X}}$ = Pooled Mean $= \frac{1}{n} \sum_{i=1}^{n} \bar{X}_i$

$$s_e = s_{\text{pooled error}} = \sqrt{\frac{\sum_{i=1}^{n} s_{ei}^2}{n}}$$

$$s_{TV} = s_{\text{true value}} = \sqrt{s_{\bar{X}_i}^2 - \frac{s_e^2}{n_1}} \quad \text{(measure of spread)}$$

$$s_{RDG} = s_{\text{reading}} = \sqrt{s_e^2 + s_{TV}^2}$$

The program determines additional statistics including

$3s_e$, $3s_{TV}$, $3s_{RDG}$

$\bar{\bar{X}} + 3s_{TV}$, $\bar{\bar{X}} - 3s_{TV}$

$\bar{\bar{X}} + 3s_{RDG}$, $\bar{\bar{X}} - 3s_{RDG}$

This REPEATABILITY program was written by Mr. John Barr of Delco-Remy Division of General Motors, Anderson, Indiana.

Reference: According to Mr. Barr, "the techniques used throughout this program are derived from analysis of variance procedures. However, to my knowledge, there is no published reference available that would explain and deal with Repeatability in exactly the same manners we have used."

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM A

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: X → X

PRESS: CONTINUE

DISPLAY

0	Z
j	Y
0	X

(j=number of points entered in this sample)

NO ← Has all data from this sample been entered ? → YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

n_1	Z
$\bar{X}_1$	Y
s^2_{ei}	X

(i is sample counter)

NO ← Have all samples been entered? → YES

USER INSTRUCTIONS (Con't)

PRESS: END

ENTER PROGRAM B

PRESS: CONTINUE

ENTER DATA: n_1 → X

PRESS: CONTINUE

DISPLAY

n	Z
$\bar{\bar{X}}$	Y
0	X

PRESS: CONTINUE

DISPLAY

s_{error}	Z
s_{TV}	Y
s_{RDG}	X

PRESS: CONTINUE

DISPLAY

$3s_{error}$	Z
$3s_{TV}$	Y
$3s_{RDG}$	X

PRESS: CONTINUE

DISPLAY

$\bar{\bar{X}}$	Z
$\bar{\bar{X}} - 3s_{TV}$	Y
$\bar{\bar{X}} + 3s_{TV}$	X

PRESS: CONTINUE

DISPLAY

$\bar{\bar{X}}$	Z
$\bar{\bar{X}} - 3s_{RDG}$	Y
$\bar{\bar{X}} + 3s_{RDG}$	X

To run another case, return to the beginning of the USER INSTRUCTIONS.

EXAMPLE

This example was supplied by Mr. Barr.

Consider an example concerning switch operating forces where we are trying to determine the part of the variation of readings due to repeatability and the part that is true variation of switches. Shown below are force measurements on ten switches where each switch was checked three times.

Switch No.	Repeat Readings I	II	III
1	4.0	4.25	4.0
2	3.5	3.5	3.5
3	4.75	5.25	5.0
4	4.5	4.75	4.5
5	3.25	3.5	3.5
6	2.5	3.0	3.0
7	3.5	3.5	3.75
8	4.0	4.0	4.0
9	2.5	2.75	2.5
10	2.75	3.25	3.0

Results:

$n_1 = 3.$
$\bar{X}_1 = 4.083333$
$s_{e1}^2 = .02083$

$n_1 = 3.$
$\bar{X}_2 = 3.50000$
$s_{e2}^2 = 0.00000$

$n_1 = 3.$
$\bar{X}_3 = 5.00000$
$s_{e3}^2 = .06250$

$n_1 = 3.$
$\bar{X}_4 = 4.58333$
$s_{e4}^2 = .02083$

$n_1 = 3.$
$\bar{X}_5 = 3.41667$
$s_{e5}^2 = .02083$

$n_1 = 3.$
$\bar{X}_6 = 2.83333$
$s_{e6}^2 = .08333$

$n_1 = 3.$
$\bar{X}_7 = 3.58333$
$s_{e7}^2 = .02083$

$n_1 = 3.$
$\bar{X}_8 = 4.00000$
$s_{e8}^2 = 0.00000$

$n_1 = 3.$
$\bar{X}_9 = 2.58333$
$s_{e9}^2 = .02083$

$n_1 = 3.$
$\bar{X}_{10} = 3.00000$
$s_{e10}^2 = .06250$

EXAMPLE (Con't)

n	=	10.
$\bar{\bar{X}}$	=	3.65833
s_{error}	=	.17678
s_{TV}	=	.76144
s_{RDG}	=	.78169
$3s_{error}$	=	.53033
$3s_{TV}$	=	2.28431
$3s_{RDG}$	=	2.34506
$\bar{\bar{X}}$	=	3.65833
$\bar{\bar{X}} - 3s_{TV}$	=	1.37403
$\bar{\bar{X}} + 3s_{TV}$	=	5.94264
$\bar{\bar{X}}$	=	3.65833
$\bar{\bar{X}} - 3s_{RDG}$	=	1.31327
$\bar{\bar{X}} + 3s_{RDG}$	=	6.00339

00	CLR	20		40	DN	25		80	0	00	
01	STP	41	ENTRY	41	UP	27		81	0	00	
02	IFG	43		42	X	36		82	0	00	
03	1	01		43	UP	27		83	0	00	
04	8	10		44	c	16		84	0	00	
05	PNT	45		45	+	33		85	0	00	
06	PNT	45		46	YTO	40		86	0	00	
07	UP	27		47	c	16		87	0	00	
08	X	36		48	b	14		88	0	00	
09	AC+	60		49	RUP	22		89	0	00	
0a	YEX	24		4a	+	33		8a	0	00	
0b	7	07		4b	YTO	40		8b	0	00	
0c	1	01		4c	b	14		8c	0	00	
0d	+	33		4d	YEX	24		8d	0	00	
10	CLX	37		50	a	13		90	0	00	
11	UP	27		51	1	01		91	0	00	
12	RDN	31		52	+	33		92	0	00	
13	YTO	40		53	YTO	40		93	0	00	
14	7	07		54	a	13		94	0	00	
15	GTO	44		55	CLR	20		95	0	00	
16	0	00		56	YEX	24		96	0	00	
17	1	01		57	7	07		97	0	00	
18	f	15		58	DN	25		98	0	00	
19	XEY	30		59	YEX	24		99	0	00	
1a	DIV	35		5a	8	10		9a	0	00	
1b	XEY	30		5b	RDN	31		9b	0	00	
1c	UP	27		5c	YEX	24		9c	0	00	
1d	X	36		5d	9	11		9d	0	00	
20	XTO	23		60	RUP	22		a0	0	00	
21	f	15		61	PNT	45		a1	0	00	
22	DN	25		62	RUP	22		a2	0	00	
23	X	36		63	PNT	45		a3	0	00	
24	e	12		64	RUP	22		a4	0	00	
25	XEY	30		65	PNT	45	S	a5	0	00	
26	-	34		66	PNT	45		a6	0	00	
27	1	01		67	PNT	45		a7	0	00	
28	CHS	32		68	PNT	45		a8	0	00	
29	RUP	22		69	CLR	20		a9	0	00	
2a	+	33		6a	GTO	44		aa	0	00	
2b	RDN	31		6b	0	00		ab	0	00	
2c	DIV	35		6c	1	01		ac	0	00	
2d	f	15		6d	0	00		ad	0	00	
30	XTO	23		70	0	00					
31	9	11		71	0	00					
32	YTO	40		72	0	00					
33	8	10		73	0	00					
34	RDN	31		74	0	00					
35	YTO	40		75	0	00					
36	7	07		76	0	00					
37	RUP	22		77	0	00					
38	XEY	30		78	0	00					
39	YEX	24		79	0	00					
3a	d	17		7a	0	00					
3b	+	33		7b	0	00					
3c	YEX	24		7c	0	00					
3d	d	17		7d	0	00					

b0	0	00		10	YEX	24		50	3	03	
b1	0	00		11	e	12		51	X	36	
b2	0	00		12	YEX	24		52	YEX	24	
b3	0	00		13	c	16		53	f	15	
b4	0	00		14	a	13		54	X	36	
b5	0	00		15	DIV	35		55	RUP	22	
b6	0	00		16	XEY	30		56	X	36	
b7	0	00		17	RUP	22		57	f	15	
b8	0	00		18	CNT	47		58	RUP	22	
b9	0	00		19	RUP	22		59	CNT	47	
ba	0	00		1a	PNT	45		5a	RUP	22	
bb	0	00		1b	RUP	22		5b	PNT	45	
bc	0	00		1c	PNT	45		5c	RUP	22	
bd	0	00		1d	RUP	22		5d	PNT	45	
c0	0	00		20	PNT	45	S	60	RUP	22	
c1	0	00		21	PNT	45		61	PNT	45	S
c2	0	00		22	YTO	40		62	PNT	45	
c3	0	00		23	c	16		63	XTO	23	
c4	0	00		24	DN	25		64	e	12	
c5	0	00		25	UP	27		65	c	16	
c6	0	00		26	X	36		66	XEY	30	
c7	0	00		27	DN	25		67	-	34	
c8	0	00		28	X	36		68	UP	27	
c9	0	00		29	b	14		69	DN	25	
ca	0	00		2a	XEY	30		6a	+	33	
cb	0	00		2b	-	34		6b	+	33	
cc	0	00		2c	1	01		6c	c	16	
cd	0	00		2d	CHS	32		6d	RDN	31	
d0	0	00		30	RUP	22		70	CNT	47	
d1	0	00		31	+	33		71	RUP	22	
d2	0	00		32	RDN	31		72	PNT	45	
d3	0	00		33	DIV	35		73	RUP	22	
d4	0	00		34	e	12		74	PNT	45	
d5	0	00		35	-	34		75	RUP	22	
d6	0	00		36	YTO	40		76	PNT	45	S
d7	0	00		37	e	12		77	PNT	45	
d8	0	00		38	d	17		78	DN	25	
d9	0	00		39	+	33		79	e	12	
da	0	00		3a	YTO	40		7a	-	34	
db	0	00		3b	f	15		7b	UP	27	
dc	0	00		3c	√	76		7c	DN	25	
dd	END	46		3d	UP	27		7d	+	33	

00	YEX	24	Program B	40	e	12	
01	d	17		41	√	76	
02	a	13		42	RUP	22	
03	DIV	35		43	√	76	
04	YTO	40		44	CNT	47	
05	d	17		45	RUP	22	
06	CLR	20		46	PNT	45	
07	STP	41	ENTRY	47	RUP	22	
08	PNT	45		48	PNT	45	
09	PNT	45		49	RUP	22	
0a	UP	27		4a	PNT	45	S
0b	d	17		4b	PNT	45	
0c	XEY	30		4c	XTO	23	
0d	DIV	35		4d	f	15	

```
80    +     33
81    c     16
82   RDN    31
83   CNT    47
84   RUP    22
85   PNT    45
86   RUP    22
87   PNT    45
88   RUP    22
89   PNT    45
8a   PNT    45
8b   PNT    45
8c   PNT    45
8d   END    46
```

SECTION X PLOTTER PROGRAMS

			Page Number
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HISTOGRAM GENERATION (WITH PLOT)

This program generates and plots a histogram of ten windows given a data set of positive numbers. In addition, it determines the mean (M_x) and the variance(σ_x^2) of the raw data, and the mean (M_h) and the variance (σ_h^2) of the normalized histogram data. Since the raw data is normalized by the program to values $0 \leq h \leq 10$, the new mean and variance are given by

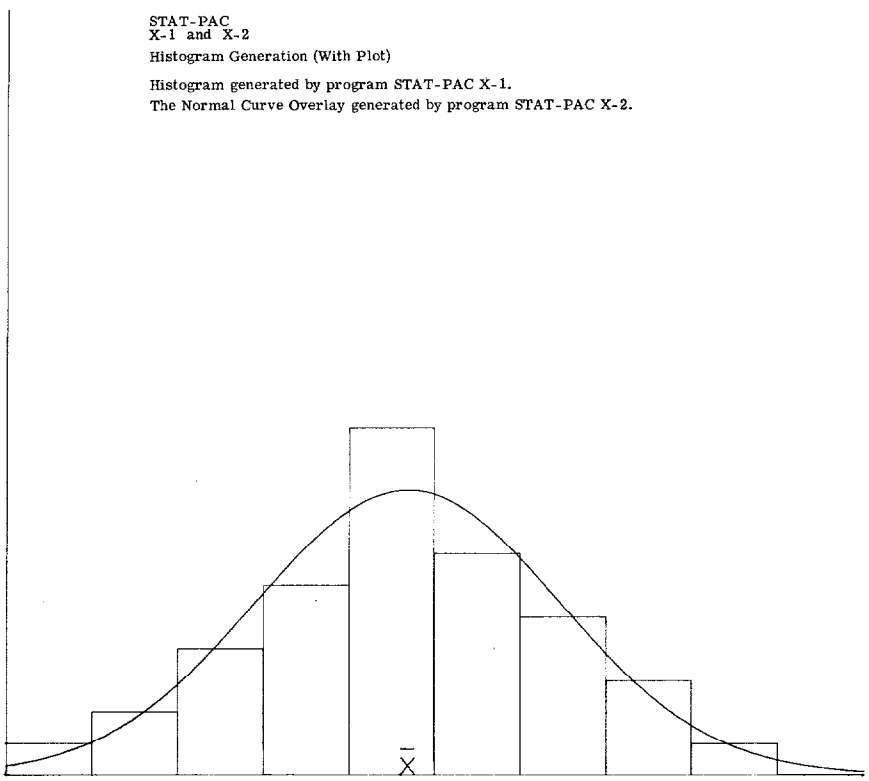
$$M_h = \frac{M_x}{W}$$

$$\sigma_h^2 = \frac{\sigma_x^2}{W^2}$$

where W is the histogram window width (normalizing factor)*. The program plots the histogram and stores M_h and σ_h^2 for use by program STAT-PAC X-2 which can be used to plot a normal curve over the histogram.

This program uses Indirect Addressing and is self-destructing of the registers +(0,0) through +(d,d). Thus, to rerun, the A side must be re-entered in the calculator.

NOTE: To generate a histogram with 1 cm. wide windows, place 2's in locations (-)(6)(c), (-)(7)(6), and (-)(8)(5).



* The window width W is chosen such that all normalized data entries X/W will lie between 0 and 10. Thus, if the data ranges from 0 → 200, a W of of 20 would be proper.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

Using the origin controls, locate the pen at X = 1 in., Y = 1 in.

SET: Decimal Wheel at 6 or less

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: W → X

PRESS: CONTINUE

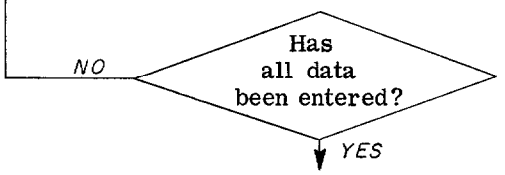
DISPLAY

N	Z
N	Y
1	X

N is the number of data points entered.

ENTER DATA: X_{N+1} → X

PRESS: CONTINUE



PRESS: SET FLAG

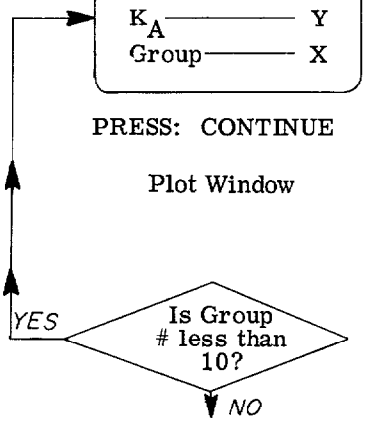
PRESS: CONTINUE

DISPLAY

%	Z	(% of data points in Group)
K_A	Y	(No. of data points in Group)
Group	X	(Group #)

PRESS: CONTINUE

Plot Window



USER INSTRUCTIONS (Con't)

DISPLAY

σ_x^2	Z
M_x	Y
0	X

PRESS: CONTINUE

DISPLAY

σ_h^2	Z
M_h	Y
1	X

EXAMPLE

The data set is:

104, 92, 83, 78, 58, 135, 146, 24, 74, 85, 81, 128, 140, 113, 79, 78, 53, 42, 34, 85, 96, 110, 133, 158, 171, 108, 84, 90, 73, 11, 51, 118, 68, 139, 92, 109, 89, 124, 91, 116.

The data varies between 0 and 200 so W is chosen to be 20.

Result

Group	K_A	%	
1	1	2.5	
2	2	5.	
3	4	10	
4	6	15.	
5	11	27.5	N = 40
6	7	17.5	
7	5	12.5	
8	3	7.5	
9	1	2.5	
10	0	0	

$\sigma_x^2 = 1252.644$ $\sigma_h^2 = 3.132$

$M_x = 93.575$ $M_h = 4.679$

The histogram plot is given with the normal curve superimposed. The normal curve resulted from running program 09100-70904 following completion of the Histogram Generation program.

00	CLR	20	Plus Page	40	CNT	47	80	CNT 47
01	XTO	23		41	CNT	47	81	CNT 47
02	9	11		42	CNT	47	82	CNT 47
03	XTO	23		43	CNT	47	83	CNT 47
04	8	10		44	CNT	47	84	CNT 47
05	XTO	23		45	CNT	47	85	CNT 47
06	7	07		46	CNT	47	86	CNT 47
07	XTO	23		47	CNT	47	87	CNT 47
08	6	06		48	CNT	47	88	CNT 47
09	XTO	23		49	CNT	47	89	CNT 47
0a	5	05		4a	CNT	47	8a	CNT 47
0b	XTO	23		4b	CNT	47	8b	CNT 47
0c	4	04		4c	CNT	47	8c	CNT 47
0d	XTO	23		4d	CNT	47	8d	CNT 47
10	3	03		50	CNT	47	90	CNT 47
11	STP	41	ENTRY	51	CNT	47	91	CNT 47
12	PNT	45		52	CNT	47	92	CNT 47
13	PNT	45		53	CNT	47	93	CNT 47
14	XTO	23		54	CNT	47	94	CNT 47
15	a	13		55	CNT	47	95	CNT 47
16	CLX	37		56	CNT	47	96	CNT 47
17	XTO	23		57	CNT	47	97	CNT 47
18	2	02		58	CNT	47	98	CNT 47
19	XTO	23		59	CNT	47	99	CNT 47
1a	0	00		5a	CNT	47	9a	CNT 47
1b	GTO	44		5b	CNT	47	9b	CNT 47
1c	c	16		5c	CNT	47	9c	CNT 47
1d	0	00		5d	CNT	47	9d	CNT 47
20	CNT	47		60	CNT	47	a0	CNT 47
21	CNT	47		61	CNT	47	a1	CNT 47
22	CNT	47		62	CNT	47	a2	CNT 47
23	CNT	47		63	CNT	47	a3	CNT 47
24	CNT	47		64	CNT	47	a4	CNT 47
25	CNT	47		65	CNT	47	a5	CNT 47
26	CNT	47		66	CNT	47	a6	CNT 47
27	CNT	47		67	CNT	47	a7	CNT 47
28	CNT	47		68	CNT	47	a8	CNT 47
29	CNT	47		69	CNT	47	a9	CNT 47
2a	CNT	47		6a	CNT	47	aa	CNT 47
2b	CNT	47		6b	CNT	47	ab	CNT 47
2c	CNT	47		6c	CNT	47	ac	CNT 47
2d	CNT	47		6d	CNT	47	ad	CNT 47
30	CNT	47		70	CNT	47		
31	CNT	47		71	CNT	47		
32	CNT	47		72	CNT	47		
33	CNT	47		73	CNT	47		
34	CNT	47		74	CNT	47		
35	CNT	47		75	CNT	47		
36	CNT	47		76	CNT	47		
37	CNT	47		77	CNT	47		
38	CNT	47		78	CNT	47		
39	CNT	47		79	CNT	47		
3a	CNT	47		7a	CNT	47		
3b	CNT	47		7b	CNT	47		
3c	CNT	47		7c	CNT	47		
3d	CNT	47		7d	CNT	47		

K_0	K_8
K_1	K_9
K_2	W
K_3	N
K_4	
K_5	
K_6	$\Sigma X^2/\sigma_h^2$
K_7	$\Sigma X/M_x/M_h$

b0	CNT	47	Plus Page
b1	CNT	47	
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	

c0	XTO	23	
c1	1	01	
c2	1	01	
c3	CHS	32	
c4	RDN	31	
c5	DN	25	
c6	1	01	
c7	+	33	
c8	UP	27	
c9	DN	25	
ca	STP	41	ENTRY
cb	CNT	47	
cc	IFG	43	
cd	d	17	

d0	d	17	
d1	PNT	45	
d2	PNT	45	
d3	UP	27	
d4	X	36	
d5	AC+	60	
d6	XEY	30	
d7	a	13	
d8	DIV	35	
d9	GTO	44	
da	-	34	
db	0	00	
dc	2	02	
dd	GTO	44	

00	3	03	Minus Page	40	0	00		80	0	00	
01	0	00		41	0	00		81	XEY	30	
02	DN	25		42	0	00		82	FMT	42	
03	INT	64		43	0	00		83	DN	25	
04	UP	27		44	0	00		84	UP	27	
05	EEX	26		45	0	00		85	5	05	
06	9	11		46	0	00		86	0	00	
07	+	33		47	0	00		87	0	00	
08	YTO	40		48	0	00		88	DIV	35	
09	-	34		49	0	00		89	1	01	
0a	1	01		4a	0	00		8a	0	00	
0b	CLX	37		4b	0	00		8b	X>Y	53	
0c	CLX	37		4c	0	00		8c	3	03	
0d	YE	24		4d	0	00		8d	4	04	
10	0	00		50	YTO	40		90	0	00	
11	0	00		51	-	34		91	UP	27	
12	0	00		52	1	01		92	FMT	42	
13	0	00		53	b	14		93	DN	25	
14	0	00		54	DIV	35		94	5	05	
15	0	00		55	EEX	26		95	EEX	26	
16	0	00		56	2	02		96	3	03	
17	0	00		57	X	36		97	XEY	30	
18	0	00		58	1	01		98	FMT	42	
19	0	00		59	RUP	22		99	DN	25	
1a	0	00		5a	+	33		9a	UP	27	
1b	0	00		5b	XFR	67		9b	FMT	42	
1c	0	00		5c	-	34		9c	DN	25	
1d	0	00		5d	1	01		9d	RCL	61	
20	IFG	43		60	XEY	30		a0	UP	2 7	
21	2	02		61	PNT	45	S	a1	b	14	
22	a	13		62	PNT	45		a2	DIV	35	
23	CLX	37		63	RUP	22		a3	YTO	40	
24	1	01		64	UP	27		a4	f	15	
25	+	33		65	7	07		a5	X	36	
26	SFL	54		66	5	05		a6	f	15	
27	GTO	44		67	X	36		a7	X	36	
28	0	00		68	RUP	22		a8	DN	25	
29	d	17		69	XEY	30		a9	-	34	
2a	GTO	44		6a	1	01		aa	b	14	
2b	+	33		6b	-	34		ab	DIV	35	
2c	c	16		6c	5	05		ac	f	15	
2d	5	05		6d	0	00		ad	UP	27	
30	YTO	40		70	0	00					
31	b	14		71	X	36					
32	CLX	37		72	DN	25			K_A		
33	UP	27		73	FMT	42					
34	EEX	26		74	DN	25					
35	9	11		75	UP	27					
36	+	33		76	5	05					
37	YTO	40		77	0	00					
38	-	34		78	0	00					
39	4	04		79	+	33					
3a	-	34		7a	DN	25					
3b	0	00		7b	FMT	42					
3c	UP	27		7c	DN	25					
3d	YE	24		7d	XEY	30					

b0	0	00		Minus
b1	PNT	45	s	Page
b2	PNT	45		
b3	a	13		
b4	DIV	35		
b5	XEY	30		
b6	RDN	31		
b7	DIV	35		
b8	DIV	35		
b9	DN	25		
ba	XEY	30		
bb	XTO	23		
bc	f	15		
bd	YTO	40		

c0	e	12	
c1	UP	27	
c2	1	01	
c3	PNT	45	s
c4	PNT	45	
c5	0	00	
c6	UP	27	
c7	UP	27	
c8	FMT	42	
c9	UP	27	
ca	END	46	

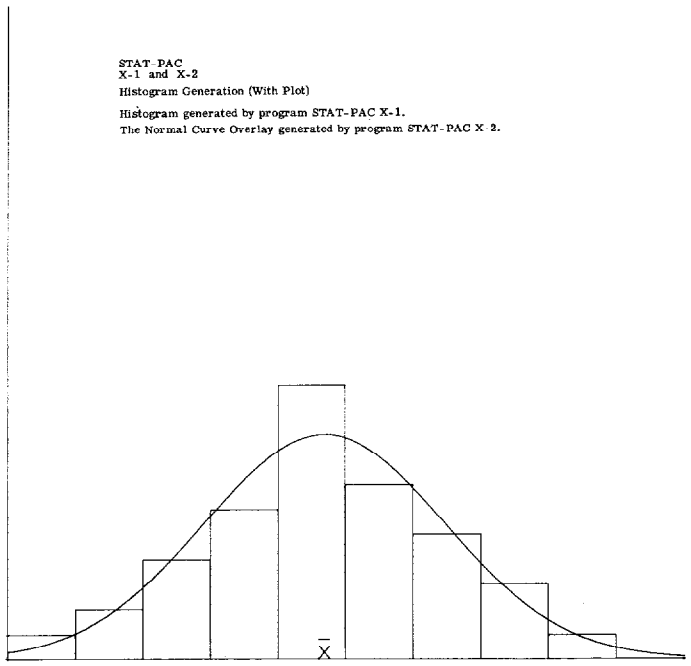
NORMAL CURVE OVERLAY

This program generates a normal curve given mean M_h and variance σ^2_h. The program determines Y from:

$$Y = \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{-(h - M_h)^2}{2\sigma^2_h}}$$

by varying h from 0 to 10 in increments of 0.1. The program requires that M_h and σ^2_h be stored in the f and e registers respectively prior to execution. This program was intended to be used in conjunction with program STAT-PAC X-1 Histogram Generation (With Plot). To plot in units of centimeters, place a 2 in locations (2)(5) (2)(c), and (4)(a).

USER INSTRUCTIONS	EXAMPLE
PRESS: STOP Using the origin controls, locate the pen at X = 1 in., Y = 1 in. SET: Decimal Wheel at 6 or less PRESS: END ENTER PROGRAM PRESS: CONTINUE	M_h = 4.679 σ_h^2 = 3.132 See plot below



00	e	12	40	X>Y	53	80	DN	25
01	√	76	41	1	01	81	UP	27
02	UP	27	42	0	00	82	8	10
03	π	56	43	0	00	83	0	00
04	UP	27	44	UP	27	84	+	33
05	2	02	45	UP	27	85	DN	25
06	X	36	46	FMT	42	86	FMT	42
07	DN	25	47	UP	27	87	DN	25
08	√	76	48	f	15	88	FMT	42
09	X	36	49	UP	27	89	UP	27
0a	YTO	40	4a	5	05	8a	0	00
0b	c	16	4b	0	00	8b	UP	27
0c	0	00	4c	0	00	8c	UP	27
0d	UP	27	4d	X	36	8d	FMT	42
10	YTO	40	50	4	04	90	UP	27
11	d	17	51	0	00	91	END	46
12	f	15	52	-	34			
13	-	34	53	0	00			
14	DN	25	54	XEY	30			
15	UP	27	55	FMT	42			
16	X	36	56	DN	25			
17	e	12	57	UP	27			
18	UP	27	58	8	10			
19	2	02	59	0	00			
1a	X	36	5a	+	33			
1b	DN	25	5b	EEX	26			
1c	DIV	35	5c	2	02			
1d	DN	25	5d	XEY	30			
20	CHS	32	60	FMT	42			
21	EXP	74	61	DN	25			
22	UP	27	62	FMT	42			
23	c	16	63	UP	27			
24	DIV	35	64	XEY	30			
25	7	07	65	0	00			
26	5	05	66	XEY	30			
27	0	00	67	FMT	42			
28	0	00	68	DN	25			
29	X	36	69	UP	27			
2a	d	17	6a	8	10			
2b	UP	27	6b	0	00			
2c	5	05	6c	-	34			
2d	0	00	6d	EEX	26			
30	0	00	70	2	02			
31	X	36	71	XEY	30			
32	DN	25	72	FMT	42			
33	FMT	42	73	DN	25			
34	DN	25	74	FMT	42			
35	d	17	75	UP	27			
36	UP	27	76	XEY	30			
37	.	21	77	UP	27			
38	1	01	78	5	05			
39	+	33	79	0	00			
3a	1	01	7a	+	33			
3b	0	00	7b	DN	25			
3c	.	21	7c	XEY	30			
3d	1	01	7d	FMT	42			

$\sigma_h\sqrt{2\pi}$

h

σ_h^2

M_h

This program generates a normal curve given the mean and variance of the normal distribution. The curve plotted is given by:

$$Y(h) = \frac{e^{-\frac{(h-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

where μ = Mean of distribution

σ^2 = Variance of distribution

The program is written to vary h between 0 and 10, in increments of 0.1. To decrease the plotting time, change Δh in locations (3)(d) and (4)(0).

To convert the plot from units of inches to centimeters, place 2's in locations (2)(b) and (3)(4).

To place axes on the plot with program STAT-PAC X-19, AXES PLOT, use the following input values:

$Y_{shift} = X_{shift} = 0$

$Y_{scale} = \frac{1}{15} = .06667$

$X_{scale} = 1$

$Y_{origin} = X_{origin} = 0$

$Y_{tic} = .1$

$X_{tic} = 1$

USER INSTRUCTIONS

PRESS: STOP

Using the origin controls, locate the pen at X = 1 in., Y = 1 in. (2.54 cm., 2.54 cm.).

SET: Decimal wheel at 6 or less.

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA:

σ^2 → Y
μ → X

PRESS: CONTINUE

The normal curve is now plotted.

To run another case

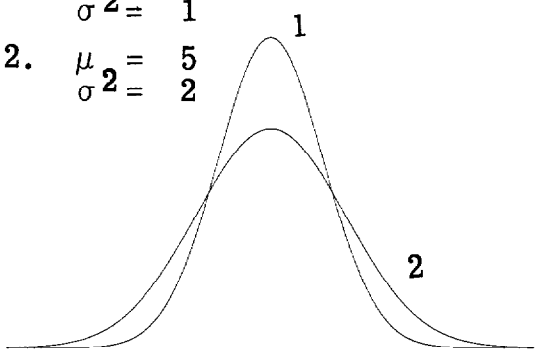
PRESS: END

STAT-PAC
X-3

Normal Curve Plot (General)

EXAMPLE

1. $\mu = 5$
   $\sigma^2 = 1$
2. $\mu = 5$
   $\sigma^2 = 2$



00	CLR	20
01	1	01
02	STP	41
03	PNT	45
04	PNT	45
05	AC+	60
06	DN	25
07	√	76
08	UP	27
09	π	56
0a	UP	27
0b	2	02
0c	X	36
0d	DN	25

ENTRY

40	1	01
41	+	33
42	1	01
43	0	00
44	.	21
45	1	01
46	X>Y	53
47	1	01
48	6	06
49	CLR	20
4a	FMT	42
4b	UP	27
4c	END	46

10	√	76
11	X	36
12	YTO	40
13	c	16
14	0	00
15	UP	27
16	YTO	40
17	d	17
18	f	15
19	-	34
1a	DN	25
1b	UP	27
1c	X	36
1d	e	12

20	UP	27
21	2	02
22	X	36
23	DN	25
24	DIV	35
25	DN	25
26	CHS	32
27	EXP	74
28	UP	27
29	c	16
2a	DIV	35
2b	7	07
2c	5	05
2d	0	00

30	0	00
31	X	36
32	d	17
33	UP	27
34	5	05
35	0	00
36	0	00
37	X	36
38	DN	25
39	FMT	42
3a	DN	25
3b	d	17
3c	UP	27
3d	.	21

$\sqrt{2\pi}\,\sigma$

h

σ^2

M

CUMULATIVE PROBABILITY FUNCTION
INTEGRAL OF THE FORM: $F(x) = \int_a^x f(u)\,du$ WITH PLOT

This program evaluates the integral of a function f(u) between any lower limit a and a successively incremented upper limit.

A modification of Simpson's one third rule is used to perform the integration. The following equations are used:

$$\text{For } j = 0, \quad \int_a^{x_j} f(u)\,du = h/6 \left[1/3\ f(a) + 4/3\ f(x_{j+1}) + 1/3\ f(x_{j+2}) \right]$$

$$\text{For } j \geq 3, \quad \int_{x_{j-1}}^{x_j} f(u)\,du = h/24 \left[f(x_{j-3}) + 5f(x_{j-2}) + 19\ f(x_{j-1}) + 9f(x_j) \right]$$

Notes:

Due to the manner in which the program is written, the function will initially be integrated over an area formed by twice the increment; thereafter, integration will proceed by one increment at a time.

To integrate with some constant (a) as a lower limit (other than zero), in the program steps in the f(u) subroutine add a to u to form u' and form f(u'). For a lower limit of zero simply form f(u). In each case the upper limit will exceed the lower by some multiple of the increment depending on the number of times x is incremented by depression of the CONTINUE key.

To integrate from some lower limit to a specified upper limit without continually depressing the CONTINUE key, enter a Pause at step (5) (7). Then just before the incremented upper limit (shown in a flashing display) reaches the desired upper limit, depress PAUSE--this will stop the program after the next integration.

If a plotter is available a plot of the function being integrated may be obtained by inserting a plot subroutine at (-9) (0). This plot subroutine scales the calculated X and F(X) by multiplying them by X_{coef} and Y_{coef} respectively.

Reference: Numerical Analysis
by
Faiser S. Kunz
McGraw-Hill Book Company, Inc.

DEPRESS: X Y Z on the 9120A
Using origin controls, position pen at (0, Y)*
PRESS: END

ENTER PROGRAM: Side A followed by Side B
PRESS: GO TO

PRESS: –

PRESS: 0

PRESS: 0

SET: [PROGRAM]

Starting at (-0) (0) enter the program steps to form the function f(u) to be integrated; u is located in the Y register. After forming f(u) place it in the Y register. The last step of the subroutine must be:

RETURN

NOTE: Registers (-0) thru (-8) are available for programming and storage of f(u).

SET: [RUN]

PRESS: END

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

h ⟶ Z
Y_{coef} ⟶ Y
X_{coef} ⟶ X

PRESS: CONTINUE

DISPLAY

$\int_a^x$ f(u) du —— Z
(- a) ——— Y
0 ——— X

Each successive "CONTINUE" increments the upper limit and evaluates the integral using the increment upper limit. The integral is plotted after each x incrementation.

If no plot is desired, place a RETURN in (-9)(0) and CONTINUES in (0)(1) and (0)(2).

EXAMPLE

Obtain the cumulative distribution function of the normally distributed random variable X of variance 1 and mean value of 0

The probability density function of X is

$$P(X) = \frac{1}{\sqrt{2\pi}} e^{-\frac{X^2}{2}}$$

The cumulative distribution function is given by

$$Q(X) = \int_{-\infty}^{x} P(u)\, du$$

The lower limit ($-\infty$) can be replaced by a = -4 without loss of accuracy.

The program steps to generate P(u) are given on page 2 .

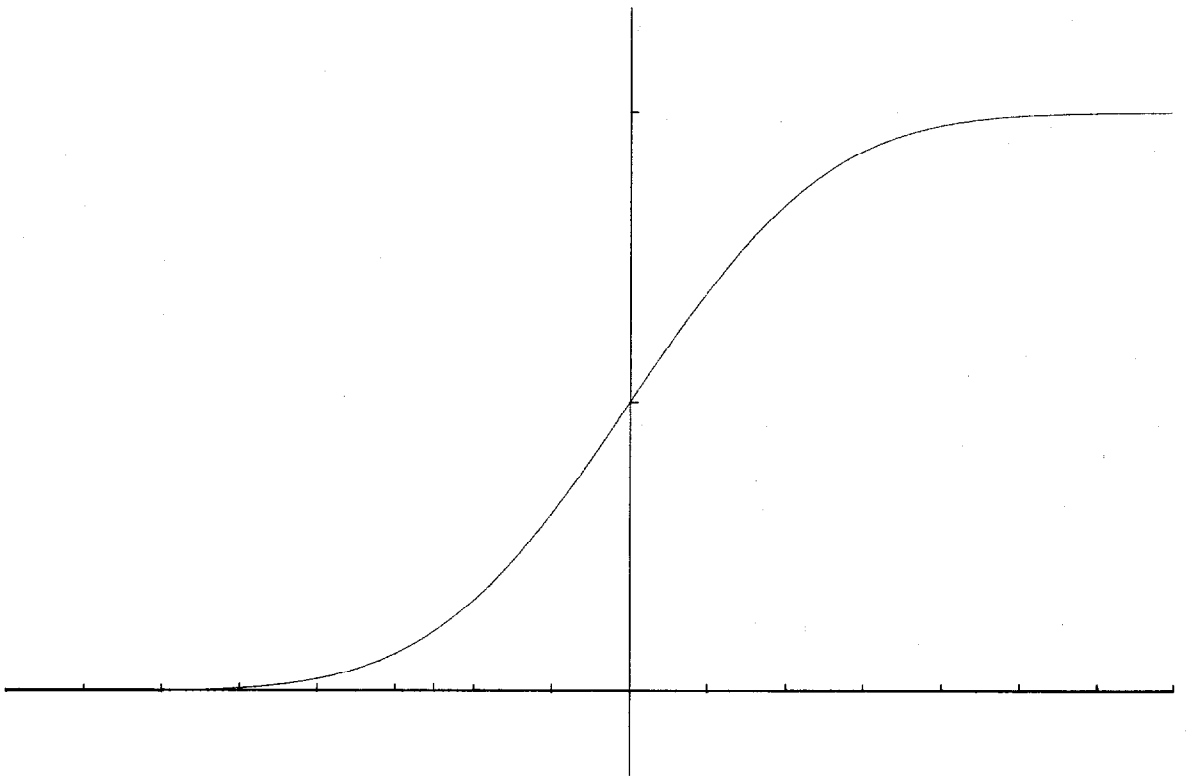
Data: Ycoef = 3760, Xcoef = 1000, h = .1

Results

X + 4	$\int_{-4}^{x} P(u)du$	X
0.5	.000201	-3.5
1.0	.001318	-3.0
1.5	.006178	-2.5
2.0	.022719	-2.0
2.5	.066775	-1.5
3.0	.158622	-1.0
3.5	.308504	-0.5
4.0	.499968	-0.0
4.5	.691432	0.5
5.0	.841314	1.0
5.5	.933162	1.5
6.0	.977218	2.0
6.5	.993758	2.5
7.0	.998618	3.0
7.5	.999736	3.5
8.0	.999937	4.0

*The Y (inches) must be determined by trial.

9100B ONLY
STAT-PAC
X-4 ▲



Seq	Key	Code
00	CLR	20
01	FMT	42
02	DN	25
03	STP	41
04	PNT	45
05	PNT	45
06	XTO	23
07	-	34
08	f	15
09	YTO	40
0a	-	34
0b	e	12
0c	DN	25
0d	YTO	40
10	d	17
11	CLR	20
12	GTO	44
13	SUB	77
14	-	34
15	0	00
16	0	00
17	2	02
18	4	04
19	DIV	35
1a	e	12
1b	UP	27
1c	0	00
1d	X=Y	50
20	6	06
21	3	03
22	1	01
23	X=Y	50
24	7	07
25	0	00
26	2	02
27	X=Y	50
28	7	07
29	8	10
2a	0	00
2b	XEY	30
2c	9	11
2d	RUP	22
30	X	36
31	RDN	31
32	AC+	60
33	1	01
34	9	11
35	RDN	31
36	YE	24
37	a	13
38	RDN	31
39	X	36
3a	RDN	31
3b	AC+	60
3c	5	05
3d	RDN	31

Plus Page ENTRY

Seq	Key	Code
40	YE	24
41	b	14
42	RDN	31
43	X	36
44	RDN	31
45	AC-	63
46	DN	25
47	1	01
48	YE	24
49	c	16
4a	XEY	30
4b	AC+	60
4c	RCL	61
4d	UP	27
50	d	17
51	X	36
52	RUP	22
53	X	36
54	d	17
55	-	34
56	CLX	37
57	PSE	57
58	CNT	47
59	GTO	44
5a	SUB	77
5b	-	34
5c	9	11
5d	0	00
60	GTO	44
61	8	10
62	3	03
63	DN	25
64	YTO	40
65	c	16
66	8	10
67	X	36
68	1	01
69	XEY	30
6a	AC+	60
6b	GTO	44
6c	8	10
6d	3	03
70	DN	25
71	YTO	40
72	b	14
73	3	03
74	2	02
75	GTO	44
76	6	06
77	7	07
78	DN	25
79	YTO	40
7a	a	13
7b	8	10
7c	X	36
7d	1	01

Seq	Key	Code
80	GTO	44
81	4	04
82	a	13
83	e	12
84	UP	27
85	d	17
86	X	36
87	GTO	44
88	1	01
89	2	02
8a	CNT	47
8b	CNT	47
8c	CNT	47
8d	CNT	47
90	CNT	47
91	CNT	47
92	CNT	47
93	CNT	47
94	CNT	47
95	CNT	47
96	CNT	47
97	CNT	47
98	CNT	47
99	CNT	47
9a	CNT	47
9b	CNT	47
9c	CNT	47
9d	CNT	47
a0	CNT	47
a1	CNT	47
a2	CNT	47
a3	CNT	47
a4	CNT	47
a5	CNT	47
a6	CNT	47
a7	CNT	47
a8	CNT	47
a9	CNT	47
aa	CNT	47
ab	CNT	47
ac	CNT	47
ad	CNT	47

h

j

$f(X_j)$

```
b0  CNT  47
b1  CNT  47
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47
```

Plus Page

```
c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47
```

```
d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

Example

```
00   4    04
01   CHS  32
02   +    33
03   DN   25
04   UP   27
05   X    36
06   2    02
07   DIV  35
08   DN   25
09   CHS  32
0a   EXP  74
0b   UP   27
0c   π    56
0d   UP   27
```

```
10   2    02
11   X    36
12   DN   25
13   √    76
14   DIV  35
15   RTN  77
```

00	4	04	Minus Page	40	CNT	47	80	CNT 47
01	CHS	32		41	CNT	47	81	CNT 47
02	+	33		42	CNT	47	82	CNT 47
03	DN	25		43	CNT	47	83	CNT 47
04	UP	27		44	CNT	47	84	CNT 47
05	X	36		45	CNT	47	85	CNT 47
06	2	02		46	CNT	47	86	CNT 47
07	DIV	35	Example	47	CNT	47	87	CNT 47
08	DN	25		48	CNT	47	88	CNT 47
09	CHS	32		49	CNT	47	89	CNT 47
0a	EXP	74		4a	CNT	47	8a	CNT 47
0b	UP	27		4b	CNT	47	8b	CNT 47
0c	π	56		4c	CNT	47	8c	CNT 47
0d	UP	27		4d	CNT	47	8d	CNT 47
10	2	02		50	CNT	47	90	XFR 67
11	X	36		51	CNT	47	91	- 34
12	DN	25		52	CNT	47	92	f 15
13	√	76		53	CNT	47	93	X 36
14	DIV	35		54	CNT	47	94	RUP 22
15	RTN	77		55	CNT	47	95	XEY 30
16	CNT	47		56	CNT	47	96	XFR 67
17	CNT	47		57	CNT	47	97	- 34
18	CNT	47		58	CNT	47	98	e 12
19	CNT	47		59	CNT	47	99	X 36
1a	CNT	47		5a	CNT	47	9a	XEY 30
1b	CNT	47		5b	CNT	47	9b	RUP 22
1c	CNT	47		5c	CNT	47	9c	FMT 42
1d	CNT	47		5d	CNT	47	9d	DN 25
20	CNT	47		60	CNT	47	a0	RTN 77
21	CNT	47		61	CNT	47	a1	END 46
22	CNT	47		62	CNT	47		
23	CNT	47		63	CNT	47		
24	CNT	47		64	CNT	47		
25	CNT	47		65	CNT	47		
26	CNT	47		66	CNT	47		
27	CNT	47		67	CNT	47		
28	CNT	47		68	CNT	47		
29	CNT	47		69	CNT	47		
2a	CNT	47		6a	CNT	47		
2b	CNT	47		6b	CNT	47		
2c	CNT	47		6c	CNT	47		
2d	CNT	47		6d	CNT	47		
30	CNT	47		70	CNT	47		
31	CNT	47		71	CNT	47		
32	CNT	47		72	CNT	47		
33	CNT	47		73	CNT	47		
34	CNT	47		74	CNT	47		
35	CNT	47		75	CNT	47		
36	CNT	47		76	CNT	47		
37	CNT	47		77	CNT	47		
38	CNT	47		78	CNT	47		
39	CNT	47		79	CNT	47		
3a	CNT	47		7a	CNT	47		
3b	CNT	47		7b	CNT	47		
3c	CNT	47		7c	CNT	47		
3d	CNT	47		7d	CNT	47		

Y_{coef}

X_{coef}

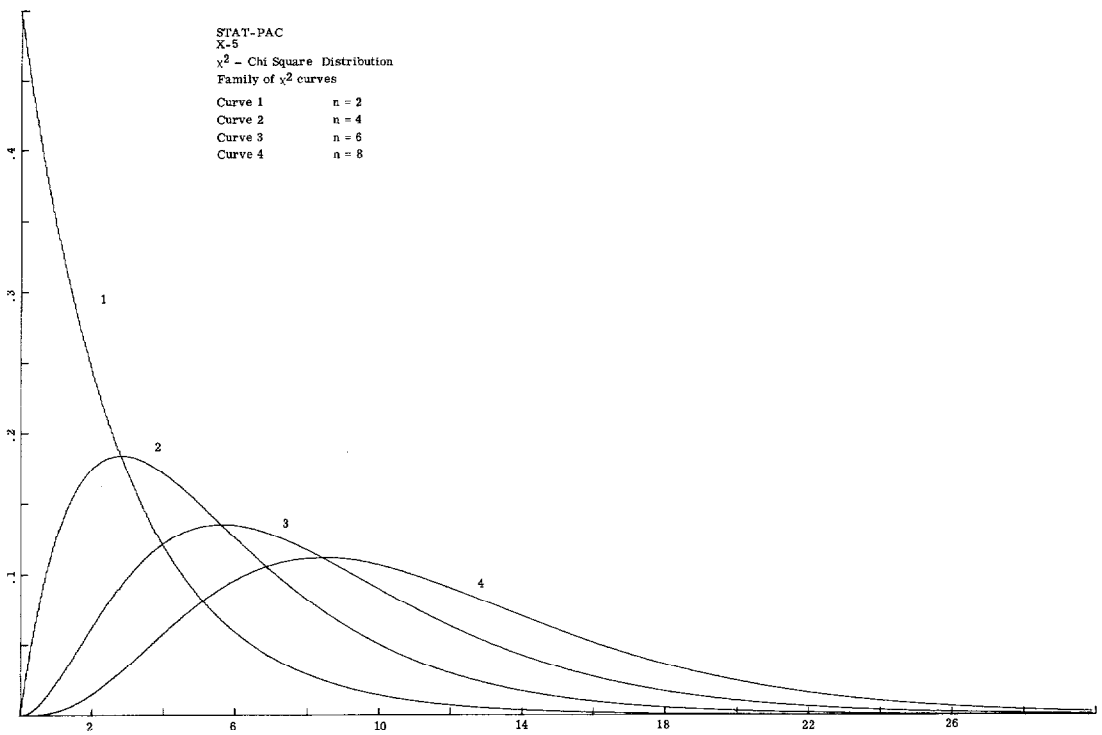
CHI-SQUARE DISTRIBUTION

This program will plot the distribution

$$Y(\chi^2) = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})}(\chi^2)^{\frac{\nu}{2}-1}\, e^{-\frac{1}{2}\chi^2}$$

for specified even values of ν , the number of degrees of freedom desired.

The axes will also be plotted.



PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

SET: Decimal Wheel at 6 or less

PRESS: END

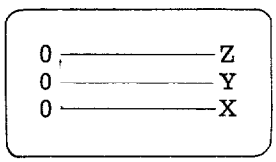
ENTER PROGRAM

PRESS: CONTINUE

The axes will now be drawn

NOTE: X axis increments ≈ 1.4
Y axis increments = .05

→ DISPLAY



ENTER ν : ν (Degrees of Freedom) ⟶ X

NOTE: $\nu = 2, 4, 6 \ldots$

PRESS: CONTINUE

← The distribution will now be plotted. When it is completed, the calculator will return to accept a new ν.

If it is not desired to wait for the termination of a distribution:

PRESS: STOP

PRESS: GO TO (5)(6)

PRESS: CONTINUE

NOTE: To plot distributions for higher degrees of freedom, decrease the scaling factor in locations b-9, b-a, b-b.

Execution of this program destroys the first two registers of program steps. To rerun the program the magnetic card must be re-entered.

00	CLR	20
01	5	05
02	0	00
03	0	00
04	-	34
05	AC+	60
06	DN	25
07	DN	25
08	FMT	42
09	DN	25
0a	UP	27
0b	IFG	43
0c	SFL	54
0d	UP	27

10	RCL	61
11	AC-	63
12	+	33
13	AC+	60
14	7	07
15	5	05
16	IFG	43
17	SFL	54
18	INT	64
19	IFG	43
1a	SFL	54
1b	5	05
1c	EEX	26
1d	3	03

20	X<Y	52
21	4	04
22	a	13
23	DN	25
24	IFG	43
25	SFL	54
26	XEY	30
27	FMT	42
28	DN	25
29	XEY	30
2a	IFG	43
2b	SFL	54
2c	XEY	30
2d	UP	27

30	5	05
31	0	00
32	+	33
33	RDN	31
34	XEY	30
35	IFG	43
36	SFL	54
37	XEY	30
38	FMT	42
39	DN	25
3a	XEY	30
3b	IFG	43
3c	SFL	54
3d	XEY	30

40	RUP	22
41	-	34
42	DN	25
43	XEY	30
44	IFG	43
45	SFL	54
46	XEY	30
47	GTO	44
48	0	00
49	8	10
4a	IFG	43
4b	5	05
4c	6	06
4d	CLR	20

50	SFL	54
51	FMT	42
52	UP	27
53	GTO	44
54	0	00
55	1	01
56	CLR	20
57	FMT	42
58	UP	27
59	STP	41
5a	XTO	23
5b	1	01
5c	XEY	30
5d	1	01

60	XEY	30
61	UP	27
62	2	02
63	DIV	35
64	LN	65
65	XEY	30
66	X	36
67	XEY	30
68	EXP	74
69	XTO	23
6a	0	00
6b	1	01
6c	-	34
6d	0	00

70	X=Y	50
71	7	07
72	9	11
73	DN	25
74	X	36
75	UP	27
76	GTO	44
77	6	06
78	b	14
79	YEX	24
7a	0	00
7b	DN	25
7c	X	36
7d	YTO	40

80	0	00
81	EEX	26
82	3	03
83	CHS	32
84	UP	27
85	.	21
86	0	00
87	6	06
88	-	34
89	AC+	60
8a	RCL	61
8b	+	33
8c	YTO	40
8d	e	12

90	DN	25
91	LN	65
92	UP	27
93	UP	27
94	YEX	24
95	1	01
96	YTO	40
97	1	01
98	2	02
99	-	34
9a	DN	25
9b	X	36
9c	DN	25
9d	EXP	74

ENTRY

a0	UP	27
a1	e	12
a2	UP	27
a3	X	36
a4	2	02
a5	CHS	32
a6	DIV	35
a7	DN	25
a8	EXP	74
a9	X	36
aa	DN	25
ab	YEX	24
ac	0	00
ad	YTO	40

b0	0	00
b1	XEY	30
b2	DIV	35
b3	EEX	26
b4	4	04
b5	X	36
b6	e	12
b7	UP	27
b8	X	36
b9	3	03
ba	5	05
bb	0	00
bc	X	36
bd	7	07
c0	5	05
c1	0	00
c2	0	00
c3	X<Y	52
c4	c	16
c5	c	16
c6	DN	25
c7	FMT	42
c8	DN	25
c9	GTO	44
ca	8	10
cb	a	13
cc	CLR	20
cd	FMT	42
d0	UP	27
d1	GTO	44
d2	5	05
d3	6	06
d4	END	46

LINEAR REGRESSION AND PLOT

This program calculates the equation of the straight line of best fit of a set of data points. It also plots each data point and the straight line determined by the calculated equation. The best fit is determined by minimizing the sum of the squares of the deviations of the data points from the line.

The program calculates m and C for the equation:

$$Y = mX + C$$

The program also calculates a correlation coefficient r, an indication of goodness of fit. Note: $-1 \leq r \leq 1$ where the sign corresponds to the slope m. If $r = 0$, there is no correlation, and if $r = \pm 1$, there is perfect correlation or a perfect fit.

The defining equations are:

$$m = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_i - \overline{X})^2}$$

$$C = \overline{Y} - m\overline{X}$$

where $\overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$ and $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$

$$r = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \overline{X})^2 \sum_{i=1}^{n} (Y_i - \overline{Y})^2}} = \frac{n\sum_{i=1}^{n} X_i Y_i - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{\sqrt{\left[n\sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2\right]\left[n\sum_{i=1}^{n} Y_i^2 - (\sum_{i=1}^{n} Y_i)^2\right]}}$$

Reference: Mathematical Statistics
by John E. Fruend
Prentice-Hall, 1962

USER INSTRUCTIONS

PRESS: STOP

Note: Use the origin controls to locate the pen in the lower left corner of the paper (on intersection of border lines if graph paper is being used).

ENTER PROGRAM A: (Starting Address is (0)(0)

SET: Decimal wheel at 6 or less

PRESS: END

PRESS: CONTINUE

ENTER TRANSLATION CONSTANTS*:

$Y_{shift} \rightarrow Y$

$X_{shift} \rightarrow X$

PRESS: CONTINUE

ENTER SCALING CONSTANTS*:

$Y_{scale} \rightarrow Y$

$X_{scale} \rightarrow X$

*See General Plotter Instructions

PRESS: CONTINUE

DISPLAY

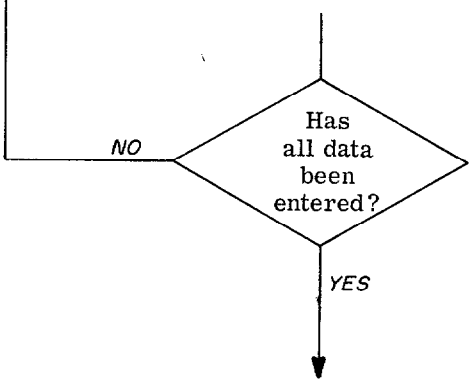
i	Z
0	Y
0	X

ENTER DATA: $Y_i \rightarrow Y$

$X_i \rightarrow X$

PRESS: CONTINUE

Note: (X_i, Y_i) is now plotted



USER INSTRUCTIONS (con't)

ENTER PROGRAM B: (Starting Address is (0)(0))

PRESS: END

PRESS: CONTINUE

DISPLAY

r	Z
C	Y
m	X

PRESS: CONTINUE

ENTER SCALING CONSTANTS: $Y_{scale} \rightarrow Y$

$X_{scale} \rightarrow X$

PRESS: CONTINUE

Note: The straight line of best fit is now plotted.

As the pen approaches any of the boundaries,
PRESS: STOP

EXAMPLE

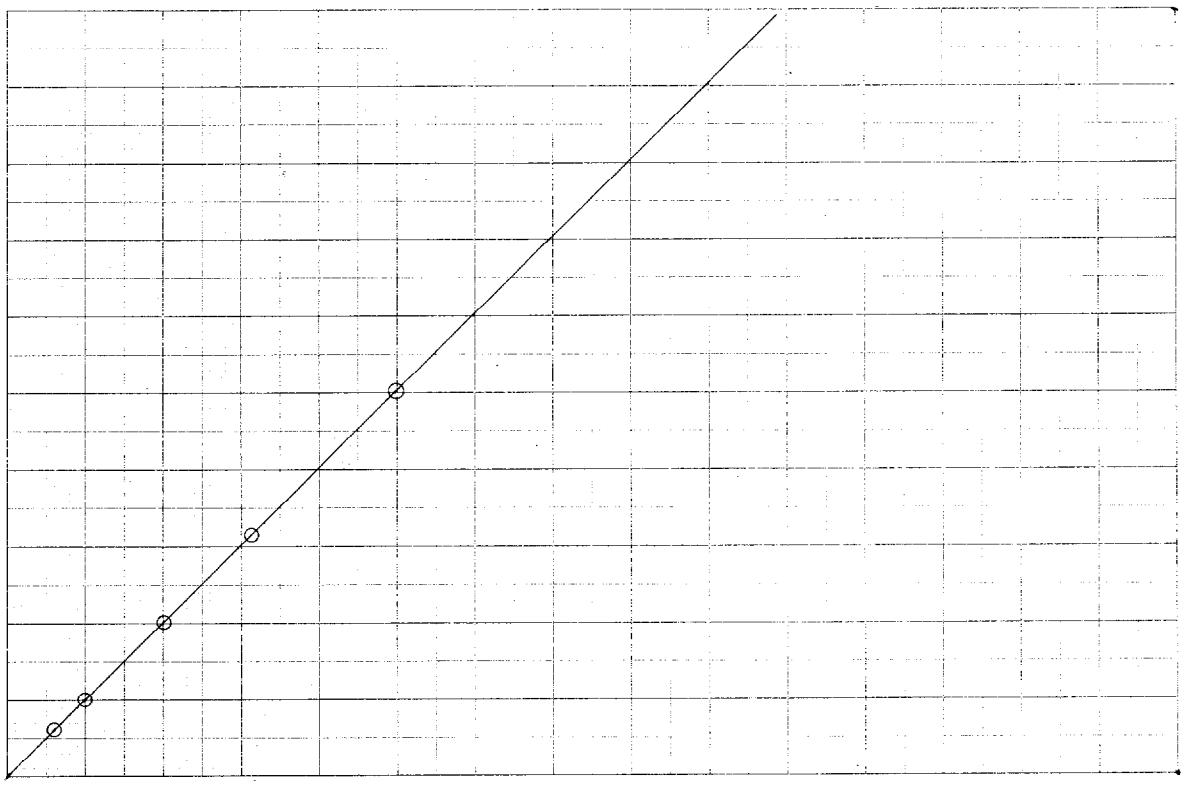
$Y_{shift} = 0$
$X_{shift} = 0$

$Y_{scale} = 1$
$X_{scale} = 1$

X	Y
1	1
2	2
5	5
.6	.6
π	π

Results

r	=	1
c	=	0
m	=	1



00	CLR	20		40	YEX	24
01	XTO	23		41	9	11
02	a	13		42	YTO	40
03	XTO	23		43	9	11
04	b	14		44	RDN	31
05	XTO	23		45	-	34
06	c	16		46	XEY	30
07	XTO	23		47	YEX	24
08	d	17		48	8	10
09	STP	41	ENTRY	49	YTO	40
0a	YTO	40		4a	8	10
0b	9	11		4b	RDN	31
0c	XTO	23		4c	-	34
0d	8	10		4d	5	05

10	CLR	20		50	0	00
11	STP	41	ENTRY	51	0	00
12	YTO	40		52	X	36
13	7	07		53	RUP	22
14	XTO	23		54	X	36
15	0	00		55	XEY	30
16	a	13		56	YEX	24
17	UP	27		57	0	00
18	1	01		58	YTO	40
19	+	33		59	0	00
1a	YTO	40		5a	RDN	31
1b	a	13		5b	DIV	35
1c	CLX	37		5c	XEY	30
1d	UP	27		5d	YEX	24

20	STP	41	ENTRY	60	7	07
21	AC+	60		61	YTO	40
22	UP	27		62	7	07
23	X	36		63	RDN	31
24	XEY	30		64	DIV	35
25	YEX	24		65	DN	25
26	d	17		66	XEY	30
27	+	33		67	FMT	42
28	YEX	24		68	UP	27
29	d	17		69	FMT	42
2a	DN	25		6a	DN	25
2b	X	36		6b	GTO	44
2c	XEY	30		6c	1	01
2d	YEX	24		6d	6	06

30	b	14		70	END	46
31	+	33				
32	YEX	24				
33	b	14				
34	DN	25				
35	RDN	31				
36	X	36				
37	XEY	30				
38	YEX	24				
39	c	16				
3a	+	33				
3b	YEX	24				
3c	c	16				
3d	RUP	22				

00	DN	25
01	1	01
02	-	34
03	YTO	40
04	a	13
05	e	12
06	XEY	30
07	DIV	35
08	YEX	24
09	f	15
0a	DIV	35
0b	YTO	40
0c	e	12
0d	X	36

10	e	12
11	X	36
12	d	17
13	XEY	30
14	-	34
15	YTO	40
16	d	17
17	c	16
18	UP	27
19	f	15
1a	UP	27
1b	X	36
1c	a	13
1d	X	36

20	DN	25
21	-	34
22	YTO	40
23	c	16
24	b	14
25	UP	27
26	f	15
27	UP	27
28	e	12
29	X	36
2a	a	13
2b	X	36
2c	DN	25
2d	-	34

30	d	17
31	UP	27
32	DN	25
33	√	76
34	DIV	35
35	c	16
36	√	76
37	DIV	35
38	d	17
39	RUP	22
3a	XEY	30
3b	DIV	35
3c	YTO	40
3d	a	13

40	e	12
41	X	36
42	f	15
3	XEY	30
44	-	34
45	a	13
46	STP	41
47	YTO	40
48	b	14
49	CLR	20
4a	XTO	23
4b	c	16
4c	FMT	42
4d	UP	27

50	STP	41
51	AC+	60
52	DN	25
53	f	15
54	X	36
55	5	05
56	0	00
57	0	00
58	DIV	35
59	RDN	31
5a	YEX	24
5b	8	10
5c	YTO	40
5d	8	10

60	+	33
61	a	13
62	X	36
63	b	14
64	+	33
65	RUP	22
66	YEX	24
67	9	11
68	YTO	40
69	9	11
6a	RDN	31
6b	-	34
6c	e	12
6d	DIV	35

70	DN	25
71	X	36
72	c	16
73	FMT	42
74	DN	25
75	RUP	22
76	+	33
77	YTO	40
78	c	16
79	GTO	44
7a	5	05
7b	3	03
7c	END	46

LINEAR REGRESSION (PRINT PLOT)

This program calculates the equation of the straight line of best fit of a set of data points. It also plots each data point and the straight line determined by the calculated equation. The best fit is determined by minimizing the sum of the squares of the deviations of the data points from the line.

The program calculates m and C for the equation:

$$Y = mX + C$$

The program also calculates a correlation coefficient r, an indication of goodness of fit. Note: $-1 \le r \le 1$ where the sign corresponds to the slope m. If $r = 0$, there is no correlation, and if $r = \pm 1$, there is perfect correlation or a perfect fit.

The defining equations are:

$$m = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n} (X_i - \bar{X})^2}$$

$$C = \bar{Y} - m\bar{X}$$

where $\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$ and $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$

$$r = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2 \sum_{i=1}^{n} (Y_i - \bar{Y})^2}} = \frac{n \sum_{i=1}^{n} X_i Y_i - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{\sqrt{\left[n \sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2\right] \left[n \sum_{i=1}^{n} Y_i^2 - (\sum_{i=1}^{n} Y_i)^2\right]}}$$

Reference: Mathematical Statistics, John E. Freund, Prentice-Hall, 1962.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: CLEAR

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

PRESS: END

ENTER PROGRAM A

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER SHIFT DATA:

$Y_{shift} \longrightarrow Y$
$X_{shift} \longrightarrow X$

PRESS: CONTINUE

ENTER SCALING DATA:

$Y_{scale} \longrightarrow Y$
$X_{scale} \longrightarrow X$

PRESS: CONTINUE

DISPLAY

1	Z
0	Y
0	X

(n - number of the data point to be entered)

ENTER DATA:

$Y_n \longrightarrow Y$
$X_n \longrightarrow X$

PRESS: CONTINUE

When all the data points have been entered:

USER INSTRUCTIONS (Con't)

PRESS: END

ENTER PROGRAM B

PRESS: CONTINUE

DISPLAY

r	Z	Correlation Coefficient
c	Y	Intercept
m	X	Slope

ENTER SCALING DATA:

$Y_{scale} \longrightarrow Y$
$X_{scale} \longrightarrow X$

PRESS: CONTINUE

(The least fit straight line will now be plotted.) To terminate, PRESS: STOP .

EXAMPLE

$Y_{shift} = 0$
$X_{shift} = 0$

$Y_{scale} = 1$
$X_{scale} = 1$

X	Y
1	1
2	2
5	5
.6	.6
π	π

Results:

r	=	1
c	=	0
m	=	1

For plot, see page 25.

00	XTO	23		40	YEX	24	00	DN	25
01	a	13		41	9	11	01	1	01
02	XTO	23		42	YTO	40	02	-	34
03	b	14		43	9	11	03	YTO	40
04	XTO	23		44	RDN	31	04	a	13
05	c	16		45	-	34	05	e	12
06	XTO	23		46	XEY	30	06	XEY	30
07	d	17		47	YEX	24	07	DIV	35
08	STP	41	ENTRY	48	8	10	08	YEX	24
09	YTO	40		49	YTO	40	09	f	15
0a	9	11		4a	8	10	0a	DIV	35
0b	XTO	23		4b	RDN	31	0b	YTO	40
0c	8	10		4c	-	34	0c	e	12
0d	STP	41	ENTRY	4d	5	05	0d	X	36
10	YTO	40		50	0	00	10	e	12
11	7	07		51	0	00	11	X	36
12	XTO	23		52	X	36	12	d	17
13	0	00		53	RUP	22	13	XEY	30
14	a	13		54	X	36	14	-	34
15	UP	27		55	XEY	30	15	YTO	40
16	1	01		56	YEX	24	16	d	17
17	+	33		57	0	00	17	c	16
18	YTO	40		58	YTO	40	18	UP	27
19	a	13		59	0	00	19	f	15
1a	CLX	37		5a	RDN	31	1a	UP	27
1b	UP	27		5b	DIV	35	1b	X	36
1c	STP	41	ENTRY	5c	XEY	30	1c	a	13
1d	PNT	45		5d	YEX	24	1d	X	36
20	PNT	45		60	7	07	20	DN	25
21	AC+	60		61	YTO	40	21	-	34
22	UP	27		62	7	07	22	YTO	40
23	X	36		63	RDN	31	23	c	16
24	XEY	30		64	DIV	35	24	b	14
25	YEX	24		65	DN	25	25	UP	27
26	d	17		66	XEY	30	26	f	15
27	+	33		67	FMT	42	27	UP	27
28	YEX	24		68	UP	27	28	e	12
29	d	17		69	FMT	42	29	X	36
2a	DN	25		6a	DN	25	2a	a	13
2b	X	36		6b	GTO	44	2b	X	36
2c	XEY	30		6c	1	01	2c	DN	25
2d	YEX	24		6d	4	04	2d	-	34
30	b	14		70	END	46			
31	+	33							
32	YEX	24							
33	b	14							
34	DN	25							
35	RDN	31							
36	X	36							
37	XEY	30							
38	YEX	24							
39	c	16							
3a	+	33							
3b	YEX	24							
3c	c	16							
3d	RUP	22							

Program B

30	d	17
31	UP	27
32	DN	25
33	√	76
34	DIV	35
35	c	16
36	√	76
37	DIV	35
38	d	17
39	RUP	22
3a	XEY	30
3b	DIV	35
3c	YTO	40
3d	a	13

70	DN	25
71	X	36
72	c	16
73	FMT	42
74	DN	25
75	RUP	22
76	+	33
77	YTO	40
78	c	16
79	GTO	44
7a	5	05
7b	3	03
7c	END	46

40	e	12
41	X	36
42	f	15
43	XEY	30
44	-	34
45	a	13
46	PNT	45
47	YTO	40
48	b	14
49	CLR	20
4a	XTO	23
4b	c	16
4c	FMT	42
4d	UP	27

S

50	STP	41
51	AC+	60
52	DN	25
53	f	15
54	X	36
55	5	05
56	0	00
57	0	00
58	DIV	35
59	RDN	31
5a	YEX	24
5b	8	10
5c	YTO	40
5d	8	10

ENTRY

60	+	33
61	a	13
62	X	36
63	b	14
64	+	33
65	RUP	22
66	YEX	24
67	9	11
68	YTO	40
69	9	11
6a	RDN	31
6b	-	34
6c	e	12
6d	DIV	35

POWER CURVE REGRESSION AND PLOT

This program computes the least squares fit and correlation coefficient of N pairs of data points for a power curve of the form:

$$Y = aX^b$$

The program will also plot each data point and the curve after the unknowns have been calculated.

The equation is linearized into $\ln Y = b \ln X + \ln a$

where
$$b = \frac{N\sum(\ln X \ln Y) - \sum \ln X \sum \ln Y}{N\sum(\ln X)^2 - (\sum \ln X)^2}$$

and
$$r = \frac{N\sum(\ln X \ln Y) - (\sum \ln X)(\sum \ln Y)}{\sqrt{\left[N\sum(\ln X)^2 - (\sum \ln X)^2\right]\left[N\sum(\ln Y)^2 - (\sum \ln Y)^2\right]}}$$

$$\ln a = \frac{\sum \ln Y}{N} - \frac{\sum \ln X}{N} b$$

Note: $X_i > 0$, $Y_i > 0$, $i = 1, \cdots, N$ and $X_{shift} > 0$

Reference: Statistical Theory and Methodology in Science and Engineering
by K. A. Brownlee
John Wiley and Sons, 1965

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A Printer

PRESS: STOP
Using the origin controls, locate the pen in the lower left corner of the paper.

ENTER PROGRAM A: (Starting Address is (0)(0))

PRESS: END

PRESS: CONTINUE

ENTER TRANSLATION CONSTANTS*:

Y_{shift} → Y

X_{shift} → X Note: $X_{shift} > 0$

PRESS: CONTINUE

ENTER SCALING CONSTANTS*:

Y_{scale} → Y

X_{scale} → X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
i	X

Each data point will be plotted as entered.

ENTER DATA: Y_i → Y

X_i → X

PRESS: PRINT

NO ← Last entry? → YES

PRESS: SET FLAG

PRESS: CONTINUE

ENTER PROGRAM B: (Starting Address is (0)(0))

PRESS: END

PRESS: CONTINUE

USER INSTRUCTIONS (con't)

DISPLAY

r	Z
b	Y
a	X

PRESS: PRINT

PRESS: CONTINUE

The Plotter will now plot the least squares curve. To interrupt the plot or stop it when finished, PRESS: STOP.

To re-run program, return to beginning of USER INSTRUCTIONS, ENTER PROGRAM A and proceed as before.

*See general Plotter instructions.

EXAMPLE

X	Y
1.2	15.25
1.7	6.68
2.3	4.54
2.9	1.74
4.4	.87
4.8	.51
5.7	.33
7.2	.23
8.4	.17

Let:

$X_{shift} = 1.1$ $X_{scale} = \frac{8.4-1.1}{15} \approx .5 \frac{\text{units}}{\text{inch}}$

$Y_{shift} = 0$ $Y_{scale} = \frac{15.25-.17}{10} \approx 2 \frac{\text{units}}{\text{inch}}$

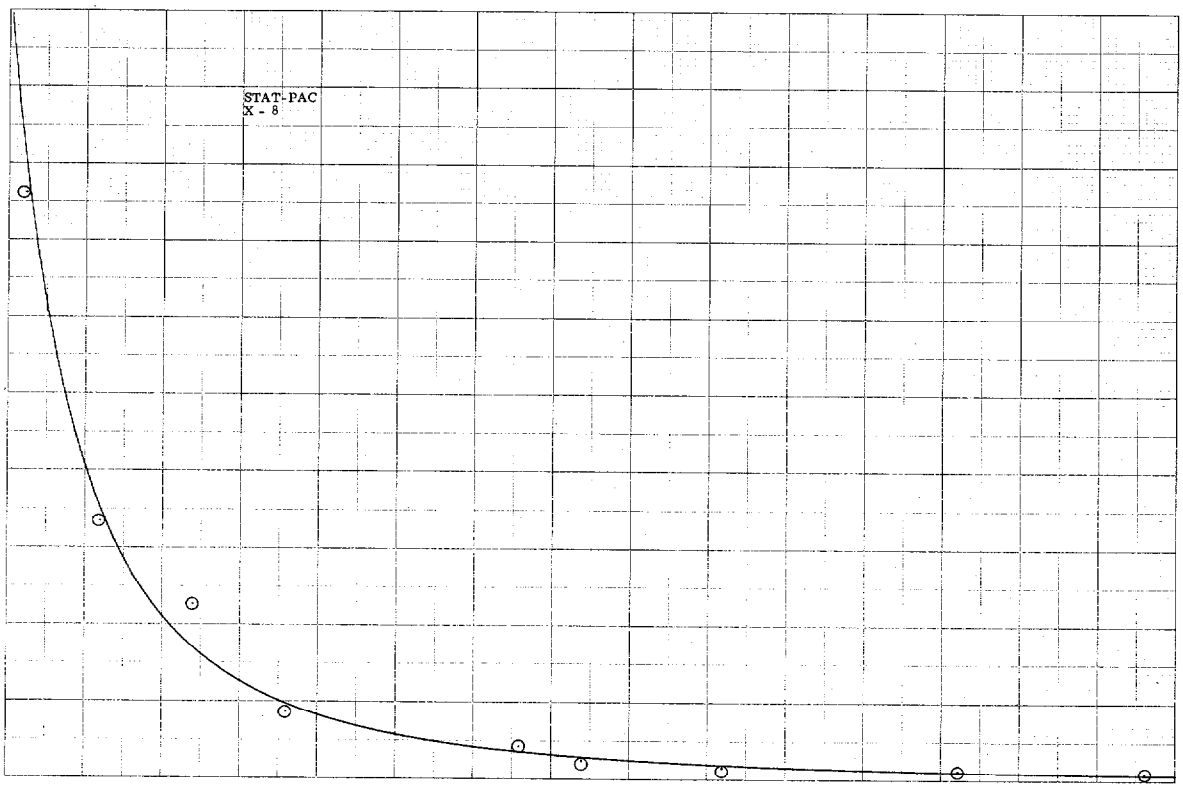
Solution:

r = -.9952

b = -2.384

a = 24.949

$Y = 24.949 X^{-2.384}$


STAT-PAC
X - 8

00	CLR	20		40	RUP	22		80	0	00	
01	STP	41	ENTRY	41	XEY	30		81	0	00	
02	XTO	23		42	-	34		82	0	00	
03	a	13		43	c	16		83	0	00	
04	YTO	40		44	DIV	35		84	0	00	
05	b	14		45	5	05		85	0	00	
06	CLR	20		46	0	00		86	0	00	
07	STP	41	ENTRY	47	0	00		87	0	00	
08	XTO	23		48	X	36		88	0	00	
09	c	16		49	RUP	22		89	0	00	
0a	YTO	40		4a	X	36		8a	0	00	
0b	d	17		4b	CLX	37		8b	0	00	
0c	CLR	20		4c	RDN	31		8c	0	00	
0d	1	01		4d	XEY	30		8d	0	00	
10	XTO	23		50	FMT	42		90	0	00	
11	0	00		51	DN	25		91	0	00	
12	STP	41	ENTRY	52	FMT	42		92	0	00	
13	LN	65		53	UP	27		93	0	00	
14	XEY	30		54	YEX	24		94	0	00	
15	LN	65		55	0	00		95	0	00	
16	AC+	60		56	IFG	43		96	0	00	
17	UP	27		57	6	06		97	0	00	
18	X	36		58	1	01		98	0	00	
19	XEY	30		59	1	01		99	0	00	
1a	YEX	24		5a	+	33		9a	0	00	
1b	8	10		5b	DN	25		9b	0	00	
1c	+	33		5c	GTO	44		9c	0	00	
1d	YEX	24		5d	1	01		9d	0	00	
20	8	10		60	0	00		a0	0	00	
21	DN	25		61	e	12		a1	0	00	
22	X	36		62	UP	27		a2	0	00	
23	XEY	30		63	X	36		a3	0	00	
24	YEX	24		64	RUP	22		a4	0	00	
25	9	11		65	YEX	24		a5	0	00	
26	+	33		66	7	07		a6	0	00	
27	YEX	24		67	X	36		a7	0	00	
28	9	11		68	RDN	31		a8	0	00	
29	DN	25		69	-	34		a9	0	00	
2a	RDN	31		6a	YEX	24		aa	0	00	
2b	X	36		6b	f	15		ab	0	00	
2c	XEY	30		6c	DN	25		ac	0	00	
2d	YEX	24		6d	STP	41		ad	0	00	
30	7	07		70	0	00					
31	+	33		71	0	00					
32	YEX	24		72	0	00					
33	7	07		73	0	00					
34	DN	25		74	0	00					X_{shift}
35	EXP	74		75	0	00					Y_{shift}
36	XEY	30		76	0	00					X_{scale}
37	EXP	74		77	0	00					Y_{scale}
38	UP	27		78	0	00					X
39	b	14		79	0	00					ΔX
3a	-	34		7a	0	00					
3b	d	17		7b	0	00					
3c	DIV	35		7c	0	00					
3d	a	13		7d	0	00					

b0	0	00
b1	0	00
b2	0	00
b3	0	00
b4	0	00
b5	0	00
b6	0	00
b7	0	00
b8	0	00
b9	0	00
ba	0	00
bb	0	00
bc	0	00
bd	0	00

10	UP	27
11	YEX	24
12	9	11
13	DN	25
14	X	36
15	RUP	22
16	XEY	30
17	YEX	24
18	8	10
19	YTO	40
1a	0	00
1b	e	12
1c	X	36
1d	DN	25

50	RCL	61
51	AC-	63
52	+	33
53	AC+	60
54	DN	25
55	LN	65
56	YEX	24
57	1	01
58	YTO	40
59	1	01
5a	X	36
5b	DN	25
5c	EXP	74
5d	YEX	24

c0	0	00
c1	0	00
c2	0	00
c3	0	00
c4	0	00
c5	0	00
c6	0	00
c7	0	00
c8	0	00
c9	0	00
ca	0	00
cb	0	00
cc	0	00
cd	0	00

20	-	34
21	UP	27
22	DN	25
23	f	15
24	CHS	32
25	DIV	35
26	DN	25
27	UP	27
28	YEX	24
29	e	12
2a	X	36
2b	XEY	30
2c	YEX	24
2d	0	00

60	0	00
61	YTO	40
62	0	00
63	X	36
64	b	14
65	-	34
66	d	17
67	DIV	35
68	e	12
69	UP	27
6a	a	13
6b	-	34
6c	c	16
6d	DIV	35

d0	0	00
d1	0	00
d2	0	00
d3	0	00
d4	0	00
d5	0	00
d6	0	00
d7	0	00
d8	0	00
d9	0	00
da	0	00
db	0	00
dc	0	00
dd	END	46

30	-	34
31	XEY	30
32	YEX	24
33	8	10
34	XEY	30
35	DIV	35
36	XEY	30
37	EXP	74
38	YEX	24
39	9	11
3a	RDN	31
3b	DIV	35
3c	e	12
3d	RUP	22

70	5	05
71	0	00
72	0	00
73	X	36
74	RUP	22
75	X	36
76	DN	25
77	XEY	30
78	FMT	42
79	DN	25
7a	GTO	44
7b	5	05
7c	0	00
7d	END	46

Program B

00	UP	27
01	X	36
02	XEY	30
03	YEX	24
04	8	10
05	RDN	31
06	XEY	30
07	X	36
08	RDN	31
09	-	34
0a	f	15
0b	X	36
0c	DN	25
0d	√	76

40	XTO	23
41	0	00
42	YTO	40
43	1	01
44	STP	41
45	CLR	20
46	c	16
47	UP	27
48	8	10
49	DIV	35
4a	a	13
4b	XEY	30
4c	-	34
4d	AC+	60

GENERAL POWER CURVE REGRESSION AND PLOT

This program computes the least squares fit and correlation coefficient of N pairs of data points for a power curve of the form:

$$Y = aX^b + C$$

The program will also plot each data point and the curve after the unknowns have been calculated.

The equation is linearized into $\ln Y = b \ln X + \ln a$

where

$$b = \frac{N\Sigma(\ln X \ln Y) - \Sigma \ln X \, \Sigma \ln Y}{N\Sigma(\ln X)^2 - (\Sigma \ln X)^2}$$

and

$$r = \frac{N\Sigma(\ln X \ln Y) - (\Sigma \ln X)(\Sigma \ln Y)}{\sqrt{\left[N\Sigma(\ln X)^2 - (\Sigma \ln X)^2\right]\left[N\Sigma(\ln Y)^2 - (\Sigma \ln Y)^2\right]}}$$

$$\ln a = \frac{\Sigma \ln Y}{N} - \frac{\Sigma \ln X}{N} b$$

NOTE: $X_i > 0$, $Y_i > 0$, $i = 1, \ldots, N$

C is the Y-Intercept Value at X = 0 or at X = ∞. The value for C may be entered directly if it is known. If C is unknown the user is advised to first run one of the exponential regression programs ($Y = ae^{bX}$) utilizing the first 3 or 4 minimum Y-value data points. The resulting value for a may then be entered as the value for C.

Reference: Statistical Theory and Methodology in Science and Engineering by K.A. Brownlee, John Wiley and Sons, 1965.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

SET: Decimal Wheel at 6 or less

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

ENTER DATA:

$C \longrightarrow Z \quad (C \neq 0)$
$Y_{shift} \longrightarrow Y$
$X_{shift} \longrightarrow X$

PRESS: CONTINUE

ENTER DATA:

$\Delta Y \longrightarrow Z$ *
$Y_{scale} \longrightarrow Y$
$X_{scale} \longrightarrow X$

PRESS: CONTINUE

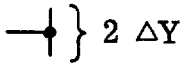
DISPLAY

0 ——— Z
i ——— Y
0 ——— X

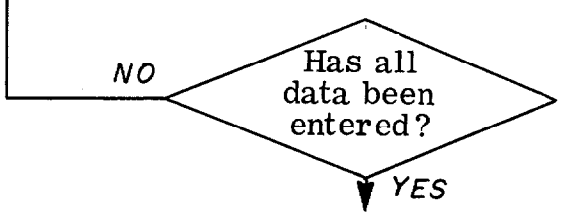
ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

NOTE: Plot points are of the form:



PRESS: CONTINUE



USER INSTRUCTIONS (Con't)

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

r ——— Z
b ——— Y
a ——— X

r, b, and a will be printed out

PRESS: CONTINUE

The plotter will now plot the least squares curve. To interrupt the plot, PRESS: STOP. The plot is self-terminating.

To run another case, return to the beginning of the USER INSTRUCTIONS, enter the program and proceed as before.

*ΔY is the desired tolerance on Y.

EXAMPLE

$C = 10$
$Y_{shift} = 0$
$X_{shift} = 0$
$\Delta Y = 3$
$Y_{scale} = 30$
$X_{scale} = 1$

i	X	Y
1	1	18
2	2	30
3	3	44
4	4	56
5	5	73
6	6	96
7	7	121
8	8	151
9	9	196
10	10	276

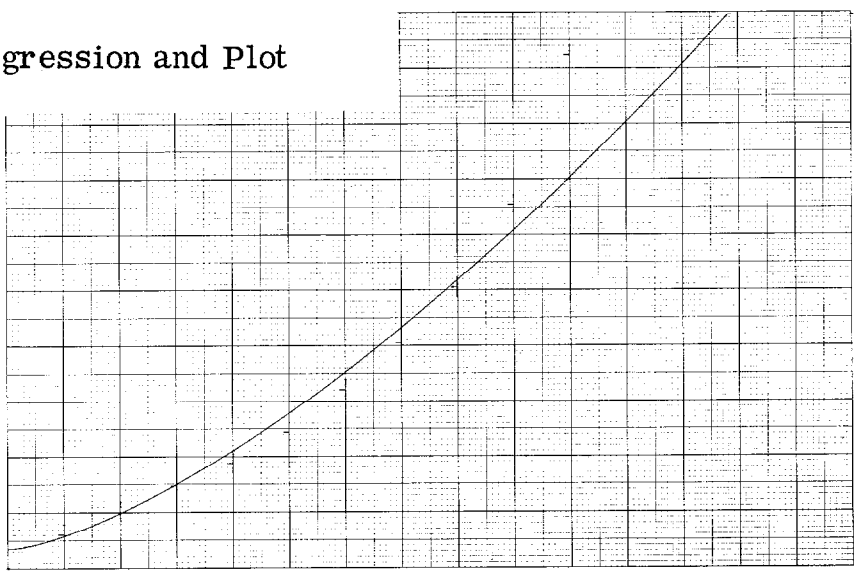
Results:

r = .99
b = 1.45
a = 7.07

STAT-PAC
X-9

General Power Regression and Plot

$Y = aX^b + C$



00	CLR	20		Plus Page	40	UP	27		80	0	00
01	STP	41	ENTRY		41	X	36		81	0	00
02	PNT	45			42	XEY	30		82	0	00
03	XTO	23			43	YE	24		83	0	00
04	a	13			44	8	10		84	0	00
05	DN	25			45	+	33		85	0	00
06	YTO	40			46	YE	24		86	0	00
07	-	34			47	8	10		87	0	00
08	f	15			48	DN	25		88	0	00
09	XEY	30			49	X	36		89	0	00
0a	-	34			4a	XEY	30		8a	0	00
0b	YTO	40			4b	YE	24		8b	0	00
0c	b	14			4c	7	07		8c	0	00
0d	CLR	20			4d	+	33		8d	0	00
10	STP	41	ENTRY		50	YE	24		90	0	00
11	PNT	45			51	7	07		91	0	00
12	PNT	45			52	DN	25		92	0	00
13	RDN	31			53	RDN	31		93	0	00
14	X	36			54	X	36		94	0	00
15	YTO	40			55	XEY	30		95	0	00
16	-	34			56	YE	24		96	0	00
17	e	12			57	9	11		97	0	00
18	RDN	31			58	+	33		98	0	00
19	5	05			59	YE	24		99	0	00
1a	0	00			5a	9	11		9a	0	00
1b	0	00			5b	DN	25		9b	0	00
1c	DIV	35			5c	EXP	74		9c	0	00
1d	YTO	40			5d	XEY	30		9d	0	00
20	c	16			60	EXP	74		a0	0	00
21	XEY	30			61	UP	27		a1	0	00
22	RDN	31			62	b	14		a2	0	00
23	DIV	35			63	-	34		a3	0	00
24	YTO	40			64	d	17		a4	0	00
25	d	17			65	DIV	35		a5	0	00
26	CLR	20			66	GTO	44		a6	0	00
27	1	01			67	-	34		a7	0	00
28	RUP	22			68	0	00		a8	0	00
29	YTO	40			69	0	00		a9	0	00
2a	0	00			6a	GTO	44		aa	0	00
2b	STP	41	ENTRY		6b	-	34		ab	0	00
2c	IFG	43			6c	3	03		ac	0	00
2d	6	06			6d	9	1		ad	0	00
30	a	13			70	0	00				
31	PNT	45			71	0	00				
32	PNT	45			72	0	00				
33	LN	65			73	0	00				
34	RUP	22			74	0	00				
35	XFR	67			75	0	00				
36	-	34			76	0	00				
37	f	15			77	0	00				
38	XEY	30			78	0	00				
39	RDN	31			79	0	00				
3a	-	34			7a	0	00				
3b	DN	25			7b	0	00				
3c	LN	65			7c	0	00				
3d	AC+	60			7d	0	00				

											$\Sigma(\ln X_i)^2$
											X_{shift}
											Y_{shift}
											X_{scale}
											Y_{scale}
											$\Sigma \ln Y \quad X$
									$\Sigma(\ln Y_i \ln X_i)$		$\Sigma \ln X \quad \Delta X$

b0	0	00	Plus Page
b1	0	00	
b2	0	00	
b3	0	00	
b4	0	00	
b5	0	00	
b6	0	00	
b7	0	00	
b8	0	00	
b9	0	00	
ba	0	00	
bb	0	00	
bc	0	00	
bd	0	00	
c0	0	00	
c1	0	00	
c2	0	00	
c3	0	00	
c4	0	00	
c5	0	00	
c6	0	00	
c7	0	00	
c8	0	00	
c9	0	00	
ca	0	00	
cb	0	00	
cc	0	00	
cd	0	00	
d0	0	00	
d1	0	00	
d2	0	00	
d3	0	00	
d4	0	00	
d5	0	00	
d6	0	00	
d7	0	00	
d8	0	00	
d9	0	00	
da	0	00	
db	0	00	
dc	0	00	
dd	0	00	

00	a	13	Minus Page
01	RUP	22	
02	XEY	30	
03	-	34	
04	c	16	
05	DIV	35	
06	XFR	67	
07	-	34	
08	e	12	
09	XEY	30	
0a	RDN	31	
0b	+	33	
0c	RDN	31	
0d	XEY	30	

10	FMT	42	
11	DN	25	
12	RDN	31	
13	XEY	30	
14	-	34	
15	-	34	
16	RDN	31	
17	XEY	30	
18	FMT	42	
19	DN	25	
1a	RDN	31	
1b	+	33	
1c	XEY	30	
1d	RUP	22	

20	FMT	42	
21	DN	25	
22	UP	27	
23	5	05	
24	0	00	
25	-	34	
26	CLX	37	
27	RDN	31	
28	FMT	42	
29	DN	25	
2a	FMT	42	
2b	UP	27	
2c	YE	24	
2d	0	00	

30	1	01	
31	+	33	
32	CLX	37	
33	UP	27	
34	RUP	22	
35	GTO	44	
36	+	33	
37	2	02	
38	8	10	
39	1	01	
3a	-	34	
3b	e	12	
3c	UP	27	
3d	X	36	

40	RUP	22	
41	YE	24	
42	9	11	
43	X	36	
44	RDN	31	
45	-	34	
46	YE	24	
47	f	15	
48	DN	25	
49	UP	27	
4a	X	36	
4b	XEY	30	
4c	YE	24	
4d	8	10	

50	RDN	31	
51	XEY	30	
52	X	36	
53	RDN	31	
54	-	34	
55	f	15	
56	X	36	
57	DN	25	
58	$\sqrt{}$	76	
59	UP	27	
5a	YE	24	
5b	7	07	
5c	DN	25	
5d	X	36	

60	RUP	22	
61	XEY	30	
62	YE	24	
63	8	10	
64	YTO	40	
65	0	00	
66	e	12	
67	X	36	
68	DN	25	
69	-	34	
6a	UP	27	
6b	DN	25	
6c	f	15	
6d	CHS	32	

70	DIV	35	
71	DN	25	
72	UP	27	
73	YE	24	
74	e	12	
75	X	36	
76	XEY	30	
77	YE	24	
78	0	00	
79	-	34	
7a	XEY	30	
7b	YE	24	
7c	8	10	
7d	XEY	30	

80	DIV	35	
81	XEY	30	
82	EXP	74	
83	YE	24	
84	7	07	
85	RDN	31	
86	DIV	35	
87	e	12	
88	RUP	22	
89	XTO	23	
8a	0	00	
8b	YTO	40	
8c	1	01	
8d	PNT	45	S

90	CLR	20	
91	c	16	
92	UP	27	
93	8	10	
94	DIV	35	
95	5	05	
96	0	00	
97	0	00	
98	X	36	
99	a	13	
9a	XEY	30	
9b	-	34	
9c	AC+	60	
9d	RCL	61	

a0	AC-	63	
a1	+	33	
a2	AC+	60	
a3	DN	25	
a4	LN	65	
a5	UP	27	
a6	XFR	67	
a7	1	01	
a8	X	36	
a9	DN	25	
aa	EXP	74	
ab	UP	27	
ac	XFR	67	
ad	0	00	

			Δ Y
			C

b0	X	36
b1	b	14
b2	-	34
b3	d	17
b4	DIV	35
b5	e	12
b6	UP	27
b7	a	13
b8	-	34
b9	c	16
ba	DIV	35
bb	7	07
bc	5	05
bd	0	00

Minus
Page

c0	0	00
c1	X<Y	52
c2	d	17
c3	5	05
c4	RUP	22
c5	XEY	30
c6	5	05
c7	EEX	26
c8	3	03
c9	X<Y	52
ca	9	11
cb	d	17
cc	DN	25
cd	XEY	30

d0	FMT	42
d1	DN	25
d2	GTO	44
d3	9	11
d4	d	17
d5	CLR	20
d6	FMT	42
d7	UP	27
d8	END	46

EXPONENTIAL REGRESSION AND PLOT

This program computes the least squares fit and a correlation coefficient of n pairs of data points for an exponential function of the form:

$$y = ae^{bx}$$

The program also plots each point and the least squares curve after the unknowns have been calculated.

The equation is linearized into

$$\ln y = \ln a + bx$$

or

$$Y = A + bx$$

Using a linear regression method,

$$b = \frac{n\sum x\,Y - \sum x \sum Y}{n\sum x^2 - (\sum x)^2}$$

$$A = \frac{\sum Y - b\sum x}{n}$$

$$a = e^{A}$$

the correlation coefficient is given by

$$r = \frac{n\sum x\,Y - \sum x \sum Y}{\sqrt{\left[n\sum x^2 - (\sum x)^2\right]\left[n\sum Y^2 - (\ Y)^2\right]}}$$

Note: $Y_i > 0 \quad i = 1, \cdots, n$

Reference: Statistical Theory and Methodology in Science and Engineering
by K. A. Brownlee

John Wiley and Sons, 1965

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A
Using the origin controls, locate the pen in the lower left corner.

ENTER PROGRAM A: (Starting Address is (0)(0))

SET: Decimal wheel at 6 or less

PRESS: END

PRESS: CONTINUE

ENTER TRANSLATION CONSTANTS:

Y_{shift} → Y

X_{shift} → X

PRESS: CONTINUE

ENTER SCALING CONSTANTS:

Y_{scale} → Y

X_{scale} → X

PRESS: CONTINUE

DISPLAY

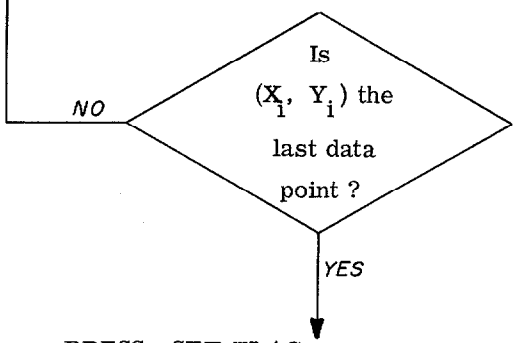
0	Z
0	Y
i	X

Each data point will be plotted as entered.

ENTER DATA: Y_i → Y

X_i → X

PRESS: PRINT



PRESS: SET FLAG

PRESS: CONTINUE

USER INSTRUCTIONS (con't)

ENTER PROGRAM B: (Starting Address is (0)(0))

PRESS: END

PRESS: CONTINUE

DISPLAY

a	Z
b	Y
r	X

PRESS: PRINT

PRESS: CONTINUE

The least squares curve $y = ae^{bx}$ will now be plotted. To interrupt the plot or stop it when finished, PRESS: STOP.

To re-run program return to beginning of USER INSTRUCTIONS, ENTER PROGRAM B and proceed as before.

EXAMPLE

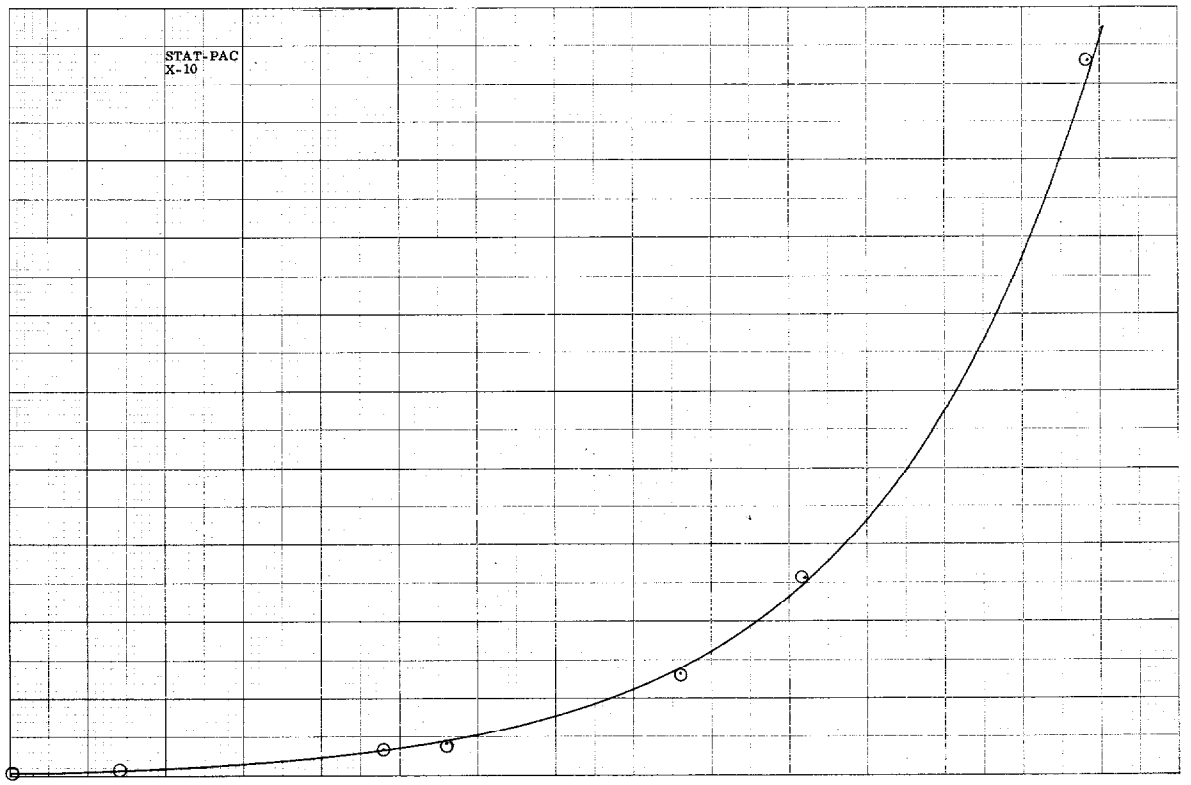
X	Y
.5	7.12
1.2	11.67
2.9	38.85
3.3	48.03
4.8	138.70
5.6	262.00
7.4	935.64

Let: $X_{shift} = .5$ $\quad X_{scale} \geq \frac{7.4 - .5}{15} \approx .5 \frac{\text{units}}{\text{inch}}$

$Y_{shift} = 5$ $\quad Y_{scale} \geq \frac{935.64 - 5}{10} \approx 100 \frac{\text{units}}{\text{inch}}$

Solution: a = 4.925
b = .705
r = .9998

$Y = 4.925e^{.705X}$


STAT-PAC
X-10

00	CLR	20		40	d	17		80	0	00	
01	STP	41	ENTRY	41	DIV	35		81	0	00	
02	XTO	23		42	5	05		82	0	00	
03	a	13		43	0	00		83	0	00	
04	YTO	40		44	0	00		84	0	00	
05	b	14		45	X	36		85	0	00	
06	CLR	20		46	RUP	22		86	0	00	
07	STP	41	ENTRY	47	X	36		87	0	00	
08	XTO	23		48	CLX	37		88	0	00	
09	c	16		49	RDN	31		89	0	00	
0a	YTO	40		4a	FMT	42		8a	0	00	
0b	d	17		4b	DN	25		8b	0	00	
0c	CLR	20		4c	FMT	42		8c	0	00	
0d	1	01		4d	UP	27		8d	0	00	
10	XTO	23		50	YEX	24		90	0	00	
11	0	00		51	0	00		91	0	00	
12	STP	41	ENTRY	52	IFG	43		92	0	00	
13	XEY	30		53	5	05		93	0	00	
14	LN	65		54	b	14		94	0	00	
15	AC+	60		55	1	01		95	0	00	
16	UP	27		56	+	33		96	0	00	
17	X	36		57	DN	25		97	0	00	
18	XEY	30		58	GTO	44		98	0	00	
19	YEX	24		59	1	01		99	0	00	
1a	7	07		5a	0	00		9a	0	00	
1b	+	33		5b	DN	25		9b	0	00	
1c	YEX	24		5c	UP	27		9c	0	00	
1d	7	07		5d	YEX	24		9d	END	46	

Program B

20	DN	25		60	8	10		00	YTO	40	
21	X	36		61	X	36		01	8	10	
22	XEY	30		62	e	12		02	X	36	
23	YEX	24		63	UP	27		03	DN	25	
24	9	11		64	X	36		04	YEX	24	
25	+	33		65	DN	25		05	f	15	
26	YEX	24		66	-	34		06	XEY	30	
27	9	11		67	UP	27		07	UP	27	
28	DN	25		68	YEX	24		08	X	36	
29	RDN	31		69	7	07		09	RDN	31	
2a	X	36		6a	XEY	0		0a	-	34	
2b	XEY	30		6b	YEX	24		0b	DN	25	
2c	YEX	24		6c	8	0		0c	YEX	24	
2d	8	10		6d	STP	41		0d	f	15	
30	+	33		70	0	00					
31	YEX	24		71	0	00					
32	8	10		72	0	00					
33	RUP	22		73	0	00					
34	EXP	74		74	0	00					X_{shift}
35	RDN	31		75	0	00					Y_{shift}
36	a	13		76	0	00					X_{scale}
37	-	34		77	0	00					Y_{scale}
38	c	16		78	0	00					x
39	DIV	35		79	0	00					Δx
3a	b	14		7a	0	00					
3b	RUP	22		7b	0	00					
3c	XEY	30		7c	0	00					
3d	-	34		7d	0	00					

10	YTO	40
11	0	00
12	X	36
13	DN	25
14	√	76
15	YEX	24
16	8	10
17	YTO	40
18	8	10
19	XEY	30
1a	YEX	24
1b	9	11
1c	X	36
1d	UP	27

20	RCL	61
21	X	36
22	DN	25
23	-	34
24	UP	27
25	YEX	24
26	9	11
27	DN	25
28	DIV	35
29	YEX	24
2a	0	00
2b	DN	25
2c	DIV	35
2d	e	12

30	UP	27
31	DN	25
32	CHS	32
33	X	36
34	f	15
35	+	33
36	XEY	30
37	YEX	24
38	8	10
39	XEY	30
3a	DIV	35
3b	DN	25
3c	EXP	74
3d	YEX	24

40	0	00
41	XTO	23
42	1	01
43	RDN	31
44	STP	41
45	CLR	20
46	c	16
47	UP	27
48	6	06
49	DIV	35
4a	a	13
4b	XEY	30
4c	-	34
4d	AC+	60

50	RCL	61
51	AC-	63
52	+	33
53	AC+	60
54	DN	25
55	YEX	24
56	0	00
57	YTO	40
58	0	00
59	X	36
5a	DN	25
5b	EXP	74
5c	YEX	24
5d	1	01

60	YTO	40
61	1	01
62	X	36
63	b	14
64	-	34
65	d	17
66	DIV	35
67	e	12
68	UP	27
69	a	13
6a	-	34
6b	c	16
6c	DIV	35
6d	5	05

70	0	00
71	0	00
72	X	36
73	RUP	22
74	X	36
75	DN	25
76	XEY	30
77	FMT	42
78	DN	25
79	GTO	44
7a	5	05
7b	0	00
7c	END	46

CUBIC (THIRD DEGREE) REGRESSION PLOT

This program plots the data points and the cubic curve associated with a third degree regression curve fit. The raw data points may be plotted before or after program STAT-PAC IV-9 is run.

The curve being plotted is given by:

$$f(X) = Y = a_0 + a_1X + a_2X^2 + a_3X^3$$

where the a_i are determined by program STAT-PAC IV-9. The user specifies the following:

X_{max}	=	Largest Value of X_i, to Be Plotted at X = 10 in; or 25.4 cm.
X_{min}	=	Smallest Value of X_i, to Be Plotted at X = 0 in; or 0 cm.
Y_{max}	=	Largest Value of Y_i, to Be Plotted at Y = 5 in; or 12.7 cm.
Y_{min}	=	Smallest Value of Y_i, to Be Plotted at Y = 0 in; or 0 cm.
X_{final}	=	The Point to Which f(X) is Desired for Prediction
$X_{initial}$	=	The Point from Which f(X) is Desired to Be Plotted. ($X_{initial} \geq X_{min}$)

The point { X_{max}, Y } is plotted at X = 10 in., or 25.4 cm. in order to leave 5 in., or 12.7 cm. of plotting surface remaining so that f(X) may be plotted beyond the limits of known (X_i, Y_i) for prediction purposes. The plot position of the points X_{max}, and Y_{max} may be changed by altering program steps (+)(0)(6) through (+)(0)(8) for X_{max}, and (+)(1)(7) through (+)(1)(a) for Y_{max}.

USER INSTRUCTIONS

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

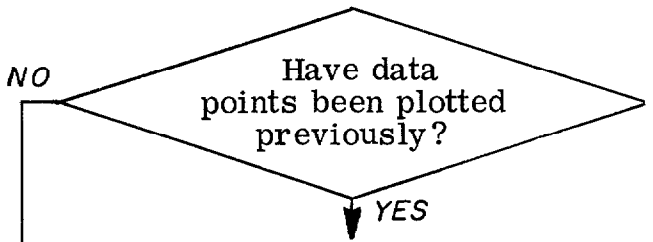
ENTER DATA:

$X_{max} \longrightarrow Y$
$X_{min} \longrightarrow X$

PRESS: CONTINUE

ENTER DATA

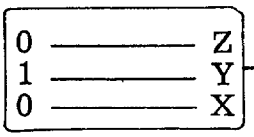
$Y_{max} \longrightarrow Y$
$Y_{min} \longrightarrow X$



PRESS: SET FLAG

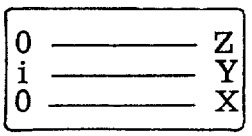
PRESS: CONTINUE

DISPLAY



PRESS: CONTINUE

DISPLAY

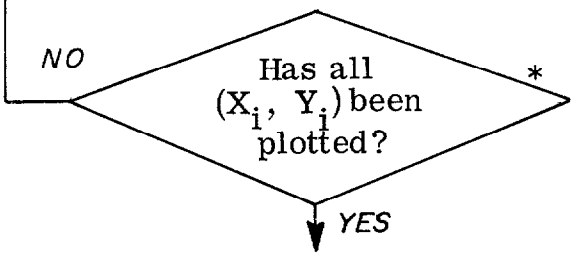


USER INSTRUCTIONS (Con't)

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

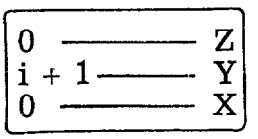
PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY



ENTER DATA:

$a_3 \longrightarrow Z$
$a_2 \longrightarrow Y$
$a_1 \longrightarrow X$

PRESS: CONTINUE

ENTER DATA:

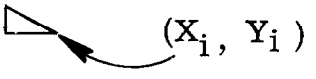
$X_{final} \longrightarrow Z$
$X_{initial} \longrightarrow Y$
$a_0 \longrightarrow X$

PRESS: CONTINUE

The cubic curve is now plotted. The plot will automatically stop when X reaches X_{final}.

To run another case

*The point (X_i, Y_i) is plotted as



EXAMPLE

Using the data from the example of program STAT-PAC IV-9.

X	Y
-2	-23
-1	2
0	5
1	4
2	17

Let:

$X_{max} = 2$
$X_{min} = -2$

$Y_{max} = 17$
$Y_{min} = -23$

$a_3 = 3$
$a_2 = -2$
$a_1 = -2$

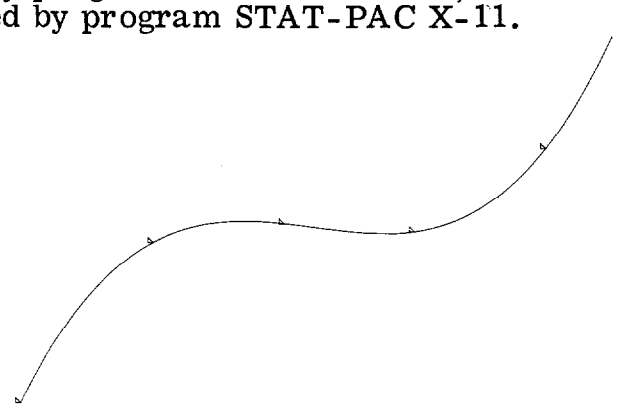
$X_{final} = 2.5$
$X_{initial} = -2$
$a_0 = 5$

The resulting plot is shown

STAT-PAC
X-11

Cubic (Third Degree) Regression Plot
$Y = 5 - 2X - 2X^2 + 3X^3$

Cubic coefficients determined by program STAT-PAC IV-9,
points and fitted curve generated by program STAT-PAC X-11.



00	CLR	20	Plus Page	40	-	34		80	CNT	47	
01	STP	41	ENTRY	41	d	17		81	CNT	47	
02	PNT	45		42	X	36		82	CNT	47	
03	PNT	45		43	DN	25		83	CNT	47	
04	-	34		44	XEY	30		84	CNT	47	
05	UP	27		45	FMT	42		85	CNT	47	
06	5	05		46	DN	25		86	CNT	47	
07	EEX	26		47	UP	27		87	CNT	47	
08	3	03		48	5	05		88	CNT	47	
09	RUP	22		49	0	00		89	CNT	47	
0a	DIV	35		4a	-	34		8a	CNT	47	
0b	DN	25		4b	RDN	31		8b	CNT	47	
0c	AC+	60		4c	FMT	42		8c	CNT	47	
0d	0	00		4d	DN	25		8d	CNT	47	
10	UP	27		50	XEY	30		90	CNT	47	
11	UP	27		51	RUP	22		91	CNT	47	
12	STP	41	ENTRY	52	+	33		92	CNT	47	
13	PNT	45		53	RDN	31		93	CNT	47	
14	PNT	45		54	XEY	30		94	CNT	47	
15	-	34		55	FMT	42		95	CNT	47	
16	UP	27		56	DN	25		96	CNT	47	
17	2	02		57	RUP	22		97	CNT	47	
18	5	05		58	+	33		98	CNT	47	
19	0	00		59	XEY	30		99	CNT	47	
1a	0	00		5a	RDN	31		9a	CNT	47	
1b	RUP	22		5b	-	34		9b	CNT	47	
1c	DIV	35		5c	DN	25		9c	CNT	47	
1d	DN	25		5d	XEY	30		9d	CNT	47	
20	YTO	40		60	FMT	42		a0	CNT	47	
21	c	16		61	DN	25		a1	CNT	47	
22	XTO	23		62	FMT	42		a2	CNT	47	
23	d	17		63	UP	27		a3	CNT	47	
24	0	00		64	0	00		a4	CNT	47	
25	UP	27		65	UP	27		a5	CNT	47	
26	1	01		66	b	14		a6	CNT	47	
27	UP	27		67	UP	27		a7	CNT	47	
28	0	00		68	1	01		a8	CNT	47	
29	IFG	43		69	+	33		a9	CNT	47	
2a	7	07		6a	0	00		aa	CNT	47	
2b	0	00		6b	GTO	44		ab	CNT	47	
2c	STP	41	ENTRY	6c	2	02		ac	CNT	47	
2d	IFG	43		6d	c	16		ad	CNT	47	
30	7	07		70	STP	41	ENTRY				a_0
31	0	00		71	PNT	45					
32	PNT	45		72	PNT	45					a_3
33	PNT	45		73	XTO	23					
34	RUP	22		74	b	14					a_2/X
35	XTO	23		75	YTO	40					
36	b	14		76	a	13					n/a_1
37	e	12		77	0	00					Y_{min}
38	-	34		78	RDN	31					
39	f	15		79	DN	25					Y_f
3a	X	36		7a	GTO	44					
3b	c	16		7b	-	34					X_{min}
3c	XEY	30		7c	0	00					
3d	RDN	31		7d	0	00					X_f

b0	CNT	47	Plus Page
b1	CNT	47	
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	

c0	CNT	47
c1	CNT	47
c2	CNT	47
c3	CNT	47
c4	CNT	47
c5	CNT	47
c6	CNT	47
c7	CNT	47
c8	CNT	47
c9	CNT	47
ca	CNT	47
cb	CNT	47
cc	CNT	47
cd	CNT	47

d0	CNT	47
d1	CNT	47
d2	CNT	47
d3	CNT	47
d4	CNT	47
d5	CNT	47
d6	CNT	47
d7	CNT	47
d8	CNT	47
d9	CNT	47
da	CNT	47
db	CNT	47
dc	CNT	47
dd	CNT	47

00	XTO	23	Minus Page	40	d	17	
01	9	11		41	X	36	
02	DN	25		42	XFR	67	
03	STP	41	ENTRY	43	-	34	
04	PNT	45		44	f	15	
05	PNT	45		45	UP	27	
06	XTO	23		46	e	12	
07	8	10		47	-	34	
08	YTO	40		48	f	15	
09	-	34		49	X	36	
0a	f	15		4a	DN	25	
0b	DN	25		4b	FMT	42	
0c	YTO	40		4c	DN	25	
0d	-	34		4d	XFR	67	
10	e	12		50	-	34	
11	-	34		51	f	15	
12	EEX	26		52	UP	27	
13	2	02		53	XFR	67	
14	DIV	35		54	-	34	
15	YTO	40		55	d	17	
16	-	34		56	+	33	
17	d	17		57	XFR	67	
18	XFR	67		58	-	34	
19	-	34		59	e	12	
1a	f	15		5a	X<Y	52	
1b	UP	27		5b	6	06	
1c	b	14		5c	5	05	
1d	XEY	30		5d	YTO	40	
20	X	36		60	-	34	
21	UP	27		61	f	15	
22	X	36		62	GTO	44	
23	XEY	30		63	1	01	
24	YE	24		64	8	10	
25	a	13		65	CLR	20	
26	XEY	30		66	FMT	42	
27	X	36		67	UP	27	
28	XEY	30		68	GTO	44	
29	YE	24		69	+	33	
2a	a	13		6a	0	00	
2b	RUP	22		6b	0	00	
2c	+	33		6c	END	46	
2d	XEY	30					
30	RUP	22					
31	UP	27					
32	X	36					
33	X	36					
34	XFR	67					
35	9	11					
36	X	36					
37	DN	25					
38	+	33					
39	XFR	67					
3a	8	10					
3b	+	33					
3c	c	16					
3d	-	34					

x_0

x_{final}

$\triangle x$

LEAST SQUARES REGRESSION
(Linear, Parabolic, Power, Exponential)

This program allows the user to calculate and plot the equations of four different regression curves with a single entry of his discrete data.

This regression package is divided into six individual programs as described below and has an error correcting option.

I. MAIN PROGRAM

All data and scaling factors for the four regression programs are entered through this program. Also entered is a desired tolerance on the regression equations. As each data point is entered, a vertical line is plotted above and below this point to allow the user to see if the particular regression curves are within this tolerance.

The main section of the program calculates and accumulates the sums which are needed to generate the least squares coefficients.

II. MAIN PROGRAM CORRECTOR

This is an optional program which may be run if while running the MAIN PROGRAM the user has entered incorrect data points. The program simply deletes these incorrect points from the series of accumulated sums and plots an "X" over such points.

III. LINEAR REGRESSION & PLOT

This program calculates a & b for the equation: $Y = bx + a$.
The defining equations are:

$$b = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_i - \overline{X})^2}$$

$$a = \overline{Y} - b\overline{X}$$

where: $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$; $\overline{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i$

IV. PARABOLIC REGRESSION & PLOT

This program calculates a_0, a_1, a_2 for the equation $Y = a_0 + a_1X + a_2X^2$.
The coefficients are found by simultaneously solving the following normal equations:

$$\Sigma Y = a_0 n + a_1 \Sigma X + a_2 \Sigma X^2$$
$$\Sigma XY = a_0 \Sigma X + a_1 \Sigma X^2 + a_2 \Sigma X^3$$
$$\Sigma X^2 Y = a_0 \Sigma X^2 + a_1 \Sigma X^3 + a_2 \Sigma X^4$$

V. POWER CURVE REGRESSION & PLOT

This program calculates the correlation coefficient and a and b for a power curve of the form $Y = aX^b$.

The equation is linearized into $\ln Y = b \ln X + \ln a$

where $$b = \frac{n \Sigma (\ln X \ln Y) - \Sigma \ln X \, \Sigma \ln Y}{n \Sigma (\ln X)^2 - (\Sigma \ln X)^2}$$

and $$r = \frac{n \Sigma \ln X \ln Y - (\Sigma \ln X)(\Sigma \ln Y)}{\sqrt{[\, n \Sigma (\ln X)^2 - (\Sigma \ln X)^2]\, [\, n \Sigma (\ln Y)^2 - (\Sigma \ln Y)^2 \,]}}$$

$$\ln a = \frac{\Sigma \ln Y}{n} - \frac{\Sigma \ln X}{n} \, b$$

VI. EXPONENTIAL REGRESSION & PLOT

This program claculates the correlation coefficient and a and b for an exponential function of the form $Y = ae^{bX}$.

The equation is linearized into $\ln Y = \ln a + bX$
or $Y = A + bX$

Using a linear regression method,

$$b = \frac{n \Sigma XY - \Sigma X \Sigma Y}{n \Sigma X^2 - (\Sigma X)^2}$$

$$A = \frac{\Sigma Y - b \Sigma X}{n}$$

$$a = e^A$$

the correlation coefficient is given by

$$r = \frac{n \Sigma XY - \Sigma X \Sigma Y}{\sqrt{[\, n \Sigma X^2 - (\Sigma X)^2 \,]\, [\, n \Sigma Y^2 - (\Sigma Y)^2 \,]}}$$

To convert the program for plotting in centimeters, make the following changes:

Program	Step	Change to
I	(-)(0)(8)	1
	(-)(0)(9)	9
	(-)(0)(a)	7
	(-)(c)(5)	2
	(-)(c)(6)	0
II	(-)(9)(0)	5
	(-)(9)(1)	0
	(-)(9)(2)	.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

SET: Decimal Wheel at 6 or less

PRESS: END

ENTER MAIN PROGRAM:
Side A followed by Side B

PRESS: CONTINUE

ENTER SHIFT FACTORS:

$Y_{shift} \longrightarrow Y$
$X_{shift} \longrightarrow X$

PRESS: CONTINUE

ENTER SCALE FACTORS:

$Y_{scale} \longrightarrow Y$
$X_{scale} \longrightarrow X$

PRESS: CONTINUE

ENTER DESIRED TOLERANCE ON Y:

$\Delta Y \longrightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
i	Y
0	X

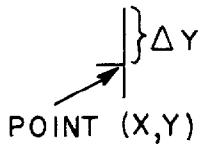
ENTER COORDINATES:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

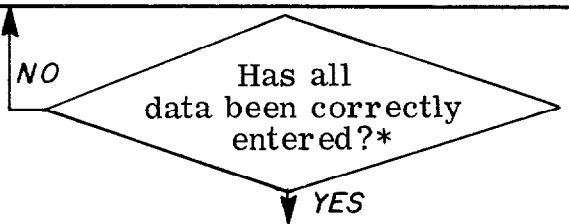
NOTE: $X_i > 0$, $Y_i > 0$

PRESS: CONTINUE

[Point is plotted as



USER INSTRUCTIONS (Con't)



PRESS: SET FLAG
PRESS: CONTINUE

DISPLAY

0	Z
0	Y
n	X

(n = total number of data points)

*Check the printer output to determine whether any incorrect data points have been entered in the program. If an error has been made, enter the correct values until all correct (X_i, Y_i) sets have been entered.

If all entered data is correct, proceed to the regression programs.

Otherwise:

PRESS: GO TO (-)(0)(0)

ENTER MAIN PROGRAM CORRECTOR

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

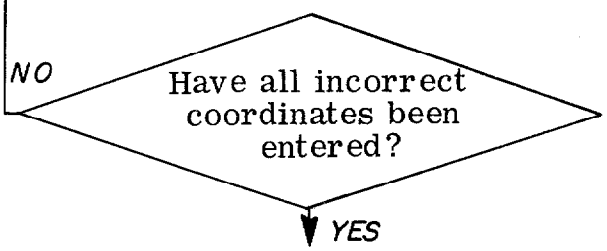
DISPLAY

0	Z
10i	Y
0	X

ENTER i^{th} SET OF COORDINATES TO BE DELETED:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
10K	X

(K = number of data points deleted)

Any of the four regression programs may now be run in any order any number of times. Individual instructions are given below. All curves are self-terminating.

POWER CURVE: $Y = aX^b$

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY

r	Z
b	Y
a	X

PRESS: CONTINUE

The curve is now plotted.

EXPONENTIAL CURVE: $Y = ae^{bX}$

(Same as POWER CURVE)

LINEAR CURVE: $Y = bX + a$

(Same as POWER CURVE except without r)

PARABOLIC CURVE: $Y=a_0+a_1X+a_2X^2$

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM A

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

a_0	Z
a_1	Y
a_2	X

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM B

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

The curve is now plotted.

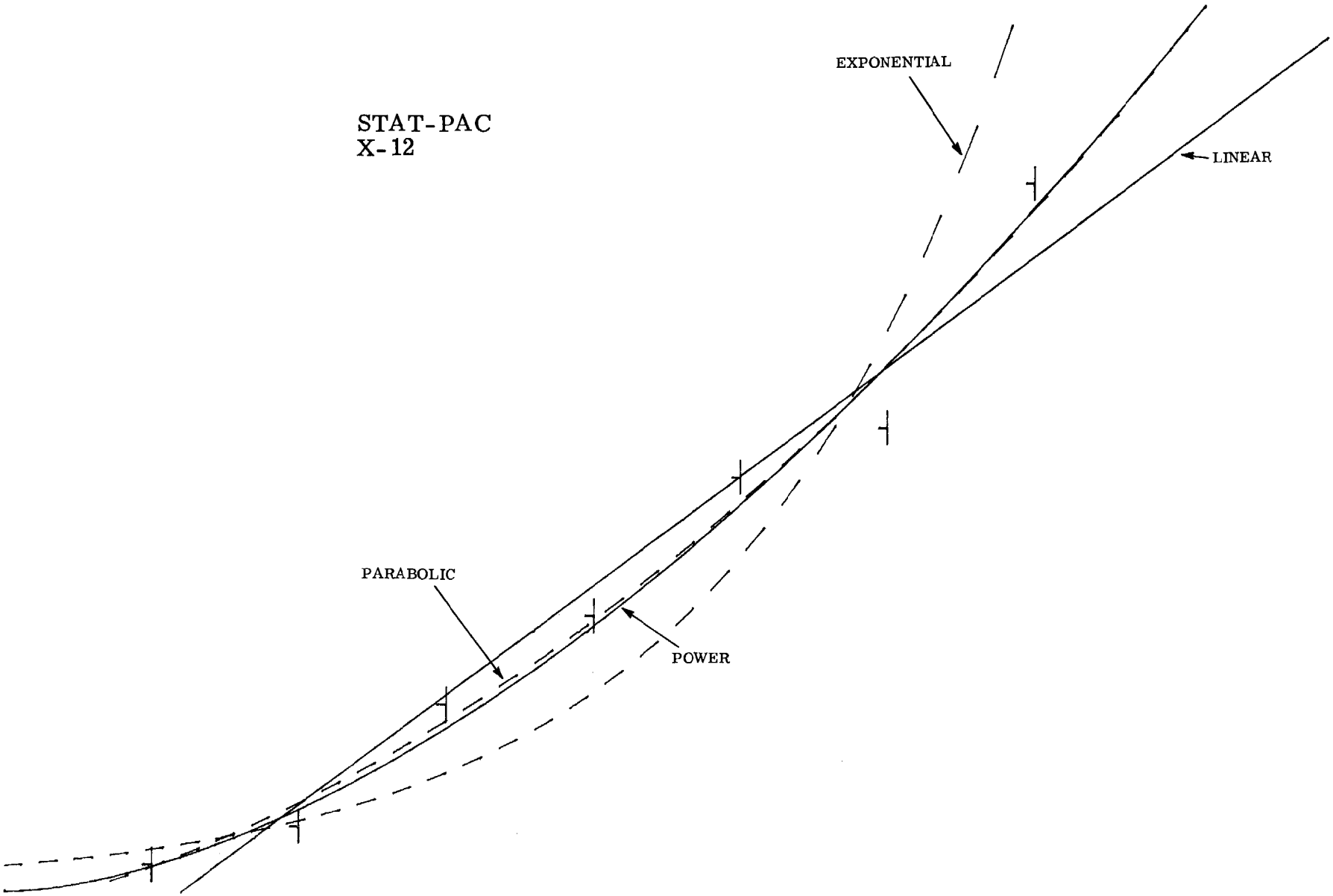
EXAMPLE

$X_{shift} = Y_{shift} = 0$
$X_{scale} = .6$
$Y_{scale} = 16$
$\Delta Y = 3$

X	Y
1	5
2	12
3	34
4	50
5	75
6	84
7	128

PARABOLA: $1.64X^2 + 6.64X - 4.00$
EXPONENTIAL: $4.63e^{.51}$, r = .96
POWER: $4.66X^{1.68}$, r = .994
LINEAR: 19.79X - 23.7

STAT-PAC
X-12



00	0	00	Plus Page	40	0	00		80	0	00
01	0	00		41	0	00		81	0	00
02	0	00		42	0	00		82	0	00
03	0	00		43	0	00		83	0	00
04	0	00		44	0	00		84	0	00
05	0	00		45	0	00		85	0	00
06	0	00		46	0	00		86	0	00
07	0	00		47	0	00		87	0	00
08	0	00		48	0	00		88	0	00
09	0	00		49	0	00		89	0	00
0a	0	00		4a	0	00		8a	0	00
0b	0	00		4b	0	00		8b	0	00
0c	0	00		4c	0	00		8c	0	00
0d	0	00		4d	0	00		8d	0	00
10	0	00		50	0	00		90	0	00
11	0	00		51	0	00		91	0	00
12	0	00		52	0	00		92	0	00
13	0	00		53	0	00		93	0	00
14	0	00		54	0	00		94	0	00
15	0	00		55	0	00		95	0	00
16	0	00		56	0	00		96	0	00
17	0	00		57	0	00		97	0	00
18	0	00		58	0	00		98	0	00
19	0	00		59	0	00		99	0	00
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1b	0	00		5b	0	00		9b	0	00
1c	0	00		5c	0	00		9c	0	00
1d	0	00		5d	0	00		9d	0	00
20	0	00		60	0	00		a0	0	00
21	0	00		61	0	00		a1	0	00
22	0	00		62	0	00		a2	0	00
23	0	00		63	0	00		a3	0	00
24	0	00		64	0	00		a4	0	00
25	0	00		65	0	00		a5	0	00
26	0	00		66	0	00		a6	0	00
27	0	00		67	0	00		a7	0	00
28	0	00		68	0	00		a8	0	00
29	0	00		69	0	00		a9	0	00
2a	0	00		6a	0	00		aa	0	00
2b	0	00		6b	0	00		ab	0	00
2c	0	00		6c	0	00		ac	0	00
2d	0	00		6d	0	00		ad	0	00
30	0	00		70	0	00				
31	0	00		71	0	00				
32	0	00		72	0	00				
33	0	00		73	0	00				
34	0	00		74	0	00				
35	0	00		75	0	00				
36	0	00		76	0	00				
37	0	00		77	0	00				
38	0	00		78	0	00				
39	0	00		79	0	00				
3a	0	00		7a	0	00				
3b	0	00		7b	0	00				
3c	0	00		7c	0	00				
3d	0	00		7d	0	00				

b0	0	00	Plus Page
b1	0	00	
b2	0	00	
b3	0	00	
b4	0	00	
b5	0	00	
b6	0	00	
b7	0	00	
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ba	0	00	
bb	0	00	
bc	0	00	
bd	0	00	
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c3	0	00	
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d0	0	00	
d1	0	00	
d2	0	00	
d3	0	00	
d4	0	00	
d5	0	00	
d6	0	00	
d7	0	00	
d8	0	00	
d9	0	00	
da	0	00	
db	0	00	
dc	0	00	
dd	CLR	20	

00	STP	41	ENTRY Minus Page	40	YE	24		80	+	33	
01	XTO	23		41	4	04		81	YE	24	
02	d	17		42	X	36		82	5	05	
03	YTO	40		43	XEY	30		83	DN	25	
04	c	16		44	YE	24		84	UP	27	
05	CLR	20		45	1	01		85	X	36	
06	STP	41	ENTRY	46	+	33		86	XEY	30	
07	UP	27		47	YE	24		87	YE	24	
08	5	05		48	1	01		88	9	11	
09	0	00		49	DN	25		89	+	33	
0a	0	00		4a	X	36		8a	YE	24	
0b	DIV	35		4b	XEY	30		8b	9	11	
0c	YTO	40		4c	YE	24		8c	XEY	30	
0d	b	14		4d	-	34		8d	X	36	
10	RUP	22		50	f	15		90	XEY	30	
11	DIV	35		51	+	33		91	YE	24	
12	YTO	40		52	YE	24		92	8	10	
13	a	13		53	-	34		93	+	33	
14	CLR	20		54	f	15		94	YE	24	
15	STP	41	ENTRY	55	DN	25		95	8	10	
16	UP	27		56	UP	27		96	XEY	30	
17	a	13		57	X	36		97	X	36	
18	X	36		58	XEY	30		98	XEY	30	
19	YTO	40		59	YE	24		99	YE	24	
1a	-	34		5a	-	34		9a	7	07	
1b	0	00		5b	e	12		9b	+	33	
1c	CLR	20		5c	+	33		9c	YE	24	
1d	XTO	23		5d	YE	24		9d	7	07	
20	-	34		60	-	34		a0	d	17	
21	e	12		61	e	12		a1	-	34	
22	XTO	23		62	DN	25		a2	b	14	
23	-	34		63	EXP	74		a3	DIV	35	
24	f	15		64	XEY	30		a4	c	16	
25	1	01		65	LN	65		a5	RUP	22	
26	XTO	23		66	UP	27		a6	XEY	30	
27	0	00		67	X	36		a7	-	34	
28	RUP	22		68	XEY	30		a8	a	13	
29	STP	41	ENTRY	69	YE	24		a9	X	36	
2a	IFG	43		6a	2	02		aa	XFR	67	
2b	d	17		6b	+	33		ab	-	34	
2c	8	10		6c	YE	24		ac	0	00	
2d	PNT	45		6d	2	02		ad	+	33	
30	PNT	45		70	DN	25					
31	AC+	60		71	EXP	74					
32	UP	27		72	X	36					
33	LN	65		73	XEY	30					
34	YE	24		74	YE	24					
35	3	03		75	6	06					
36	+	33		76	+	33					
37	YE	24		77	YE	24					
38	3	03		78	6	06					
39	RUP	22		79	XEY	30					
3a	LN	65		7a	X	36					
3b	YE	24		7b	XEY	30					
3c	4	04		7c	YE	24					
3d	+	33		7d	5	05					

```
b0   RDN   31
b1   XEY   30
b2   FMT   42
b3   DN    25
b4   RDN   31
b5   XEY   30
b6    -    34
b7    -    34
b8   RDN   31
b9   XEY   30
ba   FMT   42
bb   DN    25
bc   RDN   31
bd    +    33

c0   XEY   30
c1   RUP   22
c2   FMT   42
c3   DN    25
c4   UP    27
c5    5    05
c6    0    00
c7    -    34
c8   CLX   37
c9   RDN   31
ca   FMT   42
cb   DN    25
cc   FMT   42
cd   UP    27

d0   YE    24
d1    0    00
d2    1    01
d3    +    33
d4   DN    25
d5   GTO   44
d6    2    02
d7    6    06
d8    1    01
d9    -    34
da   YTO   40
db    0    00
dc   DN    25
dd   END   46
```

Minus
Page

00	1	01
01	0	00
02	1	01
03	XTO	23
04	-	34
05	d	17
06	RUP	22
07	STP	41
08	IFG	43
09	c	16
0a	a	13
0b	PNT	45
0c	PNT	45
0d	AC-	63

10	UP	27
11	LN	65
12	YE	24
13	3	03
14	-	34
15	YE	24
16	3	03
17	RUP	22
18	LN	65
19	YE	24
1a	4	04
1b	-	34
1c	YE	24
1d	4	04

20	X	36
21	XEY	30
22	YE	24
23	1	01
24	-	34
25	YE	24
26	1	01
27	DN	25
28	X	36
29	XEY	30
2a	YE	24
2b	-	34
2c	f	15
2d	-	34

30	YE	24
31	-	34
32	f	15
33	DN	25
34	UP	27
35	X	36
36	XEY	30
37	YE	24
38	-	34
39	e	12
3a	-	34
3b	YE	24
3c	-	34
3d	e	12

40	DN	25
41	EXP	74
42	XEY	30
43	LN	65
44	UP	27
45	X	36
46	XEY	30
47	YE	24
48	2	02
49	-	34
4a	YE	24
4b	2	02
4c	DN	25
4d	EXP	74

50	X	36
51	XEY	30
52	YE	24
53	6	06
54	-	34
55	YE	24
56	6	06
57	XEY	30
58	X	36
59	XEY	30
5a	YE	24
5b	5	05
5c	-	34
5d	YE	24

60	5	05
61	DN	25
62	UP	27
63	X	36
64	XEY	30
65	YE	24
66	9	11
67	-	34
68	YE	24
69	9	11
6a	XEY	30
6b	X	36
6c	XEY	30
6d	YE	24

70	8	10
71	-	34
72	YE	24
73	8	10
74	XEY	30
75	X	36
76	XEY	30
77	YE	24
78	7	07
79	-	34
7a	YE	24
7b	7	07
7c	YE	24
7d	0	00

80	1	01
81	-	34
82	YE	24
83	0	00
84	d	17
85	-	34
86	b	14
87	DIV	35
88	c	16
89	RUP	22
8a	XEY	30
8b	-	34
8c	a	13
8d	X	36

90	1	01
91	0	00
92	0	00
93	-	34
94	RUP	22
95	XEY	30
96	+	33
97	RDN	31
98	FMT	42
99	DN	25
9a	RUP	22
9b	-	34
9c	-	34
9d	XEY	30

a0	RDN	31
a1	+	33
a2	+	33
a3	XEY	30
a4	RUP	22
a5	FMT	42
a6	DN	25
a7	RUP	22
a8	+	33
a9	+	33
aa	RDN	31
ab	FMT	42
ac	UP	27
ad	FMT	42

b0	DN	25
b1	RUP	22
b2	-	34
b3	-	34
b4	RUP	22
b5	XEY	30
b6	-	34
b7	-	34
b8	CLX	37
b9	RDN	31
ba	XEY	30
bb	FMT	42
bc	DN	25
bd	FMT	42
c0	UP	27
c1	YE	24
c2	-	34
c3	d	17
c4	1	01
c5	+	33
c6	DN	25
c7	GTO	44
c8	0	00
c9	3	03
ca	1	01
cb	-	34
cc	DN	25
cd	END	46

00	RCL	61	40	-	34	80	X<Y	52
01	XFR	67	41	CLX	37	81	9	11
02	0	00	42	RDN	31	82	c	16
03	DIV	35	43	XTO	23	83	XEY	30
04	YTO	40	44	-	34	84	RDN	31
05	-	34	45	c	16	85	c	16
06	d	17	46	YTO	40	86	-	34
07	UP	27	47	-	34	87	a	13
08	f	15	48	d	17	88	X	36
09	XEY	30	49	PNT	45	89	CLX	37
0a	DIV	35	4a	PNT	45	8a	X>Y	53
0b	YTO	40	4b	STP	41	8b	5	05
0c	-	34	4c	b	14	8c	c	16
0d	c	16	4d	UP	27	8d	5	05
10	XEY	30	50	EEX	26	90	EEX	26
11	X	36	51	2	02	91	3	03
12	RDN	31	52	X	36	92	X<Y	52
13	X	36	53	d	17	93	5	05
14	e	12	54	XEY	30	94	c	16
15	RUP	22	55	-	34	95	DN	25
16	X	36	56	XTO	23	96	XEY	30
17	DN	25	57	-	34	97	FMT	42
18	-	34	58	b	14	98	DN	25
19	XFR	67	59	YTO	40	99	GTO	44
1a	-	34	5a	-	34	9a	5	05
1b	d	17	5b	0	00	9b	c	16
1c	UP	27	5c	YE	24	9c	CLX	37
1d	f	15	5d	-	34	9d	UP	27
20	X	36	60	0	00	a0	UP	27
21	DN	25	61	XFR	67	a1	FMT	42
22	-	34	62	-	34	a2	UP	27
23	XFR	67	63	b	14	a3	END	46
24	6	06	64	+	33			
25	+	33	65	YTO	40			
26	f	15	66	-	34			
27	UP	27	67	0	00			
28	XFR	67	68	XFR	67			
29	-	34	69	-	34			
2a	c	16	6a	d	17			
2b	X	36	6b	X	36			
2c	XFR	67	6c	XFR	67			
2d	9	11	6d	-	34			
30	XEY	30	70	c	16			
31	-	34	71	+	33			
32	DN	25	72	XFR	67			
33	DIV	35	73	-	34			
34	UP	27	74	0	00			
35	DN	25	75	UP	27			
36	XFR	67	76	d	17			
37	-	34	77	-	34			
38	c	16	78	b	14			
39	X	36	79	DIV	35			
3a	XFR	67	7a	7	07			
3b	-	34	7b	5	05			
3c	d	17	7c	0	00			
3d	XEY	30	7d	0	00			

00	XFR	67	40	1	01	80	-	34
01	3	03	41	√	76	81	1	01
02	UP	27	42	DIV	35	82	XEY	30
03	X	36	43	XFR	67	83	LN	65
04	XFR	67	44	-	34	84	X	36
05	2	02	45	0	00	85	XFR	67
06	UP	27	46	UP	27	86	-	34
07	XFR	67	47	XFR	67	87	0	00
08	0	00	48	3	03	88	XEY	30
09	X	36	49	X	36	89	EXP	74
0a	RDN	31	4a	XFR	67	8a	X	36
0b	XEY	30	4b	4	04	8b	XFR	67
0c	-	34	4c	XEY	30	8c	-	34
0d	YTO	40	4d	-	34	8d	d	17
10	-	34	50	XFR	67	90	UP	27
11	0	00	51	0	00	91	d	17
12	XFR	67	52	DIV	35	92	-	34
13	-	34	53	XFR	67	93	b	14
14	e	12	54	-	34	94	DIV	35
15	RUP	22	55	0	00	95	7	07
16	X	36	56	XEY	30	96	5	05
17	XFR	67	57	EXP	74	97	0	00
18	4	04	58	XTO	23	98	0	00
19	UP	27	59	-	34	99	X<Y	52
1a	X	36	5a	0	00	9a	b	14
1b	DN	25	5b	YTO	40	9b	7	07
1c	-	34	5c	-	34	9c	XEY	30
1d	YTO	40	5d	1	01	9d	RDN	31
20	-	34	60	PNT	45	a0	c	16
21	1	01	61	PNT	45	a1	-	34
22	XFR	67	62	STP	41	a2	a	13
23	4	04	63	b	14	a3	X	36
24	UP	27	64	UP	27	a4	CLX	37
25	XFR	67	65	EEX	26	a5	X>Y	53
26	3	03	66	2	02	a6	7	07
27	X	36	67	X	36	a7	3	03
28	XFR	67	68	d	17	a8	5	05
29	1	01	69	XEY	30	a9	EEX	26
2a	UP	27	6a	-	34	aa	3	03
2b	XFR	67	6b	XTO	23	ab	X<Y	52
2c	0	00	6c	-	34	ac	7	07
2d	X	36	6d	c	16	ad	3	03
30	RDN	31	70	YTO	40			
31	XEY	30	71	-	34			
32	-	34	72	d	17			
33	XFR	67	73	YE	24			
34	-	34	74	-	34			
35	0	00	75	d	17			
36	DIV	35	76	XFR	67			
37	YTO	40	77	-	34			
38	-	34	78	c	16			
39	0	00	79	+	33			
3a	√	76	7a	YTO	40			
3b	X	36	7b	-	34			
3c	XFR	67	7c	d	17			
3d	-	34	7d	XFR	67			

b0	DN	25
b1	XEY	30
b2	FMT	42
b3	DN	25
b4	GTO	44
b5	7	07
b6	3	03
b7	CLX	37
b8	UP	27
b9	UP	27
ba	FMT	42
bb	UP	27
bc	END	46

00	XFR	67
01	0	00
02	UP	27
03	XFR	67
04	-	34
05	f	15
06	X	36
07	f	15
08	UP	27
09	XFR	67
0a	4	04
0b	X	36
0c	DN	25
0d	-	34

10	YTO	40
11	-	34
12	0	00
13	XFR	67
14	0	00
15	UP	27
16	XFR	67
17	9	11
18	X	36
19	f	15
1a	UP	27
1b	X	36
1c	DN	25
1d	-	34

20	YTO	40
21	-	34
22	1	01
23	XFR	67
24	0	00
25	UP	27
26	XFR	67
27	-	34
28	e	12
29	X	36
2a	XFR	67
2b	4	04
2c	UP	27
2d	X	36

30	DN	25
31	-	34
32	XFR	67
33	-	34
34	1	01
35	X	36
36	DN	25
37	√	76
38	UP	27
39	XFR	67
3a	-	34
3b	0	00
3c	XEY	30
3d	DIV	35

40	YTO	40
41	-	34
42	2	02
43	XFR	67
44	-	34
45	0	00
46	UP	27
47	XFR	67
48	-	34
49	1	01
4a	DIV	35
4b	f	15
4c	UP	27
4d	DN	25

50	X	36
51	XFR	67
52	4	04
53	XEY	30
54	-	34
55	XFR	67
56	0	00
57	DIV	35
58	XFR	67
59	-	34
5a	2	02
5b	RDN	31
5c	EXP	74
5d	XTO	23

60	-	34
61	0	00
62	YTO	40
63	-	34
64	1	01
65	PNT	45
66	PNT	45
67	STP	41
68	b	14
69	UP	27
6a	EEX	26
6b	2	02
6c	X	36
6d	d	17

70	XEY	30
71	-	34
72	XTO	23
73	-	34
74	2	02
75	YTO	40
76	-	34
77	3	03
78	YE	24
79	-	34
7a	3	03
7b	XFR	67
7c	-	34
7d	2	02

80	+	33
81	YTO	40
82	-	34
83	3	03
84	XFR	67
85	-	34
86	1	01
87	X	36
88	XFR	67
89	-	34
8a	0	00
8b	XEY	30
8c	EXP	74
8d	X	36

90	XFR	67
91	-	34
92	3	03
93	UP	27
94	d	17
95	-	34
96	b	14
97	DIV	35
98	7	07
99	5	05
9a	0	00
9b	0	00
9c	X<Y	52
9d	c	16

a0	2	02
a1	XEY	30
a2	RDN	31
a3	c	16
a4	-	34
a5	a	13
a6	X	36
a7	CLX	37
a8	X>Y	53
a9	7	07
aa	8	10
ab	5	05
ac	EEX	26
ad	3	03

```
b0    X<Y   52
b1     7    07
b2     8    10
b3    DN    25
b4    XEY   30
b5    IFG   43
b6     b    14
b7     b    14
b8    FMT   42
b9    UP    27
ba    SFL   54
bb    FMT   42
bc    DN    25
bd    GTO   44

c0     7    07
c1     8    10
c2    CLX   37
c3    UP    27
c4    UP    27
c5    FMT   42
c6    UP    27
c7    END   46
```

STAT-PAC X-12 Parabolic Program 1 9100B ONLY

00	XFR	67	40	f	15	80	DIV	35
01	8	10	41	X	36	81	UP	27
02	UP	27	42	YTO	40	82	DN	25
03	f	15	43	-	34	83	e	12
04	X	36	44	1	01	84	XEY	30
05	2	02	45	UP	27	85	XTO	23
06	X	36	46	XFR	67	86	-	34
07	XFR	67	47	8	10	87	4	04
08	9	11	48	X	36	88	XFR	67
09	X	36	49	XFR	67	89	9	11
0a	UP	27	4a	9	11	8a	RUP	22
0b	X	36	4b	UP	27	8b	X	36
0c	X	36	4c	X	36	8c	RDN	31
0d	DN	25	4d	DN	25	8d	-	34
10	-	34	50	-	34	90	YTO	40
11	f	15	51	e	12	91	-	34
12	UP	27	52	X	36	92	2	02
13	X	36	53	DN	25	93	XFR	67
14	XFR	67	54	YE	24	94	6	06
15	7	07	55	-	34	95	XEY	30
16	X	36	56	1	01	96	XFR	67
17	RDN	31	57	-	34	97	8	10
18	-	34	58	YTO	40	98	RUP	22
19	XFR	67	59	-	34	99	X	36
1a	9	11	5a	2	02	9a	DN	25
1b	RUP	22	5b	XFR	67	9b	-	34
1c	X	36	5c	5	05	9c	XFR	67
1d	XFR	67	5d	XEY	30	9d	-	34
20	0	00	60	XFR	67	a0	0	00
21	X	36	61	9	11	a1	X	36
22	RDN	31	62	X	36	a2	XFR	67
23	+	33	63	XFR	67	a3	-	34
24	XFR	67	64	8	10	a4	2	02
25	8	10	65	UP	27	a5	UP	27
26	RUP	22	66	XFR	67	a6	f	15
27	XEY	30	67	6	06	a7	X	36
28	X	36	68	X	36	a8	RDN	31
29	X	36	69	DN	25	a9	-	34
2a	RDN	31	6a	-	34	aa	YTO	40
2b	-	34	6b	XFR	67	ab	-	34
2c	YTO	40	6c	-	34	ac	3	03
2d	-	34	6d	2	02	ad	XFR	67
30	0	00	70	XEY	30			
31	XFR	67	71	UP	27			
32	5	05	72	XFR	67			
33	UP	27	73	0	00			
34	f	15	74	X	36			
35	X	36	75	RUP	22			
36	XFR	67	76	YE	24			
37	9	11	77	-	34			
38	UP	27	78	0	00			
39	XFR	67	79	RUP	22			
3a	6	06	7a	XEY	30			
3b	X	36	7b	-	34			
3c	DN	25	7c	DN	25			
3d	-	34	7d	XEY	30			

```
b0    9    11
b1   XEY   30
b2   XFR   67
b3    -    34
b4    0    00
b5    X    36
b6   DN    25
b7   RDN   31
b8    X    36
b9   RDN   31
ba    -    34
bb   XFR   67
bc    -    34
bd    3    03

c0   XEY   30
c1   DIV   35
c2   YTO   40
c3    -    34
c4    3    03
c5   DN    25
c6    X    36
c7   XFR   67
c8    -    34
c9    2    02
ca   XEY   30
cb    -    34
cc   XFR   67
cd    -    34

d0    0    00
d1   DIV   35
d2   YTO   40
d3    -    34
d4    0    00
d5   XFR   67
d6    -    34
d7    3    03
d8   UP    27
d9   XFR   67
da    -    34
db    4    04
dc   PNT   45
dd   END   46
```

00	XTO	23
01	-	34
02	a	13
03	YTO	40
04	-	34
05	b	14
06	DN	25
07	YTO	40
08	-	34
09	c	16
0a	b	14
0b	UP	27
0c	EEX	26
0d	2	02
10	X	36
11	d	17
12	XEY	30
13	-	34
14	XTO	23
15	-	34
16	9	11
17	YTO	40
18	-	34
19	8	10
1a	YE	24
1b	-	34
1c	8	10
1d	XFR	67
20	-	34
21	9	11
22	+	33
23	YTO	40
24	-	34
25	8	10
26	DN	25
27	UP	27
28	UP	27
29	X	36
2a	XFR	67
2b	-	34
2c	a	13
2d	X	36
30	XFR	67
31	-	34
32	b	14
33	RUP	22
34	X	36
35	XFR	67
36	-	34
37	c	16
38	+	33
39	DN	25
3a	+	33
3b	XFR	67
3c	-	34
3d	8	10
40	UP	27
41	d	17
42	-	34
43	b	14
44	DIV	35
45	7	07
46	5	05
47	0	00
48	0	00
49	X<Y	52
4a	6	06
4b	d	17
4c	XEY	30
4d	RDN	31
50	c	16
51	-	34
52	a	13
53	X	36
54	CLX	37
55	X>Y	53
56	1	01
57	a	13
58	5	05
59	EEX	26
5a	3	03
5b	X<Y	52
5c	1	01
5d	a	13
60	DN	25
61	XEY	30
62	IFG	43
63	6	06
64	8	10
65	FMT	42
66	UP	27
67	SFL	54
68	FMT	42
69	DN	25
6a	GTO	44
6b	1	01
6c	a	13
6d	CLX	37
70	UP	27
71	UP	27
72	FMT	42
73	UP	27
74	END	46

This program computes the value of the log-normal distribution for a given set of parameters, a, μ , σ , over a specified range of X.

Definitions:

$$f(X) = \frac{1}{(X - a)\,\sigma\sqrt{2\pi}} \quad \exp\left[-\frac{(\ln [X - a] - \mu)^2}{2\sigma^2}\right]$$

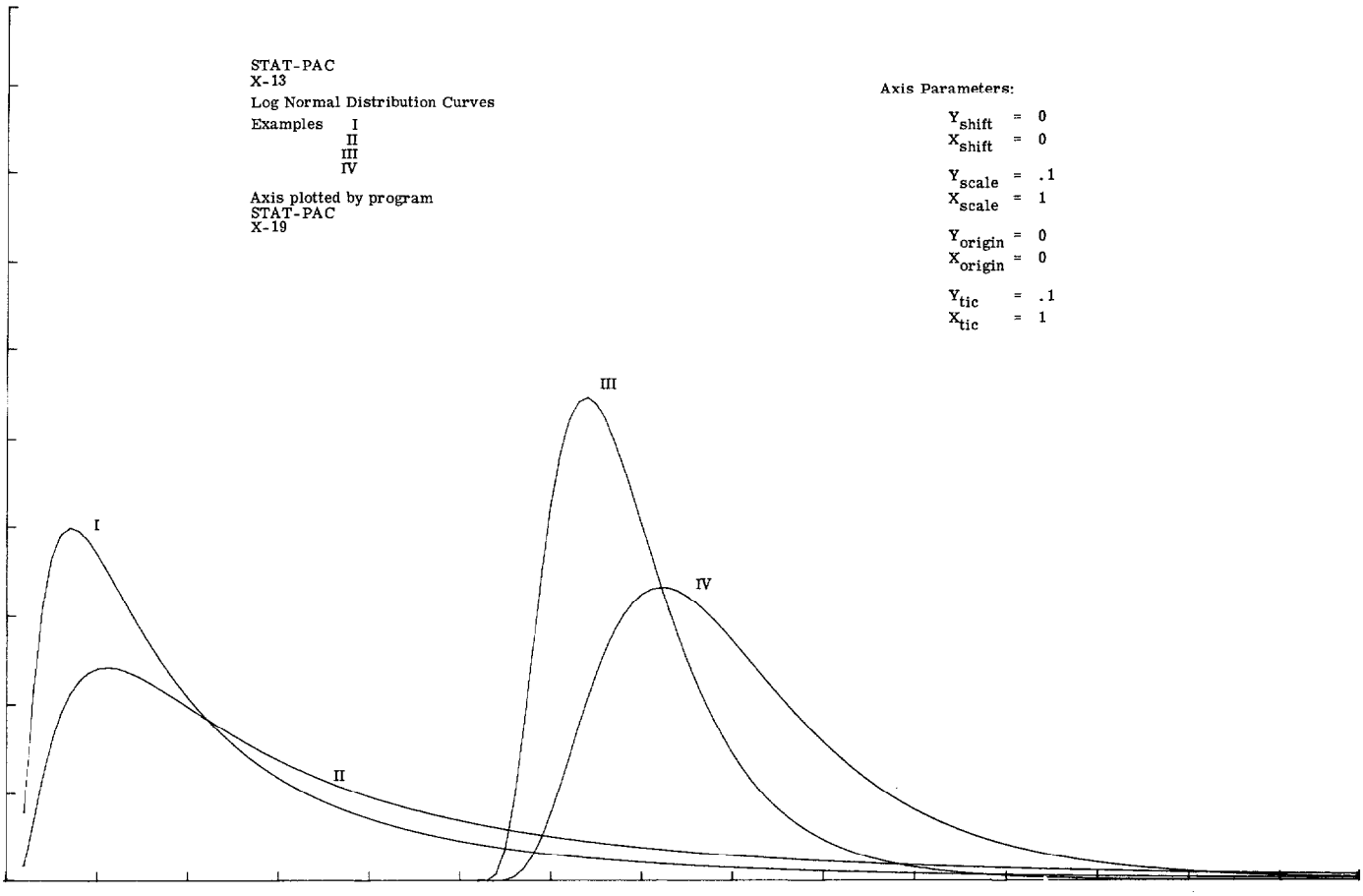
where

$$X > a$$

The program determines f(X) over a range

$$X_i \leq X \leq X_f$$

in increments of ΔX.



USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA:

a	Z
σ	Y
μ	X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA:

ΔX	Z
X_f	Y
X_i	X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
3	X

ENTER DATA:

Y_{plot}	Y
X_{plot}	X

*

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

f(X)	Z
X	Y
ΔX	X

f(X) is now plotted.

When $X = X_f$

*$Y_{plot} = 500/Y_{scale}$

$X_{plot} = 500/X_{scale}$

EXAMPLES

I.

a	=	0
σ	=	1
μ	=	.5
ΔX	=	.1
X_f	=	15
X_i	=	.1

II.

a	=	0
σ	=	1
μ	=	1
ΔX	=	.1
X_f	=	15
X_i	=	.1

III.

a	=	5
σ	=	.5
μ	=	.5
ΔX	=	.1
X_f	=	15
X_i	=	5.1

IV.

a	=	5
σ	=	.5
μ	=	1
ΔX	=	.1
X_f	=	15
X_i	=	5.1

For all cases

$Y_{plot} = 5000$

$X_{plot} = 500$

00	CLR	20		40	UP	27	
01	1	01		41	e	12	
02	STP	41	ENTRY	42	DIV	35	
03	PNT	45		43	c	16	
04	PNT	45		44	UP	27	
05	AC+	60		45	d	17	
06	DN	25		46	-	34	
07	YTO	40		47	DN	25	
08	d	17		48	DIV	35	
09	0	00		49	π	56	
0a	UP	27		4a	$\sqrt{}$	76	
0b	UP	27		4b	DIV	35	
0c	2	02		4c	2	02	
0d	STP	41	ENTRY	4d	$\sqrt{}$	76	
10	PNT	45		50	DIV	35	
11	PNT	45		51	c	16	
12	XTO	23		52	UP	27	
13	c	16		53	a	13	
14	YTO	40		54	PNT	45	
15	b	14		55	PNT	45	
16	DN	25		56	+	33	
17	YTO	40		57	YTO	40	
18	a	13		58	c	16	
19	0	00		59	XEY	30	
1a	UP	27		5a	YEX	24	
1b	UP	27		5b	9	11	
1c	3	03		5c	YTO	40	
1d	STP	41	ENTRY	5d	9	11	
20	PNT	45		60	X	36	
21	PNT	45		61	RUP	22	
22	XTO	23		62	YEX	24	
23	9	11		63	8	10	
24	YTO	40		64	YTO	40	
25	8	10		65	8	10	
26	c	16		66	X	36	
27	UP	27		67	XEY	30	
28	d	17		68	RUP	22	
29	-	34		69	FMT	42	
2a	DN	25		6a	DN	25	
2b	LN	65		6b	c	16	
2c	UP	27		6c	UP	27	
2d	f	15		6d	b	14	
30	-	34		70	X>Y	53	
31	DN	25		71	2	02	
32	UP	27		72	8	10	
33	X	36		73	X=Y	50	
34	2	02		74	2	02	
35	DIV	35		75	8	10	
36	e	12		76	FMT	42	
37	UP	27		77	UP	27	
38	X	36		78	CLR	20	
39	DN	25		79	FMT	42	
3a	DIV	35		7a	UP	27	
3b	DN	25		7b	END	46	
3c	CHS	32					
3d	EXP	74					

	$Y_p = 500 / Y_{scale}$
	$X_p = 500 / X_{scale}$
	ΔX
	X_f
	X_i
	a
	σ
	μ

GOMPERTZ CURVE PLOT

This program plots the data values used in calculating a Gompertz Curve (STAT-PAC IV-12) and plots the Gompertz curve best fitting these data points.

The Gompertz curve is given by:

$$Y = Ka^{(b^X)}$$

or by:

$$\log Y = \log K + b^X \log a.$$

The parameters log K, b, and log a are determined by program STAT-PAC IV-12.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

SET: Decimal Wheel at 6 or less

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

X_{max} ⟶ Z
Y_{max} ⟶ Y
Y_{min} ⟶ X

*

PRESS: CONTINUE

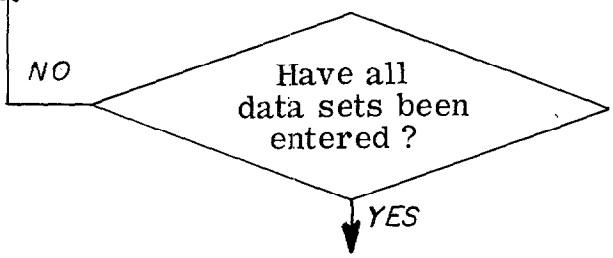
DISPLAY

0 ——— Z
i ——— Y
0 ——— X

ENTER DATA:

Y_i ⟶ Y
X_i ⟶ X

PRESS: CONTINUE



PRESS: SET FLAG

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

DISPLAY

0 ——— Z
n ——— Y
0 ——— X

(n = number of data points entered)

ENTER DATA:

log a ⟶ Z
b ⟶ Y
log K ⟶ X

PRESS: CONTINUE

The Gompertz curve will be plotted.

To terminate: PRESS: STOP

To run another case:

PRESS: END

* The point X_{max} will be plotted at X = 8 inches (20.3 cm.)

The point Y_{min} will be plotted at Y = 0 inches.

The point $Y_{max} - Y_{min}$ will be plotted at Y = 5 inches (12.7 cm.)

EXAMPLE

See next page.

EXAMPLE

The table below lists the sales (100 lb. bags) of commercial fertilizer at Farson's General Feed Store, Wray, Colorado. The store's management would like to predict its fertilizer volume to determine if a branch feed store should be opened in nearby Haxton.

Year	Qtr.	X	Qtr'ly Sales (Y) (100 lb. bags)
1966 -	1	0	124
	2	1	160
	3	2	163
	4	3	144
1967 -	1	4	195
	2	5	222
	3	6	241
	4	7	313
1968 -	1	8	305
	2	9	356
	3	10	363
	4	11	458
1969 -	1	12	406
	2	13	441
	3	14	550

N = 5

Results from program STAT-PAC IV-12

log a = -1.1
b = .94
log K = 3.16

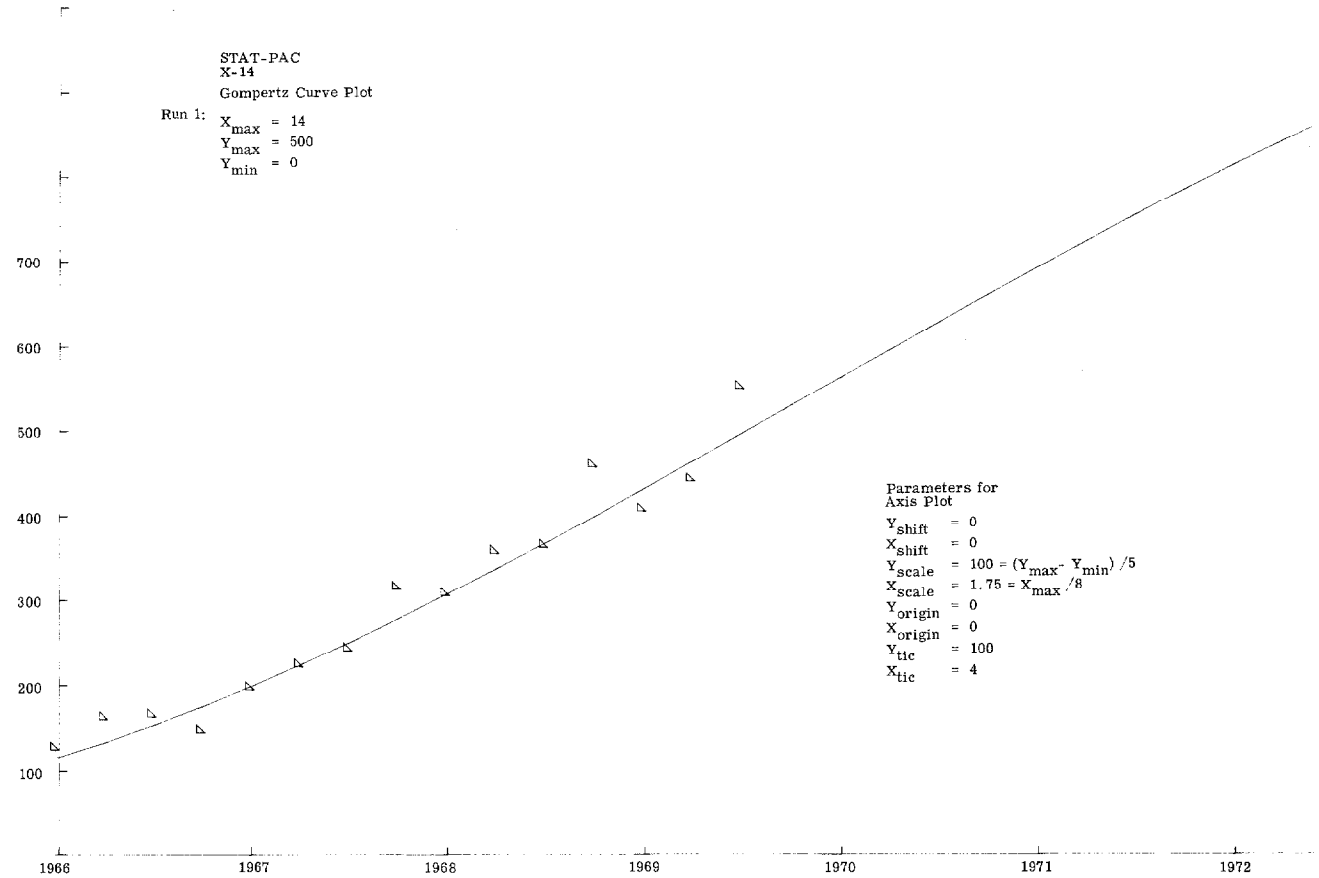
Run No. 1

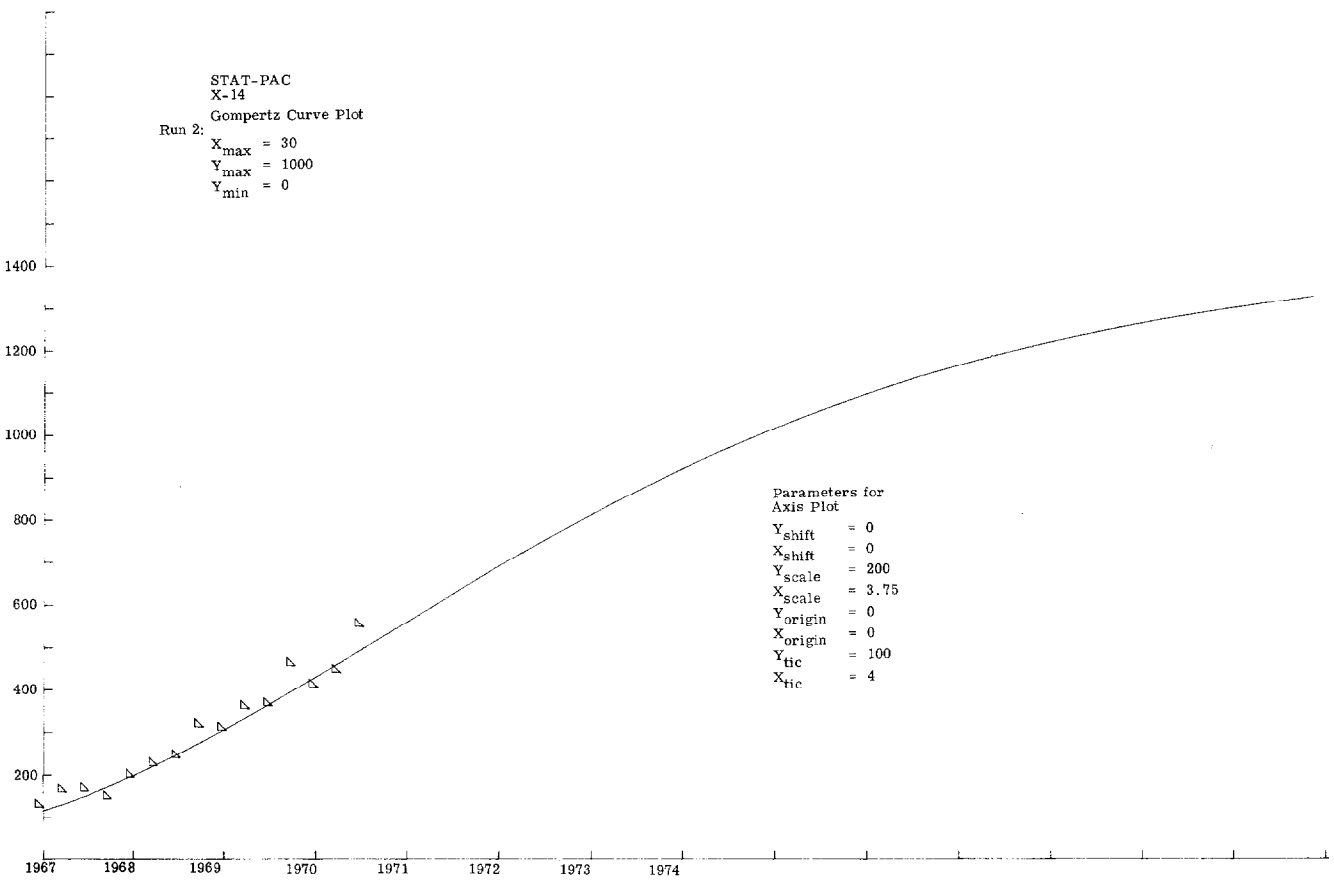
X_{max} = 14
Y_{max} = 500
Y_{min} = 0

Notice that by changing X_{max} from 14 to 30, a better estimate of future fertilizer sales is obtained.

Run No. 2

X_{max} = 30
Y_{max} = 1000
Y_{min} = 0

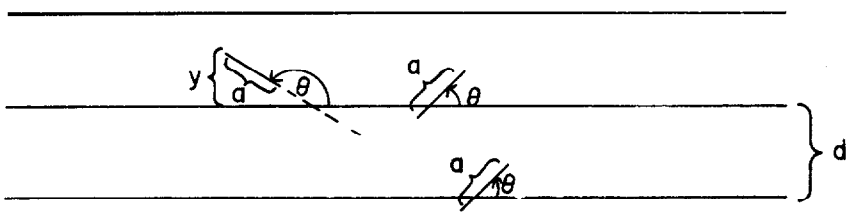

STAT-PAC
X-14
Gompertz Curve Plot
Run 1:
X_{max} = 14
Y_{max} = 500
Y_{min} = 0
Parameters for
Axis Plot
Y_{shift} = 0
X_{shift} = 0
Y_{scale} = 100 = (Y_{max}- Y_{min}) /5
X_{scale} = 1.75 = X_{max} /8
Y_{origin} = 0
X_{origin} = 0
Y_{tic} = 100
X_{tic} = 4
100
200
300
400
500
600
700
1966
1967
1968
1969
1970
1971
1972


STAT-PAC
X-14
Gompertz Curve Plot
Run 2:
X_{max} = 30
Y_{max} = 1000
Y_{min} = 0
Parameters for
Axis Plot
Y_{shift} = 0
X_{shift} = 0
Y_{scale} = 200
X_{scale} = 3.75
Y_{origin} = 0
X_{origin} = 0
Y_{tic} = 100
X_{tic} = 4
200
400
600
800
1000
1200
1400
1967
1968
1969
1970
1971
1972
1973
1974

00	CLR	20	ENTRY	40	RDN	31		80	DN	25	
01	STP	41		41	+	33		81	EXP	74	
02	PNT	45		42	XEY	30		82	UP	27	
03	PNT	45		43	RUP	22		83	a	13	
04	XTO	23		44	FMT	42		84	-	34	
05	a	13		45	DN	25		85	e	12	
06	-	34		46	XEY	30		86	X	36	
07	2	02		47	RUP	22		87	DN	25	
08	5	05		48	-	34		88	XEY	30	
09	0	00		49	RDN	31		89	UP	27	
0a	0	00		4a	+	33		8a	f	15	
0b	XEY	30		4b	DN	25		8b	X	36	
0c	DIV	35		4c	FMT	42		8c	RDN	31	
0d	4	04		4d	DN	25		8d	FMT	42	
10	EEX	26		50	FMT	42		90	DN	25	
11	3	03		51	UP	27		91	RUP	22	
12	RUP	22		52	0	00		92	DIV	35	
13	DIV	35		53	UP	27		93	1	01	
14	DN	25		54	d	17		94	.	21	
15	AC+	60		55	UP	27		95	+	33	
16	0	00		56	1	01		96	DN	25	
17	UP	27		57	+	33		97	GTO	44	
18	1	01		58	0	00		98	6	06	
19	UP	27		59	GTO	44		99	c	16	
1a	0	00		5a	2	02		9a	END	46	
1b	CNT	47		5b	0	00					
1c	CNT	47		5c	1	01					
1d	CNT	47		5d	-	34					
20	STP	41	ENTRY	60	0	00	ENTRY				
21	IFG	43		61	STP	41					
22	5	05		62	PNT	45					
23	c	16		63	PNT	45					
24	PNT	45		64	XTO	23					
25	PNT	45		65	d	17					
26	RUP	22		66	YTO	40					
27	XTO	23		67	c	16					
28	d	17		68	DN	25					
29	f	15		69	YTO	40					
2a	X	36		6a	b	14					
2b	RUP	22		6b	0	00					
2c	XEY	30		6c	UP	27					
2d	a	13		6d	UP	27					
30	-	34		70	c	16					
31	e	12		71	LN	65					
32	X	36		72	X	36					
33	DN	25		73	DN	25					
34	XEY	30		74	EXP	74					Y_{min}
35	FMT	42		75	UP	27					log a
36	DN	25		76	b	14					b
37	UP	27		77	X	36					n log K
38	5	05		78	d	17					
39	0	00		79	+	33					Y_f
3a	-	34		7a	1	01					
3b	RDN	31		7b	EXP	74					X_f
3c	FMT	42		7c	LOG	75					
3d	DN	25		7d	DIV	35					

BUFFON NEEDLE EXPERIMENT WITH PLOT

This program simulates the well known Buffon Needle Experiment described in the reference. The problem is to determine the probability that a needle of length a will intersect a line on a grid of lines separated by distance d . The geometry of the experiment is seen below:



The needle of length $a \leq d$ is randomly tossed on the grid. By considering the angle θ between the needle and a line of the grid, and the distance Y between the upper tip of the needle and the line it intersects or the nearest line below it, the possible relations can be derived:

$$0 \leq \theta \leq \pi$$
$$0 \leq Y \leq d$$

The needle intersects a line when:

$$0 \leq Y \leq a \sin \theta \ .$$

It can be shown that the probability of an intersection is theoretically given by:

$$P_T = \frac{2a}{\pi d}$$

This formula involving π has intrigued mathematicians since it allows one to experimentally determine π .

This program randomly chooses a θ from a uniform distribution such that:

$$0 \leq \theta \leq \pi$$

then independently chooses a Y from a uniform distribution such that:

$$0 \leq Y \leq 1$$

The grid distance d is defined to be 1.

Thus $P_T = \frac{2a}{\pi}$.

The value of Y is then compared against

$$Y \leq a \sin \theta .$$

If Y is less than or equal to $\underline{a \sin \theta}$, an intersection has occurred. The number of failing throws are then used to evaluate the experimental probability:

$$P_E = \frac{\#\ \text{successful}}{\text{successful} + \text{failed}}$$

The program displays:

\# trials

P_E

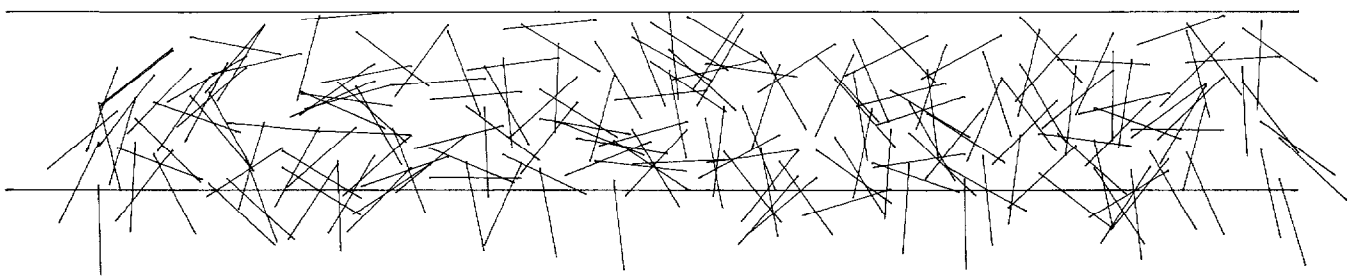
P_T

and proceeds to toss another needle.

STAT-PAC
X-15

Buffon Needle Experiment (With Plot)

Monte-Carlo simulation of throwing needles onto a grid.



Reference: Theory of Probability, M. E. Munroe, McGraw-Hill, 1951.

USER INSTRUCTIONS

SET: RADIANS

SET: Decimal Wheel at 6 or less

PRESS: STOP

Using Origin Controls, set Origin at X = 0 in., Y = 4 in., or (X = 0 cm., Y = 10 cm.).

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA:

R_Y	→ Z	*
R_θ	→ Y	
a	→ X	

PRESS: CONTINUE

DISPLAY

Trials	Z
P_E	Y
P_T	X

The program continues indefinitely.

*R_Y is an initializing value for the routine which generates the random values of Y. R_Y should be $0 \leq R_Y \leq 1$.

R_θ is a similar initializer for generating R_θ. R_θ should not equal R_Y. R_θ should be $0 \leq R_\theta \leq \pi$.

EXAMPLE

$R_Y = .75$

$R_\theta = .50$

$a = .50$

Results after 2100 trials:

$P_E = .312$

$P_T = .318$

00	CLR	20	Plus Page	40	2	02		80	CNT	47	
01	EEX	26		41	XTO	23		81	CNT	47	
02	3	03		42	9	11		82	CNT	47	
03	XEY	30		43	1	01		83	CNT	47	
04	FMT	42		44	STP	41	ENTRY	84	CNT	47	
05	DN	25		45	PNT	45		85	CNT	47	
06	CNT	47		46	PNT	45		86	CNT	47	
07	EEX	26		47	XTO	23		87	CNT	47	
08	3	03		48	d	17		88	CNT	47	
09	FMT	42		49	RUP	22		89	CNT	47	
0a	DN	25		4a	XTO	23		8a	CNT	47	
0b	2	02		4b	a	13		8b	CNT	47	
0c	EEX	26		4c	2	02		8c	CNT	47	
0d	3	03		4d	X	36		8d	CNT	47	
10	FMT	42		50	π	56		90	CNT	47	
11	DN	25		51	DIV	35		91	CNT	47	
12	3	03		52	YTO	40		92	CNT	47	
13	EEX	26		53	c	16		93	CNT	47	
14	3	03		54	π	56		94	CNT	47	
15	FMT	42		55	RUP	22		95	CNT	47	
16	DN	25		56	+	33		96	CNT	47	
17	4	04		57	DN	25		97	CNT	47	
18	EEX	26		58	UP	27		98	CNT	47	
19	3	03		59	X	36		99	CNT	47	
1a	FMT	42		5a	DN	25		9a	CNT	47	
1b	DN	25		5b	UP	27		9b	CNT	47	
1c	5	05		5c	X	36		9c	CNT	47	
1d	EEX	26		5d	DN	25		9d	CNT	47	
20	3	03		60	UP	27		a0	CNT	47	
21	FMT	42		61	X	36		a1	CNT	47	
22	DN	25		62	DN	25		a2	CNT	47	
23	6	06		63	UP	27		a3	CNT	47	
24	EEX	26		64	INT	64		a4	CNT	47	
25	3	03		65	-	34		a5	CNT	47	
26	FMT	42		66	YTO	40		a6	CNT	47	
27	DN	25		67	b	14		a7	CNT	47	
28	7	07		68	π	56		a8	CNT	47	
29	EEX	26		69	X	36		a9	CNT	47	
2a	3	03		6a	GTO	44		aa	CNT	47	
2b	FMT	42		6b	-	34		ab	CNT	47	
2c	DN	25		6c	0	00		ac	CNT	47	
2d	FMT	42		6d	0	00		ad	CNT	47	
30	UP	27		70	CNT	47					θ
31	IFG	43		71	CNT	47					
32	3	03		72	CNT	47					X_{plot}
33	9	11		73	CNT	47					
34	CLR	20		74	CNT	47					
35	SFL	54		75	CNT	47					
36	GTO	44		76	CNT	47					
37	0	00		77	CNT	47					
38	4	04		78	CNT	47					
39	CLR	20		79	CNT	47					
3a	FMT	42		7a	CNT	47					
3b	UP	27		7b	CNT	47					
3c	5	05		7c	CNT	47					
3d	EEX	26		7d	CNT	47					a sin θ

```
b0    CNT  47     Plus
b1    CNT  47     Page
b2    CNT  47
b3    CNT  47
b4    CNT  47
b5    CNT  47
b6    CNT  47
b7    CNT  47
b8    CNT  47
b9    CNT  47
ba    CNT  47
bb    CNT  47
bc    CNT  47
bd    CNT  47

c0    CNT  47
c1    CNT  47
c2    CNT  47
c3    CNT  47
c4    CNT  47
c5    CNT  47
c6    CNT  47
c7    CNT  47
c8    CNT  47
c9    CNT  47
ca    CNT  47
cb    CNT  47
cc    CNT  47
cd    CNT  47

d0    CNT  47
d1    CNT  47
d2    CNT  47
d3    CNT  47
d4    CNT  47
d5    CNT  47
d6    CNT  47
d7    CNT  47
d8    CNT  47
d9    CNT  47
da    CNT  47
db    CNT  47
dc    CNT  47
dd    CNT  47
```

9100B ONLY

Minus Page

00	DN	25
01	XTO	23
02	8	10
03	SIN	70
04	UP	27
05	d	17
06	X	36
07	YTO	40
08	7	07
09	a	13
0a	UP	27
0b	π	56
0c	+	33
0d	DN	25
10	UP	27
11	X	36
12	DN	25
13	UP	27
14	X	36
15	DN	25
16	UP	27
17	X	36
18	DN	25
19	UP	27
1a	INT	64
1b	-	34
1c	YTO	40
1d	a	13
20	DN	25
21	X<Y	52
22	3	03
23	3	03
24	X=Y	50
25	3	03
26	3	03
27	e	12
28	UP	27
29	1	01
2a	+	33
2b	YTO	40
2c	e	12
2d	f	15
30	GTO	44
31	3	03
32	b	14
33	f	15
34	UP	27
35	1	01
36	+	33
37	YTO	40
38	f	15
39	e	12
3a	XEY	30
3b	+	33
3c	UP	27
3d	DN	25

40	XEY	30
41	DIV	35
42	c	16
43	PSE	57
44	CNT	47
45	a	13
46	UP	27
47	EEX	26
48	3	03
49	X	36
4a	XFR	67
4b	9	11
4c	FMT	42
4d	DN	25
50	XFR	67
51	7	07
52	UP	27
53	EEX	26
54	3	03
55	X	36
56	DN	25
57	-	34
58	XFR	67
59	8	10
5a	COS	73
5b	UP	27
5c	d	17
5d	X	36
60	EEX	26
61	3	03
62	X	36
63	XFR	67
64	9	11
65	XEY	30
66	-	34
67	DN	25
68	FMT	42
69	DN	25
6a	XFR	67
6b	9	11
6c	UP	27
6d	EEX	26
70	2	02
71	+	33
72	7	07
73	EEX	26
74	3	03
75	X>Y	53
76	7	07
77	c	16
78	5	05
79	EEX	26
7a	2	02
7b	UP	27
7c	0	00
7d	YTO	40

80	9	11
81	XEY	30
82	FMT	42
83	UP	27
84	b	14
85	RDN	31
86	GTO	44
87	+	33
88	5	05
89	4	04
8a	END	46

CONTROL ELLIPSE

Individual or separate control of related variables will result in errors of "over" and "under" control. These errors become more pronounced if the variables are correlated. Control of individual observations, assuming that they come from a bivariate normal distribution, may be done by means of a control ellipse or by means of Hotelling's T^2 (1) control chart. The results obtained by each method are identical.

This program accepts the sample variances and the covariance, sample size, and F_α obtained from the F tables with a significance level of α, and outputs the approximation to χ^2 denoted T^2, the analytical calculations for the ellipse, and the plot. The plot indicates pictorially the nature of the out-of-control conditions.

The following equations are used:

$$T^2 = \frac{2(N-1)}{(n-2)} \quad F_\alpha$$

$$\lambda = \frac{(S_X^2 + S_Y^2) \pm \sqrt{(S_X^2 + S_Y^2)^2 - 4[S_X^2 S_Y^2 - (S_{XY})^2]}}{2}$$

λ_1 (+)

λ_2 (-)

ϕ — The angle of rotation = $\arctan\left[\frac{\lambda_1 - S_X^2}{S_{XY}}\right]$

b = Length of the semi-minor axis = $\sqrt{\lambda_2 T^2}$

a = Length of the semi-major axis = $\sqrt{\lambda_1 T^2}$

The plot is generated by calculating r, the radius from the following equation:

$$r = \frac{ep}{1 - e\cos\theta}$$

where:
- e – eccentricity
- p – magnification factor
- θ – angle through which r is rotated to plot the complete ellipse (360^o)

References: (1) Hotelling, Harold, "Multivariate Quality Control, "Techniques of Statistical Analysis, Ed. by Eisenhart, Hastay, and Wallis, McGraw-Hill, New York, 1947.

(2) "Quality Control Methods", J. Edward Jackson, Industrial Quality Control, Vol. 12 (7), January, 1956.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

SET: DEGREES

SET: Decimal Wheel at 6 or less

Using the origin controls, locate the pen in the middle of the paper. This point corresponds to $\overline{X}$, $\overline{Y}$.

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

X, Y Scale → Z
F_α → Y
N → X

Problem units/inch (cm.)

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
T^2 ——— X

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

S_{XY} → Z
S_Y^2 → Y
S_X^2 → X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

λ_1 ——— Z
λ_2 ——— Y
ϕ ——— X

PRESS: CONTINUE

DISPLAY

b ——— Z
a ——— Y
0 ——— X

PRESS: CONTINUE

The calculator returns to the second data entry point to allow the user to input new values for S_{XY}, S_Y^2 and S_X^2 and plot the new corresponding ellipse.

To rerun the program from the beginning:

PRESS: END

NOTE: To run the program and plot on metric paper, GO TO location (+)(2)(2) and enter a 2.

The plot is incrementing by 3^o. To speed-up the plot, GO TO location (+)(7)(6) and enter the appropriate increment.

EXAMPLE

See next page.

EXAMPLE

The Cumminsky Chemical Company of Williams, Michigan has two chemicals, A and B, that they are trying to control simultaneously since the quality of the final product, C, depends on the joint effect of these two chemicals.

The following sample data has been determined:

$X_{scale} = Y_{scale} = 10$

$F_{\alpha} = 3.12$ (@ $\alpha = .05$)

$N = 75$

$S_{XY} = 68.87$ CASE 1

$S_{Y^2} = 108.00$

$S_{X^2} = 102.68$

The calculator computes and prints out:

$T^2 = 6.325479$

$\lambda_1 = 174.261350$

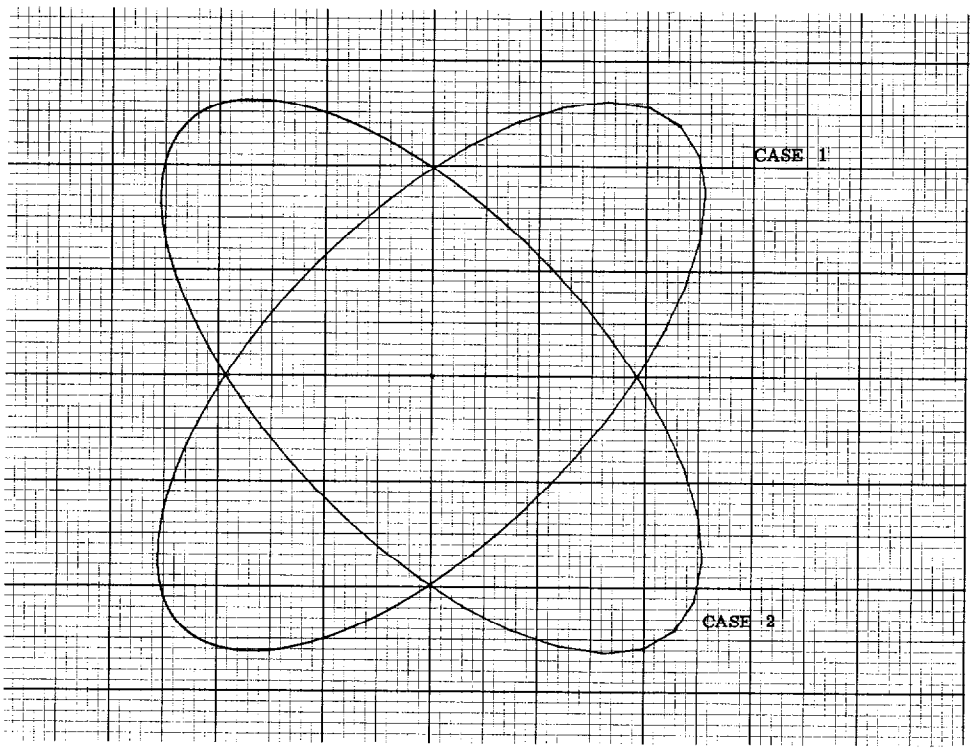
$\lambda_2 = 36.418650$

angle of rotation = 46.105932^{o}

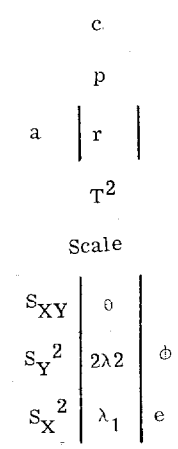
semi-minor axis length = 15.177794

semi-major axis length = 33.200702

CASE 2 is obtained by using $S_{XY} = -68.87$.



00	CLR	20	Plus Page	40	UP	27		80	CNT	47	
01	STP	41	ENTRY	41	X	36		81	CNT	47	
02	PNT	45		42	RDN	31		82	CNT	47	
03	PNT	45		43	XEY	30		83	CNT	47	
04	RDN	31		44	-	34		84	CNT	47	
05	YTO	40		45	DN	25		85	CNT	47	
06	c	16		46	√	76		86	CNT	47	
07	RDN	31		47	-	34		87	CNT	47	
08	1	01		48	YTO	40		88	CNT	47	
09	-	34		49	e	12		89	CNT	47	
0a	YTO	40		4a	+	33		8a	CNT	47	
0b	d	17		4b	+	33		8b	CNT	47	
0c	2	02		4c	2	02		8c	CNT	47	
0d	X	36		4d	DIV	35		8d	CNT	47	
10	DN	25		50	f	15		90	CNT	47	
11	X	36		51	UP	27		91	CNT	47	
12	d	17		52	DN	25		92	CNT	47	
13	UP	27		53	-	34		93	CNT	47	
14	1	01		54	d	17		94	CNT	47	
15	-	34		55	DIV	35		95	CNT	47	
16	DN	25		56	DN	25		96	CNT	47	
17	DIV	35		57	ARC	72		97	CNT	47	
18	YTO	40		58	TAN	71		98	CNT	47	
19	b	14		59	YTO	40		99	CNT	47	
1a	CLR	20		5a	f	15		9a	CNT	47	
1b	b	14		5b	UP	27		9b	CNT	47	
1c	PNT	45		5c	YE	24		9c	CNT	47	
1d	PNT	45		5d	e	12		9d	CNT	47	
20	c	16		60	2	02		a0	CNT	47	
21	UP	27		61	DIV	35		a1	CNT	47	
22	5	05		62	e	12		a2	CNT	47	
23	0	00		63	PNT	45		a3	CNT	47	
24	0	00		64	PNT	45		a4	CNT	47	
25	DIV	35		65	b	14		a5	CNT	47	
26	YTO	40		66	X	36		a6	CNT	47	
27	c	16		67	RDN	31		a7	CNT	47	
28	CLR	20		68	√	76		a8	CNT	47	
29	STP	41	ENTRY	69	RDN	31		a9	CNT	47	
2a	PNT	45		6a	X	36		aa	CNT	47	
2b	PNT	45		6b	DN	25		ab	CNT	47	
2c	AC+	60		6c	√	76		ac	CNT	47	
2d	X	36		6d	XTO	23		ad	CNT	47	
30	DN	25		70	a	13					
31	YTO	40		71	UP	27					
32	d	17		72	CLX	37					
33	RDN	31		73	PNT	45	s				
34	X	36		74	PNT	45					
35	DN	25		75	DN	25					
36	-	34		76	RDN	31					
37	4	04		77	X	36					
38	X	36		78	DN	25					
39	f	15		79	RDN	31					
3a	UP	27		7a	GTO	44					
3b	e	12		7b	-	34					
3c	+	33		7c	0	00					
3d	DN	25		7d	0	00					



```
b0    CNT  47      Plus
b1    CNT  47      Page
b2    CNT  47
b3    CNT  47
b4    CNT  47
b5    CNT  47
b6    CNT  47
b7    CNT  47
b8    CNT  47
b9    CNT  47
ba    CNT  47
bb    CNT  47
bc    CNT  47
bd    CNT  47

c0    CNT  47
c1    CNT  47
c2    CNT  47
c3    CNT  47
c4    CNT  47
c5    CNT  47
c6    CNT  47
c7    CNT  47
c8    CNT  47
c9    CNT  47
ca    CNT  47
cb    CNT  47
cc    CNT  47
cd    CNT  47

d0    CNT  47
d1    CNT  47
d2    CNT  47
d3    CNT  47
d4    CNT  47
d5    CNT  47
d6    CNT  47
d7    CNT  47
d8    CNT  47
d9    CNT  47
da    CNT  47
db    CNT  47
dc    CNT  47
dd    CNT  47
```

Minus Page

00	X	36
01	DN	25
02	XEY	30
03	-	34
04	DN	25
05	√	76
06	XTO	23
07	8	10
08	UP	27
09	a	13
0a	DIV	35
0b	YTO	40
0c	f	15
0d	X	36
10	+	33
11	UP	27
12	CLX	37
13	XTO	23
14	d	17
15	COS	73
16	RUP	22
17	XTO	23
18	a	13
19	f	15
1a	X	36
1b	1	01
1c	XEY	30
1d	-	34
20	f	15
21	DIV	35
22	a	13
23	X	36
24	YTO	40
25	9	11
26	UP	27
27	d	17
28	XEY	30
29	RCT	66
2a	YTO	40
2b	-	34
2c	f	15
2d	XTO	23
30	-	34
31	e	12
32	UP	27
33	e	12
34	COS	73
35	X	36
36	e	12
37	SIN	70
38	RUP	22
39	X	36
3a	DN	25
3b	-	34
3c	e	12
3d	COS	73
40	UP	27
41	XFR	67
42	8	10
43	X	36
44	DN	25
45	-	34
46	e	12
47	COS	73
48	UP	27
49	XFR	67
4a	-	34
4b	f	15
4c	X	36
4d	XFR	67
50	-	34
51	e	12
52	RUP	22
53	XTO	23
54	-	34
55	e	12
56	e	12
57	SIN	70
58	X	36
59	DN	25
5a	+	33
5b	e	12
5c	SIN	70
5d	UP	27
60	XFR	67
61	8	10
62	X	36
63	DN	25
64	-	34
65	XFR	67
66	-	34
67	e	12
68	UP	27
69	c	16
6a	DIV	35
6b	XEY	30
6c	RDN	31
6d	DIV	35
70	DN	25
71	XEY	30
72	FMT	42
73	DN	25
74	d	17
75	UP	27
76	3	03
77	+	33
78	3	03
79	6	06
7a	2	02
7b	X<Y	52
7c	9	11
7d	7	07
80	DN	25
81	XTO	23
82	d	17
83	COS	73
84	UP	27
85	f	15
86	X	36
87	1	01
88	XEY	30
89	-	34
8a	f	15
8b	XEY	30
8c	DIV	35
8d	XFR	67
90	9	11
91	X	36
92	d	17
93	XEY	30
94	GTO	44
95	2	02
96	9	11
97	CLR	20
98	FMT	42
99	UP	27
9a	GTO	44
9b	+	33
9c	2	02
9d	8	10
a0	END	46

X	X'
Y	Y'

GENERAL PARAMETRIC PLOT
(SELF - SCALING)

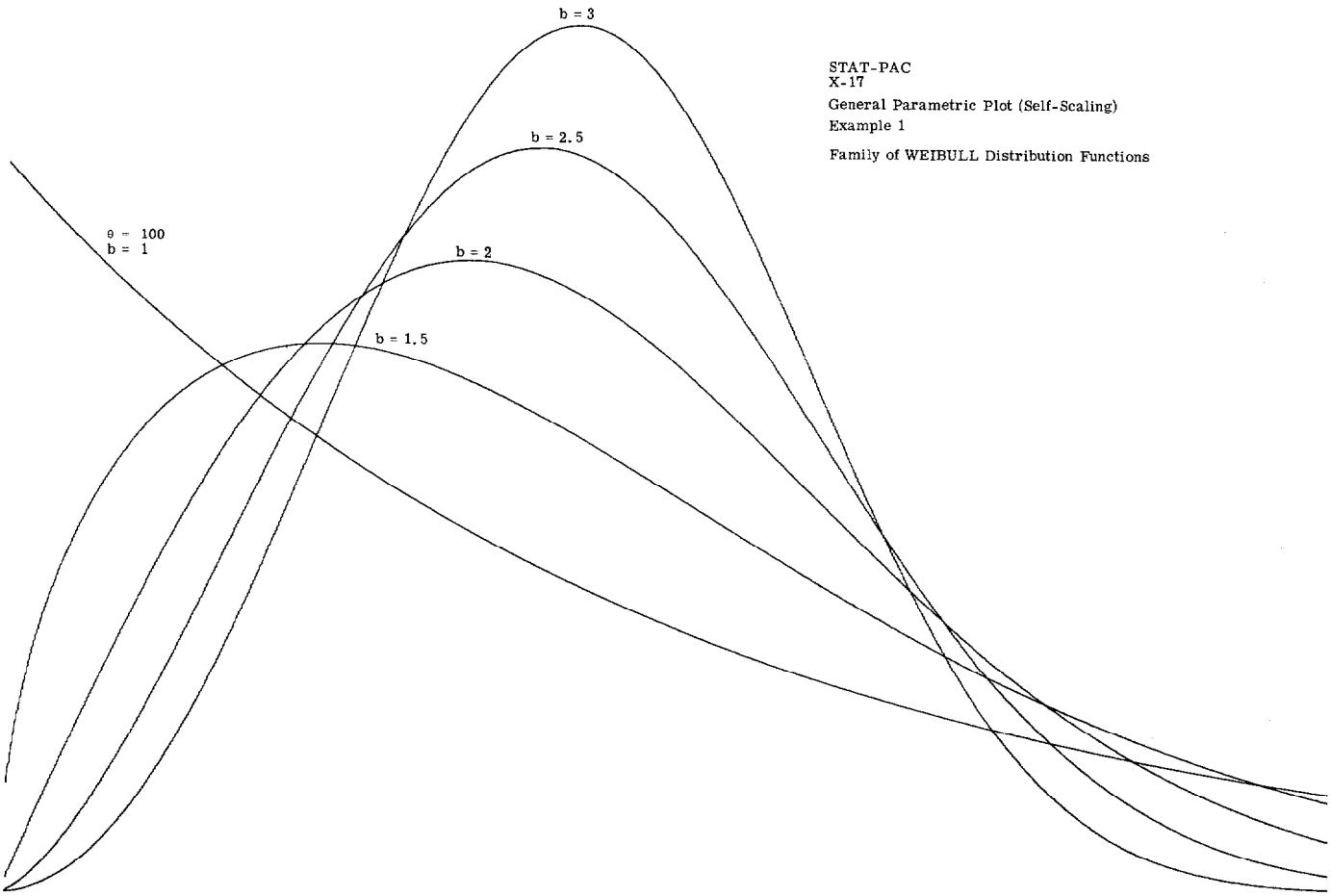
This program permits the plotting of a family of curves F(X, a, b) where a and b are parametrically varied. The program scales itself and outputs the plotter scaling constants so that axes may be placed on the plot.

The entire family may be plotted using the same set of scaling constants, or each may be individually scaled to plot full range.

The function to be plotted f(X) = F(X, a, b) is placed into the calculator by the user as a subroutine. The subroutine then evaluates F(X, a, b) where X is varied from

$$X_{min} \longrightarrow X_{max} \quad .$$

The parameters a and b remain constant until the curve is completed. Then the user may parametrically vary a and b to generate a family of curves.



USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: STOP

SET: Decimal Wheel at 6 or less

Using the origin controls, locate the pen in the lower left corner of the paper.

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: GO TO (-)(3)(0)

SET: | PROGRAM | |

Enter the subroutine which generates F(X, a, b) leaving F(X, a, b) in the Y register. The last step should be RETURN. Registers (-)(3) thru (-)(d) are available. The flag is not available. The quantities X, a, and b are found:

X —— registers X and e
a —— register (-) f
b —— register (-) e

Do not alter registers e, (-)f, or (-) e.

SET: | | RUN |

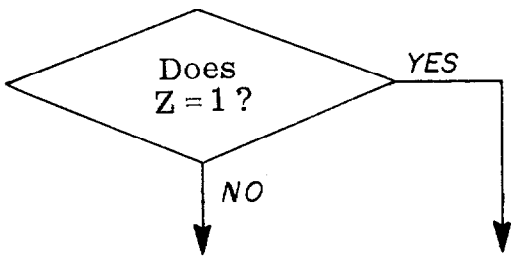
PRESS: END

PRESS: CONTINUE

ENTER DATA:

0 or 1 ⟶ Z *
b ⟶ Y
a ⟶ X

PRESS: CONTINUE



USER INSTRUCTIONS (Con't)

ENTER DATA:

X_{max} ⟶ Y
X_{min} ⟶ X

PRESS: CONTINUE

The calculator now evaluates F(X, a, b) over the range $X_{min} \rightarrow X_{max}$ searching for F_{max} and F_{min}.

F(X, a, b) is now plotted.

DISPLAY

1 —— Z
Y_{shift} —— Y
X_{shift} —— X

PRESS: CONTINUE

DISPLAY

2 —— Z
Y_{scale} —— Y
X_{scale} —— X

*If 1 is placed in the Z register F(X, a, b) is plotted using scaling constants of the previous run.

EXAMPLE

Generate a family of Weibull Distribution Functions given by:

$$F(X, b, \theta) = \frac{bX^{(b-1)}}{\theta^b} \quad e^{-\left(\frac{X}{\theta}\right)^b}$$

for the cases: (b) (a)

1. $\theta = 100$, $b = 3.0$
2. $\theta = 100$, $b = 2.5$
3. $\theta = 100$, $b = 2.0$
4. $\theta = 100$, $b = 1.5$
5. $\theta = 100$, $b = 1.0$

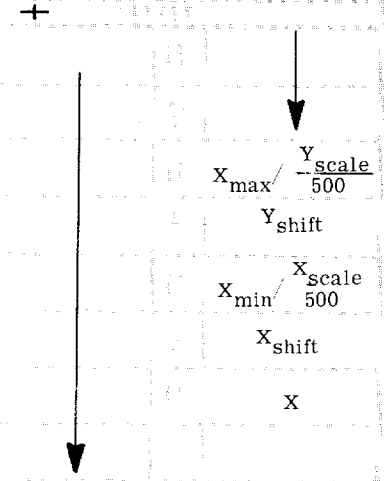
$X_{min} = 1$, $X_{max} = 200$

For cases 2, 3, 4, and 5 use the same scaling constants as for case 1. (i.e., place a 1 in the Z register.)

			Plus	Page
00	CLR	20		
01	STP	41	ENTRY	
02	PNT	45		
03	PNT	45		
04	XTO	23		
05	-	34		
06	f	15		
07	YTO	40		
08	-	34		
09	e	12		
0a	DN	25		
0b	0	00		
0c	X=Y	50		
0d	1	01		
10	b	14		
11	d	17		
12	UP	27		
13	f	15		
14	-	34		
15	YTO	40		
16	e	12		
17	GTO	44		
18	-	34		
19	0	00		
1a	3	03		
1b	1	01		
1c	STP	41	ENTRY	
1d	PNT	45		
20	PNT	45		
21	XTO	23		
22	d	17		
23	YTO	40		
24	c	16		
25	-	34		
26	5	05		
27	0	00		
28	CNT	47		
29	DIV	35		
2a	d	17		
2b	XEY	30		
2c	-	34		
2d	AC+	60		
30	EEX	26		
31	9	11		
32	9	11		
33	XTO	23		
34	b	14		
35	CHS	32		
36	XTO	23		
37	a	13		
38	RCL	61		
39	+	33		
3a	YTO	40		
3b	e	12		
3c	c	16		
3d	X<Y	52		

40	8	10
41	0	00
42	DN	25
43	SFL	54
44	GTO	44
45	SUB	77
46	-	34
47	3	03
48	0	00
49	IFG	43
4a	7	07
4b	3	03
4c	b	14
4d	-	34
50	a	13
51	DIV	35
52	e	12
53	UP	27
54	d	17
55	-	34
56	c	16
57	DIV	35
58	7	07
59	6	06
5a	0	00
5b	0	00
5c	X<Y	52
5d	6	06
60	8	10
61	DN	25
62	FMT	42
63	DN	25
64	GTO	44
65	-	34
66	0	00
67	3	03
68	0	00
69	UP	27
6a	UP	27
6b	FMT	42
6c	UP	27
6d	GTO	44
70	-	34
71	0	00
72	c	16
73	b	14
74	X>Y	53
75	YTO	40
76	b	14
77	a	13
78	X<Y	52
79	YTO	40
7a	a	13
7b	GTO	44
7c	3	03
7d	7	07

80	RCL	61
81	AC-	63
82	UP	27
83	3	03
84	CNT	47
85	DIV	35
86	d	17
87	XEY	30
88	-	34
89	AC+	60
8a	UP	27
8b	5	05
8c	0	00
8d	CNT	47
90	DIV	35
91	YTO	40
92	c	16
93	a	13
94	UP	27
95	b	14
96	-	34
97	5	05
98	EEX	26
99	3	03
9a	GTO	44
9b	-	34
9c	0	00
9d	0	00
a0	CNT	47
a1	CNT	47
a2	CNT	47
a3	CNT	47
a4	CNT	47
a5	CNT	47
a6	CNT	47
a7	CNT	47
a8	CNT	47
a9	CNT	47
aa	CNT	47
ab	CNT	47
ac	CNT	47
ad	CNT	47

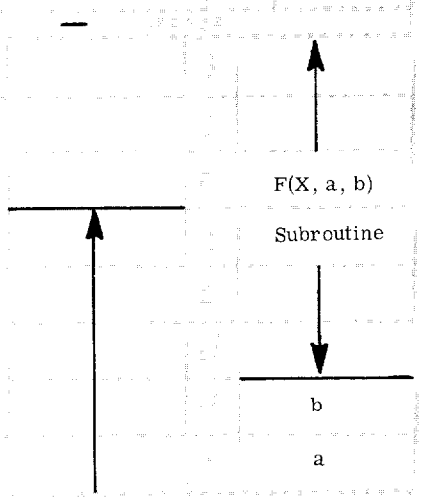


Plus Page

b0	CNT	47		10	b	14		50	UP	27	
b1	CNT	47		11	UP	27		51	XFR	67	
b2	CNT	47		12	d	17		52	-	34	
b3	CNT	47		13	PNT	45	s	53	e	12	
b4	CNT	47		14	PNT	45		54	DIV	35	
b5	CNT	47		15	5	05		55	XFR	67	
b6	CNT	47		16	EEX	26		56	-	34	
b7	CNT	47		17	2	02		57	f	15	
b8	CNT	47		18	UP	27		58	XEY	30	
b9	CNT	47		19	a	13		59	LN	65	
ba	CNT	47		1a	XEY	30		5a	X	36	
bb	CNT	47		1b	X	36		5b	DN	25	
bc	CNT	47		1c	UP	27		5c	EXP	74	
bd	CNT	47		1d	c	16		5d	CHS	32	
c0	CNT	47		20	X	36		60	EXP	74	
c1	CNT	47		21	2	02		61	X	36	
c2	CNT	47		22	RDN	31		62	RTN	77	
c3	CNT	47		23	PNT	45	s				
c4	CNT	47		24	PNT	45					
c5	CNT	47		25	0	00					
c6	CNT	47		26	UP	27					
c7	CNT	47		27	UP	27					
c8	CNT	47		28	GTO	44					
c9	CNT	47		29	+	33					
ca	CNT	47		2a	0	00					
cb	CNT	47		2b	1	01					
cc	CNT	47		2c	CNT	47					
cd	CNT	47		2d	CNT	47					
d0	CNT	47		30	UP	27					
d1	CNT	47		31	XFR	67					
d2	CNT	47		32	-	34					
d3	CNT	47		33	f	15					
d4	CNT	47		34	UP	27					
d5	CNT	47		35	1	01					
d6	CNT	47		36	-	34					
d7	CNT	47		37	DN	25					
d8	CNT	47		38	XEY	30					
d9	CNT	47		39	LN	65					
da	CNT	47		3a	X	36					
db	CNT	47		3b	DN	25					
dc	CNT	47		3c	EXP	74					
dd	CNT	47		3d	UP	27					

Minus Page

00	DIV	35	40	XFR	67		
01	YTO	40	41	-	34		
02	a	13	42	f	15		
03	RCL	61	43	X	36		
04	AC=	63	44	UP	27		
05	+	33	45	XFR	67		
06	AC+	60	46	-	34		
07	DN	25	47	e	12		
08	GTO	44	48	LN	65		
09	+	33	49	X	36		
0a	4	04	4a	DN	25		
0b	4	04	4b	EXP	74		
0c	1	01	4c	DIV	35		
0d	UP	27	4d	e	12		



Step	Key	Code
30	UP	27
31	XFR	67
32	-	34
33	f	15
34	UP	27
35	1	01
36	-	34
37	DN	25
38	XEY	30
39	LN	65
3a	X	36
3b	DN	25
3c	EXP	74
3d	UP	27
40	XFR	67
41	-	34
42	f	15
43	X	36
44	UP	27
45	XFR	67
46	-	34
47	e	12
48	LN	65
49	X	36
4a	DN	25
4b	EXP	74
4c	DIV	35
4d	e	12
50	UP	27
51	XFR	67
52	-	34
53	e	12
54	DIV	35
55	XFR	67
56	-	34
57	f	15
58	XEY	30
59	LN	65
5a	X	36
5b	DN	25
5c	EXP	74
5d	CHS	32
60	EXP	74
61	X	36
62	RTN	77
63	END	46

Example #1

Weibull Family

Step	Key	Code
30	SIN	70
31	UP	27
32	.	21
33	9	11
34	7	07
35	7	07
36	X	36
37	e	12
38	UP	27
39	+	33
3a	DN	25
3b	SIN	70
3c	UP	27
3d	.	21
40	4	04
41	5	05
42	3	03
43	X	36
44	DN	25
45	-	34
46	e	12
47	UP	27
48	3	03
49	X	36
4a	DN	25
4b	SIN	70
4c	UP	27
4d	.	21
50	2	02
51	6	06
52	2	02
53	X	36
54	DN	25
55	+	33
56	e	12
57	UP	27
58	4	04
59	X	36
5a	DN	25
5b	SIN	70
5c	UP	27
5d	.	21
60	1	01
61	5	05
62	1	01
63	X	36
64	DN	25
65	-	34
66	RTN	77
67	END	46

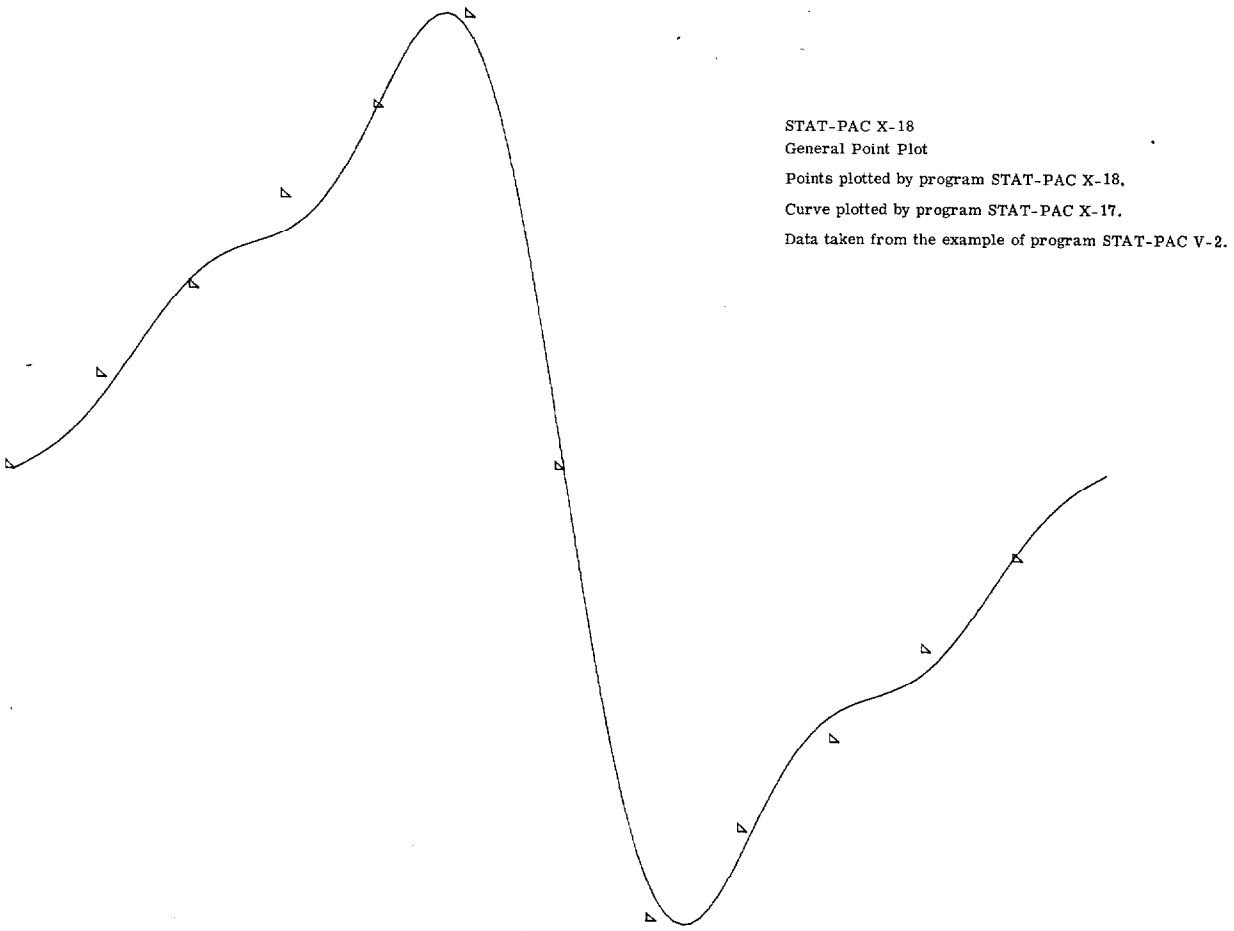
Example #2

Fourier Series used in Example from STAT-PAC X-18

GENERAL POINT PLOT

This program plots a series of points (X_i, Y_i). The program is written to plot $(X_{max} - X_{min})$ at 15 inches (38.1 cm.). To convert the X axis to units of centimeters, place 3000 in locations (0)(6) through (0)(9). This will cause $(X_{max} - X_{min})$ to be plotted at 15 cm.

The Y axis is programmed to plot $(Y_{max} - Y_{min})$ at 5 inches. To convert to units of centimeters, place 2000 in locations (1)(8) through (1)(b).



USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: STOP

SET: Decimal Wheel at 6 or less

Using the origin controls, locate the pen in the lower left corner of the paper.

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

ENTER DATA:

$X_{max} \longrightarrow Y$
$X_{min} \longrightarrow X$

PRESS: CONTINUE

ENTER DATA:

$Y_{max} \longrightarrow Y$
$Y_{min} \longrightarrow X$

PRESS: CONTINUE

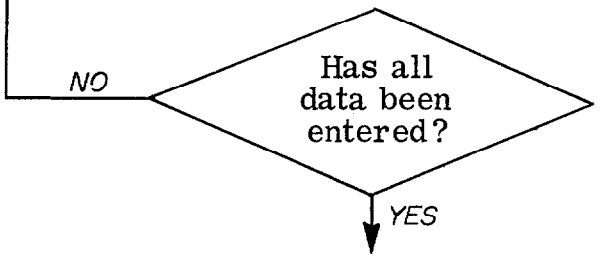
DISPLAY

$0 \longrightarrow Z$
$i \longrightarrow Y$
$0 \longrightarrow X$

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

To run another case

EXAMPLE

Plot the data from the example of program STAT-PAC V-2.

X	Y	i
0	0	1
$\frac{\pi}{6}$	.262	2
$2\frac{\pi}{6}$	.524	3
$3\frac{\pi}{6}$	.786	4
$4\frac{\pi}{6}$	1.047	5
$5\frac{\pi}{6}$	1.309	6
$6\frac{\pi}{6}$	0	7
$7\frac{\pi}{6}$	-1.309	8
$8\frac{\pi}{6}$	-1.047	9
$9\frac{\pi}{6}$	-.786	10
$10\frac{\pi}{6}$	-.524	11
$11\frac{\pi}{6}$	-.262	12

$X_{max} = 15(\frac{\pi}{6})$

$X_{min} = 0$

$Y_{max} = 1.309$

$Y_{min} = -1.309$

The curve fitted thru these data points is obtained by the use of Program STAT-PAC X-17 (General Parametric Plot).

EXAMPLE (Con't)

Plot the Fourier Series obtained from the example of program STAT-PAC V-2 .

$f(X) = .977 \sin X - .453 \sin 2X + .262 \sin 3X - .151 \sin 4X$

$X_{max} = 15 \left(\frac{\pi}{6}\right)$

$X_{min} = 0$

$a = 0$

$b = 0$

00	CLR	20		40	X	36
01	STP	41	ENTRY	41	DN	25
02	PNT	45		42	XEY	30
03	PNT	45		43	FMT	42
04	-	34		44	DN	25
05	UP	27		45	UP	27
06	7	07		46	5	05
07	5	05		47	0	00
08	0	00		48	-	34
09	0	00		49	RDN	31
0a	RUP	22		4a	FMT	42
0b	DIV	35		4b	DN	25
0c	DN	25		4c	XEY	30
0d	AC+	60		4d	RUP	22
10	0	00		50	+	33
11	UP	27		51	RDN	31
12	UP	27		52	XEY	30
13	STP	41	ENTRY	53	FMT	42
14	PNT	45		54	DN	25
15	PNT	45		55	RUP	22
16	-	34		56	+	33
17	UP	27		57	XEY	30
18	5	05		58	RDN	31
19	0	00		59	-	34
1a	0	00		5a	DN	25
1b	0	00		5b	XEY	30
1c	RUP	22		5c	FMT	42
1d	DIV	35		5d	DN	25
20	DN	25		60	FMT	42
21	YTO	40		61	UP	27
22	c	16		62	0	00
23	XTO	23		63	UP	27
24	d	17		64	b	14
25	0	00		65	UP	27
26	UP	27		66	1	01
27	1	01		67	+	33
28	UP	27		68	0	00
29	0	00		69	GTO	44
2a	STP	41	ENTRY	6a	2	02
2b	IFG	43		6b	a	13
2c	6	06		6c	CLR	20
2d	c	16		6d	FMT	42
30	PNT	45		70	UP	27
31	PNT	45		71	GTO	44
32	RUP	22		72	0	00
33	XTO	23		73	0	00
34	b	14		74	END	46
35	e	12				
36	-	34				
37	f	15				
38	X	36				
39	c	16				
3a	XEY	30				
3b	RDN	31				
3c	-	34				
3d	d	17				

AXES PLOT

This program will plot the abscissa and ordinate axes. It was written to be used in conjunction with the other plotter programs to facilitate interpretation and interpolation of plotted data.

The required inputs are:

X_{shift}, Y_{shift} — These should be the same as used in the previously run problem.

X_{scale}, Y_{scale} — These should also be the same as used in the previously run problem.

X_{origin}, Y_{origin} — Simply the value you wish the axes cross-point to take on. Must be expressed in terms of the scaling constants.

X_{tic}, Y_{tic} — The desired distance between tic marks. Must also be expressed in terms of the scaling constants.

The axes will then cross at scaled point

$$\left(\frac{500}{X_{scale}} \left[X_{origin} - X_{shift} \right], \frac{500}{Y_{scale}} \left[Y_{origin} - Y_{shift} \right] \right)$$

and the scaled distances between tic marks on the X and Y axes will be

$$\left[\frac{500}{X_{scale}} \right] \left[X_{tic} \right] \text{and} \left[\frac{500}{Y_{scale}} \right] \left[Y_{tic} \right] \text{ respectively.}$$

NOTE: All entries should be made in terms of the problem variables. To convert this program to recognize centimeters rather than inches as the basic unit of scaling, program a 2 in location 0-9 and a 6 in location 5-9.

USER INSTRUCTIONS

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper, or if plotting axes on a graph which has just been plotted, leave the pen at the location dictated by pressing the STOP key.

SET: Decimal Wheel at 6 or less

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

ENTER TRANSLATION CONSTANTS:

$Y_{shift} \longrightarrow Y$
$X_{shift} \longrightarrow X$

PRESS: CONTINUE

ENTER SCALING CONSTANTS:

$Y_{scale} \longrightarrow Y$
$X_{scale} \longrightarrow X$

PRESS: CONTINUE

ENTER DESIRED ORIGIN*:

$Y_{origin} \longrightarrow Y$
$X_{origin} \longrightarrow X$

NOTE: $Y_{origin} \geq Y_{shift}$
$X_{origin} \geq X_{shift}$

PRESS: CONTINUE

ENTER TIC MARK INCREMENTS:

$Y_{tic} \longrightarrow Y$
$X_{tic} \longrightarrow X$

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

*NOTE: Y_{origin}, X_{origin}, Y_{tic} and X_{tic} are expressed in terms of the scaling constants (i.e., the problem variables).

The axes will now be plotted.

To rerun the program, re-enter the magnetic card and proceed as before.

EXAMPLE

See next page.

EXAMPLE

Let:

Y_{shift} = -500 units

X_{shift} = -500 units

Y_{origin} = 1500 units

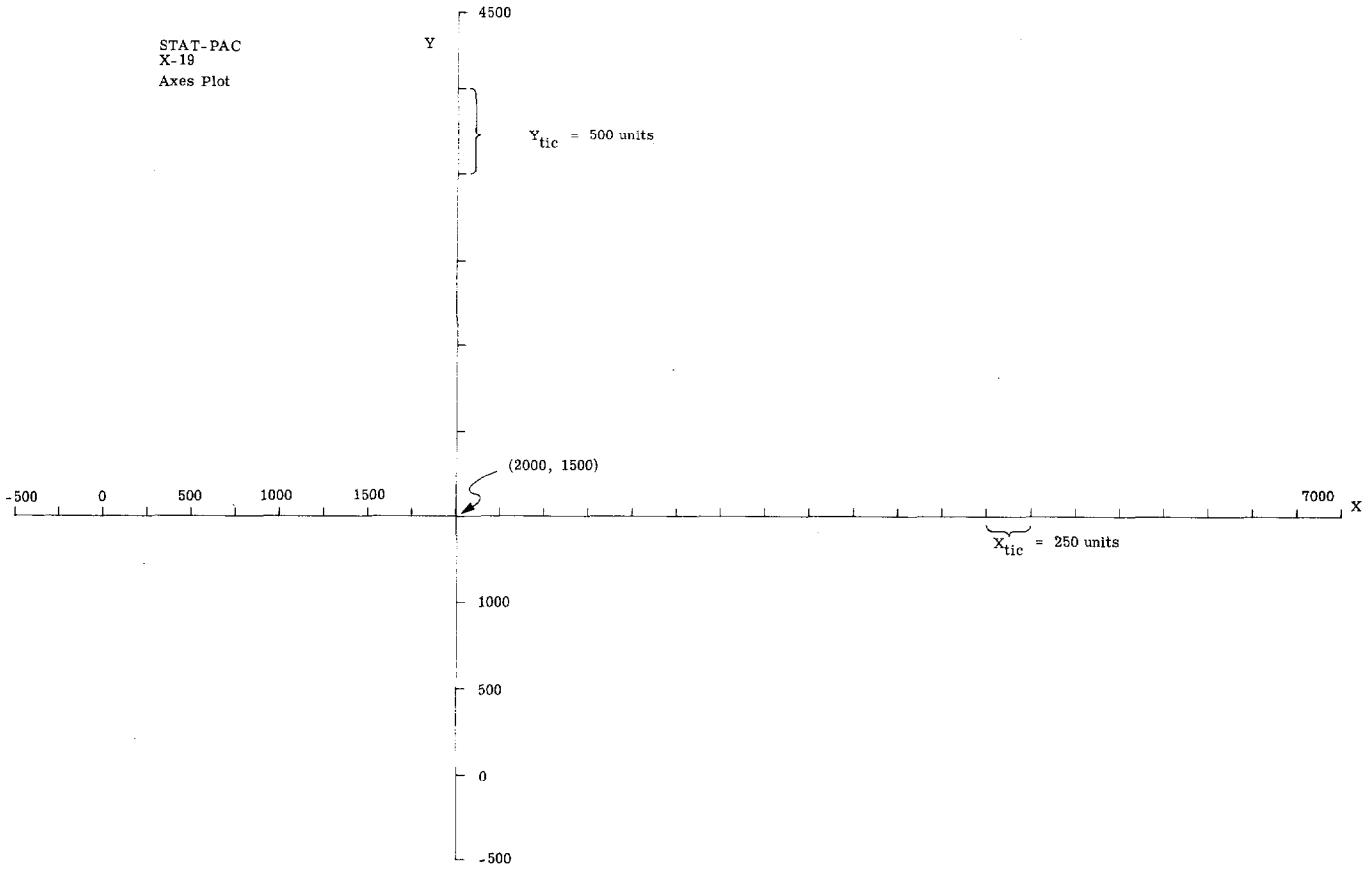
X_{origin} = 2000 units

Y_{scale} = 500 $\frac{\text{units}}{\text{inch}}$

X_{scale} = 500 $\frac{\text{units}}{\text{inch}}$

Y_{tic} = 500 $\frac{\text{units}}{\text{tic mark}}$

X_{tic} = ~~250~~ 200 $\frac{\text{units}}{\text{tic mark}}$



Adr	Key	Code		Adr	Key	Code		Adr	Key	Code
00	CLR	20		40	3	03		80	XEY	30
01	STP	41	ENTRY	41	c	16		81	IFG	43
02	XTO	23		42	X=Y	50		82	SFL	54
03	c	16		43	ARC	72		83	XEY	30
04	YTO	40		44	-	34		84	RUP	22
05	d	17		45	-	34		85	-	34
06	CLR	20		46	AC+	60		86	DN	25
07	STP	41	ENTRY	47	DN	25		87	XEY	30
08	UP	27		48	CLX	37		88	IFG	43
09	5	05		49	IFG	43		89	SFL	54
0a	0	00		4a	SFL	54		8a	XEY	30
0b	0	00		4b	XEY	30		8b	GTO	44
0c	DIV	35		4c	FMT	42		8c	4	04
0d	YTO	40		4d	DN	25		8d	c	16
10	b	14		50	UP	27		90	RUP	22
11	RUP	22		51	IFG	43		91	XEY	30
12	DIV	35		52	SFL	54		92	IFG	43
13	YTO	40		53	UP	27		93	SFL	54
14	0	00		54	RCL	61		94	XEY	30
15	CLR	20		55	AC-	63		95	FMT	42
16	STP	41	ENTRY	56	+	33		96	DN	25
17	UP	27		57	AC+	60		97	FMT	42
18	c	16		58	7	07		98	UP	27
19	-	34		59	5	05		99	IFG	43
1a	b	14		5a	IFG	43		9a	a	13
1b	DIV	35		5b	SFL	54		9b	8	10
1c	YEX	24		5c	INT	64		9c	UP	27
1d	d	17		5d	IFG	43		9d	RCL	61
20	DN	25		60	SFL	54		a0	AC-	63
21	-	34		61	5	05		a1	d	17
22	XEY	30		62	EEX	26		a2	RDN	31
23	YEX	24		63	3	03		a3	b	14
24	0	00		64	X<Y	52		a4	SFL	54
25	YTO	40		65	9	11		a5	GTO	44
26	0	00		66	0	00		a6	3	03
27	X	36		67	DN	25		a7	a	13
28	YTO	40		68	IFG	43		a8	CLR	20
29	c	16		69	SFL	54		a9	FMT	42
2a	CLR	20		6a	XEY	30		aa	UP	27
2b	STP	41	ENTRY	6b	FMT	42		ab	END	46
2c	UP	27		6c	DN	25				
2d	YEX	24		6d	XEY	30				
30	0	00		70	IFG	43				
31	RDN	31		71	SFL	54				
32	X	36		72	XEY	30				
33	YEX	24		73	UP	27				
34	b	14		74	5	05				
35	c	16		75	0	00				
36	RDN	31		76	+	33				
37	DIV	35		77	RDN	31				
38	d	17		78	XEY	30				
39	XEY	30		79	IFG	43				
3a	X<Y	52		7a	SFL	54				
3b	ARC	72		7b	XEY	30				
3c	-	34		7c	FMT	42				
3d	X<Y	52		7d	DN	25				

$\overline{X}$ CONTROL CHART PLOT

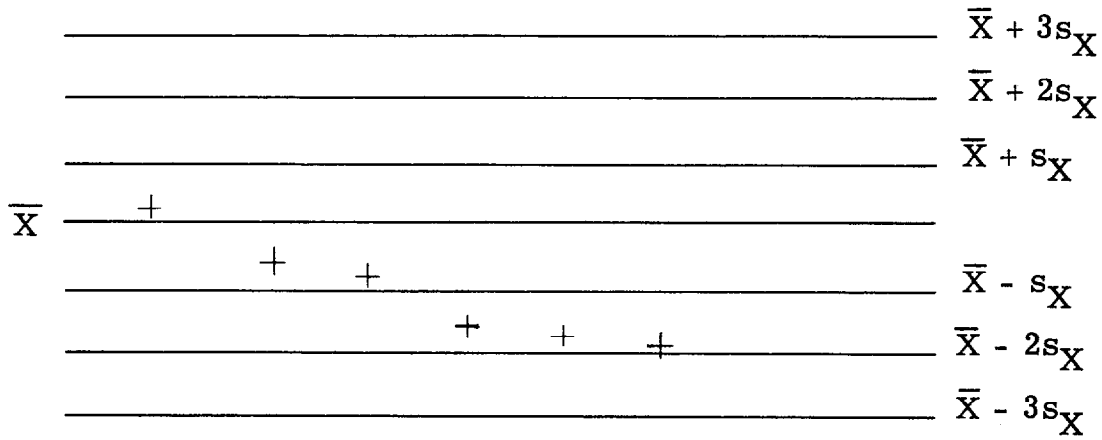
This program constructs an $\overline{X}$ Control Chart and plots observed samples on the chart. The control chart consists of lines at $\overline{X}$, $\overline{X} \pm s_X$, $\overline{X} \pm 2s_X$, and $\overline{X} \pm 3s_X$.

In general the levels $\overline{X} \pm 2s_X$ are warning levels, and $\overline{X} \pm 3s_X$ are termed control limits. The user determines.

$\overline{X}$ — Expected Value of Population

s_X — Expected Standard Deviation of Population

Δt — t Axis Increment



$\overline{X}$ CONTROL CHART

This program has an option of updating the control chart as observations are available.

USER INSTRUCTIONS

SET: Decimal Wheel at 6 or less

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

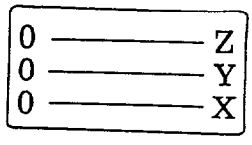
DEPRESS: X on the 9120A

PRESS: END

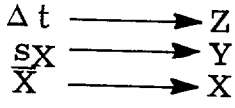
ENTER PROGRAM

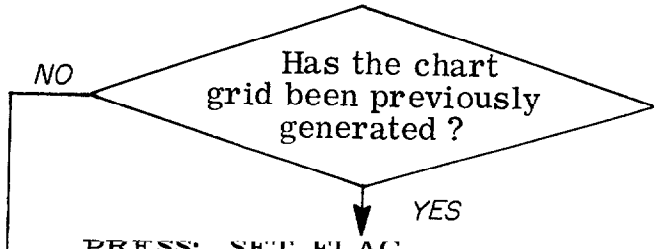
PRESS: CONTINUE

DISPLAY



ENTER DATA:





PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY



(n represents the entry to be input; if this is not the first pass at the chart, place the correct n → Y).

ENTER DATA: $X_n \longrightarrow X$

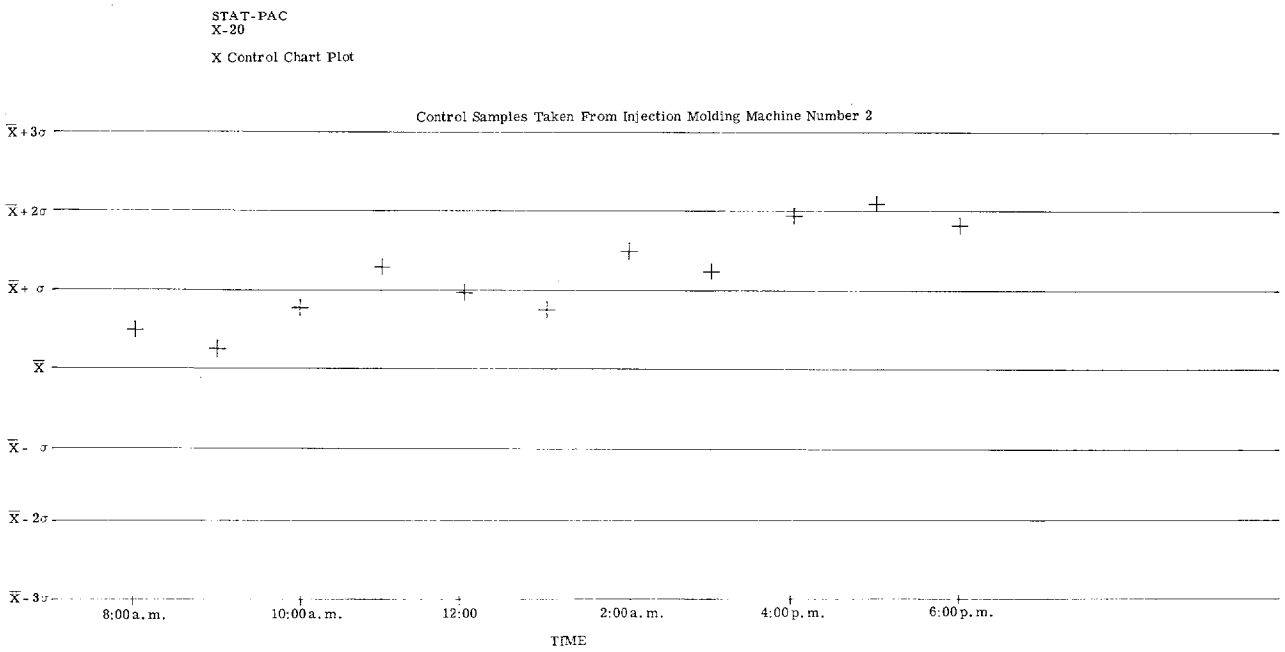
PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

If program Axis Plot STAT-PAC X-19 is to be run, use:

$$Y_{shift} = -5s_X - \bar{X}$$
$$Y_{scale} = s_X$$

$$X_{shift} = 0$$
$$X_{scale} = \Delta t$$



EXAMPLE

$\Delta t = 1$

$s_X = 1$

$\bar{X} = 0$

$X_1 = .50$
$X_2 = .25$
$X_3 = .78$
$X_4 = 1.30$
$X_5 = .97$
$X_6 = .76$
$X_7 = 1.50$
$X_8 = 1.25$
$X_9 = 1.94$
$X_{10} = 2.10$
$X_{11} = 1.82$

00	CLR	20		40	2	02		80	FMT	42	
01	STP	41	ENTRY	41	GTO	44		81	DN	25	
02	AC+	60		42	1	01		82	RUP	22	
03	RUP	22		43	6	06		83	-	34	
04	XTO	23		44	0	00		84	RUP	22	
05	d	17		45	UP	27		85	XEY	30	
06	PNT	45		46	FMT	42		86	+	33	
07	0	00		47	UP	27		87	RDN	31	
08	RUP	22		48	UP	27		88	XEY	30	
09	PNT	45		49	1	01		89	FMT	42	
0a	RUP	22		4a	XEY	30		8a	UP	27	
0b	PNT	45		4b	STP	41	ENTRY	8b	FMT	42	
0c	PNT	45		4c	XEY	30		8c	DN	25	
0d	PNT	45		4d	PNT	45		8d	RDN	31	
10	IFG	43		50	XEY	30		90	XEY	30	
11	4	04		51	PNT	45		91	-	34	
12	4	04		52	PNT	45		92	-	34	
13	EEX	26		53	YTO	40		93	DN	25	
14	3	03		54	c	16		94	XEY	30	
15	RUP	22		55	UP	27		95	FMT	42	
16	+	33		56	e	12		96	DN	25	
17	4	04		57	UP	27		97	FMT	42	
18	EEX	26		58	5	05		98	UP	27	
19	3	03		59	X	36		99	0	00	
1a	X<Y	52		5a	DN	25		9a	UP	27	
1b	4	04		5b	+	33		9b	c	16	
1c	4	04		5c	f	15		9c	UP	27	
1d	b	14		5d	-	34		9d	1	01	
20	FMT	42		60	e	12		a0	+	33	
21	DN	25		61	DIV	35		a1	0	00	
22	2	02		62	5	05		a2	GTO	44	
23	EEX	26		63	EEX	26		a3	4	04	
24	3	03		64	2	02		a4	b	14	
25	FMT	42		65	X	36		a5	END	46	
26	DN	25		66	c	16					
27	UP	27		67	UP	27					
28	+	33		68	d	17					
29	DN	25		69	DIV	35					
2a	FMT	42		6a	5	05					
2b	DN	25		6b	EEX	26					
2c	6	06		6c	2	02					
2d	EEX	26		6d	X	36					
30	3	03		70	DN	25					
31	FMT	42		71	FMT	42					
32	DN	25		72	DN	25					
33	7	07		73	UP	27					
34	.	21		74	5	05					
35	5	05		75	0	00					
36	EEX	26		76	-	34					
37	3	03		77	RDN	31					
38	FMT	42		78	FMT	42					
39	DN	25		79	DN	25					
3a	FMT	42		7a	RUP	22					
3b	UP	27		7b	+	33					
3c	5	05		7c	+	33					
3d	EEX	26		7d	RDN	31					

n

Δt

S_x

$\bar{x}$

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