

HEWLETT-PACKARD

HP-33E
SURVEYING
Applications



For Continuous Memory Models

Although this book refers specifically to the HP-33E or HP-38E, the programs and calculations contained herein apply equally well to the HP-33C or HP-38C. The user should note, however, that the display format and data register contents are retained by the calculator even though it has been turned off. It may be desirable to reset or clear these conditions before running programs or making calculations.



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HP-33E

Surveying Applications

March 1978

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Introduction

This Surveying Applications book was written to help you get the most from your HP-33E calculator. The programs were chosen to provide useful calculations for many of the common problems encountered in surveying.

They will provide you with immediate capabilities in your everyday calculations and you will find them useful as guides to programming techniques for writing your own customized software.

You will find general information on how to key in and run programs under "A Word about Program Usage" in the Applications book you received with your calculator.

We hope that this Surveying book will be a valuable tool in your work and would appreciate your comments about it.

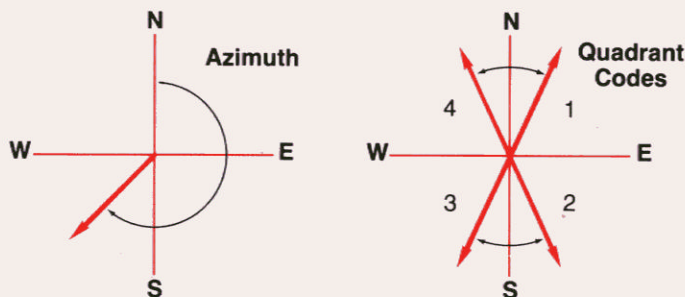
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Traversing

Azimuth-Bearing Conversions

Angle conventions for azimuths and quadrant bearings as used in this application book are shown below:



Thus azimuths are measured from the north meridian following North American surveying conventions. Bearings are measured from the meridian in the quadrant in which the line falls. Quadrant codes are shown in the above sketch.

Often it is desirable to have a quick, easy method to convert to or from azimuths and bearings. In this application book, for example, some inputs and outputs may be in azimuths rather than bearings, or vice versa, when you desire the alternate form. The following simple keystroke routines are helpful in making these conversions:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Azimuths to Bearings:			
1	Azimuth = 0° to 90°	AZ (D.MS)	No Calculation	BRG (D.MS)
				QD = 1
2	Azimuth = 90° to 180°	180	ENTER +	
		AZ (D.MS)	9 + H -	
			f + H.MS	BRG (D.MS)
				QD = 2
3	Azimuth = 180° to 270°	AZ (D.MS)	ENTER + 180 -	BRG (D.MS)
				QD = 3
4	Azimuth = 270° to 360°	360	ENTER +	
		AZ (D.MS)	9 + H -	
			f + H.MS	BRG (D.MS)
				QD = 4
	Bearings to Azimuths:			
5	Quadrant = 1	BRG (D.MS)	No Calculation	AZ (D.MS)
6	Quadrant = 2	180	ENTER +	
		BRG (D.MS)	9 + H -	
			f + H.MS	AZ (D.MS)
7	Quadrant = 3	BRG (D.MS)	ENTER + 180 +	AZ (D.MS)
8	Quadrant = 4	360	ENTER +	
		BRG (D.MS)	9 + H -	
			f + H.MS	AZ (D.MS)

If you have a number of conversions to perform the following program will be more convenient and faster. Lines 01 thru 24 convert bearings to azimuths. Lines 25 thru 39 convert azimuths to bearings. You may want to separate the two parts and only key in one section, if all your conversions are in one direction.

KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
$\boxed{x \div y}$	01- 21
\boxed{g} $\boxed{\div H}$	02- 15 6
$\boxed{x \div y}$	03- 21
$\boxed{ENTER\#}$	04- 31
$\boxed{ENTER\#}$	05- 31
2	06- 2
$\boxed{+}$	07- 71
\boxed{g} \boxed{INT}	08- 15 32
1	09- 1
8	10- 8
0	11- 0
\boxed{STO} 0	12- 23 0
\boxed{x}	13- 61
$\boxed{x \div y}$	14- 21
\boxed{RCL} 0	15- 24 0
\boxed{x}	16- 61
\boxed{f} \boxed{COS}	17- 14 8
$\boxed{R\#}$	18- 22
$\boxed{R\#}$	19- 22
$\boxed{R\#}$	20- 22

\boxed{x}	21- 61
$\boxed{-}$	22- 41
\boxed{f} $\boxed{\div HMS}$	23- 14 6
\boxed{GTO} 00	24- 13 00
\boxed{g} $\boxed{\div H}$	25- 15 6
$\boxed{ENTER\#}$	26- 31
\boxed{f} \boxed{SIN}	27- 14 7
\boxed{g} $\boxed{SIN^{-1}}$	28- 15 7
\boxed{g} $\boxed{x < 0}$	29- 15 41
\boxed{CHS}	30- 32
\boxed{f} $\boxed{\div HMS}$	31- 14 6
$\boxed{R/S}$	32- 74
$\boxed{R\#}$	33- 22
9	34- 9
0	35- 0
$\boxed{+}$	36- 71
1	37- 1
$\boxed{+}$	38- 51
\boxed{g} \boxed{INT}	39- 15 32
\boxed{GTO} 00	40- 13 00

REGISTERS

R_0 180	R_1	R_2	R_3
R_4	R_5	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	To convert bearing to azimuth:			
	a Input bearing	BRG (D.MS)	ENTER	
	b Input quadrant code	QD	GSB 01	AZ (D.MS)
3	To convert azimuth to bearing:			
	Input azimuth	AZ (D.MS)	GSB 25	BRG (D.MS)
			R/S	QD

Example 1:

Convert azimuth of $162^{\circ} 15' 32''$ to bearing/quadrant.

Keystrokes**Display**162.1532 **GSB** 25**17.4428**

BRG (D.MS)

R/S**2.0000**

QD

Convert azimuth of $39^{\circ} 42' 26''$ to bearing/quadrant.

Keystrokes**Display**39.4226 **GSB** 25**39.4226**

BRG (D.MS)

R/S**1.0000**

QD

Example 2:

Convert bearing S $34^{\circ} 56' 37''$ W to an azimuth.

Keystrokes**Display**34.5637 **ENTER** 3**GSB** 01**214.5637**

AZ (D.MS)

Convert bearing N $85^{\circ} 24' 47''$ W to an azimuth.

Keystrokes**Display**85.2447 **ENTER** 4**GSB** 01**274.3513**

AZ (D.MS)

Bearing Traverse

This program uses bearings and horizontal distances or slope distances to calculate coordinates in a surveying traverse. Starting from a known point, the calculations proceed point by point around the traverse. The total horizontal distance traversed is calculated as well as the area enclosed by the traverse (if it is a closed traverse).

Formulas Used:

1. $HD = SD \sin (ZA)$
2. $N_{k+1} = N_k + HD \cos AZ$ $LAT_k = N_{k+1} - N_k$
3. $E_{k+1} = E_k + HD \sin AZ$ $DEP_k = E_{k+1} - E_k$
4. $Area = \sum_{k=1}^n LAT_k \left(\frac{1}{2} DEP_k + \sum_{j=1}^{k-1} DEP_j \right)$

where: N, E = Northing, easting of a point

Subscript k refers to current point

n equals number of points in the survey

AZ = Azimuth of a course

HD = Horizontal distance

SD = Slope distance

ZA = Zenith angle

- All angular inputs and outputs are in the form degrees, minutes and seconds (D.MS).
- This program uses zenith angles to calculate horizontal distance from slope distance. If your instrument measures vertical angles rather than zenith angles, convert the vertical angle to a zenith angle by the following formula:

$$\text{Zenith angle} = 90^\circ - \text{Vertical angle}$$

(Remember to convert D.MS input to decimal degrees before subtracting from 90°).

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00	$\boxed{R/S}$	25- 74
\boxed{RCL} 1	01- 24 1	\boxed{GTO} 31	26- 13 31
\boxed{STO} 5	02- 23 5	$\boxed{x \div y}$	27- 21
$\boxed{R/S}$	03- 74	\boxed{g} $\boxed{\rightarrow H}$	28- 15 6
$\boxed{x \div y}$	04- 21	\boxed{f} \boxed{SIN}	29- 14 7
\boxed{g} $\boxed{\rightarrow H}$	05- 15 6	\boxed{x}	30- 61
$\boxed{x \div y}$	06- 21	\boxed{STO} $\boxed{+}$ 3	31- 23 51 3
\boxed{ENTER}	07- 31	\boxed{RCL} 0	32- 24 0
\boxed{ENTER}	08- 31	$\boxed{x \div y}$	33- 21
2	09- 2	\boxed{f} $\boxed{\rightarrow R}$	34- 14 4
$\boxed{\div}$	10- 71	\boxed{STO} $\boxed{+}$ 5	35- 23 51 5
\boxed{g} \boxed{INT}	11- 15 32	\boxed{STO} $\boxed{+}$ 1	36- 23 51 1
\boxed{RCL} 7	12- 24 7	$\boxed{x \div y}$	37- 21
\boxed{x}	13- 61	\boxed{STO} $\boxed{+}$ 6	38- 23 51 6
$\boxed{x \div y}$	14- 21	\boxed{STO} $\boxed{+}$ 2	39- 23 51 2
\boxed{RCL} 7	15- 24 7	2	40- 2
\boxed{x}	16- 61	$\boxed{\div}$	41- 71
\boxed{f} \boxed{COS}	17- 14 8	\boxed{RCL} 6	42- 24 6
$\boxed{R+}$	18- 22	$\boxed{-}$	43- 41
$\boxed{R+}$	19- 22	\boxed{x}	44- 61
$\boxed{R+}$	20- 22	\boxed{STO} $\boxed{+}$ 4	45- 23 51 4
\boxed{x}	21- 61	\boxed{RCL} 1	46- 24 1
$\boxed{-}$	22- 41	$\boxed{R/S}$	47- 74
\boxed{STO} 0	23- 23 0	\boxed{RCL} 2	48- 24 2
\boxed{f} $\boxed{\rightarrow HMS}$	24- 14 6	\boxed{GTO} 03	49- 13 03

REGISTERS

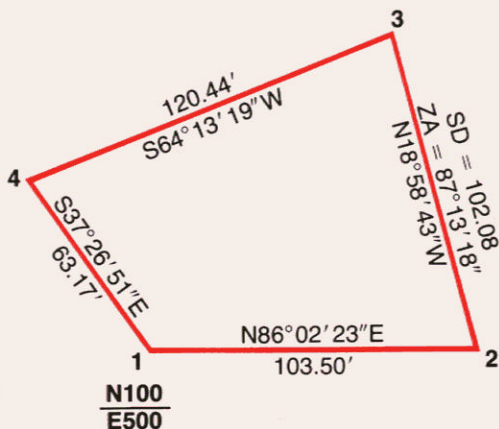
R_0 AZ	R_1 Current N	R_2 Current E	R_3 Σ HD
R_4 Area	R_5 LAT	R_6 DEP	R_7 180

10 Traversing

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize and store	180	f REG STO 7	180
	Starting point	N_1	STO 1	N_1
		E_1	STO 2	E_1
			GSB 01	N_1
3	Input bearing	BRG (D.MS)	ENTER +	
	and quadrant code	QD	R/S	AZ_1 (D.MS)
	or			
3a	azimuth	AZ (D.MS)	g →H GSB 23	AZ_1 (D.MS)
4	If horizontal distance	HD	R/S	N_1
			R/S	E_1
	or			
4a	If slope distance,			
	Input zenith angle	AZ (D.MS)	ENTER +	
	and slope distance	SD	GSB 27	N_1
			R/S	E_1
5	Repeat steps 3-4 for			
	successive courses			
6	Display total horizontal			
	distance traversed		RCL 3	Σ HD
7	Display area for closed			
	traverse (ignore sign)		RCL 4	Area

Example:

Starting with point 1 with coordinates N100, E500, traverse the figure above and compute the coordinates of the other points.



Keystrokes

Display

f REG 180 STO 7

100 STO 1

500 STO 2

GSB 01

86.0223 ENTER

1 R/S

103.5 R/S

R/S

18.5843 ENTER

4 R/S

87.1318 ENTER

102.08 GSB 27

R/S

64.1319 ENTER

3 R/S

120.44 R/S

R/S

37.2651 ENTER

2 R/S

63.17 R/S

R/S

RCL 3

RCL 4

86.0223

107.1482

603.2529

341.0117

203.5657

570.0939

244.1319

151.1880

461.6395

142.3309

101.0366

500.0490

389.0700

-8,855.4931

 AZ_2 (D.MS) N_2 E_2 AZ_3 (D.MS) N_3 E_3 AZ_4 (D.MS) N_4 E_4 AZ_5 (D.MS) N_1 E_1 Σ HD

Area

Field Angle Traverse

This program calculates coordinates of a traverse from field angles and horizontal or slope distances. The total horizontal distance traversed and the enclosed area (for a closed traverse) are also calculated.

In running this program, the user inputs the northing and easting of his starting point, the reference azimuth, and then the direction and distance from each point in the traverse to the next point. The direction may be input either as a deflection right or left, or as an angle right or left. The distance may be input either as horizontal distance, or as slope distance with zenith angle.

Equations:

$$HD = SD \sin (ZA)$$

$$N_{k+1} = N_k + HD \cos AZ$$

$$E_{k+1} = E_k + HD \sin AZ$$

$$LAT_k = N_{k+1} - N_k$$

$$DEP_k = E_{k+1} - E_k$$

$$Area = \sum_{k=1}^n LAT_k \left(\frac{1}{2} DEP_k + \sum_{j=1}^{k-1} DEP_j \right)$$

where: N, E = Northing, easting of a point

Subscript k refers to current point

Subscript n equals number of points in the survey

AZ = Azimuth of a course

HD = Horizontal distance

SD = Slope distance

ZA = Zenith angle

- All angular inputs and outputs are in the form degrees, minutes and seconds (D.MS).
- This program uses zenith angles to calculate horizontal distance from slope distance. If your instrument measures vertical angles rather than zenith angles, convert the vertical angle to a zenith angle by the

following formula:

$$\text{Zenith angle} = 90^\circ - \text{Vertical angle}$$

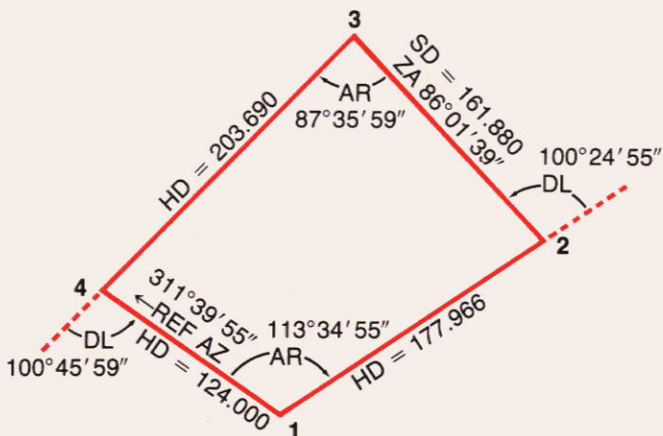
(Remember to convert D.MS input to decimal degrees before subtracting from 90°)

KEY ENTRY	DISPLAY
\boxed{f} CLEAR $\boxed{\text{PRGM}}$	00
$\boxed{9}$ $\boxed{\rightarrow H}$	01- 15 6
1	02- 1
8	03- 8
0	04- 0
$\boxed{+}$	05- 51
$\boxed{\text{STO}}$ 0	06- 23 0
$\boxed{\text{RCL}}$ 1	07- 24 1
$\boxed{\text{STO}}$ 5	08- 23 5
0	09- 0
$\boxed{\text{STO}}$ 3	10- 23 3
$\boxed{\text{STO}}$ 4	11- 23 4
$\boxed{\text{R/S}}$	12- 74
$\boxed{9}$ $\boxed{\rightarrow H}$	13- 15 6
1	14- 1
8	15- 8
0	16- 0
$\boxed{+}$	17- 51
\boxed{f} $\boxed{\rightarrow \text{HMS}}$	18- 14 6
$\boxed{9}$ $\boxed{\rightarrow H}$	19- 15 6
$\boxed{\text{RCL}}$ 0	20- 24 0
$\boxed{+}$	21- 51
$\boxed{\text{STO}}$ 0	22- 23 0
\boxed{f} $\boxed{\rightarrow \text{HMS}}$	23- 14 6
$\boxed{\text{R/S}}$	24- 74

KEY ENTRY	DISPLAY
$\boxed{\text{GTO}}$ 30	25- 13 30
$\boxed{x \rightarrow y}$	26- 21
$\boxed{9}$ $\boxed{\rightarrow H}$	27- 15 6
\boxed{f} $\boxed{\text{SIN}}$	28- 14 7
$\boxed{\times}$	29- 61
$\boxed{\text{STO}}$ $\boxed{+}$ 3	30- 23 51 3
$\boxed{\text{RCL}}$ 0	31- 24 0
$\boxed{x \rightarrow y}$	32- 21
\boxed{f} $\boxed{\rightarrow \text{R}}$	33- 14 4
$\boxed{\text{STO}}$ $\boxed{+}$ 1	34- 23 51 1
$\boxed{\text{STO}}$ $\boxed{+}$ 5	35- 23 51 5
$\boxed{x \rightarrow y}$	36- 21
$\boxed{\text{STO}}$ $\boxed{+}$ 6	37- 23 51 6
$\boxed{\text{STO}}$ $\boxed{+}$ 2	38- 23 51 2
2	39- 2
$\boxed{\div}$	40- 71
$\boxed{\text{RCL}}$ 6	41- 24 6
$\boxed{-}$	42- 41
$\boxed{\times}$	43- 61
$\boxed{\text{STO}}$ $\boxed{+}$ 4	44- 23 51 4
$\boxed{\text{RCL}}$ 1	45- 24 1
$\boxed{\text{R/S}}$	46- 74
$\boxed{\text{RCL}}$ 2	47- 24 2
$\boxed{\text{GTO}}$ 12	48- 13 12

REGISTERS			
R ₀ AZ	R ₁ Current N	R ₂ Current E	R ₃ Σ HD
R ₄ Area	R ₅ LAT	R ₆ DEP	R ₇

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize and		f REG	
	Input the starting point	N _i	STO 1	
	coordinates	E _i	STO 2	
3	Input the reference azimuth	Ref. AZ (D.MS)	GSB 01	0.0000
4a	If angle right	AR (D.MS)	R/S	
4b	If angle left	AL (D.MS)	CHS R/S	
4c	If deflection right	DR (D.MS)	GSB 19	
4d	If deflection left	DL (D.MS)	CHS GSB 19	
5a	If horizontal distance	HD	R/S	N _i
			R/S	E _i
	or,			
5b	If slope distance, input			
	zenith angle and	ZA (D.MS)	ENTER	
	slope distance	SD	GSB 26	N _i
			R/S	E _i
6	Repeat steps 4-5 for			
	successive courses			
7	Display total horizontal			
	distance traversed		RCL 3	Σ HD
8	Display area for closed			
	traverse (ignore sign)		RCL 4	Area



Begin $\frac{N\ 150.000}{E400.000}$

Keystrokes	Display	
150 [STO] 1		
400 [STO] 2		
311.3955 [GSB] 01	0.0000	
113.3455 [R/S]		
177.966 [R/S]	224.5150	N ₂
[R/S]	561.6150	E ₂
100.2455 [CHS]		
[GSB] 19		
86.0139 [ENTER]		
161.880 [GSB] 26	356.5285	N ₃
[R/S]	468.5999	E ₃
87.3559 [R/S]		
203.690 [R/S]	232.3373	N ₄
[R/S]	307.1498	E ₄
100.4559 [CHS]		
[GSB] 19		
124.0 [R/S]	149.9048	N ₁
[R/S]	399.7829	E ₁
[RCL] 3	667.1471	Σ HD
[RCL] 4	-26,558.8326	Area

You may wish to key in and run the *Compass Rule Adjustment* program at this point since data accumulated and stored by this program will already be in the registers ready for use in the example problem for the *Compass Rule Adjustment*.

Inverse from Coordinates

This program uses coordinates to calculate distances and bearings between points of a traverse. The area and the sum of the distances inversed are also computed.

$$HD = \sqrt{(N_i - N_{i-1})^2 + (E_i - E_{i-1})^2}$$

$$AZ = \tan^{-1} \frac{E_i - E_{i-1}}{N_i - N_{i-1}}$$

$$\text{Area} = \frac{1}{2} [(N_2 + N_1)(E_2 - E_1) + (N_3 + N_2)(E_3 - E_2) + \dots (N_n + N_1)(E_1 - E_n)]$$

where: N, E = Northing, easting of a point

Subscript i refers to current point

Subscript n refers to next to last point

Numeric subscript refers to point number

HD = Horizontal distance

AZ = Azimuth of a course

- Calculation of area by inverting a closed traverse may be inaccurate in cases where the coordinates are quite large. This may be minimized by using the smallest appropriate coordinates.

KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{f} REG	01- 14 33
\boxed{STO} 2	02- 23 2
$\boxed{x \div y}$	03- 21
\boxed{STO} 0	04- 23 0
\boxed{STO} 1	05- 23 1
$\boxed{R/S}$	06- 74
\boxed{RCL} 2	07- 24 2
$\boxed{-}$	08- 41
\boxed{STO} $\boxed{+}$ 2	09- 23 51 2
\boxed{STO} 5	10- 23 5
$\boxed{x \div y}$	11- 21
\boxed{RCL} 1	12- 24 1
$\boxed{-}$	13- 41
\boxed{STO} $\boxed{+}$ 1	14- 23 51 1
\boxed{g} $\boxed{\rightarrow P}$	15- 15 4
\boxed{STO} $\boxed{+}$ 3	16- 23 51 3
$\boxed{R/S}$	17- 74
$\boxed{x \div y}$	18- 21
\boxed{g} $\boxed{x > 0}$	19- 15 51
\boxed{GTO} 25	20- 13 25
3	21- 3
6	22- 6
0	23- 0
$\boxed{+}$	24- 51

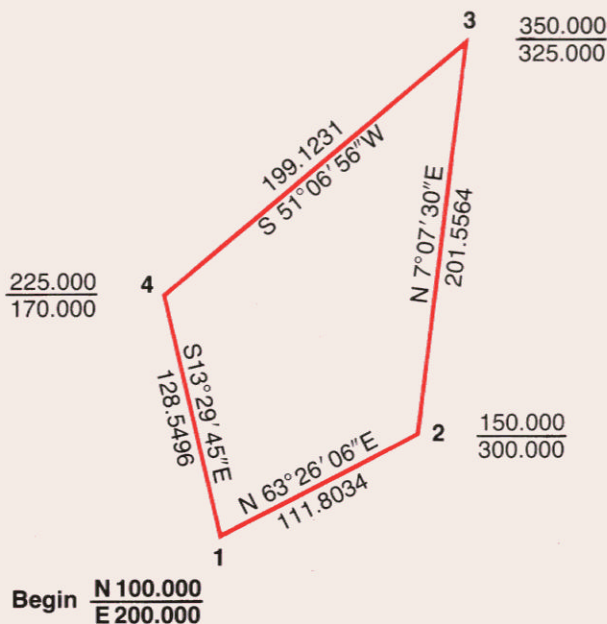
KEY ENTRY	DISPLAY
\boxed{ENTER}	25- 31
\boxed{ENTER}	26- 31
9	27- 9
0	28- 0
$\boxed{\div}$	29- 71
1	30- 1
$\boxed{+}$	31- 51
\boxed{g} \boxed{INT}	32- 15 32
$\boxed{x \div y}$	33- 21
\boxed{f} \boxed{SIN}	34- 14 7
\boxed{g} $\boxed{SIN^{-1}}$	35- 15 7
\boxed{g} $\boxed{x < 0}$	36- 15 41
\boxed{CHS}	37- 32
\boxed{f} $\boxed{\rightarrow HMS}$	38- 14 6
\boxed{RCL} 0	39- 24 0
\boxed{RCL} 1	40- 24 1
\boxed{STO} 0	41- 23 0
$\boxed{+}$	42- 51
\boxed{RCL} 5	43- 24 5
$\boxed{\times}$	44- 61
2	45- 2
$\boxed{\div}$	46- 71
\boxed{STO} $\boxed{+}$ 4	47- 23 51 4
$\boxed{R \div}$	48- 22
\boxed{GTO} 06	49- 13 06

REGISTERS

R_0 Prev. N	R_1 Current N	R_2 Current E	$R_3 \Sigma$ HD
R_4 Area	$R_5 \Delta E$	R_6	R_7

18 Traversing

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input starting coordinates	N_1	ENTER +	
		E_1	GSB 01	N_1
3	Input next coordinates and display distance	N_1	ENTER +	
		E_1	R/S	HD
4	Compute bearing and quadrant code		R/S	BRG (D.MS)
			R +	QD
5	Repeat steps 3-4 for successive courses			
6	Display total distance			
	inversed		RCL 3	Σ HD
7	Display area of closed figure			
	(ignore the sign)		RCL 4	Area



Keystrokes

Display

100 **ENTER** 200**GSB** 01150 **ENTER** 300**R/S****R/S****R**350 **ENTER** 325**R/S****R/S****R**

111.8034

63.2606

1.0000

201.5564

7.0730

1.0000

HD

BRG (D.MS)

QD

HD

BRG (D.MS)

QD

Keystrokes

225 **ENTER** 170**R/S****R/S****R↓**100 **ENTER** 200**R/S****R/S****R↓****RCL** 3**RCL** 4

Display

199.1231**51.0656****3.0000****128.5496****13.2945****2.0000****641.0325****-20,937.5000**

HD

BRG (D.MS)

QD

HD

BRG (D.MS)

QD

HD

Area

Compass Rule Adjustment*

This program adjusts a traverse by the compass rule. It is intended to be used immediately following the bearing or field traverse programs. In this case, if the calculator has not been turned off or the registers cleared or altered, the necessary data will already be stored in registers 1 thru 3.

If this program is not used immediately after the bearing or field angle traverse or if the storage registers have been altered or the calculator turned off since the traverse was run, enter the following data into the specified storage registers.

Register	Parameters to be Stored
1	Calculated ending northing
2	Calculated ending easting
3	Total distance traversed
4	Correct closing northing
5	Correct closing easting

The *Inverse From Coordinates* program may be used to obtain adjusted bearings, distances and area.

Formulas Used:

$$C_L = \frac{(\Delta N) (HD)}{\Sigma HD} \qquad C_D = \frac{(\Delta E) (HD)}{\Sigma HD}$$

where: C_L = Correction to latitude of a course
 C_D = Correction to departure of a course
 ΔN = Closing latitude
 ΔE = Closing departure
 HD = Length of course to be corrected
 ΣHD = Total length of traverse

* Also known as the Bowditch adjustment

KEY ENTRY	DISPLAY
\square CLEAR \square PRGM	00
\square RCL 5	01- 24 5
\square STO 6	02- 23 6
\square RCL 2	03- 24 2
\square -	04- 41
\square RCL 3	05- 24 3
\square \div	06- 71
\square STO 7	07- 23 7
\square RCL 4	08- 24 4
\square RCL 1	09- 24 1
\square -	10- 41
\square RCL 3	11- 24 3
\square \div	12- 71
\square STO 0	13- 23 0
\square RCL 4	14- 24 4
\square STO 3	15- 23 3
\square R/S	16- 74
\square STO 2	17- 23 2
\square $x \div y$	18- 21
\square STO 1	19- 23 1
\square RCL 4	20- 24 4
\square -	21- 41

KEY ENTRY	DISPLAY
\square STO \square + 3	22- 23 51 3
\square $x \div y$	23- 21
\square RCL 5	24- 24 5
\square -	25- 41
\square STO \square + 6	26- 23 51 6
\square \square \rightarrow P	27- 15 4
\square STO 5	28- 23 5
\square RCL 7	29- 24 7
\square \times	30- 61
\square STO \square + 6	31- 23 51 6
\square RCL 5	32- 24 5
\square RCL 0	33- 24 0
\square \times	34- 61
\square STO \square + 3	35- 23 51 3
\square RCL 1	36- 24 1
\square STO 4	37- 23 4
\square RCL 2	38- 24 2
\square STO 5	39- 23 5
\square RCL 3	40- 24 3
\square R/S	41- 74
\square RCL 6	42- 24 6
\square GTO 16	43- 13 16

REGISTERS

R_0 $\Delta N/\Sigma$ HD	R_1 Closing N	R_2 Closing E	R_3 Σ HD, N_{ADJ}
R_4 Beg. N	R_5 Beg. E	R_6 E_{ADJ}	R_7 $\Delta E/\Sigma$ HD

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Store closure data:			
	a) Calculated ending northing		STO 1	
	b) Calculated ending easting		STO 2	
	c) Total distance traversed		STO 3	
	Note: These three steps may			
	be skipped if Traverse			
	program has just been run &			
	calculator has not been			
	turned off.			
	d) Correct closing northing		STO 4	
	e) Correct closing easting		STO 5	
3	Initialize		GSB 01	
4	Input coordinates of un-			
	adjusted points & obtain adj.			
	coordinates.	N_i	ENTER	
		E_i	R/S	Adj. N_i
			R/S	Adj. E_i
	Note: Coordinates must be			
	reentered in same sequence			
	as originally traversed, start-			
	ing at the second point.			
5	For next point return to step 4.			
	For new case go to step 2.			

Example:

Adjust the coordinates of points calculated in the *Field Angle Traverse*.

Values given below are stored automatically by running the above traverse:

Register	Value	
1	149.9048	Calculated ending northing
2	399.7829	Calculated ending easting
3	667.1471	Total distance traversed

The following values must be stored manually.

4	150	Correct closing northing
5	400	Correct closing easting

Using these values, the compass rule method of adjustment yields the following results:

Point No.	Unadjusted Coordinates	Adjusted Coordinates
2	$N = 224.5150$ $E = 561.6150$	$N = 224.5404$ $E = 561.6729$
3	$N = 356.5285$ $E = 468.5999$	$N = 356.5769$ $E = 468.7104$
4	$N = 232.3373$ $E = 307.1498$	$N = 232.4148$ $E = 307.3265$
Ending & Beginning	$N = 149.9048$ $E = 399.7829$	$N = 150.0000$ $E = 400.0000$

Keystrokes

Display

If traverse program has not been run:

149.9048 **[STO]** 1

399.7829 **[STO]** 2

667.1471 **[STO]** 3

(Skip above steps if traverse has just been run and data is in registers.)

150 **[STO]** 4

400 **[STO]** 5

[GSB] 01

224.515 **[ENTER+]**

561.615 **[R/S]**

224.5404

Adj. N₂

[R/S]

561.6729

Adj. E₂

356.5285 **[ENTER+]**

468.5999 **[R/S]**

356.5769

Adj. N₃

[R/S]

468.7104

Adj. E₃

232.3373 **[ENTER+]**

307.1498 **[R/S]**

232.4148

Adj. N₄

[R/S]

307.3265

Adj. E₄

149.9048 **[ENTER+]**

399.7829 **[R/S]**

150.0000

Ending & Beginning

[R/S]

400.0000

Sideshots

This program may be used alone or in conjunction with one of the traverse programs. Used as stand-alone program, the reference bearing from a backsight is entered along with the coordinates of the occupied point. If used with a traverse program, these steps are omitted and data stored by the traverse program is used. In either case, the stored data is not destroyed, and the traverse operation may be carried out from the point occupied.

Slope angles are assumed to be entered as zenith angles. If your instrument measures vertical angles convert to zenith angles by subtracting the vertical angle from 90° .

Formulas Used:

$$HD = SD \sin (ZA)$$

$$N = N_p + \Delta N$$

$$E = E_p + \Delta E$$

where: N, E = Northing, easting of sideshot

N_p, E_p = Northing, easting of occupied point

HD = Horizontal distance

SD = Slope distance

ZA = Zenith angle

AZ = Azimuth to sideshot

$$\Delta N = HD \cos AZ$$

$$\Delta E = HD \sin AZ$$

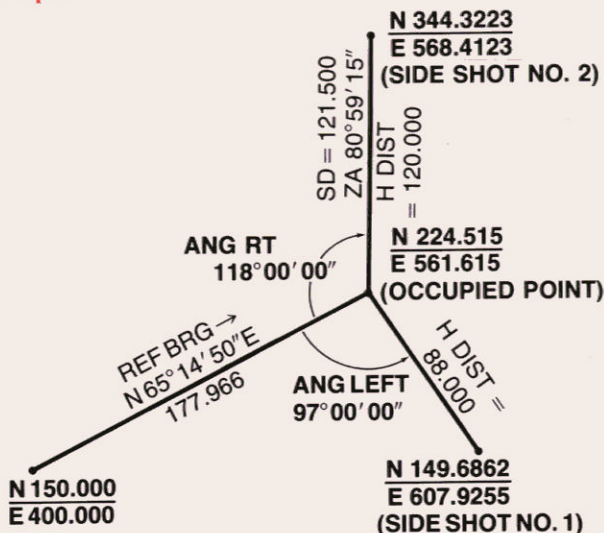
KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{STO} 2	01- 23 2
$\boxed{R+}$	02- 22
\boxed{STO} 1	03- 23 1
$\boxed{R/S}$	04- 74
$\boxed{x\div y}$	05- 21
\boxed{g} $\boxed{\leftrightarrow H}$	06- 15 6
$\boxed{x\div y}$	07- 21
2	08- 2
$\boxed{\div}$	09- 71
$\boxed{ENTER+}$	10- 31
\boxed{g} \boxed{INT}	11- 15 32
\boxed{f} $\boxed{x\neq y}$	12- 14 61
\boxed{GTO} 19	13- 13 19
$\boxed{R+}$	14- 22
$\boxed{R+}$	15- 22
\boxed{CHS}	16- 32
$\boxed{R+}$	17- 22
$\boxed{R+}$	18- 22
$\boxed{R+}$	19- 22
\boxed{g} \boxed{INT}	20- 15 32
1	21- 1
8	22- 8
0	23- 0
\boxed{STO} 7	24- 23 7

KEY ENTRY	DISPLAY
$\boxed{\times}$	25- 61
$\boxed{+}$	26- 51
\boxed{STO} 0	27- 23 0
$\boxed{R/S}$	28- 74
\boxed{g} $\boxed{\leftrightarrow H}$	29- 15 6
\boxed{RCL} 7	30- 24 7
$\boxed{+}$	31- 51
\boxed{GTO} 34	32- 13 34
\boxed{g} $\boxed{\leftrightarrow H}$	33- 15 6
\boxed{RCL} 0	34- 24 0
$\boxed{+}$	35- 51
$\boxed{R/S}$	36- 74
\boxed{GTO} 42	37- 13 42
$\boxed{x\div y}$	38- 21
\boxed{g} $\boxed{\leftrightarrow H}$	39- 15 6
\boxed{f} \boxed{SIN}	40- 14 7
$\boxed{\times}$	41- 61
\boxed{f} $\boxed{\leftrightarrow R}$	42- 14 4
\boxed{RCL} 1	43- 24 1
$\boxed{+}$	44- 51
$\boxed{R/S}$	45- 74
$\boxed{x\div y}$	46- 21
\boxed{RCL} 2	47- 24 2
$\boxed{+}$	48- 51
\boxed{GTO} 28	49- 13 28

REGISTERS

R_0 Ref. AZ	R_1 Current N	R_2 Current E	R_3 Σ HD
R_4 Area	R_5 LAT	R_6 DEP	R_7 180

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input coordinates of occupied point.	N_p	ENTER +	
		E_p	GSB 01	N_p
3	Input reference bearing and quadrant of occupied point	BRG (D.MS)	ENTER +	
		QD	R/S	AZ (D.d)*
	Note: Steps 2 & 3 may be skipped if using in conjunction with traverse program. If so press:		GTO 28	
4	Input angle right	AR (D.MS)	R/S	
4a	or, angle left	AL (D.MS)	CHS R/S	
4b	or, deflection right	DR (D.MS)	GSB 33	
4c	or, deflection left	DL (D.MS)	CHS GSB 33	
5	Input horizontal distance	HD		
5a	or, if slope shot, zenith angle & slope distance	ZA (D.MS)	ENTER +	
		SD	GTO 38	
6	Calculate sideshot coordinates		R/S	N
	* AZ is displayed as decimal degree (D.d).		R/S	E

Example:**Keystrokes****Display**

If running traverse program; key in sideshot program, then:

[GTO] 28

If not running traverse program:

224.515 [ENTER+]

561.615 [GSB] 01

65.145 [ENTER+]

[R/S]

65.2472

AZ(D.d)

In either case:

97 [CHS] [R/S] 88

[R/S]

149.6862

N } PT. 1

[R/S]

607.9255

118 [R/S] 80.5915

[ENTER+] 121.5

[GTO] 38 [R/S]

344.3223

N } PT 2

[R/S]

568.4123

Intersections

Bearing-Bearing Intersection

This program calculates coordinates of the point of intersection of two lines for which the bearing of each line is known and the coordinates of a point on each line are known.

Formulas Used:

$$N = N_1 + \text{Dist} (\cos AZ_1)$$

$$E = E_1 + \text{Dist} (\sin AZ_1)$$

$$\text{Dist} = \frac{\text{Dist}_{12} \sin (AZ_2 - AZ_{12})}{\sin (AZ_2 - AZ_1)}$$

where: AZ_1 = Azimuth of line 1

AZ_2 = Azimuth of line 2

AZ_{12} = Azimuth of line from point 1 to point 2

N_1, E_1 = Northing, easting of point 1

N_2, E_2 = Northing, easting of point 2

N, E = Northing, easting of intersection point

Dist = Distance from point 1 to intersection

Dist_{12} = Distance from point 1 to point 2

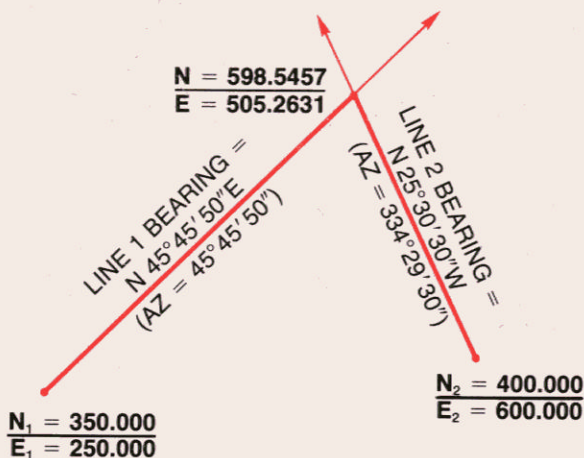
KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{STO} 2	01- 23 2
$\boxed{R\div}$	02- 22
\boxed{STO} 1	03- 23 1
$\boxed{R/S}$	04- 74
\boxed{STO} 4	05- 23 4
$\boxed{R\div}$	06- 22
\boxed{STO} 3	07- 23 3
$\boxed{R/S}$	08- 74
\boxed{g} $\boxed{+H}$	09- 15 6
\boxed{STO} 6	10- 23 6
$\boxed{R\div}$	11- 22
\boxed{g} $\boxed{+H}$	12- 15 6
\boxed{STO} 5	13- 23 5
\boxed{RCL} 4	14- 24 4
\boxed{RCL} 2	15- 24 2
$\boxed{-}$	16- 41
\boxed{RCL} 3	17- 24 3
\boxed{RCL} 1	18- 24 1
$\boxed{-}$	19- 41
\boxed{g} $\boxed{\rightarrow P}$	20- 15 4

KEY ENTRY	DISPLAY
$\boxed{x\div y}$	21- 21
\boxed{RCL} 6	22- 24 6
$\boxed{x\div y}$	23- 21
$\boxed{-}$	24- 41
\boxed{f} \boxed{SIN}	25- 14 7
$\boxed{\times}$	26- 61
\boxed{RCL} 6	27- 24 6
\boxed{RCL} 5	28- 24 5
$\boxed{-}$	29- 41
\boxed{f} \boxed{SIN}	30- 14 7
$\boxed{\div}$	31- 71
\boxed{RCL} 5	32- 24 5
$\boxed{x\div y}$	33- 21
\boxed{f} $\boxed{\rightarrow R}$	34- 14 4
\boxed{RCL} 1	35- 24 1
$\boxed{+}$	36- 51
$\boxed{R/S}$	37- 74
$\boxed{x\div y}$	38- 21
\boxed{RCL} 2	39- 24 2
$\boxed{+}$	40- 51
\boxed{GTO} 00	41- 13 00

REGISTERS			
R_0	$R_1 N_1$	$R_2 E_1$	$R_3 N_2$
$R_4 E_2$	$R_5 AZ_1$	$R_6 AZ_2$	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input coordinate of point 1	N_1	ENTER*	
		E_1	GSB 01	
3	Input coordinates of point 2	N_2	ENTER*	
		E_2	R/S	
4	Convert bearing 1 to azimuth* & input	AZ_1 (D.MS)	ENTER*	
5	Convert bearing 2 to azimuth* & input	AZ_2 (D.MS)		
6	Calculate coordinates of intersection		R/S	N
			R/S	E
7	For a new case go to step 2.			
	* See <i>Azimuth-Bearing Conversions</i> program			

Example:



Keystrokes

Display

350 **ENTER** 250**GSB** 01400 **ENTER** 600 **R/S**45.455 **ENTER**334.293 **R/S****R/S**

598.5457

505.2631

N

E

Bearing-Distance Intersection

This program calculates the coordinates of the point of intersection of two lines—one of known bearing through known coordinates and the other of known length from a point of known coordinates. Both solutions are computed.

The far solution is obtained by entering the bearing *from* point 1 and the near solution by entering the bearing *into* point 1.

Formulas Used:

$$AZ_{12} = \tan^{-1} \frac{E_2 - E_1}{N_2 - N_1}$$

$$h = \text{Dist}_{12} \sin \phi$$

$$b = \sqrt{\text{Dist}_2^2 - h^2}$$

$$N = N_1 + [(\text{Dist}_{12} \cos \phi) + b] \cos (AZ_1)$$

$$E = E_1 + [(\text{Dist}_{12} \cos \phi) + b] \sin (AZ_1)$$

where: AZ_{12} = Azimuth of line from point 1 to point 2

AZ_1 = Azimuth of line 1

ϕ = Angle between line 1 and line from point 1 to point 2

h = Perpendicular distance from point 2 to line 1

b = Distance from point of intersection to the point where the perpendicular (h) intersects line 1

Dist_2 = Length of line 2 (the known distance)

N_1, E_1 = Northing, easting of point 1

N_2, E_2 = Northing, easting of point 2

Dist_{12} = Distance from point 1 to point 2

$$\text{Reverse AZ} = \begin{cases} AZ_1 + 180^\circ & (AZ_1 < 180^\circ) \\ AZ_1 - 180^\circ & (AZ_1 > 180^\circ) \end{cases}$$

KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{STO} 2	01- 23 2
$\boxed{R\leftrightarrow}$	02- 22
\boxed{STO} 1	03- 23 1
$\boxed{R/S}$	04- 74
\boxed{STO} 4	05- 23 4
$\boxed{R\leftrightarrow}$	06- 22
\boxed{STO} 3	07- 23 3
$\boxed{R/S}$	08- 74
\boxed{STO} 6	09- 23 6
$\boxed{R\leftrightarrow}$	10- 22
\boxed{g} $\rightarrow H$	11- 15 6
\boxed{STO} 5	12- 23 5
\boxed{RCL} 4	13- 24 4
\boxed{RCL} 2	14- 24 2
$\boxed{-}$	15- 41
\boxed{RCL} 3	16- 24 3
\boxed{RCL} 1	17- 24 1
$\boxed{-}$	18- 41
\boxed{g} $\rightarrow P$	19- 15 4
\boxed{STO} 7	20- 23 7
$\boxed{R\leftrightarrow}$	21- 22
\boxed{g} $\langle X < 0$	22- 15 41
\boxed{GSB} 46	23- 12 46
\boxed{RCL} 5	24- 24 5

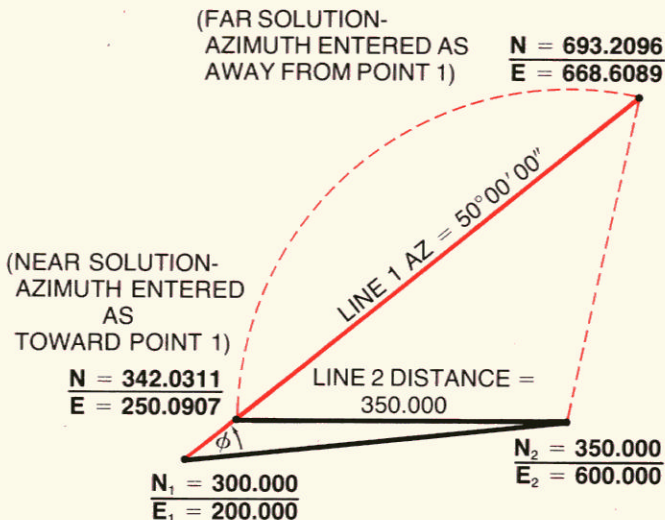
KEY ENTRY	DISPLAY
$\boxed{-}$	25- 41
\boxed{RCL} 7	26- 24 7
\boxed{f} $\rightarrow R$	27- 14 4
$\boxed{x^2y}$	28- 21
\boxed{g} $\langle x^2$	29- 15 0
\boxed{RCL} 6	30- 24 6
\boxed{g} $\langle x^2$	31- 15 0
$\boxed{x^2y}$	32- 21
$\boxed{-}$	33- 41
\boxed{f} \sqrt{x}	34- 14 0
$\boxed{+}$	35- 51
\boxed{RCL} 5	36- 24 5
$\boxed{x^2y}$	37- 21
\boxed{f} $\rightarrow R$	38- 14 4
\boxed{RCL} 1	39- 24 1
$\boxed{+}$	40- 51
$\boxed{R/S}$	41- 74
$\boxed{x^2y}$	42- 21
\boxed{RCL} 2	43- 24 2
$\boxed{+}$	44- 51
\boxed{GTO} 00	45- 13 00
\boxed{RCL} 0	46- 24 0
$\boxed{+}$	47- 51
\boxed{g} \boxed{RTN}	48- 15 12

REGISTERS

R_0 360	R_1 N_1	R_2 E_1	R_3 N_2
R_4 E_2	R_5 AZ_1	R_6 Dist 2	R_7 Dist 1 \rightarrow 2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize	360	STO 0	
3	Input coordinates of point 1	N_1	ENTER +	
		E_1	GSB 01	N_1
4	Input coordinates of point 2	N_2	ENTER +	
		E_2	R/S	N_2
5	* For solution #1:			
	Input azimuth from point 1			
	to intersection	AZ_1 (D.MS)	ENTER +	
5a	or, * For solution #2:			
	Input reverse			
	azimuth	AZ (D.MS)	ENTER +	
	If $AZ_1 < 180^\circ$	180	+	
	If $AZ_1 > 180^\circ$	180	-	
6	Input distance from point 2			
	to intersection and calculate			
	intersection	Dist.	GSB 09	N
			R/S	E
7	For second solution go			
	to step 5a.			
8	For a new case start at step 3			
	* There can be 2 solutions:			
	To obtain solution #1 (far)			
	Enter azimuth as away from			
	point 1. To obtain solution			
	#2 (near) enter azimuth as			
	into point 1 ($AZ_1 \pm 180^\circ$)			

Example:



Keystrokes

Display

360 **[STO]** 0300 **[ENTER+]** 200**[GSB]** 01350 **[ENTER+]** 600 **[R/S]**50 **[ENTER+]** 350**[GSB]** 09**[R/S]**50 **[ENTER+]** 180**[+]** 350 **[GSB]** 09**[R/S]**

693.2096

668.6089

N	}	Solution #1 (far)
E		

342.0311

250.0907

N	}	Solution #2 (near)
E		

Distance-Distance Intersection

Given two lines, each of known length and originating from two known points, this program computes the intersection coordinates. There are two possible solutions; this program calculates the one found by proceeding in a clockwise direction from the first known point to the second known point. The other solution is found by reversing the entry of the known point coordinates.

Formulas Used:

$$\phi = \cos^{-1} \frac{\text{Dist}_{12}^2 + \text{Dist}_1^2 - \text{Dist}_2^2}{2 (\text{Dist}_1) (\text{Dist}_{12})}$$

$$\text{AZ} = \tan^{-1} \frac{E_2 - E_1}{N_2 - N_1}$$

$$N = N_1 + \text{Dist}_1 \cos (\text{AZ} - \phi)$$

$$E = E_1 + \text{Dist}_1 \sin (\text{AZ} - \phi)$$

where: ϕ = Angle between line 1 and line 1 \rightarrow 2

Dist_{12} = Distance from point 1 to point 2

Dist_1 = Known distance along line 1

Dist_2 = Known distance along line 2

N_1, E_1 = Northing, easting of point 1

N, E = Northing, easting of intersection point

AZ = Azimuth of line from point 1 to point 2

KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{STO} 2	01- 23 2
$\boxed{R\div}$	02- 22
\boxed{STO} 1	03- 23 1
$\boxed{R/S}$	04- 74
\boxed{STO} 4	05- 23 4
$\boxed{R\div}$	06- 22
\boxed{STO} 3	07- 23 3
$\boxed{R/S}$	08- 74
\boxed{STO} 6	09- 23 6
$\boxed{R\div}$	10- 22
\boxed{STO} 5	11- 23 5
\boxed{RCL} 4	12- 24 4
\boxed{RCL} 2	13- 24 2
$\boxed{-}$	14- 41
\boxed{RCL} 3	15- 24 3
\boxed{RCL} 1	16- 24 1
$\boxed{-}$	17- 41
\boxed{g} $\rightarrow P$	18- 15 4
\boxed{STO} 7	19- 23 7
\boxed{g} $\boxed{x^2}$	20- 15 0
\boxed{RCL} 5	21- 24 5

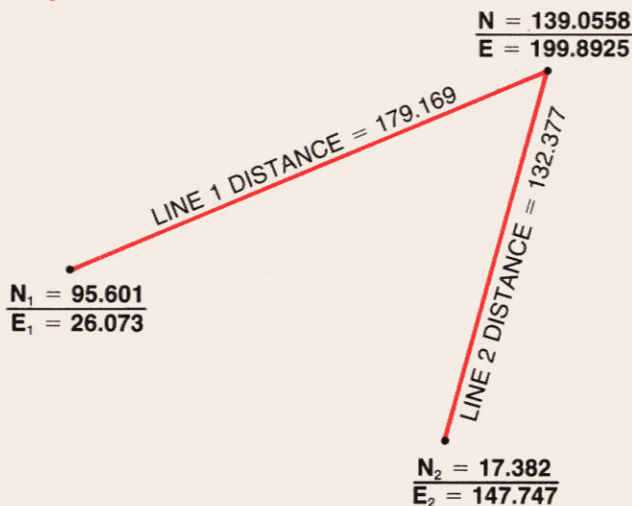
KEY ENTRY	DISPLAY
\boxed{g} $\boxed{x^2}$	22- 15 0
$\boxed{+}$	23- 51
\boxed{RCL} 6	24- 24 6
\boxed{g} $\boxed{x^2}$	25- 15 0
$\boxed{-}$	26- 41
2	27- 2
$\boxed{\div}$	28- 71
\boxed{RCL} 7	29- 24 7
\boxed{RCL} 5	30- 24 5
$\boxed{\times}$	31- 61
$\boxed{\div}$	32- 71
\boxed{g} $\boxed{\cos^{-1}}$	33- 15 8
$\boxed{-}$	34- 41
\boxed{RCL} 5	35- 24 5
\boxed{f} $\rightarrow R$	36- 14 4
\boxed{RCL} 1	37- 24 1
$\boxed{+}$	38- 51
$\boxed{R/S}$	39- 74
$\boxed{x\div y}$	40- 21
\boxed{RCL} 2	41- 24 2
$\boxed{+}$	42- 51
\boxed{GTO} 00	43- 13 00

REGISTERS

R_0	$R_1 N_1$	$R_2 E_1$	$R_3 N_2$
$R_4 E_2$	R_5 Dist 1	R_6 Dist 2	R_7 Dist 1 \rightarrow 2

40 Intersections

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input coordinates of point 1	N_1	ENTER +	
		E_1	GSB 01	N_1
3	Input coordinates of point 2	N_2	ENTER +	
		E_2	R/S	N_2
4	Input distance 1 & distance 2			
	& calculate coordinates of			
	the point of intersection	Dist 1	ENTER +	
		Dist 2	R/S	N
			R/S	E
5	*For alternate solution go to			
	step 2 and input point 2, then			
	step 3 and input point 1, then			
	step 4.			
6	For a new case start at step 2			
	* Calculated solution is			
	always clockwise from			
	point 1 to point 2. For			
	alternate solution, reverse			
	the order of input, starting			
	at point 2.			

Example:**Note:**

Computed solution is always clockwise from point 1 to 2. For alternate solution, start at point 2.

Keystrokes**Display**95.601 **ENTER**26.073 **GSB** 0117.382 **ENTER**147.747 **R/S**179.169 **ENTER**132.377 **R/S****139.0558**

$$\left. \begin{array}{l} N \\ E \end{array} \right\} \text{Solution \#1}$$
199.8925**R/S**17.382 **ENTER**147.747 **GSB** 0195.601 **ENTER**26.073 **R/S**132.377 **ENTER**179.169 **R/S****-80.5716**

$$\left. \begin{array}{l} N \\ E \end{array} \right\} \text{Solution \#2}$$
58.7034**R/S**

Offset from a Point to a Line

Given a point with known coordinates (the base point) on a line of known azimuth and another point of known coordinates offset from the line (the offset point), this program calculates offset distance from the point to the line, the distance from the base point to the point of intersection; the coordinates of the point of intersection and the azimuth from the base point to the offset point and from the offset point to the point of intersection.

Formulas Used:

$$\text{Dist}_{BO} = \sqrt{(N_B - N_O)^2 + (E_B - E_O)^2}$$

$$\alpha = \text{AZ}_{BI} - \text{AZ}_{BO}$$

$$\text{Dist}_{BI} = \text{Dist}_{BO} \cos \alpha$$

$$\text{Dist}_{OI} = \text{Dist}_{BO} \sin \alpha$$

$$N_I = N_B + \text{Dist}_{BI} \cos \text{AZ}_{BI}$$

$$E_I = E_B + \text{Dist}_{BI} \sin \text{AZ}_{BI}$$

where: N_B, E_B = Coordinates of basepoint

N_O, E_O = Coordinates of offset point

N_I, E_I = Coordinates of point of intersection

Dist_{BO} = Distance from base to offset point

Dist_{BI} = Distance from base to point of intersection

Dist_{OI} = Distance from offset to point of intersection

AZ_{BI} = Azimuth of base line from P_B

α = Angle between base line and line from base to offset

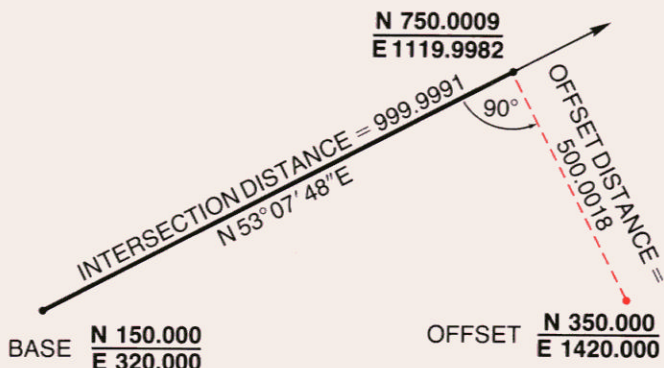
KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{STO} 1	01- 23 1
$\boxed{x \div y}$	02- 21
\boxed{STO} 0	03- 23 0
\boxed{g} \boxed{RTN}	04- 15 12
\boxed{RCL} 1	05- 24 1
$\boxed{-}$	06- 41
$\boxed{x \div y}$	07- 21
\boxed{RCL} 0	08- 24 0
$\boxed{-}$	09- 41
\boxed{g} $\boxed{+P}$	10- 15 4
\boxed{STO} 3	11- 23 3
$\boxed{R+}$	12- 22
\boxed{g} $\boxed{x < 0}$	13- 15 41
\boxed{GSB} 43	14- 12 43
\boxed{f} $\boxed{+HMS}$	15- 14 6
$\boxed{R/S}$	16- 74
\boxed{g} $\boxed{+H}$	17- 15 6
\boxed{STO} 2	18- 23 2
$\boxed{x \div y}$	19- 21
\boxed{g} $\boxed{+H}$	20- 15 6
$\boxed{-}$	21- 41
\boxed{g} \boxed{ABS}	22- 15 34
\boxed{STO} 4	23- 23 4

KEY ENTRY	DISPLAY
\boxed{RCL} 3	24- 24 3
\boxed{f} $\boxed{+R}$	25- 14 4
\boxed{STO} 6	26- 23 6
$\boxed{x \div y}$	27- 21
\boxed{STO} 7	28- 23 7
\boxed{RCL} 2	29- 24 2
\boxed{RCL} 6	30- 24 6
\boxed{f} $\boxed{+R}$	31- 14 4
\boxed{STO} $\boxed{+}$ 0	32- 23 51 0
$\boxed{R+}$	33- 22
\boxed{STO} $\boxed{+}$ 1	34- 23 51 1
\boxed{RCL} 0	35- 24 0
$\boxed{R/S}$	36- 74
\boxed{RCL} 1	37- 24 1
$\boxed{R/S}$	38- 74
\boxed{GSB} 01	39- 12 01
$\boxed{R+}$	40- 22
$\boxed{R+}$	41- 22
\boxed{GTO} 05	42- 13 05
3	43- 3
6	44- 6
0	45- 0
$\boxed{+}$	46- 51
\boxed{g} \boxed{RTN}	47- 15 12

REGISTERS			
R_0 N's	R_1 E's	R_2 AZ _{BI}	R_3 D _{BO}
R_4 α	R_5	R_6 D _{BI}	R_7 D _{OI}

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input coordinates of the base point (P_B)	N_B	ENTER +	
		E_B	GSB 01	N_B
3	Input coordinates of the offset point (P_O) and read the azimuth from P_B to P_O	N_O	ENTER +	
		E_O	R/S	AZ_{BO} (D.MS)
4	Convert the bearing of the base line (P_B to intersection) to azimuth* and input	AZ_{BI} (D.MS)		
5	Calculate coordinates of point of intersection		R/S	N_I
			R/S	E_I
6	Reinput coordinates of offset point (P_O) and calculate azimuth from P_O to intersection	N_O	ENTER +	
		E_O	R/S	AZ_{OI} (D.MS)
7	Read distance from base point to intersection		RCL 6 9 ABS	D_{BI}
8	Read distance from offset point to intersection		RCL 7 9 ABS	D_{OI}
9	For new case go to step 2.			
	* See Azimuth-Bearing Conversions program.			

Example:



Keystrokes

Display

150 **ENTER**320 **GSB** 01350 **ENTER** 1420**R/S**53.0748 **R/S****R/S**350 **ENTER** 1420**R/S****RCL** 6**RCL** 7

79.4143

750.0009

1,119.9982

323.0748

999.9991

500.0018

AZ_{BO} (D.MS)N_IE_IAZ_{OI} (D.MS)D_{BI}D_{OI}

Curves

Curve Solutions

Given the central angle and radius, or central angle and tangent distance this program calculates the chord length, arc length, and either the tangent distance or radius. It also calculates the sector and segment areas.

Formulas Used:

$$C = 2R \sin (\Delta/2)$$

$$L = \Delta R \text{ (}\Delta \text{ in radians)}$$

$$T = R \tan (\Delta/2)$$

$$\text{Sector area} = LR/2$$

$$\text{Segment area} = \text{Sector area} - \frac{1}{2} R^2 \sin (\Delta)$$

where: R = Radius

C = Chord length

L = Arc length

T = Tangent distance

Δ = Central angle

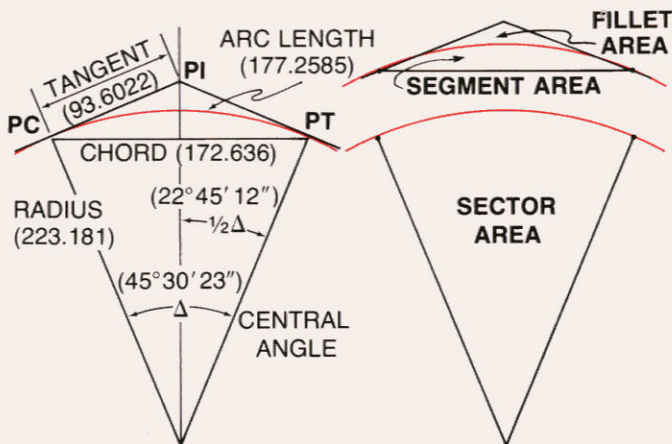
KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{g} $\boxed{+H}$	01- 15 6
\boxed{STO} 0	02- 23 0
2	03- 2
$\boxed{\div}$	04- 71
\boxed{STO} 1	05- 23 1
$\boxed{R/S}$	06- 74
\boxed{STO} 2	07- 23 2
\boxed{RCL} 1	08- 24 1
\boxed{f} \boxed{SIN}	09- 14 7
\boxed{x}	10- 61
2	11- 2
\boxed{x}	12- 61
$\boxed{R/S}$	13- 74
\boxed{RCL} 0	14- 24 0
\boxed{RCL} 2	15- 24 2
\boxed{f} $\boxed{+RAD}$	16- 14 5
\boxed{x}	17- 61
$\boxed{R/S}$	18- 74
\boxed{RCL} 2	19- 24 2
\boxed{RCL} 1	20- 24 1
\boxed{f} \boxed{TAN}	21- 14 9

KEY ENTRY	DISPLAY
\boxed{x}	22- 61
$\boxed{R/S}$	23- 74
\boxed{RCL} 1	24- 24 1
\boxed{f} \boxed{TAN}	25- 14 9
$\boxed{\div}$	26- 71
\boxed{GTO} 06	27- 13 06
\boxed{STO} 2	28- 23 2
\boxed{g} $\boxed{x^2}$	29- 15 0
\boxed{RCL} 1	30- 24 1
\boxed{f} $\boxed{+RAD}$	31- 14 5
\boxed{x}	32- 61
$\boxed{R/S}$	33- 74
\boxed{RCL} 2	34- 24 2
\boxed{g} $\boxed{x^2}$	35- 15 0
2	36- 2
$\boxed{\div}$	37- 71
\boxed{RCL} 0	38- 24 0
\boxed{f} \boxed{SIN}	39- 14 7
\boxed{x}	40- 61
$\boxed{-}$	41- 41
\boxed{GTO} 00	42- 13 00

REGISTERS

$R_0 \Delta$	$R_1 \Delta/2$	$R_2 R$	R_3
R_4	R_5	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input central angle	Δ , (D.MS)	GSB 01	
3	Input radius and calculate: chord, arc length and tangent dist.	R	R/S	Chord
			R/S	Arc length
			R/S	Tangent dist
	or,			
3a	Input tangent distance and calculate: radius, chord and arc length	T	GSB 24	Radius
			R/S	Chord
			R/S	Arc length
4	For new case go to step 3			
	To Calculate Areas:			
5	Input central angle (if not already input)	Δ , (D.MS)	GSB 01	$\Delta/2$ (D.d)
6	Input radius and calculate sector area	R	GSB 28	Sector area
7	Calculate segment area		R/S	Segment area
8	For a new case go to step 6			

**Example:**

Given central angle and tangent distance from above curve; calculate the radius, chord and arc length.

Keystrokes45.3023 **[GSB]** 0193.6022 **[GSB]** 24**[R/S]****[R/S]****Display****22.7532****223.1810****172.6360****177.2585** $\Delta/2$ (D.d)

Radius

Chord

Arc length

For same curve, calculate sector and segment areas:

223.181 **[GSB]** 28**[R/S]****19,780.3597****2,014.9969**

Sector area

Segment area

Elevations Along A Vertical Curve

This program calculates the elevation at any specified station along a vertical curve as well as the elevation at the highest or lowest point on the curve and the station at that point. Program inputs are: beginning and ending grades, length of curve, the station and elevation at the beginning of the curve and the station at which elevation is desired.

In the program, stations are entered in the form xxxx.xx for station xx + xx.xx. For example, 20 + 10.00 is entered as 2010.00.

Formulas Used:

Elevation at any station = $\frac{1}{2} AZ^2 + G_1 Z + E_0$

Distance in stations from beginning station to station of lowest elevation
= $-G_1/A$

where: E_0 = Elevation at beginning of curve

Z = Distance in 100 foot stations—measured from beginning of curve

G_1 = Grade in % at beginning of curve

G_2 = Grade in % at end of curve

$A = 100 (G_2 - G_1)/L$

L = Length of curve in feet

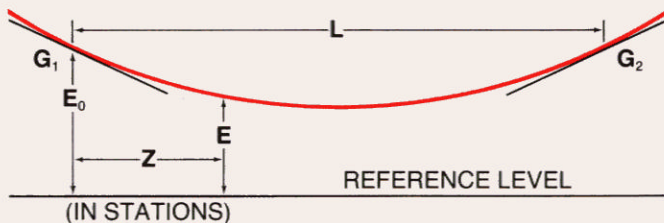
KEY ENTRY	DISPLAY
$\frac{f}{\square}$ CLEAR \square PRGM	00
$\square x^2 y$	01- 21
STO 1	02- 23 1
$\square -$	03- 41
5	04- 5
0	05- 0
$\square \times$	06- 61
STO 3	07- 23 3
CLX	08- 34
R/S	09- 74
STO 6	10- 23 6
R/S	11- 74
STO $\square \div$ 3	12- 23 71 3
$\square \star$	13- 22
STO 2	14- 23 2
R/S	15- 74
RCL 6	16- 24 6
$\square -$	17- 41
EEX	18- 33
2	19- 2
$\square \div$	20- 71
STO 4	21- 23 4
ENTER*	22- 31
9 $\square x^2$	23- 15 0

KEY ENTRY	DISPLAY
RCL 3	24- 24 3
$\square \times$	25- 61
$\square x^2 y$	26- 21
RCL 1	27- 24 1
$\square \times$	28- 61
$\square +$	29- 51
RCL 2	30- 24 2
$\square +$	31- 51
GTO 15	32- 13 15
RCL 1	33- 24 1
CHS	34- 32
2	35- 2
$\square \div$	36- 71
RCL 3	37- 24 3
$\square \div$	38- 71
GTO 21	39- 13 21
RCL 4	40- 24 4
EEX	41- 33
2	42- 2
$\square \times$	43- 61
RCL 6	44- 24 6
$\square +$	45- 51
GTO 15	46- 13 15

REGISTERS

R_0	R_1 Grade 1	R_2 Beg. Elev.	R_3 A/2
R_4 sta. #	R_5	R_6 Beg. sta.	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input beginning and ending grades	$G_1, \%$	ENTER	
		$G_2, \%$	GSB 01	
3	Input beginning station	Beg. Sta.	R/S	Beg. Sta.
4	Input beginning elevation and curve length	E_0	ENTER	
		L	R/S	E_0
5	Input station and calculate elevation	Sta.	R/S	E
6	Calculate max or min elevation		GSB 33	$E_{\text{max or min}}$
7	Display station (step 6 may be executed any time after initial data is input.)		GSB 40	Sta.
8	For a new curve go to step 2			

Example:

G_1 (beginning grade) = -1.065%

G_2 (ending grade) = 1.600%

L (length of curve) = 340 ft.

E_0 (elevation at G_1) = 614 ft.

Beginning station = 17 + 00.00

Station	Elevation (E)
18 + 00.00	613.3269
19 + 00.00	613.4376
20 + 00.00	614.3322
20 + 40.00	614.9095

Station of lowest elevation = 18 + 35.8724

Lowest elevation = 613.2765

Keystrokes

Display

1.065	CHS	ENTER	
1.6	GSB	01	0.0000
1700	R/S	614	
ENTER	340	R/S	614.0000 E_0
1800	R/S		613.3269 E
1900	R/S		613.4376 E
2000	R/S		614.3322 E
2040	R/S		614.9095 E
GSB	33		613.2765 E_{\min}
GSB	40		1,835.8724 Stat. at E_{\min}

Other

Earthwork: Volume by Average End Area

This program calculates earthwork volumes by average end area. The required information is the elevation and offset distance (distance from centerline) for each point on the cross-section and the interval between cross-sections. The program calculates accumulated volume to the present station, volume from the previous station, and area of the cross-section.

Formulas Used:

$$VOL = (AREA_i + AREA_{i-1}) \frac{INT}{2}$$

$$AREA = \frac{1}{2} [EL_1 (D_2 - D_n) + \dots + EL_n (D_1 - D_n - 1)]$$

where: VOL = Average volume between two stations

AREA = Cross-sectional area at a station

INT = Interval between stations

EL = Elevation at a point on a cross-section

D = Horizontal distance (offset) from centerline

i = Subscript referring to current point

n = Subscript referring to last point

Numeric subscript refers to point or station number

- Volumes are calculated in cubic yards, areas in square feet. If you desire to have volumes calculated in cubic feet delete 54 at steps 26 & 27 and insert 2 in its place.
- It makes no difference what point you start with on the cross-section, and the elevations and distances may be measured from any base lines as long as the same lines are used for the whole section. Also, you may work around the section clockwise (CW) or counter clockwise (CCW).

KEY ENTRY	DISPLAY
f CLEAR PRGM	00
CLX	01- 34
f REG	02- 14 33
R/S	03- 74
STO x 1	04- 23 61 1
RCL 1	05- 24 1
STO - 2	06- 23 41 2
R+	07- 22
x²y	08- 21
STO x 0	09- 23 61 0
RCL 0	10- 24 0
STO + 2	11- 23 51 2
R+	12- 22
STO 1	13- 23 1
R+	14- 22
STO 0	15- 23 0
RCL 2	16- 24 2
2	17- 2
÷	18- 71
STO 3	19- 23 3

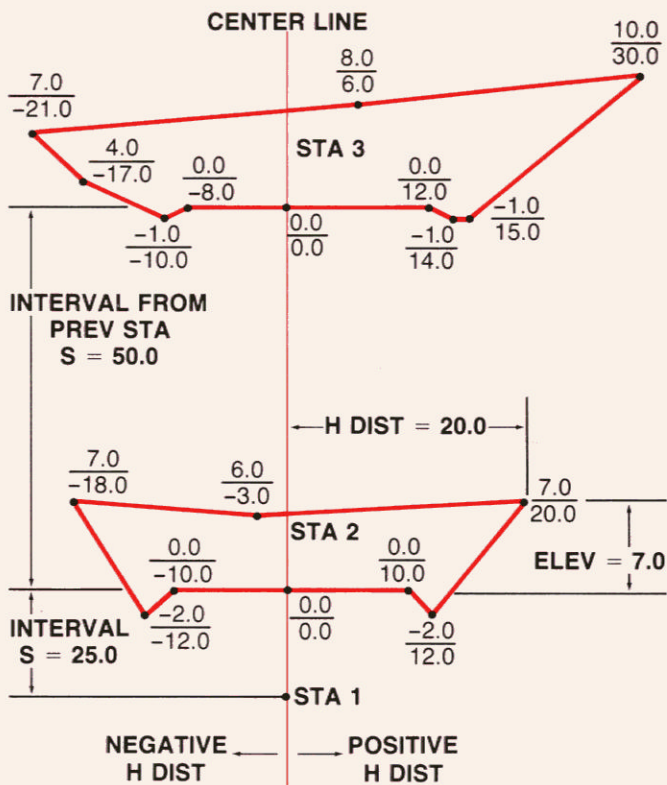
KEY ENTRY	DISPLAY
GTO 03	20- 13 03
RCL 4	21- 24 4
RCL 3	22- 24 3
9 ABS	23- 15 34
STO 4	24- 23 4
+	25- 51
5	26- 5
4	27- 4
÷	28- 71
x	29- 61
STO 5	30- 23 5
STO + 6	31- 23 51 6
RCL 6	32- 24 6
R/S	33- 74
CLX	34- 34
STO 3	35- 23 3
STO 2	36- 23 2
STO 1	37- 23 1
STO 0	38- 23 0
GTO 03	39- 13 03

REGISTERS

R ₀ EL	R ₁ D	R ₂ DMD	R ₃ Area
R ₄ Area	R ₅ Volume	R ₆ Total volume	R ₇

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		GSB 01	0.0000
3	If station has zero end area, go to step 6			
4	Input elevation and horizontal or offset dist.	EL (ft.)	ENTER	
		D (ft.)	R/S	
5	Repeat step 4, working around the section (clockwise or counterclockwise) until first EL & D have been reinput			
6	Input interval from previous station and calculate total volume	Int (ft.)	GSB 21	Tot. Vol. (cu. yds.)
	(Note: Input 0 interval for first station)			
7	Read volume of interval		RCL 5	Int. Vol. (cu. yds.)
8	Read area of cross-section		RCL 4	Area (sq. ft.)
9	Initialize for next section		R/S	
10	Go to step 3 for next section			
11	Go to step 2 for a new case			

Example:



Keystrokes

Display

(Station 1)

0 **GSB** 21

0

Tot. Vol.

(Station 2)

GSB 010 **ENTER** 0 **R/S**0 **ENTER** 10 **R/S**2 **CHS** **ENTER** 12

Keystrokes

R/S 7 ENTER+ 20
 R/S 6 ENTER+
 3 CHS R/S 7
 ENTER+ 18 CHS
 R/S 2 CHS ENTER+
 12 CHS R/S 0
 ENTER+ 10 CHS
 R/S 0 ENTER+ 0
 R/S 25 GSB 21
 RCL 5
 RCL 4

Display

100.0000
 100.0000
 216.0000

Tot. Vol. (cu. yds.)
 Int. Vol. (cu. yds.)
 Area (sq. ft.)

Initialize for next station:

R/S

(Station 3)

0 ENTER+ 0 R/S
 0 ENTER+ 12 R/S
 1 CHS ENTER+
 14 R/S 1 CHS
 ENTER+ 15 R/S 10
 ENTER+ 30 R/S
 8 ENTER+ 6 R/S
 7 ENTER+ 21 CHS
 R/S 4 ENTER+
 17 CHS R/S 1
 CHS ENTER+ 10
 CHS R/S 0 ENTER+
 8 CHS R/S 0
 ENTER+ 0 R/S
 50 GSB 21
 RCL 5
 RCL 4

597.6852
 497.6852
 321.5000

Tot. Vol. (cu. yds.)
 Int. Vol. (cu. yds.)
 Area (sq. ft.)

Coordinate Transformation

This program translates, rotates, and rescales coordinates. The traverse rotation angle is entered as a negative value for counterclockwise rotation and positive for clockwise rotation. The translation factors are calculated by entering old and new grid system coordinates for the same point; rotation is also about this point.

Formulas Used:

$$AZ_R = \phi + \tan^{-1} \frac{E_i - E_p}{N_i - N_p}$$

$$HD_S = S \sqrt{(N_i - N_p)^2 + (E_i - E_p)^2}$$

$$N = HD_S \cos (AZ_R) + N_{T1}$$

$$E = HD_S \sin (AZ_R) + E_{T1}$$

where: AZ_R = Rotated azimuth

ϕ = Rotation angle

N_i, E_i = Northing, easting of current point before transformation

N_p, E_p = Original northing, easting of pivot point

HD_S = Scaled horizontal distance

S = Scale factor

N, E = Northing, easting after transformation

N_{T1}, E_{T1} = Northing, easting of pivot point after transformation

KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{STO} 2	01- 23 2
$\boxed{x\div y}$	02- 21
\boxed{STO} 1	03- 23 1
$\boxed{R/S}$	04- 74
$\boxed{R\div}$	05- 22
$\boxed{-}$	06- 41
\boxed{STO} 3	07- 23 3
$\boxed{R\div}$	08- 22
$\boxed{x\div y}$	09- 21
$\boxed{-}$	10- 41
\boxed{STO} 4	11- 23 4
$\boxed{R/S}$	12- 74
\boxed{g} \rightarrow H	13- 15 6
\boxed{STO} 5	14- 23 5
1	15- 1
$\boxed{R/S}$	16- 74
\boxed{STO} 6	17- 23 6
$\boxed{R/S}$	18- 74
\boxed{RCL} 2	19- 24 2
$\boxed{-}$	20- 41
$\boxed{x\div y}$	21- 21
\boxed{RCL} 1	22- 24 1

KEY ENTRY	DISPLAY
$\boxed{-}$	23- 41
\boxed{g} \rightarrow P	24- 15 4
\boxed{RCL} 6	25- 24 6
$\boxed{\times}$	26- 61
$\boxed{x\div y}$	27- 21
\boxed{RCL} 5	28- 24 5
$\boxed{-}$	29- 41
$\boxed{x\div y}$	30- 21
\boxed{f} \rightarrow R	31- 14 4
\boxed{RCL} 1	32- 24 1
$\boxed{+}$	33- 51
\boxed{RCL} 3	34- 24 3
$\boxed{-}$	35- 41
$\boxed{x\div y}$	36- 21
\boxed{RCL} 2	37- 24 2
$\boxed{+}$	38- 51
\boxed{RCL} 4	39- 24 4
$\boxed{-}$	40- 41
$\boxed{x\div y}$	41- 21
$\boxed{R/S}$	42- 74
$\boxed{x\div y}$	43- 21
\boxed{GTO} 18	44- 13 18

REGISTERS

R_0	$R_1 N_p$	$R_2 E_p$	$R_3 N_p - N_{T1}$
$R_4 E_p - E_{T1}$	$R_5 \phi$	R_6 Scale Factor	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input coordinates of pivot point in old system	N_{old}	ENTER •	
		E_{old}	GSB 01	N_{old}
3	Input coordinates of pivot point in new system	N_{new}	ENTER •	
		E_{new}	R/S	
4	Input rotation angle (+ for clockwise, - for counter-clockwise)	\pm angle	R/S	1
5	(Optional) If scale factor is other than 1, input it	Scale factor		
6	Store scale factor (1 by default if step 5 not executed)		R/S	
7	Input coordinates of point to be transformed and read new coordinates	N_{old}	ENTER •	
		E_{old}	R/S	N_{new}
			R/S	E_{new}
8	Return to step 7 for the next point			
9	For a new case, go to step 2			

Example:

Coordinates before transformation are those computed by *Compass Rule Adjustment* program.

Coordinates In Old System	Coordinates In New System
N 150.000*	N 100.000*
E 400.000	E 350.000
N 224.540	N 165.9765
E 561.673	E 515.3526
N 356.577	N 302.6979
E 468.710	E 429.4272
N 232.414	N 187.1512
E 307.327	E 261.7672

* Rotated about this point

Rotation Angle = $-3^{\circ}00'00''$

Scale Factor = 1.00

Keystrokes**Display**

150 **ENTER** 400

GSB 01

150.0000

100 **ENTER** 350

R/S 3 **CHS** **R/S**

1.0000

R/S

1.0000

224.540 **ENTER**

561.673 **R/S**

165.9765

R/S

515.3526

356.577 **ENTER**

468.710 **R/S**

302.6979

R/S

429.4272

N_{new}

E_{new}

N_{new}

E_{new}

Etc.

NOTES

NOTES

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