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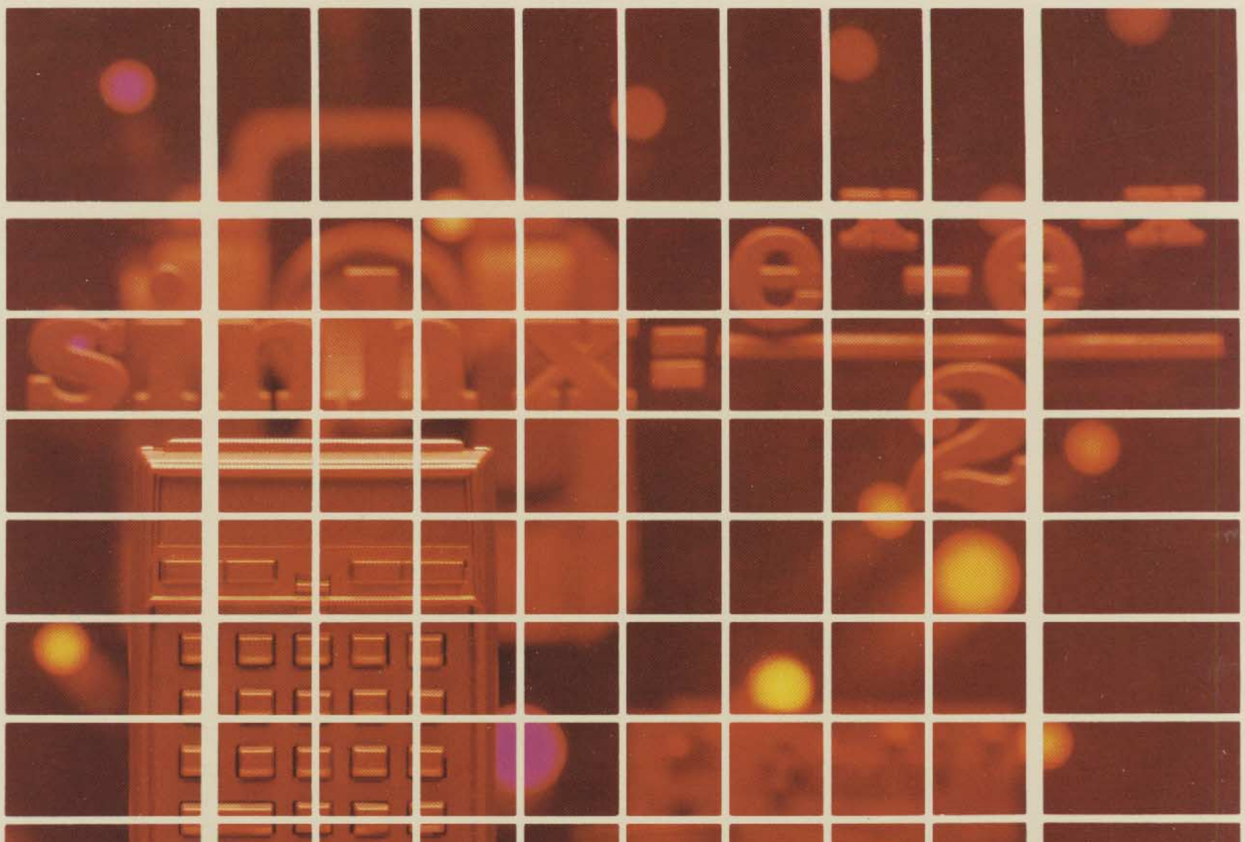
HEWLETT-PACKARD

HP-41

USERS' LIBRARY SOLUTIONS

Control Systems



NOTICE

INTRODUCTION

This HP-41C Solutions book was written to help you get the most from your calculator. The programs were chosen to provide useful calculations for many of the common problems encountered.

They will provide you with immediate capabilities in your everyday calculations and you will find them useful as guides to programming techniques for writing your own customized software. The comments on each program listing describe the approach used to reach the solution and help you follow the programmer's logic as you become and expert on your HP calculator.

KEYING A PROGRAM INTO THE HP-41C

There are several things that you should keep in mind while you are keying in programs from the program listings provided in this book. The output from the HP 82143A printer provides a convenient way of listing and an easily understood method of keying in programs without showing every keystroke. This type of output is what appears in this handbook. Once you understand the procedure for keying programs in from the printed listings, you will find this method simple and fast. Here is the procedure:

1. At the end of each program listing is a listing of status information required to properly execute that program. Included is the SIZE allocation required. Before you begin keying in the program, press [XEQ] [ALPHA] SIZE [ALPHA] and specify the allocation (three digits; e.g., 10 should be specified as 010).

   Also included in the status information is the display format and status of flags important to the program. To ensure proper execution, check to see that the display status of the HP-41C is set as specified and check to see that all applicable flags are set or clear as specified.

2. Set the HP-41C to PRGM mode (press the [PRGM] key) and press ■ [GTO] [•] [•] to prepare the calculator for the new program.

3. Begin keying in the program. Following is a list of hints that will help you when you key in your programs from the program listings in this handbook.

   a. When you see " (quote marks) around a character or group of characters in the program listing, those characters are ALPHA. To key them in, simply press [ALPHA], key in the characters, then press [ALPHA] again. So "SAMPLE" would be keyed in as [ALPHA] "SAMPLE" [ALPHA].

   b. The diamond in front of each LBL instruction is only a visual aid to help you locate labels in the program listings. When you key in a program, ignore the diamond.

   c. The printer indication of divide sign is /. When you see / in the program listing, press [÷].

   d. The printer indication of the multiply sign is ⁎. When you see ⁎ in the program listing, press [×].

   e. The ⊢ character in the program listing is an indication of the [APPEND] function. When you see ⊢, press ■ [APPEND] in ALPHA mode (press ■ and the K key).

   f. All operations requiring register addresses accept those addresses in these forms:

      nn (a two-digit number)
      IND nn (INDIRECT: ■, followed fy a two-digit number)
      X, Y, Z, T, or L (a STACK address: [•] followed by X, Y, Z, T, or L)
      IND X, Y, Z, T or L (INDIRECT stack: ■ [•] followed by X, Y, Z, T, or L)

      Indirect addresses are specified by pressing ■ and then the indirect address. Stack addresses are specified by pressing [•] followed by X, Y, Z, T, or L. Indirect stack addresses are specified by pressing ■ [•] and X, Y, Z, T, or L.

Printer Listing	Keystrokes	Display
01♦LBL "SAMPLE"	■ [LBL] [ALPHA] SAMPLE [ALPHA]	*01 LBL^T SAMPLE*
02 "THIS IS A "	[ALPHA] *THIS IS A* [ALPHA]	*02^T THIS IS A*
03 "⊢SAMPLE"	[ALPHA] ■ [APPEND] SAMPLE	*03^T ⊢ SAMPLE*
04 AVIEW	■ AVIEW [ALPHA]	*04 AVIEW*
05 6	6	*05 6*
06 ENTER↑	[ENTER↑]	*06 ENTER ↗*
07 -2	2 [CHS]	*07 -2*
08 /	[÷]	*08 /*
09 ABS	[XEQ] [ALPHA] ABS [ALPHA]	*09 ABS*
10 STO IND L	[STO] ■ [•] L	*10 STO IND L*
11 "R3="	[ALPHA] R3= ■ [ARCL] 03	*11^T R3=*
12 ARCL 03	■ [AVIEW]	*12 ARCL 03*
13 AVIEW	[ALPHA]	*13 AVIEW*
14 RTN	■ [RTN]	*14 RTN*

TABLE OF CONTENTS

FREQUENCY RESPONSE OF A TRANSFER FUNCTION

For transfer function of the form:

$$G\,(S) = \frac{K_1(Z_2S + 1)}{S^{N_3}(Z_4S + 1)\ (Z_5S + 1)\ (\frac{S^2}{\omega_7^2} + \frac{2Z_6S}{\omega_7} + 1)}$$

The program computes $\angle G\,(j\omega)$, $|G(j\omega)|$ and log $|G(j\omega)|$ for any input frequency ω.

Parameters K_1, Z_2, N_3, Z_4, Z_5, Z_6 and ω_7 are stored in registers 01, 02, 03, 04, 05, 06, and 07 respectively.

NOTE: For type 0 systems, enter $N_3 = 0$. Z_2, Z_4 and/or Z_5 can be entered as 0. If there is no quadratic term, enter Z_6 as 0 and ω_7 very large compared to $\frac{1}{Z_5}$, where Z_5 is the smallest first order term used (other than zero).

Example:

Find $|G(j\omega)|$, $\angle G(j\omega)$ and log $|G(j\omega)|$ for $G(S) = \frac{12(S + 0.6)}{S(S + 1)(S^2 + 6S + 36)}$ frequency 0.01 rad/sec.

First put G(S) into proper form: $G(S) = \frac{.2(1.67S + 1)}{S(S + 1)[(\frac{S}{6})^2 + (\frac{S}{6}) + 1]}$

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 010	
[XEQ] [ALPHA] FREQ [ALPHA]	K1=?
.2 [R/S]	Z2=?
1.67 [R/S]	N3=?
1 [R/S]	Z4=?
1 [R/S]	Z5=?
0 [R/S]	Z6=?
.5 [R/S]	W7=?
6 [R/S]	W=?
.01 [R/S]	G∠JW⅃=20.00
[R/S]	∠G∠JW⅃=-89.71
[R/S]	LOG G=1.30

User Instructions

SIZE: 010

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program			
2	Begin execution, enter data		[XEQ] FREQ	K1=?
		K_1	[R/S]	Z2=?
		Z_2	[R/S]	N3=?
		N_3	[R/S]	Z4=?
		Z_4	[R/S]	Z5=?
		Z_5	[R/S]	Z6=?
		Z_6	[R/S]	W7=?
		ω_7	[R/S]	W=?
3	Answer is automatic	ω	[R/S]	G∠JW⦣=
			[R/S]	∠G∠JW⦣=
			[R/S]	LOG G∠JW⦣=
4	For another frequency		[XEQ] W2	W=?
5	Repeat step 3.			

Program Listings

Store data

```
01◆LBL "FREQ"
02 FIX 2
03 "K1=?"
04 PROMPT
05 STO 01
06 "Z2=?"
07 PROMPT
08 STO 02
09 "N3=?"
10 PROMPT
11 STO 03
12 "Z4=?"
13 PROMPT
14 STO 04
15 "Z5=?"
16 PROMPT
17 STO 05
18 "Z6=?"
19 PROMPT
20 STO 06
21 "W7=?"
22 PROMPT
23 STO 07
24◆LBL "W2"
25 "W=?"
26 PROMPT
27 STO 08
```

$\left[\frac{S^2}{\omega_7^2} + \frac{2Z_6}{\omega_7}\right] + 1$

```
28 RCL 06
29 RCL 07
30 /
31 2
32 *
33 RCL 08
34 *
35 1
36 RCL 08
37 RCL 07
38 /
39 X↑2
40 -
41 R-P
```

$Z_5S + 1$

```
42 RCL 05
43 RCL 08
44 *
45 1
46 R-P
47 XEQ 01
```

$Z_4S + 1$

```
48 RCL 04
49 RCL 08
50 *
51 1
52 R-P
```

S^{N_3}

```
53 XEQ 01
54 RCL 08
55 RCL 03
56 Y↑X
57 *
58 X<>Y
59 RCL 03
60 90
61 *
62 +
63 CHS
64 X<>Y
65 1/X
```

$K_1(Z_2S + 1)$

```
66 RCL 02
67 RCL 08
68 *
69 RCL 01
70 *
71 RCL 01
72 R-P
73 XEQ 01
```

Display

```
74 "G<JW>="
75 ARCL X
76 AVIEW
77 STOP
78 STO 09
79 X<>Y
80 "∡G<JW>="
81 ARCL X
82 AVIEW
83 STOP
84 RCL 09
85 LOG
86 "LOG G="
87 ARCL X
88 AVIEW
89 STOP
```

Multiply complex numbers

```
90◆LBL 01
91 X<>Y
92 RDN
93 *
94 RDN
95 +
96 R↑
97 RTN
98 .END.
```

00

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00		50	
	K_1		
	Z_2		
	N_3		
	Z_4		
05	Z_5	55	
	Z_6		
	ω_7		
	ω		
	$\lvert G(j\omega)\rvert$		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 010 TOT. REG. 34 USER MODE
ENG ___ FIX 2 SCI ___ ON ___ OFF X
DEG X RAD ___ GRAD ___

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

FREQUENCY RESPONSE OF A
TRANSFER FUNCTION
PROGRAM REGISTERS NEEDED: 25

HEWLETT PACKARD
SOLUTION BOOK:
CONTROL SYSTEM

ROW 1 (1 - 3)

ROW 2 (3 - 9)

ROW 3 (9 - 15)

ROW 4 (15 - 20)

ROW 5 (21 - 24)

ROW 6 (25 - 34)

ROW 7 (35 - 47)

ROW 8 (47 - 56)

ROW 9 (57 - 68)

ROW 10 (69 - 74)

ROW 11 (74 - 80)

ROW 12 (80 - 86)

ROW 13 (86 - 96)

ROW 14 (97 - 98)

BODE OF TRANSFER FUNCTION THAT HAS EACH POLE AND ZERO GIVEN

Given a laplace transfer function:

$$F(s) = K\,\frac{(s + z_1 + jz_1')\ (s + z_2 + jz_2')\ \ldots\ (s + z_N + jz_N')}{(s + p_1 + jp_1')\ (s + p_2 + jp_2')\ \ldots\ (s + z_M + jz_M')}$$

$F(jw)$ = Magnitude $\measuredangle\, \theta$

$$\text{where magnitude} = |K|\ \frac{\sqrt{(z_1' + \omega)^2 + z_1^2}\ \sqrt{(z_2' + \omega)^2 + z_2^2}\ \ldots\ \sqrt{(z_N' + \omega)^2 + z_N^2}}{\sqrt{(p_1' + \omega)^2 + p_1^2}\ \sqrt{(p_2' + \omega)^2 + p_2^2}\ \ldots\ \sqrt{(p_M' + \omega)^2 + p_M^2}}$$

$$\text{and } \theta = \mathrm{Tan}^{-1}\left(\frac{z_1' + \omega}{z_1}\right) + \mathrm{Tan}^{-1}\left(\frac{z_2' + \omega}{z_2}\right) + \ldots + \mathrm{Tan}^{-1}\left(\frac{z_N' + \omega}{z_N}\right) - \mathrm{Tan}^{-1}\left(\frac{p_1' + \omega}{p_1}\right)$$

$$+ \mathrm{Tan}^{-1}\left(\frac{p_2' + \omega}{p_2}\right) + \ldots + \mathrm{Tan}^{-1}\left(\frac{p_M' + \omega}{p_M}\right) + 2\,\frac{|K| - K}{K}\,\mathrm{Tan}^{-1} 1$$

and decibels = 20 $\log_{10}$ magnitude.

N.B. $\text{zero}_i = -(z_i + jz_i')$

$\text{pole}_i = -(p_i + jp_i')$

NOTE: K must be real and non-zero but may be negative.

Example:

Find the gain (in decibels and volts/volt) and phase shift in degrees for:

$$F(s) = 0.09091\left(\frac{s + 62890}{s + 5717}\right)$$

at 3,333 Hz.

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 008	
[XEQ] [ALPHA] BODE1 [ALPHA]	FREQ?
3333 [R/S]	N?
1 [R/S]	Z?
62890 [R/S]	ZP?
0 [R/S]	M?
1 [R/S]	P?
5717 [R/S]	PP?
0 [R/S]	K?
.09091 [R/S]	dB=-11.1
[R/S]	MAG=2.7759E-1
[R/S]	∡=-56.3

User Instructions

SIZE: 008

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program.			
2	Begin execution:		[XEQ] BODE1	FREQ?
	input frequency in Hertz	f	[R/S]	N?
	input number of highest zero	N	[R/S]	Z?
	input Z_1	Z_1	[R/S]	ZP?
	input Z_1'	Z_1'	[R/S]	Z?
	input Z_2	Z_2	[R/S]	ZP?
	input Z_2'	Z_2'	[R/S]	Z?
	⋮	⋮	⋮	⋮
	continue until Z_N and Z_N' are in.			
				M?
	input number of highest pole	M	[R/S]	P?
	input P_1	P_1	[R/S]	PP?
	input P_1'	P_1'	[R/S]	P?
	⋮	⋮	⋮	⋮
	continue until P_N and P_N' are in.			
				K?
	input K.	K	[R/S]	
3	Solution - decibels			dB=
	- magnitude		[R/S]	MAG=
	- phase angle		[R/S]	$\measuredangle$=

Program Listings

Initialization

```
 01♦LBL "BOD
E1"
 02 CLRG
 03 "FREQ.?"
 04 PROMPT
 05 STO 06
 06 2
 07 *
 08 PI
 09 *
 10 STO 05
 11 1
 12 STO 01
 13 "N?"
 14 PROMPT
 15 1
 16 -
 17 1000
 18 /
 19 STO 00
 20♦LBL B
 21 "Z?"
 22 PROMPT
 23 "ZP?"
 24 PROMPT
 25 RCL 03
 26 1
 27 +
 28 STO 03
 29 STO 07
 30 RDN
 31 RCL 05
 32 +
 33 X<>Y
 34 R-P
 35 XEQ 00
 36 ISG 00
 37 GTO B
 38 "M?"
 39 PROMPT
 40 1
 41 -
 42 1000
 43 /
 44 STO 00
 45♦LBL C
 46 "P?"
 47 PROMPT
 48 "PP?"
 49 PROMPT
 50 RCL 04
```

```
 51 1
 52 -
 53 STO 04
 54 STO 07
 55 RDN
 56 RCL 05
 57 +
 58 CHS
 59 X<>Y
 60 R-P
 61 1/X
 62 XEQ 00
 63 ISG 00
 64 GTO C
 65 0
 66 "K?"
 67 PROMPT
 68 R-P
 69 XEQ 00
 70 RCL 01
 71 LOG
 72 20
 73 *
```

Output

```
 74 FIX 1
 75 "dB="
 76 XEQ d
 77 RCL 01
 78 SCI 4
 79 "MAG="
 80 XEQ d
 81 RCL 02
 82 FIX 1
 83 "∡="
 84♦LBL d
 85 ARCL X
 86 AVIEW
 87 STOP
 88 RTN
```

Subroutine to update magnitude

```
 89♦LBL 00
 90 ST* 01
 91 RDN
 92 RCL 02
 93 +
 94 1
 95 P-R
 96 R-P
 97 RDN
 98 STO 02
 99 RTN
100 .END.
```

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	N, M max	50	
	Magnitude M		
	Phase Ang. θ		
	Zero counts i < N		
	pole counts -1>-M		
05	radians/second	55	
	first input freq		
	last updated norm		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 008 TOT. REG. 32 USER MODE
ENG ___ FIX ___ SCI ___ ON ___ OFF X
DEG x RAD ___ GRAD ___

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

BODE OF TRANSFER FUNCTION THAT
HAS EACH POLE AND ZERO GIVEN
PROGRAM REGISTERS NEEDED: 25

HEWLETT PACKARD
SOLUTION BOOK:
CONTROL SYSTEM

ROW 1 (1 - 3)

ROW 2 (3 - 12)

ROW 3 (13 - 20)

ROW 4 (20 - 27)

ROW 5 (28 - 37)

ROW 6 (37 - 43)

ROW 7 (44 - 50)

ROW 8 (51 - 62)

ROW 9 (62 - 69)

ROW 10 (69 - 75)

ROW 11 (76 - 80)

ROW 12 (80 - 87)

ROW 13 (88 - 99)

ROW 14 (100 - 100)

FREQUENCY RESPONSE OF A THIRD OVER A FOURTH ORDER TRANSFER

by Craig A. Lincoln

Given a transfer function of the form:

$$F(s) = \frac{(N3)s^3 + (N2)s^2 + (N1)s + (N0)}{(D4)s^4 + (D3)s^3 + (D2)s^2 + (D1)s + (D0)}$$

$$\text{the magnitude} = \sqrt{\frac{(N0 - N2\ \omega^2)^2 + (N1\ \omega - N3\ \omega^3)^2}{(D0 - D2\ \omega^2 + D4\ \omega^4)^2 + (D1\ \omega - D3\ \omega^3)^2}}$$

where $\omega = 2\pi f$ and magnitude in dB = 20 $\log_{10}$ magnitude

$$\text{The phase angle} = \text{ATAN}\left(\frac{N1\ \omega - N3\ \omega^3}{N0 - N2\ \omega^2}\right) - \text{ATAN}\left(\frac{D1\ \omega - D3\ \omega^3}{D0 - D2\ \omega^2 + D4\ \omega^4}\right)$$

Phase angle in degrees unless angular mode is changed.

NOTE: N3 and D4 must be -1, 0 or +1.
All coefficient values must be real.
Low frequency must be $> 10^{-40}$.
Multiplier must be > 1.

References: 1. Kuo, B. C., Automatic Control Systems, Prentice-Hall, 1982.
2. University of Illinois, E.E. Curriculum, Class Notes.

EXAMPLE:

Find the gain (in decibels and volts/volt) and phase shift (in degrees) for:

$$F(s) = \frac{8s^3 - 1}{3s^3 + 8}$$

at 0.01, 0.1 and 1.0 Hz

Rewriting the function yields:

$$F(s) = \frac{S^3 - 0.125}{0.375\ S^3 + 1}$$

N3 = 1.0	D4 = 0	D0 = 1
N2 = 0	D3 = 0.375	Low Freq. = 0.01
N1 = 0	D2 = 0	High Freq. = 1.0
N0 = -0.125	D1 = 0	Multiplier = 10

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 012	
[XEQ] [ALPHA] FR2 [ALPHA]	HZ OR RAD?
HZ [R/S]	dB? <Y,N>
Y [R/S]	MAG? <Y,N>
Y [R/S]	∡? <Y,N>
Y [R/S]	N3?
1 [R/S]	N2?
0 [R/S]	N1?
0 [R/S]	N0?
-.125 [R/S]	D4?
0 [R/S]	D3?
.375 [R/S]	D2?
0 [R/S]	D1?
0 [R/S]	D0?
1 [R/S]	LOW FREQ?
.01 [R/S]	MULTIPLIER
10 [R/S]	HIGH FREQ?
1 [R/S]	FREQ=0.0100
[R/S]*	dB=-18.0618
[R/S]*	MAG=0.1250
[R/S]*	∡=-179.8810

Keystrokes:	Display:
[R/S]*	FREQ=0.1000
[R/S]*	dB=-11.1638
[R/S]*	MAG=0.2766
[R/S]*	∡=-111.4306
[R/S]*	FREQ=1.000
[R/S]*	dB=8.5189
[R/S]*	MAG=2.6665
[R/S]*	∡=-0.6448

For the given transfer function, find the magnitude, dB, and phase angle at 0.0 radians (assumes the above example has been run).

Note: Use 10^{-40} instead of zero. 0.0 radians = 0.0 Hertz

Keystrokes:	Display:
[XEQ] [ALPHA] RG [ALPHA]	LOW FREQ?
[EEX] [CHS] 40	MULTIPLIER
2 [R/S]	HIGH FREQ?
0 [R/S]	FREQ=1.0000E-40
[R/S]*	dB=-18.0618
[R/S]*	MAG=0.1250
[R/S]*	∡=180.0000

User Instructions

SIZE: 012

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program			
2	Begin execution		[XEQ] "FR2"	
	If SIZE too small			XEQ SIZE 012
	Reset to 012		[XEQ] "SIZE" 012	
	Continue		[R/S]	HZ OR RAD?
3	Enter units	HZ or RAD	[R/S]	dB? <Y,N>
4	Output in dB	Y or N	[R/S]	MAG? <Y,N>
5	Magnitude Output	Y or N	[R/S]	∡? <Y,N>
6	Phase Angle Output	Y or N	[R/S]	N3?
7	Enter Numerator Coefficient	N3	[R/S]	N2?
8	" " "	N2	[R/S]	N1?
9	" " "	N1	[R/S]	N0?
10	" " "	N0	[R/S]	D4?
11	Enter Denominator Coefficient	D4	[R/S]	D3?
12	" " "	D3	[R/S]	D2?
13	" " "	D2	[R/S]	D1?
14	" " "	D1	[R/S]	D0?
15	" " "	D0	[R/S]	LOW FREQ?
16	Enter lowest frequency	Low	[R/S]	MULTIPLIER
17	Enter multiplier	Mult	[R/S]	HIGH FREQ?
18	Enter high frequency	High	[R/S]*	
	NOTE: For the response to a single fre-			
	quency, enter that frequency as low, 2 as			
	the multiplier, and 0 for high.			
19	To enter a new range		[XEQ] "RG"	
	Repeat steps 16-19.			

*This keystroke not necessary if printer is in use.

Program Listings

01♦LBL "FR2"	
02 SF 21	Initialization
03 SF 25	
04 RCL 11	
05 "XEQ SIZE 012"	Check for correct size
06 FC?C 25	
07 PROMPT	
08 DEG	
09 FIX 4	Set display mode
10 SF 00	
11 CF 01	Initialize all user-defined flags
12 SF 02	
13 CF 03	
14 CF 04	
15 CF 05	
16 CF 06	
17 CF 07	
18 AON	
19 "HZ OR RAD?"	Prompt for frequency units
20 PROMPT	
21 AOFF	
22 ASTO X	
23 "RAD"	
24 ASTO Y	
25 X=Y?	
26 SF 04	Set flag 04 if rad/sec
27 "dB?"	
28 XEQ 04	
29 X=Y?	Prompt for desired output
30 SF 05	
31 "MAG?"	Set the appropriate flags
32 XEQ 04	
33 X=Y?	
34 SF 06	
35 "∡?"	
36 XEQ 04	
37 X=Y?	
38 SF 07	
39 "N3?"	
40 PROMPT	Prompt for each of the numerator coefficients
41 X=0?	
42 CF 00	
43 X<0?	
44 SF 01	
45 "N2?"	
46 PROMPT	
47 STO 01	
48 "N1?"	

49 PROMPT	
50 STO 02	
51 "N0?"	
52 PROMPT	
53 STO 03	
54 "D4?"	
55 PROMPT	Prompt for each of the denominator coefficients
56 X=0?	
57 CF 02	
58 X<0?	
59 SF 03	
60 "D3?"	
61 PROMPT	
62 STO 04	
63 "D2?"	
64 PROMPT	
65 STO 05	
66 "D1?"	
67 PROMPT	
68 STO 06	
69 "D0?"	
70 PROMPT	
71 STO 07	
72♦LBL "RG"	Prompt for frequency range and step function value
73 "LOW FREQ?"	
74 PROMPT	
75 X=0?	
76 GTO "RG"	
77 STO 10	
78 1	
79 "MULTIPLIER"	
80 PROMPT	
81 X<=Y?	
82 GTO "RG"	
83 STO 09	
84 "HIGH FREQ?"	
85 PROMPT	
86 STO 11	
87 GTO 01	
88♦LBL 16	Compute the next frequency to be used (used for calculating results for range of frequency).
89 RCL 11	
90 RCL 10	
91 RCL 09	
92 *	
93 STO 10	
94 X>Y?	
95 STOP	
96♦LBL 01	

Program Listings

```
 97 ADV
 98 ADV
 99 RCL 10          Display the cur-
100 "FREQ="          rent frequency
101 ARCL X
102 AVIEW
103 FS? 04
104 GTO 00          Convert units to
105 2                rad/sec if input
106 *                was given in
107 PI               Hertz
108 *
109♦LBL 00
110 STO 08
111 RCL 02          Calculate linear
112 X<>Y             magnitude
113 /
114 FS? 00
115 RCL 08
116 FS? 01          (Equation given
117 CHS              in Program
118 FS? 00           Description)
119 -
120 RCL 03
121 RCL 08
122 X↑2
123 /
124 RCL 01
125 -
126 R-P
127 STO 00
128 RDN
129 RCL 06
130 RCL 08
131 /
132 RCL 04
133 RCL 08
134 *
135 -
136 RCL 07
137 RCL 08
138 X↑2
139 /
140 FS? 02
141 LASTX
142 FS? 03
143 CHS
144 FS? 02
145 +
146 RCL 05
147 -
```

```
148 RCL 00
149 RDN
150 R-P
151 RDN
152 RDN
153 STO 00
154 RDN
155 X<>Y
156 /
157 ENTER↑
158 LOG             Magnitude in dB
159 20
160 *
161 "dB="
162 FS? 05          Display magnitude
163 XEQ 03           in decibels
164 X<>Y
165 "MAG="
166 FS? 06          Display linear
167 XEQ 03           magnitude
168 FC? 07
169 GTO 16
170 R↑              Calculate and
171 R↑               display the
172 RCL 00           phase angle
173 -
174 CHS
175 LASTX
176 X<>Y
177 1
178 P-R
179 R-P
180 RDN
181 "∡="
182 XEQ 03
183 GTO 16
184♦LBL 03          Display
185 ARCL X           subroutine
186 AVIEW
187 RTN
188♦LBL 04          Subroutine used
189 AON              in prompting for
190 "⊢<Y,N>"         desired output
191 PROMPT
192 AOFF
193 ASTO X
194 "Y"
195 ASTO Y
196 END
```

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	Used for temp. storage in calc. mag.
01	N2
02	N1
03	N0
04	D3
05	D2
06	D1
07	D0
08	Current Frequency
09	Multiplier
10	Lowest Frequency
11	Highest Frequency

STATUS

SIZE	012	TOT. REG.	67	USER MODE	
ENG		FIX	4	SCI	ON / OFF
DEG	X	RAD		GRAD	

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
00	S	N3 $\neq$ 0	N3 = 0
01	C	N3 < 0	N3 $\geq$ 0
02	S	D4 $\neq$ 0	D4 = 0
03	C	D4 < 0	D4 $\geq$ 0
04	C	RAD/SEC	HERTZ
05	C	dB output on	dB output off
06	C	Mag output on	Mag output off
07	C	$\measuredangle$ output on	$\measuredangle$ output off
21	S	Printer enable	Printer disable
25	S	Error ignore on	Error ignore off

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

FREQ RESPONSE OF A THIRD OVER
A FOURTH ORDER TRANSFER
PROGRAM REGISTERS NEEDED: 55

HEWLETT PACKARD
SOLUTIONS BOOK:
CONTROL SYSTEMS

ROW 1 (1 : 5)

ROW 2 (5 : 6)

ROW 3 (6 : 13)

ROW 4 (14 : 19)

ROW 5 (19 : 23)

ROW 6 (23 : 28)

ROW 7 (28 : 33)

ROW 8 (34 : 39)

ROW 9 (39 : 45)

ROW 10 (46 : 52)

ROW 11 (53 : 60)

ROW 12 (60 : 66)

ROW 13 (66 : 72)

ROW 14 (72 : 73)

ROW 15 (74 : 79)

ROW 16 (79 : 83)

ROW 17 (84 : 86)

ROW 18 (87 : 97)

ROW 19 (98 : 103)

ROW 20 (104 : 114)

ROW 21 (115 : 125)

ROW 22 (126 : 138)

ROW 23 (139 : 148)

ROW 24 (149 : 160)

ROW 25 (161 : 165)

ROW 26 (165 : 170)

ROW 27 (171 : 181)

ROW 28 (182 : 189)

ROW 29 (190 : 194)

ROW 30 (195 : 196)

BODE OF THIRD-ORDER OVER THIRD-ORDER TIMES S^N TRANSFER FUNCTION

$$F(s) = \frac{Qs^3 + Gs^2 + Hs + K}{s^N(Rs^3 + Ls^2 + Ms + P)}$$

where: Q and R are options, limited to [-1, 0, +1], G, H, K, L, M, and P may assume any real value, and N is a non-negative integer [0, 1, 2, ...].

$$\text{Magnitude} = \frac{\sqrt{(K - G\omega^2)^2 + (H\omega - Q^3)^2}}{\omega N \sqrt{(P - L\omega^2)^2 + (M\omega - R^3)^2}}$$

where: $\omega = 2\pi F$

$$\text{Decibels} = 20 \log_{10} \text{magnitude}$$

$$\text{Phase} = \text{Tan}^{-1}\left(\frac{H\omega - Q\omega^3}{K - G\omega^2}\right) - N\sin^{-1} 1.0 - \text{Tan}^{-1}\left(\frac{M\omega - R\omega^3}{P - L\omega^2}\right)$$

in degrees (unless angular mode is set to radians or grads).

NOTE: Frequency, f, cannot equal zero. Power, N, is a non-negative integer. Coefficient of s^3 in numerator or denominator is -1, 0, or +1. Display of phase will be between -180° and +180° ($-\pi$ and $+\pi$ radians or -200 and +200 grads).

Example:

Find gain (in decibels and volts/volt) and phase shift in degrees for:

$$F(s) = \frac{8s^3 - 1}{s^2(s^2 - 2)}$$

at f = 0.1 Hz and 1.0 Hz.

Rewritten:

$$F(s) = \frac{s^3 - 0.125}{s^2(0.125s^2 - 0.25)}$$

Q = 1	G = 0
H = 0	K = -0.125
N = 2	R = 0
L = 0.125	M = 0
P = -0.25	f = 0.1 and 1.0

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 010	
[XEQ] [ALPHA] BODE3 [ALPHA]	Q?
1 [R/S]	G?
0 [R/S]	H?
0 [R/S]	K?
.125 [CHS] [R/S]	N?
2 [R/S]	R?
0 [R/S]	L?
.125 [R/S]	M?
0 [R/S]	P?
.25 [CHS] [R/S]	FREQ.?
.1 [R/S]	dB=7.4229
[R/S]	MAG=2.3504
[R/S]	∡=-116.7449
[R/S]	FREQ.?
1 [R/S]	dB=1.6690
[R/S]	MAG=1.2118
[R/S]	∡=-90.0289

User Instructions

SIZE: 010

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program.			
2	Begin execution and input constants		[XEQ] BODE3	Q?
	(remember, Q and R must have values +1,	Q	[R/S]	G?
	0, or -1).	G	[R/S]	H?
		H	[R/S]	K?
		K	[R/S]	N?
		N	[R/S]	R?
		R	[R/S]	L?
		L	[R/S]	M?
		M	[R/S]	P?
		P	[R/S]	FREQ.?
3	Input frequency (Hertz).	f	[R/S]	
4	Solution - decibels			dB=
	- magnitude		[R/S]	MAG=
	- phase angle		[R/S]	∡ =
5	For a new frequency.		[R/S]	FREQ.?
6	Go to step 3.			

Program Listings

Initialization

```
01♦LBL "BODE3"
02 SF 00
03 CF 01
04 SF 02
05 CF 03
06 "Q?"
07 PROMPT
08 X=0?
09 CF 00
10 X<0?
11 SF 01
12 "G?"
13 PROMPT
14 STO 01
15 "H?"
16 PROMPT
17 STO 02
18 "K?"
19 PROMPT
20 STO 03
21 "N?"
22 PROMPT
23 STO 04
24 "R?"
25 PROMPT
26 X=0?
27 CF 02
28 X<0?
29 SF 03
30 "L?"
31 PROMPT
32 STO 05
33 "M?"
34 PROMPT
35 STO 06
36 "P?"
37 PROMPT
38 STO 07
39♦LBL A
40 "FREQ.?"
41 PROMPT
```

Convert to radians

```
42 2
43 *
44 PI
45 *
46 STO 08
47 RCL 06
48 FC? 02
49 GTO 02
50 RCL 08
```

Denominator magnitude

```
51 X↑2
52 FS? 03
53 CHS
54 -
55♦LBL 02
56 RCL 07
57 RCL 08
58 /
59 RCL 05
60 RCL 08
61 *
62 -
63 R-P
64 RCL 08
65 RCL 04
66 Y↑X
67 *
68 1
69 ASIN
70 RCL 04
71 *
72 X<>Y
73 RDN
74 +
75 R↑
76 STO 09
77 RDN
78 RCL 03
79 RCL 08
80 /
81 RCL 01
82 RCL 08
83 *
84 -
85 RCL 02
86 FC? 00
87 GTO 01
88 RCL 08
89 X↑2
90 FS? 01
91 CHS
92 -
93♦LBL 01
94 X<>Y
95 RCL 09
96 RDN
97 R-P
98 R↑
99 /
100 RDN
101 -
```

Program Listings

```
102 CHS
103 R↑
104 ENTER↑
105 LOG
106 20
107 *
108 "dB="
109 XEQ d
110 X<>Y
111 "MAG="
112 XEQ d
113 R↑
114 1
115 P-R
116 R-P
117 RDN
118 "∡="
119 XEQ d
120 GTO A
121♦LBL d
122 ARCL X
123 AVIEW
124 STOP
125 RTN
126 .END.
```

Display

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00		50	
	G		
	H		
	K		
	N		
05	L	55	
	M		
	P		
	W		
	Temp storage		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 010 TOT. REG. 37 USER MODE
ENG ____ FIX 4 SCI ____ ON ____ OFF X
DEG ____ RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
00		$Q \neq 0$	$Q = 0$
01		$Q < 0$	$Q \geq 0$
02		$R \neq 0$	$R = 0$
03		$R < 0$	$R \geq 0$

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

BODE OF 3RD–ORDER/3RD–ORDER *
S^N TRANSFER FUNCTION
PROGRAM REGISTERS NEEDED: 28

HEWLETT PACKARD
SOLUTION BOOK:
CONTROL SYSTEM

ROW 1 (1 – 3)

ROW 2 (4 – 11)

ROW 3 (11 – 18)

ROW 4 (18 – 26)

ROW 5 (27 – 33)

ROW 6 (34 – 40)

ROW 7 (40 – 49)

ROW 8 (49 – 60)

ROW 9 (61 – 73)

ROW 10 (74 – 86)

ROW 11 (86 – 96)

ROW 12 (97 – 108)

ROW 13 (108 – 112)

ROW 14 (112 – 119)

ROW 15 (120 – 126)

ROUTH TEST FOR CONTINUOUS AND DISCRETE TIME SYSTEM STABILITY

This program is designed to test the location of the roots of a characteristic equation in the form:

$$A_0S^n + A_1S^{n-1} + A_2S^{n-2} \ . \ . \ . \ A_{n-1}S + An = 0$$

using the repeated test:

$$K = A_0/A_1 \quad \text{for } i = 1, 3, 5 \ . \ . \ . \ \begin{cases} A_{i-1} = A \\ A_i = A_{i+1} - KA_{i-2} \end{cases}$$

The number of right hand plane (RHP) roots is equal to $\Sigma(\#\text{ times } K < 0)$.

NOTES: A must be $\leqslant$ 20.

Routh test with various roots along imaginary axis may indicate instability. Shift axis slightly to check actual location of roots.

Axis shift:

$$\begin{bmatrix} A_1 = A_1 + A_0d \\ A_2 = A_2 + A_1d \\ \vdots \quad \vdots \quad \vdots \\ A_n = A_n + A_{n-1}d \end{bmatrix} \quad \begin{bmatrix} A_1 = A_1 + A_0d \\ A_2 = A_2 + A_1d \\ \vdots \quad \vdots \quad \vdots \\ A_{n-1} = A_{n-1} + A_{n-2}d \end{bmatrix} \begin{bmatrix} A_1 = A_1 + A_0d \\ A_2 = A_2 + A_1d \end{bmatrix} \quad [A_1 = A_1 + A_0d]$$

Example:

Does $S^4 + 2s^3 + S^2 + 4S + 4 = 0$ represent a stable system?

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 026	
[XEQ] [ALPHA] ROUTH [ALPHA]	N?
4 [R/S]	A0?
1 [R/S]	A1?
2 [R/S]	A2?
1 [R/S]	A3?
4 [R/S]	A4?
4 [R/S]	TEST? SHIFT?
[XEQ] [ALPHA] TEST [ALPHA]	1.000 00
	2.000 00
	-1.000 00
	1.200 01
	4.000 00
	RHP ROOTS 2

Because the RHP roots is 2, the system is unstable.

User Instructions

SIZE: 026

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program.			
2	Begin execution and enter equation.		[XEQ] ROUTH	N?
		n	[R/S]	A0?
		A_0	[R/S]	A1?
		A_1	[R/S]	A2?
		A_2	[R/S]	⋮
		⋮	[R/S]	A(n)?
		A_n	[R/S]	TEST? SHIFT?
3	For Routh test:		[XEQ] TEST	A0
	first column of routh array is displayed			⋮
	using PSE, stopping to display the #RHP			RHP ROOT
	roots.			
4	Axis Shift: the coefficients to relocate			
	equation to reflect an axis shift		[XEQ] SHIFT	d?
	input d	d	[R/S]	A0
	new coefficients are displayed using PSE.			A1
				⋮
				A(n)

Program Listings

```
01♦LBL "ROU
TH"
```

Initialization

```
02 .024
03 ENTER↑
04 0
05♦LBL 05
06 STO IND
Y
07 ISG Y
08 GTO 05
09 "N?"
10 PROMPT
11 STO 24
12 1 E3
13 /
14 STO 25
15 CF 29
16 FIX 0
17♦LBL 04
18 "A"
19 RCL 25
20 INT
21 "├"
22 ARCL X
23 "├?"
24 PROMPT
25 STO IND
25
26 ISG 25
27 GTO 04
28 "TEST? S
HIFT?"
29 PROMPT
30♦LBL "TES
T"
```

Routh test

```
31 SCI 3
32♦LBL 00
33 RCL 00
34 PSE
35 RCL 01
36 X≠0?
37 GTO 06
38 CLX
39 1 E-50
40 STO 01
41♦LBL 06
42 /
43 STO 22
44 X>0?
45 GTO 07
46 1
47 ST+ 23
48♦LBL 07
49 1
50 STO 25
51♦LBL 01
52 RCL IND
25
53 DSE 25
54 STO IND
25
55 STO IND
25
56 2
57 ST+ 25
58 RCL IND
25
59 ISG 25
60♦LBL 50
61 RCL IND
25
62 RCL 22
63 *
64 -
65 DSE 25
66 DSE 25
67 STO IND
25
68 2
69 ST+ 25
70 RCL 25
71 RCL 24
72 X>Y?
73 GTO 01
74 1
75 ST- 24
76 RCL IND
25
77 0
78 STO IND
25
79 DSE 25
80 RDN
81 STO IND
25
82 RCL 24
83 X>0?
84 GTO 00
85 RCL 00
86 PSE
87 "RHP ROO
TS "
```

Program Listings

88 FIX 0	
89 ARCL 23	
90 AVIEW	
91 STOP	
92♦LBL "SHIFT"	Axis shift
93 "d?"	
94 PROMPT	
95 SCI 4	
96 STO 23	
97 RCL 24	
98 STO 22	
99♦LBL 02	
100 0	
101 STO 25	
102♦LBL 03	
103 RCL IND 25	
104 RCL 23	
105 *	
106 ISG 25	
107 "A"	
108 ST+ IND 25	
109 RCL 25	
110 RCL 22	
111 X≠Y?	
112 GTO 03	
113 1	
114 -	
115 STO 22	
116 X≠0?	
117 GTO 02	
118 SCI 5	
119 0	
120 STO 25	
121♦LBL 08	
122 RCL IND 25	Display
123 STOP	
124 ISG 25	
125 "A"	
126 RCL 24	
127 RCL 25	
128 X<=Y?	
129 GTO 08	
130 END	

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	A_0	50	
	A_1		
	A_2		
	·		
	·		
05	·	55	
	·		
	·		
	·		
	·		
10	·	60	
	·		
	·		
	·		
	·		
15	·	65	
	·		
	·		
	·		
	·		
20	A_{19}	70	
	A_{20}		
	temp. storage		
	temp. storage		
	temp. storage		
25	n	75	
	reserved		
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 026 TOT. REG. 64 USER MODE

ENG ____ FIX ____ SCI ____ ON ____ OFF ____

DEG ____ RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
29		Digit separators appear	No separators

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

ROUTH TEST FOR CONTINUOUS AND
DISCRETE TIME SYSTEM STABILITY
PROGRAM REGISTERS NEEDED: 38

HEWLETT PACKARD
SOLUTION BOOK:
CONTROL SYSTEM

ROW 1 (1 - 3)

ROW 2 (4 - 10)

ROW 3 (11 - 17)

ROW 4 (18 - 23)

ROW 5 (23 - 25)

ROW 6 (25 - 29)

ROW 7 (30 - 37)

ROW 8 (38 - 45)

ROW 9 (46 - 53)

ROW 10 (53 - 60)

ROW 11 (60 - 68)

ROW 12 (68 - 76)

ROW 13 (76 - 84)

ROW 14 (84 - 87)

ROW 15 (87 - 92)

ROW 16 (92 - 97)

ROW 17 (98 - 106)

ROW 18 (106 - 113)

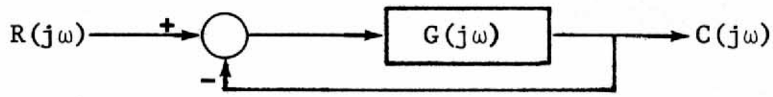
ROW 19 (114 - 122)

ROW 20 (122 - 129)

ROW 21 (130 - 130)

CONVERT FREQUENCY RESPONSE -- OPEN LOOP, CLOSED LOOP

For a linear, unity feedback control system this program converts open loop frequency response data $\angle G(j\omega)$ (or log $|G(j\omega)|$ or 20 log $|G(j\omega)|$) to closed loop data $\angle C/R(j\omega)$ and $|C/R(j\omega)|$ (or log $C/R(j\omega)$ or 20 log - $(j\omega)$).



where $\frac{C}{R}(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)}$

This program also converts from closed loop data to open loop data.

$$\left.\begin{array}{l} \angle \frac{C}{R}(j\omega) \text{ and } \left|\frac{C}{R}(j\omega)\right| \\ \angle G(j\omega) \text{ and } |G(j\omega)| \end{array}\right\}$$

the relationship used is: $g(j\omega) = \dfrac{\frac{C}{R}(j\omega)}{1 - \frac{C}{R}(j\omega)}$

Note:

A. When the input phase angle cosine ($\angle C/R(j\omega)$ or $\angle G(j\omega)$) is zero (e.g., $\angle C/R(j\omega) = -90$), the conversion is only accurate through the 6th decimal place.

Example:

For the system: $G(j\omega) = \dfrac{4}{j\omega(1 + .25j\omega)\ (1 + .0625j\omega)}$

For ω = .1, 1 and 10 rad/sec, $\measuredangle G(j\omega)$ and $|G(j\omega)|$ are:

ω rad/sec	$\measuredangle G(j\omega)$ degrees	$\lvert G(j\omega)\rvert$
.1	-91.79	39.99
1	-107.61	3.87
10	-190.20	.13

Determine closed loop frequency response for ω = .1, 1, 10 rad/sec.

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 007	
[XEQ] [ALPHA] O-C [ALPHA]	$\measuredangle$ G?
91.79 [CHS] [R/S]	C?
39.99 [R/S]	C/R=1.00
[R/S]	$\measuredangle$ C/R=-1.43
[XEQ] [ALPHA] O-C [ALPHA]	$\measuredangle$ G?
107.61 [CHS] [R/S]	G?
3.87 [R/S]	C/R=1.05
[R/S]	$\measuredangle$ C/R=-14.96
[XEQ] [ALPHA] O-C [ALPHA]	$\measuredangle$ G?
190.2 [CHS] [R/S]	G?
.13 [R/S]	C/R=0.15
[R/S]	$\measuredangle$ G/R=-191.71

User Instructions

SIZE: 007

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program.			
2	To convert from open loop to closed loop			
	response,		[XEQ] O - C	∡G?
	input angle of G(jω)	∡G(jω)	[R/S]	G?
	input magnitude G(jω)	G(jω)	[R/S]	C/R=
			[R/S]	∡C/R=
3	To convert from closed loop to open loop			
	response,		[XEQ] C - O	∡C/R?
	input angle C/R(jω)	∡C/R(jω)	[R/S]	C/R?
	input magnirude C/R(jω)	C/R(jω)	[R/S]	G=
			[R/S]	∡G=
4	For a new case		[R/S]	
	go to step 2 or 3.			

Program Listings

Open to Closed

```
01♦LBL "O-C"
02 SF 03
03 0
04 STO 06
05♦LBL 05
06 "∡C/R?"
07 FS? 03
08 "∡G?"
09 PROMPT
10 STO 04
11 RCL 06
12 -
13 1
14 "C/R?"
15 FS? 03
16 "G?"
17 PROMPT
18 GTO C
```

Closed to Open

```
19♦LBL "C-O"
20 CF 03
21 180
22 STO 06
23 GTO 05
```

Complex Algebra

```
24♦LBL C
25 X<>Y
26 RDN
27 STO 05
28 P-R
29 1
30 +
31 R-P
32 RCL 05
33 X<>Y
34 /
35 STO 01
36 X<>Y
37 CHS
38 RCL 04
39 +
40 STO 02
41 X<>Y
42 "G="
43 FS? 03
44 "C/R="
45 XEQ d
46 RCL 02
47 "∡G="
48 FS? 03
49 "∡C/R="
50 XEQ d
51 FS? 03
52 GTO "O-C"
```

Display

```
53 GTO "C-O"
54♦LBL d
55 FIX 2
56 ARCL X
57 AVIEW
58 STOP
59 RTN
60 .END.
```

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00		50	
	comp. mag.		
	comp. plan		
	entered mag.		
05	entered phase	55	
	0 or 180		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 007 TOT. REG. 25 USER MODE
ENG ___ FIX 2 SCI ___ ON ___ OFF X
DEG X RAD ___ GRAD ___

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
03		0 - C	C - 0

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

CONVERT FREQUENCY RESPONSE
-- OPEN LOOP & CLOSED LOOP
PROGRAM REGISTERS NEEDED: 19

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SOLUTION BOOK:
CONTROL SYSTEM

ROW 1 (1 - 6)

ROW 2 (6 - 10)

ROW 3 (11 - 16)

ROW 4 (17 - 20)

ROW 5 (21 - 29)

ROW 6 (30 - 42)

ROW 7 (42 - 46)

ROW 8 (47 - 50)

ROW 9 (50 - 53)

ROW 10 (53 - 60)

AID TO ROOT LOCUS PLOTS I - REAL POLES

Given the forward transfer function of unity gain feedback system:

$$KG(s) = \frac{K(s + z_1)\ (s + z_2)}{s(s + p_1)\ (s + p_2)\ (s + p_3)\ (s + p_4)}$$

where $s = \sigma + i\omega$ is the complex frequency variables and z_1, z_2, p_1, p_2, p_3 and p_4 are real numbers, the program helps in finding the roots of $1 + KG(s) = 0$, which determines the poles of the closed-loop system. It follows that at any point in the s-plane, which is a root of the above equation for some value of K, $G(\sigma + i\omega) = 1/K\ 180°$.

Since the rules for construction of approximate root locus plots are well known, this program can be used to obtain the exact location of the roots in certain regions of the s-plane. The user selects a value of σ, say σ_1 and assumes a trial value for ω, say ω_1. The program then determines $G^{-1}(\sigma_1 + i\omega_1) = ke^{i\phi}$.

If ϕ is not within ± 0.01 of 180°, a new trial value of $\omega = \omega_1$ is obtained. The equation for searching the correct value of is:

$$\omega = \omega_1\ (4 - \frac{\phi}{60}),\ \text{where } \phi \text{ is in degrees.}$$

Note:

The search question is based on the assumption that ω_1 is greater than zero. This is not a limitation since the root locus is always symmetrical about the real axis of the s-plane. For display purposes, σ is represented as "a" by the 41C.

Example:

$$KG(s) = \frac{K(s + 1)\ (s + 2)}{s(s + 3)\ (s + 4)\ (s + 5)\ (s + 6)}$$

It is desired to obtain the complex roots for $\sigma = 0$, $\sigma = -1$, and $\sigma = -2.5$.

	initial ω
1. $\sigma = 0$	$\omega = 5$
2. $\sigma = -1$	$\omega = 6$
3. $\sigma = -2.5$	$\omega = 4$

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 009	
[XEQ] [ALPHA] ROOTS 1 [ALPHA]	Z1?
1 [R/S]	Z2?
2 [R/S]	P1?
3 [R/S]	P2?
4 [R/S]	P3?
5 [R/S]	P4?
6 [R/S]	a?
0 [R/S]	W?
5 [R/S]	K=999.506
[R/S]	W=8.545
[R/S]	a?
1 [CHS] [R/S]	W?
6 [R/S]	K=519.846
[R/S]	W=6.824
[R/S]	a?
2.5 [CHS] [R/S]	W?
4 [R/S]	K=140.519
[R/S]	W=4.350

User Instructions

SIZE: 009

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program.			
2	Begin execution and input:		[XEQ] ROOTS 1	Z1?
	Z_1	Z_1	[R/S]	Z2?
	Z_2	Z_2	[R/S]	P1?
	P_1	P_1	[R/S]	P2?
	P_2	P_2	[R/S]	P3?
	P_3	P_3	[R/S]	P4?
	P_4	P_4	[R/S]	a?
3	σ	σ	[R/S]	W?
	initial guess for σ.	ω_1	[R/S]	
4	Solution will be displayed when			
	$179.99 < \phi < 180.01$.			K=
			[R/S]	W=
5	For a new root		[R/S]	a?
6	Go to step 3.			

Program Listings

01◆LBL "ROOTS 1"	
02 "Z1?"	
03 PROMPT	Initialization
04 STO 01	
05 "Z2?"	
06 PROMPT	
07 STO 02	
08 "P1?"	
09 PROMPT	
10 STO 03	
11 "P2?"	
12 PROMPT	
13 STO 04	
14 "P3?"	
15 PROMPT	
16 STO 05	
17 "P4?"	
18 PROMPT	
19 STO 06	
20◆LBL A	
21 "a?"	
22 PROMPT	
23 "W?"	
24 PROMPT	
25◆LBL B	
26 STO 08	
27 X<>Y	
28 STO 07	
29 R-P	
30 RCL 06	
31 XEQ E	
32 RCL 05	
33 XEQ E	
34 RCL 04	
35 XEQ E	
36 RCL 03	
37 XEQ E	
38 RCL 02	
39 XEQ D	
40 RCL 01	
41 XEQ D	
42 X<>Y	
43 STO 00	
44 180	
45 -	
46 ABS	
47 .01	
48 X>Y?	
49 GTO C	
50 RCL 00	
51 60	
52 /	
53 4	
54 -	
55 CHS	
56 RCL 08	
57 *	
58 STO 08	
59 RCL 07	
60 RCL 08	
61 GTO B	
62◆LBL C	
63 RCL Z	
64 "K="	K
65 XEQ d	
66 RCL 08	ω
67 "W="	
68 XEQ d	
69 GTO A	
70◆LBL d	
71 FIX 3	
72 ARCL X	Display
73 AVIEW	
74 STOP	
75 RTN	
76 STOP	
77◆LBL E	
78 RCL 07	
79 +	
80 RCL 08	
81 X<>Y	
82 R-P	
83 X<>Y	
84 RDN	
85 *	
86 RDN	
87 +	
88 R↑	
89 RTN	
90◆LBL D	
91 RCL 07	
92 +	
93 RCL 08	
94 X<>Y	
95 R-P	
96 X<>Y	
97 RDN	
98 /	
99 RDN	
100 -	
101 CHS	

Program Listings

```
102 R↑
103 RTN
104 RCL 08
105 STOP
106 .END.
```

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	ϕ	50	
	Z1		
	Z2		
	P1		
	P2		
05	P3	55	
	P4		
	$\sigma 1$		
	$\omega 1$		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 009 TOT. REG. 34 USER MODE

ENG ____ FIX 3 SCI ____ ON ____ OFF X

DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

AID TO ROOT LOCUS PLOTS I
– REAL POLES
PROGRAM REGISTERS NEEDED: 26

HEWLETT PACKARD
SOLUTION BOOK:
CONTROL SYSTEM

ROW 1 (1 – 2)

ROW 2 (2 – 8)

ROW 3 (8 – 14)

ROW 4 (15 – 21)

ROW 5 (22 – 31)

ROW 6 (31 – 37)

ROW 7 (37 – 44)

ROW 8 (44 – 51)

ROW 9 (52 – 62)

ROW 10 (62 – 67)

ROW 11 (68 – 73)

ROW 12 (74 – 85)

ROW 13 (86 – 97)

ROW 14 (98 – 106)

AID TO ROOT LOCUS PLOTS II - COMPLEX POLES

Given the forward transfer function of a unity-gain feedback system:

$$KG(s) = \frac{K(s + z_1)(s + z_2)}{s(s + p_1)(s + p_2)(s + \alpha + i\beta)(s + \alpha - i\beta)}$$

where $s = \sigma + i\omega$ is the complex frequency variable, and z_1, z_2, p_1, p_2, α and β are real numbers, the program helps in finding the roots of $1 + KS(s) = 0$, which determines the poles of the closed-loop system. It follows that at any point in the s-plane, which is a root of the above equation for some value of K, the argument of G(s) is equal to 180 degrees.

Since the rules for the construction of the approximate root locus plot are well known, this program can be used to obtain the exact location of the roots in critical regions of the s-plane. The user would select a value of σ, say σ_1 and assume a trial value for ω, say ω_1. The program then determines the modules M, and the argument, ϕ of $G^{-1}(s)$ at this point. If ϕ is not within the ±0.01 of 180° a new trial value of ω_2 is obtained using the equation $\omega_2 = \omega_1(4 - \phi/180)$.

The process is repeated until ϕ is within ±0.01 of 180°. The convergence may be slow when ω approaches zero.

Note:

The search equation is based on the assumption that ω is positive. This is not a limitation since the root-locus plot is always symmetrical about the real-axis of the s-plane. For display purposes, σ is represented as "a" by the 41C.

Example:

$$KG(s) = \frac{K}{s(s + 12 + 5i)\ (s + 12 - 5i)} = \frac{K(s + 1)\ (s + 2)}{s(s + 1)\ (s + 2)\ (s + 12 + 5i)\ (s + 12 - 5i)}$$

It is desired to obtain the exact locations of the roots for $\sigma = -1, -2,$ and -4.

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 009	
[XEQ] [ALPHA] ROOTS 2 [ALPHA]	Z1?
1 [R/S]	Z2?
2 [R/S]	P1?
1 [R/S]	P2?
2 [R/S]	A?
12 [R/S]	B?
5 [R/S]	a?
1 [CHS] [R/S]	W?
12 [R/S]	K=2,749.323
[R/S]	W=11.134
[R/S]	a?
2 [CHS] [R/S]	W?
10 [R/S]	K=1,780.106
[R/S]	W=9.220
[R/S]	a?
4 [CHS] [R/S]	W?
5.5 [R/S]	K=656.209
[R/S]	W=5.002

User Instructions

SIZE: 009

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program.			
2	Begin execution and input:		[XEQ] ROOTS 2	Z1?
	Z_1	Z_1	[R/S]	Z2?
	Z_2	Z_2	[R/S]	P1?
	P_1	P_1	[R/S]	P2?
	P_2	P_2	[R/S]	A?
	α	α	[R/S]	B?
	β	β	[R/S]	a?
3	σ	σ	[R/S]	W?
	initial guess for ω.	ω_1	[R/S]	
4	Solution will be displayed when			
	$179.99 < \phi < 180.01$			K=
			[R/S]	W=
5	For a new root		[R/S]	a?
6	Go to step 3.			

Program Listings

```
 01◆LBL "ROO
TS 2"
 02 "Z1?"
 03 PROMPT            Initialization
 04 STO 01
 05 "Z2?"
 06 PROMPT
 07 STO 02
 08 "P1?"
 09 PROMPT
 10 STO 03
 11 "P2?"
 12 PROMPT
 13 STO 04
 14 "A?"
 15 PROMPT
 16 STO 05
 17 "B?"
 18 PROMPT
 19 STO 06
 20◆LBL A
 21 "a?"
 22 PROMPT
 23 "W?"
 24 PROMPT
                      -------------
 25◆LBL B
 26 STO 08
 27 X<>Y
 28 STO 07
 29 RCL 05
 30 +
 31 X<>Y
 32 RCL 06
 33 +
 34 RCL 08
 35 RCL 06
 36 -
 37 R↑
 38 R-P
 39 RDN
 40 RDN
 41 XEQ E
 42 RCL 04
 43 RCL 07
 44 +
 45 RCL 08
 46 XEQ E
 47 RCL 03
 48 RCL 07
 49 +
 50 RCL 08
```

```
 51 XEQ E
 52 RCL 07
 53 RCL 08
 54 XEQ E
 55 RCL 02
 56 XEQ D
 57 RCL 01
 58 XEQ D
 59 X<>Y
 60 STO 00
 61 180
 62 -
 63 ABS
 64 .01
 65 X>Y?
 66 GTO C
 67 RCL 00
 68 60
 69 /                ω2
 70 4
 71 -
 72 CHS
 73 RCL 08
 74 *
 75 STO 08
 76 RCL 07
 77 RCL 08
 78 GTO B
 79◆LBL C
 80 FIX 3
 81 RCL Z            K
 82 "K="
 83 XEQ d
 84 RCL 08
 85 "W="            ω
 86 XEQ d
 87 GTO A
                    -------------
 88◆LBL d
 89 ARCL X
 90 AVIEW            Display
 91 STOP
 92 RTN
                    -------------
 93◆LBL D
 94 RCL 07
 95 +
 96 RCL 08
 97 X<>Y
 98 R-P
 99 X<>Y
100 RDN
101 /
```

Program Listings

```
102 RDN
103 -
104 CHS
105 R↑
106 RTN
107♦LBL E
108 X<>Y
109 R-P
110 X<>Y
111 RDN
112 *
113 RDN
114 +
115 R↑
116 RTN
117 .END.
```

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	ϕ	50	
	Z1		
	Z2		
	P1		
	P2		
05	α	55	
	β		
	σ1		
	ω1		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 009 TOT. REG. 35 USER MODE
ENG ___ FIX 3 SCI ___ ON ___ OFF X
DEG X RAD ___ GRAD ___

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
		NONE	

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

AID TO ROOT LOCUS PLOTS II
- COMPLEX POLES
PROGRAM REGISTERS NEEDED: 27

HEWLETT PACKARD
SOLUTION BOOK:
CONTROL SYSTEM

ROW 1 (1 - 2)

ROW 2 (2 - 8)

ROW 3 (8 - 15)

ROW 4 (16 - 23)

ROW 5 (23 - 33)

ROW 6 (34 - 44)

ROW 7 (45 - 53)

ROW 8 (54 - 60)

ROW 9 (61 - 67)

ROW 10 (68 - 78)

ROW 11 (78 - 83)

ROW 12 (84 - 89)

ROW 13 (89 - 100)

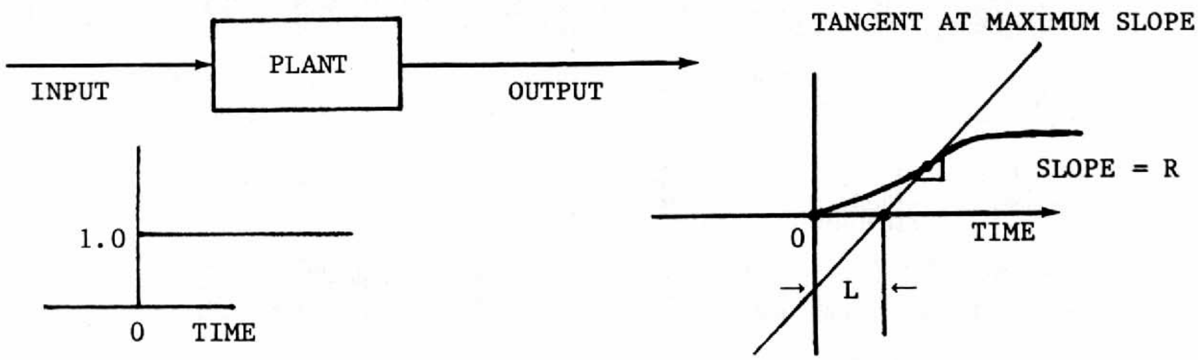
ROW 14 (101 - 112)

ROW 15 (113 - 117)

CLASSICAL CONTROL GAINS

This program computes the Ziegler-Nichols recommended settings for P, PI, and PID control. Data is required from one of two tests: the general control form is $G_c = k_c[1 + (1/T_i s) + T_D s]$.

A. <u>OPEN LOOP TEST</u> - Input step function, measure response, draw tangent at point of maximum response slope, scale L & R.

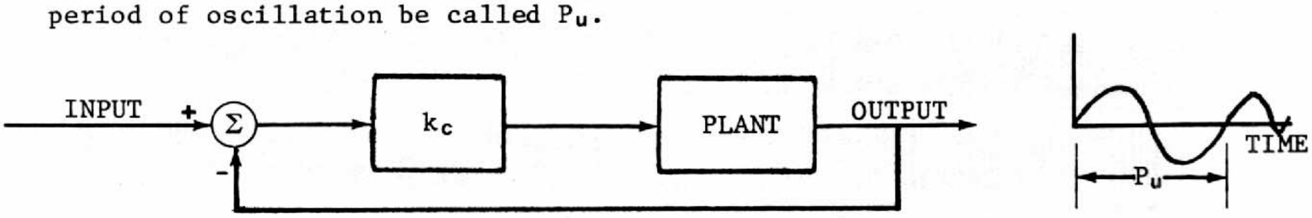


R = maximum slope
L = time intercept of maximum slope line

Then for P control, $k_c = 1/RL$; PI control, $k_c = .9/RL$, $T_i = 3.3L$; for PID control, $k_c = 1.2/RL$, $T_i = 2L$, $T_D = L/2$.

Note: Plant must be greater than first order for open loop test (s is Laplace operator).

B. <u>CLOSED LOOP TEST</u> - Increase k_c until plant is near instability (oscillating output). Let the magnitude of k_c at that point be called k_u, and the period of oscillation be called P_u.



Then for P control, let $k_c = .5\ k_u$

PI control, let $k_c = .45k_u$, $T_i = 0.83P_u$

PID control, let $k_c = 0.6k_u$, $T_i = .5P_u$, $T_D = P_{u/8}$

General control form is $G_c = k_c[1 + (1/T_i s) + T_D s]$

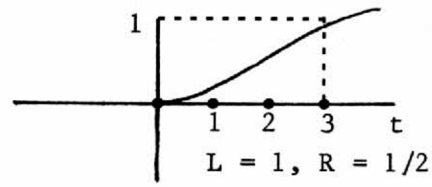
Note: Plant must be greater than second order for closed loop test.

Examples:

1. For the following open loop test data, compute the control coefficients for:

   a) P control
   b) PI control
   c) PID control

   (unit step input)



2. During a closed loop P control test, the plant output became oscillatory with k = k = 10, the period of oscillations was 50 sec. Compute coefficients for:

   a) P control
   b) PI control
   c) PID control

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 009	
[XEQ] [ALPHA] OPEN [ALPHA]	R?
0.5 [R/S]	L?
1 [R/S]	P
	KC=2.00
[R/S]	PI
	KC=1.80
[R/S]	TI=3.30
[R/S]	PID
	KC=2.40
[R/S]	TI=2.00
[R/S]	TO=0.50
[XEQ] [ALPHA] CLOSED [ALPHA]	KU?
10 [R/S]	PU?
50 [R/S]	P
	KC=5.00
[R/S]	PI
	KC=4.50
[R/S]	TI=41.50
[R/S]	PID
	KC=6.00
[R/S]	TI=25.00
[R/S]	TO=6.25

User Instructions

SIZE: 009

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program.			
2	For open loop test:		[XEQ] OPEN	R?
	input R	R	[R/S]	L?
	input L	L	[R/S]	P
				KC=
			[R/S]	PI
				KC=
			[R/S]	TI=
			[R/S]	PID
				KC=
			[R/S]	TI=
			[R/S]	TO=
3	For closed loop test:		[XEQ] CLOSED	KU?
	input Ku	Ku	[R/S]	PU?
	input Pu	Pu	[R/S]	P
				KC=
			[R/S]	PI
				KC=
			[R/S]	TI=
			[R/S]	PID
				KC=
			[R/S]	TI=
			[R/S]	TO=

Program Listings

Open loop test

```
01◆LBL "OPEN"
02 "R?"
03 PROMPT
04 "L?"
05 PROMPT
06 STO 02
07 *
08 1/X
09 STO 01
10 STO 03
11 .9
12 *
13 STO 04
14 RCL 01
15 1.2
16 *
17 STO 05
18 RCL 02
19 2
20 *
21 STO 07
22 1.65
23 *
24 STO 06
25 RCL 02
26 2
27 /
28 STO 08
29 CLX
30 GTO C
```

Closed loop test

```
31◆LBL "CLOSED"
32 "KU?"
33 PROMPT
34 "PU?"
35 PROMPT
36 STO 02
37 RDN
38 STO 01
39 2
40 /
41 STO 03
42 .9
43 *
44 STO 04
45 RCL 01
46 .6
47 *
48 STO 05
49 RCL 02
50 2
51 /
52 STO 07
53 4
54 /
55 STO 08
56 RCL 02
57 .83
58 *
59 STO 06
60 CLX
```

Display

```
61◆LBL C
62 "  P"
63 AVIEW
64 PSE
65 RCL 03
66 "KC="
67 XEQ d
68 "  PI"
69 AVIEW
70 PSE
71 RCL 04
72 "KC="
73 XEQ d
74 RCL 06
75 "TI="
76 XEQ d
77 "  PID"
78 AVIEW
79 PSE
80 RCL 05
81 "KC="
82 XEQ d
83 RCL 07
84 "TI="
85 XEQ d
86 RCL 08
87 "T0="
88◆LBL d
89 ARCL X
90 AVIEW
91 STOP
92 RTN
93 .END.
```

00

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00		50	
	1/Rc or Kc		
	L or Pu		
	P-KC		
	PI-KC		
05	PID-KC	55	
	PI-Ti		
	PID-Ti		
	PID-To		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 009 TOT. REG. 34 USER MODE

ENG ____ FIX 2 SCI ____ ON ____ OFF X

DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

CLASSICAL CONTROL GAINS

PROGRAM REGISTERS NEEDED: 26

HEWLETT PACKARD
SOLUTION BOOK:
CONTROL SYSTEM

ROW 1 (1 - 4)

ROW 2 (4 - 14)

ROW 3 (15 - 22)

ROW 4 (23 - 31)

ROW 5 (31 - 34)

ROW 6 (34 - 43)

ROW 7 (44 - 55)

ROW 8 (56 - 62)

ROW 9 (63 - 68)

ROW 10 (68 - 74)

ROW 11 (75 - 77)

ROW 12 (78 - 84)

ROW 13 (84 - 89)

ROW 14 (89 - 93)

FIRST ORDER REGULATOR

Given a system: $\dot{x} = \frac{dx}{dt} = ax + bu$

where x = system state

u = system control

a, b constant.

This program solves the regulator problem, i.e., determines the optimal feedback gain to minimize the following performance index:

$$\text{performance index} = J = 1/2 \int_0^\infty (qx^2 + ru^2)\,dt$$

the solution is: $u = -cs$

where $c = \frac{b}{r}S$ and S is the positive solution to the Riccati equation:

$$0 = -2as + \frac{s^2b^2}{r} - q$$

Then: $\dot{x} = \bar{a}x$

where $\bar{a} = a - bc$

and $x = x_0 e^{-t/\tau}$

where $\tau = -\frac{1}{a}$

Note: $a \geqslant 0$, $r > 0$, and $b \neq 0$ must be satisfied to assure proper operation.

Example:

$$\dot{x} = ax + bu, \quad J = 1/2\int_0^\infty \left[\left(\frac{x}{x_{max}}\right)^2 + \left(\frac{u}{u_{max}}\right)^2\right] dt$$

1. $a = -1,\ b = 1,\ q = \left(\frac{1}{x_{max}}\right)^2 = 0,\ r = \left(\frac{1}{u_{max}}\right)^2 = 1$
2. $a = 1,\ b = 1,\ q = 0,\ r = 1$
3. $a = 1,\ b = 1,\ q = 1,\ r = 1$
4. $a = 1,\ b = 2,\ q = 3,\ r = 4$

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 005	
[XEQ] [ALPHA] REG 1 [ALPHA]	a?
1 [CHS] [R/S]	b?
1 [R/S]	Q?
0 [R/S]	R?
1 [R/S]	C=0.00
[R/S]	S=0.00
[R/S]	a=-1.00
[R/S]	T=1.00
[XEQ] [ALPHA] REG 1 [ALPHA]	a?
1 [R/S]	b?
1 [R/S]	Q?
0 [R/S]	R?
1 [R/S]	C=2.00
[R/S]	S=2.00
[R/S]	a=-1.00
[R/S]	T=1.00
[R/S]	Q?
1 [R/S]	R?
1 [R/S]	C=2.41
[R/S]	S=2.41
[R/S]	a=-1.41
[R/S]	T=0.71

User Instructions

				SIZE: 005
STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program.			
2	Begin execution and input:		[XEQ] REG 1	a?
	a	a	[R/S]	b?
	b	b	[R/S]	Q?
	q	q	[R/S]	R?
	r	r	[R/S]	
3	Solutions: c			C=
	s		[R/S]	S=
	$\bar{a}$		[R/S]	a=
	τ		[R/S]	T=
4	To change only q and r:		[R/S]	Q?
	input q	q	[R/S]	R?
	input r	r	[R/S]	
	go to step 3.			

Program Listings

```
01+LBL "REG
1"
02 "a?"
03 PROMPT          Initialization
04 STO 01
05 "b?"
06 PROMPT
07 STO 02
08+LBL A
09 "Q?"
10 PROMPT
11 STO 03
12 "R?"
13 PROMPT
-------------------
14 STO 04
15 RCL 01
16 RCL 02
17 /
18 X↑2
19 RCL 03
20 RCL 04
21 /
22 +
23 SQRT
24 RCL 01
25 RCL 02
26 /
27 +
28 "C="            C
29 XEQ d
30 RCL 04
31 *
32 RCL 02
33 /
34 "S="            S
35 XEQ d
36 RCL 01
37 X↑2
38 RCL 02
39 X↑2
40 RCL 03
41 *
42 RCL 04
43 /
44 +
45 SQRT
46 CHS
47 "a="            a
48 XEQ d
49 1/X
50 CHS
51 "T="
52 XEQ d
53 GTO A
54+LBL d
55 ARCL X
56 AVIEW
57 STOP
58 RTN
59 .END.
```

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00		50	
	a		
	b		
	q		
	r		
05		55	
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 005 TOT. REG. 18 USER MODE
ENG ____ FIX 2 SCI ____ ON ____ OFF X
DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

FIRST ORDER REGULATOR

HEWLETT PACKARD
SOLUTION BOOK:
CONTROL SYSTEM

PROGRAM REGISTERS NEEDED: 14

ROW 1 (1 – 3)

ROW 2 (4 – 11)

ROW 3 (12 – 22)

ROW 4 (23 – 31)

ROW 5 (32 – 40)

ROW 6 (41 – 49)

ROW 7 (50 – 55)

ROW 8 (55 – 59)

SECOND ORDER REGULATOR

Given a system and a quadratic performance index as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix} u$$

$$J = 1/2\int_0^\infty \left\{ [x_1 \quad x_2] \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + u^2 r \right\} dt$$

where: x_1, x_2 are the system states
u is the control
J is the performance index to be minimized.

The optimal control is given by:

$$u = -c_1,\ x_1 - c_2x_2 \quad \text{and} \quad [\dot{x}] = [A][x] \qquad [A] = [A] - \begin{bmatrix} o \\ b \end{bmatrix} [c_1 - c_2]$$

where:

$$c_1 = \frac{1}{b}(c_1 * a_{11}c_2*) \qquad c_2 = \frac{1}{b}(a_{12}c_2*)$$

$$c_1* = a_1 + \sqrt{a_1^2 + q_1\frac{b^2}{r}} \qquad c_2* = a_2 + \sqrt{a_2^2 + 2c_1* + q_3\frac{b^2}{r}}$$

$$a_1 = a_{12}a_{21} - a_{11}a_{22} \qquad q_1 = q_{11} - 2\ \frac{q_{12}a_{11}}{a_{12}} + \frac{q_{22}a_{11}^2}{a_{12}^2}$$

$$a_2 = a_{11} + a_{22} \qquad q_3 = q_{22}$$

$$q_{11},\ q_{12},\ q_{12} \geqslant 0; \quad q_{11} + q_{12} + q_{13} = 0$$

Example:

$$\ddot{y} + \theta\dot{y} + \phi = u \qquad \text{second order system}$$

Let $x_1 = y$, $x_2 = \dot{y}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\phi & -\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad \text{suppose } \phi = 2,\ \theta = 3$$

Find the optimal control that minimizes:

1. $J = 1/2 \int_0^\infty (x_1^2 + x_2^2 + u^2)\,dt$, i.e., $q_{11} = 1$, $q_{12} = 0$, $q_{22} = 1$, $r = 1$.
2. $J = 1/2 \int_0^\infty (4x_1^2 + u^2)\,dt$, i.e., $q_{11} = 4$, $q_{12} = 0$, $q_{22} = 0$, $r = 1$.

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \qquad b = 1,\ r = 1$$

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 010	
[XEQ] [ALPHA] REG 2 [ALPHA]	A11?
0 [R/S]	A12?
1 [R/S]	A21?
2 [CHS] [R/S]	A22?
3 [CHS] [R/S]	b?
1 [R/S]	Q11?
1 [R/S]	Q12?
0 [R/S]	Q22?
1 [R/S]	R?
1 [R/S]	C1=0.236
[R/S]	C2=0.236
[R/S]	Q11?
4 [R/S]	Q12?
0 [R/S]	Q22?
0 [R/S]	R?
1 [R/S]	C1=0.828
[R/S]	C2=0.264

User Instructions

				SIZE: 010
STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program.			
2	Begin execution:		[XEQ] REG 2	A11?
	input A matrix	a11	[R/S]	A12?
		a12	[R/S]	A21?
		a21	[R/S]	A22?
		a22	[R/S]	b?
	input b.	b	[R/S]	Q11?
3	Input Q	Q11	[R/S]	Q12?
		Q12	[R/S]	Q22?
		Q22	[R/S]	R?
	input r.	r	[R/S]	C1=
			[R/S]	C2=
4	For a change of Q and R:		[R/S]	Q11?
5	Go to step 3.			
6	Closed loop dynamics:		[XEQ] A	A11=
	matrix [A]		[R/S]	A12=
			[R/S]	A21=
			[R/S]	A22=

Program Listings

```
01♦LBL "REG·
2"
02 "A11?"
03 PROMPT              Initialization
04 STO 01
05 "A12?"
06 PROMPT
07 STO 02
08 "A21?"
09 PROMPT
10 STO 03
11 "A22?"
12 PROMPT
13 STO 04
14 "b"
15 PROMPT
16 STO 05
17♦LBL C
18 "Q11?"
19 PROMPT
20 "Q12?"
21 PROMPT
22 "Q22?"
23 PROMPT
24 "R?"
25 PROMPT
-------------------------
26 STO 07
27 /
28 STO 08
29 LASTX
30 *
31 RCL 01
32 *
33 RCL 02
34 /
35 X<>Y
36 2
37 *
38 -
39 RCL 01
40 *
41 RCL 02
42 /
43 +
44 RCL 07
45 /
46 STO 07
47 RCL 02
48 RCL 03
49 *
50 RCL 01
```

```
51 RCL 04
52 *
53 -
54 STO 06
55 RCL 07
56 SQRT
57 RCL 05
58 *
59 RCL 06
60 R-P
61 RCL 06
62 +
63 STO 06
64 2
65 *
66 RCL 05
67 X↑2
68 RCL 08
69 *
70 +
71 SQRT
72 RCL 01
73 RCL 04
74 +
75 STO 07
76 R-P
77 RCL 07
78 +
79 STO 07
80 RCL 01
81 *
82 RCL 06
83 +
84 STO 06
85 RCL 05
86 /
87 "C1="                 C1
88 XEQ d
89 RCL 07
90 RCL 02
91 *
92 STO 07
93 RCL 05
94 /
95 "C2="                 C2
96 XEQ d
97 GTO C
-------------------------
98♦LBL A                [A]
99 RCL 02
100 RCL 03
101 RCL 06
```

Program Listings

```
102 -
103 RCL 04
104 RCL 07
105 -
106 RCL 01
107 "A11="
108 XEQ d
109 R↑
110 "A12="
111 XEQ d
112 R↑
113 "A21="
114 XEQ d
115 R↑
116 "A22="
117♦LBL d
118 FIX 3
119 ARCL X
120 AVIEW
121 STOP
122 RTN
123 .END.
```

Display

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00		50	
	a11		
	a12		
	a21		
	a22		
05	b	55	
	temp		
	temp		
	temp		
	R→P		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 010 TOT. REG. 38 USER MODE
ENG ___ FIX 3 SCI ___ ON ___ OFF X
DEG ___ RAD ___ GRAD ___

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

SECOND ORDER REGULATOR

PROGRAM REGISTERS NEEDED: 29

HEWLETT PACKARD
SOLUTION BOOK:
CONTROL SYSTEM

ROW 1 (1 - 2)

ROW 2 (2 - 8)

ROW 3 (8 - 14)

ROW 4 (15 - 20)

ROW 5 (20 - 25)

ROW 6 (26 - 38)

ROW 7 (39 - 51)

ROW 8 (52 - 64)

ROW 9 (65 - 77)

ROW 10 (78 - 87)

ROW 11 (88 - 95)

ROW 12 (96 - 103)

ROW 13 (104 - 110)

ROW 14 (110 - 113)

ROW 15 (114 - 118)

ROW 16 (119 - 123)

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CONTROL SYSTEMS

FREQUENCY RESPONSE OF A TRANSFER FUNCTION
BODE OF TRANSFER FUNCTION THAT HAS EACH POLE AND ZERO GIVEN
BODE OF THIRD-ORDER OVER FOURTH-ORDER TRANSFER FUNCTION
BODE OF THIRD-ORDER OVER THIRD-ORDER TIMES S^N TRANSFER FUNCTION
ROUTH TEST FOR CONTINUOUS AND DISCRETE TIME SYSTEM STABILITY
CONVERT FREQUENCY RESPONSE —— OPEN LOOP, CLOSED LOOP
AID TO ROOT LOCUS PLOTS I —— REAL POLES
AID TO ROOT LOCUS PLOTS II —— COMPLEX POLES
CLASSICAL CONTROL GAINS
FIRST ORDER REGULATOR
SECOND ORDER REGULATOR

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