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INTRODUCTION

Programs for your HP-65 Math Pac 1 have been selected from the areas of algebra, trigonometry, geometry and numerical analysis.

Each program includes a general description, formulas used in the program solution, numerical examples, and user instructions. Program listings and register allocations are given in the back of the Pac.

Some related individual programs were combined on one card when it seemed they might be useful together. In this way more programs could be included in the Pac.

We hope you find the HP-65 Math Pac 1 a useful tool for your computational work, and welcome your comments, requests and suggestions—these are our most important source of future user-oriented programs.

FORMAT OF USER INSTRUCTIONS

The following is an example of a set of User Instructions.

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Clear registers		A	
3	Perform 3-4 for $i=1, \dots, n$	a_i		
4		b_i	B	
5			C	Answer
	(To run a new case, go to 2)			

To follow the instructions, start with line 1 and read from left to right, performing indicated operations as you proceed. Lines having no numbers contain special notes to the user and are inside parentheses in the INSTRUCTIONS column. The message "To run a new case, go to 2" following line 5 in the above example is a special note.

Lines are read in sequential order except where the INSTRUCTIONS column directs otherwise. For example, "go to 2" means to jump to line 2. Repeated processes—used in most cases for a long string of input/output data—are outlined with a bold border together with a "Perform" instruction. In the above example, "Perform 3-4 for $i=1, \dots, n$ " means to execute the loop (line 3 and line 4) n times. The first time, the dummy variable i takes the value 1; the second time i takes the value 2; etc.

Normally, as in the above example, the first instruction is "Enter program" which means load the preprogrammed magnetic card (for instructions of loading a card, see "Entering A Program" on P. 7). Some instructions are self-contained and can be carried out by just reading the INSTRUCTIONS column alone, e.g., "Enter program". But some instructions depend on the information supplied by the DATA and/or KEYS columns. In line 2 of the example above, "Clear registers" appears in the INSTRUCTIONS column and **A** appears in the KEYS column, which means you have to clear the working registers by pressing the **A** key.

The DATA column specifies the input data to be supplied. Invalid arguments which result in division by zero, finding square root of a negative number, etc. will result in flashing zeros. Arguments out of the designated program range will result in incorrect answers or flashing zeros. When a computed value exceeds the calculator range, an overflow or underflow occurs and halts the program.

The KEYS column specifies the keys to be pressed. $\uparrow$ is the symbol used to denote the **ENTER** key. All other key designations are identical to those appearing on the HP-65. Ignore any blank positions in the KEYS column.

The DISPLAY column may show counters, intermediate or final results. In line 5 of the example, the answer will be displayed after pressing the **C** key.

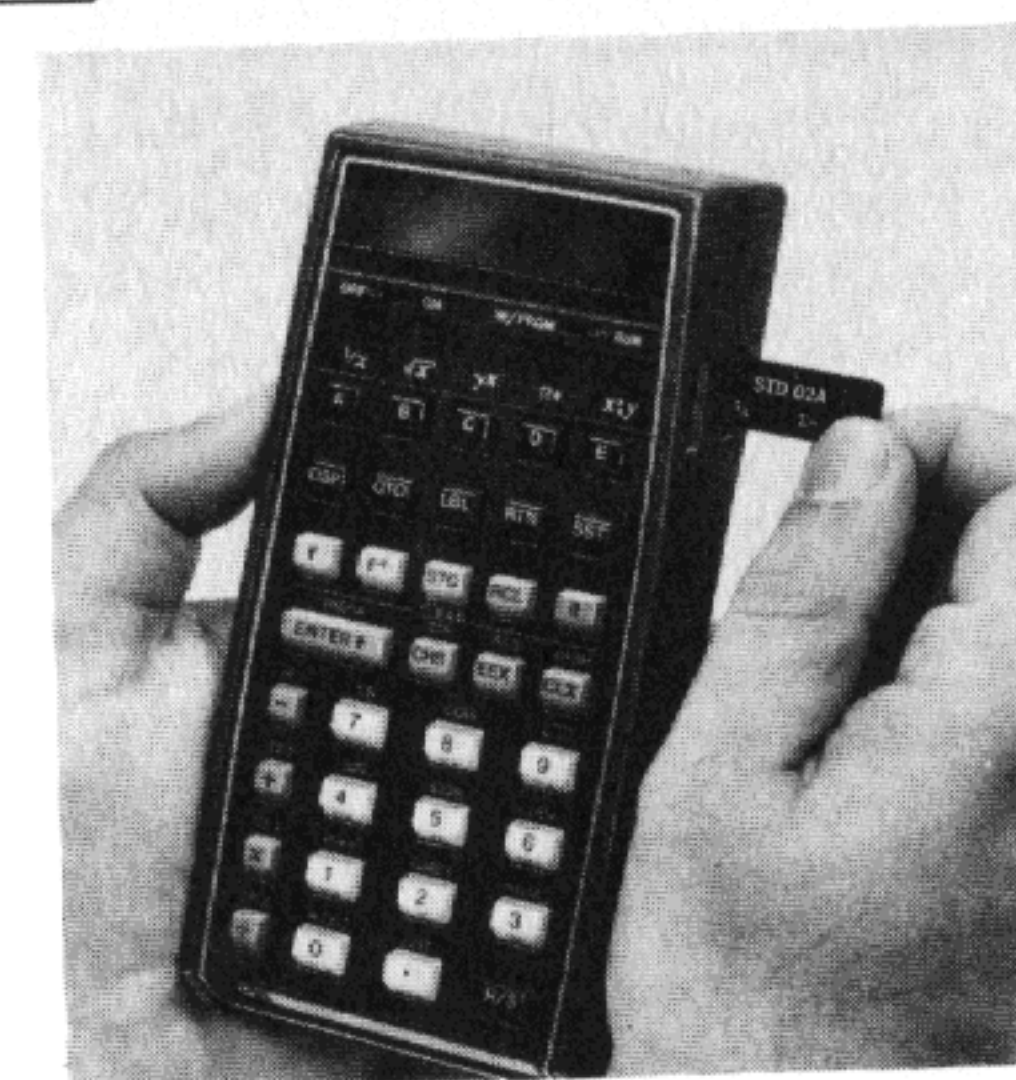
ENTERING A PROGRAM

From the card case supplied with this application pac, select a program card.

Set W/PRGM-RUN switch to RUN.

Turn the calculator ON. You should see 0.00

Gently insert the card (printed side up) in the right, lower slot as shown. When the card is part way in, the motor engages it and passes it out the left side of the calculator. Sometimes the motor engages but does not pull the card in. If this happens, push the card a little farther into the machine. Do not impede or force the card; let it move freely. (The display will flash if the card reads improperly. In this case, press **CLX** and reinsert the card.)



When the motor stops, remove the card from the left side of the calculator and insert it in the upper "window slot" on the right side of the calculator.

The program is now stored in the calculator. It remains stored until another program is entered or the calculator is turned off.



FACTORS OF AN INTEGER

FACTORS OF AN INTEGER

MATH 1-01A

This program finds all prime factors— $p_1, p_2, p_3, \dots$ —of a positive integer n ($\leq 2 \times 10^9$) and hence, determines if n is a prime number.

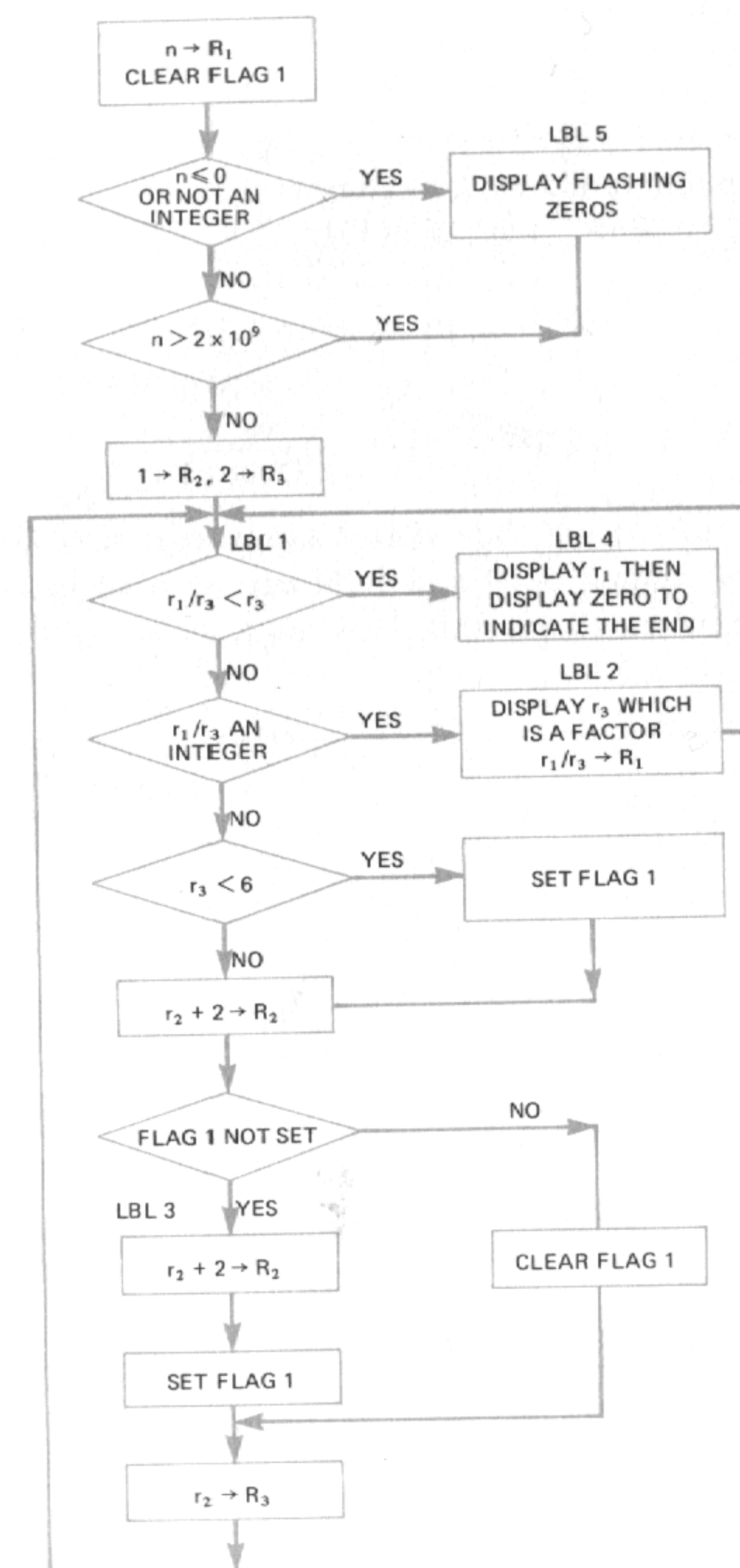
Note: Flashing zeros indicate that either

- (1) n is not an integer
- or (2) n is not positive
- or (3) $n > 2 \times 10^9$

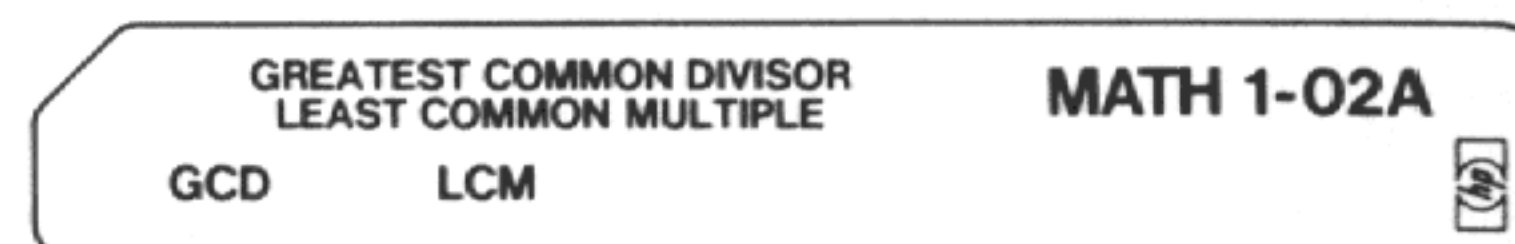
Examples:

1. $124 = 2 \times 2 \times 31$
2. 523 is a prime.
3. $4807 = 11 \times 19 \times 23$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		n	A	p_1
3	Perform 3 until $p_i = 0$		R/S	p_i
	($p_i = 0$ is an indication that all factors have been found.)			



GREATEST COMMON DIVISOR LEAST COMMON MULTIPLE



This program computes the greatest common divisor (GCD) and the least common multiple (LCM) for integers a, b. The program also finds integral coefficients s and t such that

$$\text{GCD}(a, b) = sa + tb$$

Note:

$$\text{LCM}(a, b) = \frac{ab}{\text{GCD}(a, b)}$$

The program does not use this relationship so that the two subprograms for finding GCD and LCM can be used independently in other main programs. Subroutine E is used for both subprograms.

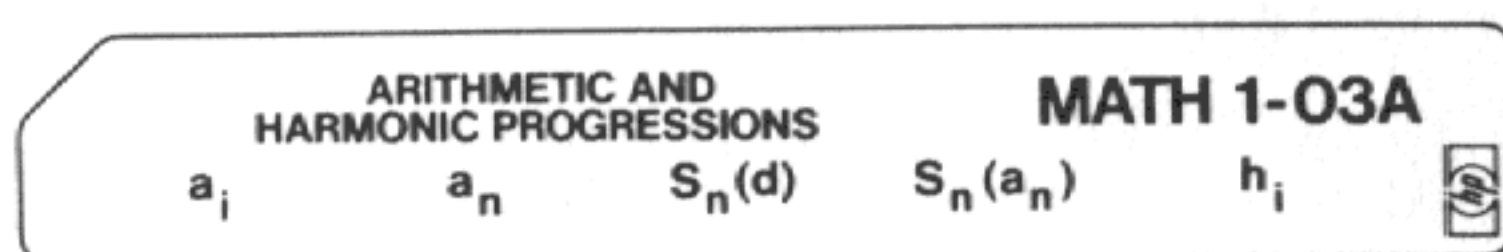
Examples:

1. $a = 240, b = 1144$
 $\text{GCD}(a, b) = 8.00$
 $s = 62.00$
 $t = -13.00$

2. $a = 240, b = 1144$
 $\text{LCM}(a, b) = 34320.00$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Obtain GCD (a, b)	a		
3		b		GCD
4				s
5				t
6	Obtain LCM (a, b)	a		
7		b		LCM

ARITHMETIC AND HARMONIC PROGRESSIONS



This program can be used for the following:

1. To display an arithmetic progression a_i ($i = 1, 2, 3, \dots$)

$$a_i = a_1 + (i - 1) d$$

2. To find the n^{th} term a_n of the progression

$$a_n = a_1 + (n - 1) d$$

3. To find the sum $S_n(d)$ of the first n terms of an arithmetic progression if the first term a_1 and the difference d are known

$$S_n(d) = na_1 + \frac{1}{2} n(n-1) d$$

4. To find the sum $S_n(a_n)$ if the first and the n^{th} terms are known

$$S_n(a_n) = \frac{n}{2} (a_1 + a_n)$$

5. To display the terms h_i ($i = 1, 2, 3, \dots$) of a harmonic progression

$$h_i = \frac{a}{b + (i - 1) c}$$

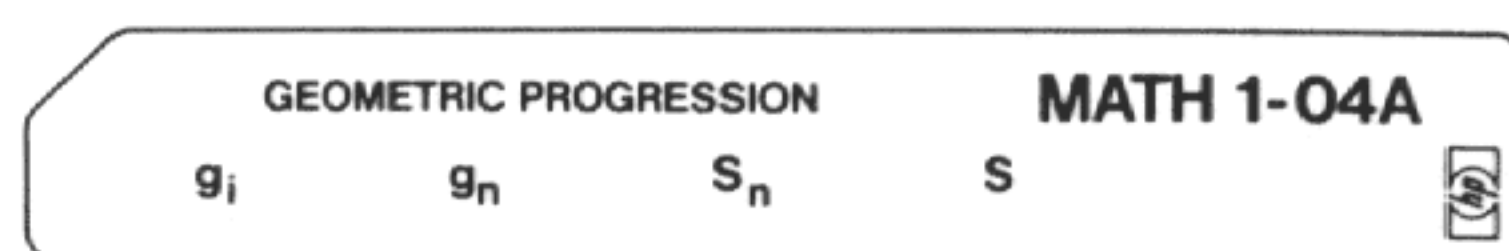
where $b + (i - 1) c \neq 0$ for $i = 1, 2, 3, \dots$

Examples:

1. $a_1 = 0$, $d = 17$, the arithmetic progression will be displayed as 0.00, 17.00, 34.00, 51.00, ...
2. The 25th term of the arithmetic progression with $a_1 = 2$, $d = 3.14$ is 77.36. ($S_{25} = 992.00$)
3. If $a_1 = 3.5$, $d = 2.15$ and $n = 11$, then $S_{11} = 156.75$
4. If $a_1 = 3.5$, $a_{11} = 25$, $n = 11$, then $S_{11} = 156.75$.
5. The harmonic progression with $a = 1$, $b = 2$, $c = 3$ is 0.50, 0.20, 0.13, 0.09, ...

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Display a_i	a_1		
3		d		a_1
4	Perform 4 for $i = 2, 3, \dots$			a_i
5	Calculate a_n (and S_n)	a_1		
6		d		
7		n		a_n
8				S_n
9	Calculate sum (given a_1, d, n)	a_1		
10		d		
11		n		$S_n(d)$
12	Calculate sum (given a_1, a_n, n)	a_1		
13		a_n		
14		n		$S_n(a_n)$
15	Display h_i	a		
16		b		
17		c		h_1
18	Perform 18 for $i = 2, 3, \dots$			h_i

GEOMETRIC PROGRESSION



This program can be used for the following:

1. To display a geometric progression g_i ($i = 1, 2, 3, \dots$)

$$g_i = ar^{i-1} \quad (a = g_1 \text{ is the first term})$$

2. To find the n^{th} term g_n of the progression

$$g_n = ar^{n-1}$$

3. To find the sum S_n of the first n terms of the progression

$$S_n = \frac{a(1-r^n)}{1-r}$$

4. To find the infinite sum S if the ratio r of two successive terms has absolute value less than 1.

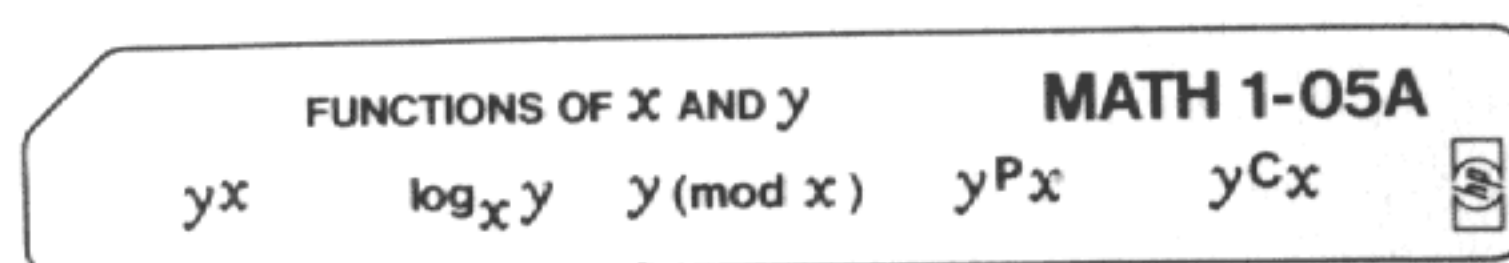
$$S = \frac{a}{1-r} \quad \text{if } -1 < r < 1$$

Note: If $|r| \geq 1$, use of the **[D]** key will cause flashing zeros in the display.

Examples:

1. Powers of 8 are 8.00, 64.00, 512.00, 4096.00, ...
2. The 14^{th} term of the geometric progression with $a = 2$, $r = -3.14$ is -5769197.69 .
3. If $a = 1$, $r = -2.1$, $n = 6$, then $S_6 = -27.34$.
4. If $a = 2$, $r = 0.5$, then $S = 4.00$.

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Display the progression	a	↑	
3		r	A	g_1
4	Perform 4 for $i = 2, 3, \dots$		R/S	g_i
5	Obtain the n^{th} term	a	↑	
6		r	↑	
7		n	B	g_n
8	Obtain the sum S_n	a	↑	
9		r	↑	
10		n	C	S_n
11	Obtain the infinite sum	a	↑	
12		r	D	S

FUNCTIONS OF x AND y 

The program can be used to find:

1. y^x for any real y and x . If y is negative then x must be an integer.

2. Logarithm of y (base x)

$$\log_x y = \frac{\ln y}{\ln x}$$

3. $y \pmod{x} = y - x \left[\text{integer part of } \left(\frac{y}{x} \right) \right]$

4. Permutation ${}_y P_x$

$${}_y P_x = \frac{y!}{(y-x)!}$$

where x, y are positive integers and $x \leq y$.

Program requires $y \leq 69$.

5. Combination ${}_y C_x$ (binomial coefficient)

$${}_y C_x = \frac{y!}{x!(y-x)!}$$

where x, y are positive integers and $x \leq y$.

Program requires $y \leq 69$.

Examples:

1. $(-32.24)^3 = -33510.82$

2. $\log_7 5 = 0.83$

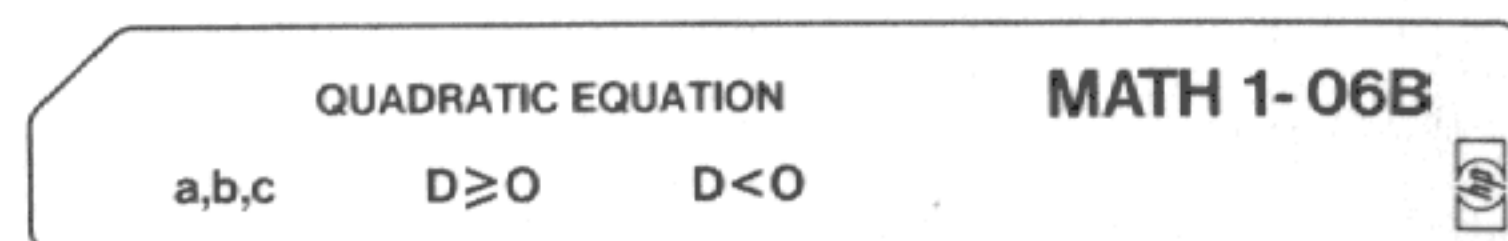
3. $52 \pmod{12} = 4.00$

4. ${}_7 P_5 = 2520.00$

5. ${}_7 C_5 = 21.00$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Compute y^x	y	$\uparrow$	
3		x	A	y^x
4	Compute $\log_x y$	y	$\uparrow$	
5		x	B	$\log_x y$
6	Compute $y \pmod{x}$	y	$\uparrow$	
7		x	C	$y \pmod{x}$
8	Compute ${}_y P_x$	y	$\uparrow$	
9		x	D	${}_y P_x$
10	Compute ${}_y C_x$	y	$\uparrow$	
11		x	E	${}_y C_x$

QUADRATIC EQUATION



The roots x_1, x_2 of $ax^2 + bx + c = 0$

are given by
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

However, better significance can be obtained in some cases by first computing the root with the largest absolute value using the following formula

$$x_1 = \frac{-ab}{|ab|} \left(\left| \frac{b}{2a} \right| + \sqrt{\frac{b^2 - 4ac}{4a^2}} \right)$$

then the smaller root by $x_2 = \frac{c}{x_1 a}$

If $D = (b^2 - 4ac)/4a^2$

is positive or zero, the roots are real. Otherwise, they are complex, being

$$u \pm iv = \frac{-b}{2a} \pm \frac{\sqrt{4ac - b^2}}{2a} i$$

Note: Subroutines D and E which are not used in this program will be used in conjunction with *Math 1-08A, Fourth Degree Polynomial Equation*.

Examples:

1. $2x^2 + 5x + 3 = 0$

($D = 0.06 > 0$)

$x_1 = -1.50$

$x_2 = -1.00$

2. $2x^2 + 3x + 4 = 0$

($D = -1.44 < 0$)

$x_1 = -0.75 + 1.20i$

$x_2 = -0.75 - 1.20i$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		a		
3		b		
4		c		D
5	If $D \geq 0$, roots are real			root 1
6				root 2
7	If $D < 0$, roots are complex			u
8				v

CUBIC EQUATION

CUBIC EQUATION

MATH 1-07A

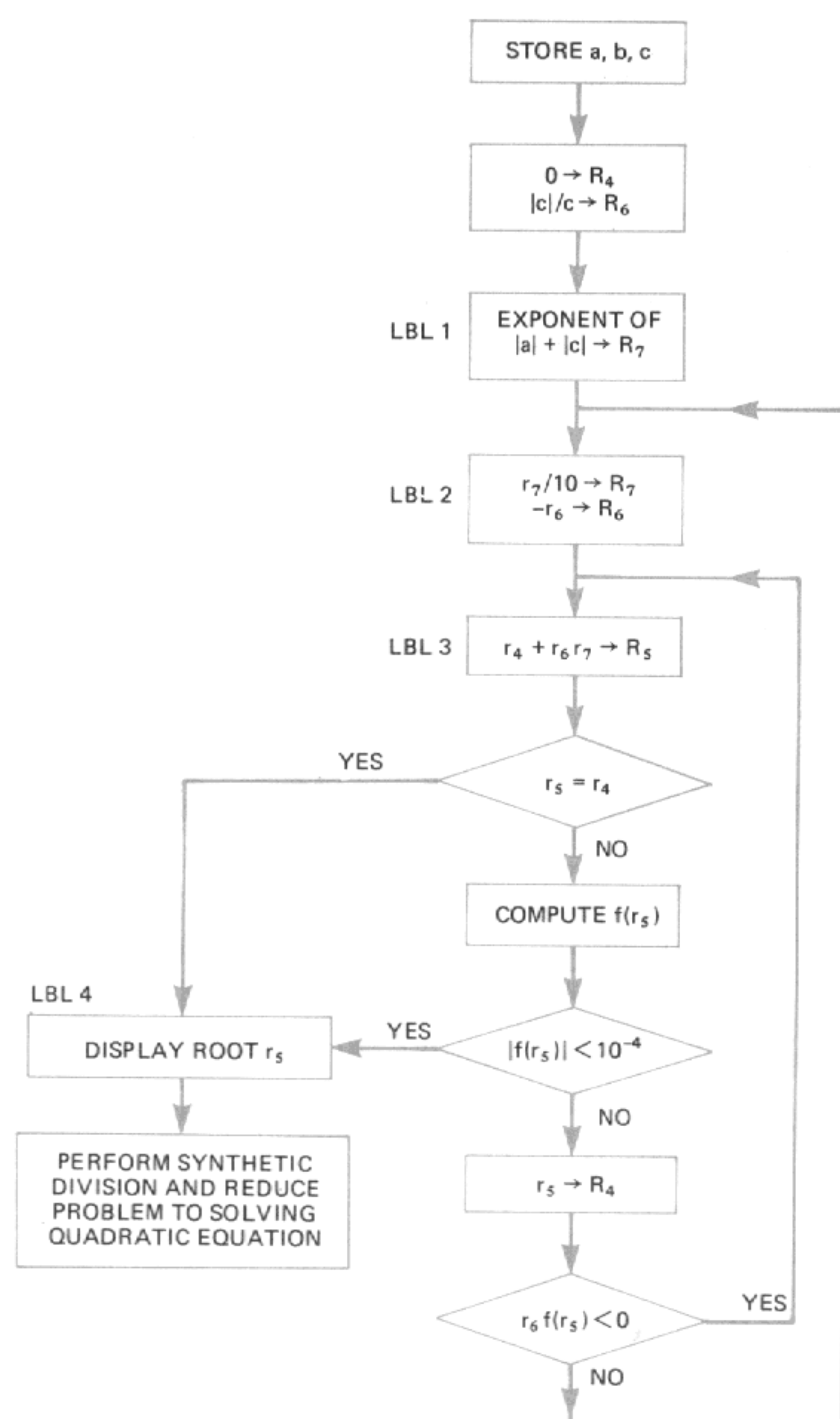


This program finds a real root of the cubic equation

$$f(x) = x^3 + ax^2 + bx + c = 0$$

where a, b, c are real.

Then by synthetic division, the problem is reduced to solving a quadratic equation.



Note: Program requires $c \neq 0$.

For if $c = 0$, zero is a real root and by factoring out x , we can use the quadratic formula to find the other two roots.

Examples:

1. $x^3 - 6x^2 + 11x - 6 = 0$
 $x_1 = 3.00, x_2 = 2.00, x_3 = 1.00$ (D = 0.25)
2. $x^3 - 4x^2 + 8x - 8 = 0$
 $x_1 = 2.00, x_2 = 1.00 + 1.73i, x_3 = 1.00 - 1.73i$ (D = -3.00)
3. $x^3 - 10x^2 - 2.25x + 22.5 = 0$
 $x_1 = -1.50, x_2 = 10.00, x_3 = 1.50$ (D = 18.06)

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program <i>Math 1-07A</i>			
2		a	STO 1	
3		b	STO 2	
4		c	STO 3	
5			A	root 1
6			R/S	
7	Enter program <i>Math 1-06B</i>		A	D
8	If $D < 0$, go to 11			
9	$D \geq 0$, roots are real		B	root 2
10			R/S	root 3
11	$D < 0$, roots $u \pm iv$ are complex		C	u
12			R/S	v

FOURTH DEGREE POLYNOMIAL EQUATION

FOURTH DEGREE
POLYNOMIAL EQUATION

MATH 1-08A



This program can be used in conjunction with *Math 1-07A, Cubic Equation*, and *Math 1-06B, Quadratic Equation*, to find the roots (real and/or imaginary) of a fourth degree polynomial equation of the form:

$$x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$$

where a_i ($i = 0, 1, 2, 3$) are real.

Brown's Method is used. First, solve the cubic equation:

$$f(y) = y^3 + b_2y^2 + b_1y + b_0 = 0$$

where $b_2 = -a_2$

$$b_1 = a_3a_1 - 4a_0$$

$$b_0 = a_0(4a_2 - a_3^2) - a_1^2$$

Suppose $f(y)$ has roots y_1, y_2, y_3 and let y_0 be the largest real root of $f(y)$.

Then the fourth degree equation is reduced to two quadratic equations:

$$x^2 + (A + C)x + (B + D) = 0$$

$$x^2 + (A - C)x + (B - D) = 0$$

where $A = \frac{a_3}{2}$, $B = \frac{y_0}{2}$

$$D = \sqrt{B^2 - a_0}$$

$$C = \begin{cases} \left(AB - \frac{a_1}{2} \right) / D & \text{if } D \neq 0 \\ \sqrt{A^2 - a_2 + y_0} & \text{if } D = 0 \end{cases}$$

Roots can be found by solving the quadratic equations.

Note: In order to get more accurate answers, intermediate results (e.g., y_0, a_3, a_2, a_1, a_0) should be recorded and entered to as many decimal places as possible.

Reference: Numerical methods in Engineering, Salvadori and Baron, Prentice-Hall, 1961

Examples:

1. $x^4 - 2x^3 + 3x^2 - 2x + 2 = 0$

$b_0 = 12.00 \quad y_1 = -2.00$

$D_2 = 0.25 \quad y_2 = 3.00$

$y_3 = 2.00$ (hence $y_0 = y_2 = 3$)

$D_3 = -1.00$

(a) Root 1 = i

(b) Root 2 = -i

$D_4 = -1.00$

(c) Root 3 = $1.00 + 1.00i$

(d) Root 4 = $1.00 - 1.00i$

2. $4x^4 - 8x^3 - 13x^2 - 10x + 22 = 0$

Rewrite equation as: $x^4 - 2x^3 - \frac{13}{4}x^2 - 2.5x + \frac{22}{4} = 0$

$b_0 = -99.75$

$y_1 = 4.75$

$D_2 = -5.00$ (hence $y_0 = y_1 = 4.75$)

$D_3 = -1.00$

(a) Root 1 = $-1.00 + 1.00i$

(b) Root 2 = $-1.00 - 1.00i$

$D_4 = 1.25$

(c) Root 3 = 3.12

(d) Root 4 = 0.88

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program <i>Math 1-08A</i>			
2		a_3	R/S	
3		a_2	R/S	
4		a_1	R/S	
5		a_0	R/S	b_0
6	If $b_0 \neq 0$, go to 17			
7	$b_0 = 0$ hence $y_1 = 0$			
8	Enter program <i>Math 1-06B</i>			
9			D A	D_1
10	If $D_1 < 0$, $y_0 = 0$, go to 13			
11	$D_1 \geq 0$, y_2, y_3 are real		B	y_2
12	Record y_2, y_3		R/S	y_3
13	$y_0 = \text{Max. of real roots}$	y_0	STO 4	
14	Enter program <i>Math 1-08A</i>			

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
15	Compute $A \pm C, B \pm D$		B	
16	Go to 32			
17	Enter program <i>Math 1-07A</i>			
18			A	y_1
19	Record y_1		R/S	
20	Enter program <i>Math 1-06B</i>			
21			A	D_2
22	If $D_2 < 0$, $y_0 = y_1$, go to 25			
23	$D_2 \geq 0$, y_2, y_3 are real		B	y_2
24	Record y_2, y_3		R/S	y_3
25	$y_0 = \text{Max. of real roots}$	y_0	STO 4	
26	Enter program <i>Math 1-08A</i>			
27		a_3	R/S	
28		a_2	R/S	
29		a_1	R/S	
30		a_0	R/S	
31	Compute $A \pm C, B \pm D$		B	
32	Enter program <i>Math 1-06B</i>			
33			A	D_3
34	If $D_3 < 0$, go to 38			
35	$D_3 \geq 0$, roots are real		B	root
36			R/S	root
37	Go to 40			
38	$D_3 < 0$, roots are $u \pm iv$		C	u
39			R/S	v
40	Solve the 2 nd quadratic		E A	D_4
41	If $D_4 < 0$, go to 44			
42	$D_4 \geq 0$, roots are real		B	root
43			R/S	root
	(All roots are found)			
44	$D_4 < 0$, roots are $s \pm it$		C	s
45			R/S	t

FIFTH DEGREE POLYNOMIAL EQUATION

FIFTH DEGREE
POLYNOMIAL EQUATION

MATH 1-09A

This program finds one real root of

$$f(x) = x^5 + \alpha_4 x^4 + \alpha_3 x^3 + \alpha_2 x^2 + \alpha_1 x + \alpha_0 = 0$$

where α_i ($i = 0, 1, 2, 3, 4$) are real.

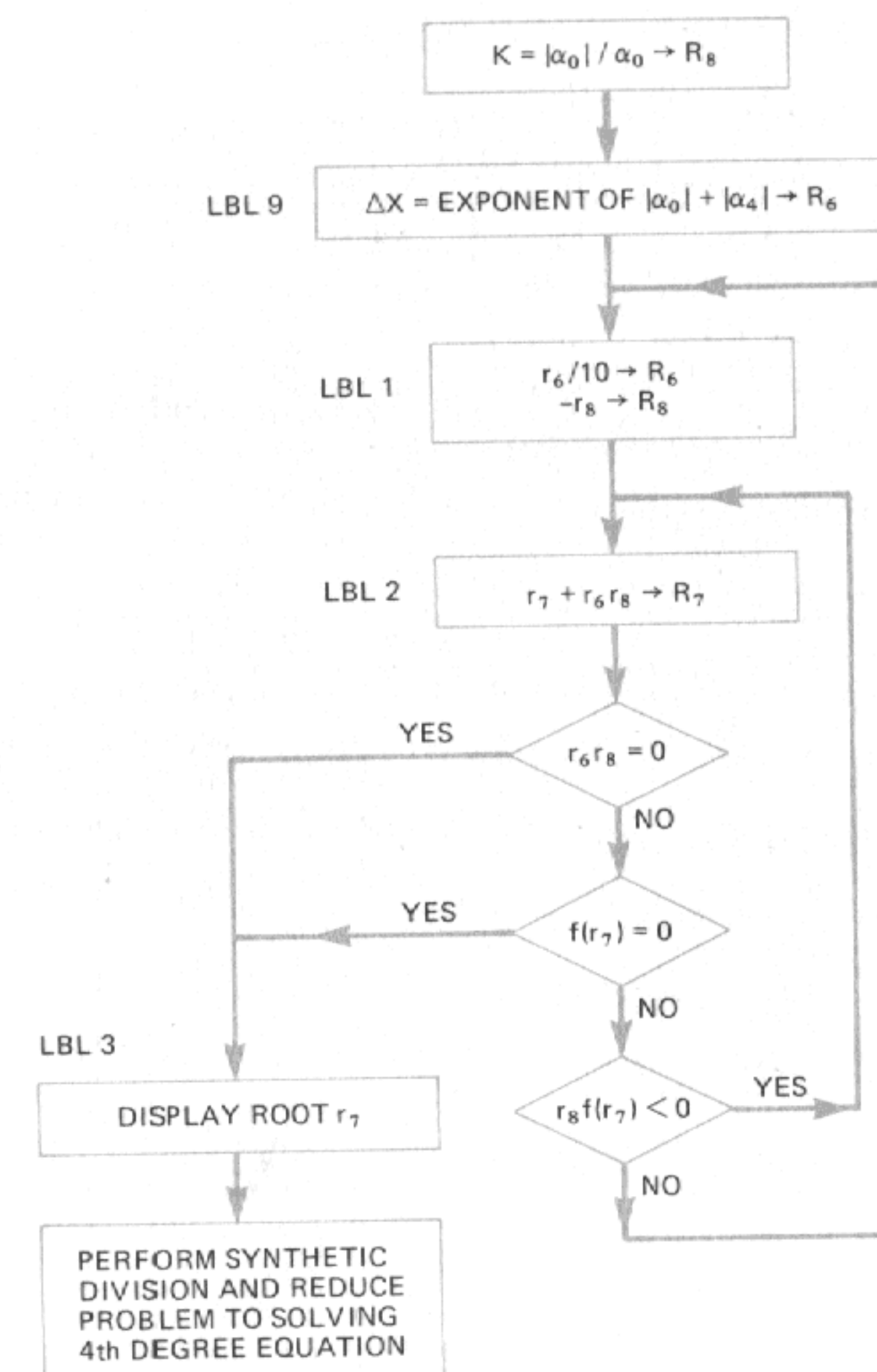
The real root is found by an iterative process, then the fifth degree equation is reduced to a fourth degree equation

$$x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0$$

by synthetic division.

Math 1-08A, Fourth Degree Polynomial Equation then can be used to find the other four roots (real and/or imaginary) of $f(x)$.

Note: Program requires $\alpha_0 \neq 0$. For if $\alpha_0 = 0$, zero is a root, problem is reduced to solving fourth degree polynomial equation.



Examples:

1. $x^5 - x^4 - 101x^3 + 101x^2 + 100x - 100 = 0$

(a) Root 1 = 10.00

$$a_3 = 9.00, a_2 = -11.00, a_1 = -9.00, a_0 = 10.00$$

$$b_0 = -1331.00, y_1 = 11.00, D_2 = 0.00, y_2 = -11.00, \\ y_3 = -11.00 \quad (\text{hence } y_0 = y_1 = 11)$$

$$D_3 = 0.00$$

(b) Root 2 = 1.00

(c) Root 3 = 1.00

$$D_4 = 20.25$$

(d) Root 4 = -10.00

(e) Root 5 = -1.00

2. $x^5 - 23x^4 + 12x^2 + 13x + 69 = 0$ (Note: $\alpha_3 = 0$)

(a) Root 1 = -1.31474954 (Press **DSP** **□** **8**)

$$a_3 = -24.31474954$$

$$a_2 = 31.96780575$$

$$a_1 = -30.02965787$$

$$a_0 = 52.48147884$$

$$b_0 = -2.521832950 \ 04$$

$$y_1 = 36.58750070$$

$$D_2 < 0 \quad (\text{hence } y_0 = y_1 = 36.5875007)$$

$$D_3 < 0$$

(b) Root 2 = -0.09426686 + 1.21928470i

(c) Root 3 = -0.09426686 - 1.21928470i

$$D_4 > 0$$

(d) Root 4 = 22.97594875

(e) Root 5 = 1.52733450

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program <i>Math 1-09A</i>			
2	Clear registers		f REG	
3	Store coefficients	α_4	STO 1	
4		α_3	STO 2	
5		α_2	STO 3	
6		α_1	STO 4	
7	($\alpha_0 \neq 0$)	α_0	STO 5	
8			R/S	root
9	Perform synthetic division		R/S	
10	Recall and record coefficients		RCL 8	a_3
11			RCL 7	a_2
12			RCL 6	a_1
13			RCL 5	a_0
14	Enter program <i>Math 1-08A</i>			
15			A	b_0
	(If $b_0 = 0$ go to line 7 of the			
	Instructions for <i>Math 1-08A</i> ,			
	<i>Fourth Degree Polynomial</i>			
	<i>Equation</i> . If $b_0 \neq 0$ go to line			
	17 of that program.)			

SIMULTANEOUS EQUATIONS IN TWO UNKNOWNNS

SIMULTANEOUS EQUATIONS
IN TWO UNKNOWNNS

MATH 1-10A



$$\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$$

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{D}$$

$$y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{D}$$

where determinant $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \neq 0$

Note: Flashing zeros indicate $D = 0$.

Example:

$$\begin{cases} 7.32x - 9.08y = 3.14 \\ 12.39x + 7y = 0.05 \end{cases}$$

$$D = 163.74$$

$$\begin{cases} x = 0.14 \\ y = -0.24 \end{cases}$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		a		
3		b		
4		e		
5		c		
6		d		
7		f		D
8	If $D \neq 0$			x
9				y

SIMULTANEOUS EQUATIONS IN THREE UNKNOWNNS

SIMULTANEOUS EQUATIONS
IN THREE UNKNOWNNS

MATH 1-11A



$$\begin{cases} a_1x + b_1y + c_1z = d_1 & (1) \\ a_2x + b_2y + c_2z = d_2 & (2) \\ a_3x + b_3y + c_3z = d_3 & (3) \end{cases}$$

Renumber the equations if necessary such that $a_1 \neq 0$.

This program uses the following algorithm:

$$x + \frac{b_1}{a_1}y + \frac{c_1}{a_1}z = \frac{d_1}{a_1} \quad (1)'$$

$$\left(\frac{b_1a_2}{a_1} - b_2\right)y + \left(\frac{c_1a_2}{a_1} - c_2\right)z = \frac{d_1a_2}{a_1} - d_2 \quad (2)'$$

$$\left(\frac{b_1a_3}{a_1} - b_3\right)y + \left(\frac{c_1a_3}{a_1} - c_3\right)z = \frac{d_1a_3}{a_1} - d_3 \quad (3)'$$

Solve (2)' and (3)' for y, z if $D_2 \neq 0$, where

$$D_2 = \begin{vmatrix} \frac{b_1a_2}{a_1} - b_2 & \frac{c_1a_2}{a_1} - c_2 \\ \frac{b_1a_3}{a_1} - b_3 & \frac{c_1a_3}{a_1} - c_3 \end{vmatrix}$$

Then solve for x from (1)'.

Example:

$$\begin{cases} 3.14x + 10.02y - 7z = 1 \\ 0.25x + 30.3y - 9.1z = 2 \\ -3.5x + 27.4y + 8z = 3 \end{cases}$$

$$D_2 = 335.31$$

$$\begin{cases} x = 0.29 \\ y = 0.11 \\ z = 0.14 \end{cases}$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		a_1	RTN	
3		b_1	↑	
4		c_1	↑	
5		d_1	R/S	
6		a_2	↑	
7		b_2	↑	
8		c_2	R/S	
9		a_3	↑	
10		b_3	↑	
11		c_3	R/S	
12		d_2	↑	
13		d_3	R/S	D_2
14	If $D_2 \neq 0$		R/S	x
15			R/S	y
16			R/S	z

SYNTHETIC DIVISION

SYNTHETIC DIVISION

MATH 1-12A



This program performs synthetic division on a polynomial of degree n (with real coefficients)

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

by $x - x_0$ so that

$$a_n x^n + \dots + a_1 x + a_0$$

$$= (x - x_0)(b_{n-1} x^{n-1} + b_{n-2} x^{n-2} + \dots + b_1 x + b_0) + R$$

where $b_{n-1} = a_n$

$$b_k = b_{k+1} x_0 + a_{k+1} \quad \text{for } k = n-2, \dots, 0$$

$$R = b_0 x_0 + a_0$$

Note: Program requires $n \leq 7$.

If $n < 7$, let

$$a_7 = \dots = a_{n+1} = 0$$

Examples:

$$1. \quad x^5 - 4x^4 + 7x^3 - 10x^2 + 8 = (x - 2)(x^4 - 2x^3 + 3x^2 - 4x - 8) - 8$$

$$2. \quad x^5 - 4x^4 + 7x^3 - 10x^2 + 8 \\ = (x - 2.65)(x^4 - 1.35x^3 + 3.42x^2 - 0.93x - 2.47) + 1.47$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	If $n < 7$, let $a_7 = \dots = a_{n+1} = 0$	a_7		
3		a_6		
4		a_5		
5		a_4	A	
6		a_3		
7		a_2		
8		a_1		
9		a_0	R/S	
10		x_0	B	b_6
11	Perform 11 for $i = 5, 4, \dots, 0$		R/S	b_i
12			R/S	R
	(For a new x_0 , go to 10)			

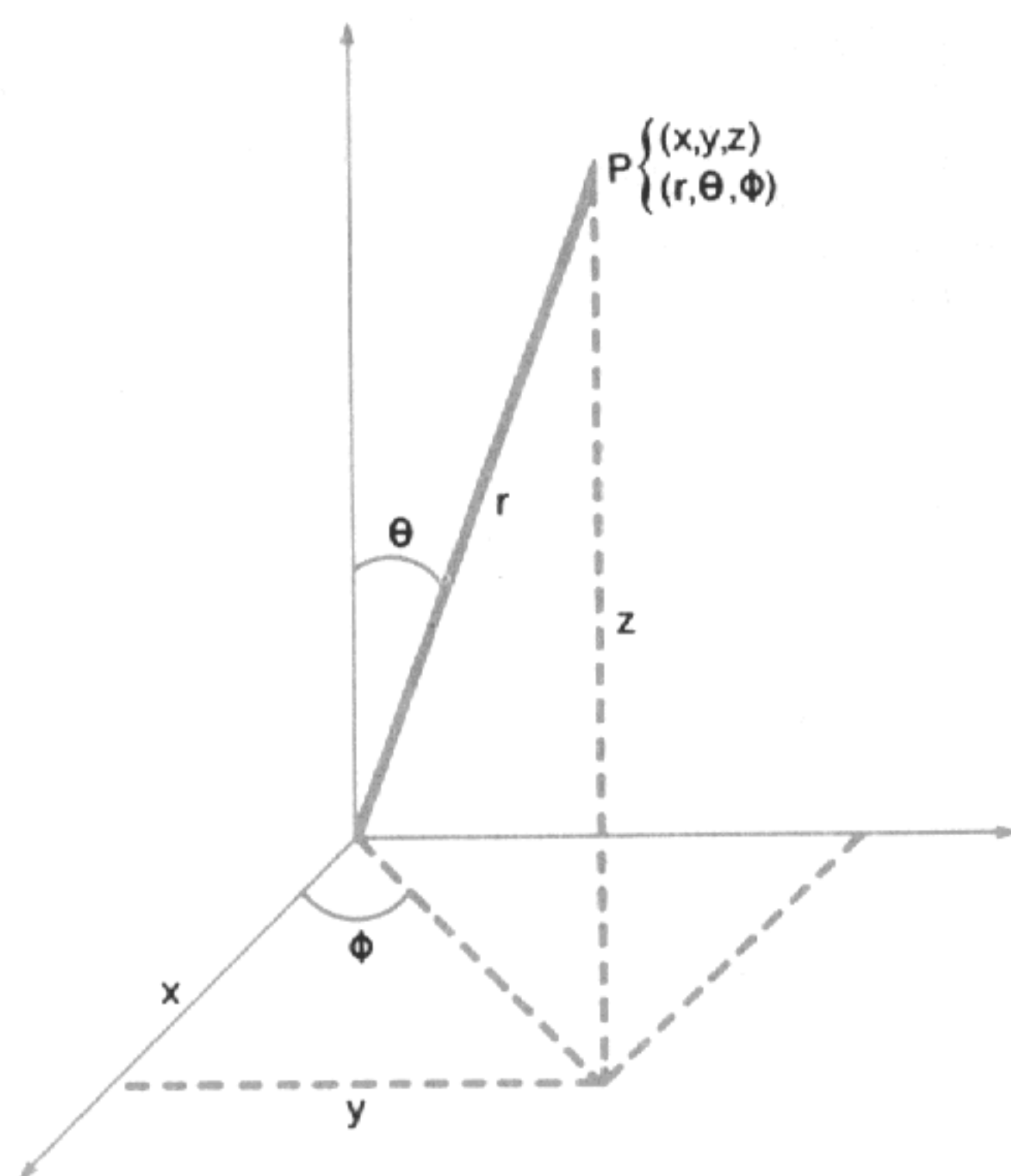
RECTANGULAR, SPHERICAL CONVERSIONS

RECTANGULAR,
SPHERICAL CONVERSIONS
R→S S→R

MATH 1-13A

$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \phi = \tan^{-1} \left(\frac{y}{x} \right) \\ \theta = \cos^{-1} (z / \sqrt{x^2 + y^2 + z^2}) \end{cases}$$



Example:

$$\begin{cases} x = 3 \\ y = 4 \\ z = 5 \end{cases}$$

$$\begin{cases} r = 7.07 \\ \phi = 53.13^\circ = 0.93 \text{ radians} = 59.03 \text{ grads} \\ \theta = 45.00^\circ = 0.79 \text{ radians} = 50.00 \text{ grads} \end{cases}$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Convert R→S	z		
3		y		
4		x		r
5				ϕ
6				θ
7	Put r, ϕ , θ in X, Y, Z registers			r
8	Convert S→R	θ		
9		ϕ		
10		r		x
11				y
12				z
13	Put x, y, z in X, Y, Z registers			x

TRANSLATION AND/OR ROTATION OF COORDINATE AXES

TRANSLATION AND/OR ROTATION OF COORDINATE AXES				MATH 1-14A	
x_0, y_0	T	θ	R	T & R	

1. Suppose point P has coordinates (x, y) with respect to the coordinate system having x, y axes. If we translate the system to a new system with origin at (x_0, y_0) , then point P will have coordinates (x', y') with respect to the new system.

$$\begin{cases} x' = x - x_0 \\ y' = y - y_0 \end{cases}$$

2. Suppose point P has coordinates (x, y) with respect to the coordinate system having x, y axes. If we rotate the axes with an angle θ , point P will have new coordinates (x'', y'') .

$$\begin{cases} x'' = x \cos \theta + y \sin \theta \\ y'' = -x \sin \theta + y \cos \theta \end{cases}$$

3. Suppose point P has coordinates (x, y) with respect to the coordinate system having x, y axes. If we translate the system to a new system with origin at (x_0, y_0) , then rotate the axes with an angle θ , point P will have coordinates (x''', y''') with respect to the new system.

$$\begin{cases} x''' = (x - x_0) \cos \theta + (y - y_0) \sin \theta \\ y''' = -(x - x_0) \sin \theta + (y - y_0) \cos \theta \end{cases}$$

Examples:

- $P = (5, -10)$
 $(x_0, y_0) = (2, 3)$
After translation, $P = (3, -13)$.
- $P_1 = (5, -10), P_2 = (3, -13)$
 $\theta = 45^\circ$
After rotation, $P_1 = (-3.54, -10.61), P_2 = (-7.07, -11.31)$.
- $P = (5, -10)$
After a translation to a new origin $(2, 3)$ and a rotation of 45° ,
 $P = (-7.07, -11.31)$.

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Perform translation	y_0	↑	
3		x_0	A	
4		y	↑	
5		x	B	x'
6			g x↔y	y'
	(For a different point, go to 4.)			
7	Perform rotation	θ	C	
8		y	↑	
9		x	D	x''
10			g x↔y	y''
	(For a different point, go to 8.)			
11	Perform translation and rotation	y_0	↑	
12		x_0	A	
13		θ	C	
14		y	↑	
15		x	E	x'''
16			g x↔y	y'''
	(For a different point, go to 14.)			

ANGLE CONVERSIONS

ANGLE CONVERSIONS				MATH 1-15A
DEG	RAD	GRD	MIL	

This program can be used to convert an angle in one angular unit to any other unit. Angles can be expressed in degrees, radians, grads or mils.

$$360^\circ = 2\pi \text{ radians} = 400 \text{ grads} = 6400 \text{ mils}$$

Examples:

1. $270^\circ = 300 \text{ grads}$
2. $1600 \text{ mils} = 90^\circ$
3. $2 \text{ radians} = 127.32 \text{ grads}$
4. $360^\circ = 6.28 \text{ radians}$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Clear flag 1		f^{-1} SF 1	
3	Convert x from degrees to grads	x	A C	
4	Convert x from mils to degrees	x	D A	
5	Convert x from radians to grads	x	B C	
6	Convert x from degrees to			
7	radians	x	A B	
	etc.			

SECONDARY VALUES OF $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$

SECONDARY VALUES OF $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$			MATH 1-16A
$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$	

Inverse trigonometric functions are multiple-valued functions, the HP-65 gives principal values as answers.

This program computes the secondary values of arc sine, arc cosine and arc tangent.

Secondary value for arc sin = 180° - principal value

Secondary value for arc cos = 360° - principal value

Secondary value for arc tan = 180° + principal value

Examples:

1. In DEG mode: $\sin^{-1}(.7) = 135.57^\circ$
 $\cos^{-1}(.7) = 314.43^\circ$
2. In RAD mode: $\cos^{-1}(.7) = 5.49$
 $\tan^{-1}(.7) = 3.75$
3. In GRD mode: $\sin^{-1}(-0.87) = 267.18$
 $\tan^{-1}(-0.87) = 154.42$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Clear flags		E	
3	If machine is in RAD mode		f SF1	
4	If machine is in GRD mode		f SF2	
5	Obtain $\sin^{-1} x$	x	A	$\sin^{-1} x$
6	Obtain $\cos^{-1} x$	x	B	$\cos^{-1} x$
7	Obtain $\tan^{-1} x$	x	C	$\tan^{-1} x$
	(For a new case in different			
	mode, go to 2.)			

TRIGONOMETRIC FUNCTIONS

TRIGONOMETRIC FUNCTIONS				MATH 1-17A
COT	SEC	CSC	ARC	

$$\cot x = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\operatorname{arc} \cot x = \operatorname{arc} \tan \frac{1}{x}$$

$$\operatorname{arc} \sec x = \operatorname{arc} \cos \frac{1}{x}$$

$$\operatorname{arc} \csc x = \operatorname{arc} \sin \frac{1}{x}$$

Restriction: x can not be a discontinuous point of the function or flashing zeros will result.

Examples:

- In DEG mode: $\cot 37^\circ = 1.33$ $\sec 45^\circ = 1.41$
- In RAD mode: $\cot 2 = -0.46$ $\operatorname{arc} \sec 2 = 1.05$
- In GRAD mode: $\csc 100 = 1.00$ $\operatorname{arc} \csc 1 = 100.00$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Clear flag 1		E	
3	Obtain cot x	x	A	cot x
4	Obtain sec x	x	B	sec x
5	Obtain csc x	x	C	csc x
6	Obtain arc cot x	x	D A	arc cot x
7	Obtain arc sec x	x	D B	arc sec x
8	Obtain arc csc x	x	D C	arc csc x

HYPERBOLIC FUNCTIONS

HYPERBOLIC FUNCTIONS				MATH 1-18A
SINH	COSH	TANH	1/x	

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} \quad (x \neq 0)$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{coth} x = \frac{1}{\tanh x} \quad (x \neq 0)$$

Examples:

- $\sinh 1.5 = 2.13$
- $\cosh 5.9 = 182.52$
- $\tanh 1.3 = 0.86$
- $\operatorname{csch} 0.95 = 0.91$
- $\operatorname{sech} (-3) = 0.10$
- $\operatorname{coth} (-1.99) = -1.04$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Obtain sinh x	x	A	sinh x
3	Obtain cosh x	x	B	cosh x
4	Obtain tanh x	x	C	tanh x
5	Obtain csch x	x	A D	csch x
6	Obtain sech x	x	B D	sech x
7	Obtain coth x	x	C D	coth x

INVERSE HYPERBOLIC FUNCTIONS

INVERSE HYPERBOLIC FUNCTIONS

MATH 1-19A

 SINH^{-1} COSH^{-1} TANH^{-1} $1/x$


$$\sinh^{-1} x = \ln [x + (x^2 + 1)^{1/2}]$$

$$\cosh^{-1} x = \ln [x + (x^2 - 1)^{1/2}] \quad (x \geq 1)$$

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x} \quad (x^2 < 1)$$

$$\operatorname{csch}^{-1} x = \sinh^{-1} \frac{1}{x} \quad (x \neq 0)$$

$$\operatorname{sech}^{-1} x = \cosh^{-1} \frac{1}{x} \quad (0 < x \leq 1)$$

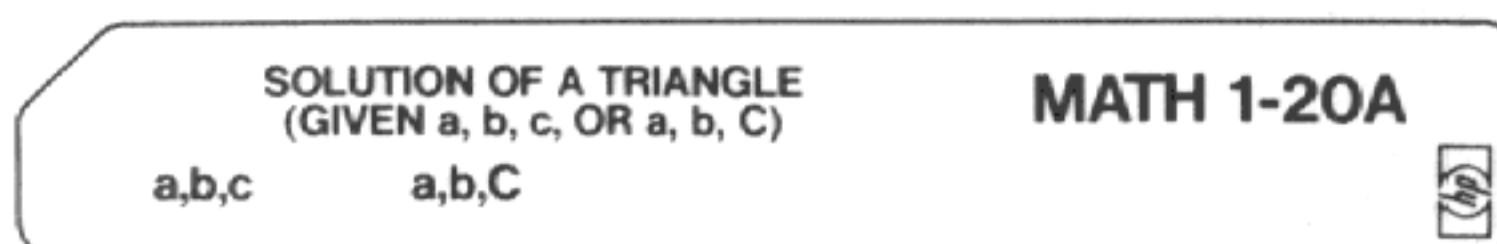
$$\operatorname{coth}^{-1} x = \tanh^{-1} \frac{1}{x} \quad (x^2 > 1)$$

Examples:

1. $\sinh^{-1} 3.5 = 1.97$
2. $\cosh^{-1} 100 = 5.30$
3. $\tanh^{-1} (-0.7) = -0.87$
4. $\operatorname{csch}^{-1} 3 = 0.33$
5. $\operatorname{sech}^{-1} 0.5 = 1.32$
6. $\operatorname{coth}^{-1} 5.4 = 0.19$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Obtain $\sinh^{-1} x$	x	A	$\sinh^{-1} x$
3	Obtain $\cosh^{-1} x$	x	B	$\cosh^{-1} x$
4	Obtain $\tanh^{-1} x$	x	C	$\tanh^{-1} x$
5	Obtain $\operatorname{csch}^{-1} x$	x	D A	$\operatorname{csch}^{-1} x$
6	Obtain $\operatorname{sech}^{-1} x$	x	D B	$\operatorname{sech}^{-1} x$
7	Obtain $\operatorname{coth}^{-1} x$	x	D C	$\operatorname{coth}^{-1} x$

SOLUTION OF A TRIANGLE (GIVEN a, b, c, OR a, b, C)



The following formulas are used to solve an oblique triangle in this program and also in *Math 1-21A*, *Math 1-22A*.

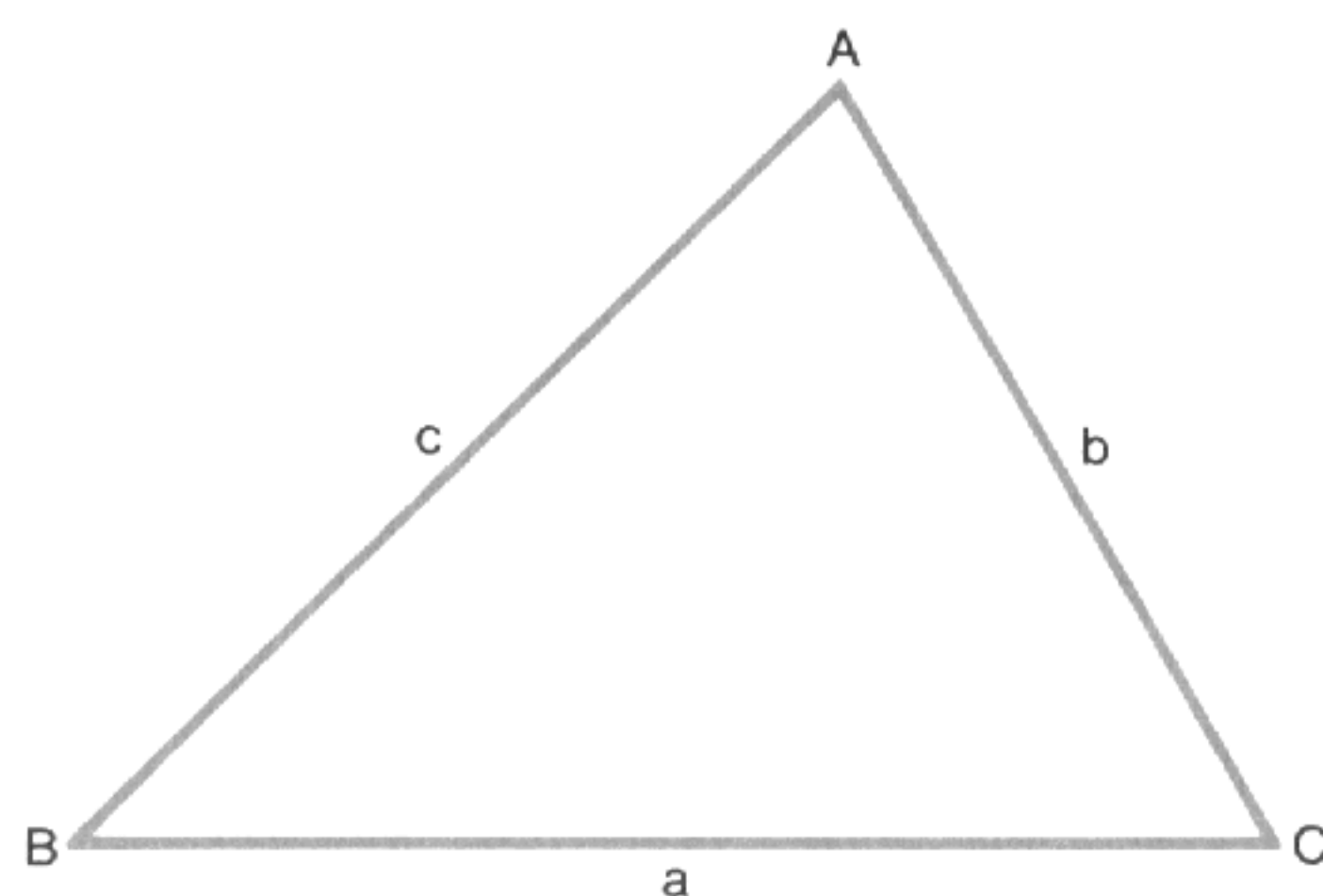
Law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosine

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$A + B + C = 180^\circ$$



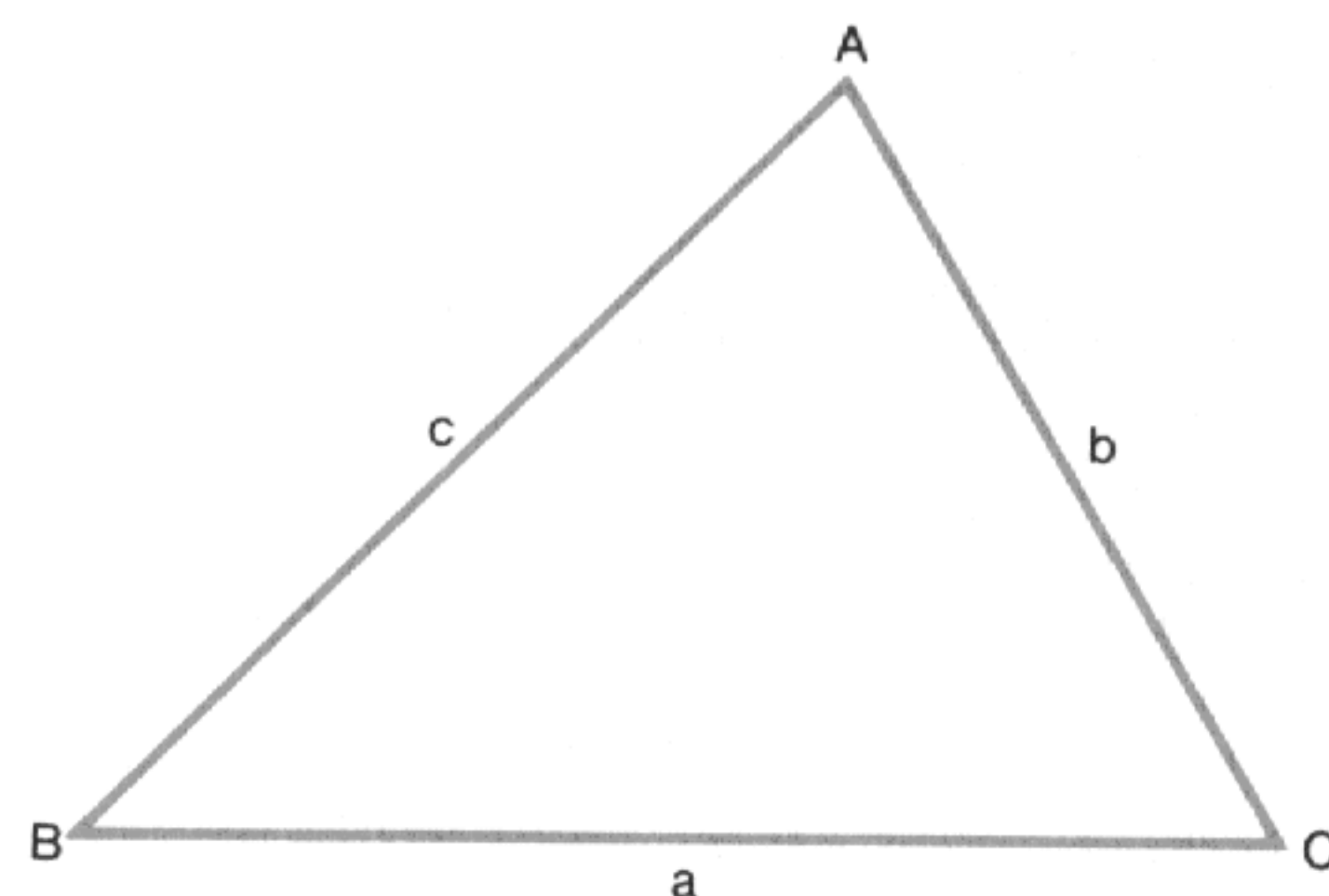
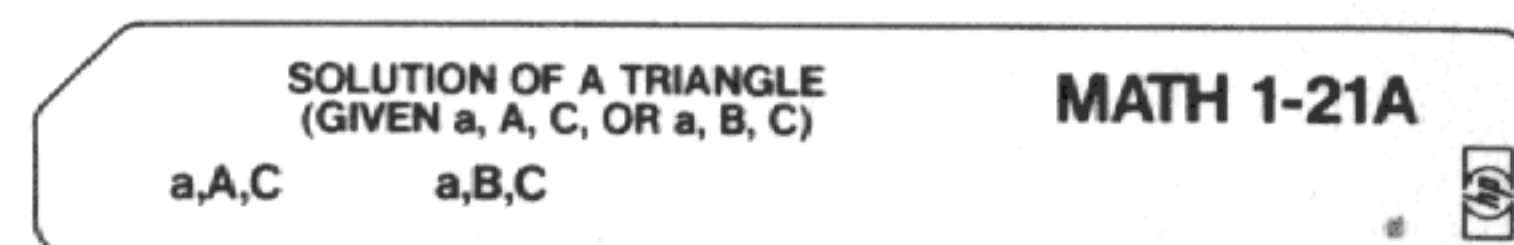
- Notes:
1. In some cases of obtuse triangles, this program generates one incorrect angle such that the sum of the three angles does not add up to 180° . Draw a sketch of the given triangle to locate the incorrect angle. Adding the difference between 180° and the sum of the three angles to this angle will yield the correct answer. This is due to the fact that the program uses inverse trigonometric functions of the HP-65 which always returns the principal values. For cases mentioned above, secondary values are required (see *Math 1-16A*, *Secondary Values of $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$*).
 2. If machine is in DEG mode, all angles are assumed to be in decimal degrees.

Examples:

1. Given $a = 30.3$, $b = 40.4$, $c = 62.6$ then
 $C = 123.99^\circ$
 $B = 32.35^\circ$
 $A = 23.66^\circ$
2. Given $a = 132$, $b = 224$, $C = 28^\circ 40'$ (convert angle C to decimal degrees first), then
 $c = 125.35$
 $A = 30.34^\circ$
 $B = 120.99^\circ$ (secondary value, display shows 59.01)

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	If a, b, c are given	a		
3		b		
4		c		C
5				B
6				A
7	If a, b, C are given	a		
8		b		
9		C		c
10				A
11				B

SOLUTION OF A TRIANGLE (GIVEN a, A, C, OR a, B, C)



Note: All angles are assumed to be in decimal degrees.

Examples:

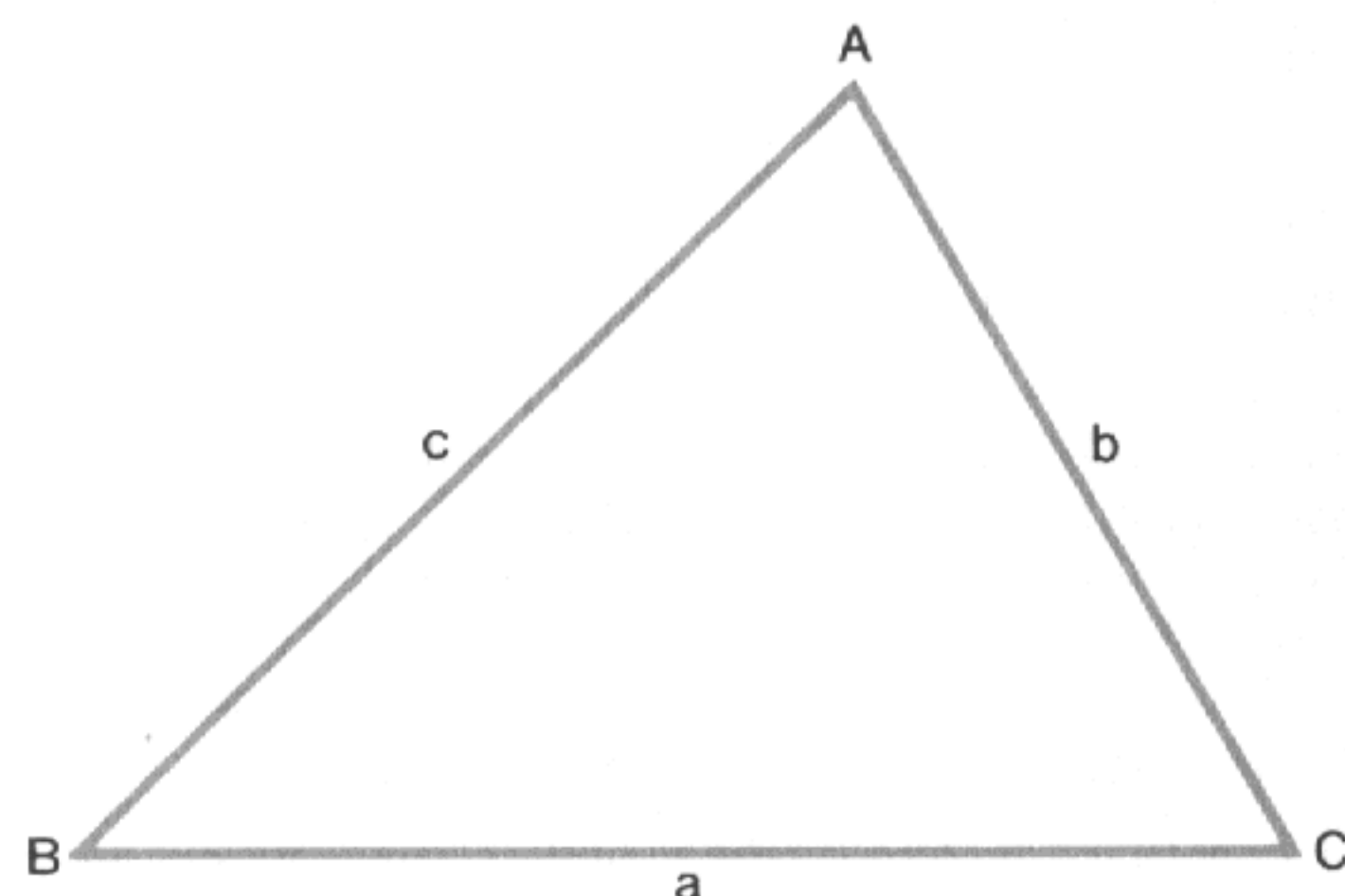
- Given $a = 17.5$, $A = 41^\circ 14'$, $C = 62^\circ 12'$ (convert A, C to decimal degrees first) then
 $B = 76.57^\circ$
 $b = 25.82$
 $c = 23.49$
- Given $a = 25.2$, $B = 35^\circ 20'$, $C = 68^\circ 30'$, (convert B, C to decimal degrees first) then
 $A = 76.17^\circ$
 $c = 24.15$
 $b = 15.01$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	If a, A, C are given	a		
3		A		
4		C		B
5				b
6				c
7	If a, B, C are given	a		
8		B		
9		C		A
10				c
11				b
	(Machine now is in DEG mode.)			

SOLUTION OF A TRIANGLE (GIVEN B, b, c)

SOLUTION OF A TRIANGLE
(GIVEN B, b, c)

MATH 1-22A



- Notes:
1. In some cases of obtuse triangles, this program generates one incorrect angle such that the sum of the three angles does not add up to 180° . Draw a sketch of the given triangle to locate the incorrect angle. Adding the difference between 180° and the sum of the three angles to this angle will yield the correct answer. This is due to the fact that the program uses inverse trigonometric functions of the HP-65 which always returns the principal values. For cases mentioned above, secondary values are required (see *Math 1-16A, Secondary Values of $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$*).
 2. If machine is in DEG mode, all angles are assumed to be in decimal degrees.
 3. If B is acute ($<90^\circ$) and $b < c$, two sets of solutions exist.
 4. Flashing zeros for angle C indicates no solution exists.

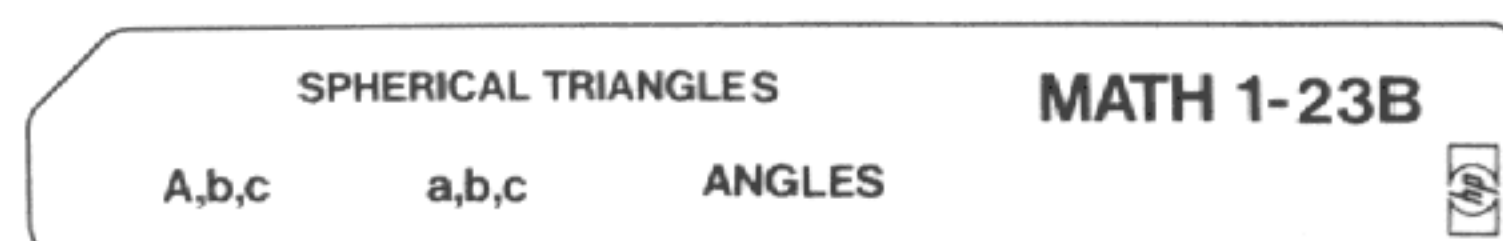
Example:

Given $B = 33^\circ 40'$, $b = 31.5$, $c = 51.8$, then

$$\begin{cases} C = 65.73^\circ \\ A = 80.60^\circ \\ a = 56.06 \end{cases} \quad \begin{cases} C_1 = 114.27^\circ \\ A_1 = 32.06^\circ \\ a_1 = 30.16 \end{cases}$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Clear flags		A	
3	If machine is in RAD mode		f SF 1	
4	If machine is in GRD mode		f SF 2	
5		B		
6		b		
7		c	B	C
8			R/S	A
9			R/S	a
10	Obtain second set of solutions		R/S	C_1
11			R/S	A_1
12			R/S	a_1

SPHERICAL TRIANGLES



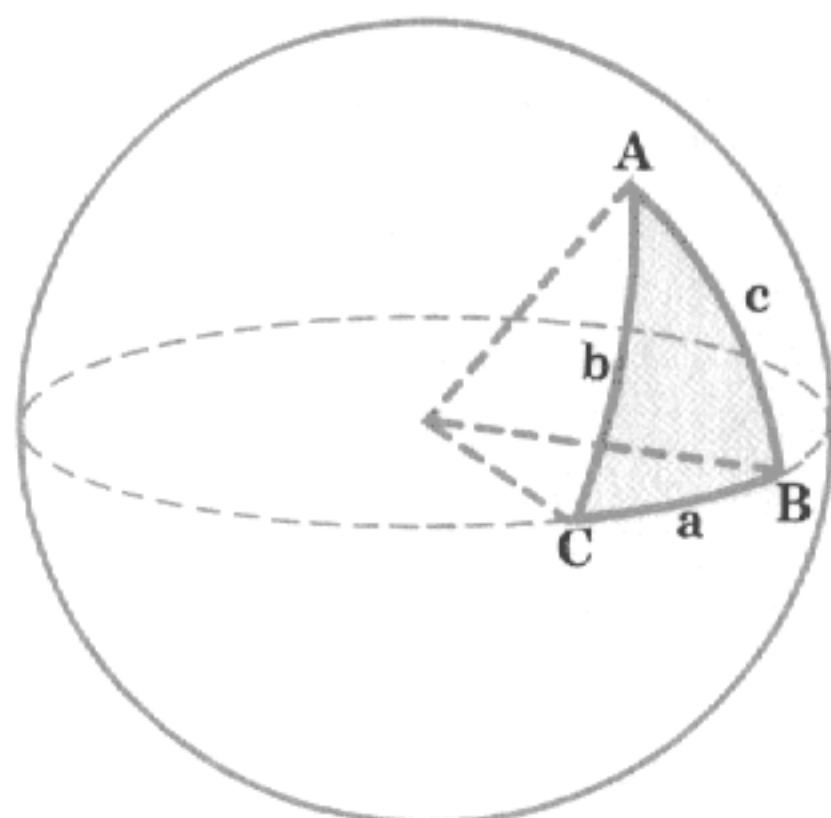
If A,B,C are the three angles of a spherical triangle and a,b,c the opposite sides, this program solves the triangle for any one of the cases:

1. A,b,c are given
2. a,b,c are given
3. a,B,C are given
4. A,B,C are given .

The laws of cosines are used:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$



Notes: 1. Area may be determined from the equation:

$$\text{Area} = r^2 (A + B + C - \pi)$$

where r is the radius of the sphere and A,B,C are in radians.

2. Program works in any angular mode. If the calculator is in DEG mode, all angles are in decimal degrees.

Examples:

1. If $A = 30^\circ$, $b = 50.5^\circ$, $c = 47.3^\circ$
then $a = 22.71^\circ$, $B = 87.88^\circ$, $C = 72.13^\circ$.
2. If $a = 0.2$ radians, $b = 0.91$ radians, $c = 0.93$ radians
then $A = 0.25$ radians, $B = 1.40$ radians, $C = 1.59$ radians.
3. If $a = 1.12^\circ$, $B = 21.63^\circ$, $C = 158.05^\circ$
then $A = 0.52^\circ$, $b = 51.90^\circ$, $c = 52.94^\circ$.
4. If $A = 47$ grads, $B = 160$ grads, $C = 60$ grads
then $a = 62.51$ grads, $b = 148.25$ grads, $c = 101.70$ grads.

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Initialize		RTN R/S	0.00
3	Case 1: given A, b, c	A		
		b		
		c	R/S	0.00
			A	a
	Go to 5			
4	Case 2: given a, b, c	a		
		b		
		c	R/S	0.00
			B	a
5	Compute solution		R/S	b
			R/S	c
			R/S	A
			R/S	B
			R/S	C
6	For a new case, go to 2			
7	Case 3: given a, B, C	a		
		B		
		C	R/S	0.00
			C A	A
	Go to 9			
8	Case 4: given A, B, C	A		
		B		
		C	R/S	0.00
			C B	A
9	Compute solution		R/S	B
			R/S	C
			R/S	a
			R/S	b
			R/S	c
10	For a new case, go to 2			

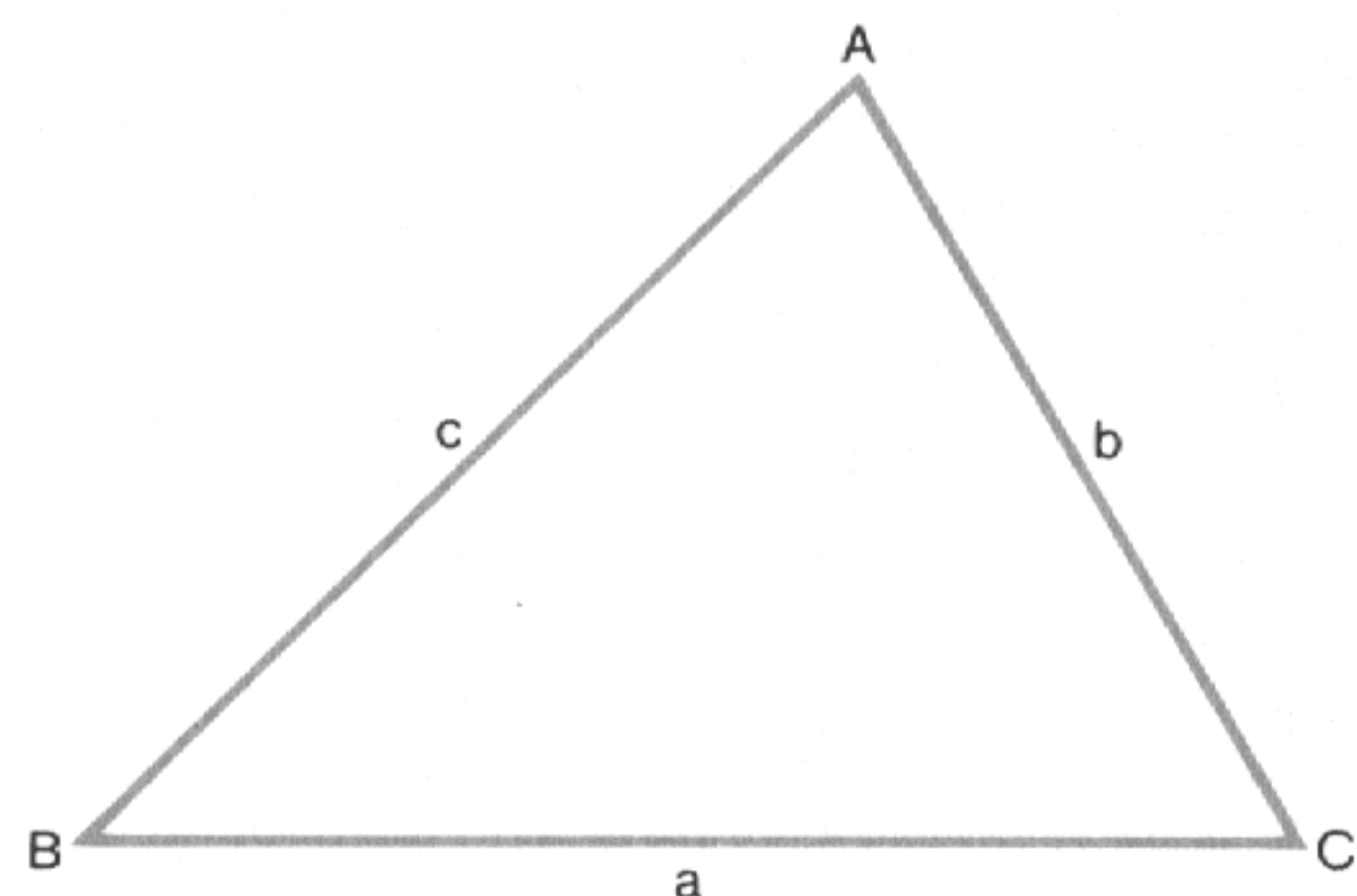
AREA OF A TRIANGLE

AREA OF A TRIANGLE		MATH 1-24A	
a,b,c	a,b,C	a,B,C	x ₁ ,x ₂ ,x ₃ y ₁ ,y ₂ ,y ₃

Case 1. Three sides a, b, c are given.

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{1}{2}(a+b+c).$$



Case 2. a, b and C are given.

$$\text{area} = \frac{1}{2} ab \sin C$$

Case 3. Side a and adjacent angles B, C are known.

$$\text{area} = \frac{a^2 \sin B \sin C}{2 \sin(B+C)}$$

Case 4. Three vertices (x₁, y₁), (x₂, y₂), (x₃, y₃) are given.

$$\text{area} = |D|$$

$$\text{where } D = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Note: If calculator is in DEG mode, angles are assumed to be in decimal degrees.

Examples:

1. Given a = 5.31, b = 7.09, c = 8.86 then
area = 18.82

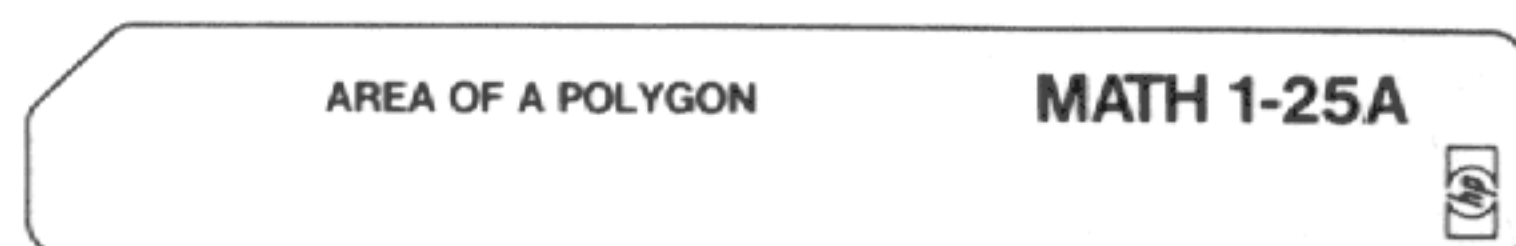
2. Given a = 5.3174, b = 7.0898, C = 45° then
area = 13.33

3. Given a = 14.625, B = 70.54°, C = 62.96° then
area = 123.82

4. If (0, 0), (4, 0), (4, 3) are vertices of a triangle, then
area = 6.00

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Case 1	a		
3		b		
4		c		area
5	Case 2	a		
6		b		
7		C		area
8	Case 3	a		
9		B		
10		C		area
11	Case 4	x ₁		
12		x ₂		
13		x ₃		
14		y ₁		
15		y ₂		
16		y ₃		area

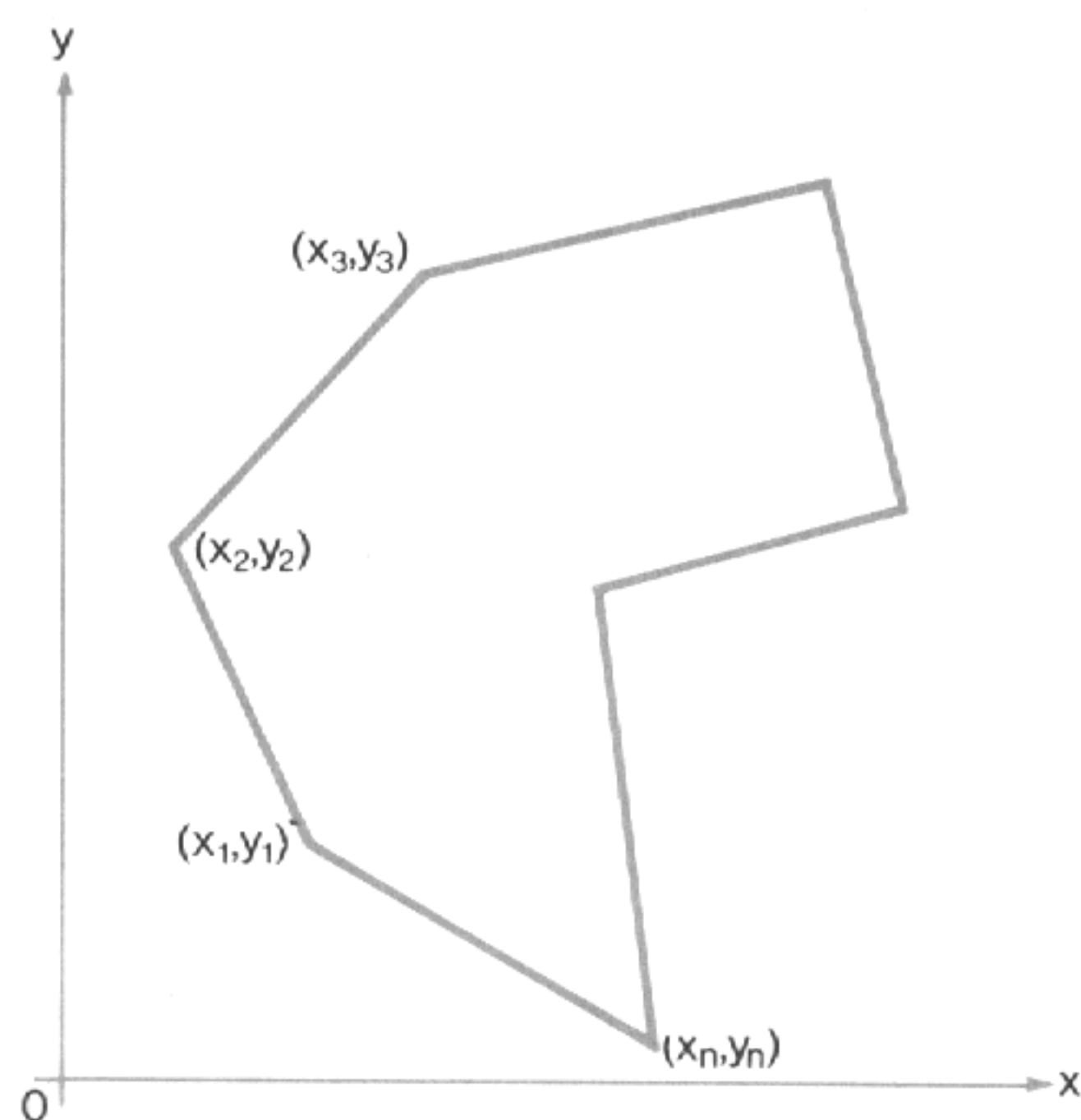
AREA OF A POLYGON



This program calculates the area of a polygon of n sides with $n \geq 3$.

If (x_i, y_i) are rectangular coordinates of the vertices of the polygon ($i = 1, 2, \dots, n$), then

$$\text{area} = \frac{1}{2} [(x_1 + x_2)(y_1 - y_2) + (x_2 + x_3)(y_2 - y_3) + \dots + (x_n + x_1)(y_n - y_1)]$$

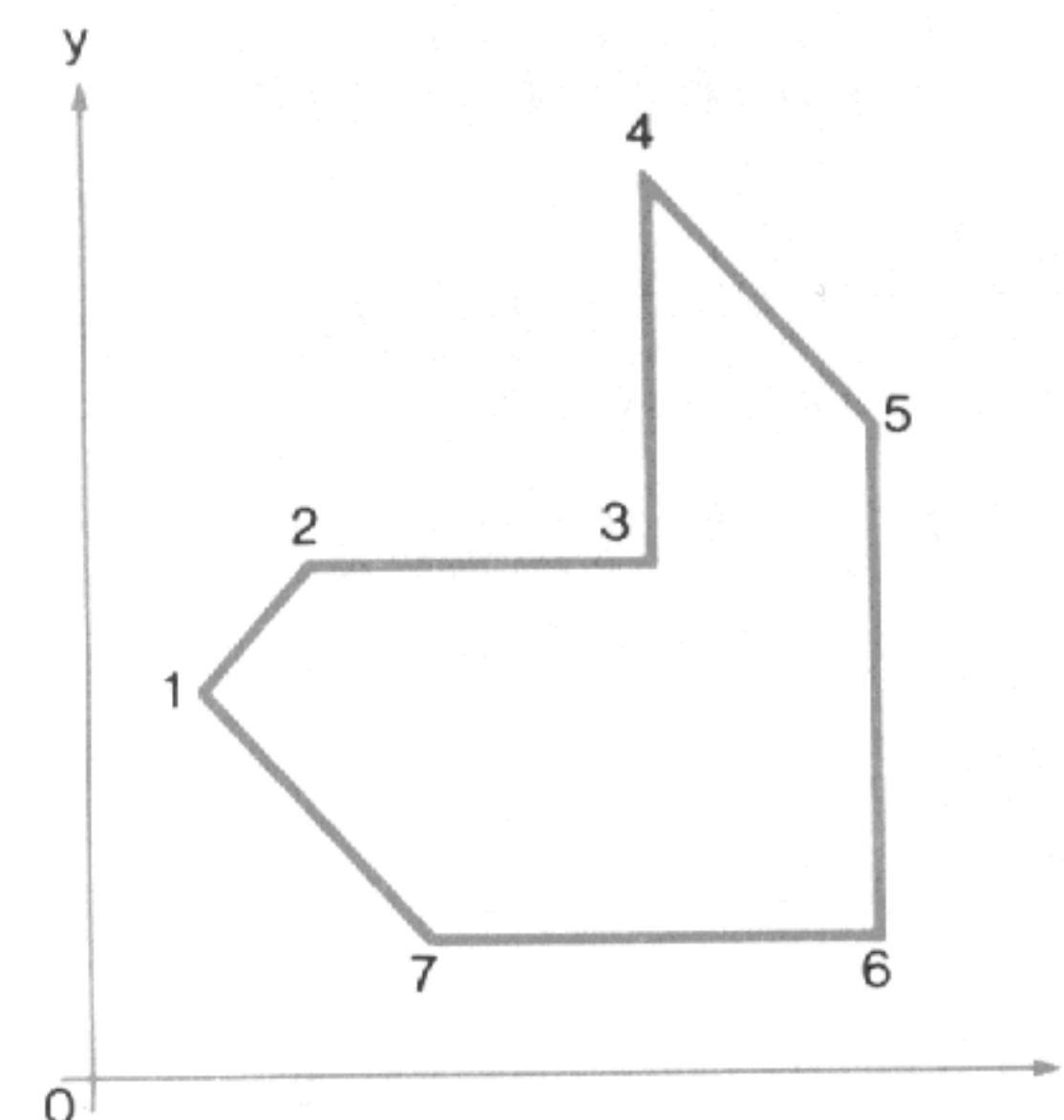


Note: When traversing counterclockwise, the area will be displayed as a negative number.

Example:

Point	Coordinates (x, y)
1	(1, 3)
2	(2, 4)
3	(5, 4)
4	(5, 7)
5	(7, 5)
6	(7, 1)
7	(3, 1)

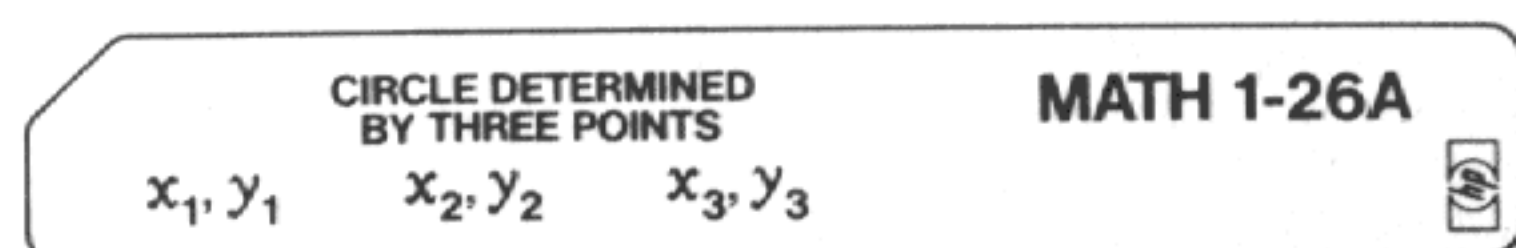
area = 19.50



LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		x_1		
3		y_1		
4		x_2		
5		y_2		
6*	Perform 6-7 for $i = 3, 4, \dots, n-1$	x_i		
7		y_i		
8*		x_n		
9		y_n		area

*When entering new coordinates, be careful not to lose the values already in the X and Y registers.

CIRCLE DETERMINED BY THREE POINTS



Given three distinct points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) on a circle, this program solves for the radius and center point of the circle using the following formulas:

$$y_0 = \frac{K_2 - K_1}{N_2 - N_1}, \quad x_0 = K_2 - N_2 y_0$$

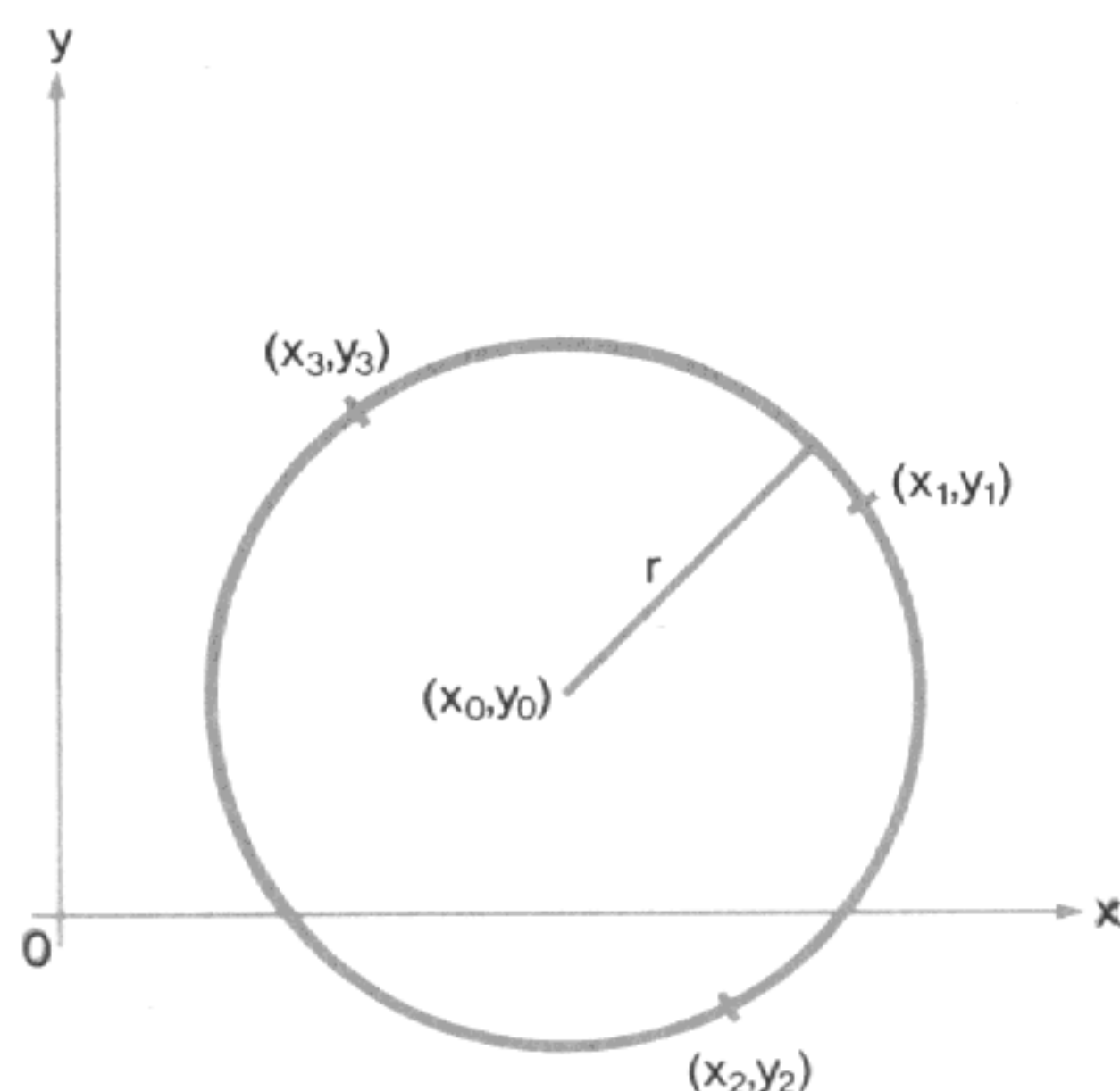
$$r = \sqrt{(x_3 - x_0)^2 + (y_3 - y_0)^2}$$

where $K_1 = \frac{(x_2 - x_1)(x_2 + x_1) + (y_2 - y_1)(y_2 + y_1)}{2(x_2 - x_1)}$

$$K_2 = \frac{(x_3 - x_1)(x_3 + x_1) + (y_3 - y_1)(y_3 + y_1)}{2(x_3 - x_1)}$$

$$N_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$N_2 = \frac{y_3 - y_1}{x_3 - x_1}$$



Restrictions: $x_1 \neq x_2, x_1 \neq x_3$
 $N_1 \neq N_2$ (three points can not be on a line)

Examples:

1. The three points $(1, 1)$, $(3.5, -7.6)$, $(12, 0.8)$ determine a circle with center $(6.45, -2.08)$ and radius 6.26.
2. Three points $(0, 1)$, $(-1, 0)$, $(0, -1)$ determine the unit circle with center at the origin.

Note: $(-1, 0)$ must be chosen as (x_1, y_1) .

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		x_1		
3		y_1		
4		x_2		
5		y_2		
6		x_3		
7		y_3		x_0
8				y_0
9				r

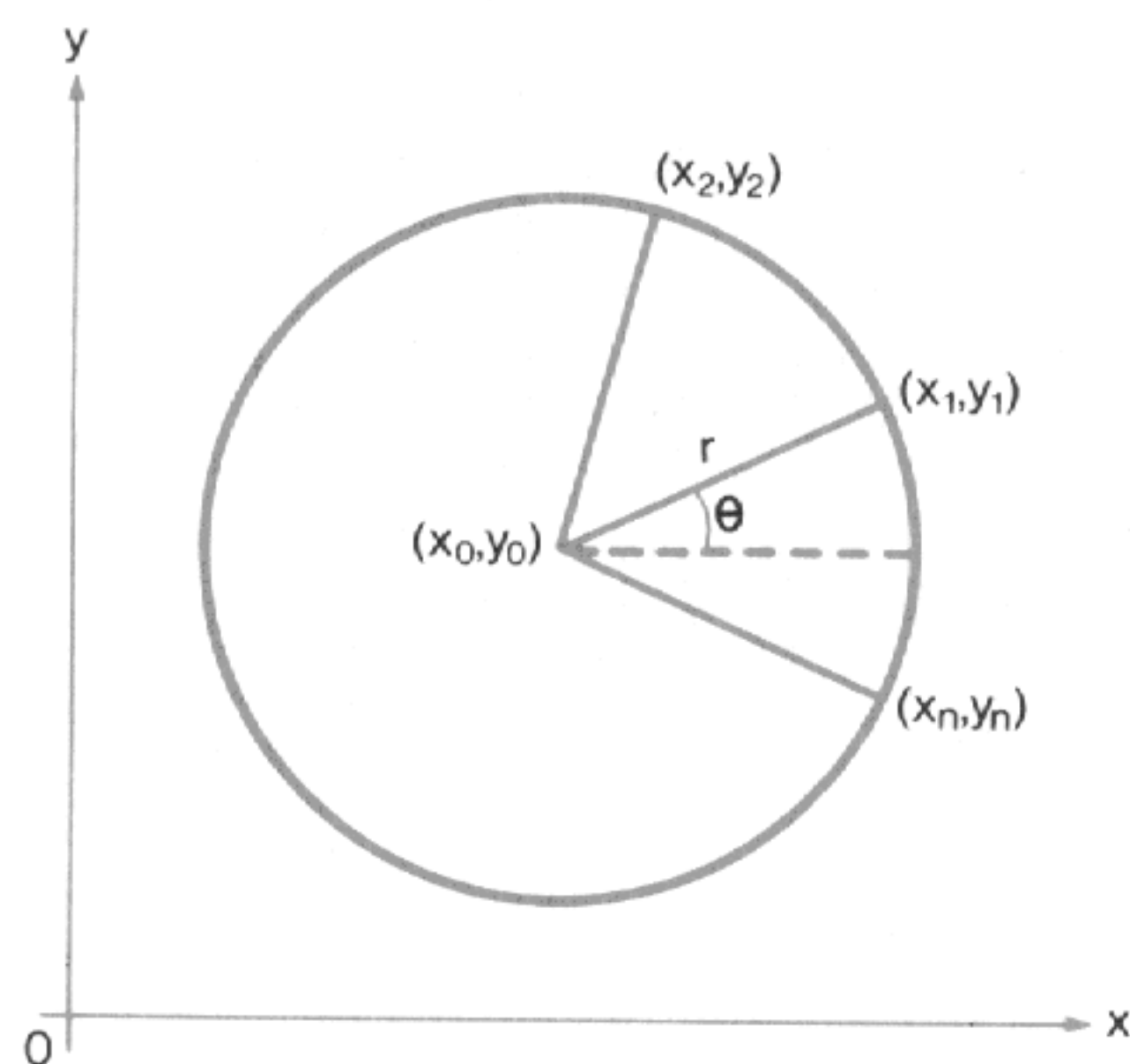
EQUALLY SPACED POINTS ON A CIRCLE

EQUALLY SPACED POINTS
ON A CIRCLE

MATH 1-27A



Given a circle with radius r and center (x_0, y_0) , the program computes the rectangular coordinates of equally spaced points (x_i, y_i) , $(i = 1, 2, \dots, n)$ on the circle if angle θ and number of points n are known. The position of the first point (x_1, y_1) on the circle is determined by the angle θ .



$$\begin{cases} x_{k+1} = x_0 + r \cos(\theta + ck) \\ y_{k+1} = y_0 + r \sin(\theta + ck) \end{cases}$$

where $k = 0, 1, 2, \dots, n-1$

$$c = \begin{cases} \frac{360}{n} & \text{if in DEG mode} \\ \frac{2\pi}{n} & \text{if in RAD mode} \\ \frac{400}{n} & \text{if in GRD mode} \end{cases}$$

Examples:

1. $\theta = \frac{\pi}{4}$, $r = 1$, $n = 5$, $x_0 = 4.28$, $y_0 = 3.1$

$$(x_1, y_1) = (4.99, 3.81)$$

$$(x_2, y_2) = (3.83, 3.99)$$

$$(x_3, y_3) = (3.29, 2.94)$$

$$(x_4, y_4) = (4.12, 2.11)$$

$$(x_5, y_5) = (5.17, 2.65)$$

2. $\theta = 36^\circ$, $r = 3.21$, $n = 3$, $x_0 = -3.4$, $y_0 = 1.8$

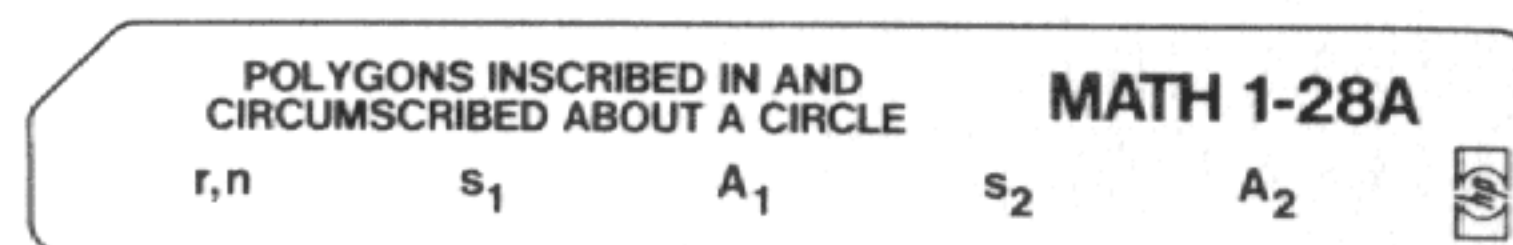
$$(x_1, y_1) = (-0.80, 3.69)$$

$$(x_2, y_2) = (-6.33, 3.11)$$

$$(x_3, y_3) = (-3.06, -1.39)$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2			A	
3		θ		
4		r		
5		n	R/S	
6		y_0		
7		x_0	R/S	x_1
8			R/S	y_1
9	Perform 9-10 for $i = 2, 3, \dots, n$		R/S	x_i
10			R/S	y_i

POLYGONS INSCRIBED IN AND CIRCUMSCRIBED ABOUT A CIRCLE



This program finds:

1. The side s_1 and area A_1 of an n -sided regular polygon inscribed in a circle of radius r

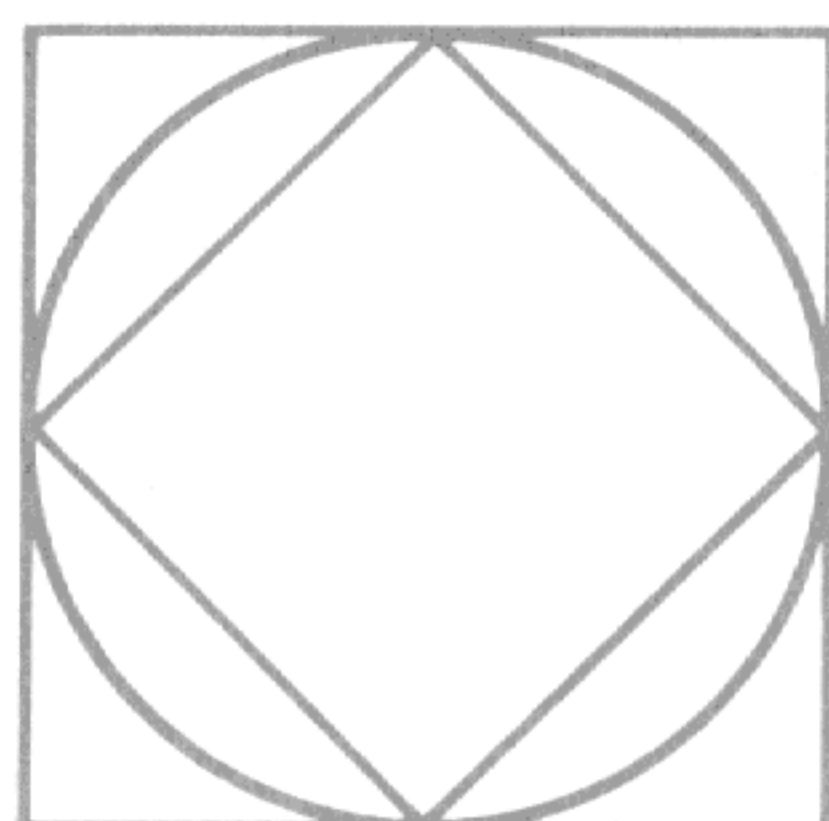
$$s_1 = 2r \sin \left(\frac{180^\circ}{n} \right)$$

$$A_1 = \frac{1}{2} nr^2 \sin \left(\frac{360^\circ}{n} \right)$$

2. The side s_2 and area A_2 of an n -sided regular polygon circumscribed about a circle of radius r

$$s_2 = 2r \tan \left(\frac{180^\circ}{n} \right)$$

$$A_2 = nr^2 \tan \left(\frac{180^\circ}{n} \right)$$



Example:

If $r = 5$, $n = 6$, then

$$s_1 = 5.00$$

$$A_1 = 64.95$$

$$s_2 = 5.77$$

$$A_2 = 86.60$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		r	↑	
3		n	A	
4	Compute s_1		B	s_1
5	Compute A_1		C	A_1
6	Compute s_2		D	s_2
7	Compute A_2		E	A_2
	(Subroutine A sets machine in			
	DEG mode)			

UNIT CONVERSIONS: C → F; ft, in → cm; lb → kg

UNIT CONVERSIONS			MATH 1-29A
C → F	ft, in → cm	lb → kg	INV

This program can be used to perform unit conversions between:

1. Centigrade and Fahrenheit

$$\text{Fahrenheit degrees} = \frac{9}{5} \text{ Centigrade degrees} + 32$$

2. feet, inches and centimeters

$$1 \text{ inch} = 2.54 \text{ centimeters}$$

3. pounds and kilograms

$$1 \text{ pound} = 0.45359237 \text{ kilograms}$$

Examples:

1. $28^{\circ}\text{C} = 82.40^{\circ}\text{F}$
2. $165 \text{ cm} = 5'4.96''$
3. $51.34 \text{ kg} = 113.19 \text{ lb}$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Clear flag 1		E	
3	Convert C→F	C	A	F
4	Convert ft, in→cm	ft	↑	
5		in	B	cm
6	Convert lb→kg	lb	C	kg
7	Convert F→C	F	D A	C
8	Convert cm→ft, in	cm	D B	ft
9			R/S	in
10	Convert kg→lb	kg	D C	lb

UNIT CONVERSIONS: mi→km; gal→ltr; yd→m; ac→ft²

UNIT CONVERSIONS			MATH 1-30A
mi → km	gal → ltr	yd → m	ac → ft ² INV

This program can be used to perform unit conversions between:

1. miles and kilometers

$$1 \text{ mile} = 1.609344 \text{ kilometers}$$

2. gallons and liters

$$1 \text{ gallon} = 3.785411784 \text{ liters}$$

3. yards and meters

$$1 \text{ yard} = 0.9144 \text{ meters}$$

4. acres and square feet

$$1 \text{ acre} = 43560 \text{ ft}^2$$

Examples:

1. $12.34 \text{ mi} = 19.86 \text{ km}$
2. $20 \text{ ltr} = 5.28 \text{ gal}$
3. $1000 \text{ m} = 1093.61 \text{ yd}$
4. $1.82 \text{ ac} = 79279.20 \text{ ft}^2$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	Clear flag 1		f ⁻¹ SF 1	
3	Convert mi→km	mi	A	km
4	Convert gal→ltr	gal	B	ltr
5	Convert yd→m	yd	C	m
6	Convert ac→ft ²	ac	D	ft ²
7	Convert km→mi	km	E A	mi
8	Convert ltr→gal	ltr	E B	gal
9	Convert m→yd	m	E C	yd
10	Convert ft ² →ac	ft ²	E D	ac

POLYNOMIAL EVALUATION (REAL)

POLYNOMIAL EVALUATION (REAL)

MATH 1-31A



This program evaluates polynomials of the form

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

for real coefficients a_k ($k = 0, 1, \dots, n$) and real x_0 , where $n \leq 8$.

Example:

$$f(x) = 11 - 7x - 3x^2 + 5x^4 + x^5 \quad (\text{Note: } a_3 = 0)$$

$$f(2.5) = 267.72$$

$$f(-5) = -29.00$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		a_0	A	
3	Perform 3 for $i = 1, \dots, n$	a_i	R/S	
4		x_0	B	$f(x_0)$
	(For new value of x_0 , go to 4)			

LINEAR AND LAGRANGIAN INTERPOLATIONS

 LINEAR AND LAGRANGIAN
INTERPOLATIONS

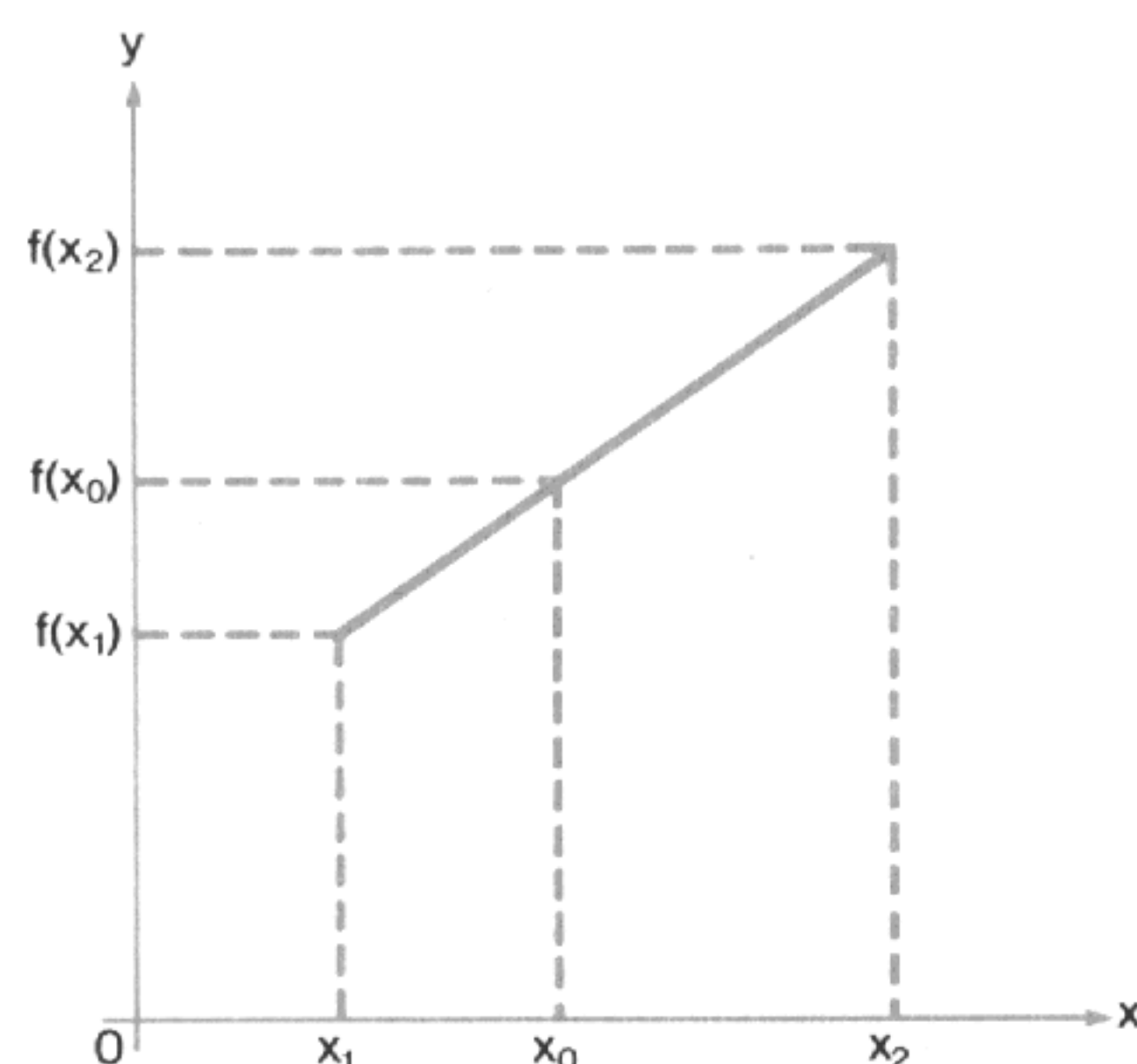
MATH 1-32A



1. Linear Interpolation

If $f(x)$ is a function of x and $x_1 < x_0 < x_2$, $f(x_0)$ can be approximated by

$$f(x_0) \cong \frac{(x_2 - x_0) f(x_1) + (x_0 - x_1) f(x_2)}{x_2 - x_1}$$



2. This program also evaluates for interpolation argument x the Lagrangian interpolating polynomial $P_2(x)$ of degree two passing through the points (x_0, y_0) , (x_1, y_1) , (x_2, y_2) .

$$P_2(x) = \sum_{i=0}^2 L_i(x) y_i$$

where

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^2 \frac{(x - x_j)}{(x_i - x_j)}, \quad i = 0, 1, 2$$

Examples:

1.	i	1	2
	x	1.2	1.3
	f(x)	0.30119	0.27253

$$f(1.27) = 0.28113$$

$$f(1.29) = 0.27540$$

2.	i	0	1	2
	x	1	3	10
	y	-5	1	25

$$x = 1.7, \quad y = -2.94$$

$$x = 9, \quad y = 21.29$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	For linear interpolation	x_1		
3		$f(x_1)$		
4		x_2		
5		$f(x_2)$		
6		x_0		$f(x_0)$
	(For a new value of x_0 , go to 6.)			
7	For Lagrangian interpolation	x_0		
8		x_1		
9		x_2		
10		y_0		
11		y_1		
12		y_2		
13		x		y
	(For a new x , go to 13.)			

FINITE DIFFERENCE INTERPOLATION

FINITE DIFFERENCE INTERPOLATION

MATH 1-33A



This program interpolates for data points in the region of tabulated data for uniformly spaced abscissas, with spacing h . The equation used is the backward-interpolation formula of Gauss which uses four pairs of data points and sets up the polynomial for cubic interpolation.

The equation used is:

$$y = y_3 + u\delta y_{-1/2} + \frac{1}{2}u(u+1)\delta^2 y_0 + \frac{1}{3!}u(u+1)(u-1)\delta^3 y_{-1/2}$$

The difference table is:

u	x	y			
-2	x_1	y_1			
-1	x_2	y_2	$y_2 - y_1$		
0	x_3	y_3	$y_3 - y_2$	$y_3 - 2y_2 + y_1$	
1	x_4	y_4	$y_4 - y_3$	$y_4 - 2y_3 + y_2$	$y_4 - 3y_3 + 3y_2 - y_1$

where $\delta y_{-1/2} = y_3 - y_2$

$$\delta^2 y_0 = y_4 - 2y_3 + y_2$$

$$\delta^3 y_{-1/2} = y_4 - 3y_3 + 3y_2 - y_1$$

and $u = \frac{x - x_3}{h}$

Example:

i	1	2	3	4
x_i	-1	1	3	5
y_i	-1	2	9	30

(Note: $h = 2$)

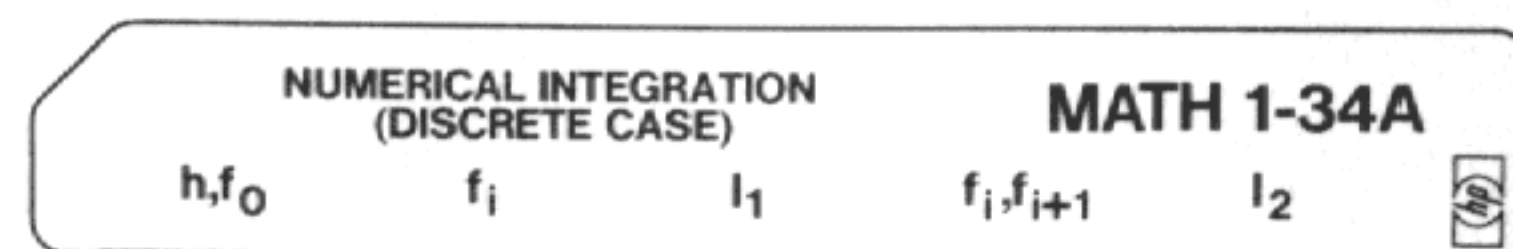
$$x = -0.5, \quad y = -0.08$$

$$x = 2.567, \quad y = 6.64$$

$$x = 4.8, \quad y = 26.99$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		y_1	↑	
3		y_2	↑	
4		y_3	↑	
5		y_4	A	
6		x_3	↑	
7		h	B	
8		x	C	y
	(For a new x , go to 8.)			

NUMERICAL INTEGRATION (DISCRETE CASE)



Let $x_0, x_1, \dots, x_n$ be equally spaced points, $x_i = x_0 + ih$ for $i = 1, 2, \dots, n$, at which corresponding values $f(x_0), f(x_1), \dots, f(x_n)$ of a function $f(x)$ are known. Using only this information, with no explicit expression for $f(x)$ itself,

$$\int_{x_0}^{x_n} f(x) dx$$

may be approximated using

(1) The trapezoidal rule:

$$\int_{x_0}^{x_n} f(x) dx \cong \frac{h}{2} \left[f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right] = I_1$$

(2) Simpson's rule:

$$\int_{x_0}^{x_n} f(x) dx \cong \frac{h}{3} \left[f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right] = I_2$$

In order to apply Simpson's rule, n must be even.

Notation used on magnetic card: $f_i = f(x_i)$, $i = 0, 1, 2, \dots, n$

Examples:

i	0	1	2	3	4	5	6	7	8
x_i	0	.25	.5	.75	1	1.25	1.5	1.75	2
$f(x_i)$	2	2.8	3.8	5.2	7	9.2	12.1	15.6	20

$$h = 0.25$$

1. Trapezoidal rule

$$\int_0^2 f(x) dx \cong I_1 = 16.68$$

2. Simpson's rule

$$\int_0^2 f(x) dx \cong I_2 = 16.58$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2	For the trapezoidal rule	h		
3		$f(x_0)$		
4	Perform 4 for $i = 1, 2, \dots, n-1$	$f(x_i)$		
5		$f(x_n)$		I_1
6	For Simpson's rule	h		
7		$f(x_0)$		
8	Perform 8-9 for $i = 1, 3, \dots, n-3$	$f(x_i)$		
9		$f(x_{i+1})$		
10		$f(x_{n-1})$		
11		$f(x_n)$		I_2

SIMPSON'S RULE FOR NUMERICAL INTEGRATION

SIMPSON'S RULE FOR NUMERICAL INTEGRATION

MATH 1-35A

The definite integral $\int_a^b f(x) dx$ can be approximated by Simpson's rule:

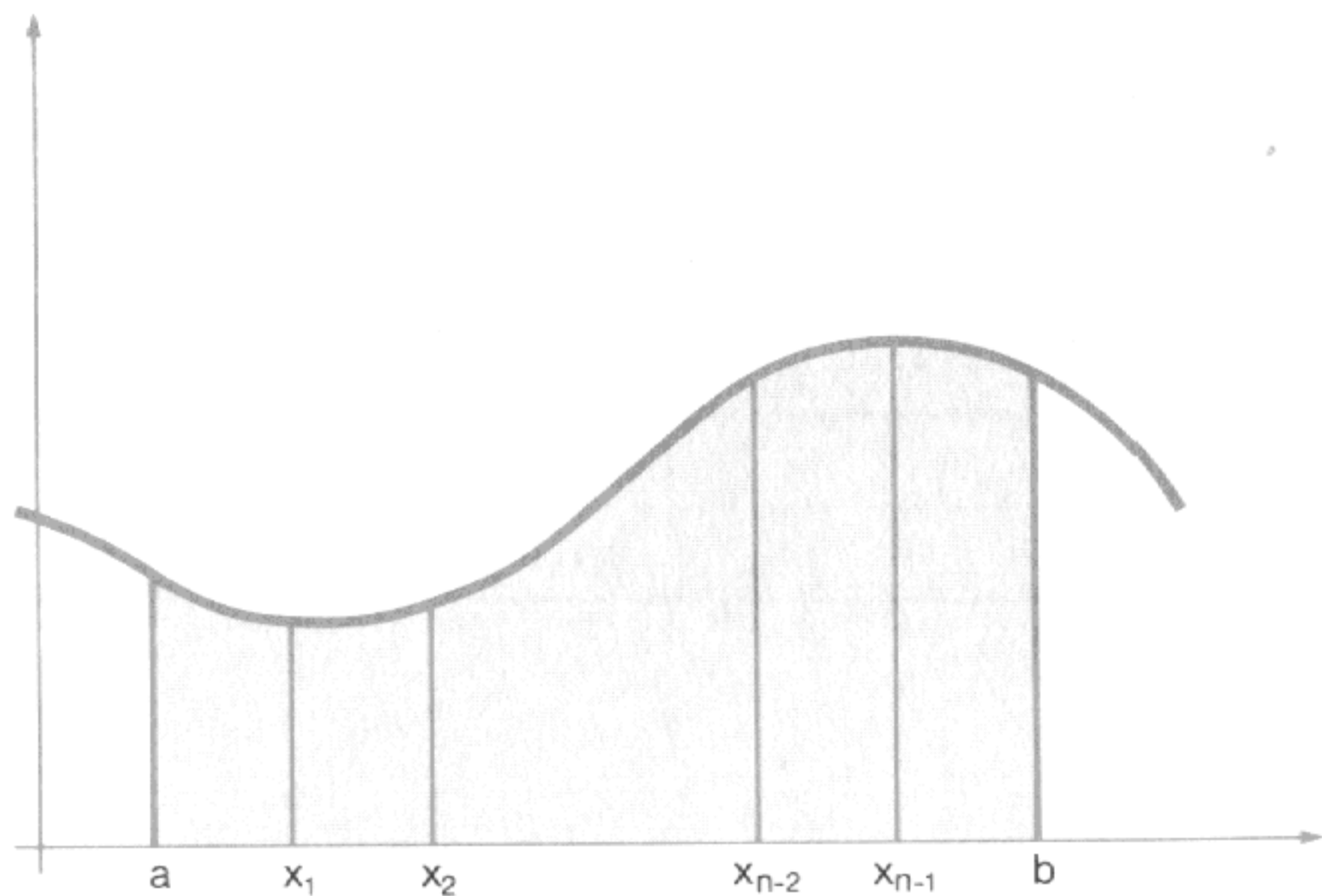
$$\int_a^b f(x) \, dx \cong \frac{h}{3} \left[f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right]$$

where $x_0 = a$, $x_i = x_0 + ih$, $x_n = b$

$$h = \frac{b - a}{n} \quad (i = 1, 2, \dots, n - 1)$$

This program approximates the integral $\int_a^b f(x) dx$ for given $f(x)$ and finite a, b such that $a \leq b$.

The number n must be a positive even integer ≥ 4 . $f(x)$ should not have any singularities in the integration interval.



$f(x)$ must be programmed in the calculator by the user. Assuming the value x is in the X register, 24 memory locations, stack registers and storage registers R_6 , R_7 are available for $f(x)$. Register R_9 is also available for temporary storage only.

Note: If n is odd, error will be indicated by flashing zeros.

Examples:

$$1. \quad \int_0^2 x^2 \, dx \cong 2.67 \quad (n=6)$$

Keys for f(x):

Correct answer is $\frac{8}{3}$

$$2. \quad \int_{-1}^8 x^2 \, dx \cong 171.00 \quad (n = 10)$$

Correct answer is 171

$$3. \quad \int_0^{2\pi} \frac{dx}{1 - \cos x + 0.25} \cong 8.22$$

(n = 10, set machine to RAD mode)

Keys for f(x): f COS 1 g x↔y - .25 + g 1/x

Correct answer is $\frac{2\pi}{0.75}$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2			GTO A	
3	Switch to W/PRGM mode			
4	Enter f(x)		RTN	
5	Switch to RUN mode			
6		a	↑	
7		b	↑	
8	(n must be even)	n	RTN R/S	
	(For different values of a, b or			
	n go to 6. For different			
	function f(x), go to 2.)			

FIRST ORDER DIFFERENTIAL EQUATION

FIRST ORDER
DIFFERENTIAL EQUATION

MATH 1-36A



This program may be used to solve a wide variety of first order differential equations of the form

$$y' = f(x, y)$$

with initial values x_0, y_0 .

The solution is a numerical solution which calculates y_i for $x_i = x_0 + ih$ ($i = 1, 2, 3, \dots$). h is an increment specified by the user.

The program uses the third-order Runge-Kutta method:

$$y_{i+1} = y_i + \frac{h}{6} (k_1 + 4k_2 + k_3)$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2} k_1\right)$$

$$k_3 = f(x_i + h, y_i + 2hk_2 - hk_1)$$

$f(x, y)$ must be programmed in the calculator by the user. Assuming x, y are in X and Y registers, 37 memory locations, the stack registers and registers R_6, R_7, R_8, R_9 are available for $f(x, y)$.

Example:

$$y' = \frac{x+1}{x} + \frac{2y}{x}$$

$$x_0 = 1, \quad y_0 = -\frac{1}{2}, \quad h = 0.5$$

Keys for $f(x, y)$: **STO** **6** **1** **+** **9** **x $\div$ y** **2** **x** **+** **RCL** **6** **$\div$**

x	1.5	2	2.5	3	3.5	4	4.5	5
y	0.23	1.46	3.18	5.40	8.11	11.32	15.02	19.21

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2			GTO B	
3	Switch to W/PRGM mode			
4	Enter $f(x, y)$		RTN	
5	Switch to RUN mode			
6		h	$\uparrow$	
7		y_0	$\uparrow$	
8		x_0	A	y_1
9	Perform 9 for $i = 2, 3, \dots$		R/S	y_i
	(For a new set of initial conditions, go to 6. For a new case, go to 2.)			

ROOTS OF $f(x)=0$ IN AN INTERVALROOTS OF $f(x)=0$ IN AN INTERVAL

MATH 1-37A



This program uses the principle of interval-halving to find real roots of an equation $f(x) = 0$ in a closed interval $[a, b]$ where the equation may be algebraic (e.g., $5x^4 - 3x + 1 = 0$), rational (e.g., $x^{3/2} + \sqrt{x} - 2 = 0$), or transcendental (e.g., $3 \cos x - 4x = 0$).

The user specifies the continuous, real function f , an interval $[a, b]$, an accuracy tolerance ϵ , and a search increment Δx . The program then begins at the left of the interval and compares the functional values at the ends of the interval $[a, a + \Delta x]$. If $f(a)$ and $f(a + \Delta x)$ are of opposite sign, this interval will be searched for a root. Otherwise, or even after a root is found, the program proceeds in the same manner with the interval $[a + \Delta x, a + 2\Delta x]$, etc. At most one root will be found by the program for each of these small intervals.

Key in and store the function $f(x)$ in the calculator assuming the value x is in the X register. 17 memory locations and the stack registers are available for $f(x)$. Register R_9 is also available for temporary storage only.

Examples:

1. The real roots of $x^3 - 8x^2 + 5x + 14 = 0$ in the interval $[-10, 10]$ using $\Delta x = 1$ and $\epsilon = 10^{-6}$ are -1.00 , 2.00 and 7.00 .

Keys for $f(x)$: $\uparrow \uparrow \uparrow 8 - \times 5 + \times 14 +$

2. The real root of $x^{5/2} - 2\sqrt{x} = 0$ in the interval $[1, 10]$ using $\Delta x = 1$ and $\epsilon = 10^{-6}$ is 1.41 .

Keys for $f(x)$: $f \sqrt{x} \uparrow \uparrow 5 g y^x g x \div y 2 \times -$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2			GTO A	
3	Switch to W/PRGM mode			
4	Enter $f(x)$		RTN	
5	Switch to RUN mode	ϵ	STO 5	
6		Δx	STO 6	
7		a	$\uparrow$	
8		b	RTN R/S	root
9	Perform 9 until display = $b + \Delta x$		R/S	root
	(Display = $b + \Delta x$ is an indication			
	of the end of search, $b + \Delta x$ is			
	not necessarily a root.)			

DETERMINANT AND CHARACTERISTIC EQUATION OF A 3x3 MATRIX

DETERMINANT AND CHARACTERISTIC
EQUATION OF A 3 x 3 MATRIX

MATH 1-38A



$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

Matrix A has characteristic equation

$$|A - \lambda I| = -\lambda^3 + d_1\lambda^2 + d_2\lambda + d_3 = 0$$

where

$$d_1 = a_1 + b_2 + c_3$$

$$d_2 = a_3c_1 + a_2b_1 + b_3c_2 - a_1b_2 - a_1c_3 - b_2c_3$$

$$d_3 = |A|$$

$$= a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2$$

- Notes:** 1. d_3 is the determinant of matrix A.
2. *Math 1-07A, Cubic Equation* can be used to find the eigenvalues.

Example:

$$\text{Matrix } A = \begin{pmatrix} 8 & 4 & 1 \\ 0 & 3 & 0 \\ 1 & 5 & -1 \end{pmatrix}$$

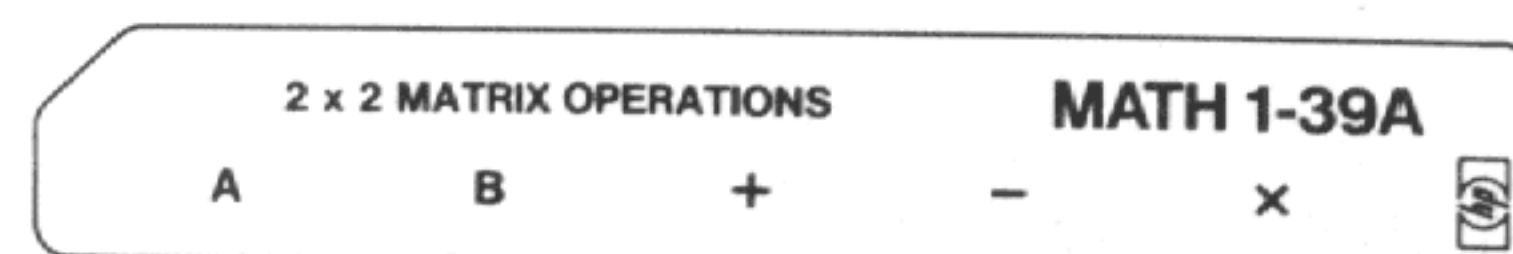
has characteristic equation

$$-\lambda^3 + 10\lambda^2 - 12\lambda - 27 = 0$$

Determinant of A = $d_3 = -27.00$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		a_1		
3		a_2		
4		a_3		
5		b_1		
6		b_2		
7		b_3		
8		c_1		
9		c_2		
10		c_3		d_1
11				d_2
12				d_3

2 x 2 MATRIX OPERATIONS



Suppose

$$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}, \quad B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$$

then

$$A + B = \begin{pmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{pmatrix} = \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}$$

$$A - B = \begin{pmatrix} a_1 - b_1 & a_2 - b_2 \\ a_3 - b_3 & a_4 - b_4 \end{pmatrix} = \begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix}$$

$$AB = \begin{pmatrix} a_1 b_1 + a_2 b_3 & a_1 b_2 + a_2 b_4 \\ a_3 b_1 + a_4 b_3 & a_3 b_2 + a_4 b_4 \end{pmatrix} = \begin{pmatrix} e_1 & e_2 \\ e_3 & e_4 \end{pmatrix}$$

Example:

$$A = \begin{pmatrix} -1.2 & 5.8 \\ 7.31 & -4.39 \end{pmatrix} \quad B = \begin{pmatrix} 10.21 & 15.8 \\ -9.33 & 7.24 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 9.01 & 21.60 \\ -2.02 & 2.85 \end{pmatrix}$$

$$A - B = \begin{pmatrix} -11.41 & -10.00 \\ 16.64 & -11.63 \end{pmatrix}$$

$$AB = \begin{pmatrix} -66.37 & 23.03 \\ 115.59 & 83.71 \end{pmatrix}$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program			
2		a ₁		
3		a ₂		
4		a ₃		
5		a ₄		
6		b ₁		
7		b ₂		
8		b ₃		
9		b ₄		
10	Compute A + B			c ₁
11	Perform 11 for i = 2, 3, 4			c _i
12	Compute A - B			d ₁
13	Perform 13 for i = 2, 3, 4			d _i
14	Compute AB			e ₁
15	Perform 15 for i = 2, 3, 4			e _i

3 x 3 MATRIX INVERSION

3 x 3 MATRIX INVERSION

MATH 1-40A



$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

has an inverse

$$A^{-1} = \begin{pmatrix} \alpha_1 & \alpha_4 & \alpha_7 \\ \alpha_2 & \alpha_5 & \alpha_8 \\ \alpha_3 & \alpha_6 & \alpha_9 \end{pmatrix}$$

where

$$\begin{aligned} \alpha_1 &= (b_2 c_3 - b_3 c_2) / \text{Det} \\ \alpha_2 &= (a_3 c_2 - a_2 c_3) / \text{Det} \\ \alpha_3 &= (a_2 b_3 - a_3 b_2) / \text{Det} \\ \alpha_4 &= (b_3 c_1 - b_1 c_3) / \text{Det} \\ \alpha_5 &= (a_1 c_3 - a_3 c_1) / \text{Det} \\ \alpha_6 &= (a_3 b_1 - a_1 b_3) / \text{Det} \\ \alpha_7 &= (b_1 c_2 - b_2 c_1) / \text{Det} \\ \alpha_8 &= (a_2 c_1 - a_1 c_2) / \text{Det} \\ \alpha_9 &= (a_1 b_2 - a_2 b_1) / \text{Det} \end{aligned}$$

if determinant Det of A is non-zero.

Note: This program must be used in conjunction with *Math 1-38A*, *Determinant and Characteristic Equation of a 3 x 3 Matrix*.

Example:

$$A = \begin{pmatrix} -1 & 0 & 3 \\ 7 & 1 & -1 \\ 2 & 3 & 0 \end{pmatrix}$$

Det = 54.00

$$A^{-1} = \begin{pmatrix} 0.06 & 0.17 & -0.06 \\ -0.04 & -0.11 & 0.37 \\ 0.35 & 0.06 & -0.02 \end{pmatrix}$$

LINE	INSTRUCTIONS	DATA	KEYS	DISPLAY
1	Enter program <i>Math 1-38A</i>			
2		a ₁	↑	
3		a ₂	↑	
4		a ₃	A	
5		b ₁	↑	
6		b ₂	↑	
7		b ₃	B	
8		c ₁	↑	
9		c ₂	↑	
10		c ₃	C D	Det
11	Enter program <i>Math 1-40A</i>			
12			R/S	α ₁
13	Perform 13 for i = 2, ..., 9		R/S	α _i

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FACTORS OF AN INTEGER

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	04	4	01	1
11	A	35 08	g R↓	23	LBL
32	f ⁻¹	31	f	04	4
51	SF1	83	INT	34 01	RCL 1
33 01	STO 1	35 00	g LST X	84	R/S
00	0	35 23	g x=y	00	0
35 07	g x↔y	22	GTO	84	R/S
35 22	g x≤y	02	2	23	LBL
22	GTO	34 03	RCL 3	05	5
05	5	06	6	00	0
41	↑	35 24	g x>y	81	÷
31	f	31	f	23	LBL
83	INT	51	SF1	03	3
35 21	g x≠y	02	2	61	+
22	GTO	34 02	RCL 2	33 02	STO 2
05	5	61	+	31	f
02	2	33 02	STO 2	51	SF1
43	EEX	02	2	33 03	STO 3
09	9	35 07	g x↔y	22	GTO
34 01	RCL 1	32	f ⁻¹	01	1
35 24	g x>y	61	TF1	35 01	g NOP
22	GTO	22	GTO	35 01	g NOP
05	5	03	3	35 01	g NOP
01	1	32	f ⁻¹	35 01	g NOP
33 02	STO 2	51	SF1	35 01	g NOP
02	2	33 03	STO 3	35 01	g NOP
33 03	STO 3	22	GTO	35 01	g NOP
23	LBL	01	1	35 01	g NOP
01	1	23	LBL	35 01	g NOP
34 01	RCL 1	02	2	35 01	g NOP
34 03	RCL 3	34 03	RCL 3	35 01	g NOP
81	÷	84	R/S		
34 03	RCL 3	35 08	g R↓		
35 24	g x>y	33 01	STO 1		
22	GTO	22	GTO		

R₁ Used
R₂ Used
R₃ Used

R₄
R₅
R₆

R₇
R₈
R₉ Used

GREATEST COMMON DIVISOR
LEAST COMMON MULTIPLE

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	71	x	24	RTN
11	A	61	+	23	LBL
35 07	g x↔y	33 04	STO 4	03	3
33 01	STO 1	22	GTO	15	E
35 07	g x↔y	01	1	35 21	g x≠y
33 02	STO 2	23	LBL	22	GTO
01	1	02	2	03	3
33 03	STO 3	34 01	RCL 1	34 03	RCL 3
44	CLX	35 24	g x>y	34 01	RCL 1
33 04	STO 4	22	GTO	81	÷
35 23	g x=y	04	4	35	g
22	GTO	42	CHS	06	ABS
02	2	84	R/S	24	RTN
33 03	STO 3	34 03	RCL 3	23	LBL
33 05	STO 5	42	CHS	15	E
01	1	84	R/S	34 01	RCL 1
33 04	STO 4	34 04	RCL 4	34 01	RCL 1
33 06	STO 6	42	CHS	34 02	RCL 2
23	LBL	24	RTN	33 01	STO 1
01	1	23	LBL	81	÷
15	E	04	4	31	f
35 23	g x=y	84	R/S	83	INT
22	GTO	34 03	RCL 3	42	CHS
02	2	84	R/S	33 07	STO 7
34 06	RCL 6	34 04	RCL 4	34 02	RCL 2
34 03	RCL 3	24	RTN	71	x
33 06	STO 6	23	LBL	61	+
34 07	RCL 7	12	B	33 02	STO 2
71	x	33 02	STO 2	00	0
61	+	35 07	g x↔y	24	RTN
33 03	STO 3	33 01	STO 1		
34 05	RCL 5	71	x		
34 04	RCL 4	33 03	STO 3		
33 05	STO 5	00	0		
34 07	RCL 7	35 23	g x=y		

R₁ Used
R₂ Used
R₃ Used

R₄ Used
R₅ Used
R₆ Used

R₇ Used
R₈
R₉ Used

ARITHMETIC AND HARMONIC PROGRESSIONS

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	13	C	81	$\div$
11	A	33 01	STO 1	84	R/S
35 07	$g x \rightleftharpoons y$	71	x	23	LBL
33 01	STO 1	34 01	RCL 1	02	2
35 08	$g R \downarrow$	01	1	44	CLX
41	$\uparrow$	51	—	61	+
41	$\uparrow$	71	x	61	+
41	$\uparrow$	02	2	41	$\uparrow$
34 01	RCL 1	81	$\div$	35	g
23	LBL	35 07	$g x \rightleftharpoons y$	04	$1/x$
01	1	34 01	RCL 1	34 01	RCL 1
84	R/S	71	x	71	x
61	+	61	+	84	R/S
22	GTO	24	RTN	22	GTO
01	1	23	LBL	02	2
23	LBL	14	D	35 01	g NOP
12	B	33 01	STO 1	35 01	g NOP
33 04	STO 4	35 08	$g R \downarrow$	35 01	g NOP
35 08	$g R \downarrow$	61	+	35 01	g NOP
33 02	STO 2	34 01	RCL 1	35 01	g NOP
35 08	$g R \downarrow$	71	x	35 01	g NOP
33 01	STO 1	02	2	35 01	g NOP
35 08	$g R \downarrow$	81	$\div$	35 01	g NOP
35 08	$g R \downarrow$	24	RTN	35 01	g NOP
01	1	23	LBL	35 01	g NOP
51	—	15	E	35 01	g NOP
71	x	33 03	STO 3	35 01	g NOP
61	+	35 08	$g R \downarrow$	35 01	g NOP
84	R/S	33 02	STO 2	35 01	g NOP
34 01	RCL 1	35 08	$g R \downarrow$	35 01	g NOP
34 02	RCL 2	33 01	STO 1	35 01	g NOP
34 04	RCL 4	34 03	RCL 3	35 01	g NOP
13	C	34 02	RCL 2	35 01	g NOP
24	RTN	34 01	RCL 1	35 01	g NOP
23	LBL	34 02	RCL 2	35 01	g NOP

R ₁	Used	R ₄	Used	R ₇
R ₂	Used	R ₅		R ₈
R ₃	Used	R ₆		R ₉

GEOMETRIC PROGRESSION

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	34 03	RCL 3	81	$\div$
11	A	71	x	32	f^{-1}
41	$\uparrow$	34 04	RCL 4	83	INT
41	$\uparrow$	01	1	00	0
35 09	$g R \uparrow$	51	—	35 23	$g x = y$
23	LBL	81	$\div$	22	GTO
01	1	24	RTN	09	9
84	R/S	23	LBL	34 02	RCL 2
71	x	14	D	34 01	RCL 1
22	GTO	35	g	35	g
01	1	06	ABS	05	y^x
23	LBL	01	1	42	CHS
12	B	35 22	$g x \leq y$	24	RTN
35 08	$g R \downarrow$	00	0	23	LBL
35 08	$g R \downarrow$	81	$\div$	09	9
33 03	STO 3	35 00	g LST X	34 02	RCL 2
35 08	$g R \downarrow$	51	—	34 01	RCL 1
35 08	$g R \downarrow$	35 07	$g x \rightleftharpoons y$	35	g
01	1	35 08	$g R \downarrow$	05	y^x
51	—	81	$\div$	24	RTN
15	E	24	RTN	35 01	g NOP
34 03	RCL 3	23	LBL	35 01	g NOP
71	x	15	E	35 01	g NOP
24	RTN	33 01	STO 1	35 01	g NOP
23	LBL	35 07	$g x \rightleftharpoons y$	35 01	g NOP
13	C	33 02	STO 2	35 01	g NOP
35 08	$g R \downarrow$	00	0	35 01	g NOP
33 04	STO 4	35 07	$g x \rightleftharpoons y$	35 01	g NOP
35 08	$g R \downarrow$	35 24	$g x > y$	35 01	g NOP
33 03	STO 3	22	GTO	35 01	g NOP
35 08	$g R \downarrow$	09	9	35 01	g NOP
35 08	$g R \downarrow$	42	CHS	35 01	g NOP
15	E	33 02	STO 2	35 01	g NOP
01	1	34 01	RCL 1	35 01	g NOP
51	—	02	2		

R ₁	n	R ₄	r	R ₇
R ₂	Used	R ₅		R ₈
R ₃	a	R ₆		R ₉ Used

FUNCTIONS OF x AND y

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	42	CHS	14	D
11	A	24	RTN	33 01	STO 1
33 01	STO 1	23	LBL	35 07	$g x \rightleftharpoons y$
35 07	$g x \rightleftharpoons y$	01	1	35	g
33 02	STO 2	34 02	RCL 2	03	n!
00	0	34 01	RCL 1	35 00	$g LST X$
35 07	$g x \rightleftharpoons y$	35	g	34 01	RCL 1
35 23	$g x = y$	05	y^x	51	—
35 08	$g R \downarrow$	24	RTN	35	g
24	RTN	23	LBL	03	n!
35 24	$g x > y$	12	B	81	$\div$
22	GTO	31	f	24	RTN
01	1	07	LN	23	LBL
42	CHS	35 07	$g x \rightleftharpoons y$	15	E
33 02	STO 2	31	f	14	D
34 01	RCL 1	07	LN	34 01	RCL 1
32	f^{-1}	35 07	$g x \rightleftharpoons y$	35	g
83	INT	81	$\div$	03	n!
00	0	24	RTN	81	$\div$
35 21	$g x \neq y$	23	LBL	24	RTN
81	$\div$	13	C	35 01	g NOP
24	RTN	33 01	STO 1	35 01	g NOP
34 01	RCL 1	35 07	$g x \rightleftharpoons y$	35 01	g NOP
02	2	33 02	STO 2	35 01	g NOP
81	$\div$	35 07	$g x \rightleftharpoons y$	35 01	g NOP
32	f^{-1}	81	$\div$	35 01	g NOP
83	INT	31	f	35 01	g NOP
00	0	83	INT	35 01	g NOP
35 23	$g x = y$	34 01	RCL 1	35 01	g NOP
22	GTO	71	x	35 01	g NOP
01	1	34 02	RCL 2	35 01	g NOP
34 02	RCL 2	35 07	$g x \rightleftharpoons y$	35 01	g NOP
34 01	RCL 1	51	—	35 01	g NOP
35	g	24	RTN	35 01	g NOP
05	y^x	23	LBL	35 01	g NOP

R₁ x
R₂ y
R₃

R₄
R₅
R₆

R₇
R₈
R₉ Used

QUADRATIC EQUATION

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	71	x	35 01	g NOP
11	A	84	R/S	35 01	g NOP
33 03	STO 3	34 03	RCL 3	35 01	g NOP
35 08	$g R \downarrow$	35 07	$g x \rightleftharpoons y$	35 01	g NOP
42	CHS	81	$\div$	35 01	g NOP
33 02	STO 2	24	RTN	35 01	g NOP
35 07	$g x \rightleftharpoons y$	23	LBL	35 01	g NOP
41	$\uparrow$	13	C	35 01	g NOP
33 01	STO 1	42	CHS	35 01	g NOP
61	+	31	f	35 01	g NOP
81	$\div$	09	$\sqrt{x}$	35 01	g NOP
41	$\uparrow$	35 07	$g x \rightleftharpoons y$	35 01	g NOP
32	f^{-1}	84	R/S	35 01	g NOP
09	$\sqrt{x}$	35 07	$g x \rightleftharpoons y$	35 01	g NOP
34 03	RCL 3	24	RTN	35 01	g NOP
34 01	RCL 1	23	LBL	35 01	g NOP
81	$\div$	14	D	35 01	g NOP
33 03	STO 3	01	1	35 01	g NOP
51	—	34 01	RCL 1	35 01	g NOP
24	RTN	34 02	RCL 2	35 01	g NOP
23	LBL	24	RTN	35 01	g NOP
12	B	23	LBL	35 01	g NOP
31	f	15	E	35 01	g NOP
09	$\sqrt{x}$	01	1	35 01	g NOP
35 07	$g x \rightleftharpoons y$	34 07	RCL 7	35 01	g NOP
35	g	34	RCL	35 01	g NOP
06	ABS	09	9	35 01	g NOP
61	+	24	RTN	35 01	g NOP
34 02	RCL 2	35 01	g NOP	35 01	g NOP
34 01	RCL 1	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
41	$\uparrow$	35 01	g NOP	35 01	g NOP
35	g	35 01	g NOP	35 01	g NOP
06	ABS	35 01	g NOP	35 01	g NOP
81	$\div$	35 01	g NOP	35 01	g NOP

R₁ a
R₂ -b
R₃ c, c/a

R₄
R₅
R₆

R₇
R₈
R₉

CUBIC EQUATION

CODE KEYS

23 LBL
11 A
00 0
33 04 STO 4
34 03 RCL 3
35 g
06 ABS
34 03 RCL 3
81 ÷
33 06 STO 6
34 03 RCL 3
35 g
06 ABS
34 01 RCL 1
35 g
06 ABS
61 +
43 EEX
42 CHS
02 2
23 LBL
01 1
01 1
00 0
71 x
35 22 $g\ x \leq y$
22 GTO
01 1
33 07 STO 7
23 LBL
02 2
34 07 RCL 7
01 1
00 0
81 ÷

R₁ Used
R₂ Used
R₃ Used

CODE KEYS

33 07 STO 7
34 06 RCL 6
42 CHS
33 06 STO 6
23 LBL
03 3
34 07 RCL 7
34 06 RCL 6
71 x
34 04 RCL 4
61 +
33 05 STO 5
34 04 RCL 4
35 23 $g\ x = y$
22 GTO
04 4
34 05 RCL 5
34 01 RCL 1
61 +
34 05 RCL 5
71 x
34 02 RCL 2
61 +
34 05 RCL 5
71 x
34 03 RCL 3
61 +
33 08 STO 8
35 g
06 ABS
43 EEX
42 CHS
04 4
35 24 $g\ x > y$
22 GTO

R₄ Used
R₅ Used
R₆ Used

CODE KEYS

04 4
34 05 RCL 5
33 04 STO 4
34 08 RCL 8
34 06 RCL 6
71 x
00 0
35 24 $g\ x > y$
22 GTO
03 3
22 GTO
02 2
23 LBL
04 4
34 05 RCL 5
84 R/S
34 01 RCL 1
61 +
33 08 STO 8
34 05 RCL 5
71 x
34 02 RCL 2
61 +
33 03 STO 3
01 1
34 08 RCL 8
34 03 RCL 3
84 R/S
35 01 g NOP
35 01 g NOP

R₇ Used
R₈ Used
R₉ Used

FOURTH DEGREE POLYNOMIAL EQUATION

CODE KEYS

33 08 STO 8
84 R/S
33 07 STO 7
84 R/S
33 06 STO 6
84 R/S
33 05 STO 5
23 LBL
11 A
34 07 RCL 7
42 CHS
33 01 STO 1
34 08 RCL 8
34 06 RCL 6
71 x
34 05 RCL 5
04 4
71 x
51 -
33 02 STO 2
34 07 RCL 7
04 4
71 x
34 08 RCL 8
32 f^{-1}
09 $\sqrt{x}$
51 -
34 05 RCL 5
71 x
34 06 RCL 6
32 f^{-1}
09 $\sqrt{x}$
51 -
33 03 STO 3
84 R/S

R₁ b₂
R₂ b₁
R₃ b₀

CODE KEYS

23 LBL
12 B
02 2
33 STO
81 ÷
08 8
34 04 RCL 4
02 2
81 ÷
33 04 STO 4
32 f^{-1}
09 $\sqrt{x}$
34 05 RCL 5
51 -
31 f
09 $\sqrt{x}$
33 05 STO 5
00 0
35 23 $g\ x = y$
22 GTO
01 1
34 08 RCL 8
34 04 RCL 4
71 x
34 06 RCL 6
02 2
81 ÷
51 -
34 05 RCL 5
81 ÷
33 06 STO 6
22 GTO
02 2
23 LBL
01 1

R₄ y₀, B
R₅ a₀, D
R₆ a₁, C

CODE KEYS

34 08 RCL 8
32 f^{-1}
09 $\sqrt{x}$
34 07 RCL 7
51 -
34 04 RCL 4
02 2
71 x
61 +
31 f
09 $\sqrt{x}$
33 06 STO 6
23 LBL
02 2
34 08 RCL 8
61 +
33 07 STO 7
34 04 RCL 4
34 05 RCL 5
61 +
33 STO
09 9
01 1
34 08 RCL 8
34 06 RCL 6
51 -
34 04 RCL 4
34 05 RCL 5
51 -
24 RTN

R₇ a₂, A + C
R₈ a₃, A
R₉ B + D

FIFTH DEGREE POLYNOMIAL EQUATION

CODE	KEYS	CODE	KEYS	CODE	KEYS
35	g	34 08	RCL 8	35 24	g $x > y$
06	ABS	71	x	22	GTO
33 06	STO 6	34 07	RCL 7	02	2
35 00	g LST X	61	+	22	GTO
81	$\div$	33 07	STO 7	01	1
33 08	STO 8	35 00	g LST X	23	LBL
34 01	RCL 1	35 23	g $x = y$	03	3
35	g	22	GTO	34 07	RCL 7
06	ABS	03	3	84	R/S
34 06	RCL 6	35 07	g $x \geq y$	34 01	RCL 1
61	+	41	$\uparrow$	61	+
83	$\cdot$	41	$\uparrow$	33 08	STO 8
01	1	41	$\uparrow$	34 07	RCL 7
23	LBL	34 01	RCL 1	71	x
09	9	61	+	34 02	RCL 2
01	1	71	x	61	+
00	0	34 02	RCL 2	33 01	STO 1
71	x	61	+	34 07	RCL 7
35 22	g $x \leq y$	71	x	71	x
22	GTO	34 03	RCL 3	34 03	RCL 3
09	9	61	+	61	+
33 06	STO 6	71	x	33 06	STO 6
23	LBL	34 04	RCL 4	34 07	RCL 7
01	1	61	+	71	x
34 06	RCL 6	71	x	34 04	RCL 4
01	1	34 05	RCL 5	61	+
00	0	61	+	33 05	STO 5
81	$\div$	00	0	34 01	RCL 1
33 06	STO 6	35 23	g $x = y$	33 07	STO 7
34 08	RCL 8	22	GTO	84	R/S
42	CHS	03	3		
33 08	STO 8	44	CLX		
23	LBL	34 08	RCL 8		
02	2	71	x		
34 06	RCL 6	00	0		

R ₁	α_4	R ₄	α_1	R ₇	x_i
R ₂	α_3	R ₅	α_0	R ₈	k
R ₃	α_2	R ₆	$ \alpha_0 , \Delta x$	R ₉	Used

SIMULTANEOUS EQUATIONS
IN TWO UNKNOWNNS

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	34 02	RCL 2	35 01	g NOP
11	A	34 04	RCL 4	35 01	g NOP
33 01	STO 1	71	x	35 01	g NOP
35 08	g R $\downarrow$	51	-	35 01	g NOP
33 02	STO 2	34 07	RCL 7	35 01	g NOP
35 08	g R $\downarrow$	81	$\div$	35 01	g NOP
33 03	STO 3	84	R/S	35 01	g NOP
24	RTN	35 07	g $x \geq y$	35 01	g NOP
23	LBL	24	RTN	35 01	g NOP
12	B	35 01	g NOP	35 01	g NOP
33 04	STO 4	35 01	g NOP	35 01	g NOP
35 08	g R $\downarrow$	35 01	g NOP	35 01	g NOP
33 05	STO 5	35 01	g NOP	35 01	g NOP
35 08	g R $\downarrow$	35 01	g NOP	35 01	g NOP
33 06	STO 6	35 01	g NOP	35 01	g NOP
34 03	RCL 3	35 01	g NOP	35 01	g NOP
35 09	g R $\uparrow$	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
34 06	RCL 6	35 01	g NOP	35 01	g NOP
34 01	RCL 1	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
51	-	35 01	g NOP	35 01	g NOP
34 03	RCL 3	35 01	g NOP	35 01	g NOP
34 05	RCL 5	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
34 02	RCL 2	35 01	g NOP	35 01	g NOP
34 06	RCL 6	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
51	-	35 01	g NOP	35 01	g NOP
84	R/S	35 01	g NOP	35 01	g NOP
33 07	STO 7	35 01	g NOP	35 01	g NOP
81	$\div$	35 01	g NOP	35 01	g NOP
34 01	RCL 1	35 01	g NOP	35 01	g NOP
34 05	RCL 5	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP

R ₁	e	R ₄	f	R ₇	D
R ₂	b	R ₅	d	R ₈	
R ₃	a	R ₆	c	R ₉	

SIMULTANEOUS EQUATIONS IN
THREE UNKNOWNNS

CODE	KEYS	CODE	KEYS	CODE	KEYS
35 09	g R↑	35 09	g R↑	71	x
81	÷	51	—	34 01	RCL 1
33 03	STO 3	33 07	STO 7	34 06	RCL 6
44	CLX	34 08	RCL 8	71	x
35 00	g LST X	34 06	RCL 6	61	+
81	÷	71	x	34 03	RCL 3
33 02	STO 2	34	RCL	35 07	g x↔y
44	CLX	09	9	51	—
35 00	g LST X	34 05	RCL 5	84	R/S
81	÷	71	x	34 06	RCL 6
33 01	STO 1	51	—	84	R/S
84	R/S	84	R/S	34 08	RCL 8
11	A	34 07	RCL 7	84	R/S
33	STO	34 06	RCL 6	23	LBL
09	9	71	x	11	A
35 07	g x↔y	34	RCL	33 06	STO 6
33 08	STO 8	09	9	35 08	g R↓
34 04	RCL 4	34 04	RCL 4	33 05	STO 5
33 07	STO 7	71	x	35 08	g R↓
84	R/S	51	—	33 04	STO 4
11	A	35 07	g x↔y	34 01	RCL 1
33 06	STO 6	81	÷	71	x
35 07	g x↔y	33 06	STO 6	34 05	RCL 5
33 05	STO 5	44	CLX	51	—
84	R/S	34 08	RCL 8	34 04	RCL 4
41	↑	34 04	RCL 4	34 02	RCL 2
34 03	RCL 3	71	x	71	x
34 04	RCL 4	34 07	RCL 7	34 06	RCL 6
71	x	34 05	RCL 5	51	—
35 07	g x↔y	71	x	24	RTN
51	—	51	—		
33 04	STO 4	35 07	g x↔y		
34 03	RCL 3	81	÷		
34 07	RCL 7	33 08	STO 8		
71	x	34 02	RCL 2		

R ₁	b ₁ /a ₁	R ₄	Used	R ₇	Used
R ₂	c ₁ /a ₁	R ₅	Used	R ₈	a, z
R ₃	d ₁ /a ₁	R ₆	Used	R ₉	b

SYNTHETIC DIVISION

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	71	x	35 01	g NOP
11	A	34 04	RCL 4	35 01	g NOP
33 04	STO 4	61	+	35 01	g NOP
35 08	g R↓	84	R/S	35 01	g NOP
33 03	STO 3	34 01	RCL 1	35 01	g NOP
35 08	g R↓	71	x	35 01	g NOP
33 02	STO 2	34 05	RCL 5	35 01	g NOP
35 08	g R↓	61	+	35 01	g NOP
33	STO	84	R/S	35 01	g NOP
09	9	34 01	RCL 1	35 01	g NOP
84	R/S	71	x	35 01	g NOP
33 08	STO 8	34 06	RCL 6	35 01	g NOP
35 08	g R↓	61	+	35 01	g NOP
33 07	STO 7	84	R/S	35 01	g NOP
35 08	g R↓	34 01	RCL 1	35 01	g NOP
33 06	STO 6	71	x	35 01	g NOP
35 08	g R↓	34 07	RCL 7	35 01	g NOP
33 05	STO 5	61	+	35 01	g NOP
24	RTN	84	R/S	35 01	g NOP
23	LBL	34 01	RCL 1	35 01	g NOP
12	B	71	x	35 01	g NOP
33 01	STO 1	34 08	RCL 8	35 01	g NOP
34	RCL	61	+	35 01	g NOP
09	9	24	RTN	35 01	g NOP
84	R/S	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
34 02	RCL 2	35 01	g NOP	35 01	g NOP
61	+	35 01	g NOP	35 01	g NOP
84	R/S	35 01	g NOP	35 01	g NOP
34 01	RCL 1	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
34 03	RCL 3	35 01	g NOP	35 01	g NOP
61	+	35 01	g NOP	35 01	g NOP
84	R/S	35 01	g NOP	35 01	g NOP
34 01	RCL 1	35 01	g NOP	35 01	g NOP

R ₁	x ₀	R ₄	a ₄	R ₇	a ₁
R ₂	a ₆	R ₅	a ₃	R ₈	a ₀
R ₃	a ₅	R ₆	a ₂	R ₉	a ₇

RECTANGULAR, SPHERICAL CONVERSIONS

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	84	R/S	35 08	g R↓
11	A	35 08	g R↓	24	RTN
33 01	STO 1	35 08	g R↓	35 01	g NOP
35 08	g R↓	24	RTN	35 01	g NOP
33 02	STO 2	23	LBL	35 01	g NOP
35 08	g R↓	12	B	35 01	g NOP
33 03	STO 3	33 01	STO 1	35 01	g NOP
32	f^{-1}	35 08	g R↓	35 01	g NOP
09	$\sqrt{x}$	33 02	STO 2	35 01	g NOP
35 09	g R↑	35 08	g R↓	35 01	g NOP
32	f^{-1}	33 03	STO 3	35 01	g NOP
09	$\sqrt{x}$	31	f	35 01	g NOP
61	+	05	COS	35 01	g NOP
34 01	RCL 1	34 01	RCL 1	35 01	g NOP
32	f^{-1}	71	x	35 01	g NOP
09	$\sqrt{x}$	34 02	RCL 2	35 01	g NOP
61	+	31	f	35 01	g NOP
31	f	04	SIN	35 01	g NOP
09	$\sqrt{x}$	34 03	RCL 3	35 01	g NOP
33 04	STO 4	31	f	35 01	g NOP
34 03	RCL 3	04	SIN	35 01	g NOP
35 07	$g x \rightarrow y$	34 01	RCL 1	35 01	g NOP
81	$\div$	71	x	35 01	g NOP
32	f^{-1}	71	x	35 01	g NOP
05	COS	35 00	g LST X	35 01	g NOP
34 02	RCL 2	34 02	RCL 2	35 01	g NOP
34 01	RCL 1	31	f	35 01	g NOP
31	f	05	COS	35 01	g NOP
01	R→P	71	x	35 01	g NOP
35 08	g R↓	84	R/S	35 01	g NOP
34 04	RCL 4	35 08	g R↓	35 01	g NOP
84	R/S	84	R/S	35 01	g NOP
35 08	g R↓	35 08	g R↓	35 01	g NOP
84	R/S	84	R/S	35 01	g NOP
35 08	g R↓	35 08	g R↓	35 01	g NOP

R ₁	x, r	R ₄	r	R ₇	
R ₂	y, ϕ	R ₅		R ₈	
R ₃	z, θ	R ₆		R ₉	Used

TRANSLATION AND/OR ROTATION
OF COORDINATE AXES

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	34 03	RCL 3	35 01	g NOP
11	A	31	f	35 01	g NOP
33 01	STO 1	05	COS	35 01	g NOP
35 07	$g x \rightarrow y$	34 04	RCL 4	35 01	g NOP
33 02	STO 2	71	x	35 01	g NOP
24	RTN	34 03	RCL 3	35 01	g NOP
23	LBL	31	f	35 01	g NOP
12	B	04	SIN	35 01	g NOP
35 07	$g x \rightarrow y$	34 05	RCL 5	35 01	g NOP
34 02	RCL 2	71	x	35 01	g NOP
51	—	61	+	35 01	g NOP
35 07	$g x \rightarrow y$	24	RTN	35 01	g NOP
34 01	RCL 1	23	LBL	35 01	g NOP
51	—	15	E	35 01	g NOP
24	RTN	12	B	35 01	g NOP
23	LBL	14	D	35 01	g NOP
13	C	24	RTN	35 01	g NOP
33 03	STO 3	35 01	g NOP	35 01	g NOP
24	RTN	35 01	g NOP	35 01	g NOP
23	LBL	35 01	g NOP	35 01	g NOP
14	D	35 01	g NOP	35 01	g NOP
33 04	STO 4	35 01	g NOP	35 01	g NOP
35 07	$g x \rightarrow y$	35 01	g NOP	35 01	g NOP
33 05	STO 5	35 01	g NOP	35 01	g NOP
34 03	RCL 3	35 01	g NOP	35 01	g NOP
31	f	35 01	g NOP	35 01	g NOP
05	COS	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
35 07	$g x \rightarrow y$	35 01	g NOP	35 01	g NOP
42	CHS	35 01	g NOP	35 01	g NOP
34 03	RCL 3	35 01	g NOP	35 01	g NOP
31	f	35 01	g NOP	35 01	g NOP
04	SIN	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
61	+	35 01	g NOP	35 01	g NOP

R ₁	x ₀	R ₄	x	R ₇	
R ₂	y ₀	R ₅	y	R ₈	
R ₃	θ	R ₆		R ₉	Used

ANGLE CONVERSIONS

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	35 07	$g \times \div y$	81	$\div$
11	A	35 08	$g R \downarrow$	31	f
33 01	STO 1	81	$\div$	61	TF1
31	f	32	f^{-1}	22	GTO
61	TF1	51	SF1	03	3
22	GTO	24	RTN	71	x
00	0	23	LBL	15	E
15	E	13	C	34 01	RCL 1
34 01	RCL 1	33 01	STO 1	24	RTN
24	RTN	83	$\cdot$	23	LBL
23	LBL	09	9	03	3
00	0	31	f	35 07	$g \times \div y$
35 07	$g \times \div y$	61	TF1	35 08	$g R \downarrow$
32	f^{-1}	22	GTO	81	$\div$
51	SF1	02	2	32	f^{-1}
24	RTN	71	x	51	SF1
23	LBL	15	E	24	RTN
12	B	34 01	RCL 1	23	LBL
33 01	STO 1	24	RTN	15	E
01	1	23	LBL	31	f
08	8	02	2	51	SF1
00	0	35 07	$g \times \div y$	24	RTN
35	g	35 08	$g R \downarrow$	35 01	g NOP
02	π	81	$\div$	35 01	g NOP
81	$\div$	32	f^{-1}	35 01	g NOP
31	f	51	SF1	35 01	g NOP
61	TF1	24	RTN	35 01	g NOP
22	GTO	23	LBL	35 01	g NOP
01	1	14	D	35 01	g NOP
71	x	33 01	STO 1	35 01	g NOP
15	E	09	9	35 01	g NOP
34 01	RCL 1	41	$\uparrow$	35 01	g NOP
24	RTN	01	1	35 01	g NOP
23	LBL	06	6	35 01	g NOP
01	1	00	0	35 01	g NOP

R ₁	x	R ₄	R ₇
R ₂		R ₅	R ₈
R ₃		R ₆	R ₉

SECONDARY VALUES OF
 $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	61	+	15	E
11	A	14	D	32	f^{-1}
35	g	24	RTN	51	SF1
41	DEG	23	LBL	32	f^{-1}
32	f^{-1}	14	D	71	SF2
04	SIN	31	f	24	RTN
01	1	61	TF1	35 01	g NOP
08	8	22	GTO	35 01	g NOP
00	0	03	3	35 01	g NOP
35 07	$g \times \div y$	31	f	35 01	g NOP
51	-	81	TF2	35 01	g NOP
14	D	22	GTO	35 01	g NOP
24	RTN	02	2	35 01	g NOP
23	LBL	24	RTN	35 01	g NOP
12	B	23	LBL	35 01	g NOP
35	g	02	2	35 01	g NOP
41	DEG	83	$\cdot$	35 01	g NOP
32	f^{-1}	09	9	35 01	g NOP
05	COS	81	$\div$	35 01	g NOP
03	3	35	g	35 01	g NOP
06	6	43	GRD	35 01	g NOP
00	0	24	RTN	35 01	g NOP
35 07	$g \times \div y$	23	LBL	35 01	g NOP
51	-	03	3	35 01	g NOP
14	D	35	g	35 01	g NOP
24	RTN	02	π	35 01	g NOP
23	LBL	71	x	35 01	g NOP
13	C	01	1	35 01	g NOP
35	g	08	8	35 01	g NOP
41	DEG	00	0	35 01	g NOP
32	f^{-1}	81	$\div$	35 01	g NOP
06	TAN	35	g		
01	1	42	RAD		
08	8	24	RTN		
00	0	23	LBL		

R ₁	R ₄	R ₇
R ₂	R ₅	R ₈
R ₃	R ₆	R ₉ Used

TRIGONOMETRIC FUNCTIONS

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	35	g	35 01	g NOP
11	A	04	$1/x$	35 01	g NOP
31	f	32	f^{-1}	35 01	g NOP
61	TF1	06	TAN	35 01	g NOP
22	GTO	15	E	35 01	g NOP
01	1	24	RTN	35 01	g NOP
31	f	23	LBL	35 01	g NOP
06	TAN	02	2	35 01	g NOP
35	g	35	g	35 01	g NOP
04	$1/x$	04	$1/x$	35 01	g NOP
24	RTN	32	f^{-1}	35 01	g NOP
23	LBL	05	COS	35 01	g NOP
12	B	15	E	35 01	g NOP
31	f	24	RTN	35 01	g NOP
61	TF1	23	LBL	35 01	g NOP
22	GTO	03	3	35 01	g NOP
02	2	35	g	35 01	g NOP
31	f	04	$1/x$	35 01	g NOP
05	COS	32	f^{-1}	35 01	g NOP
35	g	04	SIN	35 01	g NOP
04	$1/x$	15	E	35 01	g NOP
24	RTN	24	RTN	35 01	g NOP
23	LBL	23	LBL	35 01	g NOP
13	C	14	D	35 01	g NOP
31	f	31	f	35 01	g NOP
61	TF1	51	SF1	35 01	g NOP
22	GTO	24	RTN	35 01	g NOP
03	3	23	LBL	35 01	g NOP
31	f	15	E	35 01	g NOP
04	SIN	32	f^{-1}	35 01	g NOP
35	g	51	SF1	35 01	g NOP
04	$1/x$	24	RTN		
24	RTN	35 01	g NOP		
23	LBL	35 01	g NOP		
01	1	35 01	g NOP		

R ₁	R ₄	R ₇
R ₂	R ₅	R ₈
R ₃	R ₆	R ₉ Used

HYPERBOLIC FUNCTIONS

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	61	+	35 01	g NOP
11	A	81	$\div$	35 01	g NOP
32	f^{-1}	24	RTN	35 01	g NOP
07	LN	23	LBL	35 01	g NOP
41	$\uparrow$	14	D	35 01	g NOP
35	g	35	g	35 01	g NOP
04	$1/x$	04	$1/x$	35 01	g NOP
51	—	24	RTN	35 01	g NOP
02	2	35 01	g NOP	35 01	g NOP
81	$\div$	35 01	g NOP	35 01	g NOP
24	RTN	35 01	g NOP	35 01	g NOP
23	LBL	35 01	g NOP	35 01	g NOP
12	B	35 01	g NOP	35 01	g NOP
32	f^{-1}	35 01	g NOP	35 01	g NOP
07	LN	35 01	g NOP	35 01	g NOP
41	$\uparrow$	35 01	g NOP	35 01	g NOP
35	g	35 01	g NOP	35 01	g NOP
04	$1/x$	35 01	g NOP	35 01	g NOP
61	+	35 01	g NOP	35 01	g NOP
02	2	35 01	g NOP	35 01	g NOP
81	$\div$	35 01	g NOP	35 01	g NOP
24	RTN	35 01	g NOP	35 01	g NOP
23	LBL	35 01	g NOP	35 01	g NOP
13	C	35 01	g NOP	35 01	g NOP
32	f^{-1}	35 01	g NOP	35 01	g NOP
07	LN	35 01	g NOP	35 01	g NOP
33	STO	35 01	g NOP	35 01	g NOP
09	9	35 01	g NOP	35 01	g NOP
41	$\uparrow$	35 01	g NOP	35 01	g NOP
35	g	35 01	g NOP	35 01	g NOP
04	$1/x$	35 01	g NOP	35 01	g NOP
51	—	35 01	g NOP	35 01	g NOP
34	RCL	35 01	g NOP	35 01	g NOP
09	9	35 01	g NOP	35 01	g NOP
35 00	g LST X	35 01	g NOP		

R ₁	R ₄	R ₇
R ₂	R ₅	R ₈
R ₃	R ₆	R ₉ Used

INVERSE HYPERBOLIC FUNCTIONS

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	81	$\div$	35 01	g NOP
11	A	31	f	35 01	g NOP
41	$\uparrow$	07	LN	35 01	g NOP
32	f^{-1}	02	2	35 01	g NOP
09	$\sqrt{x}$	81	$\div$	35 01	g NOP
01	1	24	RTN	35 01	g NOP
61	+	23	LBL	35 01	g NOP
31	f	14	D	35 01	g NOP
09	$\sqrt{x}$	35	g	35 01	g NOP
61	+	04	$1/x$	35 01	g NOP
31	f	24	RTN	35 01	g NOP
07	LN	35 01	g NOP	35 01	g NOP
24	RTN	35 01	g NOP	35 01	g NOP
23	LBL	35 01	g NOP	35 01	g NOP
12	B	35 01	g NOP	35 01	g NOP
41	$\uparrow$	35 01	g NOP	35 01	g NOP
32	f^{-1}	35 01	g NOP	35 01	g NOP
09	$\sqrt{x}$	35 01	g NOP	35 01	g NOP
01	1	35 01	g NOP	35 01	g NOP
51	-	35 01	g NOP	35 01	g NOP
31	f	35 01	g NOP	35 01	g NOP
09	$\sqrt{x}$	35 01	g NOP	35 01	g NOP
61	+	35 01	g NOP	35 01	g NOP
31	f	35 01	g NOP	35 01	g NOP
07	LN	35 01	g NOP	35 01	g NOP
24	RTN	35 01	g NOP	35 01	g NOP
23	LBL	35 01	g NOP	35 01	g NOP
13	C	35 01	g NOP	35 01	g NOP
41	$\uparrow$	35 01	g NOP	35 01	g NOP
01	1	35 01	g NOP	35 01	g NOP
35 07	$g x \rightarrow y$	35 01	g NOP	35 01	g NOP
61	+	35 01	g NOP	35 01	g NOP
01	1	35 01	g NOP	35 01	g NOP
35 00	g LST X	35 01	g NOP	35 01	g NOP
51	-	35 01	g NOP	35 01	g NOP

R ₁	R ₄	R ₇
R ₂	R ₅	R ₈
R ₃	R ₆	R ₉

SOLUTION OF A TRIANGLE
(GIVEN a, b, c, or a, b, C)

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	84	R/S	04	SIN
11	A	34 03	RCL 3	35 07	$g x \rightarrow y$
33 03	STO 3	34 01	RCL 1	81	$\div$
35 08	$g R \downarrow$	71	x	33 03	STO 3
33 02	STO 2	32	f^{-1}	34 01	RCL 1
35 08	$g R \downarrow$	04	SIN	71	x
33 01	STO 1	24	RTN	32	f^{-1}
32	f^{-1}	23	LBL	04	SIN
09	$\sqrt{x}$	12	B	84	R/S
34 02	RCL 2	33 03	STO 3	34 03	RCL 3
32	f^{-1}	35 08	$g R \downarrow$	34 02	RCL 2
09	$\sqrt{x}$	33 02	STO 2	71	x
61	+	35 08	$g R \downarrow$	32	f^{-1}
34 03	RCL 3	33 01	STO 1	04	SIN
32	f^{-1}	32	f^{-1}	24	RTN
09	$\sqrt{x}$	09	$\sqrt{x}$	35 01	g NOP
51	-	34 02	RCL 2	35 01	g NOP
02	2	32	f^{-1}	35 01	g NOP
81	$\div$	09	$\sqrt{x}$	35 01	g NOP
34 01	RCL 1	61	+	35 01	g NOP
34 02	RCL 2	34 01	RCL 1	35 01	g NOP
71	x	34 02	RCL 2	35 01	g NOP
81	$\div$	71	x	35 01	g NOP
32	f^{-1}	34 03	RCL 3	35 01	g NOP
05	COS	31	f	35 01	g NOP
84	R/S	05	COS	35 01	g NOP
31	f	71	x	35 01	g NOP
04	SIN	02	2	35 01	g NOP
34 03	RCL 3	71	x	35 01	g NOP
81	$\div$	51	-	35 01	g NOP
33 03	STO 3	31	f	35 01	g NOP
34 02	RCL 2	09	$\sqrt{x}$	35 01	g NOP
71	x	84	R/S	35 01	g NOP
32	f^{-1}	34 03	RCL 3	35 01	g NOP
04	SIN	31	f	35 01	g NOP

R ₁	a	R ₄	R ₇
R ₂	b	R ₅	R ₈
R ₃	c or C	R ₆	R ₉ Used

SOLUTION OF A TRIANGLE (GIVEN a, A, C or a, B, C)

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	35	g	35 01	g NOP
11	A	41	DEG	35 01	g NOP
35	g	33 02	STO 2	35 01	g NOP
41	DEG	35 08	g R↓	35 01	g NOP
33 01	STO 1	33 01	STO 1	35 01	g NOP
35 08	g R↓	35 08	g R↓	35 01	g NOP
33 02	STO 2	33 03	STO 3	35 01	g NOP
35 09	g R↑	34 02	RCL 2	35 01	g NOP
61	+	31	f	35 01	g NOP
01	1	04	SIN	35 01	g NOP
08	8	71	x	35 01	g NOP
00	0	01	1	35 01	g NOP
35 07	g x↔y	08	8	35 01	g NOP
51	-	00	0	35 01	g NOP
84	R/S	34 02	RCL 2	35 01	g NOP
33 03	STO 3	34 01	RCL 1	35 01	g NOP
31	f	61	+	35 01	g NOP
04	SIN	51	-	35 01	g NOP
71	x	84	R/S	35 01	g NOP
34 02	RCL 2	31	f	35 01	g NOP
31	f	04	SIN	35 01	g NOP
04	SIN	33 02	STO 2	35 01	g NOP
81	÷	81	÷	35 01	g NOP
84	R/S	84	R/S	35 01	g NOP
34 01	RCL 1	34 01	RCL 1	35 01	g NOP
31	f	31	f	35 01	g NOP
04	SIN	04	SIN	35 01	g NOP
71	x	34 03	RCL 3	35 01	g NOP
34 03	RCL 3	71	x	35 01	g NOP
31	f	34 02	RCL 2	35 01	g NOP
04	SIN	81	÷	35 01	g NOP
81	÷	24	RTN		
24	RTN	35 01	g NOP		
23	LBL	35 01	g NOP		
12	B	35 01	g NOP		

R₁ Used
R₂ Used
R₃ Used

R₄
R₅
R₆

R₇
R₈
R₉

Used

SOLUTION OF A TRIANGLE (GIVEN B, b, c)

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	35	g	35 01	g NOP
11	A	02	π	35 01	g NOP
32	f ⁻¹	31	f	35 01	g NOP
51	SF1	81	TF2	35 01	g NOP
32	f ⁻¹	35 08	g R↓	35 01	g NOP
71	SF2	35 01	g NOP	35 01	g NOP
24	RTN	34 02	RCL 2	35 01	g NOP
23	LBL	34 03	RCL 3	35 01	g NOP
12	B	61	+	35 01	g NOP
33 01	STO 1	35 07	g x↔y	35 01	g NOP
35 08	g R↓	51	-	35 01	g NOP
33 04	STO 4	42	CHS	35 01	g NOP
35 08	g R↓	84	R/S	35 01	g NOP
33 02	STO 2	35 00	g LST X	35 01	g NOP
31	f	35 07	g x↔y	35 01	g NOP
04	SIN	31	f	35 01	g NOP
34 01	RCL 1	04	SIN	35 01	g NOP
71	x	34 01	RCL 1	35 01	g NOP
34 04	RCL 4	71	x	35 01	g NOP
81	÷	34 03	RCL 3	35 01	g NOP
32	f ⁻¹	31	f	35 01	g NOP
04	SIN	04	SIN	35 01	g NOP
84	R/S	81	÷	35 01	g NOP
33 03	STO 3	84	R/S	35 01	g NOP
23	LBL	44	CLX	35 01	g NOP
05	5	34 03	RCL 3	35 01	g NOP
02	2	51	-	35 01	g NOP
00	0	84	R/S	35 01	g NOP
00	0	33 03	STO 3	35 01	g NOP
41	↑	22	GTO	35 01	g NOP
01	1	05	5		
08	8	35 01	g NOP		
00	0	35 01	g NOP		
31	f	35 01	g NOP		
61	TF1	35 01	g NOP		

R₁ c
R₂ B
R₃ C

R₄ b
R₅
R₆

R₇
R₈
R₉ Used

SPHERICAL TRIANGLES

CODE	KEYS	CODE	KEYS	CODE	KEYS
44	CLX	31	f	35 09	g R↑
32	f ⁻¹	61	TF 1	35 09	g R↑
51	SF 1	42	CHS	33 03	STO 3
84	R/S	35 01	g NOP	31	f
33 03	STO 3	61	+	05	COS
35 08	g R↓	32	f ⁻¹	34 01	RCL 1
33 02	STO 2	05	COS	31	f
35 08	g R↓	33 01	STO 1	05	COS
33 01	STO 1	23	LBL	34 02	RCL 2
44	CLX	12	B	31	f
84	R/S	14	D	05	COS
23	LBL	33 04	STO 4	71	x
13	C	14	D	31	f
31	f	33 05	STO 5	61	TF 1
51	SF 1	14	D	42	CHS
84	R/S	33 06	STO 6	35 01	g NOP
23	LBL	34 01	RCL 1	51	—
11	A	84	R/S	34 01	RCL 1
34 01	RCL 1	34 02	RCL 2	31	f
34 02	RCL 2	84	R/S	04	SIN
31	f	34 03	RCL 3	81	÷
04	SIN	84	R/S	34 02	RCL 2
32	f ⁻¹	34 04	RCL 4	31	f
01	R→P	84	R/S	04	SIN
34 03	RCL 3	34 05	RCL 5	81	÷
31	f	84	R/S	32	f ⁻¹
04	SIN	34 06	RCL 6	05	COS
71	x	24	RTN	24	RTN
34 02	RCL 2	23	LBL	35 01	g NOP
31	f	14	D	35 01	g NOP
05	COS	34 01	RCL 1		
34 03	RCL 3	34 02	RCL 2		
31	f	33 01	STO 1		
05	COS	34 03	RCL 3		
71	x	33 02	STO 2		

R₁ UsedR₂ UsedR₃ UsedR₄ A or aR₅ B or bR₆ C or cR₇R₈R₉ Used

AREA OF A TRIANGLE

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	23	LBL	33 06	STO 6
11	A	13	C	35 08	g R↓
14	D	33 01	STO 1	33 05	STO 5
35 07	g x↔y	35 07	g x↔y	35 08	g R↓
35 08	g R↓	31	f	33 04	STO 4
61	+	04	SIN	34 06	RCL 6
61	+	35 00	g LST X	51	—
02	2	35 08	g R↓	34 02	RCL 2
81	÷	35 07	g x↔y	71	x
41	↑	31	f	34 06	RCL 6
41	↑	04	SIN	34 05	RCL 5
41	↑	71	x	51	—
34 01	RCL 1	35 07	g x↔y	34 01	RCL 1
51	—	32	f ⁻¹	71	x
71	x	09	√x	61	+
35 07	g x↔y	71	x	34 05	RCL 5
34 02	RCL 2	02	2	34 04	RCL 4
51	—	81	÷	51	—
71	x	35 07	g x↔y	34 03	RCL 3
35 07	g x↔y	34 01	RCL 1	71	x
34 03	RCL 3	61	+	61	+
51	—	31	f	02	2
71	x	04	SIN	81	÷
31	f	81	÷	35	g
09	√x	24	RTN	06	ABS
24	RTN	23	LBL	24	RTN
23	LBL	14	D	35 01	g NOP
12	B	33 03	STO 3	35 01	g NOP
31	f	35 08	g R↓	35 01	g NOP
04	SIN	33 02	STO 2	35 01	g NOP
71	x	35 08	g R↓		
71	x	33 01	STO 1		
02	2	24	RTN		
81	÷	23	LBL		
24	RTN	15	E		

R₁ UsedR₂ UsedR₃ UsedR₄ UsedR₅ UsedR₆ UsedR₇R₈R₉ Used

AREA OF A POLYGON

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	15	E	35 01	g NOP
11	A	34 05	RCL 5	35 01	g NOP
33 04	STO 4	02	2	35 01	g NOP
35 08	g R↓	81	÷	35 01	g NOP
33 03	STO 3	24	RTN	35 01	g NOP
35 08	g R↓	23	LBL	35 01	g NOP
33 02	STO 2	15	E	35 01	g NOP
35 08	g R↓	35 07	g x↔y	35 01	g NOP
33 01	STO 1	35 08	g R↓	35 01	g NOP
34 03	RCL 3	61	+	35 01	g NOP
61	+	33 06	STO 6	35 01	g NOP
34 02	RCL 2	35 08	g R↓	35 01	g NOP
34 04	RCL 4	35 07	g x↔y	35 01	g NOP
51	—	51	—	35 01	g NOP
71	x	34 06	RCL 6	35 01	g NOP
33 05	STO 5	71	x	35 01	g NOP
34 04	RCL 4	33	STO	35 01	g NOP
34 03	RCL 3	61	+	35 01	g NOP
23	LBL	05	5	35 01	g NOP
01	1	34 04	RCL 4	35 01	g NOP
84	R/S	34 03	RCL 3	35 01	g NOP
33 04	STO 4	24	RTN	35 01	g NOP
35 07	g x↔y	35 01	g NOP	35 01	g NOP
33 03	STO 3	35 01	g NOP	35 01	g NOP
15	E	35 01	g NOP	35 01	g NOP
22	GTO	35 01	g NOP	35 01	g NOP
01	1	35 01	g NOP	35 01	g NOP
23	LBL	35 01	g NOP	35 01	g NOP
12	B	35 01	g NOP	35 01	g NOP
33 04	STO 4	35 01	g NOP	35 01	g NOP
35 07	g x↔y	35 01	g NOP	35 01	g NOP
33 03	STO 3	35 01	g NOP	35 01	g NOP
15	E	35 01	g NOP	35 01	g NOP
34 02	RCL 2	35 01	g NOP	35 01	g NOP
34 01	RCL 1	35 01	g NOP	35 01	g NOP

R ₁	x ₁	R ₄	Used	R ₇
R ₂	y ₁	R ₅	Used	R ₈
R ₃	Used	R ₆	Used	R ₉

CIRCLE DETERMINED BY THREE POINTS

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	34 05	RCL 5	34 02	RCL 2
11	A	35 07	g x↔y	51	—
33 02	STO 2	51	—	34 04	RCL 4
35 07	g x↔y	84	R/S	34 02	RCL 2
33 01	STO 1	33 01	STO 1	61	+
24	RTN	34 02	RCL 2	71	x
23	LBL	84	R/S	61	+
12	B	34 04	RCL 4	34 03	RCL 3
33 04	STO 4	51	—	34 01	RCL 1
35 07	g x↔y	35 07	g x↔y	51	—
33 03	STO 3	34 03	RCL 3	02	2
15	E	51	—	71	x
33 07	STO 7	31	f	81	÷
14	D	01	R→P	24	RTN
33 08	STO 8	84	R/S	35 01	g NOP
24	RTN	23	LBL	35 01	g NOP
23	LBL	14	D	35 01	g NOP
13	C	34 04	RCL 4	35 01	g NOP
33 04	STO 4	34 02	RCL 2	35 01	g NOP
35 07	g x↔y	51	—	35 01	g NOP
33 03	STO 3	34 03	RCL 3	35 01	g NOP
15	E	34 01	RCL 1	35 01	g NOP
33 05	STO 5	51	—	35 01	g NOP
14	D	81	÷	35 01	g NOP
33 06	STO 6	24	RTN	35 01	g NOP
34 08	RCL 8	23	LBL	35 01	g NOP
51	—	15	E	35 01	g NOP
35 07	g x↔y	34 03	RCL 3	35 01	g NOP
34 07	RCL 7	34 01	RCL 1	35 01	g NOP
51	—	51	—	35 01	g NOP
35 07	g x↔y	34 03	RCL 3	35 01	g NOP
81	÷	34 01	RCL 1	35 01	g NOP
33 02	STO 2	61	+	35 01	g NOP
34 06	RCL 6	71	x	35 01	g NOP
71	x	34 04	RCL 4		

R ₁	x ₁ , x ₀	R ₄	y ₂ , y ₃	R ₇	K ₁
R ₂	y ₁ , y ₀	R ₅	K ₂	R ₈	N ₁
R ₃	x ₂ , x ₃	R ₆	N ₂	R ₉	Used

EQUALLY SPACED POINTS ON A CIRCLE

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	35 08	g R↓	35 01	g NOP
11	A	33 01	STO 1	35 01	g NOP
35	g	84	R/S	35 01	g NOP
02	π	33 03	STO 3	35 01	g NOP
02	2	35 07	g x $\rightarrow$ y	35 01	g NOP
71	x	33 04	STO 4	35 01	g NOP
33 01	STO 1	34 01	RCL 1	35 01	g NOP
04	4	23	LBL	35 01	g NOP
00	0	01	1	35 01	g NOP
00	0	34 02	RCL 2	35 01	g NOP
33 02	STO 2	32	f ⁻¹	35 01	g NOP
03	3	01	R $\rightarrow$ P	35 01	g NOP
06	6	34 03	RCL 3	35 01	g NOP
00	0	61	+	35 01	g NOP
33 05	STO 5	84	R/S	35 01	g NOP
43	EEX	35 07	g x $\rightarrow$ y	35 01	g NOP
02	2	34 04	RCL 4	35 01	g NOP
31	f	61	+	35 01	g NOP
04	SIN	84	R/S	35 01	g NOP
01	1	34 01	RCL 1	35 01	g NOP
35 23	g x=y	34 05	RCL 5	35 01	g NOP
34 02	RCL 2	61	+	35 01	g NOP
33 05	STO 5	33 01	STO 1	35 01	g NOP
44	CLX	22	GTO	35 01	g NOP
35 24	g x>y	01	1	35 01	g NOP
34 01	RCL 1	35 01	g NOP	35 01	g NOP
33 05	STO 5	35 01	g NOP	35 01	g NOP
84	R/S	35 01	g NOP	35 01	g NOP
41	$\uparrow$	35 01	g NOP	35 01	g NOP
34 05	RCL 5	35 01	g NOP	35 01	g NOP
35 07	g x $\rightarrow$ y	35 01	g NOP	35 01	g NOP
81	$\div$	35 01	g NOP	35 01	g NOP
33 05	STO 5	35 01	g NOP	35 01	g NOP
35 08	g R↓	35 01	g NOP	35 01	g NOP
33 02	STO 2	35 01	g NOP	35 01	g NOP

R ₁	$2\pi, \theta$	R ₄	y_0	R ₇	
R ₂	400, r	R ₅	360, c	R ₈	
R ₃	x_0	R ₆		R ₉	Used

POLYGONS INSCRIBED IN AND CIRCUMSCRIBED ABOUT A CIRCLE

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	34 03	RCL 3	35 01	g NOP
11	A	71	x	35 01	g NOP
33 03	STO 3	02	2	35 01	g NOP
01	1	81	$\div$	35 01	g NOP
08	8	24	RTN	35 01	g NOP
00	0	23	LBL	35 01	g NOP
35 07	g x $\rightarrow$ y	14	D	35 01	g NOP
81	$\div$	34 01	RCL 1	35 01	g NOP
33 01	STO 1	31	f	35 01	g NOP
35 07	g x $\rightarrow$ y	06	TAN	35 01	g NOP
33 02	STO 2	34 02	RCL 2	35 01	g NOP
35	g	71	x	35 01	g NOP
41	DEG	02	2	35 01	g NOP
24	RTN	71	x	35 01	g NOP
23	LBL	24	RTN	35 01	g NOP
12	B	23	LBL	35 01	g NOP
34 01	RCL 1	15	E	35 01	g NOP
31	f	34 01	RCL 1	35 01	g NOP
04	SIN	31	f	35 01	g NOP
34 02	RCL 2	06	TAN	35 01	g NOP
71	x	34 02	RCL 2	35 01	g NOP
02	2	32	f ⁻¹	35 01	g NOP
71	x	09	$\sqrt{x}$	35 01	g NOP
24	RTN	71	x	35 01	g NOP
23	LBL	34 03	RCL 3	35 01	g NOP
13	C	71	x	35 01	g NOP
34 01	RCL 1	24	RTN	35 01	g NOP
02	2	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
31	f	35 01	g NOP	35 01	g NOP
04	SIN	35 01	g NOP	35 01	g NOP
34 02	RCL 2	35 01	g NOP	35 01	g NOP
32	f ⁻¹	35 01	g NOP	35 01	g NOP
09	$\sqrt{x}$	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP

R ₁	180/n	R ₄		R ₇	
R ₂		R ₅		R ₈	
R ₃		R ₆		R ₉	Used

UNIT CONVERSIONS:

C→F; ft, in→cm; lb→kg

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	02	2	04	4
11	A	71	x	05	5
31	f	61	+	03	3
61	TF 1	02	2	05	5
22	GTO	83	•	09	9
01	1	05	5	02	2
41	↑	04	4	03	3
01	1	71	x	07	7
83	•	24	RTN	31	f
08	8	23	LBL	61	TF 1
71	x	02	2	22	GTO
03	3	41	↑	03	3
02	2	03	3	71	x
61	+	00	0	24	RTN
24	RTN	83	•	23	LBL
23	LBL	04	4	03	3
01	1	08	8	81	÷
41	↑	81	÷	15	E
03	3	41	↑	24	RTN
02	2	31	f	23	LBL
51	—	83	INT	14	D
01	1	33	01 STO 1	31	f
83	•	51	—	51	SF 1
08	8	01	1	24	RTN
81	÷	02	2	23	LBL
15	E	71	x	15	E
24	RTN	34	01 RCL 1	32	f ⁻¹
23	LBL	15	E	51	SF 1
12	B	84	R/S	24	RTN
31	f	35	07 g x⇌y	35	01 g NOP
61	TF 1	24	RTN		
22	GTO	23	LBL		
02	2	13	C		
35	07 g x⇌y	41	↑		
01	1	83	•		

R₁ Used
R₂
R₃

R₄
R₅
R₆

R₇
R₈
R₉

UNIT CONVERSIONS:

mi→km; gal→ltr; yd→m; ac→ft²

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	04	4	35	01 g NOP
11	A	04	4	35	01 g NOP
41	↑	22	GTO	35	01 g NOP
01	1	02	2	35	01 g NOP
83	•	23	LBL	35	01 g NOP
06	6	14	D	35	01 g NOP
00	0	41	↑	35	01 g NOP
09	9	04	4	35	01 g NOP
03	3	03	3	35	01 g NOP
04	4	05	5	35	01 g NOP
04	4	06	6	35	01 g NOP
22	GTO	00	0	35	01 g NOP
23	LBL	23	LBL	35	01 g NOP
12	B	02	2	35	01 g NOP
41	↑	31	f	35	01 g NOP
03	3	61	TF1	35	01 g NOP
83	•	22	GTO	35	01 g NOP
07	7	01	1	35	01 g NOP
08	8	71	x	35	01 g NOP
05	5	24	RTN	35	01 g NOP
04	4	23	LBL	35	01 g NOP
01	1	15	E	35	01 g NOP
01	1	31	f	35	01 g NOP
07	7	51	SF1	35	01 g NOP
08	8	24	RTN	35	01 g NOP
04	4	23	LBL	35	01 g NOP
22	GTO	01	1	35	01 g NOP
02	2	81	÷	35	01 g NOP
23	LBL	32	f ⁻¹	35	01 g NOP
13	C	51	SF1	35	01 g NOP
41	↑	84	R/S		
83	•	35	01 g NOP		
09	9	35	01 g NOP		
01	1	35	01 g NOP		

R₁
R₂
R₃

R₄
R₅
R₆

R₇
R₈
R₉

POLYNOMIAL EVALUATION (REAL)

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	61	+	35 01	g NOP
11	A	71	x	35 01	g NOP
31	f	34 06	RCL 6	35 01	g NOP
43	REG	61	+	35 01	g NOP
33 01	STO 1	71	x	35 01	g NOP
84	R/S	34 05	RCL 5	35 01	g NOP
33 02	STO 2	61	+	35 01	g NOP
84	R/S	71	x	35 01	g NOP
33 03	STO 3	34 04	RCL 4	35 01	g NOP
84	R/S	61	+	35 01	g NOP
33 04	STO 4	71	x	35 01	g NOP
84	R/S	34 03	RCL 3	35 01	g NOP
33 05	STO 5	61	+	35 01	g NOP
84	R/S	71	x	35 01	g NOP
33 06	STO 6	34 02	RCL 2	35 01	g NOP
84	R/S	61	+	35 01	g NOP
33 07	STO 7	71	x	35 01	g NOP
84	R/S	34 01	RCL 1	35 01	g NOP
33 08	STO 8	61	+	35 01	g NOP
84	R/S	24	RTN	35 01	g NOP
33	STO	35 01	g NOP	35 01	g NOP
09	9	35 01	g NOP	35 01	g NOP
84	R/S	35 01	g NOP	35 01	g NOP
23	LBL	35 01	g NOP	35 01	g NOP
12	B	35 01	g NOP	35 01	g NOP
41	↑	35 01	g NOP	35 01	g NOP
41	↑	35 01	g NOP	35 01	g NOP
41	↑	35 01	g NOP	35 01	g NOP
34	RCL	35 01	g NOP	35 01	g NOP
09	9	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP		
34 08	RCL 8	35 01	g NOP		
61	+	35 01	g NOP		
71	x	35 01	g NOP		
34 07	RCL 7	35 01	g NOP		

R ₁	a ₀	R ₄	a ₃	R ₇	a ₆
R ₂	a ₁	R ₅	a ₄	R ₈	a ₇
R ₃	a ₂	R ₆	a ₅	R ₉	a ₈

LINEAR AND LAGRANGIAN INTERPOLATIONS

CODE	KEYS	CODE	KEYS	CODE	KEYS
33 04	STO 4	35 07	g x↔y	34 02	RCL 2
35 08	g R↓	51	—	51	—
33 03	STO 3	71	x	34 07	RCL 7
35 08	g R↓	33 06	STO 6	34 03	RCL 3
33 02	STO 2	24	RTN	51	—
35 08	g R↓	23	LBL	71	x
33 01	STO 1	14	D	34 04	RCL 4
84	R/S	41	↑	71	x
23	LBL	34 06	RCL 6	34 07	RCL 7
12	B	81	÷	34 01	RCL 1
33 05	STO 5	33 06	STO 6	51	—
34 01	RCL 1	44	CLX	34 07	RCL 7
51	—	34 02	RCL 2	34 03	RCL 3
34 04	RCL 4	34 01	RCL 1	51	—
71	x	51	—	71	x
34 03	RCL 3	81	÷	34 05	RCL 5
34 05	RCL 5	34 02	RCL 2	71	x
51	—	34 03	RCL 3	61	+
34 02	RCL 2	51	—	34 07	RCL 7
71	x	81	÷	34 01	RCL 1
61	+	33 05	STO 5	51	—
34 03	RCL 3	44	CLX	34 07	RCL 7
34 01	RCL 1	34 01	RCL 1	34 02	RCL 2
51	—	34 02	RCL 2	51	—
81	÷	51	—	71	x
24	RTN	81	÷	34 06	RCL 6
23	LBL	34 01	RCL 1	71	x
13	C	34 03	RCL 3	61	+
33 03	STO 3	51	—	24	RTN
35 07	g x↔y	81	÷	35 01	g NOP
33 02	STO 2	33 04	STO 4		
51	—	24	RTN		
35 07	g x↔y	23	LBL		
33 01	STO 1	15	E		
34 03	RCL 3	33 07	STO 7		

R ₁	Used	R ₄	Used	R ₇	Used
R ₂	Used	R ₅	Used	R ₈	
R ₃	Used	R ₆	Used	R ₉	

FINITE DIFFERENCE INTERPOLATION

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	24	RTN	24	RTN
11	A	23	LBL	35 01	g NOP
33 04	STO 4	13	C	35 01	g NOP
35 08	g R↓	41	↑	35 01	g NOP
33 03	STO 3	34	RCL	35 01	g NOP
35 08	g R↓	09	9	35 01	g NOP
33 02	STO 2	51	—	35 01	g NOP
35 08	g R↓	34 08	RCL 8	35 01	g NOP
33 01	STO 1	81	÷	35 01	g NOP
24	RTN	33 01	STO 1	35 01	g NOP
23	LBL	34 05	RCL 5	35 01	g NOP
12	B	71	x	35 01	g NOP
33 08	STO 8	34 03	RCL 3	35 01	g NOP
35 08	g R↓	61	+	35 01	g NOP
33	STO	34 01	RCL 1	35 01	g NOP
09	9	34 01	RCL 1	35 01	g NOP
34 03	RCL 3	01	1	35 01	g NOP
34 02	RCL 2	61	+	35 01	g NOP
51	—	71	x	35 01	g NOP
33 05	STO 5	33 02	STO 2	35 01	g NOP
34 03	RCL 3	02	2	35 01	g NOP
61	+	81	÷	35 01	g NOP
34 04	RCL 4	34 06	RCL 6	35 01	g NOP
35 07	g x↔y	71	x	35 01	g NOP
51	—	61	+	35 01	g NOP
33 06	STO 6	34 02	RCL 2	35 01	g NOP
34 03	RCL 3	34 01	RCL 1	35 01	g NOP
51	—	01	1	35 01	g NOP
34 02	RCL 2	51	—	35 01	g NOP
02	2	71	x	35 01	g NOP
71	x	06	6		
61	+	81	÷		
34 01	RCL 1	34 07	RCL 7		
51	—	71	x		
33 07	STO 7	61	+		

R ₁	y ₁ , u	R ₄	y ₄	R ₇	δ ³ y _{-½}
R ₂	y ₂ , (u+1)u	R ₅	δy _{-½}	R ₈	h
R ₃	y ₃	R ₆	δ ² y ₀	R ₉	x ₃

NUMERICAL INTEGRATION (DISCRETE CASE)

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	24	RTN	35 01	g NOP
11	A	23	LBL	35 01	g NOP
33 02	STO 2	15	E	35 01	g NOP
35 07	g x↔y	35 07	g x↔y	35 01	g NOP
33 01	STO 1	04	4	35 01	g NOP
24	RTN	71	x	35 01	g NOP
23	LBL	61	+	35 01	g NOP
12	B	34 02	RCL 2	35 01	g NOP
41	↑	61	+	35 01	g NOP
61	+	34 01	RCL 1	35 01	g NOP
33	STO	71	x	35 01	g NOP
61	+	03	3	35 01	g NOP
02	2	81	÷	35 01	g NOP
24	RTN	24	RTN	35 01	g NOP
23	LBL	35 01	g NOP	35 01	g NOP
13	C	35 01	g NOP	35 01	g NOP
41	↑	35 01	g NOP	35 01	g NOP
34 02	RCL 2	35 01	g NOP	35 01	g NOP
61	+	35 01	g NOP	35 01	g NOP
34 01	RCL 1	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
02	2	35 01	g NOP	35 01	g NOP
81	÷	35 01	g NOP	35 01	g NOP
24	RTN	35 01	g NOP	35 01	g NOP
23	LBL	35 01	g NOP	35 01	g NOP
14	D	35 01	g NOP	35 01	g NOP
41	↑	35 01	g NOP	35 01	g NOP
61	+	35 01	g NOP	35 01	g NOP
35 07	g x↔y	35 01	g NOP	35 01	g NOP
04	4	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP		
61	+	35 01	g NOP		
33	STO	35 01	g NOP		
61	+	35 01	g NOP		
02	2	35 01	g NOP		

R ₁	h	R ₄		R ₇	
R ₂	Used	R ₅		R ₈	
R ₃		R ₆		R ₉	

SIMPSON'S RULE FOR NUMERICAL INTEGRATION

CODE	KEYS	CODE	KEYS	CODE	KEYS
33 03	STO 3	11	A	03	3
35 08	g R↓	04	4	81	÷
33 02	STO 2	71	x	84	R/S
35 08	g R↓	33	STO	23	LBL
33 01	STO 1	61	+	11	A
35 09	g R↑	05	5	35 01	g NOP
35 07	g x↔y	34 01	RCL 1	35 01	g NOP
51	—	34 04	RCL 4	35 01	g NOP
34 03	RCL 3	61	+	35 01	g NOP
81	÷	33 01	STO 1	35 01	g NOP
33 04	STO 4	11	A	35 01	g NOP
34 01	RCL 1	02	2	35 01	g NOP
11	A	71	x	35 01	g NOP
33 05	STO 5	33	STO	35 01	g NOP
34 03	RCL 3	61	+	35 01	g NOP
02	2	05	5	35 01	g NOP
81	÷	35	g	35 01	g NOP
01	1	83	DSZ	35 01	g NOP
51	—	22	GTO	35 01	g NOP
33 08	STO 8	01	1	35 01	g NOP
41	↑	34 01	RCL 1	35 01	g NOP
31	f	34 04	RCL 4	35 01	g NOP
83	INT	61	+	35 01	g NOP
35 23	g x=y	11	A	35 01	g NOP
22	GTO	04	4	35 01	g NOP
01	1	71	x	35 01	g NOP
00	0	33	STO	35 01	g NOP
81	÷	61	+	35 01	g NOP
84	R/S	05	5	35 01	g NOP
23	LBL	34 02	RCL 2	35 01	g NOP
01	1	11	A		
34 01	RCL 1	34 05	RCL 5		
34 04	RCL 4	61	+		
61	+	34 04	RCL 4		
33 01	STO 1	71	x		

R ₁	a	R ₄	h	R ₇	
R ₂	b	R ₅	ff	R ₈	Used
R ₃	n	R ₆		R ₉	Used

FIRST ORDER DIFFERENTIAL EQUATION

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	51	—	35 01	g NOP
11	A	34 01	RCL 1	35 01	g NOP
33 01	STO 1	34 03	RCL 3	35 01	g NOP
35 08	g R↓	61	+	35 01	g NOP
33 02	STO 2	12	B	35 01	g NOP
35 08	g R↓	34 05	RCL 5	35 01	g NOP
33 03	STO 3	04	4	35 01	g NOP
35 08	g R↓	71	x	35 01	g NOP
35 08	g R↓	61	+	35 01	g NOP
23	LBL	34 04	RCL 4	35 01	g NOP
01	1	61	+	35 01	g NOP
12	B	06	6	35 01	g NOP
33 04	STO 4	81	÷	35 01	g NOP
34 03	RCL 3	34 03	RCL 3	35 01	g NOP
71	x	71	x	35 01	g NOP
02	2	34 02	RCL 2	35 01	g NOP
81	÷	61	+	35 01	g NOP
34 02	RCL 2	84	R/S	35 01	g NOP
61	+	33 02	STO 2	35 01	g NOP
34 03	RCL 3	34 01	RCL 1	35 01	g NOP
02	2	34 03	RCL 3	35 01	g NOP
81	÷	61	+	35 01	g NOP
34 01	RCL 1	33 01	STO 1	35 01	g NOP
61	+	22	GTO	35 01	g NOP
12	B	01	1	35 01	g NOP
33 05	STO 5	23	LBL	35 01	g NOP
34 03	RCL 3	12	B	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
02	2	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP	35 01	g NOP
34 02	RCL 2	35 01	g NOP	35 01	g NOP
61	+	35 01	g NOP	35 01	g NOP
34 04	RCL 4	35 01	g NOP	35 01	g NOP
34 03	RCL 3	35 01	g NOP	35 01	g NOP
71	x	35 01	g NOP		

R ₁	Used	R ₄	Used	R ₇	
R ₂	Used	R ₅	Used	R ₈	
R ₃	Used	R ₆		R ₉	

ROOTS OF $f(x) = 0$ IN AN INTERVAL

CODE	KEYS	CODE	KEYS	CODE	KEYS
33 07	STO 7	02 2		22 GTO	
35 07	$g x \geq y$	23 LBL		00 0	
33 01	STO 1	00 0		23 LBL	
23 LBL		34 01	RCL 1	03 3	
02 2		34 02	RCL 2	34 04	RCL 4
11 A		61 +		84 R/S	
33 03	STO 3	02 2		34 08	RCL 8
00 0		81 $\div$		33 01	STO 1
35 23	$g x = y$	33 04	STO 4	22 GTO	
34 01	RCL 1	11 A		02 2	
84 R/S		35 g		23 LBL	
34 01	RCL 1	06 ABS		11 A	
34 06	RCL 6	34 05	RCL 5	35 01	g NOP
61 +		35 24	$g x > y$	35 01	g NOP
33 02	STO 2	22 GTO		35 01	g NOP
33 08	STO 8	03 3		35 01	g NOP
11 A		34 01	RCL 1	35 01	g NOP
34 03	RCL 3	11 A		35 01	g NOP
71 x		33 03	STO 3	35 01	g NOP
00 0		34 04	RCL 4	35 01	g NOP
35 24	$g x > y$	11 A		35 01	g NOP
22 GTO		34 03	RCL 3	35 01	g NOP
00 0		71 x		35 01	g NOP
34 02	RCL 2	00 0		35 01	g NOP
33 01	STO 1	35 24	$g x > y$	35 01	g NOP
34 06	RCL 6	22 GTO		35 01	g NOP
61 +		04 4		35 01	g NOP
33 02	STO 2	34 04	RCL 4	35 01	g NOP
34 07	RCL 7	33 01	STO 1	35 01	g NOP
35 07	$g x \geq y$	22 GTO		35 01	g NOP
35 24	$g x > y$	00 0			
84 R/S		23 LBL			
35 01	g NOP	04 4			
34 01	RCL 1	34 04	RCL 4		
22 GTO		33 02	STO 2		

R_1	Used	R_4	Used	R_7	b
R_2	Used	R_5	ϵ	R_8	Used
R_3	Used	R_6	Δx	R_9	Used

DETERMINANT AND CHARACTERISTIC EQUATION OF A 3×3 MATRIX

CODE	KEYS	CODE	KEYS	CODE	KEYS
23 LBL		34 04	RCL 4	71 x	
11 A		71 x		34 05	RCL 5
33 06	STO 6	61 +		71 x	
35 08	$g R \downarrow$	34 RCL		61 +	
33 08	STO 8	09 9		34 06	RCL 6
35 08	$g R \downarrow$	34 05	RCL 5	34 02	RCL 2
33 01	STO 1	71 x		71 x	
24 RTN		61 +		34 07	RCL 7
23 LBL		34 01	RCL 1	71 x	
12 B		34 02	RCL 2	51 -	
33 STO		71 x		34 08	RCL 8
09 9		51 -		34 04	RCL 4
35 08	$g R \downarrow$	34 01	RCL 1	71 x	
33 02	STO 2	34 02	RCL 2	34 03	RCL 3
35 08	$g R \downarrow$	61 +		71 x	
33 04	STO 4	34 03	RCL 3	51 -	
24 RTN		71 x		34 01	RCL 1
23 LBL		51 -		34 RCL	
13 C		84 R/S		09 9	
33 03	STO 3	23 LBL		71 x	
35 08	$g R \downarrow$	14 D		34 05	RCL 5
33 05	STO 5	34 01	RCL 1	71 x	
35 08	$g R \downarrow$	34 02	RCL 2	51 -	
33 07	STO 7	71 x		24 RTN	
84 R/S		34 03	RCL 3	35 01	g NOP
34 01	RCL 1	71 x		35 01	g NOP
34 02	RCL 2	34 08	RCL 8	35 01	g NOP
61 +		34 RCL		35 01	g NOP
34 03	RCL 3	09 9		35 01	g NOP
61 +		71 x		35 01	g NOP
84 R/S		34 07	RCL 7		
34 06	RCL 6	71 x			
34 07	RCL 7	61 +			
71 x		34 06	RCL 6		
34 08	RCL 8	34 04	RCL 4		

R_1	a_1	R_4	b_1	R_7	c_1
R_2	b_1	R_5	c_2	R_8	a_2
R_3	c_1	R_6	a_3	R_9	b_3

2 x 2 MATRIX OPERATIONS

CODE	KEYS	CODE	KEYS	CODE	KEYS
23	LBL	34 08	RCL 8	34 08	RCL 8
11	A	61	+	71	x
33 04	STO 4	24	RTN	61	+
35 08	g R↓	23	LBL	84	R/S
33 03	STO 3	14	D	34 03	RCL 3
35 08	g R↓	34 01	RCL 1	34 05	RCL 5
33 02	STO 2	34 05	RCL 5	71	x
35 08	g R↓	51	—	34 04	RCL 4
33 01	STO 1	84	R/S	34 07	RCL 7
24	RTN	34 02	RCL 2	71	x
23	LBL	34 06	RCL 6	61	+
12	B	51	—	84	R/S
33 08	STO 8	84	R/S	34 03	RCL 3
35 08	g R↓	34 03	RCL 3	34 06	RCL 6
33 07	STO 7	34 07	RCL 7	71	x
35 08	g R↓	51	—	34 04	RCL 4
33 06	STO 6	84	R/S	34 08	RCL 8
35 08	g R↓	34 04	RCL 4	71	x
33 05	STO 5	34 08	RCL 8	61	+
24	RTN	51	—	24	RTN
23	LBL	24	RTN	35 01	g NOP
13	C	23	LBL	35 01	g NOP
34 01	RCL 1	15	E	35 01	g NOP
34 05	RCL 5	34 05	RCL 5	35 01	g NOP
61	+	34 01	RCL 1	35 01	g NOP
84	R/S	71	x	35 01	g NOP
34 02	RCL 2	34 02	RCL 2	35 01	g NOP
34 06	RCL 6	34 07	RCL 7	35 01	g NOP
61	+	71	x	35 01	g NOP
84	R/S	61	+	35 01	g NOP
34 03	RCL 3	84	R/S		
34 07	RCL 7	34 01	RCL 1		
61	+	34 06	RCL 6		
84	R/S	71	x		
34 04	RCL 4	34 02	RCL 2		

R ₁	a ₁	R ₄	a ₄	R ₇	b ₃
R ₂	a ₂	R ₅	b ₁	R ₈	b ₄
R ₃	a ₃	R ₆	b ₂	R ₉	

3 x 3 MATRIX INVERSION

CODE	KEYS	CODE	KEYS	CODE	KEYS
34	RCL	34 03	RCL 3	34 01	RCL 1
09	9	34 04	RCL 4	11	A
34 03	RCL 3	11	A	84	R/S
34 02	RCL 2	42	CHS	23	LBL
11	A	84	R/S	11	A
84	R/S	35 08	g R↓	71	x
35 08	g R↓	34 06	RCL 6	35 07	g x↗y
34 06	RCL 6	34 03	RCL 3	34 05	RCL 5
34 03	RCL 3	34 01	RCL 1	71	x
34 08	RCL 8	11	A	51	—
11	A	84	R/S	35 07	g x↗y
42	CHS	12	B	81	÷
84	R/S	34 06	RCL 6	24	RTN
35 08	g R↓	34	RCL	23	LBL
34 05	RCL 5	09	9	12	B
34 02	RCL 2	34 01	RCL 1	35 08	g R↓
33 05	STO 5	11	A	34 05	RCL 5
35 08	g R↓	42	CHS	34 04	RCL 4
33 02	STO 2	84	R/S	33 05	STO 5
35 08	g R↓	12	B	35 08	g R↓
34 06	RCL 6	34 07	RCL 7	33 04	STO 4
34	RCL	34 02	RCL 2	35 08	g R↓
09	9	34 04	RCL 4	24	RTN
34 08	RCL 8	11	A	35 01	g NOP
11	A	84	R/S	35 01	g NOP
84	R/S	35 08	g R↓	35 01	g NOP
35 08	g R↓	34 08	RCL 8	35 01	g NOP
34 05	RCL 5	34 02	RCL 2	35 01	g NOP
34 07	RCL 7	34 01	RCL 1	35 01	g NOP
33 05	STO 5	11	A	35 01	g NOP
35 08	g R↓	42	CHS		
33 07	STO 7	84	R/S		
35 08	g R↓	12	B		
11	RCL	34 08	RCL 8		
09	9	34 07	RCL 7		

R ₁	Used	R ₄	Used	R ₇	Used
R ₂	Used	R ₅	Used	R ₈	Used
R ₃	Used	R ₆	Used	R ₉	Used

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