Physics

HP COMPUTER CURRICULUM

Geometrical Optics

STUDENT LAB BOOK

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Hewlett-Packard
Computer Curriculum Series

physics
STUDENT LAB BOOK

geometrical optics

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This material is designed to be used with any Hewlett-Packard system with the BASIC programming language such as the 9830A, Educational BASIC, and the 2000 and 3000 series systems.

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INTRODUCTION

This Physics Lab Book was developed to provide you the opportunity to use a computer as a problem solving tool. You will write computer programs which will enable you to investigate the physics field of geometric optics. Using just one program, you will be able to perform many different experiments and make your own generalizations.

To use Geometrical Optics, you will need the following. First, you should have a background in algebra and some trigonometry. Ordinarily the material covered in Geometrical Optics requires differential calculus, but since you will have a computer to do your calculations, you will not need this. Secondly, the Lab Book assumes that you already know how to write a simple computer program using the BASIC language. If you do not already have some experience in programming using BASIC, you will want to study this before you attempt the material. Consult the BASIC Manual for the computer you are using. Last, use of this Lab Book requires that you have access to a computer for at least two hours per week. If more time is available, you may be able to experiment further on your own, either to improve your program or to investigate other aspects of physics that interest you.

As you will discover, there is no one "right" way to use a computer as a problem solving tool. There are many different ways to solve one problem by programming. Experiment and learn as you go. You'll find you are learning something new each time, both about your subject matter and about using the computer to solve problems.

This book was designed to help you by providing several different kinds of material. First there are the exercises with the preparatory explanatory material and the drawings. These exercises are sequenced so that you can apply what you have learned in the previous problem in solving the next one. Often you can take your preceding program and simply add to it to create a program that will provide answers to the more general or more advanced problem.

An example program and flow chart follow the regular exercises. You may wish to review this flow chart and program before you begin using the Lab Book. The example demonstrates planning a solution (the flow chart) and the coding of the solution (the program). When you do begin using the Lab Book, you may choose to flow chart your solutions first. This is good programming practice. Drawing the flow chart provides a check of your logic, and the finished flow chart can then be used as a guide in each step of writing your program.

The section of advanced problems is provided for any students interested in further work in this area. You may wish to tackle this after you have completed Exercise 20.

NOTES

GEOMETRICAL OPTICS

Why can you see around a corner using a mirror? Why does a pencil standing in a glass of water appear to "bend" at the surface of the water? Why does a magnifying glass concentrate the light from the sun into an intense spot that is hot enough to ignite paper? We have all seen these things at one time or another and probably have wondered how or why they happen.

In general, physics is concerned with the explanation of occurrences such as these in terms of general laws or ideas. The better the explanation, the more things that can be explained. The examples above all fall within the part of physics known as *geometrical optics.* Geometrical optics is founded upon the twin ideas of reflection and refraction. If we can find a satisfactory explanation of reflection and refraction we will have a theoretical basis to understand most of geometrical optics.

There is an *idea* that can be used to discover the laws of reflection and refraction. It turns out that variations on the same idea can be used to explain many topics in mechanics, another branch of physics. This idea is generally not discussed in introductory physics texts. The reason is because the usual mathematical treatment of the idea requires advanced techniques that are past the capabilities of the beginner. However, with the computer we can perform an end run around the mathematical difficulties and examine *the idea* and its consequences.

Before proceeding, we should review some concepts about coordinate geometry. If you do not need this review, go on to the next section.

NOTES

REVIEW OF COORDINATE GEOMETRY

We can locate a point on a coordinate system in terms of an ordered pair of numbers (x, y). In the diagram in Figure 1, the point A is located by x = 2 and y = 1. Likewise, B is located by x = 5 and y = 5. On such a coordinate system the values of x and y may be either positive or negative.

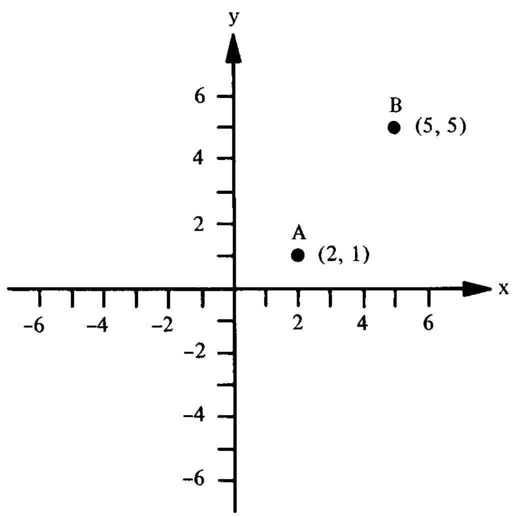


Figure 1

Exercise 1 – Coordinates

In the spaces provided in the table, record the coordinates of the points in the diagram in Figure 2.

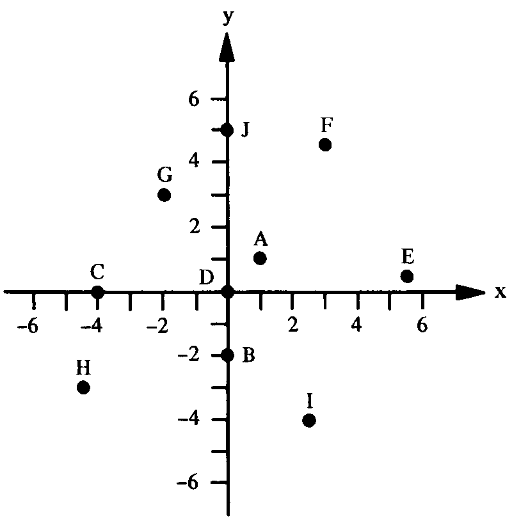


Figure 2

Point	x	y
A		
B		
C		
D		
E		
F		
G		
H		
I		
J		

Suppose we wanted to find the distance between two points. In Figure 3 the point A has coordinates (3, 2) while B is at (6, 6). We can construct a triangle as shown to find the distance between A and B. We use the familiar Pythagorean Theorem which states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides of the triangle, or

$$(AB)^2 = (AC)^2 + (CB)^2$$

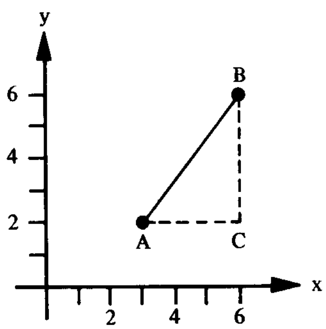


Figure 3

We can find the desired length AB by taking the square root of both sides to get

$$AB = \sqrt{(AC)^2 + (CB)^2}$$

In this example AC = (6 – 3) = 3, CB = (6 – 2) = 4. If these values are substituted into the equation for AB we find

$$AB = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

But AC is the difference in x coordinates of the two points, while CB is the difference between y coordinates. If we locate one point with the coordinates (x_1, y_1), and a second point at (x_2, y_2), you can easily see that the distance d between the two points is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Exercise 2 – Distance Between Points

Write a program which will compute the distance between two points. The input to the program is the coordinates of the two points. The output is the distance between the two points. Test your program on points for which you know the answer. Use your program to fill out the table below.

x_1	y_1	x_2	y_2	d
1	1	5	6	
−5	2	1	−2	
−3.5	−1.2	2.8	4.1	
0	0	0	1.4283	
0	−6.732	0	0	
−2	−3	−3	−2	
.923	−.149	−.758	.358	
0	1.414	1.414	0	
2.8	1.5	−4.1	−2.1	
0	3.6	0	−2.4	

FERMAT'S PRINCIPLE

Now we are ready to start exploring *the idea* in geometrical optics. It can be stated as follows:

> ***If a ray of light passes between two points, out of all the possible paths that could be taken, the light ray will follow the one for which the transit time is a minimum compared to nearby paths.***

This is a remarkable statement! How does nature explore all the possible paths and direct the light ray along that very particular one such that it will pass between the two points in the least possible time? This becomes even more remarkable when we see that the path selection must be done *in advance.* Richard Feynman, the world renowned physicist, stated that nature seems to be able to "smell" the correct path in advance. We cannot explain the *how* or *why* of this, but must remain content with the fact that it *does* happen.

To proceed, we must find how to compute the transit time for a ray of light following a particular straight path. If v is the speed of light and d is the distance, then t, the transit time is given by

$$t = d/v \qquad (1)$$

The value of v depends upon the medium through which the light is passing. If we divide the velocity of light in a vacuum ($c = 3 \times 10^{10}$ cm/sec) by the velocity in a medium, the result is

$$n = c/v = \text{index of refraction for the medium} \qquad (2)$$

For every medium, we can find the value of n. As examples, water has $n = 1.33$ while most types of glass have an index of refraction of about 1.50.

Now we can return to the transit time, and express it in a slightly different form. Since $1/v = n/c$ we have $t = nd/c$. If along the actual light path t is a minimum, then nd/c must also be a minimum. But since c is a constant, nd must be a minimum when compared to any other path. The quantity nd is called the *optical path.* If a ray passes through a distance d_1 in a medium with index of refraction

n_1, and a distance d_2 in a medium with index of refraction n_2, and so on, the total optical path is the sum of the parts, or

$$\text{total optical path} = n_1 d_1 + d_2 d_2 + n_3 d_3 + \ldots \qquad (3)$$

The point that we have been driving at is that having the least transit time for a ray passing between two points is equivalent to saying that the total optical path has the least value. Now we will rephrase *the idea* of geometrical optics in the form most suitable for our discovery exploration.

A ray of light will follow that path between two points for which the total optical path is the smallest as compared to nearby paths.

This is known as Fermat's principle and was discovered in 1650. We have not stated the principle completely above. However, to attempt to be complete at this point might hide what we are trying to accomplish. Therefore, we will return later and complete the principle.

REFLECTION

Consider the diagram in Figure 4. Suppose a ray of light is to pass from point A to point B. Quite obviously the least transit time path is on a straight line joining the two points. However, suppose we require that the ray pass from A to B by reflection from the x axis. If we consider only integer values of x falling between the two points, all the possible reflection paths are shown. (We have not considered reflection points outside the interval x = 1 to x = 11. However, this is certainly a reasonable assumption.)

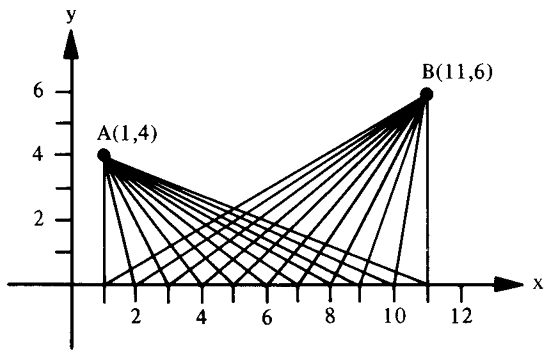


Figure 4

Exercise 3 – Rays Between Fixed Points

Write a program which will check each of the possible paths in Figure 4, and find the one which would be followed according to Fermat's principle. Have the program output the value of x where the reflection will take place. Assume that $n = 1$.

Exercise 4 – Rays Between Changing Points

Modify the program in Exercise 3 to permit the input of the coordinates of the two points. We still want as output the value of x where the reflection will take place. Check only integer values of x between the two points. Use only positive values of y. Assume n = 1.

In Exercises 3 and 4 you found a way to locate the integer value of x closest to the reflection point. Suppose that for a given problem the output is that the reflection takes place at x = 3. We have checked only integer values of x and so know that the actual value probably isn't 3.

However, we also can see that the actual value must fall somewhere between 2 and 4. If we looked at the values of x of 2.0, 2.1, . . . up to 3.9, 4.0 we could narrow the search down to a zone 0.2 wide in which the true value lies. Then we could repeat the process and find the reflection point within 0.02 and so on.

Exercise 5 – Precision Computation

Rewrite your program of Exercise 4 to find the reflection point to M significant digits. As input, the program should require the coordinates of the two points, and M. Try out the program on a set of points with M = 5. Assume that n = 1.

Now that we can locate the reflection point as accurately as we please, we can proceed to learn more about the reflection process. If at the point of reflection we construct a line perpendicular (normal) to the reflecting surface, we can compute the angle from the normal to the ray, both before and after reflection. If x_r is the x coordinate of the reflection point, convince yourself that the tangent of the angle the ray makes with the normal *before* reflection is

$$\tan(\theta_1) = (x_r - x_1)/y_1 . \tag{4}$$

By the same line of reasoning, we can compute the tangent of the angle the ray makes with the normal *after* reflection.

$$\tan(\theta_2) = (x_2 - x_r)/y_2 . \tag{5}$$

Exercise 6 – Angles of Incidence and Reflection

Write a program to accept as input the coordinates of two points. The program should compute and output the angle of incidence (the angle between the ray and normal before reflection), and the angle of reflection (the angle between the ray and the normal after reflection). Arrange your program to compute to six significant figures. Restrict values of y to positive values and as in the previous examples, assume n = 1.

Exercise 7 – Discovery

Use the program from Exercise 6 to investigate a large number of different problems. Print the results in tabular form.

Now we are ready to "discover" the first law which follows from *the idea* as stated in Fermat's principle. You are probably far ahead by now and know what we are driving for.

Exercise 8 – Generalization

Generalize the results of Exercise 7 into the form of a law. This law should be titled the Law of Specular Reflection. *Look up the law in your physics textbook and compare to your version.*

Law of Specular Reflection (your generalization).

Law of Specular Reflection (from textbook).

Exercise 9 – Corner Reflection

Consider the diagram in Figure 5. We want to investigate the path taken if a ray leaves point 1, is reflected from the line OB, is reflected again from the line OA, and finally passes through point 2. Write a computer program which uses Fermat's principle to find the exact path which will be followed. Input several sets of points and examine the results. See if you can draw a conclusion from your results.

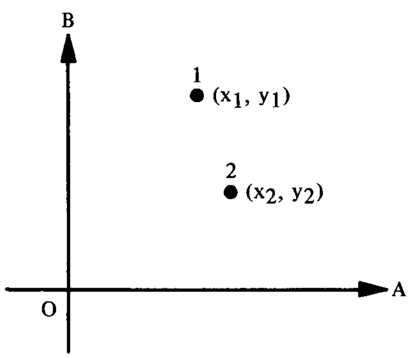


Figure 5

REFRACTION

In all the exercises to this point we have assumed that n (the index of refraction) was a constant along all possible paths of the light ray. Clearly, if the ray passes from one medium to another while traveling between our two points, this is no longer true. Now, when we use Fermat's Principle, the optical path will be determined both by distances and the values of the index of refraction in the various path segments.

Study the drawing in Figure 6. Here we have shown possible paths from point A in air (n = 1) to B in water (n = 1.33). The dividing line between the two substances is the x axis. We can use one of the programs already developed, with minor modifications, to investigate how rays pass from one medium to another. Before proceeding, we should note that in general, the index of refraction for a substance depends upon the wave length of the light ray. To simplify our discussion we will assume that the light rays have a constant wave length, and all values of n which are used correspond to that wavelength.

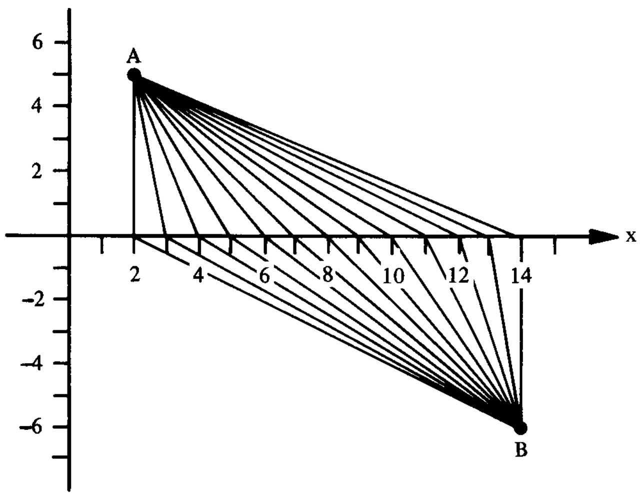


Figure 6

Exercise 10 – Refraction With Precision Computation

Modify the program in Exercise 5 to investigate the problem in Figure 6. Let M (the number of significant digits) = 5. Use as input the coordinates of A and B. (Note that the y coordinate of B will be negative.) The program should output the x coordinate where the ray passes from air into water as determined by Fermat's Principle. Run the program for several sets of points A and B, and record the results. Try to generalize these results.

Exercise 11 – Refraction Between Different Media

Run the program in Exercise 10 for each of the cases in the table below. In each case, assume that A is at (–10, 10) and B is at (10, –10). Examine carefully and try to generalize your results.

Medium 1		Medium 2	
Substance	**n_1**	**Substance**	**n_2**
Oil	1.48	Water	1.33
Water	1.33	Glass	1.65
Air	1.00	Oil	1.48
Glass	1.65	Air	1.00
Air	1.00	Air	1.00

Exercise 12 – Angles of Incidence and Refraction

Modify the program in Exercise 11 to print out additionally the angles the ray makes with the normal to the boundary between the two media both before and after passing from one medium to the next. Run your program for the same input data as in Exercise 11. What can you see in the results?

We are looking for some general **relationship** which describes how a ray will pass from one medium into another. Clearly, the results of Exercise 12 do not show any such relationship. However, the index of refraction *must* have something to do with the desired result, as well as the angles. At this point, left to your own devices, and with lots of time to engage in trial and error investigations, you could probably discover the correct relationship. To shorten the time required, a certain amount of direction will be provided.

Exercise 13 – Discovery

Write a program which uses the results of Exercise 12 to generate the following information for each case.

$n_1\theta_1$	$n_1 \tan\theta_1$	$n_1 \cos\theta_1$	$n_1 \sin\theta_1$
$n_2\theta_2$	$n_2 \tan\theta_2$	$n_2 \cos\theta_2$	$n_2 \sin\theta_2$

In each case, θ_1 refers to the angle made with the normal prior to the passage from medium 1 to 2, and θ_2 is the angle after. Examine the results for the relation.

Exercise 14 – Generalization

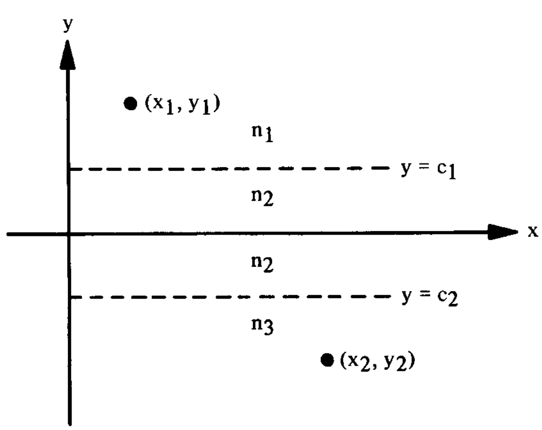
Put the results of Exercise 13 into the form of a law. The law should be titled Snell's Law. Look up this law in your physics text and compare to your version.

Snell's Law (your generalization).

Snell's Law (from textbook).

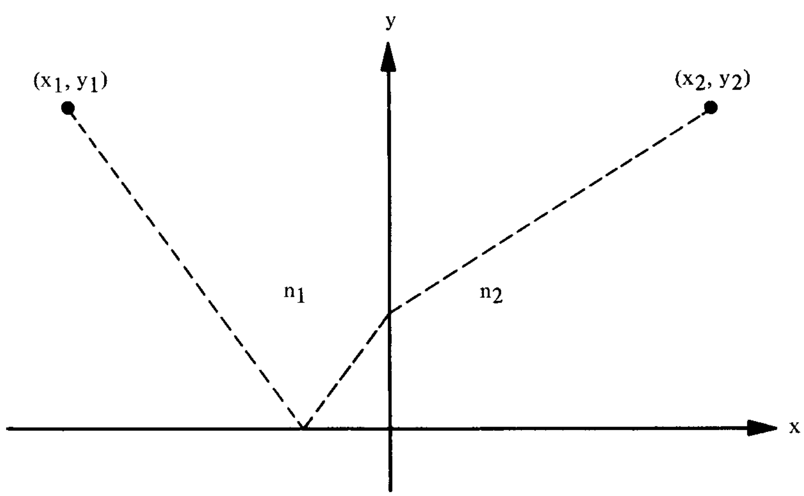
Exercise 15 – Double Refraction

Write a program to investigate how a ray of light will pass from point 1 to point 2 in the diagram below.



Exercise 16 – A Complex Problem

Write a program to investigate how a ray of light will pass from point 1 to point 2 in the diagram below. Assume that the ray must be reflected from the line $y = 0$ before passing from the medium to the left of the y axis to the medium on the right.



Now that the laws of reflection and refraction have been discovered, we should return to a complete discussion of Fermat's Principle as was promised. We will now restate the principle in its complete form.

> ***A ray of light will follow that path between two points for which the total optical path is either a maximum, a minimum, or a stationary value when compared to nearby paths.***

We have discussed only cases for which the optical path was a minimum. To complete the discussion we should illustrate cases where the optical path is either a maximum or has a stationary value. Suppose we have an ellipse (your instructor can show you how to draw an ellipse on the chalkboard and locate its focus points) and require that a ray of light leave one of the focus points and then pass through the other focus after one reflection from the ellipse. The nature of the ellipse is such that the total distance from a focus to any point on the ellipse, then to the other focus is a constant. Moreover, at the point at which the reflection takes place, the angle of incidence equals the angle of reflection in agreement with the law we have just discovered. This is an example of the optical path having a stationary value when compared to nearby paths.

Consider the diagram in Figure 7.

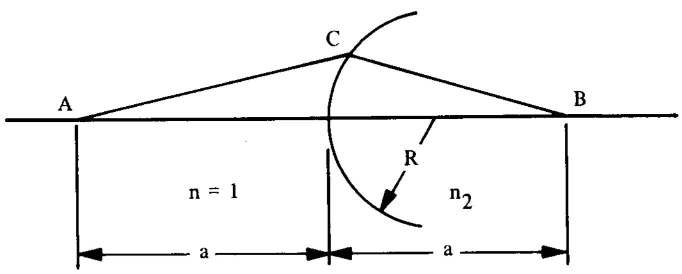


Figure 7

Using calculus we can show that if the ray passes from A to C then to B, that the optical path will be a maximum if

$$a > R\left(\frac{n_2 + n_1}{n_2 - n_1}\right) \qquad (6)$$

RAY TRACING

We mentioned earlier that most of geometrical optics is founded upon the twin ideas of reflection and refraction. Having discovered the basis for these ideas in Fermat's principle we can proceed to some applications which are particularly interesting using the computer. Up to this point we have considered in detail only the case where light is either reflected or refracted at plane surfaces. Now we will examine what takes place if the surfaces are curved.

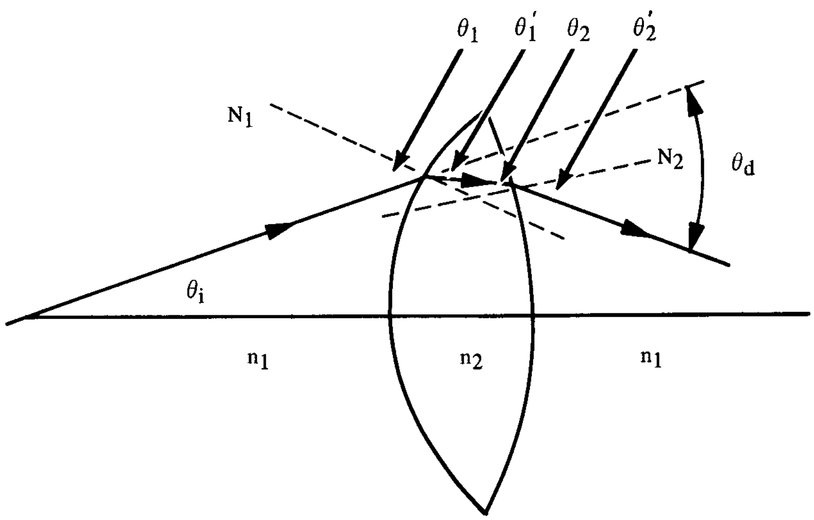


Figure 8

In Figure 8, we have portrayed a light ray moving from left to right, initially making an angle θ_i with the horizontal. It then encounters the first curved glass surface. If we construct the normal N_1 we can relate the angles of incidence and refraction and the two indices of refraction by Snell's Law.

$$n_1 \operatorname{Sin}\theta_1 = n_2 \operatorname{Sin}\theta_1' \tag{7}$$

Then the ray passes to the second curved surface where similarly we can write:

$$n_2 \text{Sin}\theta_2 = n_1 \text{Sin}\theta_2' \tag{8}$$

The net effect of the ray passing through the two surfaces is that it has been deviated from its original angle by an angle θ_d. Of course, what we are seeing is the action of a lens. We will not pursue this particular question further here as it is covered completely in all introductory physics texts.

We are interested, however, in a special case called the *thin lens approximation.* Here, the curvature of the glass surfaces and the dimensions of the lens are such that the lens can be thought to exist in a plane. This leads to the situation portrayed in Figure 9.

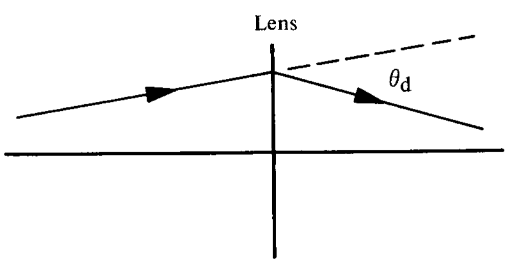


Figure 9

As before, the ray approaches from the left, passes through the lens, and is deviated by an amount θ_d. In this case we are interested only in the net effect of the lens, i.e., that the incident ray has been deviated. We are not concerned with how this happens, what the angles inside the lens are etc. If the ray is deviated towards the horizontal (which we will henceforth refer to as the *optical axis*) it is a converging lens. If the ray is deviated away from the optical axis, the lens is diverging.

Suppose now that we have a ray initially parallel to the optical axis which then passes through either a converging or diverging lens as shown in Figure 10.

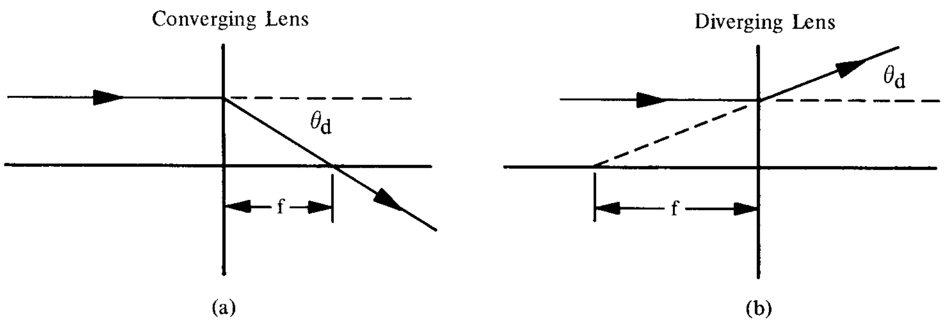


Figure 10

In both cases, the distance from the lens to where the ray either crosses or appears to have crossed the optical axis is known as the focal length f. The focal length for the converging lens is positive, and is negative for the diverging lens. You can easily see that as the focal length decreases, the deviation angle increases.

Now we can write some simple equations which when used with the computer become very powerful. First, we will restrict all light rays to those that make small angles with the optical axis. These are known as *paraxial* rays. Next, we will consider only optical systems made up out of thin lenses and spaces between them. There are no restrictions on the number of lenses or spaces.

The key to ray tracing is the fact that at any given point in an optical system (a combination of lenses and spaces) a light ray can be described by two numbers. The first is the distance away from the optical axis. The second is the angle the ray makes with the optical axis. We will use y to describe the distance, and θ for the angle (assumed to be in radians).

We must be able to describe the effect of either a lens or a space upon these two numbers. The space is easier to describe so we will dispose of it first. Figure 11 portrays a light ray passing through a distance s along the optical axis. Clearly the angle has not changed.

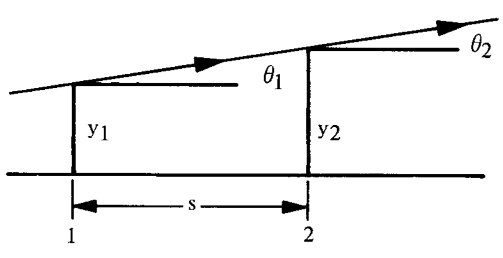


Figure 11

Thus $\theta_2 = \theta_1$. As long as θ is quite small we can say that $y_2 - y_1 = s\theta_1$. For a space s then, we have the two equations

$$\theta_2 = \theta_1 \quad \text{and} \quad y_2 = y_1 + s\theta_1. \tag{9}$$

Before we can write the corresponding equations for a lens we must establish the relation between the focal length of a lens and the deviation angle θ_d. Figure 12 illustrates the relation we are looking for.

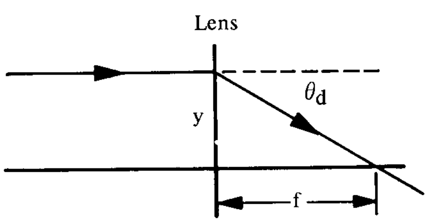


Figure 12

We know that if the ray was initially parallel to the optical axis, it will cross the axis a distance f after the lens (in the case of a converging lens) where f is the focal length. If y was the distance of the ray from the optical axis before it encountered the lens, and provided that θ_d is small we can write

$$\theta_d = y/f \tag{10}$$

Figure 13 shows the effect of a lens upon an arbitrary ray.

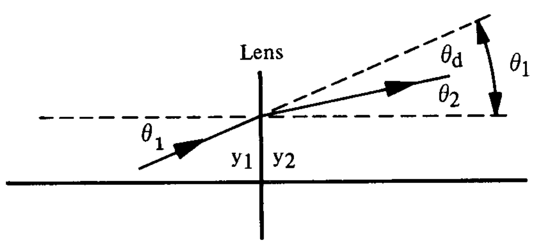


Figure 13

In this case the distance from the optical axis is unchanged, or

$$y_2 = y_1.$$

From the diagram it is clear that

$$\theta_2 = \theta_1 - \theta_d.$$

But

$$\theta_d = y_1/f.$$

Therefore

$$\theta_2 = \theta_1 - y_1/f.$$

Now we can summarize the equations we have been developing. For a space s

$$\begin{aligned} \theta_2 &= \theta_1, \\ y_2 &= y_1 + s\theta_1. \end{aligned} \tag{11}$$

For a lens with focal length f

$$\begin{aligned} \theta_2 &= \theta_1 - y_1/f, \\ y_2 &= y_1. \end{aligned} \tag{12}$$

Exercise 17 – A Single Lens

Use the equations just developed to follow a light ray which initially has $y = 1$, $\theta = -0.1$ *through a distance* $s = 20$ *and a thin lens with* $f = 10$. *Do the calculation by hand and draw a diagram of the results.*

Exercise 18 – Two Lenses

Repeat Exercise 17, except add a second space, $s = 10$, *and a second lens with* $f = -20$.

Exericse 19 – A Program For N Lenses

Write a computer program which will carry out the ray tracing done in Exercises 17 and 18. Arrange your program so that an arbitrary number of spaces and lenses can be accommodated. The program should accept the initial values of y and θ, and N pairs of numbers in the form (n, A). N is the number of optical elements (either spaces or lenses). For a space, have n = 1 and A = s. For a lense have n = 2 and A = f. In your program, you can test the value of n at each step to determine which set of equations should be utilized. The program should output the values of y and θ at the end of the optical system. Try out your program on the ray: $y = 0$, $\theta = 0.1$, $N = 4$; *(1,10), (2,20), (1,20), 2,10). Plot a diagram of the results.*

Exercise 20 – Intersection of Rays

Trace the rays $y = 1$, $\theta = 0$, *and* $y = 1$, $\theta = -0.1$ *through the optical system in Exercise 19. Where do the two output rays either intersect, or appear to intersect?*

We can now discuss the question of optical objects and images. An *object* is a source of light rays which are given off in all directions. If two different rays which start from the same point on the object either intersect or appear to have intersected after passing through an optical system an *image* of the point on the object is formed. If the rays actually intersect at the image point, the image is *real*. If the rays only appear to have intersected, the image is *virtual*. If the image point is on the same side of the optical axis as the corresponding object point, the image is *erect*. Otherwise, the image is *inverted*. The image can be magnified or decreased in *size*. We will not follow this topic any further as the subject is covered thoroughly in introductory physics texts.

Exercise 21 – Description of Image

Let us use an arrow for an optical object. The object is located along a line perpendicular to the optical axis. The point of the arrow is one unit away from the optical axis, and the tail is on the axis. An optical system is described by $N = 2$; (1, 20), (2, 10). Use the program in Exercise 19 to trace two different rays originating on the point of the object through the system. Describe the resultant image. Is the image real or virtual, erect or inverted, magnified or decreased in size? Where is the image?

It will now be wise to recall Fermat's principle as applied to the optical systems we have been studying. No matter how complicated or involved the optical system, the path actually taken by a light ray through the system is such that the *optical path* is a *minimum* (generally).

Now that we have developed computer tools to handle many interesting problems in geometrical optics, your instructor may assign additional problems which you either can work with programs you have already written, or with programs you are now capable of writing.

OPTIONAL MATERIAL

The final material in this unit should be considered optional in nature. It should prove very interesting to those students who have been introduced to the matrix notation and fundamental matrix arithmetic. If you have not studied this part of mathematics your instructor can suggest references to assist you.

In matrix notation, the equations for a space take the form

$$\begin{bmatrix} \theta_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ y_1 \end{bmatrix} \qquad (13)$$

The equations for the lens put into the same form are

$$\begin{bmatrix} \theta_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & -1/f \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ y_1 \end{bmatrix} \qquad (14)$$

Thus, the *space matrix* is

$$S = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \qquad (15)$$

and the *lens matrix* is

$$L = \begin{bmatrix} 1 & -1/f \\ 0 & 1 \end{bmatrix} \qquad (16)$$

Now, consider the optical system in Figure 14. Given a ray at position 1, we want to compute the description of the same ray when it emerges at position 5. Using the space and lens matrices we have the following.

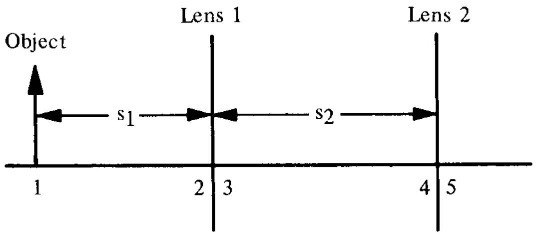


Figure 14

$$\begin{bmatrix}\theta_2\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ s_1 & 1\end{bmatrix}\begin{bmatrix}\theta_1\\ y_1\end{bmatrix}$$

$$\begin{bmatrix}\theta_3\\ y_3\end{bmatrix} = \begin{bmatrix}1 & -1/f_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}\theta_2\\ y_2\end{bmatrix}$$

$$= \begin{bmatrix}1 & -1/f_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ s_1 & 1\end{bmatrix}\begin{bmatrix}\theta_1\\ y_1\end{bmatrix}$$

$$\begin{bmatrix}\theta_4\\ y_4\end{bmatrix} = \begin{bmatrix}1 & 0\\ s_2 & 1\end{bmatrix}\begin{bmatrix}\theta_3\\ y_3\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ s_2 & 1\end{bmatrix}\begin{bmatrix}1 & -1/f_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ s_1 & 1\end{bmatrix}\begin{bmatrix}\theta_1\\ y_1\end{bmatrix}$$

$$\begin{bmatrix}\theta_5\\ y_5\end{bmatrix} = \begin{bmatrix}1 & -1/f_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}\theta_4\\ y_4\end{bmatrix}$$

$$= \begin{bmatrix}1 & -1/f_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ s_2 & 1\end{bmatrix}\begin{bmatrix}1 & -1/f_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ s_1 & 1\end{bmatrix}\begin{bmatrix}\theta_1\\ y_1\end{bmatrix}$$

However, the product of four 2 X 2 matrices in the last expression is itself a 2 X 2 matrix which we can call the *system matrix* M. Now the whole problem has the simple form

$$\begin{bmatrix} \theta_{out} \\ \\ y_{out} \end{bmatrix} = M \begin{bmatrix} \theta_{in} \\ \\ y_{in} \end{bmatrix} \qquad (17)$$

where M is the product of all the space and lens matrices in the system, the product taken in reverse order.

Exercise 22 – A Matrix Program

Write a computer program using MAT commands to compute the system matrix for a series of N components, each component being a space or a lens. Test your program on the system in Exercise 19. You should, of course, get identical results.

It is interesting to note that a complete description of the system is contained in the four numbers in the system matrix M. All the information concerning the image corresponding to a given object can be determined from these four numbers. This concept of a *system matrix* turns out to be an extremely powerful idea in several branches of physics.

Exercise 23 – Description of Image

Develop a set of rules using the four numbers in a system matrix to determine if the image is real or virtual, erect or inverted, and the magnification.

NOTES

APPLICATION PROGRAM

We have included this completely documented Application Program to illustrate one manner in which you might use the computer to solve a problem.

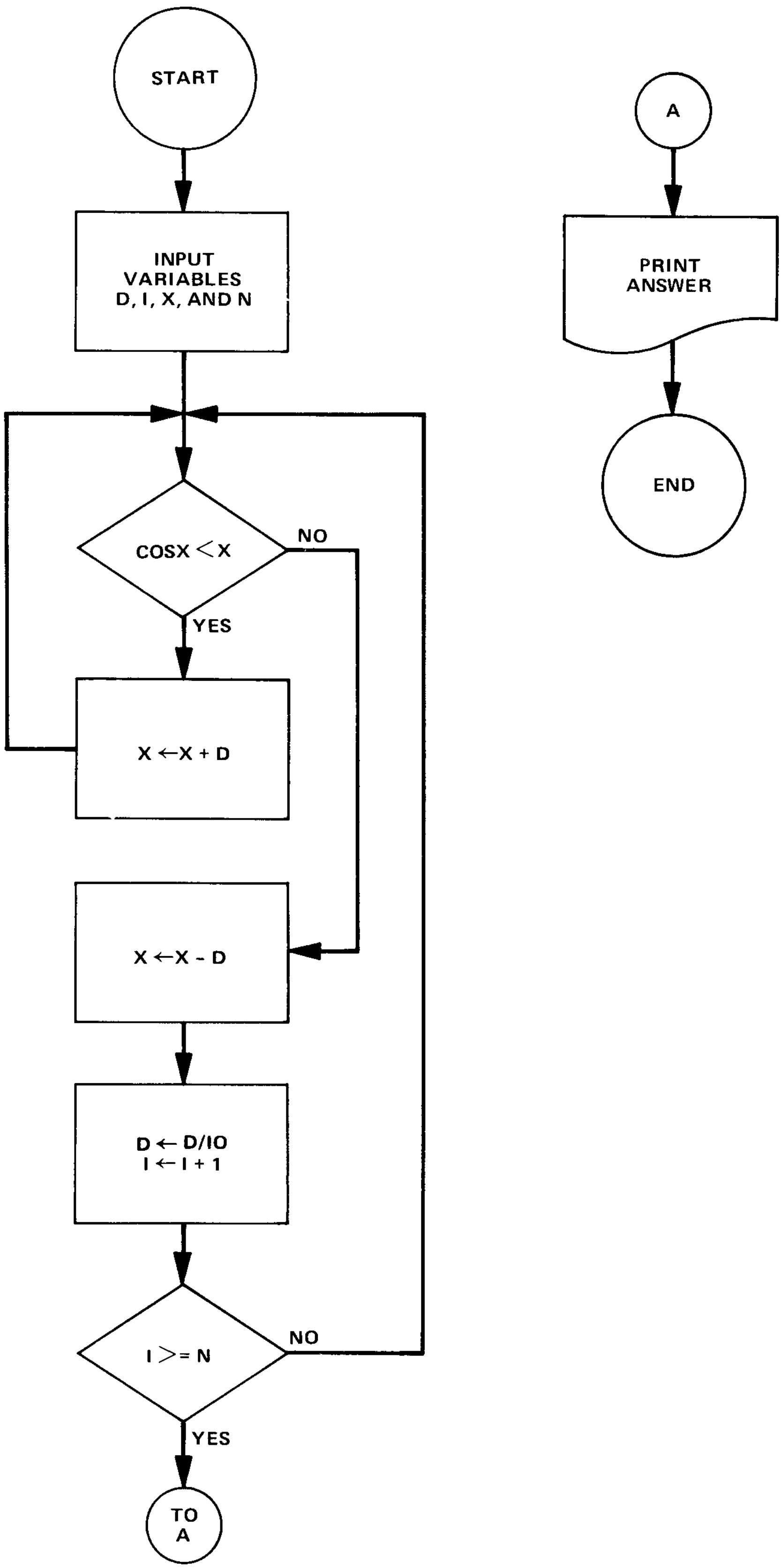
The program is listed with a definition of the problem, a flow chart, a line by line description of the program, a listing and a run.

As an application program to illustrate the use of flow charting we will examine a harmless looking problem which nevertheless requires a computer for solution. The problem is to find the root of the equation

$$x = \text{Cos}(x)$$

that lies between $x = 0$ and $x = 1$, correct to N significant digits.

If we plot $y = x$, and $y = \cos(x)$, then find the x value where they intersect, we will have the desired solution. We know that at $x = 0$ the curve $y = \text{Cos}(x)$ is above the curve $y = x$. Our strategy will be to step across towards $x = 1$ until the two curves cross, then back up one step and decrease the step size by a factor of 10, and start the process again. We will go through the process N times at which point we will know the root to the desired degree of accuracy.


START
INPUT VARIABLES D, I, X, AND N
COSX < X
NO
YES
X ← X + D
X ← X - D
D ← D/10
I ← I + 1
I >= N
NO
YES
TO A
A
PRINT ANSWER
END

Line	Description
100 110	States object of problem and establishes input feature.
120 130 140	Sets variables D, I, X and N. (D is an increment value on X, I generates the program N times working toward the intersection of the equations, consequently decreasing D and producing the desired significance.)
150 151 160 170	Examines IF statement and computes new values of X
180 190	Alters D and I values.
200 201 210	Examines IF statement with respect to I and N and goes back through loop or prints answer.
999	End.

```
100 REM ROOT OF X = COS(X)
110 INPUT N
120 LET D=.1
130 LET I=0
140 LET X=0
150 IF X<COS(X) THEN 160
151 GOTO 170
160 LET X=X+D
161 GOTO 150
170 LET X=X-D
180 LET D=D/10
190 LET I=I+1
200 IF I >= N THEN 210
201 GOTO 150
210 PRINT "ROOT AT "INT(X*10↑N)/10↑N
999 END


RUN

? 2
ROOT AT    .73



RUN

? 5
ROOT AT    .73908
```