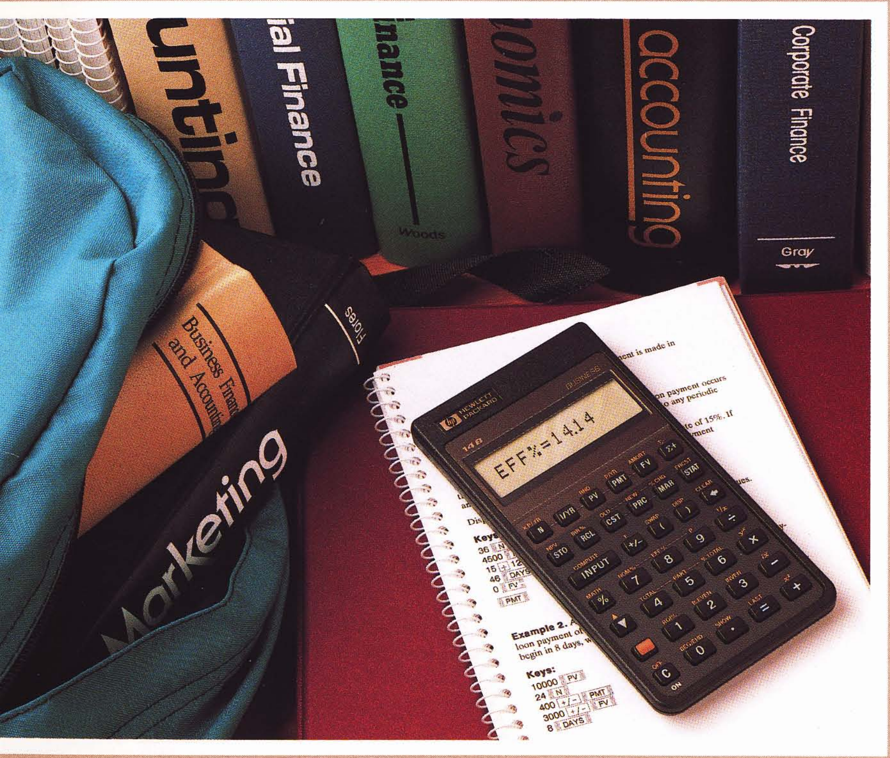
HEWLETT-PACKARD

Step-by-Step Solutions
For Your HP Calculator

Business Student Applications



HP-14B

HEWLETT PACKARD

Business Student Applications

Step-by-Step Solutions for Your HP-14B Calculator

HEWLETT PACKARD

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Contents

How To Use This Book

The *Business Student Applications* solutions book provides instructions and keystroke examples to help you solve a variety of business problems. In areas of accounting, finance, statistics, marketing, and others, this book and your HP-14B make the life of a business student *easier*.

This solutions book is designed to assist you with more specific business problem-solving than you may have found in the owner's manual. It also helps you better use the built-in HP-14B applications, like Time Value of Money and Net Present Value.

To get the most out of this book you should be familiar with the following topics from chapter 1, "Getting Started," in your owner's manual:

- Clear features (see "Using the CLEAR Menu").
- Registers (see "Using Calculator Memory").
- Primary and secondary keystrokes (see "Using the Keyboard").
- Messages in the display (see "What You See in the Display").
- Decimal places (see "Controlling the Display Format").

Our thanks to Elbert B. Greynolds, Jr., of Southern Methodist University for developing the material in this book.

1

Accounting

Cost Allocations Based on Percentages

When costs cannot be identified with a product, department, or service, accountants often allocate them. One common allocation method is based on percentage relationships.

Example. A department store allocates electricity costs to its four departments based on square feet. If the total electricity costs are $1,500, what is the amount allocated to each department?

Department	Square Feet
1	3,000
2	5,000
3	7,000
4	2,000

This problem is solved in two steps.

Step 1. Calculate the cost per square foot by dividing the total cost by the number of square feet.

Keys:	Display:	Description:
1500 [÷]	1,500.00÷	Enters total electricity cost and begins divide operation.
[(] 3000 [+] 5000 [+] 7000 [+] 2000 [)]	÷17,000.00	Calculates sum of square feet.
[=] [STO] 0	0.09	Calculates cost per square foot, and stores in register 0.

Step 2. Allocate the cost by multiplying the square feet in each department by the cost per square foot.

3000 [x] [RCL] 0 [=]	264.71	Enters square feet in department 1 and calculates electricity cost allocation.
5000 [x] [RCL] 0 [=]	441.18	Enters square feet in department 2 and calculates electricity cost allocation.
7000 [x] [RCL] 0 [=]	617.65	Enters square feet in department 3 and calculates electricity cost allocation.
2000 [x] [RCL] 0 [=]	176.47	Enters square feet in department 4 and calculates electricity cost allocation.

Imputed Interest Rates on Long-Term Non-Interest-Bearing Notes

Businesses sometimes sell a product or service in exchange for a non-interest-bearing note with a life greater than 1 year.* In this case the purchaser is borrowing money from the seller to buy the product or service. Typically, the note amount will be for more than the cash purchase amount. The difference is treated in standard accounting practice as interest income (to the seller) or expense (to the buyer). Current accounting practice requires that this interest amount be recorded as income over the life of the note.

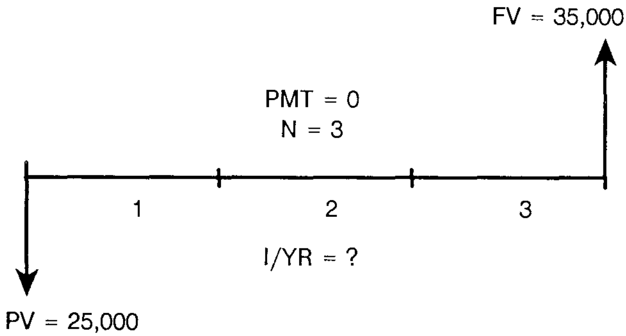
Sometimes a note of this type is paid off in one lump sum at the end of its life; other times the note is paid off through a series of periodic payments. The procedure here covers the first case. A similar procedure starting on page 41 covers the case involving periodic payments.

The following procedure calculates the imputed interest rate (sometimes referred to as the implicit interest rate) when the cash price for the property, goods, or services is known.

Example. The ABC Company sold a machine to the XYZ Company for $35,000. The XYZ company signed a note agreeing to pay the amount at the end of 3 years. Normally ABC Company sells this machine for $25,000. What is the imputed interest rate resulting from this transaction?

* This procedure is not applicable to Treasury Bills or to any discounted note with a non-integer life.

As shown on the cash-flow diagram, the future value, present value, and time period are known. Solve for the interest rate.



Keys:	**Display:**	**Description:**
1 ■ [P/YR]	P/YR=1.00	Sets 1 compounding period per year.
3 [N]	N=3.00	Enters years.
25000 [+/-] [PV]	PV=-25,000.00	Stores cash price of machine.
35000 [FV]	FV=35,000.00	Stores face value of note.
0 [PMT]	PMT=0.00	Enters 0 for payment.
[I/YR]	I/YR=11.87	Calculates imputed interest rate per year.

Expected Value of Investment

Since cash flows associated with investments can be uncertain—there may be a range of possible outcomes—they may have probabilities assigned to them. The weighted average, with the probabilities serving as weights, becomes the expected value of the cash flows. When choosing between investment alternatives, businesses generally select the investment with the largest expected value.*

Example. You must decide between two projects, each having a 1-year life. After careful research, you have developed the following probability distribution for each project. Calculate the expected value for each project.

Project A		Project B	
Probability	**Cash Flow**	**Probability**	**Cash Flow**
10%	$1,000	15%	$1,200
25%	$2,000	20%	$1,800
50%	$2,500	45%	$2,500
15%	$3,000	20%	$3,000

You solve this by computing the weighted average of the probabilities and cash flows for each project using the Statistics application.

Step 1. Enter the probabilities and cash flows for Project A and calculate the expected value.

Keys:	**Display:**	**Description:**
■ [CLEAR] {Σ}	*value*	Clears statistics registers.

* Charles T. Horngren and George Foster, *Cost Accounting: A Managerial Emphasis*, 6th ed. (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1987), p. 618.

1000 [INPUT]	1,000.00:	Enters first outcome.
10 [Σ+]	n=1.00	Enters probability for outcome.
2000 [INPUT] 25 [Σ+]	n=2.00	Enters remaining values.
2500 [INPUT] 50 [Σ+]	n=3.00	
3000 [INPUT] 15 [Σ+]	n=4.00	
[STAT] {$\bar{x}$,$\bar{y}$} {$\bar{x}$w}	$\bar{x}$w=2,300.00	Calculates expected value of Project A.

Step 2. Enter the probabilities and cash flows for Project B and calculate the expected value.

■[CLEAR] {Σ}	2,300.00	Clears statistics registers.
1200 [INPUT] 15 [Σ+]	n=1.00	Enters first value for Project B.
1800 [INPUT] 20 [Σ+]	n=2.00	Enters remaining values.
2500 [INPUT] 45 [Σ+]	n=3.00	
3000 [INPUT] 20 [Σ+]	n=4.00	
[STAT] {$\bar{x}$,$\bar{y}$} {$\bar{x}$w}	$\bar{x}$w=2,265.00	Calculates expected value of Project B.

Project A has the larger expected value.

Cost-Volume-Profit Analysis (CVP)

Cost-volume-profit (CVP) analysis is a technique for analyzing the relationships among fixed costs, variable costs, and profit (income). This technique assumes that fixed costs are recovered before any profit is earned, that the units sold equal the units produced, and that the sales mix is constant. By modifying the Break-Even (*B.EVEN*) application inputs, you can include taxes or a sales mix in the analysis.

CVP With Taxes

The HP-14B built-in break-even equation is

$$Units \times Price = Units \times Variable\ Cost + Fixed\ Costs + Profit\ Before\ Tax$$

Taxes are incorporated in the analysis by substituting the following relationship for profit before tax.

$$Profit\ Before\ Tax = \frac{Profit\ After\ Tax}{(1 - Tax\ Rate)}$$

Example. Your product sells for $25.00 per unit. The fixed costs are $15,000. Variable costs are $18.00 per unit and the tax rate is 30% of profit.

Part 1. Calculate the number of units that must be sold to earn $900 after tax.

Keys:	Display:	Description:
■ [B.EVEN]	#SLD?*value*	Enters *B.EVEN* application. The first value requested is number sold, which is what you want to calculate.

[▼]	PRC?*value*	Moves to next value in list (price).
25 [INPUT]	FIXC?*value*	Enters 25 as price. Prompts for fixed costs.
15000 [INPUT]	VARC?*value*	Enters 15,000 for fixed costs. Prompts for variable cost.
18 [INPUT]	PROF?*value*	Enters 18 as variable cost per unit. Prompts for profit.
900 [÷] [(] 1 [-] 30 [%] [)] [STO] 0 [INPUT]	#SLD?*value*	Enters profit before tax. Also, stores 1 minus the tax rate in register 0 for later use. The next value requested is units sold, which is unknown.
■ [COMPUTE]	#SLD=2,326.53	Computes units sold. You have to sell 2,327 units to earn $900 after taxes.

Part 2. If 3,000 units are sold, what is the profit after tax?

Keys:	**Display:**	**Description:**
■ [B.EVEN]	#SLD?2,326.53	Re-enters *B.EVEN* application. The first value displayed is the previous number sold, which you want to change.
3000 [INPUT]	PRC?25.00	Enters 3,000 as number of units sold. The *B.EVEN* application displays the next value.

[▼] [▼] [▼]	PROF?1,285.71	Moves profit before tax into display to prepare to compute that value.
■[COMPUTE]	PROF=6,000.00	Computes new profit before tax.
[×] [RCL] 0 [=]	4,200.00	Calculates profit after tax by multiplying by 1 minus tax rate, which is stored in register 0.

CVP Analysis With Multiple Products

The *B.EVEN* application can be used for CVP analysis with two or more products. Use the weighted average sales price and variable costs as inputs. The units in the application then represent total units sold. The number of units sold of each product is equal to the weighted average sales mix multiplied by the total units, giving the average unit contribution margin.

Example. Part 1. You manufacture and sell two products. Product A sells for $15 per unit with a variable cost of $9 per unit; Product B sells for $20 per unit with a variable cost of $12 per unit. Your fixed costs are $67,500.

Assuming you sell 5 units of Product A for each 3 units of Product B, how many units must be sold to break even?

Step 1. Calculate and store the weighted average sales mix for each product.

Keys:	**Display:**	**Description:**
5 [÷] [(] 5 [+] 3 [=]		Divides the A unit mix by total units of A and B.
[STO] 1	0.63	Stores resulting weighted average percentage for Product A in register 1.

[-] 1 [=] [+/-] [STO] 2	0.38	Subtracts weighted average percentage for Product A from 1 to calculate weighted average percentage for Product B. Stores result in register 2.

Step 2. Calculate the weighted average sales price for both products.

■ [B.EVEN]	#SLD?*value*	Enters *B.EVEN* application. The first value requested is number sold, which is what you want to calculate.
[▼]	PRC?*value*	Moves to price.
15 [×] [RCL] 1 [+] [(] 20 [×] [RCL] 2 [=]	PRC?16.88	Calculates weighted average selling price.
[INPUT]	FIXC?*value*	Enters weighted average selling price. Prompts for fixed costs.

Step 3. Enter fixed costs.

67500 [INPUT]	VARC?*value*	Enters fixed costs. The next value requested is variable costs.

Step 4. Calculate and enter the weighted average variable cost.

9 [×] [RCL] 1 [+] [(] 12 [×] [RCL] 2 [=]	VARC?10.13	Calculates weighted average variable cost.
[INPUT]	PROF?*value*	Enters weighted average variable costs. Prompts for profit.

Step 5. Calculate the number of units to break even.

0 [INPUT]	#SLD?*value*	Enters 0 for profit because you are calculating the break-even point. The next value requested is units, which is unknown.
■[COMPUTE] [STO] 0	#SLD=10,000.00	Computes total units sold. Stores result in register 0.
[×] [RCL] 1 [=]	6,250.00	Multiplies total units sold by Product A's weighted average percentage to determine how many units of A must be sold.
[RCL] 0 [×] [RCL] 2 [=]	3,750.00	Multiplies total units sold by Product B's weighted average percentage to determine how many units of B must be sold.

With a 5:3 sales mix, 6,250 units of Product A and 3,750 units of Product B must be sold to break even.

Part 2. If 12,000 total units are sold at the 5:3 sales mix, what is the total income?

Keys:	Display:	Description:
■[B.EVEN]	#SLD?10,000.00	Re-enters the *B.EVEN* application. The first value in the list is the previous number of units, which you want to change.

12000 [INPUT]	PRC?16.88	Enters units sold. The next value displayed is weighted average selling price, which you do not want to change.
[▼] [▼] [▼]	PROF?0.00	Moves to profit, your unknown value.
■[COMPUTE]	PROF=13,500.00	Computes profit.

Assuming 12,000 total units are sold with a 5:3 sales mix, the profit is $13,500.

Depreciation

Depreciation, from an accounting point of view, is the allocation of an asset's cost to future income periods over which the asset is used. Three common methods of calculating depreciation are shown in this section: straight-line, sum-of-the-years' digits, and declining-balance. These procedures are valid for whole year and partial year depreciation. The following terms are common to all of the depreciation methods:

- *Cost* (*C*): Acquisition cost of the asset.
- *Life* (*L*): Number of years for depreciating the asset.
- *Salvage* (*S*): Estimated value at the end of the asset's life.
- *Months* (*M*): Months of depreciation taken the first year.
- *Depreciable Value*: Difference between cost and salvage.
- *Net Book Value*: Difference between cost and accumulated depreciation.

Straight-Line Depreciation

Under the straight-line method, the periodic depreciation expense equals the asset cost minus the estimated salvage value, divided by the life. For situations in which partial year depreciation is taken in the first year, the total number of years for depreciating the asset is extended 1 year, with a smaller, final depreciation expense being taken in the last year.

Example. Your company purchased a machine for $17,000. Its useful life is 6 years and the salvage value is $2,000. During the first year, 8 months of depreciation will be taken. Calculate the straight-line depreciation schedule.

Keys:	Display:	Description:
17000 [−] 2000 [=]	15,000.00	Subtracts salvage value from asset cost to calculate depreciable value.

[÷] 6 [=]	2,500.00	Divides depreciable value by asset life (in years) and calculates depreciation for years 2 through 6. (If 12 months of depreciation are taken in first year, then stop here.)
[STO] 0	2,500.00	Stores whole year depreciation in register 0.
8 [÷] 12 [=]	0.67	Enters number of months of depreciation in first year and divides by 12 to calculate the fraction of a year.
[STO] 1	0.67	Stores the fraction of a year in register 1.
[×] [RCL] 0 [=]	1,666.67	Calculates depreciation for year 1.
1 [−] [RCL] 1 [×] [RCL] 0 [=]	833.33	Calculates depreciation for final (partial) year.

Sum-of-the-Years'-Digits Depreciation

Sum-of-the-years'-digits (SOYD) depreciation is an accelerated method. The yearly depreciation allocation is determined by dividing the number of remaining years by the sum of the years' digits and then multiplying by the depreciable value (cost less salvage value).

Example: Whole Year Depreciation. On January 1, a company purchased an asset for $17,000, with a useful life of 6 years and a $2,000 salvage value. Assuming a full year of depreciation for the first year, prepare an SOYD depreciation schedule.

Step 1. Calculate the sum of the digits.

Keys:	**Display:**	**Description:**
6 [N]	N=6.00	Enters life in register *N*.
[×] [(] [RCL] [N] [+] 1 [÷] 2 [=] [STO] 0	21.00	Calculates sum of the digits and stores result in register 0.

Step 2. Enter cost, salvage value, and life, and calculate intermediate values.

17000 [−] 2000 [÷]	15,000.00÷	Subtracts salvage value from asset cost to calculate depreciable value.
[RCL] 0 [=] [STO] 0	714.29	Divides depreciable value by sum of the digits and stores result in register 0.

Step 3. Multiply by the asset life to calculate the first year's depreciation.

[×] 6 [=]	4,285.71	Multiplies by life of asset and calculates first year's SOYD depreciation.

Step 4. Calculate the remaining periods' depreciation.

[−] [RCL] 0 [=]	3,571.43	Calculates second year's depreciation.
[−] [RCL] 0 [=]	2,857.14	Calculates third year's depreciation.
[−] [RCL] 0 [=]	2,142.86	Calculates fourth year's depreciation.
[−] [RCL] 0 [=]	1,428.57	Calculates fifth year's depreciation.
[−] [RCL] 0 [=]	714.29	Calculates sixth year's depreciation.

Example: Partial Year Depreciation. This procedure allows partial year depreciation using months, as shown in these equations:

$$\text{First year's depreciation} = \frac{M \times L}{12} \times \frac{(C - S) \times 2}{(L \times (L + 1))}$$

$$\text{Following years' depreciation} = -\left(Y + \frac{M}{12} - L - 2\right) \times \frac{(C - S) \times 2}{L(L + 1)}$$

Note that Y is the year for calculating depreciation. Using the equation for *following years' depreciation*, you can calculate the depreciation expenses for the second and later years in any order.

On May 1 a company paid $17,000 for an asset with a useful life of 6 years and a salvage value of $2,000. Assuming partial depreciation for the first year (8 months), prepare an SOYD depreciation schedule for years 1, 2, 6, and 7.

Step 1. Calculate the sum of the digits.

Keys:	Display:	Description:
6 [N]	N=6.00	Enters life in register *N*.
[×] [(] [RCL] [N] [+] 1 [÷] 2 [=] [STO] 0	21.00	Calculates sum of the digits and stores result in register 0.

Step 2. Enter cost, salvage value, and life, and calculate intermediate values.

17000 [−] 2000 [=]	15,000.00	Subtracts salvage value from asset cost to calculate depreciable value.
[÷] [RCL] 0 [=] [STO] 0	714.29	Divides depreciable value by sum of the digits and stores result in register 0.

8 [÷] 12 [=] [STO] 1	0.67	Enters number of months depreciation in first year, divides by 12, and stores result in register 1.
[+/-] [+] [RCL] [N] [+] 2 [=] [STO] 2	7.33	Calculates intermediate value, stores result in register 2.

Step 3. Calculate the first year's depreciation.

[RCL] 1 [×] [RCL] [N] [×] [RCL] 0 [=]	2,857.14	Calculates first year's depreciation.

Step 4. Calculate the other years' depreciation values.

2 [-] [RCL] 2 [×] [RCL] 0 [=] [+/-]	3,809.52	Enters year (2) and calculates depreciation.
6 [-] [RCL] 2 [×] [RCL] 0 [=] [+/-]	952.38	Enters year (6) and calculates depreciation.
7 [-] [RCL] 2 [×] [RCL] 0 [=] [+/-]	238.10	Calculates depreciation for seventh year.

Declining-Balance Depreciation

The declining-balance method is an accelerated depreciation method. The depreciation expense is computed by multiplying the net book value by a declining-balance factor, then dividing by the asset's depreciable life. For example, a double declining-balance factor is twice the straight-line depreciation rate, or 200%. The depreciation expense is computed until the salvage value exceeds the net book value, or until the end of the asset's life is reached, whichever comes first. For partial period depreciation, prorate the first year's expense amount, adjust the net book value, and then continue the computation routine until the salvage value exceeds the net book value (or until you reach the end of the asset life).

Example. Part 1. On June 1 a company paid $25,000 for an asset with a useful life of 5 years and a salvage value of $4,500. Assuming partial depreciation for the first year (7 months), prepare a depreciation schedule using the double declining-balance method.

Step 1. Divide the declining-balance factor by the asset life and store the factor.

Keys:	**Display:**	**Description:**
200 [÷] 5 [=] [STO] 0	40.00	Enters double-declining factor, divides by life of asset, and stores result in register 0.

Step 2. Calculate the first year's depreciation.

25000 [STO] 1	25,000.00	Enters and stores the asset's cost in register 1.
[×] [RCL] 0 [%] [=]	10,000.00	Calculates full year depreciation expense for year 1.

Step 3. Calculate the partial first year depreciation.

[×] 7 [÷] 12 [=] [+/−]	-5,833.33	Enters months of depreciation taken in year 1 and calculates partial depreciation.
[+] [RCL] 1 [=]	19,166.67	Determines net book value at end of (partial) year 1.

Step 4. Calculate remaining periods' depreciation.

[−] [RCL] 0 [%]	-7,666.67	Calculates depreciation expense for year 2.

[=]	11,500.00	Calculates net book value at end of year 2.
[-] [RCL] 0 [%]	-4,600.00	Calculates depreciation expense for year 3.
[=]	6,900.00	Calculates net book value at end of year 3.
[-] [RCL] 0 [%]	-2,760.00	Calculates depreciation expense for year 4.
[=]	4,140.00	Calculates net book value at end of year 4. Since this is less than salvage value, the depreciation expense must be recalculated.
6900 [-] 4500 [=]	2,400.00	Determines final depreciation expense by subtracting salvage value from net book value at end of year 3.

Because the asset's net book value is less than the salvage value during the fourth year (4,140 is less than 4,500), there is no depreciation expense in year 5. You must compare the calculated net book value with the salvage value to determine the last year for computing depreciation expense. In cases of zero salvage value, the difference between the net book value at the next to last period and the salvage value is the final period's depreciation expense.

Part 2. Prepare a double declining-balance depreciation schedule assuming whole-year depreciation for the same asset.

Keys:	**Display:**	**Description:**
200 [÷] 5 [=] [STO] 0	40.00	Enters double-declining factor, divides by life of asset, and stores result in register 0.

25000 [-] [RCL] 0 [%]	-10,000.00	Calculates depreciation expense for year 1.
[=]	15,000.00	Calculates net book value at end of year 1.
[-] [RCL] 0 [%]	-6,000.00	Calculates depreciation expense for year 2.
[=]	9,000.00	Calculates net book value at end of year 2.
[-] [RCL] 0 [%]	-3,600.00	Calculates depreciation expense for year 3.
[=]	5,400.00	Calculates net book value at end of year 3.
[-] [RCL] 0 [%]	-2,160.00	Calculates depreciation expense for year 4.
[=]	3,240.00	Calculates net book value at end of year 4. Since this is less than salvage value, the depreciation expense must be recalculated.
5400 [-] 4500 [=]	900.00	Determines final depreciation expense by subtracting salvage value from net book value at end of year 3.

Sinking Funds

A sinking fund is money deposited on a regular basis into a custodial account by a company according to a contractual agreement. Often such funds are established for repayment of a bond issue or some other debt issued by the company. Normally, the funds are established with the first deposit being made at the end of a quarter or year. While deposit amounts are usually constant, they occasionally vary from period to period.

This section demonstrates the use of both the TVM and the cash-flow features of the HP-14B in solving sinking fund problems.

Sinking Funds With Equal Deposits

Example. Part 1. A company issues $5 million of bonds that mature in 10 years. According to the bond's indenture agreement, the company must set up a sinking fund that will redeem the bonds in 10 years. The sinking fund is a custodial account that pays 8% annual interest, compounded quarterly. The payments will be made quarterly.

Assuming the first payment occurs 1 quarter after the bonds are issued, what is the payment amount? Solve this as an end of period payment annuity.

Keys:	Display:	Description:
■ [BEG/END] {END}		Sets End mode and 4
4 ■ [P/YR]	P/YR=4.00	payment periods per year.
8 [I/YR]	I/YR=8.00	Enters interest rate.
10 ■ [xP/YR]	N=40.00	Enters total payments.
0 [PV]	PV=0.00	Clears present value register.

5000000 [FV]	FV=5,000,000.00	Stores desired balance in sinking fund after 10 years.
[PMT]	PMT=-82,778.74	Calculates required quarterly payment.

Part 2. What interest rate is required if the company deposits $90,000 at the end of each quarter? Since only the payment amount changes, enter the new payment and calculate the required interest rate.

Keys:	Display:	Description:
90000 [+/-] [PMT]	PMT=-90,000.00	Stores quarterly payment.
[I/YR]	I/YR=6.44	Calculates required interest rate with revised payment amount.

Sinking Funds With Unequal Deposits

Occasionally, sinking funds are established with deposits that vary according to a fixed schedule. Determining the payment amount is relatively easy using a combination of cash flow and time value of money calculations.

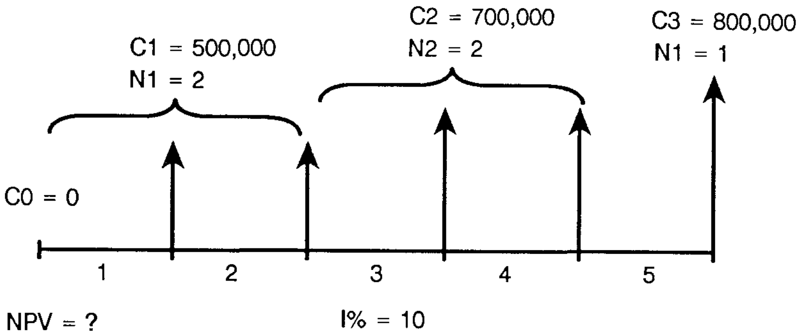
Example: Calculate Future Value and Interest Rate. A company has set up a sinking fund with the following schedule for year-end payments.

Year	Amount ($)
1	500,000
2	500,000
3	700,000
4	700,000
5	800,000

Part 1. If interest is 10% compounded annually, what is the future value of this sinking fund?

Solving for the future value requires two steps:

1. Use the *NPV* function to calculate the present value of the deposits.
2. Use the TVM keys to determine the future value of the deposits.



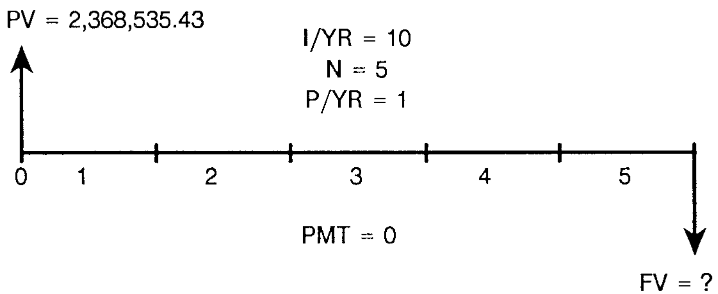
When working with future values and solving for interest rates, always enter the last cash flow as a separate group occurring one time. This step is necessary because you net the final cash flow and the ending balance when calculating *IRR%*.

Step 1. Calculate the present value of the deposits.

Keys:	**Display:**	**Description:**
■ [BEG/END] {END} 1 ■ [P/YR]	P/YR=1.00	Sets End mode and 1 payment period per year.
■ [NPV]	I%=*value*	Begins net present value calculation.
10 [INPUT]	CL CFLO? Y N	Enters interest rate. If a cash-flow list is stored, this prompt appears.

{Y}	C0?	Clears cash-flow list and prompts for initial cash flow.
0 [INPUT]	C1?	Enters 0 for initial cash-flow amount. Prompts for dollar amount of cash flow 1.
500000 [INPUT] 2 [INPUT]	C2?	Enters amount and number of deposits in first cash-flow group.
700000 [INPUT] 2 [INPUT]	C3?	Enters amount and number of deposits in second cash-flow group.
800000 [INPUT] [INPUT]	C4?	Enters third cash-flow group.
■ [COMPUTE]	NPV= 2,368,535.43	Calculates present value of the cash flows.

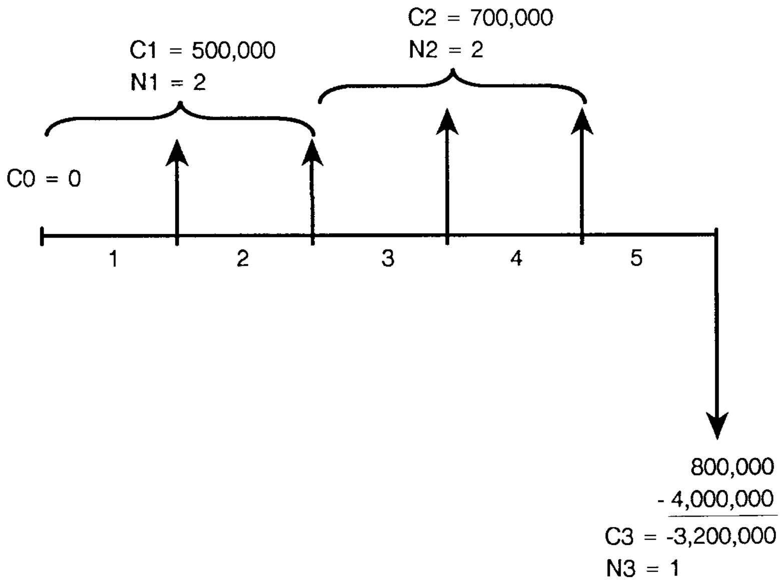
Step 2. Calculate the future value of the cash flows using the TVM keys. Now that the cash flows are converted to an equivalent value at the beginning of the first period, you can calculate their value at any future point in time.



10 [I/YR]	I/YR=10.00	Enters interest rate for TVM calculations.
5 [N]	N=5.00	Enters number of years for deposits.
0 [PMT]	PMT=0.00	Enters 0 as the payment.
[FV]	FV= -3,814,550.00	Calculates future value of the deposits.

Part 2. What annual interest rate is required for the fund to grow to $4,000,000 at the end of 5 years?

Step 1. Adjust the final cash flow by subtracting the desired balance from it.



Keys:	Display:	Description:
■ [IRR%]	CL CFLO? Y N	Prompts for clearing of existing cash-flow list.
{N}	C0?0.00	Chooses to retain current cash-flow list.
[▼] ... [▼]	C3?800,000.00	Moves to the last cash flow.
[-] 4000000 [INPUT]	N3?1.00	Subtracts ending fund balance from last period's cash flow and enters the result.

Step 2. Calculate the interest rate.

■ [COMPUTE]	IRR%=12.70	Calculates the required interest rate.

Sinking Funds With Increasing Payments

Example. The ABC Company must establish a sinking fund that equals $2,000,000 after 5 years. The payment schedule varies from year to year as shown below, with the increases expressed as an index based on the first year's payment. Payments are made at the end of each year.

Year	Index
1	1.00
2	1.10
3	1.20
4	1.30
5	1.30

Calculate the deposit amounts assuming an annual 7.5% interest rate.

Solving this problem incorporates the procedures employed in the previous example, only now the present value is in terms of 1 dollar. You solve this problem in four steps:

1. Calculate the present value of the deposits at the beginning of year 1, expressing each deposit as an index of the first amount.
2. For the present value calculated in step 1, determine the future value after 5 years.
3. Divide the amount calculated in step 2 into the sinking fund ending balance and determine the first deposit.
4. Multiply the first deposit by each period's index to determine the remaining deposit amounts.

Step 1. Calculate the present value of the deposit index values.

Keys:	**Display:**	**Description:**
■[CLEAR] {ALL} {Y}	0.00	Clears all registers in calculator.
■[BEG/END] {END} 1 ■[P/YR]	P/YR=1.00	Sets End mode and 1 payment period per year.
■[NPV]	I%=0.00	Begins NPV procedure.
7.5 [INPUT]	C0?	Enters interest rate for cash-flow calculations and asks for cash flow at time 0.
0 [INPUT]	C1?	Enters 0 for cash-flow amount at beginning of first year.
1 [INPUT] [INPUT]	C2?	Enters cash-flow index at end of year 1.
1.1 [INPUT] [INPUT]	C3?	Enters cash-flow index at end of year 2.
1.2 [INPUT] [INPUT]	C4?	Enters cash-flow index at end of year 3.

1.3 [INPUT] 2 [INPUT]	C5?	Enters cash-flow index at end of years 4 and 5 and number of times it occurs.
■[COMPUTE]	NPV=4.73	Calculates present value of index values.

Step 2. Calculate the future value of the indices after 5 years.

5 [N]	N=5.00	Enters the number of years.
7.5 [I/YR]	I/YR=7.50	Enters interest rate for TVM calculations.
[FV]	FV=-6.79	Calculates future value of indices after 5 years.

Step 3. Calculate the first deposit amount.

2000000 [÷]	2,000,000.00÷	Enters sinking fund amount.
[RCL] [FV] [=] [+/-]	294,713.76	Calculates the amount of the first deposit.
[STO] 0	294,713.76	Stores value in register 0.

Step 4. Calculate the remaining deposit amounts.

[×] 1.1 [=]	324,185.14	Enters index for year 2 and calculates deposit amount.
[RCL] 0 [×] 1.2 [=]	353,656.51	Enters index for year 3 and calculates deposit amount.
[RCL] 0 [×] 1.3 [=]	383,127.89	Enters index for years 4 and 5 and calculates deposit amounts.

Estimating Overhead

Overhead costs are manufacturing costs (other than direct material and direct labor) that cannot be identified with a product. Estimating overhead is usually difficult because of the diverse nature of the items included in the category and the limitations of accounting systems in identifying fixed and variable cost components. As a result, estimation techniques are often employed to estimate fixed and variable overhead. Two common methods are the high-low method and statistical curve fitting.

High-Low Method

This method assumes a linear relationship for overhead and fits a straight line using two data points. The linear equation's slope is called the variable cost, with the intercept treated as the constant component. Because the highest and lowest values are usually selected, the technique is called the high-low method.

Example. Your firm's records show that 4,000 units were produced in March, with overhead totaling $35,000. In April 7,000 units were produced and overhead costs equaled $50,000. Using the high-low method, calculate the overhead prediction equation. Estimate the total overhead for production levels of 6,000 and 6,500 units.

This problem is solved using the following equations:

$$Variable\ Cost = \frac{(Cost\ 1st\ Period\ -\ Cost\ 2nd\ Period)}{(Units\ 1st\ Period\ -\ Units\ 2nd\ Period)}$$

$$Fixed\ Cost = Cost\ 1st\ Period\ -\ Variable\ Cost \times Units\ 1st\ Period$$

$$Total\ Cost\ Period\ N = Fixed\ Cost + Variable\ Cost \times Units\ Period\ N$$

Keys:	Display:	Description:
35000 [-] 50000 [÷]	-15,000.00÷	Calculates difference between 1st and 2nd period costs.
[(] 4000 [-] 7000 [=] [STO] 0	5.00	Divides by difference between 1st and 2nd period units. Stores variable cost in register 0.
[×] 4000 [+/-] [+] 35000 [=] [STO] 1	15,000.00	Enters 1st period's units and costs, and calculates fixed cost. Stores fixed costs in register 1.
6000 [×] [RCL] 0 [+] [RCL] 1 [=]	45,000.00	Enters 6,000 units and calculates estimated total costs.
6500 [×] [RCL] 0 [+] [RCL] 1 [=]	47,500.00	Enters 6,500 units and calculates estimated total costs.

Statistical Curve Fitting

Statistical curve fitting techniques are often used for estimating overhead because more information is provided about the resulting equation. The coefficient of determination (r^2) provides information about the fit of the equation to the data; the closer to 1, the better the fit. The calculator computes *r*, the correlation coefficient, which is squared to derive the coefficient of determination. While accountants often use linear regression, other curves such as the exponential, logarithmic, or power curve can also be used for estimating overhead.*

* See your owner's manual for a description of the curve-fitting equations.

Example. You are estimating overhead and have the following information available for the first 9 months of the year.

Direct Labor	*(x)* Hours	*(y)* Overhead Costs
January	2,700	$24,800
February	2,400	$22,000
March	4,700	$35,600
April	3,100	$26,900
May	5,600	$44,000
June	4,300	$34,000
July	4,100	$31,500
August	3,700	$29,700
September	3,900	$31,000

Assume that overhead costs vary as a function of direct labor hours on a linear basis. Calculate the overhead cost assuming a linear relationship. Also, estimate overhead for October, assuming 5,300 labor hours are worked.

Part 1. Calculate the linear regression equation values.

Keys:	Display:	Description:
■[CLEAR] {Σ}	*value*	Clears the statistics registers.
2700 [INPUT] 24800 [Σ+]	n=1.00	Enters January's data.
2400 [INPUT] 22000 [Σ+]	n=2.00	Enters February's data.
4700 [INPUT] 35600 [Σ+]	n=3.00	Enters remaining data pairs.
3100 [INPUT] 26900 [Σ+]	n=4.00	
5600 [INPUT] 44000 [Σ+]	n=5.00	

4300 [INPUT] 34000		
[Σ+]	n=6.00	
4100 [INPUT] 31500		
[Σ+]	n=7.00	
3700 [INPUT] 29700		
[Σ+]	n=8.00	
3900 [INPUT] 31000		
[Σ+]	n=9.00	
[STAT] {FIT} {L} {m}	m=6.40	Selects linear model for curve fit and displays slope. The variable overhead is $6.40 per direct labor hour.
[STAT] {FIT} {.L.} {b}	b=6,517.78	Selects linear model and displays intercept.
[STAT] {FIT} {.L.} {r}	r=0.99	Selects linear model and displays correlation coefficient.
■ [x^2]	0.98	Calculates coefficient of determination.
[STAT] {FIT} {.L.} {ŷ}	x?	Selects linear model and chooses estimation of *y* values.
5300 [INPUT]	ŷ=40,448.92	Estimates total overhead for 5,300 direct labor hours worked.

Part 2. Use the best curve fit method to forecast overhead if 5,300 direct labor hours are worked in October.

Keys:	**Display:**	**Description:**
■ [FRCST]	x̂ ŷ	Briefly displays best fitting model as exponential and prompts for variable to estimate.
{ŷ} 5300 [INPUT]	ŷ=41,073.02	Estimates total overhead for 5,300 direct labor hours worked.
[STAT] {FIT} {.EX.} {r} ■ [x^2]	0.99	Calculates coefficient of determination.

The exponential model provides a better fit, as indicated by the higher coefficient of determination. However, both models forecast values that are within $630 dollars of each other.

Valuation of Liabilities and Amortization of Interest Expense

Sometimes, in exchange for assets or services, companies give notes without a stated interest rate (or with a stated rate that is artificially low). In such cases, the present value of the cash flows computed using an imputed interest rate equals the net book value of the note. The portions of each payment that are recognized as either an expense or a repayment of principal are calculated in an amortization schedule using the effective interest method.

The following procedure uses the TVM and Amortization applications to calculate the imputed interest rate and to generate an amortization schedule.

Example. The ABC Company acquires a machine from the XYZ Company by signing a note that requires monthly payments of $1,500 for 3 years, with a final extra payment of $20,000. The machine's cash price at the time of purchase is $56,000.

Part 1. Calculate the imputed interest rate.

Keys:	Display:	Description:
■ [BEG/END] {END} 12 ■ [P/YR]	P/YR=12.00	Sets End mode and 12 payment periods per year.
3 ■ [xP/YR]	N=36.00	Enters total payments.
56000 [PV]	PV=56,000.00	Stores current cash price of machine.
1500 [+/-] [PMT]	PMT=-1,500.00	Enters monthly lease payment.
20000 [+/-] [FV]	FV=-20,000.00	Stores final extra payment at end of loan term.
[I/YR]	I/YR=15.06	Calculates imputed interest rate on loan.

Part 2. Calculate an annual amortization schedule for the note over its life.

Keys:	**Display:**	**Description:**
■ [AMORT]	PERIODS?*value*	Displays current setting for number of periods.
12 [INPUT]	INT=-7,744.69	Enters number of periods to be amortized. Displays amount of interest expense for first year.
[▼]	PRN=-10,255.31	Displays amount of principal paid in first year.
[▼]	BAL=45,744.69	Displays liability book value at end of year 1.
■ [AMORT]	PERIODS?12.00	Do you wish to amortize 12 more months?
[INPUT]	INT=-6,089.09	Displays amount of interest incurred during second year.
[▼]	PRN=-11,910.91	Displays principal paid in year 2.
[▼]	BAL=33,833.78	Displays liability book value at end of year 2.
■ [AMORT]	PERIODS?12.00	Do you wish to amortize 12 more months?
[INPUT]	INT=-4,166.21	Displays interest paid in year 3.
[▼]	PRN=-13,833.79	Displays principal paid in year 3.
[▼]	BAL=19,999.99	Displays liability book value at end of year 3, which is also the amount of final payment.

The difference between the stated $20,000 and the displayed value is due to rounding.

Capital Lease Amortization

A capital lease is one in which the lessee has essentially purchased the property or asset. The asset value equals the present value of the lease payments, computed using the imputed interest rate. For accounting purposes, the present value of the payments also represents a liability and must be amortized over the lease term. Using the interest rate imputed to the lease and the compound interest method of amortization, each payment has two components: a reduction of the liability and a payment of interest.* The ■[AMORT] procedure calculates such an amortization schedule for payments occurring at either the beginning or the end of each period.

Example. The ABC Company leased equipment for a period of 7 years, agreeing to pay $75,000 in advance on January 1, and to pay $75,000 annually on December 31 of each of the next 6 years. The equipment has a zero residual value after 7 years. This is a capital lease with the liability amortized using the effective interest method. The imputed annual interest rate is 12%.

Part 1. Calculate the lease liability. (The lease liability equals the present value of the payments, discounted at the imputed interest rate.)

Keys:	Display:	Description:
■[BEG/END] {BEG} 1 ■[P/YR]	P/YR=1.00	Sets Begin mode and 1 payment period per year.
7 [N]	N=7.00	Stores number of payments.
12 [I/YR]	I/YR=12.00	Stores imputed interest rate.

* Financial Accounting Standards Board, Statement No. 13.

75000 [+/-] [PMT]	PMT=-75,000.00	Stores lease payment.
0 [FV]	FV=0.00	Stores residual value after 7 years.
[PV]	PV=383,355.55	Calculates present value of lease.

Part 2. Prepare an amortization schedule for the first 2 years.

Keys:	**Display:**	**Description:**
■[AMORT]	PERIODS?*value*	Displays current setting for number of periods.
1 [INPUT]	INT=0.00	Enters number of coupon periods to amortize. Displays interest for period 1. (Since payment is made up front, no interest is accrued.)
[▼]	PRN=-75,000.00	Displays reduction of lease liability by first payment.
[▼]	BAL=308,355.55	Displays lease liability after first payment.
■[AMORT] [INPUT]	INT=-37,002.67	Displays interest.
[▼]	PRN=-37,997.33	Displays reduction of lease liability by second payment.
[▼]	BAL=270,358.22	Displays lease liability after second payment.

Bond Discount/Premium Amortization

The effective interest method is the recommended accounting procedure for amortizing bond discounts or premiums. Use the effective interest rate (yield to maturity) and the ■[AMORT] procedure to prepare an amortization schedule. The value shown for the principal paid is the amount of discount or premium amortized, while the remaining balance represents the net book value of the bonds.

Example. A company issued 11%, semiannual coupon bonds with a face value of $100,000. The bonds were sold on a coupon date and will be redeemed in 10 years. The company received $94,265.04 for the bonds. Calculate the cost to the borrower and amortize the discount expense for the first two coupon payments using the effective interest method.

Keys:	Display:	Description:
■[BEG/END] {END} 2 ■[P/YR]	P/YR=2.00	Sets End mode and 2 payment periods per year.
10 ■[xP/YR]	N=20.00	Enters years and stores number of coupon payments.
94265.04 [PV]	PV=94,265.04	Stores amount received for bonds.
100000 [+/-] [FV]	FV=-100,000.00	Stores par value of bonds.
[×] 11 [%] [÷] 2 [PMT]	PMT=-5,500.00	Multiplies par value by bond coupon rate and divides by number of coupon periods per year to determine semiannual coupon payment.
[I/YR]	I/YR=12.00	Calculates effective interest rate.

■[AMORT]	PERIODS?*value*	Displays current setting for number of periods.
1 [INPUT]	INT=-5,655.90	Enters number of coupon periods to amortize. Displays bond interest for period 1.
[▼]	PRN=155.90	Displays amount of bond discount to amortize at end of period 1.
[▼]	BAL=94,420.94	Displays net book value of bonds at end of period 1.
■[AMORT] [INPUT]	INT=-5,665.26	Displays bond interest for period 2.
[▼]	PRN=165.26	Displays amount of bond discount to amortize at end of period 2.
[▼]	BAL=94,586.20	Displays net book value of bonds at end of period 2.

At the end of the last coupon period, the net book value of the bonds will equal the face value.

2

Financial Analysis

Common-Size Statement Analysis

Expressing expenses on an income statement as a percentage of sales (or balance sheet items as a percentage of total assets) is a useful technique for financial analysis. Use the Percent of Total application to make the calculations.

Example. Prepare a common-size vertical income statement analysis for the income statement shown below.

Income Statement

	Dollars ($)	*(You Calculate This Column)* Common-Size
Sales	80,000	100.00%
Cost of Sales	55,000	68.75%
Gross Margin	25,000	31.25%
Selling Expenses	9,000	11.25%
Admin. Expenses	8,000	10.00%
Total Expenses	17,000	21.25%
Income	8,000	10.00%

Keys:	Display:	Description:
80000 ■ [TOTAL]	TOTL=80,000.00	Stores sales in *TOTAL* register.
55000 ■ [PART]	PART=55,000.00	Stores cost of sales in *PART* register.
■ [%TOTAL]	%TOTL=68.75	Calculates cost of sales as a percentage of sales.
25000 ■ [PART] ■ [%TOTAL]	%TOTL=31.25	Calculates gross margin as a percentage of sales.
9000 ■ [PART] ■ [%TOTAL]	%TOTL=11.25	Calculates selling expenses as a percentage of sales.

8000 ■[PART] ■[%TOTAL]	%TOTL=10.00	Calculates administrative expenses as a percentage of sales.
17000 ■[PART] ■[%TOTAL]	%TOTL=21.25	Calculates total expenses as a percentage of sales.
8000 ■[PART] ■[%TOTAL]	%TOTL=10.00	Calculates income as a percentage of sales.

Budget Variances

Comparing actual performance to the budget is a very useful technique for evaluating performance. The Percent Change application of the HP-14B calculates the percentage change between two values, or calculates the difference as a percentage of the budgeted amount. In cost accounting this difference is called a *variance*. (When expressed as a percentage, it's called *percentage variance*.)

Example. The budgeted income statement and the actual results for the current year are shown below. Calculate the percentage over or under budget for each income statement item.

Income Statement

	Budget($)	**Actual($)**	*(You Calculate This Column)* **Percent Change**
Sales	75,000	80,000	6.67%
Cost of Sales	54,000	55,000	1.85%
Gross Margin	21,000	25,000	19.05%
Selling Expenses	9,500	9,000	-5.26%
Admin. Expenses	7,700	8,000	3.90%
Total Expenses	17,200	17,000	-1.16%
Income	3,800	8,000	110.53%

Keys:	**Display:**	**Description:**
75000 ■[OLD] 80000 ■[NEW] ■[%CHG]	%CHG=6.67	Enters budget and actual amounts and calculates percentage change in sales.
54000 ■[OLD] 55000 ■[NEW] ■[%CHG]	%CHG=1.85	Calculates percentage change between planned and actual cost of sales.

21000 ■[OLD] 25000 ■[NEW] ■[%CHG]	%CHG=19.05	Calculates percentage change between planned and actual gross margin.
9500 ■[OLD] 9000 ■[NEW] ■[%CHG]	%CHG=-5.26	Calculates percentage change between planned and actual selling expenses.
7700 ■[OLD] 8000 ■[NEW] ■[%CHG]	%CHG=3.90	Calculates percentage change between planned and actual administrative expenses.
17200 ■[OLD] 17000 ■[NEW] ■[%CHG]	%CHG=-1.16	Calculates percentage change between planned and actual total expenses.
3800 ■[OLD] 8000 ■[NEW] ■[%CHG]	%CHG=110.53	Calculates percentage change between planned and actual income.

Profitability Ratios

The *INVEN* application built into the HP-14B is designed for computing inventory turnover ratios. The basic form of the equation is

$$INVEN = \frac{SOLD}{(BEGI + ENDI) \div 2} = \frac{Cost\ of\ Goods\ Sold}{Average\ Inventory}$$

and can be generalized as

$$Ratio = \frac{Flow}{Average\ Stock}$$

A flow divided by an average stock is really a fundamental ratio that can be applied to a variety of financial analysis situations. By doing so, you can extend the solver capability of the *INVEN* application to other ratios. Here are a few examples:

$$\frac{Sales}{Average\ Working\ Capital} = Working\ Capital\ Turnover$$

$$\frac{Sales}{Average\ Assets} = Asset\ Turnover$$

$$\frac{Profit}{Average\ Assets} = Return\ on\ Assets$$

$$\frac{Profit}{Average\ Equity} = Return\ on\ Equity$$

To use the *INVEN* application for these different ratios, you must know the relationship between the *INVEN* variables and the variables in the ratio you are calculating (see the following table). The general equation that applies to all these ratios is

$$\frac{Flow}{(Beg.\ Stock + End.\ Stock) \div 2} = Ratio$$

The following table shows the relationships between the variables in the different ratios:

Ratio	Variables		
INVEN	*BEGI*	*ENDI*	*SOLD*
General Ratio	*Beg. Stock*	*End. Stock*	*Flow*
Working Capital (WC) Turnover	*Beg. WC*	*End. WC*	*Sales*
Asset Turnover	*Beg. Assets*	*End. Assets*	*Sales*
Return on Assets	*Beg. Assets*	*End. Assets*	*Profit*
Return on Equity	*Beg. Equity*	*End. Equity*	*Profit*

You can expand this table to include your own ratios or others from your finance and accounting textbooks.

Use these steps to solve a ratio problem using the *INVEN* application:

1. Press ■[INVEN] to start the application.
2. Enter three of the four variables, substituting the *INVEN* variables for the variables in your ratio as shown in the previous table.
3. Use the arrow keys to display the unknown variable.
4. Press ■[COMPUTE] to solve for the unknown.

Example. Last year you purchased stock in a company as a long-term investment, and this year you are evaluating the company's performance. Some key information is as follows:

Sales	$7,800
Cost of Goods Sold	6,000
Earnings Before Interest and Taxes (EBIT)	500
Interest	200
Income Taxes	175
Income After Taxes	125

	Beg. Year	End Year
Total Assets	$4,675	$4,800
Equity	2,200	2,100
Working Capital	1,400	1,300
Inventory	1,140	1,360

Part 1. Calculate the asset turnover ratio.

Keys:	Display:	Description:
■[INVEN]	BEGI?*value*	Enters *INVEN* application. The first value requested is beginning balance.
4675 [INPUT]	ENDI?*value*	Enters beginning assets and prompts for ending balance.
4800 [INPUT]	SOLD?*value*	Enters ending assets and prompts for equation's numerator (sales).
7800 [INPUT]	INVEN?*value*	Enters sales and prompts for inventory (which you need to calculate).
■[COMPUTE]	INVEN=1.65	The asset turnover ratio is 1.65.

Part 2. Calculate the return on assets ratio.

Keys:	Display:	Description:
■[INVEN] [▼] [▼]	SOLD?7,800.00	Re-enters *INVEN* application and moves to *SOLD* prompt.
500 [-] 175 [=]	SOLD?325.00	Subtracts income taxes from *EBIT*.
[INPUT] ■[COMPUTE] [×] 100 [=]	6.86	Computes return on assets as a percentage.

Part 3. Calculate the working capital turnover ratio.

Keys:	Display:	Description:
■[INVEN] 1400 [INPUT]	ENDI?4,800.00	Re-enters *INVEN* application and inputs beginning working capital. Displays previous value of *ENDI*.
1300 [INPUT] 7800 [INPUT] ■[COMPUTE]	INVEN=5.78	Enters ending working capital and sales, then computes working capital turnover ratio.

Return on Investment

The Return on Investment (*ROI*) application can be used to calculate the ratio of profit after taxes to assets required to earn the net income.

Example. Your company requires a 20% *ROI* for the first year of an investment. A project is proposed that requires an investment of $50,000 and should generate revenues of $24,500 the first year. What amount of income, as a percentage of sales, must be earned to meet your requirements for this project?

Keys:	Display:	Description:
■ [ROI%]	REV?*value*	Enters *ROI* application. The first value requested is revenue.
24500 [INPUT]	PROF%?*value*	Enters revenue and prompts for profit (income), which you want to calculate.
[▼] 50000 [INPUT]	ROI%?*value*	Moves down to *INVS* and enters the investment amount. The next value requested is *ROI%*.
20 [INPUT]	REV?24,500.00	Enters required *ROI%* and moves back to top of input value list.
[▼] ■ [COMPUTE]	PROF%=40.82	Moves to profit value and computes the percentage of sales that is required to earn an *ROI* of 20%.

You must earn 40.82% on sales to have a 20% *ROI*.

3

Finance

Compound Interest and Annuity Tables

The compound interest and annuity tables found in most accounting and finance textbooks can easily be duplicated by the HP-14B. While textbook tables usually show three or four decimal places, the calculator generates answers with up to 12 decimal places (depending on how you display the number), which may result in a difference between the textbook answer and the calculator answer. The calculator answer is correct.

The basic procedure for generating the contents of compound interest and annuity tables involves setting your calculator to 1 payment per year and then using the TVM keys to solve for present or future value.

The keystroke examples that demonstrate how to duplicate the tables assume your calculator is set to display four decimal places. (To change your display, press ■ [DISP] {FIX} 4. When you're ready to change your display back to two decimal places, press ■ [DISP] {FIX} 2.) Also, the examples generate a positive answer to match that found in the tables.

Future Value of $1 Due at the End of N Periods

One type of table shows the future value (*FV*) of a lump-sum payment (*PV*). The tables that show future value are often called "Future Value of $1 Due at the End of N Periods," or "Future Value of $1."

With these tables, the *PV* is always assumed to occur at the beginning of the first compounding period, and the *FV* is always assumed to occur at the end of the final compounding period. The annuity setting for beginning or end of period payments is ignored in these calculations because the *PMT* is 0.

Example 1. What is the future value of $1 at the end of 8 years using a 10% discount rate and assuming annual compounding?

Solve this problem by entering 1 for *P/YR*, 8 for *N*, 10 for *I/YR*, −1 for *PV*, 0 for *PMT*, and then calculating *FV*.

Keys:	Display:	Description:
1 ■[P/YR]	P/YR=1.0000	Sets number of compounding periods per year.
8 [N]	N=8.0000	Enters total number of compounding periods.
10 [I/YR]	I/YR=10.0000	Enters annual compounding rate.
1 [+/-] [PV]	PV=-1.0000	Stores −1 in *PV*.
0 [PMT]	PMT=0.0000	Zeros out *PMT* register.
[FV]	FV=2.1436	Calculates future value of $1.

Example 2. What is the future value of $1 after 12 months if the interest rate is compounded monthly at 2%? (Because interest is given as a monthly rate and compounded monthly, part of the solution for this problem involves interpreting *I/YR* as interest rate per *month* and *P/YR* as compounding periods per *month*.)

Solve this problem by entering 1 for *P/YR* (interpreted as compounding periods per *month*), 12 for *N*, 2 for *I/YR* (interpreted as interest rate per *month*), −1 for *PV*, 0 for *PMT*, and then calculating *FV*.

Keys:	Display:	Description:
1 ■[P/YR]	P/YR=1.0000	Sets number of compounding periods per month.

12 [N]	N=12.0000	Enters total number of compounding periods.
2 [I/YR]	I/YR=2.0000	Enters monthly compounding rate.
1 [+/-] [PV]	PV=-1.0000	Stores −1 in *PV*.
0 [PMT]	PMT=0.0000	Zeros out *PMT*.
[FV]	FV=1.2682	Calculates future value of $1.

Present Value of $1 Due at the End of N Periods

One table shows the present value (*PV*) of a lump-sum amount received in the future (*FV*). The tables that show present value are often called "Present Value of $1 Due at the End of N Periods," or "Present Value of $1."

With these tables, the *PV* is always assumed to occur at the beginning of the first compounding period, and the *FV* is always assumed to occur at the end of the final compounding period. The annuity setting for beginning or end of period payments is ignored in these calculations because the *PMT* is 0.

Example 1. What is the present value of $1 received at the end of 10 years using a 12% discount rate and assuming annual compounding? This problem has annual compounding, so *P/YR* is interpreted as compounding periods per year and is entered as 1.

Solve this problem by entering 1 for *P/YR*, 10 for *N*, 12 for *I/YR*, −1 for *FV*, 0 for *PMT*, and then calculating *PV*.

Keys:	Display:	Description:
1 ■[P/YR]	P/YR=1.0000	Sets number of compounding periods per year.

10 [N]	N=10.0000	Enters number of compounding periods.
12 [I/YR]	I/YR=12.0000	Enters annual interest rate.
1 [+/-] [FV]	FV=-1.0000	Stores −1 in *FV*.
0 [PMT]	PMT=0.0000	Zeros out *PMT*.
[PV]	PV=0.3220	Calculates present value of $1.

Example 2. What is the present value of $1 received at the end of 20 months if the monthly interest rate is 2% with monthly compounding? (Because interest is given as a monthly rate and compounded monthly, part of the solution for this problem involves interpreting *I/YR* as interest rate per *month* and *P/YR* as compounding periods per *month*.)

Solve this problem by entering 1 for *P/YR* (interpreted as compounding periods per *month*), 20 for *N*, 2 for *I/YR* (interpreted as interest rate per *month*), −1 for *FV*, 0 for *PMT*, and then calculating *PV*.

Keys:	Display:	Description:
1 ■[P/YR]	P/YR=1.0000	Sets number of compounding periods per month.
20 [N]	N=20.0000	Enters number of compounding periods.
2 [I/YR]	I/YR=2.0000	Enters monthly interest rate.
1 [+/-] [FV]	FV=-1.0000	Stores −1 in *FV*.
0 [PMT]	PMT=0.0000	Zeros out *PMT*.
[PV]	PV=0.6730	Calculates present value of $1.

Future Value of $1 Received Periodically

The tables that show the future value of an annuity are often called "Future Value of an Annuity of $1 per Period for *N* Periods," "Future Value of an Annuity of $1 in Arrears," or "Future Value of an Ordinary Annuity." These tables assume that each regular annuity payment occurs at the end of each payment period, which is End mode for the calculator. For problems requiring beginning of period payments, change the payment setting to Begin mode.

Example 1. Part 1. What is the future value of $1 received at the end of each year for 10 years if interest is compounded at 12% annually?

Solve this problem by first setting the payment mode to End, next entering 1 for *P/YR*, 10 for *N*, 12 for *I/YR*, 0 for *PV*, −1 for *PMT*, and then calculating the *FV*.

Keys:	Display:	Description:
■ [BEG/END] {END} 1 ■ [P/YR]	P/YR=1.0000	Sets End mode and enters number of compounding periods per year.
10 [N]	N=10.0000	Enters total number of compounding periods.
12 [I/YR]	I/YR=12.0000	Enters annual interest rate.
0 [PV]	PV=0.0000	Stores 0 in *PV*.
1 [+/-] [PMT]	PMT=-1.0000	Enters −1 as *PMT*.
[FV]	FV=17.5487	Calculates future value of $1 received at end of each year for 10 years.

Part 2. What is the future value if the payments occur at the beginning of each year?

Keys:	Display:	Description:
■ [BEG/END] {BEGIN}		Changes to Begin mode
[FV]	FV=19.6546	and recalculates future value amount.

Example 2. What is the future value of $1 received at the end of each month for 18 months if the interest rate is compounded monthly at 2%? (Because interest is given as a monthly rate and compounded monthly, part of the solution for this problem involves interpreting *I/YR* as interest rate per *month* and *P/YR* as compounding periods per *month*.)

Solve this problem by first setting the payment mode to End, then entering 1 for *P/YR* (interpreted as compounding periods per *month*), 18 for *N*, 2 for *I/YR* (interpreted as interest rate per *month*), 0 for *PV*, −1 for *PMT*, and then calculating *FV*.

Keys:	Display:	Description:
■ [BEG/END] {END}		Sets End mode and
1 ■ [P/YR]	P/YR=1.0000	enters number of compounding periods per month.
18 [N]	N=18.0000	Enters total number of payments.
2 [I/YR]	I/YR=2.0000	Enters monthly compounding rate.
0 [PV]	PV=0.0000	Stores 0 in *PV*.
1 [+/-] [PMT]	PMT=-1.0000	Enters −1 as *PMT*.
[FV]	FV=21.4123	Calculates future value of $1 received at end of each month for 18 months.

Present Value of $1 Received Periodically

The tables that show the present value of an annuity are often called "Present Value of an Annuity of $1 per Period for *N* Periods," "Present Value of an Annuity of $1 in Arrears," and "Present Value of an Ordinary Annuity." These tables assume that each regular annuity payment occurs at the end of each payment period, which is End mode for the calculator. For problems requiring beginning of period payments, change the payment setting to Begin mode.

Example 1. Part 1. What is the present value of $1 received at the end of each year for 14 years if interest is compounded annually at 12%?

Solve this problem by first setting the payment mode to End, next entering 1 for *P/YR*, 14 for *N*, 12 for *I/YR*, 0 for *FV*, −1 for *PMT*, and then calculating *PV*.

Keys:	Display:	Description:
■ [BEG/END] {END} 1 ■ [P/YR]	P/YR=1.0000	Sets End mode and enters number of compounding periods per year.
14 [N]	N=14.0000	Enters total number of payments.
12 [I/YR]	I/YR=12.0000	Enters annual compounding rate.
0 [FV]	FV=0.0000	Stores 0 in *FV*.
1 [+/-] [PMT]	PMT=-1.0000	Enters −1 as *PMT*.
[PV]	PV=6.6282	Calculates present value of $1 received at end of each year for 14 years.

Part 2. What is the present value if each payment occurs at the beginning of each year?

Keys:	Display:	Description:
■[BEG/END] {BEGIN}		Changes to Begin mode
[PV]	PV=7.4235	and recalculates present value.

Example 2. What is the present value of $1 received at the end of each month for 8 months if interest is compounded at 2% monthly? (Because interest is given as a monthly rate and compounded monthly, part of the solution for this problem involves interpreting *I/YR* as interest rate per *month* and *P/YR* as compounding periods per *month*.)

Solve this problem by first setting the payment mode to End, next entering 1 for *P/YR* (interpreted as compounding periods per *month*), 8 for *N*, 2 for *I/YR* (interpreted as interest rate per *month*), 0 for *FV*, −1 for *PMT*, and then calculating *PV*.

Keys:	Display:	Description:
■[BEG/END] {END}		Sets End mode and
1 ■[P/YR]	P/YR=1.0000	enters number of compounding periods per month.
8 [N]	N=8.0000	Enters total number of payments.
2 [I/YR]	I/YR=2.0000	Enters monthly compounding rate.
0 [FV]	FV=0.0000	Stores 0 in *FV*.
1 [+/-] [PMT]	PMT=-1.0000	Enters monthly payment.
[PV]	PV=7.3255	Calculates present value of $1 received at end of each month for 8 months.

Bonds Sold on a Coupon Date

A bond is a contract to pay interest, usually semiannually in the U.S., at a given (coupon) rate, and to pay the principal of the bond at some specified future date. The value or price of a bond is the present value of the coupon payments plus the present value of the principal or redemption value, all at a given interest rate (yield to maturity). The procedure described in this chapter is valid only for bonds purchased on a coupon date.

Bond price and yield problems are solved in six steps:

1. Select End mode and enter the number of coupon payments per year in *P/YR*.
2. Store the total number of coupon payments in the *N* register.
3. Store the bond's redemption value in the *FV* register.
4. Store the periodic coupon payment in the *PMT* register. Calculate the periodic coupon payment by multiplying the coupon rate times the par value and dividing by the number of coupon payments per year.
5. If calculating yield, enter the price in the *PV* register and then calculate *I/YR*.
6. If calculating price, enter the yield in the *I/YR* register and calculate *PV*.

Example. Part 1. A $1,000 bond, due in 10 years, has an 11% coupon rate, with the first coupon payment due in 6 months. What price is required for this bond to yield 10.5%, compounded semiannually?

Keys:	**Display:**	**Description:**
■ [BEG/END] {END}		Sets End mode and 2
2 ■ [P/YR]	P/YR=2.00	coupon periods per year.
10 ■ [xP/YR]	N=20.00	Calculates total number of payments and stores result in *N*.

1000 [FV]	FV=1,000.00	Stores the value.
11 [%] [÷] 2 [x] [RCL] [FV] [=] [PMT]	PMT=55.00	Enters coupon rate, converts it to a decimal, divides by number of coupon periods per year, and multiplies by redemption value to compute the periodic payment. The result is stored in *PMT*.
10.5 [I/YR]	I/YR=10.50	Enters required annual yield.
[PV]	PV=-1,030.51	Calculates bond price.

Part 2. If $1,025 is paid for the bond, what is the yield to maturity? (Since the data for everything but the bond price is entered, enter the bond price and calculate the yield to maturity.)

Keys:	Display:	Description:
1025 [+/-] [PV]	PV=-1,025.00	Enters bond price.
[I/YR]	I/YR=10.59	Calculates yield to maturity.

Part 3. If the bond is redeemed for $1,050, and its purchase price was $1,025, what is the yield?

Keys:	Display:	Description:
1050 [FV]	FV=1,050.00	Enters redemption price.
[I/YR]	I/YR=10.87	The yield is 10.87%.

Bond Duration

Bond duration measures the weighted average time to maturity of the bond. The calculation involves weighting each time period by the present value of the cash flow in that period.

For a periodic coupon bond purchased on a coupon date, the duration of the bond can be calculated using the TVM application of the HP-14B. Use the following procedure:

1. Set End mode.
2. Store the number of coupon payments per year in *P/YR*.
3. Calculate the periodic coupon payment amount and store it in *PMT* and register 1.
4. Store the remaining number of coupon payments in *N*.
5. Store the redemption value in *FV*.
6. Store the annual yield as a percent in *I/YR*.
7. Store the bond price as a negative number in register 2.
8. Calculate the following number and store it in *PMT*:

$$\frac{100}{I/YR} + \frac{1}{P/YR}$$

9. Calculate the following number and store it in *FV*:

$$\left(\frac{\textit{Redemption Value}}{\textit{Payments Per Year} \times \textit{Coupon Payment Amount}} - \frac{100}{I/YR}\right) \times N$$

10. Press [PV].
11. Calculate the bond duration in years as follows:

$$\frac{PV \times \textit{Coupon Payment Amount}}{\textit{Bond Price}}$$

Example. A $1,000, semiannual coupon bond has an annual coupon of 10% and a yield to maturity of 9%. The bond is purchased on a coupon date, and there are 10 years remaining to maturity. What are the bond price and the bond duration?

Step 1. Calculate the bond price.

Keys:	Display:	Description:
■[BEG/END] {END} 2 ■[P/YR]	P/YR=2.00	Sets End mode and enters number of coupon periods per year.
1000 [FV]	FV=1,000.00	Stores redemption value.
[×] 10 [%] [÷] [RCL] ■[P/YR] [=] [PMT] [STO] 1	PMT=50.00	Enters annual coupon payment and divides by number of coupon periods per year to give periodic coupon payment. Stores result in register 1 for later use.
10 ■[xP/YR]	N=20.00	Stores total number of coupon payments.
9 [I/YR]	I/YR=9.00	Enters bond's annual yield.
[PV] [STO] 2	PV=-1,065.04	Calculates bond price and stores it in register 2.

Step 2. Calculate the bond duration.

Keys:	Display:	Description:
100 [÷] [RCL] [I/YR] [+] [(] 1 [÷] [RCL] ■[P/YR] [)] [PMT]	PMT=11.61	Calculates and stores adjusted *PMT*.
[RCL] [FV] [÷] [RCL] 1 [÷] [RCL] ■[P/YR] [−] [(] 100 [÷] [RCL] [I/YR] [=] [×] [RCL] [N] [=]		Calculates and stores adjusted *FV*.

[FV]	FV=-22.22	
[PV]	PV=-141.82	Calculates intermediate value.
[x] [RCL] 1 [÷] [RCL] 2 [=]	6.66	Calculates duration in years.

Yield on Investments With Varying Return Rates

The procedure in this section enables you to calculate the yield of an investment having returns that vary from period to period. The model used here assumes that an initial cash outlay is made and that the annual returns are reinvested in the investment each year. The yield is computed by compounding the investment forward using the annual return rates. The result of the compounding is entered as *FV*, the investment as *PV*, the number of years as *N*, and the interest rate (which is the yield) is then calculated.

Example. An investor invests $1,000 at the beginning of year 1 in a fund. The annual returns for each year are as follows:

Year 1	5%
Year 2	15%
Year 3	-3%
Year 4	8%

What is the annual yield or compound growth rate on this investment?

You solve this in three steps.

1. Enter the amount of the investment as a negative number in the *PV* register.
2. Enter each year's return, and use the [%] key to compound the investment forward year by year.
3. After the final compounding, enter the amount in the *FV* register as a positive number, enter the number of years in the *N* register, and solve for the annual return.

This procedure is valid for investments requiring an initial cash outlay and returning a final amount at the end of a given number of years.

Keys:	Display:	Description:
■[BEG/END] {END} 1 ■[P/YR]	P/YR=1.00	Sets End mode and 1 dividend payment per year.
1000 [+/-] [PV]	PV=-1,000.00	Enters initial investment.
[+] 5 [%] [=]	-1,050.00	Adds return for year 1.
[+] 15 [%] [=]	-1,207.50	Adds return for year 2.
[-] 3 [%] [=]	-1,171.28	Subtracts return for year 3.
[+] 8 [%] [=]	-1,264.98	Adds return for year 4.
[+/-] [FV]	FV=1,264.98	Stores investment amount at end of term.
0 [PMT]	PMT=0.00	Enters 0 for *PMT*.
4 [N]	N=4.00	Enters number of years.
[I/YR]	I/YR=6.05	Calculates annual yield.

The annual yield of 6.05% is also the compound growth rate of this investment.

Stock Valuation Models

Predicting stock prices is an uncertain process. One theory is that a stock's price equals the present value of its future dividends and its future selling price. While no one can predict these future cash flows with accuracy, there are three basic stock valuation models covered in most finance textbooks. These models are the zero growth dividend model, the constant growth dividend model, and the supernormal growth dividend model.

Zero Growth Dividend Model

Assuming a stock's dividend and price remain constant, the stock's price equals the present value of a perpetuity. The equation used is

$$PRICE = \frac{Dividend}{Return\ Rate}$$

where:

- *Dividend* is the dividend paid in perpetuity.
- *Return Rate* is the expected rate of return as a decimal.

Example 1. A stock is expected to pay $1.50 annual dividends in perpetuity. If you believe an annual return of 8% is reasonable, what is the maximum amount you would pay for the stock?

Keys:	Display:	Description:
1.5 [÷] 8 [%] [=]	18.75	Divides dividend by rate of return to calculate stock price.

Constant Growth Dividend Model

If dividends are expected to grow at a constant rate indefinitely, then the stock's price is estimated using

$$PRICE = \frac{Dividend}{Return\ Rate\ -\ Growth\ Rate}$$

where:

- *Dividend* is the dividend paid for the current year.
- *Return Rate* is the expected rate of return as a decimal.
- *Growth Rate* is the expected growth rate of dividends as a decimal.

This model computes the present value of a perpetuity with a constant growth rate. For this equation to be valid, the return rate must be greater than the growth rate.

Example 2. You are interested in a stock that pays a $2 dividend. You expect the dividend to increase by 5% each year perpetually, and you want to earn 9% on the stock. What is the stock's value to you?

Keys:	Display:	Description:
2	2	Enters the dividend.
[÷] [(] 9 [−] 5 [)] [%]	÷0.04	Enters return rate and growth rate, calculates difference, then converts to a decimal.
[=]	50.00	Calculates stock value.

Supernormal Growth Dividend Model

This model assumes dividends per share grow at an accelerated rate for a period of years and then return to a normal growth rate. The equations for estimating the stock's value calculate the present value of the dividend stream and the present value of the stock's estimated terminal market value at the end of the growth period. The sum of the two present values equals the stock's intrinsic value under this set of assumptions. This model is also described as the "Step Growth Model," the "Two-Stage Growth Model," and the "Temporary Super Normal Growth Model." *

The following equations are organized to facilitate using the TVM keys. You calculate the equivalent interest rate (*i*) and the values for the *PMT* and *FV* registers, store the number of payments, and solve for the present value.

$$\textit{Equivalent Interest Rate } i = \left(\frac{1 + K}{1 + GR}\right) - 1$$

$$\textit{PV of Dividend Stream} = DIV \times \left[\frac{1 - (1 + i)^{-N}}{i}\right]$$

$$\textit{PV of Terminal VALUE} = DIV \times \left(\frac{1 + GN}{K - GN}\right) \times (1 + i)^{-N}$$

$$\textit{Stock's VALUE} = \textit{PV of Dividend Stream} + \textit{PV of Terminal VALUE}$$

where:

- *DIV* is the dividend per share for the year prior to the current year.
- *K* is the investor's required rate of return as a decimal.
- *GR* is the accelerated growth rate as a decimal.
- *GN* is the normal growth rate as a decimal.
- *N* is the number of years for accelerated growth.

* J. Fred Weston and Eugene F. Brigham, *Essentials of Managerial Finance*, 8th ed. (Chicago: The Dryden Press, 1987), p. 150.

Example 3. You are interested in purchasing a stock that last year paid a dividend of $1.25 per share. You believe the dividends and earnings will increase at the rate of 20% annually for the next 8 years. At the end of the period, you expect the stock price to grow 9% annually, which is the normal growth rate for similar stocks. Your required earnings rate is 13%. What would you expect to pay for a share of this stock?

Step 1. Calculate the equivalent interest rate for use with the TVM application.

Keys:	**Display:**	**Description:**
■ [BEG/END] {END} 1 ■ [P/YR]	P/YR=1.00	Sets End mode and 1 dividend payment per year.
13 [%] [+] 1 [÷]	1.13÷	Enters investor's required earnings rate, converts it to a decimal, and adds 1.
[(] 20 [%] [+] 1 [)]	÷1.20	Enters accelerated growth rate, converts to a decimal, and adds 1.
[-] 1 [×] 100 [I/YR]	I/YR=-5.83	Calculates equivalent rate for use with TVM application.
8 [N]	N=8.00	Enters number of years for accelerated growth.

Step 2. Enter the dividend.

1.25 [PMT]	PMT=1.25	Stores dividend in *PMT*.

Step 3. Calculate the amount to store in the *FV* register.

[+] 9 [%] [÷]	1.36÷	Enters normal growth percentage and increases dividend by that amount.
[(] [(] 13 [−] 9 [)] [%] [FV]	FV=34.06	Enters required rate of return and subtracts normal grow rate. Then converts to a decimal and stores result in *FV*.

Step 4. Calculate the estimated stock price.

[PV]	PV=-68.32	Calculates estimated price for the stock.

Capital Asset Pricing Model

The capital asset pricing model describes the relationship between expected risk and expected return. The measure of a stock's volatility relative to the rest of the market is called the beta coefficient, which is the slope of the security market line. Beta values can be found in stock market price publications (available at libraries), or if you have access to the proper information, you can calculate the beta value, as shown on page 80. (The beta values needed in the next two examples are given to you.)

Estimating the Rate of Return on Common Stock

This approach for determining the expected rate of return on a stock is based on the beta value for the firm's common stock. The equation used is

$$R = Ri + B(Rm - Ri)$$

where:

- B is the beta coefficient.
- R is the expected percentage rate of return on the stock.
- Rm is the expected percentage rate of return on the market portfolio.
- Ri is the risk-free interest rate.

Example 1. The beta coefficient for a company is 1.2. The risk-free rate of return (the current rate of Treasury Bill securities) is 7%. Calculate the expected rate of return for common stock using the capital asset pricing model, assuming the expected rate of return for the market as a whole is 9%.

Keys:	Display:	Description:
9 [−] 7 [=]	2.00	Enters expected market rate of return and subtracts risk-free interest rate.

[×] 1.2	×1.2	Multiplies by beta coefficient.
[+] 7 [=]	9.40	Adds risk-free interest rate to calculate expected rate of return on the stock.

Portfolio Beta Coefficients

The beta coefficient of a portfolio equals the weighted average of the individual betas. By using the HP-14B Statistics application, you can easily calculate the portfolio's beta coefficient.

Example 2. Part 1. A portfolio consists of four common stocks. The market value of each stock holding and the stock's beta are listed below. Calculate the beta of the portfolio.

Stock	Beta	Market Value
AA	1.2	$2,000
BB	1.1	7,000
CC	1.9	3,000
DD	1.3	6,000

Keys:	Display:	Description:
■[CLEAR] {Σ}	*value*	Clears statistics registers.
1.2 [INPUT]	1.20:	Enters beta of stock AA.
2000 [Σ+]	n=1.00	Enters market value of stock AA.
1.1 [INPUT] 7000 [Σ+]	n=2.00	Enters remaining data.
1.9 [INPUT] 3000 [Σ+]	n=3.00	
1.3 [INPUT] 6000 [Σ+]	n=4.00	
[STAT] {x̄, ȳ} {x̄w}	x̄w=1.31	Calculates portfolio beta coefficient.

Part 2. If Stock DD is replaced with $8,000 of stock EE that has a beta of 1.8, what is the beta of the new portfolio?

Keys:	Display:	Description:
1.3 [INPUT] 6000 ■[Σ-]	n=3.00	Removes beta and market value of stock DD from portfolio.
1.8 [INPUT] 8000 [Σ+]	n=4.00	Enters new stock EE beta and market value.
[STAT] {x̄,ȳ} {x̄w}	x̄w=1.51	Calculates revised portfolio beta coefficient.

Calculating Beta Coefficients

One method for estimating a stock's beta coefficient involves calculating the beta using past information and then assuming that the number calculated will be valid in the future. Generally, the beta is estimated by calculating the slope of the linear regression line. The calculation uses periodic rates of return of a market index as the independent, or x, variable and the periodic rates of return of the stock as the dependent, or y, variable.

Example 3. Part 1. The table below lists the market and stock returns for the ABC Company. Calculate the beta coefficient, intercept, and correlation coefficient.

Year	*(X)* Market Return	*(Y)* Stock Return
1	3.8%	2.6%
2	12.3	16.1
3	17.0	7.2
4	-12.7	-4.1
5	-26.5	-13.3
6	35.2	31.1

Keys:	Display:	Description:
■[CLEAR] {Σ}	*value*	Clears statistics registers.
3.8 [INPUT]	3.80:	Enters market return for first period.
2.6 [Σ+]	n=1.00	Enters stock's return for first period.
12.3 [INPUT] 16.1 [Σ+]	n=2.00	Enters returns for remaining periods.
17 [INPUT] 7.2 [Σ+]	n=3.00	
12.7 [+/-] [INPUT] 4.1 [+/-] [Σ+]	n=4.00	
26.5 [+/-] [INPUT] 13.3 [+/-] [Σ+]	n=5.00	
35.2 [INPUT] 31.1 [Σ+]	n=6.00	
[STAT] {FIT} {L} {m}	m=0.68	Calculates beta coefficient, which is the slope.
[STAT] {FIT} {.L.} {b}	b=3.31	Calculates intercept of prediction equation.
[STAT] {FIT} {.L.} {r}	r=0.95	Calculates correlation coefficient.

Part 2. Estimate the stock return if the market return is 25%.

Keys:	Display:	Description:
[STAT] {FIT} {.L.} {ŷ} 25 [INPUT]	ŷ=20.26	Enters market return and estimates stock's return.

4

Statistics

Permutations

A permutation is an ordered subset of a set of distinct objects. The number of possible permutations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$$_mP_n = \frac{m!}{(m - n)!}$$

where, for the HP-14B, m and n are integers and $0 \leq n \leq 253$.

However, when n equals m, then $_mP_n = m!$ (because $0! = 1$).

Example 1. How many ways can 10 people be seated in a car if only 4 seats are available?

Keys:	Display:	Description:
10 [-] 4 [=] ■[MATH] {n!}	720.00	Enters number of people and subtracts number of seats. Computes factorial.
[STO] 0	720.00	Stores for later use.
10 ■[MATH] {n!}	3,628,800.00	Enters number of people, and computes factorial.
[÷] [RCL] 0 [=]	5,040.00	Calculates permutation. There are 5,040 ways to seat 10 people in groups of 4 at a time in the car.

Example 2. Six candidates are scheduled to give short presentations at a debate, and you are in charge of determining the order of presentation. How many different ways are there in which the speakers can be arranged?

Since the candidates are not grouped, n is equal to m. Therefore, the number of arrangements is simply $m!$ (which is calculated using $n!$).

Keys:	**Display:**	**Description:**
6 ■ [MATH] {n!}	720.00	Enters number of objects and computes factorial.

Combinations

A combination is a selection of one or more of a set of distinct objects without regard to order. The number of possible combinations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$$ {}_{m}C_{n} = \frac{m!}{(m - n)!\,n!} $$

where m and n are integers and $0 \leq n \leq 253$.

Example. A university intramural football league has nine teams. How many games must be played in order for each team to play each other one time? In other words, find the number of combinations of nine (m) objects taken two (n) at a time.

Keys:	Display:	Description:
9 [-] 2 [=] ■[MATH] {n!}	5,040.00	Enters m and subtracts n. Calculates the factorial.
[×] 2 ■[MATH] {n!} [=]	10,080.00	Multiplies by factorial of n.
[STO] 0	10,080.00	Stores for later use.
9 ■[MATH] {n!}	362,880.00	Enters m and computes factorial.
[÷] [RCL] 0 [=]	36.00	Calculates number of combinations. There will be exactly 36 games played.

Learning Curve

Learning curves are useful in analyzing new production processes to determine how productivity will improve over time. The model discussed in this section is the cumulative average learning curve model for direct labor hours:

$$y = ax^b$$

$$xy = x\ ax^b$$

$$b = \frac{\ln\left(\frac{Rate\ \%}{100}\right)}{\ln(2)}$$

where:

- x is the cumulative units.
- y is the average labor hours required to make x cumulative units.
- xy is the total amount of labor hours.
- a is the amount of time required for making the first unit.
- b is the learning curve coefficient.
- *Rate%* is the learning curve rate expressed as a percentage.

Example. Part 1. An electronics manufacturer begins a pilot run on a new instrument. From past experience, he expects the process to have a 90% learning curve rate. If the first unit requires 8.75 labor hours to produce, what is the expected number of labor hours required to produce 100 cumulative units?

Keys:	Display:	Description:
90 [%] ■[MATH] {LN}	-0.11	Enters learning curve rate, converts to decimal, and calculates natural log.
[÷] 2 ■[MATH] {LN} [=]	-0.15	Divides by natural log of 2 and calculates learning curve coefficient.
[STO] 0	-0.15	Stores for later use.
8.75 [STO] 1	8.75	Enters amount of hours for first unit and stores in register 1.
100 ■[y^x] [RCL] 0 [×] [RCL] 1 [=]	4.35	Enters cumulative units and calculates cumulative average labor hours.
[×] 100 [=] [STO] 2	434.51	Enters cumulative units and calculates total labor hours for 100 units. Stores in register 2 for later use.

Part 2. Calculate the total labor hours required for the first 200 units.

Keys:	Display:	Description:
200 ■[y^x] [RCL] 0 [×] [RCL] 1 [=]	3.91	Enters cumulative units and calculates cumulative average labor hours.
[×] 200 [=]	782.12	Enters 200 cumulative units and calculates total labor hours.

Part 3. Calculate the number of hours required to produce the second 100 units.

Keys:	**Display:**	**Description:**
[-] [RCL] 2 [=]	347.61	Recalls total hours for first 100 units and subtracts to determine hours required for second 100 units.

Standard Error

Linear regression is often used to estimate recurring events, such as manufacturing overhead costs. The linear equation provides the intercept and slope of the cost function. The *coefficient of correlation* (r) provides a measure of the linear relationship, while the *coefficient of determination* (r^2) provides a measure of how good the line fits. Another very useful statistic is the *standard error of the residuals* (s_e), which measures the dispersion of the data points around the regression line. This value is also called the *standard error of the estimate.* Once the standard error of the estimate is determined, the *standard error of the slope* (s_b) can be calculated, which estimates the variability of the slope estimate.

The equations for s_e and s_b are shown in a way that facilitates solution by the calculator.

$$s_e = \left\{ \frac{(1 - r^2) \times s_y^2 \times (n - 1)}{(n - 2)} \right\}^{\frac{1}{2}}$$

$$s_b = \frac{s_e}{\{s_x^2 \times (n - 1)\}^{\frac{1}{2}}}$$

where:

- s_e is the standard error of the estimate.
- r^2 is the coefficient of determination (the correlation coefficient squared).
- $s_y{}^2$ is the variance of y data points.
- $s_x{}^2$ is the variance of x data points.
- n is the number of observations.
- s_b is the standard error of the slope.

Example. The amount of miscellaneous supplies used in a manufacturing process is a linear function of direct labor hours. You want to predict the amount of miscellaneous supplies for budgeting purposes and also to have some idea of how much variability to expect in the results. A sample of labor hours and miscellaneous supply costs for the past year is shown below.

Sample	*(X)* Labor Hours	*(Y)* Miscellaneous Supplies
1	1,000	$5,520
2	1,050	6,408
3	2,000	10,356
4	2,100	12,458
5	3,000	14,958
6	4,500	24,404
7	3,200	15,608

Calculate the regression line, the coefficient of determination r^2, the standard error of the estimate, and the standard error of the slope.

Step 1. Enter data points.

Keys:	**Display:**	**Description:**
■ [CLEAR] {Σ}	*value*	Clears statistics registers.
1000 [INPUT] 5520 [Σ+]	n=1.00	Enters first data point set.
1050 [INPUT] 6408 [Σ+]	n=2.00	Enters remaining data points.
2000 [INPUT] 10356 [Σ+]	n=3.00	
2100 [INPUT] 12458 [Σ+]	n=4.00	
3000 [INPUT] 14958 [Σ+]	n=5.00	

4500 [INPUT] 24404 [Σ+]	n=6.00	
3200 [INPUT] 15608 [Σ+]	n=7.00	

Step 2. Calculate *r* and r^2, the intercept, and the slope.

[STAT] {FIT} {L} {r}	r=0.99	Calculates correlation coefficient.
■ [x^2]	0.98	Calculates coefficient of determination.
[−] 1 [=] [+/−] [STO] 0	0.02	Subtracts 1 from coefficient of determination and stores in register 0 for later use.
[STAT] {FIT} {.L.} {b}	b=627.46	Calculates intercept.
[STAT] {FIT} {.L.} {m}	m=5.06	Calculates slope.

Step 3. Calculate the standard error of the estimate.

[STAT] {Σ} {n} [−] 1 [=] [STO] 1	6.00	Recalls number of observations, subtracts 1, and stores in register 1 for later use.
[STAT] {s} {sy}	sy=6,417.93	Calculates standard deviation of the *y* data points.
■ [x^2]	41,189,836.00	Calculates variance of the *y* data points.
[×] [RCL] 0 [×] [RCL] 1 [=]	5,052,525.82	Multiplies *y* variance by contents of registers 0 and 1.

[÷] [(] [RCL] 1 [−] 1 [=] ■[√x] [STO] 2	1,005.24	Calculates standard error of estimate. Stores in register 2 for later use.

Step 4. Calculate the standard error of the slope.

[STAT] {s} {sx}	sx=1,254.47	Calculates standard error of the x data points.
■[x^2]	1,573,690.48	Calculates variance of the x data points.
[×] [RCL] 1 [=] ■[√x] [÷] [RCL] 2 [=] ■[1/x]	0.33	Calculates standard error of the slope.

5

Capital Budgeting

Introduction

Capital budgeting is the process of making decisions about long-range asset acquisition, investments, and financing to help a company achieve its objectives. Managers often evaluate capital budgeting projects using the selection methods shown below.

- Internal Rate of Return (*IRR*): The interest rate that gives a net present value of zero.
- Net Present Value (*NPV*): The present value of the future cash flows discounted at the company's required rate of return (cost of capital).
- Excess Present Value Index: The net present value expressed as a percentage of the investment.
- Payback: The number of years required to recover the amount of an investment.
- Equivalent Annuity Value: A level, periodic payment that discounts to an amount equal to the net present value.

Two basic capital budgeting models are illustrated in this chapter. The first model omits income taxes from the analysis. The second model includes income taxes and demonstrates the role of depreciation in the analysis.

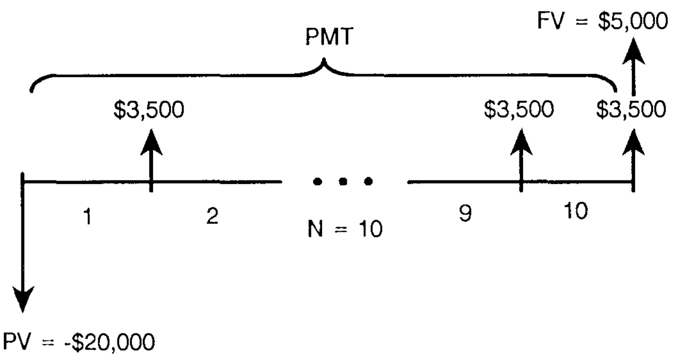
In this chapter the terminal value refers to the estimated resale value of the asset at the end of the decision period, while salvage value refers to the amount used in the depreciation calculation. The two values are not always equal.

Capital Budgeting Model — No Income Taxes

Example. You estimate that you'll save $3,500 at the end of each year for 10 years by purchasing a new special purpose machine to replace an old machine. The old machine has no market value and will be discarded if the new machine is purchased. The machine costs $20,000 and has a projected terminal value (resale value) of $5,000 after 10 years. Currently, your company's minimum required rate of return on capital budgeting projects is 12% annually.

Calculate the net present value, excess present value index, internal rate of return, equivalent annuity value, and payback. Ignore income taxes.

As shown in the cash-flow diagram, this example represents an annuity with end of period payments.



Part 1. Calculate Internal Rate of Return. Enter the known cash flows, then calculate the *IRR*.

Keys:	Display:	Description:
■[BEG/END] {END} 1 ■[P/YR]	P/YR=1.00	Sets End mode and 1 payment period per year.
10 [N]	N=10.00	Stores number of years.
3500 [PMT]	PMT=3,500.00	Stores annual savings.
5000 [FV]	FV=5,000.00	Stores terminal value.
20000 [+/-] [PV] [STO] 0	PV=-20,000.00	Stores outlay cost in registers *PV* and 0.
[I/YR]	I/YR=13.53	Calculates *IRR*.

Part 2. Calculate Net Present Value. Enter the required earnings rate and calculate the total present value of the cash flows. Subtract the outlay cost from the total present value to calculate the net present value.

Keys:	Display:	Description:
12 [I/YR]	I/YR=12.00	Stores minimum rate of return or discount rate.
[PV]	PV=-21,385.65	Calculates total present value of annual savings and terminal value.
[+/-] [+] [RCL] 0 [=] [STO] 1	1,385.65	Calculates *NPV* by netting total present value and outlay cost together.* Stores it in register 1 for later use.

* Because the annual payments and terminal value are treated as inflows (positive), the present value is automatically calculated as a negative number by the calculator. But since the total present value of the future cash inflows represents a positive value, the +/- key is pressed to convert the calculated negative value to a positive value.

Part 3. Calculate Excess Present Value Index. Divide the net present value by the outlay cost and calculate the excess present value index.

Keys:	Display:	Description:
[RCL] [PV] [÷] [RCL] 0 [=]	1.07	Divides present value of cash flows by outlay cost and calculates excess present value index.

Part 4. Calculate Payback. Divide the cost by the annual savings and compute payback.

Keys:	Display:	Description:
[RCL] 0 [PV]	PV=-20,000.00	Stores outlay cost in *PV*.
[÷] [RCL] [PMT] [+/-] [=]	5.71	Calculates payback in years.

Part 5. Calculate Equivalent Annuity Value. Calculate the equivalent annual payment that discounts to the net present value.

Keys:	Display:	Description:
[RCL] 1 [+/-] [PV]	PV=-1,385.65	Enters net present value in *PV*.
0 [FV]	FV=0.00	Clears *FV* register.
[PMT]	PMT=245.24	Calculates net present value's equivalent annuity value.

Capital Budgeting Model—Income Taxes Included

Including taxes in the capital budgeting analysis requires calculating after-tax cash flows. The method used in this example calculates the tax effect of the net cash inflows or savings before depreciation, and calculates the tax avoided because depreciation is deducted from income. This method assumes other taxable income is available to compensate for any losses generated.

Example. A company is considering the purchase of a new machine that should save the annual amounts shown in the table below. The machine being replaced has no market value and will be discarded. The new machine costs $30,000, has a salvage value of $6,000, and is depreciated over 5 years. The estimated terminal value for resale at the end of 5 years is projected to be $7,000. The company depreciates assets using the sum-of-the-years'-digits method. Taxes average 35%. The company requires an 11% annual earnings rate on this type of investment. Assume all cash flows occur at the end of each year.

The estimated annual savings are shown below along with the annual depreciation deductions, which have already been calculated.

			You Calculate These			
	Savings Before Tax	**Dep.**	**Savings After Tax**	**Dep. Shelter**	**Sale of Asset**	**After Tax CFs**
0						-$30,000
1	$9,000	$8,000	$5,850	$2,800		$8,650
2	9,100	6,400	5,915	2,240		8,155
3	9,300	4,800	6,045	1,680		7,725
4	9,700	3,200	6,305	1,120		7,425
5	9,900	1,600	6,435	560	$ 6,650	13,645

Part 1. Calculate *NPV* and *IRR*.

Step 1. Store the income tax and 1 minus the tax.

Keys:	**Display:**	**Description:**
1 [−]	`1.00-`	Enters 1 and prepares to subtract tax rate.
35 [%] [STO] 1	`-0.35`	Enters tax rate, converts to decimal percentage, and stores in register 1.
[=] [STO] 0	`0.65`	Subtracts tax from 1 and stores in register 0.

Step 2. Enter the *NPV* application. Calculate the net cash flow at the beginning of the analysis period (the net amount invested less any after-tax proceeds from the sale of old assets).

■ [NPV]	`I%?`*value*	Begins net present value procedure.
11 [INPUT]	`CL CFLO?  Y  N`	Enters required earnings rate. If a cash-flow list is stored, this prompt appears.
{Y}	`C0?`	Clears cash-flow list and displays prompt for initial cash flow.
30000 [+/−] [INPUT]	`C1?`	Enters outlay as negative value, and displays prompt for first cash flow.

Step 3. Calculate the net operating cash flows per period and store.

$$\textit{Net Operating Cash Flow per Period} = \textit{Net Cash Flows} \times (1 - \textit{Tax Rate}) + \textit{Non-Cash Flows} \times (\textit{Tax Rate})$$

9000 [×] [RCL] 0 [+]	5,850.00+	Enters first year's savings before tax and calculates after-tax amount.
[(] 8000 [×] [RCL] 1 [)]	+2,800.00	Enters first year's depreciation and calculates after-tax effect.
[=]	C1?8,650.00	Calculates first year's total after-tax cash flow.
[INPUT]	N1?1.00	Enters first year's after-tax cash flow. Prompts for number of repetitions.
[INPUT]	C2?	Enters the number of repetitions. Prompts for second year's cash flows.
9100 [×] [RCL] 0 [+] [(] 6400 [×] [RCL] 1 [INPUT] [INPUT]	C3?	Enters second year's cash flow and depreciation, and inputs number of repetitions. Prompts for third period's cash flows.
9300 [×] [RCL] 0 [+] [(] 4800 [×] [RCL] 1 [INPUT] [INPUT]	C4?	Enters third year's values. Prompts for fourth period's cash flows.
9700 [×] [RCL] 0 [+] [(] 3200 [×] [RCL] 1 [INPUT] [INPUT]	C5?	Enters fourth year's values. Prompts for fifth period's cash flows.
9900 [×] [RCL] 0 [+] [(] 1600 [×] [RCL] 1 [=] [STO] 2	C5?6,995.00	Stores in register 2 the sum of fifth period's after-tax savings and depreciation.

Step 4. Calculate the net cash flow from selling the asset at the end of the decision period.

Cash Flow From Disposal of Asset =

Terminal Value − (*Terminal Value* − *Salvage*) × (*Tax Rate*)

7000 [-] [(] [(] 7000 [-] 6000 [)] [×] [RCL] 1 [=]	C5?6,650.00	Enters terminal and salvage values and calculates after-tax cash flow from sale of asset.
[+] [RCL] 2 [=]	C5?13,645.00	Adds final period's after-tax cash flows.
[INPUT] [INPUT]	C6?	Enters last period's cash flows and number of repetitions.

Step 5. Calculate *NPV* and *IRR*.

■[COMPUTE]	NPV=3,048.75	Computes net present value.
■[IRR%] {N} ■[COMPUTE]	IRR%=14.73	Computes internal rate of return.
[RCL] [PV] [÷] 30000 [+] 1 [=]	1.10	Enters initial outlay and calculates excess present value index.

Part 2. Calculate the excess present value index and the equivalent annuity value.

Keys:	Display:	Description:
■[BEG/END] {END} 1 ■[P/YR]	P/YR=1.00	Selects End mode and enters 1 payment period per year.
5 [N]	N=5.00	Enters number of years.
11 [I/YR]	I/YR=11.00	Enters annual required interest rate.
0 [FV]	FV=0.00	Clears *FV* register.
[PMT]	PMT=-824.90	Calculates equivalent annuity value.

6

Time Value of Money Applications

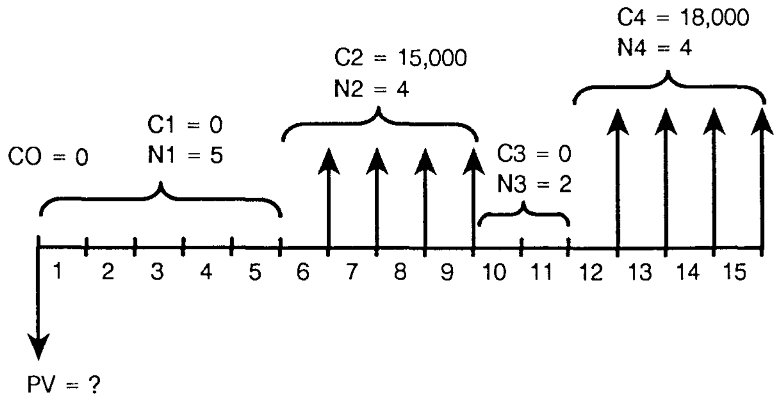
Deferred Cash Flows

Transactions often occur in which payments begin only after a specified number of periods. In other words, the payments are deferred. These payments may be of equal or unequal amounts and may occur in sequence or have skipped intervals. Using the HP-14B cash-flow feature, you can compute the present value, interest rate, and payment schedule for any of these situations. The basic procedure involves drawing a cash-flow diagram and then entering the cash flows into your HP-14B.

Example 1. You just won a lottery and want to establish a savings account that will pay for your children's college expenses. Your daughter will begin college at the end of 6 years, and your son will enter college at the end of 12 years. Your daughter's annual college costs will be $15,000 (with the first payment occurring at the end of year 6), and your son's annual college costs will be $18,000 (with the first payment occurring at the end of year 12).

Part 1. Assuming each child attends college for 4 years, how much should you invest today if you can earn 6% interest, compounded annually?

The cash-flow diagram looks like this:



Solve this problem by entering the cash-flow groups into the cash-flow list and then calculating the present value of the cash flows. As you work through this problem, notice how the cash flows and number of occurrences match the cash-flow diagram.

Keys:	Display:	Description:
■ [NPV]	I%?*value*	Begins net present value procedure.
6 [INPUT]	CL CFLO? Y N	Enters required earnings rate. If a cash-flow list is stored, this prompt asks if you want to clear it.
{Y}	C0?	Clears cash-flow list and displays prompt for initial cash flow.
0 [INPUT]	C1?	Enters 0 for initial cash flow. Prompts for next cash flow.
0 [INPUT] 5 [INPUT]	C2?	Enters amount and number of cash flows for years 1 through 5.
15000 [INPUT] 4 [INPUT]	C3?	Enters amount and number of cash flows for years 6 through 9.
0 [INPUT] 2 [INPUT]	C4?	Enters amount and number of cash flows for years 10 and 11.
18000 [INPUT] 4 [INPUT]	C5?	Enters amount and number of cash flows for years 12 through 15.
■ [COMPUTE]	NPV=71,696.67	Computes present value of deferred cash flows with a 6% annual rate.

Part 2. If you can earn 8% compounded annually, how much should you invest?

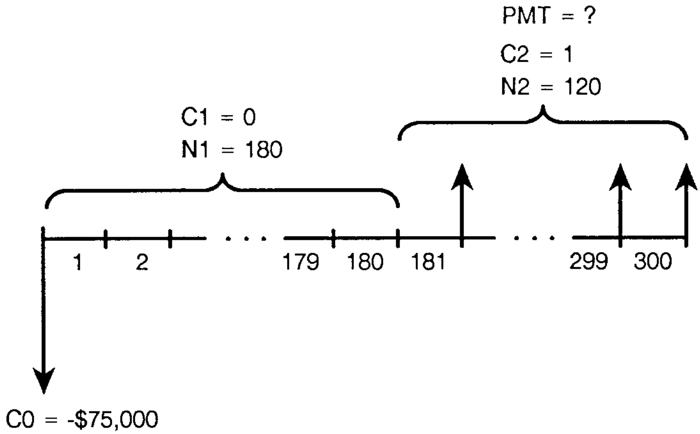
Keys:	Display:	Description:
■ [NPV]	I%?6.00	Begins net present value procedure with current interest rate displayed.
8 [INPUT]	CL CFLO? Y N	Enters new interest rate.
{N}	C0?0.00	Answers *no* to the prompt to erase cash-flow list.
■ [COMPUTE]	NPV=59,381.93	Computes present value of deferred cash flows assuming an 8% annual rate.

Part 3. If you invest $50,000, what interest rate is necessary to earn the required future cash withdrawals?

Keys:	Display:	Description:
■ [IRR%]	CL CFLO? Y N	Enters *IRR%* application.
{N}	C0?0.00	Answers *no* to the prompt to erase cash-flow list.
50000 [+/-] [INPUT]	C1?0.00	Enters amount invested as a negative number.
■ [COMPUTE]	IRR%=9.89	Computes interest rate required to withdraw scheduled amounts with a $50,000 investment.

Example 2. Your parents are setting up an investment plan for their retirement in 15 years. If they invest $75,000 today at 7% annual interest with monthly compounding, how much can they withdraw at the end of each month for 10 years once they retire? Ignore taxes in this example.

The cash-flow diagram is shown below.



Part 1. Solve this problem using factors as follows:

1. Calculate the present value of a deferred annuity with payments of $1.
2. Divide the amount invested by the present value calculated in step 1 to determine the monthly payment.

Keys:	**Display:**	**Description:**
■ [CLEAR] {ALL} {Y}	0.00	Clears the calculator memory.
■ [NPV]	I%?0.00	Begins net present value procedure.
7 [÷] 12 [=] [INPUT]	C0?	Divides annual interest rate by number of compounding periods per

		year to give interest rate per period.
0 [INPUT]	C1?	Enters 0 for initial cash flow and prompts for next cash flow.
0 [INPUT] 180 [INPUT]	C2?	Enters amount and number of cash flows for first 180 months.
1 [INPUT] 120 [INPUT]	C3?	Enters $1 and number of payments received.
■[COMPUTE]	NPV=30.23	Computes the present value of deferred $1 payments.
75000 [÷] [RCL] [PV] [=]	2,480.90	Enters the amount invested and calculates monthly payment.

Part 2. Solve the same problem using the TVM keys:

1. Calculate the amount of money your parents will have in the account when they retire in 15 years (*FV*).
2. Calculate the amount they can withdraw each month after retirement (*PMT*), depleting the account in 10 years.

Keys:	**Display:**	**Description:**
■[BEG/END] {END} 12 ■[P/YR]	P/YR=12.00	Sets End mode and 12 payments per year.
75000 [+/-] [PV]	PV=-75,000.00	Enters present value of investment.
7 [I/YR]	I/YR=7.00	Enters annual interest rate.
15 ■[xP/YR]	N=180.00	Enters total number of compounding periods.
0 [PMT]	PMT=0.00	Enters payment of 0.

[FV]	FV=213,671.00	Calculates future value of investment after 15 years.
[+/-] [PV]	PV=-213,671.00	Enters amount in account at end of 15 years as new present value.
0 [FV]	FV=0.00	Enters 0 as new future value (since they want no money left in the account after 10 years).
10 ■[xP/YR]	N=120.00	Enters number of withdrawals.
[PMT]	PMT=2,480.90	Calculates monthly payment your parents will receive for 10 years.

Nominal versus Effective Interest Rates

Annual interest rates are typically stated in the United States as nominal rates. The periodic interest rate per compounding period equals the annual rate divided by the number of compounding periods per year. For example, a mortgage with a 12% nominal annual rate and monthly payments has a periodic rate of 1% per month. However, the annual effective rate assumes 1 compounding period per year and is therefore higher.

Occasionally, discount rates or required investment rates are stated as annual effective rates, while the cash flows occur more than once a year. To compare investments with differing compounding periods, the annual effective interest rate must be converted to the equivalent annual nominal interest rate.

Example. Part 1. You are evaluating an investment that earns $800 at the end of each quarter for 6 years. Assuming you must earn a 10% annual effective interest rate, what is the present value of the cash flows?

Step 1. Convert the annual effective rate to the equivalent nominal rate with quarterly compounding.

Keys:	Display:	Description:
■ [BEG/END] {END} 4 ■ [P/YR]	P/YR=4.00	Sets End mode and 4 payment periods per year.
10 ■ [EFF%]	EFF%=10.00	Stores annual effective interest rate.
4 ■ [P]	P=4.00	Stores number of compounding periods per year for nominal rate.
■ [NOM%]	NOM%=9.65	Calculates annual nominal rate with quarterly compounding.

Step 2. Calculate the present value of the quarterly cash flows.

[I/YR]	I/YR=9.65	Stores annual nominal rate.
6 ■[xP/YR]	N=24.00	Enters years and calculates number of quarterly payments.
800 [PMT]	PMT=800.00	Stores quarterly cash flow.
0 [FV]	FV=0.00	Clears *FV* register.
[PV]	PV=-14,449.09	Calculates present value.

Part 2. If you can purchase the investment for $13,000, what is the yield expressed as an annual effective interest rate? Solve this by calculating the yield with quarterly compounding and converting it to the equivalent annual effective interest rate.

Keys:	**Display:**	**Description:**
13000 [+/-] [PV]	PV=-13,000.00	Stores purchase price of investment.
[I/YR]	I/YR=13.55	Calculates *IRR%* as a nominal rate with quarterly compounding.
■[NOM%] 4 ■[P] ■[EFF%]	EFF%=14.25	Enters nominal rate and number of compounding periods per year, and calculates annual effective rate of return.

Lease With Irregular Skipped Payments

Some leases require "creative" payment schedules. This model shows you how to calculate the present value of leases with uneven payments, including situations in which some periodic payments are skipped. The example in this section uses the *NPV* application in the HP-14B.

Example. An agent is leasing a building to the ABC Company. The company wants several rent-free periods. The lease term is 36 months, and payments are made at the beginning of each month. Assume a 10% discount rate with monthly compounding and calculate the present value of the following lease schedule:

Payment Schedule (Months)	Rent Per Month ($)
6	0
6	2,000
4	0
8	2,500
2	0
10	3,000

Because the payments occur at the beginning of each period, the first payment occurs at time 0 and is entered for C0, which in this case is $0. The amount entered for C1 is $0 for 5 periods, with $2,000 entered for C2 for 6 periods. Once the initial and first period's cash flows are placed, the remaining cash flows are entered as shown in the schedule above.

Keys:	Display:	Description:
■ [NPV]	I%? *value*	Begins net present value procedure.
10 [÷] 12 [=]	I%?0.83	Divides annual interest rate by number of payments per year and calculates monthly interest rate.

[INPUT]	CL CFLO? Y N	If previous cash flows are present, this prompt is displayed.
{Y}	C0?	Clears old list and prompts for initial cash flow.
0 [INPUT]	C1?	Enters initial cash flow and prompts for first cash flow.
0 [INPUT]	N1?1.00	Enters first cash-flow group amount. Prompts for number of repetitions as next input.
5 [INPUT]	C2?	Enters number of repetitions and displays prompt for next cash flow.
2000 [INPUT] 6 [INPUT]	C3?	Enters second cash flow and number of repetitions.
0 [INPUT] 4 [INPUT]	C4?	Enters remaining cash flows.
2500 [INPUT] 8 [INPUT]	C5?	
0 [INPUT] 2 [INPUT]	C6?	
3000 [INPUT] 10 [INPUT]	C7?	
■ [COMPUTE]	NPV=51,496.67	Calculates present value of lease payment schedule.

7

Marketing

Price To Achieve Target Return on Sale

A common pricing technique involves asking and answering the following question: What sales price will earn a particular return on sales? The equation for calculating the selling price per unit with income expressed as a percentage of sales is a modification of the break-even equation that is built into the HP-14B:

$$PRC \times \#SLD = (VARC \times \#SLD) + FIXC + TR \times (PRC \times \#SLD)$$

$$PRC = \frac{(VARC \times \#SLD) + FIXC}{\#SLD \times (1 - TR)}$$

where:

- *TR* is the percentage target return expressed as a decimal.
- *PRC* is the selling price per unit.
- *VARC* is the variable cost per unit.
- *FIXC* is the fixed cost.
- *#SLD* is the number of units produced and sold.

Example. Part 1. You think you can sell 2,000 units of a product that has a variable cost per unit of $8. The total fixed costs are $5,000, and your target return is 25% of sales. What selling price is required to meet your target return?

Keys:	Display:	Description:
■ [B.EVEN]	#SLD?*value*	Enters *B.EVEN* application. The first value requested is number sold.
2000 [INPUT]	PRC?*value*	Enters number of units sold and moves to next value in list, which is price.

[▼]	FIXC?*value*	Moves past price to fixed costs.
5000 [INPUT]	VARC?*value*	Enters fixed costs and moves to next value in list, which is variable cost.
8 [INPUT]	PROF?*value*	Enters variable cost per unit. The next value requested is profit.
0 [INPUT]	#SLD?2,000.00	Enters 0 for profit per unit.
[▼]	PRC?*value*	Moves to price prompt.
■[COMPUTE]	PRC=10.50	Computes price at break-even point.

Divide the price calculated in the previous step by 1 minus the target return rate to determine the selling price.

[PRC]	PRC=10.50	Stores break-even price for later use.
[÷] [(] 1 [-] 25 [%] [=]	14.00	Calculates required sales price.

Part 2. What selling price is required if the target return is 20%?

Keys:	**Display:**	**Description:**
[RCL] [PRC] [÷]	10.50÷	Recalls break-even price and prepares it to be divided by one minus new target.
[(] 1 [-] 20 [%] [=]	13.13	Enters new target return percentage rate and calculates sales price.

Price Elasticity of Demand

Elasticity of demand is a measure of how sensitive the market demand is for a product relative to price changes in the product. If a small price change results in a large change in demand, the demand is said to be highly elastic. The formula used to calculate the price elasticity of demand follows:*

$$E = \left(\frac{OQ - NQ}{OQ}\right) \times \left(\frac{OP}{OP - NP}\right)$$

where:

- *E* is the price elasticity of demand.
- *OQ* is the old quantity demanded.
- *NQ* is the new quantity demanded.
- *OP* is the old price.
- *NP* is the new price.

When the quantity increases as a result of a decrease in price, the elasticity is a negative number. However, it is normal to show it as a positive value. Therefore, in the example that follows you'll change the sign of the elasticity accordingly.

Example. The price of a product was lowered from $150 to $100. Sales increased from 11,000 units to 15,000 units. Assuming that the price change was the only factor affecting sales, calculate the estimated elasticity of demand.

Solve this problem in three steps:

1. Use the Percent Change application to calculate the change in price and store results.

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* Eric N. Berkowitz, Roger A. Kerin, and William Rudelius, *Marketing* (St. Louis: Times Mirrow/Mosby College Publishing, 1986), p. 297.

2. Use the Percent Change application to calculate the change in quantity.
3. Divide the percentage change in quantity by the percentage change in price.

Keys:	**Display:**	**Description:**
150 ■ [OLD]	OLD=150.00	Enters old price.
100 ■ [NEW]	NEW=100.00	Enters new price.
■ [%CHG] [STO] 0	%CHG=-33.33	Calculates percentage change in price. Stores in register 0 for later use.
11000 ■ [OLD]	OLD=11,000.00	Enters old quantity sold.
15000 ■ [NEW]	NEW=15,000.00	Enters new quantity sold.
■ [%CHG]	%CHG=36.36	Calculates percentage change in quantity.
[÷] [RCL] 0 [=]	-1.09	Calculates elasticity by dividing percentage change in quantity by percentage change in price (stored in register 0).
[+/-]	1.09	Changes the sign to show elasticity as a positive value.

Calculating Markup and Discount

Prices are often set using a markup on cost, while discounts are usually a percentage of price. You can use the Percent Change and Margin applications for markup and discount problems.

Example. A merchant pays $25.00 for a radio and then marks it up 40% on his cost to set the selling price. Later, he discounts the radio 15% from the selling price. What was the cost to the customer after the markup and then after the discount?

Solve this problem using the Percent Change application for computing any of the values based on markups, and using the Margin application for computing any values based on discounts.

Keys:	Display:	Description:
25 ■ [OLD]	OLD=25.00	Enters dollar amount of cost.
40 ■ [%CHG]	%CHG=40.00	Enters markup as a percentage.
■ [NEW]	NEW=35.00	Calculates selling price (cost to customer) with markup.
15 [MAR]	MAR=15.00	Enters discount as a percentage of sales price.
[CST]	CST=29.75	Calculates selling price (cost to customer) after discount.

Simple Moving Averages

Moving averages are often useful in forecasting sales. In a moving average, a specified number of data points are averaged. When there is a new piece of input data, the oldest piece of data is discarded to make room for the most recent data. This replacement scheme makes the moving average a valuable tool in following trends. The fewer the number of data points, the more sensitive the averages become. With a large number of data points, the average responds more slowly to new data.

Example. A company wants to calculate a 3 month moving average for the units sold each month. Volumes sold for the first 5 months are listed below:

January	5,000
February	6,100
March	5,500
April	6,500
May	7,000

Solve this problem using the Statistics application, following these steps:

1. Clear the statistics registers by pressing ■[CLEAR] {Σ}.
2. Enter the first set of values by pressing [Σ+].
3. Calculate the average.
4. Enter the oldest (first) value entered in step 2 and press [Σ-] to remove it.
5. Enter the new data point and press [Σ+].
6. Calculate the average.
7. Repeat steps 4, 5, and 6 for the remaining data points.

Keys:	Display:	Description:
■[CLEAR] {Σ}	*value*	Clears statistics registers.
5000 [Σ+]	n=1.00	Enters first data point.
6100 [Σ+]	n=2.00	Enters second data point.
5500 [Σ+]	n=3.00	Enters third data point.
[STAT] {x̄,ȳ} {x̄}	x̄=5,533.33	Calculates moving average for first 3 months.
5000 ■[Σ-]	n=2.00	Enters oldest data point (month 1) and removes it from list.
6500 [Σ+]	n=3.00	Enters next data point, month 4.
[STAT] {x̄,ȳ} {x̄}	x̄=6,033.33	Calculates moving average for months 2, 3, and 4.
6100 ■[Σ-]	n=2.00	Enters oldest data point (month 2) and removes it from list.
7000 [Σ+]	n=3.00	Enters next data point, month 5.
[STAT] {x̄,ȳ} {x̄}	x̄=6,333.33	Calculates moving average for months 3, 4, and 5.

Forecasting Revenue With Curve Fitting Models

The owner's manual for your HP-14B covers the basic trend line forecast model based on time and history. The curve fitting models can also be extended to other areas, such as estimating future revenues based on cumulative sales when you believe the price per unit is not constant.

Example. Part 1. You are an analyst trying to estimate future revenue in an industry that typically sees prices drop drastically as production volume increases. The quarterly cumulative sales volume in units and dollars is shown below for the ABC Company.

Qtr	Cumulative Units	Total Cumulative Revenue ($K)
1	100	208
2	200	245
3	400	523
4	600	517
5	1,400	1,213
6	2,000	1,169
7	3,500	2,182
8	5,000	2,200
9	7,000	3,795

What is the best curve fit model for this data?

Keys:	Display:	Description:
■ [CLEAR] {Σ}	*value*	Clears statistics registers.
100 [INPUT]	100.00:	Enters cumulative units for quarter 1.
208 [Σ+]	n=1.00	Enters cumulative sales for quarter 1.
200 [INPUT] 245 [Σ+]	n=2.00	Enters quarter 2 data points.

400 [INPUT] 523 [Σ+]	n=3.00	Enters remaining data.
600 [INPUT] 517 [Σ+]	n=4.00	
1400 [INPUT] 1213 [Σ+]	n=5.00	
2000 [INPUT] 1169 [Σ+]	n=6.00	
3500 [INPUT] 2182 [Σ+]	n=7.00	
5000 [INPUT] 2200 [Σ+]	n=8.00	
7000 [INPUT] 3795 [Σ+]	n=9.00	
■[FRCST]	MODEL=POWER	Enters Forecast application. The model fitted is briefly displayed. For this data it is a power curve.
	x̂ ŷ	Prompts for prediction of *x* or *y* value.

Part 2. Forecast sales for 8,000 cumulative units.

{ŷ} 8000 [INPUT] [STO] 0	ŷ=3,522.34	Enters cumulative number of units sold and forecasts total sales. Stores in register 0 for later use.

Part 3. Forecast sales for 9,000 cumulative units.

[▼] 9000 [INPUT]	ŷ=3,815.51	Enters new *x*-value and calculates new forecast.

Part 4. Determine the marginal revenue.

[-] [RCL] 0 [=]	293.17	Calculates marginal revenue for sales from 8,000 to 9,000 units.

Part 5. Calculate the correlation coefficient.

[STAT] {FIT} {E} {r}	r=0.99	Calculates correlation coefficient.

Step-by-Step Solutions for Your HP-14B Calculator

Business Student Applications contains a variety of applications, equations, and keystrokes to provide solutions for business students.

- **Accounting**
  Cost Allocations Based on Percentages • Imputed Interest Rates on Long-Term Non-Interest-Bearing Notes • Expected Value of Investment • Cost-Volume-Profit Analysis (CVP) • Depreciation • Sinking Funds • Estimating Overhead • Valuation of Liabilities and Amortization of Interest Expense • Capital Lease Amortization • Bond Discount/Premium Amortization
- **Financial Analysis**
  Common-Size Statement Analysis • Budget Variances • Profitability Ratios • Return on Investment
- **Finance**
  Compound Interest and Annuity Tables • Bonds Sold on a Coupon Date • Bond Duration • Yield on Investments With Varying Return Rates • Stock Valuation Models • Capital Asset Pricing Model
- **Statistics**
  Permutations • Combinations • Learning Curve • Standard Error
- **Capital Budgeting**
  Capital Budgeting Model – No Income Taxes • Capital Budgeting Model – Income Taxes Included
- **Time Value of Money Applications**
  Deferred Cash Flows • Nominal versus Effective Interest Rates • Lease With Irregular Skipped Payments
- **Marketing**
  Price To Achieve Target Return on Sale • Price Elasticity of Demand • Calculating Markup and Discount • Simple Moving Averages • Forecasting Revenue With Curve Fitting Models

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