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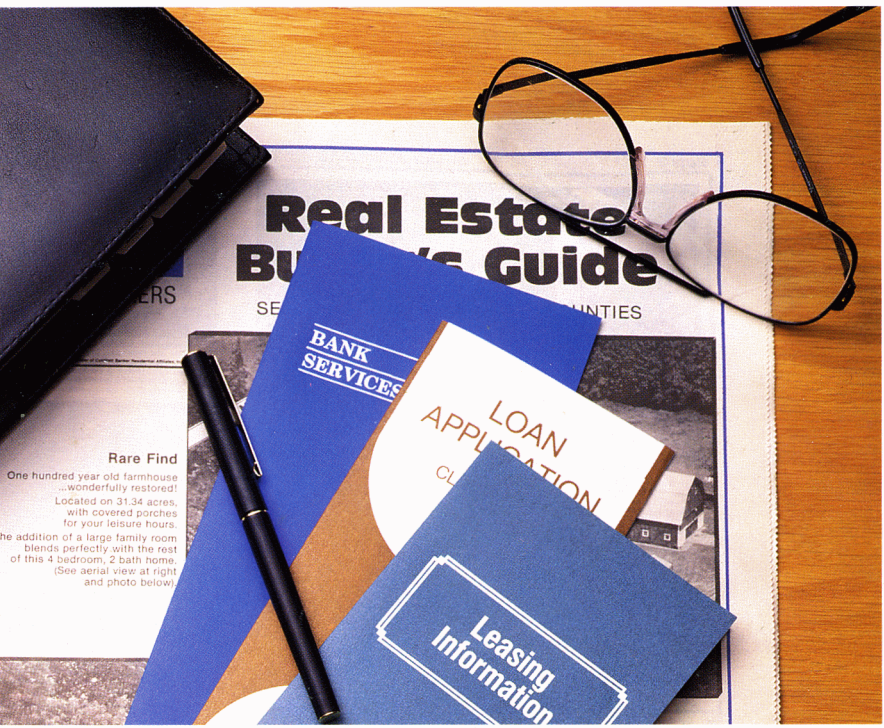
Step-by-Step Solutions
For Your HP Calculator

Real Estate, Banking, and Leasing



HP-17B
HP-19B
HP-27S

HEWLETT
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1000 N.E. Circle Blvd.
Corvallis, OR 97330, U.S.A.

Real Estate, Banking, and Leasing

Step-by-Step Solutions for Your HP-17B, HP-19B, or HP-27S Calculator

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Portable Computer Division
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Contents

How To Use This Book

The *Real Estate, Banking, and Leasing* book provides sets of keystrokes and routines to assist you in making decisions related to time and money. These routines can be used by real estate agents, investors, appraisers, loan officers, analysts, and others who make financial decisions. This book is designed to show you how your HP business calculator can help in these areas.

Before you use the solutions in this book, you should be familiar with the following concepts from the owner's manual:

- The basics of your calculator – how to do arithmetic calculations, move from menu to menu, and use the menu keys to do calculations.
- The Time Value of Money (TVM) menu and the cash-flow sign convention (cash paid out is entered as a negative number and cash received is entered as a positive number).
- How to enter cash flows in a cash-flow list. (This function is not available on the HP-27S.)
- How to enter and use equations in the Solver.

Keys and Menu Selection

A key on the calculator keyboard is represented like this: [EXIT]. A *shifted* function appears with a shift key, like this: ■ [CLEAR DATA]. A menu label is represented like this: [PV] (found in the TVM menu). The arrow keys are represented by [↑] and [↓].

This book can be used with the HP-17B, HP-19B, and HP-27S calculators. Generally, the same keystrokes are used on all three calculators to perform the specific application. However, there are some differences, which are summarized in the following table. Note that the interest conversion and cash-flow menus are not available on the HP-27S.

Keystroke Differences

HP-17B	HP-19B	HP-27S
To display the time value of money (TVM) menu:		
FIN TVM	FIN TVM	■ TVM
To store a Solver equation and its menu:		
SOLVE NEW type equation INPUT CALC	SOLVE type equation CALC	■ SOLVE NEW type equation INPUT CALC
To edit a Solver equation:		
EDIT edit equation INPUT CALC	EDIT edit equation CALC	EDIT edit equation INPUT CALC
To display the cash-flow (CFLO) menu:		
CFLO	CFLO	Not available
To display the interest conversion (ICONV) menu:		
ICNV	ICONV	Not available

Display Formats

The examples in this book show numbers displayed to two decimal places. If your display setting is otherwise, the answers in your display will not match exactly what is in this book. Refer to your owner's manual for more information about changing the number of decimal places in the display.

Entering Equations

When entering equations into your HP calculator, follow the instructions in the Solver chapter of your owner's manual. Here are hints to help you in common error situations:

1. If the calculator displays INVALID EQUATION when you press ≡ CALC ≡, the calculator does not understand something in the equation. When the equation returns to the display, the cursor blinks where the calculator detected the error. Check the equation in the display against the equation in the book. Make sure the parentheses match and that the operators are where they should be.
2. If the calculator accepts the equation but your answer does not match the example, check the values stored in the variables by recalling them (press RCL, then the menu key). If the values are correct, return to the SOLVE menu and check the equation. (Press EXIT to return to the SOLVE menu and press ≡ EDIT ≡ to view and edit the equation.) Check the equation against the one in this book for accuracy. When you find an error, edit the equation.
3. If the calculator displays INSUFFICIENT MEMORY when you press INPUT or ≡ CALC ≡, you must clear portions of memory. Refer to your owner's manual for additional information.

1

Real Estate

Basic Mortgage Components

Many of the techniques illustrated in this book require that you know the five basic components of a mortgage: *N, I%YR, PV, PMT,* and *FV*. For a particular problem, some of these values may not be known. However, if any four elements are known, the remaining unknown value can be calculated.

Storing Financial Data

Menu Label	Value Stored
N	Total number of payments.
I%YR	Annual interest rate as a percent.
PV	Initial loan balance.*
PMT	Periodic payment.*
FV	Future value or balloon payment.*

* Use the cash-flow sign convention (money paid out is negative, money received is positive).

In addition, the following settings are necessary for the payment frequency and mode.

Setting Frequency and Mode

Menu Label	Value Stored
P/YR	Number of payments per year.
END	Sets End mode.
BEG	Sets Begin mode.

Solving for Values

Unknown Value	Values Required to Solve
N	*I%YR, PV, PMT, FV,* P/YR, End or Begin*
I%YR	*N, PV, PMT, FV,* P/YR, End or Begin*
PV	*N, I%YR, PMT, FV,* P/YR, End or Begin*
PMT	*N, I%YR, PV, FV,* P/YR, End or Begin*
FV	*N, I%YR, PV, PMT, P/YR, End or Begin*

* *FV* is zero if there is no balloon payment.

Example 1: Calculating a Monthly Payment. You are considering a 30-year, $72,500 mortgage at 10.25% interest. What is the monthly payment amount?

Solution. The payment amount is the unknown value. The loan amount, total number of payments, and annual interest rate are known. (The future value is zero.)

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
30 × 12 N	N=360.00	Stores total number of payments.
72500 PV	PV=72,500.00	Stores mortgage amount.
10.25 I%YR	I%YR=10.25	Stores annual interest rate.

PMT	PMT = −649.67	Calculates monthly payment.

Part 2. What is your monthly payment assuming a 15-year mortgage?

15 × 12 N	N = 180.00	Stores total number of payments.
PMT	PMT = −790.21	Calculates monthly payment.

Example 2: Calculating a Maximum Purchase Price. The maximum monthly mortgage payment that you can afford is $750. Interest rates are currently 10.5%, and you can make a $7,000 down payment. With a 30-year mortgage, what is the maximum purchase price you can afford?

Solution. The loan amount is the unknown value. The monthly payment, annual interest rate, and total number of payments are known. (The future value is zero.)

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
750 +/− PMT	PMT = −750.00	Stores monthly payment.
10.5 I%YR	I%YR = 10.50	Stores annual interest rate.
30 × 12 N	N = 360.00	Stores total number of payments.
PV	PV = 81,990.57	Calculates loan amount.
+ 7000 =	88,990.57	Calculates maximum purchase price.

Example 3: Calculating an Interest Rate. A property has an existing loan of $100,000 with monthly payments of $1,106.20 for 30 years. What is the annual interest rate of the loan?

Solution. The interest rate is the unknown value. The loan amount, remaining number of payments, and monthly payment are known. (The future value is zero.)

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
30 × 12 N	N=360.00	Stores total number of payments.
100000 PV	PV=100,000.00	Stores loan amount.
1106.20 +/− PMT	PMT=−1,106.20	Stores monthly payment. (Remember to use the sign convention.)
I%YR	I%YR=13.00	Calculates annual interest rate.

Example 4: Calculating a Balloon Payment. What is the balloon payment due at the end of year 10 for a $750,000 loan with monthly payments of $9,483.33 and a 15% annual interest rate?

Solution. The balloon payment is the future value.

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
10 × 12 N	N = 120.00	Stores total number of payments.
15 I%YR	I%YR = 15.00	Stores annual interest rate.
750000 PV	PV = 750,000.00	Stores loan amount.
9483.33 +/− PMT	PMT = −9,483.33	Stores payment amount.
FV	FV = −720,185.74	Calculates amount of balloon payment.*

Example 5: Calculating Payment and Balance. A broker lists a property that has an assumable loan. The original loan amount was $150,000 at 7% annual interest, fully amortized with monthly payments for 25 years. The loan originated 11 years and 8 months ago. What is the loan balance?

Solution. First, calculate the monthly payment, assuming full amortization in 25 years. Then use the four known values (*N*, *I%YR*, *PV*, and *PMT*) to calculate the amount of the balloon payment (*FV*) due after 11 years and 8 months (140 payments).

* The balloon payment amount occurs coincident with, but does not include, the last periodic payment amount.

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
25 × 12 N	N=300.00	Stores total number of payments.
7 I%YR	I%YR=7.00	Stores annual interest rate.
150000 PV	PV=150,000.00	Stores original loan amount.
PMT	PMT=−1,060.17	Calculates monthly payment amount.
11 × 12 + 8 N	N=140.00	Calculates and stores number of payments already made.
FV	FV=−110,080.32	Calculates current loan balance.

Example 6: Calculating a Quarterly Payment. Mr. Seller takes a $200,000 mortgage at 12% annual interest with quarterly payments and a $150,000 balloon payment due at the end of 5 years. What is the quarterly payment?

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER 4 P/YR END EXIT		Sets 4 payments per year; End mode.
5 × 4 N	N=20.00	Stores total number of payments.
12 I%YR	I%YR=12.00	Stores annual interest rate.

200000 PV	PV=200,000.00	Stores loan amount.
150000 +/- FV	FV=-150,000.00	Stores balloon payment amount.
PMT	PMT=-7,860.79	Calculates quarterly payment amount.

Example 7: Calculating a Loan Term. Your 30-year, $65,000 mortgage at 10.25% interest has a monthly payment of $582.47. If you increase your payment by $75 a month, how long will it take to pay off the loan?

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
10.25 I%YR	I%YR=10.25	Stores annual interest rate.
65000 PV	PV=65,000.00	Stores loan amount.
582.47 + 75 = +/- PMT	PMT=-657.47	Stores new payment amount.
N	N=218.79	Calculates total number of payments.
÷ 12 =	18.23	The loan is paid off in the 19th year.

Example 8: Calculating a Balance. A loan at 15% annual interest, with monthly payments of $1,283.62, has a balloon payment of $100,000 due at the end of year 10. What is the remaining balance if the loan is paid in full at the end of the sixth year?

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
10 − 6 = × 12 N	N=48.00	Stores total number of remaining payments.
15 I%YR	I%YR=15.00	Stores annual interest rate.
1283.62 +/− PMT	PMT=−1,283.62	Stores monthly payment amount.
100000 +/− FV	FV=−100,000.00	Stores amount of balloon payment.
PV	PV=101,208.02	Calculates loan balance at the end of year 6.

Example 9: Calculating a Current Value. You purchased your home 10 years ago for $47,500. If homes in your area have appreciated 4% per year, how much is your home currently worth? (Use *PV* for the past value and *FV* for the current value.)

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER 1 P/YR END EXIT		Sets 1 compounding period per year; End mode.
10 N	N=10.00	Stores total number of years.
47500 +/− PV	PV=−47,500.00	Stores original purchase price.
4 I%YR	I%YR=4.00	Stores annual appreciation rate.
FV	FV=70,311.60	Calculates home value.

Example 10: Calculating an Appreciation Rate. You have accepted an offer to sell your home for $76,900. You originally purchased the house 6 years ago for $48,500. What is the annual rate of appreciation? (Use *PV* for the past value and *FV* for the present value.)

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER 1 P/YR END EXIT		Sets 1 compounding period per year; End mode.
76900 FV	FV=76,900.00	Stores selling price.
6 N	N=6.00	Stores number of years.
48500 +/- PV	PV=−48,500	Stores purchase price.
I%YR	I%YR=7.99	Calculates annual appreciation rate.

Homeowner's Monthly Payment Estimator

When comparison shopping for a mortgage, it is often useful to estimate the monthly payment. This equation calculates the approximate mortgage payment for the first month given the purchase price, down payment, interest rate, term of the loan, annual property taxes, and annual homeowner's insurance. (Subsequent monthly payments would be different.) The equation also assumes that mortgage interest and property taxes are allowable deductions on your income tax, so your income tax rate can be stored and the after-tax monthly payment calculated.

The calculation assumes that the assessed value is 100% of the sale price and does not include financing of the closing costs.

Entering and Using the MOPMT Equation:

1. Enter the MOPMT equation into the Solver.

```
MOPMT:IF(S(EPMT):(PRICE-DOWN)÷
USPV(I%YR÷12:N)+(PTAX+INS)÷12-EPMT:
EPMT-((PRICE-DOWN)×I%YR÷1200
+PTAX÷12)×%TAX÷100-ATAX)
```

2. Display the MOPMT equation's menu.
3. Store values in the following variables:
   - Purchase price in ≣ PRICE ≣.
   - Down payment in ≣ DOWN ≣.
   - Annual mortgage interest rate in ≣ I%YR ≣.
   - Total number of monthly payments in ≣ N ≣.
   - Amount of annual property taxes in ≣ PTAX ≣.
   - Annual homeowner's insurance in ≣ INS ≣.
   - Income tax rate as a percentage in ≣ %TAX ≣.
4. Press ≣ EPMT ≣ to calculate the estimated monthly payment, including principal and interest, property taxes, and insurance.
5. Press ≣ ATAX ≣ to calculate the after-tax monthly payment.

Example: Part 1. You are considering a $65,000 house in a neighborhood with a $25 per thousand tax rate. Homeowner's insurance would cost $250 per year. If you put $6,500 down, and receive a 10.75% loan for 30 years, what would be your monthly payment?

Display the MOPMT equation's menu.

Keys:	Display:	Description:
65000 PRICE	PRICE=65,000.00	Stores known values.
6500 DOWN	DOWN=6,500.00	
10.75 I%YR	I%YR=10.75	
30 × 12 N	N=360.00	
65 × 25 PTAX	PTAX=1,625.00	
MORE		
250 INS	INSUR=250.00	
EPMT	EPMT=702.34	Calculates estimated monthly payment.

Part 2. If your tax rate is 20%, what is your actual monthly payment?

20 %TAX	%TAX=20.00	Stores income tax rate.
ATAX	ATAX=570.44	Calculates after-tax monthly payment.

Blended Interest Rate With Two Mortgages

Note

If the mortgages do not have the same term, this procedure cannot be used on the HP-27S.

When two mortgages at different interest rates exist on a piece of property, you may wish to find the interest rate on the entire transaction, or the *blended* interest rate. To calculate the blended rate, you must know the amount and term of each mortgage, and either the interest rate of each mortgage, or the periodic payment amount. Remember to use the cash-flow sign convention (money paid out is negative, money received is positive).

1. Display the TVM menu.
2. Clear the TVM variables, store the number of payments per year in ≣ P/YR ≣, and set the payment mode (Begin or End).
3. Store the total number of payments in the first mortgage in ≣ N ≣.
4. Store the annual interest rate in ≣ I%YR ≣.
5. Store the mortgage amount in ≣ PV ≣.
6. Press ≣ PMT ≣ to calculate the payment on the first mortgage.
7. Repeat steps 3-6 using information for the second mortgage to determine the payment amount of the second mortgage.
8. Add the mortgages together and calculate the total periodic payments.
9. If the mortgages have the same term (*N*), store the combined values from step 8 in the TVM menu (in *N*, *PV*, and *PMT*) and calculate *I%YR*.
10. If the mortgages do not have the same term, use the CFLO menu and calculate the *IRR%*.

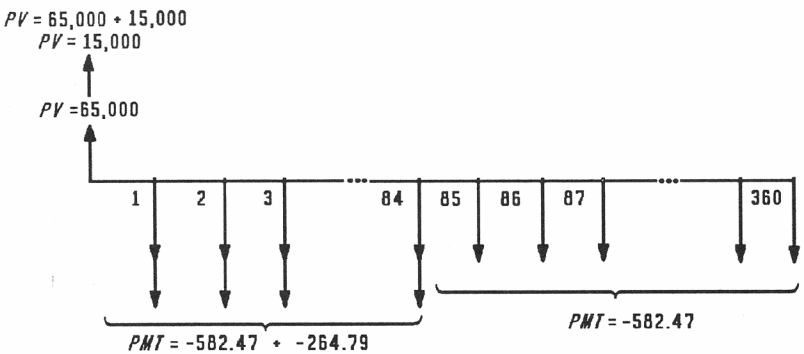
Example 1: Mortgages With the Same Term. You are assuming an existing 9%, $48,000 mortgage with a remaining term of 20 years. You also need to obtain a $20,000 second mortgage at 11.5%, for 20 years. What is the blended interest rate?

Keys:	Display:	Description:
[TVM]		Displays TVM menu.
■ [CLEAR DATA]		Clears TVM variables.
[OTHER] ■ [CLEAR DATA] [EXIT]		Sets 12 payments per year; End mode.
20 [×] 12 [N] 9 [I%YR] 48000 [PV]	N=240.00 I%YR=9.00 PV=48,000.00	Stores known values on first mortgage.
[PMT]	PMT=−431.87	Calculates monthly payment on first mortgage.
[STO] 0	PMT=−431.87	Stores payment in register 0.
11.5 [I%YR] 20000 [PV]	I%YR=11.50 PV=20,000.00	Stores known values on second mortgage.
[PMT]	PMT=−213.29	Calculates monthly payment on second mortgage.
[+] [RCL] 0 [PMT]	PMT=−645.15*	Calculates and stores total monthly payment.
[RCL] [PV] [+] 48000 [PV]	PV=68,000.00	Calculates and stores total debt.
[I%YR]	I%YR=9.75	Calculates blended interest rate.

* If the payment amounts are rounded to two decimal places, the value for *PMT* is -645.16.

Example 2: Mortgages With Unequal Terms. You are purchasing a home and have obtained a 30-year, 10.25%, $65,000 mortgage. The seller has agreed to carry a 12%, $15,000 second mortgage for 7 years. What is the blended rate of the two mortgages?

The cash-flow diagram looks like this:



Keys:	Display:	Description:
[TVM]		Displays TVM menu.
■ [CLEAR DATA]		Clears TVM variables.
[OTHER] ■ [CLEAR DATA] [EXIT]		Sets 12 payments per year; End mode.
30 [×] 12 [N] 10.25 [I%YR] 65000 [PV]	N=360.00 I%YR=10.25 PV=65,000.00	Stores known values on first mortgage.
[PMT] [STO] 0	PMT=−582.47	Calculates and stores payment on first mortgage.
7 [×] 12 [N] 12 [I%YR] 15000 [PV]	N=84.00 I%YR=12.00 PV=15,000.00	Stores known values on second mortgage.

PMT STO 1	PMT = −264.79	Calculates and stores payment on second mortgage.
EXIT CFLO		Displays CFLO menu.
■ CLEAR DATA YES *or* GET *NEW	FLOW(0) = ?*	Clears current list or gets a new one.
65000 + 15000 INPUT	FLOW(1) = ?*	Stores total of loans as initial cash flow.
RCL 0 + RCL 1 INPUT 7 × 12 INPUT	FLOW(2) = ?	Adds payments and stores result as first cash flow; this payment occurs for 7 years.
RCL 0 INPUT 30 − 7 × 12 INPUT	FLOW(3) = ?	Stores payment for the first mortgage for the remainder of the term.

The next step (pressing EXIT) is necessary if you have the HP-17B.
EXIT

CALC		Displays CFLO CALC menu.
IRR%	IRR% = 0.87	Calculates monthly interest rate.
× 12 =	10.39	Calculates annual interest rate.

* On the HP-19B, these prompts are: INIT = and FLOW(1) =.

Periodic Payment When Final Payment Is Known

The TVM menu assumes that the balloon payment amount is in addition to any periodic payment. If your convention is to state a final payment that is the sum of the last periodic payment and the balloon payment, use the following equation. Monthly payments are assumed.

Remember to use the cash-flow sign convention (money paid out is negative, money received is positive).

Entering and Using the FIPMT Equation:

1. Enter the FIPMT equation into the Solver.

   ```
   FIPMT: (N+I%YR)×0=PV+PMT×USPV(I%YR÷12:
   N-1)+FPMT×SPPV(I%YR÷12:N)
   ```

2. Display the FIPMT equation's menu.
3. Store or calculate the following variables:
   - The number of the period where the final payment occurs in [N].
   - The annual interest rate in [I%YR].
   - The loan amount in [PV].
   - The periodic payment amount in [PMT].
   - The amount of the final payment in [FPMT].

Example. A $10,000, 12% loan has 35 monthly payments plus a $5,000 balloon payment at the end of the 36th month. What is the monthly payment amount?

Display the FIPMT equation's menu.

Keys:	Display:	Description:
36 [N]	N=36.00	Stores known values.
12 [I%YR]	I%YR=12.00	
10000 [PV]	PV=10,000.00	

5000 [+/−] [FPMT]	FPMT = −5,000.00	
[PMT]	PMT = −221.21	Calculates monthly payment.

Price of a Discounted Mortgage

Mortgages can be bought or sold at prices lower (*discounted*) or higher (*at a premium*) than the remaining balance of the loan. You can calculate the price to pay for a mortgage is you know the amount of the mortgage, the periodic payment, the timing and amount of the balloon payment, and the desired yield rate. The price to pay for an existing mortgage is the present value of the remaining periodic payments and the balloon payment, discounted at the investor's required yield.

Remember to use the cash-flow sign convention (money paid out is negative, money received is positive).

1. Display the TVM menu.
2. Clear the TVM variables, store the number of payments per year in [P/YR], and set the payment mode (Begin or End).
3. Store the total number of payments in [N].
4. Store the desired annual yield (interest rate) in [I%YR].
5. Store the periodic payment amount in [PMT]. If you do not know the payment amount, it must be calculated using the actual interest rate stated in the loan. See examples 2 and 3 in this section.
6. If a balloon payment exists, store the amount in [FV]. The balloon payment occurs coincident with, and does not include, the last periodic payment. If you do not know the amount of the balloon payment, it must be calculated. Refer to the example in the owner's manual.
7. Press [PV] to calculate the purchase price of the mortgage.

Example 1: Loan With Balloon. A lender wishes to induce the borrower to prepay a low interest-rate loan. There are 72 payments remaining of $137.17 and a balloon payment at the end of the sixth year of $2,000. If the lender is willing to discount the future payments at 9%, how much would he accept as full repayment of the loan?

Keys:	Display:	Description:
[TVM]		Displays TVM menu.
■ [CLEAR DATA]		Clears TVM variables.
[OTHER] ■ [CLEAR DATA] [EXIT]		Sets 12 payments per year; End mode.
72 [N]	N=72.00	Stores number of payments.
9 [I%YR]	I%YR=9.00	Stores discount rate.
137.17 [PMT]	PMT=137.17	Stores monthly payment.
2000 [FV]	FV=2,000.00	Stores balloon payment amount.
[PV]	PV=−8,777.61	Calculates amount necessary to prepay the loan.

Example 2: Mortgage With Balloon. A 9.5% mortgage with 26 years remaining and a balance of $49,350 is available for purchase. Determine the price to pay for this mortgage if the desired yield is 12%. (Since the payment amount is not given, it must be calculated.)

Keys:	Display:	Description:
[TVM]		Displays TVM menu.
■ [CLEAR DATA]		Clears TVM variables.
[OTHER] ■ [CLEAR DATA] [EXIT]		Sets 12 payments per year; End mode.
26 [×] 12 [N]	N=312.00	Stores number of payments.
9.5 [I%YR]	I%YR=9.50	Stores annual interest rate.
49350 [+/−] [PV]	PV=−49,350.00	Stores current mortgage balance.
[PMT]	PMT=427.17	Calculates monthly payment to be received.

12 I%YR	I%YR = 12.00	Stores desired yield.
PV	PV = −40,801.57	Calculates purchase price to achieve the desired yield.

Example 3: Interest-Only Loan. How much should you pay for a $10,000, 5-year, 10% interest-only loan with annual payments to yield 13%? (Because this is an interest-only loan, the balloon payment is the entire loan amount, or $10,000.)

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER 1 P/YR END EXIT		Sets 1 payment per year; End mode.
5 N	N = 5.00	Stores number of payments.
PMT 10000 × 10 %	PMT = 1,000.00	Calculates and stores annual payment.
10000 FV	FV = 10,000.00	Stores balloon.
13 I%YR	I%YR = 13.00	Stores desired yield.
PV	PV = −8,944.83	Calculates purchase price.

Yield of a Discounted Mortgage

The annual yield of a mortgage bought at a discount or premium can be calculated given the original mortgage amount, interest rate, periodic payment, balloon payment amount (if any) and the price paid for the mortgage.

Remember to use the cash-flow sign convention (money paid out is negative, money received is positive).

1. Display the TVM menu.
2. Clear the TVM variables, store the number of payments per year in ≣ P/YR ≣, and set the payment mode (Begin or End).
3. Store the total number of payments in ≣ N ≣.
4. Store the purchase price of the mortgage in ≣ PV ≣.
5. Store the periodic payment amount in ≣ PMT ≣. If you do not know the payment amount, it must be calculated. See examples 1 and 2 in this section.
6. If a balloon payment exists, store the amount in ≣ FV ≣. The balloon payment occurs coincident with, but does not include, the last periodic payment. If you do not know the amount of the balloon payment, it must be calculated. Refer to example 3.
7. Press ≣ I%YR ≣ to calculate the annual yield.

Example 1. A 10% mortgage has 142 monthly payments of $526.54. If the mortgage can be purchased for $38,900, what is the yield?

Keys:	Display:	Description:
≣ TVM ≣		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
≣ OTHER ≣ ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
142 ≣ N ≣	N = 142.00	Stores number of payments.

526.54 PMT	PMT=526.54	Stores monthly payment.
38900 +/- PV	PMT=-38,900.00	Stores purchase price.
I%YR	I%YR=12.52	Calculates percent annual yield.

Example 2: Yield of a Discounted Mortgage. An investor wishes to purchase a $100,000 mortgage taken out at 9% for 20 years. Since the mortgage was issued, 42 monthly payments have been made. What is the yield if the purchase price of the mortgage is $79,000? (Since the payment amount is not given, it must be calculated.)

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
20 × 12 N	N=240.00	Stores the number of payments.
9 I%YR	I%YR=9.00	Stores annual interest rate.
100000 +/- PV	PV=-100,000.00	Stores mortgage amount.
PMT	PMT=899.73	Calculates the monthly payment to be received.
RCL N	N=240.00	Recalls number of payments.
- 42 N	N=198.00	Calculates and stores number of payments remaining.
79000 +/- PV	PV=-79,000.00	Stores purchase price of mortgage.
I%YR	I%YR=11.65	Calculates percent annual yield.

Example 3: Mortgage With Balloon Payment. Using the information given in example 1, calculate the annual yield if the loan is to be paid in full at the end of the fifth year, that is, 5 years from when the mortgage was issued. (In this case, both the payment amount and the balloon payment amount must be calculated.)

Keys:	Display:	Description:
These steps calculate the payment amount. They are necessary only if you have not done example 2.		
[TVM]		Displays TVM menu.
■ [CLEAR DATA]		Clears TVM variables.
[OTHER] ■ [CLEAR DATA] [EXIT]		Sets 12 payments per year; End mode.
20 [×] 12 [N]	N=240.00	Stores the number of payments.
9 [I%YR]	I%YR=9.00	Restores annual interest rate.
100000 [+/−] [PV]	PV=−100,000.00	Restores mortgage amount.
[PMT]	PMT=899.73	Calculates the monthly payment to be received.
Now, calculate the balloon payment.		
5 [×] 12 [N]	N=60.00	Stores number of payments in 5 years.
[FV]	FV=88,707.05	Calculates balloon payment amount due in 5 years.
Finally, calculate the yield on the remaining payments.		
[RCL] [N] [−] 42 [N]	N=18.00	Stores number of remaining payments (until balloon).

79000 [+/−] [PV]	PV = −79,000.00	Stores price of mortgage.
[I%YR]	I%YR = 20.72	Calculates percent annual yield.

APR of a Loan With Fees

The *annual percentage rate*, APR, incorporates fees usually charged when a mortgage is issued, which raises the interest rate. The actual amount received by the borrower (PV) is reduced, while the periodic payments remain the same. You can calculate the annual percentage rate (APR) if you know the life or term of the mortgage, the interest rate, the mortgage amount, and the basis of the fee charged (how the fee is calculated).

Remember to use the cash flow sign convention (money paid out is negative, money received is positive).

1. Display the TVM menu.
2. Clear the TVM variables, store the number of payments per year in ≡ P/YR ≡, and set the payment mode (Begin or End).
3. Store the total number of payments in ≡ N ≡.
4. Store the payment amount in ≡ PMT ≡.
5. Store the balloon payment plus any prepayment penalties in ≡ FV ≡.
6. Subtract any origination fees from the loan amount and store the result (the net proceeds) in ≡ PV ≡.
7. Press ≡ I%YR ≡ to calculate the annual percentage rate.

Example 1. A borrower is charged two points for the issuance of a mortgage. (One point is equal to 1% of the mortgage amount.) If the mortgage amount is $60,000 for 30 years and the interest rate is 11.5%, with monthly payments, what APR is the borrower paying?

Keys:	Display:	Description:
≡ TVM ≡		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
≡ OTHER ≡ ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
30 × 12 ≡ N ≡	N=360.00	Stores number of payments.

11.5 I%YR	I%YR=11.50	Stores annual interest rate.
60000 PV	PV=60,000.00	Stores loan amount.
PMT	PMT=−594.17	Calculates monthly payment.
RCL PV − 2 % PV	PV=58,800.00	Stores actual amount received by borrower.
I%YR	I%YR=11.76	Calculates annual percentage rate.

Example 2. Using the information given in example 1, calculate the APR if the mortgage fee is stated as $150 instead of as a percentage.

Keys:	**Display:**	**Description:**

These steps are necessary if you have not done example 1.

TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
30 × 12 N	N=360.00	Stores number of payments.
11.5 I%YR	I%YR=11.50	Restores annual interest rate.
60000 PV	PV=60,000.00	Restores loan amount.
PMT	PMT=−594.17	Calculates monthly payment.

Now, adjust *PV* and calculate the new APR.

RCL PV − 150 PV	PV=59,850.00	Stores actual amount received.

I%YR	I%YR=11.53	Calculates APR.

Example 3. Using the information given in example 1 again, what is the APR if the mortgage fee is stated as 2 points plus $150?

Keys:	**Display:**	**Description:**

These steps are necessary if you have not done examples 1 or 2.

TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
30 × 12 N	N=360.00	Stores number of payments.

Now, continue with the problem.

11.5 I%YR	I%YR=11.50	Restores annual interest rate.
60000 PV	PV=60,000.00	Restores loan amount.
PMT	PMT=−594.17	Calculates monthly payment.
RCL PV − 2 % − 150 PV	PV=58,650.00	Stores actual amount borrowed.
I%YR	I%YR=11.80	Calculates APR.

Example 4. A 30-year, $50,000 loan at 15% interest has fees of 2 points plus $150. Assuming that monthly payments are made and that the loan is paid in full at the end of the seventh year, what is the APR?

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
30 × 12 N	N=360.00	Stores number of payments.
15 I%YR	I%YR=15.00	Restores annual interest rate.
50000 PV	PV=50,000.00	Restores loan amount.
PMT	PMT=−632.22	Calculates monthly payment.
7 × 12 N	N=84.00	Stores number of payments until balloon.
FV	FV=−48,937.43	Calculates balloon payment.
RCL PV − 2 % − 150 PV	PV=48,850.00	Stores actual amount borrowed.
I%YR	I%YR=15.54	Calculates APR.

Example 5. A $1,000,000, 10-year, 12% interest-only loan has an origination fee of 3 points. What is the yield to the lender? Assume that monthly payment are made. (The monthly payment amount must first be calculated. The balloon payment is the entire loan amount, $1,000,000.)

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
10 × 12 N	N=120.00	Stores number of payments.

1000000 × 12 % ÷	120,000.00÷	Calculates amount of annual interest.
12 PMT	PMT = 10,000.00	Calculates and stores monthly payment.
1000000 FV	FV = 1,000,000.00	Stores balloon payment.
− 3 % = +/− PV	PV = −970,000.00	Calculates and stores actual amount borrowed.
I%YR	I%YR = 12.53	Calculates APR.

Mortgage Measure Calculations

Debt Coverage Ratio (DCR)

The *debt coverage ratio* (*DCR*) is calculated by dividing the net operating income by the annual debt service. Lenders use the *DCR* to measure safety in loan underwriting, since it compares property income to the amount needed to meet the loan payments. Given constant net operating income, the *DCR* decreases as the annual debt service increases.

Entering and Using the DCR Equation:

1. Enter the *DCR* equation into the Solver.

   DCR=NOI÷ADS
2. Display the *DCR* equation's menu.
3. Store two of the following variables:
   - Debt coverage ratio in ≣ DCR ≣.
   - Net operating income in ≣ NOI ≣.
   - Annual debt service in ≣ ADS ≣.
4. Press the menu key of the unknown variable to calculate its value.

Example 1:. The *NOI* from a property is estimated to be $6,400 per year. There are two mortgages on the property. The first has monthly payments of $325 and the second has quarterly payments of $647. What is the *DCR*?

Display the *DCR* equation's menu.

Keys:	Display:	Description:
6400 ≣ NOI ≣	NOI=6,400.00	Stores NOI.
325 × 12 + (647 × 4 ≣ ADS ≣	ADS=6,488.00	Stores total annual debt service.
≣ DCR ≣	DCR=0.99	Calculates *DCR*.

Example 2. A lender specifies a minimum *DCR* of 1.15 for a property with a *NOI* of $100,000. What is the maximum monthly loan payment amount the lender will agree to receive from the property?

Keys:	Display:	Description:
100000 [NOI]	NOI = 100,000.00	Stores *NOI*.
1.15 [DCR]	DCR = 1.15	Stores *DCR*.
[ADS]	ADS = 86,956.52	Calculates *ADS*.
[÷] 12 [=]	7,246.38	Divides the *ADS* by 12 to calculate the monthly payment amount.

Example 3. A lender agrees to a 15% loan with monthly payments and a 30-year amortization. The minimum *DCR* is 1.25. What is the maximum loan amount for a property with a *NOI* of $150,000 per year?

Keys:	Display:	Description:
1.25 [DCR]	DCR = 1.25	Stores *DCR*.
150000 [NOI]	NOI = 150,000.00	Stores *NOI*.
[ADS]	ADS = 120,000.00	Calculates *ADS*.
[÷] 12 [=] [STO] 0	10,000.00	Calculates monthly payment and stores amount in register 0.
■ [MAIN] [FIN] [TVM]		Displays the TVM menu to calculate the loan amount.
■ [CLEAR DATA]		Clears TVM variables.
[OTHER] ■ [CLEAR DATA] [EXIT]		Sets 12 payments per year; End mode.
[RCL] 0 [PMT]	PMT = 10,000.00	Stores monthly payment amount.
15 [I%YR] 30 [×] 12 [N]	I%YR = 15.00 N = 360.00	Stores remaining values.
[PV]	PV = −790,861.42	Calculates loan amount.

Annual Loan Constant (ALC)

The annual loan constant, ALC (also called loan constant or annual debt constant) is defined as the ratio of the annual debt service (*ADS*) to the amount of the loan. Thus, ALC is the annual debt service for each $1.00 of the loan amount. Usually, ALC is expressed as a percentage, rounded to two decimal places (*ALC%*). *ALC%* is the annual debt service for each $100 of the loan amount.

Entering and Using the ALC% Equation:

1. Enter the *ALC%* equation into the Solver.

   `ALC%=ADS÷LOAN×100`

2. Display the *ALC%* equation's menu.
3. Store two of the following variables:
   - Annual loan constant in ≣ ALC% ≣.
   - Annual debt service in ≣ ADS ≣.
   - Loan amount in ≣ LOAN ≣.
4. Press the menu key of the unknown variable to calculate its value.

Example 1. A $250,000 loan has an annual debt service of $30,858.36. What is the *ALC%*?

Display the *ALC%* equation's menu.

Keys:	Display:	Description:
250000 ≣ LOAN ≣	LOAN=250,000.00	Stores known values.
30858.36 ≣ ADS ≣	ADS=30,858.36	
≣ ALC% ≣	ALC%=12.34	Calculates annual loan constant.

Example 2. A loan of $275,000 has an annual loan constant of 12.85%. What is the monthly payment amount?

Keys:	Display:	Description:
275000 LOAN 12.85 ALC%	LOAN=275,000.00 ALC%=12.85	Stores known values.
ADS	ADS=35,337.50	Calculates annual debt service.
÷ 12 =	2,944.79	Calculates monthly payment amount.

Example 3. What is the annual loan constant for a $100,000, 25-year loan with quarterly payments and a 14% annual interest rate?

Keys:	Display:	Description:
■ MAIN		Displays the MAIN menu.
FIN TVM		Displays the TVM menu to calculate the ADS.
■ CLEAR DATA OTHER 4 P/YR END EXIT		Clears the financial variables, sets 4 payments per year; End mode.
100000 +/- PV 25 × 4 N 14 I%YR	PV=−100,000.00 N=100.00 I%YR=14.00	Stores known values.
PMT	PMT=3,615.93	Calculates quarterly payment.
× 4 =	14,463.71	Calculates annual debt service.
■ MAIN SOLVE		Displays the SOLVE menu and ALC% equation.*

* At this point, instead of displaying the SOLVE menu, you could calculate the *ALC%* with these keystrokes: ÷ RCL PV +/- × 100 =.

CALC		Displays ALC% menu.
STO ADS	ADS = 14,463.71	Stores known values.
100000 LOAN	LOAN = 100,000.00	
ALC%	ALC% = 14.46	Calculates annual loan constant.

Adjustable-Rate Mortgages

Note

Steps 8 and 9 of this procedure (calculating the APR of an adjustable-rate mortgage) cannot be used on the HP-27S.

An *adjustable-rate mortgage* is a mortgage loan that provides for the adjustment of its interest rate as market interest rates change. As the interest rate changes, the amount of the periodic payment changes to reflect the new interest rate.

Given the terms of the original mortgage, the changes in the interest rate, and the time frame in which the changes occur, this procedure calculates the amount of each periodic payment. Once each payment is known, the annual percentage rate (APR) of the entire transaction can be calculated.

Remember to use the cash-flow sign convention (money paid out is negative, money received is positive).

1. Display the TVM menu.
2. Clear the TVM variables, store the number of payments per year in ≣ P/YR ≣, and set the payment mode (Begin or End).
3. Calculate the amount of the initial monthly payment.
4. Calculate the loan balance (*FV*) just before payments increase the first time, change the sign, and store the result in ≣ PV ≣.
5. Change the interest rate, adjust the term, store 0 in ≣ FV ≣, and recalculate the monthly payment.
6. Calculate the loan balance before payments increase the next time, change the sign, and store the result in ≣ PV ≣.
7. Repeat steps 5 and 6 until all payments have been calculated.
8. Use ≣ IRR% ≣ in the CALC menu for CFLO lists to calculate the periodic IRR%. (The CFLO menu does not exist on the HP-27S.)
9. Multiply by the number of payment periods per year to calculate the APR.

Example 1. A $50,000, 30-year, adjustable-rate mortgage has the following terms:

12% interest in the first year.
13% interest in the second and third years.
15% interest for the remaining term.

What are the monthly payments? (Assume the lender's point of view.)

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
30 × 12 N	N=360.00	Stores total number of payments.
12 I%YR	I%YR=12.00	Stores initial annual interest rate.
50000 +/− PV	PV=−50,000.00	Stores loan amount.
PMT STO 1*	PMT=514.31	Calculates and stores payment amount in first year.
12 N	N=12.00	Stores number of payments at initial interest rate.
FV	FV=49,818.56	Calculates the loan balance after 12 payments.
+/− PV	PV=−49,818.56	Stores remaining balance as new loan amount.

* Up to 10 storage registers are available.

29 [×] 12 [N]	N=348.00	Stores remaining number of payments.
13 [I%YR]	I%YR=13.00	Stores new interest rate.
0 [FV]	FV=0.00	Sets loan balance to zero.
[PMT] [STO] 2	PMT=552.70	Calculates and stores payment in second and third years.
24 [N]	N=24.00	Stores number of payments at new interest rate.
[FV]	FV=49,464.37	Calculates remaining balance after the next 24 payments.
[+/−] [PV]	PV=−49,464.37	Stores remaining balance as new loan amount.
27 [×] 12 [N]	N=324.00	Stores remaining number of payments.
15 [I%YR]	I%YR=15.00	Stores new annual interest rate.
0 [FV]	FV=0.00	Sets loan balance to zero.
[PMT] [STO] 3	PMT=629.55	Calculates and stores payment for remaining term.

Example 2. Given the payments in example 1, calculate the APR.

Keys:	Display:	Description:
≡EXIT≡ ≡CFLO≡		Displays the CFLO menu.
■ CLEAR DATA ≡YES≡ *or* ≡GET≡ ≡*NEW≡	FLOW(0)=?*	Clears current list or gets a new one.
50000 +/− INPUT	FLOW(1)=?*	Stores initial cash flow.
RCL 1 INPUT 12 INPUT	FLOW(2)=?	Stores first cash flow group.
RCL 2 INPUT 24 INPUT	FLOW(3)=?	Stores second cash flow group.
RCL 3 INPUT 27 × 12 INPUT	FLOW(4)=?	Stores third cash flow group.

The next step (pressing EXIT) is necessary if you have the HP-17B.
EXIT

≡CALC≡		Displays the CALC menu.
≡IRR%≡	IRR%=1.18	Calculates monthly IRR%.
× 12 =	14.13	Calculates APR.

* On the HP-19B, these prompts are: INIT= and FLOW(1)=.

Graduated-Payment Mortgages

The *graduated-payment mortgage* is designed to meet the needs of homebuyers who currently cannot afford high mortgage payments, but who have the potential of increased earnings in the years to come.

Under a graduated-payment mortgage plan, the payments increase by a fixed percentage at the end of each year for a specified number of years. Thereafter, the payment amount remains constant. The result is that in the early years the borrower makes a payment that is less than a traditional mortgage payment at the same interest rate, but in the later years makes larger payments than he would have with a traditional loan.

The equations below calculate the first year's monthly payment and payments for subsequent years given the total number of payments, the annual interest rate, the loan amount, the annual percentage that the payments increase, and the number of years that the payments increase. The *GPMT* equation can be modified to accommodate other than monthly payments by changing the constant 12 to the number of payments per year.

Entering and Using the GPMT and PMT Equations:

1. Enter the *GPMT* and *PMT* equations into the Solver.*

```
GPMT=PV÷(USPV(I%YR÷12:12)×USFV
((((1+%INC÷100)÷(1+I%YR÷1200)^12)
-1)×100:#YRS)+((USPV(I%YR÷12:
N-12×#YRS)×(1+%INC÷100)^#YRS)÷SPFV
(I%YR÷12:12×#YRS)))

PMT=RND(GPMT:2)×(1+%INC÷100)^
(MIN(YR#:#YRS+1)-1)
```

2. Display the *GPMT* equation's menu.

---

* On the HP-17B and HP-27S, the ^ operator can be entered with ■ [y^x].

3. Store values in the following variables:
   - Loan amount in [PV].
   - Annual interest rate (as a percentage) in [I%YR].
   - Annual percentage increase in the monthly payment in [%INC].
   - Number of years during which payments increase in [#YRS].
   - Total number of payments in [N].
4. Press [GPMT] to calculate the monthly payment amount for the first year.
5. Display the *PMT* equation's menu.
6. Store the year number in [YR#].
7. Press [PMT] to calculate the monthly payment amount in the specified year.*

Example. A couple purchased a new house with a graduated-payment mortgage. The loan is for $50,000 over a period of 30 years at an initial annual interest rate of 12.5%. The monthly payments will be graduating at an annual rate of 5% for each of the first 5 years and then will be level for the remaining 25 years. What is the amount of the monthly payment in the first year? How much are the monthly payments in each subsequent year?

Display the *GPMT* equation's menu.

Keys:	Display:	Description:
50000 [PV]	PV=50,000.00	Stores known values.
30 [×] 12 [N]	N=360.00	
12.5 [I%YR]	I%YR=12.50	
5 [%INC]	%INC=5.00	
5 [#YRS]	#YRS=5.00	

* The *PMT* equation shares variables with the *GPMT* equation. You must store variables in the *GPMT* equation and calculate the amount of the monthly graduated-payment before calculating subsequent payments with the *PMT* equation.

GPMT	GPMT=448.88	Calculates monthly payment in first year.
EXIT ↓ CALC		Displays menu for the *PMT* equation.
2 YR# PMT	PMT=471.32	Calculates monthly payment for year 2.
3 YR# PMT	PMT=494.89	Calculates monthly payment for year 3.
4 YR# PMT	PMT=519.63	Calculates monthly payment for year 4.
5 YR# PMT	PMT=545.62	Calculates monthly payment for year 5.
6 YR# PMT	PMT=572.90	Calculates monthly payment for years 6 to 30.

Amortization Schedule With Unequal Payments

It is possible to generate an amortization schedule in situations where either the periodic payment amount or the annual interest rate (or both) change. You must know the amount of the loan, the interest rates, the amount of each payment, and the number of payments at each interest rate.

Remember to use the cash-flow sign convention (money paid out is negative, money received is positive).

1. Display the TVM menu.
2. Clear the TVM variables, store the number of payments per year in ≣ P/YR ≣, and set the payment mode (Begin or End).
3. Store the initial loan information (the loan amount, annual interest rate, and periodic payment amount) in the TVM menu. (If the payment amount is not known, it can be calculated by also storing the total number of payments in ≣ N ≣ and then pressing ≣ PMT ≣.)
4. Press ≣ OTHER ≣ ≣ AMRT ≣ to display the amortization menu.
5. Key in the number of payments to amortize and press ≣ #P ≣. Press ≣ INT ≣ and ≣ BAL ≣ to display the interest and remaining balance. Repeat this step until all payments at this amount and interest rate are amortized.
6. With the remaining balance in the calculator line, press [EXIT] [EXIT] [STO] ≣ PV ≣ to store the remaining balance as the new loan amount in the TVM menu.
7. Store the adjusted annual interest rate in ≣ I%YR ≣ and the new periodic payment amount in ≣ PMT ≣.
8. Repeat steps 4–7 until the schedule is complete.

Example 1. A $50,000, 30-year, adjustable-rate mortgage has the following terms:

12 payments at 12% interest; PMT = $514.31.
24 payments at 13% interest; PMT = $552.70.
324 payments at 15% interest; PMT = $629.55.

Generate a yearly amortization schedule for the first 3 years. Assume the lender's point of view.

Keys:	Display:	Description:
TVM		Displays the TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
12 I%YR 50000 +/- PV 514.31 PMT	I%YR = 12.00 PV = −50,000.00 PMT = 514.31	Stores initial loan information.
OTHER AMRT		Displays amortization menu.
12 #P	#P = 12 PMTS: 1 − 12	

The next step (pressing INT) is necessary if you have the HP-17B or HP-27S.

INT	INTEREST = 5,990.23	
BAL	BALANCE = −49,818.51	
EXIT EXIT		Returns to TVM menu.
STO PV	PV = −49,818.51	Stores remaining balance as new loan amount.
13 I%YR 552.70 PMT	I%YR = 13.00 PMT = 552.70	Stores adjusted interest rate and payment amount.

[OTHER] [AMRT]		Displays amortization menu.
12 [#P]	#P=12 PMTS: 1–12	Stores number of payments to amortize.

The next step (pressing [INT]) is necessary if you have the HP-17B or HP-27S.

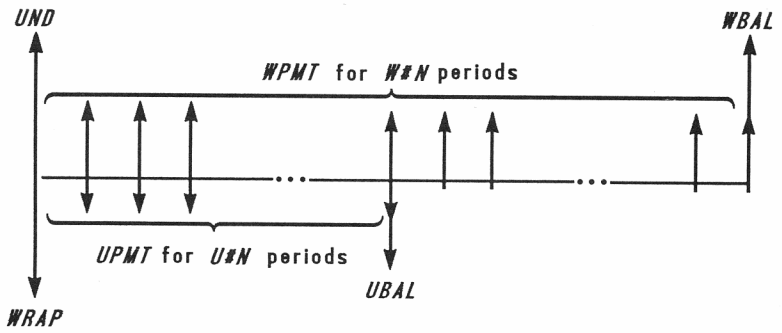
[INT]	INTEREST=6,466.76
[BAL]	BALANCE=–49,652.87
12 [#P]	#P=12 PMTS: 13–24
[INT]	INTEREST=6,443.90
[BAL]	BALANCE=–49,464.37

Wrap-Around Loans

A wrap-around loan is a loan that is partially funded by an existing underlying loan. Instead of refinancing the existing loan, the wrap-around borrower receives the difference between the wrap-around loan amount and the underlying loan balance. The borrower gives up the original loan and receives a new one. The wrap-around lender collects the loan payment on the wrap-around loan and makes the payment on the underlying loan. The wrap-around lender's net investment is the difference between the wrap-around loan amount and the underlying loan balance. In return for this net investment, the wrap-around lender receives the difference between the loan payment on the wrap-around loan and the loan payment of the underlying loan, plus the difference in balloon payments if the loans are repaid before full amortization.

The most common wrap-around loan problems calculate the percentage yield on the lender's net investment or the interest rate required on the total wrap-around loan to achieve a given yield on the lender's net investment. The following information must be known about both the wrap-around loan and the underlying loan: total number of payments, payment amount, loan amount, and amount of any balloon payments. If any of this information is unknown, you need to calculate it using the TVM menu.

The cash-flow diagram of a wrap-around loan (from the wrap-around lender's point of view) can be drawn as follows:



where:
WRAP = total amount of wrap-around loan
UND = remaining balance of underlying loan
WPMT = wrap-around loan's monthly payment
W#N = number of monthly payments in wrap-around loan
UPMT = underlying loan's monthly payment
U#N = number of monthly payments in underlying loan
WBAL = wrap-around loan's balloon payment
UBAL = underlying loan balloon payment

The following equation calculates the variables of a wrap-around loan with monthly payments. The equation can be modified to accommodate other than monthly payments by changing the constant 12 to the number of payments per year.

Entering and Using the WRAP Equation:

1. Calculate any unknown mortgage values (either for the wrap-around loan or the underlying loan) using the TVM menu.
2. Enter the WRAP equation into the Solver.

```
WRAP:WRAP-WPMT×(USPV(%YLD÷12:W#N))
-WBAL×(SPPV(%YLD÷12:W#N))=UND-UPMT×
(USPV(%YLD÷12:U#N))-UBAL×(SPPV(%YLD÷
12:U#N))
```

3. Display the WRAP equation's menu.
4. Store or calculate the following variables:
   - Total amount of wrap-around loan in ≣ WRAP ≣.
   - Monthly payment of wrap-around loan in ≣ WPMT ≣.
   - Annual yield to the lender in ≣ %YLD ≣.
   - Number of monthly payments in wrap-around loan in ≣ W#N ≣.
   - Balloon payment of wrap-around loan in ≣ WBAL ≣.
   - Remaining balance of underlying loan in ≣ UND ≣.
   - Monthly payment of underlying loan in ≣ UPMT ≣.
   - Number of monthly payments in underlying loan in ≣ U#N ≣.
   - Balloon payment of underlying loan in ≣ UBAL ≣.

Example 1. A customer has an existing loan with a balance of $125,010, a remaining term of 200 months, and a $1,051.61 monthly payment. He wishes to obtain a $200,000, 9.5% wrap-around with 240 monthly payments of $1,681.71, and a balloon payment at the end of the 240th month of $129,963.35. If you, as a lender, accept the proposal, what is your rate of return?

Display the WRAP equation's menu.

Keys:	Display:	Description:
200000 WRAP	WRAP = 200,000.00	Stores wrap-around loan values.
1681.71 WPMT	WPMT = 1,681.71	
240 W#N	W#N = 240.00	
129963.35 WBAL	WBAL = 129,963.35	
MORE		Displays second set of menu labels.
125010 UND	UND = 125,010.00	Stores underlying loan values.
200 U#N	U#N = 200.00	
1051.61 UPMT	UPMT = 1,051.61	
0 UBAL	UBAL = 0.00	
MORE		Displays first set of menu labels.
%YLD	%YLD = 11.84*	Calculates annual yield to lender.

Example 2. Your firm has determined that the yield on a wrap-around loan should be 12% annually. Continuing from example 2, what monthly payment must be received to achieve this yield on a $200,000 wrap-around loan? What interest rate is the borrower paying? (Assume that the balloon payment on the wrap-around does not change, only the payment amount changes.)

---

* The Solver searches for an iterative solution and displays intermediate estimates.

Continue from example 1.

Keys:	Display:	Description:
12 [%YLD]	%YLD = 12.00	Stores desired annual yield.
[WPMT]	WPMT = 1,693.97	Calculates amount of wrap-around monthly payment.
[STO] 0 [RCL] [WBAL] [STO] 1	WBAL = 129,963.35	Stores wrap monthly payment and remaining balance in registers 0 and 1 respectively.
■ [MAIN] [FIN] [TVM]		Displays TVM menu to calculate interest rate paid by borrower.
■ [CLEAR DATA] [OTHER] ■ [CLEAR DATA] [EXIT]		Clears TVM variables; sets 12 payments per year; End mode.
[RCL] 0 [+/−] [PMT] 240 [N] 200000 [PV] [RCL] 1 [+/−] [FV]	PMT = −1,693.97 N = 240.00 PV = 200,000.00 FV = −129,963.35	Stores information on wrap-around loan.
[I%YR]	I%YR = 9.58	Calculates annual interest paid by borrower.

Example 3. A mortgage loan on an income property has a current balance of $200,132.06. When the loan originated 8 years ago, it had a 20-year term with level monthly payments and an interest rate of at 6.75%.

A lender has agreed to "wrap" a $300,000 second mortgage at 10%, with full amortization in equal monthly payments over 12 years. What is the yield to the lender on the net cash advanced? (The monthly payment amount for each mortgage must be calculated before using the WRAP equation.)

Keys:	Display:	Description:
TVM		Displays the TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
20 − 8 = × 12 N 200132.06 +/− PV 6.75 I%YR	N = 144.00 PV = −200,132.06 I%YR = 6.75	Stores known values of underlying loan.
PMT STO 0	PMT = 2,031.55	Calculates underlying monthly payment and stores value in register 0.
300000 +/− PV 10 I%YR	PV = −300,000.00 I%YR = 10.00	Stores known values of wrap-around loan.
PMT STO 1	PMT = 3,585.23	Calculates wrap-around monthly payment and stores value in register 1.
■ MAIN SOLVE CALC		Displays the WRAP equation's menu.
300000 WRAP RCL 1 WPMT 12 × 12 W#N 0 WBAL	WRAP = 300,000.00 WPMT = 3,585.23 W#N = 144.00 WBAL = 0.00	Stores wrap-around loan values.
MORE		Displays second set of menu labels.
200132.06 UND 12 × 12 U#N RCL 0 UPMT 0 UBAL	UND = 200,132.06 U#N = 144.00 UPMT = 2,031.55 UBAL = 0.00	Stores underlying loan values.

MORE %YLD	%YLD = 15.85*	Displays first set of menu labels and calculates annual nominal yield to the lender.

* The Solver searches for an iterative solution and displays intermediate estimates.

Biweekly Mortgage Payments

One way to pay off your mortgage at a faster rate is to make biweekly mortgage payments. Instead of making one payment each month, you pay one-half of the monthly payment every two weeks. This way you make 26 or 27 payments each year (depending on the date of each payment), increase your equity, and pay less interest.*

Remember to use the cash-flow sign convention (money paid out is negative, money received is positive).

1. Display the TVM menu.
2. Clear the TVM variables, store 12 in [P/YR], and set End mode.
3. Store the total number of monthly payments in [N].
4. Store the annual interest rate in [I%YR].
5. Store the loan amount in [PV].
6. Press [PMT] to calculate the monthly payment.
7. Divide the monthly payment by 2 to calculate the biweekly payment amount. Store the biweekly payment amount in [PMT].
8. For other TVM calculations using the biweekly payment amount, you will need to store the number of payments made each year in [P/YR] before continuing. (See example 2.)

Example: Part 1. On a $75,000, 30-year, 13.5% mortgage, what would be the amount of the biweekly payment?

* This procedure is based on information provided by City Savings Bank of Meriden, Meriden, Connecticut.

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
75000 PV 30 × 12 N 13.5 I%YR	PV = 75,000.00 N = 360.00 I%YR = 13.50	Stores known values.
PMT	PMT = −859.06	Calculates monthly payment amount.
÷ 2 PMT	PMT = −429.53	Calculates and stores biweekly payment amount.

Part 2. With the biweekly payment amount just calculated, how long will it take to pay off the mortgage?

OTHER 26 P/YR EXIT		Stores number of payments each year.
N	N = 457.85	Calculates number of biweekly payments.
÷ 26 =	17.61	The loan is paid off between the 17th and 18th years.

Part 3. Continuing from examples 1 and 2: If 27 payments are made in the first year, how much interest is paid? What is the remaining balance?

OTHER AMRT		Displays amortization menu.
27 #P	#P=27 PMTS: 1–27	Amortizes payments 1 to 27.

The next steps (pressing INT and BAL) are necessary if you have the HP-17B or HP-27S.

INT	INTEREST= –10,438.07
BAL	BALANCE=73,840.76

Canadian Mortgages

Note

The following keystroke procedure cannot be done on the HP-27S. To do Canadian mortgage calculations, use the equation on page 65.

In Canada, interest is compounded semi-annually with payments made monthly. This results in a different monthly mortgage factor than is used in the United States and preprogrammed into the calculator. This difference can be handled by first calculating the Canadian mortgage factor, and then using that value as *I%YR*. Another option is to use the Solver to calculate Canadian mortgages. This is described on page 65.

To calculate the Canadian mortgage factor:

1. Display the ICNV (interest conversion) menu. Press PER to select periodic compounding.
2. Store 2 in P.
3. Store the annual interest rate in NOM%.
4. Press EFF%.
5. Store 12 in P.
6. Press NOM% to calculate the Canadian mortgage factor.

The following examples show how this factor is used as the value for I%YR in Canadian mortgage problems.

Example 1: Calculate Payment. What is the monthly payment required to fully amortize a 30-year, $30,000 Canadian mortgage if the interest rate is 12%?

Display the ICNV menu (the ICONV menu on the HP-19B).

Keys:	Display:	Description:
ICNV PER		Displays PER menu.
2 P 12 NOM%		
EFF%	EFF% = 12.36	
12 P NOM%	NOM% = 11.71	Calculates Canadian mortgage factor.
■ MAIN FIN TVM		Displays TVM menu.
STO I%YR	I%YR = 11.71	Stores Canadian mortgage factor.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
30 × 12 N 30000 PV 0 FV	N = 360.00 PV = 30,000.00 FV = 0.00	Stores known mortgage values.
PMT	PMT = −301.92	Calculates monthly payment.

Example 2: Calculate Balance. A Canadian mortgage has monthly payments of $612.77 at 8.75% interest. The principal amount is $75,500. What will be the outstanding balance at the end of 10 years?

Keys:	Display:	Description:
ICNV PER		Displays PER menu.
2 P 8.75 NOM%		
EFF%	EFF% = 8.94	
12 P NOM%	NOM% = 8.59	Calculates Canadian mortgage factor.
EXIT EXIT TVM		Displays TVM menu.
STO I%YR	I%YR = 8.59	Stores Canadian mortgage factor.

OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
612.77 +/- PMT 10 × 12 N 75500 PV	PMT = −612.77 N = 120.00 PV = 75,500.00	Stores known values.
FV	FV = −61,877.18	Calculates balance remaining at the end of ten years.

The following Solver equation creates a menu of variables to calculate Canadian mortgages. This can be used by the HP-17B, HP-19B, and HP-27S.

Entering and Using the CANADA Equation:

1. Enter the CANADA equation into the Solver.*

   ```
   CANADA:PV=-PMT×USPV(((1+CI%YR÷200)^
   (1÷6)-1)×100:N)-FV×SPPV(((1+CI%YR÷200)^
   (1÷6)-1)×100:N)
   ```

2. Display the CANADA equation's menu.
3. Store or calculate the following variables:
   - Loan amount or present value in PV.
   - Periodic payment amount in PMT.
   - Annual interest rate as a percent in CI%YR.
   - Total number of payments in N.
   - Remaining balance or future value in FV.

* On the HP-17B and HP-27S, the ^ operator can be entered with ■ y^x.

Example 3: Calculate Payment. What is the monthly payment required to fully amortize a 30-year, $50,000 Canadian mortgage if the interest rate is 9%?

Display the CANADA equation's menu.

Keys:	Display:	Description:
30 [×] 12 [N]	N=360.00	Stores known values.
9 [CI%YR]	CI%YR=9.00	
50000 [PV]	PV=50,000.00	
0 [FV]	FV=0.00	
[PMT]	PMT=−396.42	Calculates monthly payment.

Example 4: Calculate Interest. A Canadian mortgage has monthly payments of $612.77 with a maturity of 25 years. The principal amount is $75,500. What is the annual interest rate?

Keys:	Display:	Description:
612.77 [+/−] [PMT]	PMT=−612.77	Stores known values.
75500 [PV]	PV=75,500.00	
25 [×] 12 [N]	N=300.00	
0 [FV]	FV=0.00	
[CI%YR]	CI%YR=8.75*	Calculates annual interest rate.

* The Solver searches for an iterative solution and displays intermediate estimates.

Modified Internal Rate of Return

Note

These procedures cannot be done on the HP-27S.

When there is more than one sign change (positive to negative or negative to positive) in a series of cash flows, there is a potential for more than one IRR%. The modified internal rate of return (*MIRR*) procedure is an alternative that can be used when your cash-flow situation has multiple sign changes. The procedure utilizes reinvestment and borrowing rates that you specify. Negative cash flows are discounted at a "safe" rate that reflects the return on an investment in a liquid account. The figure generally used is a short-term security (T-bill) or bank passbook rate. Positive cash flows are reinvested at a "reinvestment" rate that reflects the return on an investment of comparable risk. An average return rate on recent market investments might be used.*

Use the following steps to calculate the *MIRR*. For several *MIRR* calculations, you may wish to use the Solver equation on page 70.

1. In the CFLO menu, calculate the present value of the negative cash flows (*NPV*) at the safe rate and store the result in register 0. Key in 0 for any cash flow that is positive.
2. Calculate the future value of the positive cash flows (*NFV*) at the reinvestment rate and store the result in register 1. Key in 0 for any cash flow that is negative.
3. In the TVM menu, store the total number of periods in [N], the present value of the negative cash flows in [PV], and the future value of the positive cash flows in [FV].

* Reinvestment rates and safe rates are based on current and expected capital market yields and the investor's financial management strategy. We assume that after-tax cash flows and yields will be used when applying this technique.

4. Press ≣ I%YR ≣ to calculate the periodic interest rate. Multiply by the number of periods per year to calculate the annual *MIRR*.

Example 1. An investor is considering the investment summarized below. His safe rate is 9% and he expects to reinvest positive cash flows at 14%. What is the *MIRR*?

Year	Cash Flow ($)
0	−75,000
1	−9,500
2	27,000
3	−11,000
4	50,000
5	83,000

Keys:	Display:	Description:
≣ CFLO ≣		Displays CFLO menu.
■ CLEAR DATA ≣ YES ≣ *or* ≣ GET ≣ ≣ *NEW ≣	FLOW(0)=?*	Clears current list or gets a new one.
75000 +/− INPUT	FLOW(1)=?*	Stores the initial cash flow.
9500 +/− INPUT INPUT	FLOW(2)=?	Stores *FLOW(1)*.
0 INPUT INPUT	FLOW(3)=?	Stores 0 as *FLOW(2)*.
11000 +/− INPUT INPUT	FLOW(4)=?	Stores *FLOW(3)*.

* On the HP-19B, these prompts are: INIT= and FLOW(1)=.

The next step (pressing [EXIT]) is necessary if you have the HP-17B.
[EXIT]

[CALC]		Displays the CALC menu.
9 [I%]	I% = 9.00	Stores safe interest rate.
[NPV]	NPV = −92,209.61	Calculates *NPV* of negative cash flows.
[STO] 0		Stores *NPV* in register 0.
[EXIT]		Displays the CFLO menu.
■ [CLEAR DATA] [YES]		Clears current list or gets a new one.
or		
[GET] [*NEW]	FLOW(0) = ?	
0 [INPUT]	FLOW(1) = ?	Stores 0 as initial cash flow.
0 [INPUT] [INPUT]	FLOW(2) = ?	Stores 0 as *FLOW(1)*.
27000 [INPUT] [INPUT]	FLOW(3) = ?	Stores *FLOW(2)*.
0 [INPUT] [INPUT]	FLOW(4) = ?	Stores 0 as *FLOW(3)*.
50000 [INPUT] [INPUT]	FLOW(5) = ?	Stores *FLOW(4)*.
83000 [INPUT] [INPUT]	FLOW(6) = ?	Stores *FLOW(5)*.

The next step (pressing [EXIT]) is necessary if you have the HP-17B.
[EXIT]

[CALC]		Displays the CALC menu.
14 [I%]	I% = 14.00	Stores reinvestment rate.
[NFV]	NFV = 180,001.69	Calculates *NFV* of positive cash flows.
[STO] 1		Stores *NFV* in register 1.

■ MAIN FIN TVM		Displays the TVM menu.
■ CLEAR DATA OTHER 1 P/YR END EXIT		Clears TVM variables, sets 1 compounding period per year, End mode.
5 N	N=5.00	Stores number of years in investment.
RCL 0 PV	PV=−92,209.61	Stores present value of negative cash flows.
RCL 1 FV	FV=180,001.69	Stores future value of positive cash flows.
I%YR	I%YR=14.31	Calculates annual *MIRR*.

The following equation creates a menu of variables to calculate the *MIRR*. This equation cannot be used by the HP-27S.

Entering and Using the MODIRR Equation:

1. Enter the MODIRR equation into the Solver, substituting an appropriate cash-flow list name for *name*.*

   ```
   MODIRR:(1+MIRR÷100)^Σ(L:1:SIZEC(name):1
   :#T(name:L))=−Σ(J:0:SIZEC(name):1:
   MAX(FLOW(name:J):0)×USFV(RISK:#T(name:J))
   ×SPFV(RISK:Σ(L:J+1:SIZEC(name):1:#T(name
   :L))))÷(MIN(FLOW(name:0):0)+Σ(J:1:SIZEC
   (name):1:MIN(FLOW(name:J):0)×USPV(SAFE:
   #T(name:J))×SPPV(SAFE:Σ(L:1:J−1:1:#T
   (name:L))))
   ```

2. Enter all of the cash flows into a CFLO list. Name the list the same name used in the Solver equation.

---

* On the HP-17B and HP-27S, the ^ operator can be entered with ■ y^x. To enter the Σ symbol, press WXYZ OTHER MORE Σ.

3. Display the menu of variables for the Solver equation.
4. Store the following variables:
   - Periodic safe rate in ≡ SAFE ≡.
   - Periodic risk rate in ≡ RISK ≡.
5. Press ≡ MIRR ≡ to calculate the periodic *MIRR*. To calculate the annual *MIRR*, multiply by the number of periods per year.

Example 2. Use the Solver equation to calculate the *MIRR* for the cash flows in example 1 (page 68).

Keys:	Display:	Description:
≡ CFLO ≡		Displays CFLO menu.
■ CLEAR DATA ≡ YES ≡		Clears the current list or gets a new one.
or		
≡ GET ≡ ≡ *NEW ≡	FLOW(0)=?	
75000 +/− INPUT	FLOW(1)=?	Stores initial cash flow.
9500 +/− INPUT		Stores cash flows.
INPUT	FLOW(2)=?	
27000 INPUT	FLOW(3)=?	
INPUT		
11000 +/− INPUT		
INPUT	FLOW(4)=?	
50000 INPUT	FLOW(5)=?	
INPUT		
83000 INPUT	FLOW(6)=?	
INPUT		

The next step (pressing EXIT) is necessary if you have the HP-17B.

EXIT		
≡ NAME ≡		Displays CFLO NAME menu.
INV INPUT		Names the list INV.

Type the MODIRR equation, substituting INV for *name*. Press ≡ CALC ≡ to display the menu of variables.

14 ≡ RISK ≡	RISK=14.00	Stores known values.
9 ≡ SAFE ≡	SAFE=9.00	
≡ MIRR ≡	MIRR=14.31	Calculates *MIRR*.

Private Mortgage Insurance

Private mortgage insurance (*PMI*) is usually calculated based on a percentage of the outstanding loan balance. Since the loan balance changes, the *PMI* calculation is done once each year.

Use the following procedure to calculate the amount of private mortgage insurance.

1. For year 1:
   - Key in the loan amount and press [×].
   - Key in the insurance rate as a percentage and press [%] [=]. The annual insurance amount is displayed.
   - Divide by the number of payments per year to calculate the monthly insurance premium.
2. For subsequent years:
   - Calculate the balance at the end of the previous year and press [×].
   - Key in the insurance rate as a percentage and press [%].
   - Divide by the number of payments per year.

Example: Part 1. A 30-year, $75,000 loan at 15% interest has private mortgage insurance of .25% of the loan balance. Calculate the monthly private mortgage insurance (*PMI*) payment for year 1.

Keys:	Display:	Description:
75000 [×]	75,000.00 ×	Enters loan amount.
.25 [%] [=]	187.50	Calculates year 1 *PMI* payment.
[÷] 12 [=]	15.63	Calculates monthly *PMI* payment for year 1.

Part 2. Calculate the monthly *PMI* for year 2:

TVM		Displays TVM menu.
■ CLEAR DATA OTHER ■ CLEAR DATA EXIT		Clears TVM variables, sets 12 payments per year, End mode.
30 × 12 N 15 I%YR 75000 +/− PV	N = 360.00 I%YR = 15.00 PV = −75,000.00	Stores known mortgage values.
PMT	PMT = 948.33	Calculates monthly mortgage payment.
12 N FV	FV = 74,860.68	Calculates balance at end of first year.
× .25 % =	187.15	Calculates year 2 *PMI* payment.
÷ 12 =	15.60	Calculates monthly *PMI* payment for year 2.

Part 3. Calculate the monthly *PMI* payment for year 3.

24 N FV	FV = 74,698.97	Calculates balance at end of second year.
× .25 % =	186.75	Calculates year 3 *PMI* payment.
÷ 12 =	15.56	Calculates monthly *PMI* payment for year 3.

Part 4. Calculate the monthly *PMI* payment for year 30.

29 × 12 N FV	N = 348.00 FV = 10,506.88	Calculates balance at end of year 29.
× .25 % =	26.27	Calculates year 30 *PMI* payment.
÷ 12 =	2.19	Calculates monthly *PMI* payment for year 30.

Reverse Annuity Mortgage Payment

A reverse annuity mortgage allows people over 62 years of age to use the equity in their homes to generate regular monthly income. The following procedure calculates the monthly payment one could receive from a reverse annuity mortgage. Remember to use the cash flow sign convention (money paid out is negative; money received is positive).

1. Display the TVM menu.
2. Clear the TVM variables, store the number of payments per year in ≣ P/YR ≣, and set either Begin or End mode.
3. Store the total number of payments in ≣ N ≣.
4. Store the annual interest rate in ≣ I%YR ≣.
5. If an initial payment is received, store the amount in ≣ PV ≣.
6. Store the total loan amount in ≣ FV ≣.
7. Press ≣ PMT ≣ to calculate the periodic payment.

Example: Part 1. You plan to take out a reverse annuity mortgage and you want to know how much money you will receive at the beginning of each month, given the following: loan amount is $64,000 (the equity you have in your home), 5-year term, 13% interest rate, and an initial payment of $2,500.

Keys:	Display:	Description:
≣ TVM ≣		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
≣ OTHER ≣ ■ CLEAR DATA ≣ BEG ≣ EXIT		Sets 12 payments per year; Begin mode.
5 × 12 ≣ N ≣	N=60.00	Stores the number of payments.
13 ≣ I%YR ≣	I%YR=13.00	Stores the annual interest rate.

2500 PV	PV=2,500.00	Stores the initial payment.
64000 +/− FV	FV= −64,000.00	Stores the total amount of the loan.
PMT	PMT=698.41	Calculates the monthly payment.

Part 2. Suppose you need only $500 per month. For how many months could you receive these payments?

500 PMT	PMT=500.00	Stores the new payment amount.
N	N=75.31	You could receive these payments for 75 months (6 years, 3 months).

Source: Reverse Annuity Mortgage Program, San Francisco Development Fund, San Francisco, Ca.

NPV and Discount Rate Sensitivity

Note

This keystroke procedure cannot be used on the HP-17B or HP-27S.

The cash-flow plotting capability of the HP-19B can be used to determine graphically at which discount rates an investment makes sense. If the net present value is positive at the discount rate, the investment is acceptable. Use the following procedure.

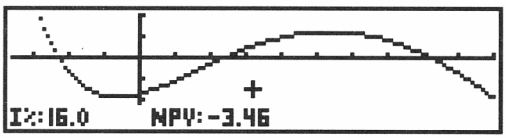
1. Display the CFLO menu and enter the cash flows into a CFLO list.
2. Press ≡ PLOT ≡ to plot *NPV* versus *I%* for the current CFLO list.

Example 1. You are looking at the following real estate investment opportunity. If your discount rate is 9%, is this an attractive investment? What if your discount rate is 25%?

Year	Cash Flow ($)
0	-714
1	2,440
2	-2,730
3	1,000

Keys:	Display:	Description:
≡ CFLO ≡		Displays CFLO menu.
■ CLEAR DATA ≡ YES ≡ *or* ≡ GET ≡ ≡ *NEW ≡	INIT =	Clears the current list or gets a new one.

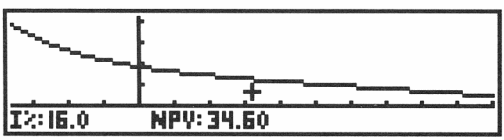
714 [+/−] [INPUT]	FLOW(1) =	Stores cash flows.
2440 [INPUT] [INPUT]	FLOW(2) =	
2730 [+/−] [INPUT]		
[INPUT]	FLOW(3) =	
1000 [INPUT]		
≡ PLOT ≡		



Now use the arrow keys ([↑], [↓], [←], and [→]) to move the graphics cursor. The plot shows that when the discount rate is between approximately 13% and 40%, the investment is attractive.

Example 2. If the last cash flow in the previous example is changed to $1,100, is the investment attractive at a 20% discount rate?

Keys:	**Display:**	**Description:**
[EXIT]		Displays the CFLO menu.
[↑]		Moves the pointer to FLOW(3).
1100 [INPUT]		Stores new cash flow.
≡ PLOT ≡		



The investment is attractive at any discount rate.

2

Banking

Savings Plans

These examples are presented as guidelines for evaluating savings plans when the payment period coincides with the compounding period.* Since the Time Value of Money (TVM) menu is used, remember the cash-flow sign convention when entering dollar amounts (money paid out is entered as a negative number, money received is entered as a positive number).

1. Clear the TVM variables, store the number of payments per year in [P/YR], and set either Begin or End mode.
2. Store values in at least three of the following variables. (Both N and I%YR must be a part of a problem. Either both values are known, or one is known and the other is to be calculated.)
   - Number of periodic deposits in [N].
   - Annual interest rate in [I%YR].
   - Initial investment in [PV].
   - Periodic deposit in [PMT].
   - Future value in [FV].
3. Press the menu key of the unknown variable to calculate its value.

Example 1: Balance of a Savings Account After Initial Deposit and Regular Deposits. You have just opened a savings account with a $200 deposit. If you deposit $50 a month, and the account earns 5.25% compounded monthly, how much will you have in the account in 3 years?

* If the periodic deposits do not coincide with the compounding periods, the account must be evaluated in another manner. Use the Compounding Periods Different From Payment Periods procedure on page 92.

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
3 × 12 N	N=36.00	Stores known values.
5.25 I%YR	I%YR=5.25	
200 +/− PV	PV=−200.00	
50 +/− PMT	PMT=−50.00	
FV	FV=2,178.94	Calculates amount in savings account in 3 years.

Example 2: Number of Deposits or Withdrawals to Reach a Specified Balance. Part 1. Your savings account presently contains $18,000 and earns 5.25% compounded monthly. You wish to withdraw $300 a month until the account is depleted. How long will this take?

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
5.25 I%YR	I%YR=5.25	Stores known values.
18000 +/− PV	PV=−18,000.00	
300 PMT	PMT=300.00	
N	N=69.75	Calculates number of months. (The 70th withdrawal will be less than $300.)

Part 2. If you wish to reduce the account to $5,000, how many withdrawals can you make?

5000 FV	FV=5,000.00	Stores remaining balance.
N	N=52.41	Calculates number of monthly withdrawals. (The 53rd withdrawal reduces the account balance to less than $5,000.)

Example 3: Amount to Deposit Today to Have a Certain Future Balance. How much money would you have to invest today if you want $10,000 in 10 years? Assume the interest rate is 9%, compounded annually.

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER 1 P/YR END EXIT		Sets 1 payment per year; End mode.
10000 FV 9 I%YR 10 N	FV=10,000.00 I%YR=9.00 N=10.00	Stores known values.
PV	PV=−4,224.11	Calculates the amount to deposit today to have $10,000 in the future.

Example 4: Monthly Deposits to Reach a Future Balance. You plan to replace your car in 3 years, and you want to have $6,000 to help pay for the new one. How much should you save each month, beginning today, to accumulate $6,000 in 3 years? Assume 7.5% interest, compounded monthly.

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA BEG EXIT		Sets 12 payments per year; Begin mode.
3 × 12 N 7.5 I%YR 6000 FV	N=36.00 I%YR=7.50 FV=6,000.00	Stores known values.
PMT	PMT=−148.21	Calculates the monthly payment.

Example 5: Periodic Deposits and Withdrawals. Part 1. You are presently depositing $50 at the end of each month into a local savings and loan, earning 5.25% compounded monthly. Your account balance is $1,023.25. How much will you accumulate in 5 months?

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
5 N 5.25 I%YR 1023.25 +/− PV 50 +/− PMT	N=5.00 I%YR=5.25 PV=−1,023.25 PMT=−50.00	Stores known values.
FV	FV=1,298.03	Calculates account balance after five months.

Part 2. At the beginning of the sixth month, you withdraw $80. What is the new balance?

− 80 =	1,218.03	Calculates new balance.

Part 3. At the end of months 6, 7, and 8, you deposit $65. How much will you have in the account at the end of month 8?

[+/−] [PV]	PV = −1,218.03	Stores the beginning balance.
65 [+/−] [PMT] 3 [N]	PMT = −65.00 N = 3.00	Stores known values.
[FV]	FV = 1,429.94	Calculates balance after an additional three months.

Part 4. You decide not to make deposits for the next 2 months. What is the balance in the account after those 2 months?

[+/−] [PV]	PV = −1,429.94	Stores the beginning balance.
2 [N] 0 [PMT]	N = 2.00 PMT = 0.00	Stores known values.
[FV]	FV = 1,442.48	Calculates balance after an additional two months.

Example 6: Deposits Needed for Future Withdrawals. You wish to retire in 25 years with a monthly income of $2,000, continuing for 20 years. How much should you deposit each month in an account that earns 8.5% interest to achieve your retirement objectives?

A two-step solution is appropriate. First, calculate the present value of the desired retirement income. This represents the amount you need in the account when you retire. Then calculate the monthly deposit necessary to accumulate this amount.

Keys:	Display:	Description:
Step 1:		
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER ■ CLEAR DATA EXIT		Sets 12 payments per year; End mode.
20 × 12 N 8.5 I%YR 2000 PMT	N=240.00 I%YR=8.50 PMT=2,000.00	Stores retirement income values.
PV	PV=−230,461.68	Calculates amount needed in fund at retirement.
+/− FV	FV=230,461.68	Stores retirement fund amount.
Step 2:		
0 PV	PV=0.00	Stores 0 in *PV* as there is no deposit made today.
25 × 12 N	N=300.00	Stores total number of deposits.
PMT	PMT=−223.30	Calculates monthly deposit.

Example 7: Compounding on 365/360-Day Basis. Part 1. Your savings account earns 5.5% interest compounded on a 365/360-day basis (use actual number of days for *N*, use 360 days for number of compounding periods per year). If the account presently contains $1,200, how much will be in the account in 1 year (365 days)?

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER 360 P/YR END EXIT		Sets 360 payments per year; End mode.

365 N	N = 365.00	Stores known values.
5.5 I%YR	I%YR = 5.50	
1200 +/− PV	PV = −1,200.00	
FV	FV = 1,268.81	Calculates account balance.

Part 2. How does this compare with daily compounding on a 360-day basis?

360 N	N = 360.00	Stores number of compounding periods.
FV	FV = 1,267.84	Calculates account balance on 360-day basis.

Savings Account Compounded Daily

The equation below determines the value of a savings account when interest is compounded daily. You can calculate the total amount in the account after a series of transactions on specified dates.

Entering and Using the SAVFV Equation:

1. Enter the *SAVFV* equation into the Solver.

   ```
   SAVFV=(PV+PMT)×SPFV(I%YR÷365:
   DDAYS(DATE1:DATE2:1))
   ```

2. Display the *SAVFV* equation's menu.
3. Store five of the following variables:
   - Value of the account on the second date in ≣ SAVFV ≣.
   - Value of the account on the first date in ≣ PV ≣.
   - Payment to the account in ≣ PMT ≣.
   - Annual interest rate as a percent in ≣ I%YR ≣.
   - First date in ≣ DATE1 ≣.
   - Second date in ≣ DATE2 ≣.
4. Press the menu key to calculate the unknown value.

Example. An account earns 5.25%, compounded daily. Calculate the amount in this account after the following transactions:

September 15, 1987	$125.00 deposit
October 9, 1987	$60.00 deposit
October 14, 1987	$70.00 deposit
November 18, 1987	$50.00 withdrawal
December 1, 1987	$175.00 deposit
January 22, 1988	$100.00 withdrawal

Enter the dates in MM.DDYYYY format.

Display the *SAVFV* equation's menu.

Keys:	Display:	Description:
125 PV	PV = 125.00	Stores the amount deposited on the first date.
0 PMT	PMT = 0.00	Stores 0 as the payment amount.
5.25 I%YR	I%YR = 5.25	Stores the nominal annual interest rate.
9.151987 DATE1	DATE1 = 9.15	Stores the first date.
10.091987 DATE2	DATE2 = 10.09	Stores the second date.
SAVFV	SAVFV = 125.43	Calculates the value of the account on the second date.
STO PV	PV = 125.43	Stores the balance of the savings account.
60 PMT	PMT = 60.00	Stores amount of deposit.
RCL DATE2 STO DATE1	DATE1 = 10.09	Stores *DATE2* as the first date.
10.141987 DATE2	DATE2 = 10.14	Stores the second date.
SAVFV	SAVFV = 185.57	Calculates the value of the account on the second date.
STO PV	PV = 185.57	Stores the balance of the savings account.
70 PMT	PMT = 70.00	Stores amount of deposit.
RCL DATE2 STO DATE1	DATE1 = 10.14	Stores *DATE2* as the first date.
11.181987 DATE2	DATE2 = 11.18	Stores the second date.
SAVFV	SAVFV = 256.86	Calculates the value of the account on the second date.
STO PV	PV = 256.86	Stores the balance of the savings account.

50 [+/−] [PMT]	PMT = −50.00	Stores amount of withdrawal.
[RCL] [DATE2] [STO] [DATE1]	DATE1 = 11.18	Stores *DATE2* as the first date.
12.011987 [DATE2]	DATE2 = 12.01	Stores the second date.
[SAVFV]	SAVFV = 207.24	Calculates the value of the account on the second date.
[STO] [PV]	PV = 207.24	Stores the balance of the savings account.
175 [PMT]	PMT = 175.00	Stores amount of deposit.
[RCL] [DATE2] [STO] [DATE1]	DATE1 = 12.01	Stores *DATE2* as the first date.
1.221988 [DATE2]	DATE2 = 1.22	Stores the second date.
[SAVFV]	SAVFV = 385.11	Calculates the value of the account on the second date.
[−] 100 [=]	285.11	Final amount in the savings account.

Simple Interest

With simple interest, only the principal (the original loan amount) earns interest for the entire life of the transaction. The interest earned, plus the principal, is repaid in one lump sum. The following equations calculate the amount of simple interest on a 360-day basis and a 365-day basis. Once the interest is calculated, the total payment (loan amount plus interest) can also be calculated.

Entering and Using the SINT Equation:

1. Enter the *SINT* equation into the Solver.

   `SINT=#DYS÷YEAR×LOAN×I%YR÷100`

2. Display the *SINT* equation's menu.
3. Store or calculate the following variables:
   - Amount of simple interest in ≣ SINT ≣.
   - Total number of days in ≣ #DYS ≣.
   - Number of days in the calendar year (either 360 or 365) in ≣ YEAR ≣.
   - Loan amount in ≣ LOAN ≣.
   - Annual interest rate in ≣ I%YR ≣.
4. Press [+] [RCL] ≣ LOAN ≣ [=] to calculate the total payment (interest plus principal).

Example. A friend has requested that you lend him $1,200 for 90 days. You lend him the money at 8% simple annual interest, to be calculated on a 360-day basis. How much interest will he owe you in 90 days? What is the total amount owed?

Display the *SINT* equation's menu.

Keys:	Display:	Description:
1200 ≣ LOAN ≣	LOAN = 1,200.00	Stores known values.
90 ≣ #DYS ≣	#DYS = 90.00	
8 ≣ I%YR ≣	I%YR = 8.00	

360 YEAR	YEAR=360.00	
SINT	SINT=24.00	Calculates amount of simple interest.
+ RCL LOAN		Calculates total payment amount.
=	1,224.00	

Compounding Periods Different From Payment Periods

Note

This keystroke procedure cannot be used on the HP-27S. Refer to page 176 of the *HP-27S Owner's Manual* for the equivalent Solver equations for converting interest rates.

Savings account deposits and withdrawals may not occur at the same time as the bank's compounding periods. The TVM menu, however, assumes these two periods are the same. This procedure shows you how to adjust the interest rate so that you can use the TVM menu in situations when the compounding period is different from the payment period.

When the bank's interest rate is known, adjust the annual interest rate to correspond to the payment period, then use the TVM menu to calculate the unknown value.

1. Display the periodic interest rate conversion menu (≣ ICNV ≣ ≣ PER ≣ on the HP-17B, ≣ ICONV ≣ ≣ PER ≣ on the HP-19B).
2. Store the following variables. The bank provides this information.
   - Nominal annual interest rate in ≣ NOM% ≣.
   - Number of compounding periods per year in ≣ P ≣.
3. Press ≣ EFF% ≣ to calculate the effective annual interest rate.
4. Store the number of payments or withdrawals per year in ≣ P ≣.
5. Press ≣ NOM% ≣ to calculate the nominal rate that corresponds to the payment period.
6. Press EXIT EXIT ≣ TVM ≣ to display the TVM menu.
7. Press STO ≣ I%YR ≣ to store the adjusted nominal rate.
8. Store the number of payments per year in ≣ P/YR ≣ and set either Begin or End mode.
9. Store or calculate the following variables. Remember to use the cash-flow sign convention (money paid out is negative; money received is positive).

- Total number of periodic deposits in N.
- Initial deposit in PV.
- Amount of periodic deposit or withdrawal in PMT.
- Future value in FV.

When the interest rate is the unknown variable, calculate I%YR in the TVM menu (this is the nominal rate that corresponds to the payment period). Then use the interest conversion menu to calculate the nominal annual interest rate corresponding to the compounding period.

Example 1: Balance of a Savings Account. Starting today, you make monthly deposits of $25 into an account paying 5% interest compounded daily (365-day basis). At the end of 7 years, how much will you receive from the account?

Keys:	Display:	Description:
ICNV PER		Displays the periodic conversion menu.
5 NOM% 365 P	NOM% = 5.00 P = 365.00	Stores the known values.
EFF%	EFF% = 5.13	Calculates the effective interest rate for daily compounding.
12 P	P = 12.00	Stores the number of deposits per year.
NOM%	NOM% = 5.01*	Calculates the equivalent nominal interest rate for monthly compounding.
EXIT EXIT TVM		Displays the TVM menu.

* Because compounding is less frequent, a higher nominal interest rate is needed to achieve the same effective rate.

STO I%YR	I%YR=5.01	Stores the interest rate.
OTHER 12 P/YR BEG EXIT		Sets 12 payments per year; Begin mode.
7 × 12 N 25 +/- PMT 0 PV	N=84.00 PMT=−25.00 PV=0.00	Stores the known values.
FV	FV=2,519.61	Calculates the value of the account in 7 years.

Example 2: Amount to Deposit Today to Have a Certain Future Balance. You wish to make weekly deposits for eight years into a savings account paying 5.25% interest compounded quarterly. How much should you deposit each week to accumulate $6,000?

Keys:	Display:	Description:
ICNV PER		Displays the periodic interest conversion menu.
5.25 NOM% 4 P	NOM%=5.25 P=4.00	Stores the known values.
EFF%	EFF%=5.35	Calculates the effective rate for quarterly compounding.
52 P	P=52.00	Stores the number of deposits per year.
NOM%	NOM%=5.22	Calculates the equivalent nominal interest rate for weekly compounding.
EXIT EXIT TVM		Displays the TVM menu.
STO I%YR	I%YR=5.22	Stores the interest rate.
OTHER 52 P/YR BEG EXIT		Sets 52 payments per year; Begin mode.

8 [×] 52 [N]	N=416.00	Stores the known values.
6000 [FV]	FV=6,000.00	
0 [PV]	PV=0.00	
[PMT]	PMT=−11.62	Calculates amount of weekly deposit.

Example 3: Length of Time to Accumulate a Balance. You make weekly deposits of $10 into an account paying 5.25% compounded daily (365-day basis). How long will it take to accumulate $1,000?

Keys:	Display:	Description:
[ICNV] [PER]		Displays the periodic interest conversion menu.
5.25 [NOM%]	NOM%=5.25	Stores the known values.
365 [P]	P=365.00	
[EFF%]	EFF%=5.39	Calculates the effective rate for daily compounding.
52 [P]	P=52.00	Stores the number of deposits per year.
[NOM%]	NOM%=5.25*	Calculates the equivalent nominal interest rate for weekly compounding.
[EXIT] [EXIT] [TVM]		Displays the TVM menu.
[STO] [I%YR]	I%YR=5.25	Stores the interest rate.
[OTHER]		Sets 52 payments per year; Begin mode.
52 [P/YR]		
[BEG] [EXIT]		

* To see the difference between the two nominal rates, display more than two decimal places.

10 [+/−] [PMT] 1000 [FV] 0 [PV]	PMT = −10.00 FV = 1,000.00 PV = 0.00	Stores the known values.
[N]	N = 95.22	Calculates the number of weeks. (The 96th deposit places the balance over $1,000.)

Example 4: Calculating Interest Rate. Your bank statement indicates that you earned $4.63 in interest for 1 month. Your beginning balance was $975.46. What interest rate is your bank quoting, assuming daily compounding on a 365-day basis?

Keys:	Display:	Description:
[TVM]		Displays the TVM menu.
■ [CLEAR DATA]		Clears the TVM variables.
[OTHER] 12 [P/YR] [BEG] [EXIT]		Sets 12 payments per year; Begin mode.
1 [N] 975.46 [+/−] [PV] [+/−] [+] 4.63 [FV]	N = 1.00 PV = −975.46 FV = 980.09	Stores the known values.
[I%YR]	I%YR = 5.70	Calculates the periodic interest rate.
[EXIT] [ICNV] [PER]		Displays the periodic interest conversion menu.
[STO] [NOM%]	NOM% = 5.70	Stores the nominal interest rate.
12 [P]	P = 12.00	Stores the number of deposits per year.
[EFF%]	EFF% = 5.85	Calculates the effective rate for monthly compounding.

365 P	P=365.00	Stores the number of compounding periods per year.
NOM%	NOM%=5.68	Calculates the nominal interest rate quoted by the bank.

Increasing Annuities

These equations calculate the present and future values of an annuity (a series of payments) that increases at a constant rate at equal intervals of time. The first equation (PVINCR), calculates the present value of an increasing annuity. Once the present value has been calculated, the second equation (FVINCR) calculates the future value of the annuity.

The equations assume that payments are made at the end of each period. In addition, the total number of payments (*#YRS* × *P/YR*) divided by the number of periods before payments increase (*#PER*) must be an integer.

Entering and Using the PVINCR and FVINCR Equations:

1. Enter the PVINCR and FVINCR equations into the Solver.*

```
PVINCR:PMT×USPV(I%YR÷P/YR:#PER)×
USFV(((1+%INC÷100)÷(1+I%YR÷P/YR÷100)^
#PER-1)×100:#YRS×P/YR÷#PER)-PVINCR

FVINCR:SPFV(I%YR÷P/YR:#YRS×P/YR)×
PVINCR-FVINCR
```

2. Display the PVINCR equation's menu.
3. Store the following variables:
   - Periodic payment amount in ≣ PMT ≣.
   - Annual interest rate as a percent in ≣ I%YR ≣.
   - Number of payments per year in ≣ P/YR ≣.
   - Number of periods before payments increase in ≣ #PER ≣.
   - Percentage of each increase in ≣ %INC ≣.
   - Total number of years in ≣ #YRS ≣.
4. Press ≣ PVIN ≣ to calculate the present value of the increasing annuity.

---

* To key in the / character on the HP-17B and HP-27S, press ≣ WXYZ ≣ ≣ OTHER ≣ ≣ MORE ≣ ≣ / ≣.

5. Once the PVINCR is calculated, press [FVIN] to calculate the future value of the increasing annuity.*

Example 1. A client has a 20-year annuity that pays $110 per month for the first year. The monthly payment increases 5.5% each year. Assuming a discount rate of 11.5%, what is the present value of the series of payments?

Display the PVINCR equation's menu.

Keys:	Display:	Description:
110 [PMT]	PMT=110.00	Stores known values.
11.5 [I%YR]	I%YR=11.50	
12 [P/YR]	P/YR=12.00	
12 [#PER]	#PER=12.00	
5.5 [%INC]	%INC=5.50	
[MORE]		Displays second set of menu labels.
20 [#YRS]	#YRS=20.00	Stores known values.
[PVIN]	PVINCR=14,793.46	Calculates present value of increasing annuity.

Example 2. Starting at the end of this year, you plan to make yearly deposits into an account that earns 13% interest, compounded annually. Each year you plan to increase the amount of your deposit by 8%. If the first deposit is $1,200, how much will you accumulate over the next 10 years?

Display the PVINCR equation's menu.

---

* The calculation of FVINCR requires that values be stored in [I%YR], [P/YR], [#YRS], and [PVIN].

Keys:	Display:	Description:
1200 PMT	PMT = 1,200.00	Stores known values.
13 I%YR	I%YR = 13.00	
1 P/YR	P/YR = 1.00	
1 #PER	#PER = 1.00	
8 %INC	%INCR = 8.00	
MORE		Displays second set of menu labels.
10 #YRS	#YRS = 10.00	Stores known values.
PVIN	PVINCR = 8,736.14	Calculates present value of increasing annuity.
EXIT ↓ CALC		Displays FVINCR menu.
FVIN	FVINCR = 29,655.42	Calculates the future value of the increasing annuity.

Example 3. In 30 years you wish to have $250,000 to retire. At the end of each month, you plan to deposit a sum into an account that earns 8% interest, compounded monthly. Each year you will increase the amount of the deposit by 3%. How much is your first deposit?

Display the FVINCR equation's menu.

Keys:	Display:	Description:
250000 FVIN	FVINCR = 250,000.00	Stores known values.
8 I%YR	I%YR = 8.00	
12 P/YR	P/YR = 12.00	
30 #YRS	#YRS = 30.00	
PVIN	PVINCR = 22,860.84	Calculates the present value.
EXIT ↑ CALC		Displays PVINCR menu.
12 #PER	#PER = 12.00	Stores remaining known values.
3 %INC	%INC = 3.00	
PMT	PMT = 125.08	Calculates initial monthly payment.

Deposits Needed to Meet a Future Cash Flow Need

Note

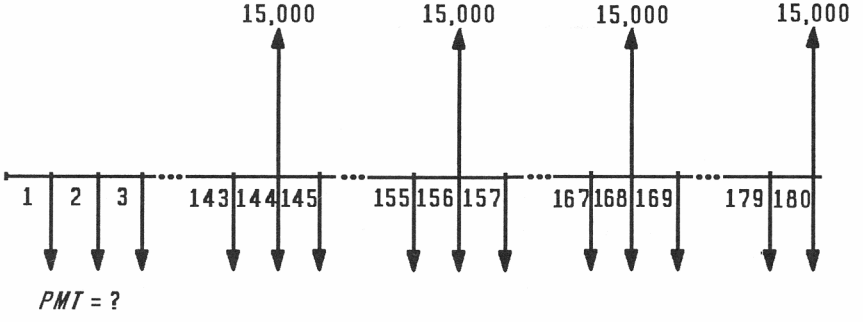
This procedure cannot be used on the HP-27S.

Sometimes you want to know how much money you need to save now to accumulate money for a future series of outflows. An example of this situation is making periodic savings deposits for college. The following procedure helps determine how much you need to save each period. You need to know when you need the money, how much is needed, and at what interest rate you can invest.

1. Display the CFLO menu.
2. Press ■ CLEAR DATA ≣ YES ≣ to clear the list. (If you don't want to delete the list, name your old list and get a new list.)
3. Store 0 as the initial cash flow.
4. Store 0 as *FLOW(1)*, and the number of payment periods until the withdrawals begin as the number of times.
5. Store the withdrawal amount as *FLOW(2)*. Continue entering cash flows of 0 and withdrawals through the last withdrawal.
6. Press EXIT ≣ CALC ≣ to display the CFLO CALC menu.
7. Store the periodic interest rate in ≣ I% ≣.
8. Do **a** and/or **b**:
   - **a.** Press ≣ NUS ≣ to calculate the amount of the periodic payments you need to make (starting at the end of the period).
   - **b.** Press ≣ NPV ≣ to calculate the lump sum you would need to deposit now.

Example 1: Periodic Deposits. Your daughter will be going to college in 12 years and you are starting a fund for her education. She will need $15,000 at the beginning of each year for four years. The fund earns 9%, compounded monthly, and you plan to make monthly deposits, starting at the end of the current month. How much should you deposit each month to meet her educational expenses?

The cash-flow diagram looks like this:



Keys:	Display:	Description:
[CFLO]		Displays CFLO menu.
■ [CLEAR DATA] [YES]		Clears current list or gets a new one.
or		
[GET] [*NEW]	FLOW(0) = ?*	
0 [INPUT]	FLOW(1) = ?*	Stores initial cash flow.
0 [INPUT] 12 [×] 12 [−] 1 [INPUT]	FLOW(2) = ?	Stores cash flows until withdrawals begin as *FLOW(1)*.

* On the HP-19B, these prompts are: INIT = and FLOW(1) =.

15000 [INPUT] [INPUT]	FLOW(3)=?	Stores first withdrawal as *FLOW(2)*.
0 [INPUT] 11 [INPUT]	FLOW(4)=?	Stores cash flows of zero for the rest of the year as *FLOW(3)*.
15000 [INPUT] [INPUT]	FLOW(5)=?	Stores second withdrawal as *FLOW(4)*.
0 [INPUT] 11 [INPUT]	FLOW(6)=?	Stores cash flows of zero for the rest of the year as *FLOW(5)*.
15000 [INPUT] [INPUT]	FLOW(7)=?	Stores third withdrawal as *FLOW(6)*.
0 [INPUT] 11 [INPUT]	FLOW(8)=?	Stores cash flows of zero for the rest of the year as *FLOW(7)*.
15000 [INPUT] [INPUT]	FLOW(9)=?	Stores fourth withdrawal as *FLOW(8)*.

The next step (pressing [EXIT]) is necessary if you have the HP-17B.
[EXIT]

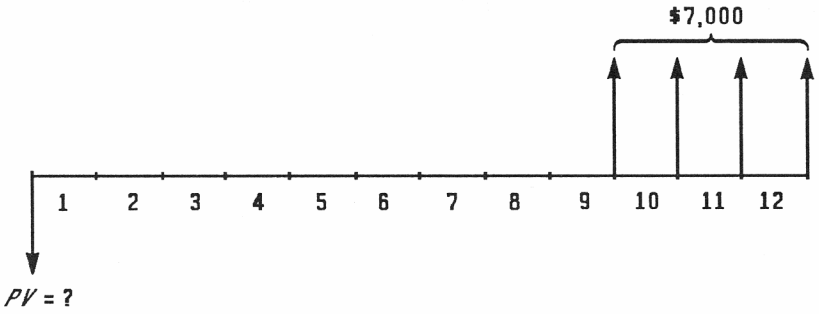
[CALC]		Displays CFLO CALC menu.
9 [÷] 12 [I%]	I%=0.75	Stores monthly interest rate.
[NUS]	NUS=182.30	Calculates monthly payment.

Part 2. Alternatively, how much would you need to deposit today in one lump sum to fund her college education?

[NPV]	NPV=17,973.48	Solves for net present value of the withdrawals.

Example 2: Single Deposit. You wish to put money aside for your daughter's college education. You estimate that when she is of college age, 9 years from now, she will need $7,000 at the beginning of each year for four years to cover college tuition and expenses. You wish to establish a fund that earns 6% annually. How much do you need to deposit in the fund today to meet your daughter's educational expenses?

The cash-flow diagram looks like this:



Keys:	Display:	Description:
≡ CFLO ≡		Displays CFLO menu.
■ CLEAR DATA ≡ YES ≡ *or* ≡ GET ≡ ≡ *NEW ≡	FLOW(0) = ?	Clears current list or gets a new one.
0 INPUT	FLOW(1) = ?	Stores initial cash flow.
0 INPUT 8 INPUT	FLOW(2) = ?	Stores first through eighth cash flows.
7000 INPUT 4 INPUT	FLOW(3) = ?	Stores ninth through twelfth cash flows.

The next step (pressing EXIT) is necessary if you have the HP-17B.
EXIT

CALC		Displays the CALC menu.
6 I%	I% = 6.00	Stores annual interest rate.
NPV	NPV = 15,218.35	Calculates amount of one-time deposit needed for college fund.

Odd-Period Calculations

The TVM menu deals with financial transactions in which each payment period is the same length. However, situations exist in which the first payment period is not the same length as the remaining periods. The period from the date that interest begins accruing to the date of the first payment, when not equal to the regular payment period, is sometimes referred to as an *odd first period*.

The following procedure calculates *N*, *I%*, *PV*, *PMT*, or *FV* for transactions involving an odd first period. Simple interest accrues during the odd period and is added to the loan at the beginning of the first full period. The equation is valid for 0 to 59 days, and monthly payments with a 30-day month is assumed.*

Entering and Using the ODD Equation:

1. Enter the ODD equation into the Solver.†

   ```
   ODD: PV×(I%÷100×FP(DAYS÷30)+1)=
   -IF(DAYS<30:(1+I%÷100)×PMT:PMT)×
   USPV(I%:N)-FV×SPPV(I%:N)
   ```

2. Display the ODD equation's menu.
3. Store or calculate the following variables. Remember to use the cash-flow sign convention (money paid out is negative, money received is positive).
   - Loan amount in [PV].
   - Periodic interest rate as a percent in [I%].

---

* This procedure duplicates the odd-period calculations (using simple interest) of the HP-12C, with one difference. You do not need to specify Begin or End mode. If the number of days until the first payment is less than 30, Begin mode is assumed. If the number of days until the first payment is between 30 and 59, inclusive, End mode is assumed.

† To key in the < symbol on the HP-17B and HP-27S, press [WXYZ] [OTHER] [<].

- Actual number of days until the first payment is made in [DAYS].
- Periodic payment amount in [PMT].
- Total number of payments in [N].
- Balloon payment amount in [FV]. (The balloon payment occurs at the end of the *N*th period and is in addition to any periodic payment.)

Example 1. A 36-month loan for $4,500 has an interest rate of 15%. If the first payment is made in 46 days, what is the monthly payment amount?

Display the ODD equation's menu.

Keys:	Display:	Description:
36 [N]	N=36.00	Stores known values.
4500 [PV]	PV=4,500.00	
15 [÷] 12 [I%]	I%=1.25	
46 [DAYS]	DAYS=46.00	
0 [FV]	FV=0.00	
[PMT]	PMT=−157.03	Calculates monthly payment amount.

Example 2. A $10,000 loan has 24 monthly payments of $400, plus a balloon payment of $3,000 at the end of the 24th month. If the payments begin in 8 days, what interest rate is being charged?

Keys:	Display:	Description:
10000 [PV]	PV=10,000.00	Stores known values.
24 [N]	N=24.00	
400 [+/−] [PMT]	PMT=−400.00	
3000 [+/−] [FV]	FV=−3,000.00	
8 [DAYS]	DAYS=8.00	

[I%]	I% = 1.64*	Calculates monthly interest rate.
[×] 12 [=]	19.67	Calculates annual interest rate.

* The Solver searches for an iterative solution and displays intermediate estimates.

Loan With a Constant Amount Paid Towards Principal

This type of loan is structured so that the principal is repaid in equal installments, with the interest for each period paid in addition. Therefore, each periodic payment has a constant amount applied toward the principal and a varying amount of interest.

The following equation computes the amount of each periodic payment and the remaining balance.

Entering and Using the CNSTPMT Equation:

1. Enter the CNSTPMT equation into the Solver.

```
CNSTPMT:IF(S(TPMT):CPMT+I%÷100×
(PV-(PMT#-1)×CPMT)-TPMT:
PV-PMT#×CPMT-BAL)
```

2. Display the CNSTPMT equation's menu.
3. Store values in the following variables:
   - Amount of constant payment to principal in ≣ CPMT ≣.
   - Periodic interest rate as a percent in ≣ I% ≣.
   - Loan amount in ≣ PV ≣.
   - Payment number in ≣ PMT# ≣.
4. Press ≣ TPMT ≣ to calculate the total amount of the payment (principal plus interest).
5. Press ≣ BAL ≣ to calculate the balance remaining after the specified payment is made.

Example. A $60,000 loan at 10% has equal semi-annual principal payments of $5,000 for 6 years. What is the amount of each payment in year 1? What is the balance after the eighth payment?

Display the CNSTPMT equation's menu.

Keys:	Display:	Description:
5000 CPMT	CPMT = 5,000.00	Stores known values.
10 ÷ 2 I%	I% = 5.00	
60000 PV	PV = 60,000.00	
1 PMT#	PMT# = 1.00	
TPMT	TPMT = 8,000.00	Calculates first payment.
2 PMT#	PMT# = 2.00	Stores payment number.
TPMT	TPMT = 7,750.00	Calculates second payment.
8 PMT#	PMT# = 8.00	Stores payment number.
BAL	BAL = 20,000.00	Calculates balance after eighth payment.

Rule of 78 Interest Rebate

This procedure calculates the *interest rebate*, as well as the remaining principal balance due for a prepaid consumer loan using the Rule of 78.* The interest rebate is the portion of interest that you save by paying off the loan early. The known values are the current installment number, the total number of installments for which the loan was written, and the total finance charge (amount of interest).

Entering and Using the RULE78 Equation:

1. Enter the RULE78 equation into the Solver.

   ```
   RULE78:IF(S(RBATE):(#MO-PMT#)×
   (2×(#MO-PMT#+1)÷(#MO×(#MO+1))×FCHG)÷2+
   0×PMT-RBATE:(#MO-PMT#)×PMT-RBATE-BAL)
   ```

2. Display the RULE78 equation's menu.
3. Store values in the following variables:
   - Total number of months in loan in ⌊#MO⌋.
   - Payment number when prepayment occurs in ⌊PMT#⌋.
   - Total finance charge in ⌊FCHG⌋.
   - Amount of the monthly payment in ⌊PMT⌋.
4. Press ⌊RBATE⌋ to calculate the rebate amount (the amount of unearned interest).
5. Press ⌊BAL⌋ to calculate the remaining balance.

Example. A 48-month, $8,500 loan having a finance charge of $4,080 is being repaid at $262.08 per month. Calculate the rebate and balance due after the 35th regular payment.

---

* The sum of the numbers 1 through 12 is 78. Although the loan term does not have to be 12 months, the name Rule of 78 is used for this method.

Display the RULE78 equation's menu.

Keys:	Display:	Description:
48 [#MO]	#MO=48.00	Stores known values.
35 [PMT#]	PMT#=35.00	
4080 [FCHG]	FCHG=4080.00	
262.08 [PMT]	PMT=262.08	
[RBATE]	RBATE=315.71	Calculates the unearned interest (rebate).
[BAL]	BAL=3,091.33	Calculates the remaining balance.

Add-On Interest Rates and APR

An add-on interest rate determines what portion of the principal will be added on as the finance charge for a loan. This sum is then divided by the number of months in the loan to determine the monthly payment. For example, a $6,000, 10% add-on rate for one year means that you add 10 percent of $6,000 on to the amount of the loan. This amount is usually called the "finance charge." The total loan amount is $6,600, and the monthly payment is $550.00 (6600 ÷ 12). If the loan is for two years, add a finance charge of $1,200 (600 × 2), giving a monthly payment of $300.00 (7200 ÷ 24).

The following equation converts an add-on interest rate (*RATE*) to an APR (*I%YR*), or converts an APR to an add-on rate. The equation can be modifed to accommodate other than monthly payments by changing the constant 12 to the number of payments per year. In that case, *#MO* would reflect the total number of payments.

Entering and Using the ADDON Equation:

1. Enter the ADDON equation into the Solver.

   ```
   ADDON:#MO÷(1+(#MO÷12)×(RATE÷100))=
   USPV(I%YR÷12:#MO)
   ```

2. Display the ADDON equation's menu.
3. Store or calculate the following variables:
   - Total number of months in the loan in ≣ #MO ≣.
   - Add-on interest rate as a percentage in ≣ RATE ≣.
   - Annual interest rate (as a percent) in ≣ I%YR ≣.

Example 1. What is the equivalent add-on rate for a 24-month loan with an APR of 13.5%?

Display the ADDON equation's menu.

Keys:	Display:	Description:
24 [#MO]	#MO=24.00	Stores known values.
13.5 [I%YR]	I%YR=13.50	
[RATE]	RATE=7.33	Calculates add-on rate.

Example 2: Part 1. Calculate the APR and monthly payment of a 9.5%, $15,000 add-on loan that has a term of 36 months.

Display the ADDON equation's menu.

Keys:	Display:	Description:
36 [#MO]	#MO=36.00	Stores known values.
9.5 [RATE]	RATE=9.50	
[I%YR]	I%YR=17.08*	Calculates APR.
[STO] 0	I%YR=17.08	Stores value in register 0.

Part 2. Once the APR has been calculated, display the TVM menu to calculate the monthly payment:

■ [MAIN] [FIN] [TVM]	17.08	Displays TVM menu.
■ [CLEAR DATA]		Clears TVM variables.
[OTHER] ■ [CLEAR DATA] [EXIT]		Sets 12 payments per year; End mode.
[RCL] 0 [I%YR]	I%YR=17.08	Stores known values.
36 [N]	N=36.00	
15000 [PV]	PV=15,000.00	
[PMT]	PMT=−535.42	Calculates monthly payment.

* The Solver searches for an iterative solution and displays intermediate estimates.

Interest at Maturity Notes

A *note* is a written agreement to pay a sum of money, plus interest, at a certain date. Notes do not have periodic coupons, since all interest is paid at maturity.

The price and yield of a note can be calculated using the following equations. The price is quoted as a percentage. Three calendar options are available: 30/360, actual/360, and actual/actual.*

Entering and Using the Note Equations:

1. Enter the 360NOTE and/or 365NOTE equations into the Solver.
   - For notes on a 30/360 day basis:

     ```
     360NOTE:(DDAYS(ISSUE:SETT:3)+0×MAT)÷
     BASIS×%I+PRICE=(1+(DDAYS(ISSUE:MAT:3)÷
     BASIS)×%I÷100)÷(1+(DDAYS(SETT:MAT:3)÷
     BASIS)×YLD%÷100)×100
     ```

   - For notes on an actual/360 or actual/actual day basis:

     ```
     365NOTE:(DDAYS(ISSUE:SETT:1)+0×MAT)÷
     BASIS×%I+PRICE=(1+(DDAYS(ISSUE:MAT:1)÷
     BASIS)×%I÷100)÷(1+(DDAYS(SETT:MAT:1)÷
     BASIS)×YLD%÷100)×100
     ```

2. Display the menu for the appropriate equation.
3. Store values in the following variables:
   - Issue date in ISSUE.
   - Settlement date in SETT.
   - Maturity date in MAT.

---

* If you are not sure of the calendar basis for your particular note, refer to *Standard Securities Calculation Methods*, Lynch and Mayle, Securities Industry Association, New York, NY, 1986.

- Number of days in a calendar year (either 360 or 365) in [BASIS].*

4. Do **a** or **b**.

   a. To calculate the price, store the annual yield (as a percentage) in [YLD%] and press [PRICE].

   b. To calculate the annual yield, store the purchase price in [PRICE] and press [YLD%].

Example 1: Price of Tax Exempt Note. Calculate the price of the following tax exempt note. Assume a 30/360 day calendar basis.

Issue date: January 4, 1988.
Settlement date: March 21, 1988.
Maturity date: July 5, 1988.
Interest: 5%.
Yield: 6.2%.

Display the 360NOTE equation's menu.

Keys:	Display:	Description:
1.041988 [ISSUE]	ISSUE=1.04	Stores known values.
3.211988 [SETT]	SETT=3.21	
7.051988 [MAT]	MAT=7.05	
360 [BASIS]	BASIS=360.00	
5 [%I]	%I=5.00	
[MORE]		
6.2 [YLD%]	YLD%=6.20	
[PRICE]	PRICE=99.64	Calculates note price.

* For notes on a 30/360 or actual/360 day basis, store 360 in [BASIS]. For notes on an actual/actual day basis, store 365 in [BASIS]. You may wish to modify the above equations to eliminate the *BASIS* variable.

Example 2: Yield of Tax Exempt Note. If the note in the previous example was purchased for 99.5, what is the annual yield?

Continuing from example 1.

Keys:	Display:	Description:
99.5 PRICE	PRICE=99.50	Stores purchase price.
YLD%	YLD%=6.69	Calculates note yield.

Discounted Notes

A *discounted note* is a note that is purchased below its face value. In other words, the note is purchased at a discount. The face value of the note, plus interest, is paid at maturity.

The following equations find the price and/or yield of a discounted note. The equations assume that the calendar basis is actual/360.

Entering and Using the Discounted Note Equations:

1. Enter the DPRICE and DYIELD equations into the Solver.

   ```
   DPRICE:PRICE=RV-DISC×RV×DDAYS(SETT:
   MAT:1)÷36000
   ```

   ```
   DYIELD:YLD%=(RV-PRICE)÷PRICE×36000÷
   DDAYS(SETT:MAT:1)
   ```

2. If you need to calculate price, display the DPRICE equation's menu. Store values in the following variables:
   - Redemption value per $100 in RV.
   - Discount rate as a percent in DISC.
   - Settlement date (in MM.DDYYYY format) in SETT.
   - Maturity date (in MM.DDYYYY format) in MAT.

   Press PRICE to calculate the purchase price.

3. Display the DYIELD equation's menu. If you did not just calculate *PRICE*, store *RV, SETT,* and *MAT* as described in step 2. Also store the *PRICE*.

4. Press YLD% to calculate the yield.

Example 1: Treasury Bill. Calculate the price and yield of the following U.S. Treasury Bill: settlement date October 14, 1987; maturity date March 17, 1988; discount rate 8.7%.

Display the DPRICE equation's menu.

Keys:	Display:	Description:
10.141987 SETT	SETT=10.14	Stores known values.
3.171988 MAT	MAT=3.17	
8.7 DISC	DISC=8.70	
100 RV	RV=100.00	
PRICE	PRICE=96.25	Calculates price.
EXIT ↓ CALC		Displays DYIELD equation's menu.
YLD%	YLD%=9.04	Calculates yield.

Example 2: Discounted Note. Determine the yield of the following discounted note: settlement date June 25, 1987; maturity date September 10, 1987; price 99.45; redemption value 101.33.

Display the DYIELD equation's menu.

Keys:	Display:	Description:
6.251987 SETT	SETT=6.25	Stores known values.
9.101987 MAT	MAT=9.10	
99.45 PRICE	PRICE=99.45	
101.33 RV	RV=101.33	
YLD%	YLD%=8.84	Calculates yield.

3

Leasing

Leasing Calculations

Leasing calculations typically call for payments in advance. For one payment in advance, you can simply use Begin mode, since this puts payments at the beginning of each period. The first payment is therefore due "in advance" - that is, when the lease is initiated. For more than one payment in advance, use the ADVPMT equation on page 126.

This part gives examples typical in the leasing profession. Since the Time Value of Money (TVM) menu is used, remember the cash-flow sign convention (money paid out is entered as a negative number, money received is entered as a positive number).

1. Clear the TVM variables, store the number of payments per year in [P/YR], and set Begin mode.
2. Store values in at least three of the following variables. (Both *N and I%YR* must be a part of a problem. Either both values are known, or one is known and the other is to be computed.)
   - Number of periodic payments in [N].
   - Annual yield to the lessor in [I%YR].
   - Initial value of the leased item in [PV].
   - Periodic payment in [PMT].
   - Residual value of the leased item in [FV].
3. Press the menu key of the unknown variable to calculate its value.

Example 1: Calculate Lease Payment. A piece of machinery valued at $100,000 is being leased for 5 years. At the end of the leasing period, the machinery can be purchased for $20,000. What monthly payment is necessary to yield the lessor 15% annually. The first payment is due at the inception of the lease.

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER 12 P/YR BEG EXIT		Sets 12 payments per year; Begin mode.
100000 +/− PV 5 × 12 N 20000 FV 15 I%YR	PV = −100,000.00 N = 60.00 FV = 20,000.00 I%YR = 15.00	Stores known values.
PMT	PMT = 2,126.61	Calculates monthly lease payment.

Part 2. If the purchase price in 5 years is reduced to $10,000, what monthly payment is necessary to yield the lessor 15%?

10000 FV	FV = 10,000.00	Stores new purchase option.
PMT	PMT = 2,238.12	Calculates monthly lease payment.

Example 2: Calculate Annual Interest Rate. A $11,900 truck can be leased for 60 months at $214.10 per month. At the end of the leasing period, the truck may be purchased for $2,600. If the purchase option is exercised, what interest rate is the lessor earning? The first payment is due today.

Keys:	Display:	Description:
TVM		Displays TVM menu.
■ CLEAR DATA		Clears TVM variables.
OTHER 12 P/YR BEG EXIT		Sets 12 payments per year; Begin mode.

11900 [+/-] [PV]	PV = −11,900.00	Stores known values.
60 [N]	N = 60.00	
214.10 [PMT]	PMT = 214.10	
2600 [FV]	FV = 2,600.00	
[I%YR]	I%YR = 9.48	Calculates annual interest rate.

Example 3: Calculate Value of Lease. A 24-month equipment lease is available for $600 per month. The lease also includes a $1,000 buy-back option at the end of the lease term. Assuming a 14% interest rate, what is the value of the lease? The first payment is due today.

Keys:	Display:	Description:
[TVM]		Displays TVM menu.
■ [CLEAR DATA]		Clears TVM variables.
[OTHER] 12 [P/YR] [BEG] [EXIT]		Sets 12 payments per year; Begin mode.
24 [N]	N = 24.00	Stores known values.
600 [+/-] [PMT]	PMT = −600.00	
1000 [+/-] [FV]	FV = −1,000.00	
14 [I%YR]	I%YR = 14.00	
[PV]	PV = 13,399.45	Calculates value of lease.

Example 4: Calculate Residual Value. An asset worth $30,000 is being leased for 4 years at $700 per month, with an option to buy at the end of the lease. If the lessor wishes at least a 12% yield, what is the minimum value of the buy-back option? The first payment is due today.

Keys:	Display:	Description:
[TVM]		Displays TVM menu.
■ [CLEAR DATA]		Clears TVM variables.
[OTHER] 12 [P/YR] [BEG] [EXIT]		Sets 12 payments per year; Begin mode.

30000 [+/−] [PV]	PV = −30,000.00	Stores known values.
4 [×] 12 [N]	N = 48.00	
700 [PMT]	PMT = 700.00	
12 [I%YR]	I%YR = 12.00	
[FV]	FV = 5,082.40	Calculates residual value.

Part 2. If the asset is purchased in 4 years for $6,200, what is the yield to the lessor?

6200 [FV]	FV = 6,200.00	Stores purchase option.
[I%YR]	I%YR = 13.12	Calculates annual yield.

Example 5: Calculate Lease Term. A $9,000, 10% lease has monthly payments of $245. How many payments remain? The first payment was made at the inception of the lease.

Keys:	Display:	Description:
[TVM]		Displays TVM menu.
■ [CLEAR DATA]		Clears TVM variables.
[OTHER]		Sets 12 payments per year; Begin mode.
12 [P/YR] [BEG]		
[EXIT]		
10 [I%YR]	I%YR = 10.00	Stores known values.
9000 [PV]	PV = 9,000.00	
245 [+/−] [PMT]	PMT = 245.00	
[N]	N = 43.60	Calculates number of remaining payments.

Multiple Advance Payments

Situations may exist in which one or more payments are made in advance (leasing is a good example). These agreements call for the extra payments to be made when the transaction is closed. A residual value (salvage value) can exist at the end of the normal term.

The following equation calculates the monthly payment amount (*PMT*) *and the annual yield* (*I%YR*) when one or more payments are made in advance. The equation can be modified to accommodate other than monthly payments by changing the constant 12 to the number of payments per year. In that case, *PMT, N,* and *#ADV* would apply to the periodic payment. Remember to use the cash-flow sign convention (money paid out is negative, money received is positive).

Entering and Using the ADVPMT Equation:

1. Enter the ADVPMT equation into the Solver.

   ```
   ADVPMT:PMT=(-PV-FV×(SPPV(I%YR÷12:N)))÷
   (USPV(I%YR÷12:N-#ADV)+#ADV)
   ```

2. Display the ADVPMT equation's menu.
3. Store or calculate the following variables:
   - Monthly payment amount in PMT.
   - Loan amount in PV.
   - Amount of the balloon payment in FV.
   - Annual interest rate as a percent in I%YR.
   - Total number of monthly payments in N.
   - Number of monthly payments made in advance in #ADV.

Example 1. Equipment worth $750 is leased for 12 months. The equipment is assumed to have no salvage value at the end of the lease. The lessee has agreed to make three payments at the time of closing. What monthly payment is necessary to yield the lessor 10% annually?

Display the ADVPMT equation's menu.

Keys:	Display:	Description:
750 [+/-] [PV]	PV = −750.00	Stores known values.
12 [N]	N = 12.00	
0 [FV]	FV = 0.00	
3 [#ADV]	#ADV = 3.00	
10 [I%YR]	I%YR = 10.00	
[PMT]	PMT = 64.45	Calculates monthly payment.

Example 2. Continuing from example 1: what is the payment amount if the yearly interest rate is 15%?

Keys:	Display:	Description:
15 [I%YR]	I%YR = 15.00	Stores new interest rate.
[PMT]	PMT = 65.43	Calculates monthly payment to achieve 15% yield.

Example 3. A lease has been written to run for 60 months with a monthly payment of $600. The leased equipment has a value of $25,000. The lessee has agreed to make 3 payments at the time of closing. What is the annual yield to the lessor?

Keys:	Display:	Description:
60 [N]	N = 60.00	Stores known values.
3 [#ADV]	#ADV = 3.00	
600 [PMT]	PMT = 600.00	
25000 [+/-] [PV]	PV = −25,000.00	
0 [FV]	FV = 0.00	

I%YR	I%YR=17.33*	Calculates percent annual yield.

Example 4. Equipment worth $5,000 is leased for 36 months at $145 per month. The lessee has agreed to pay the first and last payments in advance. At the end of the lease, the equipment can be purchased for $1,500. What is the annual yield to the lessor if the equipment is purchased?

Keys:	Display:	Description:
5000 +/- PV	PV=-5,000.00	Stores known values.
36 N	N=36.00	
145 PMT	PMT=145.00	
2 #ADV	#ADV=2.00	
1500 FV	FV=1,500.00	
I%YR	I%YR=18.10*	Calculates annual yield to lessor.

* The Solver searches for an iterative solution and displays intermediate estimates.

Skipped Payments

Note

This procedure cannot be used on the HP-27S.

Sometimes a loan (or lease) can be negotiated in which a specific number of monthly payments are going to be skipped each year. Seasonality is usually the reason for such an agreement. For example, because of heavy rainfall, a bulldozer cannot be operated in Oregon during December, January, and February. A lessee wishes to make payments only when his machinery is being used. He makes nine payments per year, but the interest continues to compound during the months in which a payment is not made.

The following procedure calculates the monthly payment amount necessary to amortize the loan in the specified amount of time. The only restriction is that the term of the loan must be an integral number of years.

1. Display the cash-flow menu.
2. For the first year of the transaction, enter \$1.00 for each payment that is made, and \$0 for each skipped payment. Store the periodic interest rate in *I%* and calculate *NFV*. (This is an equivalent annual cash flow that occurs at the end of the first year.) Store the result in register 0.
3. Display the interest conversion menu (ICNV) and calculate the effective annual interest rate, using the annual interest rate as *NOM%*.
4. Display the TVM menu, store the effective interest rate in [I%YR], store 1 in [P/YR], and set End mode.
5. Store the total number of years in [N], the value from register 0 in [PMT], and 0 in [FV]
6. Press [PV] to calculate the present value of the annualized payments.

7. Key in the loan amount and press [÷] [RCL] [PV] [=] to calculate the monthly payment amount.

Reference: Greynolds, Aronofsky, Frame, *Financial Analysis Using Calculators*, McGraw-Hill, 1980.

Example. A bulldozer worth $100,000 is purchased in September. The first payment is due 1 month later, and payments continue for 5 years. Because of the weather, the machinery will not be used during the winter months, and the purchaser does not wish to make payments during January, February, and March (months 4 thru 6). If the annual interest rate is 14%, what monthly payment is necessary to amortize the loan?

Keys:	Display:	Description:
[CFLO]		Displays the CFLO menu.
■ [CLEAR DATA] [YES] *or* [GET] [*NEW]	FLOW(0)=?*	Clears the current cash-flow list or gets a new one.
0 [INPUT]	FLOW(1)=?*	Stores 0 as initial cash flow.
1 [INPUT] 3 [INPUT]	FLOW(2)=?	Stores first group of cash flows.
0 [INPUT] 3 [INPUT]	FLOW(3)=?	Stores second group of cash flows.
1 [INPUT] 6 [INPUT]	FLOW(4)=?	Stores third group of cash flows.

The next step (pressing [EXIT]) is necessary if you have the HP-17B.
[EXIT]

---

* On the HP-19B, these prompts are: INIT= and FLOW(1)=.

CALC		Displays CALC menu.
14 ÷ 12 I%	I% = 1.17	Stores monthly discount rate.
NFV	NFV = 9.55	Calculates net future value of cash flows in first year.
STO 0	NFV = 9.55	Stores *NFV* in register 0.
EXIT EXIT ICNV		Displays ICNV menu.
PER		Displays PER menu.
14 NOM%	NOM% = 14.00	Stores known values.
12 P	P = 12.00	
EFF%	EFF% = 14.93	Calculates effective annual interest rate.
EXIT EXIT TVM		Displays TVM menu.
STO I%YR	I%YR = 14.93	Stores annual interest rate.
OTHER 1 P/YR END EXIT		Stores 1 payment per year and sets End mode.
5 N	N = 5.00	Stores known values.
RCL 0 PMT	PMT = 9.55	
0 FV	FV = 0.00	
PV	PV = −32.05	Calculates present value of $1 cash flows.
100000 ÷ RCL PV =	−3,119.89	Calculates monthly payment amount.

A

Equations Used

Graduated-Payment Mortgages

$$GPMT = PV \div \left\{ \left[\frac{1 - (1 + I/1200)^{-12}}{I/1200} \right] \left[\frac{\left(\frac{1 + \%I/100}{(1 + I/1200)^{12}} \right)^{\#YR} - 1}{\frac{1 + \%I/100}{(1 + I/1200)^{12}} - 1} \right] + \frac{(1 + \%I/100)^{\#YR} \left[\frac{1 - (1 + I/1200)^{-[N - (12 \times \#YR)]}}{I/1200} \right]}{(1 + I/1200)^{12 \times \#YR}} \right\}$$

where:
PV = loan amount
$\%I$ = percentage increase in monthly payment
I = annual interest rate (as a percent)
$\#YR$ = number of years that payments increase
N = total number of payments
$GPMT$ = monthly payment

Reference:

Greynolds, Aronofsky, and Frame, *Financial Analysis Using Calculators,* McGraw-Hill Book Company, 1980.

Wrap-Around Loans

$$WRAP - \frac{WPMT\left[1-\left(1+\frac{\%YLD}{1200}\right)^{-(W\#N)}\right]}{\frac{\%YLD}{1200}} - WBAL\left(1+\frac{\%YLD}{1200}\right)^{-(W\#N)} =$$

$$UND - \frac{UPMT\left[1-\left(1+\frac{\%YLD}{1200}\right)^{-(U\#N)}\right]}{\frac{\%YLD}{1200}} - UBAL\left(1+\frac{\%YLD}{1200}\right)^{-(U\#N)}$$

where:
$WRAP$ = total amount of wrap-around loan
$WPMT$ = wrap-around loan monthly payment
$\%YLD$ = annual yield to lender (as a percent)
$W\#N$ = number of monthly payments in wrap-around loan
$WBAL$ = wrap-around loan balloon payment
UND = remaining balance of underlying loan
$UPMT$ = underlying loan monthly payment
$U\#N$ = number of monthly payments in underlying loan
$UBAL$ = underlying loan balloon payment

Canadian Mortgages

$$PV = -PMT\left[\frac{1-(1+r)^{-N}}{r}\right] - FV\,(1+r)^{-N}$$

where: $$r = \left[\left(1+\frac{CI\%YR}{200}\right)^{1/6} - 1\right]$$

N = total number of monthly payments
$CI\%YR$ = annual interest rate (as a percent)
PV = loan amount
PMT = monthly payment
FV = balloon payment

Modified Internal Rate of Return

$$MIRR = 100\left[\left(\frac{NFV_P}{-NPV_N}\right)^{1/n} - 1\right]$$

where:
n = total number of compounding periods
NFV_P = net future value of positive cash flows
NPV_N = net present value of negative cash flows

Increasing Annuities

$$PVINCR = PMT\left[\frac{1 - \left(1 + \frac{I\%YR}{100 \times P/YR}\right)^{-P/YR}}{\frac{I\%YR}{100 \times P/YR}}\right] \times \left[\frac{\left(\frac{1 + \frac{\%INC}{100}}{\left[1 + \left(\frac{I\%YR}{P/YR \times 100}\right)\right]^{\#PER}}\right)^{\frac{\#YRS \times P/YR}{\#PER}} - 1}{\frac{1 + \frac{\%INC}{100}}{\left[1 + \left(\frac{I\%YR}{P/YR \times 100}\right)\right]^{\#PER}} - 1}\right]$$

$$FVINCR = PVINCR\left(1 + \frac{I\%YR}{100 \times P/YR}\right)^{\#YRS \times P/YR}$$

where:
$PVINCR$ = present value of increasing annuity
PMT = periodic payment amount
$I\%YR$ = annual interest rate (as a percent)
P/YR = number of payments per year
$\#PER$ = number of periods at a given interest rate
$\%INC$ = percentage increase in payment
$\#YRS$ = total number of years
$FVINCR$ = future value of increasing annuity

Odd-Period Calculations

$$PV\left[1 + i \times \frac{DAYS}{30}\right] = -(1 + i \times S) \times PMT \times \left[\frac{1 - (1 + i)^{-N}}{i}\right] - FV(1 + i)^{-N}$$

where:
PV = loan amount
i = periodic interest rate as a decimal
$DAYS$ = actual number of days until the first payment
PMT = periodic payment amount
N = total number of payments
FV = balloon payment amount
$S = 1$ if $DAYS < 30$
$S = 0$ if $DAYS \geq 30$

Loan With a Constant Amount Paid Towards Principal

$$TPMT = CPMT + \frac{I\%}{100}(PV - CPMT\ (PMT\# - 1))$$

$$BAL = PV - CPMT\ (PMT\#)$$

where:
$TPMT$ = total payment amount
$CPMT$ = amount of constant payment to principal
$I\%$ = periodic interest rate as a percent
PV = loan amount
$PMT\#$ = payment number
BAL = balance remaining

Rule of 78 Interest Rebate

$$RBATE = (\#MO - PMT\#)\left(\frac{2\ (\#MO - PMT\# + 1)}{\#MO\ (\#MO + 1)}\right)\left(\frac{FCHG}{2}\right)$$

$$BAL = (\#MO - PMT\#)\ PMT - RBATE$$

where:
$RBATE$ = rebate amount (unearned interest)
$\#MO$ = number of months
$PMT\#$ = payment number where prepayment occurs
$FCHG$ = total finance charge

Add-On Interest Rates and APR

$$\frac{\#MO}{\left(1 + \frac{\#MO}{12}\right)\left(\frac{RATE}{100}\right)} = \frac{1 - \left(1 + \frac{I\%YR}{1200}\right)^{\#MO}}{\frac{I\%YR}{1200}}$$

where: $\#MO$ = total number of months
$RATE$ = add-on rate as a percent
$I\%YR$ = APR (annual percentage rate)

Advance Payments

$$PMT = \frac{-PV - FV\,(1 + i)^{-N}}{\left[\frac{1 - (1 + i)^{-(N - \#ADV)}}{i} + \#ADV\right]}$$

where: PMT = payment amount
PV = loan amount
FV = balloon payment amount
i = periodic interest rate (as a decimal)
N = total number of payments
$\#ADV$ = number of payments made in advance

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