HP-80
APPLICATION NOTES
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ANNUAL PERCENTAGE RATE (APR) FOR LOANS WITH A BALLOON PAYMENT; RATE OF RETURN FOR EVEN CASH FLOWS TERMINATED BY AN UNEVEN CASH FLOW

GENERAL

This note will be useful to the HP-80 user wishing to calculate the ***Annual Percentage Rate*** (APR, and sometimes called the actuarial rate) ***of loans terminated by a balloon payment, or the rate of return (often called yield) of a series of even cash flows terminated by an uneven cash flow.***

The following symbolic values are used in demonstrating the general keystroke solutions.

A = number of payment periods in a year

B = number of years (and/or fraction of a year as appropriate)

C = Annual Percentage Rate (APR), or rate of return expressed as a percent

D = periodic payment amount (cash flow)

E = initial principal amount of loan (initial investment)

G = balloon payment amount or remaining balance

ANNUAL PERCENTAGE RATE FOR LOANS WITH A BALLOON PAYMENT

When the initial principal (E), total number of payments (A × B), payment amount (D) and balloon payment (G) amount are known, the HP-80 can be used to solve for the Annual Percentage Rate (APR) of the loan. These keystrokes are valid regardless of the way in which the loan was initially constructed (i.e. direct reduction, add-on rate etc.)

Keystrokes (when the balloon payment occurs in the same time period as the last payment)

1. Divide the balloon by 100 and store the result.

   G [SAVE ↑] 100 [÷] [STO]

2. Calculate the number of payments, multiply by 365, divide by 2 and press [n].

   A [SAVE ↑] B [×] 365 [×] 2 [÷] [n]

3. Multiply the payment amount by 2, divide by the number stored in step 1. and press [PMT].

   D [SAVE ↑] 2 [×] [RCL] [÷] [PMT]

4. Divide the loan amount by the number stored in step 1. and press [PV].

   E [RCL] [÷] [PV]

5. Calculate the APR.

   (gold key) [i] A [×] 2 [÷] ——→ C

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Example:

Find the annual interest rate (APR) on a $2100 (E) loan requiring equal monthly (A = 12) payments of $42.52 (D) for 2 years (B) and a balloon payment of $1500 (G) due in addition to the final payment at month 24.

Procedure:		*See Displayed:*
1.	1500 [SAVE ↑] 100 [÷] [STO]	15.00
2.	12 [SAVE ↑] 2 [×] 365 [×] 2 [÷] [n]	4380.00
3.	42.52 [SAVE ↑] 2 [×] [RCL] [÷] [PMT]	5.67
4.	2100 [RCL] [÷] [PV]	140.00
5.	(gold key) [i] 12 [×] 2 [÷]	11.53% (APR)

Keystrokes: (when the balloon payment occurs 1 period after the last payment).

1. Subtract one payment from the balloon payment, divide by 100 and store the result.
   G [SAVE ↑] D [−] 100 [÷] [STO]
2. Calculate the total number of payments, including the balloon payment, multiply by 365, divide by 2 and press [n].
   A [SAVE ↑] B [×] 1 [+] 365 [×] 2 [÷] [n]
3. Enter payment amount, multiply by 2, divide by the number stored in step 1. and press [PMT].
   D [SAVE ↑] 2 [×] [RCL] [÷] [PMT]
4. Enter the loan amount, divide by the number stored in step 1. and press [PV].
   E [RCL] [÷] [PV]
5. Solve for the APR.
   (gold key) [i] A [×] 2 [÷] → C

Example:

Assuming that the balloon payment of $1500 in the previous example was due one month after the last payment (i.e. at month 25), and all other factors remained the same, what would the APR be?

Procedure:		*See Displayed:*
1.	1500 [SAVE ↑] 42.52 [−] 100 [÷] [STO]	14.57
2.	12 [SAVE ↑] 2 [×] 1 [+] 365 [×] 2 [÷] [n]	4562.50
3.	42.52 [SAVE ↑] 2 [×] RCL [÷] [PMT]	5.83
4.	2100 [RCL] [÷] [PV]	144.08
5.	(gold key) [i] 12 [×] 2 [÷]	11.19% (APR)

RATE OF RETURN

The keystrokes below calculate the rate of return for situations that involve some initial payment followed by even cash flows (equal amounts, equally spaced in time) and a final uneven cash flow (balloon payment) coincident with the last payment.

Keystrokes:

The keystrokes shown here are exactly the same as those shown for calculating the APR of a loan with a balloon payment when the balloon payment occurs at the same time as the last periodic payment.

1. G [SAVE ↑] 100 [÷] [STO]
2. A [SAVE ↑] B [×] 365 [×] 2 [÷] [n]
3. D [SAVE ↑] 2 [×] [RCL] [÷] [PMT]
4. E [RCL] [÷] [PV]
5. (gold key) [i] A [×] 2 [÷] ——→ C

Example:

Assume that something (property, rights to a franchise, etc.) is purchased for $200,000 (E), and that it generates an annual income (A = 1) of $12,000 (D) for 6 (B) years. If it is sold at the end of the 6 years for $300,000 (G), what is the annual rate of return?

Procedure:		***See Displayed:***
1.	300000 [SAVE ↑] 100 [÷] [STO]	3000.00
2.	1 [SAVE ↑] 6 [×] 365 [×] 2 [÷] [n]	1095.00
3.	12000 [SAVE ↑] 2 [×] [RCL] [÷] [PMT]	8.00
4.	200000 [RCL] [÷] [PV]	66.67
5.	(gold key) [i] 1 [×] 2 [÷]	12.14% (rate of return)

Notes:

1. When calculating the rate of return, if the periodic payments (D) represent a cash outflow instead of an income this value would be entered as a negative number.

2. The HP-80 yield to maturity calculation for bonds is used to solve the problems in this issue. Solving for APR or rate of return is analogous to solving for bond yield. The property price or initial loan amount corresponds to the bond price. Income from property or loan payments correspond to bond coupons, and the balloon payment on a loan, or sale price in the rate of return calculation compares to the redemption (face) value of a bond.

   The HP-80 bond calculations have built-in assumptions which are specifically tied to bond calculations. Some of these are:

   - Bond coupons are paid semiannually
   - Time is entered in days
   - Bond price is expressed as a percent of redemption value

   To change these assumptions and use another set of conditions, the data must be adjusted. This is the reason for all the numerical data entries (i.e., 365 [×], 2 [÷]) and the [RCL] [÷] sequence in the general symbolic keystroke solutions shown.

3. Since the bond yield algorithm is being used, the same operating limits as expressed in Appendix D of the *HP-80 Owner's Handbook* would apply. For these applications these limits can be expressed as follows:

   The absolute value of the number entered for [PMT] ***must be greater than .125 and less than the value entered for*** [PV] ***. The value entered for*** [PV] ***must be greater than 20 and less than 5000.***

4. It should be noted that for the calculations in this note the periodic payments (D) are assumed to occur ***at the end*** of the payment periods (ordinary annuity).