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REAL ESTATE APPLICATIONS HANDBOOK

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HP-12C

Real Estate Applications Handbook

Mark H. Goldman
Stephen D. Messner

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Preface

The purpose of this handbook is to provide a set of programs and keystroke procedures for the HP-12C calculator that will assist real estate practitioners in making real estate investment decisions. The programs and procedures in this handbook can be used by brokers, investors, appraisers, analysts, and others who need to make financial decisions about real estate transactions. Most of the routines in this handbook involve the estimation of the value of investment real estate and the measurement of investment performance. These techniques are also useful for structuring real estate transactions and comparing investment alternatives.

To make effective use of the procedures and programs in this handbook, you will need to be familiar with the operation and programming of the HP-12C Programmable Financial Calculator and you should also have access to the *HP-12C Owner's Handbook and Problem-Solving Guide* and the *HP-12C Solutions Handbook.*

Contents

Introduction 6

Section 1: Real Estate Financing Analysis 8
- Basic Mortgage Components 8
- Mortgage Measure Calculations 10
  - Debt Coverage Ratio 10
  - Loan Payment Amount Using *DCR* 11
  - Loan Amount Using *DCR* 11
  - Annual Loan Constant 12
  - Loan Interest Rate for a Known *ALC* 13
  - Calculating *n* From the Annual Loan Constant 14
  - Loan Amount Using *ALC* and *DCR* 15
  - Loan Payment From the *ALC* 15
- Partially Amortized Loan Calculations 16
  - Balloon Payment 16
  - Payment Required for a Specified Balloon 17
  - Remaining Balance of a Partially Amortized Loan 18
- Yield on a Loan With Fees or a Prepayment Penalty 18
- Price to Pay for a Mortgage 20
- Discount Point Calculations 21
  - Number of Discount Points on a Loan 21
  - Number of Discount Points and Net Loan Proceeds 22
  - Annual Percentage Rate of a Loan With Discount Points 22
- Loan Amortization Program 25
- Before-Tax Yield on Wrap-Around Loans 26
  - Yield on the Wrap-Around Lender's Net Investment 27
  - Wrap-Around Loan Interest Rate 30
- Quick Estimate of After-Tax Cash Flows 34
- After-Tax Yield to the Wrap-Around Lender 38
- Refinancing Versus a Second Mortgage 39
- Participation Loan Calculations 42
  - Estimating Debt Service for an Income Participation Loan 43
  - Lender's Yield on a Participation Loan 45

Section 2: Appraisal Applications 48
- Mortgage-Equity (Ellwood) Analysis 48

Band of Investment Analysis 52
Appreciation/Depreciation Required for a Specified Equity Yield 54
Equity Yield Rate 57
Compound Interest and Discount Factors 59

Section 3: Investment Analysis 62
Cash Flow Before Taxes or Cash Throw Off 62
Cash Flow After Taxes 65
The Accelerated Cost Recovery System 69
Percentage Lease Rent 71
Present Value of Cash Flows or Investment Value 72
Present Value of an Increasing Annuity 73
Payments of an Increasing Annuity 76
Investment Value of Unequal Cash Flows 77

Section 4: Differential Cash Flow Analysis 79

Section 5: Financial Management Rate of Return 82
Calculating *FMRR* 82
Using *FMRR* 85

Section 6: Miscellaneous Feasibility Measures 89
Cash-On-Cash or Equity Dividend Rate 89
Gross Income Multiplier or Gross Rent Multiplier 89
Calculating the Gross Income Multiplier 90
Estimating Price Using the Gross Rent Multiplier 90
Overall Capitalization Rate 91
Equity Return Rate 91
Profitability Index 92

Appendix A: Calculation of the Number of Payments or Compounding Periods 94
Using the [n] Key 94
Computing the Exact Value of *n* 95

Appendix B: Accelerated Cost Recovery System Tables 98
Real Estate Tables 98
Low Income Housing 99
All Other Real Estate 99
Personal Property Tables 100
For Property Placed in Service from 1981 to 1984 100
For Property Placed in Service in 1985 101
For Property Placed in Service After December 31, 1985 .. 102

Introduction

This applications handbook is intended to supplement the *HP-12C Owner's Handbook and Problem-Solving Guide* and the *HP-12C Solutions Handbook* (referred to later in this handbook as the owner's handbook and the solutions handbook respectively). We will refer you to these two books when a topic is discussed that we can't fully develop here.

Where there are applications in this book that are similar to those in the other two books, we will give a short discussion of the relationships. We invite and encourage you to experiment with the various solutions available so that you can learn which methods are most efficient and useful for you. Without attempting to provide a textbook on the topics discussed, we will attempt to provide enough explanatory detail so that you can select the most appropriate method.

Using Calculator Programs

Many of the applications in this handbook involve a calculator program. These programs can save you the drudgery of repeating long sequences of keystrokes for repetitive problems. Also, once the program is correctly stored in the calculator, it will reduce the possibility of errors caused by pressing the wrong key at some point in the calculation. Refer to section 8 of the owner's handbook for essential information about entering and using programs. Remember to check whether you have entered the program correctly by working the examples that accompany the programs before using them for your own problems.

Conventions

Several applications in this book involve taxation. These routines calculate an estimate of taxes for generalized cases, not actual tax liabilities.

The HP-12C cash flow sign convention is used throughout this book. (Refer to page 41 in the owner's handbook.) Cash receipts are assumed to carry a positive sign and cash expenditures are assumed to carry a negative sign. Be sure to always use the correct

cash flow signs when using the procedures and programs in this handbook. Also, unless otherwise stated in the context of a particular problem, all cash flows are assumed to occur at the *end* of the period (press [g] [END]) and the display format is rounded to two decimal places (press [f] [2]).

There are many real estate financing problems in which a period other than one year is used. In virtually all of these cases, the interest rate is originally stated as an annual rate, or—when the interest rate is to be calculated—an annual rate is desired at the end. Remember that in all financial calculations used in this book, the period and interest rate must match within a problem—whether it is months, calendar quarters, or years—so be sure to convert when necessary.

We are aware that conversion of monthly interest rates is a topic of some debate and is not universally accomplished by a single method. The method that we use to convert a periodic rate to an annual rate is the conventional method of multiplying the periodic rate by the number of periods per year. We call the result, unless specifically noted otherwise, the "annual interest rate." For example, a monthly interest rate of 1.3% equals an annual interest rate of 15.6%. In the examples and problems we will refer to this interest rate as either the "investor's yield" or the "borrower's interest rate cost," depending on the viewpoint of the transaction. We will not obscure the discussion by using the adjective "effective," which is often used in a different sense.

Technical Assistance

The keystroke procedures and program material in this handbook are supplied with the assumption that the user has a working knowledge of the concepts and terminology used. Hewlett-Packard's technical support of this product is limited to explanations of operating procedures used in the handbook and verification of answers given in the examples. If you have technical problems when using this handbook, consult your *HP-12C Owner's Handbook and Problem-Solving Guide.* Should you need further assistance, you may write to:

Hewlett-Packard
Corvallis Division Customer Support
1000 N.E. Circle Blvd.
Corvallis, OR 97330

Section 1

Real Estate Financing Analysis

Basic Mortgage Components

Many of the analytical techniques illustrated in this book require that you know certain basic mortgage components. For a particular problem certain of these values may not be known. However, if any three elements are known (four for mortgages with balloon payments) the remaining unknown value may be calculated. Each of these calculations is illustrated in the owner's manual (section 3, "Basic Financial Functions").

We have summarized the basic financial functions below for quick reference.

Storing Financial Data
(Use correct cash flow sign.)

Key	Value Stored	Register
[n]	Number of payments (n)	n
[i]	Periodic interest rate as a percent (i)	i
[PV]	Initial loan balance (PV)	PV
[PMT]	Periodic payment (PMT)	PMT
[FV]	Future value or balloon payment (FV)	FV

Unknown Value	Known Values Required to Solve
[n]*	[i] [PV] [PMT] [FV]†
[i]	[n]* [PV] [PMT] [FV]†
[PV]	[n]* [i] [PMT] [FV]†
[PMT]	[n]* [i] [PV] [FV]†
[FV]	[n]* [i] [PV] [PMT]

* Refer to appendix A for further information on the calculation of n.

† FV will be zero if there is no balloon payment; refer to "Partially Amortized Loan Calculations," page 16.

Example 1: A broker lists a property that has an assumable loan. The original loan amount was $150,000 at 7% annual interest, fully amortized with monthly payments for 25 years. The loan was taken 11 years and 8 months ago. What is the loan balance?

Solution: The broker needs to know the monthly payment to calculate the loan balance. The known basic mortgage components are the original term (n), the periodic interest (i), and the original balance (PV). The monthly payment (PMT) is unknown. We can calculate the balance as a balloon payment (FV) due after 11 years and 8 months. To compute the balloon payment the broker must calculate the monthly payment, then he can use the four known values (n, i, PV, and PMT) to compute the balloon payment.

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
25 [g] [12×]	300.00	Mortgage term.
7 [g] [12÷]	0.58	Interest rate.
150000 [PV]	150,000.00	Original loan amount.
[PMT]	-1,060.17	Monthly payment.
11 [ENTER]	11.00	
12 [×]	132.00	
8 [+] [n]	140.00	Number of payments made.
[FV]	-110,080.32	Loan balance.

Example 2: A property has an existing loan of $100,000 with monthly payments of $1,127.84 for 25 years. What is the annual interest rate of the loan?

Solution: The interest rate is the unknown value. The loan amount, remaining term, and monthly payment are known.

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
25 [g] [12×]	300.00	Mortgage term.
100000 [PV]	100,000.00	Original loan amount.
1127.84 [CHS] [PMT]	-1,127.84	Monthly payment.
[i]	1.08	Monthly interest rate.
12 [×]	13.00	Annual interest rate.

Mortgage Measure Calculations

Debt Coverage Ratio (*DCR*)

The debt coverage ratio is calculated by dividing Net Operating Income by Annual Debt Service (*ADS*). Lenders use the *DCR* as a measure of safety in loan underwriting since it is a comparison of property income to the amount needed to meet loan payment obligations. Given constant *NOI*, the *DCR* decreases as the *ADS* increases. The debt coverage ratio is calculated as follows:

$$DCR = \frac{NOI}{ADS} \qquad \text{(Equation 1-1a)}$$

therefore:

$$ADS = \frac{NOI}{DCR} \qquad \text{(Equation 1-1b)}$$

Example: The *NOI* from a property is estimated to be $6,400 per year. There are two mortgages on the property. The first has monthly payments of $325 and the second has quarterly payments of $647. What is the *DCR* (use equation 1-1a):

Solution: The annual *NOI* must be divided by the total annual mortgage payments (*ADS*). You can use the automatic memory stack of the HP-12C to solve this example by entering the numbers in the order given in the example:

Keystrokes	Display	
6400 ENTER	6,400.00	*NOI.*
325 ENTER	325.00	
12 ×	3,900.00	Annual payment on first mortgage.
647 ENTER	647.00	
4 ×	2,588.00	Annual payment on second mortgage.
+	6,488.00	Total annual debt service.
÷	0.99	*DCR.*

Loan Payment Amount Using *DCR*

You can calculate the loan payment amount by using equation 1-1b, then dividing by the number of payments per year:

Enter information as follows:

1. Key in the *NOI*, then press ENTER.
2. Key in the *DCR*.
3. Solve for the *ADS* by pressing ÷.
4. Key in the number of payments per year.
5. Solve for the loan payment amount by pressing ÷.

Example: A lender specifies a minimum *DCR* of 1.15 for a property with an *NOI* of $100,000. What is the maximum monthly loan payment amount the lender will receive from the property?

Solution: Calculate the *ADS* using equation 1-1b, then divide by 12 to get the monthly payment.

Keystrokes	Display	
100000 ENTER	100,000.00	*NOI.*
1.15 ÷	86,956.52	*ADS.*
12 ÷	7,246.38	Monthly loan payment amount.

Loan Amount Using *DCR*

The loan amount is the present value of the loan payments discounted at the periodic interest rate of the loan. You calculate the loan payment amount using the method above. Then the loan amount is calculated as follows:

1. Press f CLEAR FIN to clear the financial registers.
2. Calculate the loan payment amount (refer to preceding section).
3. Enter the loan payment amount using PMT (use appropriate sign convention).
4. Enter the number of loan payemnts using n.
5. Enter the periodic interest rate using i.
6. Press PV to solve for the loan amount.

Example: A lender will make a loan at 15% annual interest with monthly payments and a 30 year amortizaton. The minimum *DCR* is 1.25. What is the maximum loan amount for a property with an *NOI* of $150,000 per year?

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
150000 ENTER	150,000.00	*NOI*.
1.25 ÷	120,000.00	*ADS*.
12 ÷ CHS PMT	-10,000.00	Monthly payment.
30 g 12×	360.00	Number of payments.
15 g 12÷	1.25	Periodic interest rate.
PV	790,861.42	Loan amount.

Annual Loan Constant (*ALC*)

The annual loan constant (also called loan constant or annual debt constant) is calculated by dividing the *ADS* by the loan amount:

$$ALC = \frac{ADS}{\text{Loan Amount}} \qquad \text{(Equation 1-2a)}$$

The annual loan constant is defined as the ratio of *ADS* to the loan amount for a loan of $1.

If you know the *ALC* and the loan amount, you can calculate the annual debt service (*ADS*) as follows:

$$ADS = ALC \times \text{Loan Amount} \qquad \text{(Equation 1-2b)}$$

If you know the *ALC* and the *ADS*, you can calculate the loan amount as follows:

$$\text{Loan Amount} = \frac{ADS}{ALC} \qquad \text{(Equation 1-2c)}$$

The loan constant is usually converted to a percentage rounded to two decimal places by using a loan amount of $100 to calculate *ALC*.

The *ALC* can be calculated using equation 1-2a as follows:

1. Press f CLEAR FIN to clear the fiancial registers.
2. Key in the term of the loan, then press n.
3. Key in the periodic interest rate, then press i.
4. Solve for the payment to amortize a $100 loan by pressing 100 CHS PV PMT.
5. Key in the number of payments per year.
6. Solve for the annual loan constant by pressing ×.

Example: What is the annual loan constant for a 25 year loan with quarterly payments and a 14% annual interest rate (use a loan amount of 100 so that the result will be a percent)?

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
25 ENTER 4 × n	100.00	Number of payments.
14 ENTER 4 ÷ i	3.50	Periodic interest rate.
100 CHS PV	-100.00	Loan amount.
PMT	3.62	Periodic payment.
4 ×	14.46	Annual loan constant as a percent.

Loan Interest Rate for a Known *ALC*

You can use the following procedure to calculate the loan interest rate:

1. Press f CLEAR FIN to clear the financial registers.
2. Enter the total number of payments using n or g 12×.
3. Press 1 ENTER.
4. Key in the annual loan constant as a percent, then press % to convert to a decimal value.
5. Key in the number of payments per year, then press ÷ CHS PMT.

6. Press 1 [PV].
7. Press [i].
8. Enter the number of payments per year, then press [×] to solve for the annual interest rate.

Example: What is the annual interest rate of a 25 year loan with monthly payments and an annual loan constant of 15.00?

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
25 [g] [12×]	300.00	Number of payments.
1 [ENTER]	1.00	
15 [%]	0.15	Annual loan constant as a decimal.
12 [÷] [CHS] [PMT]	−0.01	Monthly payments for a loan of $1.
1 [PV]	1.00	Loan amount.
[i]	1.22	Monthly interest rate.
12 [×]	14.60	Annual interest rate.

Calculating *n* From the Annual Loan Constant

You can use the following procedure to calculate the approximate number of payments, *n*, rounded up to the next integer, from an annual loan constant. (Refer to appendix A for further discussion of calculation of *n*.)

1. Press [f] CLEAR [FIN] to clear the financial registers.
2. Enter the periodic interest rate using [g] [12÷] or [i].
3. Press 1 [PV].
4. Press 1 [ENTER].
5. Key in the *ALC* as a percent, then press [%] to convert to a decimal value.
6. Key in the number of payments per year, then press [÷] [CHS] [PMT].
7. Solve for the approximate number of payment periods by pressing [n].

Example: What is the approximate number of payments for a loan with 14.50% annual interest, monthly payments, and an *ALC* of 14.91%?

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
14.5 [g] [12÷]	1.21	Periodic interest rate.
1 [PV]	1.00	Loan amount.
1 [ENTER]	1.00	
14.91 [%]	0.15	Annual loan constant as a decimal.
12 [÷] [CHS] [PMT]	−0.01	
[n]	300.00	Approximate number of payments.

Loan Amount Using *ALC* and *DCR*

The following procedure uses equations 1-1a and 1-2c to calculate the loan amount:

1. Key in the *DCR*, then press [ENTER].
2. Key in the *ALC* as a percent, then press [%].
3. Key in the *NOI* from the property.
4. Solve for the loan amount by pressing [x⇄y] [÷].

Example: A lender will make a loan with an *ALC* of 14.75% and a *DCR* of 1.15. What is the maximum loan amount for a property with an *NOI* of $36,000 per year?

Keystrokes	Display	
1.15 [ENTER]	1.15	*DCR.*
14.75 [%]	0.17	
36000	36,000.	*NOI.*
[x⇄y] [÷]	212,232.87	Loan amount.

Loan Payment From the *ALC*

You can calculate the periodic payment by using equation 1-2b to compute the *ADS*, then dividing by the number of payments per year.

Enter information as follows:

1. Key in the loan amount and press [ENTER].
2. Key in the *ALC* as a percent.
3. Solve for the *ADS* by pressing [%].
4. Key in the number of payments per year.
5. Solve for the loan payment amount by pressing [÷].

Example: A loan of $275,000 has an annual loan constant of 12.85%. What is the monthly payment?

Keystrokes	Display	
275000 [ENTER]	275,000.00	Loan amount.
12.85 [%]	35,337.50	Annual debt service.
12 [÷]	2,944.79	Monthly payment.

Partially Amortized Loan Calculations

A partially amortized loan is defined as a loan with periodic payments less than the payments required to fully amortize the loan during the loan term. A "balloon payment" or lump sum payment will be due at the end of the loan term. The balloon payment is the unamortized loan principal and will be added to the last regular payment for loans where payments are made at the end of each period. A balloon payment may be a specified amount or, if loan payments are calculated for an amortization period longer than the loan term, it will be the loan balance.

Balloon Payment

Use the following sequence to calculate a balloon payment:

1. Press [f] CLEAR [FIN] to clear the financial registers.
2. Enter the number of loan payments, using [n] or [g] [12×].
3. Enter the periodic interest rate, using [i] or [g] [12÷].
4. Key in the loan amount, then press [PV].
5. Key in the payment, then press [PMT] (use the correct cash flow sign convention).
6. Solve for the amount of the balloon payment by pressing [FV].

Example: What is the balloon payment due at the end of year 10 for a $750,000 loan with monthly payments of $9,483.33 and a 15% annual interest rate?

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
10 [g] [12×]	120.00	Number of payments.
15 [g] [12÷]	1.25	Periodic interest rate.
750000 [PV]	750,000.00	Loan amount.
9483.33 [CHS] [PMT]	−9,483.33	Payment amount.
[FV]	−720,185.74	Balloon payment.

Payment Required for a Specified Balloon

Use the following procedure to calculate the loan payment when the amount of the balloon payment is known:

1. Press [f] CLEAR [FIN] to clear the financial registers.
2. Enter the number of payments, using [n] or [g] [12×].
3. Enter the periodic interest rate, using [i] or [g] [12÷].
4. Enter the loan amount, using [PV].
5. Enter the specified balloon payment, using [FV] (use the correct cash flow sign convention).
6. Solve for the payment amount by pressing [PMT].

Example: Mr. Seller takes a $200,000 purchase money mortgage at 12% annual interest with quarterly payments and with a $150,000 balloon payment due at the end of five years. What is the quarterly payment?

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
5 [ENTER] 4 [×] [n]	20.00	Number of payments.
12 [ENTER] 4 [÷] [i]	3.00	Periodic interest rate.
200000 [PV]	200,000.00	Loan amount.
150000 [CHS] [FV]	−150,000.00	Balloon payment amount.
[PMT]	−7,860.79	Quarterly payment amount.

Remaining Balance of a Partially Amortized Loan

When the original loan balance is unknown, you can calculate the remaining balance due on a partially amortized loan using the following procedure:

1. Press [f] CLEAR [FIN] to clear the financial registers.
2. Enter the *remaining* number of payments, using [n].
3. Enter the periodic interest rate, using [i].
4. Enter the payment, using [PMT] (use the correct cash flow sign convention).
5. Enter the balloon payment, using [FV] (use the correct cash flow convention).
6. Solve for the remaining balance by pressing [PV].

Example: A ten year loan at 15% annual interest, with monthly payments of $1,283.62, has a balloon payment of $100,000 due at the end of year 10. What is the remaining balance if the loan is to be paid in full at the end of the sixth year (EOY 6)?

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
10 [ENTER] 6 [−]	4.00	Remaining term, EOY 6.
[g] [12×]	48.00	Number of payments remaining.
15 [g] [12÷]	1.25	Periodic interest rate.
1283.62 [CHS] [PMT]	−1,283.62	Monthly payment (negative cash flow).
100000 [CHS] [FV]	−100,000.00	Balloon payment (also negative cash flow).
[PV]	101,208.02	Loan balance at the end of year 6.

Yield on a Loan With Fees or a Prepayment Penalty

Loan fees and prepayment penalties will increase the cost or yield of a loan. Calculation of the interest rate on loans is demonstrated in the owners' handbook. However, fees and prepayment penalties

change the yield to the lender and the cost to the borrower. You can use the following procedure to solve for the yield to the lender:

1. Press [f] CLEAR [FIN] to clear the financial registers.
2. Enter the number of payments, using [n] or [g] [12×].
3. Subtract any origination fees from the loan amount and, using [PV], enter the result as the net proceeds.
4. Enter the loan payment amount, using [PMT] (use correct cash flow sign convention).
5. Enter the loan balance plus total prepayment penalties, using [FV] (use correct cash flow sign convention).
6. Solve for the periodic yield by pressing [i].
7. Enter the number of payments per year.
8. Solve for the annual yield by pressing [×].

Example: A lender charges an origination fee of 3 points* plus $200 to make a $50,000 loan with monthly payments of $550.54. The loan balance of $41,917.27 plus a prepayment penalty of 2% of the loan balance is paid after 8 years. What is the yield to the lender?

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
8 [g] [12×]	96.00	Number of payments.
50000 [ENTER]	50,000.00	Loan amount.
3 [%]	1,500.00	3 points.
200 [+]	1,700.00	Total origination fees.
[−] [PV]	48,300.00	Net loan proceeds.
550.54 [CHS] [PMT]	−550.54	Periodic payment.
41917.27 [CHS] [ENTER]	−41,917.27	Loan balance after 8 years.
2 [%]	−838.35	Prepayment penalty.

* "Points" is a term used in mortgage lending to describe a type of origination fee. A point equals one percent of the loan amount.

Keystrokes	Display	
[+] [FV]	-42,755.62	Loan balance plus total prepayment penalties.
[i]	1.07	Periodic yield.
12 [×]	12.85	Annual yield.

Price to Pay for a Mortgage

The price to pay for an existing mortgage is equal to the present value of the remaining income stream, including the periodic payments and the balloon payment if any, discounted at the investor's required yield.

To calculate the price to pay for a mortgage:

1. Clear the financial registers by pressing [f] CLEAR [FIN].
2. Enter the number of remaining periods using [n] or [g] [12×].
3. Enter the periodic discount rate (investors required yield) using [i] or [g] [12÷].
4. Enter the periodic payment, using [PMT] (use the correct cash flow sign convention).
5. Enter the balloon payment (if any), using [FV] (use the correct cash flow sign convention).
6. Solve for the price to pay for the mortgage by pressing [PV].

Example: An investor wishes to earn 18% annual interest on second mortgages. How much should he pay for a second mortgage with 62 remaining monthly payments of $200 and a balloon payment of $20,000 due at the end of the 62nd month?

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
62 [n]	62.00	Remaining monthly payments.
18 [g] [12÷]	1.50	Discount rate (required yield).
200 [PMT]	200.00	Periodic payments.
20000 [FV]	20,000.00	Balloon payment.
[PV]	-15,981.92	Price to pay for mortgage.

Notice that $15,981.92 is not the remaining balance due on the loan. The rate of interest in the loan contract would have to be known to solve for the remaining balance (refer to the example for calculating the remaining balance of a partially amortized loan on page 18).

Discount Point Calculations

Often there is a difference between the interest rate stated in a loan contract and the interest rate required by the market for loan transactions. A measure of this difference, where the market rate is higher than the contract rate is called "discount points". A discount point is 1% of the loan face amount.

This section presents several programs involving discount points. They supplement the applications involving points in section 12 of the owner's handbook in that, in the following examples, points are either calculated by the program or are entered directly by the user.

Number of Discount Points on a Loan

The number of discount points on a loan is equal to the difference, in percent, between the face amount of the loan and the specified target yield or the price to pay for the mortgage (refer to previous section). To solve for discount points on a loan when the net proceeds is known:

1. Key in the loan amount then press [ENTER].
2. Key in the net proceeds of the loan (the amount the borrower actually receives; refer to previous section for calculation if not known).
3. Press [Δ%] [CHS] to solve for the number of discount points.

Example: What is the number of discount points that will be charged for an $85,000 loan where the net loan proceeds required for the lender to achieve the specified yield is $80,750?

Keystrokes	Display	
85000 [ENTER]	85,000.00	Face amount of loan.
80750	80,750.00	Net loan proceeds.
[Δ%] [CHS]	5.00	Number of points.

Number of Discount Points and Net Loan Proceeds

The amount, in dollars, of a loan discount is equal to the face amount of the loan times the number of discount points. Recall that a discount point is 1% of the loan amount. Net loan proceeds are simply the face amount of the loan less the discount. To solve for the net loan proceeds when the number of points is known:

1. Key in the face amount of the loan and press [ENTER].
2. Key in the number of points.
3. Solve for the amount of the discount by pressing [%].
4. Solve for the net loan proceeds by pressing [−].

Example: What is the amount of the discount (in dollars) and the net loan proceeds for a $95,000 loan if the lender requires a discount of 3 points?

Keystrokes	Display	
95000 [ENTER]	95,000.00	Face amount of loan.
3 [%]	2,850.00	Amount of discount.
[−]	92,150.00	Net loan proceeds.

Annual Percentage Rate of a Loan With Discount Points

The annual percentage rate (*APR*) of a loan with discount points depends on the stated interest rate of the loan, the amortization term of the loan, the remaining duration of the loan, and the number of discount points. The following program will calculate the annual interest rate of loans with monthly payments. Different loan terms may be entered to compare alternative loans. First key in the following program:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[RCL] [PV]	09– 45 13
[f] CLEAR [PRGM]	00–	[RCL] 4	10– 45 4
[g] [12×]	01– 43 11	[%]	11– 25
[RCL] 1	02– 45 1	[−]	12– 30
[g] [12÷]	03– 43 12	[PV]	13– 13
[RCL] 2	04– 45 2	[i]	14– 12
[PV]	05– 13	1	15– 1
[RCL] 3	06– 45 3	2	16– 2
[PMT]	07– 14	[×]	17– 20
[FV]	08– 15	[f] [P/R]	

REGISTERS			
n: Used	i: Used	PV: Used	PMT: Used
FV: Loan Bal. After Calc.	R_1: Nominal Int. Rate	R_2: Face Amount of Loan	R_3: Monthly Loan Amt.
R_4: Number of Disc. Pts.			

1. Key in the program.
2. Key in the annual interest rate of the loan, then press [STO] 1.
3. Key in the face amount of the loan*, then press [STO] 2.
4. Key in the monthly loan payment*, then press [STO] 3.
5. Key in the number of discount points, then press [STO] 4.
6. Key in the number of years until the loan is paid off (this should not exceed the amortization term of the loan), then press [R/S] to calculate and display the *APR*.

*Use the correct cash flow sign convention.

7. Display the loan balance by pressing [RCL] [FV] after running the program.
8. To repeat the calculations for a different loan duration, go to step 6.
9. To change other loan data and repeat the calculations, repeat any of steps 2 through 5 required and go to step 6.

As the program is written, only monthly payment loans and whole numbered years can be used. To change the program to allow any period definition, replace step 01- in the program with [n], replace step 03- with [i], and do not enter steps 15-, 16-, and 17- (just press [f] [P/R] after entering step 14- of the program).

When running the modified program, key in the *periodic* interest rate in step 2 and the number of *periods* in step 6.

Example: What is the *APR* of a $100,000 loan with monthly payments of $1,280.83 at a nominal annual interest rate of 15% when 3 points are charged by the lender and the loan balance is paid after 3 years? After 10 years? What if 4 points were charged?

Keystrokes	Display	
15 [STO] 1	**15.00**	Nominal (contract) interest rate.
100000 [STO] 2	**100,000.00**	Face value of loan.
1280.83 [CHS] [STO] 3	**-1,280.83**	Periodic payment.
3 [STO] 4	**3.00**	Number of points.
3 [R/S]	**16.28**	*APR* for payback in 3 years.
10 [R/S]	**15.61**	*APR* for payback in 10 years.
4 [STO] 4	**4.00**	Change loan discount (step 5).
3 [R/S]	**16.71**	*APR* for payback in 3 years at 4 points.
10 [R/S]	**15.82**	*APR* for payback in 10 years at 4 points.

Loan Amortization Program

The loan amortization function key is discussed and illustrated at the end of Section 3 in the owner's handbook. The simple program below is included to provide an easy way to generate an annual loan amoritization schedule for a loan with monthly payments.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[R/S]	04– 31
[f] CLEAR [PRGM]	00–	[x≷y]	05– 34
1	01– 1	[R/S]	06– 31
2	02– 2	[RCL] [PV]	07– 45 13
[f] [AMORT]	03– 42 11	[f] [P/R]	

REGISTERS			
n: Used	i: Periodic Int. Rate	PV: Loan Amount	PMT: Periodic Payment
FV: Unused			

1. Key in the program.
2. Enter the periodic interest rate, using [i] or [g] [12÷].
3. Enter the loan amount, using [PV].
4. Enter the periodic payment, using [PMT] (use correct cash flow sign convention).
5. If the first year of the loan is a full 12 months (12 payments made) go to step 6, otherwise key in the number of periods in the first year and press [f] [AMORT] [x≷y] to solve for accumulated principal reduction and [RCL] [PV] to view loan balance.
6. Press [R/S] to view the accumulated interest for the full year.

7. Press [R/S] to view the annual principal reduction.
8. Press [R/S] to view the loan balance at the end of the year.
9. Repeat steps 6, 7 and 8 as many times as needed.

Example: Create an annual amortization schedule for a $100,000 loan at 12% interest with monthly payments of $1,028.61. Three payments are made during the first year.

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
12 [g] [12÷]	1.00	Periodic interest rate.
100000 [PV]	100,000.00	Loan amount.
1028.61 [CHS] [PMT]	-1028.61	Periodic payment.
3 [f] [AMORT]	-2,999.13	Interest, EOY 1 (3 months).
[x≷y]	-86.70	Principal, EOY 1.
[RCL] [PV]	99,913.30	Balance, EOY 1.
[R/S]	-11,969.48	Interest, EOY 2.
[R/S]	-373.84	Principal, EOY 2.
[R/S]	99,539.46	Balance, EOY 2.
[R/S]	-11,922.06	Interest, EOY 3.
[R/S]	-421.26	Principal, EOY 3.
[R/S]	99,118.20	Balance, EOY 3.

Before-Tax Yield on Wrap-Around Loans

A wrap-around loan is a loan that is partially funded by an existing underlying loan. Instead of refinancing the existing loan, the wrap-around borrower receives the difference between the wrap-around loan amount and the underlying loan balance. The wrap-around lender collects the loan payment on the wrap-around loan and continues payment on the underlying loan. The wrap-around lender's net investment is the difference between the wrap-around loan amount and the underlying loan balance. In return for this net investment, the wrap-around lender receives the difference between the loan payment on the wrap-around loan and the loan payment on the underlaying loan, plus the difference in balloon payments if the loans are repaid before full amortization.

The wrap-around loan analysis here is different from that in the solutions handbook (pages 7 to 11). The solution in the handbook solves for yield if the wrap-around is held to full maturity. The application here allows you to select a term of analysis that is less than the full term of the wrap-around loan and the yields that are calculated vary depending on the term you select. The reason the yield varies is that loans held to less than full maturity involve balloon payments. The programs in this application take those balloon payments into account when calculating the yield.

There are two cases to be solved in the before-tax wrap-around yield situation. The first is the yield on the lender's net investment, given the terms of the wrap-around loan and the underlying loan. The second case is the interest rate required on the total wrap-around loan to achieve a given yield on the lender's net investment. The interest rate on the total wrap-around loan depends on the interest rate of the underlying loan and the yield specified by the wrap-around lender on his net investment. Therefore, the interest rate on the total wrap-around loan is sometimes called the composite or blend rate.

The assumptions for both cases are that the remaining terms of both loans exceed the time period under analysis, that there is no penalty clause for early repayment of the loans, and that the period between payments is the same for both loans (that is, both monthly, or both annually, etc.).

Yield on the Wrap-Around Lender's Net Investment

The first case to be solved is to calculate the yield to the lender on his net investment.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		−	15– 30
f CLEAR PRGM	00–	FV	16– 15
RCL 3	01– 45 3	RCL 5	17– 45 5
i	02– 12	RCL 2	18– 45 2
RCL 4	03– 45 4	−	19– 30
PV	04– 13	STO 8	20– 44 8
RCL 5	05– 45 5	PMT	21– 14
PMT	06– 14	RCL 4	22– 45 4
FV	07– 15	RCL 1	23– 45 1
RCL 0	08– 45 0	−	24– 30
i	09– 12	STO 7	25– 44 7
RCL 1	10– 45 1	PV	26– 13
PV	11– 13	i	27– 12
RCL 2	12– 45 2	STO 6	28– 44 6
PMT	13– 14	f P/R	
FV	14– 15		

REGISTERS			
n: Number of Periods	i: Used	PV: Used	PMT: Used
FV: Used	R_0: Underlying Int. Rate	R_1: Underlying Loan Amt.	R_2: Underlying Payment
R_3: Wrap-Around Int. Rate	R_4: Wrap-Around Loan Amt.	R_5: Wrap-Around Payment	R_6: Net Invest. Int. Rate
R_7: Net Investment	R_8: Net Payment		

1. Key in the program.
2. Key in the number of periods under analysis, then press n.
3. Key in the periodic interest rate of the underlying loan, then press STO 0.
4. Key in the underlying loan balance, then press STO 1.
5. Key in the underlying loan payment, then press CHS STO 2.
6. Key in the periodic interest rate of the wrap-around loan, then press STO 3.
7. Key in the wrap-around loan amount, then press STO 4.
8. Key in the wrap-around loan payment, then press CHS STO 5.
9. Solve for the before-tax yield on the lender's net investment by pressing R/S.

This program permits the user to change any of the variables of the problem to solve for the yield on the lender's net investment under alternative assumptions. Steps 2 through 8 store all required inputs and can be repeated in any order to change inputs.

Example: Sam Seller sells a property for $250,000 with a $30,000 down payment. He makes a wrap-around loan to the buyer for $220,000 at a composite rate of 13% annual interest with monthly payments of $2,577.47. Mr. Seller will continue the payments of $1,258.79 on the underlying loan balance of $139,908.66 at 9% interest. What is Mr. Seller's before-tax yield if the balances are repaid at the end of the tenth year?

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
120 n	120.00	Number of periods under analysis.
9 g 12÷ STO 0	0.75	Underlying loan rate.
139908.66 STO 1	139,908.66	Underlying loan balance.
1258.79 CHS STO 2	−1,258.79	Underlying loan payment.
13 g 12÷ STO 3	1.08	Wrap-around interest rate.

Keystrokes	Display	
220000 STO 4	220,000.00	Wrap-around loan amount.
2577.47 CHS STO 5	-2,577.47	Wrap-around loan payment amount.
R/S	1.62	Periodic yield.
12 ×	19.48	Annual yield on the net investment.

Wrap-Around Loan Interest Rate

The second case to be solved is that of calculating the composite interest rate that the wrap-around loan must have in order to achieve the specified yield on the wrap-around lender's net investment. The following program allows you to structure the terms of the wrap-around loan.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		+	15– 40
f CLEAR PRGM	00–	FV	16– 15
RCL 0	01– 45 0	RCL 8	17– 45 8
i	02– 12	RCL 2	18– 45 2
RCL 1	03– 45 1	+	19– 40
PV	04– 13	STO 5	20– 44 5
RCL 2	05– 45 2	PMT	21– 14
PMT	06– 14	RCL 7	22– 45 7
FV	07– 15	RCL 1	23– 45 1
RCL 6	08– 45 6	+	24– 40
i	09– 12	STO 4	25– 44 4
RCL 7	10– 45 7	PV	26– 13
PV	11– 13	i	27– 12
RCL 8	12– 45 8	STO 3	28– 44 3
PMT	13– 14	f P/R	
FV	14– 15		

REGISTERS			
n: Number of Periods	i: Used	PV: Used	PMT: Used
FV: Used	R_0: Underlying Int. Rate	R_1: Underlying Loan Amt.	R_2: Underlying Payment
R_3: Wrap-Around Int. Rate	R_4: Wrap-Around Loan Amt.	R_5: Wrap-Around Payment	R_6: Net Invest. Int. Rate
R_7: Net Investment	R_8: Net Payment		

1. Key in the program.
2. Key in the number of periods under analysis, then press n.
3. Key in the periodic interest rate of the underlying loan, then press STO 0.
4. Key in the underlying loan balance, then press STO 1.
5. Key in the underlying loan payment, then press CHS STO 2.
6. Key in the wrap-around lender's specified periodic yield on the net investment, then press STO 6.
7. Key in the lender's net investment amount (wrap-around loan amount minus the underlying loan amount), then press STO 7.
8. Key in the net payment to the wrap-around lender (wrap-around loan payment minus the underlying loan payment), then press CHS STO 8.
9. Solve for the composite interest rate on the total wrap-around loan by pressing R/S.

As with the previous program, you can change any of the variables in the problem to solve for the wrap-around composite interest rate under alternative assumptions.

Example: Consider Mr. Seller from the previous example. Assume that his required before-tax yield on the investment is 18%. What composite interest rate must appear on the wrap-around loan? The period of the analysis is 10 years; the underlying loan rate is 9%, the underlying loan balance is $139,908.66; and the underlying loan monthly payment is $1.258.79. Mr. Seller requires an 18% annual rate of return on the net investment. Since the wrap-around loan is for $220,000, the net investment of the lender (Mr. Seller) is:

$$\$220{,}000.00 - 139{,}908.66 = \$80{,}091.34$$

Likewise, since the total wrap-around loan payment is $2577.47, the net payment to the lender is:

$$\$2{,}577.47 - 1{,}258.79 = \$1{,}318.68$$

We use these net figures in running the program.

Keystrokes	Display	
f CLEAR FIN		Clears financial register.
120 n	120.00	Number of periods.
9 g 12÷ STO 0	0.75	Underlying loan rate.
139908.66 STO 1	139,908.66	Underlying loan balance.
1258.79 CHS STO 2	-1,258.79	Underlying loan payment.
18 g 12÷ STO 6	1.50	Wrap-around lender's yield on net investment.
80091.34 STO 7	80,091.34	Lender's net investment.
1318.68 CHS STO 8	-1,318.68	Lender's net payment received.
R/S	1.02	Periodic composite interest rate.
12 ×	12.20	Annual composite interest rate of wrap-around loan.

Because this program solves for the composite interest rate of the total wrap-around loan (given the loan amount, the payment amount, and the calculated balloon payment at the end of the analysis period), the term to full amortization of the balloon is *not* determined. To solve for the full amortization period, use the following procedure:

Keystrokes	Display	
RCL 4 PV	220,000.00	Stores wrap-around loan amount in PV.
RCL 5 PMT	-2,577.47	Stores wrap-around loan payment in PMT.
CLx FV	0.00	Stores ending balance of fully amortized loan in FV.
RCL 3 i	1.02	Stores wrap-around loan interest rate in i.
n	200.00	Computes number of periods to fully amortize loan.

If you use the above procedure to calculate the number of periods, be sure to restore the number of periods in the analysis to the n register (step 2 in the program procedure list) before running the program again. Note that you must look carefully at the answer to the composite rate calculation. You entered a yield on the net investment and the HP-12C calculated the composite rate that would be required on the total wrap-around loan. To achieve a high net investment yield, the composite rate required on the total wrap-around loan can be so high that the wrap-around loan payment (stored in memory register R_5) is not sufficiently large to cover the interest on the loan. If your application does not permit this situtation, called a "negative amortization" loan, you must adjust the payment size to achieve a permissible solution—or lower the required yield on the net investment.

By using the procedures described in section 11 of the owner's handbook, you can have both of the above programs in memory at the same time. Since the registers used by both programs hold the same information, you can work with either program without having to reenter information that doesn't change (refer to the register maps that follow each program listing).

Quick Estimate of After-Tax Cash Flows

The following program and keystroke procedure will produce an estimate of after-tax cash flows. The results of these calculations can then be used to calculate the resulting after-tax *IRR* for a wrap-around lender (discussed in next section).

Cash flows after tax are derived as follows:

Total Debt Service Received by Wrap-Around Lender
− Debt Service Paid by Wrap-Around Lender
= Cash Flow Before Taxes

Total Interest Earned by Wrap-Around Lender
− Interest Paid by Wrap-Around Lender
= Net Taxable Interest Income
× Tax Rate
= Tax on Taxable Income

Cash Flow Before Taxes
− Tax on Taxable Income
= Cash Flow After Taxes

The after-tax *IRR* is calculated from: the net investment by the wrap-around lender at the origination of the wrap-around loan, the cash flows after taxes, and the net loan proceeds when the wrap-around loan balance is repaid.

You can use the following program for estimating, by the method above, after-tax cash flows to the wrap-around lender:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		RCL 3	20– 45 3
f CLEAR PRGM	00–	PMT	21– 14
RCL 5	01– 45 5	RCL 0	22– 45 0
STO 9	02– 44 9	f AMORT	23– 42 11
RCL 2	03– 45 2	RCL PV	24– 45 13
STO · 0	04–44 48 0	STO · 0	25–44 48 0
RCL 4	05– 45 4	x≷y	26– 34
i	06– 12	RCL 8	27– 45 8
RCL 9	07– 45 9	−	28– 30
PV	08– 13	RCL 7	29– 45 7
RCL 6	09– 45 6	%	30– 25
PMT	10– 14	RCL 3	31– 45 3
RCL 0	11– 45 0	RCL 6	32– 45 6
f AMORT	12– 42 11	−	33– 30
STO 8	13– 44 8	RCL 0	34– 45 0
RCL PV	14– 45 13	×	35– 20
STO 9	15– 44 9	x≷y	36– 34
RCL 1	16– 45 1	−	37– 30
i	17– 12	R/S	38– 31
RCL · 0	18–45 48 0	g GTO 05	39–43,33 05
PV	19– 13	f P/R	

REGISTERS			
n: Used	i: Used	PV: Used	PMT: Used
FV: Unused	R_0: Number of Pmts./Year	R_1: Under. Per. Int. Rate	R_2: Underlying Loan Bal.
R_3: Underlying Loan Pmt.	R_4: Wrap Per. Int. Rate	R_5: Wrap Loan Amount	R_6: Wrap Payment
R_7: Tax Rate as a Percent	R_8: Amount to Interest	R_9: Used	$R_{.0}$: Used

1. Key in the program.
2. Press [f] CLEAR [FIN] to clear the financial registers.
3. Key in the number of payments per year, then press [STO] 0 (the number of payments per year must be the same for both the wrap-around loan *and* the underlying loan).
4. Key in the periodic interest rate of the underlying loan, then press [STO] 1.
5. Key in the underlying loan balance, then press [STO] 2.
6. Key in the underlying loan payment, then press [CHS] [STO] 3.
7. Key in the periodic interest rate of the wrap-around loan, then press [STO] 4.
8. Key in the wrap-around loan amount, then press [STO] 5.
9. Key in the wrap-around loan payment, then press [CHS] [STO] 6.
10. Key in the marginal tax rate as a percent, then press [STO] 7.
11. Press [R/S] to calculate the after-tax cash flow to the wrap-around lender.
12. Repeat step 11 for each successive year.
13. To re-run the program from the first year with different data, press [f] CLEAR [PRGM], enter changed inputs in steps 2 through 10, then go to step 11.

Note: After the cash flow is displayed, the underlying balance may be found in $R_{.0}$ and the wrap-around loan balance in R_9.

Note: Because of the features of [f] [AMORT], answers using [f] 2 decimal display may be different from answers using [f] 9. Refer to the first footnote under "Amortization" in section 3 of the owner's handbook.

Example: What is the after-tax cash flow for a $150,000.00 wrap-around loan at 13% interest with monthly payments of $1,691.75? The underlying loan balance is $90,000.00 at 8% interest with monthly payments of $1,150.00. The wrap-around lender's marginal tax rate is 45%.

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
12 [STO] 0	12.00	Number of payments per year.
8 [ENTER] 12 [÷] [STO] 1	0.67	Periodic interest rate of underlying loan.
90000 [STO] 2	90,000.00	Balance of underlying loan.
1150 [CHS] [STO] 3	−1,150.00	Periodic payment of underlying loan.
13 [ENTER] 12 [÷] [STO] 4	1.08	Periodic interest rate of wrap-around loan.
150000 [STO] 5	150,000.00	Loan amount of wrap-around loan.
1691.75 [CHS] [STO] 6	−1,691.75	Periodic payment of wrap-around loan.
45 [STO] 7	45.00	Lender's marginal tax rate.
[R/S]	876.90	Cash flow after taxes, EOY 1.
[R/S]	673.99	Cash flow after taxes, EOY 2.
[R/S]	457.13	Cash flow after taxes, EOY 3.

Keystrokes	Display	
R/S	225.57	Cash flow after taxes, EOY 4.
R/S	-21.43	Cash flow after taxes, EOY 5.
RCL 9	144,400.10	Wrap-around loan remaining balance, EOY 5.
RCL . 0	49,587.68	Underlying loan remaining balance, EOY 5.
−	94,812.42	Net loan balance due to wrap-around lender, EOY 5.

Notice that the wrap-around lender has a negative cash flow, after taxes, at the end of year 5 of the loan. This occurs because the tax on the net taxable interest income exceeds the wrap-around lender's cash flow before taxes.

Running the program does not change the data registers used to store the original data. To rerun the program, press f CLEAR PRGM R/S and repeat the output procedure. Any of the data can be changed before pressing f CLEAR PRGM R/S to run the program with new data.

After-Tax Yield to the Wrap-Around Lender *

The after-tax yield to the wrap-around lender is equal to the *IRR* of the after-tax cash flows produced by the loan. Use the f IRR function to calculate this yield. You can calculate the after-tax cash flows by using the program in the previous section.

1. Key in the wrap-around lender's initial investment using the correct cash flow sign convention, then press g CFo.
2. Key in each cash flow after taxes, then press g CFj.
3. Repeat step 2 as needed. (Include the net balance due the wrap-around lender in the last year of the analysis.)
4. Solve for the wrap-around lender's after-tax yield by pressing f IRR.

Example: Using the cash flows calculated in the preceding example, what is the after tax yield to the wrap-around lender?

Keystrokes	Display	
150000 ENTER	150,000.00	Wrap-around loan amount.
90000 − CHS g CFo	-60,000.00	Wrap-around lender's initial investment.
876.90 g CFj	876.90	First cash flow (year 1).
673.99 g CFj	673.99	Second cash flow (year 2).
457.13 g CFj	457.13	
225.57 g CFj	225.57	
21.43 CHS ENTER	-21.43	Last cash flow (year 5).
94812.42 + g CFj	94,790.99	Net loan balance.
f IRR	10.26	After tax yield.

Refinancing Versus a Second Mortgage (A Mini-Case Study)

Analyzing the cost of refinancing is similar to analyzing a wrap-around loan. The following mini-case study is presented to illustrate how the cost of refinancing can be calculated for comparison with the cost of a second mortgage. Unlike most of the other examples in this handbook, this example will require intermediate calculations. This will increase the length and complexity of the keystroke sequence needed to find the final solution. Of course, other considerations such as total debt service and loan origination fees affect the final selection of financing method. Based only on the cost of borrowing money, the borrower would choose the loan with the lowest interest rate cost. Therefore, the interest rate cost for alternative financing must be calculated for comparison.

Example: An investor owns a property that has appreciated in value for several years. The existing loan balance of $201,908.37 is at 8% per year and monthly payments are $1,929.54. The property may be refinanced with a new loan of $250,000 at 12% interest with monthly payments of $2,633.06 and a balloon payment at the end

of 10 years. A lender will charge 2 points as an origination fee on the new loan amount. Another lender will make a second mortgage loan of $50,000 at 18% interest per year with monthly payments of $805.21 plus a balloon payment of $31,709.41 due at the end of year 10. The second mortgage lender will charge a 4 point origination fee. What is the interest rate cost for the net loan proceeds of each alternative?

Approach: Treat the new loan alternative as a wrap-around loan and calculate the yield to the wrap-around lender. Then compare the result with the interest rate cost of the second mortgage (including discount points).

Although the keystroke procedure below is long and complex, we have not suggested that you use a program to solve this mini-case problem. This is because there is not likely to be another problem so exactly like this one that the keystroke sequence will be exactly duplicated. The purpose of the mini-case, unlike the other problems in this book, is not to provide a cookbook set of steps to a solution, but instead to give the user the "feel" of working through a relatively unstructured set of data to an eventual solution.

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
10 g 12×	120.00	
8 g 12÷	0.67	
201908.37 PV	201,908.37	
1929.54 CHS PMT	-1,929.54	
FV STO 0	-95,162.25	Save balance of existing loan, EOY 10.
10 g 12×	120.00	
12 g 12÷	1.00	
250000 PV	250,000.00	
2633.06 CHS PMT	-2,633.06	
FV	-219,391.05	Balance of new loan, EOY 10.
RCL 0 − FV	-124,228.80	Additional loan balance, EOY 10 over existing loan.

Keystrokes	Display	
RCL PMT		
1929.54 + PMT	-703.52	Additional loan payment amount.
250000 ENTER	250,000.00	New loan amount.
2 %	5,000.00	New loan discount.
−	245,000.00	Net loan proceeds before repayment of existing loan.
201908.37 − PV	43,091.63	Net cash proceeds of refinancing.
i	2.01	Monthly interest rate.
12 ×	24.17	Annual interest rate cost on net proceeds of refinancing.
10 g 12×	120.00	
50000 ENTER	50,000.00	Second mortgage amount.
4 %	2,000.00	Discount on second mortgage.
− PV	48,000.00	Net loan proceeds.
805.21 CHS PMT	-805.21	
31709.41 CHS FV	-31,709.41	
i	1.58	Monthly interest rate.
12 ×	18.97	Annual interest rate cost of second mortgage.

Summary of Results

Result	New Loan (Refinance)	Second Mortgage
Interest Rate on Net Proceeds	24.17%	18.97%
Net Cash Proceeds	$43,091.63	$48,000.00
Net Increase in Monthly Payments	$703.52	$805.21
Total Monthly Payments	$2,633.06	$2,734.75 (805.21 + 1,929.54)
Total Loan Balance(s) EOY 10	$219,391.05	$126,871.66 (31,709.41 + 95,162.25)

The investor may base a decision on any one or a combination of the data above. Based solely on interest rate cost, the second mortgage would be the better alternative.

Participation Loan Calculations

Lenders have had difficulty determining a fixed interest rate for long term loans in the recent volatile capital market. Extreme variations in market interest rates and in inflation seem to have virtually eliminated the availability of long term fixed-rate real estate loans. However, many real estate lenders are willing to make loans if they can participate in the income or equity appreciation from the property. The effective cost of borrowing using a participation loan is the sum of the debt service at the nominal terms of the loan and the lender's participation income. This sum equals the lender's total return.

We will discuss two types of problems involving participation loans. The first involves estimating the debt service of an income participation loan. The second is that of estimating the lender's yield in the case where there is income or equity participation.

Estimating Debt Service for an Income Participation Loan

An income participation loan requires that the borrower pay the lender a portion of the income from the property in addition to a specified minimum debt service. The following program will estimate the income participation and total debt service for a property with income that increases at a constant rate. Provision is made for cases in which the participation is payable when the property income exceeds a specified minimum amount. The income participation may be based on Effective Gross Income, Net Operating Income, or other specified income figure.

The program used to calculate debt service for an income participation loan is also used, later in this book, to calculate rent from a percentage lease (page 71).

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		RCL 3	13- 45 3
f CLEAR PRGM	00-	%	14- 25
RCL 1	01- 45 1	R/S	15- 31
STO 5	02- 44 5	RCL 0	16- 45 0
RCL 5	03- 45 5	+	17- 40
RCL 2	04- 45 2	R/S	18- 31
g x≤y	05- 43 34	RCL 5	19- 45 5
g GTO 12	06-43,33 12	RCL 4	20- 45 4
0	07- 0	%	21- 25
R/S	08- 31	+	22- 40
RCL 0	09- 45 0	STO 5	23- 44 5
R/S	10- 31	g GTO 03	24-43,33 03
g GTO 19	11-43,33 19	f P/R	
−	12- 30		

REGISTERS			
n: Unused	i: Unused	PV: Used	PMT: Unused
FV: Unused	R_0: Minimum Debt Serv.	R_1: First Year Income	R_2: Minimum Income
R_3: Particip. Percentage	R_4: % Increase in Income	R_5: Property Income	

1. Key in the program.
2. Key in the minimum debt service, then press [STO] 0.
3. Key in the property income for first year, then press [STO] 1.
4. Key in the minimum income applied to property income, then press [STO] 2. Key in 0 [STO] 2 if there is no minimum income.
5. Key in the participation percentage, then press [STO] 3.
6. Key in the annual percent increase in income, then press [STO] 4.
7. Press [R/S] to calculate the income participation amount.
8. Press [R/S] to calculate the total debt service.
9. Repeat steps 7 and 8 for each subsequent year.
10. For a new problem or different input, press [f] CLEAR [PRGM] to reset the program to the first line, then go to step 2.

Example: A lender requires a minimum debt service of $60,000.00 per year plus 20% of any income over $75,000. Income from the property is expected to be $65,000 the first year and to increase 10% per year. Estimate the total debt service for the first five years of the loan.

Keystrokes	Display	
60000 [STO] 0	60,000.00	Minimum debt service.
65000 [STO] 1	65,000.00	First year property income.
75000 [STO] 2	75,000.00	Minimum income break point.
20 [STO] 3	20.00	Participation percentage.
10 [STO] 4	10.00	Annual percentage increase in property income.
[R/S]	0.00	Income participation, EOY 1.
[R/S]	60,000.00	Total debt service, EOY 1.
[R/S]	0.00	Income participation, EOY 2.
[R/S]	60,000.00	Total debt service, EOY 2.
[R/S]	730.00	Income participation, EOY 3.
[R/S]	60,730.00	Total debt service, EOY 3.
[R/S]	2,303.00	Income participation, EOY 4.
[R/S]	62,303.00	Total debt service, EOY 4.
[R/S]	4,033.30	Income participation, EOY 5.
[R/S]	64,033.30	Total debt service, EOY 5.

Lender's Yield on a Participation Loan

The lender's before-tax yield on a loan is equal to the borrower's interest rate cost for the loan even if it is a participation loan. Therefore, the cost of borrowing is equal to the *IRR* on the net loan proceeds. The cash flows include the minimum debt service plus the income participation (if any) and the loan balance plus the equity participation (if any) at the end of the investment holding period. Income participation loans were discussed in the preceding section. An equity participation loan involves the payment of some portion of the equity in the property to the lender at some time in the future. It may be calculated as a portion of equity, net appreciation in property value, or some other way. Both income

and equity participation may be present in a loan. The lender's yield is calculated as follows:

1. Key in the net loan proceeds, then press [g] [CFo].
2. Key in the total of the minimum annual debt service plus income participation each year, then press [g] [CFj].
3. Repeat step 2 as needed, *except* for the year of sale.
4. For the year of sale, key in the sum of the minimum debt service, plus the income participation, plus the loan balance, plus equity participation, then press [g] [CFj].
5. Solve for the estimate* of before-tax annual yield by pressing [f] [IRR].

Example: The lender in the income participation problem on page 44 will also receive an equity participation of 50% of net property appreciation. The property cost $650,000 and is sold after 5 years for $1,000,000. The original loan amount was $475,000 at 12% interest, payable at $5,000 per month ($60,000 per year). A loan origination fee of one point was charged at the originaton of the loan. Use the information from the income participation example on page 44 plus these new figures to calculate the lender's yield.

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
5 [g] [12×]	60.00	Number of payments.
12 [g] [12÷]	1.00	Periodic interest rate of loan.
475000 [PV]	475,000.00	Original loan amount.
5000 [CHS] [PMT]	-5,000.00	Periodic loan payment.
[FV] [STO] [·] 0	-454,582.58	Loan balance, EOY 5 (saved in register R_0 for use later in procedure).
[RCL] [PV] [CHS]	-475,000.00	Loan amount.
1 [%]	-4,750.00	Origination fee.

*This routine assumes annual cash flows are used. The calculated yield will be slightly different if payments are not annual because of the difference in compounding periods.

Keystrokes	Display	
[-] [g] [CFo]	-470,250.00	Net loan proceeds.
60000 [g] [CFj]	60,000.00	Total debt service, EOY 1.
[g] [CFj]	60,000.00	Total debt service, EOY 2.
60730 [g] [CFj]	60,730.00	Total debt service, EOY 3.
62303 [g] [CFj]	62,303.00	Total debt service, EOY 4.
1000000 [ENTER]	1,000,000.00	Sale price.
650000 [-]	350,000.00	Net appreciation.
50 [%]	175,000.00	Equity participation.
[RCL] [-] 0 [CHS] [+]	629,582.58	Total loan balance plus equity participation due lender upon sale.
64033.30	64,033.30	Total debt service, EOY 5.
[+] [g] [CFj]	693,615.88	Grand total due lender upon sale.
[f] [IRR]	17.75	Lender's yield.

This example assumes that the lender collects the loan origination fee. The lender's yield is unchanged if a mortgage broker collects a fee. However, the cost to the borrower is slightly increased and is different from the lender's yield. If a fee is paid to a third party, subtract the fees from the loan amount and store the net loan proceeds into R_0. Then press [f] [IRR] to calculate the interest rate cost to the borrower.

Section 2

Appraisal Applications

Mortgage-Equity (Ellwood) Analysis

Mortgage-Equity Analysis* is a technique used to appraise and analyze income properties on a before tax basis. Mortgage terms, principal amortization, equity yield, and the investment holding period are used to calculate the Basic Capitalization Rate, r. Then r is applied to the Net Operating Income, NOI, of the property to estimate value, V, using the income approach to make the estimate. The expected change in property value during the holding period is then entered into the analysis to calculate the Overall Capitalization Rate, R (or Overall Rate). Value is then estimated from the following familiar formula:

$$V = \frac{NOI}{R} \qquad \text{(Equation 2-1)}$$

The following program calculates the Ellwood mortgage coefficient, c, basic rate, r, and the overall rate, R according to the following formulas:

$$c = \left(y + p \times \frac{1}{s_n}\right) - f \qquad \text{(Equation 2-2a)}$$

$$r = y - m \times c \qquad \text{(Equation 2-2b)}$$

$$R = y - m \times c \begin{pmatrix} + \text{ dep.} \\ - \text{ app.} \end{pmatrix} \times \frac{1}{s_n} \qquad \text{(Equation 2-2c)}$$

*This method is often called by the name of the late L.W. Ellwood. His book, *Ellwood Tables for Real Estate Appraising and Financing*, 3rd Edition (Chicago: American Institute of Real Estate Appraisers, 1970) is the complete reference. William N. Kinnard, *Income Property Valuation*, (Lexington, Massachusetts: Heath Lexington Books, 1971), also presents a full explanation of the method.

where

y = Equity yield rate.

p = The portion of the mortgage principal paid during the holding period.

$\frac{1}{s_n}$ = Sinking fund factor for the holding period at the equity yield rate.

f = Mortgage constant.

m = Mortgage loan-to-value ratio.

$\begin{matrix} + \text{dep.} \\ - \text{app.} \end{matrix}$ = Total change in value during the holding period.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		f CLEAR FIN	18- 42 34
f CLEAR PRGM	00-	1	19- 1
f CLEAR FIN	01- 42 34	FV	20- 15
1	02- 1	RCL 1	21- 45 1
PV	03- 13	n	22- 11
RCL 6	04- 45 6	RCL 2	23- 45 2
g 12×	05- 43 11	i	24- 12
RCL 5	06- 45 5	PMT	25- 14
g 12÷	07- 43 12	×	26- 20
PMT	08- 14	STO − 0	27-44 30 0
1	09- 1	1	28- 1
2	10- 2	RCL 2	29- 45 2
×	11- 20	%	30- 25
STO 0	12- 44 0	STO + 0	31-44 40 0
RCL 1	13- 45 1	RCL 0	32- 45 0
g 12×	14- 43 11	R/S	33- 31
FV	15- 15	RCL 2	34- 45 2
1	16- 1	RCL 0	35- 45 0
+	17- 40	RCL 4	36- 45 4

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
×	37– 20	RCL 3	43– 45 3
−	38– 30	%	44– 25
1	39– 1	x≷y	45– 34
%	40– 25	R↓	46– 33
R/S	41– 31	+	47– 40
RCL PMT	42– 45 14	f P/R	

REGISTERS			
n: Used	i: Used	PV: Used	PMT: Used
FV: Used	R_0: Used	R_1: Holding Period	R_2: Equity Yield Rate
R_3: Change in Value	R_4: Loan-to-Value Ratio	R_5: Mortgage Int. Rate	R_6: Mortgage Amort. Rate

1. Key in the program.
2. Key in the holding period in years, then press STO 1. (Assumed to be less than or equal to the mortgage amortization term.)
3. Key in the equity yield rate as a percent, then press STO 2.
4. Key in the net change in value as a percent, then press STO 3. Use CHS STO 3 to enter an expected decline in value as a negative number.
5. Key in the loan-to-value ratio as a percent, then press STO 4.

6. Key in the annual mortgage interest rate, then press STO 5.
7. Key in the mortgage amortization term, then press STO 6.
8. Press R/S to calculate the mortgage coefficient, *c*.
9. Press R/S to calculate the basic rate, *r*.
10. Press R/S to calculate the Overall Rate, *R*.
11. To rerun the program with different values, repeat any of steps 2 through 7 necessary to make changes and then repeat steps 8, 9, and 10.

Note: If you aren't interested in the mortgage coefficient, *c*, delete the R/S in line 33–. Likewise, if you don't wish to see the basic rate, *r*, delete the R/S in line 41–.

The program assumes that the mortgage has monthly payments.

Example: What are the mortgage coefficient, basic rate, and overall rate for a property which is expected to increase in value 25% over a 10 year holding period? The equity yield rate is 15%. A loan for 75% of the value is available at 12% interest with monthly payments for 25 years. Make the same calculations assuming that the property is financed with a loan for 80% of the value at 13% interest with monthly payments for 30 years.

Keystrokes	Display	
10 STO 1	10.00	Holding period.
15 STO 2	15.00	Equity yield rate.
25 STO 3	25.00	Change in value.
75 STO 4	75.00	Loan-to-value ratio.
12 STO 5	12.00	Annual mortgage interest rate.
25 STO 6	25.00	Mortgage amortization term.
f 6	25.000000	Sets display to six decimal places.
R/S	0.029643	*c*, mortgage coefficient.
R/S	0.127767	*r*, basic rate.
R/S	0.115454	*R*, overall rate.

Keystrokes	Display	
80 STO 4	80.000000	Alternative loan-to-value ratio.
13 STO 5	13.000000	Alternative loan interest rate.
30 STO 6	30.000000	Alternative loan amortization term.
R/S	0.020004	Revised *c*.
R/S	0.133996	Revised *r*.
R/S	0.121683	Revised *R*.

Band of Investment Analysis

Discount rates are sometimes estimated using the weighted average of the return on equity investment* and the mortgage constant. The mortgage constant is the sum of the return on the mortgage amount and the return of the mortgage principal. This approach to calculating the discount rate is called band of investment analysis. The following program will calculate the discount rate using the band of investment approach. It is assumed that the mortgage requires monthly payments.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		g 12÷	07– 43 12
f CLEAR PRGM	00–	PMT	08– 14
f CLEAR FIN	01– 42 34	1	09– 1
1	02– 1	2	10– 2
PV	03– 13	×	11– 20
RCL 1	04– 45 1	RCL 0	12– 45 0
g 12×	05– 43 11	%	13– 25
RCL 2	06– 45 2	STO 4	14– 44 4

*The equity dividend rate (or cash-on-cash rate) is sometimes used instead of the actual equity rate.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
1	15– 1	%	20– 25
RCL 0	16– 45 0	RCL 4	21– 45 4
%	17– 25	−	22– 30
−	18– 30	f P/R	
RCL 3	19– 45 3		

REGISTERS			
n: Used	i: Used	PV: Used	PMT: Used
FV: Unused	R_0: Loan-to-Value Ratio	R_1: Mortgage Amort. Term	R_2: Mortgage Int. Rate
R_3: Equity Return Rate			

1. Key in the program.
2. Key in the loan-to-value ratio as a percentage, then press STO 0.
3. Key in the mortgage amortization term in years, then press STO 1.
4. Key in the annual mortgage interest rate as a percentage, then press STO 2.
5. Key in the equity return rate as a percentage, then press STO 3.
6. Press R/S to calculate the discount rate.
7. To rerun the program using alternative data, repeat any of steps 2 through 5 necessary to change data, then repeat step 6.

Example: Use band of investment analysis to calculate the discount rate for a property financed with a loan for 70% of the value at 11½% interest with monthly payments for 20 years. The estimated market return on equity is 18%.

Keystrokes	Display	
70 STO 0	**70.00**	Loan-to-value ratio.
20 STO 1	**20.00**	Mortgage amortization term.
11.5 STO 2	**11.50**	Annual mortgage interest rate.
18 STO 3	**18.00**	Equity return rate.
f 6	**18.000000**	Sets display to 6 significant places.
R/S	**0.143580**	Discount rate.
12 STO 2	**12.000000**	Changes mortgage interest rate.
25 STO 1	**25.000000**	Changes mortgage amortization term.
R/S	**0.142471**	Revised discount rate.

Appreciation/Depreciation Required for a Specified Equity Yield

You can use the following procedure to calculate the change in value, as a percent, required during an investment holding period to achieve a specified equity yield. The result is useful in mortgage-equity analysis.

1. Clear the financial registers by pressing f CLEAR FIN.
2. Key in the mortgage amortization term, then press n or g 12×.
3. Key in the periodic mortgage interest rate, then press i or g 12÷.

4. Key in the loan-to-value ratio as a percent, then press PV.
5. Press PMT.
6. Key in the number of mortgage payments during the holding period, then press n or g 12×.
7. Press FV STO 0 to calculate and save the mortgage balance in R_0.
8. Key in the capitalization rate as a percentage, then press RCL PMT 12* × + PMT.
9. Key in the holding period, then press n.
10. Key in the required equity yield, then press i.
11. Press RCL PV 100 − PV FV to calculate the required proceeds of resale as a percentage of the purchase price.
12. Press RCL 0 − to calculate the resale price as a percentage of the purchase price.
13. Press 100 x≷y Δ% to calculate the appreciation/depreciation as a percentage of the purchase price.

Example: What is the appreciation in value, as a percentage, of a property financed with a loan for 80% of the value at 12.75% interest with monthly payments for 30 years? The property will be sold in five years. The investor requires an equity yield of 16%. The capitalization rate† is 10.50%.

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*12 represents the number of mortgage payments per year. Use 4 for quarterly payments, etc.

†The captalization rate can be calculated as follows:

$$\text{Capitalization Rate} = \frac{\text{Net Operating Income}}{\text{Purchase Price}} \times 100$$

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
30 g 12×	360.00	Mortgage amortization term.
12.75 g 12÷	1.06	Mortgage interest rate.
80 PV	80.00	Loan-to-value ratio.
PMT	-0.87	
5 g 12×	60.00	Number of mortgage payments during the holding period.
FV STO 0	-78.39	Saves the mortgage balance.
10.5	10.5	
RCL PMT 12 ×	-10.43	
+ PMT	0.07	
5 n	5.00	Holding period.
16 i	16.00	Required equity yield.
RCL PV	80.00	
100 - PV	-20.00	Percent equity investment.
FV	41.54	Required proceeds of sale as a percentage of purchase price.
RCL 0	-78.39	
-	119.93	Resale price as a percentage of purchase price.
100 x≷y Δ%	19.93	Required change in value as percentage of purchase price.

Note: The loan-to-value ratio and the capitalization rate may be replaced with actual dollar amounts. If this is done, replace 100 with the actual purchase price in steps 11– and 13–.

Equity Yield Rate

The equity yield rate (*y* in the Ellwood Mortgage-Equity formulation) can be calculated from the equity investment, cash throw off (*CTO*), and cash proceeds of resale (*CPR*) for properties with level net operating income (*NOI*). Cash throw off (also called cash flow before taxes) is equal to the *NOI* minus *ADS*. Cash proceeds of resale is equal to the sale price minus the mortgage balance. In a simple case, the equity yield rate can be calculated as follows:

1. Key in the holding period, then press n.
2. Key in the equity investment, then press CHS PV.
3. Key in the *CTO*, then press PMT.
4. Key in the *CPR*, then press FV.
5. Press i to solve for the equity yield rate.

Example: What is the equity yield rate for a property with level *NOI*, held for five years? *CTO* is equal to $12,000. The equity investment was $100,000. *CPR* is $125,000.

Keystrokes	Display	
5 n	5.00	Holding period.
100000 CHS PV	-100,000.00	Equity investment.
12000 PMT	12,000.00	*CTO*.
125000 FV	125,000.00	*CPR*.
i	15.66	Calculates the equity yield rate.

Note: f CLEAR FIN was not required in this example because all four financial variables are entered during the procedure. Whenever four of the five financial variables are entered in a procedure, you can omit the f CLEAR FIN step.

The equity yield rate can also be calculated from the capitalization rate, loan-to-value ratio, and the change in value as a percentage of the purchase price. The following example illustrates this procedure:

1. Press [f] CLEAR [FIN] to clear the financial registers.
2. Key in the loan amortization term, then press [n] or [g] [12×].
3. Key in the loan interest rate, then press [i] or [g] [12÷].
4. Key in the loan-to-value ratio as a percentage, then press [PV].
5. Press [PMT].
6. Key in the number of loan payments during holding period, then press [n] or [g] [12×].
7. Press [FV].
8. Key in the holding period in years, then press [n].
9. Press [RCL] [PV] 100 [−] [PV] to calculate the equity investment.
10. Key in the capitalization rate, then press [RCL] [PMT] 12* [×] [+] [PMT] to calculate *CTO*.
11. Key in the percent change in value, then press [ENTER] 100 [+] [RCL] [FV] [+] [FV] to solve for *CPR*.
12. Press [i] to solve for the equity yield rate.

Example: What is the equity yield rate for a property that is held for five years? It is financed with a loan for 75% of the value at 12.25% interest with monthly payments for 25 years. The capitalization rate is calculated to be 9.75% and the property is expected to increase in value by 30% over the 5 year holding period.

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
25 [g] [12×]	300.00	Loan amortization term.
12.25 [g] [12÷]	1.02	Loan interest rate.
75 [PV]	75.00	Loan-to-value ratio.
[PMT]	−0.80	
5 [g] [12×]	60.00	

*This assumes monthly mortgage payments. Replace 12 with the number of mortgage payments per year for other than monthly payments.

Keystrokes	Display	
[FV]	−71.86	
5 [n]	5.00	Holding period.
[RCL] [PV]	75.00	
100 [−] [PV]	−25.00	Calculates the equity investment.
9.75	9.75	
[RCL] [PMT] 12 [×]	−9.65	
[+] [PMT]	0.10	Calculates *CTO*.
30 [ENTER] 100 [+]	130.00	
[RCL] [FV]	−71.86	
[+] [FV]	58.14	Calculates *CPR*.
[i]	18.69	Equity yield rate.

Compound Interest and Discount Factors

Compound interest and discount factors have been available in the form of published tables for many years; and for many practitioners these tables have been the primary source of answers to many loan and financing problems. The HP-12C can be used to replicate and even extend any of these tables. The procedures in this section allow you to duplicate any table value and to extend the tables to include interest rates and numbers of periods not available in the table.

As they are most commonly presented, the tables contain six sets of factors, where n and i are always known and the three remaining values (*PMT*, *PV*, and *FV*) are taken in pairs. For each pair, one value is given as a standard value (like $1) and the second is computed from the known values. For example, *PV* might be solved for a *PMT* of $1. The tables then consist of six sets of pairs, where *PMT*, *PV*, and *FV* are compared in various combinations.

You can use the following general procedure to solve for any of the values in the tables:

1. Press [f] CLEAR [FIN] to clear the financial registers.
2. Key in the number of compounding periods, then press [n] or [g] [12×].
3. Key in the periodic interest rate, then press [i] or [g] [12÷].

4. Key in 1 [CHS], then press [PV], [PMT] or [FV] as needed. (Refer to the table below for specific combinations.)
5. Solve for the desired factor by pressing [PV], [PMT] or [FV] as needed.

Calculation of Compound Interest and Discount Factors

To Calculate:	Ellwood Tables Column Number	Key 1 Into:	To Solve Press:
Value of $1 at Compound Interest.	1	[PV]	[FV]
Accumulation of $1 Per Period.	2	[PMT]	[FV]
Sinking Fund Factor.	3	[FV]	[PMT]
Present Value Reversion of $1.	4	[FV]	[PV]
Present Value, Ordinary Annuity, $1 Per Period.	5	[PMT]	[PV]
Installment to Amortize $1.	6	[PV]	[PMT]

Use [g] [END] to calculate factors for tables in which payments are made at the end of each period and [g] [BEG] for tables in which payments are made at the beginning of each period. Also, so that results will be consistent with most published tables, set the display to show six decimal places by pressing [f] 6.

Example 1: What is the sinking fund factor for 7 years, with semiannual compounding, at 10% interest? Also what is the sinking fund factor for quarterly compounding?

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
[f] 6		Sets display to six places.
7 [ENTER] 2 [×] [n]	14.000000	Total number of semiannual compounding periods.
10 [ENTER] 2 [÷] [i]	5.000000	Periodic interest rate.
1 [CHS] [FV]	−1.000000	
[PMT]	0.051024	Sinking fund factor.
7 [ENTER] 4 [×] [n]	28.000000	Total number of quarterly compounding periods.
10 [ENTER] 4 [÷] [i]	2.500000	Quarterly interest rate.
[PMT]	0.025088	Sinking fund factor.

Example 2: What is the factor for the value of $1 at compound interest for 8 years and 4 months? The interest rate is 10.88% compounded monthly.

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
8 [ENTER] 12 [×]	96.000000	
4 [+] [n]	100.000000	Total number of compounding periods.
10.88 [g] [12÷]	0.906667	Periodic interest rate.
1 [CHS] [PV]	−1.000000	
[FV]	2.465960	Factor for the value of $1 at compound interest.

Section 3

Investment Analysis

Cash Flow Before Taxes or Cash Throw Off

Cash flow before taxes, also called cash throw off in appraisal literature, can be calculated as follows:

Gross Scheduled Rental Income	(*GSRI*)
− Vacancy and Credit Losses	(*V&CL*)
+ Other Income	
= Effective Gross Income	
− Operating Expenses	
= Net Operating Income	(*NOI*)
− Annual Debt Service	(*ADS*)
= Cash Flow Before Taxes	

You can use the following program to forecast net operating income and cash flow before taxes for properties whose gross scheduled rental income, other income, and operating expenses are expected to increase at different rates. Vacancy and credit losses and annual debt service are assumed to remain constant.

The *HP-12C Solutions Handbook* contains programs and procedures for estimating cash flows and resale proceeds after taxes (pp. 14-22). The programs in the solutions handbook are longer than the one below because they include adjustments for taxes. They are different also, in that they do not allow for an escalation rate for other income.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		%	15– 25
f CLEAR PRGM	00–	+	16– 40
RCL 0	01– 45 0	STO 0	17– 44 0
RCL 4	02– 45 4	RCL 2	18– 45 2
%	03– 25	RCL 3	19– 45 3
−	04– 30	%	20– 25
RCL 2	05– 45 2	+	21– 40
+	06– 40	STO 2	22– 44 2
RCL 5	07– 45 5	RCL 5	23– 45 5
−	08– 30	RCL 6	24– 45 6
R/S	09– 31	%	25– 25
RCL 7	10– 45 7	+	26– 40
−	11– 30	STO 5	27– 44 5
R/S	12– 31	g GTO 01	28–43,33 01
RCL 0	13– 45 0	f P/R	
RCL 1	14– 45 1		

REGISTERS			
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: *GSRI*	R_1: % Increase in *GSRI*	R_2: Other Income
R_3: % Incr. in Other Inc.	R_4: % *V&CL*	R_5: Operating Expense	R_6: % Incr. in Op. Expense
R_7: *ADS*			

1. Key in the program.
2. Key in the Gross Scheduled Rental Income (*GSRI*) for year 1, then press [STO] 0.
3. Key in the annual increase in *GSRI* as a percentage, then press [STO] 1.
4. Key in other income for year 1, then press [STO] 2.
5. Key in the annual increase in other income as a percentage, then press [STO] 3.
6. Key in the vacancy and credit losses as a percentage of *GSRI*, then press [STO] 4.
7. Key in the operating expenses for year 1, then press [STO] 5.
8. Key in the annual increase in operating expenses as a percentage, then press [STO] 6.
9. Key in the annual debt service, then press [STO] 7.
10. Press [R/S] to calculate the net operating income for the first year.
11. Press [R/S] to calculate the cash flow before taxes for the first year.
12. Repeat steps 10 and 11 as needed for successive years.

To use alternative assumptions:

13. Reset the program counter by pressing [f] CLEAR [PRGM].
14. Repeat steps 2, 4, and 7 to reset R_0, R_2 and R_5 to first year values.
15. Repeat steps 3, 5, 6, 8, and 9 to change data in R_1, R_3, R_4, R_6, and R_7 as desired.
16. Repeat steps 10 and 11 as needed.

Example: The gross scheduled rental income of a property is forecast to be $25,000 the first year and is expected to increase 12% per year. Other income is $1,000 in year 1 and is expected to remain constant. Vacancy and credit losses are estimated at 4% of gross scheduled rental income. Operating expenses are estimated to be $10,000 in year 1 and to increase 10% per year. Annual debt service is $13,650. What are the net operating income (*NOI*) and cash flows before taxes (*CFBT*) for the next three years?

Keystrokes	Display	
25000 [STO] 0	25,000.00	*GSRI* for year 1.
12 [STO] 1	12.00	Annual increase in *GSRI*.
1000 [STO] 2	1,000.00	Other income for year 1.
0 [STO] 3	0.00	No annual increase expected.
4 [STO] 4	4.00	Vacancy and credit losses.
10000 [STO] 5	10,000.00	Operating expenses for year 1.
10 [STO] 6	10.00	Annual increase in operating expenses.
13650 [STO] 7	13,650.00	Annual debt service.
[R/S]	15,000.00	*NOI*, EOY 1.
[R/S]	1,350.00	*CFBT*, EOY 1.
[R/S]	16,880.00	*NOI*, EOY 2.
[R/S]	3,230.00	*CFBT*, EOY 2.
[R/S]	19,005.60	*NOI*, EOY 3.
[R/S]	5,355.60	*CFBT*, EOY 3.

Cash Flows After Taxes (Quick Estimate)

Cash flow after taxes is equal to the cash flows before taxes* less taxes (or savings) allocated to the investment income. Typically, for income properties, taxable income is computed as follows:

Net Operating Income
− Interest
− Cost Recovery (Depreciation)
= Taxable Income
× Marginal Tax Rate†
= Taxes on Investment Income

* Cash flow before taxes was defined in the previous section.

† Because of "graduated" tax rates, as your income increases your tax *rate* also typically increases. Your *marginal* tax rate is the rate of tax on each additional dollar of taxable income in your "tax bracket." This rate is higher than the average tax rate over total income.

The taxes (or tax savings) are estimated by multiplying the taxable income from the investment times the investor's marginal tax rate. Cash flow after taxes is:

Cash Flows Before Taxes
− Taxes on Investment Income
= Cash Flows After Taxes

The following program* will produce a quick estimate of cash flow after taxes (*CFAT*) for an income property which uses straight-line cost recovery and which is financed by a loan with monthly payments:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		2	12– 2
f CLEAR PRGM	00–	×	13– 20
1	01– 1	RCL 2	14– 45 2
2	02– 2	+	15– 40
f AMORT	03– 42 11	x≷y	16– 34
RCL 1	04– 45 1	−	17– 30
−	05– 30	R/S	18– 31
RCL 2	06– 45 2	RCL 2	19– 45 2
+	07– 40	RCL 3	20– 45 3
RCL 4	08– 45 4	%	21– 25
%	09– 25	STO + 2	22–44 40 2
RCL PMT	10– 45 14	g GTO 01	23–43,33 01
1	11– 1	f P/R	

*A more detailed program for cash flows after taxes may be found in the *HP-12C Solutions Handbook*, page 14.

REGISTERS			
n: Used	i: Loan Int. Rate	PV: Loan Amount	PMT: Monthly Payment
FV: Unused	R_0: Used	R_1: SL Cost Recovery	R_2: *NOI*
R_3: Δ*NOI*	R_4: Tax Rate (%)		

1. Key in the program.
2. Key in the amount of straight line (*SL*) cost recovery, then press STO 1.
3. Key in the *NOI* for the first year, then press STO 2.
4. Key in the annual change in the *NOI* as a percent (this may be negative for declining *NOI*), then press STO 3.
5. Key in the tax rate as a percent, then press STO 4.
6. Key in the annual mortgage interest rate, then press g 12÷.
7. Key in the loan amount, then press PV.
8. Key in the monthly loan payment, then press CHS PMT.
9. Press R/S to solve for the cash flow after taxes.
10. Repeat step 9 for each subsequent year desired.

Note: If you wish to use the accelerated cost recovery method instead of the straight-line cost recovery method, you can replace program step 04– with a R/S. When running the program, you must then calculate the cost recovery deduction manually after the program stops at step 04–. You then press R/S to continue. Accelerated cost recovery is discussed in more detail in the next topic.

Example: What are the estimated cash flows after taxes for a property under the following assumptions?

Purchase price:	$500,000.
Unimproved value:	$80,000.
Basis of improvements:	$420,000.
Financing:	$400,000, monthly payments of $4,114.45, 12% interest.
Cost recovery:	15 years, straight-line method.
Net operating income:	$65,000, will increase 5% per year.
Marginal tax rate:	40%.
Holding period:	5 years.

Keystrokes	Display	
420000 [ENTER]	**420,000.00**	Purchase price.
15 [÷] [STO] 1	**28,000.00**	*SL* cost recovery deduction.
65000 [STO] 2	**65,000.00**	*NOI*.
5 [STO] 3	**5.00**	Annual percent increase in *NOI*.
40 [STO] 4	**40.00**	Tax rate.
12 [g] [12÷]	**1.00**	Interest rate.
400000 [PV]	**400,000.00**	Loan amount.
4114.45 [CHS] [PMT]	**–4,114.45**	Monthly payment.
[R/S]	**19,995.36**	*CFAT*, EOY 1.
[R/S]	**21,871.72**	*CFAT*, EOY 2.
[R/S]	**23,836.26**	*CFAT*, EOY 3.
[R/S]	**25,892.63**	*CFAT*, EOY 4.
[R/S]	**28,044.64**	*CFAT*, EOY 5.
[RCL] [PV]	**390,652.96**	Loan balance EOY 5.

The loan balance as of the last year is in PV. This is illustrated since this amount is required to estimate the proceeds of sale at the end of the projected holding period.

Be sure to reset the loan amount for the first year, in PV, and the *NOI* for the first year, in R_2, when repeating the analysis under alternative investment assumptions. Any of the other data may be changed as desired by repeating steps 2, 4, 5, 6, or 8 as needed.

The Accelerated Cost Recovery System (ACRS)

The Economic Recovery Tax Act of 1981 made sweeping changes in the tax accounting procedures used to recover the cost of assets. Depreciation methods were replaced with the Accelerated Cost Recovery System (ACRS). Cost recovery deductions replace depreciation deductions for qualifying property placed into service after 1980. Straight-line cost recovery is calculated the same way as straight-line depreciation. Therefore, the [f] [SL] function may be used to calculate straight-line cost recovery deductions (refer to section 4 of the owner's manual). Note that cost recovery deductions for real property are pro-rated by the number of months an asset is owned during the tax year, which is different from the treatment of personal property.

Accelerated cost recovery deductions are calculated from tables published by the Internal Revenue Service. The tables are in appendix B. The factors provided in the tables are percentages of the total cost recovery basis that may be deducted each year. The following program will calculate the cost recovery deduction and the remaining basis of the property, including land.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[%]	02– 25
[f] CLEAR [PRGM]	00–	[STO] [–] 0	03–44 30 0
[RCL] 1	01– 45 1	[f] [P/R]	

REGISTERS			
n: Used	i: Used	PV: Used	PMT: Used
FV: Used	R_0: Total Rem. Basis	R_1: Cost Rec. Basis	

1. Key in the program.
2. Key in the total basis of the property, then press [STO] 0.
3. Key in the cost recovery basis, then press [STO] 1.
4. Key in the percentage of cost recovery deduction for each tax year, then press [R/S] to calculate the cost recovery deduction.
5. Repeat step 4 as desired. Calculate cost recovery deductions for each year sequentially.
6. The adjusted basis may be found in R_0.

Example: An apartment building, costing $825,000 was placed in service during the third month of Ms. Taxpayer's tax year. The cost recovery basis is $750,000. What are the accelerated cost recovery deduction and adjusted basis at the end of each year for the first two years?

Keystrokes	Display	
825000 [STO] 0	**825,000.00**	Total basis.
750000 [STO] 1	**750,000.00**	Cost recovery basis.
10 [R/S]	**75,000.00**	Cost recovery deduction, EOY 1.
[RCL] 0*	**750,000.00**	Adjusted basis, EOY 1.
11 [R/S]	**82,500.00**	Cost recovery deduction, EOY 2.
[RCL] 0*	**667,500.00**	Adjusted basis, EOY 2.

*This step may be bypassed if desired.

Percentage Lease Rent

You can use the program on page 43 for estimating debt service for an income participation loan to estimate the rent from a percentage lease. A percentage lease is a lease that requires a minimum rental amount plus a percentage of any gross sales over a specified minimum sales amount.

1. Key in the program from page 43.
2. Key in the minimum rent, then press [STO] 0.
3. Key in the expected sales volume for the first year, then press [STO] 1.
4. Key in the minimum sales volume, then press [STO] 2. If the lease requires only percentage rent with no minimum sales volume, then press 0 [STO] 2.
5. Key in the overage percentage, then press [STO] 3. If breakpoint sales is not used, key in 100 as the overage percentage.
6. Key in the expected annual increase in sales volume, then press [STO] 4.
7. Press [R/S] to calculate the overage rental amount.
8. Press [R/S] to calculate the total rent.
9. Repeat steps 7 and 8 for each subsequent year.

Note: When changing data for alternative assumptions, step 3 must always be repeated and [f] CLEAR [PRGM] must be pressed to reset the program to the first line. You may then proceed as usual.

Example: Acme Retailer has a percentage lease that requires a minimum annual rent of $12,000 per year plus 7% of gross sales in excess of $180,000 per year. Sales are expected to increase 12% per year from a first year estimate of $165,000. What is the rent for the first three years?

Keystrokes	Display	
12000 STO 0	12,000.00	Minimum rent.
165000 STO 1	165,000.00	Expected sales volume for first year.
180000 STO 2	180,000.00	Minimum sales volume.
7 STO 3	7.00	Overage percentage.
12 STO 4	12.00	Expected annual increase in sales.
R/S	0.00	Overage rent, EOY 1.
R/S	12,000.00	Total rent, EOY 1.
R/S	336.00	Overage rent, EOY 2.
R/S	12,336.00	Total rent, EOY 2.
R/S	1,888.32	Overage rent, EOY 3.
R/S	13,888.32	Total rent, EOY 3.

The minimum sales volume called for in this program can be any amount. However, the minimum sales volume is often equal to an amount called "breakpoint" sales volume, in which minimum rent is equal to the actual sales volume times an overage percentage. Breakpoint is calculated by the following formula:

$$\text{Breakpoint} = \left[\frac{\text{Minimum Rent}}{\text{Overage Percentage}}\right] \times 100$$

Present Value of Cash Flows or Investment Value

Investment value is equal to the present value of all anticipated cash flow benefits discounted at the investor's required rate of return (opportunity cost of capital). For an investment property, the investment value is equal to the present value of anticipated cash flows to the owners plus the present value of anticipated cash flow to the lender(s). The latter is equal to the loan amount minus discount points and origination fees, if any. As the discount rate decreases, the investment value increases. In other words, the investor willing to take a lower yield will pay a greater price for a specific cash flow.

Present Value of an Increasing Annuity*

You can use the following program to calculate the present value of an ordinary annuity that increases at a constant rate and at equal intervals. The program can be used to discount a series of cash flows that escalate over time (as might be caused by inflation).

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		RCL 3	14– 45 3
f CLEAR PRGM	00–	RCL 2	15– 45 2
f CLEAR FIN	01– 42 34	÷	16– 10
1	02– 1	n	17– 11
RCL 0	03– 45 0	RCL 4	18– 45 4
%	04– 25	PMT	19– 14
+	05– 40	FV	20– 15
RCL 2	06– 45 2	f CLEAR FIN	21– 42 34
$x^{\leftrightarrow}$	07– 21	PMT	22– 14
1	08– 1	RCL 0	23– 45 0
RCL 1	09– 45 1	i	24– 12
%	10– 25	RCL 2	25– 45 2
+	11– 40	n	26– 11
Δ%	12– 24	PV	27– 13
i	13– 12	f P/R	

*This is a special application of the mathematics for a standard graduated payment mortgage. Refer to Elbert B. Greynolds, Jr., et. al., *Financial Analysis Using Calculators: Time Value of Money* (New York: McGraw-Hill, 1980) p. 418.

REGISTERS			
R_0: Periodic Disc. Rate	R_1: % Inc. of Payments	R_2: # of Equal Payments	R_3: Tot. # of Periods
R_4: First Payment			

1. Key in the program. (Be sure calculator is in "END" of period mode.)
2. Key in the periodic discount rate, then press [STO] 0.
3. Key in the percentage increase of payments, then press [STO] 1.
4. Key in the number of equal payments before each increase, then press [STO] 2.
5. Key in the total number of periods, then press [STO] 3.
6. Key in the first payment amount, then press [STO] 4.
7. Press [R/S] to solve for the present value of the increasing ordinary annuity.
8. If payments are received at the beginning of each period; press [RCL] 0 [%] [+] to calculate the present value of the increasing annuity due.
9. To calculate the present value under alternative assumptions, repeat steps 2, 3, 4, 5, and 6 as required to change values in registers R_0 through R_4 as needed, then repeat steps 7 and 8.

Example 1: A leasehold interest will produce $1,000 at the beginning of each month for the first year. The monthly income will increase 10% at the end of each year for 8 years. An investor requires a 15% rate of return on this type of investment. What is the present value of the payments?

Keystrokes	Display	
15 [ENTER]	15.00	Annual discount rate.
12 [÷] [STO] 0	1.25	Periodic discount rate.
10 [STO] 1	10.00	Percentage increase of payments.
12 [STO] 2	12.00	Number of equal payments before each increase.
8 [ENTER]	8.00	Years.
12 [×] [STO] 3	96.00	Total number of periods.
1000 [STO] 4	1,000.00	First payment.
[R/S]	73,990.43	*PV* for end of period payments.
[RCL] 0 [%] [+]	74,915.31	*PV* for beginning of period payments.

Example 2: Using the figures from example 1 above, what would be the present value if the investor required an 18% rate of return on his investment?

Keystrokes	Display	
18 [ENTER]	18.00	New annual discount rate.
12 [÷] [STO] 0	1.50	New periodic discount rate.
[R/S]	66,375.38	*PV* for end of period payments.
[RCL] 0 [%] [+]	67,371.01	*PV* for beginning of period payments.

Interpretation: An investor who requires a 15% rate of return would be willing to pay $74,915.31 for this leasehold interest. An investor who requires an 18% rate of return would only pay $67,371.01 for this leasehold interest.

You can also use this program to calculate the payments required to achieve a specified yield if all of the other data (R_0-R_3) are given.

Payments of an Increasing Annuity

The preceding program can be used to calculate the payments required to achieve a specified yield if the price paid for the annuity is known.

1. Key in the program on page 73. (Be sure the calculator is set to "END" mode.)
2. Key in the periodic discount rate, then press STO 0.
3. Key in the percentage increase of payments, then press STO 1.
4. Key in the number of equal payments before each increase, then press STO 2.
5. Key in the total number of periods, then press STO 3.
6. Press 1 STO 4.
7. Press R/S 1/x to solve for the factor.
8. Key in the present value, then press × to solve for the first payment amount at the end of each period.
9. Optional step for payments made at the beginning of each period: press 1 RCL 0 % + ÷ to solve for first payment amount.
10. Press RCL 1 % + to solve for each successive payment amount.
11. Repeat step 10 as needed.

Example: Mr. Seller takes a $500,000 purchase money mortgage on a shopping center at 10% interest. Quarterly payments are to increase annually by 15%. The loan is to be paid in full after 5 years. What are the quarterly payments necessary for each year?

Keystrokes	Display	
10 ENTER	10.00	Annual interest.
4 ÷ STO 0	2.50	Periodic discount rate.
15 STO 1	15.00	Percentage increase of payments.
4 STO 2	4.00	Number of equal payments before each increase.

Keystrokes	Display	
5 ENTER	5.00	Number of years.
4 × STO 3	20.00	Total number of payments.
1 STO 4	1.00	
R/S	20.45	Factor, *PV* of annuity starting at $1.
1/x	0.05	Factor for payment.
500000 ×	24,448.37	Quarterly payments, year 1.
RCL 1 % +	28,115.63	Quarterly payments, year 2.
RCL 1 % +	32,332.97	Quarterly payments, year 3.
RCL 1 % +	37,182.92	Quarterly payments, year 4.
RCL 1 % +	42,760.36	Quarterly payments, year 5.

Investment Value of Unequal Cash Flows

The f NPV keystroke can be used to calculate the present value of any series of unequal cash flows if CFo is equal to 0. Use the following procedure.

1. Press 0 g CFo.
2. Key in the cash flow, then press g CFj.
3. If the cash flow repeats, key in the number of times it repeats, then press g Nj as needed. If the cash flow does not repeat, go to step 4.
4. Repeat steps 2 and 3 as needed.
5. Key in the periodic discount rate percent, then press i.
6. Press f NPV to calculate the present value of the cash flows.

The calculated present value can be used in further calculations, as demonstrated below.

Example: A broker prepares a cash flow analysis of an investment property. Assume that the property is financed with a $500,000 mortgage less a 3 point origination fee. Cash flows after taxes are forecast to be:

Year	Cash Flow ($)
1	18,000
2	20,000
3	25,000
4	26,000
5	27,500 + 175,000

The net proceeds of resale are shown in year 5. Investors will require an after-tax yield of about 15% for this type of investment. What is the investment value of this property?

Keystrokes	Display	
0 [g] [CFo]	**0.00**	Sets initial cash flow to 0.
18000 [g] [CFj]	**18,000.00**	First year cash flow.
20000 [g] [CFj]	**20,000.00**	Second year cash flow.
25000 [g] [CFj]	**25,000.00**	Third year cash flow.
26000 [g] [CFj]	**26,000.00**	Fourth year cash flow.
27500 [ENTER]	**27,500.00**	
175000 [+] [g] [CFj]	**202,500.00**	Fifth year cash flow, including net proceeds of resale.
15 [i]	**15.00**	Discount rate.
[f] [NPV]	**162,756.83**	*PV* of owner's cash flows.
500000 [ENTER]	**500,000.00**	Loan amount.
3 [%]	**15,000.00**	Origination fees.
[−]	**485,000.00**	Net loan proceeds.
[+]	**647,756.83**	Investment value.

Interpretation: Under these assumptions, an investor who requires a 15% rate of return would invest, at most, a total of $162,756.83 in this property. The net loan proceeds are estimated to be $485,000.00. Therefore, the investor would be willing to pay a total of $647,756.83 for this property, the investment value.

Section 4

Differential Cash Flow Analysis

Differential cash flow analysis* is used to compare cash flows to determine which alternative will maximize future wealth, measured in dollars. This procedure is useful in a wide variety of situations including comparison of installment sale offers, decisions to exchange properties, determination of a cost recovery method, choosing among financing alternatives, and buy-lease decisions. The analysis is applied to alternatives considered in pairs. The differential cash flow is the difference between the two cash flows in each pair.

Differential cash flow analysis is done in four basic steps:

1. Reduce each alternative to a series of cash flow forecasts. Be consistent with regard to before-tax or after-tax cash flows.
2. Calculate the differential cash flows between two alternatives for each year.
3. Calculate the *IRR* of the differential cash flows.
4. Compare the calculated *IRR* with the return (before-tax or after-tax) of comparable investment alternatives for the differential cash flows.

Example: A property owner can buy an "energy-saver" replacement air-conditioning system for $50,000 or a standard unit for $35,000 (initial costs are stated net of investment tax credits). After-tax operating costs are summarized below. Both units have an expected life of seven years.

* A complete discussion of differential cash flow analysis may be found in Messner, et. al., *Marketing Investment Real Estate*, 2nd Edition (Chicago: REALTORS NATIONAL MARKETING INSTITUTE, 1982) pp. 360-389.

Year *n*	*A*: Energy Saver	*B*: Standard	Differential Cash Flows *A* − *B*
0	($50,000)	($35,000)	($15,000)
1	($10,000)	($12,500)	$ 2,500
2	($12,000)	($15,000)	$ 3,000
3	($14,400)	($18,000)	$ 3,600
4	($17,300)	($21,600)	$ 4,300
5	($20,700)	($25,900)	$ 5,200
6	($24,900)	($31,100)	$ 6,200
7	($29,900)	($37,000)	$ 7,400

You can calculate the differential, $A - B$, as part of the process of entering the differential cash flows into the CFj registers. The following solution illustrates this.

Keystrokes	Display	
50000 CHS ENTER	−50,000.00	
35000 CHS − g CFo	−15,000.00	Initial differential cash flow.
10000 CHS ENTER	−10,000.00	
12500 CHS − g CFj	2,500.00	Differential, EOY 1.
12000 CHS ENTER	−12,000.00	
15000 CHS − g CFj	3,000.00	Differential, EOY 2.
14400 CHS ENTER	−14,400.00	
18000 CHS − g CFj	3,600.00	Differential, EOY 3.
17300 CHS ENTER	−17,300.00	
21600 CHS − g CFj	4,300.00	Differential, EOY 4.
20700 CHS ENTER	−20,700.00	
25900 CHS − g CFj	5,200.00	Differential, EOY 5.
24900 CHS ENTER	−24,900.00	

Keystrokes	Display	
31100 CHS − g CFj	6,200.00	Differential, EOY 6.
29900 CHS ENTER	−29,900.00	
37300 CHS − g CFj	7,400.00	Differential, EOY 7.
f IRR	19.10	After tax yield on additional investment.

Conclusion: The additional $15,000 invested in the energy saving equipment produced after-tax savings from reduced energy costs. The differential cash flow ($A - B$) is viewed as an investment. The after-tax yield on the additional investment is 19.10%. Therefore, if the $15,000 can be invested at an after-tax rate of return greater than 19.10%, then the standard system should be purchased. Otherwise the "energy-saver" system is the alternative that will maximize future wealth.

The *IRR* of the differential cash flows is the discount rate at which the *NPV*'s of both alternatives are equal.

Section 5

Financial Management Rate of Return (*FMRR*)

Calculating *FMRR*

The *FMRR** measures the geometric mean rate of growth of invested capital. What this means is that *FMRR* indicates how much money an investment will produce and accumulate to at some time in the future. When using this technique, assumptions made regarding reinvestment of positive cash flows and discounting of negative cash flows must be consistent with the investor's preferences for liquidity and risk. Negative cash flows are discounted at a "safe" rate when estimating the amount to invest in a low-risk, high-liquidity investment that will meet anticipated capital requirements. Usually positive cash flows will be reinvested in more speculative investments with greater yields. Reinvestment rates and "safe" rates are based on current and expected capital market yields and the investor's financial management strategy. We assume that after-tax cash flows and yields will be used when applying this technique.

This section extends the analytical methods available in section 13 of the owner's handbook, which includes solutions for the Modified Internal Rate of Return (*MIRR*). The *FMRR* is derived from a more complex analytical model than the *MIRR*. The reference cited in the footnote at the bottom of this page gives a complete development of the *FMRR*.

The procedure for calculating *FMRR* is:

1. Eliminate negative cash flows by discounting at an appropriate "safe" rate as follows:
   a. Discount and subtract from preceding positive cash flow if available, then

* For an expanded discussion on *FMRR*, refer to Messner, et. al., *Marketing Investment Real Estate*, 2nd Edition (Chicago: REALTORS NATIONAL MARKETING INSTITUTE, 1982) pp. 74-80.

   b. Discount to the initial investment period (giving one initial investment amount).

2. Compound positive cash flows forward at an appropriate reinvestment rate to calculate a single terminal wealth value.

3. Calculate the *FMRR* as follows:
   a. Press [f] CLEAR [FIN] to clear the financial registers.
   b. Key in the investment holding period, then press [n].
   c. Key in the initial investment amount, then press [CHS] [PV].
   d. Key in the terminal wealth value, then press [FV].
   e. Press [i] to solve for *FMRR*.

Example: An investor is considering investment *A* which produces the cash flows after taxes summarized below. The investors "safe" rate, at which negative cash flows are discounted, is 8%, the current yield on tax-free municipal bonds. The investor expects to reinvest positive cash flows at an after-tax rate of 12%. What is the *FMRR*?

Summary of cash flows from investment *A*:

Investment *A*

n	Cash Flows
0	($75,000)
1	($ 9,500)
2	$25,000
3	($10,000)
4	$50,000
5	$87,000

Step 1: Adjust the series to eliminate negative cash flows.

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
1 n	1.00	
8 i	8.00	
10000 FV	10,000.00	
PV	-9,259.26	
25000 +	15,740.74	Net cash flow, EOY 2.
9500 FV	9,500.00	
PV	-8,796.30	
75000 CHS +	-83,796.30	Total initial investment.

The preceding adjustments are graphically summarized below:

n	Cash Flows	Adjustments	Adjusted Cash Flows
0	($75,000)	+ ($8,796.30)	= ($83,796.30)
1	($ 9,500)		0
2	$25,000	+ ($9,259.26)	= $15,740.74
3	($10,000)		0
4	$50,000		$50,000
5	$87,000		$87,000

Step 2: Compound the adjusted positive cash flows forward:

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
12 i	12.00	
15740.74 CHS PV	-15,740.74	Adjusted cash flow, EOY 2.
3 n	3.00	Years reinvested.
FV STO + 0	22,114.61	Saves *FV*, EOY 5 in R_0.
50000 CHS PV	-50,000.00	Net positive cash flow, EOY 4.
1 n	1.00	
FV STO + 0	56,000.00	Add *FV*, EOY 5 to R_0.
RCL 0	78,114.61	
87000 +	165,114.61	Terminal wealth value, EOY 5.

The preceding adjustments are summarized below:

n	Adjusted Cash Flows	Adjustments	Adjusted Cash Flows
0	($83,796.30)		($83,796.30)
1	0		0
2	$15,740.74		0
3	0		0
4	$50,000		0
5	$87,000	+56,000+22,114.61	=$165,114.61

Adjustments that reflect the investor's investment strategy were made to eliminate all intermediate cash flows.

Step 3: Calculate *FMRR*.

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
5 n	5.00	
83796.3 CHS PV	-83,796.30	
165114.61 FV	165,114.61	
i	14.53	*FMRR*.

Interpretation: Under the assumptions given, an initial investment of $83,796.30 is required to both purchase the investment and fund the anticipated first year deficit. After adjusting the cash flows, we computed a terminal wealth value of $165,114.61. The initial investment of $83,796.30 will grow at an annual compound rate of 14.53% to a future value of $165,114.61 at the end of the fifth year.

Using *FMRR*

FMRR can be used to compare investment alternatives. The *FMRR* model automatically makes adjustments for:

1. Discounting negative cash flows.
2. Reinvestment of positive cash flows.

These were demonstrated in the preceding section. When comparing investment alternatives, two additional adjustments may need to be made for differences among alternatives in:

3. The size of the initial investment, and
4. The investment holding period.

The need for the latter two adjustments can be illustrated by comparing the adjusted cash flows of investment *A* in the preceding example to the adjusted cash flows of a second alternative, investment *B*. The two alternatives are summarized below:

Adjusted Cash Flows

n	*A*	*B*
0	($83,796.30)	($100,000)
1	0	0
2	0	0
3	0	0
4	0	$166,000
5	$165,114.61	
Unadjusted *FMRR* =	14.53%	13.51%

A person considering both investments must have at least $100,000 available. If investment *A* is acquired, the difference between the initial costs of investments *A* and *B* would be available for other purposes. When making the adjustment for differences in the size of alternative investments, we assume that the difference between the amount of capital available and the amount used for the selected alternative is invested at a rate of return which produces a level of risk acceptable to the investor for the duration of the longer period. The return on this additional investment amount is then added to the terminal wealth value.

To adjust for differences in the investment holding periods, note that investment *A* has a five year holding period while investment *B* has a four year holding period. To make the adjustment, we assume that the terminal wealth value of the shorter term

investment will be reinvested at some reasonable rate of return until the end of the longer holding period under analysis. In this example, the terminal wealth (EOY 4) of investment *B* would be reinvested for one year to account for the different terms of investments *A* and *B*.

Example: What are the adjustments for the size and time differences between investments *A* and *B*? Assume that the cash flows can be reinvested at an after-tax rate of 12%.

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
83796.3 ENTER	83,796.30	Initial investment amount of *A*.
100000 − PV	−16,203.70	Difference between sizes of initial investments *A* and *B*.
5 n	5.00	Term of investment with larger initial amount.
12 i	12.00	Reinvestment rate of return.
FV	28,556.46	*FV* of reinvested amount, EOY 5.
165114.61 +	193,671.07	Adjusted terminal wealth value of *A*.
f CLEAR FIN		
166000 CHS PV	−166,000.00	Terminal wealth value of *B*, EOY 4.
1 n	1.00	Difference between investment terms.
12 i	12.00	Reinvestment rate of return.
FV	185,920.00	Adjusted terminal value of *B*, EOY 5.
f CLEAR FIN		
5 n	5.00	
100000 CHS PV	−100,000.00	

Keystrokes	Display	
193671.07 [FV]	**193,671.07**	Adjusted terminal wealth of *A*.
[i]	**14.13**	Adjusted *FMRR* of *A*.
185920 [FV]	**185,920.00**	Adjusted terminal wealth of *B*.
[i]	**13.20**	Adjusted *FMRR* of *B*.

The preceding adjustments are graphically summarized below:

n	Investment *A*	Investment *B*
0	($83,796.30) +($16,203.00) = ($100,000.00)	($100,000.00)
1		
2		
3		
4		$166,000.00
5	$165,114.61 + $28,556.46 = $193,671.07	$185,920.00
Adjusted Basis =	14.13%	13.20%

Interpretation: Under these assumptions, the investor would maximize future wealth by choosing investment *A* and pursuing an investment strategy consistent with the assumptions used in this analysis. The investments have been adjusted for negative cash flows, reinvestment of positive cash flows, time differences and size differences. After-tax cash flows and yields were used, therefore taxation has been included in the analysis. The choice can be made simply by comparing the estimated terminal wealth values of the alternative investments.

Section 6

Miscellaneous Feasibility Measures

Real estate practitioners use various measures to compare investment alternatives. Some are used as "rules of thumb" to determine if the investment merits further consideration. These measures are often known by various names and may also be calculated differently by some practitioners. Therefore, when discussing these measures, make sure that everyone involved is using the same definition and method of calculation.

Cash-On-Cash or Equity Dividend Rate

Cash-on-cash is calculated by dividing the cash flow before taxes by the initial equity investment. It is usually expressed as a percent:

$$\text{Cash-on-cash} = \frac{\text{Cash Flow Before Taxes}}{\text{Equity Investment}} \times 100$$

Example: The cash flow before taxes for a property is $15,000. The equity investment is $175,000. What is the cash-on-cash rate?

Keystrokes	Display	
175000 [ENTER]	**175,000.00**	
15000 [%T]	**8.57**	Cash-on-cash percentage rate.

Gross Income Multiplier (*GIM*) or Gross Rent Multiplier (*GRM*)

The gross income multiplier is equal to the property purchase price divided by gross income. Typically all income and rent figures are on an annual basis and the resulting multipliers are therefore also on an annual basis.

The definitions of gross income and gross rent are not standardized. Usually gross rent means gross scheduled rental income. Some practitioners may use gross income which includes adjustments for vacancy, credit losses, and other income.

Calculating the Gross Income Multiplier

The gross income multiplier is calculated by:

$$GIM = \frac{\text{Purchase Price}}{\text{Gross Income}}$$

Example: The price of a property is $750,000 and gross scheduled income is forecast to be $130,000. What is the Gross Income Multiplier?

Keystrokes	Display	
750000 ENTER	750,000.00	
130000 ÷	5.77	*GIM.*

Estimating Price Using the Gross Rent Multiplier

The Gross Rent Multiplier times the gross rent will indicate the purchase price of a property.

$$\text{Purchase Price} = GRM \times \text{Gross Rent}$$

Example: A property has gross scheduled rental income of $130,000 and the market Gross Rent Multiplier is 5.77 for comparable properties. What is the estimated purchase price?

Keystrokes	Display	
130000 ENTER	130,000.00	
5.77 ×	750,100.00	Purchase price.

Note: Since the rounded value of 5.77 is used in this calculation, the resulting purchase price is slightly different from the previous example.

Overall Capitalization Rate (*R*)

The overall capitalization rate, or cap rate, is equal to the net operating income (*NOI*) divided by the purchase price (*V*). It is discussed in detail in section 2, "Appraisal Applications". The cap rate is usually expressed as a percentage.

$$R = \frac{NOI}{V} \times 100$$

Example: What is the cap rate for a property with an *NOI* of $42,000 and a purchase price of $475,000?

Keystrokes	Display	
475000 ENTER	475,000.00	
42000 %T	8.84	Cap percentage rate.

Equity Return Rate (*ERR*)

There are a variety of definitions of *ERR*. The following formula is the one we use:

$$ERR = \left[\frac{CFBT + \text{Loan Principal Reduction} + \Delta V}{\text{Initial Equity Investment}}\right] \times 100$$

Where: $CFBT$ = Cash flow before taxes.

ΔV = Net change in value.

Example: An investor's initial equity investment is $135,000. Cash flow before taxes the first year is $12,250. The property cost a total of $540,000. The expected first year increase in value of the property is 8%. To make the purchase, the investor borrowed $405,000 ($540,000 − 135,000) at 12% interest with monthly payments of $4,165.88. What is the *ERR*?

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
12 g 12÷	1.00	Periodic interest rate.
405000 CHS PV	-405,000.00	Loan principal (entered as a negative value for computational convenience).
4165.88 PMT	4,165.88	Monthly payment.
12 f AMORT	48,520.91	Computes interest for first year.
x≷y STO 0	1,469.65	Displays and saves principal reduction for first year in R_0.
540000 ENTER	540,000.00	Initial value of property.
8 %	43,200.00	Net change in value.
RCL 0 +	44,669.65	
12250 +	56,919.65	*CFBT* + principal reduction + ΔV.
135000 x≷y	56,919.65	Initial equity investment.
%T	42.16	*ERR* percent.

Profitability Index (*PI*)

The profitability index is equal to the present value of the anticipated cash flows—discounted at the investor's required rate of return*—divided by the initial equity investment. The *PI* will equal 1 when the discount rate is equal to the *IRR* of the investment. If the *IRR* of the investment exceeds the investor's required yield, the profitability index will be greater than 1. Usually, before-tax cash flows and before-tax discount rates are used.

$$PI = \frac{\text{Present Value of Cash Flows}}{\text{Initial Equity Investment}}$$

* Refer to "Present Value of Cash Flows or Investment Value," page 72.

Example: An investor pays $100,000 for a leasehold interest that produces $15,800 at the end of each year. At the end of 7 years, the leasehold interest is sold for $75,000. What is the *PI* if the investor requires a 13% rate of return?

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
7 n	7.00	
13 i	13.00	Discount rate.
15800 PMT	15,800.00	
75000 FV	75,000.00	
PV CHS	101,756.79	*PV* of cash flows.
100000 ÷	1.02	*PI* percent.

Appendix A

Calculation of the Number of Payments or Compounding Periods

Section 3 of the owner's handbook explains the use of the basic financial function keys: [n], [i], [PV], [PMT], and [FV]. Use of these five financial keys permits the user to solve an extremely wide range of loan related problems, as demonstrated in this applications book. This appendix is included to supplement the discussion of the [n] key in section 3 of the owner's handbook by completing the development of the solution for the number of payments or compounding periods.

As explained in the owner's handbook, if *n* contains a fractional part (that is, a non-zero value to the right of the decimal) then the HP-12C interprets this as a fractional period *before* the first whole period of the loan. For example, if you key in 36.5 [n], the calculator sees this as 36 and ½ loan periods with the ½ period *at the beginning* of the loan. Because of this special meaning, when you use the [n] key to calculate *n* it is *rounded-up* to the next whole number (integer) and stored in the n register.

We will give a review example of this result in this appendix, followed by a program which you can use to calculate the mathematical result, including the fractional portion of the number.

Using the [n] Key

Example: A lender requires an annual interest rate of 16% on real estate loans. If a loan of $75,000 were to require payments of $1,125 per month, what would the number of payments be to amortize the loan completely?

Keystrokes	Display	
[f] CLEAR [FIN]		Clears financial registers.
16 [g] [12÷]	1.33	Monthly interest rate.
75000 [PV]	75,000.00	Loan amount.
1125 [CHS] [PMT]	-1,125.00	Payment amount.
[n]	166.00	Number of payments.

The number 166 in the display is the number of payments needed. It does *not* mean that 166 *full* payments of $1,125 are needed. What it *does* mean is that 165 payments of $1,125 are needed *plus* one additional payment of up to $1,125. The last payment is not likely to be as large as $1,125. The method for calculating what this last payment must be is not the same in all professions involving loans.

Computing the Exact Value of *n*

The following program will compute the exact value of *n* in periods and fractions of periods, which is the internal result of the [n] key calculation. (Refer to appendix D in the owner's handbook for the formulas used in this calculation.)

Using this program to solve for *n* differs only slightly from the standard procedure using the [n] key. The program instructions following the program listing explains these differences.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[RCL] 0	07– 45 0
[f] CLEAR [PRGM]	00–	[y^x]	08– 21
1	01– 1	[RCL] [PMT]	09– 45 14
[RCL] [i]	02– 45 12	[×]	10– 20
[%]	03– 25	[STO] 3	11– 44 3
[STO] 1	04– 44 1	[RCL] [FV]	12– 45 15
[+]	05– 40	[RCL] 1	13– 45 1
[STO] 2	06– 44 2	[×]	14– 20

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
−	15– 30	÷	21– 10
RCL PV	16– 45 13	g LN	22– 43 23
RCL 1	17– 45 1	RCL 2	23– 45 2
×	18– 20	g LN	24– 43 23
RCL 3	19– 45 3	÷	25– 10
+	20– 40	f P/R	

REGISTERS			
n: Number of Periods	i: Periodic Interest	PV: Present Value	PMT: Payment Amount
FV: Future Value.	R_0: Begin/ end Flag.	R_1: i/100	R_2: 1+i/100
R_3: PMT(1+i)^s s=0 or 1			

1. Key in the program.
2. Press f CLEAR FIN to clear the financial registers.
3. Enter the financial values using the i, PV, PMT, and FV keys exactly as you would in an ordinary financial calculation. (Use correct cash flow sign conventions.)
4. Key in a 0 if payments are at the end of the period, or a 1 if payments are at the beginning of the period, then press STO 0.
5. Solve for *n* by pressing R/S.

Example: What is the exact, decimal value of *n* in the previous example?

Keystrokes	Display	
f CLEAR FIN		Clears financial registers.
16 g 12÷	1.33	Monthly interest rate.
75000 PV	75,000.00	Loan amount.
1125 CHS PMT	−1,125.00	Payment amount.
0 STO 0	0.00	End of period payment mode.
R/S	165.89	*n* to 2 decimal places.
f 9	165.8880344	*n* to full precision of calculator.

Note that this result is essentially the same as the result using the n key, that is, 165 full payments of $1,125 plus an additional payment, smaller than $1,125, due at the end of the 166th period. As stated above there is no consistently followed convention for solving for the last payment amount. A commonly used method can be calculated by pressing the following keys immediately after solving for *n* above:

g INTG n FV RCL i % +.

This series of keystrokes solves for the remaining balance of the loan at the end of the 165th period, calculates one month's interest on the balance, and adds the interest for the month to the remaining balance to give the final payment at the end of the 166th period. In the example above, that amount is $−999.77.

Appendix B

Accelerated Cost Recovery System Tables

These tables* are included for your convenience. The remarks accompanying the tables do not encompass all tax situations.

The use of the tables is quite simple. In all cases the recovery amount for any year is the basis cost of the property multiplied by the percentage from the appropriate table for that year.

Real Estate Tables

The deduction for accelerated cost recovery for real estate is adjusted by the number of months the property is in service during the first tax year of ownership. Multiply the indicated percentage by the original basis of improvements (cost recovery basis) each year.

*Tables are reprinted with permission from Messner, et. al., *Marketing Investment Real Estate*, 2nd Edition (Chicago: REALTORS NATIONAL MARKETING INSTITUTE, 1982) pp. 240-241, 246-247.

Low Income Housing

	Month Placed in Service											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1	13	12	11	10	9	8	7	6	4	3	2	1
2	12	12	12	12	12	12	12	13	13	13	13	13
3	10	10	10	10	11	11	11	11	11	11	11	11
4	9	9	9	9	9	9	9	9	10	10	10	10
5	8	8	8	8	8	8	8	8	8	8	8	9
6	7	7	7	7	7	7	7	7	7	7	7	7
7	6	6	6	6	6	6	6	6	6	6	6	6
8	5	5	5	5	5	5	5	5	5	5	6	6
9	5	5	5	5	5	5	5	5	5	5	5	5
10	5	5	5	5	5	5	5	5	5	5	5	5
11	4	5	5	5	5	5	5	5	5	5	5	5
12	4	4	4	5	4	5	5	5	5	5	5	5
13	4	4	4	4	4	4	5	4	5	5	5	5
14	4	4	4	4	4	4	4	4	4	5	4	4
15	4	4	4	4	4	4	4	4	4	4	4	4
16			1	1	2	2	2	3	3	3	4	4

All Other Real Estate

	Month Placed in Service											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1	12	11	10	9	8	7	6	5	4	3	2	1
2	10	10	11	11	11	11	11	11	11	11	11	12
3	9	9	9	9	10	10	10	10	10	10	10	10
4	8	8	8	8	8	8	9	9	9	9	9	9
5	7	7	7	7	7	7	8	8	8	8	8	8
6	6	6	6	6	7	7	7	7	7	7	7	7
7	6	6	6	6	6	6	6	6	6	6	6	6
8	6	6	6	6	6	6	5	6	6	6	6	6
9	6	6	6	6	5	6	5	5	5	6	6	6
10	5	6	5	6	5	5	5	5	5	5	6	5
11	5	5	5	5	5	5	5	5	5	5	5	5
12	5	5	5	5	5	5	5	5	5	5	5	5
13	5	5	5	5	5	5	5	5	5	5	5	5
14	5	5	5	5	5	5	5	5	5	5	5	5
15	5	5	5	5	5	5	5	5	5	5	5	5
16			1	1	2	2	3	3	4	4	4	5

Personal Property Tables

The cost recovery deduction for personal property is equal to the original basis cost of the personal property multiplied by the indicated percentage from the table for the cost recovery year. The cost basis need not be reduced by salvage value.

For Property Placed in Service from 1981 to 1984

Recovery Percentage

	Class of Investment			
	3-year	5-year	10-year	15 year
Year				
1	25	15	8	5
2	38	22	14	10
3	37	21	12	9
4		21	10	8
5		21	10	7
6			10	7
7			9	6
8			9	6
9			9	6
10			9	6
11				6
12				6
13				6
14				6
15				6
Total	100	100	100	100

For Property Placed in Service in 1985

Recovery Percentage

	Class of Investment			
	3-year	5-year	10-year	15-yr
Year				
1	29	18	9	6
2	47	33	19	12
3	24	25	16	12
4		16	14	11
5		8	12	10
6			10	9
7			8	8
8			6	7
9			4	6
10			2	5
11				4
12				4
13				3
14				2
15				1
Total	100	100	100	100

For Property Placed in Service After December 31, 1985

Recovery Percentage

	Class of Investment			
	3-year	5-year	10-year	15-yr
Year				
1	33	20	10	7
2	45	32	18	12
3	22	24	16	12
4		16	14	11
5		8	12	10
6			10	9
7			8	8
8			6	7
9			4	6
10			2	5
11				4
12				3
13				3
14				2
15				1
Total	100	100	100	100

Order Part Number
00012-90015

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