

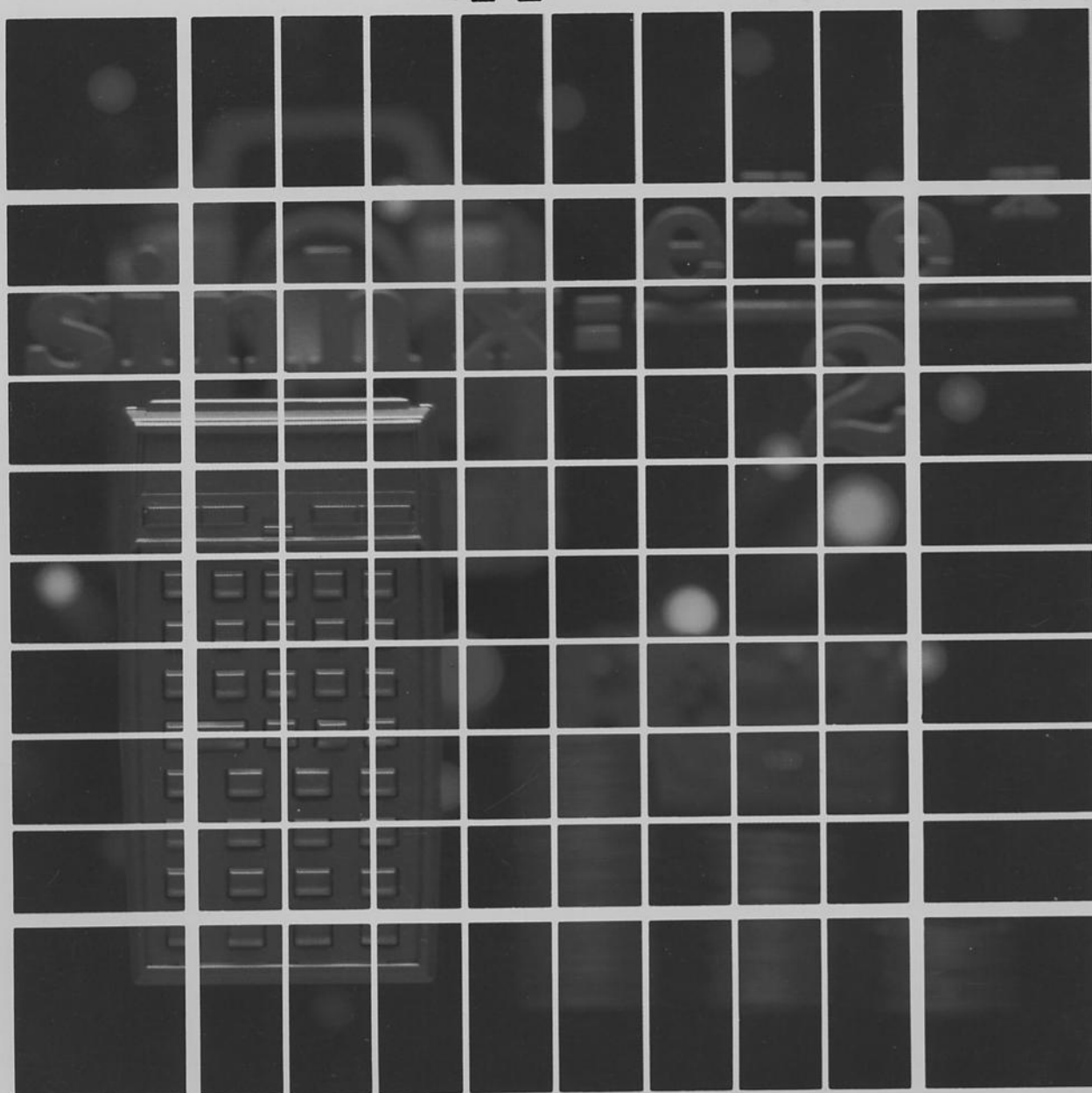
HEWLETT-PACKARD

HP-41C

USER'S

LIBRARY SOLUTIONS

Applied statistics I



- Chi-square distribution

a - n' odd

$$P(\chi^2, n') = 2 P(\chi) + 2 Z(\chi) \sum_{r=1}^{(n'-1)/2} \frac{\chi^{2r-1}}{1.5.5....(2r-1)}$$

b - n' even

$$P(\chi^2, n') = \sqrt{2\pi} Z(\chi) \left[1 + \sum_{r=1}^{(n'-2)/2} \frac{\chi^{2r}}{2.4.6....2r} \right]$$

$$\chi = \sqrt{\chi^2} \quad Z(\chi) \text{ and } P(\chi) \text{ from the normal distribution}$$

- F distribution

n'_1 = d.f. of the numerator n'_2 = d.f. of the denominator of F
 $P(F, n'_1, n'_2)$ is computed like $P(u)$, with the transformation :

$$u = \frac{(1 - 2/9 n'_2) F^{1/3} - (1 - 2/9 n'_1)}{\sqrt{\frac{2}{9 n'_1} + \frac{2}{9 n'_2} F^{2/3}}}$$

From : M.ABRAMOWITZ & I.STEGUN - Handbook of Mathematical Functions.

The exemples in the programs come from :

J.PHILIPPE - Les Méthodes statistiques en Pharmacie et en Chimie
 Masson Paris 1967

S.SIEGEL - Non Parametric statistics Mc Graw Hill New York 1975

G.W.SNEDECOR & W.G.COCHRAN - Méthodes statistiques A.C.T.A. Paris 1971

USER'S GENERAL INSTRUCTIONS

All the programs are for HP 41 C with the card reader and the printer attached, and one or two memory modules.

In the instructions, the quote mark "....." around a group of characters indicates that this group is ALPHA. The key ALPHA must be pressed before and after entering the instruction.

- the user mode should be always on
- the statistical registers are located in R14 to R19
- all the data are entered through the R/S key
- a correction subroutine is provided to correct a wrong data, initialized by E.
- the yellow prefix key $\square$ is denoted in the instructions by p.
(for instance, p.A = a)

Before keying in a new program :

- clear the previous one by : XEQ ALPHA CLP ALPHA ; ALPHA ALPHA
- be sure that all the flags of a previous program are cleared
- set the proper number of data storage registers by XEQ "SIZE" —
(the size allocation required for each program is indicated in the status information sheet)

Before executing a new program, it is initialized by : XEQ "name"; this initialization is not necessary for other runs with the same program and other sets of data.

The number of decimals is usually set at 2; if more are needed, according to the nature of the data, they can be set by p.FIX -

MEAN AND CONFIDENCE INTERVALDESCRIPTION -

From a sample of n observations, the mean is calculated :

$$\bar{y} = \frac{1}{n} \sum y_i$$

This sample is considered as drawn at random from a normal population with mean μ and standard deviation σ . From the central-limit theorem, it is known that the distribution of the mean of samples of size n from this population is also normal, with mean μ and standard error $\sigma/\sqrt{n}$. The confidence interval of the mean is :

$$\bar{y} - u_p \sigma / \sqrt{n} \leq \mu \leq \bar{y} + u_p \sigma / \sqrt{n}$$

For $u_p = u_{0,05} = 1,96$, the confidence interval calculated will include 19 times out of 20, on the long run, the true mean μ . These limits are the 95 % confidence limits.

Generally, the true value of σ is not known and must be estimated from the data, with :

$$s = \sqrt{\frac{\sum y^2 - (\sum y)^2/n}{(n-1)}}$$

In this case, the distribution of "t" is used instead of the normal distribution, and the confidence interval is :

$$\bar{y} - t_p s / \sqrt{n} \leq \mu \leq \bar{y} + t_p s / \sqrt{n}$$

t_p is the value of t for the number of degrees of freedom $(n-1)$ and the chosen probability p.

This program computes the mean and the 95 % and 99 % confidence limits, with σ -known, or estimated by s.

USER INSTRUCTIONS I

PROGRAMMABLAUF I INSTRUCTIONS D'EMPLOI I NORME OPERATIVE I

STAT.01

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	Instructions Operation Instructions Istruzioni	Variables Dateneingabe Données Dati	Function(s) Taste(n) Touche(s) Tasti	Result Resultat Résultat Risultato
1	Key in the program STAT.01, tracks 1 and 2.			
2	Initialize		XEQ"INT.CONF."	
3	<u>Standard deviation σ known</u>			
	Call		C	
	Input	σ	R/S	
3'	<u>Standard deviation s estimated</u>			
	Call		A	
4	Input the data y_1	y_1	R/S	
5	<u>Correction routine</u>			
	Call		E	
	Input the erroneous y	y_E	R/S	
	Input the correct y	y_C	R/S	
6	After the last y_1			
	Call		B	
	Display of : mean			MOY. $\bar{y}$
	standard deviation			D.S. σ or s
	standard error			E.S. $s_{\bar{y}}$
	Confidence interval			
	P = 0,95			
	Lower limit			L.I.
	Upper limit			L.S.
	P = 0,99			
	Lower limit			L.I.
	Upper limit			L.S.
7	For another set of data, go to step 3 or 3'			

EXAMPLE 1 - σ known. (*)

A sample of 10 data : 34,30 - 34,60 - 34,00 - 34,85 - 33,60 -
34,20 - 34,10 - 33,80 - 34,10 - 36,45 -

is drawn at random from a population with a standard deviation $\sigma = 0,75$

EXAMPLE 2 - s estimated

Same data

```

XEQ "INTCONF"
      XEQ A
      34.30  RUN
      34.60  RUN
      34.00  RUN
      34.85  RUN
(error) 336.00 RUN
      XEQ E
      336.00 RUN
      33.60  RUN
      34.20  RUN
      34.10  RUN
      33.80  RUN
      34.10  RUN
      36.45  RUN
      XEQ B

```

MOY.=34.40
D.S.=0.81
E.S.=0.25
D.L.=9.00

P=0.95
L.I.=33.82
L.S.=34.98

P=0.99
L.I.=33.57
L.S.=35.23

```

XEQ "INTCONF"
      XEQ C
      0.75  RUN
      34.30  RUN
      34.60  RUN
      34.00  RUN
      34.85  RUN
      33.60  RUN
      34.20  RUN
      34.10  RUN
      33.80  RUN
      34.10  RUN
      36.45  RUN
      XEQ B

```

MOY.=34.40
D.S.=0.75
E.S.=0.24
D.L.=9.00

P=0.95
L.I.=33.94
L.S.=34.86

P=0.99
L.I.=33.79
L.S.=35.01

Line Zeile Ligne Linea	Keystrokes Tastenfolge Touches Tasti	Comments Kommentar Commentaires Commenti	Line Zeile Ligne Linea	Key pressed Tastenfolge Touches Tasti	Comments Kommentar Commentaires Commenti
01	01*LBL "INTCONF"		51	RCL 03	
	02 CLRG		52	/	
	03 CF 00		53	E↑X	
	04 RTN		54	1.576	
	05*LBL C		55	+	
	06 STOP		56	FS? 00	
	07 STO 00		57	2.576	
	08 ADV		58	"P=0.99"	
	09 SF 00		59	AVIEW	
	10*LBL A		60	XEQ 02	
	11*LBL 01		61	RTN	
	12 STOP		62*LBL 02		
	13 SREG 14		63	RCL 02	
	14 Σ+		64	*	
	15 GTO 01		65	STO 04	
	16*LBL P		66	RCL 01	
	17 MEAN		67	-	
	18 "MOY."		68	CHS	
	19 XEQ 11		69	"L.I."	
	20 STO 01		70	XEQ 11	
	21 SDEV		71	LASTX	
	22 FS? 00		72	RCL 04	
	23 RCL 00		73	+	
	24 "D.S."		74	"L.S."	
	25 XEQ 11		75	XEQ 11	
	26 RCL 19		76	ADV	
	27 SQRT		77	RTN	
	28 /		78*LBL 11		
	29 STO 02		79	"I="	
	30 "E.S."		80	ARCL X	
	31 XEQ 11		81	AVIEW	
	32 RCL 19		82	RTN	
	33 1		83*LBL E		
	34 -		84	STOP	
	35 STO 03		85	Σ-	
	36 "D.L."		86	GTO 01	
	37 XEQ 11		87	.END.	
	38 ADV				
	39 2.3777				
	40 RCL 03				
	41 /				
	42 E↑X				
	43 .96				
	44 +				
	45 FS? 00				
	46 1.96				
	47 "P=0.95"				
	48 AVIEW				
	49 XEQ 02				
	50 4.65				

REGISTERS, STATUS, FLAGS

STAT.01

REGISTERBELEGUNG, FLAGS, BETRIEBSARTEN
REGISTRES, INDICATEURS, MODES OPÉRATOIRES
REGISTRI, MODI OPERATIVI, FLAGS

Registers Datenspeicher Registres de données Registri				Status Betriebsart Modes opératoires Modi operativi			
00		50		Size 026 Total Reg.			User Mode
01	$\bar{y}$			Eng ☐	Fix ☒	Sci ☐	On ☒
02	$s_{\bar{y}}$			Deg ☐	Rad ☐	Grad ☐	Off ☐
03	d.f.			Purpose Bedeutung Signification Scopo			
04	$t s_{\bar{y}}$						
05		55		Flags SET CLEAR			
				00		☒	
				01			
				02			
10		60		03			
				04			
				05			
				06			
				07			
15		65		08			
				09			
				10			
				11	Audio execute		
				12			
20		70		13			
				14			
				15			
				16			
				17			
25		75		18			
				19			
				20			
				21	Printer Enable		
				22	Number Input		
30		80		23	Alpha Input		
				24	Range Ignore		
				25	Error Ignore		
				26	Audio Enable		
				27	User Mode		
35		85		28	Decimal Point		
				29	Digit Grouping		
				Assignments Tastenbelegung/Assignations/Assegnamenti			
				Function Funktion Fonction Funzione	Key Taste Touche Tasto	Function Funktion Fonction Funzione	Key Taste Touche Tasto
40		90					
45		95					
		99					

STAT.02 -

ONE SAMPLEHISTOGRAM - NORMALITY TESTSDESCRIPTION -

When a sample is large enough ($n > 30$), it is possible to build a table of frequency distributions, from which a frequency histogram can be constructed. This histogram will give an idea of the shape of the distribution of the parent population. The number of cells of this table is chosen by the user, who selects the lower limit of the first cell y_0 and the cell size L .

If the data are not already grouped, they are input one after the other and classified by the program in the several cells. The program computes :

$$\begin{aligned} - \text{the mean : } \bar{y} &= \sum y_i / N \\ - \text{the standard deviation : } s &= \sqrt{\frac{\sum y_i^2 - (\sum y_i)^2 / N}{N - 1}} \end{aligned}$$

The normality of the distribution may be checked by computing g_1 and g_2 , estimations of γ_1 , coefficient of symmetry (or skewness) and γ_2 , coefficient of kurtosis. These coefficients are computed from the moments of order 3 and 4, as :

$$\begin{aligned} g_1 &= 1 / ns^3 \sum (y_i - \bar{y})^3 \\ g_2 &= 3 - (1 / ns^4) \sum (y_i - \bar{y})^4 \end{aligned}$$

They are both zero when the population is normal. A negative g_1 indicates skewness to the left; a positive g_1 skewness to the right. A negative g_2 indicate platykurtosis; a positive g_2 shows leptokurtosis.

The y_i may be replaced by their logarithms to see if the distribution is log.normal.

If the data are already grouped in a frequency distribution, they are input directly. In both cases, the program prints :

- the midpoint of each class
- the frequency in this class f_i
- the relative frequency $f_i \% = \frac{100 f_i}{\sum f_i}$
- the cumulative relative frequencies $\sum f_i \%$

From these data, one can check the normality by a graphical method using probability paper.

USER INSTRUCTIONS I

PROGRAMMABLAUF I
 INSTRUCTIONS D'EMPLOI I
 NORME OPERATIVE I

STAT.02

Step Schritt Pas Passo	Instructions Operation Instructions Istruzioni	Variables Dateneingabe Données Dati	Function(s) Taste(n) Touche(s) Tasti	Result Resultat Résultat Risultato
1	Key in the program STAT.02, tracks 1 to 4			
2	Initialize		XEQ "HIST."	
3	a- <u>Transform of y to log.y</u> Call		C	
3'	b- <u>Ungrouped data</u> Call		A	
4	Input : lower limit of first cell cell size	y ₀ L	R/S R/S	
5	Input the data	y ₁	R/S	
6	<u>Correction routine</u> Call		E	
	Input the erroneous y	y _E	R/S	
	Input the correct y	y _C	R/S	
7	Call		D	N
	Display of : sample size			MOY.
	mean			D.S.
	standard deviation			s ₁
	coeff. skewness			s ₂
	coeff. kurtosis			
	Frequency distribution :			y'
	class midpoint			f
	frequency			f %
	relative frequency			Σ f %
	cumulative relat. freq.			
8	c- <u>Grouped data</u> Call		B	
9	Input : sample size	N	R/S	
10	For each class, input :			
	class midpoint	y'	R/S	
	frequency	f	R/S	
11	<u>Correction routine</u> If y' is erroneous, enter f before calling	E		
	Input again the erroneous pair	y' _E f _E	R/S R/S	
	then the correct data		D	
12	Call			N
	Display of : sample size			MOY.
	mean			D.S.
	standard deviation			s ₁
	coeff. skewness			s ₂
	coeff. kurtosis			
13	For another set of data, go to step 2.			

EXAMPLE 1 - Ungrouped data

XEQ	"HIST"	N=40.000
XEQ A		MOY.=11.475
10.025	RUN	D.S.=0.689
0.350	RUN	s1=0.319
		s2=0.103
12.200	RUN	
10.700	RUN	10.200 ***
12.100	RUN	f=1.000
10.800	RUN	f%=2.500
12.900	RUN	Σf%=2.500
11.500	RUN	
11.900	RUN	10.550 ***
11.600	RUN	f=6.000
11.800	RUN	f%=15.000
11.300	RUN	Σf%=17.500
10.400	RUN	
10.400	RUN	10.900 ***
11.200	RUN	f=4.000
10.700	RUN	f%=10.000
10.700	RUN	Σf%=27.500
11.700	RUN	
11.600	RUN	11.250 ***
11.600	RUN	f=7.000
10.800	RUN	f%=17.500
12.000	RUN	Σf%=45.000
11.100	RUN	
11.900	RUN	11.600 ***
11.000	RUN	f=7.000
12.400	RUN	f%=17.500
12.300	RUN	Σf%=62.500
13.300	RUN	
11.900	RUN	11.950 ***
11.700	RUN	f=9.000
12.200	RUN	f%=22.500
11.200	RUN	Σf%=85.000
10.200	RUN	
11.800	RUN	12.300 ***
11.800	RUN	f=4.000
10.500	RUN	f%=10.000
10.900	RUN	Σf%=95.000
11.300	RUN	
11.800	RUN	12.650 ***
11.100	RUN	f=0.000
11.600	RUN	f%=0.000
11.100	RUN	Σf%=95.000
	XEQ D	
		13.000 ***
		f=1.000
		f%=2.500
		Σf%=97.500
		13.350 ***
		f=1.000
		f%=2.500
		Σf%=100.000

EXAMPLE 2 - Ungrouped data
Log. transform

XEQ	"HIST"	N=40.000
XEQ C		MOY.=1.059
XEQ A		D.S.=0.026
1.006	RUN	s1=0.138
.012	RUN	s2=-0.178
12.200	RUN	
10.700	RUN	1.012 ***
15.100	RUN	f=3.000
	XEQ E	f%=7.500
15.100	RUN	Σf%=7.500
12.100	RUN	
10.800	RUN	1.024 ***
12.900	RUN	f=4.000
11.500	RUN	f%=10.000
11.900	RUN	Σf%=17.500
11.600	RUN	
11.800	RUN	1.036 ***
11.300	RUN	f=4.000
10.400	RUN	f%=10.000
10.400	RUN	Σf%=27.500
11.200	RUN	
10.700	RUN	1.048 ***
10.700	RUN	f=7.000
11.900	RUN	f%=17.500
11.600	RUN	Σf%=45.000
11.600	RUN	
10.800	RUN	1.060 ***
12.000	RUN	f=5.000
11.100	RUN	f%=12.500
11.900	RUN	Σf%=57.500
11.000	RUN	
12.400	RUN	1.072 ***
12.300	RUN	f=9.000
13.300	RUN	f%=22.500
11.900	RUN	Σf%=80.000
11.700	RUN	
12.200	RUN	1.084 ***
11.200	RUN	f=5.000
10.200	RUN	f%=12.500
11.800	RUN	Σf%=92.500
11.800	RUN	
10.500	RUN	1.096 ***
10.900	RUN	f=1.000
11.300	RUN	f%=2.500
11.800	RUN	Σf%=95.000
11.100	RUN	
11.600	RUN	1.108 ***
11.100	RUN	f=1.000
	XEQ D	f%=2.500
		Σf%=97.500
		1.120 ***
		f=1.000
		f%=2.500
		Σf%=100.000

CJC

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EXAMPLE 3 - Grouped data (≡)

	XEQ "HIST"	124,000	RUN		134,000	RUN
	XEQ B	23,000	RUN	(error)	15,000	RUN
250,000	RUN	9,200	***		6,000	***
		35,600	***		101,200	***
116,000	RUN					
1,000	RUN	125,000	RUN			XEQ E
0,400	***	21,000	RUN		134,000	RUN
0,400	***	8,400	***		15,000	RUN
		44,000	***		134,000	RUN
117,000	RUN				5,000	RUN
2,000	RUN	126,000	RUN		2,000	***
0,800	***	29,000	RUN		97,200	***
1,200	***	11,600	***			
		55,600	***		135,000	RUN
118,000	RUN				4,000	RUN
2,000	RUN	127,000	RUN		1,600	***
0,800	***	26,000	RUN		98,800	***
2,000	***	10,400	***			
		66,000	***		136,000	RUN
119,000	RUN				3,000	RUN
5,000	RUN	128,000	RUN		1,200	***
2,000	***	22,000	RUN		100,000	***
4,000	***	8,800	***			
		74,800	***			XEQ D
120,000	RUN			N=250,000		
10,000	RUN	129,000	RUN	MOY.=126,096		
4,000	***	14,000	RUN	D.S.=3,994		
8,000	***	5,600	***	s1=0,185		
		80,400	***	s2=-0,173		
(error)	131,000	RUN				
	11,000	RUN				
	4,400	***				
	12,400	***				
		86,400	***			
	XEQ E					
131,000	RUN	131,000	RUN			
11,000	RUN	10,000	RUN			
121,000	RUN	4,000	***			
11,000	RUN	90,400	***			
4,400	***					
12,400	***	132,000	RUN			
		7,000	RUN			
122,000	RUN	2,800	***			
15,000	RUN	93,200	***			
6,000	***					
18,400	***	133,000	RUN			
		5,000	RUN			
123,000	RUN	2,000	***			
20,000	RUN	95,200	***			
8,000	***					
26,400	***					

(≡) PHILIPPE p.6

01*LBL "HIST"	58 STO 06	115 RCL 00	172 3	229 SF 13
02 CLRG	59 1	116 1	173 -	230 XEQ 11
03 CF 00	60 FS? 01	117 -	174 /	231 CF 13
04 CF 01	61 STOP	118 RCL 00	175 RCL 05	232 ST+ 04
05 RTN	62 STO 05	119 2	176 RCL 06	233 1
06*LBL 0	63 RTN	120 -	177 *	234 ST+ 03
07 SF 00	64*LBL 06	121 *	178 /	235 ST+ 10
08 RTN	65 100	122 RCL 05	179 3	236 RCL 04
09*LBL 8	66 *	123 RCL 06	180 RCL 00	237 "ZF"
10 SF 01	67 RCL 00	124 *	181 1	238 SF 13
11 STOP	68 /	125 STO 05	182 -	239 XEQ 11
12 STO 00	69 PRX	126 *	183 X+2	240 CF 13
13 ADV	70 ST+ 11	127 /	184 *	241 100
14 GTO 01	71 RCL 11	128 "G1"	185 RCL 00	242 X=Y?
15*LBL A	72 PRX	129 SF 13	186 2	243 GTO 04
16 STOP	73 ADV	130 XEQ 11	187 -	244 GTO 05
17 STO 07	74 RTN	131 CF 13	188 /	245*LBL 04
18 STOP	75*LBL D	132 RCL 00	189 RCL 00	246*LBL E
19 STO 09	76 RCL 01	133 RCL 04	190 3	247 0
20 ADV	77 RCL 00	134 *	191 -	248 STO 10
21*LBL 01	78 "N"	135 4	192 /	249 5
22 0	79 XEQ 11	136 RCL 01	193 -	250 STO 13
23 STO 10	80 /	137 RCL 03	194 "G2"	251 XEQ 02
24 5	81 "MOY."	138 *	195 SF 13	252 FC? 01
25 STO 13	82 XEQ 11	139 *	196 XEQ 11	253 GTO 00
26 XEQ 02	83 RCL 02	140 -	197 CF 13	254 100
27 FS? 01	84 RCL 01	141 6	198 FS? 01	255 *
28 XEQ 06	85 X+2	142 RCL 01	199 GTO 04	256 RCL 00
29*LBL 03	86 RCL 00	143 X+2	200 20	257 /
30 RCL 05	87 /	144 *	201 STO 10	258 ST- 11
31 ST+ IND 10	88 -	145 RCL 02	202 0	259*LBL 08
32 RCL 06	89 RCL 00	146 RCL 00	203 STO 04	260*LBL 07
33 *	90 1	147 /	204 STO 03	261 RCL 05
34 STO 05	91 -	148 *	205 RCL 07	262 ST- IND 10
35 1	92 /	149 +	206 RCL 09	263 RCL 06
36 ST+ 10	93 STO 05	150 3	207 2	264 *.
37 ST- 13	94 SQR	151 RCL 01	208 /	265 STO 05
38 RCL 12	95 "D.S."	152 4	209 +	266 1
39 X>0?	96 XEQ 11	153 Y+X	210 STO 07	267 ST+ 10
40 GTO 03	97 STO 06	154 *	211*LBL 05	268 ST- 13
41 FS? 01	98 RCL 00	155 RCL 00	212 ADV	269 RCL 13
42 GTO 01	99 RCL 03	156 X+2	213 RCL 03	270 X>0?
43 RCL 06	100 *	157 /	214 RCL 09	271 GTO 07
44 RCL 07	101 3	158 -	215 *	272 FS? 01
45 -	102 RCL 01	159 RCL 00	216 RCL 07	273 GTO 01
46 RCL 09	103 RCL 02	160 1	217 +	274 RCL 06
47 /	104 *	161 +	218 PRX	275 RCL 07
48 20	105 *	162 *	219 RCL IND 10	276 -
49 +	106 -	163 RCL 00	220 "F"	277 RCL 09
50 STO 16	107 2	164 1	221 SF 13	278 /
51 1	108 RCL 01	165 -	222 XEQ 11	279 20
52 ST+ IND 16	109 3	166 /	223 CF 13	280 +
53 GTO 01	110 Y+X	167 RCL 00	224 100	281 STO 16
54*LBL 02	111 *	168 2	225 *	282 1
55 STOP	112 RCL 00	169 -	226 RCL 00	283 ST- IND 16
56 FS? 00	113 /	170 /	227 /	284 GTO 01
57 LOG	114 +	171 RCL 00	228 "F"	285*LBL 11

PROGRAM LISTING
PROGRAMMAUFLISTUNG
LISTAGE DU PROGRAMME
LISTATO DI PROGRAMMA

286 "f="

287 ARCL X

288 AVIEW

289 RTN

290 ADV

291 END

REGISTERS, STATUS, FLAGS

REGISTERBELEGUNG, FLAGS, BETRIEBSARTEN

REGISTRES, INDICATEURS, MODES OPÉRATOIRES

REGISTRI, MODI OPERATIVI, FLAGS

19

Registers Datenspeicher Registres de données Registri				Status Betriebsart Modes opératoires Modi operativi			
00	Σf	50		Size <u>050</u>	Total Reg. _____	User Mode	
	Σfy			Eng ☐	Fix ☒	Sci ☐	On ☒
	Σfy^2			Deg ☐	Rad ☐	Grad ☐	Off ☐
	Σfy^3			Purpose Bedeutung Signification Scopo Flags SET CLEAR			
	Σfy^4						
05	13 y or log.y	55		00	Log transform	☒	
	y_0			01	Grouped data	☒	
	N			02			
	L			03			
10	0	60		04			
	$\Sigma f \%$			05			
	5			06			
				07			
15		65		08			
	class midpoint			09			
				10			
				11	Audio execute		
				12			
20	to 50 f in cells	70		13			
				14			
				15			
				16			
				17			
25		75		18			
				19			
				20			
				21	Printer Enable		
				22	Number Input		
30		80		23	Alpha Input		
				24	Range Ignore		
				25	Error Ignore		
				26	Audio Enable		
				27	User Mode		
35		85		28	Decimal Point		
				29	Digit Grouping		
				Assignments Tastenbelegung/Assignations/Assegnamenti			
				Function Funktion Fonction Funzione	Key Taste Touche Tasto	Function Funktion Fonction Funzione	Key Taste Touche Tasto
40		90					
45		95					
		99					

TWO INDEPENDANT SAMPLES

1 - COMPARISON OF VARIANCES : "F" TEST

2 - COMPARISON OF MEANS : "t" TEST

DESCRIPTION

Two independent samples, with size n_1 and n_2 , are compared.

1 - Comparison of variances.

The two variances are estimated from the data, as s_1^2 and s_2^2 .

H_0 : the two variances are estimations of the same variance σ^2

H_1 : the two variances are different : $\sigma_1^2 \neq \sigma_2^2$

$F = s_1^2/s_2^2$ (or s_2^2/s_1^2) using the largest variance as the numerator. The program display the probability $P_{(F)}$ (two-tailed test)

2 - Comparison of means.

H_0 : the two samples come from the same population (μ, σ)

H_1 : the two samples come from two populations with means μ_1 and μ_2 , but with the same variance σ^2

$H_{1(a)}$: $\mu_1 \neq \mu_2$ (two-tailed test)

$H_{1(b)}$: $\mu_1 > \mu_2$ or $\mu_1 < \mu_2$ (one-tailed test)

If F is not significant, the Student's "t" test is appropriate :

$$t = \frac{|\bar{y}_1 - \bar{y}_2|}{s_{\text{comb}} \sqrt{1/n_1 + 1/n_2}} \quad \text{with } (n_1 + n_2 - 2) \text{ degrees of freedom}$$

If F is significant ($P < 0,05$), the Student's "t" test is not convenient; the program computes :

$$t' = \frac{|\bar{y}_1 - \bar{y}_2|}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} \quad \text{with a number of d.f. such as}$$

$$\frac{1}{f} = \frac{c^2}{n_1 - 1} + \frac{(1 - c)^2}{n_2 - 1}, \quad \text{with } c = \frac{s_1^2}{n_1} / \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)$$

The user selects the two-tailed or the one-tailed test, and the program displays the probability $P_{(t)}$. If P is lower than the chosen significance level, the difference $|\bar{y}_1 - \bar{y}_2|$ is significant.

EXAMPLE 1 - F non significant (*)

	XEQ "ECHIND"	146,60	RUN	F=2,07	
	146,30	RUN	143,00	RUN	DL1=6,00
	140,50	RUN	145,30	RUN	DL2=9,00
	143,00	RUN	149,50	RUN	P=0,157
	143,90	RUN	144,10	RUN	t=3,04
	142,00	RUN	149,40	RUN	DL=15,00
(error)	1.141,50	RUN	144,00	RUN	
	141,60	RUN		XEQ B	XEQ b
	143,20	RUN	145,99	***	P=0,008
	144,50	RUN			
	140,70	RUN			
	XEQ E				
	1.141,50	RUN			
	141,50	RUN			
	XEQ B				
	142,72	***			

EXAMPLE 2 - F significant

	XEQ A	338,00	RUN	F=3,02	
348,00	RUN	350,00	RUN	DL1=17,00	
345,00	RUN	335,00	RUN	DL2=17,00	
350,00	RUN	333,00	RUN	P=0,014	
344,00	RUN	345,00	RUN	t=3,14	
342,00	RUN	331,00	RUN	DL=27,15	
346,00	RUN	354,00	RUN		XEQ b
349,00	RUN	339,00	RUN	P=0,004	
345,00	RUN	339,00	RUN		
341,00	RUN	343,00	RUN		
347,00	RUN	347,00	RUN		
344,00	RUN	340,00	RUN		
339,00	RUN	336,00	RUN		
341,00	RUN	342,00	RUN		
348,00	RUN	345,00	RUN		
352,00	RUN	338,00	RUN		
345,00	RUN	335,00	RUN		
346,00	RUN	339,00	RUN		
349,00	RUN		XEQ B		
	XEQ B	340,50	***		
345,61	***				

01*LBL "ECHIND"	58 XEQ 05	115 RCL 20	172 RCL 05	229 RCL 01
02*LBL A	59*LBL 04	116 3	173 +	230 /
03 CLRG	60 RTN	117 Y+X	174 /	231 1
04 CF 00	61*LBL 05	118 .000344	175 RCL 00	232 RCL 11
05*LBL 01	62 2	119 *	176 1/X	233 -
06 STOP	63 ENTER↑	120 +	177 RCL 04	234 X↑2
07 Σ+	64 9	121 RCL 20	178 1/X	235 RCL 05
08 GTO 01	65 /	122 4	179 +	236 /
09*LBL B	66 STO 17	123 Y+X	180 *	237 +
10 XEQ 02	67 RCL 09	124 .019527	181 SQRT	238 1/X
11 1	68 /	125 *	182 STO 00	239 STO 09
12 -	69 STO 18	126 +	183 GTO 00	240 "DL"
13 XEQ 02	70 RCL 17	127 -4	184*LBL 07	241 XEQ 19
14 MEAN	71 RCL 10	128 Y+X	185 RCL 03	242*LBL 10
15 PRX	72 /	129 2	186 RCL 00	243 RTN
16 ADV	73 STO 17	130 /	187 /	244*LBL a
17 XEQ 02	74 1	131 CHS	188 RCL 07	245 SF 02
18 SDEV	75 -	132 1	189 RCL 04	246*LBL b
19 X↑2	76 CHS	133 +	190 /	247 RCL 10
20 XEQ 02	77 RCL 00	134 FS?C 02	191 +	248 ABS
21 CLS	78 3	135 XEQ 06	192 SQRT	249 RCL 09
22 4	79 1/X	136 FIX 3	193 STO 00	250 RAD
23 RCL 25	80 Y+X	137 "P"	194*LBL 00	251 SQRT
24 X=Y?	81 *	138 XEQ 19	195 RCL 02	252 /
25 GTO 01	82 1	139 FIX 2	196 RCL 06	253 ATAN
26 RCL 07	83 RCL 18	140 ADV	197 -	254 STO 02
27 RCL 03	84 -	141 .05	198 ABS	255 RCL 09
28 X↑Y?	85 -	142 X=Y?	199 RCL 00	256 ?
29 GTO 03	86 STO 19	143 GTO C	200 /	257 /
30 /	87 RCL 17	144 GTO D	201 SF 13	258 INT
31 "F"	88 RCL 00	145 RTN	202 "T"	259 LASTX
32 XEQ 19	89 2	146*LBL 06	203 XEQ 19	260 X*Y?
33 STO 00	90 ENTER↑	147 1	204 CF 13	261 GTO 11
34 RCL 05	91 3	148 X<>Y	205 STO 10	262 0
35 "DL1"	92 /	149 -	206 FS? 00	263 STO 05
36 XEQ 19	93 Y+X	150 RTN	207 GTO 09	264*LBL 16
37 STO 09	94 *	151*LBL 02	208 RCL 01	265 RCL 02
38 RCL 01	95 RCL 18	152 STO IND 25	209 RCL 05	266 COS
39 "DL2"	96 +	153 7ISZ	210 +	267 X↑2
40 XEQ 19	97 SQRT	154 RTN	211 STO 09	268 STO 03
41 STO 10	98 RCL 19	155*LBL E	212 "DL"	269 RCL 02
42 XEQ 05	99 X<>Y	156 STOP	213 XEQ 19	270 SIN
43 GTO 04	100 /	157 Σ-	214 GTO 10	271 STO 04
44*LBL 03	101 STO 20	158 GTO 01	215*LBL 09	272 RCL 09
45 X<>Y	102 X>0?	159*LBL D	216 RCL 03	273 2
46 /	103 SF 02	160 SF 00	217 RCL 00	274 X=Y?
47 "F"	104 ABS	161*LBL C	218 /	275 GTO 14
48 XEQ 19	105 1	162 FS? 00	219 STO 11	276 /
49 STO 00	106 RCL 20	163 GTO 07	220 RCL 07	277 1
50 RCL 01	107 .196854	164 RCL 01	221 RCL 04	278 -
51 "DL1"	108 *	165 RCL 03	222 /	279 STO 25
52 XEQ 19	109 +	166 *	223 +	280 1
53 STO 09	110 RCL 20	167 RCL 05	224 RCL 11	281 STO 06
54 RCL 05	111 X↑2	168 RCL 07	225 X<>Y	282*LBL 12
55 "DL2"	112 .115194	169 *	226 /	283 RCL 03
56 XEQ 19	113 *	170 +	227 STO 11	284 *
57 STO 10	114 +	171 RCL 01	228 X↑2	285 RCL 05

PROGRAM LISTING

PROGRAMMAUFLISTUNG
LISTAGE DU PROGRAMME
LISTATO DI PROGRAMMA

286 1	343*LBL 13
287 +	344 1
288 *	345 X<>Y
289 LASTX	346 -
290 1	347 FS?C 02
291 +	348 XEQ 17
292 STO 05	349 FIX 3
293 /	350 "P"
294 ST+ 06	351 XEQ 19
295 7DSZ	352 FIX 2
296 GTO 12	353 RTN
297 RCL 06	354*LBL 17
298 RCL 04	355 2
299 *	356 /
300 FS? 01	357 RTN
301 GTO 18	358*LBL 20
302 GTO 13	359 RCL 02
303*LBL 18	360 SIN
304 RTN	361 GTO 21
305*LBL 11	362 RTN
306 RCL 02	363*LBL 19
307 2	364 "I="
308 *	365 ARCL X
309 PI	366 AVIEW
310 /	367 RTN
311 STO 07	368 END
312 3	
313 RCL 09	
314 X=Y?	
315 GTO 20	
316 1	
317 STO 05	
318 ST- 09	
319 X=Y?	
320 GTO 15	
321 SF 01	
322 XEQ 16	
323 CF 01	
324*LBL 21	
325 RCL 02	
326 COS	
327 *	
328 2	
329 *	
330 PI	
331 /	
332 RCL 07	
333 +	
334 GTO 13	
335 RTN	
336*LBL 15	
337 RCL 07	
338 GTO 13	
339*LBL 14	
340 RCL 04	
341 GTO 13	
342 RTN	

REGISTERS, STATUS, FLAGS

REGISTERBELEGUNG, FLAGS, BETRIEBSARTEN

REGISTRES, INDICATEURS, MODES OPÉRATOIRES

REGISTRI, MODI OPERATIVI, FLAGS

Registers Datenspeicher Registres de données Registri				Status Betriebsart Modes opératoires Modi operativi			
00	n_1	50		Size <u>026</u>		Total Reg. _____	User Mode
	$n_1 - 1$			Eng ☐	Fix ☒	Sci ☐	On ☒
	mean ₁			Deg ☐	Rad ☒	Grad ☐	Off ☐
	var. ₁						
05	n_2	55		Purpose Bedeutung Signification Scopo		Flags	
	$n_2 - 1$					SET	CLEAR
	mean ₂			00	t' test	☒	
	var. ₂			01	one-tailed test	☒	
	F			02	P(t) routine	☒	
10	d.f. ₁	60		03			
	d.f. ₂			04			
				05			
				06			
15	statistical	65		07			
	stack			08			
				09			
				10			
				11	Audio execute		
20		70		12			
				13			
				14			
				15			
				16			
25	ISZ reg.	75		17			
				18			
				19			
				20			
				21	Printer Enable		
30		80		22	Number Input		
				23	Alpha Input		
				24	Range Ignore		
				25	Error Ignore		
				26	Audio Enable		
35		85		27	User Mode		
				28	Decimal Point		
				29	Digit Grouping		
				Assignments Tastenbelegung / Assignations / Assegnamenti			
40		90		Function Funktion Fonction Funzione	Key Taste Touche Tasto	Function Funktion Fonction Funzione	Key Taste Touche Tasto
45		95					
		99					

INTRODUCTION

This HP-41C Solutions book was written to help you get the most from your calculator. The programs were chosen to provide useful calculations for many of the common problems encountered.

They will provide you with immediate capabilities in your everyday calculations and you will find them useful as guides to programming techniques for writing your own customized software. The comments on each program listing describe the approach used to reach the solution and help you follow the programmer's logic as you become and expert on your HP calculator.

KEYING A PROGRAM INTO THE HP-41C

There are several things that you should keep in mind while you are keying in programs from the program listings provided in this book. The output from the HP 82143A printer provides a convenient way of listing and an easily understood method of keying in programs without showing every keystroke. This type of output is what appears in this handbook. Once you understand the procedure for keying programs in from the printed listings, you will find this method simple and fast. Here is the procedure:

- At the end of each program listing is a listing of status information required to properly execute that program. Included is the SIZE allocation required. Before you begin keying in the program, press **[XEQ]** **[ALPHA]** **SIZE** **[ALPHA]** and specify the allocation (three digits; e.g., 10 should be specified as 010).

Also included in the status information is the display format and status of flags important to the program. To ensure proper execution, check to see that the display status of the HP-41C is set as specified and check to see that all applicable flags are set or clear as specified.

- Set the HP-41C to PRGM mode (press the **[PRGM]** key) and press **[GTO]** **[0]** **[.]** to prepare the calculator for the new program.
- Begin keying in the program. Following is a list of hints that will help you when you key in your programs from the program listings in this handbook.
 - When you see " (quote marks) around a character or group of characters in the program listing, those characters are ALPHA. To key them in, simply press **[ALPHA]**, key in the characters, then press **[ALPHA]** again. So "SAMPLE" would be keyed in as **[ALPHA]** "SAMPLE" **[ALPHA]**.
 - The diamond in front of each LBL instruction is only a visual aid to help you locate labels in the program listings. When you key in a program, ignore the diamond.
 - The printer indication of divide sign is /. When you see / in the program listing, press **[÷]**.
 - The printer indication of the multiply sign is ×. When you see × in the program listing, press **[×]**.
 - The † character in the program listing is an indication of the **[APPEND]** function. When you see †, press **[APPEND]** in ALPHA mode (press **[ALPHA]** and the K key).
 - All operations requiring register addresses accept those addresses in these forms:
 - nn (a two-digit number)
 - IND nn (INDIRECT: **[IND]**, followed by a two-digit number)
 - X, Y, Z, T, or L (a STACK address: **[X]** followed by X, Y, Z, T, or L)
 - IND X, Y, Z, T or L (INDIRECT stack: **[IND]** **[X]** followed by X, Y, Z, T, or L)

Indirect addresses are specified by pressing **[IND]** and then the indirect address. Stack addresses are specified by pressing **[X]** followed by X, Y, Z, T, or L. Indirect stack addresses are specified by pressing **[IND]** **[X]** and X, Y, Z, T, or L.

Printer Listing

```

01♦LBL "SAM
PLE"
02 "THIS IS
A "
03 "†SAMPLE
"
04 AVIEW
05 6
06 ENTER†
07 -2
08 /
09 ABS
10 STO IND
L
11 "R3="
12 ARCL 03
13 AVIEW
14 RTN

```

Keystrokes

```

[♦] [LBL] [ALPHA] SAMPLE [ALPHA]
[ALPHA] THIS IS A [ALPHA]
[ALPHA] [APPEND] SAMPLE
[♦] AVIEW [ALPHA]
6
[ENTER]
2 [CHS]
+
[XEQ] [ALPHA] ABS [ALPHA]
[STO] [IND] L
[ALPHA] R3= [ARCL] 03
[♦] AVIEW
[ALPHA]
[RTN]

```

Display

```

01 LBLT SAMPLE
02T THIS IS A
03T † SAMPLE
04 AVIEW
05 6
06 ENTER /
07 -2
08 /
09 ABS
10 STO IND L
11T R3=
12 ARCL 03
13 AVIEW
14 RTN

```

TWO RELATED SAMPLES"t" TEST FOR PAIRED COMPARISONSDESCRIPTION

Two samples are considered as paired when, to each measurement in one group corresponds a measurement in the other : this will be the case, for example, when two treatments are applied in sequence to the same unit (patient, animal); or when two analytical methods are used on several lots of material.

The variable used in the test is the difference $\underline{d}$ between the measurements y_1 and y_2 of each pair. The efficiency of the test is increased by eliminating the variations between the pairs.

The program computes the d_i and the mean $\bar{d}$.

H_0 : the differences d_i have a normal distribution with mean $\mu = 0$ and variance σ_d^2 .

H_1 : a- $\mu \neq 0$ (~~two~~-tailed test)

b- $\mu > 0$ or $\mu < 0$ (one-tailed test)

The test statistic is :

$$t = \frac{|\bar{d} - 0|}{s_d \sqrt{n}} \quad \text{with } (n - 1) \text{ d.f.}$$

n is the number of pairs.

The program displays the value of "t", the number of degrees of freedom and the probability $P(t)$. If P is lower than the chosen significance level, $\bar{d}$ is significantly different from zero.

PROGRAMMABLAUF I
INSTRUCTIONS D'EMPLOI I
NORME OPERATIVE I

STAT.04

Step Schritt Pas Passo	Instructions Operation Instructions Istruzioni	Variables Dateneingabe Données Dati	Function(s) Taste(n) Touche(s) Tasti	Result Résultat Risultato
1	Key in the program STAT.04, tracks 1 to 3			
2	Initialize		XEQ"ECH.PAIR."	
3	Input the data in pairs	y ₁ y ₂	R/S R/S	
	Display of d = y ₁ - y ₂			
4	Correction routine If one y is erroneous, enter the complete pair, before calling Input the erroneous pair	y _E y ₂	E R/S R/S	
	then the correct one			
5	After the last pair, call Display of : $\bar{d}$ standard error of d value of "t" number of d.f.		B	MOY E.S. t D.L.
6	For a one-tailed test, call For a two-tailed test, call Display of : probability P(t)		p.A p.B	P
7	For a new set of data, call and return to step 3.		A	

EXAMPLE 1 - One-tailed test (※)

	XEQ "ECHPAIR"			
	985,00	RUN		
	990,00	RUN		
	-5,00	***		
	1.015,00	RUN		
	980,00	RUN		
	35,00	***		
(error)	1.980,00	RUN		
	995,00	RUN		
	985,00	***		
		XEQ E		
	1.980,00	RUN		
	995,00	RUN		
	980,00	RUN		
	995,00	RUN		
	-15,00	***		
	1.000,00	RUN		
	995,00	RUN		
	5,00	***		
	1.010,00	RUN		
	990,00	RUN		
	20,00	***		
	1.005,00	RUN		
	965,00	RUN		
	40,00	***		
	995,00	RUN		
	985,00	RUN		
	10,00	***		
	1.020,00	RUN		
	995,00	RUN		
	25,00	***		
	1.045,00	RUN		
	1.005,00	RUN		
	40,00	***		

MOY.=17,22
E.S.=6,62
t=2,60
D.L.=8,00

XEQ B

P=0,016

XEQ a

01*LBL "ECHPAIR"	58 SQRT	115 *
02*LBL A	59 /	116 PI
03 CLRG	60 ATAN	117 /
04*LBL 01	61 STO 02	118 STO 07
05 STOP	62 RCL 09	119 3
06 STOP	63 2	120 RCL 09
07 -	64 /	121 X=Y?
08 PRX	65 INT	122 GTO 20
09 ADV	66 LASTX	123 1
10 SREG 14	67 X=Y?	124 STO 05
11 Z+	68 GTO 19	125 ST- 09
12 GTO 01	69 0	126 X=Y?
13*LBL B	70 STO 05	127 GTO 15
14 MEAN	71*LBL 16	128 SF 01
15 "MOY."	72 RCL 02	129 XEQ 16
16 XEQ 11	73 COS	130 CF 01
17 ABS	74 X↑2	131*LBL 21
18 STO 00	75 STO 03	132 RCL 02
19 SDEV	76 RCL 02	133 COS
20 RCL 19	77 SIN	134 *
21 SQRT	78 STO 04	135 2
22 /	79 RCL 09	136 *
23 "E.S."	80 2	137 PI
24 XEQ 11	81 X=Y?	138 /
25 RCL 00	82 GTO 14	139 RCL 07
26 X<>Y	83 /	140 +
27 /	84 1	141 GTO 13
28 SF 13	85 -	142 RTN
29 "T"	86 STO 25	143*LBL 15
30 XEQ 11	87 1	144 RCL 07
31 CF 13	88 STO 06	145 GTO 13
32 STO 10	89*LBL 12	146*LBL 14
33 RCL 19	90 RCL 03	147 RCL 04
34 1	91 *	148 GTO 13
35 -	92 RCL 05	149 RTN
36 STO 09	93 1	150*LBL 13
37 "D.L."	94 +	151 1
38 XEQ 11	95 *	152 X<>Y
39 RTN	96 LASTX	153 -
40*LBL 11	97 1	154 "FS?C02"
41 "I="	98 +	155 XEQ 17
42 ARCL X	99 STO 05	156 FIX 3
43 AVIEW	100 /	157 "P"
44 RTN	101 ST+ 06	158 XEQ 11
45*LBL E	102 7DSZ	159 FIX 2
46 STOP	103 GTO 12	160 RTN
47 STOP	104 RCL 06	161*LBL 17
48 -	105 RCL 04	162 2
49 Σ-	106 *	163 /
50 GTO 01	107 FS? 01	164 RTN
51*LBL a	108 GTO 18	165*LBL 20
52 SF 02	109 GTO 13	166 RCL 02
53*LBL b	110*LBL 18	167 1
54 RCL 10	111 RTN	168 RTN
55 ABS	112*LBL 19	169 END
56 RCL 09	113 RCL 02	
57 RAD	114 2	

REGISTERS, STATUS, FLAGS

REGISTERBELEGUNG, FLAGS, BETRIEBSARTEN

REGISTRES, INDICATEURS, MODES OPÉRATOIRES

REGISTRI, MODI OPERATIVI, FLAGS

Registers Datenspeicher Registres de données Registri				Status Betriebsart Modes opératoires Modi operativi			
00	d	50		Size	026	Total Reg.	User Mode
				Eng	☐	Fix ☒	Sci ☐
				Deg	☐	Rad ☒	Grad ☐
	routine			Purpose Bedeutung Signification Scopo Flags SET CLEAR			
	P(t)						
05		55		00			
				01	P(t) routine	☒	
				02	One-tailed t.	☒	
				03			
				04			
				05			
				06			
	statistic.			07			
15	stack	65		08			
				09			
				10			
				11	Audio execute		
				12			
20		70		13			
				14			
				15			
				16			
				17			
25	ISZ reg.	75		18			
				19			
				20			
				21	Printer Enable		
				22	Number Input		
30		80		23	Alpha Input		
				24	Range Ignore		
				25	Error Ignore		
				26	Audio Enable		
				27	User Mode		
35		85		28	Decimal Point		
				29	Digit Grouping		
				Assignments Tastenbelegung/Assignations/Assegnamenti			
				Function	Key	Function	Key
				Funktion	Taste	Funktion	Taste
				Fonction	Touche	Fonction	Touche
				Funzione	Tasto	Funzione	Tasto
40		90					
45		95					
		99					

SINGLE CLASSIFICATION ANALYSIS OF VARIANCE

DESCRIPTION

It is a comparison between a samples or groups, equal in size or not, and assumed to be :

- a - either sampled randomly from the same normal population (μ, σ^2)
- b - or from normal populations with means $\mu_1, \mu_2, \dots, \mu_a$, but with the same variance σ^2 .

In the analysis of variance (ANOVA), two estimations of the variance (or mean squares) are computed :

- one based on the within groups variations, obtained as a pooled variance (weighted average of the a variances of the groups) s_1^2
- one computed from the group means or totals (variation among groups) s_2^2

These two mean squares are compared with an "F" test :

$$F = s_2^2 / s_1^2$$

If F is not significant, one can conclude that the a groups come from the same population. If F is significant, i.e. if the mean square among groups s_2^2 is significantly larger than the mean square within groups s_1^2 (experimental error), one can conclude that an extra source of variation, due to a difference between the groups, is added to the variation due to the experimental error.

If the added variation is caused by fixed treatments chosen by the experimenter, it is a Model I ANOVA; the several means can be compared to each other (see Progr. ANOVA 4). If the added variation is a random effect, it is a Model II ANOVA, and the variation due to this effect can be estimated.

The results are generally displayed in the form of an ANOVA table

Source of variation	Nbr D.F.	Sum of Squares	Mean Square	F	P(F)
Among groups	a - 1				
Within groups (error)	N - a				
Total	N - 1				

Since the number of degrees of freedom and the sums of squares are additive, the table is conveniently computed as follows :

$$\text{- Total S.S. : } \sum y^2 - (\sum y)^2/N$$

$$\text{- Group S.S. : } \sum (T_i/n_i)^2 - (\sum y)^2/N$$

T_i = sum of the n_i responses in group i .

The program displays the results in the order of the ANOVA table.

-Equality of the variances

For the ANOVA to be valid, the several variances should be equal (homoscedasticity). This can be checked by the Bartlett's test.

$$X^2 = \frac{2,3026}{C} (N' \log s^2 - \sum n'_i \log s_i^2)$$

with $C = 1 + \frac{1}{3(a-1)} \left(\sum \frac{1}{n'_i} - \frac{1}{N'} \right)$ n'_i and s_i^2 = d.f. and variance of each sample; N' = total d.f.; s^2 = pooled variance

This statistic X^2 has a chi square distribution with $(a-1)$ d.f.

If the test is significant, one should try an appropriate transformation of the data, or use a non parametric test.

- Model II ANOVA

If F is significant, one can estimate the percentage of the total variation due to the variation among groups, and the percentage due to the experimental error. When the group size is unequal, n_0 is computed as :

$$n_0 = \frac{1}{a-1} \left(\sum n_i - \frac{\sum n_i^2}{\sum n_i} \right)$$

USER INSTRUCTIONS I

33

STAT.05

PROGRAMMABLAUF I INSTRUCTIONS D'EMPLOI I NORME OPERATIVE I

Step Schritt Pas Passo	Instructions Operation Instructions Istruzioni	Variables Dateneingabe Données Dati	Function(s) Taste(n) Touche(s) Tasti	Result Resultat Résultat Risultato
1	Key in the program STAT.05, tracks 1 to 6			
2	Initialize		XEQ"ANVA.1"	
3	Enter : number of groups	a	R/S	
4	Input the data of the first group	y ₁	R/S	
5	At the end of the group, call Display : total of the group mean variance		B	Σ MOY. VAR.
6	<u>Correction routine</u> Before calling B. Call input the erroneous y then the correct one	y _E y _C	E R/S	
7	Repeat steps 4 and 5 for the other groups At the end of the last group, display of the ANOVA : Among groups Error Total			d.l., S.C. C.M., F, P d.l., S.C. C.M. d.l., S.C.
8	<u>Bartlett's test</u> Display of : nbr d.f. limit value <u>ANOVA Model II</u>			x ² d.l. x ² 10,05
9	Call Display of : average size of groups variance groups % variance groups % error		C	n ₀ VAR.G VAR.G % VAR.E %
10	For an ANOVA Model I, see Prog. STAT.08 For a new set of data, call and return to step 3.		A	

EXAMPLE 1 - ANOVA MODEL I (≡)

EXAMPLE 2I - ANOVA MODEL II (≡)

	XEQ A		ANVA
3,00	RUN		TRAIT
			D.L.=2,00
118,00	RUN		S.C.=3.650,42
85,00	RUN		C.M.=1.825,21
80,00	RUN		F=6,72
102,00	RUN		P=0,008
75,00	RUN		
65,00	RUN		ERR
86,00	RUN		D.L.=15,00
73,00	RUN		S.C.=4.071,58
	XEQ B		C.M.=271,44
Σ=684,00			TOT
MOY.=85,50			D.L.=17,00
VAR.=292,29			S.C.=7.722,00
			BART
120,00	RUN		CHI.2=0,05
97,00	RUN		D.L.=2,00
103,00	RUN		CHI.2.0,05=6,02
(error) 1.105,00	RUN		
138,00	RUN		
96,00	RUN		
	XEQ E		
1.105,00	RUN		
105,00	RUN		
	XEQ B		
Σ=659,00			
MOY.=109,83			
VAR.=264,57			
100,00	RUN		
128,00	RUN		
113,00	RUN		
134,00	RUN		
	XEQ B		
Σ=475,00			
MOY.=118,75			
VAR.=234,25			

(≡) PHILIPPE p.124

	XEQ "ANVA1"		133,00	RUN
6,00	RUN		68,00	RUN
			90,00	RUN
103,00	RUN		125,00	RUN
85,00	RUN		4,00	RUN
138,00	RUN			XEQ B
160,00	RUN	Σ=420,00		
74,00	RUN	MOY.=84,00		
	XEQ B	VAR.=2.693,50		
Σ=560,00				
MOY.=112,00			51,00	RUN
VAR.=1.308,50			102,00	RUN
			-1,00	RUN
-33,00	RUN		68,00	RUN
54,00	RUN		80,00	RUN
18,00	RUN			XEQ B
42,00	RUN	Σ=300,00		
19,00	RUN	MOY.=60,00		
	XEQ B	VAR.=1.507,50		
Σ=100,00				
MOY.=20,00				
VAR.=1.113,50				
		ANVA		
		TRAIT		
		D.L.=5,00		
58,00	RUN	S.C.=42.024,17		
171,00	RUN	C.M.=8.404,83		
193,00	RUN	F=2,95		
210,00	RUN	P=0,032		
33,00	RUN			
	XEQ B	ERR		
Σ=665,00		D.L.=24,00		
MOY.=133,00		S.C.=68.352,00		
VAR.=6.649,50		C.M.=2.848,00		
170,00	RUN	TOT		
96,00	RUN	D.L.=29,00		
112,00	RUN	S.C.=110.376,17		
151,00	RUN			
11,00	RUN	BART		
	XEQ B	CHI.2=4,62		
Σ=540,00		D.L.=5,00		
MOY.=108,00				
VAR.=3.815,50		CHI.2.0,05=11,08		
				XEQ C
		n0=5,00		
		VAR.G=1.111,37		
		VAR.Gz=28,07		
		VAR.Ez=71,93		

(≡) PHILIPPE p.115

01+LBL "ANVA1"	58 RCL 02	115 Y+X	172 "P"	229 XEQ 11
02+LBL A	59 /	116 *	173 XEQ 11	230 ADV
03 CLRG	60 STO 01	117 1	174 FIX 2	231 2.518232
04 CF 00	61 -	118 RCL 09	175 ADV	232 RCL 25
05 CF 01	62 STO 00	119 -	176 "ERR"	233 1.282189
06 STOP	63 "ANVA"	120 -	177 AVIEW	234 *
07 STO 20	64 AVIEW	121 STO 07	178 RCL 06	235 +
08 ADV	65 "TRAIT"	122 RCL 01	179 "D.L."	236 RCL 25
09 STO 25	66 AVIEW	123 RCL 08	180 XEQ 11	237 X+2
10 30.054	67 RCL 03	124 2	181 RCL 04	238 -.00211427
11 STO 26	68 RCL 01	125 ENTER↑	182 "S.C."	239 *
12+LBL 01	69 -	126 3	183 XEQ 11	240 +
13 STOP	70 STO 03	127 /	184 RCL 05	241 RCL 25
14 ZREG 14	71 -	128 Y+X	185 "C.M."	242 LN
15 Σ+	72 STO 04	129 *	186 XEQ 11	243 1.371169
16 GTO 01	73 RCL 02	130 RCL 09	187 ADV	244 *
17+LBL B	74 RCL 20	131 +	188 "TOT"	245 +
18 RCL 15	75 -	132 SQR	189 AVIEW	246 "CHI.2.0.05"
19 ST+ 00	76 STO 06	133 RCL 07	190 RCL 02	247 XEQ 11
20 RCL 14	77 /	134 X<>Y	191 1	248 RTN
21 ST+ 01	78 STO 05	135 /	192 -	249+LBL 10
22 "Σ"	79 RCL 20	136 STO 08	193 "D.L."	250 RCL 14
23 XEQ 11	80 1	137 X>0?	194 XEQ 11	251 STO IND 26
24 RCL 19	81 -	138 SF 02	195 RCL 00	252 ISG 26
25 ST+ 02	82 STO 07	139 ABS	196 "S.C."	253 RCL 19
26 X+2	83 "D.L."	140 1	197 XEQ 11	254 STO IND 26
27 ST+ 24	84 XEQ 11	141 RCL 08	198 ADV	255 ISG 26
28 MEAN	85 RCL 03	142 .196654	199 "BART"	256 RTN
29 "MOY."	86 "S.C."	143 *	200 AVIEW	257+LBL C
30 XEQ 11	87 XEQ 11	144 +	201 RCL 05	258 RCL 20
31 SDEV	88 X<>Y	145 RCL 08	202 LOG	259 1
32 X+2	89 /	146 X+2	203 RCL 06	260 -
33 "VAR."	90 "C.M."	147 .115194	204 *	261 1/X
34 XEQ 11	91 XEQ 11	148 *	205 RCL 22	262 RCL 02
35 LOG	92 RCL 05	149 +	206 -	263 RCL 24
36 RCL 19	93 /	150 RCL 08	207 RCL 20	264 RCL 02
37 1	94 "F"	151 3	208 1	265 /
38 -	95 STO 08	152 Y+X	209 -	266 -
39 ST+ 21	96 XEQ 11	153 .000344	210 STO 25	267 *
40 *	97 2	154 *	211 3	268 SF 13
41 ST+ 22	98 ENTER↑	155 +	212 *	269 "NO"
42 LASTX	99 9	156 RCL 08	213 1/X	270 XEQ 11
43 1/X	100 /	157 4	214 RCL 23	271 STO 06
44 ST+ 23	101 STO 01	158 Y+X	215 RCL 21	272 CF 13
45 ADV	102 RCL 07	159 .019527	216 1/X	273 RCL 03
46 RCL 14	103 /	160 *	217 -	274 RCL 20
47 X+2	104 STO 09	161 +	218 *	275 1
48 RCL 19	105 RCL 01	162 -4	219 1	276 -
49 /	106 RCL 06	163 Y+X	220 +	277 /
50 ST+ 03	107 /	164 2	221 2.3026	278 RCL 05
51 XEQ 10	108 STO 01	165 /	222 X<>Y	279 -
52 CLZ	109 1	166 CHS	223 /	280 RCL 06
53 7DS2	110 -	167 1	224 *	281 /
54 GTO 01	111 CHS	168 +	225 "CHI.2"	282 "VAR.G"
55 RCL 00	112 RCL 08	169 FS?C 02	226 XEQ 11	283 XEQ 11
56 RCL 01	113 3	170 XEQ 02	227 RCL 25	284 ENTER↑
57 X+2	114 1/X	171 FIX 3	228 "D.L."	285 ENTER↑

CONTENTS

VOLUME I

- Principles of statistical tests
- User's general instructions
- Programs :

Number	Title	Initial	Nber memory modules	Nber cards & tracks
-----	-----	-----	-----	-----
STAT.01	Mean and confidence inter- val	INT. CONF.	1	1 (1 and 2)
02	Histogram - Normality test	HIST.	2	2 (1 to 4)
03	Two independent samples "F" and "t" tests	ECH. IND.	1	3 (1 to 5)
04	Two related samples "t" test	ECH. PAIR.	1	2 (1 to 3)
05	Single classification ANOVA	ANVA 1	2	3 (1 to 6)
06	Complete block ANOVA	ANVA 2	2	3 (1 to 5)
07	Latin square ANOVA	ANVA 3	2	3 (1 to 5)
08	Comparisons among means		2	3 (1 to 5)
09	Nested ANOVA	ANVA	2	3 (1 to 6)

PROGRAM LISTING
PROGRAMMAUFLISTUNG
LISTAGE DU PROGRAMME
LISTATO DI PROGRAMMA

```
286 RCL 05
287 +
288 1/Y
289 *
290 100
291 *
292 "VAR.G%"
293 XEQ 11
294 100
295 -
296 CHS
297 "VAR.E%"
298 XEQ 11
299 RTN
300*LBL E
301 STOP
302 E-
303 GTO 01
304*LBL 11
305 "I="
306 ARCL X
307 AVIEW
308 RTN
309*LBL 02
310 1
311 X<>Y
312 -
313 RTN
314 END
```

REGISTERS, STATUS, FLAGS

REGISTERBELEGUNG, FLAGS, BETRIEBSARTEN
REGISTRES, INDICATEURS, MODES OPÉRATOIRES
REGISTRI, MODI OPERATIVI, FLAGS

Registers Datenspeicher Registres de données Registri				Status Betriebsart Modes opératoires Modi operativi			
00	$\sum y^2$	50		Size <u>055</u>	Total Reg. _____	User Mode	
	$\sum y$			Eng ☐	Fix ☒	Sci ☐	On ☒
	$\sum n$			Deg ☐	Rad ☐	Grad ☐	Off ☐
	S.C. group			Purpose Bedeutung Signification Scopo			
	S.C. error			Flags			
05	var. error	55				SET	CLEAR
	d.l. error						
	(a - 1)			00			
	F			01			
				02	Routine P(F)	☒	
10		60		03			
				04			
				05			
				06			
	statistic.			07			
15	stack	65		08			
				09			
				10			
				11	Audio execute		
				12			
20	a	70		13			
	d.l.			14			
	n'log s ²			15			
	1/n'			16			
				17			
25	a	75		18			
	30			19			
				20			
				21	Printer Enable		
				22	Number Input		
30	T _i	80		23	Alpha Input		
	n _i			24	Range Ignore		
	.			25	Error Ignore		
	.			26	Audio Enable		
	.			27	User Mode		
35		85		28	Decimal Point		
				29	Digit Grouping		
				Assignments Tastenbelegung / Assignations / Assegnamenti			
40		90		Function Funktion Fonction Funzione	Key Taste Touche Tasto	Function Funktion Fonction Funzione	Key Taste Touche Tasto
45		95					
		99					

RANDOMIZED COMPLETE BLOCKS ANALYSIS OF VARIANCE

DESCRIPTION

In this design, originally devised in agricultural research, the several treatments are applied to plots of land (or blocks), the treatments in each plot being assigned randomly. It is thus possible, in the ANOVA, to subtract from the experimental error the variation between plots.

The same design has also numerous applications in other fields: for example, each block may be an individual (patient or animal) receiving several treatments one after another; or a piece of equipment in which several experiments are performed.

There are a treatments ($a \leq 12$; when $a > 12$, display : ERROR), with b blocks, and each treatment once in every block. The ANOVA table is :

Source of variation	Number of d.f.	Sum of squares	Mean square	F	P(F)
Between blocks	(b - 1)				
Between treatments	(a - 1)				
Error	(a-1)(b-1)				
Total	(ab - 1)				

The program computes :

- Total sum of squares : $\sum y^2 - C.F.$
- Block sum of squares : $\sum T_b^2/a - C.F.$
- Treatment S.S. : $\sum_a T_a^2/b - C.F.$

C.F. = correction factor = $(\sum y)^2/ab$

T_a = sum of treatments

T_b = sum of blocks

If $F = M.S. \text{ treat.} / M.S. \text{ error}$ is significant, the treatments can be compared using Program STAT.08.

Step Schritt Pas Passo	Instructions Operation Instructions Istruzioni	Variables Dateneingabe Données Dati	Function(s) Taste(n) Touche(s) Tasti	Result Resultat Résultat Risultato
1	Key in the program STAT.06, tracks 1 to 5			
2	Initialize		XEQ"ANVA.2"	
3	Enter : number of treatments number of blocks	a b	R/S R/S	
4	Input the data of the first block	y ₁	R/S	
5	At the end of the block, call Display: Total of the bloc T _b		B	Σ B
6	<u>Correction routine</u> a - to correct an erroneous y just entered, call input the erroneous y then the correct one b - if several y have been input after y _E (but before calling B) call enter the number of the treatment the erroneous y then the correct one	 y _E y _C	 p.E R/S R/S R/S	
7	Repeat steps 4 and 5 for the other blocks	1 to a	R/S	
8	At the end of the last block, display of the ANOVA	y _E y _C	R/S R/S	
9	If F(treatment) is significant, use STAT.08 For another set of data, call and go to step 3		A	

EXAMPLE 1 (±)

	XEQ "ANVA2"		Σ.T=54.00
	5.00 RUN		MOY.=10.80
	5.00 RUN		Σ.T=31.00
			MOY.=6.20
	8.00 RUN		Σ.T=41.00
	2.00 RUN		MOY.=8.20
	4.00 RUN		Σ.T=33.00
	3.00 RUN		MOY.=6.60
	9.00 RUN		Σ.T=29.00
	XEQ B		MOY.=5.80
Σ.B=26.00			
	10.00 RUN		ANVA
	6.00 RUN		BLOC
	10.00 RUN		D.L.=4.00
	5.00 RUN		S.C.=49.84
	7.00 RUN		C.M.=12.46
	XEQ B		F.=2.30
Σ.B=38.00			P=0.103
	12.00 RUN		TRAIT
	7.00 RUN		D.L.=4.00
	9.00 RUN		S.C.=83.84
	9.00 RUN		C.M.=20.96
(error)	15.00 RUN		F.=3.87
	XEQ E		P=0.022
	15.00 RUN		ERR.
	5.00 RUN		D.L.=16.00
	XEQ B		S.C.=86.56
Σ.B=42.00			C.M.=5.41
	13.00 RUN		TOT.
(error)	111.00 RUN		D.L.=24.00
	8.00 RUN		S.C.=220.24
	10.00 RUN		
	5.00 RUN		
	XEQ e		
	2.00 RUN		
	111.00 RUN		
	11.00 RUN		
	XEQ B		
Σ.B=47.00			
	11.00 RUN		
	5.00 RUN		
	10.00 RUN		
	6.00 RUN		
	3.00 RUN		
	XEQ B		
Σ.B=35.00			

01*LBL "ANVA2"	58 GTO 02	115 -	172 STO 01	229 /
02*LBL A	59 RCL 22	116 STO 00	173 1	230 CHS
03 CLRG	60 X+2	117 XEQ 05	174 -	231 1
04 STOP	61 RCL 20	118 "ERR."	175 CHS	232 +
05 STO 20	62 RCL 21	119 AVIEW	176 RCL 17	233 7DSP3
06 13	63 *	120 RCL 06	177 3	234 FS?C 02
07 X<=Y?	64 /	121 "D.L."	178 1/X	235 XEQ 04
08 GTO 09	65 STO 22	122 XEQ 11	179 Y+X	236 "P"
09 STOP	66 RCL 13	123 RCL 14	180 *	237 XEQ 11
10 STO 21	67 RCL 22	124 "S.C."	181 1	238 7DSP2
11 30.054	68 -	125 XEQ 11	182 RCL 03	239 ADV
12 STO 27	69 STO 13	126 RCL 05	183 -	240 RTN
13 ADV	70 RCL 23	127 "C.M."	184 -	241*LBL 04
14*LBL 01	71 RCL 20	128 XEQ 11	185 STO 07	242 1
15 STOP	72 /	129 ADV	186 RCL 01	243 X<>Y
16 ST+ IND 25	73 RCL 22	130 "TOT."	187 RCL 17	244 -
17 7ISZ	74 -	131 AVIEW	188 2	245 RTN
18 ΣREG 14	75 STO 23	132 RCL 20	189 ENTER↑	246*LBL 10
19 Σ+	76 RCL 24	133 RCL 21	190 3	247 RCL IND 25
20 GTO 01	77 RCL 21	134 *	191 /	248 STO IND 27
21*LBL B	78 /	135 1	192 Y+X	249 ISG 27
22 RCL 15	79 RCL 22	136 -	193 *	250 RCL 21
23 ST+ 13	80 -	137 "D.L."	194 RCL 03	251 STO IND 27
24 RCL 14	81 STO 24	138 XEQ 11	195 +	252 ISG 27
25 ST+ 22	82 RCL 13	139 RCL 13	196 SORT	253 RTN
26 "Σ.P"	83 RCL 23	140 "S.C."	197 RCL 07	254*LBL 11
27 XEQ 11	84 RCL 24	141 XEQ 11	198 X<>Y	255 "t="
28 ADV	85 +	142 RTN	199 /	256 ARCL X
29 X+2	86 -	143*LBL 05	200 STO 00	257 AVIEW
30 ST+ 23	87 STO 14	144 "D.L."	201 X>0?	258 RTN
31 CLZ	88 RCL 20	145 XEQ 11	202 SF 02	259*LBL E
32 0	89 1	146 RCL IND 25	203 ABS	260 RCL 25
33 STO 25	90 -	147 "S.C."	204 1	261 1
34 1	91 RCL 21	148 XEQ 11	205 RCL 00	262 -
35 ST+ 24	92 1	149 X<>Y	206 .196854	263 STO 25
36 RCL 21	93 -	150 /	207 *	264 STOP
37 RCL 24	94 *	151 "C.M."	208 +	265 ST- IND 25
38 X*Y?	95 STO 06	152 XEQ 11	209 RCL 00	266 Σ-
39 GTO 01	96 /	153 RCL 05	210 X+2	267 GTO 01
40 0	97 STO 05	154 /	211 .115194	268*LBL e
41 STO 24	98 ADV	155 STO 17	212 *	269 STOP
42*LBL 02	99 23	156 "F."	213 +	270 ENTER↑
43 RCL IND 25	100 STO 25	157 XEQ 11	214 RCL 00	271 1
44 "Σ.T"	101 "ANVA"	158 XEQ 03	215 3	272 -
45 XEQ 11	102 AVIEW	159 RTN	216 Y+X	273 STO 26
46 RCL 21	103 "BLOC"	160*LBL 03	217 .000344	274 STOP
47 /	104 AVIEW	161 2	218 *	275 ST- IND 26
48 "MOY."	105 RCL 21	162 ENTER↑	219 +	276 Σ-
49 XEQ 11	106 1	163 9	220 RCL 00	277 STOP
50 RCL IND 25	107 -	164 /	221 4	278 ST+ IND 26
51 X+2	108 STO 00	165 STO 01	222 Y+X	279 Σ+
52 ST+ 24	109 XEQ 05	166 RCL 00	223 .019527	280 GTO 01
53 XEQ 10	110 7ISZ	167 /	224 *	281 .END.
54 7ISZ	111 "TRAIT"	168 STO 03	225 +	
55 RCL 20	112 AVIEW	169 RCL 01	226 -4	
56 RCL 25	113 RCL 20	170 RCL 06	227 Y+X	
57 X*Y?	114 1	171 /	228 2	

REGISTERS, STATUS, FLAGS

REGISTERBELEGUNG, FLAGS, BETRIEBSARTEN

REGISTRES, INDICATEURS, MODES OPÉRATOIRES

REGISTRI, MODI OPERATIVI, FLAGS

Registers Datenspeicher Registres de données Registri				Status Betriebsart Modes opératoires Modi operativi			
00	T _a	50		Size <u>055</u> Total Reg. _____		User Mode	
	.			Eng ☐	Fix ☒	Sci ☐	On ☒
	.			Deg ☐	Rad ☐	Grad ☐	Off ☐
	.			Purpose Bedeutung Signification Scopo Flags SET CLEAR			
05	Error M.S. Error d.f.	55					
				00			
				01			
				02	P(F) routine	☒	
10		60		03			
				04			
				05			
	Total S.S.			06			
	Error S.S.			07			
15	Statist.stack	65		08			
				09			
				10			
				11	Audio execute		
				12			
20	a	70		13			
	b			14			
	C.F.			15			
	Treat. S.S.			16			
	Blocks S.S.			17			
25		75		18			
				19			
	Data for			20			
	Progr.STAT.08			21	Printer Enable		
	.			22	Number Input		
30	.	80		23	Alpha Input		
	.			24	Range Ignore		
	.			25	Error Ignore		
	.			26	Audio Enable		
				27	User Mode		
35		85		28	Decimal Point		
				29	Digit Grouping		
				Assignments Tastenbelegung / Assignations / Assegnamenti			
40		90		Function Funktion Fonction Funzione	Key Taste Touche Tasto	Function Funktion Fonction Funzione	Key Taste Touche Tasto
45		95					
		99					

LATIN SQUARE ANALYSIS OF VARIANCEDESCRIPTION

In this design, used in agricultural research, a factor is tested at n levels, and the field is divided in n row blocks and in n column blocks. Each level of the factor (treatment) is replicated only once in a given row or column. In the ANOVA, the variations due to rows and columns are eliminated from the error mean square. This design need not be restricted to agricultural research. For instance, the columns may be the subjects and the rows the order of administration of the treatments. This program can accomodate up to a 12×12 latin square. (If the number of treatments is larger than 12, a sign ERROR is displayed).

The ANOVA is the following :

Source of variation	Nber d.f.	Sum of squares	Mean square	F	P(F)
Between rows	$(n - 1)$				
Between columns	$(n - 1)$				
Between treatments	$(n - 1)$				
Error	$(n-1)(n-2)$				
Total	$(n^2 - 1)$				

The data are entered in rows and by pair : first, the number of the treatment (the treatments are numbered : 1, 2, 3..... 12), then the corresponding response. The sums of rows T_r are displayed, then the sums of columns T_c and the sums of treatments T_t .

The program computes :

$$\begin{aligned}
 \text{Total sum of squares} &: \sum y^2 - \text{C.F.} & (\text{C.F.} = (\sum y)^2/n) \\
 \text{Row S.S.} &: \sum T_r^2 / n - \text{C.F.} \\
 \text{Column S.S.} &: \sum T_c^2 / n - \text{C.F.} \\
 \text{Treatment S.S.} &: \sum T_t^2 / n - \text{C.F.}
 \end{aligned}$$

If $F = \text{Treat.M.S.} / \text{Error M.S.}$ is significant, the treatment means may be compared, using program STAT.08.

USER INSTRUCTIONS I

STAT.07

PROGRAMMABLAUF I
INSTRUCTIONS D'EMPLOI I
NORME OPERATIVE I

Step Schritt Pas Passo	Instructions Operation Instructions Istruzioni	Variables Dateneingabe Données Dati	Function(s) Taste(n) Touche(s) Tasti	Result Resultat Résultat Risultato
1	Key in the program STAT.07, tracks 1 to 5			
2	Initialize		XEQ"ANVA.3"	
3	Enter : number of treatments (= rows = columns)	n	R/S	
4	Input the data of the first row in pairs : first nbr of treatment then the result	n ^o y _i	R/S R/S	
5	At the end of the first row, call Display : row sum T _r		B	ΣR
6	<u>Correction routine</u> a - immediate . if necessary, finish to input the data of the pair, then call input again the erroneous pair then the correct one b - delayed (but before calling B) input : the number of the column the erroneous pair then the correct one	n ^o _E y _E n ^o _C y _C c n ^o _E y _E n ^o _C y _C	E R/S R/S R/S R/S p.E R/S R/S R/S R/S	
7	Repeat step 4 and 5 for the other rows			
8	At the end of the last row, Display of : column sums T _c treatment sums T _t treatment mean Display of the ANOVA			ΣC ΣT MOY.T
9	If F (treatments) is significant, the treatment means can be compared using program STAT.08 For another set of data, call and return to step 3		A	

EXAMPLE 1 - Latin square

Subject	Treatment order				
	1	2	3	4	5
1	(1) 209	(4) 183	(2) 204	(3) 216	(5) 178
2	(4) 180	(3) 198	(5) 162	(2) 188	(1) 166
3	(2) 202	(5) 198	(3) 217	(1) 193	(4) 172
4	(3) 223	(1) 202	(4) 192	(5) 179	(2) 202
5	(5) 192	(2) 202	(1) 189	(4) 165	(3) 191

```

XEQ "ANVA3"
5.00 RUN
1.00 RUN
209.00 RUN
4.00 RUN
183.00 RUN
2.00 RUN
204.00 RUN
3.00 RUN
216.00 RUN
5.00 RUN
178.00 RUN
XEQ B
ΣR=990.00
4.00 RUN
180.00 RUN
(error) 8.00 RUN
198.00 RUN
XEQ E
8.00 RUN
198.00 RUN
3.00 RUN
198.00 RUN
5.00 RUN
162.00 RUN
2.00 RUN
188.00 RUN
1.00 RUN
166.00 RUN
XEQ B
ΣR=894.00

2.00 RUN
202.00 RUN
5.00 RUN
198.00 RUN
3.00 RUN
217.00 RUN
1.00 RUN
193.00 RUN
4.00 RUN
172.00 RUN
XEQ B
ΣR=982.00
3.00 RUN
(error) 1.223.00 RUN
1.00 RUN
202.00 RUN
4.00 RUN
192.00 RUN
5.00 RUN
179.00 RUN
2.00 RUN
202.00 RUN
XEQ e
1.00 RUN
3.00 RUN
1.223.00 RUN
3.00 RUN
223.00 RUN
XEQ B
ΣR=998.00

5.00 RUN
192.00 RUN
2.00 RUN
202.00 RUN
1.00 RUN
189.00 RUN
XEQ B
ΣR=939.00
ΣC.=1.006.00
ΣC.=983.00
ΣC.=964.00
ΣC.=941.00
ΣC.=909.00
ΣT.=959.00
MOY.T=191.80
ΣT.=998.00
MOY.T=199.60
ΣT.=1.045.00
MOY.T=209.00
ΣT.=892.00
MOY.T=178.40
ΣT.=909.00
MOY.T=181.80

ANVA
RANG.
D.L.=4.00
S.C.=1.524.64
C.M.=381.16
F=8.15
P=0.002
COL.
D.L.=4.00
S.C.=1.124.24
C.M.=281.06
F=6.01
P=0.007
TRAIT.
D.L.=4.00
S.C.=3.178.64
C.M.=794.66
F=16.99
P=1.860E-4
ERR.
D.L.=12.00
S.C.=561.12
C.M.=46.76
TOT.
D.L.=24.00
S.C.=6.388.64

```


PRINCIPLES OF STATISTICAL TESTS

The most frequent applications of statistics are to test some hypothesis. A statistical test is basically intended to evaluate the risk that the user is willing to take when accepting the hypothesis, or when rejecting that hypothesis and accepting an alternative one. The method is called Hypothesis testing.

Whatever the statistical test used, the procedure usually involves several steps :

1 - State the hypothesis.

The problem to be solved is defined by two hypotheses :

- the null hypothesis H_0 , which is the hypothesis of no difference.
For instance, if we want to compare the means of two groups and if a difference is found, we state that this difference is not real, but due to the sampling error.
- the alternative hypothesis, H_1 , which will be accepted if H_0 is rejected. H_1 may be stated in two different ways :
 - . H_{1a} : the research hypothesis simply states that the means of the two groups are different $\mu_1 \neq \mu_2$; this is a two-tailed test.
 - . H_{1b} : the theory predicts the direction of the difference, if any. H_1 will be either $\mu_1 > \mu_2$ or $\mu_1 < \mu_2$; this is a one-tailed test.

2 - Choice of the statistical test.

We have many tests which can be used under various conditions. The criteria for choosing among several tests and the corresponding programs will be examined below.

3 - Choice of the significance level.

This is a limit which should be specified in advance of the data collection and which justifies the rejection of the null hypothesis. It is the risk of falsely rejecting H_0 when the null hypothesis is actually true. This error (Type I error) and the corresponding risk, commonly called α is usually set at the level $P = 0,05$.

PROGRAM LISTING

PROGRAMMAUFLISTUNG
LISTAGE DU PROGRAMME
LISTATO DI PROGRAMMA

01+LBL "ANVA3"	58 RCL IND 25	115 -	172 RCL IND 25	229 1
02+LBL A	59 "ET."	116 STO 07	173 "S.C."	230 RCL 16
03 CLRG	60 XEQ 11	117 RCL 20	174 XEQ 11	231 ,196854
04 STOP	61 X+2	118 1	175 X<>Y	232 *
05 STO 20	62 ST+ 22	119 -	176 /	233 +
06 13	63 LASTX	120 STO 09	177 "C.M."	234 RCL 16
07 X<=Y?	64 RCL 20	121 RCL 20	178 XEQ 11	235 X+2
08 GTO 09	65 /	122 2	179 RCL 05	236 ,115194
09 ADV	66 "MOY.T"	123 -	180 /	237 *
10+LBL 01	67 XEQ 11	124 *	181 STO 10	238 +
11 STOP	68 7ISZ	125 STO 06	182 "F"	239 RCL 16
12 2	69 RCL 20	126 RCL 07	183 XEQ 11	240 3
13 *	70 STO IND 25	127 X<>Y	184 XEQ 05	241 Y+X
14 28	71 7ISZ	128 /	185 7ISZ	242 ,000344
15 +	72 RCL 20	129 STO 05	186 RTN	243 *
16 STO 13	73 2	130 "ANVA"	187+LBL 05	244 +
17 STOP	74 *	131 AVIEW	188 2	245 RCL 16
18 ST+ IND 25	75 30	132 RCL 09	189 9	246 4
19 7ISZ	76 +	133 2	190 /	247 Y+X
20 ST+ IND 13	77 RCL 25	134 STO 25	191 STO 17	248 ,019527
21 ZREG 14	78 X*Y?	135 "RANG."	192 RCL 09	249 *
22 E+	79 GTO 03	136 AVIEW	193 /	250 +
23 GTO 01	80 ADV	137 XEQ 04	194 STO 14	251 -4
24+LBL B	81 RCL 27	138 "COL."	195 RCL 17	252 Y+X
25 RCL 15	82 X+2	139 AVIEW	196 RCL 06	253 2
26 ST+ 26	83 RCL 20	140 XEQ 04	197 /	254 /
27 RCL 14	84 X+2	141 "TRAIT."	198 STO 17	255 CHS
28 "SR"	85 /	142 AVIEW	199 1	256 1
29 XEQ 11	86 STO 01	143 XEQ 04	200 -	257 +
30 ST+ 27	87 RCL 26	144 "ERR."	201 CHS	258 FIX 3
31 X+2	88 RCL 01	145 AVIEW	202 RCL 10	259 FS?C 02
32 ST+ 28	89 -	146 RCL 06	203 3	260 XEQ 06
33 CLS	90 STO 00	147 "D.L."	204 1/X	261 "P"
34 0	91 RCL 20	148 XEQ 11	205 Y+X	262 XEQ 11
35 STO 25	92 RCL 20	149 RCL 07	206 *	263 FIX 2
36 1	93 /	150 "S.C."	207 1	264 ADV
37 ST+ 29	94 RCL 01	151 XEQ 11	208 RCL 14	265 RTN
38 RCL 20	95 -	152 RCL 05	209 -	266+LBL 06
39 RCL 29	96 STO 02	153 "C.M."	210 -	267 1
40 X*Y?	97 RCL 21	154 XEQ 11	211 STO 15	268 X<>Y
41 GTO 01	98 RCL 20	155 ADV	212 RCL 17	269 -
42 ADV	99 /	156 "TOT."	213 RCL 10	270 RTN
43+LBL 02	100 RCL 01	157 AVIEW	214 2	271+LBL 11
44 RCL IND 25	101 -	158 RCL 20	215 3	272 "t="
45 "SC."	102 STO 03	159 X+2	216 /	273 ARCL X
46 XEQ 11	103 RCL 22	160 1	217 Y+X	274 AVIEW
47 X+2	104 RCL 20	161 -	218 *	275 RTN
48 ST+ 21	105 /	162 "D.L."	219 RCL 14	276+LBL E
49 7ISZ	106 RCL 01	163 XEQ 11	220 +	277 RCL 25
50 RCL 20	107 -	164 RCL 00	221 SQR	278 1
51 RCL 25	108 STO 04	165 "S.C."	222 RCL 15	279 -
52 X*Y?	109 RCL 00	166 XEQ 11	223 X<>Y	280 STO 25
53 GTO 02	110 RCL 02	167 RTN	224 /	281 STOP
54 ADV	111 RCL 03	168+LBL 04	225 STO 16	282 2
55 30	112 RCL 04	169 RCL 09	226 X>0?	283 *
56 STO 25	113 +	170 "D.L."	227 SF 02	284 28
57+LBL 03	114 +	171 XEQ 11	228 ABS	285 +

```
286 STO 24
287 STOP
288 ST- IND 24
289 ST- IND 25
290 Σ-
291 GTO 01
292*LBL e
293 STOP
294 1
295 -
296 STO 23
297 STOP
298 2
299 *
300 28
301 +
302 STO 24
303 STOP
304 ST- IND 23
305 ST- IND 24
306 Σ-
307 STOP
308 2
309 *
310 28
311 +
312 STO 24
313 STOP
314 ST+ IND 23
315 ST+ IND 24
316 Σ+
317 GTO 01
318 .END.
```

STAT.07

REGISTERS, STATUS, FLAGS

REGISTERBELEGUNG, FLAGS, BETRIEBSARTEN

REGISTRES, INDICATEURS, MODES OPÉRATOIRES

REGISTRI, MODI OPERATIVI, FLAGS

Registers Datenspeicher Registres de données Registri				Status Betriebsart Modes opératoires Modi operativi			
00	T_c	Tot.S.S.	50	Size	055	Total Reg.	User Mode
		C.F.		Eng	☐	Fix	☒
		Row S.S.		Deg	☐	Rad	☐
		Column S.S.				Sci	☐
		Treat. S.S.				Grad	☐
05		Error M.S.	55	Purpose Bedeutung Signification Scopo			
		Error d.f.		Flags			
		Error S.S.					
		(n - 1)		00		SET	CLEAR
10			60	01			
				02	P(F) routine	☒	
				03			
				04			
				05			
				06			
				07			
15		Storage for P(F) routine	65	08			
				09			
				10			
				11	Audio execute		
				12			
20		$\sum n$	70	13			
		$\sum T_c^2$		14			
		$\sum T_t^2$		15			
				16			
				17			
25		$\sum y^2$	75	18			
		$\sum y$		19			
		$\sum T_r^2$		20			
				21	Printer Enable		
30		Sum treatm. _i	80	22	Number Input		
		n_i		23	Alpha Input		
		.		24	Range Ignore		
		.		25	Error Ignore		
		.		26	Audio Enable		
		.		27	User Mode		
35			85	28	Decimal Point		
				29	Digit Grouping		
				Assignments Tastenbelegung/Assignations/Assegnamenti			
40			90	Function Funktion Fonction Funzione	Key Taste Touche Tasto	Function Funktion Fonction Funzione	Key Taste Touche Tasto
45			95				
			99				

ANOVA - COMPARISONS AMONG MEANS

DESCRIPTION

After the initial significance test, a Model II ANOVA is completed by the estimation of the added variance components (See Prog.STAT.05). A Model I ANOVA is completed by examining the data in greater detail, testing which means are different from which other ones, or which groups of means are different from other such groups or from single means.

When these tests are planned before the experiment, and independently of the results, they are called "a priori" comparisons. These tests can be applied regardless of the results of the preliminary ANOVA. When the comparisons are decided after the experiments, being based on the results, they are called "a posteriori" tests, and they are performed only if the preliminary ANOVA is significant

1 - A priori tests

a - Subdivision of the treatment sums of squares and degrees of freedom into separate comparisons.

To compare k groups of any size n_i, take the sum of each group, square it, divide it by its sample size n_i and sum the k quotients thus obtained. From the sum of these quotients, subtract a correction factor, which is the grand sum of all the groups in the comparison, squared and divided by the number of items in the grand sum. This S.S. has (k-1) d.f. and the corresponding mean square is tested by an "F" test against the error mean square. If all the comparisons are independent, their S.S. and d.f. are additive.

b - Least significant difference - (L.S.D.)

This test is based on the "t" test : one computes

$$\text{L.S.D.} = t_{0,05} \sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

with s^2 = error mean square and $t_{0,05}$ = value of t for the number of d.f. of the ANOVA.

If a difference $(\bar{y}_1 - \bar{y}_2)$ is larger than the L.S.D., the two means differ significantly at the 0,05 significance level.

2 - A posteriori tests

a - Comparison of all the means to the same control

A frequent case is the separate comparison of all the treatment means $\bar{y}_i$ to the same control mean $\bar{y}_0$. This can be done with the Dunnett test. Each treatment mean is compared to $\bar{y}_0$: $(\bar{y}_0 - \bar{y}_i)$, and a value of "t" is computed as :

$$t = \frac{|\bar{y}_0 - \bar{y}_i|}{\sqrt{s^2 \left(\frac{1}{n_0} + \frac{1}{n_i} \right)}}$$

If this calculated "t" is larger than a limit value "t'" found in the Dunnett table^(*), which takes in account the number p of means $\bar{y}_i$ compared to the control, the difference is significant.

b - multiple comparisons

The overall comparison of all the means can be performed with the S.S. subdivision method described above, taking the number of d.f. of F as $(a - 1)$ and $a(n - 1)$; or with a method based on the studentized range, which has not been included in this program. (Duncan test, for instance)^(**)

The program STAT.08 can be used after programs STAT.05, 06 or 07.

(*) C.W.DUNNETT - Biometrics 1964; 20; p.488

(**) J.PHILIPPE p.121

USER INSTRUCTIONS I

STAT.08

 PROGRAMMABLAUF I
 INSTRUCTIONS D'EMPLOI I
 NORME OPERATIVE I

Step Schritt Pas Passo	Instructions Operation Instructions Istruzioni	Variables Dateneingabe Données Dati	Function(s) Taste(n) Touche(s) Tasti	Result Resultat Résultat Risultato
1	When program 05, 06 or 07 has been executed, clear the program		XEQ "CLP" " "	
2	Key in the program STAT.08, tracks 1 to 5			
	<u>Least significant difference</u>			
3	Initialize		A	
4	Input the number of the two groups to be compared	n_1^0	R/S	
		n_2^0	R/S	
5	Display of : difference $ \bar{y}_1 - \bar{y}_2 $ least sign. diff. result			DIFF PPSD SIGN. or N.S.
6	For another comparison, return to step 4			
	<u>Dunnett test</u>			
7	Initialize		B	
	Display : number of groups compared to the control d.f. of the ANOVA The program asks for (value of Dunnett t)			P D.L. t ?
8	Input	$t_{0,05}$	R/S	
9	For all comparisons, display of : $ \bar{y}_0 - \bar{y}_1 $ t calcul. result			DIFF t SIGN. or N.S.
	<u>Subdivision of treatment S.S.</u>			
10	A priori test Initialize		C	
10'	A posteriori test Initialize		p.C	
11	Input the number of groups to be compared	k	R/S	
12	Input the number(s) of the treatment(s) of the first group	$n^0 \dots$	R/S	
13	Call		D	
14	Repeat steps 12 and 13 for each of the other groups			
15	Display of : nbr of d.f. sum of squares mean square F P(F)			D.L. S.C. C.M. F P
16	For another comparison, return to step 11			

[illegible]

PROGRAMMAUFLISTUNG
 LISTAGE DU PROGRAMME
 LISTATO DI PROGRAMMA

01*LBL "STAT8"	58 /	115 "N.S."	172 GTO 09	229 ENTER↑
02*LBL A	59 STO 02	116 AVIEW	173*LBL 08	230 3
03 CF 01	60 RCL IND 00	117 GTO 06	174 RCL 20	231 /
04 2,3777	61 1/X	118*LBL 05	175 1	232 Y↑X
05 RCL 06	62 STO 03	119 "SIGN"	176 -	233 *
06 /	63 ISG 00	120 AVIEW	177 STO 11	234 RCL 03
07 E↑X	64*LBL 04	121*LBL 06	178 "D.L."	235 +
08 ,96	65 RCL IND 00	122 ADV	179 XEQ 11	236 SQRT
09 +	66 ISG 00	123 RTN	180*LBL 09	237 RCL 07
10 STO 22	67 RCL IND 00	124*LBL C	181 RCL 02	238 X<>Y
11*LBL 01	68 /	125 SF 01	182 X↑2	239 /
12 STOP	69 XEQ 03	126*LBL C	183 RCL 03	240 STO 08
13 XEQ 02	70 RCL 04	127 0	184 /	241 X>0?
14 STO 02	71 X<>Y	128 STO 00	185 RCL 04	242 SF 02
15 RCL IND 00	72 /	129 STO 01	186 X<>Y	243 ABS
16 1/X	73 SF 13	130 STO 02	187 -	244 1
17 STO 03	74 "T"	131 STO 03	188 "S.C."	245 RCL 08
18 STOP	75 XEQ 11	132 STO 04	189 XEQ 11	246 ,196854
19 XEQ 02	76 CF 13	133 STOP	190 RCL 08	247 *
20 XEQ 03	77 RCL 09	134 ADV	191 1	248 +
21 RCL 22	78 X<>Y	135 STO 08	192 -	249 RCL 08
22 *	79 XEQ 12	136 STO 25	193 /	250 X↑2
23 "PPDS"	80 ISG 00	137*LBL 07	194 "C.M."	251 ,115194
24 XEQ 11	81 GTO 04	138 STOP	195 XEQ 11	252 *
25 RCL 04	82 RTN	139 2	196 RCL 05	253 +
26 XEQ 12	83*LBL 02	140 *	197 /	254 RCL 08
27 GTO 01	84 2	141 28	198 "F"	255 3
28*LBL B	85 *	142 +	199 STO 12	256 Y↑X
29 CF 01	86 28	143 STO 10	200 XEQ 11	257 ,000344
30 RCL 20	87 +	144 RCL IND 10	201 2	258 *
31 1	88 STO 00	145 ST+ 00	202 ENTER↑	259 +
32 -	89 RCL IND 00	146 ST+ 02	203 9	260 RCL 08
33 SF 13	90 STO 01	147 1	204 /	261 4
34 "P"	91 1	148 ST+ 10	205 STO 01	262 Y↑X
35 XEQ 11	92 ST+ 00	149 RCL IND 10	206 RCL 11	263 ,019527
36 CF 13	93 RCL 01	150 ST+ 01	207 /	264 *
37 RCL 06	94 RCL IND 00	151 ST+ 03	208 STO 03	265 +
38 "D.L."	95 /	152 GTO 07	209 RCL 01	266 -4
39 XEQ 11	96 RTN	153*LBL D	210 RCL 06	267 Y↑X
40 SF 13	97*LBL 03	154 RCL 00	211 /	268 2
41 "T?"	98 RCL 02	155 X↑2	212 STO 01	269 /
42 PROMPT	99 -	156 RCL 01	213 1	270 CHS
43 STO 09	100 ABS	157 /	214 -	271 1
44 CF 13	101 STO 04	158 ST+ 04	215 CHS	272 +
45 RCL 20	102 "DIFF"	159 0	216 RCL 12	273 FIX 3
46 2	103 XEQ 11	160 STO 00	217 3	274 FS?C 02
47 *	104 RCL IND 00	161 STO 01	218 1/X	275 XEQ 10
48-29	105 1/X	162 7DSZ	219 Y↑X	276 "P"
49 +	106 RCL 03	163 GTO 07	220 *	277 XEQ 11
50 1000	107 +	164 FS? 01	221 1	278 FIX 2
51 /	108 RCL 05	165 GTO 08	222 RCL 03	279 ADV
52 30	109 *	166 RCL 08	223 -	280 GTO C
53 +	110 SQRT	167 1	224 -	281*LBL 10
54 STO 00	111 RTN	168 -	225 STO 07	282 1
55 RCL IND 00	112*LBL 12	169 STO 11	226 RCL 01	283 X<>Y
56 ISG 00	113 X>Y?	170 "D.L."	227 RCL 12	284 -
57 RCL IND 00	114 GTO 05	171 XEQ 11	228 2	285 RTN

One may choose to set α at a lower level (i.e. 0,01), if one feels that the danger of falsely rejecting H_0 is important.

Another error which may be made is to accept H_0 when it is actually false; this is the type II error, and the corresponding risk is β ($1 - \beta$) is called the power of the test; this power increases with the size of the sample(s).

4 - Sampling distribution.

It is the distribution, under H_0 , of the statistic chosen for the test (for instance: u, t, F, χ^2). The experimental value of this statistic is computed from the data, and the probability P corresponding to the calculated value, if H_0 is satisfied, is found in the proper table, or computed in the program.

5 - Decision.

If the probability associated with the observed value of the statistic is equal to or less than the previously determined significance level α , H_0 is considered as false and rejected, and H_1 is accepted. The difference is "significant". If the probability of the observed value is larger than H_0 cannot be rejected; the difference is not statistically significant.

In the first case, we know that, when rejecting H_0 , we accept a risk α to be wrong (for instance, 1 out of 20 if $\alpha = 0,05$). In the second case, not rejecting H_0 does not imply that H_0 is true; it may exist a small, but real difference δ , but the power of the test may not be sufficient to show it.

CHOICE OF THE TESTS

The choice between several tests is based on three criteria :

1 - the nature of the problem to be solved

It may be a comparison between dispersions or between means of two or more groups; the study of several factors,...

2 - the nature of the experimental data

The data may be classified in several types of measurements :

PROGRAM LISTING

PROGRAMMAUFLISTUNG
LISTAGE DU PROGRAMME
LISTATO DI PROGRAMMA

286*LBL 11
287 "I="

288 ARCL X

289 AVIEW

290 RTN

291 .END.

REGISTERS, STATUS, FLAGS

REGISTERBELEGUNG, FLAGS, BETRIEBSARTEN
REGISTRES, INDICATEURS, MODES OPÉRATOIRES
REGISTRI, MODI OPERATIVI, FLAGS

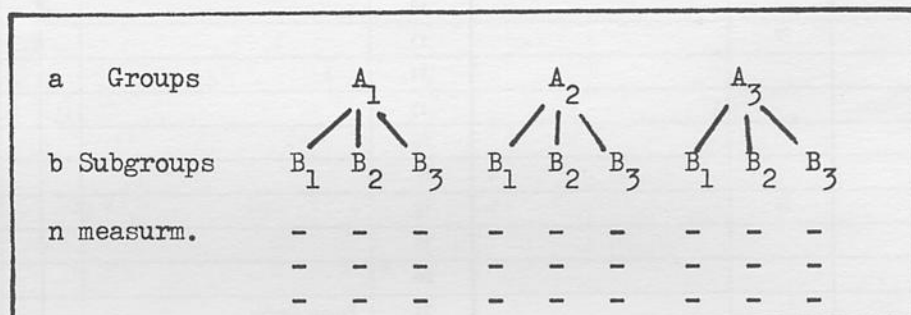
Registers Datenspeicher Registres de données Registri				Status Betriebsart Modes opératoires Modi operativi			
00	$\bar{y}$	50		Size <u>055</u> Total Reg. _____		User Mode	
	$\sum T_i$			Eng ☐	Fix ☒	Sci ☐	On ☒
	$\bar{y}_1$			Deg ☐	Rad ☐	Grad ☐	Off ☐
	$1/n_1$			Purpose Bedeutung Signification Scopo Flags SET CLEAR			
05	$\bar{y}_1 - \bar{y}_2$	55					
	var. error			00	A posteriori test	☒	
	d.f. error			01			
	k			02	P(F) routine	☒	
10	Dunnett t'	60		03			
	n° treatm.			04			
				05			
				06			
				07			
15		65		08			
				09			
				10			
				11	Audio execute		
				12			
20		70		13			
				14			
	$t_{0,05}$			15			
				16			
				17			
25	counter	75		18			
				19			
				20			
				21	Printer Enable		
				22	Number Input		
30		80		23	Alpha Input		
				24	Range Ignore		
				25	Error Ignore		
				26	Audio Enable		
				27	User Mode		
35		85		28	Decimal Point		
				29	Digit Grouping		
				Assignments Tastenbelegung/Assignations/Assegnamenti			
				Function Funktion Fonction Funzione	Key Taste Touche Tasto	Function Funktion Fonction Funzione	Key Taste Touche Tasto
40		90					
45		95					
		99					

NESTED ANALYSIS OF VARIANCE

DESCRIPTION

In a nested (or hierarchic) ANOVA, each of the a groups is divided in b subgroups, the subordinate classification being nested within the higher level classification; n measurements are collected in each subgroup. This type of ANOVA may be used, for instance, to study an industrial process or an analytical method involving several steps.

The groups representing the subordinate level of classification (subgroups) must be randomly chosen. The higher level of classification may be Model II (random), giving a pure Model II ANOVA, or Model I (fixed) giving a Mixed Model ANOVA.



The ANOVA table is as follows :

Source of variation	Number of d.l.	Sum of squares	Mean square	F
Among groups	$(a - 1)$		s_g^2	
Among subgroups within groups	$a(b - 1)$		s_{sg}^2	
Within subgroups (error)	$ab(n - 1)$		s_e^2	
Total	$abn - 1$			

The existence of a variation among subgroups within groups is first tested by $F_1 = s_{sg}^2 / s_e^2$; then the significance of the variation among groups is tested by $F_2 = s_g^2 / s_{sg}^2$.

in a pure Model II ANOVA, one can estimate the variance among groups

STAT.09

σ_A^2 , the variance of B within A $\sigma_{B(A)}^2$ and the variance of the error σ_E^2 and the percent of total variation assignable to each source of variation.

In a Mixed ANOVA, the different levels of A can be compared, using as error term s_{sg}^2 when this term is significant, or a pooled variance of s_{sg}^2 and s_e^2 when s_{sg}^2 is not significant. The difference between means can be tested, using program STAT.08.

The data are entered by subgroups, starting with the first subgroup B_1 of the first group A_1 ; display of the subgroup totals T_b and of the group totals T_A . The program computes :

$$\text{Total sum of squares : } \sum y^2 - \text{C.F.} \quad (1) \quad \text{C.F.} = (\sum y)^2 / abn$$

$$\text{Group S.S. : } \sum T_a^2 / bn - \text{C.F.} \quad (2)$$

$$\text{Subgroup S.S. : } \sum T_b^2 / n - \text{C.F.} \quad (3)$$

$$\text{Among subgroups within groups S.S.} = (3) - (2)$$

$$\text{Within subgroups (error) S.S.} = (1) - (2)$$

**PROGRAMMABLAUF I
INSTRUCTIONS D'EMPLOI I
NORME OPERATIVE I**

Step Schritt Pas Passo	Instructions Operation Instructions Istruzioni	Variables Dateneingabe Données Dati	Function(s) Taste(n) Touche(s) Tasti	Result Resultat Résultat Risultato
1	Key in program STAT.09, tracks 1 to 6			
2	Initialize		XEQ "ANVA.5"	
3	Enter : number of groups	a	R/S	
	number of subgroups	b	R/S	
	number of measurements par subgroup	n	R/S	
4	Input the data of the first subgroup of the first group	$y_1, \dots$	R/S	
5	Call		B	ΣB
6	Display : subgroup total T_b Repeat steps 4 and 5 for the other subgroups of the first group			
7	At the end of the last subgroup, display : group total T_a			ΣA
8	Repeat steps 4 to 7 for the other groups			
9	Correction routine Call		E	
	input the erroneous data	y_E	R/S	
	then the correct one	y_C	R/S	
10	At the end of the last group, display of the ANOVA			
	<u>Model II ANOVA</u> Estimates of the several variances			
11	Call Display of :		C	VAR.A & % VAR.B(A) & % VAR.E & %
12	For another set of data, call and return to step 3		A	
	<u>Mixed Model ANOVA</u>			
13	Call and go to Program STAT.08.		D	

EXAMPLE 1 - Nested ANOVA Model II (※)

XEQ "ANVA5"	2.77	RUN	ANVA
4.00	2.66	RUN	GROUP
3.00		XEQ B	D.L.=3.00
2.00			S.C.=7.56
	$\Sigma B=5.43$		C.M.=2.52
3.28	3.74	RUN	F=7.67
3.09	3.44	RUN	P=0.010
		XEQ B	
$\Sigma B=6.37$	$\Sigma B=7.18$		S/GROUP
3.52	2.55	RUN	D.L.=8.00
3.48	2.55	RUN	S.C.=2.63
		XEQ B	C.M.=0.33
$\Sigma B=7.00$	$\Sigma B=5.10$		F=49.41
2.88		RUN	P=5.915E-6
2.80		RUN	ERR.
	$\Sigma A=17.71$	XEQ B	D.L.=12.00
$\Sigma B=5.68$			S.C.=0.08
$\Sigma A=19.05$	$\Sigma B=7.65$		C.M.=0.01
2.46	4.07	RUN	TOT.
2.44	14.12	RUN	D.L.=23.00
		XEQ B	S.C.=10.27
$\Sigma B=4.90$	(error)	XEQ E	VAR. A=0.37
1.87	14.12	RUN	VAR. A $\times$ =68.53
1.92	4.12	RUN	VAR. B(A)=0.16
		XEQ B	VAR. B(A) $\times$ =30.22
$\Sigma B=3.79$	$\Sigma B=8.19$		VAR. ERR.=0.01
2.19	3.31	RUN	VAR. ERR. $\times$ =1.25
2.19	3.31	RUN	
		XEQ B	
$\Sigma B=4.38$	$\Sigma B=6.62$		
$\Sigma A=13.07$	$\Sigma A=22.46$		

(※) SNEDECOR - p: 319

EXAMPLE 2 - Nested ANOVA Mixed Model

Same data used as if A were fixed treatments

	Xt		
ΣB=6,...			XEQ A
		1,00	RUN
ΣA=22,46		2,00	RUN
		DIFF=1,00	
		PPDS=0,76	
		SIGN	
ANVA			
GROUP			
D.L.=3,00		2,00	RUN
S.C.=7,56		3,00	RUN
C.M.=2,52			
F=7,67		DIFF=0,77	
P=0,010		PPDS=0,76	
		SIGN	
S/GROUP			
D.L.=8,00		2,00	RUN
S.C.=2,63		4,00	RUN
C.M.=0,33			
F=49,41		DIFF=1,57	
P=5,915E-6		PPDS=0,76	
		SIGN	
ERR.			XEQ C
D.L.=12,00		2,00	RUN
S.C.=0,08			
C.M.=0,01		1,00	RUN
			XEQ D
TOT.		2,00	RUN
D.L.=23,00		3,00	RUN
S.C.=10,27		4,00	RUN
			XEQ D
	XEQ D		
	CLP **	D.L.=1,00	
		S.C.=0,21	
		C.M.=0,21	
		F=0,65	
		P=0,450	
PACKING	XEQ B		
p=3,00			
D.L.=8,00			
t?			
	2,88	run	2,00
			RUN
DIFF=1,00			2,00
t=3,01			RUN
SIGN			3,00
			RUN
			XEQ D
			4,00
			RUN
			XEQ D
DIFF=0,22			
t=0,67			
N.S.		D.L.=1,00	
		S.C.=5,55	
		C.M.=5,55	
DIFF=0,57		F=16,89	
t=1,72		P=0,004	
N.S.			

STAT.09

PROGRAM LISTING

 PROGRAMMAUFLISTUNG
 LISTAGE DU PROGRAMME
 LISTATO DI PROGRAMMA

01*LBL "ANVA5"	58 RCL 20	115 XEQ 11	172 "S.C."	229 1
02*LBL A	59 RCL 21	116 XEQ 10	173 XEQ 11	230 RCL 00
03 CLRG	60 RCL 22	117 "S/GROUP"	174 RTN	231 ,196854
04 STOP	61 *	118 AVIEW	175*LBL 11	232 *
05 STO 20	62 *	119 RCL 01	176 "I="	233 +
06 STOP	63 /	120 STO 00	177 ARCL X	234 RCL 00
07 STO 21	64 STO 03	121 "D.L."	178 AVIEW	235 X↑2
08 STO 25	65 RCL 15	122 XEQ 11	179 RTN	236 ,115194
09 STOP	66 X<>Y	123 RCL 08	180*LBL E	237 *
10 STO 22	67 -	124 "S.C."	181 STOP	238 +
11 30,054	68 STO 04	125 XEQ 11	182 ST- 00	239 RCL 00
12 STO 26	69 RCL 02	126 RCL 24	183 2-	240 3
13 ADV	70 RCL 03	127 "C.M."	184 GTO 01	241 Y↑X
14*LBL 01	71 -	128 XEQ 11	185*LBL 10	242 ,000344
15 STOP	72 STO 07	129 RCL 20	186 2	243 *
16 ST+ 00	73 RCL 01	130 RCL 21	187 ENTER↑	244 +
17 SREG 14	74 RCL 02	131 RCL 22	188 9	245 RCL 00
18 2+	75 -	132 1	189 /	246 4
19 GTO 01	76 STO 08	133 -	190 STO 02	247 Y↑X
20*LBL B	77 RCL 15	134 *	191 RCL 00	248 ,019527
21 RCL 00	78 RCL 01	135 *	192 /	249 *
22 "SB"	79 -	136 STO 01	193 STO 13	250 +
23 XEQ 11	80 STO 09	137 RCL 09	194 RCL 02	251 -4
24 ADV	81 "ANVA"	138 X<>Y	195 RCL 01	252 Y↑X
25 ST+ 03	82 AVIEW	139 /	196 /	253 2
26 X↑2	83 "GROUP"	140 STO 06	197 STO 02	254 /
27 RCL 22	84 AVIEW	141 RCL 24	198 1	255 CHS
28 /	85 RCL 20	142 X<>Y	199 -	256 1
29 ST+ 01	86 1	143 /	200 CHS	257 +
30 0	87 -	144 "F"	201 RCL 05	258 FIX 3
31 STO 00	88 "D.L."	145 XEQ 11	202 3	259 FS?C 02
32 7DSZ	89 XEQ 11	146 STO 05	203 1/X	260 XEQ 09
33 GTO 01	90 STO 00	147 XEQ 10	204 Y↑X	261 "P"
34 RCL 21	91 RCL 07	148 "ERR."	205 *	262 STO 27
35 STO 25	92 "S.C."	149 AVIEW	206 1	263 XEQ 11
36 XEQ 02	93 XEQ 11	150 RCL 01	207 RCL 13	264 FIX 2
37 RCL 03	94 X<>Y	151 "D.L."	208 -	265 ADV
38 "SA"	95 /	152 XEQ 11	209 -	266 RTN
39 XEQ 11	96 STO 07	153 RCL 09	210 STO 14	267*LBL 09
40 ADV	97 "C.M."	154 "S.C."	211 RCL 02	268 1
41 X↑2	98 XEQ 11	155 XEQ 11	212 RCL 05	269 X<>Y
42 RCL 22	99 RCL 20	156 RCL 06	213 2	270 -
43 RCL 21	100 RCL 21	157 "C.M."	214 ENTER↑	271 RTN
44 *	101 1	158 XEQ 11	215 3	272*LBL 02
45 /	102 -	159 ADV	216 /	273 RCL 03
46 ST+ 02	103 *	160 "TOT."	217 Y↑X	274 STO IND 26
47 0	104 STO 01	161 AVIEW	218 *	275 ISG 26
48 STO 03	105 STO 28	162 RCL 20	219 RCL 13	276 RCL 21
49 1	106 RCL 08	163 RCL 21	220 +	277 RCL 22
50 ST+ 04	107 X<>Y	164 RCL 22	221 SQR	278 *
51 RCL 04	108 /	165 *	222 RCL 14	279 STO IND 26
52 RCL 20	109 STO 24	166 *	223 X<>Y	280 ISG 26
53 X*Y?	110 RCL 07	167 1	224 /	281 RTN
54 GTO 01	111 X<>Y	168 -	225 STO 00	282*LBL C
55 ADV	112 /	169 "D.L."	226 X>0?	283 RCL 24
56 RCL 14	113 STO 05	170 XEQ 11	227 SF 02	284 RCL 06
57 X↑2	114 "F"	171 RCL 04	228 ABS	285 -

PROGRAM LISTING

PROGRAMMAUFLISTUNG
LISTAGE DU PROGRAMME
LISTATO DI PROGRAMMA

STAT.09

286 RCL 22	343 +
287 /	344 STO 06
288 STO 01	345 /
289 RCL 07	346 STO 05
290 RCL 24	347*LBL 04
291 -	348 RTN
292 RCL 22	349 .END.
293 /	
294 RCL 21	
295 /	
296 STO 02	
297 RCL 01	
298 +	
299 RCL 06	
300 +	
301 STO 03	
302 RCL 02	
303 "VAR.A"	
304 XEQ 11	
305 XEQ 05	
306 "VAR.A%"	
307 XEQ 11	
308 RCL 01	
309 "VAR.B(A)"	
310 XEQ 11	
311 XEQ 05	
312 "VAR.B(A)%"	
313 XEQ 11	
314 RCL 06	
315 "VAR.ERR."	
316 XEQ 11	
317 XEQ 05	
318 "VAR.ERR.%"	
319 XEQ 11	
320 RTN	
321*LBL 05	
322 RCL 03	
323 /	
324 100	
325 *	
326 RTN	
327*LBL D	
328 .05	
329 RCL 27	
330 X>Y?	
331 GTO 03	
332 RCL 24	
333 STO 05	
334 RCL 28	
335 STO 06	
336 GTO 04	
337*LBL 03	
338 RCL 09	
339 RCL 08	
340 +	
341 RCL 01	
342 RCL 28	

STAT.09

REGISTERS, STATUS, FLAGS

REGISTERBELEGUNG, FLAGS, BETRIEBSARTEN

REGISTRES, INDICATEURS, MODES OPÉRATOIRES

REGISTRI, MODI OPERATIVI, FLAGS

Registers Datenspeicher Registres de données Registri			Status Betriebsart Modes opératoires Modi operativi		
00	T _b ; d.f.1 a(b-1) T _a ² /bn T _a S.S. Total S.S.	50	Size 055	Total Reg. _____	User Mode
			Eng ☐	Fix ☒	Sci ☐
			Deg ☐	Rad ☐	Grad ☐
			Purpose Bedeutung Signification Scopo		Flags SET CLEAR
05	F Error M.S. Group S.S. S/gr. S.S. Error S.S.	55	00		
			01		
			02	P(F) routine	X
10		60	03		
			04		
			05		
			06		
	Statistic. stack		07		
15		65	08		
			09		
			10		
			11	Audio execute	
			12		
20	a b n S/gr. M.S.	70	13		
			14		
			15		
			16		
			17		
25		75	18		
			19		
			20		
			21	Printer Enable	
			22	Number Input	
30		80	23	Alpha Input	
			24	Range Ignore	
			25	Error Ignore	
			26	Audio Enable	
			27	User Mode	
35		85	28	Decimal Point	
			29	Digit Grouping	
			Assignments Tastenbelegung/Assignations/Assegnamenti		
			Function Funktion Fonction Funzione	Key Taste Touche Tasto	Function Funktion Fonction Funzione
40		90			
45		95			
		99			

- nominal or classificatory variables

the observations are distributed into two or more classes, and the variable is the frequencies in each category; they can also be expressed as ratios or percentages.

- ordinal or ranked variables

the data are scores or measurements which can be ordered or ranked by increasing or decreasing values.

- measurement variables

they may be continuous or discontinuous (discrete)

3 - the knowledge of the distribution

Each group of data is considered as a sample drawn at random from a population. Generally, the distribution of this population is not known, and the sample is too small to give an idea of this distribution, which must be assumed.

The different criteria determine three groups of tests :

- 1 - the variable is a measured quantity, normally distributed (or assumed normal). Parametric tests should be used.
- 2 - the data are scores, or measured variables known as non normal; the data are ranked and analysed with non parametric rank tests
- 3 - the data are frequencies; they are analysed with non parametric frequency tests.

It can be seen that the conditions of use of the parametric tests (normal distribution) are more stringent than the non parametric tests (sometimes called "distribution free"); but, for a given sample size, the parametric tests are also the more powerful.

A few suggestions for the choice of the appropriate test and the corresponding program are listed below :

A - the variable is measured and normally distributed :

1 - One sample

- mean and confidence interval
- histogram and normality test

Prog. STAT.01

02

APPLIED STATISTICS I

MEANS AND CONFIDENCE INTERVAL
HISTOGRAM – NORMALITY TEST
TWO INDEPENDENT SAMPLES «F» AND «t» TESTS
TWO RELATED SAMPLES «t» TEST
SINGLE CLASSIFICATION ANOVA
COMPLETE BLOCK ANOVA
LATIN SQUARE ANOVA
COMPARISONS AMONG MEANS
NESTED ANOVA



2 - Two samples	
a - independent	
. Comparison of variances : test "F"	
. Comparison of means : test "t"	STAT.03
b - related	
. comparison of means : test "t" by difference	04
3 - More than two samples : Analysis of variance (ANOVA)	
a - One factor	
. single classification randomized ANOVA	05
. restrictions in design :	
.. complete blocks	06
.. latin square	07
. comparison of means	08
b - Two factors	
. hierarchic classification : nested ANOVA	09
. independent : two-way ANOVA	10
c - More than two factors	
. three factors ANOVA (B nested and C crossed)	11
. three, four and five factors at two levels	12
4 - Relations between two factors	
a - two random variables : correlation	13
b - one dependent variable : linear regression	14
c - one concomitant variable : analysis of covariance (ANCOVA)	15
B - The variable is measured, but not normally distributed, or is ordinal (scores); the data can be ranked	17
1 - Two samples	
a - independent : Mann-Whitney U test	
b - related : Wilcoxon signed-rank test	
2 - More than two samples	
a - independent : Kruskal-Wallis one-way ANOVA	
b - related : Friedman two-way ANOVA	
3 - Relationship between two variables : Spearman rank correlation coefficient	
C - The variable is qualitative (frequencies)	
1 - Tests of Goodness of fit : G test	

2 - Contingency Tables

a - (2 x 2) Tables

- . small samples : Fisher exact test
- . large samples : Pearson χ^2 test with correction of continuity

b - (r x c) Tables : G test

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The probability under H_0 of the several statistics used in the tests is computed by a subroutine, to avoid the consultation of tables. The following algorithms are used for this computation :

- Normal distributiona - Ordinate at point u :

$$Z(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$$

b - Probability

$$P(u) = 1 - 1/2 (1 + c_1 u + c_2 u^2 + c_3 u^3 + c_4 u^4)^{-4}$$

$$c_1 = 0,196854 \quad c_3 = 0,000344$$

$$c_2 = 0,115194 \quad c_4 = 0,019527$$

- Student "t" distribution

$$\theta = \tan^{-1} \left(\frac{|t|}{\sqrt{n'}} \right) \quad (\text{radians}) \quad n' = \text{number of d.f.}$$

a - n' even

$$R = \sin \theta \left\{ 1 + \frac{1}{2} \cos^2 \theta + \frac{1.3}{2.4} \cos^4 \theta + \dots + \frac{1.3.5 \dots (n'-3)}{2.4.6 \dots (n'-2)} \cos^{(n'-2)} \theta \right\}$$

b - n' odd

$$n' = 1 \quad R = 2 \theta / \pi$$

$$n' > 1$$

$$R = \frac{2\theta}{\pi} + \frac{2}{\pi} \cos \theta \left\{ \sin \theta \left[1 + \frac{2}{3} \cos^2 \theta + \dots + \frac{2.4.6 \dots (n'-3)}{1.3.5 \dots (n'-2)} \cos^{(n'-3)} \theta \right] \right\}$$

$$\text{Two-tailed test: } P(t) = 1 - R$$

$$\text{One-tailed test: } P(t) = \frac{(1 - R)}{2}$$

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