~~018590~~

Program Description I

00301

Program Title: SECOND ORDER ACTIVE NETWORK POLE AND ZERO POLYNOMIAL COEFFICIENTS

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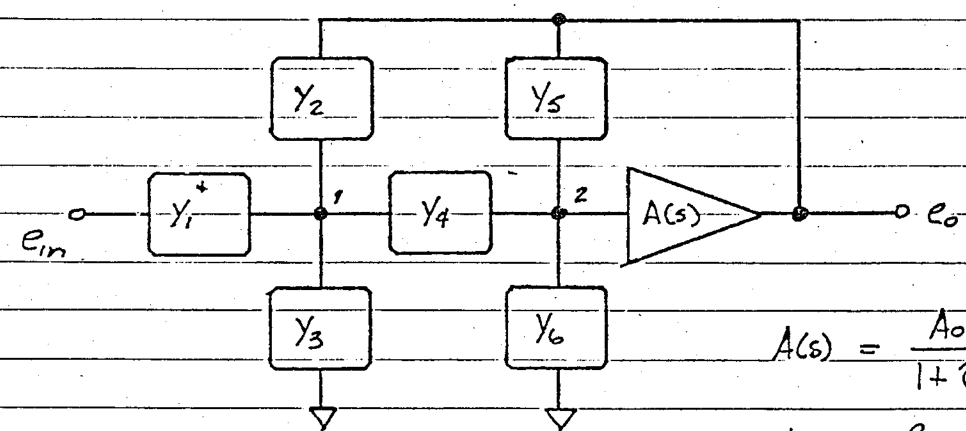
Program Description, Equations, Variables: This program will provide the coefficients of the numerator and denominator polynomials of the transfer function $H(s) = N(s)/D(s)$ of a generalized second order active network as shown in figure 1.



$$A(s) = \frac{A_0}{1+\tau s} ; \quad \tau = \frac{1}{2\pi f_0}$$

$$H(s) = \frac{e_o}{e_{in}} = \frac{N(s)}{D(s)}$$

Figure 1

* Y_1 through Y_6: each element may be either a resistor or a capacitor

Operating Limits and Warnings: Resistors range: $10^{-40} \leq R < 10^{30}$ ohms
Capacitor range: $10^{-99} \leq C \leq 10^{29}$

01859D Program Description

The matrix form nodal equations for the generalized circuit shown in figure 1 may be written.

1) noting that $e_2 = e_0/A$, the matrix is:

2)
$$\begin{bmatrix} \{y_1 + y_2 + y_3 + y_4\} & \{\frac{-y_4}{A} - y_2\} \\ \{-y_4\} & \{\frac{y_4 + y_5 + y_6}{A} - y_5\} \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ e_0 \end{bmatrix} = \begin{bmatrix} y_1 e_{in} \\ 0 \end{bmatrix}$$

the determinant of the coefficient matrix is:

3)
$$\Delta = (y_1 + y_2 + y_3 + y_4)\left(\frac{y_4 + y_5 + y_6}{A} - y_5\right) - y_4\left(\frac{y_4}{A} - y_2\right)$$

clearing fractions, and eliminating term subtraction:

4)
$$A \cdot \Delta = (y_1 + y_2 + y_3)(y_4 + y_6 + y_5(1-A)) + y_4(y_5(1-A) - Ay_2 + y_6)$$

substituting $A = A_0/1+\tau s$ and clearing fractions

5)
$$A_0 \cdot \Delta = (y_1 + y_2 + y_3)((y_4 + y_6)(1+\tau s) + y_5(1 - A_0 + \tau s))$$
$$+ y_4(y_6(1+\tau s) + y_5(1 - A_0 + \tau s) - A_0 y_2)$$

Cramer's rule is used to obtain $e_0/e_{in} = H(s)$

6)
$$e_0 = \frac{y_1 y_4}{\Delta} \cdot e_{in}$$

or

7)
$$\frac{e_0}{e_{in}} = \frac{A_0 y_1 y_4}{(y_1 + y_2 + y_3)\{(y_4 + y_6)(1+\tau s) + y_5(1 - A_0 + \tau s)\} + y_4\{y_6(1+\tau s) + y_5(1 - A_0 + \tau s) - A_0 y_2\}}$$

The first program evaluates this equation to form the numerator and denominator polynomials. The second program finds the roots of these polynomials.

PROGRAM #1 DESCRIPTION.

As stated previously, this program evaluates both the denominator and numerator polynomials of equation 7. Each y_i is assumed to be either a conductance; $y_i = g$, or a susceptance $y_i = sC$.

When data is entered, capacitors are signified by a negative mantissa. Label 8 contains the input storage subroutine. If the entry has a negative mantissa, the absolute value is stored, if the mantissa is positive, the reciprical is taken, and the resulting value multiplied by 10^{60} before storage.

When data is recalled for summation (i.e. $y_1+y_2+y_3$), the subroutine under label 0 is employed. If the stored number is greater than 10^{30}, it is assumed to be a conductance, and the ssummation is done in the stack, for values less than or equal to 10^{30}, the summation is done in the designated (i) register. To complete a summation sequence, subroutine 2 (Label 2) is employed to sum the last admittance, and to store the stack summation in R(i-1).

After each pair of admittance sums are calculated and stored, the polynomial multiplication is accomplished by the subroutine under label 6. If flag 0 is set, the polynomial coefficient registers are cleared before multiplication. this condition exists for the first product-of-sums. flag 0 is cleared for the second product-of-sums to indicate continued summation into the polynomial coefficient registers.

After the denominator has been calculated, the coefficients are divided by the s^0 coefficient for polynomial normalization before coefficient output. This same normalization affects the numerator.

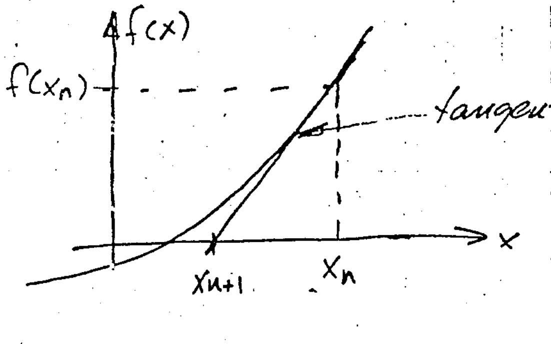
SECOND PROGRAM DESCRIPTION.

If the donominator polynomial is third order, a modified Newton-Raphson solution is used to find the real root (there will be at least one). A polynomial division is performed to reduce the third order polynomial to second order by removing the real root. The quadratic equation formula is used to find the roots of this second order polynomial. If the roots are complex, the program also calculates the natural frequency, fn, and the quality factor, Q, of this second order pair.

If the s^3 coefficient is zero, the program solution proceeds directly to the second order solution described in the above paragraph.

The modified newton-raphson root extraction routine employs an accelerated convergence procedure. this helps speed the convergence for a real root that is widely separated from the remaining second order roots. The modification consists of the addition of the factor k to the correction factor i.e.

Normal Newton-Raphson solution



tangent to curve, $f'(x_n) = \dfrac{d f(x_n)}{dx}$

$$f'(x_n) = \frac{\Delta f(x)}{\Delta x} = \frac{f(x_n) - 0}{x_n - x_{n+1}}$$

or

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

add the convergence speed up factor, k

$$x_{n+1} = x_n - k\,\frac{f(x_n)}{f'(x_n)}$$

Program Description

When the third order roots are widely spaced, the coefficient of the s^3 term dominates i.e.:

$$D(s) = b_3 s^3 + b_2 s^2 + b_1 s + 1 \approx b_3 s^3 = f(s)$$

$$f'(s) \cong 3 b_3 s^2 \text{ , hence}$$

$$S_{n+1} = S_n - \frac{b_3 S_n^3}{3 b_3 S_n^2} = S_n - \frac{S_n}{3} = 2/3\, S_n$$

$\therefore S_n / S_{n+1} = 2/3$ for above approximation.

let $n = \left| S_n / (f(s)/f'(s)) \right|$

form $m = n + 1/n$

if $m < 3.4$, set $k = 2.5$ (accelerated convergence)

otherwise set $k = 1$ normal convergence.

The advantage of the accelerated convergence may be seen in the following example.

let $f(s) = 10^{-15} s^3 + 10^{-9} s^2 + 10^{-6} s + 1 = 0$

the normal convergence (normal newton-raphson) requires 23 iterations to find the real root $s = -1 \times 10^{-6}$ to 8 significant figures starting with an initial guess for s of $-(10^{-15})^{-0.6} = -1 \times 10^{-9}$

The accelerated convergence routine requires only 13 iterations to achieve the same accuracy with the same starting conditions

01859D Program Description

Once the real third order root has been obtained, a polynomial division is performed to extract the second order polynomial that remains.

α is a root of $f(s)$ $(s+\alpha=0)$

i.e. $f(-\alpha)=0$

$$\begin{array}{r l}
 & b_3s^2+(\alpha b_3+b_2)s+(\alpha(\alpha b_3+b_2)+b_1) \\
s-\alpha \,\big) & b_3s^3+b_2s^2+b_1s+1 \\
 & b_3s^3-\alpha b_3s^2 \\
\hline
 & (\alpha b_3+b_2)s^2 \\
 & (\alpha b_3+b_2)s^2-\alpha(\alpha b_3+b_2)s \\
\hline
 & \alpha(\alpha b_3+b_2)s+ \\
 & \alpha(\alpha b_3+b_2)s+b_1-\alpha(\alpha(\alpha b_3+b_2)+b_1) \\
\hline
 & 0
\end{array}$$

by defination of α as a root

If the second order equation is described as:

$C_2s^2+C_1s+C_0$, then

$C_2=b_3$

$C_1=\alpha C_2+b_2$

$C_0=\alpha C_1+b_1$

This polynomial division is done by steps 088 to 099 of the program

Steps 99 to 102 normalize the second order polynomial so $C_0 = 1$. This places the equation in the same form as the third order equation was originally.

The quadratic equation is used to extract the second order roots. The roots are expressed by

$$S_{1,2} = \frac{-C_1 \pm \sqrt{C_1^2 - 4C_2}}{2C_2} \qquad (C_0 = 1)$$

This equation may be algebraically manipulated into a form more suited to the HP-97

$$S_{1,2} = -\frac{C_1}{2C_2} \pm \sqrt{\left(\frac{C_1}{2C_2}\right)^2 - \frac{1}{C_2}}$$

if the discriminant, $\sqrt{\left(\frac{C_1}{2C_2}\right)^2 - \frac{1}{C_2}}$ is positive, the two real roots exist. If it is 0, a double root exists, and if it is negative the roots are a complex conjugate pair.

Steps 103 through 136 perform the above function.

01859D Program Description

The natural frequency and quality factor of the second order polynomial may be calculated in the case where the roots are a complex conjugate pair.

In this instance:

$$\omega_n = \sqrt{C_0/C_2} = \sqrt{1/C_2} \quad ; C_0 = 1$$

$$f_n = \omega_n/2\pi$$

$$\frac{1}{\omega_n Q} = \frac{C_1}{C_0} \quad \therefore \quad Q = \frac{\sqrt{C_2}}{C_1} \quad ; \quad C_0 = 1$$

These calculations are performed by steps 138 through 149 of the program

Assuming the 3rd order real pole (parasitic pole) is large with respect to the second order poles, the gain term, K_0, may be defined in terms of the numerator and denominator coefficients:

$$H(s) = \frac{a_2 s^2 + a_1 s + a_0}{\left(\frac{s}{\alpha}+1\right)\left(C_2 s^2 + C_1 s + 1\right)}$$

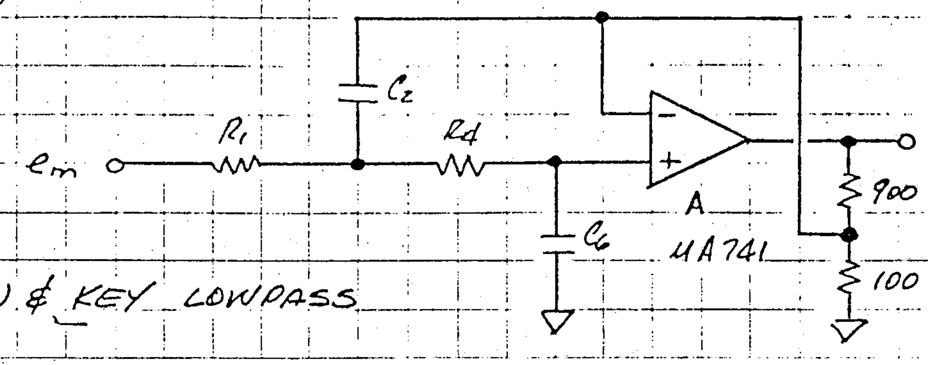
lowpass case: $K_0 = a_0$

bandpass case: $K_0 = a_1/C_1$

highpass case: $K_0 = a_2/C_2$

Steps 150 through 181 perform this function.

Sketch(es)



μA741
* $A_o = +1.0$
* $f_o = 50000$
$A = \frac{1}{1+\tau s}$
$\tau = \frac{1}{2\pi f_o}$

SALLEN & KEY LOWPASS

Sample Problem(s) Determine the transfer function of the above Sallen and Kay type active lowpass filter meeting the following characteristics

$e_o/e_{in}\Big|_{DC} = 10$

$f_o = 10000$ Hz

$Q = 1/\sqrt{2}$

$R_1 = 10000$ ohms

$R_4 = R_1$

$C_2 = \frac{Q}{\pi f_o R} = 2.251 \times 10^{-9}$ F

$C_6 = \frac{C_2}{4Q^2} = 1.125 \times 10^{-9}$ F

* The gain of 10 is accomplished by the resistive feedback divider with a gain of 0.1, hence the active filter sees a positive gain of 1.0 with -3dB point at 50000 Hz.

Solution(s) SEE NEXT PAGE FOR HP-97 PRINTOUT

Compare this topology to the multiple negative feedback lowpass topology on page 15/26. For unity gain applications, the Sallen & Key topology (above) is the preferred topology from both a component count basis and element sensitivity basis.

Reference(s) ACTIVE INDUCTORLESS FILTERS, SANJIT K. MITRA, Editor © 1971 by IEEE, John Wiley publisher

Program Description

HP-97 PRINTOUT FOR
SALLEN & KEY ACTIVE LOWPASS FILTER

LOAD FIRST PROGRAM CARD

10000.	GSBA	R_1
-2.251-09	GSBB	C_2
0.	GSBC	y_3 (open circuit)
10000.	GSBD	R_4
0.	GSBE	y_5 (open circuit)
-1.125-09	GSBa	C_6
1.	ENT↑	A_o — uA 741 characteristics
50000.	GSBd	f_o
	GSBe	
886.1-18	***	s^3 — Denominator polynomial coefficients
396.5-12	***	s^2
25.68-05	***	s^1
1.000+00	***	s^0
0.000+00	***	s^2 — numerator polynomial coefficients
0.000+00	***	s^1
1.000+00	***	s^0

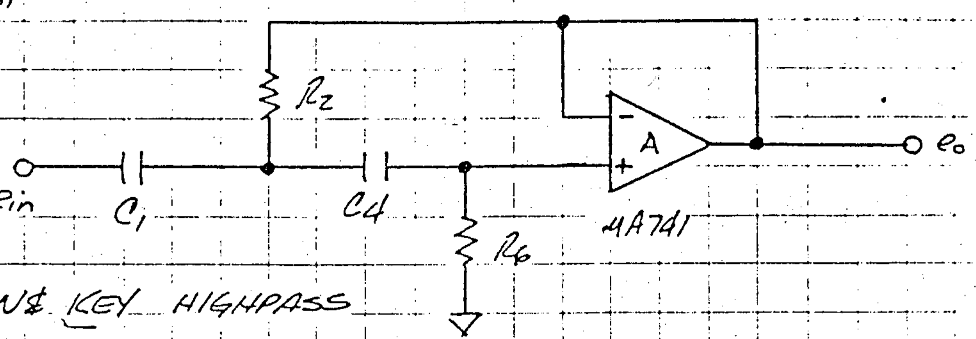
LOAD SECOND PROGRAM CARD

	GSBE	start pole & zera determination
-423.6+03	***	real pole — Denominator polynomial
41.36+03	***	complex conjugate pair
-34.15+03	***	
-41.98+03	***	
-34.15+03	***	
8.613+03	***	f_n — of complex conjugate pair
792.3-03	***	Q
1.000+00	***	K_o - constant in numerator.

this lowpass topology has all zoros o/ transmission at infinity

01859D Program Description II

Sketch(es)



SALLEN & KEY HIGHPASS

μA741
unity gain
characteristics

$A_0 = 1.0$

$f_0 = 500000$

$A = \frac{A_0}{1+\tau s}$

$\tau = \frac{1}{2\pi f_0}$

Sample Problem(s) Determine the transfer function of the active highpass Sallen and Key Topology as shown above. The filter is to meet the following characteristics

$f_0 = 10000$

$Q = 1/\sqrt{2}$

$C_1 = 10^{-9}$ Farad

$C_4 = C_1$

$R_2 = \frac{1}{4\pi f_0 Q C} = 11253\ \Omega$

$R_6 = 4Q^2 R_2 = 22507\ \Omega$

Solution(s) SEE NEXT PAGE FOR HP-97 PRINTOUT

Compare this topology to the multiple negative feedback topology shown on page 17/26. For unity gain applications, the Sallen and Key topology is the preferred topology both from a component count basis and an element sensitivity basis

Reference(s)

01859D Program Description

HP-97 PRINTOUT FOR ACTIVE HIGHPASS
USING SALLEN AND KEY TOPOLOGY

LOAD FIRST PROGRAM CARD

```
 -1.-09 GSBA   C1
 11253. GSBB   R2
     0. GSBC   y3 (open element)
 -1.-09 GSBD   C4
     0. GSBE   y5 (open element)
 22507. GSBc   R6

     1. ENT↑   Ao } unity gain uA741 characteristics
500000. GSBd   fo }

        GSBe
80.62-18  ***  s3 }
267.6-12  ***  s2 } Denominator polynomial coefficients
22.82-06  ***  s1 }
1.000+00  ***  s0 }

253.3-12  ***  s2 }
0.000+00  ***  s1 } Numerator polynomial coefficients
0.000+00  ***  s0 }
```

LOAD SECOND PROGRAM CARD

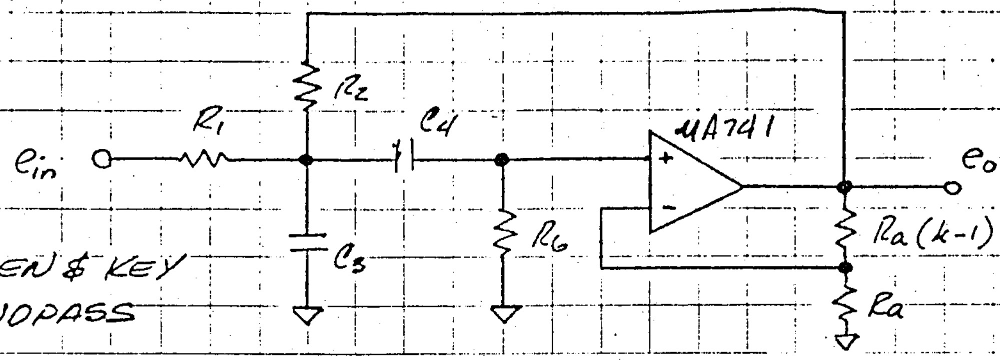
```
          GSBE start pole & zero location solution

-3.233+06  ***  real pole                          }
                                                   }
 44.40+03  ***  } complex                          }
-43.19+03  ***  } conjugate                        }  Denominator
                } pair                             }
-44.40+03  ***  }                                  }
-43.19+03  ***  }                                  }
                                                   }
 9.858+03  ***     } of complex                    }
 717.0-03  ***  Q  } conjugate pair                }

 971.7-03  ***  fo of numerator

 0.000+00  ***  } numerator zero locations
 0.000+00  ***  }
```

$$H(s) = \frac{K_o\, s^2\, (2\pi \cdot 9.858 \times 10^3)^2\, (3.233 \times 10^6)}{(s + 3.233 \times 10^6)(s + 43.19 \times 10^3 + j\,44.40 \times 10^3)(s + 43.19 \times 10^3 - j\,44.40 \times 10^3)}$$

or

$$H(s) = \frac{K_o\, s^2\, (2\pi \cdot 9.858 \times 10^3)^2\, (3.233 \times 10^6)}{(s + 3.233 \times 10^6)(s^2 + 86.38 \times 10^3 s + 11.58 \times 10^6)}$$

Sketch(es)



SALLEN & KEY
BANDPASS

Sample Problem(s) Determine the transfer function of the active band pass filter using the Sallen and Key topology. The filter specifications are:

then

$f_n = 10000$ Hz $\qquad R = R_1 = R_2 = R_6 = \dfrac{\sqrt{2}}{\omega_n C} = \dfrac{1}{\sqrt{2}\pi f_n C}$

$Q = 10 \quad (\alpha = 0.1)$

$C = 10^{-9}$ Farad. $\qquad R = 22.51\ K$

$$k = 4 - \frac{\sqrt{2}}{Q} = 3.859$$

gain at resonance, $K_o = \sqrt{8}\,Q - 1 = 27.28$

Solution(s) See next page for HP-97 printout

op amp "gain" $= +3.859$, $BW = \dfrac{500000}{3.859} = 129.6 \times 10^3$

NOTE: THIS IS NOT A PREFERRED TOPOLOGY FOR THE BANDPASS SECOND ORDER FUNCTION. SEE THE DELYIANNIS POSITIVE AND NEGATIVE FEEDBACK TOPOLOGY IN THE CITED REFERENCE (~~GEFFE~~)

Reference(s) 1) Principles of Active Filter Synthesis, Daryanani

2) Designers Guide to Active Filters by Phil Geffe (6 parts) Cahners publishing Co - EDN magazine 2/74, 3/74, 4/74, 5/74, 6/74, 7/74 editions (reprint available)

01859D Program Description

HP-97 PRINTOUT FOR ACTIVE BANDPASS FILTER USING SALLEN & KEY TOPOLOGY

LOAD FIRST PROGRAM CARD

```
22510.   GSBA  R1
22510.   GSBB  R2
-1.-05   GSBC  C3
-1.-05   GSBD  C4
0.       GSBE  open element
22510.   GSBa  R6

3.859    ENT↑  Ao }
129.6+03 GSBa  fo } UA741 characteristics

         GSBe  find transfer function

311.1-15  ***  s3 }
308.6-12  ***  s2 } Denominator polynomial
2.815-06  ***  s1 }
1.000+00  ***  s0 }

0.000+00  ***  s2 }
43.43-06  ***  s1 } Numerator polynomial
0.000+00  ***  s0 }
```

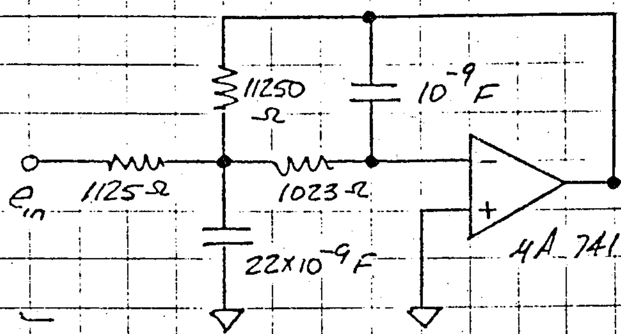
LOAD SECOND PROGRAM CARD

```
         GSBE  find pole locations

-986.1+03 ***  real pole

57.02+03  ***  complex      }
-2.935+03 ***  conjugate    }
                pair        }
-57.02+03 ***               }  Denominator
-2.935+03 ***               }
                            }
9.086+03  ***  fn } of complex  }
9.726+00  ***  Q  } conjugate   }
                    pair        }

24.12+00  ***  Ko            }
                             }  Numerator
0.000+00  ***  zero location }
```

$$H(s) = K_o \frac{\dfrac{s}{(2\pi \cdot 9086)(9.726)}}{\left(\dfrac{s}{986.1 \times 10^3}+1\right)\left(\dfrac{s^2}{(2\pi \cdot 9086)^2} + \dfrac{s}{(2\pi \cdot 9086)(9.726)} + 1\right)}$$

01859D **Program Description II**

Sketch(es)



MULTIPLE NEG FEEDBACK LOWPASS

$A_0 = -100000$ V/V
$f_0 = 5$ Hz

Sample Problem(s) Analyze the above multiple negative feedback lowpass active filter as designed by EE pac 08A program. This program assumes infinite op amp gain and gain-bandwidth.

The EE-08A pac inputs are:

$f_0 = 10000$ Hz
$A = 10$
$\alpha = 1/Q = \sqrt{2}$ (Butterworth response)
$C = .001\ \mu F = 10^{-9}$ Farad

Compare this filter analysis to the Sallen and Key lowpass topology.

Solution(s) See next page for HP-97 printout

Compare this topology to the Sallen & Key form shown on page 9/26

Reference(s)

Principles of Active Network Synthesis, Gobind Daryanani, John Wiley, Publisher, © 1976 by Bell Telephone Labs

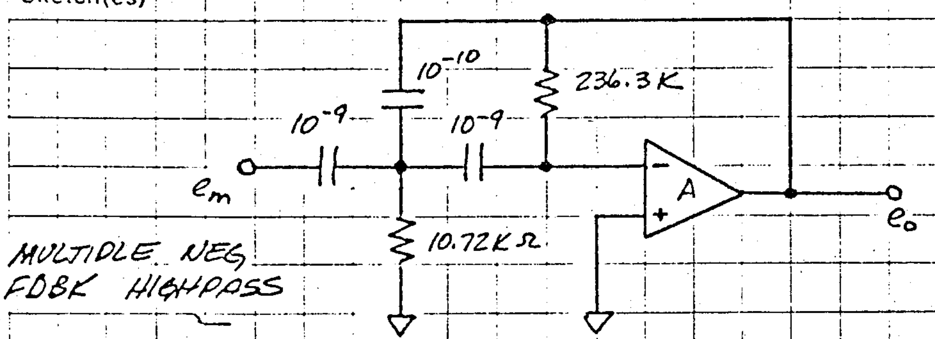
HP-97 PRINTOUT - LOWPASS
MULTIPLE NEGATIVE FEEDBACK

```
        1125. GSBA   R1
       11250. GSBB   R2
     -22.-09 GSBC    C3  (capacitor denoted by negative sign)
        1023. GSBD   R4
      -1.-09 GSBE    C5
          0. GSBa    C6 (open circuit)

   -100000. ENT↑     A0 }
         5. GSBd     f0 }  μA 741 characteristics

             GSBe    start analysis
   88.58-16  ***     s^3 }
   339.1-12  ***     s^2 }  Denominator polynomial coefficients
   26.00-06  ***     s^1 }
   1.000+00  ***     s^0 }

   0.000+00  ***     s^2 }
   0.000+00  ***     s^1 }  Numerator polynomial coefficients
  -9.999+00  ***     s^0 }

                LOAD SECOND PROGRAM CARD

             GSBE    find real roots of denominator

  -4.131+06  ***     real root                              }
                                                            }
   38.82+03  ***  }  complex                                }
  -38.70+03  ***  }  conjugate                              }  Denominator
                  }  pair                                   }  polynomial
  -38.82+03  ***  }                                         }
  -38.70+03  ***  }  notice bandedge                        }
                     shrinkage *                            }
   6.723+03  ***     fn }                                   }
   706.2-03  ***     Q  }  of complex conj pr               }


  -9.999+00  ***     D.C. gain term (K0)
```

* the bandedge is reduced because of the reduced loop gain at this frequency. $A_{excess} = \frac{GBP}{f_n \cdot A} = \frac{5 \times 10^5}{10^4 \cdot 10} = 5$

An excess gain of 5 does not approximate infinity, hence, the altered passband characteristics.

Sketch(es)



OP AMP IS μA 741

$A = A_0/1+\tau s$

$A_0 = -100000$

$f_0 = 5 Hz$

$\tau = \frac{1}{2\pi f_0}$

Sample Problem(s) Analyze the multiple negative feedback highpass active filter as designed by EE-08A program. For all designs, this program assumes infinite gain and gain-BW. This assumption is not valid with real op-amps which have finite gain and gain-bandwidth. When the loop gain is small, the cutoff frequency and Q will be displaced from the desired values.

The EE-08 inputs are

$f_0 = 10000$ Hz

$A = 10$

$\alpha = 1/Q = \sqrt{2}$ (Butterworth response)

$c = .001 \mu F = 10^{-9}$ Farad

Solution(s) SEE next page for HP-97 printout

Compare this topology to the Sallen & Key form shown on page 11/26

Reference(s)

HP-97 PRINTOUT - HIGHPASS
MULTIPLE NEGATIVE FEEDBACK

-1.-09 GSBA C_1 (denoted by negative sign)
-1.-10 GSBB C_2
10720. GSBC R_3
-1.-09 GSBD C_4
236300. GSBE R_5
0. GSBc C_6 (open circuit)

-100000. ENT↑ A_0 } µA 741 Characteristics
5. GSBd f_0 }

GSBe start analysis

886.9-18 *** s^3 } Denominator polynomial Coefficients
335.7-12 *** s^2 }
22.63-06 *** s^1 }
1.000+00 *** s^0 }

-2.533-09 *** s^2 } Numerator polynomial Coefficients
0.000+00 *** s^1 }
0.000+00 *** s^0 }

LOAD SECOND PROGRAM CARD

GSBE find polynomial roots *
-306.5+03 *** real root (f_{-3dB} = 48.8 KHz)

48.81+03 *** } complex conjugate pair
-35.99+03 *** }
-48.81+03 *** }
-35.99+03 *** }

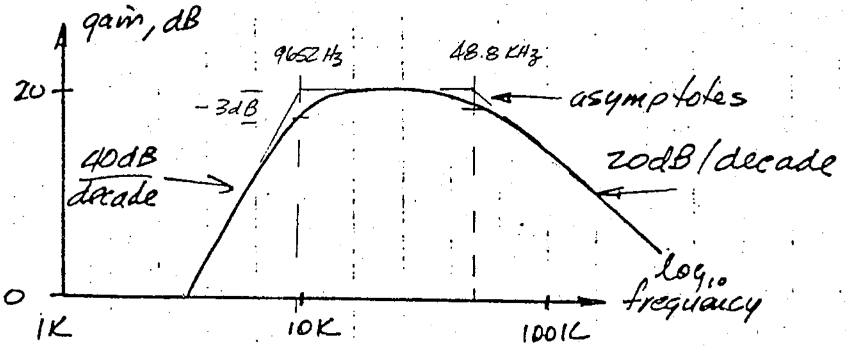
9.652+03 *** f_n } of complex conj pr
842.5-03 *** Q }

Denominator Polynomial

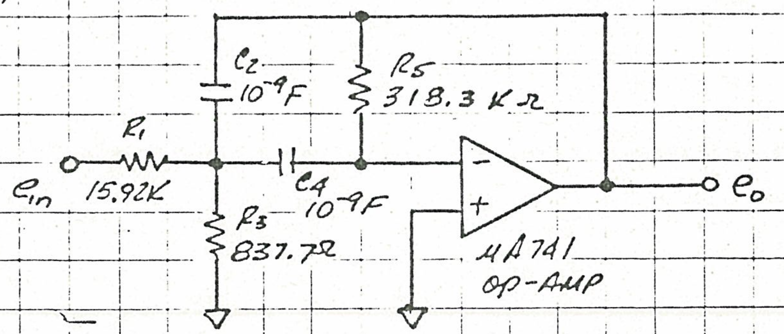
-9.317+00 *** Asymptotic gain (K_0 term)

0.000+00 *** } numerator zero locations
0.000+00 *** }

* Passband Characteristics



Sketch(es) MULTIPLE NEGATIVE FEEDBACK BANDPASS



$\mu A\ 741$ specs

$A_0 = -100000$

$f_0 = 5Hz$

$A = A_0/1+\tau s$

$\tau = \frac{1}{2\pi f_0}$

Sample Problem(s) Analyze the multiple feedback, infinite gain, bandpass active filter as designed by EE-08A using a real op-amp with finite gain and gain-bandwidth

The EE-08 inputs for the above design were:

$f_0 = 1000$

$A = 10$

$\alpha = 1/Q = 0.1$

$C = .001\ \mu F = 10^{-9}F$

Solution(s) See next page for HP-97 printout

Note: the Delyiannis positive and negative feedback resonator is a preferred topology for this application. both the component value range and the operational amplifier characteristics may be accomodated.

Reference(s) "Principles of Active Network Synthesis", Gobind Daryanani

"Designers Guide to Active Filters", Geffe

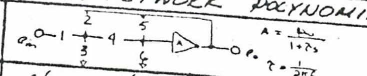
01859D Program Description

HP-97 PRINTOUT - BANDPASS
MULTIPLE NEGATIVE FEEDBACK TOPOLOGY

```
15.92+03 GSBA    R1
  -1.-09 GSBB    C2
   837.7 GSBC    R3
  -1.-09 GSBD    C4
316.3+03 GSBE    R5
      0. GSBa    C6 (open circuit)

-100000. ENT↑    Ao } uA 741 Characteristics
      5. GSBd    fo }

         GSBe    perform analysis

30.63-18   ***   s³ }
355.1-12   ***   s² } Denominator polynomial Coefficients
1.913-06   ***   s¹ }
1.000+00   ***   s⁰ }

0.000+00   ***   s² }
-15.91-06  ***   s¹ } Numerator polynomial Coefficients
0.000+00   ***   s0 }

         LOAD SECOND CARD

         GSBE    find polynomial roots

-4.400+06  ***   real root                    }
                                              }
53.04+03   ***   } complex                    }
-2.376+03  ***   } conjugate                  } Denominator
                 } pair                       } Polynomial
-53.04+03  ***   }                            }
-2.376+03  ***   }                            }
                                              }
8.450+03   ***   fn } of complex              }
11.17+00   ***   Q  } conjugate pair          }

-9.438+00  ***   midband gain (Ko)

0.000+00   ***   numerator zero locations.
```

$$H(s) = \frac{(-9.438)\,s\,(4.400\times10^6)\overbrace{(8.450\times10^3)^2(2\pi)^2}^{\omega_n^2}}{(s+4.400\times10^6)(s+53.04\times10^3+j\,2.376\times10^3)(s+53.04\times10^3-j\,2.376\times10^3)}$$

SECOND ORDER ACTIVE NETWORK POLYNOMIAL COEFFICIENTS

element 6		$A_0 \uparrow f_0$	START ANALYSIS	11/77 BKM
element 1	element 2	element 3	element 4	element 5

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS			OUTPUT DATA/UNITS
1	LOAD BOTH SIDES OF PROGRAM CARD					
2	ENTER ELEMENT 1					
	a) IF RESISTOR (Value ≠ 0)**					
	b) IF CAPACITOR (*Change Sign of Mantissa)	R	A			
		C	chs*	A		
3	ENTER ELEMENT 2					
	a) IF RESISTOR					
	b) IF CAPACITOR	R	B			
	c) IF NO ELEMENT PRESENT	C	chs*	B		
		∅	B			
4	ENTER ELEMENT 3					
	a) IF RESISTOR					
	b) IF CAPACITOR	R	C			
	c) IF NO ELEMENT PRESENT	C	chs*	C		
		∅	C			
5	ENTER ELEMENT 4					
	a) IF RESISTOR (VALUE ≠ 0)**					
	b) IF CAPACITOR	R	D			
		C	chs*	D		
6	ENTER ELEMENT 5					
	a) IF RESISTOR					
	b) IF CAPACITOR	R	E			
	c) IF NO ELEMENT	C	chs*	E		
		∅	E			
7	ENTER ELEMENT 6					
	a) IF RESISTOR					
	b) IF CAPACITOR	R	f	A		
	c) IF NO ELEMENT	C	chs*	f	A	
		∅	f	A		
	ENTER OP AMP PARAMETERS					
		Ao	↑			
		fo	f	D		
	START ANALYSIS					
			f	E		Denominator Coef's
	GO BACK AND CHANGE ANY ELEMENT					Numerator Coef's
	& RERUN 9, OR LOAD SECOND					
	CARD TO FIND DENOMINATOR POLE					
	LOCATIONS, fn, AND Q					

User Instructions

SECOND AND THIRD ORDER ROOT FINDER PGM.
USE WITH POLYNOMIAL COEFFICIENTS PROGRAM

1 | 2

START

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	LOAD BOTH SIDES OF PROGRAM CARD			
2	START ANALYSIS	E		
	a) IF 3^{rd} ORDER			
	i) IF 3 REAL ROOTS			$-a$
	$(s+a)(s+b)(s+c)$			$-b$
				$-c$
	ii) IF ONE REAL ROOT & COMPLEX			$-a$
	CONJUGATE PAIR $(s+\alpha+j\beta)(s+\alpha-j\beta)(s+a)$			
				$+j\beta$
				$-\alpha$
				$-j\beta$
				$-\alpha$
			OF SECOND	f_n
			ORDER PAIR	Q
	b) IF SECOND ORDER			
	i) IF 2 REAL ROOTS			$-a$
	$(s+a)(s+b)$			
				$-b$
				f_n
				Q
	ii) IF COMPLEX CONJUGATE PAIR			$j\beta$
	$(s+\alpha+j\beta)(s+\alpha-j\beta)$			$-\alpha$
				$-j\beta$
				$-\alpha$
				f_n
				Q

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	element 1 input
002	GSB8	23 08	
003	STO1	35 01	
004	RTN	24	
005	*LBLB	21 12	element 2 input
006	GSB8	23 08	
007	STO2	35 02	
008	RTN	24	
009	*LBLC	21 13	element 3 input
010	GSB8	23 08	
011	STO3	35 03	
012	RTN	24	
013	*LBLD	21 14	element 4 input
014	GSB8	23 08	
015	STO4	35 04	
016	RTN	24	
017	*LBLE	21 15	element 5 input
018	GSB8	23 08	
019	STO5	35 05	
020	RTN	24	
021	*LBLa	21 16 11	element 6 input
022	GSB8	23 08	
023	STO6	35 06	
024	RTN	24	
025	*LBLd	21 16 14	A_0 & f_0 input
026	Pi	16-24	
027	x	-35	calculate & store
028	2	02	$\tau = \frac{1}{2\pi f_0}$
029	x	-35	
030	1/X	52	
031	STO7	35 07	
032	R↓	-31	
033	STO0	35 00	Store A_0
034	RTN	24	
035	*LBLe	21 16 15	calculate transfer fcn
036	GSB1	23 01	calculate $y_1+y_2+y_3$
037	RCL1	36 01	
038	GSB0	23 00	
039	RCL2	36 02	
040	GSB0	23 00	
041	RCL3	36 03	
042	GSB2	23 02	
043	RCL4	36 04	calculate
044	GSB0	23 00	
045	RCL6	36 06	$y_4+y_6+y_5(1-A_0)$
046	GSB0	23 00	s^0 an s^1 terms
047	RCL5	36 05	
048	1	01	
049	RCL0	36 00	
050	-	-45	
051	STO9	35 09	
052	x	-35	
053	GSB2	23 02	
054	RCL4	36 04	
055	GSB0	23 00	
056	RCL5	36 05	

STEP	KEY ENTRY	KEY CODE	COMMENTS
057	GSB0	23 00	calculate & store s^2 & s^1 terms
058	RCL6	36 06	of y_4
059	GSB0	23 00	
060	GSB3	23 03	
061	SF0	16 21 00	calculate and store
062	GSB6	23 06	$(y_1+y_2+y_3)((y_4+y_6)(1+\tau_s)+y_5(1-A_0+\tau_s))$
063	GSB1	23 01	initialize index counter
064	RCL4	36 04	calculate & store y_4
065	GSB2	23 02	
066	RCL6	36 06	calculate and store
067	GSB0	23 00	
068	RCL5	36 05	$y_4(y_6+y_5(1-A_0)-A_0y_2)$
069	RCL9	36 09	s^0 and s^1 terms
070	x	-35	
071	GSB0	23 00	
072	RCL2	36 02	
073	RCL0	36 00	
074	x	-35	
075	CHS	-22	
076	GSB2	23 02	
077	RCL6	36 06	calculate and store
078	GSB0	23 00	s^2 and s^1 terms of
079	RCL5	36 05	
080	GSB0	23 00	$(y_5+y_6)(\tau_s)$
081	GSB3	23 03	
082	CF0	16 22 00	clg to indicate additional summing calculate and store
083	GSB6	23 06	$y_4(y_6(1+\tau_s)+y_5(1-A_0+\tau_s)-A_0y_2)$
084	P⇄S	16-51	and add to denominator
085	RCL0	36 00	normalize denominator
086	STOA	35 11	terms
087	ST÷1	35-24 01	$\frac{a_3}{a_0}s^3+\frac{a_2}{a_0}s^2+\frac{a_1}{a_0}s+1$
088	ST÷2	35-24 02	
089	ST÷3	35-24 03	
090	RCL3	36 03	recall normalized
091	PRTX	-14	denominator terms &
092	STOD	35 14	restore & print.
093	RCL2	36 02	$\frac{a_3}{a_0} \to R_D$
094	PRTX	-14	
095	STOC	35 13	$\frac{a_2}{a_0} \to R_C$
096	RCL1	36 01	
097	P⇄S	16-51	$\frac{a_1}{a_0} \to R_B$
098	PRTX	-14	
099	STOB	35 12	
100	1	01	$\frac{a_0}{a_0} = 1$
101	GSB7	23 07	SFLG0 to clear summing regs.
102	SF0	16 21 00	initialize reg & index reg
103	GSB1	23 01	
104	RCL1	36 01	calculate $y_1 = g+jb$
105	GSB2	23 02	
106	RCL4	36 04	calculate $y_2 = g+jb$
107	GSB2	23 02	
108	0	00	set y_2 s^2 term $=0$
109	STOi	35 45	
110	GSB6	23 06	calculate y_1 y_4
111	RCL0	36 00	
112	P⇄S	16-51	

REGISTERS

0	1	2	3	4	5	6	7	8	9
A_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	τ		

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
Σs^0 terms	Σs^1 terms	Σs^2 terms	Σs^3 terms		s_1^2	s_1^0	s_1^1	s_2^0	s_2^1

A	B	C	D	E	I
b_0	D_1	D_2	D_3		index

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	RCLA	36 11	normalize numerator terms
114	÷	-24	
115	STx2	35-35 02	
116	STx1	35-35 01	
117	STx0	35-35 00	
118	RCL2	36 02	Recall and print numerator terms.
119	PRTX	-14	
120	RCL1	36 01	
121	PRTX	-14	
122	RCL0	36 00	
123	P⇄S	16-51	
124	*LBL7	21 07	
125	PRTX	-14	
126	SPC	16-11	
127	RTN	24	
128	*LBL0	21 00	recall y element, decide whether it is conductance or capacitance. if conductance, form Σg in stack. if capacitance, form Σ sC in R(i)
129	ENT↑	-21	
130	ABS	16 31	
131	EEX	-23	
132	3	03	
133	0	00	
134	X>Y?	16-34	
135	GTO9	22 09	
136	R↓	-31	
137	R↓	-31	
138	EEX	-23	
139	6	06	
140	0	00	
141	÷	-24	
142	+	-55	
143	RTN	24	
144	*LBL9	21 09	capacitance term summation
145	R↓	-31	
146	R↓	-31	
147	ST+i	35-55 45	
148	R↓	-31	
149	RTN	24	
150	*LBL1	21 01	initialize $R_I = 19$ & $R_{19} = 0$
151	1	01	
152	9	09	
153	STOI	35 46	
154	0	00	
155	STOi	35 45	
156	RTN	24	
157	*LBL2	21 02	finish y element decision and store
158	GSB0	23 00	
159	DSZI	16 25 46	
160	STOi	35 45	
161	DSZI	16 25 46	
162	0	00	
163	STOi	35 45	
164	RTN	24	
165	*LBL3	21 03	multiply by ∂s to form s^2 and additional s' terms and store away
166	RCL7	36 07	
167	P⇄S	16-51	
168	STx5	35-35 05	
169	x	-35	
170	ST+7	35-55 07	
171	P⇄S	16-51	
172	RTN	24	
173	*LBL4	21 04	subroutine used in polynomial multiplication routine
174	x	-35	
175	ST+i	35-55 45	
176	RTN	24	
177	*LBL5	21 05	initialization subroutine used in polynomial multiplication
178	STOI	35 46	
179	0	00	
180	F0?	16 23 00	
181	STOi	35 45	
182	RTN	24	
183	*LBL6	21 06	polynomial multiplication
184	P⇄S	16-51	
185	0	00	
186	GSB5	23 05	s^0 term
187	RCL6	36 06	
188	RCL8	36 08	
189	GSB4	23 04	
190	1	01	s^1 term
191	GSB5	23 05	
192	RCL7	36 07	
193	RCL8	36 08	
194	GSB4	23 04	
195	RCL6	36 06	
196	RCL9	36 09	
197	GSB4	23 04	
198	2	02	s^2 term
199	GSB5	23 05	
200	RCL7	36 07	
201	RCL9	36 09	
202	GSB4	23 04	
203	RCL5	36 05	
204	RCL8	36 08	
205	GSB4	23 04	
206	3	03	s^3 term
207	GSB5	23 05	
208	RCL5	36 05	
209	RCL9	36 09	
210	GSB4	23 04	
211	P⇄S	16-51	
212	RTN	24	
213	*LBL8	21 08	input subroutine. if element is negative, it is a capacitor, chs & store.
214	X>0?	16-44	
215	GTO8	22 08	
216	CHS	-22	
217	RTN	24	
218	*LBL8	21 08	if element value is positive, it is a resistor, multiply by 10^{60} and store
219	1/X	52	
220	EEX	-23	
221	6	06	
222	0	00	
223	x	-35	
224	RTN	24	

LABELS

A enter y_1	B enter y_2	C enter y_3	D enter y_4	E enter y_5
a enter y_6	b	c	d enter A0 ↑ f0	e START ANALYSIS
0 separate Σsom R & sC	1 initialize loop (19→I)	2 terminate loop	3 calculate s^2&s' terms	4 summation subroutine
5 initialization routine for sum	6 summation routine	7 prt & space	8 input subroutine	9

FLAGS

0	used
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	■	DEG	■	FIX	☐
1	☐	☐	GRAD	☐	SCI	☐
2	☐	☐	RAD	☐	ENG	■
3	☐	☐			n 3	

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLE	21 15	Start analysis
002	RCLD	36 14	if s^3 coef is
003	X≠0?	16-42	zero, store
004	GTO0	22 00	remaining coefficients
005	RCLC	36 13	in proper locations
006	P⇄S	16-51	for second order
007	STO9	35 09	solution and then
008	RCLB	36 12	go to second order
009	STO8	35 08	solution routine
010	1	01	
011	STO7	35 07	
012	GTO2	22 02	
013	*LBL0	21 00	3rd order solution
014	1/X	52	routine.
015	.	-62	calculate initial
016	6	06	guess for real root
017	Y^X	31	
018	CHS	-22	
019	STO8	35 08	
020	*LBL1	21 01	newton-Raphson
021	RCL8	36 08	(modified) loop start
022	ENT↑	-21	
023	ENT↑	-21	
024	ENT↑	-21	
025	RCLD	36 14	Calculate
026	×	-35	$f(x)$
027	RCLC	36 13	
028	+	-55	
029	×	-35	
030	RCLB	36 12	
031	+	-55	
032	×	-35	
033	1	01	
034	+	-55	
035	STO9	35 09	
036	CLX	-51	
037	RCLD	36 14	calculate $f'(x)$
038	3	03	
039	×	-35	
040	×	-35	
041	RCLC	36 13	
042	2	02	
043	×	-35	
044	+	-55	
045	×	-35	
046	RCLB	36 12	
047	+	-55	
048	X=0?	16-43	$f'(x_n)=0$ escape
049	GTO0	22 00	
050	ST÷9	35-24 09	calculate $f(x_n)/f'(x_n)$
051	RCL8	36 08	$f(x)_{n-1}=0$ escape
052	X=0?	16-43	
053	GTO0	22 00	
054	RCL9	36 09	

STEP	KEY ENTRY	KEY CODE	COMMENTS
055	X=0?	16-43	$f(x)_n=0$ escape
056	GTO0	22 00	
057	÷	-24	calculate $\frac{f(x)_{n-1}}{f(x)_n}=k_1$
058	ABS	16 31	
059	ENT↑	-21	
060	1/X	52	calc $\lvert k\rvert+\frac{1}{\lvert k\rvert}=k_2$
061	+	-55	
062	3	03	if $k_2>3.4$, go
063	.	-62	to normal newton-
064	4	04	raphson solution,
065	X≤Y?	16-35	otherwise
066	GTO0	22 00	
067	RCL9	36 09	accelerated newton
068	2	02	raphson solution.
069	.	-62	(increment)
070	5	05	
071	×	-35	
072	GTO1	22 01	
073	*LBL0	21 00	normal Newton-Raphson
074	RCL9	36 09	increment
075	*LBL1	21 01	if $\lvert S_n-S_{n+1}\rvert>10^{-8}S_n$
076	ST-8	35-45 08	
077	ABS	16 31	
078	RCL8	36 08	
079	EEX	-23	
080	8	08	
081	÷	-24	
082	ABS	16 31	
083	X≤Y?	16-35	
084	GTO1	22 01	
085	RCL8	36 08	recall and print
086	GSB9	23 09	real 3rd order root
087	P⇄S	16-51	
088	STO6	35 06	use polynomial
089	RCLD	36 14	division to remove
090	STO9	35 09	real root in
091	×	-35	3rd order polynomial
092	RCLC	36 13	to reduce to
093	+	-55	2ND order polynomial
094	STO8	35 08	
095	RCL6	36 06	
096	×	-35	
097	RCLB	36 12	
098	+	-55	
099	STO7	35 07	
100	ST÷7	35-24 07	normalize resulting
101	ST÷8	35-24 08	second order
102	ST÷9	35-24 09	polynomial
110			

REGISTERS

0	1	2	3	4	5	6	7	8	9
A_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	?	used	used

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
ΣS^0_n	ΣS^1_n	ΣS^2_n			used	σ	c, c/a	b_3, $b/2a$	a

A	B	C	D	E	I
b_0	D_1	D_2	D_3		

STEP	KEY ENTRY	KEY CODE	COMMENTS
103	*LBL2	21 02	second order polynomial solution using quadratic formula
104	RCL9	36 09	
105	ST÷7	35-24 07	
106	2	02	
107	x	-35	
108	ST÷8	35-24 08	
109	RCL8	36 08	
110	X²	53	
111	RCL7	36 07	
112	-	-45	
113	X<0?	16-45	Imaginary soln?
114	GTO3	22 03	
115	√X	54	real root solution
116	STO5	35 05	
117	RCL8	36 08	
118	-	-45	
119	GSB9	23 09	print real root #1
120	RCL5	36 05	
121	RCL8	36 08	
122	+	-55	
123	CHS	-22	
124	GTO9	22 09	print real root #2
125	*LBL3	21 03	second order imaginary root solution
126	CHS	-22	
127	√X	54	
128	STO5	35 05	
129	PRTX	-14	print complex conjugate root #1
130	RCL8	36 08	
131	CHS	-22	
132	GSB9	23 09	
133	RCL5	36 05	print complex conjugate root #2
134	CHS	-22	
135	PRTX	-14	
136	X⇄Y	-41	
137	GSB9	23 09	
138	RCL7	36 07	calculate natural frequency, fn of complex conjugate pair
139	√X	54	
140	2	02	
141	÷	-24	

STEP	KEY ENTRY	KEY CODE	COMMENTS
142	STO5	35 05	Calculate fn (con't)
143	Pi	16-24	
144	÷	-24	
145	PRTX	-14	
146	RCL5	36 05	calculate Q of second order pair
147	RCL8	36 08	
148	÷	-24	
149	GSB9	23 09	
150	RCL9	36 09	restore second order coefficients to normalized form
151	ST×7	35-35 07	
152	2	02	
153	x	-35	
154	ST×8	35-35 08	
155	SPC	16-11	
156	SPC	16-11	
157	RCL2	36 02	Is numerator polynomial highpass form?
158	X≠0?	16-42	
159	GTO0	22 00	
160	RCL0	36 00	Is numerator polynomial lowpass form
161	X≠0?	16-42	
162	GTO8	22 08	
163	RCL1	36 01	∴ numerator polynomial is bandpass form. - calculate and print gain at resonance
164	RCL8	36 08	
165	÷	-24	
166	GSB9	23 09	
167	0	00	
168	GTO8	22 08	
169	*LBL0	21 00	high pass. calculate asymptotic gain
170	RCL9	36 09	
171	÷	-24	
172	GSB9	23 09	
173	0	00	
174	ENT↑	-21	
175	PRTX	-14	
176	*LBL8	21 08	P⇄S, print & space
177	P⇄S	16-51	
178	*LBL9	21 09	print and space
179	PRTX	-14	
180	SPC	16-11	
181	RTN	24	

LABELS

A	B	C	D	E START ANALYSIS
a	b	c	d	e
0 used	1 used	2 2ND order solution start	3 second order imaj subroutine	4
5	6	7	8	9

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☐	DEG	■	FIX	☐
1	☐	☐	GRAD	☐	SCI	☐
2	☐	☐	RAD	☐	ENG	■
3	☐	☐			n	3