PROGRAM DESCRIPTION I

Program Title LINEAR PROGRAMMING USING THE SIMPLEX ALGORITHM

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Program Description, Equations, Variables The program may be based to solve classical maximize/minimize problems of the linear programming format using the "Simplex" or "Z phase" approach. The program optimizes a set of inequalities subject to an objective function that has been formulated by the user according to rules and conventions in this document. The program repetitively finds the best pivot, transforms the matrix according to a pre-set strategy and outputs the solution(s) - if they exist. Available to the user are several options, including editing, printing of each iteration, shadow prices and auto size checking. In addition, the program operates in a 'user-friendly' manner and is quite fast for a calculator .

OVERVIEW: If all inequality constraints are graphed, the intersection

Necessary Accessories 2 memory modules (minimum configuration), printer optional

Operating Limits and Warnings Program assumes that all variables will be greater than or equal to zero in the solution. A "NON EXISTENT" display implies an unbound solution or infeasible tableau formulation; note that no more than 10 constraints and/or 10 variables can be handled by the program.

Reference(s) (1) HILLIER, F.S. and LIEBERMAN, G.J. Introduction to Operations Research, HOLDENDAY 1972.

(2) HP USERS LIBRARY programmes 3195D and 3058D

(3) MBA COURSE NOTES, UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG.

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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of constraints constitute "extreme points" in this "basic feasible area of possible solutions. Linear programming theory states that an optimum solution will lie at one of these extremities. By passing from extremity to extremity, according to a programmed strategy, the HP 41 finds that optimum point, at one of the extremities, where the objective function is either MAXIMIZED or MINIMIZED, depending on the user's specifications.

1. Conventions adopted in this program.
By way of explanation, a Simplex Matrix may be of the following form, after formulation by the user:-

Maximize $Z = X1 + 2X2 - 3X3$ (objective function)
$X1 + 6X2 = 50$ (constraint 1)

Subject to $X1 + X2 + X3 \leq 600$ (constraint 2)
$2X1 - X3 \geq 100$ (constraint 3)
$5X1 + 4X2 \leq 250$ (constraint 4)

This data must be translated to a recognizable format for use by the program. To do this, all "$\geq$" signs must be mathematically translated to "$\leq$" signs. To do this, multiply both sides of the "$\geq$" inequality by -1. All "=" constraints must be converted into 2 new constraints, i.e. one "$\geq$" and one "$\leq$". The "$\geq$" constraint of the pair is then translated into a "$\leq$" constraint by multiplying both sides by -1, as above.

For the example shown above, the first constraint is converted to 2 new constraints, as an "=" constraint cannot be handled by the program.

$X1 + 6X2 = 50$ is mathematically equivalent to:
$X1 + 6X2 \leq 50$ (constraint 1a)
and $X1 + 6X2 \geq 50$ (constraint 1b)

Constraint no. 2 in the example is fine in it's present form.
Constraint no. 3 must be translated to a "$\leq$" format, as will the new constraint 1b above.

$X1 + 6X2 \geq 50$ is equivalent to $-X1 - 6X2 \leq -50$
(multiply both sides of inequality by -1) and similarly
$2X1 - X3 \geq 100$ is equivalent to $-2X1 + X3 \leq -100$

Constraint no. 4 is also acceptable in its present form.
Therefore the "translated" set of inequalities become:

$X1 + 6X2 \leq 50$ (constraint 1)
$-X1 - 6X2 \leq -50$ (constraint 2)
$X1 + X2 + X3 \leq 600$ (constraint 3)
$-2X1 + X3 \leq -100$ (constraint 4)
$5X1 + 4X2 \leq 250$ (constraint 5)

In addition, the program requires the user to alter the original objective function to use the strategies inherent in the program. This is achieved by simply multiplying all elements in the objective function by -1. As "Z" (the value of the objective function) is initially set to zero, the converted objective function, ready for

input, looks as follows (using the above example):

$$-X1 - 2X2 + 3X3 = 0$$

To input to the HP 41, the objective function should be the last row input. Combining the constraints and objective function prior to input into the HP 41, we obtain a Matrix (called "A") of all coefficients, and we create "non basic" variables (X1, X2 and X3 in the above example), and "basic" variables (constraints 1 to 5 in the above example), shown in exhibit 1.

NON BASIC VARIABLE NAME	X1	X2	X3	Q	BASIC VARIABLE NAME
	1	6	0	50	Y1
	-1	-6	0	-50	Y2
	1	1	1	600	Y3
	-2	0	1	-100	Y4
	5	0	4	250	Y5
OBJECTIVE FUNCTION (ROW "W")	-1	-2	3	0	Z

EXHIBIT 1: MATRIX A

Column "Q", shown above, is the name given (for convenience) to signify the quantity of the constriants. Row "W" is similarly named to signify the objective function (last) row. "Z" is the current value of the objective function, initially set to zero.

2. Operation of the Two Phase Approach: Phase I.
The first phase in optimization is to check if any negative elements exist in column Q, signifying $\geq$ constraints. The first phase inspects this column and finds the largest negative value (in magnitude). If no negative value is found, the program passes to phase 2. The largest negative value in column Q gives the pivot row location. The calculator now looks along this row to find the pivot column location. It decides on this location by dividing each negative element in the pivot row into the corresponding element in row W. The largest quotient from this operation determines the pivot column location, and the pivotal procedure (explained in 4 below) is initiated.

After pivotal reduction, the phase I procedure is repeated until no negative elements remain in column Q. At this stage phase I is completed.

3. Operation of the Two Phase Approach: Phase II
In the second phase, Row W is inspected. If the problem is a MINIMIZE one, the largest positive element (in magnitude) gives the program its pivot column. Under MAXIMIZATION, the largest negative element, in magnitude, gives the program its pivot column location. If no column element is found, then an optimum, i.e. MIN/MAX, situation has been attained and the program outputs the results.

Having found the column element, the program looks for the pivotal row location by dividing every positive element in the pivot column into the corresponding element in column Q. The lowest

quotient thus obtained represents the pivot row location and reduction commences. If all elements in the pivot column are negative at this stage, a pivot row cannot be found for that column, so the next best column element is chosen and the procedure repeated, and so on. If no pivot can be found after inspecting each possibility, then a suboptimal solution is output.

4. The Pivotal Procedure

Assuming we have chosen a pivot "P" and have elements "C(i)" representing pivot column elements, "R(i)" representing pivot row elements and "E(i)" representing all elements other than pivot row or column elements, our matrix, prior to pivotal reduction, looks as follows in a 4 x 4 matrix:

N1	N2	N3	Q	
E1	C1	E2	E3	B1
R1	P	R2	R3	B2
E4	C2	E5	E6	B3
E7	C3	E8	E9	

BEFORE PIVOTAL REDUCTION

$$\begin{array}{|c|c|c|c|c|}
\hline
N1 & B2 & N3 & Q & \\
\hline
\left\{E1-\frac{C1R1}{P}\right\} & \left\{\frac{-C1}{P}\right\} & \left\{E2-\frac{C2R3}{P}\right\} & \left\{E3-\frac{C1R3}{P}\right\} & B1 \\
\hline
\frac{(R1)}{P} & \left(1/_P\right) & \frac{(R2)}{P} & \frac{(R3)}{P} & N2 \\
\hline
\left\{E4-\frac{R1C2}{P}\right\} & \left\{\frac{-C2}{P}\right\} & \left\{E5-\frac{C2R2}{P}\right\} & \left\{E6-\frac{C2R3}{P}\right\} & B3 \\
\hline
\left\{E7-\frac{R1C3}{P}\right\} & \left\{\frac{-C3}{P}\right\} & \left\{E8-\frac{C3R2}{P}\right\} & \left\{E9-\frac{C3R3}{P}\right\} & \\
\hline
\end{array}$$

AFTER PIVOTAL REDUCTION

Where N1, N2 and N3 are non basic variable names, B1, B2 and B3 ar the basic variable names. Note, we have exchanged N2 and B2, making N2 basic for this iteration.

Briefly, the procedure is to replace each element, except the pivot, in the pivot row by itself divided by the pivot; each element in the pivot column by itself divided by the pivot and multiplied by -1. Every other element is replaced by itself less the product of the adjacent pivot column element and pivot row element divided by the pivot. The pivot is replaced by its inverse. In this simplified form, a GAUSSIAN transformation is achieved via matrix manipulation. This method is well suited to an electronic calculator because of time efficiencies.

5. Example of Choosing a Pivot.

In the example on page 3, the phase one approach would be to inspect column Q for the largest negative element, if any. In the example, the program finds this element to be "- 100", thus the pivot row is the row containing the "- 100", row Y4. To find the corresponding column element, the calculator divides each negative element in this row into the corresponding element of row W. There is only one such operation in our example (vis) -1 divided by -2; thus the pivot column is the column containing the "-1", given by X1. The pivot is the "-2" element, and reduction can proceed. Note that Y4 and X1 will be exchanged, as one becomes non basic (the "leaving" variable) and one becomes basic (the "entering" variable).

(CONTINUATION PAGE)

After transformation, the matrix looks as follows (assume we are maximizing)

NON BASIC VARIABLE	Y4	X2	X3	Q	BASIC VARIABLE
	0,5	6	0,5	0	Y1
	-0,5	-6	-0,5	0	Y2
	0,5	1	1,5	550	Y3
	-0,5	0	-0,5	50	X1
	2,5	0	6,5	0	Y5
	-0,5	-2	2,5	50	Z

EXHIBIT 2: MATRIX "A" AFTER FIRST TRANSFORMATION

As column Q is now all positive elements, Second phase operates. As we are maximizing, the calculator looks at row W and chooses "-2" as the largest (magnitude) negative element. Dividing each positive element in that column into the corresponding value in column Z, we find only 2 elements qualify (vis) 0/6 and 550/1. Clearly 0/6 is the smallest, so "6" will be the pivot, bringing in X2 and taking Y1 out of the basis.

The process continues until all elements in row W are positive. (This occurs after the 3rd transformation when Z = 50, X1 = 50)

IMPORTANT NOTE:
The minimum "size" for operation of this program is 021
Thereafter every element in your matrix requires an increment of 1 to that size. Therefore for a matrix of 5 rows, 5 columns, the size is

021 + (5 x 5) = 046

The calculator has, however, a built-in routine for computing and prompting the size required. If at any stage in the program the calculator should prompt with

"XEQ SIZE nnn", then do the following

PRESS [XEQ] [ALPHA] S I Z E [ALPHA], followed by the three prompted numbers. Then press [R/S] to continue operation where you left off. You do not have to restart the program.

Note Also that if a printer is in the system, it should be set to [MAN] position for this programme!

Sample Problem (Sketch if Desired)

PROBLEM ONE

Maximize

$15X1 - 12X2 - 4X3 + 3X4 = Z$

Subject to

$8X1 - 3X2 - X3 + X4 \leq 3$

$-6X1 - X2 + X3 - X4 \geq 6$

Formulated Matrix "A"

X1	X2	X3	X4	Q	
8	-3	-1	1	3	Y1
6	1	-1	1	-6	Y2
-15	12	4	-3	0	Z

NOTE: SIZE SHOULD BE SET TO
21 + (3 X 5) = 036

SOLUTION: (ASSUME NO PRINTER)

Input	Function	Display	Comments
			*LOAD CARDS, SET SIZE, ETC.
	XE0 "LP"		*INITIALIZE REGISTERS
		MAXIMIZE?	*ANSWER "YES" TO THIS PROMPT BY "Y"
Y	R/S		
		PRINT TABS?	*ANSWER "NO" TO THIS TAB BY "N"
N	R/S		
		ROWS?	*INPUT NO. OF ROWS = 3
3	R/S	COLS?	*INPUT NO. OF COLS = 5
5	R/S	A 1.1?	*INPUT 1st MATRIX ELEMENT SECOND, ETC.
8	R/S	A 1.2?	
3 CHS	R/S	A 1.3?	
1 CHS	R/S	A 1.4?	
1	R/S	A 1.5?	
3	R/S	A 2.1?	
6	R/S	.	
.	.	.	
.	.	.	
.	.	A 3.4?	
3 CHS	R/S	A 3.5?	
0	R/S	CHECK?	*ANSWER "NO" BY "N"
N	R/S	TABLEAU 1	*FIRST TAB BEING PROCESSED
		X3 = 6.00	*SOLUTION AFTER 1 TRANSFORMATION
	R/S	Z = -24.00	
	R/S	Y2 = 4.00	

ANSWER: OPTIMAL SOLUTION IS
X1, X2, X4 = 0
X3 = 6 UNITS
MAX. Z = -24 AND
SHADOW PRICE OF
SECOND CONSTRAINT
Y2 = 4.00

Sample Problem (Sketch if Desired)

PROBLEM TWO

Minimize

$2X1 + 3X2 - 2X3 = Z$

Subject to

$4X1 - X2 + X3 \geq 10$
$3X1 - X3 \geq 4$
$2X1 + 2X2 \geq 3$

Formulated matrix "A"

X1	X2	X3	Q	
-4	1	-1	-10	Y1
-3	0	1	-4	Y2
-2	-2	0	-3	Y3
-2	-3	2	0	Z

NOTE: SIZE SHOULD BE SET TO
21 + (4 X 4) = 037

SOLUTION: (ASSUMES NO PRINTER)

Input	Function	Display	Comments
			*LOAD CARDS, SET SIZE, ETC.
	XEQ "LP"		*INITIALIZE
		MAXIMIZE?	*ANSWER "N" FOR "NO"
N	R/S	PRINT TABS?	*ANSER "N" FOR "NO"
N	R/S	ROWS?	*INPUT MATRIX ELEMENTS IN
*→ 4	R/S	COLS?	ORDER AS PROMPTED
4 CHS	R/S	A 1.2?	
1	R/S	A 1.3?	
1 CHS	R/S	A 1.4?	
10 CHS	R/S	A 2.1?	
3 CHS	R/S	.	
.	.	.	
.	.	.	
.	.	A 4.3?	
2	R/S	A 4.4?	
0	R/S	CHECK?	*ANSWER "N" FOR "NO"
N	R/S		
		TABLEAU 1	*ITERATION COUNTER
		TABLEAU 2	
		X3 = 2.00	*SOLUTION AFTER 2
	R/S	X1 = 2.00	TABLEAU TRANSFORMATIONS=
	R/S	Z = 0.00	
	R/S	Y2 = -1.43	X2 = 0, X3 = 2.00, X1 = 2.00
		Y1 = 0.57	
			AND MINIMUM Z = 0.00
			SHADOW PRICE OF 2nd
			CONSTRAINT Y2 = -1.43
← INSERT:			AND OF 1st CONSTRAINT
		COLS?	Y1 = 0.57
4	R/S	A 1.1?	

PROGRAM DESCRIPTION II

Sample Problem (Sketch if Desired)

PROBLEM THREE

Maximize

X1 + 3X2 - X3 = Z

Subect to

X1 + X2 + X3 $\leq$ 12
X1 = 3
3X1 - X3 $\geq$ 4
2X1 + 2X2 $\geq$ 4

NOTE: 2nd CONSTRAINT MUST BE TRANSLATED TO 2 INEQUALITIES, vis X1 $\geq$ 3 AND X1 $\leq$ 3

Formulated Matrix "A"

X1	X2	X3	Q	
1	1	1	12	Y1
1	0	0	3	Y2
-1	0	0	-3	Y3
-3	0	1	-4	Y4
-2	-2	0	-4	Y5
-2	-3	1	0	Z

NOTE: SIZE SHOULD BE SET TO 21 + (4 x 6) = 045 FOR THIS PROBLEM

SOLUTION: (ASSUMES A PRINTER IS CONNECTED - SEE PRINTED OUTPUT OVERLEAF)

Input	Function	Display	Comments
			*LOAD CARDS, SET SIZE, ETC.
	XEQ "LP"		*INITIALIZE, AS BEFORE
		MAXIMIZE?	*ANSWER "Y" FOR "YES"
Y	R/S	PRINT TABS?	*ANSWER "Y" FOR "YES"
Y	R/S	ROWS?	*NO. OF ROWS = 6
6	R/S	COLS?	*NO. OF COLS = 4
4	R/S	A 1.1?	INPUT ELEMENT 1,1 OF A (AND THE REST IN SEQUENCE)
1	R/S	A 1.2?	
1	R/S	A 1.3?	
.	.	.	
.	.	.	
.	.	.	
.	.	A 6.3?	
1	R/S	A 6.4?	
0	R/S	CHECK?	*ANSWER "N" FOR NO"
N	R/S		
		TABLEAU 1	(TABLEAU CURRENTLY BEING
		TABLEAU 2	PROCESSED - SEE PRINTER
		TABLEAU 3	OUTPUT ON NEXT PAGE)
		TABLEAU 4	
		X2 = 9.00	OPTIMAL SOLUTION AFTER
		X1 = 3.00	4 ITERATIONS (SEE PRINTOUT
		Z = 33.00	OVERLEAF)
		Y1 = 3.00	X3 = 0.00 UNITS
		Y3 = 1.00	X2 = 9.00 UNITS
			X1 = 3.00 UNITS
			MAXIMUM Z = 33.00
			SHADOW PRICE OF THIRD CONSTRAINT Y3 = 1.00 & FIRST CONSTRAINT Y1 = 3.00

SAMPLE PINTER OUTPUT - PROBLEM 3

KEY: <P> REFERS TO THE PIVOT CHOSEN.

R IS THE ROW NUMBER; R6 IN THIS EXAMPLE IS ROW "W"

TABLEAU 1

R 1		1.00
		1.00
		1.00
		12.00
R 2		1.00
		0.00
		0.00
		3.00
R 3		-1.00
		0.00
		0.00
		-3.00
R 4	<P>	-3.00
		0.00
		1.00
		-4.00
R 5		-2.00
		-2.00
		0.00
		-4.00
R 6		-2.00
		-3.00
		1.00
		0.00

TABLEAU 2

R 1		0.33
		1.00
		1.33
		10.67
R 2		0.33
		0.00
		0.33
		1.67
R 3		-0.33
		0.00
	<P>	-0.33
		-1.67
R 4		-0.33
		0.00
		-0.33
		1.33
R 5		-0.67
		-2.00
		-0.67
		-1.33
R 6		-0.67
		-3.00
		0.33
		2.67

TABLEAU 3

R 1		-1.00
	<P>	1.00
		4.00
		4.00
R 2		0.00
		0.00
		1.00
		0.00
R 3		1.00
		0.00
		-3.00
		5.00
R 4		0.00
		0.00
		-1.00
		3.00
R 5		0.00
		-2.00
		-2.00
		2.00
R 6		-1.00
		-3.00
		1.00
		1.00

TABLEAU 4

R 1		-1.00
		1.00
		4.00
		4.00
R 2		0.00
		0.00
		1.00
		0.00
R 3	<P>	1.00
		0.00
		-3.00
		5.00
R 4		0.00
		0.00
		-1.00
		3.00
R 5		-2.00
		2.00
		6.00
		10.00
R 6		-4.00
		3.00
		13.00
		13.00

TABLEAU END

R 1	1.00
	1.00
	1.00
	9.00
R 2	0.00
	0.00
	1.00
	0.00
R 3	1.00
	0.00
	-3.00
	5.00
R 4	0.00
	0.00
	-1.00
	3.00
R 5	2.00
	2.00
	1.00-09
	20.00
R 6	4.00
	3.00
	1.00
	33.00

SOLUTION

X2=9.00
X1=3.00

Z=33.00
Y1=3.00
Y3=1.00

00320C

USER INSTRUCTIONS

00320C

SIZE: (HP-41C) 021

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	LOAD CARDS (8 SIDES)			
2	INITIALIZE & START PROGRAM		XEQ "LP"	MAXIMIZE?
3	TO MAXIMIZE, PRESS "Y" FOR "YES"			
	TO MINIMIZE, PRESS "N" FOR "NO"	"Y"or"N"	R/S	PRINT TABS?
4	PRINT TABLEAU FOR EACH ITERATION,			
	SELECT "Y" OR "N" AS BEFORE	"Y"or"N"	R/S	ROWS?
5	INPUT NO.OF ROWS IN MATRIX "A"	i	R/S	COLS?
6	INPUT NO.OF COLUMNS IN MATRIX "A"	j	R/S	A 1,1?
7	INPUT FIRST ELEMENT OF MATRIX	A(1,1)	R/S	A i,j?
8	REPEAT THIS STEP UNTIL ALL	A(i,j)	R/S	A i,j+1?
	ELEMENTS OF MATRIX "A" HAVE BEEN		:	
	INPUT		R/S	CHECK?
9	TO ENTER CHECK ROUTINE PRESS "Y"			
	FOR "YES" IF YOU DO NOT WISH TO	Y	R/S	
	CHECK INPUT PRESS "N" FOR "NO" &	N	R/S	A 1,1=( )?
	GO TO STEP 12			
10	CHECK ELEMENT OF MATRIX. IF			
	CORRECT PRESS R/S WITHOUT INPUT.		R/S	A i,j=( )?
	TO CORRECT ELEMENT OF MATRIX, TYPE			
	IN NEW VALUE AND PRESS R/S	A i,j	R/S	A i,J+1=( )?
11	CONTINUE STEP 10 UNTIL ALL MATRIX			
	ELEMENTS HAVE BEEN CHECKED; GO TO 9			
12	CALCULATOR COMMENCES MATRIX			TABLEAU 1
	MANIPULATIONS. IF PRINTER CON-			TABLEAU 2,
	NECTED AND ANSWER TO (4) WAS "Y",			ETC.
	COMPLETE MATRIX WILL BE PRINTED.			

* WILL NOT ATTEMPT PRINTING IF PRINTER NOT CONNECTED, i.e IGNORES YOUR INPUT

** IF i OR j > 11, CALCULATOR RETURNS TO STEP 5

USER INSTRUCTIONS

(CONTINUED)

SIZE: 021

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
13	AFTER ONE OR MORE ITERATIONS OUTPUT			
	IS PRINTED. NOTE: IF PRINTER			
	CONNECTED AND ANSWER TO (4) WAS "Y",			
	THEN ENDING TABLEAU, i.e. LAST			
	TABLEAU, IS PRINTED			XK = ( )
14	TO VIEW SUCCESSIVE XK (OPTIONAL			
	VARIABLE VALUES)		R/S*	XK = ( )
15	BY SUCCESSIVE PRESSING OF R/S, Z			
	AND YK (SHADOW PRICES) ARE OUTPUT		R/S*	Z = ( )
			R/S*	YK = ( ) ETC.
16	TO REPRINT OUTPUT, PRESS R/S		R/S	
	AND GO TO STEP 13 ABOVE			
17	FOR A NEW PROBLEM, GO TO STEP 2			
	ABOVE			

* ONLY NECESSARY IF PRINTER NOT CONNECTED OR OFF

00320C

PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
01	✱LBL "LP"		
02	CF 00		
03	CF 01		
04	CF 02		
05	CF 21		
06	20		
07	XEQ 17		--. CHECK IF SIZE
08	FC?C 25		INITIALLY SET TO o21
09	PROMPT		
10	CLRG		
11	XEQ 15		--.Lbl 15 SETS UP OUTPUT
12	"MAXIMIZ E"		COUNTERS
13	XEQ 16		
14	X=Y?		---.PROMPT FOR MAXIMIZE
15	SF 00		IF ANSWER IS "N" THEN
16	"PRINT T ABS"		MINIMIZE (SF00) ----.PROMPT TO PRINT
17	XEQ 16		TABLEAUS FULLY. IF
18	X≠Y?		ANSWER IS "Y" FLAG 02
19	SF 02		IS SET
20	✱LBL 02		
21	"ROWS?"		-.PROMPT FOR NO. OF
22	PROMPT		ROWS & COLUMNS IN
23	STO 00		INITIAL MATRIX
24	"COLS?"		
25	PROMPT		
26	STO 01		
27	1 E5		
28	/		←---.STORE INCREMENT
29	STO 03		
30	✱LBL 10		
31	RCL 00		
32	11		
33	X<Y?		◄--.IF NO. OF ROWS OR
34	GTO 02		COLS > 11, RE-INPUT
35	RCL 01		AT LABEL 02
36	X<>Y		
37	X<Y?		
38	GTO 02		
39	RDN		
40	*		
41	20		
42	+		
43	STO 02		←--.STORE "Z"ELEMENT
44	XEQ 17		LOCATION NUMBER
45	FC?C 25		•CHECK IF SIZE
46	PROMPT		ADEQUATE
47	21.9		
48	STO 07		
49	RCL 00		
50	RCL 01		←-. SET UP COUNTERS
51	1 E3		FOR INPUT, STORE
52	ST/ 02		IN R04 - R07
53	ST/ Z		
54	/		
55	ISG Y		
56	ISG X		
57	STO 06		
58	STO 04		
59	RDN		
60	STO 05		
61	FIX 0		
62	✱LBL 01		--. INPUT LOOP
63	SF 29		
64	"A "		
65	ARCL 05		
66	CF 29		
67	ARCL 04		
68	RCL IND 07		
69	FC? 01		--. IF FLAG 01 SET,
70	GTO 01		THEN WE ARE IN
71	FIX 2		"CHECK" MODE
72	"⊢="		
73	ARCL X		
74	FIX 0		
75	✱LBL 01		
76	"⊢?"		
77	PROMPT		
78	STO IND 07		
79	ISG 07		
80	ISG 04		
81	GTO 01		
82	RCL 06		
83	STO 04		
84	ISG 05		
85	GTO 01		
86	"CHECK"		
87	XEQ 16		
88	X=Y?		◄-. IF ANSWER IS "N"
89	GTO 00		THEN COMMENCE
90	SF 01		CALCULATIONS
91	GTO 10		ELSE EDIT EACH
92	✱LBL 00		ELEMENT

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PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
93	RCL 02		--.COMPUTE COUNTERS
94	RCL 03		IN R19 AND R20
95	+		FOR KEEPING TRACK
96	LASTX		OF COLUMN Q AND ROW
97	1 E2		W LOCATIONS.
98	*		
99	-		
100	FRC		
101	RCL 01		
102	20		
103	+		
104	+		
105	STO 19		
106	RCL 02		
107	RCL 02		
108	1 E3		
109	*		
110	RCL 01		
111	-		
112	.999		
113	+		
114	+		
115	STO 20		
116	GTO 23		
117	◆LBL 16		--.ALPHA PROMPT
118	AON		SUBROUTINE
119	"⊢?"		
120	PROMPT		
121	AOFF		
122	ASTO X		
123	"N"		
124	ASTO Y		
125	RTN		
126	◆LBL 15		--.INITIALIZE OUTPUT
127	9.018		LABEL REGISTERS BY
128	STO 07		PLACING CODE NUMBERS
129	1.001		IN REGISTERS 09 TO 18
130	+		
131	STO 08		
132	◆LBL 06		
133	RCL 08		
134	INT		
135	STO Y		
136	10		
137	-		
138	1 E2		
139	/		
140	+		
141	STO IND 07		
142	ISG 07		
143	CLD		
144	ISG 08		
145	GTO 06		
146	RTN		
147	◆LBL 17		--.SIZE CHECK
148	SF 25		ROUTINE(BASED ON
149	1 E2		STANDARD H.P ROM
150	X<>Y		ROUTINE)
151	RCL IND X		
152	1		
153	+		
154	"XEQ SIZ E "		
155	X<=Y?		
156	"⊢0"		
157	FIX 0		
158	CF 29		
159	ARCL X		
160	SF 29		
161	RTN		
162	◆LBL 23		--.START OF PIVOT
163	"TABLEAU "		FINDER ROUTINE
			(CONTINUES UP TO
164	9.02		LINE 268)
165	◆LBL 04		
166	CF IND X		--.CLEAR FLAGS 9-
167	ISG X		20(INITIALIZE)
168	GTO 04		
169	RCL 02		
170	FRC		
171	1 E3		
172	*		
173	STO 05		--.STORE COUNTER
174	RCL 19		
175	XEQ 21		
176	X=0?		----.CHECK FOR NEGATIVE
177	GTO 03		ELEMENTS IN COL Q
178	RCL 04		IF NONE, PROCEED
179	STO 07		TO CHECK ROW W.
180	RCL 20		
181	SF 09		
182	XEQ 28		--.NEG.ELEMENT FOUND
183	RCL 04		IN Q. FIND PIVOT
184	STO 08		COLUMN

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00320C PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
185	GTO 05		.HAVING FOUND ROW/COL
186	*LBL 03		INDEX OF PIVOT, TRANS-
187	RCL 20		FORM THE MATRIX.
188	FS? 00		.CHECK FOR LARGEST - VE
189	SF 09		OR + VE ELEMENT IN W
190	XEQ 21		(DEPENDS IF MAX OR MIN)
191	CF 09		
192	X=0?		
193	GTO 27		.IF NONE FOUND, PROCEED
194	RCL 04		TO OUTPUT
195	STO 08		
196	RCL 19		
197	*LBL 28		.FIND OTHER PIVOT
198	STO Y		ELEMENT
199	RCL 05		
200	RCL 04		
201	-		.CREATE COUNTER
202	-		
203	STO 06		
204	RDN		
205	90		
206	SF 24		
207	TAN		.CREATE + INFINITY IN
208	*LBL 13		X REGISTER VIA TAN 90
209	RCL IND 06		
210	FS? 09		--.IF FLAG 09 SET WE ARE
211	CHS		DOING A PHASE ONE PASS
212	X<=0?		ONLY NEG.NOS.REQUIRED
213	GTO 00		
214	CLX		
215	RCL IND Z		
216	RCL IND 06		
217	/		
218	X<Y?		.SORT ROUTINE FOR
219	XEQ 19		SMALLEST RATIO
220	*LBL 00		
221	RDN		
222	ISG 06		
223	CLD		
224	ISG Y		
225	GTO 13		
226	FS?C 09		
227	RTN		.RETURN IF SUBROUTINE
228	RCL 04		
229	STO 07		
230	RDN		
231	X=0?		
232	SIGN		
233	1/X		--.IF 00 STILL IN X, NO
234	X≠0?		PIVOT FOUND; IF X≠0
235	GTO 05		THEN PIVOT THE MATRIX
236	RCL 01		
237	RCL 05		--.SET FLAG CORRES-
238	RCL 04		PONDING IN THE
239	-		ELEMENT OF W JUST
240	-		PROCESSED AS NO
241	10		PIVOT IN THAT COL.
242	+		
243	SF IND X		
244	GTO 03		--.FIND NEW PIVOT COL
245	*LBL 21		--.START OF ROUTINE
246	11.1		FOR FINDING LARGEST
247	STO Z		POS OR NEG.NO IN W
248	CLX		OR Q
249	*LBL 14		
250	RCL IND Y		
251	FS? IND T		--.HAS THE ELEMENT
252	CLX		BEEN TRIED BEFORE
253	FS? 09		
254	CHS		
255	X<Y?		
256	XEQ 19		--.IF LOWER THAN ELE-
257	RDN		MENT IN Y (INITIAL
258	ISG Z		LY ZERO) THEN SAVE
259	ISG Y		THE INDEX NO.
260	GTO 14		
261	RTN		
262	*LBL 19		--.SUBROUTINE TO SAVE
263	X<>Y		INDEX NOS. TEMPO-
264	RDN		RARILY IN R04
265	RCL Y		
266	INT		
267	STO 04		
268	RTN		
269	*LBL 05		--.START OF PIVOTAL
270	ISG 02		REDUCTION ROUTINE
271	RCL 02		
272	INT		
273	RCL 00		
274	RCL 01		--.CHECK IF WE ARE IN
275	*		AN ENDLESS LOOP
276	X<Y?		(UNBOUNDED)
277	GTO e		CONDITION

50

---NON ~~EXISTENT ROUTI~~ ERROR MESSAGE

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☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
278	ARCL Y		
279	AVIEW		
280	RCL 07	__	.COMPUTE COUNTERS FOR PIVOT ROW, PIVOT COLUMN AND PIVOT ITSELF
281	RCL 00		
282	RCL 01		
283	*		
284	20		
285	+		
286	RCL 08		
287	-		
288	-		
289	STO 06	__	.PIVOT REG. NO.
290	RCL 07		
291	1 E3		
292	/		
293	RCL 07		
294	RCL 01		
295	DSE X		
296	-		
297	+		
298	STO 07		
299	RCL 08		
300	1 E3		
301	/		
302	RCL 08		
303	RCL 00		
304	DSE X		
305	RCL 01		
306	*		
307	-		
308	+		
309	RCL 03		
310	+		
311	STO 08		
312	STO 05		
313	FS? 55	__	.IF PRINTER EXISTS, PRINT TABLEAU
314	XEQ 24		
315	◆LBL 11		
316	RCL 07		
317	STO 04		
318	◆LBL 12		
319	RCL 04		
320	RCL 05		
321	+		
322	INT		
323	RCL 06		
324	-		
325	X<=0?	__	.IF COLUMN NO. + ROW NO. - PIVOT NO. ≤ 0, IGNORE
326	GTO 07		
327	RCL 04		
328	INT		
329	X=Y?	__	.IF NO=COLUMN NO, IGNORE
330	GTO 07		
331	CLX		
332	RCL 05		
333	INT		
334	X=Y?	__	.IF NO=ROW NO, IGNORE
335	GTO 07		
336	CLX		
337	RCL 06		
338	X=Y?	__	.IF NO=PIVOT NO, IGNORE
339	GTO 07		
340	X<>Y		
341	RCL IND 04	__	.COMPUTE ADJACENT PIVOT COLUMN X ADJ. PIVOT ROW / PIVOT
342	RCL IND 05		
343	*		
344	RCL IND 06		
345	/		
346	ST- IND Y	__	.SUBTRACT FROM ORIGINAL ELEMENT
347	◆LBL 07		
348	ISG 04		
349	GTO 12		
350	ISG 05		
351	GTO 11		
352	RCL 07		
353	RCL IND 06		
354	-1		
355	ST* IND 06		
356	RDN		
357	XEQ 25	__	.DIVIDE EACH ELEMENT
358	RCL 08		IN PIVOT ROW BY THE
359	X<>Y		PIVOT
360	CHS		
361	XEQ 25	__	.DIVIDE EACH ELEMENT
362	RCL 08		IN PIVOT COL. BY -1
363	INT		* PIVOT
364	12	__	.COMPUT OUTPUT TABLE,
365	-		i.e. ROW/COLUMN INTER-
366	STO 08		CHANGE BY CODE NO.
367	RCL 07		
368	21		
369	-		
370	INT		
371	RCL 01		

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PROGRAM LISTING

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
372	/		
373	9		
374	+		
375	STO 07		
376	RCL IND 08		
377	INT		
378	STO IND 08		
379	CLX		
380	LASTX		
381	FRC		
382	1 E2		
383	*		
384	RCL IND 07		
385	FRC		
386	STO IND 07		
387	CLX		
388	LASTX		
389	INT		
390	1 E2		
391	/		
392	ST+ IND 08		
393	X<>Y		
394	ST+ IND 07		
395	GTO 23		.GO SEE IF A NEW PIVOT
396	*LBL 24		REQUIRED
397	CF 12		.PRINT MATRIX IF FLAG
398	CF 13		02 SET. CLEAR FLAGS
399	FC? 02		12 + 13 IF SET IN
400	RTN		PREVIOUS ROUTINES
401	SF 21		
402	PRA		
403	RCL 00		
404	1 E3		
405	/		
406	1		
407	+		
408	RCL 02		
409	FRC		
410	21		
411	ST+ Y		
412	ADV		
413	RCL 01		
414	STO L		
415	RDN		
416	SF 09		
417	*LBL 26		
418	"R "		.ROW HEADER
419	FIX 0		
420	ARCL Z		
421	FS? 09		
422	ACA		
423	FIX 2		
424	" "		
425	FS?C 09		
426	ACA		
427	RCL 06		
428	X=Y?		
429	"<P> "		.LABEL THE PIVOT
430	ACA		WHITE PRINTING
431	RDN		
432	RCL IND Y		
433	ACX		
434	RDN		
435	ADV		
436	DSE L		
437	GTO 26		
438	RCL 01		
439	STO L		
440	RDN		
441	SF 09		
442	ISG Z		
443	ADV		
444	*LBL 26		
445	ISG X		
446	DEG		
447	ISG Y		
448	GTO 26		
449	ADV		
450	ADV		
451	FIX 0		
452	CF 21		
453	RTN		
454	*LBL 25		.SUBROUTINE TO DIVIDE
455	ST/ IND Y		ROW & COLUMN ELE-
456	ISG Y		MENTS BY A NUMBER
457	GTO 25		IN X
458	RTN		
459	*LBL 27		.START OF OUTPUT
460	0		ROUTINE
461	STO 06		
462	"⊢END"		.PRINT FINAL
463	FS? 55		TABLEAU

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PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
464	XEQ 24		
465	CF 02		
466	SF 21		
467	RCL 19		
468	STO 08		
469	SF 09		
470	◆LBL 30		
471	9.1		
472	STO 07		
473	◆LBL 08		.IF INDEX/CODE IS <
474	FIX 0		10,COMPUTE AND
475	"Y"		PRINT THE "X"LABEL
476	RCL IND		FOLLOWED BY ITS
07			VALUE. IF ≥ 10,
477	FS? 09		PRINT THE "Y"LABEL
478	GTO 08		AND VALUE
479	FRC		
480	1 E2		
481	*		
482	9		
483	X<>Y		
484	X<=Y?		
485	GTO 00		
486	X<>Y		
487	MOD		
488	ARCL X		
489	XEQ 09		
490	GTO 00		
491	◆LBL 08		
492	"X"		
493	INT		
494	10		
495	X<=Y?		
496	GTO 00		
497	RDN		
498	1		.PRINT "Z" ON FIRST
499	+		PASS
500	ARCL X		
501	XEQ 09		
502	◆LBL 00		
503	"Z"		
504	ISG 07		
505	ISG 08		
506	GTO 08		
507	ADV		
508	FS? 09		
509	XEQ 09		
510	ADV		
511	RCL 20		
512	STO 08		
513	FS?C 09		.REPRINT IF R/S PRES.
514	GTO 30		.OUTPUT SUBROUTINE
515	STOP		
516	GTO 27		
517	◆LBL 09		
518	"⊢="		
519	FIX 2		
520	ARCL IND		
08			
521	AVIEW		
522	END		

70 80 90 00

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00320C

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	NO. OF ROWS
01	NO. OF COLS.
02	MAX.REG.NO.COUNT.
03	NO.OF COLS÷1000
04)	USED AS
05)	COUNTERS
06)	
07)	PIVOT LOCATIONS
08)	
09)	
10)	
11)	NUMBERS STORED
12)	AS AA.BB
13)	WHERE AA IS
14)	BASIC VARIABLE
15)	LABEL AND BB
16)	IS NON BASIC
17)	VARIABLE LABEL
18)	
19)	Q COL.COUNTER
20	W ROW COUNTER
21	USERS MATRIX "A" AND SUB-SEQUENT TRANS-FORMS (STORED
25	SEQUENTIAL LY)

STATUS

SIZE	021	TOT. REG.	149	USER MODE			
ENG		FIX 0,2	SCI		ON	OFF	X
DEG	X	RAD	GRAD				

FLAGS

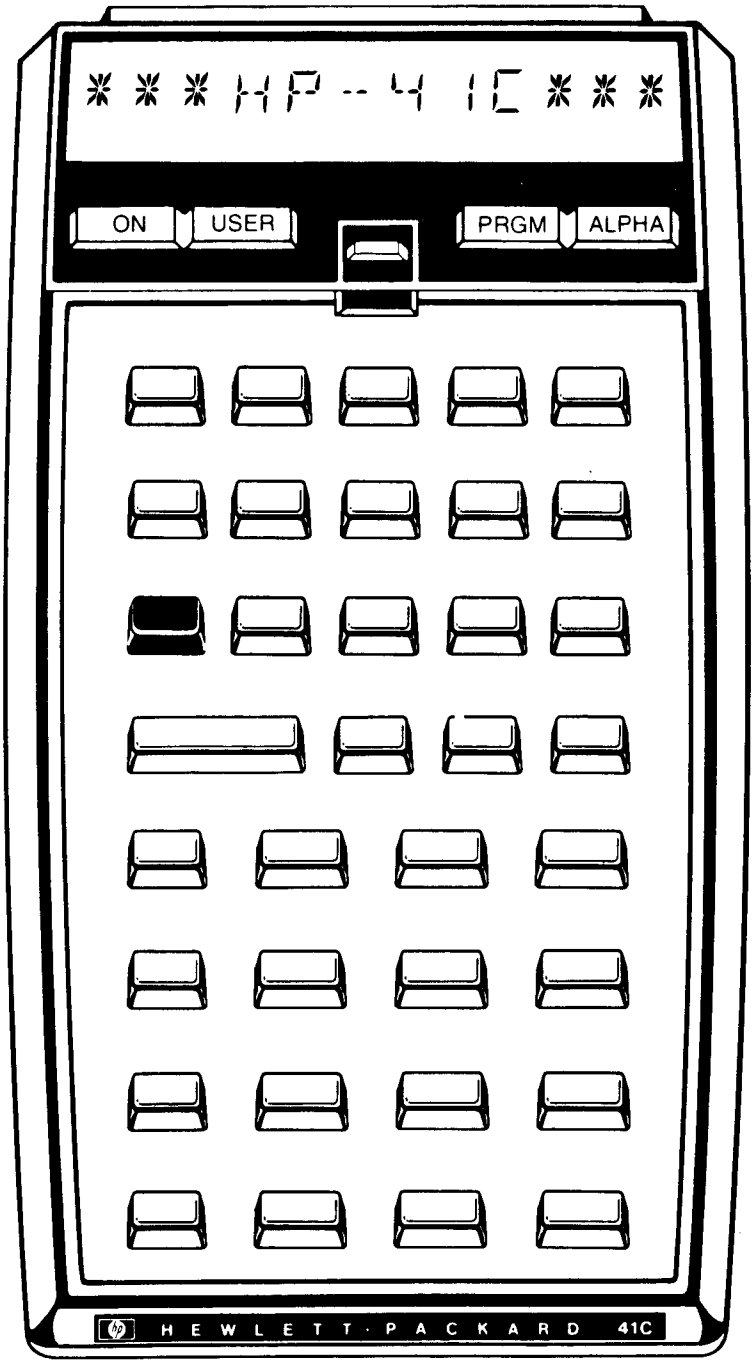
#	INIT S/C	SET INDICATES	CLEAR INDICATES
00	C	MINIMIZE MODE	MAXIMIZE MODE
01	C	CHECK MODE	NON CHECK MODE
02	C	PRINT TABS IF PRINTER	PRINT OUTPUT ONLY
09	C	(TEMP)SUBROUT. RETURN FLAG	SUBROUTINE NON-RETURN FLAG
10	C		
11	C	USED TO RECIDE	
12	C	IF W ELEMENT	
13	C	ALREADY TRIED	ELEMENT NOT
14	C	IF SET,IGNORE	USED
15	C	(SEE LABEL 21)	(SEE LABEL 21)
16	C		
17	C		
18	C		
19	C		
20	C		
21	C	PRINT DELETED UNLESS FLAG 02 SET AND PRINTER CONNECTED WHEREUPON TABLEAU PRINTED. FLAG 21 TURNS ON AT LABEL 27 TO PRINT/PROMPT OUT-PUT SOLUTION.	AT ALL TIMES,
24	C	RANGE ERROR IGNORE FOR OLD 41C's	
25	C	ERROR IGNORE	SIZE TO BE PROMP
29	S	DECIMAL POINT	SUPPRESS DECIM.
55	C/S	PRINTER EXISTS DO PRINT TABS ONLY IF FLAG 02 SET	PRINTER OUT-IGNORE.PRINT ROUTINE

ASSIGNMENTS

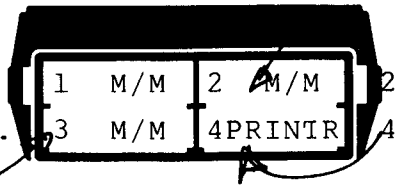
FUNCTION	KEY	FUNCTION	KEY

KEYBOARD CARD LABELING

KEYBOARD



SYSTEM CONFIGURATION

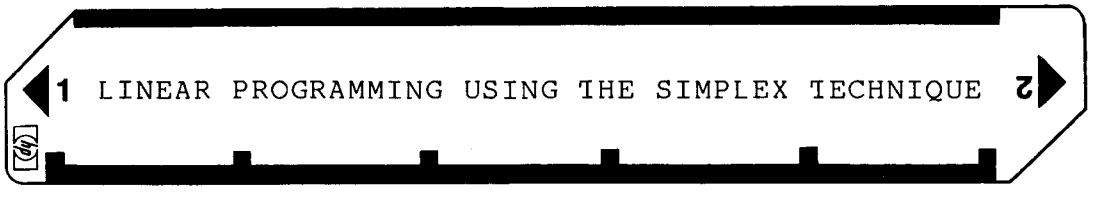


1 MEMORY MODULES COMPULSORY

3 ADDITIONAL MEMORY OPTION.

4 PRINTER OPTIONAL

CARD



ROW 1 (1 : 5)

ROW 2 (5 : 11)

ROW 3 (12 : 14)

ROW 4 (15 : 16)

ROW 5 (17 : 21)

ROW 6 (22 : 27)

ROW 7 (28 : 38)

ROW 8 (38 : 46)

ROW 9 (47 : 53)

ROW 10 (53 : 62)

ROW 11 (63 : 68)

ROW 12 (69 : 74)

ROW 13 (75 : 81)

ROW 14 (82 : 87)

ROW 15 (87 : 95)

ROW 16 (96 : 105)

ROW 17 (105 : 112)

ROW 18 (113 : 119)

ROW 19 (120 : 127)

ROW 20 (127 : 133)

ROW 21 (134 : 141)

ROW 22 (142 : 149)

ROW 23 (149 : 154)

ROW 24 (154 : 159)

ROW 25 (159 : 163)

ROW 26 (163 : 168)

ROW 27 (169 : 176)

ROW 28 (177 : 184)

ROW 29 (185 : 191)

ROW 30 (191 : 198)

ROW 31 (199 : 209)

ROW 32 (209 : 217)

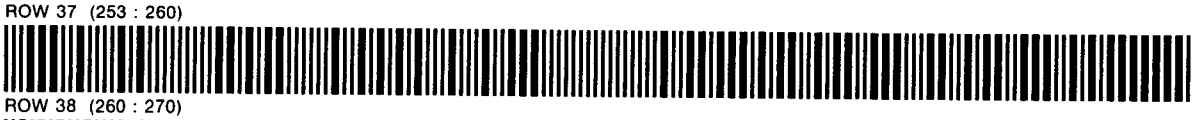
ROW 33 (218 : 225)

ROW 34 (226 : 236)

ROW 35 (237 : 245)

ROW 36 (246 : 252)

ROW 37 (253 : 260)



ROW 38 (260 : 270)

ROW 39 (270 : 279)

ROW 40 (280 : 291)

ROW 41 (291 : 300)

ROW 42 (300 : 311)

ROW 43 (312 : 321)

ROW 44 (322 : 332)

ROW 45 (333 : 342)

ROW 46 (342 : 350)

ROW 47 (350 : 357)

ROW 48 (357 : 366)

ROW 49 (367 : 377)

ROW 50 (378 : 386)

ROW 51 (386 : 394)

ROW 52 (395 : 401)

ROW 53 (401 : 410)

ROW 54 (410 : 418)

ROW 55 (418 : 424)

ROW 56 (424 : 429)

ROW 57 (429 : 436)

ROW 58 (437 : 444)

ROW 59 (444 : 451)

ROW 60 (452 : 458)

ROW 61 (459 : 464)

ROW 62 (464 : 471)

ROW 63 (471 : 478)

ROW 64 (478 : 487)

ROW 65 (488 : 494)

ROW 66 (495 : 503)

ROW 67 (503 : 509)

ROW 68 (510 : 516)

ROW 69 (517 : 522)