00390C PROGRAM DESCRIPTION I

Program Title ROOTS OF POLYNOMIALS

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Program Description, Equations, Variables Consider the polynomial of degree n (where n is less than 21) with real coefficients,

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + \ldots\ldots + a_1 z + a_0 \qquad (1)$$

The roots of $P(z) = 0$ will be either real or complex in conjugate pairs. Let a guess for a root be given by $z_k = x_k + i\, y_k$. Laguerre's method uses the following iterative algorithm to improve the guess.

$$z_{k+1} = z_k - \frac{n\, P(z_k)}{P'(z_k) \pm \sqrt{H(z_k)}} \qquad (2)$$

where

$$H(z_k) = (n-1)\left[(n-1)\, P'(z_k)^2 - n\, P(z_k)\, P''(z_k)\right] \qquad (3)$$

$$P'(z_k) = \left.\frac{dP(z_k)}{dz}\right|_{z = z_k} \qquad (4)$$

and

$$P''(z_k) = \left.\frac{d^2 P(z_k)}{dz^2}\right|_{z = z_k} \qquad (5)$$

(Continued on page 2)

Necessary Accessories One memory module

Operating Limits and Warnings The program uses 0+i0 as the initial guess. In some cases, this may result in the display "DATA ERROR". In this case one has to store a nonzero guess in R_{31} and proceed by executing B.

Reference(s) Dahlquist, B. and Bjorck, A., "Numerical Methods" (translated by N. Anderson) Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1974.

(Continuation Sheet)

In equation (2), the sign in the denominator must be chosen in such a manner that $|z_{k+1} - z_k|$ is as small as possible. This is done as follows. Let $H(z_k) = r_1 \angle \theta_1$ (in the polar form) and let $P'(z_k) = r_2 \angle \theta_2$. Then, the positive sign should be chosen in equation (2) if $|\theta_1 - \theta_2|$ is less than 90°.

The program starts by calculating $P(z_k)$. Horner's Rule is used, with complex multiplication carried out in rectangular coordinates for better accuracy. The modulus of $P(z_k)$ is then displayed, following TONE 9. If this is less than 10^{-7} (this tolerance may be altered by placing the desired number in line 43 of the program), the beep sound is heard and either a real root or a complex conjugate root-pair is displayed. Otherwise, a new guess for the root is calculated according to equation (2), and the iteration is continued.

If a real root is obtained, say x_r, then $P(z)$ is divided by $z-x_r$ to obtain a polynomial of degree n-1. If a complex pair of roots is obtained, say $x_r \pm i\, y_r$, then $P(z)$ is divided by $z^2 - 2x_r z + (x_r^2+y_r^2)$ to obtain a polynomial of degree n-2. This process is repeated until all the roots are known.

The program is compatible with the HP-82143A Printer in that with the printer in the manual mode, the values of the coefficients are printed as these are input. The roots are printed when evaluated. The program is also compatible with the program POLY in the MATH module since a_i are stored in R_i and the degree in R_{22}.

As repeated deflation of a polynomial may cause error due to round-off, it is suggested that after all the roots have been calculated, one final iteration with each root be performed on the full polynomial. Hence, it may be convenient to store the polynomial on a card if a card-reader is available.

PROGRAM DESCRIPTION II

Sample Problem (Sketch if Desired)

1. $P(z) = z^6 + 6z^5 - 56z^3 - 97z^2 + 26z + 120$

$= (z - 1)(z - 3)(z + 2)(z + 4)(z^2 + 4z + 5)$

SOLUTION:

Input	Function	Display	Comments
	User		Set user mode
	XEQ SIZE 037		Minimum size
	A	N=?	Degree of the polynomial
6	R/S	a_0 =?	Enter the coefficients
120	R/S	120.000 a_1 =?	
26	R/S	26.000 a_2 =?	
97 CHS	R/S	-97.000 a_3 =?	
56 CHS	R/S	-56.000 a_4 =?	
0	R/S	0.000 a_5 =?	
6	R/S	6.000 a_6 =?	
1	R/S	1.000, 120.00, 61.92	Intermediate values of $P(z_k)$
		9.665, 0.560,	
		1.555 -04, 0.000	
		ROOT = -2.000	Real root evaluated
		30.000, 7.587, 0.011	Intermediate values
		0.000	
		COMPLEX ROOTS	
		-2.000 I1.000	Pair of complex roots evaluated
		36.056, 15.229,	Intermediate values
		1.379, 0.003,	
		4.900 -11	
		ROOT = 1.000	Real root evaluated
		10.000, 0.000	Intermediate values
		ROOT = 3.000	Real root
		7.000, 0.000	Intermediate values
		ROOT = -4.000	Real root

Sample Problem (Sketch if Desired)

2. $P(z) = z^{12} - 0.5\, z^{6} + 1$

In this sample problem the initial guess 0 + i 0 causes difficulty since both $P'(z_0)$ and $P''(z_0)$ are zero. Since this leads to division by zero in equation (2) a display of "DATA ERROR" is obtained. Another initial guess is required.

SOLUTION: In this case the input subroutine will be bypassed

Input	Function	Display	Comments
	User		Set user mode
	XEQ SIZE 037		Minimum size
	XEQ CLRG		Set all registers to zero
1.00	STO 00, STO 12		Set a_{12} and a_0 to 1.00
0.5 CHS	STO 06		Set a_6 to -0.5
12	STO 22		Set n = 12
	XEQ B	1.000 "DATA ERROR"	The initial guess of 0 + i0 leads to division by zero
1.00	STO 31, STO 32	63.127, 0.903, 0 269	Initial guess = 1 + i1
		0 001, 5.000 -10	
		COMPLEX ROOTS	
		0.677 I0.736	Pair of complex roots
		7.891, 2.022, 0.041,	Intermediate values
		6.399 -07	
		1.640 -09	
		COMPLEX ROOTS	
		0.299 I0.954	Pair of complex roots
		8.065, 1.151, 0.028,	Intermediate values
		1.920 -06	
		2.088 -09	
		COMPLEX ROOTS	
		-0.299 I0.954	Pair of complex roots
		2.606, 0.155,	Intermediate values
		3.405 -04	
		2.236 -09	
		COMPLEX ROOTS	
		-0.677 I0.736	Pair of complex roots
		1.978, 0.153, 0.001,	Intermediate values
		1.077 -09	
		COMPLEX ROOTS	
		-0.976 I0.218	Pair of complex roots
		3.904, 1.4005 -09	Intermediate values
		COMPLEX ROOTS	
		0.976 I0.218	Pair of complex roots

USER INSTRUCTIONS

SIZE: 037

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1.	Enter the program, check status and set			
	USER mode			
2.	To start data input.		A	N = ?
		N	R/S	a_0 = ?
		a_0	R/S	a_0, a_1 = ?
		.....	.....	a_{N-1}, a_N = ?
		a_N	R/S	a_N, Inter-
				mediate values
	(Display for real root)			ROOT = ...
	(Display for complex roots)			COMPLEX ROOTS
				.... I.....
3.	Alternatively, store a_i in R_i and			
	N in R_{22}. Also store real part of			
	initial guess in R_{31} and the imaginary part			
	in R_{32}. To start the iterations		B	Intermediate
				values followed
				by the roots
				as above.
4.	Sometimes the program started in step 2 may halt with the display "DATA ERROR".			
	In this case store a nonzero guess in R_{31} and/or R_{32} and start the iterations		B	Intermediate
				values follwed
				by the roots.

00390C PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
	01 ◆LBL "ROO T N"		
	02 "N=?"		
	03 PROMPT		Input degree n
	04 STO 22		
	05 1 E3		
	06 /		
	07 STO 21		
	08 ◆LBL 13		Input coefficients
	09 CF 29		
	10 "a"		
	11 RCL 21		
	12 INT		
	13 FIX 0		
	14 ARCL X		
	15 "⊢=?"		
	16 AVIEW		
20	17 STOP		
	18 STO IND 21		
	19 FIX 3		
	20 VIEW X		
	21 ISG 21		
	22 GTO 13		
	23 SF 29		
	24 0		Initial guess 0 + i0
	25 STO 31		
30	26 STO 32		
	27 ADV		
	28 ◆LBL B		Start of iterations
	29 CLD		
	30 XEQ 02		Calculate $P(z_k)$
	31 RCL 22		
	32 *		
	33 STO 25		
40	34 X<>Y		
	35 RCL 22		
	36 *		
	37 STO 26		
	38 R-P		
	39 RCL 22		
	40 /		
	41 TONE 9		$\lvert P(z_k) \rvert$
	42 PSE		
	43 1 E-7		
50	44 X>Y?		Is $10^{-7} > \lvert P(z_k) \rvert$?

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
	45 GTO 07		Root is obtained!
	46 XEQ 01		Calculate $P''(z_k)$
	47 RCL 26		
	48 RCL 25		
	49 XEQ 08		Calculate nPP''
	50 STO 23		
	51 X<>Y		
	52 STO 24		
	53 XEQ 03		Calculate $P'(z_k)$
	54 STO 27		
	55 X<>Y		
	56 STO 28		
	57 X<>Y		
	58 RCL 28		
	59 RCL 27		
	60 XEQ 08		Calculate P'^2
	61 RCL 22		
	62 1		
	63 -		
	64 STO 29		n-1
	65 *		
	66 RCL 23		
	67 -		
	68 STO 23		$\mathrm{Re}\left[(n-1)P'^2-nPP''\right]$
	69 X<>Y		
	70 RCL 29		
	71 *		
	72 RCL 24		
	73 -		
	74 STO 24		$\mathrm{Im}\left[(n-1)P'^2-nPP''\right]$
	75 X<>Y		
	76 R-P		
	77 RCL 29		
	78 *		
	79 SQRT		$\lvert\sqrt{H(z_k)}\rvert$
	80 STO 23		
	81 X<>Y		
	82 2		
	83 /		$\angle\sqrt{H(z_k)}$
	84 STO 24		
	85 RCL 28		
	86 RCL 27		
	87 R-P		
	88 RDN		
	89 -		
	90 ABS		
	91 90		
	92 X<=Y?		

Note: Refer to "HP-41C OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for specific information on keystrokes. The Function Index is found at the very back of the Handbook. Refer to Appendix E in 67 or 97 "OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for exact keystrokes.

PROGRAM LISTING

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STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
93	SF 00		Selecting the sign of $\sqrt{H(z_k)}$
94	RCL 24		
95	RCL 23		
96	FS?C 00		
97	CHS		
98	P-R		
99	ST+ 27		
100	X<>Y		
101	ST+ 28		
102	RCL 25		Calculate $z_{k+1} = z_k - \frac{nP}{P' \pm \sqrt{H}}$ and store in R_{31}, R_{32}.
103	RCL 27		
104	*		
105	RCL 26		
106	RCL 28		
107	*		
108	+		
109	RCL 27		
110	X↑2.		
111	RCL 28		
112	X↑2		
113	+		
114	STO 29		
115	/		
116	ST- 31		
117	RCL 26		
118	RCL 27		
119	*		
120	RCL 25		
121	RCL 28		
122	*		
123	-		
124	RCL 29		
125	/		
126	ST- 32		
127	GTO B		Return for next iteration
128	*LBL 01		Subroutine for calculating $P''(z_k)$
129	SF 01		
130	*LBL 03		Subroutine for calculating $P'(z_k)$
131	XEQ 06		
132	FS? 01		
133	DSE 30		
134	*LBL 04		
135	FS? 01		
136	ISG 30		
137	CLA		
138	RCL 30		
139	RCL IND 30		
140	*		
141	FS? 01		
142	XEQ 05		
143	+		
144	DSE 30		
145	XEQ 09		
146	ISG 30		
147	CLA		
148	DSE 30		
149	GTO 04		
150	CF 01		
151	RTN		
152	*LBL 05		
153	DSE 30		
154	RCL 30		
155	*		
156	RTN		
157	*LBL 02		Subroutine for calculating $P(z_k)$
158	XEQ 06		
159	*LBL 14		
160	RCL IND 30		
161	+		
162	XEQ 09		
163	DSE 30		
164	GTO 14		
165	RCL IND 30		
166	+		
167	RTN		
168	*LBL 06		
169	RCL 22		
170	STO 30		
171	0		
172	ENTER↑		
173	0		
174	RTN		
175	*LBL 07		Display real root
176	RCL 22		
177	STO 30		

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PROGRAM LISTING

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STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
178	0		
179	STO 29		
180	RCL 32		
181	ABS		
182	STO 32		
183	1 E-9		
184	X<=Y?		
185	GTO 10		
186	BEEP		
187	"ROOT = "		
188	ARCL 31		
189	AVIEW		
190	PSE		
191	CLA		
192	RCL 31		
193	CHS		Linear factor for synthetic division
194	STO 28		
195	GTO 12		
196	*LBL 09		
197	RCL 32		Subroutine for multiplication by z_k
198	RCL 31		
199	*LBL 08		
200	STO 35		
201	RDN		
202	STO 36		Subroutine for multiplication of two complex numbers.
203	RDN		
204	STO 33		
205	RDN		
206	STO 34		
207	*		
208	RDN		
209	*		
210	R↑		
211	+		
212	RCL 33		
213	RCL 35		
214	*		
215	RCL 34		
216	RCL 36		
217	*		
218	-		
219	RTN		
220	*LBL 10		
221	BEEP		Display complex roots
222	"COMPLEX ROOTS"		
223	AVIEW		
224	CLA		
225	SF 13		
226	ARCL 31		
227	"+ I"		
228	ARCL 32		
229	AVIEW		
230	PSE		
231	CF 13		
232	CLA		
233	RCL 31		Quadratic factor for synthetic division
234	2		
235	*		
236	CHS		
237	STO 27		
238	RCL 32		
239	X↑2		
240	RCL 31		
241	X↑2		
242	+		
243	STO 28		
244	*LBL 11		Synthetic division by quadratic factor
245	RCL 28		
246	RCL IND 30		
247	*		
248	LASTX		
249	RCL 27		
250	*		
251	DSE 30		
252	ST- IND 30		
253	RDN		
254	DSE 30		
255	ST- IND 30		
256	1		
257	ST+ 30		
258	RCL 29		
259	+		
260	RCL 30		
261	X≠Y?		
262	GTO 11		
263	SF 02		
264	GTO 00		

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PROGRAM LISTING

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☐ 67 ☐ 97 ☐ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
	265*LBL 12		Synthetic division by linear factor
	266 RCL IND 30		
	267 RCL 28		
	268 *		
	269 DSE 30		
	270 ST- IND 30		
	271 RCL 30		
	272 RCL 29		
	273 X≠Y?		
	274 GTO 12		
	275*LBL 00		Shift down the coefficients by 1 register for real root and by 2 registers for complex pair
	276 1		
	277 ST+ 30		
	278 RCL IND 30		
	279 FS? 02		
	280 DSE 30		
	281 DSE 30		
	282 CLA		
	283 STO IND 30		
	284 RCL 30		
	285 RCL 22		
	286 1		
	287 ST+ 30		
	288 FS? 02		
	289 ST+ 30		
	290 RDN		
	291 X≠Y?		
	292 GTO 00		
	293 1		Decrease n by 1 for real and by 2 for complex pair
	294 FS?C 02		
	295 2		
	296 ST- 22		
	297 RCL 22		
	298 X≠0?		Continue if n ≠ 0
	299 GTO B		
	300 END		
50			

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
51			
60			
70			
80			
90			
00			

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REGISTERS, STATUS, FLAGS, ASSIGNMENTS

00390C

DATA REGISTERS

00	a_0	50	
	a_1		
	a_2		
	etc.		
05		55	
10		60	
15		65	
20		70	
	n (degree)		
25		75	
30		80	
	x_k		
	y_k		
35	Used	85	
	Used		
40		90	
45		95	

STATUS

SIZE 037 TOT. REG. 112 USER MODE

ENG ____ FIX 3 SCI ____ ON x OFF ____

DEG x RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
00	C		
01	C		
02	C		
13	C		
29	S		

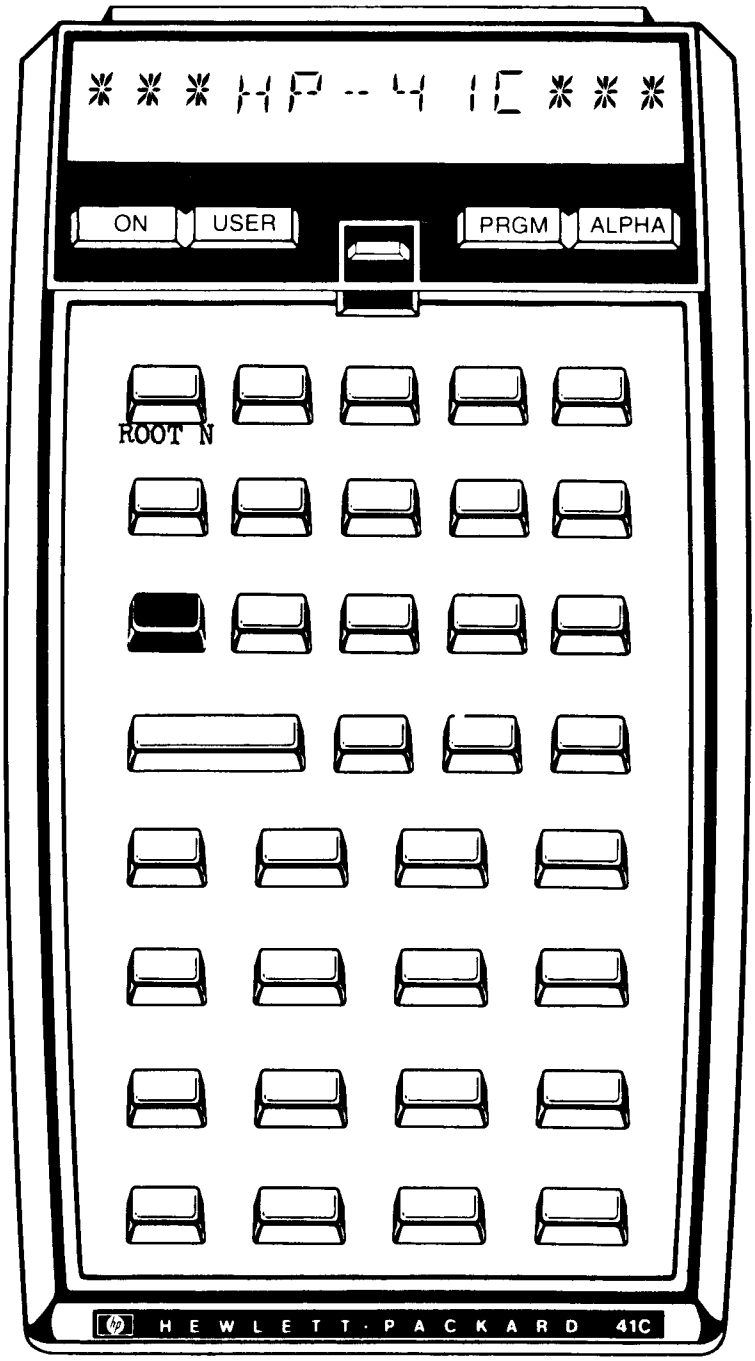
ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY
ROOT N	^		

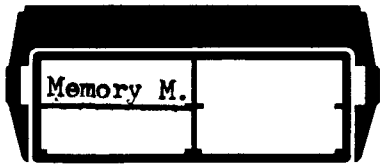
00390C

KEYBOARD CARD LABELING

KEYBOARD



SYSTEM CONFIGURATION



CARD

PROGRAM REGISTERS NEEDED: 75

ROW 1 (1 - 2)

ROW 2 (2 - 9)

ROW 3 (10 - 15)

ROW 4 (16 - 23)

ROW 5 (23 - 30)

ROW 6 (31 - 39)

ROW 7 (39 - 46)

ROW 8 (46 - 52)

ROW 9 (52 - 59)

ROW 10 (59 - 66)

ROW 11 (67 - 75)

ROW 12 (76 - 85)

ROW 13 (85 - 94)

ROW 14 (94 - 102)

ROW 15 (102 - 110)

ROW 16 (111 - 118)

ROW 17 (119 - 127)

ROW 18 (127 - 133)

ROW 19 (134 - 141)

ROW 20 (142 - 148)

ROW 21 (148 - 156)

ROW 22 (157 - 163)

ROW 23 (164 - 172)

ROW 24 (173 - 181)

ROW 25 (182 - 187)

ROW 26 (187 - 193)

ROW 27 (194 - 201)

ROW 28 (202 - 211)

ROW 29 (212 - 220)

ROW 30 (221 - 222)

ROW 31 (222 - 228)

ROW 32 (228 - 237)

ROW 33 (238 - 246)

ROW 34 (246 - 254)

ROW 35 (255 - 262)

ROW 36 (263 - 270)

ROW 37 (270 - 278)

ROW 38 (278 - 285)

ROW 39 (285 - 293)

ROW 40 (294 - 300)

ROW 41 (300 - 300)