PROGRAM DESCRIPTION I

Program Title Fast Fourier transform I

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Program Description, Equations, Variables This program can be used to evaluate the discrete Fourier transform (DFT) or inverse DFT (IDFT) of an N-point sequence of complex numbers, where N must be a power of two, $N=2^M$, and M is an integer of ≤ 7.

This program is an implementation of the radix-2 decimation-in-time algorithm. The input is first rearranged into "bit-reversed" order and then $\log_2 N$ stages of "butterflies" are performed by program lines 67 to 176.

The DFT of {x(n)} is defined as

$$X(k) = \sum_{n=0}^{N-1} x(n)\exp(-j2\pi nk/N) \; ; \; k = 0,1,\ldots,N-1$$

The IDFT is defined as

$$x(n) = (1/N)\sum_{k=0}^{N-1} X(k)\exp(j2\pi nk/N) \; ; \; n = 0,1,\ldots N-1$$

Necessary Accessories The additional memory modules according to the total registers; TOT.REG = 62 + 2N

Operating Limits and Warnings

N must be a power of 2 and ≤ 128 for the storage limit. We can use this program to grasp the FFT algorithm and investigate the properties of DFT for $N \leq 16$ reasonably. Above 16-point it takes much time to compute the FFT in this machine.

Reference(s) J.W. Cooley & J.W. Tukey, *An Algorithm for the Machine Calculation of Complex Fourier Series*, Math. Comp., vol. 19, pp. 297-301, April 1965.

L.R. Rabiner & B. Gold, *Theory and Application of Digital Signal Processing*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1975.

TO OUR CUSTOMERS:

The following information was supplied by our reviewer for your use with this program. The comments are intended to be helpful in avoiding any complications that may inadvertently arise.

THE HEWLETT PACKARD USERS' LIBRARY STAFF

"FFT" does not initially clear flag ØØ. To include this in the program, steps 1 - 4 should be replaced by:

```
01  LBL "IFF"
02  SF 00
03  GTO 03
04  LBL "FFT"
05  CF 00
06  LBL 03
07  RAD
```

If the above coding changes are made, STEP 219 should be eliminated.

The FIX = 0, 3, 4, and 8, although the output is displayed in FIX 3, and the calculator is left in FIX 3.

PROGRAM DESCRIPTION II

Sample Problem (Sketch if Desired)

Example 1: Find the DFT of a sequence, x(n) = (1, 1+j, 3, 1-j).

Example 2: Find the IDFT of a sequence, X(k) = (6, 0, 2, -4).

SOLUTION:

Input	Function	Display	Comments
* Insert a memory module, load program and clear flag 00 if set.			
	(XEQ)SIZE025		
Example 1:	(XEQ)FFT	POINTS=?	Prompting N.
4	(R/S)	X0=Re↑IM?	Prompting input points.
1	(ENTER↑)	1.0000	Input the real part of x(0),
0	(R/S)	X1=Re↑IM?	and the imaginary part.
1	(ENTER↑)	1.0000	Input all points as prompting.
1	(R/S)	X2=Re↑IM?	
3	(ENTER↑)	3.0000	
0	(R/S)	X3=Re↑IM?	
1	(ENTER↑)	1.0000	
-1	(R/S)	X0=6.000,0.000	Display of output points in the
	(R/S)	X1=0.000,0.000	form of complex-rectangular format
	(R/S)	X2=2.000,0.000	as X(k)=*Real part,Imaginary part.*
	(R/S)	X3=-4.000,0.000	
Example 2:	(XEQ)IFF	POINTS=?	
4	(R/S)	X0=Re↑IM?	
6	(ENTER↑)	6.000	
0	(R/S)	X1=Re↑IM?	
0	(ENTER↑)	0.000	
0	(R/S)	X2=Re↑IM?	
2	(ENTER↑)	2.000	
0	(R/S)	X3=Re↑IM?	
-4	(ENTER↑)	-4.000	
0	(R/S)	X0=1.000,0.000	
	(R/S)	X1=1.000,1.000	Outputs of Example 2.
	(R/S)	X2=3.000,0.000	
	(R/S)	X3=1.000,-1.000	
	(R/S)		(R/S) to clear flag00.

USER INSTRUCTIONS

				SIZE: (17 + 2N) (HP-41C)
STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Load program and initialize by CF00,if set.			
2	Set SIZE to a given points, N.		(XEQ) SIZE nnn	
3	Execute the program, FFT		(XEQ) FFT	POINTS=?
	or IFF, if IDFT.	or	(XEQ) IFF	POINTS=?
4	Input the points, N, power of 2 & ≤ 128.	N	(R/S)	X0=Re↑IM?
5	Input the sequence in complex forms as	Re.X(0)	(ENTER↑)	
	prompting. '0' must be input for the	Im.X(0)	(R/S)	X1=Re↑IM?
	imaginary part of a real point.	Re.X(1)	(ENTER↑)	
		Im.X(1)	(R/S)	X2=Re↑IM?
		.		.
		.		.
		.		.
		.		.
6	Repeat step #5 until all the points have			
	been keyed in	Im.X(N-1)	(R/S)	X0=*Re.,Im.*
7	Press (R/S) to see successive solutions		(R/S)	X1=*Re.,Im.*
	properly labeled. All the solutions are		.	.
	displayed in FIX 3 format for nice views		.	.
	The values of real part and imaginary		.	.
	part of a solution are retained in the			
	stacks, X and Y respectively.			

PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
	01◆LBL "IFF "		
	02 SF 00		
	03◆LBL "FFT "		
	04 RAD		
	05 CF 29		
	06 "POINTS= ?"		
	07 PROMPT		*Input N.*
	08 STO 04		
	09 STO 08		
	10 LOG		
	11 2		
	12 LOG		
	13 /		
	14 FIX 8		*Compute & store*
	15 RND		*$M=\log_2 N$.*
	16 STO 05		
	17 FRC		
	18 STO 00		*If N isn't power of 2,*
	19 FACT		*display "DATA ERROR".*
	20◆LBL 01		*Start input shuffling.*
	21 FIX 0		
	22 "X"		
	23 ARCL 00		
	24 "⊢=Re↑IM ?"		
	25 FIX 4		
	26 RCL 05		
	27 STO 01		
	28 RCL 00		
	29 STO 03		
	30 CLX		
	31 STO 02		
	32◆LBL 05		*Start bit reversal.*
	33 RCL 03		
	34 ENTER↑		
	35 ENTER↑		
	36 2		
	37 /		
	38 INT		
	39 STO 03		
	40 ST+ X		
	41 −		
	42 RCL 02		
	43 ST+ X		
	44 +		
	45 STO 02		
	46 DSE 01		
	47 GTO 05		
	48 17		
	49 RCL 02		
	50 ST+ X		
	51 +		
	52 STO 06		
	53 1		
	54 +		
	55 STO 07		
	56 PROMPT		
	57 STO IND 07		*Store input in bit reversed order.*
	58 X<>Y		
	59 STO IND 06		
	60 1		
	61 ST+ 00		
	62 DSE 08		
	63 GTO 01		*Loop for input shuffling.*
	64 1		
	65 STO 03		
	66 STO 02		
	67◆LBL 15		*Start FFT calculation.*
	68 2		
	69 RCL 03		
	70 Y↑X		
	71 STO 01		
	72 RCL 02		
	73 STO 08		
	74 CLX		
	75 STO 09		
	76 PI		
	77 ST+ X		
	78 FC? 00		*For IDFT, flag 00 set.*
	79 CHS		
	80 RCL 01		
	81 /		
	82 RCL 02		
	83 P-R		
	84 STO 10		
	85 X<>Y		
	86 STO 11		
	87 RCL 02		
	88 STO 12		
	89◆LBL 16		
	90 RCL 12		
	91 STO 00		

Note: Refer to "HP-41C OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for specific information on keystrokes. The Function Index is found at the very back of the Handbook. Refer to Appendix E in 67 or 97 "OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for exact keystrokes.

PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
92	◆LBL 02		*Start butterfly computation.*
93	RCL 02		
94	15		
95	RCL 00		
96	ST+ X		
97	+		
98	STO 13		
99	+		
100	STO 14		
101	LASTX		
102	RCL 01		
103	+		
104	STO 15		
105	RCL 02		
106	+		
107	STO 16		
108	RCL IND 15		
109	RCL 08		
110	*		
111	RCL IND 16		
112	RCL 09		
113	*		
114	-		
115	STO 06		
116	RCL IND 15		
117	RCL 09		
118	*		
119	RCL IND 16		
120	RCL 08		
121	*		
122	+		
123	STO 07		
124	RCL IND 14		
125	RCL 07		
126	-		
127	STO IND 16		
128	RCL IND 13		
129	RCL 06		
130	-		
131	STO IND 15		
132	RCL IND 13		
133	RCL 06		
134	+		
135	STO IND 13		
136	RCL IND 14		
137	RCL 07		
138	+		
139	STO IND 14		
140	RCL 01		
141	ST+ 00		
142	RCL 04		
143	RCL 00		
144	X<=Y?		*Loop for in-place butterfly computation.*
145	GTO 02		
146	RCL 10		
147	RCL 08		
148	*		
149	RCL 11		
150	RCL 09		
151	*		
152	-		
153	RCL 08		
154	RCL 11		
155	*		
156	RCL 09		
157	RCL 10		
158	*		
159	+		
160	STO 09		
161	X<>Y		
162	STO 08		
163	RCL 02		
164	ST+ 12		
165	RCL 01		
166	2		
167	/		
168	RCL 12		
169	X<=Y?		
170	GTO 16		
171	RCL 02		
172	ST+ 03		
173	RCL 05		
174	RCL 03		
175	X<=Y?		

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PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
176	GTO 15		
177	BEEP		*Signal the end of FFT.*
178	CLX		*Start output display.*
179	STO 00		
180	17		
181	STO 08		
182	18		
183	STO 09		
184	RCL 04		
185	STO 01		
186	◆LBL 11		
187	RCL IND 09		*Recall output point.*
188	RCL IND 08		
189	FC? 00		
190	GTO 06		
191	RCL 04		
192	/		*Divide by N for IDFT.*
193	X<>Y		
194	LASTX		
195	/		
196	X<>Y		
197	◆LBL 06		
198	FIX 0		
199	"X"		
200	ARCL 00		
201	"⊢="		
202	FIX 8		
203	RND		
204	X<>Y		*Round off the round-ing errors.*
205	RND		
206	X<>Y		
207	FIX 3		
208	ARCL X		
209	"⊢,"		
210	ARCL Y		*Display the output in complex form; X(k)=Re.,Im.,shown in FIX 3.*
211	PROMPT		
212	RCL 02		
213	ST+ 00		
214	2		
215	ST+ 08		
216	ST+ 09		
217	DSE 01		
218	GTO 11		*Output loop.*
219	CF 00		
220	.END.		

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REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	Scratch
	Scratch
	Scratch
	Scratch
	N
05	M
	Index for Re.X(n)
	Index for Im.X(n)
	Index
	Index
10	Index
	Index
	Index
	Used
	Used
15	Used
	Used
*	(Beginning of
:	data storages)
:	
20	
:	
:	
:	
:	
∇	

STATUS

SIZE 17+2N TOT. REG. 62+2N USER MODE

ENG ____ FIX 3 SCI ____ ON ____ OFF X

DEG ____ RAD x GRAD ____

FLAGS

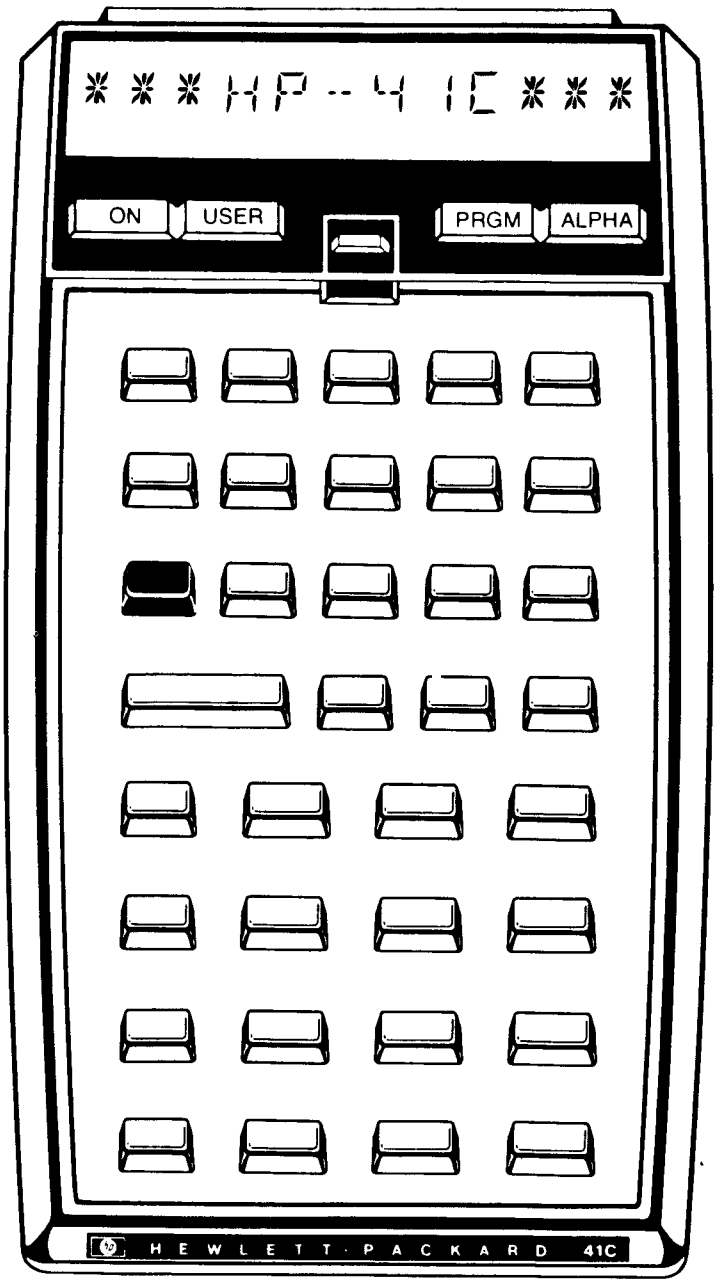
#	INIT S/C	SET INDICATES	CLEAR INDICATES
00	C	IFF(inverse DFT)	FFT(DFT)
29	C		No decimal point.

ASSIGNMENTS

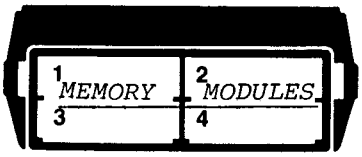
FUNCTION	KEY	FUNCTION	KEY

KEYBOARD CARD LABELING

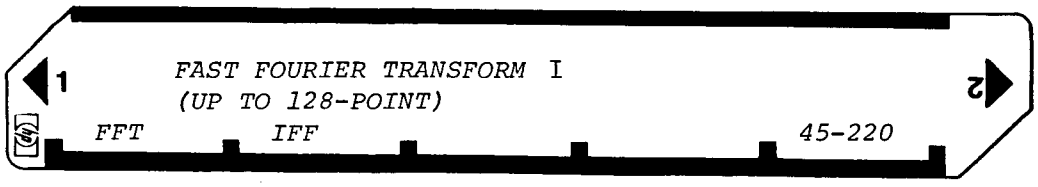
KEYBOARD



SYSTEM CONFIGURATION



CARD



FAST FOURIER TRANSFORM I

PROGRAM REGISTERS NEEDED: 46

USERS' LIBRARY
PROGRAM NUMBER: 00868C

ROW 1 (1 : 3)

ROW 2 (3 : 6)

ROW 3 (6 : 16)

ROW 4 (17 : 24)

ROW 5 (24 : 30)

ROW 6 (31 : 42)

ROW 7 (43 : 50)

ROW 8 (51 : 61)

ROW 9 (61 : 70)

ROW 10 (71 : 81)

ROW 11 (82 : 93)

ROW 12 (94 : 104)

ROW 13 (105 : 114)

ROW 14 (115 : 124)

ROW 15 (125 : 133)

ROW 16 (134 : 142)

ROW 17 (143 : 154)

ROW 18 (155 : 166)

ROW 19 (167 : 176)

ROW 20 (176 : 185)

ROW 21 (186 : 194)

ROW 22 (195 : 202)

ROW 23 (202 : 210)

ROW 24 (210 : 218)

ROW 25 (218 : 220)