00869C

PROGRAM DESCRIPTION I

Program Title PARTIAL FRACTION EXPANSION

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Program Description, Equations, Variables This program can be used to obtain the coefficients of partial fraction expansion of a rational function with the poles of multiplicity up to 4. For the rational function, the order of numerator is less than that of denominator. For a given rational function, F(S), with the partial fraction expansion,

$$F(S)=(a_kS^k + a_{k-1}S^{k-1} + \text{---} + a_1S + a_0)/[(S-P_1)^{M1}(S-P_2)^{M2}\text{---}(S-P_n)^{Mn}]$$

$$= K_{11}/(S-P_1)^{M_1} + K_{12}/(S-P_1)^{M_1-1} + \text{---} + K_{1M_1}/(S-P_1)$$

$$+ K_{21}/(S-P_2)^{M_2} + K_{22}/(S-P_2)^{M_2-1} + \text{---} + K_{2M_2}/(S-P_2)$$

$$+ \text{---------------------}$$

$$+ K_{n1}/(S-P_n)^{Mn} + K_{n2}/(S-P_n)^{Mn-1} + \text{---} + K_{nMn}/(S-P_n)\ ,$$

NOTE: In the multiple-pole expansion, the solutions are shown in the order from high degree term to low degree term of the pole.

the coefficients, K_{qr}, of the partial fraction expansion for F(S), are given by;

$$K_{qr}=\frac{1}{(r-1)!}\left\{\frac{d^{(r-1)}}{dS^{(r-1)}}[(S-P_q)^{Mn}\bullet F(S)]\right\}_{S=P_q},$$ for q=1,2,---,n & r=1,2,---,Mn where P_q is the pole with multiplicity of Mn which is less than or equal to 4 and the poles must be expressed in the complex-rectangular forms.

Necessary Accessories The additional memory modules; $\# = \text{INT}(\frac{110+4P+N}{64})$, where P is the # of poles and N is the degree of denominator polynomial of F(S).

Operating Limits and Warnings

For the multiple pole case, the solutions are approximated by using the numerical differentiations, and the resulting coefficients are accurate to the third decimal place roughly.

Reference(s)

ANALYSIS OF LINEAR NETWORKS AND SYSTEMS, by Shu-Park Chan, et al., Addison-Wesley Co..

And other various books for Controls and Systems.

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TO OUR CUSTOMERS:

The following information was supplied by our reviewer for your use with this program. The comments are intended to be helpful in avoiding any complications that may inadvertently arise.

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This program does not initialize (clear) flags 00 - 03.

Sample Problem (Sketch if Desired)

Example 1: $$F(S)=\frac{S^2+1}{(S+1)^3(S+2)}=\frac{K11}{(S+1)^3}+\frac{K12}{(S+1)^2}+\frac{K13}{S+1}+\frac{K21}{S+2}$$

Find K_{11} , K_{12} , K_{13} and K_{21} .

Example 2: $$F(S)=\frac{S^2+3S+1}{S(S^2+2S+5)}=\frac{K_{11}}{S}+\frac{K_{21}}{S+1+j2}+\frac{K_{31}}{S+1-j2}$$

Find K_{11} , K_{21} and K_{31} .

NOTE: *The coefficients of complex-conjugate poles are expressed in the rectangular forms,* KR_{nm}*=Re.,Im..*

SOLUTION:

Input	Function	Display	Comments
✲ Insert two memory modules & load program. Clear all the flags in ANNUNCIATOR.			
Example 1:	(XEQ)PF	N:DEN↑POLES=?	Prompting N & P of F(S).
4	(ENTER↑)	4.0000	Input N, the order of denominator.
2	(R/S)	SET SIZE 42	Input P, the number of poles.
	(XEQ)SIZE042(R/S)	P1=Re↑Im?	Prompting pole#1 in complex form.
-1	(ENTER↑)	-1.0000	Input the real part of pole #1.
0	(R/S)	M1=?	Input the imaginary part of pole#1.
3	(R/S)	P2=Re↑Im?	Input the multiplicity of pole#1.
-2	(ENTER↑)	-2.0000	Input pole #2.
0	(R/S)	M2=?	
1	(R/S)	DEG. OF NUM.=?	Prompting the order of numerator.
2	(R/S)	a2=?	Input the order & prompting of a_k.
1	(R/S)	a1=?	Input the numerator coefficients.
0	(R/S)	a0=?	
1	(R/S)	K1,1=2.000	The solutions are displayed.
	(R/S)	K1,2=-4.000	
	(R/S)	K1,3=5.000	
	(R/S)	K2,1=-5.000	*NOTE: If you want to calculate another PF for F(S) with the same denominator polynomial, then press (R/S), input another numerator coefficients as prompt, and so forth.*
Example 2:	(XEQ)PF	N:DEN↑POLES=?	
3	(ENTER↑)	3.000	
3	(R/S)	SET SIZE 45	
	(XEQ)SIZE045(R/S)	P1=Re↑Im?	
0	(ENTER↑)	0.000	
0	(R/S)	M1=?	
1	(R/S)	P2=Re↑Im?	
-1	(ENTER↑)	-1.000	
-2	(R/S)	M2=?	
1	(R/S)	P3=Re↑Im?	
-1	(ENTER↑)	-1.000	
2	(R/S)	M3=?	
1	(R/S)	DEG. OF NUM.=?	
2	(R/S)	a2=?	
1	(R/S)	a1=?	
3	(R/S)	a0=?	
1	(R/S)	K1,1=0.200	(continuation)

PROGRAM DESCRIPTION II

(CONTINUATION PAGE)

INPUT	FUNCTION	DISPLAY	COMMENTS
	(R/S)	KR2,1=0.400,0.450 ;	the solutions of complex conjug-
	(R/S)	KR3,1=0.400,-0.450;	ate poles are displayed in the
			complex-rectangular forms,
			KRnm=(real part),(imaginary part)

USER INSTRUCTIONS

SIZE: (HP-41C) ***

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1a	Insert memory modules or quad-memory.			
1b	Load program and initialize by clearing			
	all the flags in the ANNUNCIATOR.			
1c	Execute the program.		(XEQ) PF	N:DEN.↑POLES=?
2	Input the order of denominator of F(S)	N	(ENTER↑)	
	and the number of poles, P.	P	(R/S)	SET SIZE nnn
3	Set size and continue		(XEQ) SIZE nnn	
	(*** SIZE= 30+4P+N)		(R/S)	P1=Re↑Im?
4	Input the poles of F(S) in complex form	Re(P1)	(ENTER↑)	
	and the multiplicity of each pole in	Im(P1)	(R/S)	M1=?
	the sequential manner appeared in denom-	M1	(R/S)	P2=Re↑Im?
	inator. '1' must be input for those mult-	Re(P2)	(ENTER↑)	
	iplicity of simple poles. And $M_n \leq 4$.	Im(P2)	(R/S)	M2=?
		.	.	.
		.	.	.
		.	.	.
5	Repeat step #4 until all the poles of	.	.	.
	F(S) have been keyed in.	Mn	(R/S)	DEG. OF NUM.=?
6	Input the degree of numerator of F(S).	K	(R/S)	a_k=?
7	Key in the coefficients of numerator.	a_k	(R/S)	a_{k-1}=?
	Zero must be input for the coefficients	a_{k-1}	(R/S)	a_{k-2}=?
	which are zero.	.	.	.
	Repeat until display asks for a_0.	.	.	.
		a_1	(R/S)	a_0=?
8a	Input coefficient a_0 and press (R/S) to	a_0	(R/S)	K1,1=?
	see successive solutions labeled aptly.		.	.
	In the multiple-pole case, the coeffi-		.	.
	cients are shown in the order from high			
	degree term to low degree term of the			

USER INSTRUCTIONS

				SIZE: (HP-41C)
STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	pole.			
8b	For the complex-conjugate poles, the			
	solutions are given in the rectangular			
	forms.		(R/S)	$KR_{n,m}$=Re,Im
			. .	. .
9a	To calculate another set of the coeffi-			
	cients in the partial fraction expansion			
	of a rational function with the same			
	denominator polynomial, press (R/S) and		(R/S)	DEG. OF NUM.=?
	and go to step #6.	K	(R/S)	a_k=?
			. .	. .
9b	For a new case, go to step #1.			

PROGRAM LISTING

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STEP/LINE KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
01♦LBL "PF"		
02 CF 29		
03 "N:DEN.↑POLES=?"		Input N and P.
04 PROMPT		
05♦LBL 04		Calculation of the SIZE.
06 ENTER↑		
07 SF 25		
08 4		SIZE=30+4P+N
09 *		
10 R↑		
11 +		
12 29		
13 +		
14 STO IND X		
15 FS?C 25		
16 GTO 00		
17 1		
18 +		
19 FIX 0		
20 "SET SIZE "		
21 ARCL X		
22 PROMPT		
23 RDN		
24 GTO 04		
25♦LBL 00		
26 RDN		
27 STO 04		Store P.
28 1 E-6		Δ for double-pole
29 STO 17		
30 1 E-4		Δ for triple-pole
31 STO 18		
32 1 E-3		Δ for quad-pole
33 STO 19		
34 RCL 04		
35 *		
36 1		
37 STO 05		Store 1.00P,index for Δ
38 +		
39 STO 29		
40 STO 06		Store the index for
41 30		Xi, the real part of Pi.
42 STO 02		
43 STO 08		
44 RCL 05		
45 +		
46 STO 09		Store the index for Yi.

STEP/LINE KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
47 RCL 05		
48 +		
49 STO 10		Store the index for
50 RCL 05		Xoi, the original Xi.
51 +		
52 STO 11		Store the index(Mi).
53♦LBL 12		
54 " P"		
55 FIX 0		
56 ARCL 06		
57 FIX 4		
58 "⊢=Re↑IM?"		Input & store Pi.
59 PROMPT		
60 STO IND 09		
61 X<>Y		
62 STO IND 08		
63 STO IND 10		
64 " M"		
65 FIX 0		
66 ARCL 06		
67 FIX 4		
68 "⊢=?"		
69 PROMPT		
70 STO IND 11		
71 4		Increment & loop in-
72 ST+ 08		dex for pole, Pi.
73 ST+ 09		
74 ST+ 10		
75 ST+ 11		
76 ISG 06		
77 GTO 12		
78♦LBL 20		
79 "DEG. OF NUM.=?"		Input & store K, the
80 PROMPT		order of numerator.
81 STO 03		
82 RCL 05		
83 +		
84 STO 06		Store index for a_k.
85 RCL 04		
86 4		
87 *		
88 RCL 02		
89 +		

PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
90	STO 00		
91	♦LBL 08		
92	" a"		
93	FIX 0		
94	RCL 06		
95	RCL 05		
96	−		
97	ARCL X		
98	"⊢=?"		
99	PROMPT		
100	STO IND 00		*Input & store a_k.*
101	RCL 05		
102	ST+ 00		
103	DSE 06		
104	GTO 08		
105	RCL 02		
106	STO 08		*Store the indices for the pole data.*
107	RCL 05		
108	+		
109	STO 09		
110	RCL 05		
111	+		
112	STO 10		
113	RCL 05		
114	+		
115	STO 11		
116	RCL 29		
117	STO 06		
118	♦LBL 16		
119	RCL IND 11		
120	RCL 05		
121	X=Y?		
122	SF 01		*Set flag 01 for simple pole case.*
123	−		
124	RCL 19		
125	*		
126	RCL 05		
127	+		
128	STO 01		*Store index for the loop of pole, Pi.*
129	RCL IND 11		
130	15		
131	+		
132	STO 28		*Store index for Δ.*
133	22		

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
134	STO 20		
135	RCL 05		*Store index for F(Pi).*
136	+		
137	STO 21		
138	♦LBL 17		
139	RCL 29		
140	STO 07		*Store the indices for the pole, Pj.*
141	RCL 05		
142	STO 15		
143	CLX		
144	STO 16		
145	RCL 02		
146	STO 12		
147	RCL 05		
148	+		
149	STO 13		
150	RCL 05		
151	ST+ X		
152	+		
153	STO 14		
154	♦LBL 05		*Start of the computation; F(Pi)=N(Pi)/D(Pi) where N(Pi) & D(Pi) are the numerator part & the denominator part of F(Pi), respectively.*
155	RCL 06		
156	RCL 07		
157	X=Y?		
158	GTO 00		
159	RCL IND 09		
160	X≠0?		
161	SF 03		*Set flag 03 for complex pole case.*
162	RCL IND 13		
163	−		
164	RCL IND 08		*Complex computation of D(Pi).*
165	RCL IND 12		
166	−		
167	DEG		
168	R-P		
169	RCL IND 14		
170	Y↑X		
171	X<>Y		
172	RCL IND 14		
173	*		
174	RCL 16		
175	RCL 15		
176	ST* T		

PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
177	RDN		
178	+		
179	STO 16		*Store D(Pi).*
180	X<>Y		
181	STO 15		
182	◆LBL 00		
183	4		
184	ST+ 12		
185	ST+ 13		*Increment of loop index*
186	ST+ 14		*for D(Pi).*
187	ISG 07		
188	GTO 05		
189	RCL 03		*Start computation of N(Pi)*
190	X=0?		
191	SF 00		
192	RCL 19		
193	*		
194	RCL 05		
195	+		
196	STO 07		*Store loop index for N(Pi)*
197	RCL 04		
198	4		
199	*		
200	RCL 02		
201	+		
202	STO 00		*Store index for a_k.*
203	0		
204	RCL IND 00		
205	FS?C 00		
206	GTO 00		*Skip if K=0.*
207	◆LBL 11		
208	STO 12		
209	RCL IND 09		
210	*		
211	X<>Y		
212	STO 13		
213	RCL IND 08		*Computation of N(Pi).*
214	*		
215	+		
216	RCL IND 08		
217	RCL 12		
218	*		
219	RCL IND 09		
220	RCL 13		
221	*		
222	-		
223	RCL 05		
224	ST+ 00		
225	CLX		
226	RCL IND 00		
227	+		
228	ISG 07		
229	GTO 11		
230	◆LBL 00		
231	R-P		
232	RCL 15		
233	RCL 16		
234	ST- T		*Compute & store F(Pi).*
235	RDN		
236	/		
237	P-R		
238	STO 15		
239	X<>Y		
240	STO 16		
241	X<>Y		
242	FS?C 02		
243	RTN		*Return if multiple pole.*
244	FS?C 01		
245	GTO 00		
246	◆LBL 15		*Compute derivatives of F(S).*
247	SF 00		
248	XEQ 00		*Display output.*
249	RCL IND 28		
250	ST+ IND 08		*Store Pi=Pi+Δ*
251	RCL 15		
252	STO IND 20		
253	RCL 16		
254	STO IND 21		
255	SF 02		
256	XEQ 17		
257	RCL IND 20		*Compute the coefficients of PF for multiple pole case.*
258	RCL 01		
259	INT		
260	STO 00		
261	*		
262	-		

Note: Refer to "HP-41C OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for specific information on keystrokes. The Function Index is found at the very back of the Handbook. Refer to Appendix E in 67 or 97 "OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for exact keystrokes.

PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/LINE KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
263 X<>Y		
264 RCL IND 21		
265 RCL 00		
266 *		
267 -		
268 X<>Y		
269 XEQ IND 01		
270 R-P		
271 RCL IND 28		
272 RCL 00		
273 Y↑X		
274 RCL 00		
275 FACT		
276 *		
277 /		
278 P-R		
279 2		
280 ST+ 20		
281 ST+ 21		
282 RDN		
283 ISG 01		
284 GTO 15		
285♦LBL 00		*Display of solution.*
286 FIX 0		
287 "K"		*K= real pole case*
288 FS? 03		*KR=complex pole case.*
289 "KR"		
290 ARCL 06		
291 "⊢,"		
292 ARCL 01		
293 FIX 3		
294 "⊢="		
295 ARCL X		
296 FS? 03		
297 "⊢,"		
298 FS?C 03		
299 ARCL Y		
300 PROMPT		
301 FS?C 00		
302 RTN		
303 RCL IND 10		
304 STO IND 08		
305 4		*Increment of indices for the next pole data.*
306 ST+ 08		
307 ST+ 09		
308 ST+ 10		
309 ST+ 11		
310 ISG 06		
311 GTO 16		
312 GTO 20		*Jump to compute a F(S).*
313♦LBL 01		
314 RTN		
315♦LBL 02		*Difference calculation for triple-pole.*
316 RCL 22		
317 +		
318 X<>Y		
319 RCL 23		
320 +		
321 X<>Y		
322 RTN		
323♦LBL 03		*Difference calculation for quad-pole.*
324 RCL 24		
325 RCL 00		
326 *		
327 +		
328 X<>Y		
329 RCL 25		
330 RCL 00		
331 *		
332 +		
333 X<>Y		
334 RCL 22		
335 -		
336 X<>Y		
337 RCL 23		
338 -		
339 X<>Y		
340 END		
00		

Note: Refer to "HP-41C OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for specific information on keystrokes. The Function Index is found at the very back of the Handbook. Refer to Appendix E in 67 or 97 "OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for exact keystrokes.

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	Scratch	
	1+(Mn-1)/10^3;index	
	30	
	K;degree of numer.	
	P;number of poles	
05	1	
	i;index	
	j;index	
	Xi;index of Re(Pi)	
	Yi;index of Im(Pi)	
10	Xoi;original Xi	
	Mi;multi. index	
	Xj;index of Re(Pj)	
	Yj;index of Im(Pj)	
	Mj;multi. index	
15	Scratch	
	Scratch	
	1 E-6	
	1 E-4	
	1 E-3	
20	Index	
	Index	
	Re[F(Pi)]	
	Im[F(Pi)]	
	Re[F(Pi+Δ)]	
25	Im[F(Pi+Δ)]	
	Re[F(Pi+2Δ)]	
	Im[F(Pi+2Δ)]	
	Mi+15;index for Δ	
	1+P/1000	
30	[*Beginning of in-*	
	put data for F(S)]	
	⋮	
	▽	

STATUS

SIZE 30+4P+N TOT. REG. 109+4P+N USER MODE

ENG ___ FIX X SCI ___ ON ___ OFF ___

DEG X RAD ___ GRAD ___

FLAGS

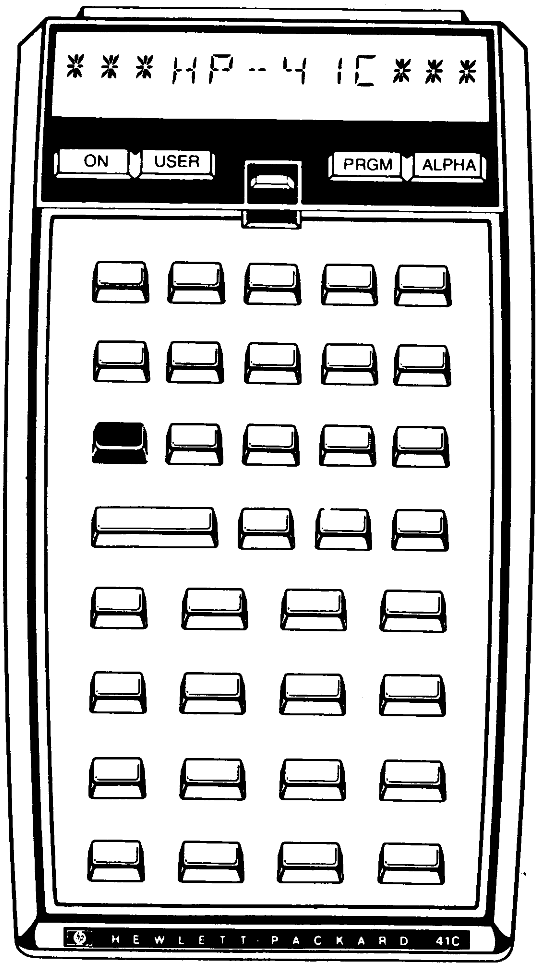
#	INIT S/C	SET INDICATES	CLEAR INDICATES
00	C	K=0, or output	
01	C	XEQ of single pole	
02	C	Compute F(Pi)	
03	C	Input of complex pole	
25	C	Ignore error	
29	C		No decimal point.

ASSIGNMENTS

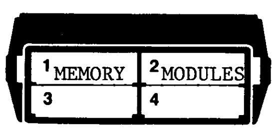
FUNCTION	KEY	FUNCTION	KEY

KEYBOARD CARD LABELING

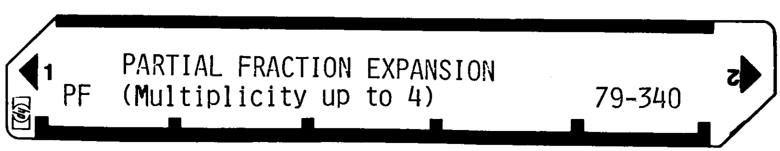
KEYBOARD



SYSTEM CONFIGURATION



CARD



PROGRAM REGISTERS NEEDED: 80

ROW 1 (1 : 3)

ROW 2 (3 : 6)

ROW 3 (7 : 15)

ROW 4 (16 : 20)

ROW 5 (20 : 28)

ROW 6 (28 : 32)

ROW 7 (32 : 41)

ROW 8 (41 : 53)

ROW 9 (54 : 58)

ROW 10 (58 : 63)

ROW 11 (64 : 68)

ROW 12 (69 : 76)

ROW 13 (76 : 79)

ROW 14 (79 : 85)

ROW 15 (86 : 95)

ROW 16 (96 : 102)

ROW 17 (103 : 113)

ROW 18 (114 : 122)

ROW 19 (123 : 132)

ROW 20 (132 : 139)

ROW 21 (140 : 151)

ROW 22 (151 : 161)

ROW 23 (161 : 169)

ROW 24 (170 : 179)

ROW 25 (179 : 187)

ROW 26 (188 : 197)

ROW 27 (198 : 207)

ROW 28 (208 : 217)

ROW 29 (218 : 227)

ROW 30 (228 : 236)

ROW 31 (237 : 245)

ROW 32 (246 : 252)

ROW 33 (252 : 258)

ROW 34 (259 : 269)

ROW 35 (270 : 280)

ROW 36 (281 : 287)

ROW 37 (288 : 293)

ROW 38 (293 : 298)

ROW 39 (299 : 306)

ROW 40 (307 : 312)

ROW 41 (312 : 322)

ROW 42 (323 : 333)

ROW 43 (334 : 340)