01299C

PROGRAM DESCRIPTION I

Program Title Multiple Linear Regression

Contributor's Name Walter W. Steffen

Address Meridian Life Ins. Co., P.O. Box 1980,

City Indianapolis **State** Indiana **Zip Code** 46206

Program Description, Equations, Variables This is a powerful multiple linear regression program that calculates the regression coefficients, correlation coefficients, standard error of estimate, net regression coefficients and partial correlation coefficients for any number of independent variables up to 8. This user-friendly program performs all the calculations when the user executes one function. The data is preserved for reuse in computing the various factors and can be stored on mag cards, adjusted by adding additional data sets or subtracting data sets before recalculating new coefficients. The principle features and parts of the program are as follows:

1. Clear registers and flags by XEQ b, after setting in User Mode.

2. Initiate by XEQ MLR (Assigned to $\sqrt{x}$), or XEQ C in User Mode. This prompts for the number of variables and then uses that information to set up the appropriate controls to store the entered data and cross products in the appropriate registers depending on the number of variables.

Necessary Accessories HP 41CV or HP 41C with Quad Memory Module Math Pac I ROM, Printer, Card Reader and/or Wand

Operating Limits and Warnings

Reference(s) Any good statistics book. For example: Methods of Correlation and Regression Analysis, Third Edition, Mordecai Ezekiel and Karl A. Fox, John Wiley and Sons.

3. XEQ "X" (Assigned Σ+) or A in User Mode, to store each data set. Prompts appear to request each data point as needed, reducing the possibility of error from forgetting which data is needed.

4. R/S after XEQ X will correct the last set of data entered by subtracting it and then permit entry of the correct set of data by XEQ X. XEQ a will permit any previously entered set of data to be subtracted.

5. After all data is entered, XEQ RG (Assigned 1/X), or B in User Mode, to calculate:

a. The regression coefficients, labeled X_1 through X_n.

b. R^2, the coefficient of Multiple determination

c. R^2_U (frequently designated as $\bar{R}^2$ in textbooks), an unbiased estimate of the percent of variance most probably associated with the particular sets of values of the independent variables in a universe where the independent variables remain fixed, but values of the dependent variable are subject to random variation.

d. $S_{Y:Z}$ (which is really $S_{Y:Z_1Z_2\ldots Z_n}$), which is the standard error of estimate, i.e., the amount of variance in Y that is not associated with the independent variables.

e. $S_{Y:ZU}$ (which is $\bar{S}_{Y:Z_1Z_2\ldots Z_n}$ in textbooks), which is an estimate of the standard error of estimate in the universe.

f. s_Y, which is the standard deviation of the dependent variable in the universe.

6. Then calculate the β coefficients (XEQ F). These represent the net regression coefficients, which is the regression coefficient when the corresponding variable is stated in its own standard deviation. Thus the regression formula:

$Y = X_1+X_2Z_1+X_3Z_2+X_4Z_3\ldots+X_9Z_8$ is transformed into:

$$\frac{Y}{s_Y} = X_1' + \beta_{YZ_1.Z_2\ldots Z_n}\frac{Z_1}{s_{Z_1}} + (\beta_{YZ_2.Z_1Z_3\ldots Z_n})\frac{Z_2}{s_{Z_2}} +\ldots$$

The relative sizes of the β's gives one a measure of the importance of each item on Y. They also reflect the "amount of change" in Y corresponding to a Unit of change in Z.

7. Y may be predicted. Prompts are made for the appropriate Z value to make the prediction. Control numbers are determined from the number of variables in order to call up the X_n's from their various registers. See page 58 of the Math Pac booklet for the location in the chart under the heading "Location of Column." For matrix of N order, they are in registers (N^2+N+15) to $(N^2+2N+14)$. For N variables (matrix order N+1) they are in registers $(N^2+3N+17)$ to $(N^2+4N+16)$.

8. Partial coefficients of correlation are calculated in LBL G. This calculation will destroy the previously calculated X_n's stored as indicated in paragraph 7. Hence, they are moved to the next higher unused registers in

LBL 25 for preservation and future use without having to recalculate the regression coefficients. However, when there are 8 variables there are no registers available and this routine cannot be used in that case without revision by deleting this moving routine and the recall routines in Labels F and Y. It would then require recalculation of LBL RG to recall the regression coefficients for prediction.

The partial correlation coefficients, r_{YZ_i}, etc, represent a measure of the importance of the variable Z_i after all the other variables except it are taken into account. It can also be defined as a measure of the extent to which that part of the variation of the dependent variable which was not explained by the other independent variables can be explained by the addition of the new factor.

In our sample problem $R^2_{Y.123}$ (Z's have been omitted to simplify notation, thus 1 is really Z_1) is .810994 and $R_{Y.123}$ = .900552; $R^2_{Y.23}$ = .557963 (secured by working backwards from the formula below for $r_{Y_{1.23}}$). Thus (1 - $R^2_{Y.123}$) = .189006 or 18.9% of the variance is left to be explained; and (1 - $R^2_{Y.23}$) = .442037, or 44.2% of the variance is left unexplained when only variable Z_2 and Z_3 are correlated. Adding the additional variable (Z_1) has increased the variance which can be explained by the difference between these two figures, or 25.3% (44.2 - 18.9). If the importance of this increase is determined by comparing it with the variance left unexplained before the new variable was added, we find that 25.3/44.2, or 57.2 percent of the variance previously unexplained has now been found to be associated with variable Z_1. Taking the square root of this number gives .756307, which is $r_{Y.23}$ (The difference between this and .756584 is because of rounding since figures were carried to only three digits in this paragraph.) This illustrates the power of performing the partial correlation coefficient calculation and the importance it can serve in a multi-variate regression analysis to determine which variables are highly significant or unimportant.

Incorporating this feature in an 8-variable analysis without destroying data-requiring its reentry (8 times for an 8-variable problem) - retaining prediction capability and net regression coefficient determination uses every "nook and cranny" of your 41C. See a paragraph below for minor revisions available when less than 8 variables are used.

Notation used to display various calculated items differs occasionally from that generally used in text books. This occurs because of (1) the difficulty in producing certain displays, such as $\bar{R}^2$ or $\bar{S}^2$, and (2) the large number of bytes required to produce $R^2_{Y.12345...n}$. Shortened notation has been adopted -- the following table of reference can be used to relate the shortened notation to the customary notation.

Shortened Notation Used	Customary Notation
R↑2 (R^2)	$R^2_{Y.12...n}$
R↑2↓U (R^2_U)	$\bar{R}^2_{Y.12...n}$
SY:Z ($S_{Y:Z}$)	$S_{Y.123...n}$
SY:Z↓U ($S_{Y:ZU}$)	$\bar{S}_{Y.12...n}$
βYZi ($\beta_{Y:Zi}$)	$\beta_{Yi.12...(i-1)(i+1)...n}$
rYZi (r_{YZi})	$r_{Yi.12...(i-1)(i+1)...n}$
Xi	$b_{Yi.12...(i-1)(i+1)...n}$

Formulas used:

(For 8 variables) $Y=X_1+X_2Z_1+X_3Z_2+X_4Z_3+X_5Z_4+X_6Z_5+X_7Z_6+X_8Z_7+X_9Z_8$ (1)

(For n variable, n<8) $Y=X_1+X_2Z_1+\ldots X_{n+1}Z_n$ (2)

Regression coefficients:

Determined by solving a matrix of order (N+1) for N variables, using the Math Pac instructions booklet.

(For n variables, n≤8) $R^2=\left[X_{n+1}\Sigma Z_nY+\ldots+X_2\Sigma Z_1Y+X_1\Sigma Y-(\Sigma Y)^2/n\right] \div \Sigma y^2$ (3)

where $\Sigma y^2=\Sigma Y^2-(\Sigma Y)^2/n$ and n = number of data sets (4)

$R^2_U(\text{or } \bar{R}^2)=1-(1-R^2)\left\{(n-1)/(n-m)\right\}$ where m = no. of regression (5)

Coefficients calculated

$$S_{Y:Z} = \sqrt{(1-R^2)\Sigma y^2/n} \quad (6)$$

$$S_{Y:ZU} = \sqrt{(1-R^2)\Sigma y^2/(n-m)} \quad (7)$$

$$s_Y=\sqrt{\left\{\Sigma Y^2-(\Sigma Y)^2/n\right\}/(n-1)} \quad (8)$$

$$\beta YZi=X_{i+1}(s_i/s_y)=X_{i+1}\sqrt{\Sigma z_i^2/\Sigma y^2} \quad (9)$$

where $\Sigma z_i^2=\Sigma Z_i^2-(\Sigma Z_i)^2/n$ (10)

$$r^2YZi=1-(1-R^2_{Y.12\ldots n})/(1-R^2_{Y.12\ldots(i-1)(i+1)\ldots n}) \quad (11)$$

$$rYZi=\sqrt{r^2YZi} \quad (12)$$

$\hat{Y}$ is determined by solving equation (1) using the determined values of X_i and the values of Z_i used to predict.

General comments regarding the program.

R_{14} is very important throughout. It is used in the Math Pac matrix evaluation and requires the matrix order (one more than the number of variables) to be stored in R_{14} to work successfully. When calculating r_{YZi} the matrix starts at order (N+1) and then changes to order N. Hence, R_{14} is carefully controlled. F3 is set when it contains the number of variables and cleared when it contains the matrix order. The Flag control does not always occur back-to-back with the storage in R_{14}; it sometimes occurs at the end of the routine. Should you stop the program operation in the middle of a routine and start it somewhere else in the program, it is desirable to check R_{14} (VIEW 14) and make the F3 and R_{14} data consistent with the above before starting the new operation.

R_1 - R_{14} are used in the Math Pac and cannot be used for other than temporary storage. They are used frequently between Math Pac runs.

If you have 8 variables, the rYZi feature cannot be used without modification. The following modifications are recommended - only when you have 8 variables and wish to use the rYZi feature.

1. Change the present steps 108, 175 and 529 to 125
2. Change the present step 155 to 124
3. Load the entire program
4. Change step 161 to END
5. Record the MLR part of the program on a mag card. Call this Part I.
6. Change step 161 back to RTN
7. Change step 126 to END
8. Record the RG part of the program on a mag card. Call this Part II.
9. CLP and put Part I in the calculator.
10. Set Size 180
11. Load data
12. CLP Part I and put Part II in the calculator.
13. Run the rest of the program (RG, LBL F, LBL G, LBL Y) routinely.
14. You may retain it as 2 parts and use it for any number of variables without changing it back. The reason it wasn't written in this manner was to permit it to be used without the inconvenience of reloading various parts as it is used because 8 variables are not used too often.

Frequently a particular user may routinely work with less than 8 variables. This user can avoid tying up the whole calculator by making a few minor changes and retain full flexibility and use of the program. The following table gives these changes according to the maximum number of variables. The numbers in the column for 8 variables are those presently used and are included as a frame of reference to be sure the correct numbers are being changed. The numbers in parenthesis in the 8 variable column are the changes to be used if the modifications suggested in the preceeding paragraphs are used.

No. of Variables	8	7	6	5	4	3	2
Size	170(180)	150	123	99	78	60	45
Steps 108, 175 and 529	115(125)	105	87	71	57	45	35
Step 155	114(124)	104	86	70	56	44	34

When less than 8 variables are customarily used, these minor changes will free-up registers for other use. These changes do <u>not</u> limit the partial correlation coefficient calculations, but provide for it (except of course when 8 variables are used without modification as mentioned earlier).

This program has been thoroughly tested, but if any user finds an error, weakness, or limitation, I shall welcome hearing about it. Any further enhancements will also be of interest to me.

Sample Problem

Example:* Calculate the regression coefficients, coefficient of correlation for the sample and the universe, standard error of estimate for the sample and the universe, standard deviation of the dependent variable, net regression coefficients and partial correlation coefficients for the following data. Then predict Y for $Z_1 = 12$, $Z_2 = 150$, and $Z_3 = 100$.

Z_1	Z_2	Z_3	Y
0	136	106	103
1	140	103	108
2	86	108	102
3	115	102	111
4	115	111	95
12	161	91	109
13	235	109	118
14	304	118	123
15	224	123	108
16	185	108	100
17	108	100	88
18	193	88	109
19	175	109	103

*Taken from Ezekiel and Fox, Methods of Correlation and Regression Analysis; Table A2.1, p.490.

SOLUTION:

	Input	Function	Display	Comments
1.		XEQ b		Clear registers and flags
2.		XEQ MLR (asg $\sqrt{x}$)	No. Z VRBLS?	Initiate for data loading
	3	R/S	125.129	
3.		XEQ X (asg $\Sigma+$)	Z_1 = ?	Load first set of data
	0	R/S	Z_2 = ?	
	136	R/S	Z_3 = ?	
	106	R/S	$\hat{Y}$ = ?	
	103	R/S	1	
		XEQ X (asg $\Sigma+$)	Z_1 = ?	Load second set of data
	1	R/S	Z_2 = ?	
	140	R/S	Z_3 = ?	
	103	R/S	$\hat{Y}$ = ?	
	108	R/S	2	
	Repeat this process for each set of data. The last set entered will be as follows.			
		XEQ X (asg $\Sigma+$)	Z_1 = ?	
	19	R/S	Z_2 = ?	Load 13th set of data
	175	R/S	Z_3 = ?	
	109	R/S	$\hat{Y}$ = ?	
	103	R/S	13	

SOLUTION:

	Input	Function	Display	Comments
3a.	As a check that the data was loaded accurately			
	PRINTER ON			
	115.129	XEQ PRREGx	R115= 13.00000000	
			R116= 134.0000000	
			R117= 2177.000000	
			R118= 1376.000000	
			R119= 1994.000000	
			R120= 25315.00000	
			R121= 14158.00000	
			R122= 409443.0000	
			R123= 233148.0000	
			R124= 146738.0000	
			R125= 146855.0000	
			R126= 1377.000000	
			R127= 14224.00000	
			R128= 235519.0000	
			R129= 145923.0000	
4.		XEQ RG	X1=116.818867	Calculate regression coefficients,
			X2=-0.809588	coefficient of correlation for
			X3=0.180353	the sample and universe, standard
			X4=-0.309440	error of estimate for the sample
			R↑2= 0.810994	and universe and standard
			R↑2↓U= 0.747992	deviation of the dependent (Y)
			SY:Z= 3.810943	variable
			SY:Z↓U= 4.580183	
			sY= 9.123793	
5.		XEQ F	βYZ3=-0.323786	Calculate net regression coefficients
			βYZ2= 1.208873	
			βYZ1=-0.634084	
6.		XEQ G	X1=100.501051	Calculate partial correlation
			X2=0.117933	coefficients
			X3=-0.135360	
			r↑2YZ1= 0.572419	
			rYZ1=-0.756584	
			X1=88.479653	
			X2=0.056061	
			X3=0.159340	
			r↑2YZ2= 0.805294	
			rYZ2= 0.897382	
			X1=87.268161	
			X2=-0.665401	
			X3=0.152355	
			r↑2YZ3= 0.297665	
			rYZ3=-0.545587	

SOLUTION:

	Input	Function	Display	Comments
7.		XEQ Y (asg LN)	Z1=?	Predict $\hat{Y}$
	12	R/S	Z2=?	
	150	R/S	Z3=?	
	100	R/S	Y= 103.212794	

Note: Steps 4, 5, 6 and 7 may be performed in any order, or one or more may be omitted.

USER INSTRUCTIONS

User Mode | SIZE: (HP-41C) 170

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
I.	Clear flags and registers		XEQ b	
II.	Initiate for ~~data~~ loading		XEQ MLR (asg $\sqrt{x}$)	No. Z VRBLS?
	Insert ~~number~~ of Z variables	N	R/S	
III.	1. Enter Data		XEQ X (asg $\Sigma+$)	Z_1=?
	Enter each Z_1 value in turn as	Z_1	R/S	Z_2=?
	prompted by number, then the Y-	Z_2	R/S etc until	Z_n=?
	value when the Y-prompt appears.	Z_n	R/S	Y=?
	At the end the display shows the	Y	R/S	n
	number of sets of data entered.			
	2. Repeat III.1. until all data is		Same as III.1.	
	entered			
	3. To correct last entry		R/S	n-1
	4. To delete or subtract a set of data		XEQ a	Z_1=?
		Z_1	R/S	Z_2=?
		Z_2	R/S etc until	Z_n=?
		Z_n	R/S	Y=?
		Y	R/S	n-1
IV.	Fit Multiple Linear Regression		XEQ RG (asg 1/X)	X_1
				X_2 etc
				X_n
				R↑2=

USER INSTRUCTIONS

User Mode | SIZE: (HP-41C) 170

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
				R↑2↓U =
				SY:X =
				SY:X↓U =
				s_Y =
V.	Calculate β coefficients, i.e., net		XEQ F	βYZ_n
	regression coefficients in terms of			βYZ_{n-1} etc
	its standard deviation.			βYZ1
VI.	Calculate partial correlation		XEQ G	X_1
	coefficients. All partial correlation			X_2 etc
	coefficients will be calculated in			X_{n-1}
	turn and each printed and labeled			r^2YZ1
	until finished. The X_1 etc. X_{n-1}			rYZ1
	output represents to regression			X_1
	coefficients for a regression fit of			X_2 etc
	Y with the dependent variables when			X_{n-1}
	one (represented by the subscript of			r^2YZ2
	rYZi) is left out.			rYZ2 etc
				X_1
				X_2 etc
				X_{n-1}
				r^2YZn
				rYZn

USER INSTRUCTIONS

User Mode | SIZE: (HP-41C) 170

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
VII.	To predict $\hat{Y}$		XEQ Y (asg LN)	Z_1=?
	The various Z-values are prompted	Z_1	R/S	Z_2=?
	and entered and the $\hat{Y}$ is printed.	Z_2	R/S etc to	Z_n=?
		Z_n	R/S	Y=(predicted value)
VIII.	Note: Steps V, VI and VII may be performed in any order when there are 7 or less independent variables. Where there are 8 variables the changes mentioned in Program Description I are necessary to perform step VI.			

PROGRAM LISTING

☐ 67 ☐ 97 ☐ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
01	◆LBL b		Clear Flags and
02	CF 00		Registers
03	CF 01		
04	CF 02		
05	CF 03		
06	CLRG		
07	RTN		
08	◆LBL "MLR "		Initiate for data entry
09	"NO. Z VRBLS?"		
10	PROMPT		
11	STO 14		
12	1 E-3		
13	STO 10		
14	*		
15	1		
16	+		
17	STO 11		Controls for
18	STO 12		cross products
19	XEQ 20		Controls for
20	STO Y		storing n,ΣZi and
21	LASTX		ΣZiZj
22	X<>Y		
23	RCL 10		
24	*		
25	+		
26	STO 13		
27	RCL 14		Control for
28	2		storing ΣY^2, ΣY
29	+		and ΣZiY
30	RCL 10		
31	*		
32	+		
33	FRC		
34	+		
35	ISG X		
36	STO 15		
37	SF 03		R_{14}= # Variables
38	RTN		
39	◆LBL a		Subtract data
40	SF 00		
41	◆LBL "X"		Enter data
42	RCL 13		Control Zi data
43	STO 00		storage
44	FIX 0		
45	CF 29		
46	"Z"		Prompt for Zi data
47	ARCL 12		
48	"⊢=?"		
49	FC? 02		By-pass to correct
50	PROMPT		last data
51	FC? 02		
52	STO IND 12		Store in Ri for cross products
53	ISG 12		
54	GTO "X"		
55	RCL 11		Restore Control #
56	STO 12		
57	1		Store n
58	FS? 00		Subtract n
59	CHS		
60	ST+ IND 00		Σn
61	ISG 00		
62	SF 04		Control for ΣZi
63	XEQ 01		Store ΣZi
64	ISG 00		
65	CF 04		
66	RCL 11		Restore
67	FRC		$C_{O_{N_{T_{R_{O_L}}}}}$
68	STO 11		
69	◆LBL 03		
70	ISG 11		
71	RCL 11		
72	STO 12		Number
73	XEQ 01		Store ΣZiZj
74	ISG 00		
75	GTO 03		
76	RCL 12		Restore Control
77	STO 11		Number
78	RCL 14		
79	-		
80	STO 12		
81	SF 29		
82	◆LBL 04		
83	ISG 00		
84	GTO 01		
85	RCL 15		Store "Y" Control
86	STO 00		number
87	"Y=?"		Prompt for Y
88	FC? 02		data
89	PROMPT		
90	FS?C 02		
91	RCL IND 11		
92	STO IND 11		

Note: Refer to "HP-41C OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for specific information on keystrokes. The Function Index is found at the very back of the Handbook. Refer to Appendix E in 67 or 97 "OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for exact keystrokes.

012990

PROGRAM LISTING

☐ 67 ☐ 97 ☐ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
93	RCL X		
94	FS? 00		
95	CHS		
96	*		Y^2
97	ST+ IND 00		ΣY^2
98	ISG 00		
99	LASTX		Y
100	ST+ IND 00		ΣY
101	ISG 00		
102	XEQ 01		Store ΣZiY
103	RCL 11		Restore Control
104	RCL 14		number
105	-		
106	STO 11		
107	STO 12		
108	115		Display
109	RCL IND X		n
110	SF 03		R_{14}=# variables
111	CF 00		
112	RTN		
113	SF 02		Correct last data
114	GTO a		set stored
115	◆LBL 01		Subroutine to
116	RCL IND 11		store ΣZi and
117	FS? 04		$\Sigma ZiZj$
118	1		
119	FS? 00		
120	CHS		
121	RCL IND 12		
122	*		
123	ST+ IND 00		
124	ISG 12		Control to RTN
125	GTO 04		when last item
126	RTN		stored
127	◆LBL 13		Calculate Control
128	1.5		numbers to store
129	ST+ 03		Y data in proper
130	RCL 14		registers to
131	X↑2		store proper Y -
132	STO Y		data in matrix
133	LASTX		
134	ST+ Z		
135	2		columns.
136	*		
137	+		
138	14		
139	+		
140	RCL 10		
141	*		
142	+		
143	15		
144	+		
145	STO 04		
146	GTO 07		
147	◆LBL 20		Calculate control
148	3		numbers to calcu-
149	RCL 14		late R^2 and store
150	+		data.
151	RCL 14		
152	*		
153	2		
154	/		
155	114		Control for R^2
156	STO 02		
157	1		
158	+		Control for
159	STO 01		n
160	+		
161	RTN		
162	◆LBL "RG"		Regression fit
163	FIX 6		
164	XEQ 27		Store Matrix
165	STO 13		Order in R_{14}
166	X↑2		
167	14		Control for
168	+		storage of Matrix
169	1 E-3		
170	STO 10		
171	*		
172	15		
173	+		
174	STO 04		
175	115		Control for
176	ENTER↑		storing matrix
177	DSE 13		rows
178	RCL 14		
179	+		
180	RCL 10		
181	*		
182	+		

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01299C

PROGRAM LISTING

☐ 67 ☐ 97 ☐ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
183	RCL 10		
184	-		
185	STO 03		
186	RCL 10		
187	-		
188	STO 01		
189	STO 09		
190	0		Control indicating
191	STO 05		which matrix is
192	STO 06		being stored (0 =
193	◆LBL 07		row 1, etc.)
194	RCL IND 03		Matrix storage subroutine
195	STO IND 04		
196	ISG 03		
197	GTO 06		
198	1		Test when matrix
199	RCL 13		storage is in last
200	X=Y?		row and completed
201	DSE 03		
202	CLD		
203	X<Y?		
204	GTO 13		
205	RCL 10		Change control to
206	*		select data for
207	ST+ 03		last part of next
208	RCL 09		row of matrix.
209	1.001		Control # for
210	+		first part of next
211	STO 09		row of matrix
212	RCL 14		
213	1		
214	-		
215	STO 13		
216	1 E-5		
217	STO 07		
218	*		
219	+		
220	STO 01		
221	RCL 06		Matrix "row
222	1		number" control
223	+		
224	STO 05		
225	STO 06		
226	ISG 04		
227	DSE 13		
228	◆LBL 10		Matrix storage
229	RCL IND 01		subroutine - 1st part of row
230	STO IND 04		
231	ISG 01		
232	GTO 11		
233	DSE 05		Switch to last
234	GTO 12		part of matrix row
235	GTO 06		storage
236	◆LBL 11		
237	ISG 04		
238	GTO 10		
239	◆LBL 12		Adjustment of
240	RCL 13		control # to select
241	RCL 10		proper data for
242	*		matrix storage.
243	RCL 07		
244	-		
245	ST+ 01		
246	DSE 13		
247	GTO 11		
248	◆LBL 06		Determine when
249	ISG 04		matrix is complete
250	GTO 07		
251	FS?C 06		Return for partial
252	RTN		corr. coef. calc.
253	SF 04		Set flags to
254	SF 07		evaluate matrix
255	XROM "PV T"		Evaluate matrix
256	CF 03		Clear "left over" F3
257	XEQ 20		Control # for R^2
258	FC? 00		calc.
259	GTO 05		
260	RCL 14		Control # for
261	2		partial correlation
262	+		coefficient
263	+		calculation
264	RCL X		
265	RCL 14		
266	-		
267	RCL IND 02		
268	INT		
269	+		
270	X=Y?		
271	GTO 02		
272	1 E3		

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PROGRAM LISTING

☐ 67 ☐ 97 ☐ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
273	/		
274	+		
275	GTO 05		
276	◆LBL 02		
277	1		
278	-		
279	◆LBL 05		Control # for regression coef.
280	STO 04		
281	RCL 10		
282	1 E-3		
283	+		
284	STO 12		
285	CLST		
286	◆LBL 14		Calculate R^2
287	RCL IND 10		
288	RCL IND 04		
289	*		
290	+		
291	DSE 04		
292	GTO 02		
293	RCL 04		Adjustment for partial corre. coef. calc.
294	INT		
295	STO 04		
296	RDN		
297	DSE 04		
298	◆LBL 02		Control of R^2 and r^2 calculation
299	DSE 10		
300	GTO 14		
301	RCL IND 04		ΣY^2
302	ISG 04		
303	CLD		
304	RCL IND 04		ΣY
305	X↑2		$(\Sigma Y)^2$
306	RCL IND 01		n
307	STO 03		
308	/		$\Sigma Y^2/n$
309	ST- Z		
310	-		Σy^2
311	STO 11		
312	/		R^2
313	FS?C 00		Return for r^2 calculation
314	RTN		
315	STO 00		
316	"R↑2"		Print R^2
317	XEQ 17		
318	CHS		
319	1		
320	+		$(1 - R^2)$
321	RCL 03		
322	LASTX		
323	-		(n - 1)
324	RCL 03		
325	RCL 14		
326	-		(n - m)
327	STO 08		
328	/		(n-1)/(n-m)
329	*		
330	CHS		
331	1		
332	+		R^2U
333	"R↑2"		Print R^2U
334	XEQ 12		
335	1		
336	RCL 00		
337	-		$(1-R^2)$
338	RCL 11		Σy^2
339	*		
340	STO 07		
341	RCL 03		
342	/		
343	SQRT		
344	"SY:Z"		$S_{Y:Z}$
345	XEQ 17		
346	RCL 07		
347	RCL 08		
348	X≠0?		
349	GTO 02		
350	RDN		Adjustment for n=m
351	RCL 03		
352	◆LBL 02		
353	/		
354	SQRT		
355	"SY:Z"		$S_{Y:ZU}$
356	XEQ 12		
357	RCL 11		
358	RCL 03		
359	1		
360	-		
361	/		s_Y
362	SQRT		
363	SF 13		Print s_Y

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01299C

PROGRAM LISTING

☐ 67 ☐ 97 ☐ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
364	"S"		
365	ACA		
366	CF 13		
367	"Y"		
368	GTO 17		
369◆	LBL 12		Print subroutine for R_U^2 and $S_{Y:ZU}$
370	ACA		
371	7		
372	ACCHR		
373	RDN		
374	"U"		
375◆	LBL 17		
376	"⊢="		Print subroutine
377	ACA		
378	FIX 6		
379	ACX		
380	PRBUF		
381	RTN		
382◆	LBL G		r^2's routine control for r^2 calc.
383	1.003		
384	STO IND 02		
385	RCL 12		Control to restore
386	INT		and preserve
387	STO 09		regression
388	1		coefficients and
389	+		R^2 while calculating r^2.
390	RCL X		
391	1 E-3		
392	*		
393	+		
394	RCL 14		
395	+		
396	STO 10		
397	XEQ 21		Restore regression coefficients and R^2 while calculating r^2.
398	RCL 00		
399	STO IND 10		
400	RCL 10		
401	STO 00		
402◆	LBL 30		Calculate r^2
403	SF 06		
404	XEQ "RG"		
405	RCL 14		Control number to
406	RCL IND 02		delete row of matrix in which Z_i (i = row # in
407	INT		
408	*		
409	15		R_{114}) appears to
410	+		calculate r_{YZi}
411	RCL 14		
412	1		
413	−		
414	RCL Y		
415	+		
416	RCL 10		
417	*		
418	+		
419	0		Deletes row i of
420	XEQ 22		matrix
421	1 E-2		
422	RCL 14		Control number
423	*		to delete column
424	LASTX		of matrix in
425	X↑2		which Z_i appears.
426	LASTX		
427	2		
428	*		
429	+		
430	14		
431	+		
432	+		
433	RCL 10		
434	*		
435	RCL IND 02		
436	INT		
437	15		
438	+		
439	+		
440	0		Delete column
441	XEQ 22		of matrix
442	RCL 14		Control number
443	X↑2		to "pack" matrix
444	14		into order (N-1)
445	+		to determine
446	RCL 10		r^2_{YZi}
447	*		
448	15		
449	+		
450	14.2		
451	XEQ 23		Pack matrix
452	RCL 14		Control number to
453	1.002		pack "Location of
454	*		Columns" section
455	ST+ Z		of matrix.

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012990

PROGRAM LISTING

☐ 67 ☐ 97 ☐ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
456	RDN		
457	LASTX		
458	RCL 14		
459	DSE X		
460	STO 14		
461	*		
462	+		
463	XEQ 23		Pack Y-data of
464	SF 00		matrix
465	XEQ 06		Evaluate matrix
466	CHS		
467	1		
468	+		$(1-R^2_{YZi})$
469	1		
470	RCL IND 00		
471	-		$(1-R^2)$
472	X<>Y		
473	/		
474	CHS		
475	1		
476	+		r^2_{YZi}
477	SF 07		Print r^2
478	XEQ 00		
479	SQRT		r
480	RCL 00		Routine to deter-
481	RCL IND 02		mine the sign of
482	+		r - same as that
483	1		of regression
484	+		coefficient
485	RDN		
486	RCL IND T		
487	SIGN		
488	*		
489	XEQ 00		Print r
490	SF 03		R_{14} = # Variables
491	ISG IND 02		Control for next r
492	GTO 30		
493	STOP		
494	◆LBL 00		Subroutine to
495	SF 13		print r^2 and r
496	"R"		
497	ACA		
498	CF 13		
499	FIX 0		
500	FS? 07		
501	"↑2YZ"		
502	FC?C 07		
503	"YZ"		
504	ARCL IND 02		
505	GTO 17		Subroutine to
506	◆LBL 22		delete unnecessary
507	STO IND Y		rows and columns of matrix to
508	ISG Y		calculate r^2
509	GTO 22		
510	RTN		
511	◆LBL 23		Routine to pack
512	RCL IND Y		matrix
513	X=0?		
514	GTO 02		
515	ISG Y		
516	STO IND Y		
517	◆LBL 02		
518	RDN		
519	ISG Y		
520	GTO 23		
521	RTN		
522	◆LBL F		β routine
523	FS? 03		Restore X_n's and
524	XEQ 25		R^2 to original
525	RCL 14		registers
526	1		Control for ΣZ_i
527	-		
528	STO 08		
529	115		
530	+		
531	STO 05		
532	LASTX		
533	RCL 08		
534	X↑2		Control for ΣZ_i^2
535	+		
536	1 E-5		
537	STO 07		
538	+		
539	STO 04		
540	RCL 12		
541	1 E-3		
542	-		
543	STO 10		

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012990

PROGRAM LISTING

☐ 67 ☐ 97 ☐ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
544	◆LBL 08		
545	RCL IND 04		Σz_i^2
546	RCL IND 05		ΣZ_i
547	X↑2		$(\Sigma Z_i)^2$
548	RCL IND 01		n
549	/		$(\Sigma Z_i)^2/n$
550	-		Σz_i^2
551	RCL 11		Σy^2
552	/		$\Sigma z_i^2/\Sigma y^2$
553	SQRT		
554	RCL IND 10		X_i
555	*		βYZi
556	5		Print β
557	ACCHR		
558	RDN		
559	FIX 0		
560	"YZ"		
561	ARCL 08		
562	XEQ 17		
563	RCL 07		Adjust control
564	ST+ 04		for ΣZ^2
565	DSE 05		Controls ΣZ_i
566	DSE 04		" Σz_i^2
567	DSE 10		" X_i
568	DSE 08		" β calc.
569	GTO 08		
570	STOP		
571	◆LBL 27		Subroutine to store
572	RCL 14		matrix order in
573	1		R_{14} & clear F3
574	FC?C 03		
575	CLX		
576	+		
577	STO 14		
578	RTN		
579	◆LBL "Y"		Prediction routine
580	FS? 03		Restore X_n's and
581	XEQ 25		R^2 to original reg.
582	RCL 12		Control for
583	INT		regression
584	LASTX		coefficients
585	FRC		
586	1 E3		
587	*		
588	X<>Y		
589	LASTX		
590	/		
591	+		
592	STO 04		
593	1.1		Control for Z
594	STO 05		prompt
595	FIX 0		
596	CF 29		
597	RCL IND 04		X_1
598	ISG 04		
599	◆LBL 09		
600	"Z"		Prompt for Z_i
601	ARCL 05		
602	"⊢=?"		
603	PROMPT		
604	RCL IND 04		X_n's
605	*		
606	+		
607	ISG 05		
608	ISG 04		
609	GTO 09		
610	FIX 6		$\hat{Y}$
611	"Y"		Print $\hat{Y}$
612	GTO 17		
613	◆LBL 25		Control numbers
614	RCL IND 00		to restore regression coef.
615	X<> 00		and R^2 to original
616	XEQ 27		registers.
617	+		
618	INT		
619	STO 09		
620	RCL 12		
621	1 E-3		
622	STO 05		
623	RCL 14		
624	2		
625	*		
626	*		
627	LASTX		
628	+		
629	1		
630	+		
631	+		
632	STO 12		

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01299C

PROGRAM LISTING

☐ 67 ☐ 97 ☐ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
633	RCL 05		
634	-		
635	STO 10		
636	◆LBL 21		Transfer regression coefficients and R^2 to proper registers for storage or return to original registers.
637	RCL IND 09		
638	STO IND 10		
639	DSE 09		
640	DSE 10		
641	GTO 21		
642	END		

CAT 1

LBL 'MLR
LBL 'X
LBL 'RG
LBL 'Y
END
1032 BYTES
.END.
04 BYTES

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
51			
60			
70			
80			
90			
00			

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
30			
40			
50			

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REGISTER CHART

"N" always refers to number of Independent Variables; "(N+1)" is the Matrix Order in this program.
On page 58 of Math Pac Booklet N is the matrix order and one greater than the N in this program.

No. of Independent Variables	Location of Ind. Variable (Z) Data	Location of Dep. Variable (Y) Data	Location of Matrix Regression Fit	Location of Y-Data in Matrix and Regression Coef.	Regression Coefficients Transferred in Partial Corr. Coef. Calculation
N	116 to 115 + N(N+3)/2	116 + N(N+3)/2 to 117+N+N(N+3)/2	15 to $(N+1)^2$ + 14	15+(N+1)(N+2) to 13 + $(N+2)^2$	14 + $(N+2)^2$ to 13+(N+2)(N+3)
1	116 - 117	118 - 120	15 - 18	21 - 22	23 - 25
2	116 - 120	121 - 124	15 - 23	27 - 29	30 - 33
3	116 - 124	125 - 129	15 - 30	35 - 38	39 - 43
4	116 - 129	130 - 135	15 - 39	45 - 49	50 - 55
5	116 - 135	136 - 142	15 - 50	57 - 62	63 - 69
6	116 - 142	143 - 150	15 - 63	71 - 77	78 - 85
7	116 - 150	151 - 159	15 - 78	87 - 94	95 - 103
8	116 - 159	160 - 169	15 - 95	105 - 113	See special note in text

Partial Correlation Coefficient Calculation

No. of Independent Variables	Size	Location of Matrix	Location of Y-Data in Matrix & Regression Coefficients
N		15 to 14 + N^2	15 + N(N+1) to 13 + $(N+1)^2$
1	121	15 - 15	17 - 17
2	125	15 - 18	21 - 22
3	130	15 - 23	27 - 29
4	136	15 - 30	35 - 38
5	143	15 - 39	45 - 49
6	151	15 - 50	57 - 62
7	160	15 - 63	71 - 77
8	170	15 - 78	87 - 94

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

01299C

Data Loading - Illustrating 8 Variables

DATA REGISTERS

00	Control	145	ΣZ4↑2
	Z1		ΣZ4Z5
	Z2		ΣZ4Z6
	Z3		ΣZ4Z7
	Z4		ΣZ4Z8
05	Z5	150	ΣZ5↑2
	Z6		ΣZ5Z6
	Z7		ΣZ5Z7
	Z8		ΣZ5Z8
	Y		ΣZ6↑2
10	.001	155	ΣZ6Z7
	Control		ΣZ6Z8
	Control		ΣZ7↑2
	Control		ΣZ7Z8
	N		ΣZ8↑2
15	Control	160	ΣY↑2
6 - 170	See Chart for		ΣY
	8 Variables -		ΣZ1Y
	R_{115}ff are		ΣZ2Y
			ΣZ3Y
115	Σn	165	ΣZ4Y
	ΣZ1		ΣZ5Y
	ΣZ2		ΣZ6Y
	ΣZ3		ΣZ7Y
	ΣZ4	169	ΣZ8Y
120	ΣZ5		
	ΣZ6		
	ΣZ7		
	ΣZ8		
	ΣZ1↑2		
125	ΣZ1Z2		
	ΣZ1Z3		
	ΣZ1Z4		
	ΣZ1Z5		
	ΣZ1Z6		
130	ΣZ1Z7		
	ΣZ1Z8		
	ΣZ2↑2		
	ΣZ2Z3		
	ΣZ2Z4		
135	ΣZ2Z5		
	ΣZ2Z6		
	ΣZ2Z7		
	ΣZ2Z8		
	ΣZ3↑2		
140	ΣZ3Z4		
	ΣZ3Z5		
	ΣZ3Z6		
	ΣZ3Z7		
	ΣZ3Z8		

STATUS

SIZE * TOT. REG. * USER MODE

ENG ____ FIX X SCI ____ ON X OFF ____

DEG ____ RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
00	C	Subtract Data	Add Data
		Evaluate r^2 Matrix	Evaluate R^2 Matrix
01			
02	C	Correct Last Entry	Prompt for data entry
03	C	$R_{14} = N$	$R_{14} = N + 1$
04	C	ΣZ_i Evaluate Matrix	$\Sigma Z_i Z_j$
05			
06	C	RTN for r^2 calc.	R^2 calc
07		Evaluate Matrix	
		Display r^2	Display r
13	C	Lower Case	Upper Case
29	C	Radix Point	No radix point

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY
LBL X	Σ+		
LBL RG	1/X		
LBL MLR	$\sqrt{X}$		
LBL Y	LN		

*See Register Chart. Total Registers = Size + 148.

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

01299C

DATA REGISTERS

	LBL's RG, F & Y	LBL G
00	R↑2	Control
	Control	
	Control	S
	Control→n	A
	Control	M
05	Control	E
	Control	
	.00001→(1-r↑	
	2)Σy↑2	A
	.00001→(1-R↑	S
	2)Σy↑2	
	n-m	
10	Control	L
	.001→Control	B
	Σy↑2	L
	Control	
	Control	F
	N→N+1	
15	See Chart on Page 21	
to		
169		

STATUS

SIZE _____ TOT. REG. _____ USER MODE

ENG _____ FIX X SCI _____ ON X OFF _____

DEG _____ RAD _____ GRAD _____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
		See Page 21	

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY
See Page 21			

PROGRAM REGISTERS NEEDED: 148

ROW 1 (1 : 8)

ROW 2 (8 : 9)

ROW 3 (9 : 13)

ROW 4 (14 : 23)

ROW 5 (24 : 35)

ROW 6 (36 : 41)

ROW 7 (42 : 48)

ROW 8 (48 : 54)

ROW 9 (55 : 63)

ROW 10 (63 : 71)

ROW 11 (72 : 80)

ROW 12 (81 : 87)

ROW 13 (88 : 94)

ROW 14 (95 : 102)

ROW 15 (102 : 110)

ROW 16 (111 : 117)

ROW 17 (118 : 125)

ROW 18 (126 : 134)

ROW 19 (134 : 144)

ROW 20 (145 : 155)

ROW 21 (155 : 162)

ROW 22 (162 : 169)

ROW 23 (169 : 177)

ROW 24 (177 : 189)

ROW 25 (190 : 198)

ROW 26 (199 : 208)

ROW 27 (209 : 216)

ROW 28 (216 : 226)

ROW 29 (227 : 233)

ROW 30 (234 : 242)

ROW 31 (243 : 250)

ROW 32 (251 : 257)

ROW 33 (257 : 266)

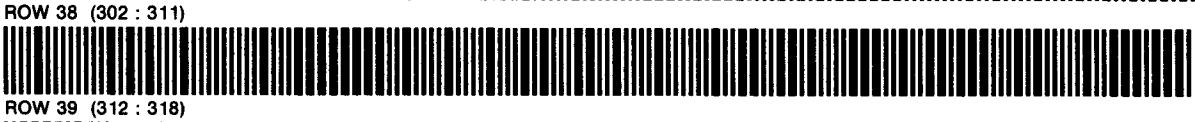
ROW 34 (267 : 275)

ROW 35 (275 : 284)

ROW 36 (285 : 293)

ROW 37 (294 : 302)

ROW 38 (302 : 311)



ROW 39 (312 : 318)

ROW 40 (319 : 331)

ROW 41 (332 : 339)

ROW 42 (340 : 346)

ROW 43 (347 : 355)

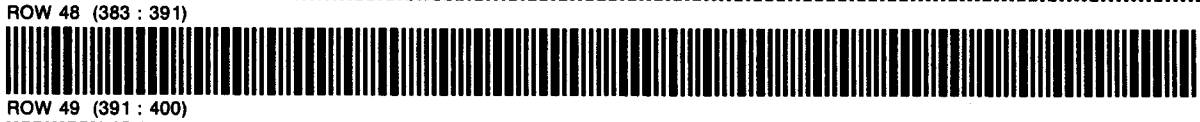
ROW 44 (355 : 364)

ROW 45 (364 : 370)

ROW 46 (371 : 377)

ROW 47 (378 : 383)

ROW 48 (383 : 391)



ROW 49 (391 : 400)

ROW 50 (401 : 407)

ROW 51 (408 : 418)

ROW 52 (419 : 426)

ROW 53 (427 : 437)

ROW 54 (437 : 446)

ROW 55 (447 : 453)

ROW 56 (453 : 460)

ROW 57 (461 : 468)

ROW 58 (469 : 478)

ROW 59 (478 : 488)

ROW 60 (489 : 495)

ROW 61 (495 : 501)

ROW 62 (501 : 505)

ROW 63 (506 : 512)

ROW 64 (512 : 520)

ROW 65 (520 : 527)

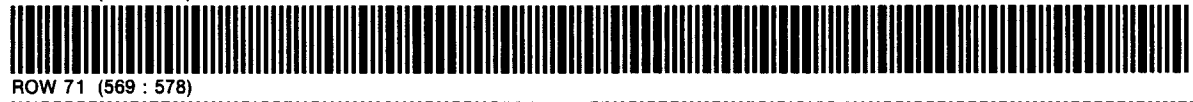
ROW 66 (528 : 536)

ROW 67 (536 : 545)

ROW 68 (545 : 554)

ROW 69 (555 : 562)

ROW 70 (562 : 568)



ROW 71 (569 : 578)

ROW 72 (579 : 584)

ROW 73 (585 : 593)

ROW 74 (594 : 601)

ROW 75 (601 : 608)

ROW 76 (608 : 614)

ROW 77 (614 : 621)

ROW 78 (621 : 633)

ROW 79 (634 : 641)

ROW 80 (641 : 642)