01530C PROGRAM DESCRIPTION I

Program Title CUBIC SPLINE INTERPOLATION

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Program Description, Equations, Variables SUPPOSE ONE IS GIVEN A SET OF REAL ABSCISSAS X(1),...X(N) AND CORRESPONDING ORDINATES Y(1),...Y(N). HERE X(1)< X(2) <...X(N), AND EACH Y(I) IS SOME OBSERVED OR MATHEMATICALLY DEFINED REAL NUMBER CORRESPONDING TO X(I). THE PROBLEM OF ONE-DIMENSIONAL INTERPOLATION IS TO CONSTRUCT A FUNCTION F SUCH THAT F[X(I)] = Y(I) FOR EACH I, SUCH THAT F[x] ASSUMES "REASONABLE" VALUES FOR ALL x BETWEEN THE DATA POINTS. THE CRITERION OF REASONABLENESS MAY VARY FROM PROBLEM TO PROBLEM AND MAY NEVER BE SATISFACTORILY UNDERSTOOD.

THE PURPOSES OF INTERPOLATION ARE MANY, BUT THEY USUALLY AMOUNT TO THE DESIRE TO HAVE A FAST ALGORITHM TO OBTAIN VALUES F[x] FOR x NOT IN THE TABLE OF DATA [X(I),Y(I)]. A SHORT TABLE OF DATA AND A SHORT INTERPOLATION SUBROUTINE MAY SUBSTITUTE FOR A VERY LONG TABLE OF FUNCTION

Necessary Accessories HP41CV OR HP41C WITH TWO TO FOUR MEMORY MODULES.

Operating Limits and Warnings THE PROGRAM USES REGISTERS 00 THROUGH (12+5*N) SO ANYTHING STORED IN THESE REGISTERS WILL BE LOST. UP TO 33 POINTS CAN BE SPLINE FIT DEPENDING ON THE NUMBER OF STORAGE REGISTERS AVAILABLE. X,Y PAIRS MUST BE INPUT IN INCREASING X ORDER.

Reference(s) AHLBERG,H.J.,E.N. NILSON AND J.L. WALSH (1967),"THE THEORY OF SPLINES AND THEIR APPLICATION",NEW YORK: ACADEMIC PRESS.

FORSYTHE, G.E, M.A. MALCOLM ABD C.B. MOLER, (1977),"COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS", ENGLEWOOD CLIFFS,N.J.: PRENTICE-HALL INC.

01530C

TO OUR CUSTOMERS:

The following information was supplied by our reviewer for your use with this program. The comments are intended to be helpful in avoiding any complications that may inadvertently arise.

THE HEWLETT PACKARD USERS' LIBRARY STAFF

This program does not consider the status of flag 21 in conjunction with its use of VIEW/AVIEW.

VALUES. SOMETIMES ONE MAY WANT TO GET F'[x] OR F''[x] AT INTERMEDIATE POINTS, OR ONE MAY WANT TO BE ABLE TO CALCULATE THE INTEGRAL OF F OVER AN ARBITRARY SUBINTERVAL (a,b) of the interval [X(1),X(N)]. THE HEART OF THE INTERPOLATION PROBLEM IS A DEFINITION OF HOW A REASONABLE FUNCTION WILL BEHAVE BETWEEN DATA POINTS. THE CUBIC SPLINE FUNCTION HAS BEEN SHOWN TO BEHAVE REASONABLY.

CUBIC SPLINE FUNCTIONS MODEL A VERY OLD MECHANICAL DEVICE. MECHANICAL SPLINES (FLEXIBLE STRIPS OF AN ELASTIC MATERIAL) HAVE LONG BEEN USED BY DRAFTSMEN AND ENGINEERS TO DRAW SMOOTH CURVES THROUGH A SET OF POINTS CALLED KNOTS. THE SPLINE ASSUMES THE SHAPE WHICH MINIMIZES ITS POTENTIAL ENERGY, AND BEAM THEORY STATES THAT THIS ENERGY IS PROPORTIONAL TO THE INTEGRAL WITH RESPECT TO ARC LENGTH OF THE SQUARE OF THE CURVATURE OF THE SPLINE. ELEMENTARY BEAM THEORY ALSO SUGGESTS THAT s(x) IS A CUBIC POLYNOMIAL BETWEEN EACH PAIR OF KNOTS AND ADJACENT POLYNOMIALS JOIN CONTINUOUSLY WITH CONTINUOUS FIRST AND SECOND DERIVATIVES.

EQUATIONS:

$$s(x) = Y(I) + B(I)*(x-X(I)) + C(I)*(x-X(I))**2 + D(I)*(x-X(I))**3$$

$$s'(x) = B(I) + 2*C(I)*(x-X(I)) + 3*D(I)*(x-X(I))**2$$

FOR $X(I) \leq x \leq X(I+1)$

$Y(I) = s(X(I))$ $\quad B(I) = s'(X(I))$ $\quad C(I) = s''(X(I)) / 2$

$D(I) = s'''(X(I)) / 6$

WHERE PRIME DENOTES DIFFERENTIATION, Ds/Dx

IF $c_1(x)$ AND $c_n(x)$ ARE THE UNIQUE CUBICS WHICH PASS THROUGH THE FIRST FOUR AND THE LAST FOUR DATA POINTS RESPECTIVELY, THE END CONDITIONS USED IN THIS PROGRAM MATCH THE THIRD DERIVATIVES OF THE SPLINE, s(x), TO THE THIRD DERIVATIVES OF THESE CUBICS AT THE END POINTS, I.E.

$s'''(X(1)) = c_1'''(X(1))$, AND

$s'''(X(N)) = c_n'''(X(N))$

THE MATRIX OF COEFFICIENTS IS SOLVED BY GAUSSIAN ELIMINATION WITHOUT SCALING OR PIVOTING. THIS IS POSSIBLE BECAUSE THE COEFFICIENT MATRIX HAS SEVERAL SPECIAL PROPERTIES -

1. THE MATRIX IS TRIDIAGONAL
2. THE MATRIX IS SYMMETRIC
3. FOR ANY CHOICE OF $X(1) < X(2) < \ldots < X(N)$ THE MATRIX IS NONSINGULAR AND DIAGONALLY DOMINANT

THE PROGRAM AS SHOWN IN THE LISTING MAY CONTAIN FEATURES WHICH SOME USERS MAY NOT FIND USEFUL IN THEIR PARTICULAR USE OF THE PROGRAM. THE TABLE BELOW SHOWS PORTIONS OF THE PROGRAM WHICH MAY BE DELETED WITHOUT AFFECTING PROGRAM OPERATION. THE BYTE SAVINGS ASSOCIATED WITH THE DELETIONS ARE ALSO SHOWN.

PROGRAM SECTION	STEPS	BYTE SAVINGS
SLOPE EVALUATION	485-503	31
LINEAR INTERPOLATION	124-127, 355-374	45
CHANGE/REVIEW ONE POINT	84-115	71
REVIEW ALL POINTS	55-83	60
	TOTAL	207
	OR 30 REGISTERS	

NOTE: IF TWO POINTS ARE INPUT INTERPOLATION/EXTRAPOLATION IS LINEAR. IF THREE POINTS ARE INPUT THE SPLINE SOLUTION IS THE SAME AS A LAGRANGE INTERPOLATING POLYNOMIAL OF DEGREE TWO. THESE FEATURES ARE ILLUSTRATED IN THE EXAMPLE PROBLEMS.

Sample Problem (Sketch if Desired)

PROBLEM 1

GIVEN THE TWO POINTS BELOW, LINEARLY INTERPOLATE THE VALUE AT X=3.0, AND EXTRAPOLATE THE VALUE AT X=10.0.

X	Y
1.0	2.0
4.0	8.0

SOLUTION:

Input	Function	Display	Comments
	[USER]		SET USER MODE
	[XEQ] SPLINE	NO. POINTS?	INITIALIZE
2	[R/S]	SET SIZE=23.	SIZE MUST BE ≥ THE SIZE SHOWN
	[XEQ] SIZE023		
	[A]	POINT 1.=?	
1.	[ENTER↑]	1.	
2.	[R/S]	POINT 2.=?	
4.	[ENTER↑]	4.	
8.	[R/S]	0.	
	[B]	WORKING	CALCULATE SPLINE
		DONE	COEFFICIENTS
3.	[C]	Y= 6.0000	INTERPOLATE
10.	[C]	Y= 20.0000	

Sample Problem (Sketch if Desired)

PROBLEM 2

GIVEN THE THREE POINTS BELOW ON THE CURVE $Y = X^2$, USE THE SPLINE INTERPOLATION PROGRAM TO OBTAIN VALUES AT X= 0.5, 5.0, and 10.0. THIS PROBLEM ILLUSTRATES THAT GIVEN THREE POINTS THE SPLINE COEFFICIENTS GIVE A SECOND DEGREE POLYNOMIAL.

X	Y
1.0	1.0
3.0	9.0
6.0	36.0

SOLUTION:

Input	Function	Display	Comments
	[USER]		SET USER MODE
	[XEQ] SPLINE	NO. POINTS?	
3	[R/S]	SET SIZE=28.	
	[XEQ] SIZE028		
	[A]	POINT 1.=?	RUN POINT INPUT ROUTINE
1.	[ENTER↑]	1.	
1.	[R/S]	POINT 2.=?	
3.	[ENTER↑]	3.	
9.	[R/S]	POINT 3.=?	
6.	[ENTER↑]	6.	
36.	[R/S]	0.	
	[c]		REVIEW ALL POINTS
		POINT 1.	
		X = 1.0000	
		Y = 1.0000	
		POINT 2.	
		X = 3.0000	
		Y = 9.0000	
		POINT 3.	
		X = 6.0000	
		Y = 36.0000	
	[B]	WORKING	CALCULATE SPLINE
		DONE	COEFFICIENTS

Sample Problem (Sketch if Desired)

PROBLEM 2 (CONT.)

SOLUTION:

Input	Function	Display	Comments
.5	[C]	Y = 0.2500	INTERPOLATE
5.	[C]	Y = 25.0000	
10.	[C]	Y = 100.0000	

Sample Problem (Sketch if Desired)

PROBLEM 3

GIVEN THE FOUR POINTS ON THE CURVE $Y = X^3$ LISTED BELOW USE THE SPLINE INTERPOLATION PROGRAM TO OBTAIN VALUES AT X= 0.50 , 4.5, AND 12.0. IN ADDITION OBTAIN THE SLOPE OF THE SPLINE AT EACH OF THESE POINTS. THIS PROBLEM ILLUSTRATES THAT GIVEN FOUR OR MORE POINTS THE SPLINE DOES INDEED PRODUCE A CUBIC POLYNOMIAL.

X	Y
1.0	1.0
4.0	64.0
6.0	216.0
10.0	1000.0

SOLUTION:

Input	Function	Display	Comments
	[USER]		SET USER MODE
	[f] [b]	NO. POINTS?	INITIALIZE
4	[R/S]	SET SIZE=33.	
	[XEQ] SIZE033		
	[A]	POINT 1.=?	RUN POINT INPUT ROUTINE
1.	[ENTER↑]	1.	
1.	[R/S]	POINT 2.=?	
4.	[ENTER↑]	4.	
65.	[R/S]	POINT 3.=?	
6.	[ENTER↑]	6.	
216.	[R/S]	POINT 4.=?	
10.	[ENTER↑]	10.	
1001.	[R/S]	0.	OOPS. POINT 4 IS WRONG
	[f] [a]	POINT 4.=?	
10.	[ENTER↑]	10.	
1000.	[R/S]	0.	
2	[f] [d]	POINT 2. X,Y =?	REVIEW POINT 2
	[R/S]	X = 4.0000 Y = 65.0000	OOPS. POINT 2 IS WRONG TOO
2	[f] [d]	POINT 2. X,Y =?	CHANGE POINT 2
4.	[ENTER↑]	4.	
64.	[R/S]	4.	

Sample Problem (Sketch if Desired)

PROBLEM 3 (CONT.)

SOLUTION:

Input	Function	Display	Comments
	[c]	POINT 1. X = 1.0000 Y = 1.0000 POINT 2. X = 4.0000 Y = 64.0000 POINT 3. X = 6.0000 Y = 216.0000 POINT 4. X = 10.0000 Y = 1000.0000	REVIEW ALL POINTS
	[B]	WORKING DONE	CALCULATE SPLINE COEFFICIENTS (54 SEC.)
.5	[C] [R/S]	Y = 0.1250 DY/DX= 0.7500	PERFORM INTERPOLATION AND SLOPE EVALUATION
4.5	[C] [R/S]	Y = 91.1250 DY/DX= 60.7500	
12.	[C] [R/S]	Y = 1728.0000 DY/DX= 432.0000	

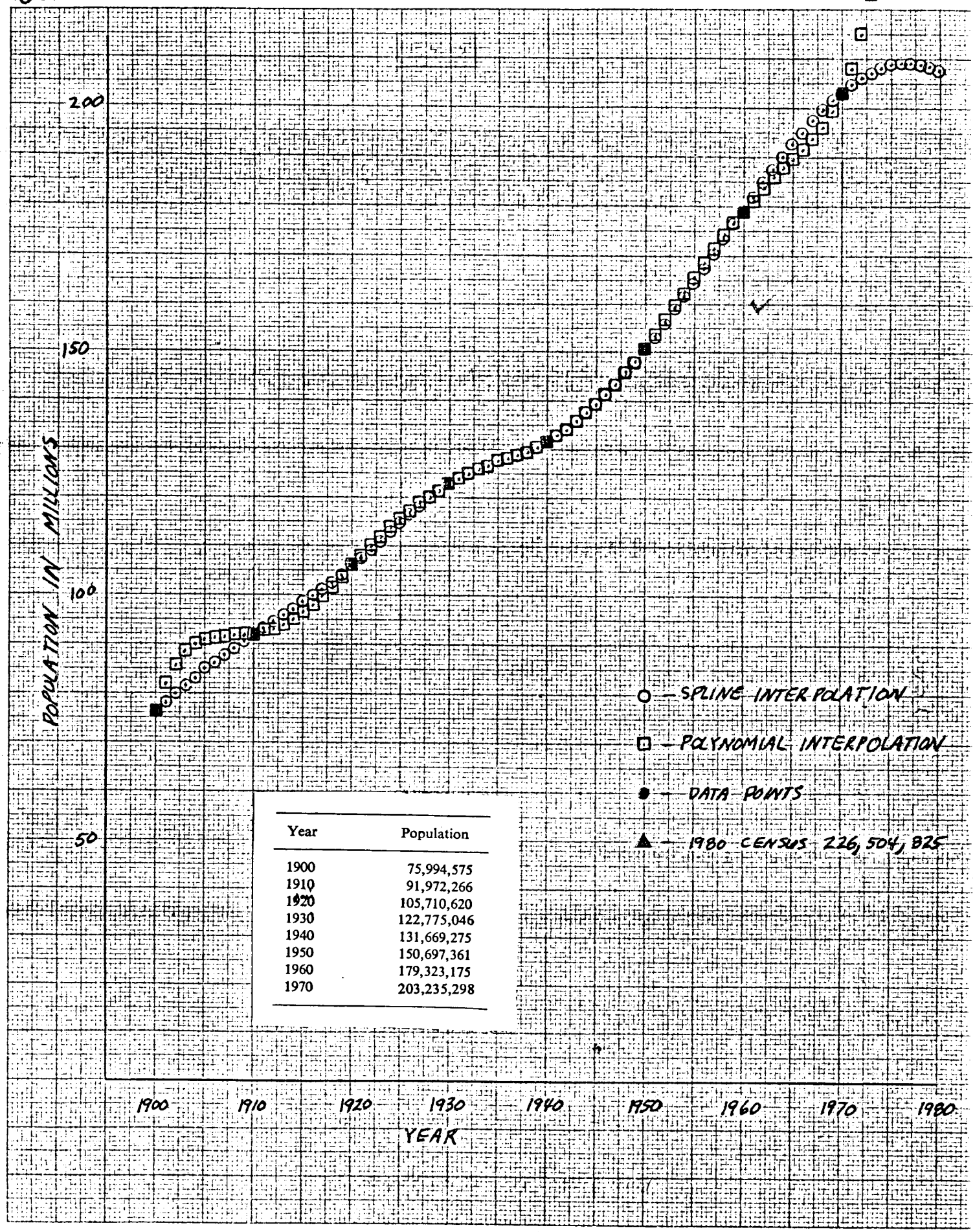
Sample Problem (Sketch if Desired)

PROBLEM 4

GIVEN THE CENSUS DATA FROM 1900 THROUGH 1970. DETERMINE THE POPULATION AND RATE OF POPULATION CHANGE PER YEAR FOR THE FOLLOWING YEARS - 1949, 1975, AND 1980. THE DATA AND THE SPLINE CURVE ARE SHOWN ON THE FOLLOWING PAGE. ALSO INCLUDED IS A POLYNOMIAL OF DEGREE SEVEN WHICH ALSO INTERPOLATES THE DATA. THIS PROBLEM ILLUSTRATES THE RELATIVE SMOOTHNESS OF THE SPLINE AND POLYNOMIAL INTERPOLATING FUNCTIONS. ALSO, THE EXTRAPOLATED SPLINE RESULT FOR THE 1980 CENSUS WHILE NOT EXACTLY PREDICTING THE CORRECT POPULATION DID COME SIGNIFICANTLY CLOSER THAN DID THE POLYNOMIAL EXTRAPOLATION.

SOLUTION:

Input	Function	Display	Comments
	[USER]		SET USER MODE
	[f] [b]	NO. POINTS?	
8	[R/S]	SET SIZE=53	
	[XEQ] SIZE053		
	[A]	POINT 1.=?	RUN POINT INPUT ROUTINE
1900.	[ENTER↑]	1900.	
75994575.	[R/S]	POINT 2.=?	
1910.	[ENTER↑]	1910.	
91972266.	[R/S]	POINT 3.=?	
1920.	[ENTER↑]	1920.	
105710620.	[R/S]	POINT 4.=?	
1930.	[ENTER↑]	1930.	
122775046.	[R/S]	POINT 5.=?	
1940.	[ENTER↑]	1940.	
131669275.	[R/S]	POINT 6.=?	
1950.	[ENTER↑]	1050.	
150697361.	[R/S]	POINT 7.=?	
1960.	[ENTER↑]	1960.	
179323175.	[R/S]	POINT 8.=?	
1970.	[ENTER↑]	1970.	
203235298.	[R/S]	0.	
	[B]	WORKING DONE	CALCULATE SPLINE COEFFICIENTS (98 SEC.)



Year	Population
1900	75,994,575
1910	91,972,266
1920	105,710,620
1930	122,775,046
1940	131,669,275
1950	150,697,361
1960	179,323,175
1970	203,235,298

Sample Problem (Sketch if Desired)

PROBLEM 4 (CONT.)

SOLUTION:

Input	Function	Display	Comments
1949.	[C] [R/S]	Y = 148,187,217.0 DY/DX= 2,451,350.307	
1975.	[C] [R/S]	Y = 208,818,291.2 DY/DX= 512,834.6840	
1980.	[C] [R/S]	Y = 207,767,335.7 DY/DX= -992,847.7920	ACTUAL 1980 CENSUS WAS 226,504,825.

USER INSTRUCTIONS

01530C

SIZE: 13 + 5*N (HP-41C)

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	LOAD SIDES 1 THROUGH 9 AND SET USER MODE.			
2	INITIALIZE CALCULATION AND SET SIZE IF NECESSARY	N	[XEQ] SPLINE OR [b] [R/S]	NO. POINTS? SET SIZE=(MM)
3	INPUT KNOTS IN INCREASING X ORDER ° INPUT X VALUE ° INPUT Y VALUE ° CONTINUE UNTIL ALL POINTS ARE INPUT ° OPTIONAL: IF A POINT IS INCORRECT THEN CONTINUE INPUTTING POINTS	 X(1) Y(1) ° °	[A] [ENTER↑] [R/S] ° ° [a]	POINT 1.=? X(1) POINT 2.=? ° ° POINT(I)=? POINT(I-1)=?
4	OPTIONAL: TO REVIEW A PARTICULAR DATA PAIR AFTER ALL POINTS ARE INPUT	I	[d] [R/S]	X,Y =? POINT (I) X = (X(I)) Y = (Y(I))
5	OPTIONAL: TO CHANGE A PARTICULAR DATA PAIR AFTER ALL POINTS ARE INPUT	I X(I) Y(I)	[d] [ENTER↑] [R/S]	X,Y =? X(I)
6	OPTIONAL: TO REVIEW ALL POINTS		[c]	POINT 1. X = (X(1)) Y = (Y(1)) ° °

USER INSTRUCTIONS

01580C

SIZE: 13 + 5*N (HP-41C)

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
7	CALCULATE SPLINE COEFFICIENTS		[redacted] [B]	WORKING DONE
8	PERFORM INTERPOLATION	x	[C]	Y = (y)
	° CALCULATE DERIVATIVE AT x		[R/S]	DY/DX=(Dy/Dx)
9	FOR A NEW x VALUE GO TO 8			
10	FOR A NEW CASE GO TO 2			

PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
01			
	01◆LBL "SPL INE"		
	02◆LBL b		INITIALIZE CALCULATION
	03 CF 00		
	04 FIX 0		
	05 "NO. POI NTS?"		
	06 PROMPT		
	07 STO 01		
	08 ENTER↑		
	09 ENTER↑		
	10 ENTER↑		
	11 13		
	12 STO 02		STORE X,Y,B,C, D REGISTER LOCATIONS
	13 +		
	14 STO 03		
	15 +		
	16 STO 04		
	17 +		
	18 STO 05		
	19 +		
	20 STO 06		
	21 1		
	22 ST- 01		
	23 X<>Y		
	24 RCL 01		
	25 +		
	26 1		
	27 +		
	28 "SET SIZ E="		
	29 ARCL X		
	30 AVIEW		
	31 XEQ 04		
	32 RTN		
	33◆LBL a		RESET INPUT POINT COUNTER
	34 XEQ 01		
	35◆LBL A		POINT INPUT ROUTINE
	36 RCL 07		
	37 12		
	38 −		
	39 "POINT "		
	40 ARCL X		
	41 "⊢=?"		
	42 PROMPT		
	43 STO IND 08		
	44 X<>Y		
	45 STO IND 07		
	46 XEQ 02		
	47 RCL 01		
	48 RCL 07		
	49 13		
	50 −		
	51 X<=Y?		
	52 GTO A		
	53 CLX		
	54 RTN		
	55◆LBL c		REVIEW ALL POINTS
	56 1		
	57 −		
	58 RCL 01		
	59 1		
	60 +		
	61 XEQ 08		
	62 XEQ 04		
	63◆LBL 00		
	64 FIX 0		
	65 " POINT "		
	66 ARCL 00		
	67 AVIEW		
	68 PSE		
	69 FIX 4		
	70 " X = "		
	71 ARCL IND 07		
	72 AVIEW		
	73 PSE		
	74 PSE		
	75 " Y = "		
	76 ARCL IND 08		
	77 AVIEW		
	78 PSE		
	79 1		
	80 XEQ 03		
	81 ISG 00		
	82 GTO 00		
	83 RTN		
	84◆LBL d		
	85 STO 00		CHANGE/REVIEW ONE POINT
	86 XEQ 04		
	87 RCL 00		
	88 1		
	89 −		

Note: Refer to "HP-41C OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for specific information on keystrokes. The Function Index is found at the very back of the Handbook. Refer to Appendix E in 67 or 97 "OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for exact keystrokes.

PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
90	XEQ 03		
91	FIX 0		
92	" POINT "		
93	ARCL 00		
94	AVIEW		
95	PSE		
96	CF 22		
97	" X,Y =? "		
98	PROMPT		
99	FC? 22		
100	GTO 09		
101	STO IND 08		
102	X<>Y		
103	STO IND 07		
104	RTN		
105	◆LBL 09		
106	FIX 4		
107	" X = "		
108	ARCL IND 07		
109	AVIEW		
110	PSE		
111	PSE		
112	" Y = "		
113	ARCL IND 08		
114	AVIEW		
115	RTN		
116	◆LBL B		
117	" WORKIN G"		CALCULATE SPLINE COEFFICIENTS
118	AVIEW		
119	CF 00		
120	RCL 01		
121	1		
122	X>Y?		
123	RTN		
124	CLX		
125	2		
126	X>Y?		
127	GTO 50		
128	XEQ 04		SET UP TRIDIAGONAL SYSTEM
129	XEQ 02		
130	RCL IND 07		
131	RCL IND 02		B=DIAGONAL C=OFFDIAGONAL D=RIGHT HAND SIDE
132	-		
133	STO IND 06		
134	RCL IND 08		
135	RCL IND 03		
136	-		
137	RCL IND 06		
138	/		
139	STO IND 10		
140	XEQ 07		
141	1		
142	+		
143	STO 00		
144	XEQ 01		
145	◆LBL 10		
146	XEQ 02		
147	XEQ 02		
148	RCL IND 07		
149	XEQ 01		
150	RCL IND 07		
151	-		
152	STO IND 11		
153	XEQ 01		
154	RCL IND 11		
155	XEQ 02		
156	RCL IND 11		
157	+		
158	ST+ X		
159	STO IND 09		
160	RCL IND 08		
161	RCL IND 11		
162	/		
163	CHS		
164	RCL IND 11		

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PROGRAM LISTING

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STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
165	1/X		
166	XEQ 02		
167	RCL IND 08		
168	*		
169	+		
170	STO IND 10		
171	XEQ 01		
172	RCL IND 10		
173	-		
174	STO IND 10		
175	ISG 00		
176	GTO 10		
177	RCL IND 06		END CONDITIONS, THIRD DERIVATIVES AT X(1) AND X(N) OBTAINED FROM DIVIDED DIFFERENCES
178	CHS		
179	STO IND 04		
180	RCL IND 11		
181	CHS		
182	XEQ 02		
183	STO IND 09		
184	CLX		
185	STO IND 05		
186	STO IND 10		
187	2		
188	RCL 01		
189	-		
190	X=0?		
191	GTO 15		
192	XEQ 04		
193	XEQ 02		
194	XEQ 02		
195	XEQ 02		
196	XEQ 05		
197	STO 12		
198	XEQ 01		
199	XEQ 05		
200	ST- 12		
201	RCL 12		
202	STO IND 05		
203	XEQ 04		
204	RCL 01		
205	XEQ 03		
206	XEQ 05		
207	STO 12		
208	XEQ 01		
209	XEQ 05		
210	ST- 12		
211	RCL 12		
212	RCL IND 11		
213	X↑2		
214	*		
215	CHS		
216	XEQ 01		
217	XEQ 01		
218	RCL IND 07		
219	CHS		
220	3		
221	XEQ 03		
222	RDN		
223	RCL IND 07		
224	+		
225	/		
226	STO IND 10		
227	XEQ 04		
228	3		
229	XEQ 03		
230	RCL IND 05		
231	RCL IND 06		
232	X↑2		
233	*		
234	RCL IND 07		
235	RCL IND 02		
236	-		
237	/		
238	STO IND 05		
239	XEQ 01		
240	*LBL 15		FOREWARD ELIMINATION
241	XEQ 01		
242	RCL 01		

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PROGRAM LISTING

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
243	1		
244	+		
245	XEQ 08		
246	1		
247	+		
248	STO 00		
249	◆LBL 20		
250	XEQ 01		
251	RCL IND		
11			
252	RCL IND		
09			
253	/		
254	STO 12		
255	RCL IND		
11			
256	*		
257	CHS		
258	XEQ 02		
259	RCL IND		
09			
260	+		
261	STO IND		
09			
262	XEQ 01		
263	RCL IND		
10			
264	XEQ 02		
265	RCL 12		
266	*		
267	CHS		
268	RCL IND		
10			
269	+		
270	STO IND		
10			
271	XEQ 02		
272	ISG 00		
273	GTO 20		
274	XEQ 01		BACK SUBSTITUTION
275	RCL IND		
10			
276	RCL IND		
09			
277	/		
278	STO IND		
10			
279	XEQ 07		
280	◆LBL 30		
281	RCL IND		
10			
282	XEQ 01		
283	RCL IND		
11			
284	*		
285	CHS		
286	RCL IND		
10			
287	+		
288	RCL IND		
09			
289	/		
290	STO IND		
10			
291	ISG 00		
292	GTO 30		
293	RCL 01		COMPUTE CUBIC POLYNOMIAL COEFFICIENTS
294	XEQ 03		
295	RCL IND		
10			
296	ST+ X		
297	XEQ 01		
298	RCL IND		
10			
299	+		
300	RCL IND		
11			
301	*		
302	STO 12		
303	XEQ 02		
304	RCL IND		
08			
305	XEQ 01		
306	RCL IND		
08			
307	-		
308	RCL IND		
11			
309	/		
310	ST+ 12		
311	XEQ 02		
312	RCL 12		
313	STO IND		
09			
314	XEQ 04		
315	XEQ 07		
316	◆LBL 40		
317	XEQ 02		

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PROGRAM LISTING

☐ 67 ☐ 97 · ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
318	RCL IND		
10			
319	XEQ 01		
320	RCL IND		
10			
321	ST+ X		
322	+		
323	RCL IND		
11			
324	*		
325	STO 12		
326	XEQ 02		
327	RCL IND		
08			
328	XEQ 01		
329	RCL IND		
08			
330	-		
331	RCL IND		
11			
332	/		
333	RCL 12		
334	-		
335	STO IND		
09			
336	XEQ 02		
337	RCL IND		
10			
338	XEQ 01		
339	RCL IND		
10			
340	-		
341	RCL IND		
11			
342	/		
343	STO IND		
11			
344	3		
345	ST* IND		
10			
346	XEQ 02		
347	ISG 00		
348	GTO 40		
349	3		
350	ST* IND		
10			
351	XEQ 01		
352	RCL IND		
11			
353	XEQ 02		
354	STO IND		
11			
355	GTO 51		
356	*LBL 50		
357	XEQ 04		TWO POINTS
358	XEQ 02		INPUT, LINEAR
359	RCL IND		INTERPOLATION
08			
360	RCL IND		
03			
361	-		
362	RCL IND		
07			
363	RCL IND		
02			
364	-		
365	/		
366	STO IND		
04			
367	CLX		
368	STO IND		
05			
369	STO IND		
06			
370	STO IND		
10			
371	STO IND		
11			
372	RCL IND		
04			
373	STO IND		
09			
374	*LBL 51		
375	BEEP		
376	FIX 4		
377	" DONE"		
378	AVIEW		
379	RTN		
380	*LBL 01		
381	-1		DECREMENT
382	XEQ 03		INDEX
383	RDN		REGISTERS
384	RTN		
385	*LBL 02		
386	1		INCREMENT INDEX
387	XEQ 03		REGISTERS
388	RDN		
389	RTN		

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PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
390	◆LBL 03		ADD X STACK VALUE TO INDEX REGISTERS
391	ST+ 07		
392	ST+ 08		
393	ST+ 09		
394	ST+ 10		
395	ST+ 11		
396	RTN		
397	◆LBL 04		INITIALIZE INDEX REGISTERS
398	RCL 02		
399	STO 07		
400	RCL 03		
401	STO 08		
402	RCL 04		
403	STO 09		
404	RCL 05		
405	STO 10		
406	RCL 06		
407	STO 11		
408	RTN		
409	◆LBL 05		
410	RCL IND 07		
411	XEQ 01		
412	XEQ 01		
413	RCL IND 07		
414	−		
415	XEQ 02		
416	RCL IND 10		
417	X<>Y		
418	/		
419	XEQ 02		
420	RTN		
421	◆LBL 07		SET UP LOOP INDICIES
422	RCL 01		
423	◆LBL 08		
424	1 E3		
425	/		
426	1		
427	+		
428	STO 00		
429	RTN		
430	◆LBL C		CUBIC SPLINE EVALUATION
431	STO 12		
432	FC? 00		
433	GTO 11		
434	RCL IND 07		CHECK IF INPUT
435	X>Y?		x VALUE IS IN THE LAST INTERVAL INTERPOLATED. IF YES GO TO EVALUATION ROUTINE
436	GTO 11		
437	RDN		
438	XEQ 02		
439	RCL IND 07		
440	RCL 12		
441	X>Y?		
442	GTO 11		
443	XEQ 01		
444	GTO 14		
445	◆LBL 11		BINARY SEARCH
446	XEQ 04		
447	RCL 12		
448	RCL IND 02		
449	X>Y?		
450	GTO 14		
451	XEQ 07		
452	◆LBL 12		
453	XEQ 02		
454	RCL 12		
455	RCL IND 07		
456	X>Y?		
457	GTO 13		
458	ISG 00		
459	GTO 12		
460	GTO 14		
461	◆LBL 13		
462	XEQ 01		
463	◆LBL 14		EVALUATE Y
464	SF 00		
465	RCL 12		
466	RCL IND 07		
467	−		
468	STO 00		
469	RCL IND 11		
470	*		
471	RCL IND 10		
472	+		
473	RCL 00		
474	*		
475	RCL IND 09		
476	+		

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PROGRAM LISTING

☐ 67 ☐ 97 ☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
	477 RCL 00		
	478 *		
	479 RCL IND 08		
	480 +		
	481 STO 12		
	482 " Y= "		
	483 ARCL 12		
	484 AVIEW		
	485 STOP		
	486 RCL IND 11		EVALUATE Dy/Dx
	487 3		
	488 *		
	489 RCL 00		
	490 *		
	491 RCL IND 10		
	492 2		
	493 *		
	494 +		
	495 RCL 00		
	496 *		
	497 RCL IND 09		
	498 +		
	499 STO 00		
	500 " DY/DX= "		
	501 ARCL 00		
	502 AVIEW		
	503 RTN		
	504 END		
40			
50			
51			
60			
70			
80			
90			
00			

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REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	USED
01	N
02	X LOCATION (13)
03	Y LOCATION (13+ N)
04	B LOCATION (13+2*N)
05	C LOCATION (13+3*N)
06	D LOCATION (13+4*N)
07	X INDEX
08	Y INDEX
09	B INDEX
10	C INDEX
11	D INDEX
12	USED

(13)	to	(12+ N)	X
(13+ N)	to	(12+2*N)	Y
(13+2*N)	to	(12+3*N)	B
(13+3*N)	to	(12+4*N)	C
(13+4*N)	to	(12+5*N)	D

STATUS

SIZE 13+5*N TOT. REG. 150+5*N USER MODE

ENG FIX SCI ON X OFF

DEG RAD GRAD 955 BYTES

FLAGS

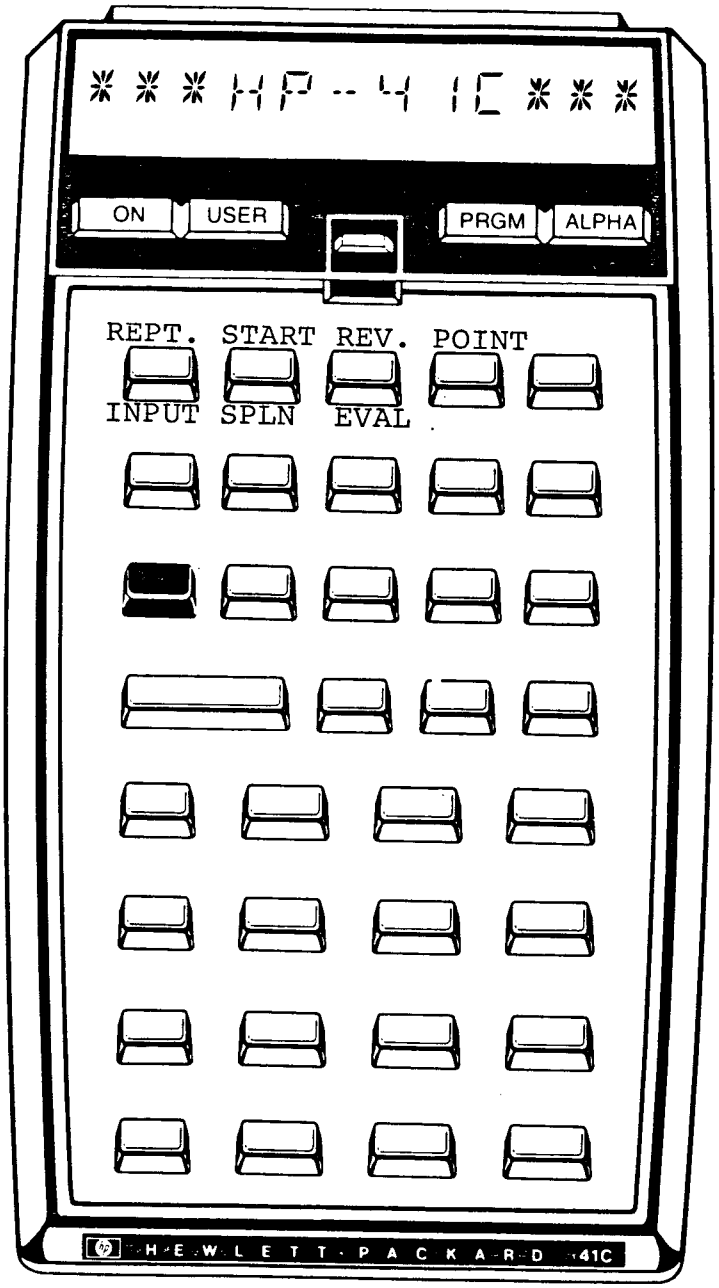
#	INIT S/C	SET INDICATES	CLEAR INDICATES
00	C	INTERPOLATION ROUTINE HAS BEEN USED	INTERPOLATION ROUTINE HAS NOT BEEN USED
22	C	DIGIT ENTERED	NO DIGIT ENTERED

ASSIGNMENTS

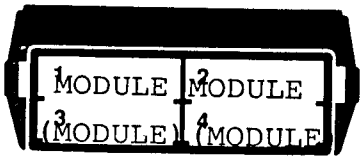
FUNCTION	KEY	FUNCTION	KEY

KEYBOARD CARD LABELING

KEYBOARD

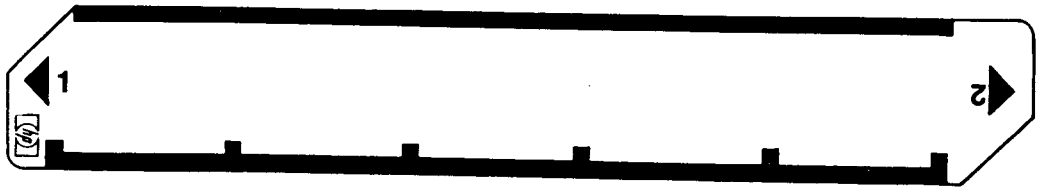


SYSTEM CONFIGURATION



MODULES 3 AND 4 ARE OPTIONAL

CARD



PROGRAM REGISTERS NEEDED: 137

ROW 1 (1 : 3)

ROW 2 (3 : 5)

ROW 3 (5 : 15)

ROW 4 (16 : 27)

ROW 5 (28 : 30)

ROW 6 (31 : 37)

ROW 7 (37 : 41)

ROW 8 (41 : 48)

ROW 9 (49 : 57)

ROW 10 (58 : 65)

ROW 11 (65 : 69)

ROW 12 (70 : 75)

ROW 13 (75 : 81)

ROW 14 (81 : 89)

ROW 15 (90 : 92)

ROW 16 (93 : 97)

ROW 17 (97 : 105)

ROW 18 (106 : 111)

ROW 19 (112 : 117)

ROW 20 (117 : 121)

ROW 21 (122 : 129)

ROW 22 (129 : 136)

ROW 23 (137 : 144)

ROW 24 (144 : 149)

ROW 25 (150 : 155)

ROW 26 (156 : 163)

ROW 27 (164 : 171)

ROW 28 (171 : 177)

ROW 29 (178 : 185)

ROW 30 (185 : 192)

ROW 31 (193 : 197)

ROW 32 (198 : 203)

ROW 33 (203 : 209)

ROW 34 (209 : 216)

ROW 35 (217 : 223)

ROW 36 (224 : 230)

ROW 37 (231 : 239)

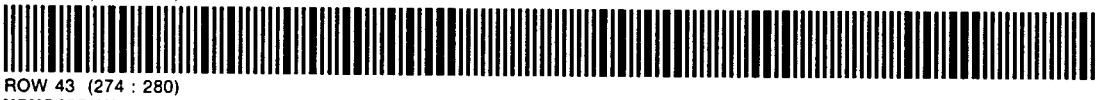
ROW 38 (239 : 245)

ROW 39 (246 : 253)

ROW 40 (254 : 261)

ROW 41 (262 : 268)

ROW 42 (269 : 274)



ROW 43 (274 : 280)

ROW 44 (281 : 288)

ROW 45 (288 : 294)

ROW 46 (295 : 301)

ROW 47 (302 : 308)

ROW 48 (308 : 314)

ROW 49 (315 : 319)

ROW 50 (320 : 327)

ROW 51 (327 : 335)

ROW 52 (335 : 341)

ROW 53 (341 : 348)

ROW 54 (348 : 353)

ROW 55 (354 : 358)

ROW 56 (359 : 366)

ROW 57 (367 : 373)

ROW 58 (374 : 378)

ROW 59 (379 : 387)

ROW 60 (387 : 395)

ROW 61 (395 : 407)

ROW 62 (408 : 414)

ROW 63 (415 : 422)

ROW 64 (423 : 432)

ROW 65 (432 : 439)

ROW 66 (439 : 446)

ROW 67 (446 : 453)

ROW 68 (453 : 460)

ROW 69 (461 : 469)

ROW 70 (469 : 479)

ROW 71 (479 : 486)

ROW 72 (486 : 497)

ROW 73 (497 : 501)

ROW 74 (501 : 504)