01733C PROGRAM SUBMITTAL

☒ New Program ☐ Revision to Program

Model No. ☐ 67 ☐ 97 ☒ 41C

Program Title NUMERICAL INTEGRATION

No. of Steps/Lines 94 Category No. 205

Category Name INTEGRATION

Abstract — 50 Word Maximum This program performs numerical integration using either Simpson's rule or the trapazoidal rule. The integrand may be an explicit function or may be specified at a discrete set of equally spaced points. For the discrete case, the user will be prompted for the function values. The program can be called as a subroutine, is compact, and uses only 7 data registers.

Necessary Accessories: none required

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If my program is accepted, my bonus choice is: (Please select two programs if your program is a revision.)

Acceptance Choice: ☐ FOUR PROGRAMS, ☒ CREDIT FOR FOUR PROGRAMS*, OR TWO PROGRAMS AND 10 BLANK CARDS.

4/19/82
1-21-82

* No partial credit will be given. Select all four programs at the same time.

Submittal Checklist: Please use the checklist below to insure submittal of all proper program documentation.

☒ Program Submittal	☒ Program Description II	☒ Program Listing(s)	☒ Registers, Status ...
☒ Program Description I	☒ User Instructions	☒ Magnetic Card(s)	☒ Keyboard, Card Labeling (optional)

ACKNOWLEDGMENT AND AGREEMENT

To the best of my knowledge, I have the right to contribute this program material without breaching any obligation concerning nondisclosure of proprietary or confidential information of other persons or organizations. I am contributing this program material on a nonconfidential nonobligatory basis to Hewlett-Packard Company ("HP") for inclusion in its program library, and I agree that HP may use, duplicate, modify, publish, and sell the program material, and authorize others to do so without obligation or liability of any kind. HP may publish my name and address, as the contributor, to facilitate user inquiries pertaining to this program material.

Signature Gary Goodman Date Jan. 8, 1982

PROGRAM DESCRIPTION I

Program Title Numerical Integration

Contributor's Name Gary Goodman

Address 2730 High Ridge Road

City Stamford **State/Country** Connecticut **Zip Code** 06903

Program Description, Equations, Variables This program performs numerical integration using Simpson's rule if an even number of divisions is chosen, or the trapazoidal rule if an odd number of divisions is chosen. The integrand may be an explicit function, programmed as an external subroutine, or it may be specified at a discrete set of equally spaced points. For the discrete cas the user will be prompted for the function value. The program can be executed as a main program or called as a subroutine.

The user must enter the function name if an explicit function is used, the lower limit of integration (a), the upper limit (b), and the number of divisions (n) into which the interval is to be divided. The function (f) is evaluated at n+1 equally spaced points: $x_i = a + ih$, $i=0,1,2,...n$; where $h = (b-a)/n$. The integral is then approximated using either of the two formulas shown on the following page.

Necessary Accessories None require, but the program will use the printer if connected.

Operating Limits and Warnings The function must be continuous on the interval and defined at the end points.

Reference(s) Scientific Analysis on the Pocket Calculator 2nd edition, by Jon M. Smith published by John Wiley & Sons, Inc. 1977

PROGRAM DESCRIPTION I

(Continued)

1. The trapezoidal rule:

$$\int_{x_0}^{x_n} f(x)\,dx \simeq \frac{h}{2}\left[f(x_0) + 2\sum_{j=1}^{n-1} f(x_j) + f(x_n)\right]$$

2. Simpson's rule:

$$\int_{x_0}^{x_n} f(x)\,dx \simeq \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

The error associated with each of these formulas is given by:

$\varepsilon \approx \frac{(b-a)h^2}{24} f''(\xi)$ for the trapazoidal rule, and

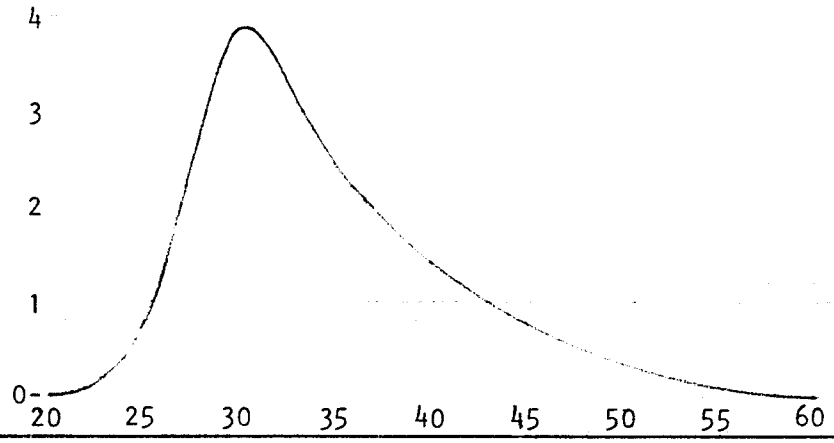
$\varepsilon \approx \frac{(b-a)h^4}{180} f''''(\xi)$ for Simpson's rule, where $(a \leq \xi \leq b)$.

Simpson's rule is generally more accurate than the trapazoidal rule even for small values of n, and as can be seen from the above error formulas, that as n increases (h decreases) the error associated with Simpson's rule decreases much more rapidly than the error associated with the trapazoidal rule. For this reason it is almost always better to use Simpson's rule when it can be applied, i.e. when n is even.

PROGRAM DESCRIPTION II

Sample Problem (Sketch if Desired)

Example 1. Find the total integral of the function whose graph is shown below. Use Simpson's rule with 10 divisions.



Example 2.
Using an Explicit Function

Evaluate the integral shown below:

$$\int_0^1 xe^{-x}dx$$

using Simpson's rule and the trapazoidal rule with 32 & 65 divisions respectively.

(The correct answer is 0.26424118)

SOLUTION:

	Input	Function	Display	Comments
1.		[XEQ] SIZE 008		minimum size
		USER		
		[GTO] ᵀINTG		Do NOT use [XEQ] ᵀINTG*
		[A]	FUNCTION?	no entry for discrete case
		[R/S]	a↗b	Enter limits of integration.
	20 [ENTER] 60	[R/S]	N?	Enter number of divisions.
	10	[R/S]	X0=20.000 F=	Enter F(20.000).
	0	[R/S]	X1=24.000 F=	" F(24.000).
	0.4	[R/S]	X2=28.000 F=	" F(28.000).
	2.7	[R/S]	X3=32.000 F=	" F(32.000).
	3.5	[R/S]	X4=36.000 F=	" F(36.000).
	2.2	[R/S]	X5=40.000 F=	" F(40.000).
	1.4	[R/S]	X6=44.000 F=	" F(44.000).
	0.9	[R/S]	X7=48.000 F=	" F(48.000).
	0.5	[R/S]	X8=52.000 F=	" F(52.000).
	0.23	[R/S]	X9=56.000 F=	" F(56.000).
	0.06	[R/S]	X10=60.000 F=	" F(60.000).
	0	[R/S]	47.333	Answer
2.	Enter the program for the explicit function. [GTO] .. , Then put calculator in program mode and enter the following subroutine. [LBL] ᵀS , [ENTER] , [CHS] , [e^x] , [×] Now take the calculator out of program mode.			Set display mode: [FIX] 9
		[GTO] ᵀINTG		note flag 01 is clear ⇒ Simpson's
		[A]	FUNCTION?	Enter function name "S".
	S	[R/S]	a↗b	Enter limits of integration.
	0 [ENTER] 1	[R/S]	N?	Enter number of divisions.
	32	[R/S]	0.264241106	Calc. Result with Simpson's rule
	65	[C]	0.264221394	Calc. Result with trap. rule note: the function name and the integration limits do not have to be reentered each time.

*[XEQ] INTG is for subroutine useage. See User Instructions.

USER INSTRUCTIONS

SIZE: (HP-41C) 007+

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1.	Load program and set USER mode.			
2.	If an explicit function is to be integrated:			
	Prepare to load function;		[GTO] ..	
	Switch to PRGM mode;		[PRGM]	
	Load function under desired label, (6 char. or less), reg 07 and up may be used by function;		LBL "_	
	Switch out of Prgm mode.		[PRGM]	
3.	Go to integration program.		[GTO] "INTG	
4.	Initialize program		[A]	FUNCTION?
5a.	If an explicit function is used, key in			
	the name of the function;	name	[R/S]	a/b
5b.	else for discrete points:		[R/S]	
6.	Input lower & upper limits of integration	a	[ENTER]	
		b	[B]	N?
7.	Enter number of divisions for integ.	N	[C]	
	Simpson's rule is used for even N, trapazoidal rule is used for odd N.			
8a.	For an explicit function:			$\int_a^b f(x)dx$
8b.	For the discrete case:			Xj=x.xxx F=
	key in the function value at x_j.	$f(x_j)$	[R/S]	
	Repeat this step for j=0,1,...,N.	$f(x_N)$	[R/S]	$\int_b^a$ etc. $f(x)dx$
9.	For a new function, go to step 2 or 4 as neccessary.			
10.	For new integration limits, go to step 6.			
11.	To change just the number of divisions, go to step 7.			
	Note: If N is not a positive integer,			
	the program will stop with the			
	message DATA ERROR.			

USER INSTRUCTIONS

SIZE: (HP-41C) 007+

For Use As a Subroutine

The calling program must:

1. Enter the name of the function in register 04 if an explicit function is to be integrated, or
   enter the number 4 in rgister 04 if the function values are to be entered as a set of discrete points,
2. Enter the lower limit of integration, a, in register 01,
3. Enter the upper limit of integration, b, in register 02,
4. Enter the number of divisions, N, in register 03,
   (If N is even, Simpson's rule will be used; if odd, the trapazoidal rule will be used. N must be a positive integer.)
5. Call the subroutine with an XEQ INTG instruction.

The result will be returned in both the x-register and register 00.

This subroutine uses registers 00 through 06 so these may not be used by the calling program except as specified above. Registers 00, 05, & 06 will be changed by the subroutine.

PROGRAM LISTING

☒ 41C

STEP/ LINE	KEY ENTRY	KEY CODE (67/97 only)	COMMENTS
01	✦LBL A		Starting point for use as main program
02	4		
03	"FUNCTIO N?"		
04	CF 23		
05	AON		
06	PROMPT		
07	AOFF		
08	STO 04		store function name
09	FS? 23		
10	ASTO 04		
11	" a↑b"		
12	PROMPT		
13	✦LBL B		
14	STO 02		store integ. limits
15	X<>Y		
16	STO 01		
17	" N?"		
18	PROMPT		
19	✦LBL C		
20	STO 03		store no. of div.
21	XEQ 00		call INTG subrtn.
22	VIEW X		display & print result
23	STOP		
24	✦LBL "INT G"		Subroutine entry point
25	✦LBL 00		
26	CF 01		
27	RCL 03		Test for valid N and select integration rule
28	OCT		
29	LOG		
30	RCL 03		
31	2		
32	MOD		
33	X≠0?		
34	SF 01		
35	RCL 02		
36	RCL 01		
37	−		
38	RCL 03		
39	/		step size
40	STO 05		
41	0		
42	STO 06		
43	RCL 01		lower limit, a
44	XEQ IND 04		f(a)
45	STO 00		
46	✦LBL 01		
47	1		increment step number
48	ST+ 06		
49	RCL 03		
50	RCL 06		test for last step
51	X=Y?		
52	GTO 02		
53	RCL 05		
54	*		
55	RCL 01		
56	+		x_i
57	XEQ IND 04		$f(x_i)$
58	ST+ X		
59	ST+ 00		
60	RCL 06		apply Simpson's formula if flag 01 is clr.
61	2		
62	MOD		
63	*		
64	FC? 01		
65	ST+ 00		
66	GTO 01		
67	✦LBL 02		
68	RCL 02		b
69	XEQ IND 04		f(b)
70	ST+ 00		
71	RCL 05		
72	FC? 01		Simpson's rule
73	3		
74	FS?C 01		Trapazoidal rule
75	2		
76	/		
77	ST* 00		
78	RCL 00		recall result
79	RTN		
80	✦LBL 04		Subroutine to prompt for f(x) at discrete pts.
81	CF 29		
82	FIX 0		
83	"X"		
84	ARCL 06		
85	"⊢="		
86	FIX 3		
87	ARCL X		
88	"⊢ F="		
89	PROMPT		
90	SCI 3		
91	ARCL X		
92	FS? 55		
93	PRA		
94	END		

Note: Refer to "HP-41C OWNER'S HANDBOOK AND PROGRAMMING GUIDE" for specific information on keystrokes. The Function Index is found at the very back of the Handbook.

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00 Σ & result
01 lower limit, a
02 upper limit, b
03 no. of div., N
04 function name
05 used
06 used
07 available for use by explicit function or main program

STATUS

SIZE 07+	TOT. REG. 30+		USER MODE
ENG	FIX	SCI	ON × OFF
DEG	RAD	GRAD	

FLAGS

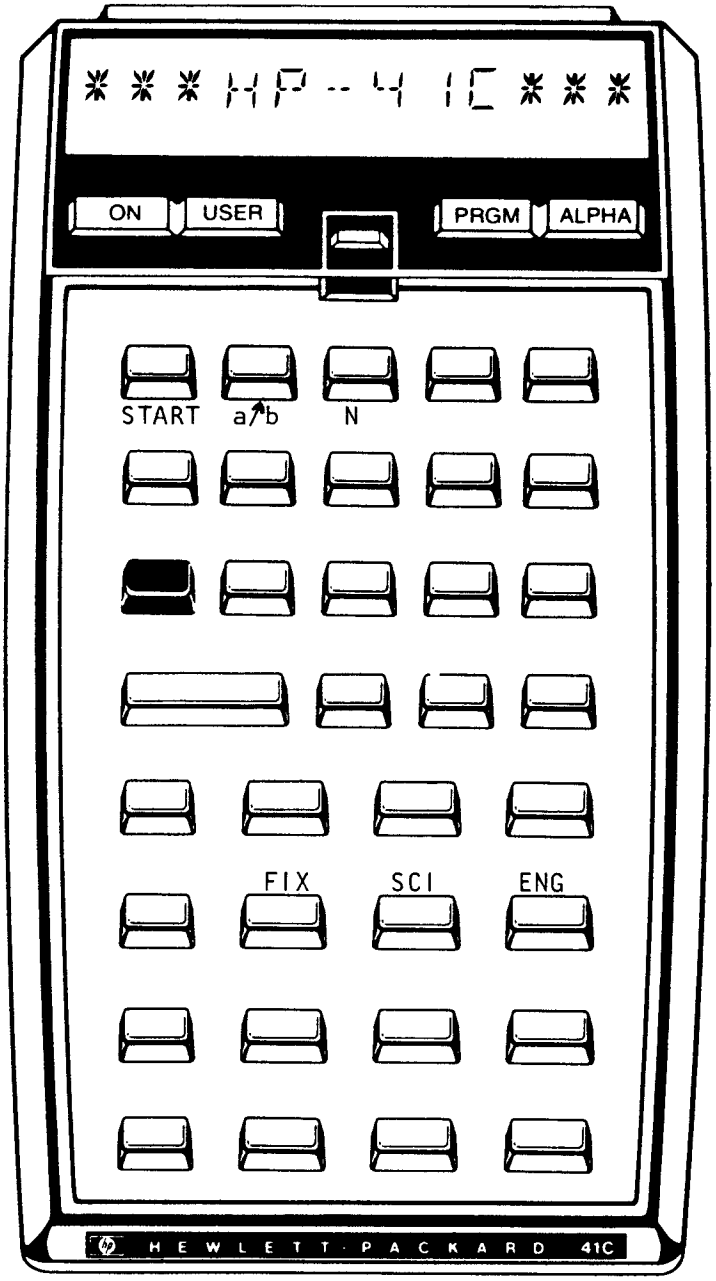
#	INIT S/C	SET INDICATES	CLEAR INDICATES
01		Trapazoidal rule	Simpson's rule
23		function name/ explicit functn	no function name/ discrete points
29			suppress dec. pt.
55		printer present	printer not attached

ASSIGNMENTS

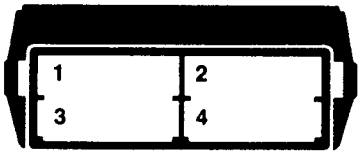
FUNCTION	KEY	FUNCTION	KEY

KEYBOARD CARD LABELING

KEYBOARD



SYSTEM CONFIGURATION



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