

Program Title CURVE FITTING

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Program Description, Equations, Variables Program fits a set of data points to:
 Straight lines, exponential curves, logarithmic curves and power curves.
 Calculates regression coefficients, correlation coefficient and projection of
 x and y values for each curve. Data need to be entered only once.

Equations used: see also page 3 and 4.

- | | |
|--------------------------|-------------------|
| 1. Linear Regression | $y = a + bx$ |
| 2. Exponential Curve Fit | $y = ae^{bx}$ |
| 3. Logarithmic Curve Fit | $y = a + b \ln x$ |
| 4. Power Curve Fit | $y = ax^b$ |

Operating Limits and Warnings Negative and zero values of x_i and/or y_i will cause machine error. This can be avoided by adding or subtracting an appropriate constant number to/from all x_i and/or y_i .

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Sketch(es)

Sample Problem(s) Listed are the sales for the first 6 months of a product's life. According to linear projection, what should be the sales after 12 months? When would sales reach the 150 unit per month mark assuming constant linear growth? What should the above results be for respectively an exponential, logarithmic or power relationship instead of a linear one?

Month	1	2	3	4	5	6
Sales	15	37	52	59	83	92

Solution(s) See page 5

Reference(s) See page 6

Equations used:

1. Linear Regression:

$$b = \frac{\sum xy - \sum x \sum y / n}{\sum x^2 - (\sum x)^2 / n}$$

$$a = \bar{y} - b\bar{x}$$

$$r = b \frac{s_x}{s_y}$$

$$\hat{y} = a + bx$$

$$\hat{x} = \frac{y - a}{b}$$

2. Exponential Curve Fit :

$$b = \frac{\sum x \ln y - \sum x \sum \ln y / n}{\sum x^2 - (\sum x)^2 / n}$$

$$a = \exp \left[\overline{\ln y - bx} \right]$$

$$r = b \frac{s_{\ln x}}{s_y}$$

$$\hat{y} = \exp \left[\ln a + bx \right]$$

$$\hat{x} = \frac{\ln y - \ln a}{b}$$

3. Logarithmic Curve Fit:

$$b = \frac{\sum y \ln x - \sum \ln x \sum y / n}{\sum (\ln x)^2 - (\sum \ln x)^2 / n}$$

$$a = \bar{y} - b \overline{\ln x}$$

$$r = b \frac{s_x}{s_{\ln y}}$$

$$\hat{y} = \left[a + b \ln x \right]$$

$$\hat{x} = \exp \left[\frac{y - a}{b} \right]$$

4. Power Curve Fit

$$b = \frac{\sum \ln x \ln y - \sum \ln x \sum \ln y / n}{\sum (\ln x)^2 - (\sum \ln x)^2 / n}$$

$$a = \exp \left[\overline{\ln y} - b \overline{\ln x} \right]$$

$$r = b \frac{s_{\ln x}}{s_{\ln y}}$$

$$\hat{y} = \exp \left[\ln a + b \ln x \right]$$

$$\hat{x} = \exp \left[\frac{\ln y - \ln a}{b} \right]$$

Where: $\bar{x} = \frac{\sum x}{n}$;

$\bar{y} = \frac{\sum y}{n}$

$$\overline{\ln x} = \frac{\sum \ln x}{n} ;$$

$$\overline{\ln y} = \frac{\sum \ln y}{n}$$

$$s_x = \left[\frac{\sum x^2 - (\sum x)^2 / n}{n-1} \right]^{\frac{1}{2}}$$

$$s_{\ln x} = \left[\frac{\sum (\ln x)^2 - (\sum \ln x)^2 / n}{n-1} \right]^{\frac{1}{2}}$$

$$s_y = \left[\frac{\sum y^2 - (\sum y)^2 / n}{n-1} \right]^{\frac{1}{2}}$$

$$s_{\ln y} = \left[\frac{\sum (\ln y)^2 - (\sum \ln y)^2 / n}{n-1} \right]^{\frac{1}{2}}$$

Sample Solution

Key Strokes

{f} {A} → 0.00

1 {ENTER} 15 {A} 2 {ENTER} 37 {A} 3 {ENTER} 52 {A} 4 {ENTER}
59 {A} 5 {ENTER} 83 {A} 6 {ENTER} 92 {A} → 16.00

{f} {B} → 1.00 (Linear Regression)
→ 3.33 *** (a)
→ 15.14 *** (b)
→ 0.99 *** (r)

12 {E} → 185.05 units
150 {D} → 9.69 months

{f} {C} → 2.00 (Exponential Curve Fit)
→ 15.17 *** (a)
→ 0.33 *** (b)
→ 0.94 *** (r)

12 {E} → 814.92 units
150 {D} → 6.90 months

{f} {D} → 3.00 (Logarithmic Curve Fit)
→ 10.20 *** (a)
→ 42.07 *** (b)
→ 0.97 *** (r)

12 {E} → 114.74 units
150 {D} → 27.75 months

{f} {E} → 4.00 (Power Curve Fit)
→ 16.38 *** (a)
→ 0.99 *** (b)
→ 0.99 *** (r)

12 {E} → 191.70 units
150 {D} → 9.37 months

References:

- Hewlett Packard HP-97 Standard Pac.

- Spiegel, M.R., Statistics, First Edition,
Schaum's Outline Series, McGraw-Hill,
New York, 1961.

CURVE FITTING

INITIALIZE 1. LIN.a,b,r 2. EXP.a,b,r 3. LOG.a,b,r 4. PWR.a,b,r

$x_i \uparrow y_i (+)$ $x_i \uparrow y_i (-)$ $y \rightarrow \hat{y}$ $x \rightarrow \hat{y}$

[illegible]

Program Listing I

03139D

STEP KEY ENTRY KEY CODE

```

001 *LBLA 21 16 11
002 CLR6 16-53
003 P2S 16-51
004 CLR6 16-53
005 0 00
006 RTN 24
007 *LELA 21 11
008 STOD 35 14
009 LN 32
010 X*Y -41
011 STOC 35 13
012 x -35
013 ST+0 35-55 00
014 RCLD 36 14
015 RCLC 36 13
016 LN 32
017 x -35
018 ST+1 35-55 01
019 GSB0 23 00
020 Z+ 56
021 GSB1 23 01
022 Z+ 56
023 RTN 24
024 *LBLB 21 12
025 STOD 35 14
026 LN 32
027 X*Y -41
028 STOC 35 13
029 x -35
030 ST-0 35-45 00
031 RCLD 36 14
032 RCLC 36 13
033 LN 32
034 x -35
035 ST-1 35-45 01
036 GSB0 23 00
037 Z- 16 56
038 GSB1 23 01
039 Z- 16 56
040 RTN 24
041 *LBL0 21 00
042 RCLD 36 14
043 RCLC 36 13
044 P2S 16-51
045 RTN 24
046 *LEL1 21 01
047 RCLD 36 14
048 LN 32
049 RCLC 36 13
050 LN 32
051 P2S 16-51
052 RTN 24
053 *LBL6 21 16 12
054 SPC 16-11
055 CF0 16 22 00
056 CF1 16 22 01
    
```

COMMENTS

```

Clear registers for
data entry and
output 0.00.

Calculate  $x_i \ln y_i$ ,
 $y_i \ln x_i$  and add to
reg.0 and 1.

Compute  $\Sigma+$  in
primary regs. and
 $\Sigma \ln+$  in secondary
regs.

Calculate  $x_i \ln y_i$ ,
 $y_i \ln x_i$  and
subtract from reg.
0 and 1.

Compute  $\Sigma-$  in
primary regs. and
 $\Sigma \ln-$  in secondary
regs.

Recall  $y_i$ ,  $x_i$  and
switch to
secondary.

Recall  $y_i$ ,  $x_i$  take
 $\ln$  and switch back
to primary.

Clear flag 0 and 1
to indicate LIN.
    
```

STEP KEY ENTRY KEY CODE

```

057 1 01
058 PRTX -14
059 GSB2 23 02
060 GSB3 23 03
061 P2S 16-51
062 RTN 24
063 *LBL2 21 02
064 RCL8 36 08
065 RCL4 36 04
066 RCL6 36 06
067 x -35
068 RCL9 36 09
069 ÷ -24
070 - -45
071 RCL5 36 05
072 RCL4 36 04
073 X2 53
074 RCL9 36 09
075 ÷ -24
076 - -45
077 ÷ -24
078 STOB 35 12
079 P2S 16-51
080 x 16 53
081 RCLB 36 12
082 x -35
083 - -45
084 RTN 24
085 *LBL3 21 03
086 STOA 35 11
087 PRTX -14
088 S 16 54
089 X*Y -41
090 ÷ -24
091 RCLB 36 12
092 PRTX -14
093 x -35
094 PRTX -14
095 RTN 24
096 *LEL0 21 16 13
097 SPC 16-11
098 SF0 16 21 00
099 CF1 16 22 01
100 2 02
101 PRTX -14
102 RCL0 36 00
103 GSB4 23 04
104 ex 33
105 GSB5 23 05
106 RTN 24
107 *LEL4 21 04
108 RCL4 36 04
109 P2S 16-51
110 RCL6 36 06
111 P2S 16-51
112 x -35
    
```

COMMENTS

```

LIN indicator

Calculate a,b,r for
LIN.

Calculate b for
LIN and PWR.

Calculate a for
LIN and lna for
PWR.

Calculate r for
LIN and PWR.

Set flag 0, clear
flag 1 to
indicate EXP.

EXP indicator

Calculate a,b,r for
EXP.

Calculate part of
b for EXP and LOG.
    
```

REGIS. LINES

0	1	2	3	4	5	6	7	8	9
$\Sigma x \ln y$	$\Sigma y \ln x$	0	0	Σx	Σx^2	Σy	Σy^2	Σxy	n
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
0	0	0	0	$\Sigma \ln x$	$\Sigma (\ln x)^2$	$\Sigma \ln y$	$\Sigma (\ln y)^2$	$\Sigma \ln x \ln y$	n
A a	B b	C x_i	D y_i	E 0	F	G	H	I	J

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
113	RCL9	36 09		169	GSB2	23 02	Calculate a,b,r for PWR
114	=	-24		170	e*	33	
115	-	-45		171	GSB3	23 03	
116	RCL5	36 05		172	RTN	24	
117	RCL4	36 04		173	*LBLD	21 14	Make projections based on y value, using fit as indicated by the status of flag 0 and 1.
118	X ²	53		174	F0?	16 23 00	
119	RCL9	36 09		175	LN	32	
120	=	-24		176	RCL4	36 11	
121	-	-45		177	F0?	16 23 00	
122	=	-24		178	LN	32	
123	STOB	35 12		179	-	-45	
124	P*S	16-51	Calculate lna for EXP and a for LOG.	180	RCLB	36 12	
125	RCL6	36 06		181	=	-24	
126	P*S	16-51		182	F1?	16 23 01	
127	RCL4	36 04		183	e*	33	
128	RCLB	36 12		184	RTN	24	Make projections based on x value, using fit as indicated by the status of flag 0 and 1.
129	x	-35		185	*LBLD	21 15	
130	-	-45		186	F1?	16 23 01	
131	RCL9	36 09		187	LN	32	
132	=	-24		188	RCLB	36 12	
133	RTN	24		189	x	-35	
134	*LBL5	21 05	Calculate r for EXP and LOG.	190	RCL4	36 11	
135	STOA	35 11		191	F0?	16 23 00	
136	PRTX	-14		192	LN	32	
137	P*S	16-51		193	+	-55	
138	S	16 54		194	F0?	16 23 00	
139	P*S	16-51		195	e*	33	
140	S	16 54		196	RTN	24	
141	X*Y	-41		197	R/S	51	
142	LSTX	16-63					
143	X*Y	-41					
144	=	-24					
145	RCLB	36 12					
146	PRTX	-14					
147	x	-35					
148	PRTX	-14					
149	RTN	24					
150	*LBLd	21 16 14					
151	SFC	16-11	Clear flag 0 and set flag 1 to indicate LOG				
152	CF0	16 22 00					
153	SF1	16 21 01	LOG indicator				
154	3	03					
155	PRTX	-14					
156	RCL1	36 01	Calculate a,b,r for LOG.				
157	P*S	16-51					
158	GSB4	23 04					
159	GSB5	23 05					
160	P*S	16-51					
161	RTN	24					
162	*LBLd	21 16 15	Set flag 0 and 1 to indicate PWR.				
163	SPC	16-11					
164	SF0	16 21 00					
165	SF1	16 21 01					
166	4	04	PWR indicator				
167	PRTX	-14					
168	P*S	16-51					

LABELS					FLAGS		SET STATUS		
A ^x Y ⁱ (+)	B ^x Y ⁱ (-)	C	D ^y X	E ^x Y	0	see p 10	FLAGS	TRIG	DISP
Initialize	1 LIN,a,b,r	2 EXP,a,b,r	3 LOG,a,b,r	4 PWR,a,b,r	1	see p 10	ON OFF	DEG ☒	FIX ☒
0 used	1 used	2 used	3 used	4 used	2		1 ☐ ☒	GRAD ☐	SCI ☐
5 used	6	7	8	9	3		2 ☐ ☒	RAD ☐	ENG ☐
							3 ☐ ☒		n 2

Explanatory note on the use of label D and E

and flag 0 and 1.

Label D is used to make projections based on a known y value using the fit as indicated by the Status of flag 0 and 1.

Label E does the same based on a known x value. The use and status of flag 0 and 1 are shown hereunder:

Equations used under label D ($y \rightarrow \hat{x}$):

1. LIN

2. EXP

3. LOG

4. PWR

$$\hat{x} = \frac{y-a}{b}$$

$$\hat{x} = \frac{\ln y - \ln a}{b}$$

$$\hat{x} = \exp \left[\frac{y-a}{b} \right]$$

$$\hat{x} = \exp \frac{\ln y - \ln a}{b}$$

Label D as it would have looked for each fit without using flag 0 and 1:

LBLD

LBLD

LBLD

LBLD

RCLA

LN

RCLA

LN

-

RCLA

-

RCLA

RCLB

LN

RCLB

LN

÷

-

÷

-

RTN

RCLB

e^x

RCLB

÷

RTN

÷

RTN

EXP

RTN

With the use of flag 0 and 1 it is not difficult to combine these 4 programs into one. It is convenient however to use the same status of flag 0 and 1 for label E.

Equations used under label E (x y):

1.LIN	2.EXP	3 LOG	4.PWR
$\hat{y} = a+bx$	$\hat{y} = \exp[lna+bx]$	$\hat{y} = a+blnx$	$\hat{y} = \exp[lna+blnx]$

Label E as it would have looked for each fit without using flag 0 and 1:

LBLE	LBLE	LBLE	LBLE
RCLB	RCLB	LN	LN
X	X	RCLB	RCLB
RCLA	RCLA	X	X
+	LN	RCLA	RCLA
RTN	+	+	LN
	e^x	RTN	+
	RTN		e^x

Status of flag 0 and 1 (+=set, -= cleared): RTN

The status of flag 0 and 1 is the same for label D and label E for each fit and is automatically declared with the chosen fit and shown hereunder.

	1.LIN	2.EXP	3.LOG.	4. PWR.
F 0	-	+	-	+
F 1	-	-	+	+

The 4 combined programs with label D and E then look as follows:

LBLD	LBLE
FO ?	F1 ?
LN	LN
RCLA	RCLB
FO?	X
LN	RCLA
-	FO ?
RCLB	LN
÷	+
F1 ?	FO ?
e^x	e^x
RTN	RTN.