Program Title 67-N-TH DEGREE POLYNOMIAL ⇐ 18 BAIRSTOW METHOD.

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Program Description, Equations, Variables $P_n(x) = a_0x^n + a_1x^{n-1} + \ldots + a_n = 0$

if s and p are sum and product of 2 roots, $x^2 - sx + p$ divides exactely $P_n(x)$. formulas:

$$\begin{cases} s_{k+1} = s_k + \frac{S}{\Delta} \\ p_{k+1} = p_k + \frac{P}{\Delta} \end{cases}$$

where $S = b_nc_{n-3} - b_{n-1}c_{n-2}$ $\qquad \Delta = c_{n-2}^2 - c_{n-1}c_{n-3}$.

$P = b_nc_{n-2} - b_{n-1}c_{n-1}$

with : $b_0 = a_0$ $\qquad c_0 = b_0$

$b_1 = a_1 + sb_0$ $\qquad c_q = b_q + sc_{q-1} - pc_{q-2}$

$b_2 = a_2 + sb_1 - pb_0$

$b_n = a_n + sb_{n-1} - pb_{n-2}$ but ⚠ $c_{n-1} = sc_{n-2} - pc_{n-3}$

test : $\dfrac{|s_{k+1} - s_k| + |p_{k+1} - p_k|}{|s_{k+1}| + |p_{k+1}|} < 10^{-8}$

when it has succeeded, 2 iterations more and then s and p are a good approximate of the sum and the product of two roots.

- the result of the division is $b_0x^{n-2} + \ldots + b_{n-2}$

Operating Limits and Warnings The time taken for the resolution depends on the degree of the polynomial and on the coefficients themselves. In some cases, convergence is impossible. If for high degree, program hasn't stopped after, say, 5 hours, the polynomial can't probably be solved by the method.

0653 Program Description II

Sketch(es)

Sample Problem(s) Solve $10^{-15}x^{17} - 10^{-12}x^{16} + 5.10^{-8}x^{12} + 10^{-6}x^{9} - 2.10^{-6}x^{8} + 5.10^{-2}x^{4} + 0{,}1x^{3} - x^{2} + 3x + 1 = 0$

Solution

- Load Side 1 and 2 of card 1
- press: E 1 R/S 3 R/S 1 CHS R/S . 1 R/S 5 EEX 2 CHS R/S R/S R/S R/S 2 CHS EEX 6 CHS R/S EEX 6 CHS R/S R/S R/S 5 EEX 8 CHS R/S R/S R/S R/S 1 CHS EEX 12 CHS R/S EEX 15 CHS R/S (display shows 17.00 =degree)
- press A
- insert side 1 of card 2 while program is running (so that it will be read when calculator stops)

Program will run ca. 1 hour and 15 minutes and displays during that time the new degree when it has found a quadratic factor

Solution(s) ... 15.00 ... 13.00 ... 11.00 ... ... 3.00 ... 1.00

(card 2 is read and solutions are immediately displayed: they appear by couple and are imaginary roots when preceded by 1. (DSP 0))

output : 999.9999499 1. $-.078068147 \pm i$ 14.95162105
1. $5.154471705 \pm i$ 2.181345798 1. $-2.005956871 \pm i$ 5.592417781
-5.277535838 -14.87382976 1. $2.158738310 \pm i$ 5.471415990
1. $-4.890014178 \pm i$ 2.860897132 1. $2.372169846 \pm i$ 2.143183000
15.03086045 -0.302126186 [0.00 (END)]

Imaginary roots are underlined e.g. [-0.78068147 +/- i 14.95162105]

Reference(s) Rem. if you calculate P(999.999499) you'll find 10^{25} ! (and not near zero !) Consider yet that P(999.9) = -10^{31} and P(1000.0) = $+10^{28}$!! so, it is meanwhile a good approximation.

reference : H.Bastin: Elements d'Analyse Numérique
Presses Universitaires de Bruxelles

USERS' CLUB HP EUROPE 5065 3 **Program Description 11**

Sketch(es)

Sample Problem(s) $P = 5x^4 + 4x^3 + 3x^2 + 2x + 1$

Solve $P = 0$

Keystrokes : (E) (π) (R/S) (2) (R/S) (3) (R/S) (4) (R/S) (5) (R/S) (A) insert card2 while running → 3 minutes →

output : 1. -0.537832275 0.358284686 1. 0.137832275 0.678154389; 0.00

HP 97 INPUT, OUTPUT (NORM.)

solution is $-0.537832275 \pm i\ 0.358284686$ (z_1)

$0.137832275 \pm i\ 0.678154389$ (z_2)

precision: $P(z_2) = 5 \times 10^{-10} \mp i\ 6 \times 10^{-10}$

$P(z_1) = 0 \pm i\ 7 \times 10^{-10}$

```
GSBE
1.00  R/S
2.00  R/S
3.00  R/S
4.00  R/S
5.00  R/S
GSBA
2.00  ***

1.  ***
-0.537832275  ***
0.358284686  ***

1.  ***
0.137832275  ***
0.678154389  ***
```

~~Solution~~(s)

Reference(s)

50653 User Instructions

1 $\sum_{i=0}^{n} a_i x^i = 0 \quad (n \leq 18)$ BAIRSTOW METHOD CARD 1 2

SOLVE $E\ a_0; a_1 \ldots$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2 of card 1			
2	load coefficients of $\sum a_i x^i$	-	E	-
		a_0 *	R/S	0.00
	(* 0 needn't be input if $a_i = 0$: press	a_1 *	R/S	1.00
	only R/S)	a_2	R/S	2.00
		a_n	R/S	n.00
				n = degree
3	solve		A	
4	insert side 1 of card 2 while program			
	is running so that it will be read			
	when calculator stops.			
5				n-2
				n-4
				n-6
				...
				1 or 2
6	card 2 is read			$root_1$ *
				$root_2$
	* - when 1 (DSP 0) appears, the			
	following numbers are imaginary:			
	→ first (real part) + second (ima-			...
	- ginary part)			
	- when not preceded by 1 (DSP 0)			
	they are real.			
	END			0.00
7	to review roots (card 2!)		B	see step 6
8	for a new case, go to step 1			

50653

Program Listing I (CARD 1)

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	* LBL 1	31 25 01	
	MRG	32 41	
	PSE	35 72	pause for loading
	R/S	84	card 2
	* LBL A	31 25 11	
	2	02	
	P↔S	31 42	
	RCL 9	34 09	
	P↔S	31 42	
010	X≤Y?	32 71	test if degree >2
	GTO 1	22 01	
	1	01	initialization of s & p
	STO C	33 13	
	STO D	33 14	
	* LBL a	32 25 11	
	GSB 9	31 22 09	
	RCL A	34 11	test to see if another iteration must be done
	RCL C	34 13	
	-	51	
020	ABS	35 64	
	RCL B	34 12	
	RCL D	34 14	
	-	51	
	ABS	35 64	
	+	61	
	RCL C	34 13	
	ABS	35 64	
	RCL D	34 14	
	ABS	35 64	
	+	61	
	÷	81	
	EEX	43	
	8	08	
	CHS	42	
	X≤Y	32 71	
	GTO a	22 31 11	
	GSB 9	31 22 09	2 more iterations.
	GSB 9	31 22 09	
	GTO D	22 14	
040	* LBL 9	31 25 09	executes algor.
	P↔S	31 42	
	RCL 9	34 09	
	P↔S	31 42	
	STO I	35 33	
	RCL C	34 13	
	RCL D	34 14	
	RCL D	34 14	
	0	00	
	STO A	33 11	init. of b_i & c_i
050	STO B	33 12	
	STO C	33 13	
	STO D	33 14	
	STO E	33 15	
	R↓	35 53	
	* LBL 0	31 25 00	
	RCL A	34 11	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	CHS	42	
	X	71	
	R↑	35 54	
060	RCL B	34 12	"hart" of program
	STO A	33 11	
	X↔Y	35 52	
	x	71	
	LST X	35 82	
	R↓	35 53	
	+	61	
	RCL (i)	34 24	
	+	61	
	STO B	33 12	
070	R↓	35 53	
	ENT↑	41	
	F?2	35 71 02	
	RTN	35 22	
	RCL D	34 14	
	STO C	33 13	
	CHS	42	
	x	71	
	R↑	35 54	
	RCL E	34 15	
080	STO D	33 14	
	X↔Y	35 52	
	x	71	
	LST X	35 82	
	R↓	35 53	
	+	61	
	RCL B	34 12	
	+	61	
	STO E	33 15	
	R↓	35 53	
090	ENT↑	41	
	DSZ I	31 33	
	GTO 0	22 00	
	SF 2	35 51 02	
	GSB 0	31 22 00	
	R↓	35 53	
	RCL E	34 15	
	RCL A	34 11	
	-	51	
	STO E	33 15	
100	RCL C	34 13	
	CHS	42	
	x	71	
	RCL D	34 14	
	X^2	32 54	
	+	61	
	ST I	35 33	△ in Reg I
	R↓	35 53	
	RCL C	34 13	
	RCL B	34 12	
110	x	71	
	STO C	33 13	
	R↓	35 53	

REGISTERS

0	1	2	3	4	5	6	7	8	9
coeff. of X^0	coeff. of X	coeff. of X^2	...						
S0	S1	S2	S3	S4	S5	S6	S7 ...	S8 coeff. of X^{18}	S9 degree

A	B	C	D	E	I
Used	used	used	used	used	ind. control

50653 Program Listing II (CARD 1)

STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCL A	34 11	
	RCL D	34 14	
	CHS	42	
	x	71	
	RCLC	34 13	
	+	61	
	STO C	33 13	S in Reg C
120	R↓	35 53	
	RCL E	34 15	
	RCL A	34 11	
	x	71	
	STO E	33 15	
	R↓	35 53	
	RCL B	34 12	
	RCL D	34 14	
	x	71	
	RCL E	34 15	
130	-	51	P
	RC I	35 34	
	÷	81	
	X↔Y	35 52	
	STO B	33 12	p_{k+1} in Reg. B
	+	61	
	STO D	33 14	p_{k+1} in Reg D
	R↓	35 53	
	STO A	33 11	s_k in Reg A
	RCL C	34 13	
140	RC I	35 34	
	÷	81	
	+	61	
	STO C	33 13	s_{k+1} in Reg C
	RTN	35 22	
*	LBL D	31 25 14	divides by X^2-sX+p
	0	00	
	STO A	33 11	
	STO B	33 12	
	P↔S	31 42	
150	RCL 0	34 09	
	P↔S	31 42	
	ST I	35 33	
*	LBL 8	31 25 08	
	RCL A	34 11	
	RCL D	34 14	
	CHS	42	
	x	71	
	RCL B	34 12	
	STO A	33 11	
160	RCL C	34 13	
	x	71	
	+	61	
	RCL (i)	34 24	
	+	61	
	STO B	33 12	
	STO (i)	33 24	put the new coeff. in the right registers
	DSZ I	31 33	
	GTO 8	22 08	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	2	02	
170	P↔S	31 42	
	STO-9	33 51 09	
	P↔S	31 42	
	ST I	35 33	
*	LBL 4	31 25 04	
	RCL (i)	34 24	
	DSZ I	31 33	
	DSZ I	31 33	
	STO (i)	33 24	
	STO (i)	33 24	
180	RC I	35 34	
	1	01	
	6	06	
	X=Y ?	32 51	
	GTO 5	22 05	
	ISZ I	31 34	
	ISZ I	31 34	
	ISZ I	31 34	
	ISZ I	31 34	
	GTO 4	22 04	
*	LBL 5	31 25 05	
190	P↔S	31 42	
	RCL C	34 13	
	STO 7	33 07	
	RCL D	34 14	
	STO 8	33 08	
	RCL 9	34 09	
	P↔S	31 42	
	-X-(print)	31 84	
	GTO A	22 11	
*	LBL E	31 25 15	coefficients input
200	CL REG	31 43	
	P↔S	31 42	
	CL REG	31 43	
*	LBL e	32 25 15	
	R/S	84	
	F ? 3	35 71 03	
	STO (i)	33 24	
	1	01	
	9	09	
	RC I	35 34	
210	P↔S	31 42	
	STO 9	33 09	
	P↔S	31 42	
	ISZ I	31 34	
	X≠Y ?	32 61	
	GTO e	22 31 15	
	GTO (i)	22 24	
	(R/S)	(84)	
220			

LABELS					FLAGS
A	B	C	D	E	0
a	b	c	d	e	1
0	1	2	3	4	2
5	6	7	8	9	3

SET STATUS		
FLAGS	TRIG	DISP
ON OFF		
0 ☐ ☒	DEG ☒	FIX ☒
1 ☐ ☒	GRAD ☐	SCI ☐
2 ☐ ☒	RAD ☐	ENG ☐
3 ☐ ☒		n 2

STEP	KEY ENTRY	KEY CODE	COMMENTS
001 ✱	LBL B	31 25 12	review roots
	DSP 9	23 09	
	P↔S	31 42	
	RCL 9	34 09	
	P↔S	31 42	
	1	01	
	X=Y ?	32 51	
008	GTO 2	22 02	
	R↓	35 53	
010	2	02	
	X≠Y ?	32 61	
	GTO 9	22 09	
	RCL 1	34 01	
	RCL 2	34 02	
	CHS	42	
	+	81	
	STO C	33 13	
018	RCL 0	34 00	
	RCL 2	34 02	
020	÷	81	
	STO D	33 14	
	GSB 6	31 22 06	
✱	LBL 3	31 25 03	
	1	01	
	8	08	
	ST I	35 33	
✱	LBL 4	31 25 04	
028	CF 2	35 61 02	
	RC I	35 34	
	2	02	
	X=Y ?	32 51	
	GTO 9	22 09	
	RCL (i)	34 24	
	X=0 ?	31 51	
	SF 2	35 51 02	
	STO D	33 14	put p_i in D
	DSZ I	31 33	
038	RCL (i)	34 24	
	X=0 ?	31 51	
040	GTO 8	22 08	
✱	LBL 5	31 25 05	
	STO C	33 13	put s_i in C
	DSZ I	31 33	
	GSB 6	31 22 06	
	GTO 4	22 04	
✱	LBL 2	31 25 02	1st degree resolution
	RCL 0	34 00	
048	RCL 1	34 01	
	CHS	42	
050	÷	81	
	SPACE	35 84	
	-X-	31 84	
	GTO 3	22 03	
✱	LBL 8	31 25 08	
	F ? 2	35 71 02	
	GTO 9	22 09	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	GTO 5	22 05	
058 ✱	LBL 9	31 25 09	"exit" label
	0	00	
060	DSP 2	23 02	
	-X-	31 84	
	R/S	84	
✱	LBL 6	31 25 06	quadratic resolution
	SPACE	35 84	
	RCL C	34 13	
	2	02	
	÷	81	
068	ENTER↑	41	
	ENTER↑	41	
070	X^2	32 54	
	RCL D	34 14	
	-	51	
	X<0	31 71	
	GTO 7	22 07	
	$\sqrt{X}$	31 54	real roots
	+	61	
	-X-	31 84	
078	R↓	35 53	
	LAST x	35 82	
080	-	51	
	-X-	31 84	
	RTN	35 22	
✱	LBL 7	31 25 07	
	ABS	35 64	
	$\sqrt{X}$	31 54	complex roots
	X↔Y	35 52	
	1	01	
088	DSP 0	23 00	
	-X-	31 84	
090	R↓	35 53	
	DSP 9	23 09	
	-X-	31 84	
	R↓	35 53	
	-X-	31 84	
	RTN	35 22	
	(R/S)	(84)	
098			
100			
108			
110			

LABELS

A	B USED	C	D	E	
a	b	c	d	e	
0	1	2 —	3 —	4 —	2 USED
5 —	6 —	7 —	8 —	9 —	3

FLAGS

0
1
2 USED
3

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☐
SCI	☐
ENG	☐
n	2