

# Program Description I

Program Title Conversion of Legendre Polynomials into power series.

Contributor's Name Dr. Henrique E. Adler

Address Rua de S. João de Brito, 136

City Oporto

Country Portugal

Postal Code \_\_\_\_\_

Program Description, Equations, Variables This program is an optional complement to program Nr. 50520 D. It converts the sum of Legendre Polynomials  $\hat{y} = \sum_0^n a_k P_k(t)$  into an ordinary power series  $\hat{y} = \sum_0^n c_k x^k$ .  $n \leq 7$ . The conversion is done in two steps:

$$\begin{array}{ll} \text{Step 1: } b_0 = a_0 - \frac{1}{2}a_2 + \frac{3}{8}a_4 - \frac{5}{16}a_6 & b_4 = \frac{35}{8}a_4 - \frac{315}{16}a_6 \\ b_1 = a_1 - \frac{3}{2}a_3 + \frac{15}{8}a_5 - \frac{35}{16}a_7 & b_5 = \frac{63}{8}a_5 - \frac{693}{16}a_7 \\ b_2 = \frac{3}{2}a_2 - \frac{15}{4}a_4 + \frac{105}{16}a_6 & b_6 = \frac{231}{16}a_6 \\ b_3 = \frac{5}{2}a_3 - \frac{35}{4}a_5 + \frac{315}{16}a_7 & b_7 = \frac{429}{16}a_7 \end{array}$$

$$\text{Step 2: } c_k = p^k \sum_K^n \binom{i}{k} (-q)^{i-k} b_i.$$

For significance of t, p and q see program description I of program Nr. 50520 D.

Projections are made using the Horner scheme:

$$\hat{y} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 \dots$$

Operating Limits and Warnings All register contents from program 50520 D must be conserved without any alteration. If, however, a register exchange occurred, this will be corrected automatically.

This program has been verified only with respect to the numerical example given in Program Description II. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Sketch(es)

Sample Problem(s) Convert the result of the sample problem page 2 of program Nr. 50520D,  $y = 4.450 + 6.086t + 1.322P_2(t) - 0.315P_3(t)$ ,  $t = 0.2x - 1$ , into a power series with argument  $x$ .

Solve that sample problem as indicated on page 2 of Nr. 50520D. Dont disturb the register contents. Load side 1 and side 2 of the card of the present program.

Keystrokes: 3 f a  $\rightarrow$  3.000

B  $\rightarrow$   $k = 0.^{xxx}$   $c_k = 0.00053^{xxx}$

1.<sup>xxx</sup> 0.04656<sup>xxx</sup>

2.<sup>xxx</sup> 0.17368<sup>xxx</sup>

3.<sup>xxx</sup> -0.00629<sup>xxx</sup>

3.00000 (end)

Solution(s)  $y = 0.00053 + 0.04656x + 0.17368x^2 - 0.00629x^3$

Make projections:  $x$  E  $\rightarrow$   $\hat{y}$

Result:

| x         | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10    |
|-----------|------|------|------|------|------|------|------|------|------|------|-------|
| $\hat{y}$ | 0.00 | 0.21 | 0.74 | 1.53 | 2.56 | 3.79 | 5.17 | 6.68 | 8.72 | 9.90 | 11.54 |

(Same as found on page 2 of Nr.50520D.)

Reference(s)

$n_{start}$  $\rightarrow k, c_k$ 
$$X \rightarrow Y$$
[illegible]

| STEP | KEY ENTRY | KEY CODE | COMMENTS                | STEP | KEY ENTRY | KEY CODE | COMMENTS |
|------|-----------|----------|-------------------------|------|-----------|----------|----------|
| 001  | * LBL a   | 32 25 11 | Register<br>Positioning |      | X=Y       | 32 51    |          |
|      | STOE      | 33 15    |                         |      | GTO 8     | 22 08    |          |
|      | RCL 9     | 34 09    |                         |      | RCL(i)    | 34 24    |          |
|      | X=O       | 31 51    |                         | 060  | 8         | 08       |          |
|      | PZS       | 31 42    |                         |      | :         | 81       |          |
|      | 1         | 01       |                         |      | STO+1     | 33 61 01 |          |
|      | 1         | 01       |                         |      | 7         | 07       |          |
|      | STI       | 35 33    |                         |      | X         | 71       |          |
|      | DSZ       | 31 33    |                         |      | STO+1     | 33 61 01 |          |
| 010  | RCL E     | 34 15    |                         |      | STO+1     | 33 61 01 |          |
|      | +         | 61       |                         |      | STO-3     | 33 51 03 |          |
|      | STOA      | 33 11    |                         |      | 9         | 09       |          |
|      | RCL(i)    | 34 24    |                         |      | X         | 71       |          |
|      | STO O     | 33 00    |                         | 070  | STO-3     | 33 51 03 |          |
|      | ISZ       | 31 34    |                         |      | STO 5     | 33 05    |          |
|      | RCL(i)    | 34 24    |                         |      | GSB 9     | 31 22 09 |          |
|      | STO 1     | 33 01    |                         |      | X=Y       | 32 51    |          |
|      | GSD 9     | 31 22 09 |                         |      | GTO 8     | 22 08    |          |
|      | X=Y       | 32 51    |                         |      | RCL(i)    | 34 24    |          |
| 020  | GTO 8     | 22 08    |                         |      | RCLI      | 35 34    |          |
|      | RCL(i)    | 34 24    |                         |      | :         | 81       |          |
|      | 2         | 02       |                         |      | 5         | 05       |          |
|      | :         | 81       |                         |      | X         | 71       |          |
|      | STO-0     | 33 51 00 |                         | 080  | STO-0     | 33 51 00 |          |
|      | 3         | 03       |                         |      | 2         | 02       |          |
|      | X         | 71       |                         |      | 1         | 01       |          |
|      | STO 2     | 33 02    |                         |      | X         | 71       |          |
|      | GSD 9     | 31 22 09 | $a_k \rightarrow b_k$   |      | STO+2     | 33 61 02 |          |
|      | X=Y       | 32 51    | $R_k \leftarrow b_k$    |      | 3         | 03       |          |
| 030  | GTO 8     | 22 08    |                         |      | X         | 71       |          |
|      | RCL(i)    | 34 24    |                         |      | STO-4     | 33 51 04 |          |
|      | STO 3     | 33 03    |                         |      | 1         | 01       |          |
|      | 2         | 02       |                         |      | 5         | 05       |          |
|      | :         | 81       |                         | 090  | :         | 81       |          |
|      | 3         | 03       |                         |      | 1         | 01       |          |
|      | X         | 71       |                         |      | 1         | 01       |          |
|      | STO-1     | 33 51 01 |                         |      | X         | 71       |          |
|      | STO+3     | 33 61 03 |                         |      | STO 6     | 33 06    |          |
|      | GSD 9     | 31 22 09 |                         |      | GSD 9     | 31 22 09 |          |
| 040  | X=Y       | 32 51    |                         |      | X=Y       | 32 51    |          |
|      | GTO 8     | 22 08    |                         |      | GTO 8     | 22 08    |          |
|      | RCL(i)    | 34 24    |                         |      | RCL(i)    | 34 24    |          |
|      | 8         | 08       |                         |      | 1         | 01       |          |
|      | :         | 81       |                         | 100  | 6         | 06       |          |
|      | STO 4     | 33 04    |                         |      | :         | 81       |          |
|      | STO+4     | 33 61 04 |                         |      | 3         | 03       |          |
|      | 3         | 03       |                         |      | 5         | 05       |          |
|      | X         | 71       |                         |      | X         | 71       |          |
|      | STO+0     | 33 61 00 |                         |      | STO-1     | 33 51 01 |          |
| 050  | STO+4     | 33 61 04 |                         |      | 9         | 09       |          |
|      | 1         | 01       |                         |      | X         | 71       |          |
|      | 0         | 00       |                         |      | STO+3     | 33 61 03 |          |
|      | X         | 71       |                         |      | 1         | 01       |          |
|      | STO-2     | 33 51 02 |                         | 110  | 1         | 01       |          |
|      | STO+4     | 33 61 04 |                         |      | X         | 71       |          |
|      | GSD 9     | 31 22 09 |                         |      | 5         | 05       |          |

## REGISTERS

|    |                |    |                |    |                |    |                |    |                |    |                |    |                |    |                |    |   |                                     |   |   |  |
|----|----------------|----|----------------|----|----------------|----|----------------|----|----------------|----|----------------|----|----------------|----|----------------|----|---|-------------------------------------|---|---|--|
| 0  | C <sub>0</sub> | 1  | C <sub>1</sub> | 2  | C <sub>2</sub> | 3  | C <sub>3</sub> | 4  | C <sub>4</sub> | 5  | C <sub>5</sub> | 6  | C <sub>6</sub> | 7  | C <sub>7</sub> | 8  |   | $\frac{9}{2} \int_{-1}^{+1} y^2 dt$ |   |   |  |
| S0 | a <sub>0</sub> | S1 | a <sub>1</sub> | S2 | a <sub>2</sub> | S3 | a <sub>3</sub> | S4 | a <sub>4</sub> | S5 | a <sub>5</sub> | S6 | a <sub>6</sub> | S7 | a <sub>7</sub> | S8 | 0 | S9                                  | 0 |   |  |
| A  | used           |    |                | B  | used           |    |                | C  | 1/p            |    |                | D  | q              |    |                | E  | n |                                     |   | I |  |

| STEP | KEY ENTRY      | KEY CODE | COMMENTS | STEP | KEY ENTRY      | KEY CODE | COMMENTS |
|------|----------------|----------|----------|------|----------------|----------|----------|
|      | :              | 81       |          |      | y <sup>x</sup> | 35 63    |          |
|      | STO-5          | 33 51 05 |          | 170  | :              | 81       |          |
|      | 1              | 01       |          |      | STO+(i)        | 33 61 24 |          |
|      | 3              | 03       |          |      | DSZ            | 31 33    |          |
|      | X              | 71       |          |      | GTO 6          | 22 06    |          |
|      | 2              | 02       |          |      | RCLD           | 34 14    |          |
|      | 1              | 01       |          |      | RCLB           | 34 12    |          |
| 120  | :              | 81       |          |      | y <sup>x</sup> | 35 63    |          |
|      | STO 7          | 33 07    |          |      | RCLA           | 34 11    |          |
|      | * LBL 9        | 31 25 09 |          |      | X              | 71       |          |
|      | ISZ            | 31 34    |          |      | STO+0          | 33 61 00 |          |
|      | RCI            | 35 34    |          | 180  | RCL E          | 34 15    |          |
|      | RCLA           | 34 11    |          |      | RCLB           | 34 12    |          |
|      | RTN            | 35 22    |          |      | GTO 7          | 22 07    |          |
|      | * LBL 8        | 31 25 08 |          |      | * LBL B        | 31 25 12 |          |
|      | RCL 1          | 34 01    |          |      | 0              | 00       |          |
|      | RCLD           | 34 14    |          |      | STI            | 35 33    |          |
| 130  | X              | 71       |          |      | * LBL 5        | 31 25 05 |          |
|      | STO+0          | 33 61 00 |          |      | DSP 0          | 23 00    |          |
|      | RCLC           | 34 13    |          |      | -X-            | 31 84    |          |
|      | STO:1          | 33 81 01 |          |      | DSP 5          | 23 05    |          |
|      | RCL E          | 34 15    |          | 190  | RCL (i)        | 34 24    |          |
|      | 1              | 01       |          |      | -X-            | 31 84    |          |
|      | * LBL 7        | 31 25 07 |          |      | ISZ            | 31 34    |          |
|      | X=Y            | 32 51    |          |      | RCL E          | 34 15    |          |
|      | GTO 3          | 22 03    |          |      | RCI            | 35 34    |          |
|      | 1              | 01       |          |      | X ≤ Y          | 32 71    |          |
| 140  | +              | 61       |          |      | GTO 5          | 22 05    |          |
|      | STOB           | 33 12    |          |      | RCL E          | 34 15    |          |
|      | STI            | 35 33    |          |      | GTO 3          | 22 03    |          |
|      | RCL(i)         | 34 24    |          |      | * LBL E        | 31 25 15 |          |
|      | STOA           | 33 11    |          | 200  | DSP 2          | 23 02    |          |
|      | RCLC           | 34 13    |          |      | RCL E          | 34 15    |          |
|      | RCI            | 35 34    |          |      | STI            | 35 33    |          |
|      | y <sup>x</sup> | 35 63    |          |      | R↓             | 35 53    |          |
|      | STO:(i)        | 33 81 24 |          |      | ENTER↑         | 41       |          |
|      | DSZ            | 31 33    |          |      | ENTER↑         | 41       |          |
| 150  | * LBL 6        | 31 25 06 |          |      | 0              | 00       |          |
|      | RCLB           | 34 12    |          |      | * LBL 4        | 31 25 04 |          |
|      | N!             | 35 81    |          |      | RCL (i)        | 34 24    |          |
|      | RCI            | 35 34    |          |      | +              | 61       |          |
|      | N!             | 35 81    |          | 210  | X              | 71       |          |
|      | :              | 81       |          |      | DSZ            | 31 33    |          |
|      | RCLB           | 34 12    |          |      | GTO 4          | 22 04    |          |
|      | RCI            | 35 34    |          |      | RCL 0          | 34 00    |          |
|      | -              | 51       |          |      | +              | 61       |          |
|      | N!             | 35 81    |          |      | * LBL 3        | 31 25 03 |          |
| 160  | RCLD           | 34 14    |          |      | R/S            | 84       |          |
|      | LSTX           | 35 82    |          |      | GTO A          | 22 11    |          |
|      | y <sup>x</sup> | 35 63    |          |      |                |          |          |
|      | :              | 81       |          |      |                |          |          |
|      | :              | 81       |          |      |                |          |          |
|      | RCLA           | 34 11    |          | 220  |                |          |          |
|      | X              | 71       |          |      |                |          |          |
|      | RCLC           | 34 13    |          |      |                |          |          |
|      | RCI            | 35 34    |          |      |                |          |          |

 $b_k \rightarrow c_k$  $R_k \leftarrow c_k$ Display  $k, c_k$ 

Projections

$$\hat{y} = \sum_0^n c_k x^k$$

Lock

| LABELS  |              |   |   |                           | FLAGS | SET STATUS                                                     |                                         |                                         |
|---------|--------------|---|---|---------------------------|-------|----------------------------------------------------------------|-----------------------------------------|-----------------------------------------|
| A Error | B → $k, c_k$ | C | D | E $x \rightarrow \hat{y}$ | 0     | FLAGS                                                          | TRIG                                    | DISP                                    |
| a n     | b            | c | d | e                         | 1     | ON OFF                                                         |                                         |                                         |
| 0       | 1            | 2 | 3 | 4                         | 2     | 0 <input type="checkbox"/> <input checked="" type="checkbox"/> | DEG <input checked="" type="checkbox"/> | FIX <input checked="" type="checkbox"/> |
| 5       | 6            | 7 | 8 | 9                         | 3     | 1 <input type="checkbox"/> <input checked="" type="checkbox"/> | GRAD <input type="checkbox"/>           | SCI <input type="checkbox"/>            |
|         |              |   |   |                           |       | 2 <input type="checkbox"/> <input checked="" type="checkbox"/> | RAD <input type="checkbox"/>            | ENG <input type="checkbox"/>            |
|         |              |   |   |                           |       | 3 <input type="checkbox"/> <input checked="" type="checkbox"/> |                                         | n <u>3</u>                              |

