



50925 Program Description I

Program Title Linear second order boundary-value problem

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Program Description, Equations, Variables

The program computes by 4th order Runge-Kutta integration the solution of the linear differential equation $y'' = f(x, y, y')$ for boundary conditions of the third kind

$$\alpha_1 y(a) + \beta_1 y'(a) + \gamma_1 = 0 \quad (1)$$

$$\alpha_2 y(b) + \beta_2 y'(b) + \gamma_2 = 0 \quad (2)$$

The program thus comprises boundary conditions of the first kind ($\beta_1 = \beta_2 = 0$) and second kind ($\alpha_1 = \alpha_2 = 0$).

Two arbitrary starting values u_i ($i = 1; 2$) are chosen as input for $y_i'(a)$. u_i also defines $y_i(a)$ according to Eq.(1). The program then computes $y_i(b)$, $y_i'(b)$, and the offense against the boundary condition (2)

$$v_i = \alpha_2 y_i(b) + \beta_2 y_i'(b) + \gamma_2$$

In the special case $\alpha_1 = 0$ the program chooses u_i as starting value for $y_i(a)$, taking $y_i'(a)$ from boundary condition (1). The program stops after computing the true initial slope $y'(a)$ or initial ordinate $y(a)$, respectively, from the expression

$$(u_1 v_2 - u_2 v_1) / (v_2 - v_1)$$

As a last step the program computes and prints the solution $y(x)$ in the range (a, b) . Accuracy of the solution only depends on step width h .

Operating Limits and Warnings

Ensure proper definition of y'' for every point of the integration interval. The display of an excessively large value $|y'(a)|$ or $|y(a)|$, respectively, may indicate exceptional boundary conditions admitting no solution at all.

It is recommended, though not required, to choose u_1 and u_2 such that they enclose the range in which $y'(a)$ or $y(a)$, respectively, are expected to lie.

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Sketch(es)

Sample Problem(s) 1) $y'' + y'/x + y = 0$ ANALYTIC SOLUTION: $y = A J_0(x) + B Y_0(x)$

a = 1	PROGRAM FOR y''		
b = 3	124	*LBLB	21 12
h = 0,2	125	RCL0	36 00
$\alpha_1 = 2$	126	1/X	52
$\alpha_2 = 1,5$	127	x	-35
$\beta_1 = 4$	128	+	-55
$\beta_2 = 2$	129	CHS	-22
$\gamma_1 = 1$			
$\gamma_2 = -4$			

2) $x^2 y'' - 3x y' + 4y = 5x$ ANALYTIC SOL.: $y = 5x + A x^2 + B x^2 \ln x$

a = 0,5	PROGRAM FOR y''		
b = 1,7	124	*LBLB	21 12
h = 0,1	125	3	03
$\alpha_1 = 0$	126	x	-35
$\alpha_2 = 1$	127	X*Y	-41
$\beta_1 = -0,8$	128	4	04
$\beta_2 = 0,5$	129	x	-35
$\gamma_1 = 2,3$	130	RCL0	36 00
$\gamma_2 = 0,7$	131	÷	-24
	132	-	-45
	133	5	05
	134	+	-55
	135	RCL0	36 00
	136	÷	-24

	AN.SOL.	PROGRAM
Solution(s) $y'(a)$	1.19055	1.19052
$y(x)$	-2.88116	-2.88104
A = -3,70052	-2.61138	-2.61133
B = -0,56050	-2.28705	-2.28701
	-1.92087	-1.92085
	-1.52572	-1.52571
	-1.11458	-1.11457
ARBITRARY	-0.70029	-0.70029
INPUT:	-0.29537	-0.29537
	0.08844	0.08843
$u_1 = -100$	0.44040	0.44035
$u_2 = +100$	0.75110	0.75109

	AN.SOL.	PROGRAM
$y(x)$	2.0271	2.0270
	2.2883	2.2883
A = -2,21510	2.4361	2.4360
B = -0,46649	2.6430	2.6436
	2.7456	2.7454
	2.7849	2.7847
	2.7635	2.7638
ARBITRARY	2.6878	2.6876
INPUT:	2.5496	2.5495
	2.3508	2.3506
$u_1 = -10$	2.0904	2.0903
$u_2 = +10$	1.7681	1.7679
	1.3630	1.3629

Reference(s) M. Abramowitz and L.A. Stegun, "Handbook of Mathematical Functions", 9th Printing, 1970, p. 896, National Bureau of Standards AMS 55.

J.-D. Welly, HP Program #50015, "Second Order Differential Equation"

BOUNDARY PROBLEM

$$\alpha_1 \neq 0: y'(a) \quad f(x, y, y') \quad y(x)$$
[illegible]



STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11		057	ST08	35 08	
002	ST08	35 12	Store u_1, u_2	058	GSB2	23 02	
003	R+	-31		059	RCL5	36 05	
004	ST0A	35 11		060	ST+0	35-55 00	
005	GSB _a	23 16 11		061	X $\neq$ Y	-41	
006	GSB1	23 01		062	GSB2	23 02	
007	ST0C	35 13		063	ST+8	35-55 08	
008	RCLB	36 12		064	GSB2	23 02	
009	GSB _a	23 16 11		065	RCL5	36 05	
010	GSB1	23 01		066	ST+0	35-55 00	
011	ST0D	35 14		067	RCL6	36 06	
012	RCLA	36 11	Compute $v_1, v_2,$	068	ENT↑	-21	
013	x	-35	$y'(a),$ and $y(a)$	069	+	-55	
014	RCLC	36 13		070	ST06	35 06	
015	RCLB	36 12		071	R↑	16-31	
016	x	-35		072	ENT↑	-21	
017	-	-45		073	+	-55	
018	RCLD	36 14		074	GSB2	23 02	
019	RCLC	36 13		075	RCL8	36 08	
020	-	-45		076	+	-55	
021	÷	-24		077	3	03	
022	RTN	24		078	÷	-24	
023	*LBL _a	21 16 11		079	RCL2	36 02	
024	RCL3	36 03		080	+	-55	
025	ST08	35 08		081	ST02	35 02	
026	X $\neq$ Y	-41		082	RCL1	36 01	
027	ST01	35 01		083	RCL7	36 07	
028	ST02	35 02		084	RCL6	36 06	
029	P $\neq$ S	16-51		085	+	-55	
030	RCL0	36 00	$\alpha_1 \neq 0:$	086	RCL5	36 05	
031	X=0?	16-43	Take input as $y_i'(a),$	087	x	-35	
032	GT06	22 16 12	compute $y_i(a)$ from	088	3	03	
033	R↓	-31	boundary condition	089	÷	-24	
034	RCL2	36 02		090	-	-45	
035	x	-35	at $x = a$	091	ST01	35 01	
036	RCL4	36 04		092	F0?	16 23 00	
037	+	-55		093	PRTX	-14	
038	RCL0	36 00		094	RCL0	36 00	
039	P $\neq$ S	16-51		095	RCL4	36 04	
040	CHS	-22		096	X $\neq$ Y?	16-34	
041	÷	-24		097	GT01	22 01	
042	ST01	35 01		098	P $\neq$ S	16-51	
043	RTN	24		099	RCL3	36 03	
044	*LBL _b	21 16 12	$\alpha_1 = 0:$	100	RCL5	36 05	
045	RCL4	36 04	Take input as $y_i(a),$	101	RCL1	36 01	
046	RCL2	36 02	compute $y_i'(a)$ from	102	P $\neq$ S	16-51	
047	P $\neq$ S	16-51	boundary condition	103	RCL1	36 01	
048	CHS	-22		104	x	-35	
049	÷	-24	at $x = a$	105	+	-55	
050	ST02	35 02		106	X $\neq$ Y	-41	
051	RTN	24		107	RCL2	36 02	
052	*LBL1	21 01	Runge-Kutta	108	x	-35	
053	RCL2	36 02	routine	109	+	-55	
054	ST07	35 07		110	RTN	24	
055	0	00		111	*LBL2	21 02	
056	ST06	35 06		112	ST+8	35-55 08	

REGISTERS									
0	1	2	3	4	5	6	7	8	9
x	y	y'	a	b	h/2	Used	Used	Used	
S0 α_1	S1 α_2	S2 β_1	S3 β_2	S4 γ_1	S5 γ_2	S6	S7	S8	S9
A		B		C		D		E	

[illegible]