

Program Title 4 SIMULTANEOUS EQUATIONS IN 4 RATIONAL VARIABLES

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Program Description, Equations, Variables SOLVES THE SYSTEM OF 4 EQUATIONS

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} a_{15} \\ a_{25} \\ a_{35} \\ a_{45} \end{pmatrix} \quad \text{WHERE ALL COEFFICIENTS } a_{ij} \text{ ARE RATIONALS, AND WHERE } x, y, z, t \text{ ARE REQUIRED IN SIMPLIFIED RATIONAL FORM.}$$

THE EQUATIONS ARE SOLVED USING GAUSS-JORDAN REDUCTION (WITHOUT PIVOTAL EXCHANGE).

PAIRS OF REGISTERS R_{2n} AND R_{2n+1} HOLD THE INTEGER-VALUED NUMERATORS & DENOMINATORS OF RATIONALS, FOR $n=0,1,\dots,11$ GIVING AN EFFECTIVE 'RATIONAL REGISTER' Q_n . RATIONAL ARITHMETIC IS PERFORMED USING A 2-LEVEL RATIONAL 'STACK' (Q-STACK). THE Q_y REGISTER HOLDS THE NUMERATOR IN R_0 & DENOMINATOR IN R_1 . THE Q_x REGISTER OF THE Q-STACK HOLDS THE NUMERATOR IN THE Y-REGISTER OF THE NORMAL STACK & THE DENOMINATOR IN THE X-REGISTER. PROGRAM USES EUCLIDEAN ALGORITHM (USING Q_0 & Q_x REGISTERS) TO SIMPLIFY RATIONAL IN Q_x , BY FINDING G.C.D. (GREATEST COMMON DIVISOR) OF NUMERATOR a & DENOMINATOR b , GIVING SIMPLIFIED RATIONAL $\frac{a_1}{b_1}$ WHERE $a = \lambda a_1$, $b = \lambda b_1$. λ IS PUT IN R_2 . PROGRAM USES SUBROUTINES TO PUT IN Q_x THE FOLLOWING: Q_n (LIFTING Q_x TO Q_y AS IN HP-47 'RCL' OPERATION), $Q_x + Q_y$, $Q_x - Q_y$, $Q_x \times Q_y$, $Q_y \div Q_x$, AND ALSO INCLUDES A 'STORE' SUBROUTINE TO STORE Q_x IN Q_n . IN ORDER TO MINIMISE RISK OF OVERFLOW (I.E. NUMERATOR OR DENOMINATOR $\geq 10^{10}$), COMMON FACTORS ARE DELETED WHERE-EVER POSSIBLE IN CALCULATION. THIS GIVES SLOW EXECUTION, BUT OFTEN GIVES RESULTS WHERE SIMPLER ALGORITHMS OVERFLOW. SEE OVERLEAF FOR ARITHMETIC ALGORITHMS USED.

Operating Limits and Warnings CANNOT HANDLE NUMERATORS OR DENOMINATORS $\geq 10^{10}$, AND PROGRAM HALTS DISPLAYING 'Error' IF SUCH ARE ENCOUNTERED AT ANY STAGE.

SOLUTION DOES NOT USE PIVOTAL EXCHANGE. IF ONE OF THE PIVOTS IS ZERO, PROGRAM WILL OUTPUT THE 'INDETERMINATE' 0/0 AS A SOLUTION. IF A UNIQUE SOLUTION EXISTS, REARRANGING THE ORDER OF THE EQUATIONS SHOULD GIVE SOLUTION. IN SOME CASES WHEN THE PROGRAM IS HALTED BY OVERFLOW, THE SOLUTION MAY BE OBTAINED IF EQUATIONS ARE INPUT IN DIFFERENT ORDER. INCONSISTENT EQUATIONS WILL RESULT IN INFINITE (I.E. $\pm 1/0$) SOLUTIONS. (THE SIGN OF ∞ IS OF NO SIGNIFICANCE)

This program has been verified only with respect to the numerical example given in Program Description II. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Sketch(es) DENOTE THE SIMPLIFICATION OF $\frac{a}{b}$ BY $\frac{a'}{b'}$ = $G(\frac{a}{b})$,
WHERE $a = \lambda a'$, $b = \lambda b'$, λ = G.C.D. OF a & b . THEN ALGORITHMS
FOR + and x ARE AS FOLLOWS, GIVEN $\frac{a}{b}, \frac{c}{d}$ IN SIMPLIFIED FORM
INITIALLY
 $\frac{a}{b}, \frac{c}{d} \rightarrow G(\frac{a}{b}) = \frac{a'}{b'}$, $G(\frac{c}{d}) = \frac{c'}{d'}$, $\frac{a}{b} \times \frac{c}{d} = \frac{a' \times c'}{b' \times d'}$
 $\frac{a}{b}, \frac{c}{d} \rightarrow G(\frac{a}{b}) = \frac{a'}{b'}$, $GCD(b, d) = \lambda \rightarrow G(\frac{a'd' + cb'}{\lambda}) = \frac{e}{\lambda'}$
 $\rightarrow \frac{a}{b} + \frac{c}{d} = \frac{e}{\lambda' \times b' \times d'}$, $\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \frac{-c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.
THESE ALGORITHMS MINIMISE OVERFLOW RISK AND GIVE
RESULT IN SIMPLIFIED FORM.

4 [ENT] 7 [R/S] →

4. ***
7. ***

4 [ENT] 5 [R/S] →

4. ***
5. ***

LOAD CARD 2 SIDE 1,

2 [ENT] 1 [R/S] →

2. ***
1. ***

11 [ENT] 8 [R/S] →

11. ***
8. ***

27 [ENT] 26 [R/S] →

27. ***
26. ***

(N.B. LONG WAIT AFTER
EACH PRINTOUT ON
CARD 2 SIDE 1)

LOAD CARD 2 SIDE 2

2 [ENT] 1 [CHS] [R/S] →

2. ***
-1. ***

7 [ENT] 2 [R/S] →

7. ***
2. ***

13 [ENT] 3 [R/S] →

13. ***
3. ***

5 [ENT] 1 [R/S] →

5. ***
1. ***

20 [ENT] 11 [R/S] →

20. ***
11. ***

LOAD CARD 3 SIDE 1

x = 3. ***
4. ***

y = -5. ***
7. ***

z = -3. ***
13. ***

t = 4. ***
11. ***

Sample Problem(s)

SOLVE

$$\frac{4}{3}x + \frac{7}{5}y + z - \frac{11}{4}t = \frac{10}{13}$$

$$\frac{2}{2}x + \frac{7}{20}y + \frac{13}{2}z + \frac{3}{2}t = \frac{1}{22}$$

$$\frac{4}{7}x + \frac{4}{5}y + 2z + \frac{11}{8}t = \frac{27}{26}$$

$$\frac{2}{-1}x + \frac{7}{2}y + \frac{13}{3}z + 5t = \frac{20}{11}$$

(solutions: $x = \frac{3}{4}, y = \frac{5}{7}, z = \frac{-3}{13}, t = \frac{4}{11}$)

LOAD CARD 1

A 4 [ENT] 3 [R/S] →

4. ***
3. ***

7 [ENT] 5 [R/S] →

7. ***
5. ***

1 [ENT] [R/S] →

1. ***
1. ***

11 [CHS] [ENT] 4 [R/S] →

-11. ***
4. ***

10 [ENT] 13 [R/S] →

10. ***
13. ***

2 [ENT] [R/S] →

2. ***
2. ***

7 [ENT] 20 [R/S] →

7. ***
20. ***

13 [ENT] 2 [R/S] →

13. ***
2. ***

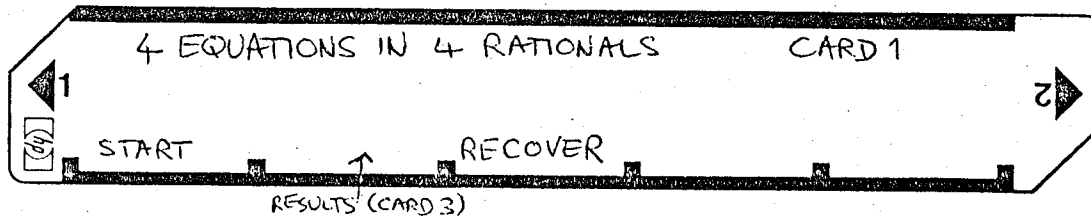
3 [ENT] 2 [R/S] →

3. ***
2. ***

1 [ENT] 22 [R/S] →

1. ***
22. ***

Reference(s)



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	LOAD SIDES 1 AND 2 OF CARD 1		☐ ☐	
2	START		☐ A ☐	
3	ENTER FIRST RATIONAL COEFFICIENT $a_{11} = \frac{m}{n}$ (m & n integers; RATIONAL NEED NOT BE IN SIMPLIFIED FORM)	m n	☐ ENT ☐ ☐ R/S ☐	m *** n ***
4	REPEAT STEP 3 FOR COEFFICIENTS $a_{12}, a_{13}, a_{14}, a_{15}, a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{31}, a_{32}$		☐ ☐	
5	DISPLAY WILL FLASH AT SECOND INTERVALS. INSERT SIDE 1 OF CARD 2 INTO READER. WHEN CARD IS READ, PROGRAM WILL CONTINUE		☐ ☐	
6	AT NEXT HALT, REPEAT STEP 3 FOR THE COEFFICIENTS a_{33}, a_{34}, a_{35} . THEN:		☐ ☐	
7	DISPLAY WILL FLASH AT SECOND INTERVALS. INSERT SIDE 2 OF CARD 2 INTO READER. WHEN CARD IS READ, PROGRAM WILL CONTINUE		☐ ☐	
8	AT NEXT HALT, REPEAT STEP 3 FOR COEFFICIENTS $a_{41}, a_{42}, a_{43}, a_{44}, a_{45}$		☐ ☐	
9	DISPLAY WILL FLASH AT SECOND INTERVALS. INSERT SIDE 1 OF CARD 3 INTO READER. WHEN CARD IS READ, PROGRAM WILL CONTINUE		☐ ☐	
10	THE 4 SOLUTIONS WILL BE PRINTED $x = \frac{x_N}{x_D} \quad y = \frac{y_N}{y_D} \quad z = \frac{z_N}{z_D} \quad t = \frac{t_N}{t_D}$ (ALL RESULTS ARE IN SIMPLIFIED FORM) (SEE 'WARNING' ON PAGE 1 FOR INTERPRETATION OF 0/0, $\pm 1/0$ OR 'Error' RESULTS)		☐ ☐	x_N *** x_D *** y_N *** y_D *** z_N *** z_D *** t_N *** t_D ***
11	(OPTIONAL: TO REPRINT OR (HP-67) RE-REVIEW RESULTS, PRESS		☐ B ☐	
	NOTE: IF CARDS DO NOT LOAD CORRECTLY WITH 'Error' DISPLAY, PRESS KEY TO CLEAR 'Error' THEN PRESS		☐ CLY ☐	
	IF DISPLAY DOES NOT THEN SHOW 'Error', RE-LOAD NEW CARD AS ABOVE AND CONTINUE AS NORMAL.		☐ C ☐	
	IF DISPLAY STILL SHOWS 'Error', RE-LOAD CARD 1, PRESS AND THEN RE-LOAD NEW CARD AND CONTINUE AS NORMAL.		☐ C ☐	
NB	IF DISPLAY CONTINUES FLASHING AT STEPS 5 & 7 AFTER LOADING CARD 2, LOAD THE CORRECT SIDE!		☐ ☐	



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Program Listing I

CARD 1

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STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLc	21 15 13	LIFT Q-STACK SUBTRACT FROM NEW DATUM	057	STOC	35 13	x OVERFLOW? $\sin 10^{10} = \text{Error}$ (if $\geq 10^{10}$) RECALL FROM Qn LIFT Q-STACK INPUT RATIONAL NB. HP-67 USERS SHOULD DELETE STEPS 100-104 SIMPLIFY
002	0	00		058	GSEW	23 16 14	
003	GSEW	23 06		059	XZY	-41	
004	GSEW	23 08	X	060	RCLD	36 14	
005	GTO1	22 01		061	XZY	-41	
006	*LBL3	21 13	÷	062	STOD	35 14	
007	GSEW	23 06		063	GSEW	23 16 14	
008	GTO3	22 03		064	F2?	16 23 02	
009	*LBL4	21 04		065	CHS	-22	
010	GSEW	23 06		066	-	-45	
011	XZY	-41		067	GSEW	23 16 13	
012	*LBL3	21 03		068	RCLC	36 15	
013	RCLC	36 13		069	GSEW	23 00	
014	XZY	-41		070	RCLC	36 13	
015	STOC	35 13		071	RCLD	36 14	
016	R↓	-31		072	X	-35	
017	GSEW	23 00		073	*LBLd	21 16 14	
018	RCLD	36 14		074	X	-35	
019	RCLC	36 13		075	*LBLe	21 16 15	
020	R↑	16-31		076	ENT↑	-21	
021	STOD	35 14		077	ABS	16 31	
022	R↑	16-31		078	EEK	-23	
023	STOC	35 13		079	1	01	
024	R↓	-31		080	0	00	
025	R↓	-31		081	XZY?	16-35	
026	GSEW	23 00		082	SIN↑	16 41	
027	XZY	-41		083	R↓	-31	
028	RCLD	36 14		084	R↓	-31	
029	GSEW	23 16 14		085	RTN	24	
030	XZY	-41		086	*LBL6	21 06	
031	RCLC	36 13		087	2	02	
032	GTOd	22 16 14		088	X	-35	
033	*LBL5	21 05		089	STOI	35 46	
034	2	02		090	R↓	-31	
035	X	-35		091	STOC	35 13	
036	STOI	35 46		092	XZY	-41	
037	R↓	-31		093	STOD	35 14	
038	XZY	-41		094	RCL↑	36 45	
039	STOI	35 45		095	ISZI	16 26 46	
040	XZY	-41		096	RCL↑	36 45	
041	ISZI	16 26 46		097	RTN	24	
042	STOI	35 45		098	*LBL8	21 08	
043	RTN	24		099	R/S	51	
044	*LBL5	21 05		100	XZY	-41	
045	SFZ	16 21 02		101	PRTX	-14	
046	*LBL2	21 02		102	XZY	-41	
047	GSEW	23 06		103	PRTX	-14	
048	*LBL1	21 01		104	SFC	16-11	
049	XZY	-41		105	*LBL0	21 00	
050	RCLC	36 13		106	1	01	
051	XZY	-41		107	2	02	
052	STOC	35 13		108	CHS	-22	
053	R↓	-31		109	STOI	35 46	
054	GSEW	23 00		110	R↓	-31	
055	RCLC	36 13		111	XZY	-41	
056	XZY	-41		112	STOI	35 01	

REGISTERS

Q0 DENOMINATOR	Q1 NUMERATOR	Q2 DENOM	Q3 NUM.	Q4 DENOM.	Q5 NUM	Q6 DENOM	Q7 NUM	Q8 DENOM	Q9 NUM
S0 Q5 DENOM	S1 Q5 NUM	S2 Q6 DENOM	S3 Q6 NUM	S4 Q7 DENOM	S5 Q7 NUM	S6 Q8 DENOM	S7 Q8 NUM	S8 Q9 DENOM	S9 Q9 NUM
A Q10 DENOMINATOR	B Q10 NUMERATOR	C Qy (USED) DENOMINATOR d	D Qy (USED) NUMERATOR c	E G.C.D. 7	I USED				

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
113	ABS	16 31		163	3	03	
114	STOE	35 15		170	G5B4	23 16 14	a ₂₄
115	XZY	-41		171	6	06	
116	STOG	35 00		172	G5B5	23 05	
117	ABS	16 31		173	4	04	
118	ROLE	36 15		174	G5B4	23 16 14	a ₂₅
119	XZY	-41		175	7	07	
120	ROLE	36 15		176	G5B5	23 05	
121	XZY	-41		177	1	01	
122	STOE	35 15		178	G5B3	23 03	
123	X#0?	16-42		179	4	04	
124	=	-24		180	G5B2	23 02	
125	INT	16 34		181	4	04	
126	ROLE	36 15		182	G5B5	23 05	
127	X	-35		183	6	06	
128	-	-45		184	G5B6	23 06	
129	X#0?	16-42		185	1	01	
130	GT01	22 45		186	G5B3	23 03	
131	RCL0	36 00		187	3	03	
132	ROLE	36 15		188	G5B2	23 02	
133	XZY?	16-34		189	3	03	
134	CHS	-22		190	G5B5	23 05	
135	STOE	35 15		191	3	03	
136	X#0?	16-43		192	G5B6	23 06	
137	RTN	24		193	1	01	
138	ST=1	35-24 01		194	G5B3	23 03	
139	ST=0	35-24 00		195	2	02	
140	RCL1	36 01		196	G5B2	23 02	
141	RCL0	36 00		197	2	02	
142	RTN	24		198	G5B4	23 16 12	ROW 3 a ₃₁
143	*LEBL0	21 13		199	2	02	
144	MRC	16-62		200	G5B3	23 03	
145	FSE	16 51		201	1	01	
146	GT00	22 13		202	G5B5	23 05	
147	*LEBLA	21 11		203	G5B8	23 08	a ₃₂
148	CF2	16 22 02		204	SF2	16 21 02	
149	G5B8	23 08		205	CF3	16 22 03	
150	9	09		206	GT00	22 13	
151	G5B4	23 16 11		207	*LEBL6	21 16 12	
152	1	01		208	G5B5	23 05	
153	G5B4	23 16 11		209	SFC	16-11	
154	2	02		210	G5B8	23 08	
155	G5B4	23 16 11		211	9	09	
156	3	03		212	GT05	22 05	
157	G5B4	23 16 11		213	*LEBL4	21 16 14	
158	4	04		214	G5B6	23 06	
159	G5B4	23 16 12		215	9	09	
160	1	01		216	G5B3	23 03	
161	G5B3	23 03		217	G5B6	23 16 13	
162	G5B6	23 16 13		218	8	08	
163	6	06		219	GT04	22 04	
164	G5B5	23 05		220	*LEBL4	21 16 11	
165	2	02		221	G5B5	23 05	
166	G5B4	23 16 14		222	G5B8	23 08	
167	5	05		223	9	09	
168	G5B5	23 05		224	GT04	22 04	

←
EUCLIDEAN
ALGORITHM
LOOP
TO
FIND
G.C.D.

MERGE LOOP
(RECOVER)

START
ROW 1
a₁₁
a₁₂
a₁₃
a₁₄
a₁₅
GAUSS-
JORDAN
REDUCTION

ROW 2 a₂₁

a₂₂

a₂₃

ROW 3 a₃₁

a₃₂

NEW ROW
BEGIN ROW 2, 3

a_{ij} → Q₅

USED, ROW 1

NEW DATUM ÷ a₁₁

LABELS					FLAGS		SET STATUS (CARD 1)		
A	B	C	D	E	0	1	FLAGS	TRIG	DISP
START		MERGE LOOP							
a ROW 1	b NEW ROW	c SUBTRACT FROM NEW DATUM	d X $\frac{LX-60}{OVERFLOW?}$	e OVERFLOW?	1		ON OFF		
0 SIMPLIFY	1 Q _x - Q _y	2 —	3 X	4 ÷	2 +? CARD 2 SIDE 1		0 ☐ ☐	DEG ☐	FIX ☒
5 STO	6 RCL	7	8 INPUT	9 L	3		1 ☐ ☐	GRAD ☐	SCI ☐
							2 ☒ ☐	RAD ☐	ENG ☐

Program Listing I

[illegible]

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	F3?	16 23 03		057	G5B5	23 05	
002	GTOA	22 11	MERGE	058	1	01	
003	GTOC	22 13	LOOP	059	G5B6	23 06	
004	ALZLA	21 11	CONTINUE	060	4	04	
005	CF2	16 22 02		061	G5B5	23 03	
006	5	05	CARD 2	062	2	02	
007	G5B3	23 03	SIDE 2	063	G5B5	23 09	
008	7	07		064	G5B6	23 16 13	
009	G5B2	23 02		065	5	05	
010	7	07		066	G5B4	23 04	
011	G5B5	23 05	BEGIN ROW 4	067	1	01	
012	SFC	16-11		068	G5B5	23 05	
013	G5B5	23 05	a41	069	3	03	
014	1	01		070	G5B5	23 03	
015	G5B5	23 05		071	4	04	
016	G5B6	23 06	a42	072	G5B2	23 02	
017	8	08		073	4	04	
018	G5B5	23 05		074	G5B5	23 05	
019	G5B6	23 06	a43	075	1	01	
020	2	02		076	G5B6	23 06	
021	G5B5	23 05		077	5	05	
022	1	01		078	CF3	16 22 03	
023	0	00		079	GTOC	22 13	
024	G5B3	23 03					
025	5	05					
026	G5B5	23 05					
027	8	08					
028	G5B6	23 06					
029	6	06					
030	G5B3	23 03					
031	5	05					
032	G5B9	23 09		200			
033	5	05					
034	G5B5	23 05					
035	1	01					
036	G5B6	23 06					
037	3	03					
038	G5B3	23 03					
039	5	05					
040	G5B9	23 09					
041	G5B6	23 16 13	a44				
042	5	05		210			
043	G5B5	23 05					
044	2	02					
045	G5B5	23 05					
046	9	09					
047	G5B3	23 03					
048	2	02					
049	G5B5	23 05					
050	6	06					
051	G5B6	23 06					
052	7	07		220			
053	G5B3	23 03					
054	2	02					
055	G5B9	23 09					
056	2	02					

REGISTERS:
AS CARD 1

LABELS					FLAGS	SET STATUS		
NAME	B	C	D	E	0	FLAGS	TRIG	DISP
a	b	c	d	e	1	ON OFF		
0	1	2	3	4	2 +?	0 ☐ ☐	DEG ☐	FIX ☒
5	6	7	8	9	3	1 ☐ ☐	GRAD ☐	SCI ☐
						2 ☐ ☒	RAD ☐	ENG ☐
						3 ☐ ☒	Any	n ☐

