Program Description I

Program Title: FOURIER SERIES - HARMONIC ANALYSIS - DISCRETE DOMAIN

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Program Description, Equations, Variables WE HAVE N DATA POINTS (x_i, y_i) THAT ARE SAMPLES OF A PERIODIC FUNCTION $y(x)$, AND WE ASSUME THAT $y(x)$ MAY BE APPROXINATED BY A FURIER SERIES OF THE SAME PERIOD AS $y(x)$. THE ARGUMENTS x_i ARE EQUALLY SPACED

$$x = 0, h, 2h, 3h, \ldots$$

BUT A MERE DIVISION YIELDS THE MORE APPROPIATE SET $x' = 0, 1, 2, 3, \ldots$.

THEN, THERE ARE 2 DIFFERENT CASES:

a) $N = 2L+1$ (N IS ODD), $x = 0, 1, 2, \ldots 2L$

$$a_k = \frac{2}{2L+1} \sum_{x=0}^{x=2L} y(x) \cos \frac{2\pi}{2L+1} kx \quad , \quad k = 0, 1, \ldots L$$

$$b_k = \frac{2}{2L+1} \sum_{x=0}^{x=2L} y(x) \operatorname{sen} \frac{2\pi}{2L+1} kx \quad , \quad x = 1, 2, \ldots L$$

$$\hat{y}(x) = \frac{1}{2} a_0 + \sum_{k=1}^{k=L} \left(a_k \cos \frac{2\pi}{2L+1} kx + b_k \operatorname{sen} \frac{2\pi}{2L+1} kx \right)$$

b) $N = 2L$ (N IS EVEN), $x = 0, 1, 2, \ldots 2L-1$

$$a_k = \frac{1}{L} \sum_{x=0}^{x=2L-1} y(x) \cos \frac{\pi}{L} kx \quad , \quad k = 0, 1, \ldots L$$

$$b_k = \frac{1}{L} \sum_{x=0}^{x=2L-1} y(x) \operatorname{sen} \frac{\pi}{L} kx \quad , \quad k = 1, 2, \ldots L-1$$

$$\hat{y}(x) = \frac{1}{2} a_0 + \sum_{k=1}^{k=L-1} \left(a_k \cos \frac{\pi}{L} kx + b_k \operatorname{sen} \frac{\pi}{L} kx \right) + \frac{1}{2} a_L \cos \pi x$$

~~Operating Limits and Warnings~~

THE PROGRAM ALSO COMPUTES THE SUM OF SQUARED ERRORS FOR EACH K

$$S_{MIN\,k} = \sum_{x=0}^{x=Nh} \left(y(x) - \hat{y}_k(x) \right)^2 = \frac{2L+1}{2} \sum_{k=M+1}^{k=L} (a_k^2 + b_k^2) \quad \text{IF N IS ODD}$$

AND A VERY SIMILAR EXPRESSION IF N IS EVEN.

$\boxed{\text{WARNING}}$ PROGRAM MUST NOT BE STOPPED DURING CALCULATIONS; IF AN ACCIDENTAL STOP HAPPENS, CHECK THAT A P≷S HAS NOT TAKEN PLACE BEFORE RESUMING EXECUTION.

PROGRAM CHARACTERISTICS

- LBLA ≡ DATA INPUT AND HARMONICS COMPUTATION

WE HAVE N DATA POINTS (x_i, y_i), $x_i = 0, h, 2h, \cdots (N-1)h$

PROCEED AS FOLLOWS: h [ENTER] N [A] $\rightarrow x_{N-1}$

$y(x_{N-1})$ [R/S] $\rightarrow x_{N-2}$, etc

and finally, $y(x_1)$ [R/S] $\rightarrow 0$

$y(0)$ [R/S] $\rightarrow a_0 \rightarrow (1) \rightarrow a_1 \rightarrow b_1$

$\rightarrow (2) \rightarrow a_2 \rightarrow b_2$

$\rightarrow (k) \rightarrow a_k \rightarrow b_k$

$\rightarrow 0.0000$

AFTER $y(0)$ HAS BEEN INPUT HARMONICS WILL BE AUTOMATICALLY DISPLAYED

- N may be any integer > 1; h must not equal zero
- IF $N \leq 19$, program calculates $INT[N/2]$ harmonics; IF $N > 19$, 9 harmonics will be calculat
- $a_1, a_2, \cdots a_9$ are stored in R_1 to R_9; $a_0, b_1, b_2, \cdots b_9$, in R_{S0} to R_{S9}
- a_0 is 2 times the mean value of $y(x)$ over the period.

- LBLa ≡ DELETION OF A MISTAKE

FOLLOW THIS SAMPLE:

$y(x_k)$ [R/S] $\rightarrow x_{k-1}$

NOW, YOU MAKE A MISTAKE $\rightarrow y(x_{k-1})$ [R/S] $\rightarrow x_{k-2}$, WHERE $y(x_{k-1})$ IS ERRONEOUS.

DELETE THE ERROR [f A] $\rightarrow x_{k-1}$

INPUT CORRECT VALUE $y(x_{k-1})$ [R/S] $\rightarrow x_{k-2}$, etc.

- this subroutine corrects any erroneous $y(x_i)$ except $y(0)$. Apart from this, any other value can be corrected any number of times.

- LBLB ≡ REVIEW

pressing [B] causes the program to display $\rightarrow a_0 \rightarrow (1) \rightarrow a_1 \rightarrow b_1$

$\rightarrow (2) \rightarrow a_2 \rightarrow b_2$

$\rightarrow (3) \rightarrow a_3 \rightarrow b_3$, etc.

- a_k, b_k are displayed rounded to 4 decimals, but remain unchanged in their storage locations.
- a maximum of 9 a_k, b_k, plus a_0 will be displayed. Depends on # stored in R_B

- LBLC ≡ SQUARED ERRORS SUMS

press, [C] $\rightarrow k \rightarrow S_k$

$\rightarrow k-1 \rightarrow S_{k-1}$

$\rightarrow \cdots \cdots$

$\rightarrow 0 \rightarrow S_0 \rightarrow 0.0000$

- IF $N \leq 19$, $\# = INT(N/2)$, $S\# = 0$, because $\hat{y}(x)$ collocates in every data point.
- $S\#$ is not displayed (is 0)

- this calculation is valid if $N \leq 19$. Otherwise, some unknown constant must be added to $S_{MIN\,k}$
- to obtain correct values, proper # must be stored in R_B. This is always the case, except if you have previously changed k value for evaluate $\hat{y}(x)$ with some k number of harmonics. see LBLE
- this values $S_{MIN\,k}$ are very useful to select the number of harmonics that are enough in $\hat{y}(x)$ evaluation within a prescribed error. The RMS value is:

$$\text{error RMS}_k = \sqrt{S_{MIN\,k}/N}$$

- LBL E ≡ $\hat{y}(x)$ evaluation

- if you want to calculate $\hat{y}(x)$ with maximum available # of harmonics

x [E] $\rightarrow \hat{y}(x)$

if $N \leq 19$, $\hat{y}(x)$ collocates in every one of the N arguments $x = 0, h, 2h, \cdots (N-1)h$

if $N > 19$, $\hat{y}(x)$ is a least-squares approximation with 9 harmonics to $y(x)$

- if you want to calculate $\hat{y}(x)$ with some k number of harmonics $(1 \leq k \leq INT[N/2]$ or 9

input k; k [f E] $\rightarrow k$

- if you want to recalculate $S_{MIN\,k}$, or display again a_k, b_k, or calculate $\hat{y}(x)$ with the maximum possible # of harmonics, reset original value

[f C] $\rightarrow$ # of harmonics

LBL D ≡ automatic evaluation

- it evaluates $\hat{y}(x)$ for $x = x_0, x_0 + \Delta, x_0 + 2\Delta, \cdots$

x_0 [ENTER] Δ [D] $\rightarrow (x_0) \rightarrow \hat{y}(x_0)$

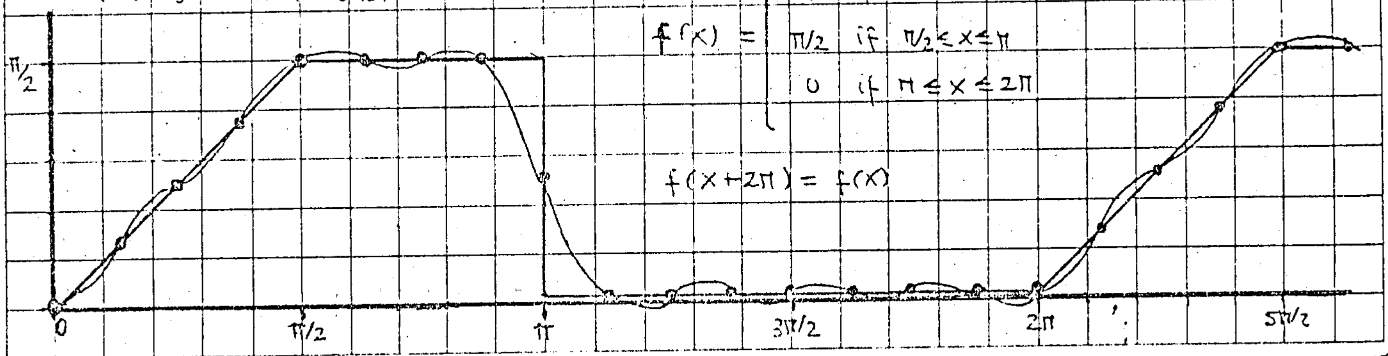
$\rightarrow (x_0+\Delta) \rightarrow \hat{y}(x_0+\Delta)$, etc. Stop pressing [R/S]

Program Description II

Sketch(es) $\hat{y}(x)$ IS ROUGHLY REPRESENTED

$$f(x) = \begin{cases} x & \text{if } 0 \le x \le \pi/2 \\ \pi/2 & \text{if } \pi/2 \le x \le \pi \\ 0 & \text{if } \pi \le x \le 2\pi \end{cases}$$

$$f(x+2\pi) = f(x)$$



Sample Problem(s) #1) SUPPOSE THAT F(X) IS DEFINED IN [0,2π] IN SUCH A WAY THAT

$$F(x) = \begin{cases} x & \text{if } 0 \le x \le \pi/2 \\ \pi/2 & \text{if } \pi/2 \le x \le \pi \\ 0 & \text{if } \pi \le x \le 2\pi \end{cases} \quad \text{(SEE SKETCH)}$$

AND IS DEFINED ELSEWHERE BY THE REQUIREMENT THAT IT BE PERIODIC WITH PERIOD 2π. ASSUME THAT THE FOLLOWING DATA ARE GIVEN. FIND THE REQUIRED HARMONICS TO FIT THOSE DATA.

x	0	π/8	π/4	3π/8	π/2	5π/8	3π/4	7π/8	π	9π/8	5π/4	11π/8	3π/2	13π/8	7π/4	15π/8
y	0	π/8	π/4	3π/8	π/2	π/2	π/2	π/2	π/4	0	0	0	0	0	0	0

- LOAD PROGRAM
- THERE ARE 16 DATA POINTS, AND THE SPACING IS $h = \pi/8$

INPUT DATA: π/8 [ENTER] 16 [A] → 5.8905 ($= x_{15} = 15\pi/8$)
0 [R/S] → 5.4978 (7π/4), 0 [R/S] → 5.1051, 0 [R/S] → 4.7124
0 [R/S] → 4.3197, 0 [R/S] → 3.9270, 0 [R/S] → 3.5343
0 [R/S] → 3.1416, π/4 [R/S] → 2.7489, π/2 [R/S] → 2,3562

Solution(s)

π/2 [R/S] → 1.9635, π/2 [R/S] → 1.5708 [R/S] → 1.1781 [R/S] → 0.7854
[R/S] → 0.3927 [R/S] → 0.0000 [R/S] → 1,1781 (a_0) →

→ (1) → −0.3224 (a_1) → 0.8160 (b_1)	→ (5) → −0.0178 → 0.0833
→ (2) → −0.1676 (a_2) → −0.2370 (b_2)	→ (6) → −0.0288 → −0.0407
→ (3) → −0.0398 (a_3) → 0.1072 (b_3)	→ (7) → −0.0128 → 0.0068
→ (4) → 0.0000 (a_4) → −0.0982 (b_4)	→ (8) → 0.0000 → 0.0000 → 0.0000

SO, y(x) IS APPROXIMATED AS: (SEE SKETCH: REPRESENTED $\hat{y}_7(x)$)

$$y(x) = \tfrac{1}{2} \cdot 1.1781 + (-0.3224\cos x + 0.8160\sin x) + (-0.1676\cos 2x - 0.2370\sin 2x) + \cdots \text{ etc.}$$

(TO BE CONTINUED ON THE OTHER SIDE OF THIS PAGE)

Reference(s) INTRODUCTION TO NUMERICAL ANALYSIS – (F.B. HILDEBRAND) –
– INTERNATIONAL SERIES IN PURE AND APPLIED MATHEMATICS – MCGRAW-HILL BOOKS

2) GIVEN THIS 7 DATA POINTS OF A PERIODIC FUNCTION, COMPUTE:

X	0	5	10	15	20	25	30
y	1	3	4	2	0	6	5

a) MEAN VALUE
b) ALL REQUIRED HARMONICS TO FIT THE DATA
c) VALUES OF $\hat{y}(x)$ WITH K = 0, 1, 2, 3 HARMONICS FOR X = 0, 5, 10, 15, 20, 25, 30
d) THE SUM OF SQUARED ERRORS WITH K = 0, 1, 2, 3 HARMONICS

— FIRSTLY, INPUT DATA: 5 [ENTER] 7 [A] → 30; 5 [R/S] → 25; 6 [R/S] → 20; 0 [R/S] → 15; 2 [R/S] → 10; 4 [R/S] → → 5; 3 [R/S] → 0; 1 [R/S] → 6.0000 (a_0) →
→ (1) → 0.5602 (a_1) → −0.7559 (b_1)
→ (2) → −2.4408 (a_2) → −0.7559 (b_2)
→ (3) → −0.1194 (a_3) → 0.7559 (b_3) → 0.0000

— SO, THE MEAN VALUE IS $\frac{a_0}{2} = 3.0000$. THE REQUIRED HARMONICS ARE $a_0, a_1, b_1, a_2, b_2, a_3, b_3$.

— NOW, LET'S CALCULATE VALUES OF $\hat{y}(x)$ WITH 3 HARMONICS, 2, 1, 0. THE VALUES WITH 0 HARMONICS ARE ALL EQUAL TO THE MEAN VALUE, 3.0000. AS AN EXAMPLE, FOR K = 2:

2 [fE] → 2.0000; 0 [ENTER] 5 [D] → (0.0000) → 1.1194 → (5.0000) → 2.5644 → (10.0000) → 4.6655, etc.

DOING THE SAME FOR K = 1, 3, WE CAN THUS FORM THE TABLE:

	x=0	x=5	x=10	x=15	x=20	x=25	x=30	SMIN K	error RMS
K=0	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	28.0000	2.0000
K=1	3.5602	2.7583	2.1384	2.1673	2.8232	3.6123	3.9403	24.9015	1.8861
K=2	1.1194	2.5644	4.6655	1.2365	0.7104	5.4834	5.2204	2.0499	0.5411
K=3	1.0000	3.0000	4.0000	2.0000	0.0000	6.0000	5.0000	——	——

— TO CALCULATE SMIN K, FIRST RESET # OF HARMONICS. [fC] → 3.0000
— NOW, [C] → (2) → 2.0499 → (1) → 24.9015 → (0) → 28.0000 → 0.0000

I WILL POINT OUT SEVERAL FACTS: — FOR K = 3, $\hat{y}(x)$ COLLOCATES IN EVERY DATA POINT; SMIN = 0
— FOR K = 0, $\hat{y}(x) = a_0/2$
— IT IS EASILY PROBED THAT $S_{MIN}(0) = (1-3)^2 + (3-3)^2 + (4-3)^2 + (2-3)^2 + (0-3)^2 + (6-3)^2 + (5-3)^2 = 28$, IS CORRECT

3) PRACTICAL EXAMPLE: BY MEANS OF AN SPECIAL GENERATOR, WE WERE ABLE TO REPRESENT IN AN OSCILLOSCOPE'S SCREEN THE GRAPHIC OF THE INTENSITY IN THE PRIMARY WINDING OF A Δ-Y TRANSFORMER (NON-LOAD CONDITIONS). FROM THE GRAPHIC WAS FORMED THE FOLLOWING TABLE: (33 DATA)

x_i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
y_i	2.00	1.85	1.60	1.50	1.52	1.65	1.10	0.70	0.35	0.05	−0.28	−0.58	−0.80	−1.00	−1.26	−1.80	−2.00	−1.90	−1.75
x_i	19	20	21	22	23	24	25	26	27	28	29	30	31	32					
y_i	−1.55	−1.55	−1.62	−1.64	−1.30	−0.65	−0.27	0.02	0.30	0.60	0.75	1.01	1.40	1.82	y(x+33) = y(x)				

WE ARE INTERESTED IN THE HARMONIC ANALYSIS OF THE INTENSITY.

— INPUT DATA: 1 [ENTER] 33 [A] → 32.0000; 1.82 [R/S] → 31; 1.40 [R/S] → 30; --- --- --- [R/S] → 0.0000; 2 [R/S] →
→ WE OBTAIN THE HARMONICS:

$a_0 = -0.1048$
$a_1 = 1.7752$, $b_1 = 0.4475$ | $a_5 = 0.1474$, $b_5 = -0.1201$
$a_2 = 0.0353$, $b_2 = 0.0030$ | $a_6 = -0.0108$, $b_6 = -0.0171$
$a_3 = 0.0047$, $b_3 = 0.0112$ | $a_7 = 0.0917$, $b_7 = 0.0051$
$a_4 = 0.0042$, $b_4 = 0.0380$ | $a_8 = 0.0332$, $b_8 = 0.0162$
| $a_9 = -0.0259$, $b_9 = -0.0117$

— ONLY THE FUNDAMENTAL, AND THE 5TH, 7TH HARMONICS ARE RELEVANT. THE 5TH ONE IS ≃ 10% OF THE FUNDAMENTAL, AND THE 7H ONE IS ≃ 5%. THE MEAN VALUE $a_0/2$ IS NEGLIGIBLE

— WE HAVE CALCULATED UP TO a_9, b_9. BUT WE NEED A SHORTER APPROXIMATION WITHOUT INTRODUCING TOO MUCH ERROR. AS N = 33 > 19, WE CAN'T GET SMIN K, BUT WE'LL STILL HAVE SOME INFORMATION ABOUT ERROR:

[C] → (8) → 0.0134 → (7) → 0.0358 → (6) → 0.1749 ------ → 0.0000

— THESE VALUES ARE INEXACT. THEY ARE EQUAL TO CORRECT VALUES − CONSTANT
— BY LOOKING AT THESE VALUES, WE DECIDE TO CHECK $\hat{y}(x)$ WITH 9, 8, 7 HARMONICS AGAINST DATA POINTS: 0 [ENTER] 1 [D] → $\hat{y}_9$; 8 [fE], 0 [ENTER] 1 [D] → $\hat{y}_8$

X	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\hat{y}_9$	2.00	1.85	1.59	1.49	1.58	1.55	1.19	0.68	0.32	0.08	−0.26	−0.62	−0.80	−0.95	−1.32	−1.77	−1.99	−1.93	−1.73
X	19	20	21	22	23	24	25	26	27	28	29	30	31	32					
$\hat{y}_9$	−1.56	−1.53	−1.64	−1.64	−1.27	−0.69	−0.25	0.02	0.30	0.58	0.77	1.00	1.40	1.82	y(x+33) = y(x)				
X	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\hat{y}_8$	2.03	1.86	1.56	1.49	1.61	1.54	1.16	0.67	0.34	0.05	−0.28	−0.59	−0.79	−0.98	−1.32	−1.74	−2.00	−1.95	−1.72
X	19	20	21	22	23	24	25	26	27	28	29	30	31	32					
$\hat{y}_8$	−1.54	−1.55	−1.66	−1.61	−1.26	−0.72	−0.25	0.05	0.29	0.55	0.79	1.02	1.37	1.81	y(x+33) = y(x)				

THE FITTING IS VERY GOOD

User Instructions

FOURIER SERIES - HARMONIC	ANALYSIS - DISCRETE	DOMAIN		
DELETE		RESET #		#HARMON. FOR $\hat{y}$
$h \uparrow N \rightarrow$ INPUT	REVIEW $\rightarrow a_K, b_K$	$\rightarrow (K) \rightarrow S_{MIN\,K}$	AUTOMATIC EVAL. $x_0 \uparrow \Delta \rightarrow x_i, \hat{y}_i$	EVALUATION $x \rightarrow \hat{y}$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	LOAD SIDE 1 AND TWO OF CARD.				
2	ENTER h, (FIRST NONZERO X VALUE), N (# OF DATA POINTS	h	ENTER↑		
		N	A		X_{N-1}
3	ENTER y_i VALUES ($y_i = y(x_i)$)	$y(x_i)$	R/S		x_{i-1}
		$y(x_{i-1})$	R/S		x_{i-2}
	AFTER ALL DATA HAVE BEEN ENTERED	---	--	--	---
	HARMONICS WILL BE DISPLAYED	$y(0)$	R/S		a_0
	(IF YOU MAKE A MISTAKE WHEN ENTERING DATA, GOTO 7)				(1)
	FINALLY, DISPLAYS 0.0000 AND STOPS.				a_1
					b_1
					(2)

					0.0000
4	IF DESIRED, AT ANY TIME, REDISPLAY HARMONICS		B		a_0
					$(K) \rightarrow a_K, b_K$
					0.0000
5	TO EVALUATE $\hat{y}(X)$: (OPTIONAL: STORE # OF HARMONICS) →	K	f	E	K
	ENTER X VALUE	X	E		$\hat{y}_K(X)$
6	AUTOMATIC EVALUATION: (OPTIONAL: STORE # OF HARMONICS) →	K	f	E	K
	ENTER X_0, Δ	X_0	ENTER		
		Δ	D		$(X_i), \hat{y}_K(X_i)$
7	TO DELETE A MISTAKE ($y(x_i)$ ERRONEOUS)		f	A	LAST X VALUE
		CORRECT $Y(X_i)$	R/S		X_{i-1}, etc.
8	TO CALCULATE $S_{MIN\,K}$				
	(IF $\boxed{fE}$ HAS BEEN USED, FIRST PRESS:		f	C	#
			C		$(K) \rightarrow S_{MIN\,K}$
					0.0000
9	TO RESET # OF HARMONICS, FOR EVALUATING				
	PURPOSES OR TO CALCULATE $S_{MIN\,K}$, PRESS		f	C	#
10	FOR ANOTHER CASE, GOTO 2				

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	* LBLA	31 25 11	DATA INPUT & a_k, b_k COMPUTATION
	CF0	35 61 00	
	CF2	35 61 02	
	RAD	35 42	
	CLREG	31 43	INITIALIZATION
	P≷S	31 42	
	CLREG	31 43	
	STO E	33 15	$N \to R_E$
	STO 0	33 00	$N \to R_0$
010	DSZ(i)	32 33	$N-1 \to R_0$
	2	02	
	X≷Y	35 52	
	÷	81	$2/N$
	π	35 73	$2\pi/N$
	×	71	
	STO D	33 14	$2\pi/N \to R_D$
	R↓	35 53	
	STO C	33 13	$h \to R_C$
	GSB c	32 22 13	COMPUTES & STORES # OF HARMONICS NEEDED
020	* LBL0	31 25 00	MAIN SUBROUTINE FOR a_k, b_k COMPUTATION
	RCL0	34 00	
	X=0	31 51	$x_i = 0$?
	SF2	35 51 02	
	F0?	35 71 00	DELETE AN ERROR ?
	RCLA	34 11	RECALL ERRONEOUS y_i
	F0?	35 71 00	DELETE ?
	CHS	42	$y_i \to -y_i$
	F0?	35 71 00	DELETE?
	GTO7	22 07	
030	RCLC	34 13	RECALL h
	×	71	$x_i = h.i$
	R/S	84	DISPLAYS x_i & INPUTS y_i
	RCLE	34 15	N
	÷	81	
	2	02	$\frac{2}{N} y_i$
	×	71	
	* LBL7	31 25 07	Σ ITERATION
	STO A	33 11	$2/N \, y_i \to R_A$
	RCLB	34 12	# OF HARMONICS TO CALCULATE
040	ST I	35 33	# → ST I
	* LBL1	31 25 01	
	RCL0	34 00	x'_i
	RCLD	34 14	$2\pi/N$
	×	71	
	RCI	35 34	Y | $y_i \cdot \frac{2}{N} \cdot \cos \frac{2\pi}{N} k x'_i$
	×	71	X | $y_i \cdot \frac{2}{N} \cdot \sin \frac{2\pi}{N} k x'_i$
	RCLA	34 11	
	P→R	31 72	
	STO+(i)	33 61 24	Σ, a_k
050	X≷Y	35 52	
	P≷S	31 42	
	STO+(i)	33 61 24	Σ, b_k
	P≷S	31 42	
	DSZ	31 33	ANOTHER HARMONIC?
	GTO 1	22 01	YES, ITERATE
	RCLA	34 11	NO, Σ, a_0

STEP	KEY ENTRY	KEY CODE	COMMENTS
	P≷S	31 42	
	STO+0	33 61 00	Σ, a_0
	P≷S	31 42	
060	F2?	35 71 02	$x_i = 0$?
	GTO B	22 12	EXIT = GOTO DISPLAY
	1	01	
	F0?	35 71 00	F0 SET = ERROR
	CLX	44	$i \to i-1$
	F0?	35 71 00	
	CF0	35 61 00	
	STO-0	33 51 00	
	GTO 0	22 00	
	* LBLB	31 25 12	DISPLAY SUBROUTINE
070	P≷S	31 42	
	RCL0	34 00	
	P≷S	31 42	
	DSP4	23 04	ROUNDING TO 4 DECIMALS.
	RND	31 24	
	-X-	31 84	DISPLAY a_0
	1	01	
	ST I	35 33	
	* LBL3	31 25 03	
	DSP0	23 00	
080	PSE	35 72	(K)
	DSP4	23 04	
	RCL(i)	34 24	DISPLAY a_K ROUNDED
	RND	31 24	
	-X-	31 84	
	P≷S	31 42	
	RCL(i)	34 24	DISPLAY b_K ROUNDED
	P≷S	31 42	
	RND	31 24	
	-X-	31 84	
090	ISZ	31 34	
	RCL B	34 12	
	RC I	35 34	
	X≤Y	32 71	ANOTHER HARMONIC?
	GTO 3	22 03	
	CLX	44	DISPLAY 0.0000 AND STOP
	RTN	35 22	
	* LBLD	31 25 14	AUTOMATIC PLOTTING
	STO A	33 11	STORE Δ
	X≷Y	35 52	
100	* LBL4	31 25 04	
	PSE	35 72	DISPLAY x_m
	GSB E	31 22 15	CALCULATE $\hat{y}_m$
	-X-	31 84	DISPLAY $\hat{y}_m$
	RCL0	34 00	
	RCLC	34 13	
	×	71	x_m
	RCLA	34 11	Δ
	+	61	$x_{m+1} = x_m + \Delta$
	GTO 4	22 04	PLOT ANOTHER POINT x_m
110	* LBLE	31 25 15	MANUAL PLOTTING, $x \to \hat{y}$
	RCLC	34 13	h
	÷	81	$x' = x/h$

REGISTERS

0	1	2	3	4	5	6	7	8	9
USED ≡ x_i	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
S0 a_0	S1 b_1	S2 b_2	S3 b_3	S4 b_4	S5 b_5	S6 b_6	S7 b_7	S8 b_8	S9 b_9

A	B	C	D	E	I
Δ	# harmumics	h	$2\pi/N$	N	USED ≡ K

STEP	KEY ENTRY	KEY CODE	COMMENTS
	STO 0	33 00	x' → R0
	RCL B	34 12	# → ST I
	ST I	35 33	
	0	00	INITIALIZES Σ
	* LBL 5	31 25 05	MAIN SUBROUTINE FOR $\hat{y}$ EVALUATION
	RC I	35 34	
	RCL 0	34 00	
120	×	71	$\frac{2\pi}{N}kx'$
	RCL D	34 14	
	×	71	
	ENTER↑	41	
	COS	31 63	$a_k \cos \frac{2\pi}{N} kx'$
	RCL(i)	34 24	
	×	71	
	x≷y	35 52	$b_k \operatorname{sen} \frac{2\pi}{N} kx'$
	SIN	31 62	
	P≷S	31 42	
130	RCL(i)	34 24	
	P≷S	31 42	
	×	71	
	+	61	$a_k \cos \frac{2\pi}{N} kx' + b_k \operatorname{sen} \frac{2\pi}{N} kx'$
	+	61	Σ
	DSZ	31 33	ANOTHER HARMONIC?
	GTO 5	22 05	YES, ITERATE
	P≷S	31 42	
	RCL 0	34 00	
	P≷S	31 42	$a_0/2$
140	2	02	
	÷	81	
	+	61	$\frac{1}{2}a_0 + \Sigma = \hat{y}$
	F1?	35 71 01	N IS ODD?
	RTN	35 22	YES, STOP AND DISPLAY $\hat{y}$
	RCL E	34 15	NO, N IS EVEN
	2	02	
	RCL B	34 12	N ≥ 10 ?
	×	71	
	x=y	32 51	
150	GTO 8	22 08	NO, GTO 8
	R↑	35 54	YES, STOP AND DISPLAY $\hat{y}$
	RTN	35 22	
	* LBL 8	31 25 08	N IS EVEN, N<9
	LAST X	35 82	$L \to R_I$
	ST I	35 33	
	R↑	35 54	
	RCL 0	34 00	
	π	35 73	$\frac{1}{2}a_L \cos \pi x'$
	×	71	
160	COS	31 63	
	RCL(i)	34 24	
	×	71	
	2	02	
	÷	81	
	−	51	$\hat{y} = \frac{1}{2}a_0 + \Sigma + \frac{1}{2}a_L \cos \pi x'$
	RTN	35 22	STOP AND DISPLAY $\hat{y}$
	* LBL C	31 25 13	$SMIN_k$ COMPUTATION
	SF3	35 51 03	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	F1?	35 71 01	N IS ODD?
170	CF3	35 61 03	YES, CF3
	RCL B	34 12	NO, # → R_I
	ST I	35 33	
	0	00	INITIALIZES Σ
	* LBL 6	31 25 06	MAIN SUBROUTINE FOR $SMIN_k$ EVALUATION
	RC I	35 34	DISPLAYS # − 1
	1	01	
	−	51	
	DSP 0	23 00	
	PSE	35 72	
180	DSP 4	23 04	
	R↓	35 53	
	RCL(i)	34 24	a_k^2
	x²	32 54	
	P≷S	31 42	
	RCL(i)	34 24	b_k^2
	P≷S	31 42	
	x²	32 54	
	+	61	$a_k^2 + b_k^2$
	RCL E	34 15	
190	×	71	$(a_k^2 + b_k^2) \cdot \frac{N}{2}$
	2	02	
	F3?	35 71 03	
	x²	32 54	
	÷	81	
	+	61	Σ
	-x-	31 84	DISPLAYS SMIN k
	DSZ	31 33	ANOTHER SMIN ?
	GTO 6	22 06	YES, ITERATE
	CLX	44	DISPLAY 0.0000 AND STOP
200	RTN	35 22	
	* LBL a	32 25 11	DELETE AN ERROR
	CF2	35 61 02	
	SF0	35 51 00	
	1	01	
	STO+0	33 61 00	
	GTO 0	22 00	
	* LBL e	32 25 15	STORES # OF HARMONICS FOR $\hat{y}$ EVALUATION
	STO B	33 12	
	RTN	35 22	
210	* LBL c	32 25 13	RESET ORIGINAL #
	CF1	35 61 01	
	RCL E	34 15	N/2
	2	02	
	÷	81	
	ENTER↑	41	
	INT	31 83	
	x≠y	32 61	N IS ODD?
	SF1	35 51 01	YES, SF1
	9	09	
220	x≷y	35 52	STORES INT($\frac{N}{2}$) OR 9 IN R_B
	x>y	32 81	
	x≷y	35 52	
	STO B	33 12	
	RTN	35 22	STOP

LABELS

A n, N → INPUT	B REVIEW → a_k, b_k	C → $SMIN_k$	D x_0↑Δ → $x_i, \hat{y}_i$	E x → $\hat{y}$
a DELETE	b	c RESET	d	e # HAR. $\hat{y}$
0 used	1 used	2	3 used	4 used
5 used	6 used	7 used	8 used	9

FLAGS

0	DELETE
1	EVEN/ODD
2	EXIT
3	SMIN k

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☐	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☒	ENG	☐
3	☐	☒			n	4