Program Description I

Program Title: ELLIPTIC LOWPASS FILTER DESIGN

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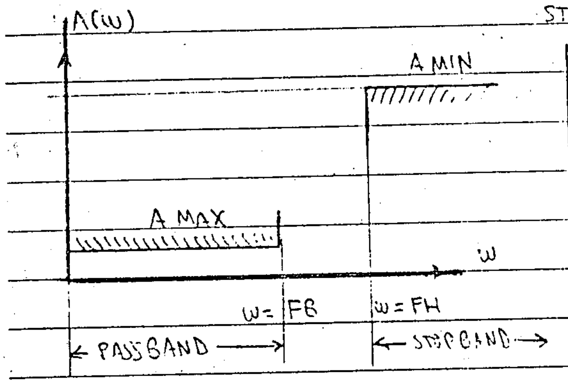
Program Description, Equations, Variables: THIS PROGRAM IS INTENDED AS A HELP WHEN DESIGNING ELLIPTIC LOWPASS FILTERS (OR HIGHPASS, BANDPASS, BANDSTOP ONCE THEY HAVE BEEN TRANSFORMED INTO A NORMALIZED LOWPASS). THE PROGRAM CAN BE USED TO FIND THE MINIMUM DEGREE OF AN ELLIPTIC LOWPASS FILTER THAT MEETS THE STANDARD REQUIREMENTS AMIN, AMAX, FB, FH, WHERE:



AMIN = MINIMUM STOPBAND LOSS

AMAX = MAXIMUM PASSBAND LOSS

FB = UPPER PASSBAND EDGE

FH = UPPER STOPBAND EDGE

IF THE DEGREE N IS $N \leq 30$, THE PROGRAM ALSO FINDS THE ZEROS & POLES OF ATTENUATION AND, USING CARD 2, CAN FIND THE LOSS AT ANY DESIRED FREQUENCY, EITHER MANUALLY OR AUTOMATICALLY. IN THE CASE YOU PREFER AN AUTOMATIC PLOT, THERE IS A CHOICE BETWEEN LINEAR OR LOGARITHMIC SWEEP.

EQUATIONS USED ARE:

– CONSIDER THE ELLIPTIC INTEGRAL OF THE 1ST KIND $\equiv U(\varphi, k) = \int_0^{\varphi} \frac{dx}{\sqrt{1-k^2 \sin^2 x}}$

– THEN, THE COMPLETE ELLIPTIC INTEGRAL IS EXPRESED AS:

$$K(k) = U(\pi/2, k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2 \sin^2 x}}$$

~~Operating Limits and Warnings~~ AND THE COMPLEMENTARY COMPLETE ELLIPTIC INTEGRAL:

$$K'(k) = K(k') \quad , \text{WHERE } k' = \sqrt{1-k^2}$$

– THE ELLIPTIC SINE: $SN(u, k) = \sin \varphi$

– THEN, WE COMPUTE $XL = FH/FB$, $\varepsilon = 10^{0.1\,AMAX} - 1$

$$L = \sqrt{\frac{10^{0.1\,AMIN} - 1}{10^{0.1\,AMAX} - 1}}$$

(TO BE CONTINUED ON THE OTHER SIDE OF THIS PAGE)

- THE MINIMUM DEGREE IS FOUND BY USING:

$$m = \mathrm{INT}\left[\frac{\mathbb{K}\left(\frac{1}{X_L}\right)\cdot \mathbb{K}'\left(\frac{1}{L}\right)}{\mathbb{K}'\left(\frac{1}{X_L}\right)\cdot \mathbb{K}\left(\frac{1}{L}\right)}\right] + 1$$

- ONCE THE NECESSARY DEGREE IS FOUND, THE ZEROS & POLES OF ATTENUATION MAY BE COMPUTED AND STORED (IF $m \leq 30$)

IN NORMALIZED FORM ≡ FOR $i = 1, 2, \ldots \mathrm{INT}\left[\frac{m}{2}\right]$

NORMALIZED ZEROS OF ATTENUATION

- N IS EVEN ≡ $x'z_i = \mathrm{sn}\left[\frac{(2i-1)\mathbb{K}\left(\frac{1}{X_L}\right)}{m}, \frac{1}{X_L}\right]$
- N IS ODD ≡ $x'z_i = \mathrm{sn}\left[\frac{2i\,\mathbb{K}\left(\frac{1}{X_L}\right)}{m}, \frac{1}{X_L}\right]$

NORMALIZED POLES OF ATTENUATION

- N ODD (OR EVEN) ≡ $x'p_i = \frac{X_L}{x'z_i}$

BOTH, ZEROS & POLES ARE DENORMALIZED BY

$$xz_i = x'z_i \times FB$$
$$xp_i = x'p_i \times FB$$

- THE LOSS AT ANY FREQUENCY ω, IS COMPUTED AS FOLLOWS:

$$A(\omega) = 10 \log\left[1 + \varepsilon R_m^2\right]$$

WHERE $R_m(x, L) = C x^{1-NI} \cdot \prod_{i=1}^{\mathrm{INT}(m/2)} \frac{x^2 - xz_i^2}{x^2 - xp_i^2}$

AND $C = \frac{1}{R_m(FB)}$, ($\omega = x$ IN $R_m(x, L)$)

- THE ROUTINES FOR COMPUTATION OF:
  - COMPLETE ELLIPTIC INTEGRAL OF THE FIRST KIND, $\mathbb{K}(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2\sin^2 x}}$
  - COMPLEMENTARY " " " " " " , $\mathbb{K}'(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-(1-k^2)\sin^2 x}}$
  - ELLIPTIC SINE ≡ $\mathrm{SN}(u, k) = \sin\varphi$

ARE ALSO AVAILABLE FROM KEYBOARD WITHOUT DISTURBING THE STORED ZEROS & POLES

- THE ALGORITHMS THAT ARE USED ARE AS FOLLOWS:

- $\boxed{\mathbb{K}(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2\sin^2 x}}}$ ⇒ DATA ≡ k, $\varphi_0 = \pi/2$, $P_0 = 1$, $X_0 = \sqrt{1-k^2}$

LOOP

$$x_{i+1} = \frac{2}{1+\sin(\cos^{-1} x_i)} - 1, \quad P_{i+1} = P_i\,(1 + x_{i+1})$$

$$\varphi_{i+1} = \frac{\varphi_i + \sin^{-1}\left[\sin(\cos^{-1} x_i)\cdot \sin\varphi_i\right]}{2}$$

IF $x_{i+1} = 0$, EXIT

THEN, $\mathbb{K}(k) = P_{i+1} \cdot \mathrm{LN}\left[\mathrm{TAN}\left(\mathrm{TAN}^{-1} 1 + \frac{\varphi_{i+1}}{2}\right)\right]$

- $\boxed{\mathrm{sn}(u, k)}$ ⇒ FIRSTLY, $\mathrm{sn}(u/8, k)$ IS COMPUTED BY MEANS OF A TAYLOR SERIES EXPANSI[ON]

$$\mathrm{sn}(u/8, k) \doteq \frac{u}{8} - (1+k^2)\frac{(u/8)^3}{3!} + (1 + 14k^2 + k^4)\cdot\frac{(u/8)^5}{5!} - (1 + 135k^2 + 135k^4 + k^6)\frac{(u/ \ldots}{7 \ldots}$$

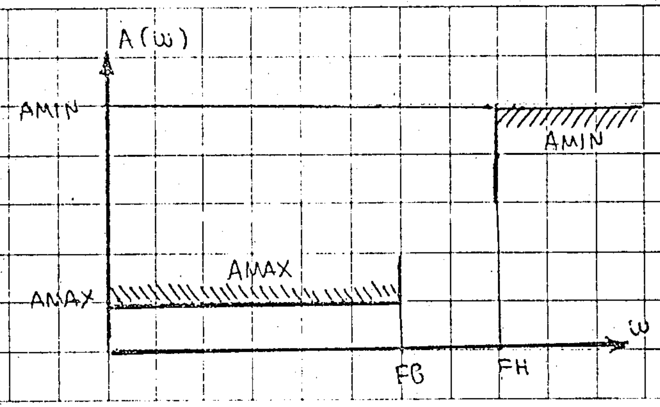
AND THEN, THE DUPLICATION FORMULA $\mathrm{sn}\,2u = \frac{2\,\mathrm{sn}\,u\sqrt{1-\mathrm{sn}^2 u}\sqrt{1-k^2\mathrm{sn}^2 u}}{1-k^2\mathrm{sn}^4 u}$

IS APPLIED 3 TIMES TO YIELD THE DESIRED VALUE $\mathrm{sn}(u, k)$

- THIS PROCEDURE IS VERY FAST (11 SECONDS FOR ANY ARGUMENT) AND QUITE ACCURATE .(

Program Description II

Sketch(es)



Sample Problem(s) NOW, LET'S SEE ALL FUNCTIONS A LITTLE CLOSER:

CARD 1 (BOTH CARDS WORKS ON RADIANS MODE)

CARD 1 DOES NOT DEPEND ON CARD 2 TO PERFORM ITS FUNCTIONS. IN FACT, CARD 2 IS ONLY FOR EVALUATING THE LOSS ONCE ZEROS & POLES HAVE BEEN COMPUTED BY CARD 1

LBL A INPUT: AMIN ENTER AMAX ENTER FH ENTER FB A → m (DEGREE)

CHARACTERISTICS

- IT TAKES ABOUT 87 SECONDS (ON THE AVERAGE)
- AMIN, AMAX, --- ARE USED, BUT ONLY FB, AMIN ARE STORED
- IT DESTROYS PREVIOUS ZEROS & POLES IN R8, R4

LBL B PRESS B (DEGREE m MUST HAVE BEEN PREVIOUSLY COMPUTED, OF COURSE)

→ (1) → XZ1 → XP1 (DSP9)
→ (2) → XZ2 → XP2 (DSP9) WHERE m = INT(m/2)
-- -- -- -- -- --

Solution(s) → (m) → XZm → XPm → 0.00

CHARACTERISTICS

- TO BE USED ONCE m HAS BEEN COMPUTED
- ONLY IF m ≤ 30 (m ODD or EVEN)
- IT TAKES ABOUT ½×13 SECONDS PER DEGREE (I.E = IF DEGREE IS 2×3, THEN CALCULATION OF ZEROS & POLES WILL LAST ABOUT 40 SECONDS)
- ZEROS ARE STORED IN R1 THROUGH R15 (POLES NEED NOT BE STORED, THEY ARE FUNCTION OF ZEROS)

Reference(s) LBL C ALLOWS REVIEWING BOTH ZEROS & POLES AT ANY TIME

PRESS C → (1) → XZ1 → XP1 (DSP9)
→ (2) → XZ2 → XP2 (DSP9)
--- -- -- --
→ (X) → XZx → XPx → 0.00 , (x = INT(m/2))

(CONTINUES ON THE OTHER SIDE OF THIS PAGE)

$\boxed{\text{LBL D}}$ COMPUTES $K(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2\sin^2 x}}$

— SIMPLY, INPUT k $\boxed{D} \rightarrow K(k)$

$\boxed{\text{LBL d}}$ COMPUTES $K'(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-(1-k^2)\sin^2 x}}$, INPUT k $\boxed{FD} \rightarrow K'(k)$

CHARACTERISTICS

- IT TAKES FROM 16 TO 32 SECONDS. ABOUT 22 ON THE AVERAGE
- k MUST NOT BE $k < 10^{-4}$ (FOR $k < 10^{-4}$, $K(k) = \pi/2$)
  OR $k = 1$ (IF $k = 1$, $K(k) = \infty$)
- USING LBLD OR LBLd DOES NOT ALTER STORED ZEROS
- ACCURACY IS 8 DECIMALS OR BETTER

$\boxed{\text{LBL e}}$ STORES k FOR $sn(u,k)$ COMPUTATIONS : k $\boxed{FE} \rightarrow k$

$\boxed{\text{LBL E}}$ COMPUTES $sn(u,k)$. k IS ASSUMED TO BE IN RE ; u $\boxed{E} \rightarrow sn(u,k)$

CHARACTERISTICS

- IT TAKES 11 SECONDS
- $0 \le k \le$ SOME UNDEFINED VALUE
- IT DOES NOT ALTER STORED ZEROS
- ACCURACY IS 8 DECIMALS OR BETTER FOR NON-OUT OF RANGE VALUES OF k

$\boxed{\text{Card 2}}$ IS AN OPTIONAL COMPLEMENT TO CARD 1. ONCE CARD 1 HAS DETERMINED DEGREE, AND ZEROS & POLES, THE LOSS FOR ANY FREQUENCY CAN BE COMPUTED. THERE ARE SEVERAL OPTIONS :

$\boxed{\text{LBL A}}$ COMPUTES THE LOSS (IN DECIBELS) FOR A GIVEN ω (NOTE: THE FIRST ω TAKES MORE TIME TO BE COMPUTED THAN ALL SUBSEQUENTS)

ω $\boxed{A} \rightarrow A(\omega)$ (DSP 4)

$\boxed{\text{LBL B}}$ SETS LINEAR SWEEP : $\boxed{B} \rightarrow 0.0000$ (LINEAR $\equiv$ $\omega_1 = \omega_0 + \Delta$, $\omega_2 = \omega_1 + \Delta$, etc

$\boxed{\text{LBL C}}$ SETS LOGARITHMIC SWEEP : $\boxed{C} \rightarrow 1.0000$ (LOGARITH $\equiv$ $\omega_1 = \omega_0 \cdot \Delta$, $\omega_2 = \omega_1 \cdot \Delta$, etc

$\boxed{\text{LBL D}}$ STARTS SWEEP : ω_0 $\boxed{\text{ENTER}}$ Δ $\boxed{D}$ $\rightarrow (\omega_0) \rightarrow A(\omega_0)$

$\rightarrow (\omega_1) \rightarrow A(\omega_1)$

$\rightarrow (\omega_2) \rightarrow A(\omega_2)$, etc.

(LINEAR SWEEP IS SELECTED WHEN CARD 2 IS LOADED)

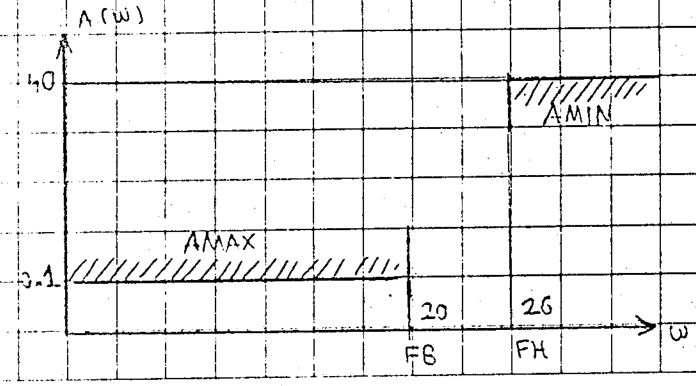
SWEEP CAN BE STOPPED BY PRESSING $\boxed{R/S}$ AFTER $A(\omega_i)$ IS PRINTED.
(THIS IS, AS SOON AS THE PRINTER WORKS OR THE DISPLAY FLASHES, PRESS $\boxed{R/S}$)
OTHERWISE, AN UNDESIRED $\boxed{R/S}$ MAY HAVE BEEN PERFORMED.

$\boxed{\text{LBL E}}$ IS A REVIEWING ROUTINE. IT REVIEWS ZEROS & POLES (SEE LBLC ON CARD 1)
IT'S INCLUDED TO AVOID RELOADING OF CARD 1 FOR REVIEWING PURPOSES.

Program Description II

Sketch(es)

AMAX = 0.1
AMIN = 40
FB = 20
FH = 26



Sample Problem(s) FIND ALL NECESSARY INFORMATION ABOUT AN ELLIPTIC LOWPASS FILTER THAT MEETS THE FOLLOWING REQUIREMENTS (SEE SKETCH)

AMAX = 0.1 dB , AMIN = 40 dB , FB = 20 , FH = 26

a) DEGREE: - LOAD CARD 1, BOTH SIDES

- INPUT FILTER REQUIREMENTS : 40 [ENTER] 0.1 [ENTER] 26 [ENTER] 20 [A] → 6.00

SO, THE MINIMUM DEGREE IS 6 , WE NEED A 6TH-ORDER E.LOWPASS FILTER

b) LOSS AS A FUNCTION OF FREQUENCY

- WE WANT TO MAKE A DRAWING OF THE LOSS AS A FUNCTION OF FREQUENCY. TO OBTAIN THIS GRAPHIC, WE MUST FIND THE MOST RELEVANT ABCISSAE:

(1) ZEROS & POLES : [B] → (1) → 6.295011340 → 82.60507346

Solution(s)

→ (2) → 15.62227766 → 33.28579938
→ (3) → 19.56561718 → 26.57723471 → 0.00

$\underbrace{\hphantom{19.56561718}}_{\text{ZEROS}} \quad \underbrace{\hphantom{26.57723471}}_{\text{POLES}}$

(2) FREQUENCIES OF MAXIMUM PASSBAND LOSS

- THESE ARE $fe_i = FB \times \mathrm{sn}\left(\frac{2iK}{n}\right)$; $i = 0, 1, 2, 3$

- PROCEED AS FOLLOWS : [DSP 4] 0 [E] → $fe_0 = 0.0000$

IE : $fe_1 = 20\,\mathrm{sn}\left[\frac{2K}{6}, \frac{20}{26}\right]$ ⇒ [RCL 0] 2 [×] 6 [÷] [E] 20 [×] → $fe_1 = 11.6623$

[RCL 0] 4 [×] 6 [÷] [E] 20 [×] → $fe_2 = 18.1792$

Reference(s) (1) APPROXIMATION METHODS FOR ELECTRONIC FILTER DESIGN-
AUTHOR: RICHARD W. DANIELS - MAC GRAW-HILL BOOK COMPANY - PAGES 51 TO 82
(2) MODERN FILTER THEORY AND DESIGN
AUTHOR: EDITED BY GABOR C. TEMES & SANJIT K. MITRA - JOHN WILEY & SONS - PAGES 31 TO 53

ECLC E 20 X → $f_{e3} = 20.0000$

(3) FREQUENCIES OF MINIMUM STOPBAND LOSS

— THESE ARE GIVEN BY $f_{Ei} = \dfrac{F_B \times F_H}{f_{ei}}$

$$f_{E1} = \frac{20 \times 26}{11.6623} = 44.5881 \quad , \quad f_{E2} = \frac{520}{18.1792} = 28.6041$$

$$f_{E3} = \frac{20 \times 26}{20} = 26.0000 \quad , \quad f_{E0} = \frac{20 \times 26}{0} \to \infty$$

— WE ARE READY FOR THE LOSS COMPUTATION. LOAD CARD 2 (ITS ONLY SIDE)

— LET'S CALCULATE SOME POINTS MANUALLY

THE ZEROS:
- 0.2950 A → 0.0000
- 15.6223 A → 0.0000
- 19.5056 A → 0.0000

THE F_i OF MAX. PASSBAND LOSS:
- 0 A → 0.1000 dB
- 11.6623 A → 0.1000 dB
- 18.1792 A → 0.1000 dB
- 20 A → 0.1000 dB

THE NEAR-POLES:
- 82.6050 A → 161.3803 dB
- 33.2857 A → 139.7363 dB
- 26.5772 A → 133.7026 dB

THE F_i OF MIN. STOPBAND LOSS:
- 44.5881 A → 46.8540
- 28.6041 A → 46.8540
- 26 A → 46.8540
- $\infty \Rightarrow$ 100 000 A → 46.8540

— AND, FINALLY, A GENERAL SWEEP. SELECT LINEAR SWEEP:

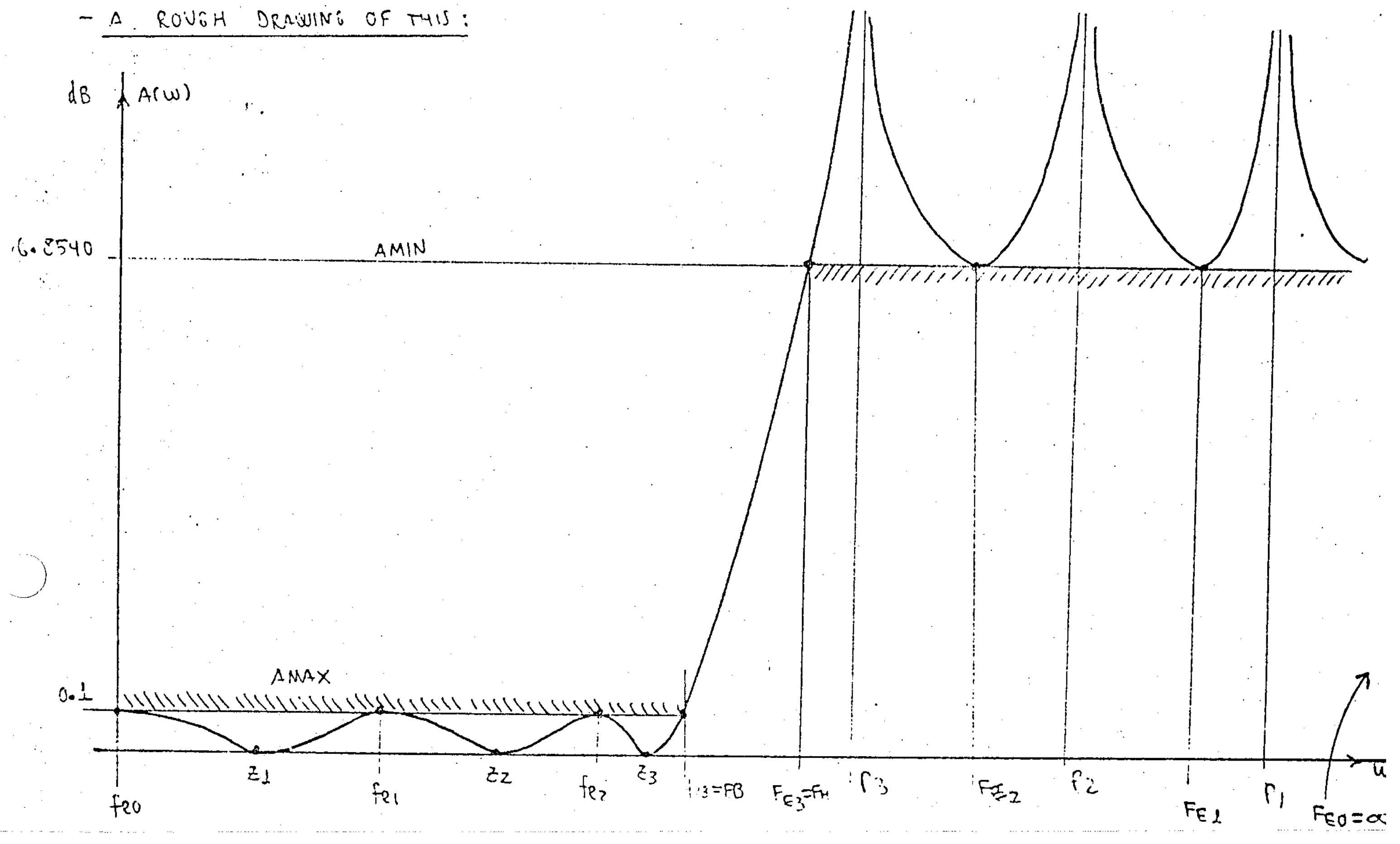
B → 0.0000 , 0 ENTER 2 D → (0.0000) → 0.1000 → (2.0000) → 0.0783
→ (4.0000) → 0.0311 → (6.0000) → 0.0006 --- etc

WE THUS FORM THE TABLE:

ω	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36
A(ω)	0.1000	0.0783	0.0311	0.0006	0.0200	0.0748	0.0987	0.0434	0.0036	0.0980	0.1000	9.1868	24.8645	46.8540	47.3712	48.6069	56.9113	63.0148	52.7390

PASSBAND (LOSS ≤ 0.1000) — STOPBAND (LOSS ≥ 46.8540)

— A ROUGH DRAWING OF THIS:



ELLIPTIC LOWPASS FILTER DESIGN - CARD 1				
			$k \to K'(k)$	STO k
DEGREE AMIN↑AMAX↑FH↑FB	→ ZEROS & POLES	REVIEW	$k \to K(k)$	$u \to sn(u,k)$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	LOAD CARD 1, BOTH SIDES				
2	INPUT AMIN, AMAX, FH, FB, AND COMPUTE DEGREE m	AMIN dB	ENTER		
		AMAX dB	ENTER		
		FH Hz	ENTER		
		FB Hz	A		DEGREE m
3	OPTIONAL: FIND ZEROS & POLES		B		$\to(1)\to Z_1\to P_1$
					$\to(2)\to Z_2\to P_2$
					etc
4	OPTIONAL: REVIEW ZEROS & POLES		C		$\to(i)\to Z_i\to P_i$
					etc
5	OPTIONAL: COMPUTE $K(k)$	k	D		$K(k)$
6	OPTIONAL: COMPUTE $K'(k)$	k	f	D	$K'(k)$
7	OPTIONAL: COMPUTE $sn(u,k)$	k	f	E	k
		u	E		$sn(u,k)$
8	OPTIONAL: LOAD CARD 2, SIDE A				
9	TO COMPUTE THE LOSS AT SOME FREQUENCY ω	ω Hz	A		$A(\omega)$ dB
10	TO PERFORM A SWEEP OF FREQUENCIES:				
	a) SELECT LINEAR OR		B		0.0000
	b) SELECT LOGARITHMIC MODE		C		1.0000
	START SWEEP:	ω_0 Hz	ENTER		
		Δ	D		$\to(\omega_i)\to A(\omega_i)$
	TO STOP SWEEP, PRESS $\boxed{R/S}$ AFTER $A(\omega_i)$				etc
11	TO REVIEW ZEROS and POLES		E		$\to(i)\to Z_i\to P_i$
					etc.
12	FOR ANOTHER CASE, GO TO APPROPRIATE STEP.				

CARD 2

ELLIPTIC LOWPASS FILTER DESIGN - CARD 2 -				
		SWEEP		
$\omega \to A(\omega)$	LINEAR	LOGARITHMIC	$\omega_0\uparrow\Delta \to A(\omega_i)$	REVIEW

UROPE

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	* LBL A	31 25 11	DEGREE COMPUTATION
	STO D	33 14	STORES FB
	X≷Y	35 52	
	÷	81	
	STO E	33 15	STORES $K = 1/X_L$
	R↓	35 53	
	GSB 8	31 22 08	COMPUTES ε
	STO A	33 11	STORES ε
	X≷Y	35 52	
010	GSB 8	31 22 08	COMPUTES $\frac{1}{L}$
	÷	81	
	√	31 54	
	GSB 9	31 22 09	COMP. & STORES $\frac{K(\frac{1}{L})}{K'(\frac{1}{L})}$
	STO C	33 13	
	RCL E	34 15	COMP. $K(1/X_L)/K'(1/X_L)$
	GSB 9	31 22 09	
	RCL C	34 13	$m = \left[\frac{K(\frac{1}{X_L}) \cdot K'(\frac{1}{L})}{K'(\frac{1}{X_L}) \cdot K(\frac{1}{L})}\right] + 1$
	÷	81	
	1	01	
020	+	61	
	INT	31 33	
	STO C	33 13	
	RTN	35 22	
	* LBL 8	31 25 08	AUXILIAR SUBROUTINE
	.	83	
	1	01	COMPUTES EITHER $10^{0.1\,AMAX} - 1$ OR $10^{0.1\,AMIN} - 1$
	×	71	
	10^x	32 53	
	1	01	
030	−	51	
	RTN	35 22	
	* LBL 9	31 25 09	AUXILIAR SUBROUTINE
	STO 9	33 09	
	GSB D	31 22 14	$K(\frac{1}{X_L})$ OR $K(\frac{1}{L})$
	STO 8	33 08	
	STO 0	33 00	
	RCL 9	34 09	
	GSB d	32 22 14	$K'(\frac{1}{X_L})$ OR $K'(\frac{1}{L})$
	STO÷8	33 81 08	
040	RCL 8	34 08	$K(1/L)/K'(\frac{1}{L})$ OR ---
	RTN	35 22	
	* LBL D	31 25 14	$K(K)$ COMPUTATION
	P≷S	31 42	
	SIN^{-1}	32 62	$\sqrt{1-x^2}$
	COS	31 63	
	P≷S	31 42	
	* LBL d	32 25 14	$K'(K)$ COMPUTATION
	P≷S	31 42	
	STO 9	33 09	STORE X_0
050	1	01	
	STO 8	33 08	STORE P_0
	SIN^{-1}	32 62	STORE $\varphi_0 = \pi/2$
	STO 7	33 07	
	RCL 9	34 09	RCL X_0
	* LBL 3	31 25 03	LOOP
	COS^{-1}	32 63	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	SIN	31 62	$\sqrt{1-x^2}$
	STI	35 33	$1 + \sin(\cos^{-1} x_i)$
	ISZ	31 34	
060	RCL 7	34 07	NEW $\varphi_i \to \varphi_{i+1}$
	SIN	31 62	
	×	71	
	SIN^{-1}	32 62	
	STO+7	33 61 07	
	2	02	
	STO÷7	33 81 07	
	RCI	35 34	
	÷	81	
	STO×8	33 71 08	$P_i \to P_{i+1}$
070	1	01	
	−	51	
	STO 9	33 09	NEW $X_i \to X_{i+1}$
	X≠0	31 61	$X_{i+1} = 0$?
	GTO 3	22 03	NO; ITERATE
	RCL 7	34 07	YES;
	2	02	
	÷	81	
	1	01	
	TAN^{-1}	32 64	$K(K) = P \times LN[TAN[----]]$
080	+	61	
	TAN	31 64	
	LN	31 52	
	RCL 8	34 08	
	×	71	
	P≷S	31 42	
	RTN	35 22	
	* LBL B	31 25 12	ZEROS & POLES
	CF0	35 61 00	
	RCL C	34 13	STORES INT($m/2$) AND SETS F0 IF $m = \dot{2}$
090	2	02	
	÷	81	
	ENTER↑	41	
	INT	31 83	
	STO B	33 12	
	STI	35 33	
	X=Y	32 51	
	SF0	35 51 00	
	* LBL C	31 25 06	LOOP
	RCI	35 34	
100	2	02	
	×	71	
	0	00	
	F0?	35 71 00	EVEN?
	e^x	32 52	YES = SUBSTRACT 1
	−	51	
	RCL C	34 13	$u_i = \frac{2i - \{^0_1\}}{m} \cdot K(\frac{1}{X_L})$
	÷	81	
	RCL 0	34 00	
	×	71	
110	GSB E	31 22 15	$sn(u_i, 1/X_L)$
	RCL D	34 14	$\times z_i$
	×	71	

REGISTERS

0	1	2	3	4	5	6	7	8	9
$K(\frac{1}{L})$	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	used, Z_8	used, Z_9

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
Z_{10}	Z_{11}	Z_{12}	Z_{13}	Z_{14}	Z_{15}	used	used	used	used

A	B	C	D	E	I
ε, used	AMIN, INT($m/2$), used	m	FB	$K = \frac{1}{X_L}$	used

STEP	KEY ENTRY	KEY CODE	COMMENTS
	STO(i)	33 24	STORE XZi
	DSZ	31 33	
	GTO 6	22 06	LOOP
	* LBL C	31 25 13	DISPLAY & REVIEW ROUTINE
	0	00	RESET INDEX TO 0
	ST I	35 33	
	* LBL 0	31 25 00	LOOP
120	ISZ	31 34	
	RCI	35 34	
	DSP 0	23 00	} DISPLAY i
	PAUSE	35 72	
	DSP 9	23 09	
	RCL D	34 14	
	x^2	32 54	
	RCL(i)	34 24	
	−x−	31 84	DISPLAY XZi
	RCL E	34 15	} COMPUTE Xpi
130	×	71	
	÷	81	
	−x−	31 84	DISPLAY XPi
	RCL B	34 12	} ALL DISPLAYED?
	RCI	35 34	
	x≠y	32 61	
	GTO 0	22 00	NO, REPEAT
	CLX	44	YES, RESET
	DSP 2	23 02	
	RTN	35 22	
140	* LBL e	32 25 15	STORE K
	STO E	33 15	
	RTN	35 22	
	* LBL E	31 25 15	sm(u,x) COMPUTATION
	P⇄S	31 42	
	SF 3	35 51 03	
	SF 2	35 51 02	
	8	08	u → u/8
	÷	81	
	RCL E	34 15	
150	x^2	32 54	
	STO 9	33 09	
	STO 7	33 07	
	STO 6	33 06	
	1	01	sm(u,k) is
	+	61	
	STO 8	33 08	APPROXIMATED
	STO×6	33 71 06	
	x^2	32 54	BY A
	RCL 7	34 07	
160	1	01	TAYLOR'
	2	02	
	×	71	SERIES
	+	61	
	STO 7	33 07	EXPANSION
	RCL 8	34 08	
	×	71	
	RCL 6	34 06	
	5	05	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	N!	35 81	$sm(u,x) = \frac{u}{8} - \frac{(1+k^2)(u/8)^3}{3!} + \frac{(1+14k^2+k^4)(u/8)^5}{5!} + \cdots$
170	STO 6	33 06	
	×	71	
	+	61	
	×	71	
	7	07	
	N!	35 81	
	÷	81	
	×	71	
	RCL 7	34 07	
	RCL 6	34 06	
180	÷	81	
	−	51	
	×	71	
	×	71	
	RCL 8	34 08	
	6	06	
	÷	81	
	+	61	
	×	71	
	×	71	
190	1	01	
	−	51	
	×	71	
	CHS	42	
	* LBL 4	31 25 04	DUPLICATION FORMULA
	STO 6	33 06	
	STO×6	33 71 06	
	2	02	
	×	71	
	1	01	
200	RCL 6	34 06	
	GSB 7	31 22 07	
	GSB 7	31 22 07	
	−	51	
	÷	81	
	F2?	35 71 02	
	GTO 4	22 04	DUPLICATE AGAIN
	F3?	35 71 03	
	GTO 4	22 04	DUPLICATE ONCE MORE
	P⇄S	31 42	
210	RTN	35 22	
	* LBL 7	31 25 07	AUXILIAR SUBROUTINE
	−	51	
	√	31 54	
	×	71	
	1	01	
	RCL 9	34 09	
	RCL 6	34 06	
	STO×6	33 71 06	
	×	71	
220	RTN	35 22	
			HP97 OWNERS: USE THESE 4 FREE STEPS TO INSERT "SPACE" AND CHANGE "PAUSE" TO "PRINT X"

LABELS

A DEGREE	B → ZEROS, POL.	C REVIEW	D K → Σ(K)	E u → sm(u,k)
a	b	c	d K → Σ'(K)	e STO K
0 used REVIEW	1	2	3 used Σ(K)	4 used sm(u,k)
5	6 used Z, P	7 used sm(u,k)	8 used DEGREE	9 used DEGREE

FLAGS

0	ODD/EVEN
1	
2	used sm(u,k)
3	used sm(u,k)

SET STATUS

FLAGS	ON	OFF
0	☐	■
1	☐	■
2	☐	■
3	☐	■

TRIG	
DEG	☐
GRAD	☐
RAD	■

DISP	
FIX	■
SCI	☐
ENG	☐
n	2

DON'T FORGET

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	31 25 11	ω → A(ω)		RCLC	34 13	} COMPUTES $INT(\frac{m}{2})$
-	DSP4	23 04	SET DISPLAY	170	2	02	
-	F1?	35 71 01	FIRST TIME?	-	÷	81	
-	GTO 0	22 00	YES,	060	ENTER↑	41	
-	*LBL3	31 25 03	RETURN LABEL		INT	31 83	
-	GSB1	31 22 01	GO TO LOOP AND RETURN	-	x=y	32 51	EVEN?
-	RCL0	34 00	} $\varepsilon.C^2$	-	SF2	35 51 02	YES, SF2
120 -	×	71		-	RTN	35 22	
-	1	01		-	*LBLB	31 25 12	LINEAR SWEEP
010	+	61	$A(\omega) = 10 \log[1+\varepsilon C^2 Rm(\omega)^2]$	-	CF0	35 61 00	
-	LOG	31 53		-	0	00	
-	1	01		180 -	RTN	35 22	
-	0	00		-	*LBLC	31 25 13	LOGARITHMIC SWEEP
-	×	71		070	SF0	35 51 00	
-	RTN	35 22		-	1	01	
-	*LBL0	31 25 00	COMPUTE CONSTANT C	-	RTN	35 22	
-	CF1	35 61 01		-	*LBLD	31 25 14	SWEEP
130 -	RCLA	34 11	} ε → R0	-	R⇄S	31 42	
-	STO 0	33 00		-	STO 9	33 09	STORE Δ
020	R↓	35 53	} ω → RA	-	x⇄y	35 52	
-	STO A	33 11		-	*LBL5	31 25 05	LOOP
-	RCLD	34 14	FB	190 -	PAUSE	35 72	PAUSES ωi
-	GSB1	31 22 01	Rm(FB) = 1/C	-	STO 8	33 08	STORES ωi
-	STO÷0	33 81 00	ε.C → R0	080	R⇄S	31 42	
-	RCLA	34 11	} RECALL ω	-	GSB A	31 22 11	COMPUTES A(ωi)
-	GTO 3	22 03		-	-x-	31 84	
-	*LBL1	31 25 01	COMPUTATION OF Rm(ω)	-	0	00	
140 -	x²	32 54		-	P⇄S	31 42	
-	STO B	33 12		-	RCL8	34 08	ωi
030	GSB4	31 22 04	RESET I AND ODD/EVEN	-	RCL9	34 09	Δ
-	STI	35 33		-	F0?	35 71 00	LOGARITHMIC?
-	RCLB	34 12	EVEN?	200 -	×	71	YES, $\omega_{i+1} = \omega_i . \Delta$
-	F2?	35 71 02		-	+	61	NO, $\omega_{i+1} = \omega_i + \Delta$
-	1	01		090	GTO 5	22 05	CONTINUE SWEEP
-	*LBL2	31 25 02		-	*LBLE	31 25 15	REVIEW ROUTINE
-	RCLB	34 12	LOOP	-	0	00	
-	RCL(i)	34 24		-	STI	35 33	
150 -	x²	32 54		-	*LBL6	31 25 06	
-	−	51		-	ISZ	31 34	
040	×	71		-	RCI	35 34	
-	RCLB	34 12		-	DSP0	23 00	DISPLAYS i
-	RCLD	34 14		210 -	PAUSE	35 72	
-	x²	32 54		-	DSP9	23 09	
-	RCL(i)	34 24	EVALUATES $\prod_{i=1}^{INT(\frac{m}{2})} \frac{X^2 - XZ_i^2}{X^2 - XP_i^2}$	100	RCLD	34 14	
-	÷	81		-	x²	32 54	
-	RCLE	34 15		-	RCL(i)	34 24	
-	÷	81		-	-x-	31 84	DISPLAYS XZi
160 -	x²	32 54		-	RCLE	34 15	
-	−	51		-	×	71	
050	÷	81		-	÷	81	
	DSZ	31 33		-	-x-	31 84	DISPLAYS XPi
	GTO2	22 02		220 -	GSB4	31 22 04	} ALL DISPLAYED?
	x²	32 54		-	RCI	35 34	
	RTN	35 22		110	x≠y	32 61	
	*LBL4	31 25 04	AUXILIAR SUBROUTINE	-	GTO 6	22 06	
	CF2	35 61 02		112	CLX	44	END OF REVIEW.

LABELS

A ω → A(ω)	B LINEAR	C LOGARITH.	D ω↑Δ → A(ωi)	E REVIEW
a	b	c	d	e
0 used	1 used	2 used	3 used	4 used
5 used	6 used	7	8	9

FLAGS

0	
1	FIRST TIME?
2	ODD/EVEN
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	■	DEG	☐	FIX	■
1	■	☐	GRAD	☐	SCI	☐
2	☐	■	RAD	■	ENG	☐
3					4	