Program Description I

Program Title: NUMERICAL ANALYSIS OF FUNCTIONS

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Program Description, Equations, Variables: THIS PROGRAM TRIES TO SUMMON ON A SINGLE CARD SEVERAL OF THE MOST USEFUL ROUTINES THAT MAY BE FOUND IN NUMERICAL ANALYSIS FOR THE PURPOSE OF ANALYZING A GIVEN FUNCTION OF X, $F(X) \equiv y(X)$. IT IS TO BE CONSIDERED AS A MUCH IMPROVED VERSION OF "CALCULUS AND ROOTS OF F(X)" SD-11 B. NOW, LET'S SEE CLOSELY EVERY FUNCTION PERFORMED BY THE PROGRAM =

$\boxed{\text{LBL A}}$ $X \to y(X)$

- TO DEFINE $y(x)$, PRESS: $\boxed{\text{GTO A}}$, SWITCH TO PRGM, PRESS THE KEYSTROKES THAT DEFINE $y(x)$, KNOWING THAT x IS IN THE DISPLAY, $\boxed{\text{RTN}}$, SWITCH TO RUN. THERE ARE UP TO 54 STEPS TO DEFINE $y(x)$ AND ONE LEVEL OF SUBROUTINE NESTING IS AVAILABLE. R9, RA, RB, AND ALL SECONDARY REGISTERS MAY BE USED FOR DEFINING PURPOSES OR WHATEVER.

ONCE $y(x)$ HAS BEEN DEFINED ⇨ x $\boxed{A}$ $\to y(X)$

$\boxed{\text{LBL B}}$ ⇨. $X \to y'(X)$

THE FIRST DERIVATIVE OF $y(x)$ IS COMPUTED USING THE APPROXIMATION

$$y'(x) \simeq \frac{y(x+\Delta) - y(x-\Delta)}{2\Delta}, \text{ WHERE } \Delta = 8.10^{-4}$$

(CONTINUES ON THE OTHER SIDE OF THIS PAGE)

Operating Limits and Warnings: - VALUES OF $y'(x)$ OR $y''(x)$ MAY RESULT IN A LOSS OF SIGNIFICANT FIGURES FOR LARGE VALUES OF x, OR $x \simeq 0$.

- LARGE VALUES OF N WHEN EVALUATING $\int_a^b y(x)\,dx$ MAY RESULT IN EXCESSIVE AMOUNTS OF COMPUTING TIME. $\int_a^\infty y(x)\,dx$ MAY CONVERGE QUITE SLOWLY FOR SOME $y(x)$.
- THE ROOT-FINDER ROUTINE IS NOT CERTAIN TO CONVERGE IF x_0 IS NOT A FAIRLY GOOD APPROXIMATION TO THE ACTUAL ROOT.

- TO FIND $y'(x)$, SIMPLY ⇨ - DEFINE $y(x)$ (SEE LBL A)

- PRESS ⇨ x $\boxed{B} \rightarrow y'(x)$

CHARACTERISTICS : - THE RESULTS ARE USUALLY QUITE ACCURATE, BUT MAY BE IN A LARGE ERROR IF x IS LARGE ENOUGH, OR VERY SMALL. GENERALLY, AN ACCURACY OF 6 OR 7 DECIMAL PLACES IS OBT.

$\boxed{LBL\ C \equiv y''(x)}$ THE SECOND DERIVATIVE OF $y(x)$ IS COMPUTED USING THE APPROXIMATION:

$$y''(x) \simeq \frac{y'(x+\Delta) - y'(x-\Delta)}{2\Delta}$$

WHERE $\Delta = 4.10^{-3}$

TO FIND $y''(x)$ ⇨ - DEFINE $y(x)$ (SEE LBL A)

- PRESS ⇨ x $\boxed{C} \rightarrow y''(x)$

IT HAS THE SAME CHARACTERISTICS OF $y'(x)$, BUT HERE THE RESULTS ARE USUALLY LESS ACCURATE THAN THOSE OF $y'(x)$ ROUTINE, ABOUT 5 - 6 DEC. PLACES

$\boxed{LBL\ D \equiv a \uparrow b \rightarrow \int_a^b y(x)\,dx}$ THE INTEGRAL OF $y(x)$ BETWEEN ARBITRARY FINITE LIMITS a, b, IS COMPUTED USING THE APPROXIMATE FORMULA : a $\boxed{ENTER}$ b $\boxed{D} \rightarrow \int_a^b y(x)\,dx$

$\int_a^b y(x)\,dx$ IS TRANSFORMED TO $\int_{-1}^{1} y^*(x)\,dx$ BY $x = \frac{b+a}{2} + \frac{b-a}{2}t$, $dx = \frac{b-a}{2}dt$

AND THE RESULTING $\int_{-1}^{1} y^*(x)\,dx \simeq \frac{5}{6}\left[y^*\left(\frac{1}{\sqrt{5}}\right) + y^*\left(-\frac{1}{\sqrt{5}}\right)\right] + \frac{1}{6}\left[y(1) + y(-1)\right] + \text{Error}$

CHARACTERISTICS = - THE Error TERM BECOMES ZERO IF $y(x)$ IS A POLYNOMIAL OF DEGREE 5 OR LESS. (ON THE OTHER HAND, THE WELL-KNOWN SIMPSON'S RULE IS EXACT ONLY FOR POLYNOMIALS OF DEGREE 3 OR LESS

$\boxed{LBL\ E \equiv x_0 \rightarrow \text{root of } y(x)}$ STARTING FROM AN INITIAL APPROXIMATION x_0, THE PROGRAM COMPUTES AN IMPROVED ONE TO THE ACTUAL ROOT OF $y(x)$ USING THE NEWTON'S METHOD :

$$x_{i+1} = x_i - \frac{y(x_i)}{y'(x_i)}$$

WHERE $y'(x_i) \simeq \frac{y(x_i+\Delta) - y(x_i-\Delta)}{2\Delta}$

THE NEW APPROXIMATION IS REFINED UNTIL $|x_m - \text{root}| < 5.10^{-8}$

CHARACTERISTICS = - CONVERGENCE IS NOT GUARANTEED FOR EVERY x_0.

- CONVERGENCE MAY BE OBSERVED, AS ALL APPROXIMATIONS AR DISPLAYED VIA THE PAUSE FUNCTION. IF SOME x_0 FAILS TO CONVERGE, TRY ANOTHER ONE. TO F THE ROOT ⇨ DEFINE $y(x)$ (SEE LBL A), x_0 $\boxed{E} \rightarrow (x_1) \rightarrow (x_2) \rightarrow \cdots \rightarrow (x_i) \rightarrow \cdots \rightarrow$ root

$\boxed{LBL\ a \equiv \text{AUTOMATIC PLOT}}$ IF YOU MUST EVALUATE $y(x)$ AT SOME EQUALLY SPACED POINTS, USE THE AUTOMATIC PLOT ROUTINE. PRESS x_0 $\boxed{ENTER}$ Δ $\boxed{f\ A} \rightarrow (x_0) \rightarrow y(x_0)$

$\rightarrow (x_1) \rightarrow y(x_1)$, etc. etc., press $\boxed{R/S}$ to s

WHERE $x_i = x_0 + i.\Delta$

CHARACTERISTICS = - TO EVALUATE $y'(x)$, $y''(x)$ INSTEAD OF $y(x)$, SIMPLY CHANGE STEP 142 GSB A TO 142 GSB B OR 142 GSB C, AS APPROPIATE.

- THIS ROUTINE IS SPECIALLY INTENDED TO QUICKLY LOCATE SOME ROOTS OF $y(x)$ OR $y'(x)$, BEFORE USING THE ROOT-FINDER ROUTINE.

$\boxed{LBL\ b \equiv \#\ \text{OF SUBINTERVALS}}$ STORES THE DESIRED # OF INTERVALS USED IN $\int_a^b y(x)\,dx$ CALCULATI # IS STORED IN Rc. PRESS m $\boxed{f\ B} \rightarrow m$

$\boxed{LBL\ c \equiv a \rightarrow \int_a^\infty y(x)\,dx}$ COMPUTES THE INTEGRAL OF $y(x)$ BETWEEN a AND ∞. CONVERGENCE TO A FINITE VALUE IS ASSUMED. THE PROGRAM COMPUTES THE FOLLOWING :

$$\int_a^\infty = \int_a^{a+1} + \int_{a+1}^{a+2} + \int_{a+2}^{a+3} + \cdots$$

THIS SERIES IS TERMINATED AS SOON AS TWO CONSECUTIVE PARTIAL SUMS DIFFERS 5.10^{-5} OR LESS. DEFINE $y(x)$, AND PRESS a $\boxed{f\ C} \rightarrow \int_a^{a+1} \rightarrow \int_{a+1}^{a+2} \rightarrow \int_{a+2}^{a+3} \rightarrow \cdots \rightarrow \int_a^\infty$

CHARACTERISTICS : - CONVERGENCE MAY BE VERY SLOW FOR SOME $y(x)$

- TO IMPROVE ACCURACY, DO ONE OR BOTH OF THE FOLLOWING:

- CHANGE STEP 159 DSP 4 TO 159 DSP 5 OR 159 DSP 6, etc.
- CHANGE STEP 155 1, TO 155 . / 156 5 } OR SOME OTHER VALUE < 1

Program Description II

[illegible]) $\boxed{\text{LBL d}}$ COMPUTES THE INTEGRAL $\int_a^b y(x)$, DIVIDING THE INTERVAL $[a, b]$ IN m SUBINTERVALS, WHERE m IS A USER'S-SPECIFIED VALUE, $m = 2, 3, 4, ---$. THIS ALLOWS THE USER TO COMPUTE A GIVEN INTEGRAL WITH ALL DESIRED ACCURACY. IT IS VERY USEFUL TO CHECK THE ACCURACY OF THE RESULTS, TOO. STORE m WITH LBL b, AND THEN $a \uparrow b \boxed{f\ D} \rightarrow \int_a^b$

$\boxed{\text{LBL 2}}$ FINDS A ROOT OF $y'(x)$. IT OPERATES EXACTLY AS $\boxed{\text{LBL E}}$, BUT REFERRED TO $y'(x)$ INSTEAD OF $y(x)$. SEE LBL E

Sample Problem(s) (1) CONSIDER THE FUNCTION $y = e^{-x^2}$. (a) PLOT THE CURVE. (b) FIND THE VALUE OF y IF $x = 1.446328$. (c) FIND THE SLOPE FOR $x = 1$ (D) FIND THE CURVATURE AT $x = 0.5$ (e) FIND $\int_{0.326}^{1.128} e^{-x^2}\,dx$ (f) FIND THE VALUE OF X IF $y = 0.9394131$ (g) FIND THE VALUE OF X SUCH AS TO MAKE y MAXIMUM. (h) FIND $\int_0^{\infty} e^{-x^2}\,dx$. (i) PLOT $y'(x)$ AND $y''(x)$. (ALL PLOTTINGS ARE FOR $X \geq 0$)

$\boxed{\text{(a) PLOTTING}}$

- LOAD PROGRAM
- DEFINE $y(x)$ ⇨ $\boxed{\text{GTO A}}$, SWITCH TO PRGM $\boxed{x^2}$ $\boxed{\text{CHS}}$ $\boxed{e^x}$ $\boxed{\text{RTN}}$, SWITCH TO RUN
- PRESS ⇨ $\boxed{\text{DSP 4}}$ 0 $\boxed{\text{ENTER}}$ .5 $\boxed{\text{f A}}$ → 0.0000 (x_0) → 1.0000 (y_0)

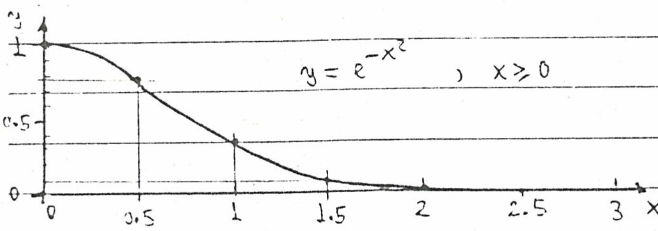
→ 0.5000 (x_1) → 0.7788 (y_1)

→ 1.0000 (x_2) → 0.3679 (y_2)

→ 1.5000 (x_3) → 0.1054 (y_3)

→ 2.0000 (x_4) → 0.0183 (y_4)

→ 2.5000 (x_5) → 0.0019 (y_5) $\boxed{\text{R/S}}$



Solution(s) $\boxed{\text{(b) } y(1.446328)}$ - PRESS ⇨ $\boxed{\text{DSP 7}}$ 1.446328 $\boxed{\text{A}}$ → 0.1234567

$\boxed{\text{(c) } y'(1)}$ PRESS ⇨ 1 $\boxed{\text{B}}$ → −0.7357588 (EXACT RESULT IS $y'(1) = -0.7357589+$)

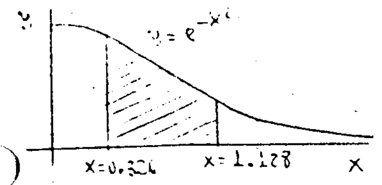
$\boxed{\text{(D) THE CURVATURE AT } x = 0.5}$ THE CURVATURE IS GIVEN BY $K(x) = \dfrac{y''(x)}{(1 + [y'(x)]^2)^{3/2}}$

PRESS ⇨ .5 $\boxed{\text{C}}$ → −0.7788047 $\boxed{\text{STO 9}}$ .5 $\boxed{\text{B}}$ → −0.7788003 $\boxed{x^2}$ 1 $\boxed{+}$ 1.5 $\boxed{y^x}$ $\boxed{\text{STO} \div 9}$ $\boxed{\text{RCL 9}}$ → −0.3824682 = THE CURVATURE AT $x = 0.5$

Reference(s)

- INTRODUCTION TO NUMERICAL ANALYSIS - F.B. HILDEBRAND MAC GRAW-HILL (INTERNATIONAL SERIES IN PURE & APPLIED MATHEMATICS)
- NUMERICAL ANALYSIS - F.B. SCHEID MAC GRAW-HILL (SCHAUM'S OUTLINE SERIES)

$\boxed{(E)\ \int_{0.326}^{1.128} e^{-x^2}\,dx}$



– WE WILL OBTAIN A VALUE FOR $\int_{0.326}^{1.128} e^{-x^2}dx$, AND THEN FIND THE ERR

– PRESS ⇨ •326 [ENTER] 1.128 [D] → <u>0.4733569</u>

– NOW, LET'S CHECK THE RESULT BY RECALCULATING THE INTEGRAL WITH 2 AND 4 SUBINTERVALS

2 [fB] •326 [ENTER] 1.128 [fD] → 0.4733466
4 [fB] •326 [ENTER] 1.128 [fD] → <u>0.4733465</u>

SO, THE MOST ACCURATE VALUE IS <u>0.4733465</u> . USING A SINGLE SUBINTERVAL GIVES 0.4733569 , WHICH IS IN ERROR BY 10^{-5}.

$\boxed{(F)\ x\,(0.9394131)}$ – TO FIND THE x- VALUE CORRESPONDING TO y = 0.9394131, WE MUST FIND A ROOT OF $e^{-x^2} = 0.9394131$. e^{-x^2} IS ALREADY DEFINED, BUT NOW $e^{-x^2} - 0.9394131$ IS NEEDED. PRESS [GTO A] , SWITCH TO PRGM , [SST] [SST] [SST] [RCL A] [–] , , SWITCH TO RUN ;

•9394131 [STO A] 1 (GUESS) [E] → 1.0000000 → 0.2232051 → → 0.2514289 → 0.2500035 → → 0.2499999 → <u>0.2499999</u>

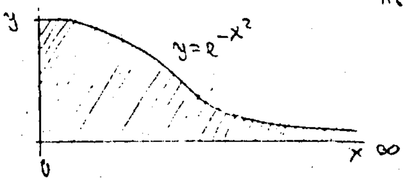
SO, <u>x = 0.2499999 +</u> . TO TEST ROOT [A] → 0.0000000

$\boxed{(G)\ \text{A MAXIMUM OF } y=e^{-x^2}}$ – THIS IS, A ZERO OF $y'(x)$.

0.4 (GUESS) [fE] → −0.1882357 → 0.0143602 → −0.0000061 → → −8.67859000 · 10^{-9} → <u>−8.67859000 · 10^{-9}</u>

SO, <u>x = −8.67859 · 10^{-9} ≃ 0</u> , WHICH CORRESPONDS TO y(x=0) = 1, <u>A MAXIMUM</u>

$\boxed{(h)\ \int_0^{\infty} e^{-x^2}dx}$; BE CAREFUL, BECAUSE LBLA COMPUTES NOW $e^{-x^2} - 0.9394131$. TO COMPUTE e^{-x^2} AGAIN , [GTO A] , SWITCH TO PRGM , [SST] [SST] [SST] [RTN] , SWITCH TO RUN



PRESS ⇨ 0 [fC] → 0.7468 $(=\int_0^1 e^{-x^2})$ → 0.1352 $(=\int_1^2 e^{-x^2})$ → → 0.0041 $(=\int_2^3 e^{-x^2})$ → 1.97... 10^{-5} $(=\int_3^4 e^{-x^2})$ → → <u>0.8862</u> [DSP 5] → <u>0.88623</u> (EXACT VALUE IS $\frac{\sqrt{\pi}}{2}$ = 0.88623)

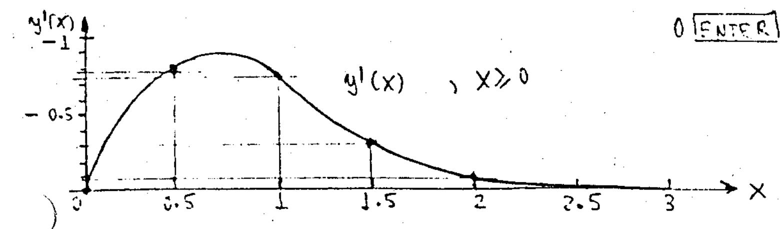
– TO IMPROVE ACCURACY ⇨ [GTO •159] [DEL] [DSP 6] [GTO •155] [DEL] •5 , SWITCH TO RUN,

SWITCH TO PRGM

PRESS 0 [fC] → 0.461281 → 0.285544 → 0.109364 → → 0.025893 → 0.003785 → 0.000341 → → 0.000019 → 0.000001 → 1.35... 10^{-8} → → 0.886227 [DSP 9] → <u>0.886226925</u>

– AS IT WAS PREVIOUSLY STATED, EXACT RESULT IS $\frac{\sqrt{\pi}}{2}$ = 0.886226926 + , SO ERROR IS ABOUT 10^{-9} –

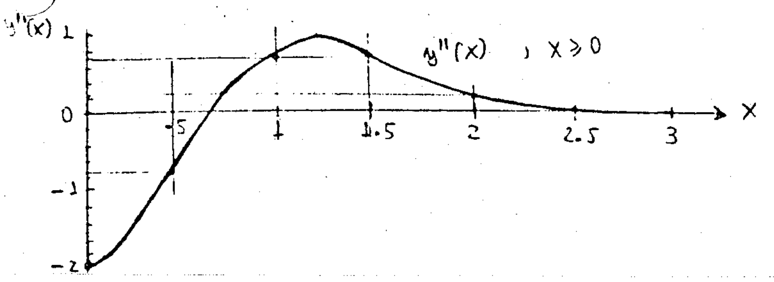
$\boxed{(i)\ y'(x)\ \&\ y''(x)\ \text{GRAPH}}$ PRESS ⇨ [GTO •142] , SWITCH TO PRGM, [DEL] [B] , SWITCH TO RUN ; [DSP 4]



0 [ENTER] •5 [fA] → 0.0000 → 0.0000
→ 0.5000 → −0.7788
→ 1.0000 → −0.7358
→ 1.5000 → −0.3162
→ 2.0000 → −0.0733 [R/S]

} GRAPH OF y'(x)

(EXACT ⇨ $y'(x) = -2x \cdot e^{-x^2}$)



PRESS ⇨ [GTO •142] , SWITCH TO PRGM, [DEL] [C] , SWITCH TO

0 [ENTER] •5 [fA] → 0.0000 → −2.0000
→ 0.5000 → −0.7788
→ 1.0000 → 0.7357
→ 1.5000 → 0.7378
→ 2.0000 → 0.2564 [R/S]

} GRAPH OF y''(x)

(EXACT ⇨ $y''(x) = (4x^2 - 2)e^{-x^2}$)

User Instructions

NUMERICAL ANALYSIS OF A FUNCTION (R9, A, B, S0 – S9)				
AUTOMATIC PLOT $x_0 \uparrow \Delta \to (x_i) \to y_i$	# OF SUBINTERV.	$a \to \int_0^\infty y(x)\,dx$	$\int_a^b$ WITH # SUB INT.	$x_0 \to$ root of y'
$x \to y(x)$	$x \to y'(x)$	$x \to y''(x)$	$a \uparrow b \to \int_a^b y(x)\,dx$	$x_0 \to$ root of y

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	LOAD PROGRAM, BOTH SIDES				
2	DEFINE F(X) ⇨ PRESS		GTO	A	
	SWITCH TO PRGM, AND ENTER THE SEQUENCE OF KEYS-				
	TROKES THAT CALCULATES F(X); X CAN BE FOUND				
	IN THE DISPLAY. AT THE END OF THE SEQUENCE INSERT				
	A RTN COMMAND. (R9, RA, RB, RS0 – RS9 MAY BE USED)				
3	ALL SUBSEQUENTS STEPS ARE OPTIONAL:				
3a	TO EVALUATE y(x) AT SOME POINT X	X	A		y(x)
3b	TO EVALUATE y(x) OVER A SET OF EQUALLY	x_0	ENTER		
	SPACED POINTS x_i, WHERE $x_i = x_0 + i \cdot \Delta$	Δ	f	A	(x_i)
	PRESS R/S TO STOP THE SWEEP				y_i
4	TO EVALUATE y'(x) AT SOME POINT X	X	B		y'(x)
5	TO EVALUATE y''(x) AT SOME POINT X	X	C		y''(x)
6	TO FIND A ROOT OF y(x), INPUT AN APPROXIMATION	x_0	E		$x_1, x_2 \ldots x_i$
					... root
7	TO FIND A ROOT OF y'(x), INPUT A GUESS	x_0	f	E	$x_1, x_2 \ldots$
					$\ldots x_i \ldots$ root
8	TO FIND THE INTEGRAL OF y(x) BETWEEN a, b				
	– WITH 1 INTERVAL, ENTER	a	ENTER		
		b	D		$\int_a^b y(x)\,dx$
	– WITH m SUBINTERVALS, ENTER m	m	f	B	m
	ENTER LIMITS	a	ENTER		
		b	f	D	$\int_a^b y(x)\,dx$
9	TO FIND THE INTEGRAL OF y(x) BETWEEN a AND ∞	a	f	C	PARTIAL SUMS...
					$\ldots \int_0^\infty y(x)\,dx$
10	TO AUTOMATICALLY EVALUATE y'(x), CHANGE STEP				
	142 GSB A TO 142 GSB B				
	AND PROCEED AS IN STEP 3b				
11	TO AUTOMATICALLY EVALUATE y''(x), CHANGE STEP				
	142 GSB A TO 142 GSB C				
	AND PROCEED AS IN STEP 3b				
12	TO IMPROVE ACCURACY OF $\int_a^\infty y(x)\,dx$, CHANGE				
	STEP 159 DSP 4 TO 159 DSP 6				
	AND STEP 155 1 TO				
	155 • } OR SOME OTHER VALUE < 1				
	156 5				

Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBL a	32 25 15	ROOT OF $y'(x)$
	1	01	
	1	01	ADDRESS = LBL B = 11
	GTO 0	22 00	
	*LBL E	31 25 15	ROOT OF $y(x)$
	1	01	
	0	00	ADDRESS = LBL A = 10
	*LBL 0	31 25 00	
	ST I	35 33	STORE THE PROPER ADDRESS
010	R↓	35 53	
	STO 6	33 06	STORE x_0
	*LBL 2	31 25 02	ROOT-FINDER LOOP
	RCL 6	34 06	RECALL AND
	PAUSE	35 72	DISPLAY x_i
	GSB (i)	31 22 24	COMPUTE $y(x_i)$ [OR $y'(x_i)$]
	X=0	31 51	IF $y(x_i)=0$, x_i IS THE ROOT
	GTO 3	22 03	
	STO 7	33 07	STORE $y(x_i)$ [OR $y'(x_i)$]
	RCL 6	34 06	
020	ISZ	31 34	COMPUTE $y'(x_i)$ [OR $y''(x_i)$]
	GSB(i)	31 22 24	
	DSZ	31 33	
	STO ÷ 7	33 81 07	$R_7 \rightarrow y(x_i)/y'(x_i)$
	RCL 7	34 07	
	STO − 6	33 51 06	$x_{i+1} = x_i - y(x_i)/y'(x_i)$
	DSP 7	23 07	← CAN BE CHANGED TO ANOTHER DISPLAY TO IMPROVE ACCURACY
	RND	31 24	$\lvert x_{i+1} - x_i \rvert < 5.10^{-Y}$?
	X≠0	31 61	
	GTO 2	22 02	YES, ITERATE
030	*LBL 3	31 25 03	DISPLAY & RETURN LABEL
	RCL 6	34 06	
	RTN	35 22	
	*LBL C	31 25 13	COMPUTATION OF $y''(x)$
	STO 3	33 03	STORE x
	4	04	
	EEX	43	$\Delta = 4.10^{-3}$
	CHS	42	
	3	03	
	STO 4	33 04	$R_4 \rightarrow 8.10^{-3} = 2\Delta$
040	STO+4	33 61 04	
	STO−3	33 51 03	$R_3 \rightarrow x-\Delta$
	+	61	
	GSB B	31 22 12	COMPUTE $y'(x+\Delta)$
	STO 5	33 05	
	RCL 3	34 03	
	GSB B	31 22 12	COMPUTE $y'(x-\Delta)$
	STO − 5	33 51 05	
	RCL 5	34 05	
	RCL 4	34 04	$y''(x) \simeq \frac{y'(x+\Delta) - y'(x-\Delta)}{2\Delta}$
050	÷	81	
	RTN	35 22	
	*LBL B	31 25 12	COMPUTATION OF $y'(x)$
	STO 0	33 00	STORE x
	8	08	
	EEX	43	$\Delta = 8.10^{-4}$
	CHS	42	
	4	04	
	STO 1	33 01	$R_1 \rightarrow 16.10^{-4} = 2\Delta$
	STO+1	33 61 01	
060	STO − 0	33 51 00	$R_0 \rightarrow x-\Delta$
	+	61	
	GSB A	31 22 11	COMPUTE $y(x+\Delta)$
	STO 2	33 02	
	RCL 0	34 00	
	GSB A	31 22 11	COMPUTE $y(x-\Delta)$
	STO−2	33 51 02	
	RCL 2	34 02	$y'(x) \simeq \frac{y(x+\Delta) - y(x-\Delta)}{2\Delta}$
	RCL 1	34 01	
	÷	81	
070	RTN	35 22	
	*LBL d	32 25 14	COMP. OF $\int_a^b y(x)dx$ WITH m SUBIN
	X⇄Y	35 52	
	STO 6	33 06	STORE a
	−	51	b−a
	RCL C	34 13	
	ST I	35 33	$\# = \frac{b-a}{m} = \Delta$
	÷	81	
	STO 7	33 07	STORE Δ
	RCL 8	34 08	CLEAR R_8
080	STO−8	33 51 08	
	*LBL 6	31 25 06	SUBINTERV. INTEGRAT. LOOP
	RCL 6	34 06	a_i
	RCL 6	34 06	
	RCL 7	34 07	
	STO+6	33 61 06	$a_{i+1} = a_i + \Delta$
	+	61	
	GSB D	31 22 14	COMPUTE $\int_{a_i}^{a_{i+1}} y(x)dx$
	STO+8	33 61 08	$\Sigma\int$
	DSZ	31 33	ANOTHER SUBINTERVAL
090	GTO 6	22 06	
	RCL 8	34 08	RECALL $\Sigma = \int_a^b y(x)dx$
	RTN	35 22	
	*LBL D	31 25 14	INTEGRAL $\int_a^b y(x)\,dx$
	STO 5	33 05	STORE b
	STO 0	33 00	
	X⇄Y	35 52	
	STO 2	33 02	STORE a
	STO−0	33 51 00	$R_0 \rightarrow b-a$
	+	61	
100	STO 1	33 01	$R_1 \rightarrow b+a$
	RCL 0	34 00	
	5	05	
	1/X	35 62	
	√	31 54	
	STO 3	33 03	$x_1 = \frac{b+a}{2} + \frac{b-a}{2} \cdot \sqrt{\frac{1}{5}}$
	×	71	
	+	61	
	2	02	
	÷	81	
110	GSB A	31 22 11	COMPUTE $y(x_1)$
	STO 4	33 04	Σ
	RCL 1	34 01	

REGISTERS

0	1	2	3	4	5	6	7	8	9
used	used	used	used	used	used	used, root	used	used	

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
		used, # OF SUBINT	used, Δ	used, Xi	address, index

Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCL 0	34 00	$x_2 = \frac{b+a}{2} - \frac{b-a}{2} \cdot \sqrt{\frac{1}{5}}$
	RCL 3	34 03	
	×	71	
	−	51	
	2	02	
	÷	81	
	GSB A	31 22 11	COMPUTE $y(x_2)$
120	STO+4	33 61 04	Σ
	5	05	
	STO×4	33 71 04	
	RCL 2	34 02	COMPUTE y(a)
	GSB A	31 22 11	
	STO+4	33 61 04	Σ
	RCL 5	34 05	COMPUTE y(b)
	GSB A	31 22 11	
	STO+4	33 61 04	Σ
	RCL 4	34 04	$\int \simeq \frac{b-a}{12} \Sigma$
130	1	01	
	2	02	
	÷	81	
	RCL 0	34 00	
	×	71	
	RTN	35 22	
	*LBL a	32 25 11	AUTOMATIC PLOT
	STO D	33 14	STORE Δ
	x≷y	35 52	
	*LBL 1	31 25 01	LOOP
140	PAUSE	35 72	DISPLAY x_i
	STO E	33 15	STORE x_i
142	GSB A	31 22 11	COMPUTE & DISPLAY $y(x_i)$
	-x-	31 84	
	RCL E	34 15	$x_{i+1} = x_i + \Delta$
	RCL D	34 14	
	+	61	
	GTO 1	22 01	ANOTHER $x_i \to x_{i+1}$
	*LBL c	32 25 13	COMPUT. OF $\int_a^\infty y(x)\,dx$
	RCL 6	34 06	CLEAR R6
150	STO−6	33 51 06	
	R↓	35 53	
	*LBL 5	31 25 05	LOOP
	ENTER↑	41	COMPUTE $\int_a^{a+1} y(x)\,dx$
	ENTER↑	41	
155	1	01	
	+	61	
	GSB D	31 22 14	
	STO+6	33 61 06	Σ
	DSP 4	23 04	DISPLAY PARTIAL SUM.
160	PAUSE	35 72	
	RND	31 24	LAST Δ EQUALS ZERO WHEN ROUNDED ?
	X=0	31 51	
	GTO 3	22 03	YES, DISPLAY Σ
	RCL 5	34 05	NO, RECAL a+1
	GTO 5	22 05	NEW a → a+1, AND LOOP
	*LBL b	32 25 12	STORE OF # OF SUBINTV.
	STO C	33 13	
	RTN	35 22	
	*LBL A	31 25 11	DEFINITION OF y(x)
170	RTN	35 22	
180			
190			
200			
210			
220			

SOME USEFUL REMARK[S]

— TO DEFINE y(x), REMEMBER THAT X CAN BE FOUND IN THE DISPLAY, AND THAT R9, RA, RB AND ALL SECONDARY REGISTERS MAY BE USED FOR DEFINING PURPOSES.

— HP-97 OWNERS:

— INSERT "SPACE" AFTER 139 LBL 1

— CHANGE 140 PAUSE TO 140 PRINT X

TO IMPROVE ACCURACY OF $\int_a^\infty y(x)dx$:

a) STEP 155 1 MAY BE CHANGED TO
155 •
156 5
OR SOME OTHER SMAL[L] VALUE.

b) STEP 159 DSP[4] MAY BE CHANGED TO 159 DSP 6 OR SO.

TO AUTOMATICALLY EVALUATE y'(x) OR y''(x), YOU MAY CHANGE STEP 142 GSB A TO 142 GSB B (y… OR 142 GSB C (y…

LABELS

A $x \to y(x)$	B $x \to y'(x)$	C $x \to y''(x)$	D $a \uparrow b \to \int_a^b y(x)dx$	E $x_0 \to$ root
a AUTO-PLOT $x_0 \uparrow \Delta \to (x_i, y(x_i))$	b # of subinterv.	c $a \to \int_a^\infty y(x)dx$	d $\int_a^b$ WITH # SUBINT.	e $x_0 \to$ root of $y'(x)$
0 used	1 used	2 used	3 used	4
5 used	6 used	7	8	9

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	■	DEG	☐	FIX	■
1	☐	■	GRAD	☐	SCI	☐
2	☐	■	RAD	■	ENG	☐
3	☐	■			n	7